K. J. SOMAIYA COLLEGE OF ENGINEERING
DEPARTMENT OF ELECTRONICS ENGINEERING
ELECTRONIC CIRCUITS
Low and High-frequency response of single-stage amplifier

Numerical 1:
For the circuit given below in figure 1,
a) Determine r_π
b) Find $A_{V_{mid}} = V_o/V_i$
c) Calculate Z_i
d) Find $A_{V_{S_{mid}}} = V_o/V_S$
e) Determine f_{LCC1} and f_{LCC2}
f) Determine lower cut-off frequency

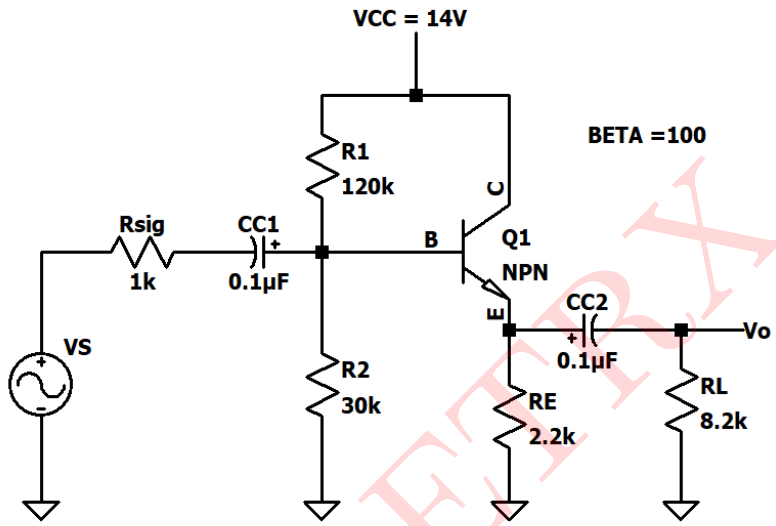

Figure 1: Circuit 1

Solution:

DC analysis:

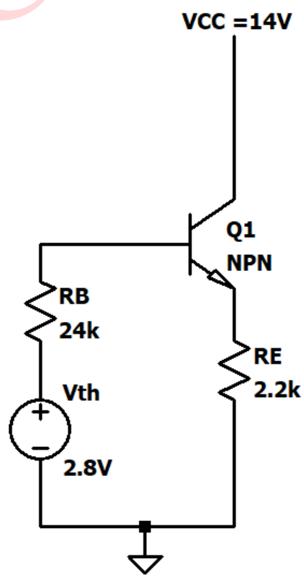

Figure 2: Thevenin's equivalent circuit

$$V_{TH} = \frac{R_2}{R_1 + R_2} \times V_{CC} = \frac{30k\Omega}{120k\Omega + 30k\Omega} \times 14V = \mathbf{2.8V}$$

$$R_B = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} = \frac{120k\Omega \times 30k\Omega}{120k\Omega + 30k\Omega} = \mathbf{24k\Omega}$$

Applying KVL to base-emitter loop,

$V_{TH} - I_B R_B - V_{BE} - I_E R_E = 0$

$V_{TH} - I_B R_B - V_{BE} - (\beta + 1) I_B R_E = 0$

$2.8V - I_B(24k\Omega) - 0.7V - (101) I_B (2.2k\Omega) = 0$

$2.1V - I_B[(24k\Omega + 101(2.2k\Omega))] = 0$

$$I_B = \frac{2.1V}{24k\Omega + 101(2.2k\Omega)} = \mathbf{8.52\mu A}$$

$\because I_C = \beta I_B = (100)(8.52\mu\text{A}) = \mathbf{0.852mA}$

Small signal parameters:

i) $g_m = \frac{I_C}{V_T} = \frac{0.852mA}{26mV} = \mathbf{32.76mA/V}$

ii) $r_o = \frac{V_A}{I_C} = \infty$

iii) $r_\pi = \beta \times \frac{V_T}{I_C} = 100 \times \frac{26mV}{0.852\mu A} = \mathbf{3.051k\Omega}$

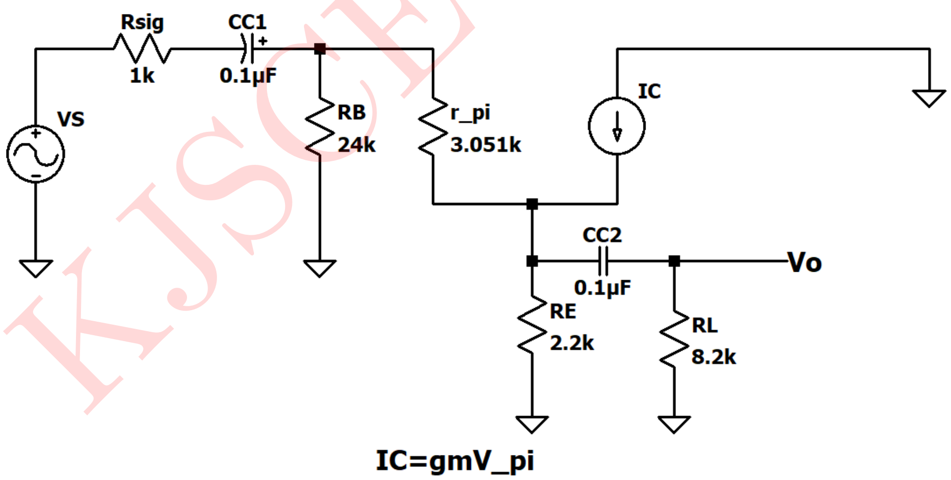

Figure 3: Low frequency AC equivalent circuit

For C_{C1}:

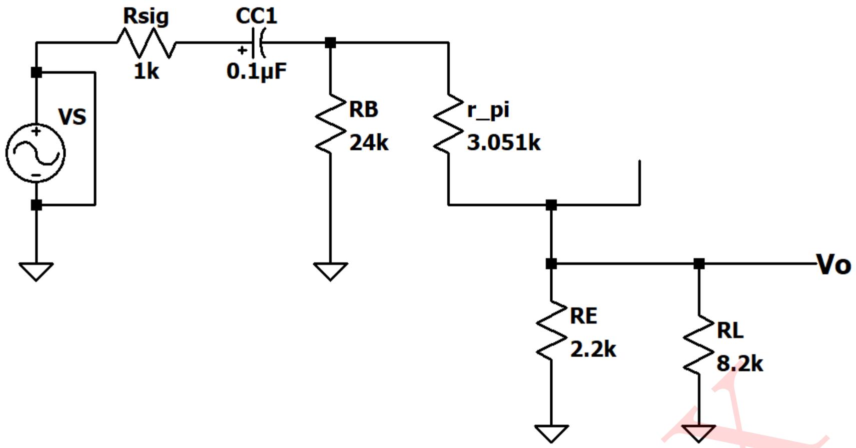

Figure 4: Small signal low frequency equivalent circuit for C_{C1} alone

$$
\begin{aligned}
R_i &= R_B \parallel [r_\pi + (1+\beta)(R_E \parallel R_L)] \\
&= 24k\Omega \parallel [3.051k\Omega + (101)(2.2k\Omega \parallel 8.2k\Omega)] \\
&= 24k\Omega \parallel \left[3.051k\Omega + 101\left(\frac{2.2k\Omega \times 8.2k\Omega}{2.2k\Omega + 8.2k\Omega}\right)\right] \\
&= 24k\Omega \parallel [3.051k\Omega + 101(1.73k\Omega)] \\
&= 24k\Omega \parallel [178.24k\Omega)] \\
&= \frac{24k\Omega \times 178.24k\Omega}{24k\Omega + 178.24k\Omega} = \mathbf{21.15k\Omega}
\end{aligned}
$$

$$
\begin{aligned}
R_{eq_{CC1}} &= R_i + R_{sig} \\
&= 21.15k\Omega + 1k\Omega = \mathbf{22.15k\Omega}
\end{aligned}
$$

$\therefore$ The lower cut-off frequency due to C_{C1} alone is,

$$
\begin{aligned}
f_{L_{CC1}} &= \frac{1}{2\pi R_{eq_{CC1}} C_{C1}} \\
&= \frac{1}{2\pi \times 22.15k\Omega \times 0.1\mu F} = \mathbf{71.88Hz}
\end{aligned}
$$

For C_{C2}:

