

K. J. SOMAIYA COLLEGE OF ENGINEERING
DEPARTMENT OF ELECTRONICS ENGINEERING
ELECTRONIC CIRCUITS
DC CIRCUITS

Numerical 1: Find i and V_o in the following circuit:

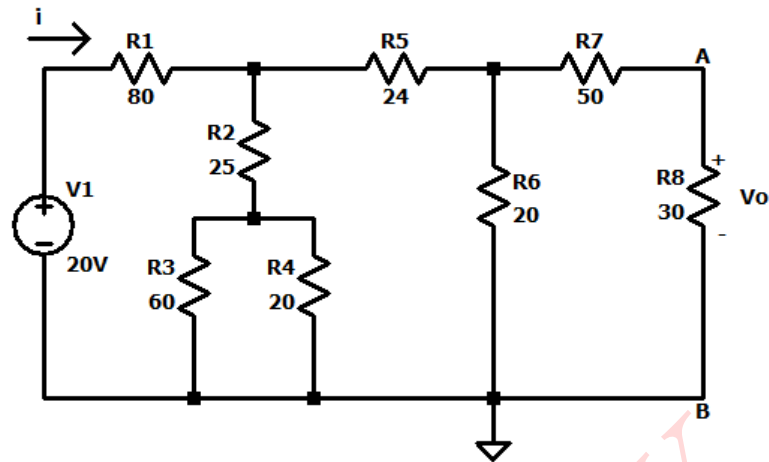


Figure 1: Circuit 1

Solution:

We will first use series - parallel reduction techniques to simplify the circuit. Then we will apply KVL to the loops generated to find i .

60Ω and 20Ω are connected in parallel.

$$\begin{aligned}\therefore R_p &= 60 \parallel 20 \\ &= \frac{60 \times 20}{60 + 20} \\ &= 15\Omega\end{aligned}$$

\therefore The circuit is simplified as shown in figure 2:

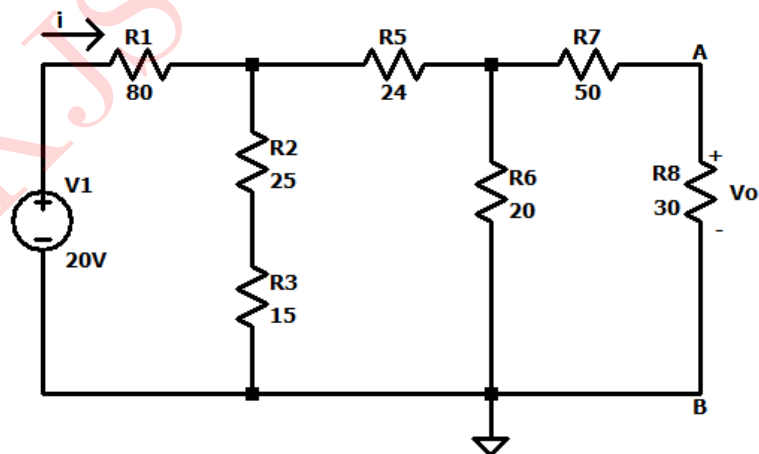


Figure 2: Simplified Circuit 1a for figure 1

25Ω and 15Ω resistors are connected in series

$$\therefore R_s = 25 + 15 = 40\Omega$$

∴ The circuit is further simplified as shown in figure 3:

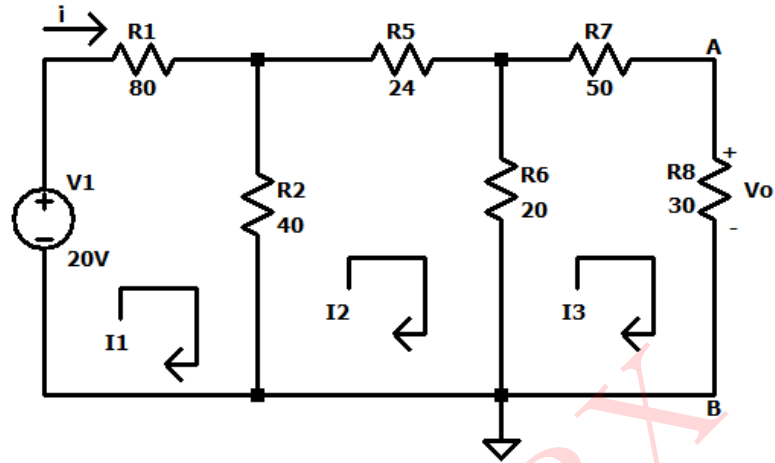


Figure 3: Simplified Circuit 1b for figure 2

We will now use Mesh Analysis to find i

Assume mesh currents I_1 , I_2 and I_3 flowing through loops 1, 2 and 3 in clockwise direction

Applying KVL to loop 1, we get:

$$\begin{aligned} 20 - 80I_1 - 40(I_1 - I_2) &= 0 \\ \therefore -120I_1 + 40I_2 &= -20 \end{aligned} \quad \text{.....(i)}$$

Applying KVL to loop 2, we get:

$$\begin{aligned} -24I_2 - 20(I_2 - I_3) - 40(I_2 - I_1) &= 0 \\ \therefore 40I_1 - 84I_2 + 20I_3 &= 0 \end{aligned} \quad \text{.....(ii)}$$

Applying KVL to loop 3, we get:

$$\begin{aligned} -20(I_3 - I_2) - 50I_3 - 30I_3 &= 0 \\ \therefore 20I_2 - 100I_3 &= 0 \end{aligned} \quad \text{.....(iii)}$$

Solving (i), (ii), (iii) we get

$$I_1 = 0.2A, I_2 = 0.1A, I_3 = 0.02A$$

$$\therefore i = I_1 = 0.2A$$

Now $I_{30\Omega} = I_3$
 $\therefore V_o = I_{30\Omega} \times 30$
 $= 0.2 \times 30$
 $= 0.6V$
 $\therefore i = 0.2A$
 $\therefore V_o = 0.6V$

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

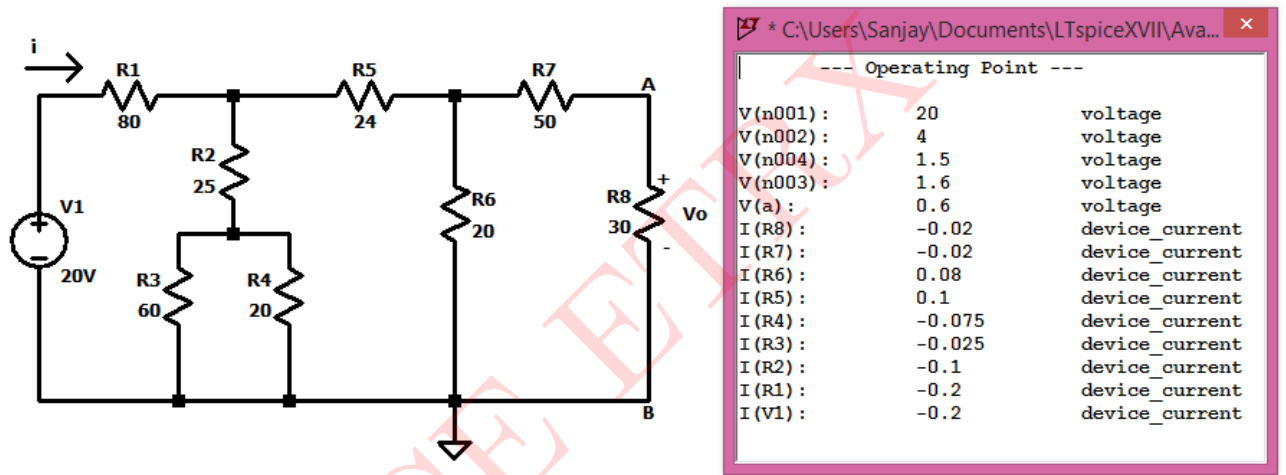


Figure 4: Circuit Schematic and Simulated Results

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
i	0.2A	0.2A
V _o	0.6V	0.6V

Table 1: Numerical 1

Numerical 2: Obtain the equivalent resistance between the terminals a-b for each of the circuits:

(a)

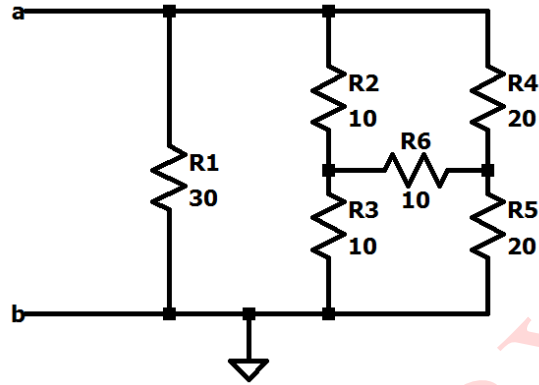


Figure 5: Circuit 2a

Solution:

We will use series - parallel and star-delta reduction techniques to obtain the equivalent resistance between terminals a and b.

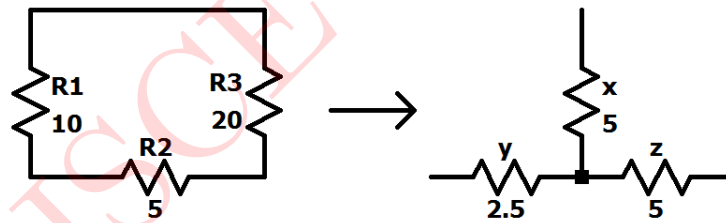


Figure 6: Delta - Star Conversion

The resistors 10Ω , 10Ω and 20Ω are connected in delta.

Converting delta to star, we have:

$$x = \frac{10 \times 20}{10 + 10 + 20} = 5\Omega$$

$$y = \frac{10 \times 10}{10 + 10 + 20} = 2.5\Omega$$

$$z = \frac{20 \times 10}{10 + 10 + 20} = 5\Omega$$

∴ The circuit is simplified as shown in figure 7:

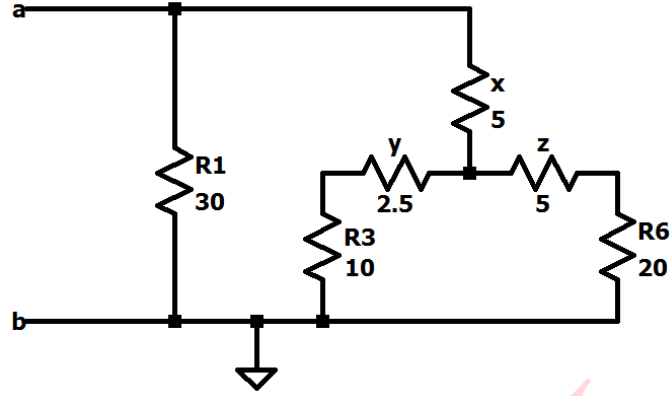


Figure 7: Simplified Circuit 2a.1 for figure 6

The resistors 10Ω and 2.5Ω are connected in series.

$$\therefore R_s = 10 + 2.5 = 12.5\Omega$$

The resistors 20Ω and 5Ω are connected in series.

$$\therefore R_s = 20 + 5 = 20\Omega$$

∴ The circuit is further simplified as shown in figure 8:

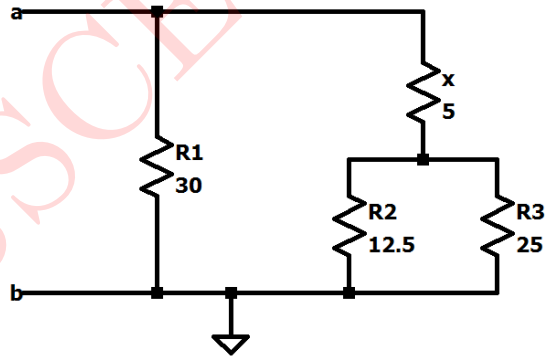


Figure 8: Simplified Circuit 2a.2 for figure 7

The resistors 12.5Ω and 25Ω are connected in parallel.

$$\begin{aligned} \therefore R_p &= 12.5 \parallel 25 \\ &= \frac{12.5 \times 25}{12.5 + 25} \\ &= 8.33\Omega \end{aligned}$$

∴ The circuit is further simplified as shown in figure 9:

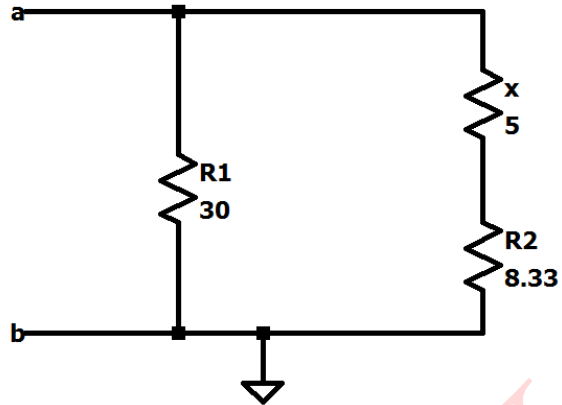


Figure 9: Simplified Circuit 2a.3 for figure 8

The resistors 8.33Ω and 5Ω are connected in series.

$$\therefore R_s = 8.33 + 5 = 13.33\Omega$$

∴ The circuit is further simplified as shown in figure 10:

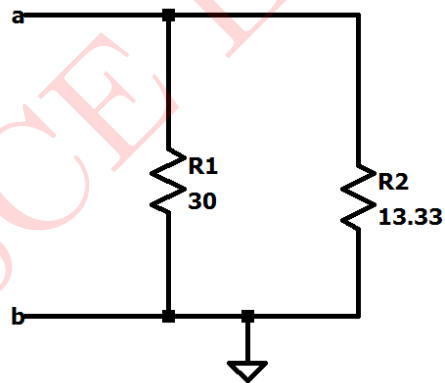


Figure 10: Simplified Circuit 2a.4 for figure 9

The resistors 30Ω and 13.33Ω are connected in parallel.

$$\begin{aligned} \therefore R_{ab} &= 30 \parallel 13.33 \\ &= \frac{30 \times 13.33}{30 + 13.33} \\ &= 9.23\Omega \end{aligned}$$

$$\therefore R_{ab} = 9.23\Omega$$

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

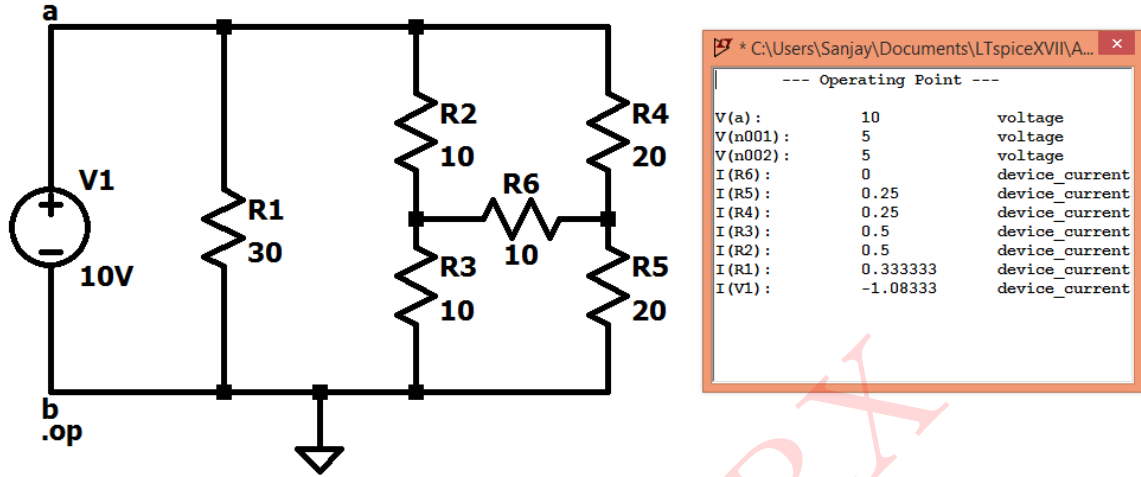


Figure 11: Circuit Schematic and Simulated Results

$$\therefore R_{ab} = \frac{V1}{I(V1)} = \frac{10}{1.0833} = 9.2308\Omega$$

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
R_{ab}	9.23Ω	9.2308Ω

Table 2: Numerical 2a

(b)

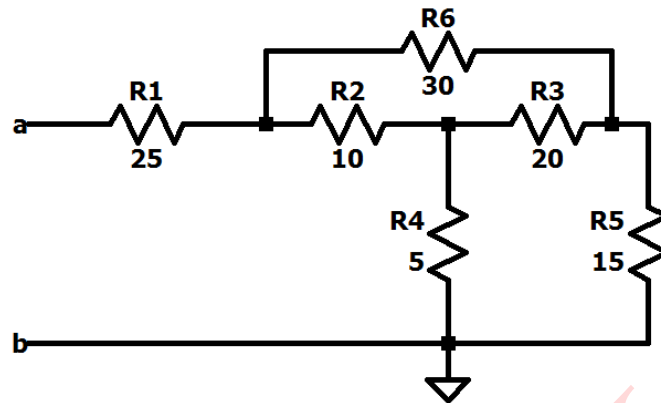


Figure 12: Circuit 2b

Solution:

We will use series - parallel and star-delta reduction techniques to obtain the equivalent resistance between terminals a and b.

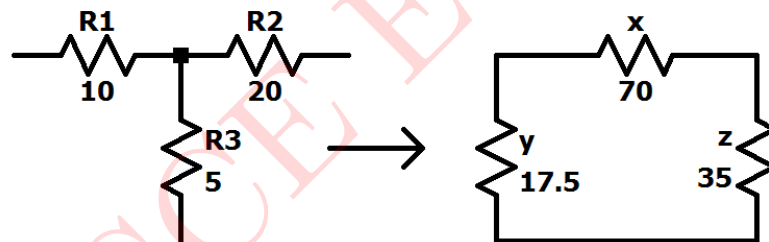


Figure 13: Star - Delta Conversion

The resistors 10Ω , 20Ω and 5Ω are connected in delta.

