# MortgageAnnuityPayments

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## 1 Introduction

This document describes the derivation of the mortgage payment formula (or annuity). Both financial instruments work in the same way but just other directions.

There are several important points:

- Monthly payment is constant over time
- In the end (in classical case)there is no balloon (big final) payment.
- This means that the ratio between capital and interest payment is changing over time.

Concept

1. $P_k$ - principal(body) part in the beginning of k-th period. i - monthly interest payment
2. Then $I_k = P_k * i$, this is your monthly interest payment for k-th period
3. Since a monthly payment (M) is constant by design, we can find $C_k = M - I_k$, the amount of principal you pay for the month
4. Knowing the initial outstanding amount $P_k$ and the capital payment $C_k$ one can calculate the new balance of your principal of your loan $P_{k+1} = P_k - C_k$
5. Repeat these steps until the $P_k \neq 0$

The formula for a monthly payment can be derieved as follows:

## 2 Monthly payment formula derivation

To derive a formula let's make a couple of iterations For the first month $N = 1$:

$$I_1 = P_1 * i$$

$$C_1 = M - I_1 = M - P_1 * i$$
$$P_2 = P_1 - C_1 = P_1 - (M - P_1 * i) = P_1 + iP_1 - M$$

For the second month $N = 2$ :

$$I_2 = P_2 * i = i(P_1 + iP_1 - M) = i(P_1(1+i) - M)$$
$$C_2 = M - i(P_1(1+i) - M) = M(1+i) - i(1+i)P_1$$
$$P_3 = P_2 - C_2 = P_1 * (1+i) - M - M(1+i) + i(1+i)P_1 = P_1(1+i)(1+i) - M(1+i) - M$$

Doing this further we arrive to the next view

$$P_{k+1} = P_1(1+i)^k - (M(1+i)^{k-1} + ... + M(1+i)^1 + M)$$

Using the geometric sum: $S_n = a(1-b^n)/(1-b)$

$$P_{k+1} = P_1(1+i)^k - \frac{M(1-(1+i)^k)}{1-(1+i)} = P_1(1+i)^k - \frac{M(1-(1+i)^k)}{-i}$$

Now $P_{k+1}$ should be 0 on the last iteration, since the total loan should be re-payed.

$$P_1(1+i)^k = \frac{M(1-(1+i)^k)}{-i}$$
$$M = \frac{-iP_1(1+i)^k}{(1-(1+i)^k)} = \frac{-iP_1}{(\frac{1}{(1+i)^k} - 1)} = \frac{-iP_1}{(1+i)^{-k} - 1} = \frac{iP_1}{1-(1+i)^{-k}}$$