VaR and ES (in progress)

panko.aliaksandr

October 2020

1 Introduction

Everyone wants to know the future! It is clear, when you know the future you can profit from taking additional risk and avoid fails, isn't it?

Exactly the same problem arises in finance. A bank earns money making investments: giving loans, buying stocks and so on. To make more money a bank invests not it's own money but also takes loans from others: private individuals (mostly deposits) and financial institutions (taking loans or issuing bonds). Of course, if one can invest money having higher returns than the borrowing costs(interest rate), borrowing makes sense.

At the same time, it a bank cannot return borrowed money back it is a huge issue not only for a bank but also for a whole financial system. In financial world financial institutions are deeply connected, so bankruptcy of one leads to following bankruptcies.

This creates for a bank an optimization problem: how to invest as much as possible making sure that a bank is able to return it's debts.

Here are some interesting observations:

1. If a bank in total owes some sum it does not mean that it must set aside the whole sum, since it has to return the sum distributed in time (not at ones). This means that a bank can set aside only,say, 10% and use the rest 90% of deposits as investment capital for itself.

2. Some investments are not successful, so a bank cannot be always sure to get expected amount of money back in time.

3. Some assets can depreciate, so a bank cannot get expected sum selling them.

These observations lead us to the next idea: what if we could analyse historical data and find some consistent pattern! For example, how many people want to withdraw money before the agreed date, how frequently borrowers default etc.

As soon as one finds a pattern he can in some way predict the future. This is exactly the purpose of modeling. The modeling is an activity aimed at defining a pattern in a process.

A modeling process consists of 3 parts:

- Define modeled variable (simple)
- Define influencing input factors (important and sometimes difficult)
- Define the dependency structure (linear, non-linear etc. very difficult, normally some assumptions are made)

2 VaR Model

The portfolio of a bank consists of many subportfolios (equity, bonds, real estate etc.). In case of money shortage a portfolio can be sold, but what would be the price? VaR model answers the question how much a portfolio can lose over specified period with specified probability).

If is obvious that 1 day 99% VaR is smaller than 10 days 99% VaR and 1 day 99% VaR is greater than 1 day 95% VaR.

So VaR modeling is associated with market risk (risk that one's portfolio drops in value, because of, say, recession and bear financial market)

3 VaR backtesting

Assume you have 1 day 99% VaR (already calculated and we do not care how many observations were used to calculate it). You want to backtest your model with 500 (around 1.5 years) observations.

In this case we expect (if the model is good) 500*0.01=5 observations with loss more than VaR. However, since this result is based on random sample (500 days profit and loss are random sample) 5 is just a mean of Binomial distribution. It sounds natural that values around 5 are also plausible, but what about,for example, 10?

We also know how to calculate the standard deviation of Binomial distribution and, say, 95% confidence interval. From this theoretical approach, we can find that with 95% of probability the number of days when a bank experiences a loss more than VaR does not exceed 9.

Then you can simply calculate how many days the losses were above the VaR (the VaR value comes from your model and actually examined) and if the number of days is more than 9 than the model is not correct with 95% of probability.

It is important to understand that the error can occur only in your VaR calculation, since parameters of Binomial distribution are known and the fact that required distribution is Binomial is also obvious.

4 VaR Model Creation

Now we know why VaR is used and how to backtest the model. The most complicated part is how to create the model.

Let's assume that we have profit and loss time-series (this we always have). Now the biggest assumption comes. The next step is to assume a population distribution of the values (Normal, Student, Gamma etc.) After that one can use Maximum likelihood estimation (MLE) procedure to fit the distribution (find the distribution parameters)

$$VaR_\alpha = F^{-1}(\alpha) * \sigma$$

It is important to understant that VaR for a portfolio should be calculated from assets (not as 1 time-series as whole). Correlation here is important, just summing up signle VaR values is clearly wrong, because doing so one would assume that all assets are perfectly correlated, when they are not.

Algorithm

Step 1: Set VaR parameters: probability of loss and confidence level, time horizon, and base currency.

Step 2: Determine market value of each position, in base currency.

Step 3: Calculate VaR of individual positions, given market volatilities.

Step 4: Calculate portfolio VaR, given correlations between all variables. (TODO add maths part here)

5 CVaR Expected Shortfall

VaR tells us which losses we can expect most of the time, but actually it is even more important for risk management to know what exactly happens in times of crisis. In other words if these 5% or 1% case occurs how much we lose? If the values is huge a bank can collapse in case of crisis.

ES or CVaR is expected value of a loss beyond VaR level.

TODO add maths section here