Converting star to delta, we have:

$$x = 10 + 20 + \frac{10 \times 20}{5} = 70\Omega$$

$$y = 10 + 5 + \frac{10 \times 5}{20} = 17.5\Omega$$

$$z = 20 + 5 + \frac{20 \times 5}{10} = 35\Omega$$

∴ The circuit is simplified as shown in figure 14:

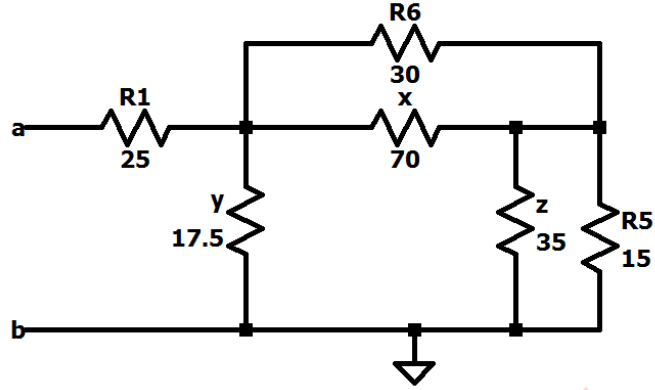


Figure 14: Simplified Circuit 2b.1 for figure 13

The resistors 30Ω and 70Ω are connected in parallel.

$$\begin{aligned}\therefore R_p &= 30 \parallel 70 \\ &= \frac{30 \times 70}{30 + 70} \\ &= 21\Omega\end{aligned}$$

Similarly, the resistors 35Ω and 15Ω are connected in parallel.

$$\begin{aligned}\therefore R_p &= 35 \parallel 15 \\ &= \frac{35 \times 15}{35 + 15} \\ &= 10.5\Omega\end{aligned}$$

∴ The circuit is further simplified as shown in figure 15:

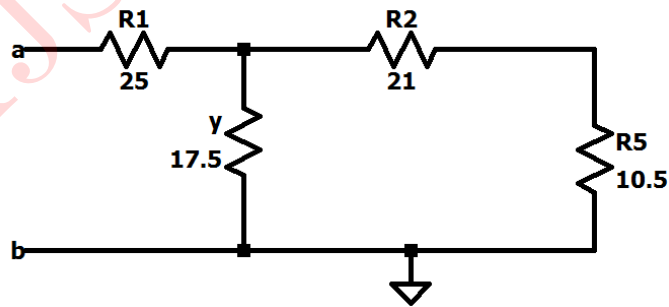


Figure 15: Simplified Circuit 2b.2 for figure 14

The resistors 21Ω and 10.5Ω are connected in series.

$$\therefore R_s = 21 + 10.5 = 31.5\Omega$$

∴ The circuit is further simplified as shown in figure 16:

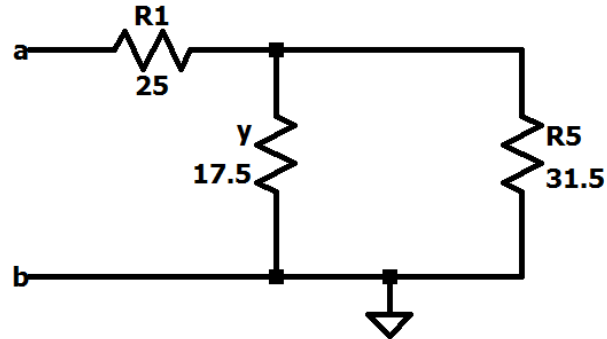


Figure 16: Simplified Circuit 2b.3 for figure 15

The resistors 17.5Ω and 31.5Ω are connected in parallel.

$$\begin{aligned}\therefore R_p &= 17.5 \parallel 31.5 \\ &= \frac{17.5 \times 31.5}{17.5 + 31.5} \\ &= 11.25\Omega\end{aligned}$$

So, the circuit is further simplified as shown in figure 17:

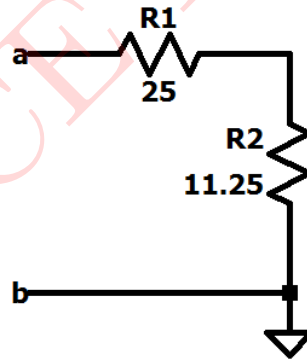


Figure 17: Simplified Circuit 2b.4 for figure 16

The resistors 25Ω and 11.25Ω are connected in series.

$$\therefore R_{ab} = 25 + 11.25 = 36.25\Omega$$

$$\therefore \mathbf{R_{ab} = 36.25\Omega}$$

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

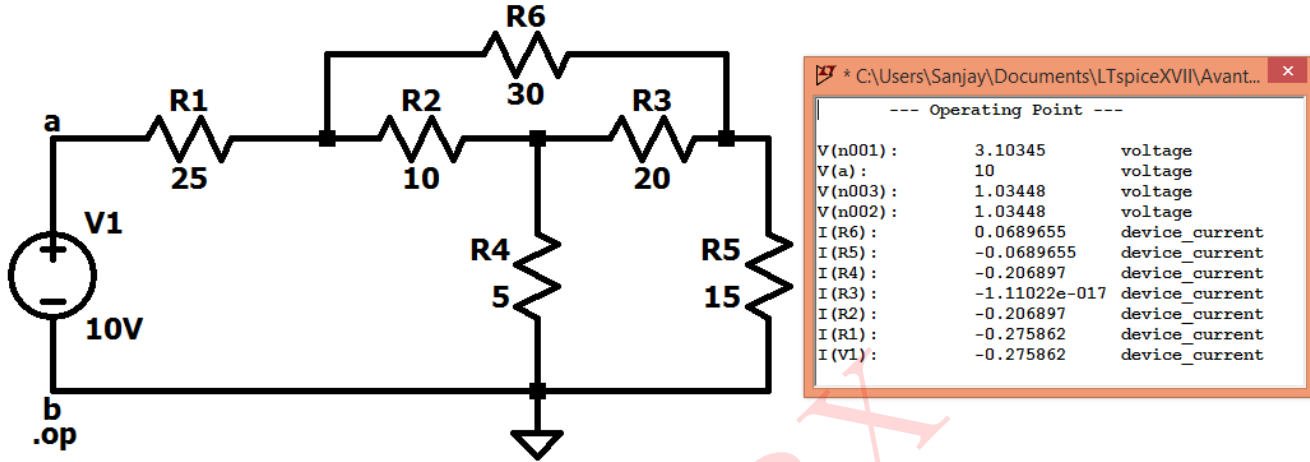


Figure 18: Circuit Schematic and Simulated Results

$$\therefore R_{ab} = \frac{V1}{I(V1)} = \frac{10}{0.275862} = 36.250\Omega$$

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
R_{ab}	36.25Ω	36.250Ω

Table 3: Numerical 2b

Numerical 3: For the circuit, use superposition theorem to find i and calculate the power delivered by the 4Ω resistor :

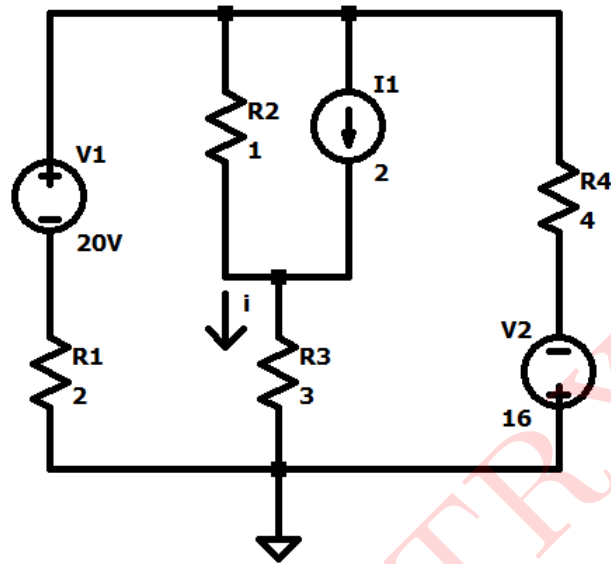


Figure 19: Circuit 3

Solution:

We will use Superposition theorem to find i .

Case 1: We will first consider the 20V source alone and replace the other sources by their internal resistances

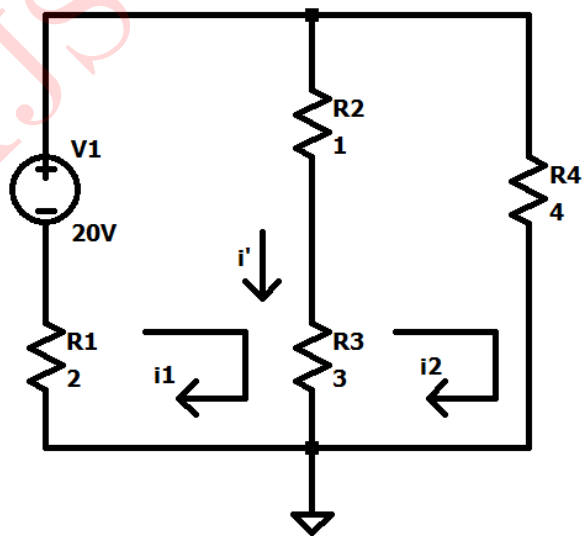


Figure 20: Case 1 - Considering only 20V source acting alone

We will now use Mesh Analysis to find i'

Assume mesh currents i_1 and i_2 flowing through loops 1 and 2 in clockwise direction

Applying KVL to loop 1, we get:

$$\begin{aligned} 20 - 1(i_1 - i_2) - 3(i_1 - i_2) - 2i_1 &= 0 \\ \therefore -6i_1 + 4i_2 &= -20 \end{aligned} \quad \text{.....(i)}$$

Applying KVL to loop 2, we get:

$$\begin{aligned} -3(i_2 - i_1) - 1(i_2 - i_1) - 4i_2 &= 0 \\ \therefore 4i_1 - 8i_2 &= 0 \end{aligned} \quad \text{.....(ii)}$$

Solving (i) and (ii) we get

$$i_1 = 5\text{A and } i_2 = 2.5\text{A,}$$

$$\therefore i' = i_1 - i_2 = 5 - 2.5 = 2.5\text{A} \downarrow$$

Case 2: We will now consider the 16V source alone and replace the other sources by their internal resistances

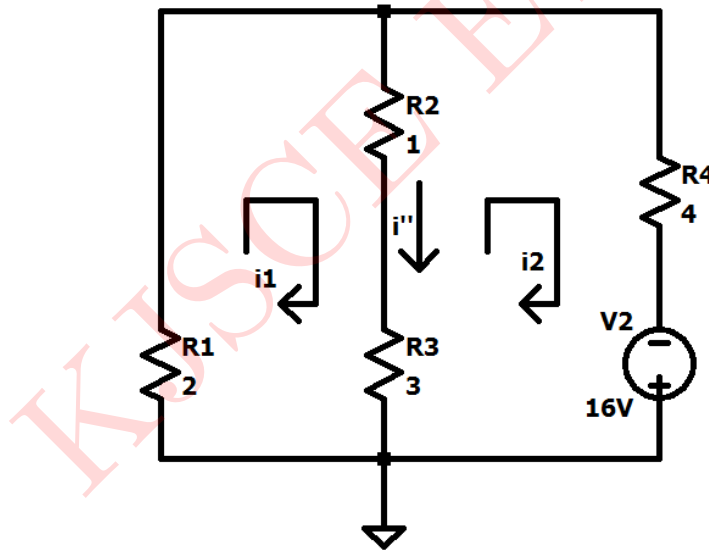


Figure 21: Case 2 - Considering only 16V source acting alone

We will now use Mesh Analysis to find i''

Assume mesh currents i_1 and i_2 flowing through loops 1 and 2 in clockwise direction

Applying KVL to loop 1, we get:

$$\begin{aligned} -2i_1 - 1(i_1 - i_2) - 3(i_1 - i_2) &= 0 \\ \therefore -6i_1 + 4i_2 &= 0 \end{aligned} \quad \text{.....(i)}$$

Applying KVL to loop 2, we get:

$$\begin{aligned} -3(i_2 - i_1) - 1(i_2 - i_1) - 4i_2 + 16 &= 0 \\ \therefore 4i_1 - 8i_2 &= -16 \end{aligned} \quad \text{.....(ii)}$$

Solving (i) and (ii) we get

$$i_1 = 2\text{A and } i_2 = 3\text{A,}$$

$$\therefore i'' = i_1 - i_2 = -1\text{A} \downarrow$$

Case 3: We will now consider the 2A source alone and replace the other sources by their internal resistances

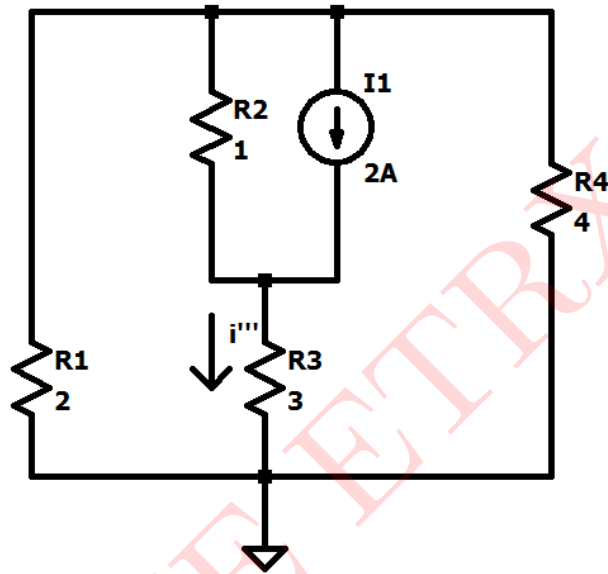


Figure 22: Case 3 - Considering only 2A source acting alone

The resistors 2Ω and 4Ω are connected in parallel.

$$\begin{aligned}\therefore R_p &= 2 \parallel 4 \\ &= \frac{2 \times 4}{2 + 4} \\ &= 1.333\Omega\end{aligned}$$

∴ The circuit is simplified as shown in figure 23:

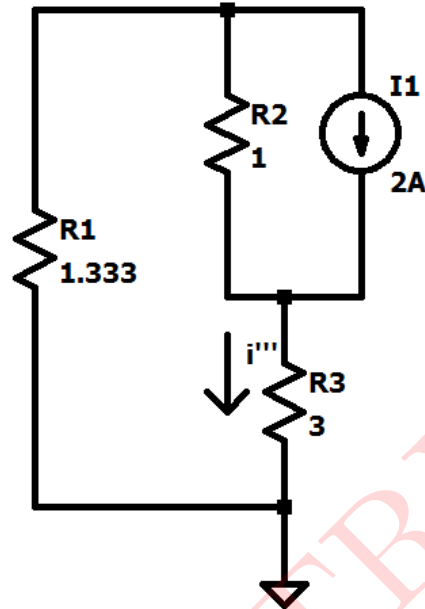


Figure 23: Simplified Circuit 3a for figure 22

∴ The circuit can be redrawn as shown in figure 24 :

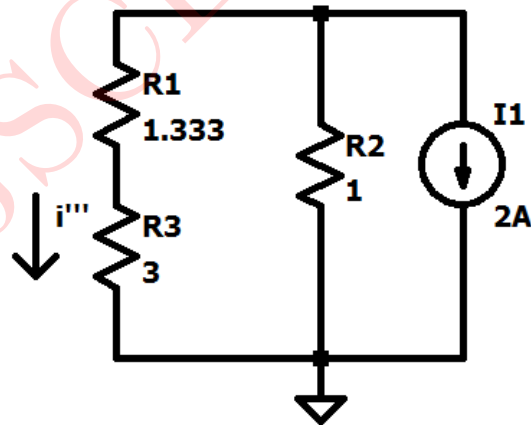


Figure 24: Simplified Circuit 3b for figure 23

By current division rule, we get:

$$\therefore i''' = \frac{2 \times 1}{1.333 + 3 + 1}$$

$$\therefore i''' = 0.3750\text{A} \downarrow$$

$$\therefore i = i' + i'' + i'''$$

$$= 2.5\downarrow + (-1)\downarrow + 0.3750\downarrow$$

$$= 1.875\text{A} \downarrow$$

$$\begin{aligned}
 \therefore P &= i^2 \times R_3 \\
 &= 1.875^2 \times 3 \\
 &= 10.546875W
 \end{aligned}$$

$$\begin{aligned}
 \therefore i &= 1.875A \\
 \therefore P &= 10.546875W
 \end{aligned}$$

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

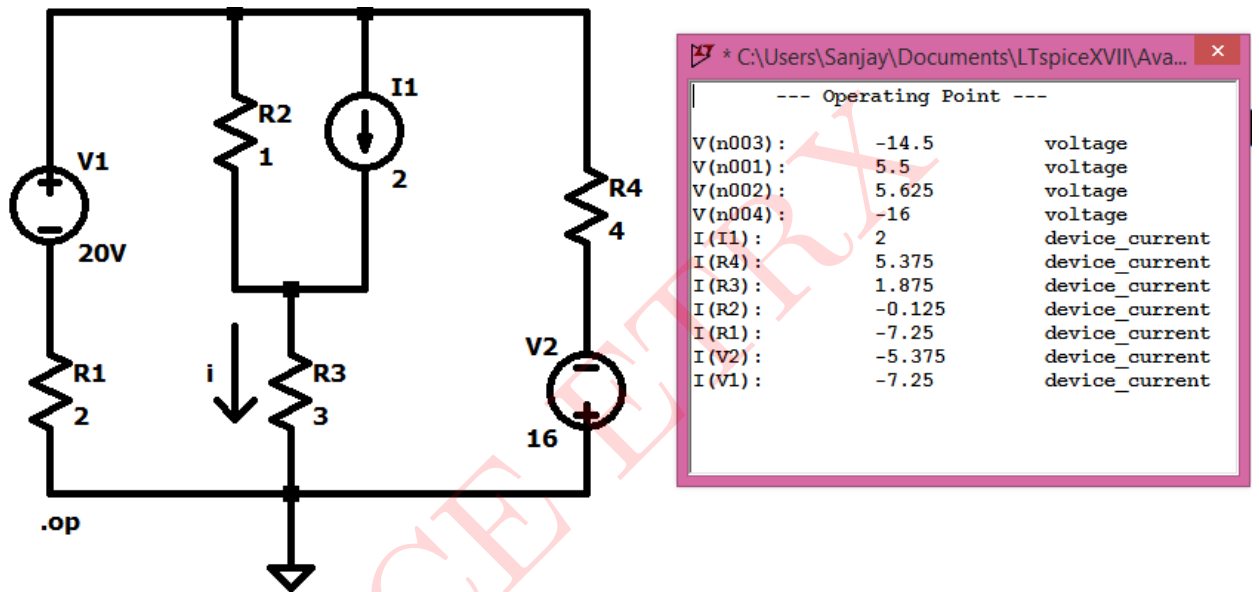


Figure 25: Circuit Schematic and Simulated Results

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
i	1.875A	1.8750A
P	10.546875W	10.546875W

Table 4: Numerical 3

Numerical 4: For the given circuit shown in figure 26, use superposition theorem to find i_o .

