Option Pricing

panko.aliaksandr

July 2021

1 Option Types

An option is a derivative financial instrument which gives the owner the right , but not the obligation, to buy or sell an asset.

Options can be classified in several ways:

- By side:
  1. Call (right to buy an asset)
  2. Put (right to sell an asset)
- By time of execution:
  1. European (can be executed on one expiration date)
  2. American (can be executed anytime)
  3. Bermudan (can be executed on a set of specific dates; average betweed European and American)
- By features:
  1. Plain vanilla (Traditional Option)
  2. Exotic (can be customized to meet the risk tolerance and desired profit of the investor, differ in their payment structures, expiration dates, and strike prices)
     - [Documentation]
     - Chooser (choose whether the option is a put or call with same parameters)
     - Compound (it is option on option)
     - Barrier (can be -in and -out. Activates/Deactivates depending on the underlying predefined price level)
     - Binary (payment is fixed, only direction matters)
     - Bermuda (average between European and American)
     - Quantity-Adjusting (quanto) (eliminates exchange rate risk)

- Look-Back (invester can choose the best historical exercises price for him)
- Asian (strice price compared against the average price (not the predefined exercise)
- Basket (several underlyings)
- Extendible (allow to extend the expiration date)
- Spread (The underlying asset for spread options is the spread or difference between the prices of two underlying assets)
- Shout (investor can "save" one intermediate price as exercise, however, if the final execution price is better this one is used)
- Range (payoff is based on the difference between the maximum and minimum price of the underlying asset during the life of the option)

2 Option Pricing

There are 3 the most common option pricing techniques:

1. Black-Scholes-Merton model (BSM)
2. Binomial tree
3. Monte-Carlo simulation method

2.1 BSM

The Black-Scholes Merton (BSM) model is a differential equation used to solve for options prices.

Assumptions:

- The returns on the underlying asset are log-normally distributed
- following a random walk (Markets are efficient) with constant drift and volatility
- The option is European
- No dividends
- No transaction costs in buying the option
- risk-free rate and volatility of the underlying are known and constant

The Black-Scholes model requires five input variables:

1. strike price
2. current stock price

3. time to expiration
4. risk-free rate
5. volatility

- Knowing an option price one can calculate the implied volatility (it is a constant) of an underlying, solving the equation for volatility. Newton's method can be used (for example) [link] to the project with the implementation
- The Black-Scholes model is not efficient for calculating implied volatility (in practice it's not constant for out-the-money, ATM and deep ITM options, looks like a smile)
- For European options on stocks, currencies, futures etc. (cannot be applied to the American-style)

2.2 Greeks

All greeks are just derivatives from BSM formula. They show the influence of a factor (like stock price or time to expiration) on an option price.

1. Delta (by stock price)
2. Theta (by time)
3. Gamma (Second by stock price)
4. Vega (by implied volatility)
5. Rho (by interest rate)
6. Minor: lambda, epsilon, vomma, vera, speed, zomma, color, ultima. (just some second derivatives)

2.3 Binomial tree

- The simplest method
- Assumes perfectly efficient markets
- Requires possible prices of the underlying asset (needs to be guessed) and the strike price of an option
- This model is a basement for Monte-Carlo methods

2.4 Monte-Carlo

In this method, we simulate the possible future stock prices and then use them to find the discounted expected option payoffs. Reference

There are 2 possible variants:

1. number of periods(exercise dates) known (Bermudan style)
2. number of periods infinite (American style)

2.4.1 Case 1: Bermudan style

- Determine the growth shocks of the stock price (there are 2 formulas) (these are probabilities of predefined moves)

$$\hat{u} = e^{(r-\delta-0.5\sigma^2)h+\sigma\sqrt{h}}$$

$$\hat{d} = e^{(r-\delta-0.5\sigma^2)h-\sigma\sqrt{h}}$$

- Finding future asset prices for all required periods
- Find the payoff of the option and discount this payoff to the present value.
- Repeat the previous steps several times(of finding prices) to get more precise results
- Average all present values found to find the fair value of the option.

2.4.2 Case 2: American style

Assumes Geometric Brownian Motion of the stock price which implies that the stock follows a random walk.

- In the Geometric Brownian Motion model, we can specify the formula for stock price change
- There is also a formula to estimate a stock price in some particular time T:

$$S(T) = S_0 e^{(r-\delta-0.5\sigma^2)T+\sigma\epsilon\sqrt{T}}$$

- Repeat many times with different random ϵ
- Find the option's payoff at the maturity and discount it to the present value.
- Take the average