

Assignment 1 - Disney VS SP500

Aliaksandr Panko

March 10, 2018

Contents

1 Objectives	1
2 Theoretical Solution	1
2.1 Nash Equilibrium	2
2.2 Mixed Nash Equilibrium	2
3 Python Implementation	3
4 Gambit software	3

1 Objectives

1. Implement a simultaneous game
2. Find Nash equilibrium (or equilibria) if exists.
3. Find mixed strategy Nash equilibrium if exists.
4. Describe python package used for the game implementation.
5. Research the Gambit software

2 Theoretical Solution

		Firm2	
		Aggressive	Passive
Firm 1	Aggressive	25, 9	33, 10
	Passive	30, 13	36, 12

2.1 Nash Equilibrium

- If firm 2 choose aggressive firm 1 choose passive
- If firm 2 choose passive firm 1 choose passive
- So firm 1 always choose passive
- If firm 1 choose aggressive firm 2 choose passive
- If firm 1 choose passive firm 2 choose aggressive
- So The Nash equilibrium is **Firm 1 choose passive and Firm 2 choose aggressive with payoffs(30,13)**

2.2 Mixed Nash Equilibrium

Let's define probabilities:

- Let Firm 1 choose aggressive (A) strategy with probability p
- Let Firm 1 choose passive (P) strategy with probability $(1-p)$
- Let Firm 2 choose aggressive (A) strategy with probability q
- Let Firm 2 choose passive (P) strategy with probability $(1-q)$

Let's calculate expectations:

- $E_1(A) = 25 * q + 33 * (1 - q)$
- $E_1(P) = 30 * q + 36 * (1 - q)$
- $E_2(A) = 9 * p + 13 * (1 - p)$
- $E_2(P) = 10 * p + 12 * (1 - p)$

Now check whether the Mixed Nash Equilibrium exists. It is clear that a firm will mix between the two strategies only if these two expected payoffs are the same. For Firm 1:

$$25q + 33(1 - q) = 30q + 36(1 - q) \rightarrow q = -\frac{3}{2}$$

It is clear that Mixed Nash Equilibrium does not exist since $q \notin [0,1]$

Just for purpose of training Firm 2:

$$9p + 13(1 - p) = 10p + 12(1 - p) \rightarrow p = \frac{1}{2}$$

3 Python Implementation

The game could be simulated using **nash** python library. To install it execute next command in anaconda prompt:

-pip install nashpy

```
import nash
game = nash.Game()
game.support_enumeration()
```

These functions allows to determine nash equilibrium strategies. **As a result I have the same result (30,13)**

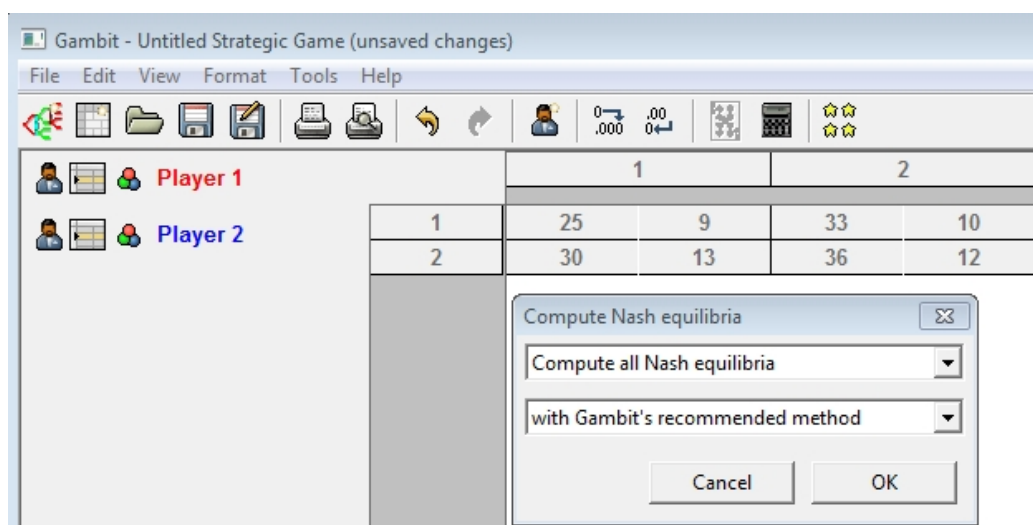
Mixed nash equilibrium is implemented using linear systems:

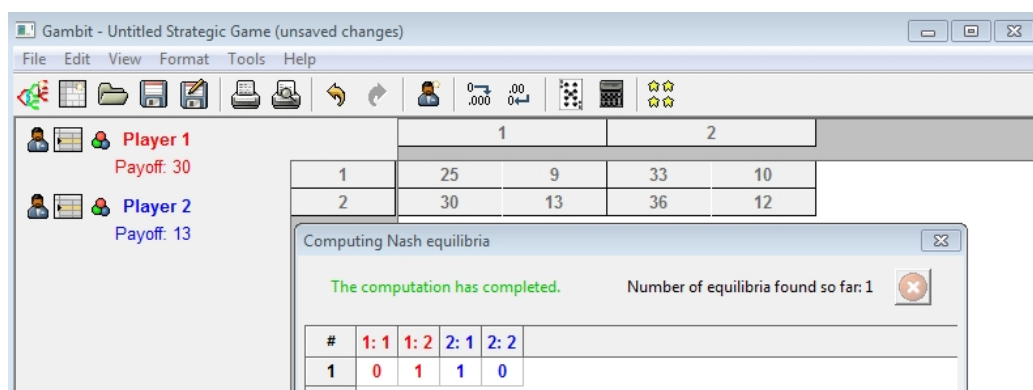
$$x = np.linalg.solve(A, b)$$

Where A and b are a matrix and a vector of coefficients (payoffs) and x is (p,q) probabilities vector. Validations for p and q ($p, q \in [0,1]$) show that **mixed nash equilibrium does not exist**.

4 Gambit software

Gambit is a library of game theory software and tools for the construction and analysis of finite extensive and strategic games. Gambit is fully-cross platform. Equilibrium-computing algorithms are available as command-line tools, callable from scripts and other programs. It also has a Python API for developing scripting applications.





As we can see the program gives us the same result!