PM: Foundations HA 2

Agafonov Nikolai, Kurennoi Ilia,
Panko Aliaksandr, Yakimchyk Nadzeya

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Task 1: Summary Statistics

As required by the task, we provide the market weight of the indices and the standard deviation of excess logreturns (annualized).

	Market Weight, %	Annualized St Dev Of Excess Returns, %
Bonds.exUS	26.66	7.98
Bonds.US	19.90	3.18
Bonds.HY	2.50	10.91
Stocks.USLG	9.88	15.03
Stocks.USLV	9.98	14.89
Stocks.USSG	1.67	19.92
Stocks.USSV	1.61	19.11
Stocks.DMexUS	19.79	17.87
Stocks.EM	8.01	22.63

As expected, the standard deviations of bonds returns are lower than those of stocks, so the risk in investing in bonds is generally lower than that in stocks. Among bonds the US bonds (Bonds.US) have the lowest standard deviation (so their risk, as expected, is the lowest) and high yield bonds (Bonds.HY) have the highest one. Among stocks US large stocks (Stocks.USLG, Stocks.USLV) have the lowest standard deviation and emerging markets stocks (Stocks.EM) (again, as expected) - the hightest.

Correlation matrix of the risky asset classes excess returns:

	Bonds.exUS	Bonds.US	Bonds.HY	Stocks.USLG	Stocks.USLV	Stocks.USSG
Bonds.exUS	1.00	0.57	0.49	0.33	0.34	0.25
Bonds.US	0.57	1.00	0.30	0.06	0.04	0.00
Bonds.HY	0.49	0.30	1.00	0.74	0.69	0.73
Stocks.USLG	0.33	0.06	0.74	1.00	0.89	0.91
Stocks.USLV	0.34	0.04	0.69	0.89	1.00	0.85
Stocks.USSG	0.25	0.00	0.73	0.91	0.85	1.00
Stocks.USSV	0.27	0.04	0.70	0.86	0.91	0.94
Stocks.DMexUS	0.52	0.15	0.83	0.88	0.86	0.82
Stocks.EM	0.52	0.19	0.82	0.79	0.73	0.75

	Stocks.USSV	Stocks.DMexUS	Stocks.EM
Bonds.exUS	0.27	0.52	0.52
Bonds.US	0.04	0.15	0.19
Bonds.HY	0.70	0.83	0.82
Stocks.USLG	0.86	0.88	0.79
Stocks.USLV	0.91	0.86	0.73
Stocks.USSG	0.94	0.82	0.75
Stocks.USSV	1.00	0.80	0.70
Stocks.DMexUS	0.80	1.00	0.90
Stocks.EM	0.70	0.90	1.00

Covariance matrix of the risky asset classes excess returns:

	Bonds.exUS	Bonds.US	Bonds.HY	Stocks.USLG	Stocks.USLV	Stocks.USSG
Bonds.exUS	6.4	1.4	4.3	3.9	4.1	4.0
Bonds.US	1.4	1.0	1.0	0.3	0.2	0.0
Bonds.HY	4.3	1.0	11.9	12.1	11.3	15.8
Stocks.USLG	3.9	0.3	12.1	22.6	19.9	27.3
Stocks.USLV	4.1	0.2	11.3	19.9	22.2	25.1
Stocks.USSG	4.0	0.0	15.8	27.3	25.1	39.7
Stocks.USSV	4.1	0.2	14.6	24.6	25.9	35.9
Stocks.DMexUS	7.4	0.8	16.2	23.6	22.8	29.3

```
Stocks.EM                9.4      1.4     20.3     26.8        24.5        33.6
             Stocks.USSV Stocks.DMexUS Stocks.EM
Bonds.exUS          4.1           7.4       9.4
Bonds.US            0.2           0.8       1.4
Bonds.HY           14.6          16.2      20.3
Stocks.USLG        24.6          23.6      26.8
Stocks.USLV        25.9          22.8      24.5
Stocks.USSG        35.9          29.3      33.6
Stocks.USSV        36.5          27.2      30.2
Stocks.DMexUS      27.2          31.9      36.3
Stocks.EM          30.2          36.3      51.2
```