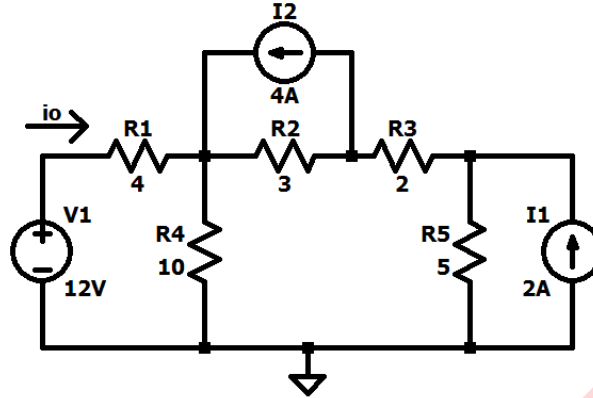


Figure 26: Circuit 4

Solution:

We will use Superposition theorem to find i_o .

Case 1: We will first consider the 12V source alone and replace the other sources by their internal resistances

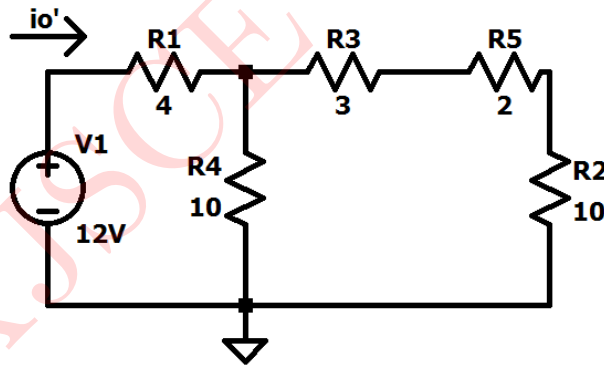


Figure 27: Case 1 - Considering only 12V source acting alone

The resistors 3Ω , 2Ω and 5Ω are connected in series.

$$\therefore R_s = 3 + 2 + 5 = 10\Omega$$

∴ The circuit is simplified as shown in figure 28:

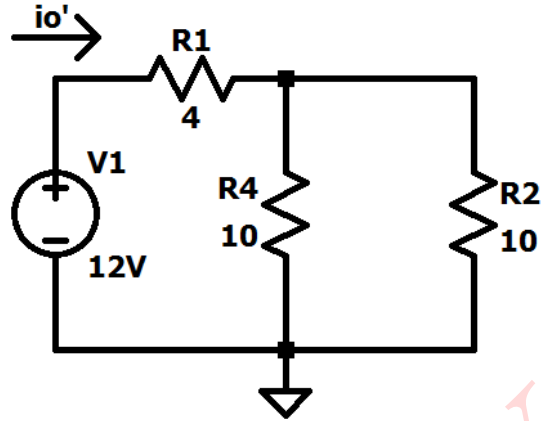


Figure 28: Simplified Circuit 4a for figure 27

The resistors 10Ω and 10Ω are connected in parallel.

$$\begin{aligned}\therefore R_p &= 10 \parallel 10 \\ &= \frac{10 \times 10}{10 + 10} \\ &= 5\Omega\end{aligned}$$

∴ The circuit is further simplified as shown in figure 29:

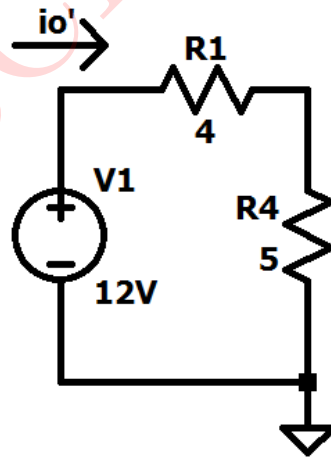


Figure 29: Simplified Circuit 4b for figure 28

$$\therefore i_o' = \frac{12}{4 + 5} = 1.3333\text{A}$$

$$\therefore i_o' = 1.3333\text{A} \rightarrow$$

Case 2: We will now consider the 4A source alone and replace the other sources by their internal resistances

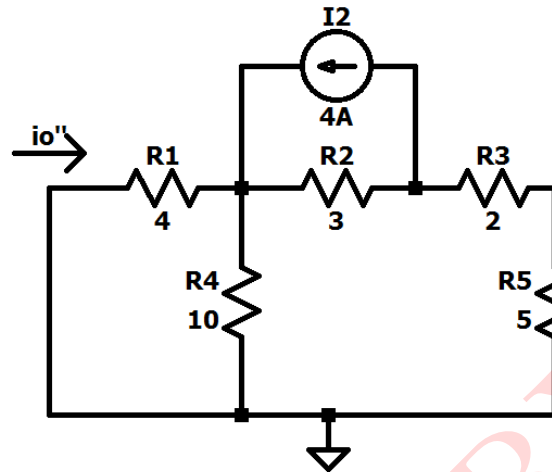


Figure 30: Case 2 - Considering only 4A source acting alone

The resistors 2Ω and 5Ω are connected in series.

$$\therefore R_s = 2 + 5 = 7\Omega$$

\therefore The circuit is simplified as shown in figure 31:

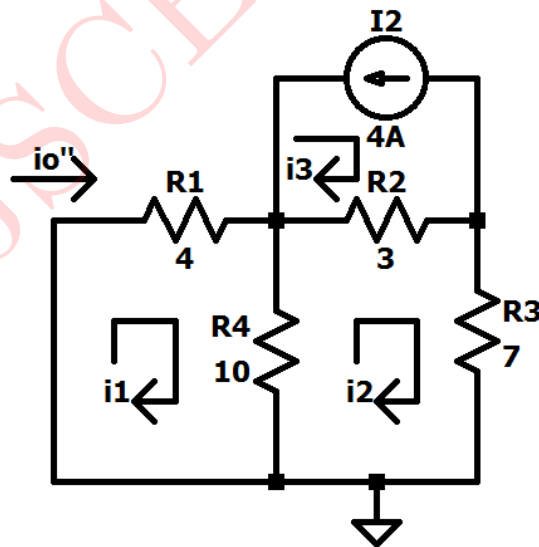


Figure 31: Simplified Circuit 4c for figure 30

We will now use Mesh Analysis to find i_o''

Assume mesh currents i_1 , i_2 and i_3 flowing through loops 1, 2 and 3 in clockwise direction

From the figure, $i_3 = -4A$ (i)

Applying KVL to loop 1, we get:

$$\begin{aligned} -4i_1 - 10(i_1 - i_2) &= 0 \\ \therefore -14i_1 + 10i_2 &= 0 \end{aligned} \quad \text{.....(ii)}$$

Applying KVL to loop 2, we get:

$$\begin{aligned} -10(i_2 - i_1) - 3(i_2 - i_3) - 7i_2 &= 0 \\ \therefore 10i_1 - 20i_2 &= 12 \end{aligned} \quad \text{.....(iii)}$$

Solving (i), (ii) and (iii) we get

$$i_1 = -0.6667\text{A and } i_2 = -0.9333\text{A}$$

$$\therefore i_o'' = i_1 = -0.6667\text{A} \rightarrow$$

Case 3: We will now consider the 2A source alone and replace the other sources by their internal resistances

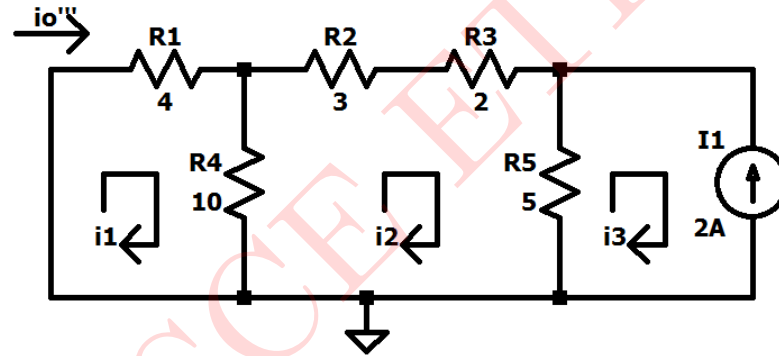


Figure 32: Case 3 - Considering only 2A source acting alone

We will now use Mesh Analysis to find i_o'''

Assume mesh currents i_1 , i_2 and i_3 flowing through loops 1, 2 and 3 in clockwise direction

$$\text{From the figure, } i_3 = -2\text{A} \quad \text{.....(i)}$$

Applying KVL to loop 1, we get:

$$\begin{aligned} -4i_1 - 10(i_1 - i_2) &= 0 \\ \therefore -14i_1 + 10i_2 &= 0 \end{aligned} \quad \text{.....(ii)}$$

Applying KVL to loop 2, we get:

$$\begin{aligned} -10(i_2 - i_1) - 3i_2 - 2i_2 - 5(i_2 - i_3) &= 0 \\ \therefore 10i_1 - 20i_2 &= 10 \end{aligned} \quad \text{.....(iii)}$$

Solving (i), (ii) and (iii) we get

$$i_1 = -0.5556\text{A and } i_2 = -0.7778\text{A}$$

$$\therefore i_o''' = i_1 = -0.5556\text{A} \rightarrow$$

$$\begin{aligned}\therefore i_o &= i_o' + i_o'' + i_o''' \\ &= 1.3333 \rightarrow + (-0.6667) \rightarrow + (-0.5556) \rightarrow \\ &= 0.1111 \text{A} \rightarrow\end{aligned}$$

$$\therefore i_o = 0.1111 \text{A}$$

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

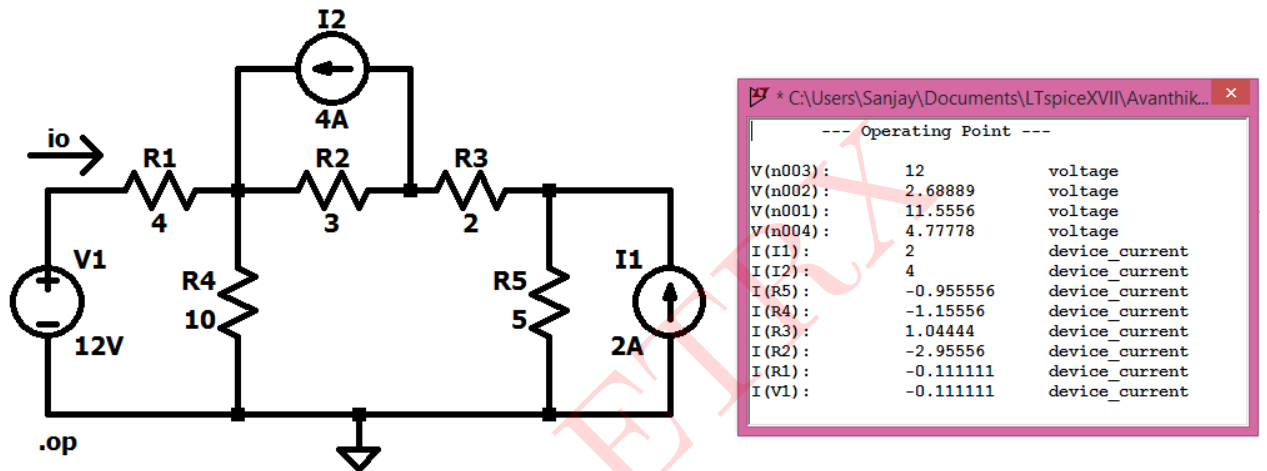


Figure 33: Circuit Schematic and Simulated Results

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
i_o	0.1111A	0.1111A

Table 5: Numerical 4

Numerical 5: Obtain the Thevenin's Equivalent at terminals a-b for the circuit shown in figure 34

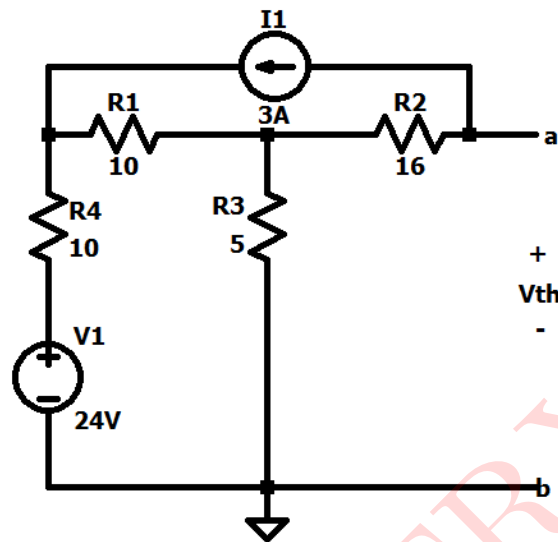


Figure 34: Circuit 5

Solution:

I. Calculation of V_{th}

We will consider open-circuit voltage $V_{ab} = V_{th}$ across terminals a-b.

We will use mesh analysis to find the currents through the loops of the circuit.

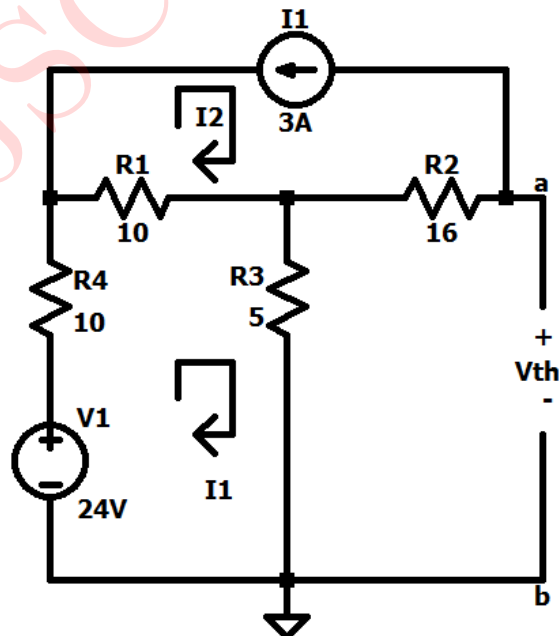


Figure 35: Circuit for calculation of V_{th}

Assume mesh currents I_1 and I_2 flowing through loops 1 and 2 in clockwise direction

From the figure 2, we can see that:

$$I_2 = -3A \quad \text{.....(i)}$$

Applying KVL to loop 1, we get:

$$24 - 10I_1 - 10(I_1 - I_2) - 5I_1 = 0$$

$$\therefore -25I_1 + 10I_2 = -24 \quad \text{.....(ii)}$$

Solving (i) and (ii) we get

$$I_1 = -0.24A \quad \text{.....(iii)}$$

Equation of V_{th} :

$$5I_1 + 16I_2 = V_{th}$$

Using (i) and (iii) we get

$$\therefore V_{th} = -49.2V$$

II. Calculation of R_{th}

Replacing all voltage and current sources by short and open circuit respectively we get,

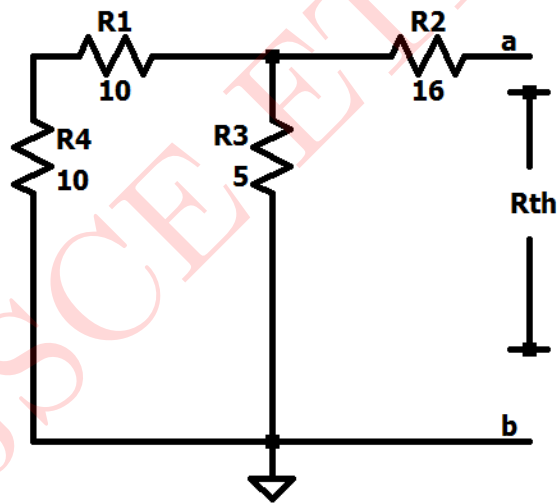


Figure 36: Circuit for calculation of R_{th}

10Ω and 10Ω resistors are connected in series

$$\therefore R_s = 10 + 10 = 20\Omega$$

∴ The circuit is simplified as shown in figure 37:

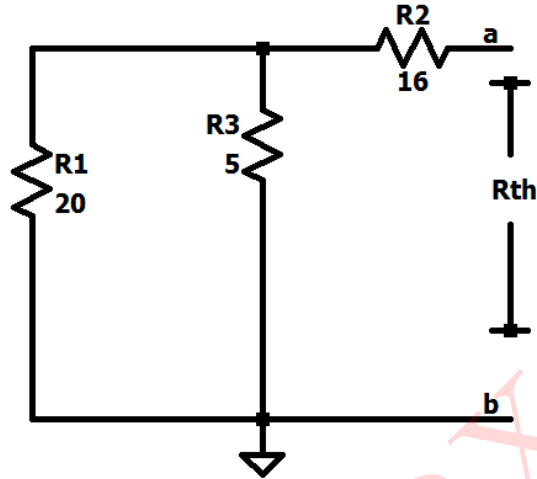


Figure 37: Simplified Circuit 5a for figure 36

The resistors 20Ω and 5Ω are connected in parallel.

$$\begin{aligned}\therefore R_p &= 20 \parallel 5 \\ &= \frac{20 \times 5}{20 + 5} \\ &= 4\Omega\end{aligned}$$

∴ The circuit is simplified as shown in figure 38:

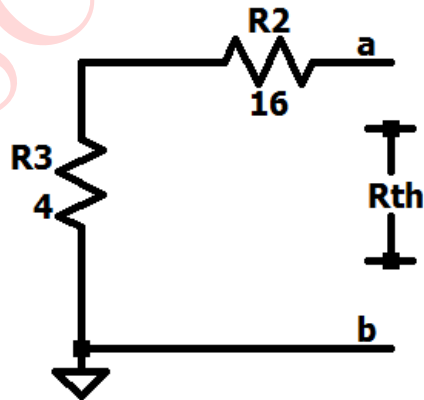


Figure 38: Simplified Circuit 5b for figure 37

The resistors 4Ω and 16Ω are connected in series.

$$\therefore R_{ab} = 4\Omega + 16\Omega = 20\Omega$$

$$\therefore R_{th} = 20\Omega$$

∴ The Thevenin's Equivalent circuit is as shown in figure 39:

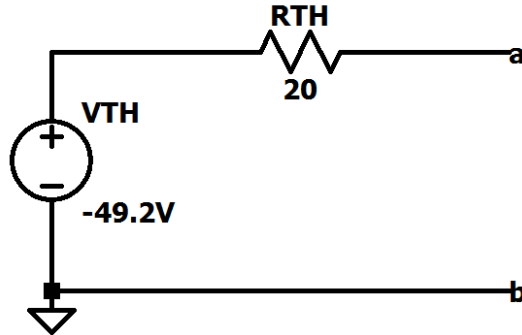


Figure 39: Thevenin's Equivalent Circuit

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

I. Simulation of circuit to find V_{th}

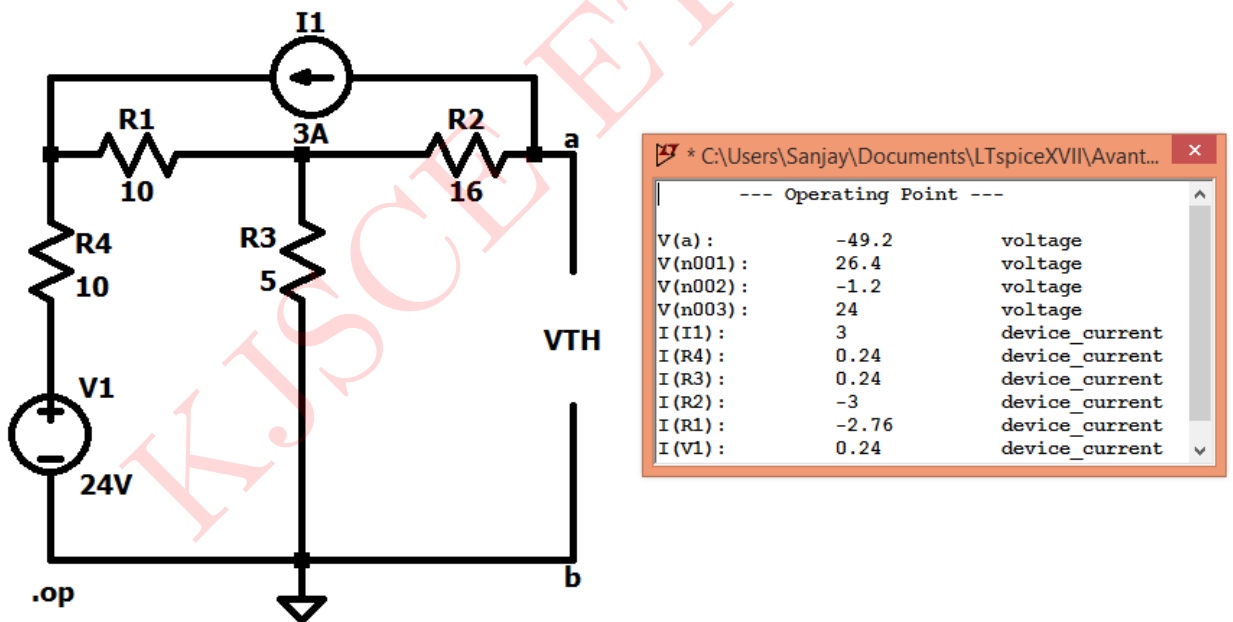


Figure 40: Circuit Schematic for V_{th} and Simulated Results

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
V_{th}	-49.2V	-49.2V

Table 6: Numerical 5:- Calculation of V_{th}

II. Simulation of circuit to find R_{th}

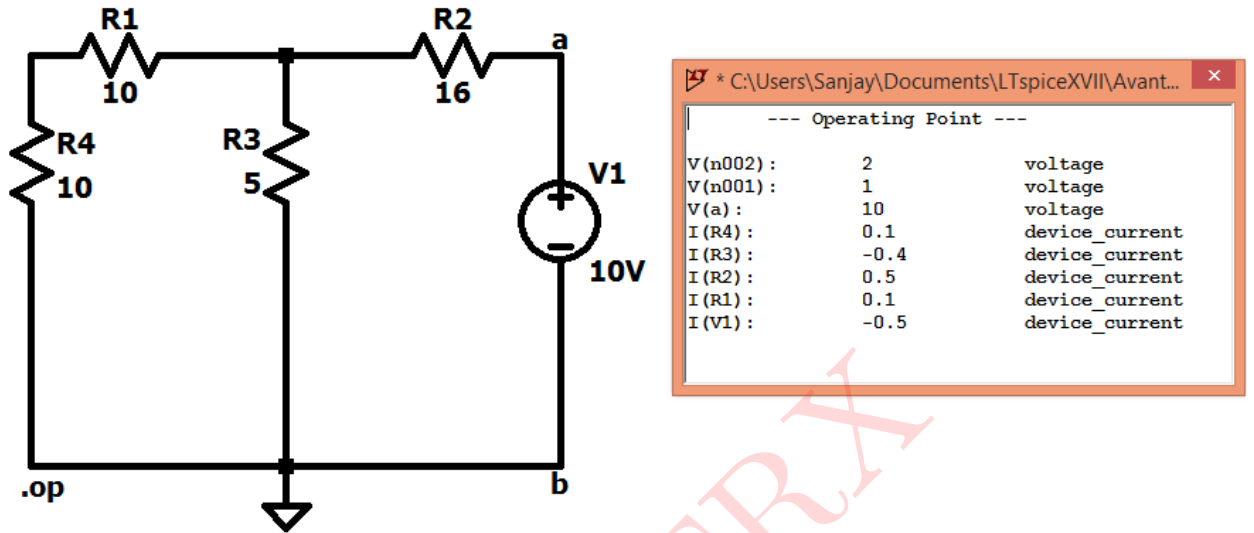


Figure 41: Circuit Schematic for R_{th} and Simulated Results

$$R_{th} = \frac{V1}{I(V1)} = \frac{10}{0.5} = 20\Omega$$

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
R_{th}	20Ω	20Ω

Table 7: Numerical 5:- Calculation of R_{th}

Numerical 6: Obtain the Thevenin's Equivalent at terminals a-b for the circuit shown in figure 42

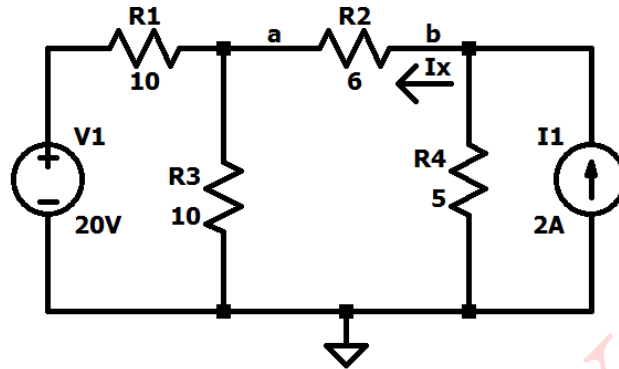


Figure 42: Circuit 6

Solution:

I. Calculation of V_{th}

We will remove the 6Ω resistor and consider open-circuit voltage $V_{ab} = V_{th}$ across terminals a-b.

We will use mesh analysis to find the currents through the loops of the circuit.

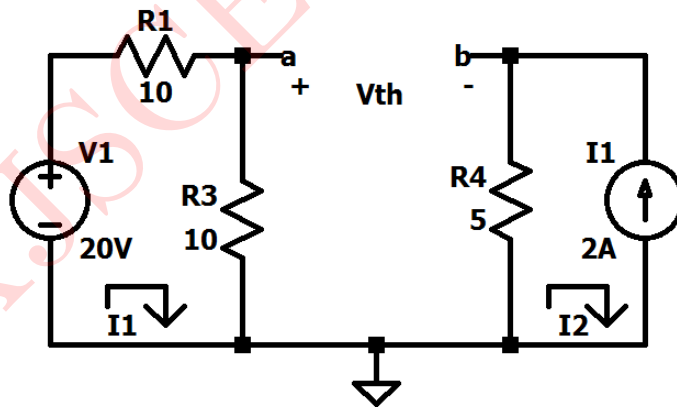


Figure 43: Circuit for calculation of V_{th}

Assume mesh currents I_1 and I_2 flowing through loops 1 and 2 in clockwise direction

From the figure 2, we can see that:

$$I_2 = -2A \quad \text{.....(i)}$$

Applying KVL to loop 1, we get:

$$\begin{aligned} 20 - 10I_1 - 10I_1 &= 0 \\ \therefore 20I_1 &= 20 \\ \therefore I_1 &= 1A \quad \text{.....(ii)} \end{aligned}$$

Equation of V_{th} :

$$-5I_2 - 10I_1 = V_{th}$$

Using (i) and (ii) we get

$$\therefore V_{th} = 0V$$

II. Calculation of R_{th}

Replacing all voltage and current sources by short and open circuit respectively we get,

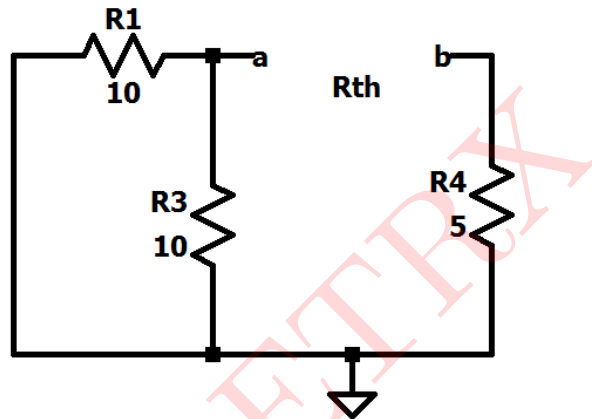


Figure 44: Circuit for calculation of R_{th}

The resistors 10Ω and 10Ω are connected in parallel.

$$\begin{aligned}\therefore R_p &= 10 \parallel 10 \\ &= \frac{10 \times 10}{10 + 10} \\ &= 5\Omega\end{aligned}$$

\therefore The circuit is simplified as shown in figure 45

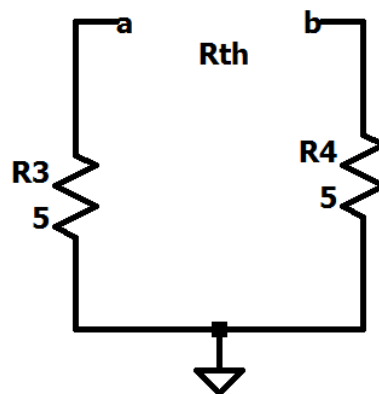


Figure 45: Simplified Circuit 6a for figure 44

The resistors 5Ω and 5Ω are connected in series.

$$\therefore R_{ab} = 5\Omega + 5\Omega = 10\Omega$$

$$\therefore R_{th} = 10\Omega$$

\therefore The Thevenin's Equivalent circuit is as shown in figure 13:

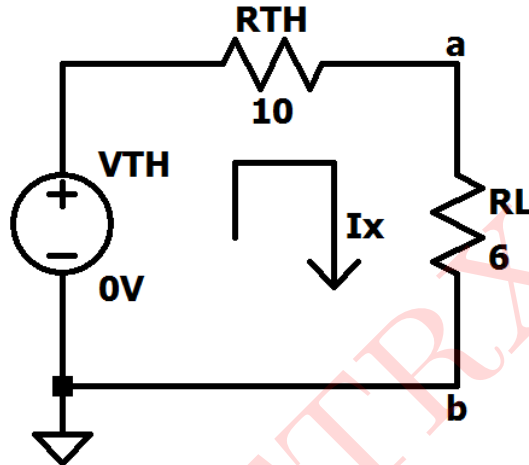


Figure 46: Thevenin's Equivalent Circuit

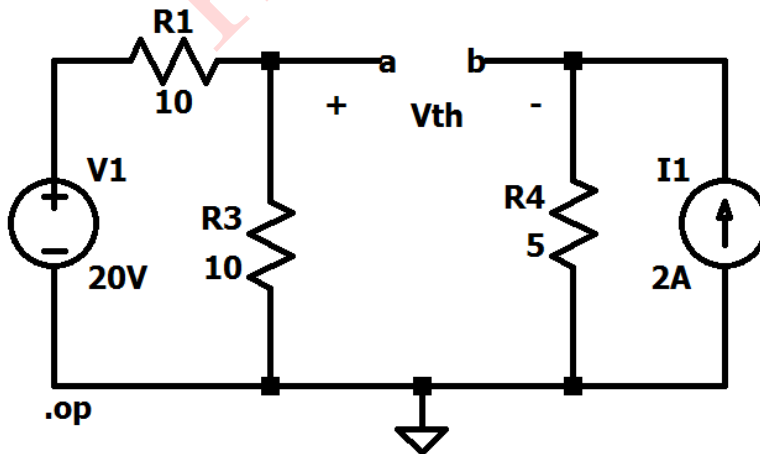
$$i_x = \frac{V_{th}}{R_{th} + R_L} = \frac{0}{10 + 6} = 0A$$

$$\therefore i_x = 0A$$

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

I. Simulation of circuit to find V_{th}



--- Operating Point ---		
V(n001):	20	voltage
V(b):	10	voltage
V(a):	10	voltage
I(I1):	2	device_current
I(R4):	-2	device_current
I(R3):	-1	device_current
I(R1):	-1	device_current
I(V1):	-1	device_current

Figure 47: Circuit Schematic for V_{th} and Simulated Results

$$V_{th} = V_a - V_b = 10 - 10 = 0V$$

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
V_{th}	0V	0V

Table 8: Numerical 6:- Calculation of V_{th}

II. Simulation of circuit to find R_{th}

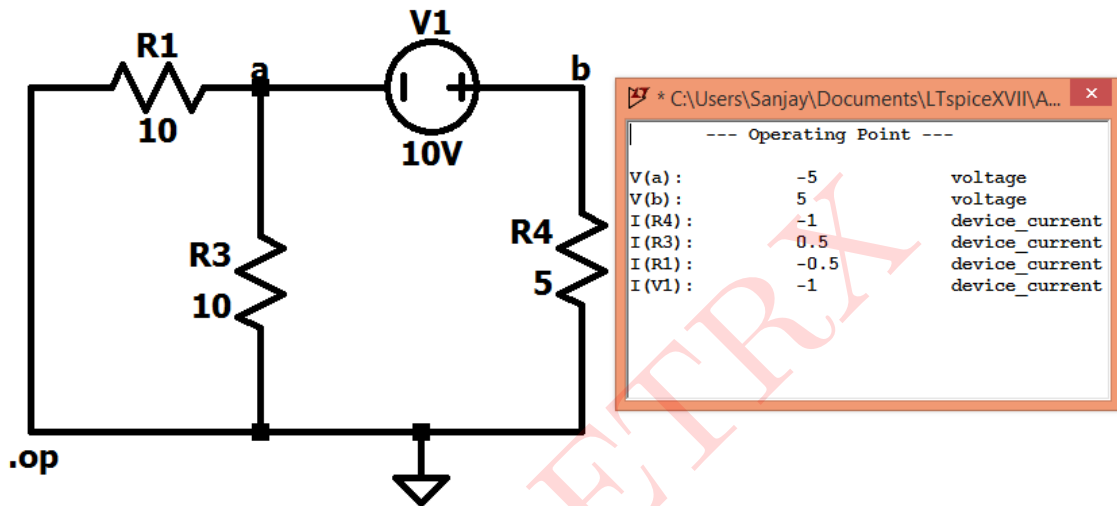


Figure 48: Circuit Schematic for R_{th} and Simulated Results

$$R_{th} = \frac{V1}{I(V1)} = \frac{10}{1} = 10\Omega$$

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
R_{th}	10Ω	10Ω

Table 9: Numerical 6:- Calculation of R_{th}

III. Simulation of circuit to find i_x

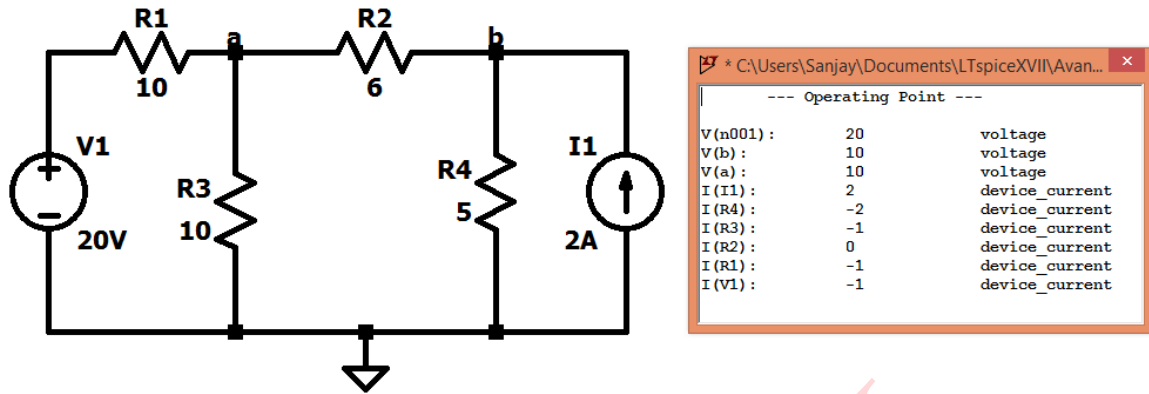


Figure 49: Circuit Schematic for i_x and Simulated Results

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
i_x	0A	0A

Table 10: Numerical 6:- Calculation of i_x

Numerical 7: With the help of Norton's Theorem, find V_o in the circuit shown in figure 50

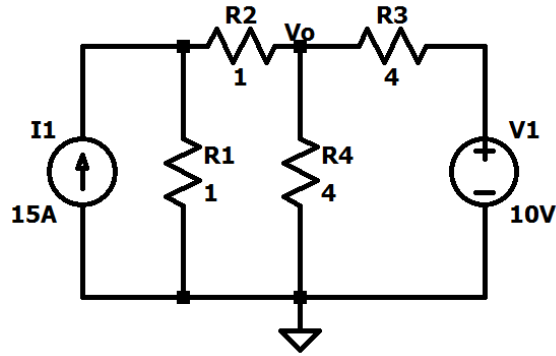


Figure 50: Circuit 7

Solution:

I. Calculation of I_{sc}

We will remove the 4Ω resistor and consider short-circuit current $I_N = I_{sc}$.