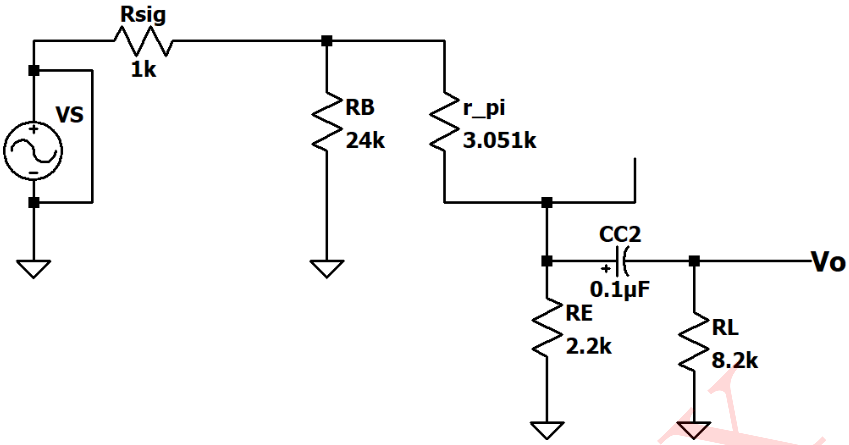

Figure 5: Small signal low frequency equivalent circuit for C_{C2} alone

$$R_o = R_E \parallel \left[\frac{r_\pi + (R_S \parallel R_B)}{1+\beta}\right]$$
$$= 2.2k\Omega \parallel \left[\frac{3.051k\Omega + (1k\Omega \parallel 24k\Omega)}{101}\right]$$
$$= 2.2k\Omega \parallel \left[\frac{3.051k\Omega + \left(\frac{1k\Omega \times 24k\Omega}{1k\Omega + 24k\Omega}\right)}{101}\right]$$
$$= 2.2k\Omega \parallel \left[\frac{3.051k\Omega + 0.96k\Omega}{101}\right]$$
$$= 24k\Omega \parallel 0.04k\Omega$$
$$= \frac{24k\Omega \times 0.04k\Omega}{24k\Omega + 0.04k\Omega} = \mathbf{0.03k\Omega}$$

$$R_{eq_{CC2}} = R_o + R_L$$
$$= 0.03k\Omega + 8.2k\Omega = \mathbf{8.239k\Omega}$$

$\therefore$ The lower cut-off frequency due to C_{C2} alone is,

$$f_{L_{CC1}} = \frac{1}{2\pi R_{eq_{CC2}} C_{C2}}$$
$$= \frac{1}{2\pi \times 8.239k\Omega \times 0.1\mu F} = \mathbf{193.27Hz}$$

Since, $f_{L_{CC2}}$ is the largest among $f_{L_{CC2}}$ and $f_{L_{CC2}}$, it is the lower cut-off frequency of the overall circuit.

$\therefore f_L = \mathbf{193.27Hz}$

Mid-frequency AC equivalent circuit:

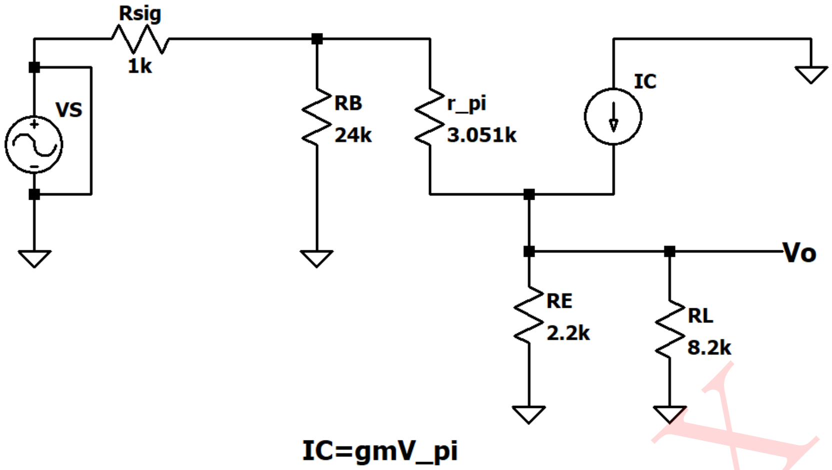

Figure 6: Mid-frequency AC equivalent circuit

Input impedance,

$$
\begin{aligned}
Z_i &= R_B \parallel [r_\pi + (1+\beta)(R_E \parallel R_L)] \\
&= 24k\Omega \parallel [3.051k\Omega + (101)(2.2k\Omega \parallel 8.2k\Omega)] \\
&= 24k\Omega \parallel \left[3.051k\Omega + 101\left(\frac{2.2k\Omega \times 8.2k\Omega}{2.2k\Omega + 8.2k\Omega}\right)\right] \\
&= 24k\Omega \parallel [3.051k\Omega + 101(1.73k\Omega)] \\
&= 24k\Omega \parallel [178.24k\Omega] \\
&= \frac{24k\Omega \times 178.24k\Omega}{24k\Omega + 178.24k\Omega} = \mathbf{21.15k\Omega}
\end{aligned}
$$

$$
\begin{aligned}
A_{V_{mid}} &= \frac{V_o}{V_{in}} \\
&= \frac{(R_E \parallel R_L)}{\frac{1}{g_m} + (R_E \parallel R_L)} \\
&= \frac{2.2k\Omega \parallel 8.2k\Omega}{\frac{1}{32.76mA/V} + (2.2k\Omega \parallel 8.2k\Omega)} \\
&= \frac{\frac{2.2k\Omega \times 8.2k\Omega}{2.2k\Omega + 8.2k\Omega}}{0.03k\Omega + \left(\frac{2.2k\Omega \times 8.2k\Omega}{2.2k\Omega + 8.2k\Omega}\right)} \\
&= \frac{1.73k\Omega}{0.03k\Omega + 1.73k\Omega} \\
&= \frac{1.73k\Omega}{1.76k\Omega} = \mathbf{0.982}
\end{aligned}
$$

$$
\begin{aligned}
A_{V_{S_{mid}}} &= \frac{V_o}{V_S} \\
&= \frac{V_o}{V_{in}} \times \frac{V_{in}}{V_S} \\
&= A_V \times \frac{V_{in}}{V_S} \\
&= A_V \times \frac{Z_i}{Z_i + R_{sig}} \\
&= 0.982 \times \frac{21.15k\Omega}{21.15k\Omega + 1k\Omega} \\
&= 0.982 \times \frac{21.15k\Omega}{22.15k\Omega} = \mathbf{0.937}
\end{aligned}
$$

$$\therefore A_{V_{S_{mid}}} \text{ in dB} = 20\log_{10}(0.937) = -\mathbf{0.565dB}$$

SIMULATED RESULTS:

Above circuit is simulated in LTspice and results are as follows:

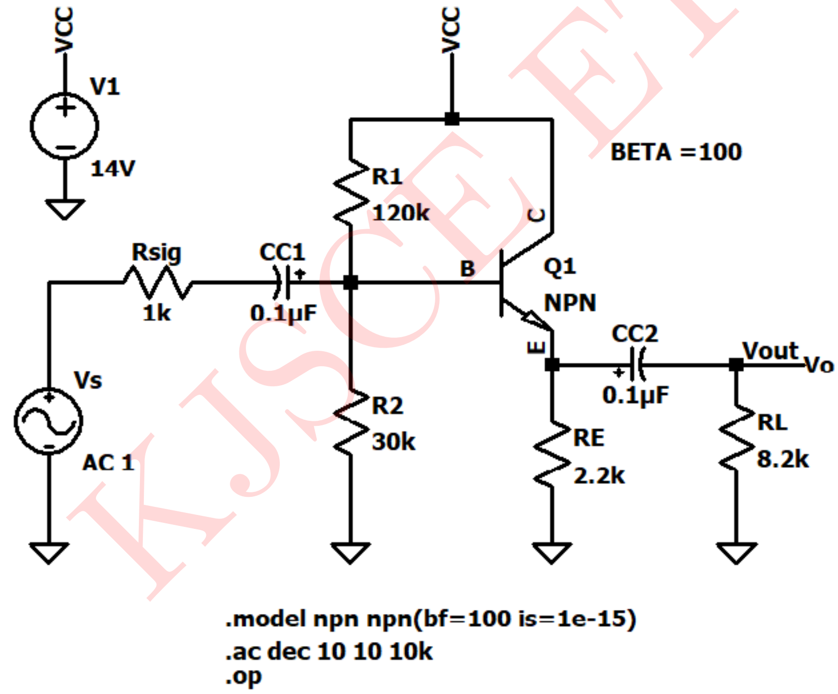

Figure 7: Circuit Schematic 1

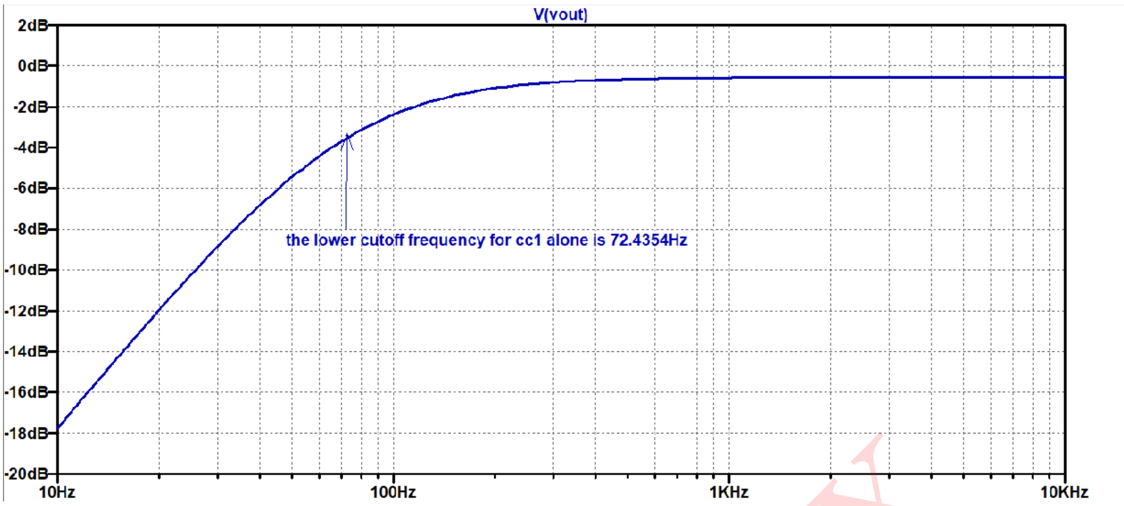

Figure 8: Lower frequency response for C_{C1}

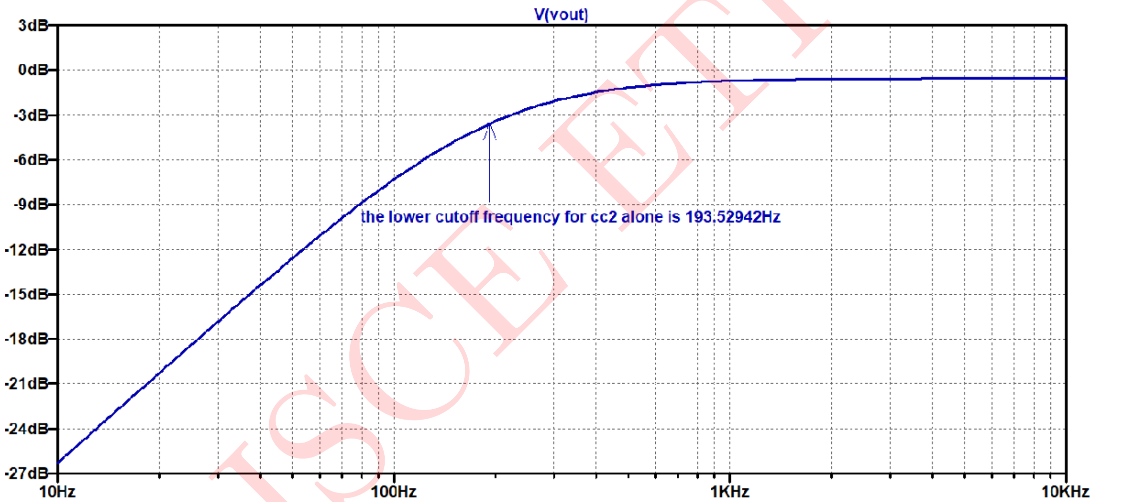

Figure 9: Lower frequency response for C_{C2}

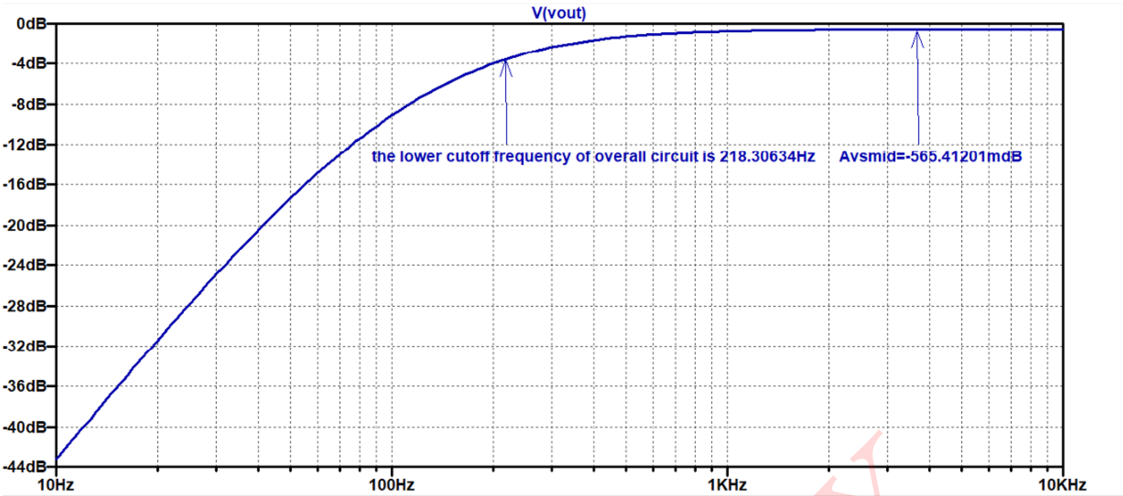

Figure 10: Lower frequency response for overall circuit

Comparison between theoretical and simulated values:

Parameters	Theoretical values	Simulated values
I_C	0.852mA	0.84874mA
Lower cut-off frequency due to C_{C1}	71.88Hz	72.435Hz
Lower cut-off frequency due to C_{C2}	193.27Hz	193.529Hz
Overall cut-off frequency f_L	193.27Hz	218.306Hz
Midband voltage gain $A_{V_{S_{mid}}}$	-0.565dB	-0.56541dB

Table 1: Numerical 1

Numerical 2:

For the circuit given below in figure 11,

a) Determine V_{GSQ} and I_{DQ}

b) Find g_{m_o} and g_m

c) Calculate the midband gain of $A_V = V_o/V_i$

d) Determine Z_i

e) Calculate $A_V = V_o/V_s$

f) Determine $f_{L_{CC1}}$, $f_{L_{CC2}}$ and $f_{L_{CS}}$

g) Determine the low cut-off frequency

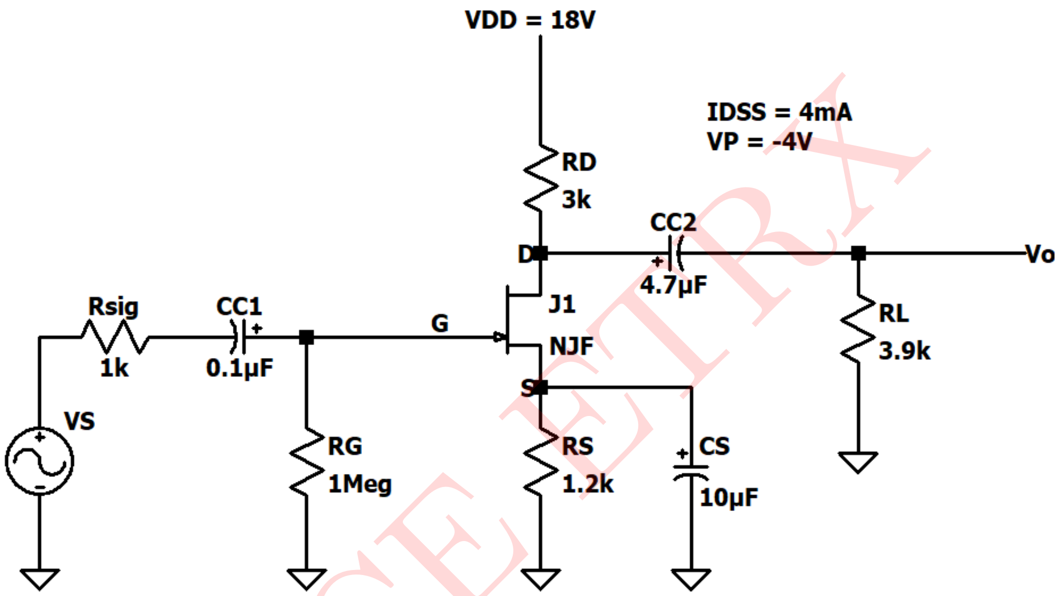

Figure 11: Circuit 2

Solution:

DC analysis:

Applying KVL in gate-source loop,

$-I_g R_G - V_{GS} - I_D R_S = 0$

$0 - V_{GS} - I_D R_S = 0 \qquad (\because I_g = 0, I_g R_G = 0)$

$\therefore V_{GS} = -I_D R_S \qquad ....(1)$

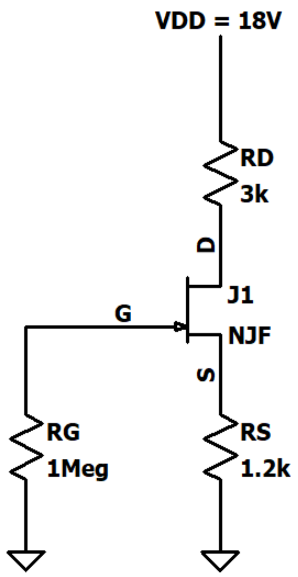

Figure 12: DC equivalent circuit

In JFET,

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

Substituting this value of I_D in equation (1)

$$\because V_{GS} = I_D R_S$$

$$\therefore V_{GS} = -I_{DSS}R_S\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$\therefore V_{GS} = -4mA(1.2k\Omega)\left(1 - \frac{V_{GS}}{-4}\right)^2$$

$$\therefore V_{GS} = -4.8\left(1 + \frac{V_{GS}}{4}\right)^2$$

$$\therefore V_{GS} = -4.8\left(1 + \frac{2V_{GS}}{4} + \frac{{V_{GS}}^2}{16}\right)$$

$$\therefore V_{GS} = -4.8 - 2.4V_{GS} - 0.3{V_{GS}}^2$$

$$\therefore 0 = -4.8 - 3.4V_{GS} - 0.3{V_{GS}}^2$$

$$\therefore V_{GS} = -1.65V \text{ or } V_{GS} = -9.68V$$

We reject the value $V_{GS} = -9.68V$ because $V_{GS} > V_P$

$$\therefore V_{GSQ} = -\ \mathbf{1.65V}$$

$$\therefore I_D = I_{DSS}\left(1 - \frac{(-1.65)}{(-4)}\right)^2$$

$$= 4mA\left(1 - \frac{1.65}{4}\right)^2$$

$$= 4mA\,(0.5875)^2$$

$$\therefore I_{DQ} = \mathbf{1.38mA}$$

Small signal parameters:

i) $g_m = \left|\dfrac{2I_{DSS}}{V_P}\right|\left(1 - \dfrac{V_{GSQ}}{V_P}\right)$

$= \dfrac{1 \times 4mA}{4V}\left(1 - \dfrac{(-1.65V)}{(-4V)}\right)$

$= \dfrac{1 \times 4mA}{4V}\left(1 - \dfrac{1.65V}{4V}\right)$

$= \dfrac{1 \times 4mA}{4V}(0.5875) = \mathbf{1.175mA/V}$

$r_d = \mathbf{50k\Omega}$

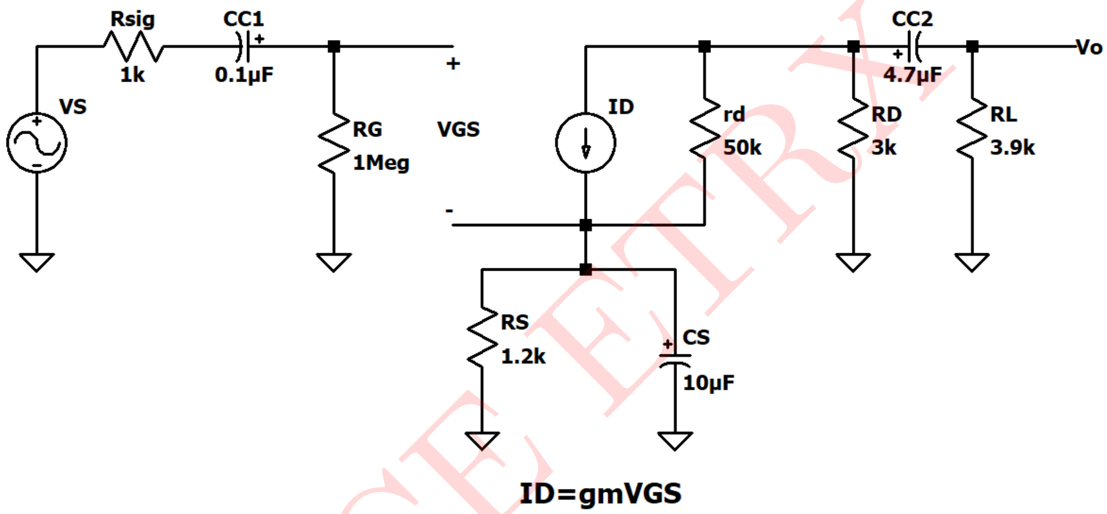

Figure 13: Low frequency AC equivalent circuit

For C_{C1}:

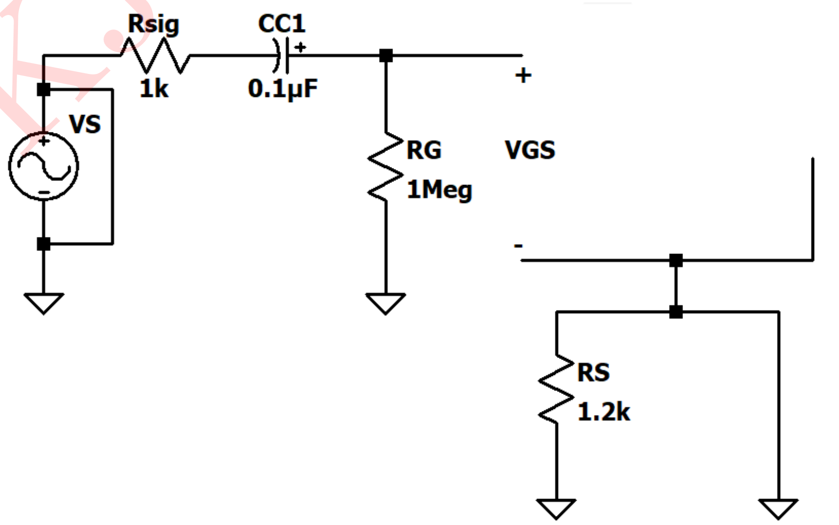

Figure 14: Small signal AC equivalent circuit for C_{C1} alone

$R_i = R_G = \mathbf{1M\Omega}$

$$R_{eqCC1} = R_i + R_{sig}$$
$$= 1M\Omega + 1k\Omega = \mathbf{1001k\Omega}$$

$\therefore$ The lower cut-off frequency due to C_{C1} alone is,

$$f_{LCC1} = \frac{1}{2\pi R_{eqCC1} C_{C1}}$$
$$= \frac{1}{2\pi \times 1001k\Omega \times 0.1\mu F} = \mathbf{1.59Hz}$$

For C_{C2}:

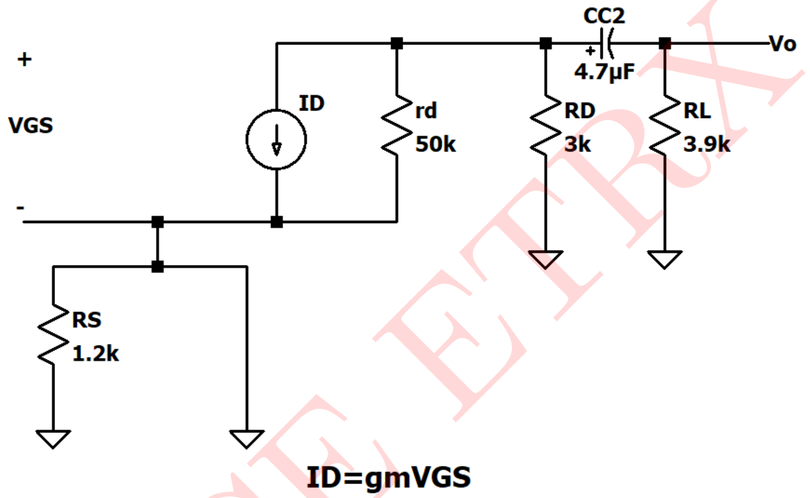

Figure 15: Small signal low frequency equivalent circuit for C_{C2} alone

$$R_o = R_D \parallel r_d$$
$$= 3k\Omega \parallel 50k\Omega$$
$$= \frac{3k\Omega \times 50k\Omega}{3k\Omega + 50k\Omega} = \mathbf{2.83k\Omega}$$

$$R_{eqCC2} = R_o + R_L$$
$$= 2.83k\Omega + 3.9k\Omega = \mathbf{6.73k\Omega}$$

$\therefore$ The lower cut-off frequency due to C_{C2} alone is,

$$f_{LCC2} = \frac{1}{2\pi R_{eqCC2} C_{C2}}$$
$$= \frac{1}{2\pi \times 6.73k\Omega \times 4.7\mu F} = \mathbf{5.03Hz}$$

