Task 3.3

April 5, 2019

```
> # Download package "Sim.DiffProc", version 2.5
> # library(devtools)
> # install_version("Sim.DiffProc", version = "2.5",
> #                 repos = "http://cran.us.r-project.org")
>
> # Libraries
> library(Sim.DiffProc)
> calculate_fair_spread <- function(p_x0, p_r, p_sigma){
+
+   # Quarterly premium payments
+   paym_times = seq(from = 0.25, to = 5, by = 0.25)
+   paym_number = 20
+
+   # Parameters for CIR function that simulates the process
+   # which remains constant
+   N = 1000
+   M = 100
+   t0 = 0
+   theta = 1
+
+   # Other parameters
+   rate = 0.01
+   LGD = 0.6
+   delta_t = 0.25
+
+   # Vectors to store intermediate results
+   gamma_integrals = rep(NA, paym_number)
+   exponents_of_rate = rep(NA, paym_number)
+   exponents_of_gamma = rep(NA, paym_number)
+
+   # Loop over all premium payments
+   for (i in 1:paym_number)
+   {
+     t = paym_times[i]
+
```

```
+     # Simulate CIR process
+     CIR_result = CIR(N = N,
+                      M = M,
+                      t0 = t0,
+                      T = t,
+                      x0 = p_x0,
+                      theta = theta,
+                      r = p_r,
+                      sigma = p_sigma)
+
+     # Average process trajectory
+     gamma_average = mean(CIR_result$X.mean)
+
+     # Monte Carlo integration
+     gamma_integrals[i] = gamma_average * t
+
+     # Apply exponent function
+     exponents_of_gamma[i] = exp(-gamma_integrals[i])
+     exponents_of_rate[i] = exp(-rate*t)
+   }
+
+   fair_spread = LGD * (exponents_of_rate[1] - exponents_of_rate[paym_number] *
+     exponents_of_gamma[paym_number] + exponents_of_gamma[1:(paym_number - 1)] %*%
+     (exponents_of_rate[2:paym_number] - exponents_of_rate[1:(paym_number - 1)])) /
+     (delta_t * exponents_of_rate %*% exponents_of_gamma)
+
+   return(fair_spread)
+ }
> # Parameters for CIR function that simulates the process
> x0 = 0.03
> r = 0.04
> sigma = 0.2
> calculate_fair_spread(p_x0 = x0,
+                       p_r = r,
+                       p_sigma = sigma)

           [,1]
[1,] 0.02307365

> # Increase x0
> x0_increased = 0.3
> calculate_fair_spread(p_x0 = x0_increased,
+                       p_r = r,
+                       p_sigma = sigma)

           [,1]
[1,] 0.05950977
```

```
> # Increase r
> r_increased = 0.4
> calculate_fair_spread(p_x0 = x0,
+                       p_r = r_increased,
+                       p_sigma = sigma)

          [,1]
[1,] 0.1802421

> # Increase sigma
> sigma_increased = 0.28
> calculate_fair_spread(p_x0 = x0,
+                       p_r = r,
+                       p_sigma = sigma_increased)

           [,1]
[1,] 0.02188583
```

As a result we got fair spread $X^* \approx 0.023$.
When we increased ψ_0 from 0.03 to 0.3 we got $X^*_{new} \approx 0.06 > X^*$.
When we increased θ from 0.04 to 0.4 we got $X^*_{new} \approx 0.179 > X^*$.
When we increased σ from 0.2 to 0.28 we got $X^*_{new} \approx 0.0228 \approx X^*$.