Probability

Definitions:

- **Random experiment** – output cannot be surely predicted
- Ω (**state space**) – set of all possible outcomes
- **Event** – subset of all possible outcomes
- **Probability of an event** – a number associated with an event which measures the likelihood of the event.
- **Prior probability** – before experiment
- **Posterior probability** – after experiment
- **Probability space** $(\Omega, \mathcal{A}, P)$ – triple, consisting of: a) space
  b) family of all events
  c) family of all $P(A), A \in \mathcal{A}$

- **Random variable** – is a function that maps outcome of the experiment to R^n, $n: \overline{1,k}$
- **σ-algebra** – subset of 2^Ω (family of all events) which has properties:
  1) $\emptyset \in \mathcal{A}$, $\Omega \in \mathcal{A}$
  2) $A \in \mathcal{A} \Rightarrow A^c \in \mathcal{A}$
  3) $A_i \in \mathcal{A} \Rightarrow \bigcup_{i=1}^{\infty} A_i \in \mathcal{A}$, $\bigcap_{i=1}^{\infty} A_i \in \mathcal{A}$
- **σ-algebra generated by C** – smallest σ-algebra, containing C
- **Borel σ-algebra** – σ-algebra generated by the open sets

- **Probability measure** – mapping $P: \mathcal{A} \to [0,1]$:
  1) $P(\Omega) = 1$
  2) $P(\bigcup_{n=1}^{\infty} A_n) \overset{\text{disjoint}}{=\!=} \sum_{n=1}^{\infty} P(A_n)$
- **Disjoint events** – events that don't have any common part
- **Conditional Probability** – every event can be splited into 2 events: Ex. P(you'll pass a test) = P(you'll pass test having answers) + P(you'll pass tests having no answers) =>

$$P(\text{pass}) = \underbrace{P(\text{pass}|\text{answ.}) + P(\text{pass}|\text{no answ.})}_{\text{conditional}}$$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- **Independent events**:

$$P(A \cap B) = P(A) \cdot P(B)$$

Ex. Results of event A(coin) don't depend on results of event B(dice)

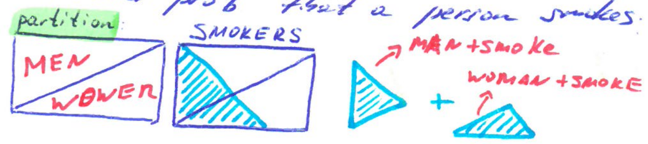
Ideas

- We cannot predict single output, but we can make conclusions about general structure of underlying random process and conclude on confidence intervals of the output.

- **Partition Equation**:

$$P(A) = \sum_n P(A|E_n) P(E_n)$$; E_n – partition

Ex: Find prob that a person smokes.



- **Bayes' Teorem**

$$P(E_n|A) = \frac{P(A|E_n) P(E_n)}{\sum_m P(A|E_m) P(E_m)}$$

Ex. Before: Find the prob that a person smokes.
Now: Find the prob that smoking person is a man