Task 2: Equilibrium returns and Markowitz optimization

Equilibrium excess returns Π:

```
                Equilibrium Excess Return, %
Bonds.exUS                             1.57
Bonds.US                               0.28
Bonds.HY                               2.79
Stocks.USLG                            3.98
Stocks.USLV                            3.87
Stocks.USSG                            4.91
Stocks.USSV                            4.65
Stocks.DMexUS                          5.19
Stocks.EM                              6.22
```

Equilibrium excess returns of the examined indices correspond to their risk shown above. Not surprizingly, the US bonds have the lowest expected excess return and the largest equilibrium excess return is provided by the emerging markets stocks.

The same vector, where the equilibrium excess return for International Bonds (i.e., Bonds ex U.S.) is replaced with an absolute view of 0.5% p.a.:

```
                Excess Return, %
Bonds.exUS                  0.50
Bonds.US                    0.28
Bonds.HY                    2.79
Stocks.USLG                 3.98
Stocks.USLV                 3.87
Stocks.USSG                 4.91
Stocks.USSV                 4.65
Stocks.DMexUS               5.19
Stocks.EM                   6.22
```

The weights obtained by Markowitz optimization:

```
                Markowitz Optimization Weights, %
Bonds.exUS                                -103.04
Bonds.US                                   172.30
Bonds.HY                                    -4.38
Stocks.USLG                                 -7.93
Stocks.USLV                                 17.84
Stocks.USSG                                 -7.15
Stocks.USSV                                 -3.49
Stocks.DMexUS                               58.43
Stocks.EM                                   17.41
```

As we can see, Markowitz optimization offers considerable short positions, which are hardly implemetable in practice.

Task 3 - Black-Litterman - absolute view

In this task:
$P\_1 = (1,0,0,0,0,0,0,0,0)$;
$V\_1 = (0.005)$;
OMEGA_3a = 0;
OMEGA_3b = Inf;
OMEGA_3c = 0.000026

The results of BL procedure with $\tau = 0.01$:

```
            Prior excess_returns, % Prior weights, %
Bonds.exUS                    1.57            26.66
Bonds.US                      0.28            19.90
Bonds.HY                      2.79             2.50
Stocks.USLG                   3.98             9.88
Stocks.USLV                   3.87             9.98
Stocks.USSG                   4.91             1.67
Stocks.USSV                   4.65             1.61
Stocks.DMexUS                 5.19            19.79
Stocks.EM                     6.22             8.01
            Posterior excess returns(OMEGA = 0), %
Bonds.exUS                                    0.50
Bonds.US                                      0.04
Bonds.HY                                      2.07
Stocks.USLG                                   3.33
Stocks.USLV                                   3.18
Stocks.USSG                                   4.25
Stocks.USSV                                   3.96
Stocks.DMexUS                                 3.95
Stocks.EM                                     4.64
            Posterior weights(OMEGA = 0), %
Bonds.exUS                          -29.27
Bonds.US                             19.90
Bonds.HY                              2.50
Stocks.USLG                           9.88
Stocks.USLV                           9.98
Stocks.USSG                           1.67
Stocks.USSV                           1.61
Stocks.DMexUS                        19.79
Stocks.EM                             8.01
            Posterior excess returns(OMEGA = Inf), %
Bonds.exUS                                    1.57
Bonds.US                                      0.28
Bonds.HY                                      2.79
Stocks.USLG                                   3.98
Stocks.USLV                                   3.87
Stocks.USSG                                   4.91
Stocks.USSV                                   4.65
Stocks.DMexUS                                 5.19
Stocks.EM                                     6.22
            Posterior weights(OMEGA = Inf), %
Bonds.exUS                           26.66
Bonds.US                             19.90
Bonds.HY                              2.50
Stocks.USLG                           9.88
Stocks.USLV                           9.98
Stocks.USSG                           1.67
Stocks.USSV                           1.61
```

```
Stocks.DMexUS                                    19.79
Stocks.EM                                         8.01
             Posterior excess returns(OMEGA = 0.000026), %
Bonds.exUS                                               0.81
Bonds.US                                                 0.11
Bonds.HY                                                 2.28
Stocks.USLG                                              3.52
Stocks.USLV                                              3.38
Stocks.USSG                                              4.44
Stocks.USSV                                              4.16
Stocks.DMexUS                                            4.31
Stocks.EM                                                5.10
             Posterior weights(OMEGA = 0.000026), %
Bonds.exUS                                      -13.04
Bonds.US                                         19.90
Bonds.HY                                          2.50
Stocks.USLG                                       9.88
Stocks.USLV                                       9.98
Stocks.USSG                                       1.67
Stocks.USSV                                       1.61
Stocks.DMexUS                                    19.79
Stocks.EM                                         8.01
```