We will use mesh analysis to find the currents through the loops of the circuit.

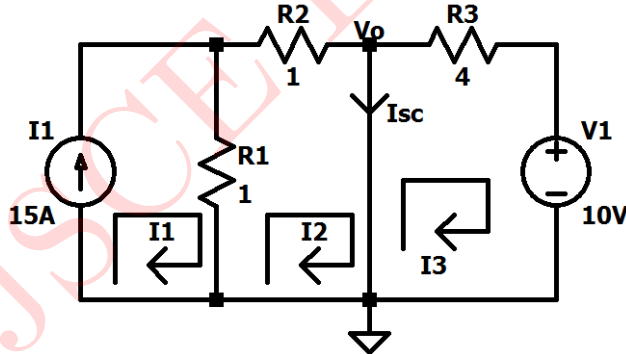


Figure 51: Circuit for calculation of I_{sc}

Assume mesh currents I_1 and I_2 flowing through loops 1 and 2 in clockwise direction

From the figure 2, we can see that:

$$I_1 = 15A \quad \text{.....(i)}$$

Applying KVL to loop 2, we get:

$$\begin{aligned} -1(I_2 - I_1) - I_2 &= 0 \\ \therefore I_1 - 2I_2 &= 0 \end{aligned} \quad \text{.....(ii)}$$

Solving (i) and (ii) we get

$$I_2 = 7.5A \quad \text{.....(iii)}$$

Applying KVL to loop 3, we get:

$$-4I_3 - 10 = 0$$

$$\therefore I_3 = -2.5\text{A}$$

.....(iv)

Using (i) and (iv) we get

$$I_{sc} = I_2 - I_3$$

$$= 7.5 - (-2.5)$$

$$\therefore I_{sc} = 10\text{A}$$

II. Calculation of R_N

Replacing all voltage and current sources by short and open circuit respectively we get,

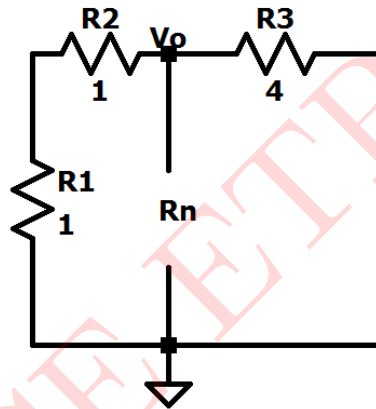


Figure 52: Circuit for calculation of R_N

1Ω and 1Ω resistors are connected in series

$$\therefore R_s = 1 + 1 = 2\Omega$$

\therefore The circuit is simplified as shown in figure 53:

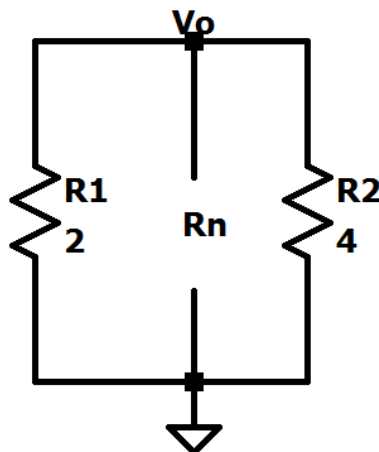


Figure 53: Simplified Circuit 7a for figure 52

The resistors 2Ω and 4Ω are connected in parallel.

$$\begin{aligned}\therefore R_p &= 2 \parallel 4 \\ &= \frac{2 \times 4}{2 + 4} \\ &= 1.3333\Omega\end{aligned}$$

$$\therefore R_N = 1.3333\Omega$$

\therefore The Norton's Equivalent circuit is as shown in figure 54:

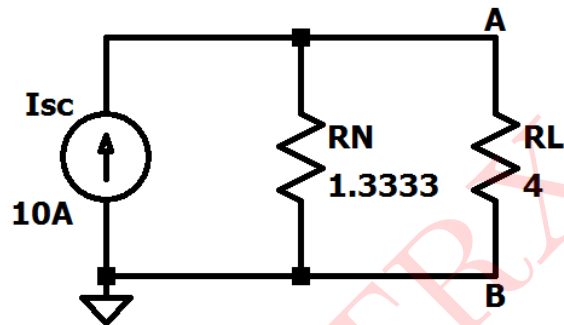


Figure 54: Norton's Equivalent Circuit

\therefore By current division rule, we have

$$I_{(RL)} = \frac{10 \times 1.3333}{1.3333 + 4} = 2.5A$$

$$\begin{aligned}\therefore V_o &= I_{(RL)} \times R_L \\ &= 2.5 \times 4 \\ &= 10V\end{aligned}$$

$$\therefore V_o = 10V$$

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

I. Simulation of circuit to find I_{sc}

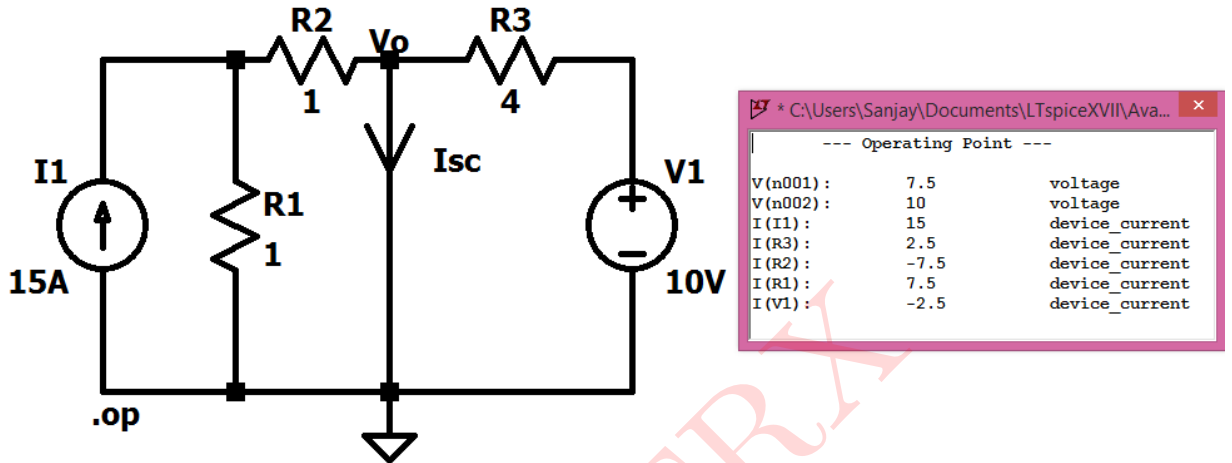


Figure 55: Circuit Schematic for I_{sc} and Simulated Results

$$I_{sc} = I_{R3} - I_{R2} = 2.5 - (-7.5) = 10A$$

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
I_{sc}	10A	10A

Table 11: Numerical 7:- Calculation of I_{sc}

II. Simulation of circuit to find R_N

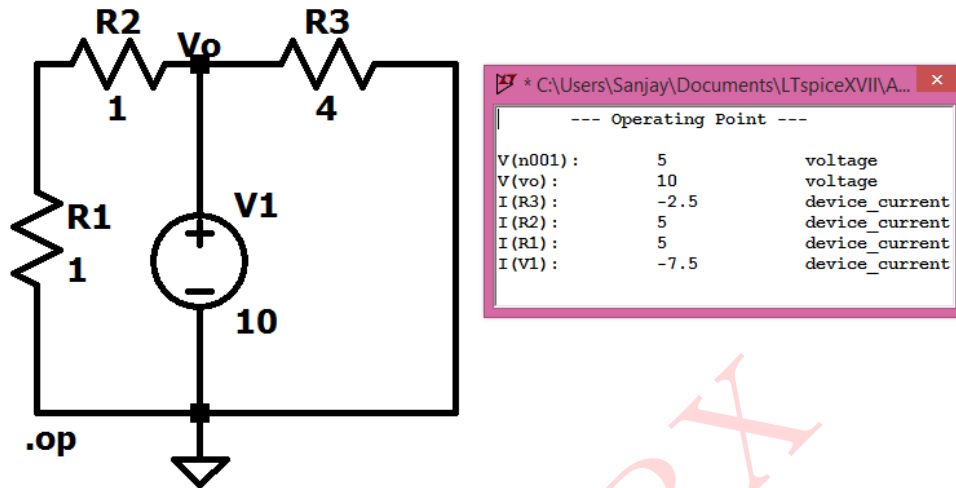


Figure 56: Circuit Schematic for R_N and Simulated Results

$$R_{th} = \frac{V1}{I(V1)} = \frac{10}{7.5} = 1.3333\Omega$$

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
R_N	1.3333Ω	1.3333Ω

Table 12: Numerical 7:- Calculation of R_N

III. Simulation of circuit to find V_o

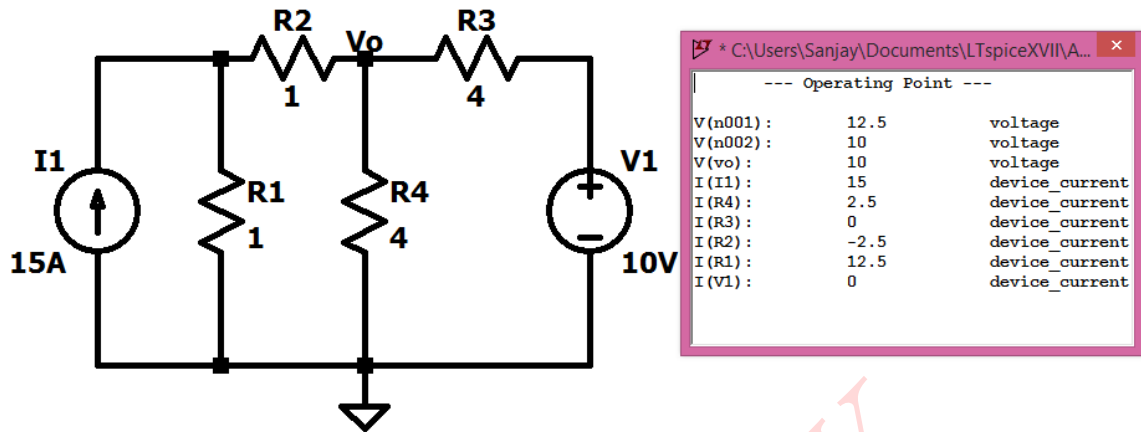


Figure 57: Circuit Schematic for V_o and Simulated Results

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
V_o	10V	10V

Table 13: Numerical 7:- Calculation of V_o

Numerical 8: For the circuit given in figure 58, calculate the current in the 6Ω resistor using Norton's theorem

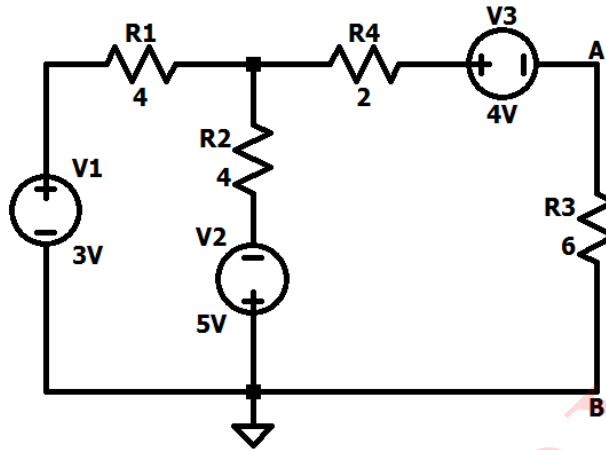


Figure 58: Circuit 8

Solution:

I. Calculation of I_{sc}

We will remove the 6Ω resistor and consider short-circuit current $I_N = I_{sc}$ across terminals A and B

We will use mesh analysis to find the currents through the loops of the circuit.

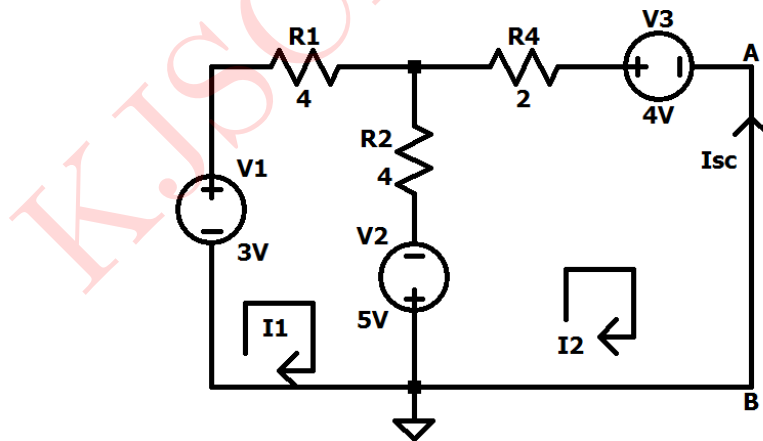


Figure 59: Circuit for calculation of I_{sc}

Assume mesh currents I_1 and I_2 flowing through loops 1 and 2 in clockwise direction

Applying KVL to loop 1, we get:

$$3 - 4I_1 - 4(I_1 - I_2) + 5 = 0$$

$$\therefore -8I_1 + 4I_2 = -8$$

.....(i)

Applying KVL to loop 2, we get:

$$-2I_2 - 4 - 5 - 4(I_2 - I_1) = 0$$

$$\therefore 4I_1 - 6I_2 = 9$$

.....(ii)

Solving (i) and (ii) we get,

$$I_1 = 0.375\text{A and } I_2 = -1.25\text{A}$$

From the figure we see that

$$I_{sc} = -I_2$$

$$\therefore I_{sc} = 1.25\text{A}$$

II. Calculation of R_N

Replacing all voltage sources by short circuit we get,

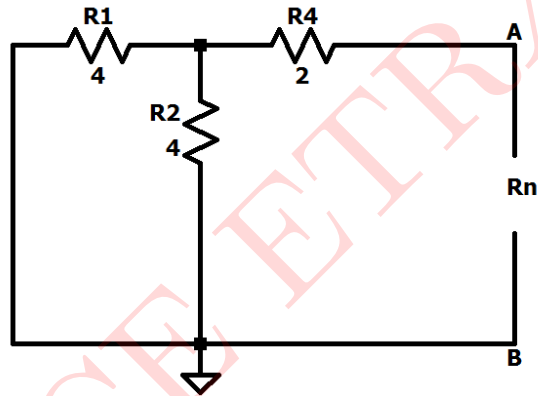


Figure 60: Circuit for calculation of R_N

The resistors 4Ω and 4Ω resistors are connected in parallel and this combination is in series with 2Ω

$$R_N = (4||4) + 2$$

$$= \frac{4 \times 4}{4 + 4} + 2$$

$$= 4\Omega$$

$$\therefore R_N = 4\Omega$$

∴ The Norton's Equivalent circuit is as shown in figure 61:

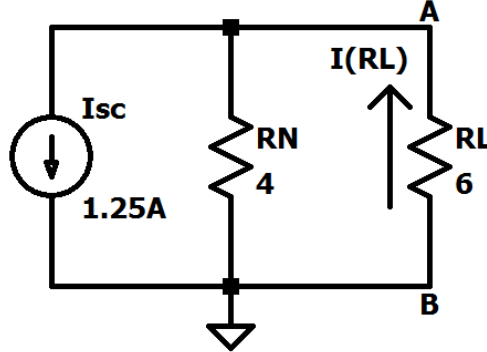


Figure 61: Norton's Equivalent Circuit

∴ By current division rule, we have

$$I_{(RL)} = \frac{1.25 \times 4}{4 + 6} = 0.5A$$

∴ $I_{(RL)} = 0.5A$

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

I. Simulation of circuit to find I_{sc}

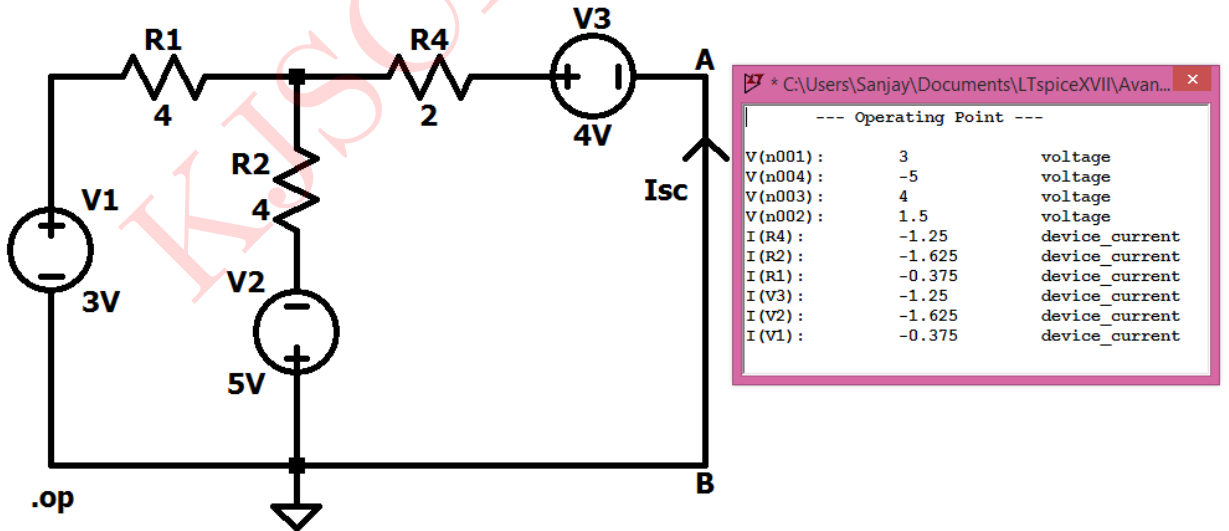


Figure 62: Circuit Schematic for I_{sc} and Simulated Results

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
I_{sc}	1.25A	1.25A

Table 14: Numerical 8:- Calculation of I_{sc}

II. Simulation of circuit to find R_N

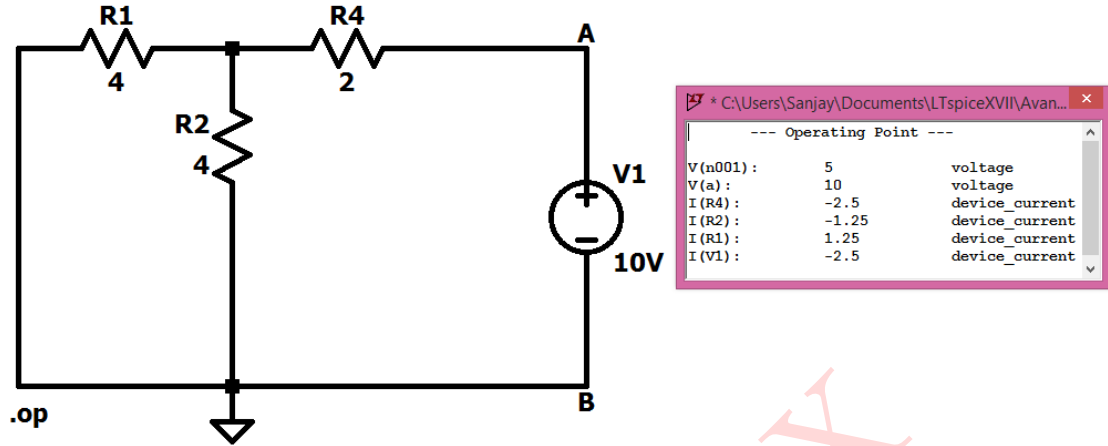


Figure 63: Circuit Schematic for R_N and Simulated Results

$$R_N = \frac{V1}{I(V1)} = \frac{10}{2.5} = 4\Omega$$

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
R_N	4Ω	4Ω

Table 15: Numerical 8:- Calculation of R_N

III. Simulation of circuit to find $I_{(R3)}$

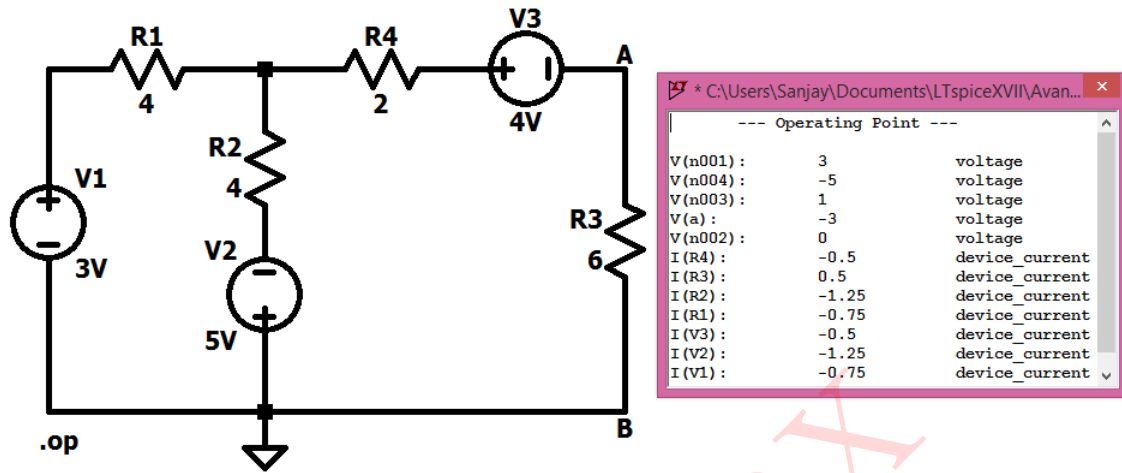


Figure 64: Circuit Schematic for I_{R3} and Simulated Results

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
I_{R3}	0.5A	0.5A

Table 16: Numerical 8:- Calculation of $I_{(R3)}$

Numerical 9: Obtain the Thevenin's and Norton's equivalent circuits at terminals a and b.

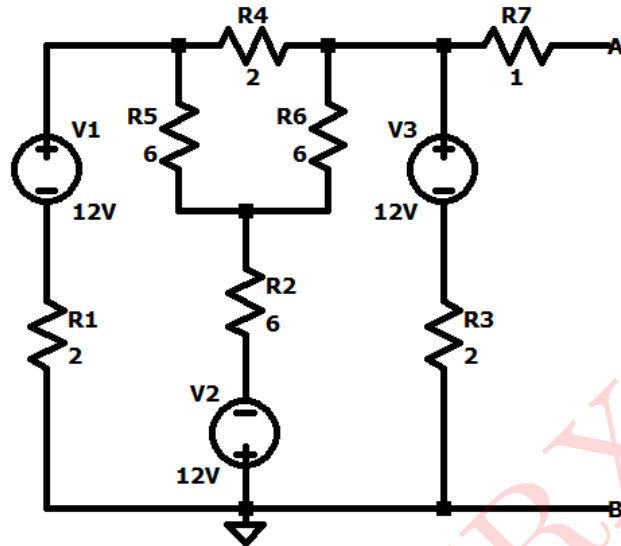


Figure 65: Circuit 9

Solution:

THEVENIN'S EQUIVALENT CIRCUIT:-

I. Calculation of V_{th}

Consider open-circuit voltage $V_{ab} = V_{th}$ across terminals a-b.

We will use mesh analysis to find the currents through the loops of the circuit.

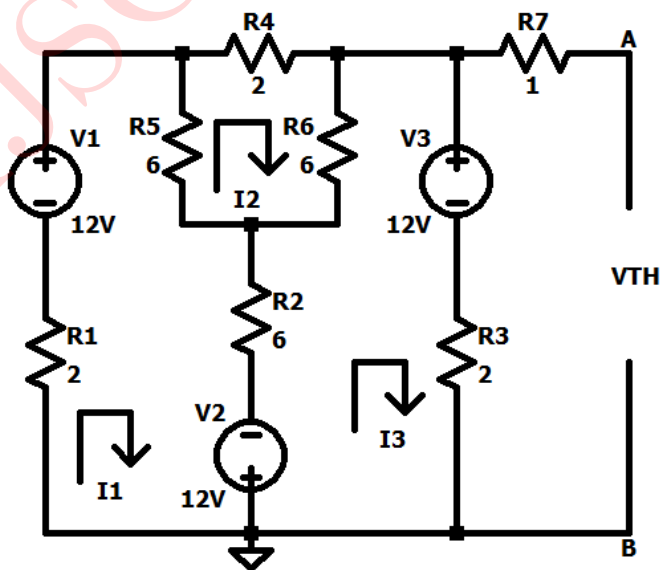


Figure 66: Circuit for calculation of V_{th}

Assume mesh currents I_1 , I_2 and I_3 flowing through loops 1, 2 and 3 in clockwise direction

Applying KVL to loop 1, we get:

$$\begin{aligned} -2I_1 + 12 - 6(I_1 - I_2) - 6(I_1 - I_3) + 12 &= 0 \\ \therefore -14I_1 + 6I_2 + 6I_3 &= -24 \end{aligned} \quad \text{.....(i)}$$

Applying KVL to loop 2, we get:

$$\begin{aligned} -2I_2 - 6(I_2 - I_3) - 6(I_2 - I_1) &= 0 \\ \therefore 6I_1 - 14I_2 + 6I_3 &= 0 \end{aligned} \quad \text{.....(ii)}$$

Applying KVL to loop 3, we get:

$$\begin{aligned} -12 - 2I_3 - 12 - 6(I_3 - I_1) - 6(I_3 - I_2) &= 0 \\ \therefore 6I_1 + 6I_2 - 14I_3 &= 24 \end{aligned} \quad \text{.....(iii)}$$

Solving (i), (ii) and (iii) we get,

$$I_1 = 1.2\text{A}, I_2 = 0\text{A} \text{ and } I_3 = -1.2\text{A}$$

Equation of V_{th} :

$$-2I_3 + 12 = V_{th}$$

$$\therefore V_{th} = 9.6\text{V}$$

II. Calculation of R_{th}

Replacing all voltage and current sources by short and open circuit respectively we get,

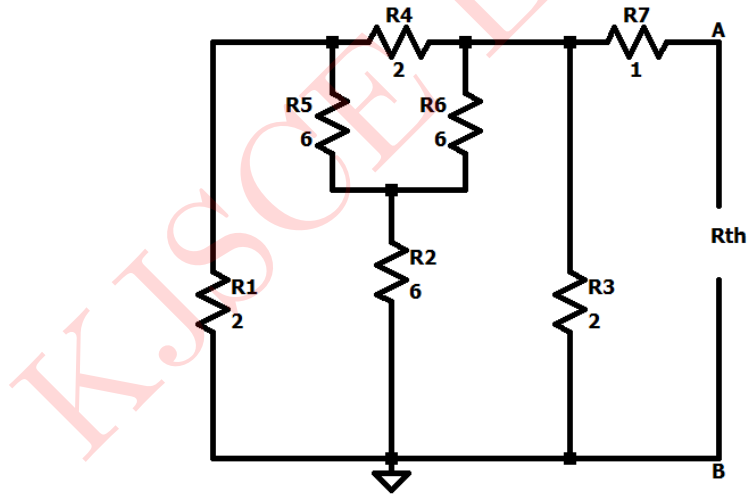


Figure 67: Circuit for calculation of R_{th}

The resistors 6Ω , 6Ω and 2Ω are connected in delta.

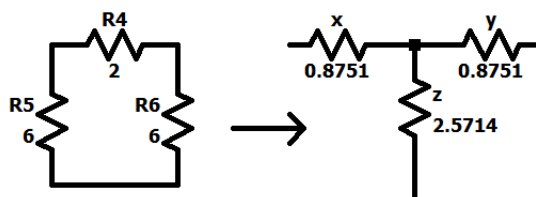


Figure 68: Delta - Star Conversion

Converting delta to star, we have:

$$x = \frac{6 \times 2}{6 + 2 + 6} = 0.8571\Omega$$

$$y = \frac{2 \times 6}{2 + 6 + 6} = 0.8571\Omega$$

$$z = \frac{6 \times 6}{6 + 20} = 2.5714\Omega$$

\therefore The circuit is simplified as shown in figure 69:

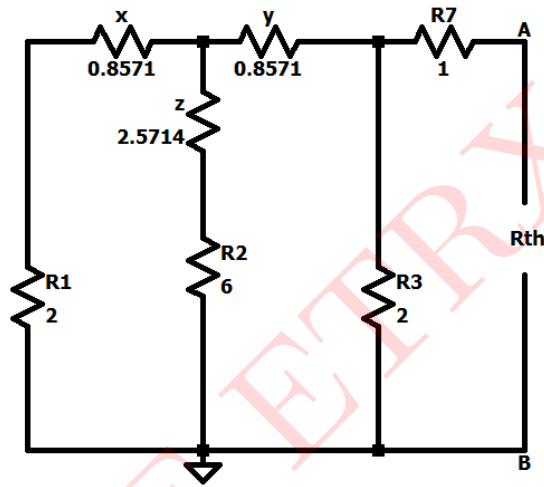


Figure 69: Simplified Circuit 9a for figure 67

The resistors 6Ω and 2.5714Ω are connected in series.

$$\therefore R_s = 6 + 2.5714 = 8.5714\Omega$$

Similarly, the resistors 2Ω and 0.8571Ω are connected in series.

$$\therefore R_s = 2 + 0.8571 = 2.8571\Omega$$

\therefore The circuit is simplified as shown in figure 70:

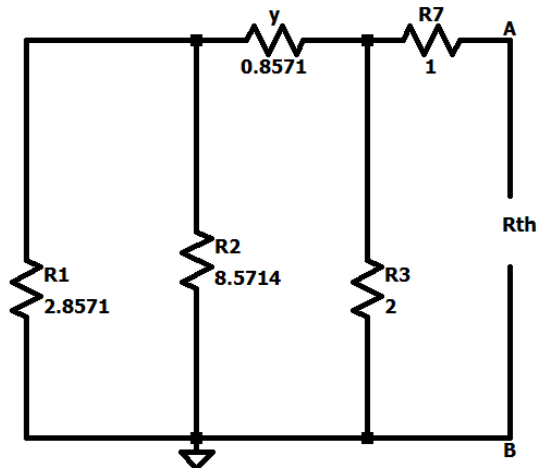


Figure 70: Simplified Circuit 9b for figure 69

The resistors 8.5714Ω and 2.8571Ω are connected in parallel.

$$\begin{aligned}\therefore R_p &= 8.5714 \parallel 2.8571 \\ &= \frac{8.5714 \times 2.8571}{8.5714 + 2.8571} \\ &= 2.1428 \Omega\end{aligned}$$

\therefore The circuit is simplified as shown in figure 71:

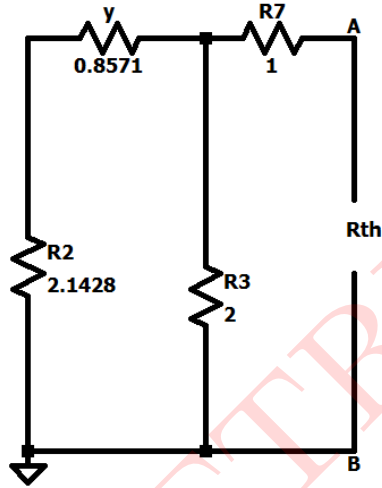


Figure 71: Simplified Circuit 9c for figure 70

The resistors 2.1428Ω and 0.8571Ω are connected in series.

$$\therefore R_s = 0.8571 + 2.1428 = 3\Omega$$

\therefore The circuit is simplified as shown in figure 72:

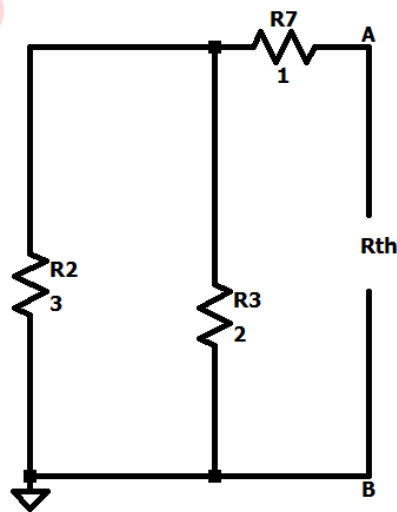


Figure 72: Simplified Circuit 9d for figure 71

The resistors 3Ω and 2Ω are connected in parallel.

$$\begin{aligned}\therefore R_p &= 3 \parallel 2 \\ &= \frac{3 \times 2}{3 + 2} \\ &= 1.2\Omega\end{aligned}$$

\therefore The circuit is simplified as shown in figure 73:

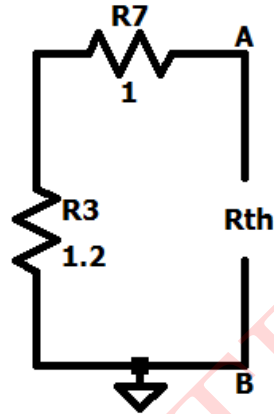


Figure 73: Simplified Circuit 9e for figure 72

The resistors 1Ω and 1.2Ω are connected in series.

$$\therefore R_s = 1 + 1.2 = 2.2\Omega$$

$$\therefore R_{th} = 2.2\Omega$$

\therefore The Thevenin's Equivalent circuit is as shown in figure 74:

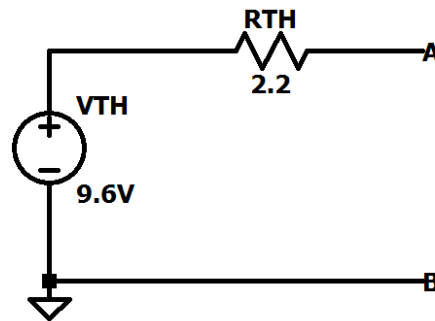


Figure 74: Thevenin's Equivalent Circuit

NORTON'S EQUIVALENT CIRCUIT :-

I. Calculation of I_{sc}

Consider short-circuit current $I_N = I_{sc}$ between terminals a and b.

