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Dynamic Credit Risk Models SS 2018

Exercises Series 3, to be handed in by 30.4.2018,

1. Copulas and credit risk. (8 points) Denote by τ_1, τ_2 the default times of two companies. Assume that the τ_i are exponentially distributed with parameters λ_i, $i = 1, 2$, ($P(\tau_i \leq t) = 1 - \exp(-\lambda_i t)$). Using Sklars theorem it is possible to construct a bivariate distribution F such that $\tau_i \sim \text{Exp}(\lambda_i)$ and such that the dependence structure of (τ_1, τ_2) is given by C_ρ^{Ga}, the Gauss copula with correlation parameter ρ. In this exercise we take $\rho = 0.3$.

a) (4 points.) You are given a random number generator which draws random numbers from the one-dimensional standard normal distribution. Develop an algorithm how to draw 100 random vectors (τ_1, τ_2) from the two-dimensional distribution F.

b) (4 points) Assume that $\lambda_1 = \lambda_2$ (homogenous portfolio). Express the probability of the following two events in terms of C_ρ^{Ga}. Answer qualitatively, how an increase in ρ affects both probabilities.

- $P(\{\tau_1 \leq T\} \cap \{\tau_2 \leq T\})$ (both firms default before T)
- $P(\{\tau_1 \leq T\} \cup \{\tau_2 \leq T\})$ (at least one firm defaults before T)

2. Moments of the credit loss distribution (5 points) Consider a portfolio containing $m = 1000$ equally rated credit risks. Assume that for every obligor the exposure is $e_i = 1M$ EURO and the default probability is $p_i = 1\%$. Calculate the expected value and standard deviation of the portfolio loss in the following two situations:
i) LGDs are deterministic and $\delta_i = 0.4$ for all i, defaults occur independently.
ii) LGDs are deterministic as in i) but defaults are dependent, the default correlation between pairs of default indicators is given by $\rho(Y_i, Y_j) = 0.005$ for $1 \leq i, j \leq m,\ i \neq j$.

3. Two-factor Gaussian threshold model (7 points) Consider a Gaussian threshold model $(\mathbf{X}, \mathbf{d})$ in which the critical variables follow the factor model

$$X_i = \sqrt{\beta_i}\tilde{F}_i + \sqrt{1 - \beta_i}\epsilon_i, \quad i = 1, \dots, m,$$

where $\tilde{F}_i, \epsilon_1, \dots, \epsilon_m$ are independent standard normal variables, $0 \leq \beta_i \leq 1$ for all i, and the systematic variables $\tilde{F}_i$ satisfy

$$\tilde{F}_i = \begin{cases} F_1, & i = 1, \dots, n, \\ \rho F_1 + \sqrt{1 - \rho^2} F_2, & i = n + 1, \dots, m, \end{cases}$$

where F_1, F_2 are independent standard normal factors, $0 \leq \rho \leq 1$ and $1 < n < m - 1$. Obviously this model has a two-group structure determining the dependence between defaults.

a) (3 points) Derive expressions for the within-group asset correlations in the two groups and the between-group asset correlation.

b) (2 points) Give the mixture representation of the model.

c) (2 points) Express the joint default probability probability $P(Y_1 = 1, Y_m = 1)$ as an integral with respect to the distribution of the factors on $\mathbb{R}^2$.