MATHS 1
Calculation Algorithms

14 Find eigenvector decomposition

14.1) Find eigenvalues: use 8

14.2) Find eigenvectors: use 13

14.3) Put eigenvalues into diagonal matrix: $D = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_n \end{pmatrix}$

14.4) Put eigenvectors (as columns and in the same order as eigenvalues in D) in matrix T

$T = \begin{pmatrix} e.vec1 & e.vec2 \dots & e.vecn \\ \vdots & \vdots & \vdots \end{pmatrix}$

14.5) Find T^{-1} use 2

14.6) $A = TDT^{-1}$

15 Create orthonormal system from $\{v_1, \dots, v_n\}$

15.1) Use Grand-Schmidt orthogonalization process:

15.1.1) $n_1 = v_1$

15.1.2) $n_2 = v_2 - \frac{\langle v_2, n_1 \rangle}{\langle n_1, n_1 \rangle} \cdot n_1$

. . . .

15.1.3) $n_k = v_k - \frac{\langle v_k, n_1 \rangle}{\langle n_1, n_1 \rangle} n_1 - \dots - \frac{\langle v_k, n_{k-1} \rangle}{\langle n_{k-1}, n_{k-1} \rangle} n_{k-1}$

15.2) Normalize n_i: $\frac{n_i}{\|n_i\|} \rightarrow$ normalized

16 Calculate inner product:

$\langle \vec{x}, \vec{y} \rangle = x^T y$

17 Find projection $\vec{x}$ on $\vec{y}$

$P_y(x) = \frac{\langle x, y \rangle}{\langle y, y \rangle} \cdot y$

18 Compute ∇f (Gradient)

$\nabla f = \begin{pmatrix} \frac{\partial f}{\partial x_1} \\ \vdots \\ \frac{\partial f}{\partial x_n} \end{pmatrix}$

19 Compute Hf (Hessian)

$Hf = \begin{pmatrix} \frac{\partial f}{\partial x_1 \partial x_1} & \cdots & \frac{\partial f}{\partial x_1 \partial x_n} \\ \vdots & & \vdots \\ \frac{\partial f}{\partial x_n \partial x_1} & & \frac{\partial f}{\partial x_n \partial x_n} \end{pmatrix}$

20 ODE

20.1) Separable case:

20.1.1) $y'(t) = \frac{dy}{dt}$

20.1.2) Separate y and t

20.1.3) Integrate both parts

20.1.4) Find constant from

20.2) Other cases: use link

21 Find max/min of a function with equality constraint

21.1) Create $L(x, \lambda) = f(x) + \lambda_i n_i$, where $n_i \rightarrow$ constraint type equality

21.2) Solve $\nabla L(x, \lambda) = 0$ use 18

this gives you stationary points

21.3) Find Hf use 19 in $\forall$ stat. point

21.4) Define definitness of Hf in $\forall$ stat point; use 22

21.5) Conclude if it max or min or nothing

22

A is positive strictly definite if and only if all eigenvalues $\lambda_i > 0$ or $\Leftrightarrow m_i > 0$, where m_i – principal minors [min & convex]

A is positive-semi-definite $\Leftrightarrow$ $\lambda_i \geq 0$ [min & convex]

A is negative definite $\Leftrightarrow$ $\lambda_i < 0$

A is negative semidef $\Leftrightarrow$ $\lambda_i \leq 0$

[max & concave]