As expected, with the certain view the posterior return of Bonds.exUS is exactly the same as the view. The returns of other assets are adjusted because of the correlation. The most considerable adjustments are demonstrated by Bonds.US, Stocks.DMexUS and Stocks.EM, which have the highest correlation with Bonds.exUS. Bonds.HY also has large correlation with Bonds.exUS, but for some reasons does not demonstrate considerable adjustment comparing the other indices correlated with Bonds.exUS. Regarding the weights only the weight of Bonds.exUS is changed, but it changes very considerably and becomes negative, from 26.66% to -29.27%.

Quite obviously, when the variance of the view goes to infinity, posterior returns and weigths are equal to prior ones.

When there is some uncertainty of the view, but a finite one, the posterior of Bonds.exUS becomes 0.81%, which is between 0.5% (the view) and 1.57% (the prior). The pattern of adjustment of returns is the same as in the case of certain views. The most considerable adjustments are demonstrated by Bonds.US, Stocks.DMexUS and Stocks.EM, which have the highest correlation with Bonds.exUS. Bonds.HY also has large correlation with Bonds.exUS, but for some reasons does not demonstrate considerable adjustment comparing the other indices correlated with Bonds.exUS. Again, the weight of Bonds.exUS is the only one which changed, but it again changes very considerably and becomes negative (although not that negative as in case of certain view), from 26.66% to -13.04%.

Let's examine whether the posterior returns and weights are affected by changes in τ. Let's choose $\tau = 0.00001$, smaller than our basic $\tau = 0.01$, and $\tau = 10$, larger than our basic $\tau = 0.01$.

The differences between BL procedure with $\tau = 0.00001$ and our basic $\tau = 0.01$:

```
             Prior excess_returns, % Prior weights, %
Bonds.exUS                        0                0
Bonds.US                          0                0
Bonds.HY                          0                0
Stocks.USLG                       0                0
Stocks.USLV                       0                0
Stocks.USSG                       0                0
Stocks.USSV                       0                0
Stocks.DMexUS                     0                0
Stocks.EM                         0                0
             Posterior excess returns(OMEGA = 0), %
Bonds.exUS                                        0
```

```
Bonds.US                                             0
Bonds.HY                                             0
Stocks.USLG                                          0
Stocks.USLV                                          0
Stocks.USSG                                          0
Stocks.USSV                                          0
Stocks.DMexUS                                        0
Stocks.EM                                            0
            Posterior weights(OMEGA = 0), %
Bonds.exUS                                    0
Bonds.US                                      0
Bonds.HY                                      0
Stocks.USLG                                   0
Stocks.USLV                                   0
Stocks.USSG                                   0
Stocks.USSV                                   0
Stocks.DMexUS                                 0
Stocks.EM                                     0
            Posterior excess returns(OMEGA = Inf), %
Bonds.exUS                                           0
Bonds.US                                             0
Bonds.HY                                             0
Stocks.USLG                                          0
Stocks.USLV                                          0
Stocks.USSG                                          0
Stocks.USSV                                          0
Stocks.DMexUS                                        0
Stocks.EM                                            0
            Posterior weights(OMEGA = Inf), %
Bonds.exUS                                    0
Bonds.US                                      0
Bonds.HY                                      0
Stocks.USLG                                   0
Stocks.USLV                                   0
Stocks.USSG                                   0
Stocks.USSV                                   0
Stocks.DMexUS                                 0
Stocks.EM                                     0
            Posterior excess returns(OMEGA = 0.000026), %
Bonds.exUS                                           0.76
Bonds.US                                             0.17
Bonds.HY                                             0.50
Stocks.USLG                                          0.46
Stocks.USLV                                          0.49
Stocks.USSG                                          0.47
Stocks.USSV                                          0.49
Stocks.DMexUS                                        0.87
Stocks.EM                                            1.12
            Posterior weights(OMEGA = 0.000026), %
Bonds.exUS                                    39.56
Bonds.US                                      0.00
Bonds.HY                                      0.00
Stocks.USLG                                   0.00
Stocks.USLV                                   0.00
Stocks.USSG                                   0.00
Stocks.USSV                                   0.00
Stocks.DMexUS                                 0.00
Stocks.EM                                     0.00
```

