Microeconomics & Optimization

1 Max/Min in R^1

$$\begin{cases} f'(x)=0 \\ f''(x) \overset{Min}{>} 0 \text{ or } f''(x) \overset{MAX}{<} 0 \end{cases}$$

2 Unconstrained in R^n

FONC: $\nabla f(x^*)=0$

SONC: $Hf(x^*) \geq 0$

sufficient: $Hf(x^*)$ strictly pos. def.

3 Constrained

3.1 Problem:

$$\begin{cases} \min\limits_{x \in X} f(x) \\ h_i(x)=0, \quad i:\overline{1,m} \\ g_j(x) \leq 0, \quad j=\overline{1,r} \end{cases}$$

3.2 Lagrange Function

$$L(x,\lambda^*,\mu^*)=f(x)+\sum_{i=1}^{m}\lambda_i^* h_i(x)+\sum_{j=1}^{r}\mu_j^* g_j(x)$$

λ^*, μ^* – lagrange multipliers

3.3 Kuhn–Tucker conditions:

$$\begin{cases} \nabla L(x)=0 \\ \lambda_i g_i = 0 \\ \lambda_i \geq 0, \; g_i(x) \leq c_j \\ H_L(x^*,\lambda^*) \text{ полож. опр} \to \min \\ \qquad\qquad \text{отриц.} \to \max \end{cases}$$

[по минорам]

$\boxed{\text{If } u(\cdot) \text{ - quasi-concave: necessary} \Leftrightarrow \text{sufficient}}$!

4 Marginal Rate of substitution

$$\frac{\frac{\partial u'}{\partial x_1}}{\frac{\partial u'}{\partial x_2}} = \frac{p_1}{p_2}$$

$\frac{\partial u'}{\partial x_1} \to MRS^1_{1,2}$ in point $x=x^*$

important! G&R p 39

5 Pareto eff. alloc. [2 goods, 2 players]

$$\begin{cases} x_1^1+x_1^2=w_1^1+w_1^2 \\ x_2^1+x_2^2=w_2^1+w_2^2 \end{cases} \text{ на складе пусто}$$

$MRS^1_{12} = MRS^2_{1,2}$ } все довольны

6 Define if good is normal or luxury:

$$\frac{\partial x^*}{\partial w} \overset{?}{>} 0$$

7 Utility Max. Problem [UMP]

$$\begin{cases} u(x) \to \max \\ px \leq w \end{cases}$$

8 Expenditure Min Problem [EMP]

$$\begin{cases} c(x) \to \min; \; [c(x)=p\cdot x] \\ u(x) \geq u_0 \end{cases}$$

$c(x)$ – expend. func.

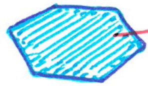
Strictly concave $\overset{(1)}{}$ ⇒ concave $\overset{(2)}{}$ ⇒ ⇒ quasi-concave $\overset{(3)}{}$

not so strict conditions for Hessian ma. !

Check slides 45

Problems Types

1. Max Utility function
2. Define type of product: normal/luxury
3. Min expenditure
4. Hicksian demand
5. Pareto efficient allocation
6. Excess demand functions
7. Walrasian equilibrium prices & allocations
8. Explain what is λ^* $\quad \lambda = \frac{\partial u^*}{\partial M}$
9. Marginal rate of substitution
10. Normal vs Luxury $\quad \frac{\partial x(p,w)}{\partial w} \overset{?}{>} 0$
11. Indirect utility function
12. Feasible set (Determine) – physically possible



→ convex set. WHY?
{a – possible, b – possible} ⇒ between possible!

13. Find Walrasian Budget Set

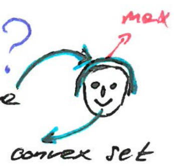
→ Physicaly possible + you can afford it

$$B(p,w)=\{x \in X \mid p\cdot x \leq w\} \subseteq X$$

14. Budget line (Determine & Draw)

$$\{x \mid p\cdot x = w\}$$

15. What is utility function?
   - Differentiable
   - quasi-concave
   - Monotone



16. Find Walrasian / Marshallian / uncompensated (the same) demand function? $\boxed{x=x(p,w)}$ – solution