We will use mesh analysis to find the currents through the loops of the circuit.

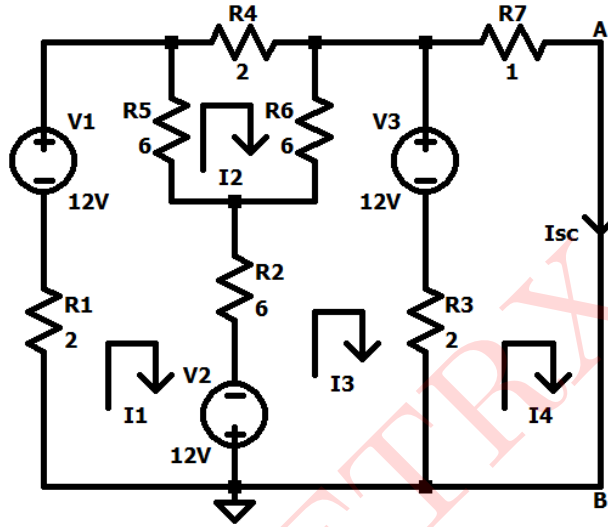


Figure 75: Circuit for calculation of I_{sc}

Assume mesh currents I_1 , I_2 , I_3 and I_4 flowing through loops 1, 2, 3 and 4 in clockwise direction

Applying KVL to loop 1, we get:

$$\begin{aligned} -2I_1 + 12 - 6(I_1 - I_2) - 6(I_1 - I_3 + 12) &= 0 \\ \therefore -14I_1 - 6I_2 + 6I_3 &= -24 \end{aligned} \quad \text{.....(i)}$$

Applying KVL to loop 2, we get:

$$\begin{aligned} -2I_2 - 6(I_2 - I_3) - 6(I_2 - I_1) &= 0 \\ \therefore 6I_1 - 14I_2 + 6I_3 &= 0 \end{aligned} \quad \text{.....(ii)}$$

Applying KVL to loop 3, we get:

$$\begin{aligned} -12 - 2(I_3 - I_4) - 12 - 6(I_3 - I_1) - 6(I_3 - I_2) &= 0 \\ \therefore 6I_1 + 6I_2 - 14I_3 + 2I_4 &= 24 \end{aligned} \quad \text{.....(iii)}$$

Applying KVL to loop 4, we get:

$$\begin{aligned} -I_4 - 2(I_4 - I_3) + 12 &= 0 \\ \therefore 2I_3 - 3I_4 &= 24 \end{aligned} \quad \text{.....(iv)}$$

Solving (i), (ii), (iii) and (iv) we get,

$$I_1 = 2.5090A, I_2 = 1.3090A, I_3 = 0.5454A \text{ and } I_4 = 4.3636A$$

$$\therefore I_{sc} = I_4 = 4.3636A$$

II. Calculation of R_N

The value of R_{th} and R_N will be same.

$$\therefore R_N = R_{th} = 2.2\Omega$$

∴ The Norton's Equivalent circuit is as shown in figure 76:

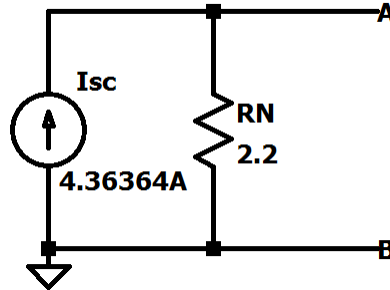


Figure 76: Norton's Equivalent Circuit

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

I. Simulation of circuit to find V_{th}

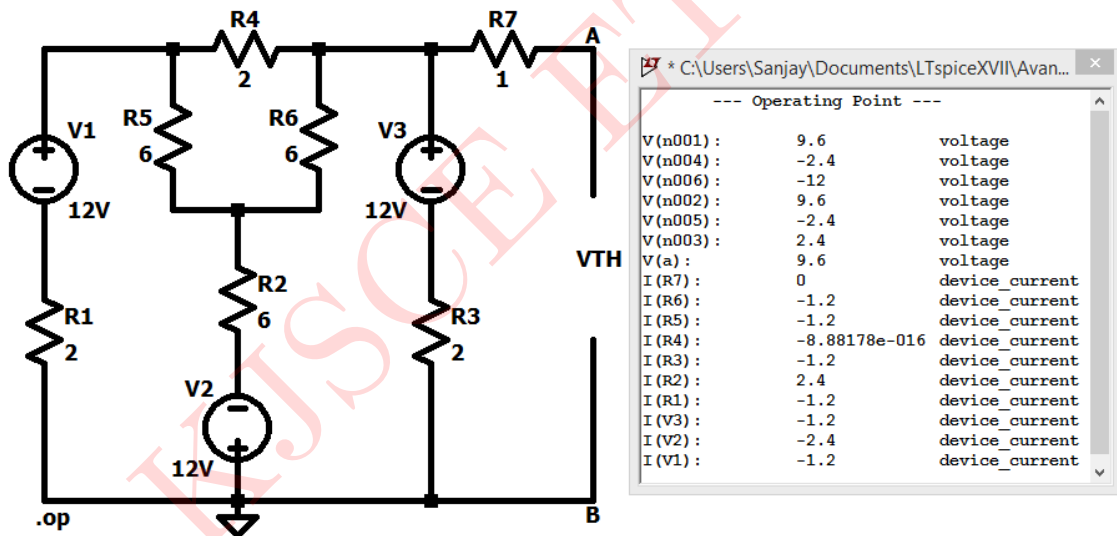


Figure 77: Circuit Schematic for V_{th} and Simulated Results

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
V_{th}	9.6V	9.6V

Table 17: Numerical 9:- Calculation of V_{th}

II. Simulation of circuit to find R_{th}

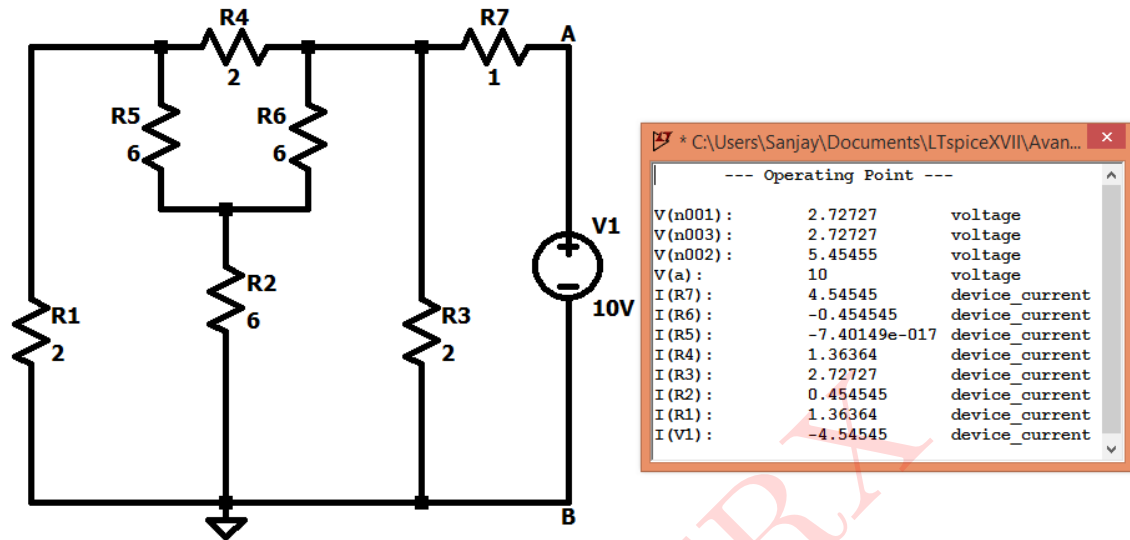


Figure 78: Circuit Schematic for R_{th} and Simulated Results

$$R_{th} = \frac{V1}{I(V1)} = \frac{10}{4.5454} = 2.2\Omega$$

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
R_{th}	10Ω	10Ω

Table 18: Numerical 9:- Calculation of R_{th}

III. Simulation of circuit to find I_{sc}

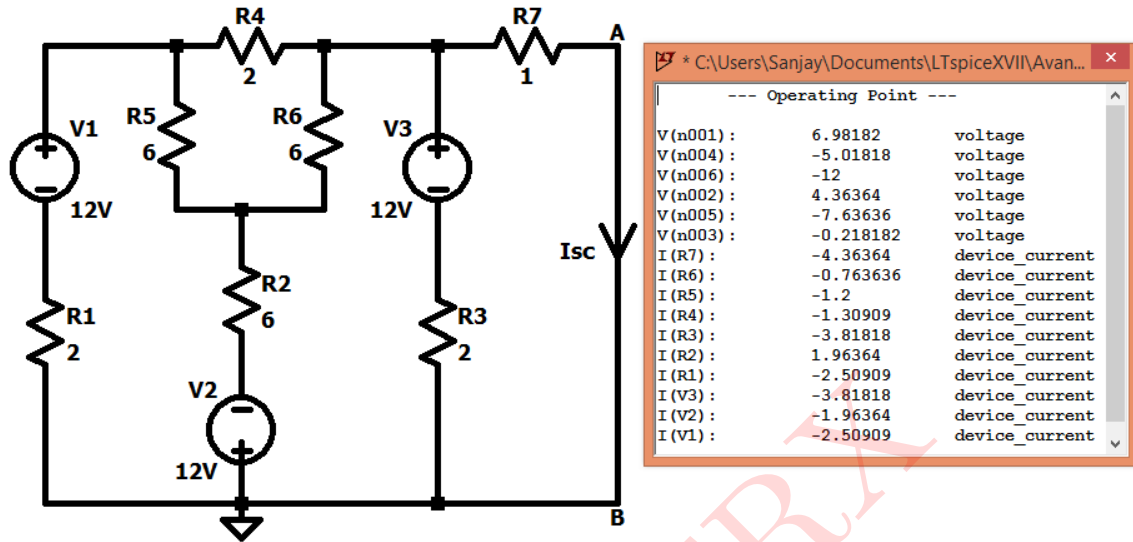


Figure 79: Circuit Schematic for I_{sc} and Simulated Results

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
I_{sc}	4.3636A	4.3636A

Table 19: Numerical 9:- Calculation of I_{sc}

Numerical 10: For the circuit given in figure 80, find the maximum value of R_L

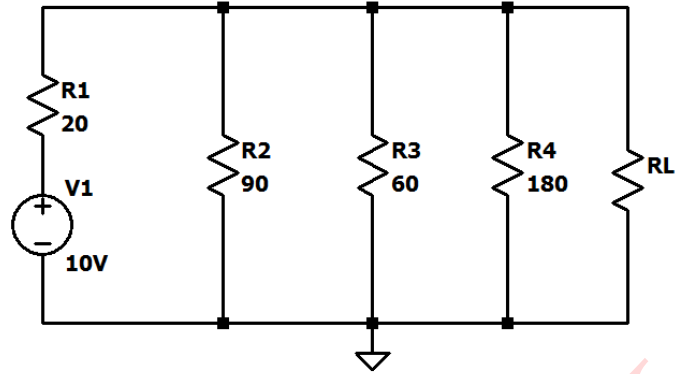


Figure 80: Circuit 10

Solution:

We will use Thevenin's Theorem to calculate V_{th} and R_{th}

I. Calculation of V_{th}

We will remove R_L and consider open-circuit voltage V_{th} across terminals.

We will use mesh analysis to find the currents through the loops of the circuit.

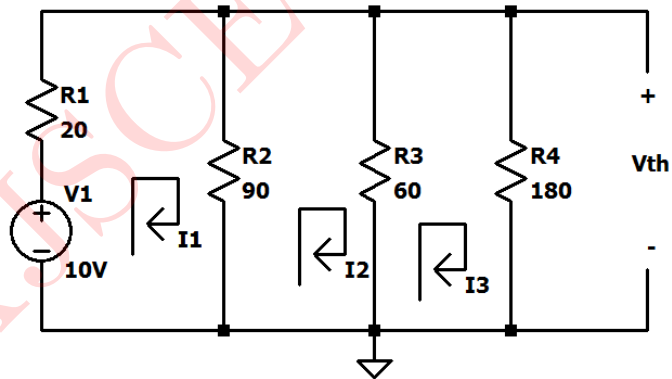


Figure 81: Circuit for calculation of V_{th}

Assume mesh currents I_1 , I_2 and I_3 flowing through loops 1, 2 and 3 in clockwise direction

Applying KVL to loop 1, we get:

$$10 - 20I_1 - 90(I_1 - I_2) = 0$$

$$\therefore -110I_1 + 90I_2 = -10$$

.....(i)

Applying KVL to loop 2, we get:

$$-90(I_2 - I_1) - 60(I_2 - I_3) = 0$$

$$\therefore 90I_1 - 150I_2 + 60I_3 = 0$$

.....(ii)

Applying KVL to loop 3 , we get:

$$-60(I_3 - I_2) - 180I_3 = 0$$

$$\therefore 60I_2 - 240I_3 = -0$$

.....(iii)

Solving (i), (ii) and (iii) we get

$$I_1 = 0.2\text{A}, I_2 = 1.3333\text{A} \text{ and } I_3 = 0.3333\text{A}$$

Equation of V_{th} :

$$10 - 20I_1 = V_{th}$$

Using (i) we get

$$\therefore V_{th} = 6\text{V}$$

II. Calculation of R_{th}

Replacing all voltage and current sources by short and open circuit respectively we get,

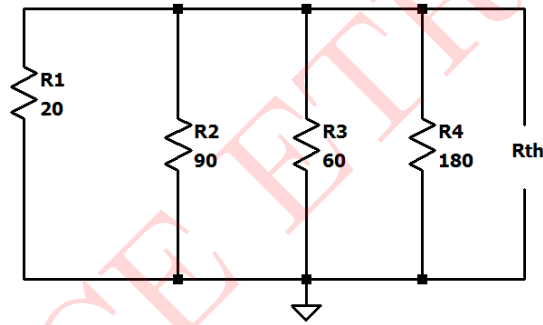


Figure 82: Circuit for calculation of R_{th}

The resistors 20Ω and 90Ω are connected in parallel.

$$\begin{aligned} \therefore R_p &= 20 \parallel 90 \\ &= \frac{20 \times 90}{20 + 90} \\ &= 16.3636\Omega \end{aligned}$$

\therefore The circuit is simplified as shown in figure 83:

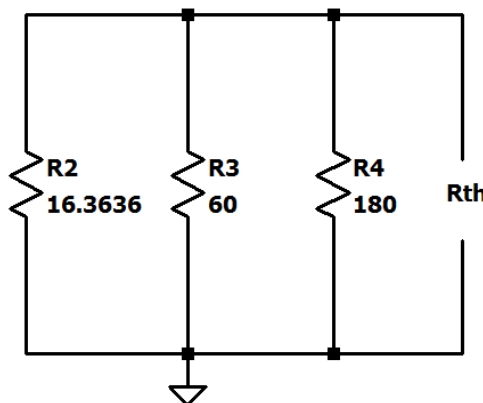


Figure 83: Simplified Circuit 10a for figure 82

The resistors 16.3636Ω and 60Ω are connected in parallel.

$$\begin{aligned}\therefore R_p &= 16.3636 \parallel 60 \\ &= \frac{16.3636 \times 60}{16.3636 + 60} \\ &= 12.8571\Omega\end{aligned}$$

\therefore The circuit is simplified as shown in figure 84:

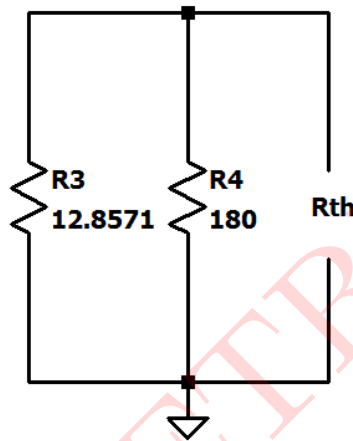


Figure 84: Simplified Circuit 10b for figure 83

The resistors 12.8571Ω and 180Ω are connected in parallel.

$$\begin{aligned}\therefore R_p &= 12.8571 \parallel 180 \\ &= \frac{12.8571 \times 180}{12.8571 + 180} \\ &= 12\Omega\end{aligned}$$

$$\therefore R_{th} = 12\Omega$$

According to Maximum Power Transfer Theorem, for maximum power transfer, $R_{th} = R_L$

$$\therefore R_L = 12\Omega$$

\therefore The Thevenin's Equivalent circuit is as shown in figure 85:

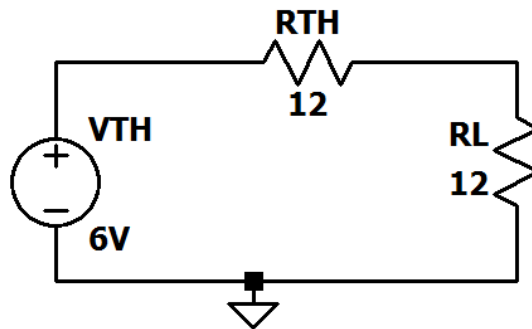


Figure 85: Thevenin's Equivalent Circuit

$$\begin{aligned}
\therefore P_{\max} &= \frac{V_{th}^2}{4 \times R_L} \\
&= \frac{6^2}{4 \times 12} \\
&= 0.75W \\
&= 750mW
\end{aligned}$$

