Probability Distributions

Discrete

1) Uniform:

$$P(A) = \frac{\text{number of favourable}}{\text{number of all}}$$

2) Hypergeometric: [uniform repeated sev. times]

Ex What is the prob. that k of the n drawn balls are red?

$$P(X=k) = \frac{C_N^k \cdot C_M^{n-k}}{C_{N+M}^n}$$

3) Binomial

Ex What is the prob. of k successes among n draws?

$$P(X=k) = C_n^k p^k (1-p)^{n-k}$$

4) Poisson [used for very rare events or to count events during time interval]

$$P(X=k) = e^{-\lambda} \cdot \frac{\lambda^k}{k!}$$

Ex What is the prob. of observing 4 births in an hour, given an average rate 1.8 b/h?

5) Geometric [number of failures before a success]

$$P(x=k) = (1-\theta)\theta^{k-1}$$

Ex What is the prob to have 2 daughter before a son?

- **Distribution function (c.d.f.)** – almost not used

$$F(a) = P(X \le a) = \int_{-\infty}^{a} p(x)\,dx$$

- **Density function** – used in reality and helps to find probability of r.v. to be in specified interval. (conf. interval)

$$P_{[a,b]} = \int_a^b f(x)\,dx$$

f – density

- **Expected Value**

$$E(x) = \int_{-\infty}^{+\infty} x f(x)\,dx \quad \text{or in discret} \quad \sum x \cdot p(x)$$

- **Variance** $\quad Var(x) = E(x^2) - (E(x))^2$

Continuous

1) Uniform

$$f(x) = \begin{cases} \frac{1}{b-a}, & x \in (a,b) \\ 0, & x \notin (a,b) \end{cases}$$

2) Gamma

$$f(x) = \begin{cases} \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, & x > 0 \\ 0, & \text{else} \end{cases}$$

3) Exponential

- Time between failures

$$f(x) = \begin{cases} \beta e^{-\beta x}, & x > 0 \\ 0, & x \le 0 \end{cases}$$

4) Normal

$$\varphi(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

5) Pareto

$$f(x) = \begin{cases} \frac{\alpha}{x^{\alpha+1}}, & x > 1 \\ 0, & x \le 1 \end{cases}$$

6) Cauchy

$$f(x) = \frac{1}{\pi} \cdot \frac{1}{1+x^2}$$

7) $I_{[a,b]}(x)$

$$F(x) = \begin{cases} 1, & x \in [a,b] \\ 0, & x \in [a,b] \end{cases}$$