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Dynamic Credit Risk Models SS 2018

Exercises Series 4, to be handed in by 7.5.2018, 9 am

1. Exchangeable mixture model. (5 points) Suppose you have a portfolio of 1000 similarly rated obligors. You decide to calibrate an exchangeable Bernoulli mixture model with a beta-distributed mixing variable to this portfolio.

a) (2 points) Determine the parameters a and b of the mixing distribution that would give a model with default probability $\pi = 0.01$ and default correlation $\rho_Y = 0.005$.

b) (3 points) Show that for any exchangeable mixture model the default correlation satisfies $\rho_Y \in [0, 1]$. Which choice of the mixing distribution gives $\rho_Y = 0$ respectively $\rho_Y = 1$ (for a fixed with default probability π)?

2. Exchangeable probit-normal mixture (6 points) The credit risk of a portfolio of m similar obligors is modelled with an exchangeable Bernoulli mixture model where the mixing variable Q has a probit-normal distribution with parameters $\mu \in \mathbb{R}$ and $\sigma > 0$.

a) (4 points) Derive the distribution function and density function of Q.

b) (2 points) Show that the kth order joint default probability is given by

$$\pi_k = \int_{-\infty}^{\infty} \Phi^k(\mu + \sigma x)\phi(x)dx,$$

where ϕ and Φ are the density and the df of the standard normal distribution.

3. Exponential tilting for the gamma distribution (4 points) Let $X \sim \text{Ga}(\alpha, \beta)$ be a gamma distributed random variable with density denoted $f(x)$. Compute the exponential tilting density $g_t(x) = \exp(tx)f(x)/M_X(t)$ where $M_X(t)$ is the moment generating function. Are there any constraints on the value of t? Can the mean of X be shifted to arbitrary values under the importance sampling density?

Exercise 3 gives bonus points.