Econometrics MODELS: GARCH

GARCH(r,s)

- is used to reflect the fact that periods with high volatility are followed by periods of low volatility.
- Assumption: (log)returns are W.N.

$$\begin{cases} r_t = \mu + \varepsilon_t \\ \varepsilon_t \sim N(0, \sigma_t^2) \end{cases}$$ → depends on t

- Equation:

$$\begin{cases} \sigma_t^2 = a_0 + \sum_1^s a_j \varepsilon_{t-j}^2 + \sum_1^r b_j \sigma_{t-j}^2 \\ \sum_1^s a_j + \sum_1^r b_j < 1, \quad a_j, b_j > 0 \end{cases}$$

ε_{t-j}^2: previous errors; σ_{t-j}^2: prev. variance

IGARCH(r,s)

- GARCH(r,s) with $\sum_1^s a_j + \sum_1^r b_j = 1$

ARCH(s) = GARCH(0,s)

- Current variance is explained only by previous errors

t-GARCH

- Used to better model fat tails
- Assumption:

$$\begin{cases} r_t = \mu + \varepsilon_t \\ \varepsilon_t \sim t(df > 2) \end{cases}$$

instead of $N(\mu, \sigma^2)$

ARCH-in-mean [ARCH-M]

- Investors are risk averse ⇒ they want higher average return in uncertain periods: DIFFERENT model for returns assumed

$$\begin{cases} r_t = \mu_0 + \mu_1 \sigma_t^2 + \varepsilon_t \\ \mu_1 > 0 \\ \varepsilon_t \sim N(0, \sigma_t^2) \end{cases}$$

→ capture higher risk

Asymmetric GARCH [threshold GARCH, GJR model]

- Good and Bad news effects differently

Equation (for GARCH(1,1))

$$\begin{cases} \sigma_t^2 = a_0 + a_1 \varepsilon_{t-1}^2 + b_1 \sigma_{t-1}^2 + j D_{t-1} \varepsilon_{t-1}^2 \\ D - \text{dummy; positive shock} \end{cases}$$

Exponential GARCH [EGARCH]

- Guaranties positive variance: $\log(\sigma_t^2)$ is modeled:

Equation:

$$\begin{cases} \log(\sigma_t^2) = a_0 + a_1 \frac{|\varepsilon_{t-1}|}{\sigma_{t-1}^2} + b_1 \log(\sigma_{t-1}^2) + j \frac{\varepsilon_{t-1}}{\sigma_{t-1}^2} \\ \text{if } j < 0, \text{ positive shocks generate less volatility then negative} \end{cases}$$

HOW IT WORKS!

use e.g. ARIMA

1. From original ts. → stationary with errors = W.N.
2. We still can have big errors in prediction because of non-constant volatility → use GARCH models → now we can predict not only „trend" but also capture all volatility patterns, making the whole model more precise