MATHS 1
Calculation Algorithms

1 Solve system of equations

$A\vec{x} = \vec{b}$

1.1) Find A^{-1}
1.2) $x = A^{-1}b$
$\Leftrightarrow (A|b) \xrightarrow{\text{use row-echelon}} (I|x)$

2 Find A^{-1}: $(A|I) \xrightarrow{\text{using row-echelon}} (I|A^{-1})$

3 Find Ker(A):

Solve: $Ax = 0$, the solution is the ker(A) using 1

4 Find Im(A):

Find $A \cdot X$ (just multiply), where $\begin{cases} A: n \times m \\ X: m \times 1 \end{cases}$

X here just any vector: $X = (x_1, \dots x_m)$

5 Find Det(A): use Laplas rule

6 Find Rank(A): determine number of independent columns using row echelon form

7 Determine whether $\overset{n\times m}{A}$ is

→ injective: $rank(A) = m$ (number of column)

→ surjective: $rank(A) = n$ (N of rows)

8 Find Eigenvalues of A

8.1) Solve equation $\det(A - \lambda I) = 0$

8.2) Solution λ_i - eigenvalues

9 Write matrix representation of f w.r.t. a basis $B = \{b_1, \dots, b_n\}$, $i = \overline{1,n}$ $\boxed{A_{BB}}$

9.1) Compute $f(b_i)$ $\forall i = \overline{1,n}$

9.2) Find coordinates of $f(b_i)$ in required basis $B = \{b_1, \dots, b_n\}$ use 10

9.3) Write coordinates of $f(b_i)$ in B as columns

$$A_{BB} = \begin{array}{c} \begin{matrix} f(b_1) & f(b_2) & \dots & f(b_n) \end{matrix} \\ \begin{bmatrix} \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \end{array}$$

10 Find coordinates of $\vec{V} = (V_1 \dots V_n)$ in basis $B = (\vec{b}_1 \dots \vec{b}_n)$

10.1) Solve next system: 1 - to use

n equations
Solvable!

$$\begin{pmatrix} V_1 \\ V_2 \\ \vdots \\ V_n \end{pmatrix} = \underbrace{\alpha_1 \vec{b}_1 + \dots + \alpha_n \vec{b}_n}_{n \text{ unknowns}} \quad [B\vec{\alpha} = \vec{V}]$$

10.2) In basis B $\vec{V}$ has coordinates: $\vec{V} = (\alpha_1, \dots, \alpha_n)$

11 Change matrix representation of f in different basis: $\boxed{A_{BB} \to A_{CC}}$

11.1) Find A_{BC} using 12

11.2) Find $A_{CB} = A_{BC}^{-1}$

11.3) $A_{CC} = A_{BC} A_{BB} A_{CB} = A_{BC} A_{BB} A_{BC}^{-1}$

12 Find matrix that changes a basis from $B(b_1, \dots b_n)$ to $C = (c_1 \dots c_n)$

12.1) $\forall b_i \in B$ use 10 and you'll have coordinates of b_i in basis C

12.2) After 12.1) you have all $\vec{b}_1 \dots \vec{b}_n$ represented in basis C

$$\vec{b}_1 = \begin{pmatrix} b_{11} \\ b_{12} \\ \vdots \\ b_{1n} \end{pmatrix}, \dots, \vec{b}_n = \begin{pmatrix} b_{n1} \\ b_{n2} \\ \vdots \\ b_{nn} \end{pmatrix}$$

Just write all $\vec{b}_i$ in basis C as a matrix by columns

$$A_{BC} = \begin{bmatrix} b_{11} & \dots & b_{n1} \\ b_{12} & \dots & b_{n2} \\ \vdots & & \vdots \\ b_{1n} & \dots & b_{nn} \end{bmatrix}$$

13 Find eigenvectors of A

13.1) Find eigenvalues of A using 8

13.2) Solve $A\vec{u}_i = \lambda_i \vec{u}_i$ $\forall \lambda_i$, where λ_i is eigenvalue

Or equivalently $(A - \lambda_i I)\vec{u}_i = 0$

Solve it using 1

13.3) The solution will be not unique, take any one (the simplest)

13.4) You need to solve as many systems as number of different λ_i