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

I. Simulation of circuit to find V_{th}

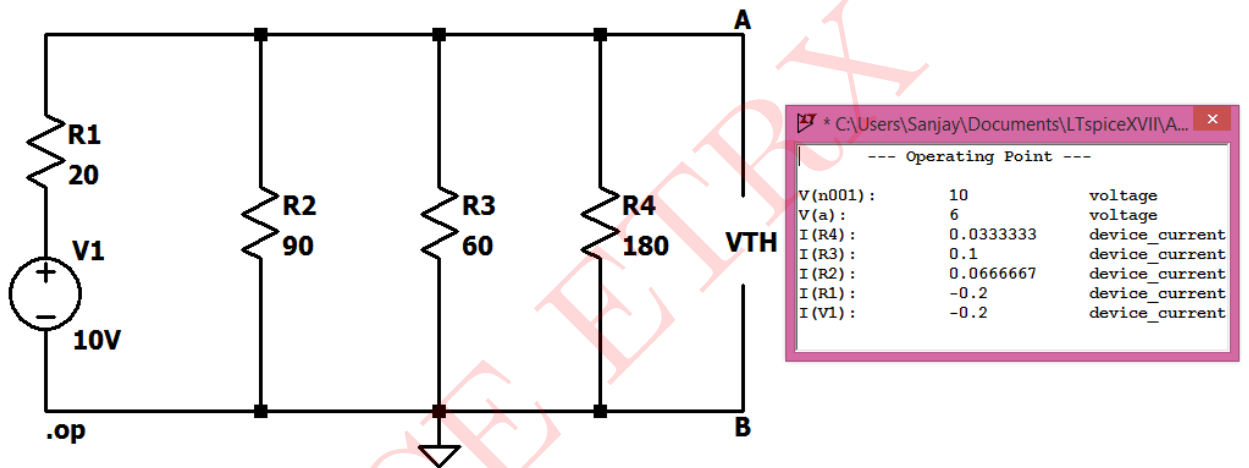


Figure 86: Circuit Schematic for V_{th} and Simulated Results

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
V_{th}	6V	6V

Table 20: Numerical 10 :- Calculation of V_{th}

II. Simulation of circuit to find R_{th}

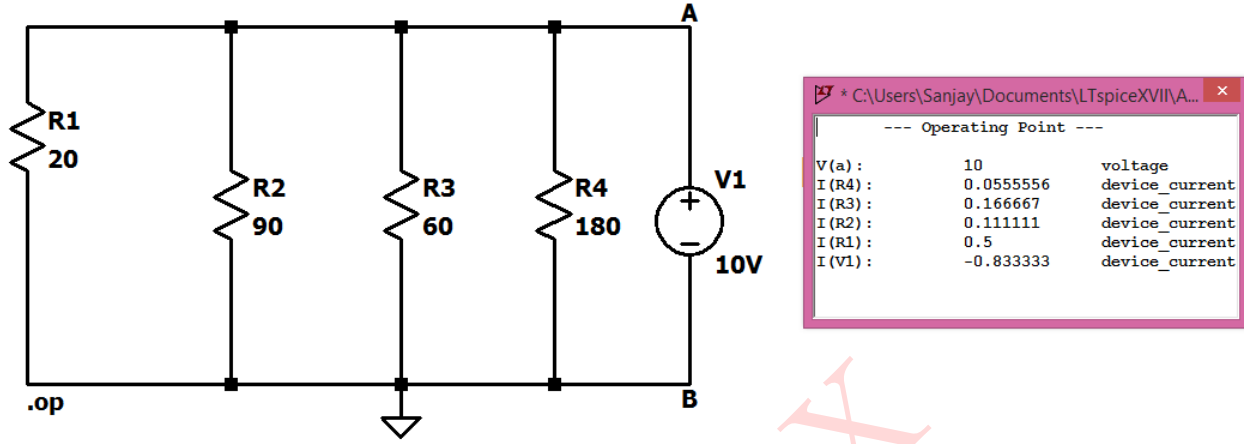


Figure 87: Circuit Schematic for R_{th} and Simulated Results

$$R_{th} = \frac{V1}{I(V1)} = \frac{10}{0.83333} = 12\Omega$$

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
R_{th}	12Ω	12Ω

Table 21: Numerical 10 :- Calculation of R_{th}

II. Simulation of circuit to find P_{\max}

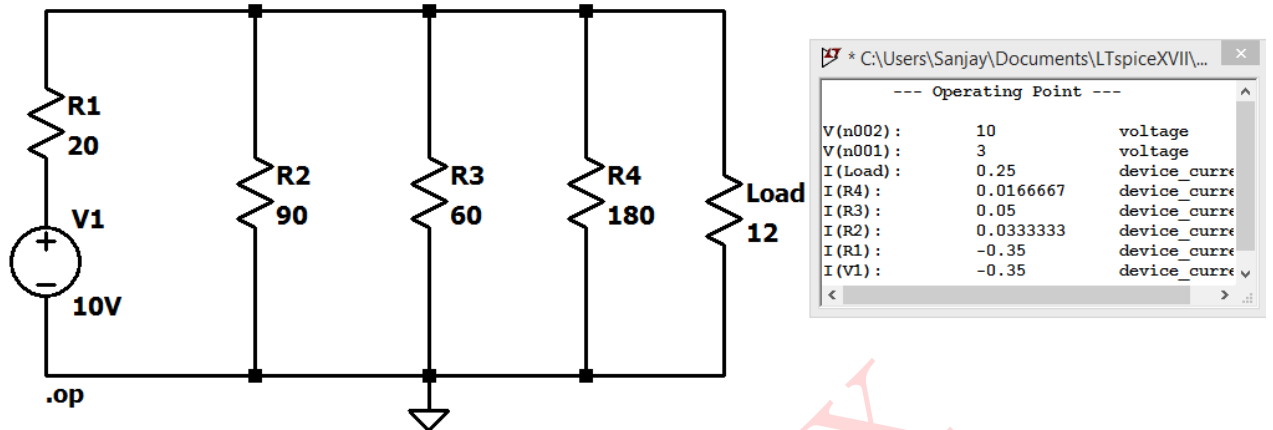


Figure 88: Circuit Schematic for P_{\max} and Simulated Results

$$P_{\max} = I^2 \times R = 0.25^2 \times 12 = 750\text{mW}$$

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
P_{\max}	750mW	750mW

Table 22: Numerical 10 :- Calculation P_{\max}

- Numerical 11:** a) For the circuit given in figure 89, obtain the Thevenin equivalent at terminals a-b.
b) Calculate the current for $R_L = 8\Omega$
c) Find R_L for maximum power deliverable to R_L
d) Determine that maximum power

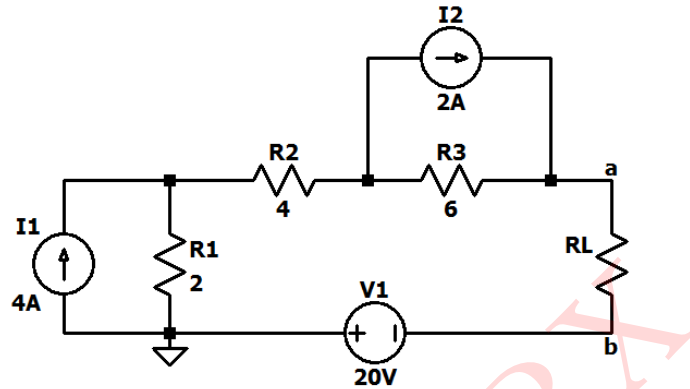


Figure 89: Circuit 11

Solution:

a) I. Calculation of V_{th}

We will remove the R_L and consider open-circuit voltage V_{th} across terminals a-b.

We will use mesh analysis to find the currents through the loops of the circuit.

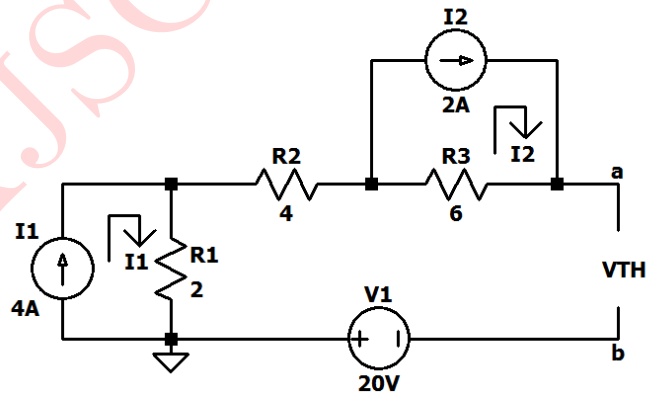


Figure 90: Circuit for calculation of V_{th}

Assume mesh currents I_1 and I_2 flowing through loops 1 and 2 in clockwise direction

From the figure, we can see that $I_1 = 4A$ and $I_2 = 2A$

Equation of V_{th} :

$$2I_1 + 10I_1 + 20 = V_{th}$$

Substituting values of I_1 and I_2 we get

$$\therefore V_{th} = 40V$$

II. Calculation of R_{th}

Replacing all current sources by open circuit we get,

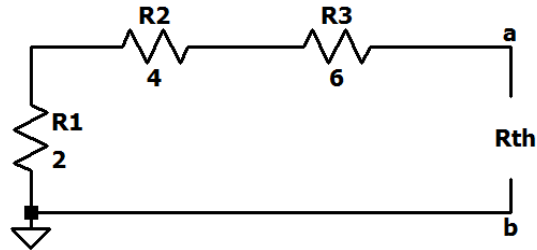


Figure 91: Circuit for calculation of R_{th}

The resistors 2Ω , 4Ω and 6Ω are connected in series.

$$\therefore R_{ab} = 2\Omega + 4\Omega + 6\Omega = 12\Omega$$

$$\therefore R_{th} = 12\Omega$$

\therefore The Thevenin's Equivalent circuit is as shown in figure 92:

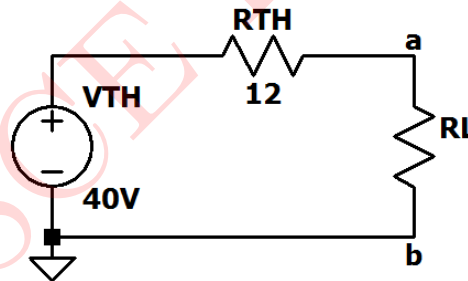


Figure 92: Thevenin's Equivalent Circuit

b) The value of R_L is to be taken as 8Ω

\therefore The Thevenin's Equivalent circuit for $R_L = 8\Omega$ is as shown in figure 92:

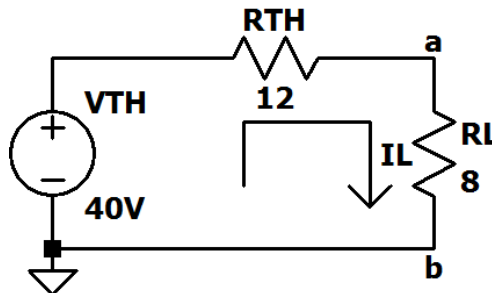


Figure 93: Thevenin's Equivalent Circuit for $R_L = 8\Omega$

$$I_L = \frac{V_{th}}{R_{th} + R_L} = \frac{40}{12 + 8} = 2A$$

$$\therefore I_L = 2A$$

c) According to Maximum Power Transfer Theorem, for maximum power to be delivered,
 $R_{th} = R_L$

$$\therefore R_L = 12\Omega$$

\therefore The Thevenin's Equivalent circuit for $R_L = 12\Omega$ is as shown in figure 94:

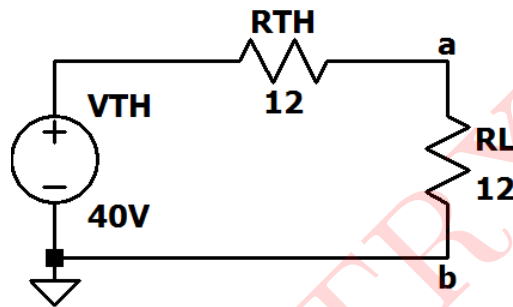


Figure 94: Thevenin's Equivalent Circuit for $R_L = 12\Omega$

$$\therefore P_{max} = \frac{V_{th}^2}{4 \times R_L}$$

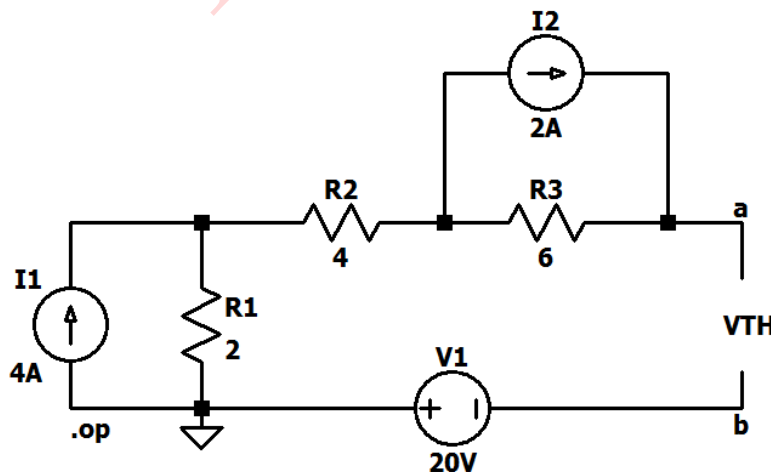
$$= \frac{40^2}{4 \times 12}$$

$$= 33.33W$$

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

I. Simulation of circuit to find V_{th}



--- Operating Point ---		
V(n002):	8	voltage
V(n001):	8	voltage
V(a):	20	voltage
V(b):	-20	voltage
I(I2):	2	device_current
I(I1):	4	device_current
I(R3):	2	device_current
I(R2):	0	device_current
I(R1):	4	device_current
I(V1):	0	device_current

Figure 95: Circuit Schematic for V_{th} and Simulated Results

$$\therefore V_{th} = V_a - V_b = 20 - (-20) = 40V$$

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
V_{th}	40V	40V

Table 23: Numerical 11:- Calculation of V_{th}

II. Simulation of circuit to find R_{th}

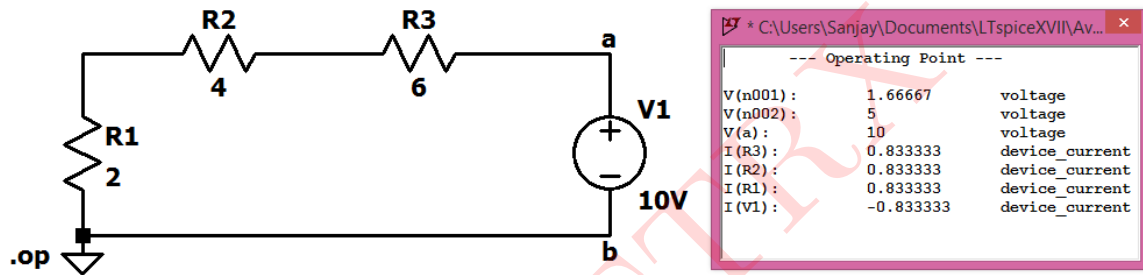


Figure 96: Circuit Schematic for R_{th} and Simulated Results

$$R_{th} = \frac{V1}{I(V1)} = \frac{10}{0.83333} = 12\Omega$$

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
R_{th}	12Ω	12Ω

Table 24: Numerical 11:- Calculation of R_{th}

III. Simulation of circuit to find P_{\max}

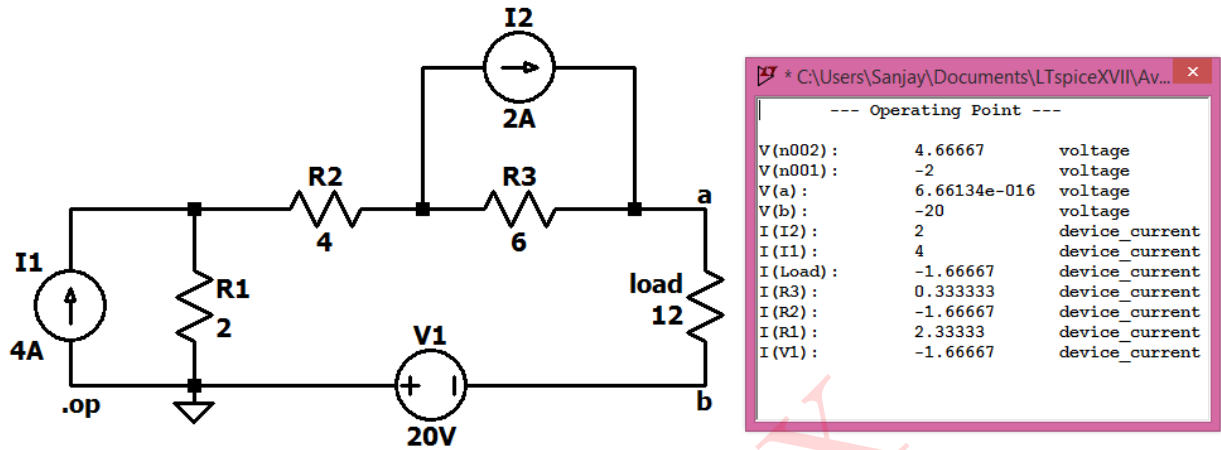


Figure 97: Circuit Schematic for P_{\max} and Simulated Results

$$P_{\max} = I^2 \times R = 1.6667^2 \times 12 = 33.33\text{W}$$

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
P_{\max}	33.33W	33.33W

Table 25: Numerical 11:- Calculation P_{\max}