As we can see, there is no differences in extreme cases (certain view and totally uncertain view), as expected from the idea of BL model. But with some finite uncertainty the posterior excess returns of all indices increase, while the weight of Bonds.exUS is 26.52% - quite close to the prior one (so the difference is +39.56 percentage points).

The differences between BL procedure with $\tau = 10$ and our basic $\tau = 0.01$:

	Prior excess_returns, %	Prior weights, %
Bonds.exUS	0	0
Bonds.US	0	0
Bonds.HY	0	0
Stocks.USLG	0	0
Stocks.USLV	0	0
Stocks.USSG	0	0
Stocks.USSV	0	0
Stocks.DMexUS	0	0
Stocks.EM	0	0

	Posterior excess returns(OMEGA = 0), %
Bonds.exUS	0
Bonds.US	0
Bonds.HY	0
Stocks.USLG	0
Stocks.USLV	0
Stocks.USSG	0
Stocks.USSV	0
Stocks.DMexUS	0
Stocks.EM	0

	Posterior weights(OMEGA = 0), %
Bonds.exUS	0
Bonds.US	0
Bonds.HY	0
Stocks.USLG	0
Stocks.USLV	0
Stocks.USSG	0
Stocks.USSV	0
Stocks.DMexUS	0
Stocks.EM	0

	Posterior excess returns(OMEGA = Inf), %
Bonds.exUS	0
Bonds.US	0
Bonds.HY	0
Stocks.USLG	0
Stocks.USLV	0
Stocks.USSG	0
Stocks.USSV	0
Stocks.DMexUS	0
Stocks.EM	0

	Posterior weights(OMEGA = Inf), %
Bonds.exUS	0
Bonds.US	0
Bonds.HY	0
Stocks.USLG	0
Stocks.USLV	0
Stocks.USSG	0
Stocks.USSV	0
Stocks.DMexUS	0
Stocks.EM	0

	Posterior excess returns(OMEGA = 0.000026), %
Bonds.exUS	-0.31

```
Bonds.US                                                  -0.07
Bonds.HY                                                  -0.21
Stocks.USLG                                               -0.19
Stocks.USLV                                               -0.20
Stocks.USSG                                               -0.19
Stocks.USSV                                               -0.20
Stocks.DMexUS                                             -0.36
Stocks.EM                                                 -0.46
            Posterior weights(OMEGA = 0.000026), %
Bonds.exUS                                      -16.21
Bonds.US                                          0.00
Bonds.HY                                          0.00
Stocks.USLG                                       0.00
Stocks.USLV                                       0.00
Stocks.USSG                                       0.00
Stocks.USSV                                       0.00
Stocks.DMexUS                                     0.00
Stocks.EM                                         0.00
```

Again, as we can see, there is no differences in extreme cases (certain view and totally uncertain view), as expected from the idea of BL model. But with some finite uncertainty the posterior excess returns of all indices decrease, while the weight of Bonds.exUS is -29.25% - quite far away from prior and very close to that in case of certain view with $\tau = 0.01$ (so the difference is -16.21 percentage points).

Task 4 - Black-Litterman - relative view

In this task:
$P\_2 = (0,0,0,1,0,0,0,-0.71,-0.29)$;
$V\_2 = (0.01)$;
$OMEGA\_4 = 0.000083$;

The results of BL procedure with $\tau = 0.01$:

```
          Prior excess_returns, % Prior weights, %
Bonds.exUS                   1.57             26.66
Bonds.US                     0.28             19.90
Bonds.HY                     2.79              2.50
Stocks.USLG                  3.98              9.88
Stocks.USLV                  3.87              9.98
Stocks.USSG                  4.91              1.67
Stocks.USSV                  4.65              1.61
Stocks.DMexUS                5.19             19.79
Stocks.EM                    6.22              8.01
            Posterior excess returns(OMEGA = 0.000083), %
Bonds.exUS                                        0.99
Bonds.US                                          0.18
Bonds.HY                                          2.01
Stocks.USLG                                       3.71
Stocks.USLV                                       3.37
Stocks.USSG                                       4.44
Stocks.USSV                                       4.15
Stocks.DMexUS                                     3.79
Stocks.EM                                         4.22
            Posterior weights(OMEGA = 0.000083), %
Bonds.exUS                                       26.66
Bonds.US                                         19.90
Bonds.HY                                          2.50
Stocks.USLG                                      58.19
Stocks.USLV                                       9.98
```

```
Stocks.USSG                                    1.67
Stocks.USSV                                    1.61
Stocks.DMexUS                                -14.61
Stocks.EM                                     -5.91
```

In equilibrium difference between US Large Growth and the capital weighted average of International Developed and Emerging Equity excess returns was -1.5%. In posterior excess returns this difference became equal to - 0.2%. This result reflects the shift towards the view. The returns of other assets are adjusted because of the correlation.

The posterior weight of US Large Growth equals to 58.19% compared to the prior weight of 9.88%. The posterior weights of International Developed and Emerging Equity are negative. The posterior weights of other assets are equal to their prior weights.

Task 5 - Black-Litterman - relative view

In this task:

$$P\_3 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -0,71 & -0,29 \end{bmatrix}$$

$$V\_3 = (0.005, 0.01)^t$$

$$OMEGA\_5 = \begin{bmatrix} 0.000026 & 0 \\ 0 & 0.000083 \end{bmatrix}$$

The results of BL procedure with $\tau = 0.01$:

```
                 Prior excess_returns, % Prior weights, %
Bonds.exUS                          1.57            26.66
Bonds.US                            0.28            19.90
Bonds.HY                            2.79             2.50
Stocks.USLG                         3.98             9.88
Stocks.USLV                         3.87             9.98
Stocks.USSG                         4.91             1.67
Stocks.USSV                         4.65             1.61
Stocks.DMexUS                       5.19            19.79
Stocks.EM                           6.22             8.01
                 Posterior excess returns, % Posterior weights, %
Bonds.exUS                          0.66             6.52
Bonds.US                            0.10            19.90
Bonds.HY                            1.83             2.50
Stocks.USLG                         3.50            53.50
Stocks.USLV                         3.17             9.98
Stocks.USSG                         4.25             1.67
Stocks.USSV                         3.95             1.61
Stocks.DMexUS                       3.48           -11.26
Stocks.EM                           3.85            -4.56
```

The posterior excess return on International Bonds is lower than the prior one reflecting the shift towards the first view.

In equilibrium difference between US Large Growth and the capital weighted average of International Developed and Emerging Equity excess returns was -1.5%. In posterior excess returns this difference became equal to - 0.1%. This result reflects the shift towards the second view. The returns of other assets are adjusted because of the correlation.

The posterior weight of International Bonds is lower than the prior one. The posterior weight of US Large Growth has increased compared to the prior weight. The posterior weights of International Developed and Emerging Equity are negative. The posterior weights of other assets are equal to their prior weights.