For C_S:

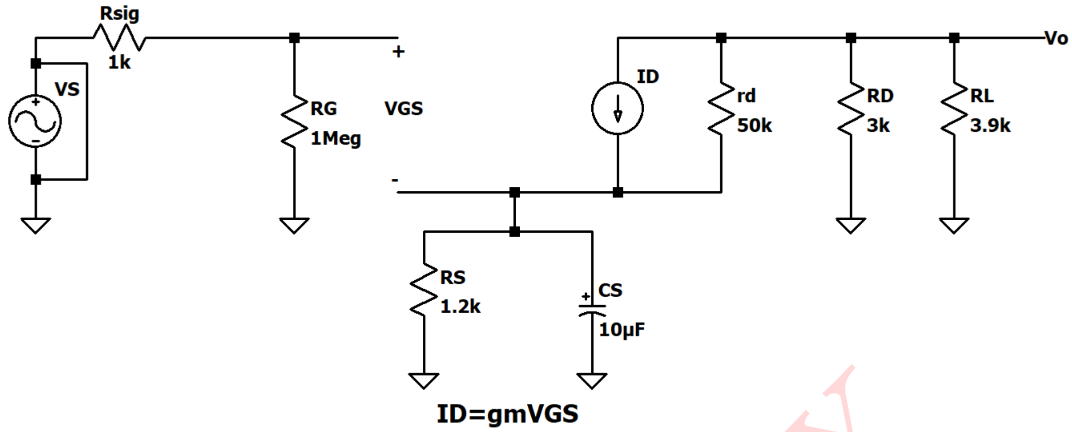

Figure 16: Small signal low frequency equivalent circuit for C_S alone

$$R_{eq_{CS}} = \frac{R_S}{1 + \dfrac{R_S(1+g_m r_d)}{(r_d + R_D \parallel R_L)}}$$

$$= \frac{1.2k\Omega}{1 + \dfrac{1.2(1+(1.175mA/V \times 50k\Omega))}{(50k\Omega + (3k\Omega \parallel 3.9k\Omega))}}$$

$$= \frac{1.2k\Omega}{1 + \dfrac{1.2(59.75)}{\left(50k\Omega + \dfrac{3k\Omega \times 3.9k\Omega}{3k\Omega + 3.9k\Omega}\right)}}$$

$$= \frac{1.2k\Omega}{1 + \dfrac{71.7k\Omega}{51.69k\Omega}}$$

$$= \frac{1.2k\Omega}{1 + 1.38k\Omega} = \mathbf{0.504k\Omega}$$

$\therefore$ The lower cut-off frequency due to C_S alone is,

$$f_{L_S} = \frac{1}{2\pi R_{eq_{CS}} C_S}$$

$$= \frac{1}{2\pi \times 0.504k\Omega \times 10\mu F} = \mathbf{31.59Hz}$$

Since $f_{L_{CS}}$ is the largest of $f_{L_{CC1}}$ and $f_{L_{CC2}}$, the lower cut-off frequency of the overall circuit(f_L) is 31.59Hz.

Mid-frequency AC equivalent circuit:

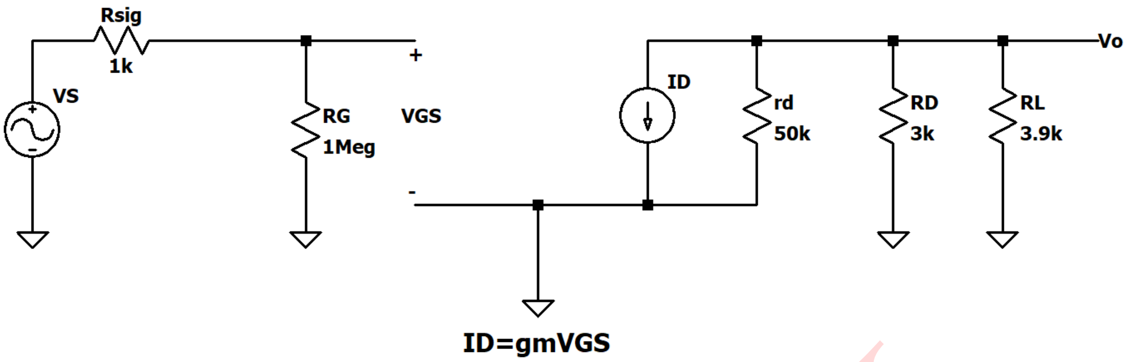

Figure 17: Mid-frequency AC equivalent circuit

$$
\begin{aligned}
A_{V_{mid}} &= \frac{V_o}{V_i} \\
&= \frac{(-g_m V_{GS})(R_L \parallel R_D \parallel r_d)}{V_{GS}} \\
&= -g_m(R_L \parallel R_D \parallel r_d) \\
&= -1.175mA/V(3.9k\Omega \parallel 3k\Omega \parallel 50k\Omega) \\
&= -1.175mA/V\left[3.9k\Omega \parallel \left(\frac{3k\Omega \times 50k\Omega}{3k\Omega + 50k\Omega}\right)\right] \\
&= -1.175mA/V[3.9k\Omega \parallel 2.83k\Omega] \\
&= -1.175mA/V\left(\frac{3k\Omega \times 50k\Omega}{3k\Omega + 50k\Omega}\right) \\
&= -1.175mA/V \times 1.63k\Omega = \mathbf{-1.92}
\end{aligned}
$$

Input impedance,
$Z_i = R_G = \mathbf{1M\Omega}$

$$
\begin{aligned}
A_{V_{S_{mid}}} &= \frac{V_o}{V_S} \\
&= \frac{V_o}{V_{in}} \times \frac{V_{in}}{V_S} \\
&= A_V \times \frac{V_{in}}{V_S} \\
&= A_V \times \frac{Z_i}{Z_i + R_{sig}} \\
&= A_V \times \frac{1M\Omega}{1M\Omega + 1k\Omega} \\
&= -1.92 \times 0.99 = \mathbf{-1.91}
\end{aligned}
$$

$\therefore A_{V_{S_{mid}}}$ in dB $= 20\log_{10}(1.91) = \mathbf{5.62dB}$

SIMULATED RESULTS:

Above circuit is simulated in LTspice and results are as follows:

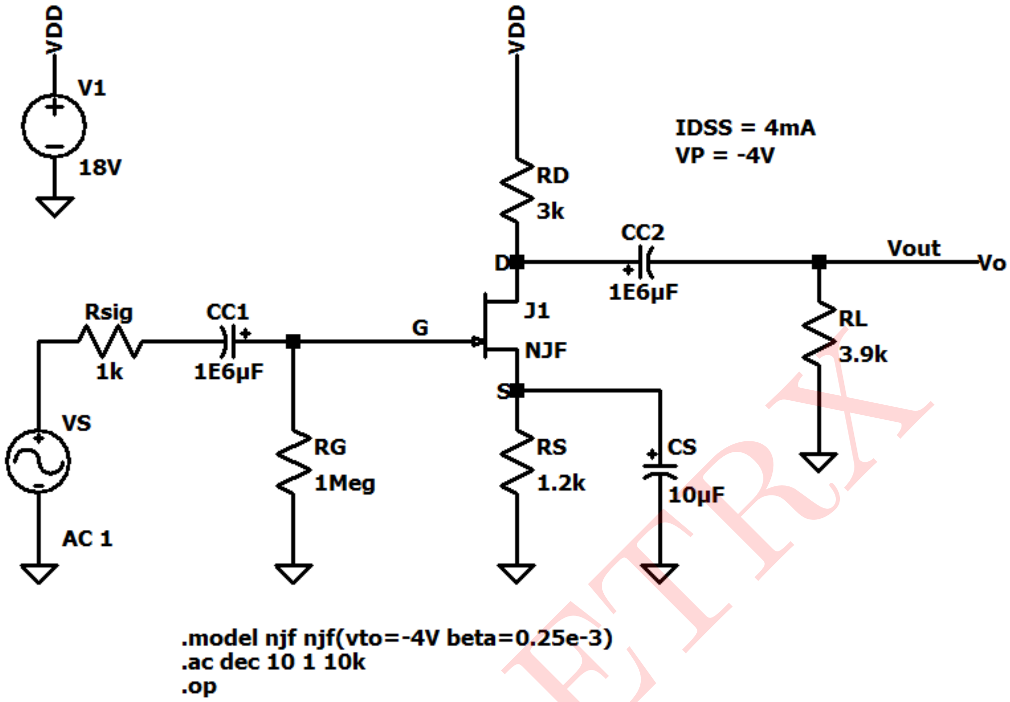

Figure 18: Circuit Schematic

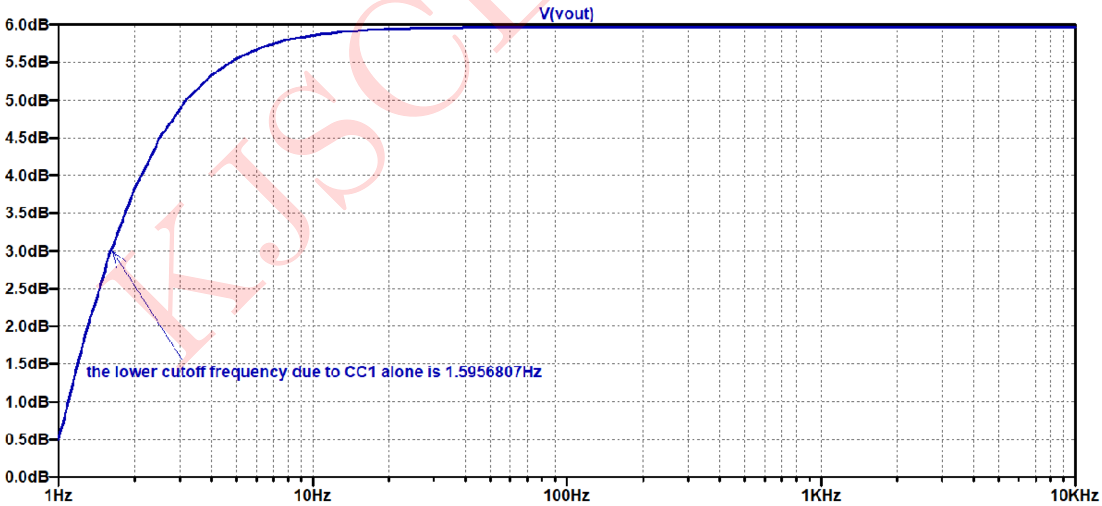

Figure 19: Lower frequency response for C_{C1}

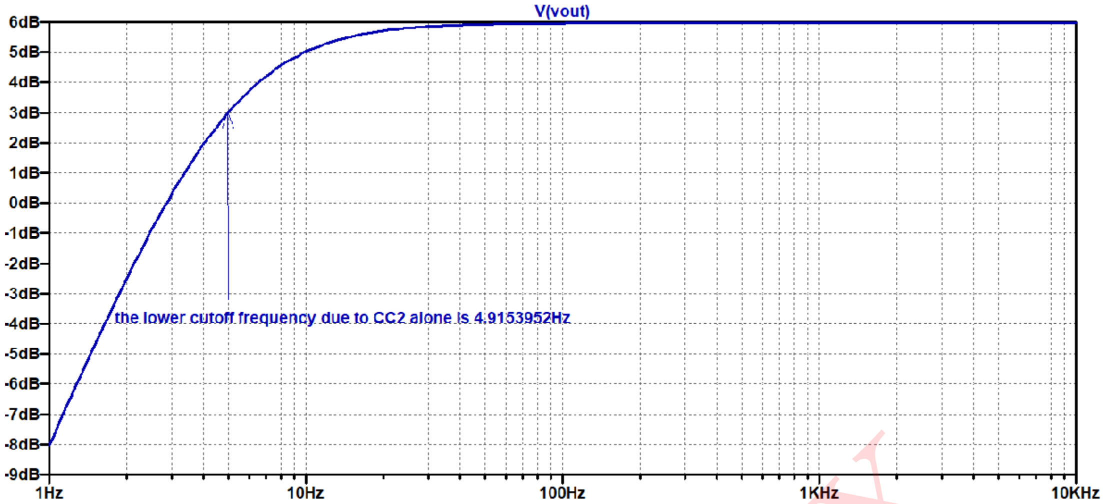

Figure 20: Lower frequency response for C_{C2}

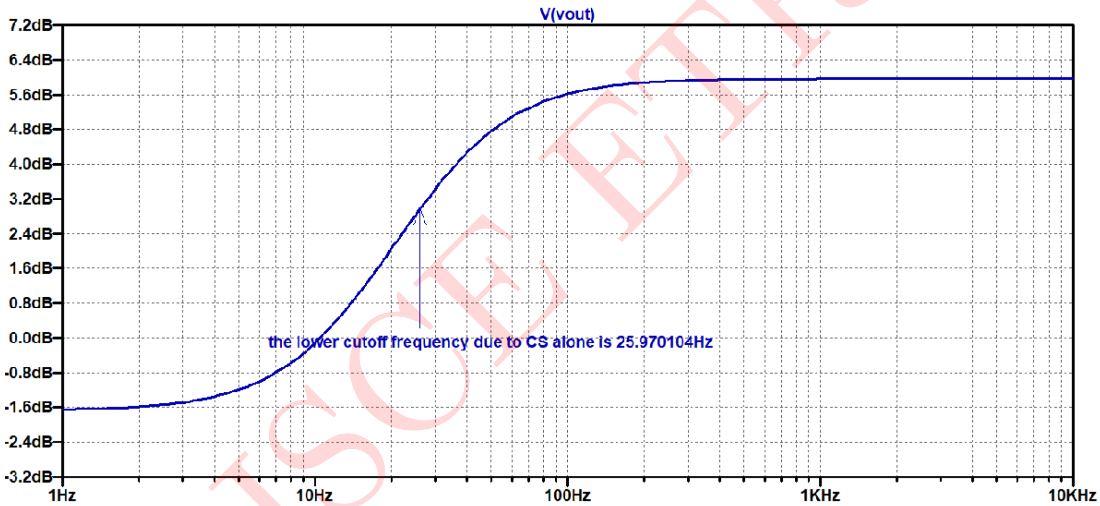

Figure 21: Lower frequency response for C_S

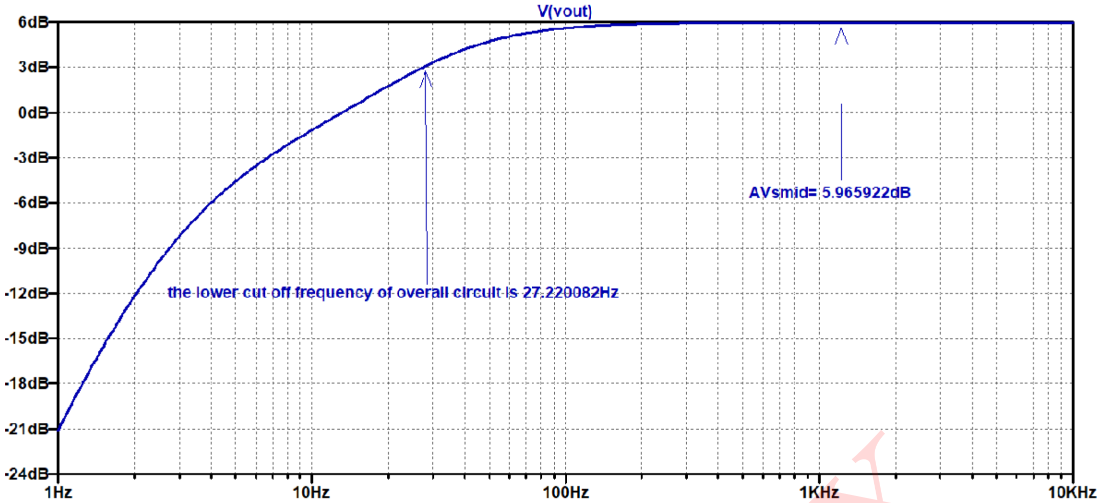

Figure 22: Lower frequency response for overall circuit

Comparison between theoretical and simulated values:

Parameters	Theoretical values	Simulated values
I_{DQ}	1.38mA	1.37mA
V_{GSQ}	-1.65V	-1.65281V
Lower cut-off frequency due to C_{C1}	1.59Hz	1.59568Hz
Lower cut-off frequency due to C_{C2}	5.03Hz	4.9153Hz
Overall cut-off frequency f_L	31.59Hz	27.2200Hz
Midband voltage gain $A_{VS_{mid}}$ in dB	5.62dB	5.9659dB

Table 2: Numerical 2

Numerical 3:
For the network given below in figure 23,
a) Determine V_{GSQ} and I_{DQ}
b) Find g_{m_o} and g_m
c) Calculate the midband gain of $A_V = V_o/V_i$
d) Determine Z_i
e) Calculate $A_V = V_o/V_s$
f) Determine f_{LCC1}, f_{LCC2} and f_{LCS}
g) Determine the low cut-off frequency
f) Determine the higher cut-off frequency

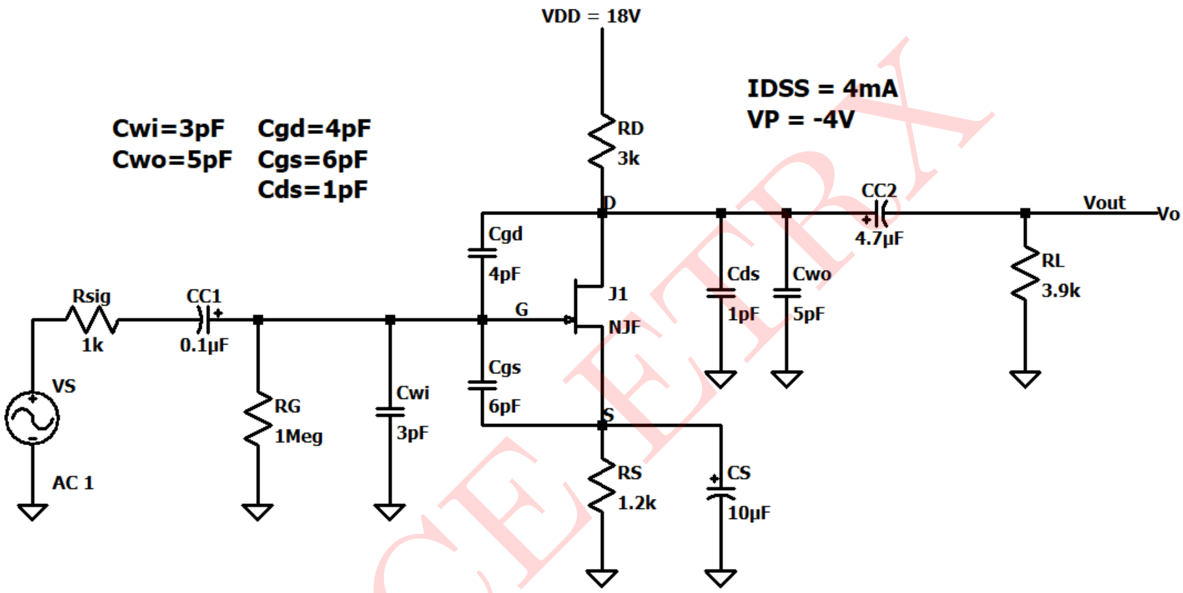

Figure 23: Circuit 3

Solution:
DC analysis:
Applying KVL in gate-source loop,

$-I_g R_G - V_{GS} - I_D R_S = 0$

$0 - V_{GS} - I_D R_S = 0 \qquad (\because I_g = 0, I_g R_G = 0)$

$\therefore V_{GS} = -I_D R_S \qquad ....(1)$

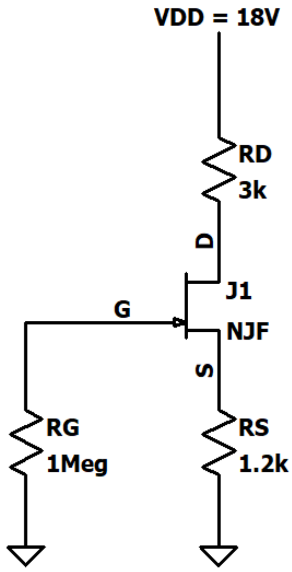

Figure 24: DC equivalent circuit

In JFET,

$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$

Substituting this value of I_D in equation (1)

$\because V_{GS} = I_D R_S$

$\therefore V_{GS} = -I_{DSS} R_S \left(1 - \frac{V_{GS}}{V_P}\right)^2$

$\therefore V_{GS} = -4mA(1.2k\Omega)\left(1 - \frac{V_{GS}}{-4}\right)^2$

$\therefore V_{GS} = -4.8\left(1 + \frac{V_{GS}}{4}\right)^2$

$\therefore V_{GS} = -4.8\left(1 + \frac{2V_{GS}}{4} + \frac{{V_{GS}}^2}{16}\right)$

$\therefore V_{GS} = -4.8 - 2.4V_{GS} - 0.3{V_{GS}}^2$

$\therefore 0 = -4.8 - 3.4V_{GS} - 0.3{V_{GS}}^2$

$\therefore V_{GS} = -1.65V$ or $V_{GS} = -9.68V$

We reject the value $V_{GS} = -9.68V$ because $V_{GS} > V_P$

$\therefore V_{GSQ} = -$ **1.65V**

$$\begin{aligned}\therefore I_D &= I_{DSS}\left(1 - \frac{(-1.65)}{(-4)}\right)^2 \\ &= 4mA\left(1 - \frac{1.65}{4}\right)^2 \\ &= 4mA\,(0.5875)^2\end{aligned}$$

$\therefore I_{DQ} =$ **1.38mA**

Small signal parameters:

i) $g_m = \left|\frac{2I_{DSS}}{V_P}\right|\left(1 - \frac{V_{GSQ}}{V_P}\right)$

$$= \frac{1 \times 4mA}{4V}\left(1 - \frac{(-1.65V)}{(-4V)}\right)$$

$$= \frac{1 \times 4mA}{4V}\left(1 - \frac{1.65V}{4V}\right)$$

$$= \frac{1 \times 4mA}{4V}(0.5875) = \mathbf{1.175mA/V}$$

$r_d = \mathbf{50k\Omega}$

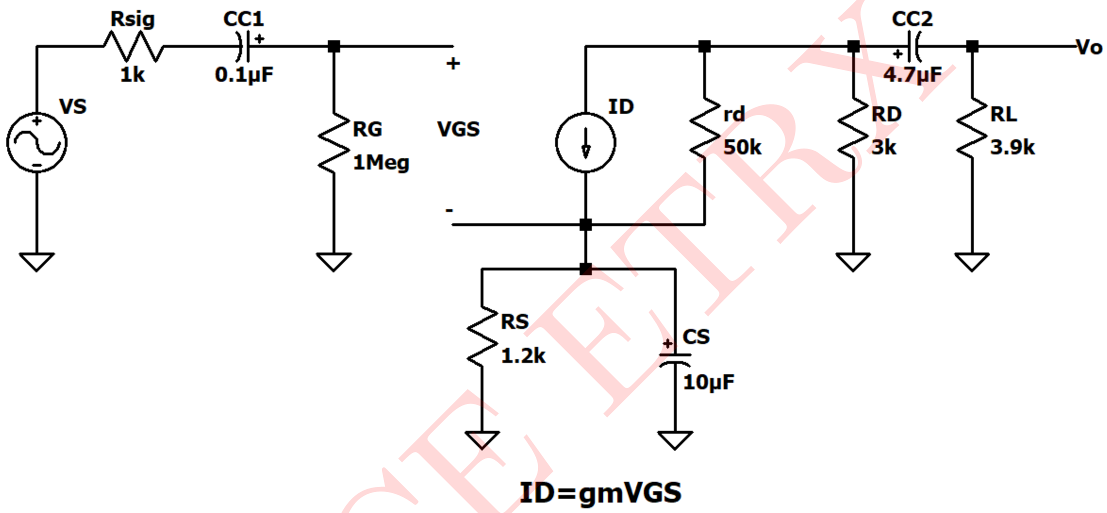

Figure 25: Low frequency AC equivalent circuit

For C_{C1}:

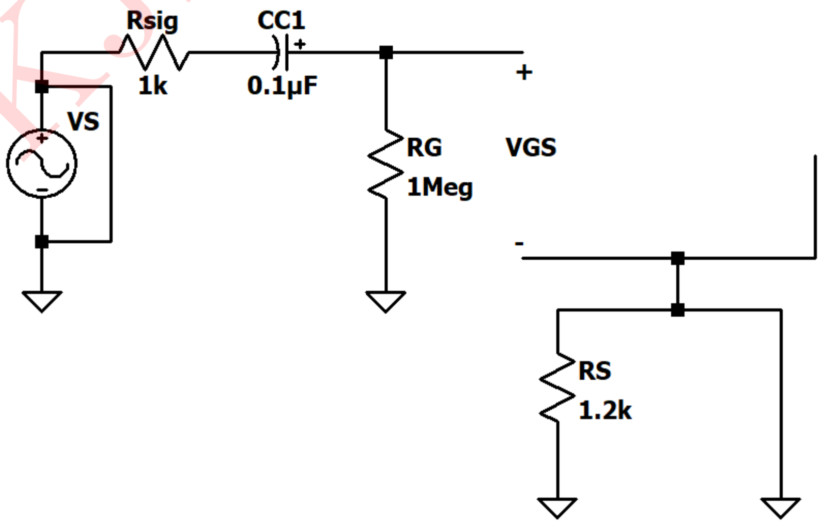

Figure 26: Small signal AC equivalent circuit for C_{C1} alone

$R_i = R_G = \mathbf{1M\Omega}$

$$\begin{aligned} R_{eqCC1} &= R_i + R_{sig} \\ &= 1M\Omega + 1k\Omega = \mathbf{1001k\Omega} \end{aligned}$$

$\therefore$ The lower cut-off frequency due to C_{C1} alone is,

$$\begin{aligned} f_{LCC1} &= \frac{1}{2\pi R_{eqCC1} C_{C1}} \\ &= \frac{1}{2\pi \times 1001k\Omega \times 0.1\mu F} = \mathbf{1.59Hz} \end{aligned}$$

For C_{C2}:

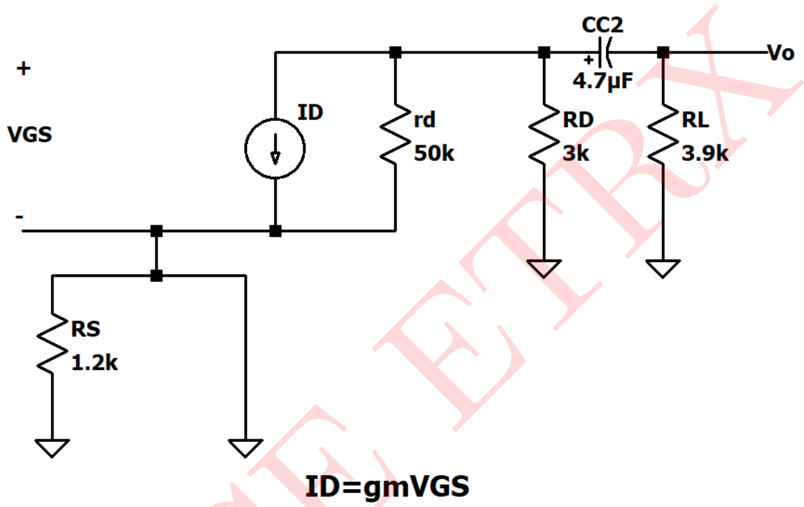

Figure 27: Small signal low frequency equivalent circuit for C_{C2} alone

$$\begin{aligned} R_o &= R_D \parallel r_d \\ &= 3k\Omega \parallel 50k\Omega \\ &= \frac{3k\Omega \times 50k\Omega}{3k\Omega + 50k\Omega} = \mathbf{2.83k\Omega} \end{aligned}$$

$$\begin{aligned} R_{eqCC2} &= R_o + R_L \\ &= 2.83k\Omega + 3.9k\Omega = \mathbf{6.73k\Omega} \end{aligned}$$

$\therefore$ The lower cut-off frequency due to C_{C2} alone is,

$$\begin{aligned} f_{LCC2} &= \frac{1}{2\pi R_{eqCC2} C_{C2}} \\ &= \frac{1}{2\pi \times 6.73k\Omega \times 4.7\mu F} = \mathbf{5.03Hz} \end{aligned}$$

For C_S:

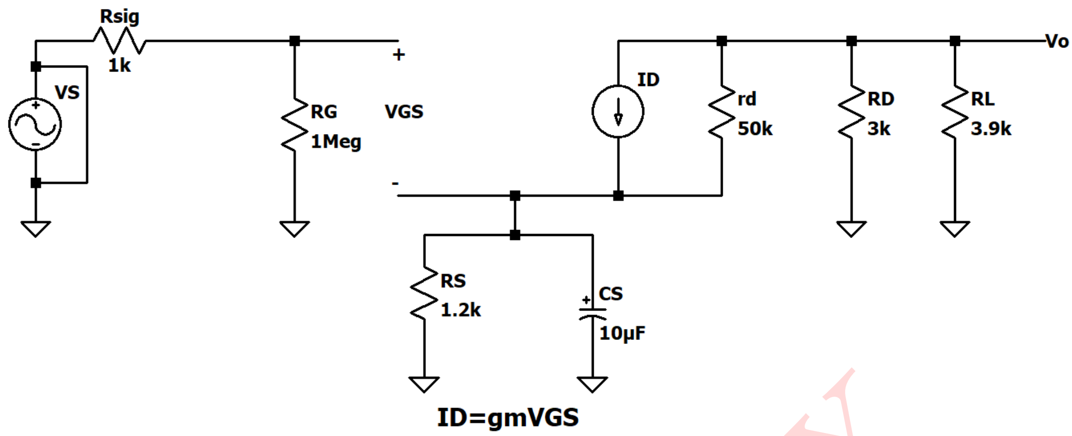

Figure 28: Small signal low frequency equivalent circuit for C_S alone

$$
\begin{aligned}
R_{eq_{CS}} &= \frac{R_S}{1 + \dfrac{R_S(1 + g_m r_d)}{(r_d + R_D \parallel R_L)}} \\
&= \frac{1.2k\Omega}{1 + \dfrac{1.2(1 + (1.175mA/V \times 50k\Omega))}{(50k\Omega + (3k\Omega \parallel 3.9k\Omega))}} \\
&= \frac{1.2k\Omega}{1 + \dfrac{1.2(59.75)}{\left(50k\Omega + \dfrac{3k\Omega \times 3.9k\Omega}{3k\Omega + 3.9k\Omega}\right)}} \\
&= \frac{1.2k\Omega}{1 + \dfrac{71.7k\Omega}{51.69k\Omega}} \\
&= \frac{1.2k\Omega}{1 + 1.38k\Omega} = \mathbf{0.504k\Omega}
\end{aligned}
$$

$\therefore$ The lower cut-off frequency due to C_S alone is,

$$
\begin{aligned}
f_{L_{CS}} &= \frac{1}{2\pi R_{eq_{CS}} C_S} \\
&= \frac{1}{2\pi \times 0.504k\Omega \times 10\mu F} = \mathbf{31.59Hz}
\end{aligned}
$$

Since $f_{L_{CS}}$ is the largest of $f_{L_{CC1}}$ and $f_{L_{CC2}}$, the lower cut-off frequency of the overall circuit(f_L) is 31.59Hz.

Mid-frequency AC equivalent circuit:

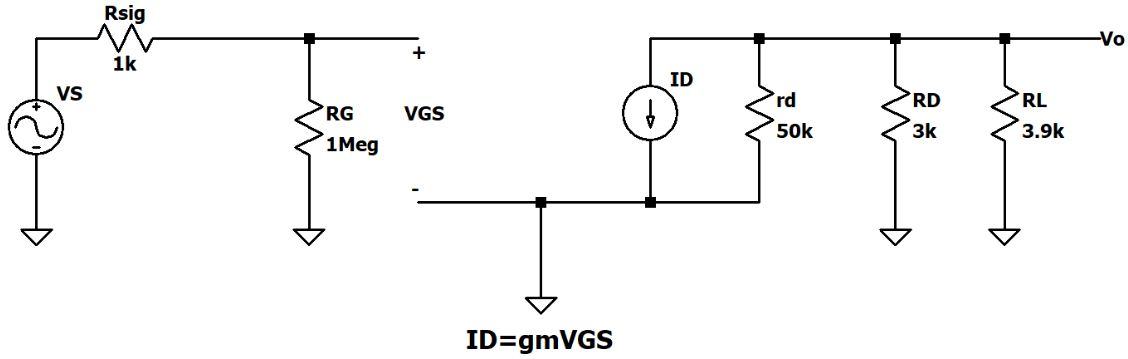

Figure 29: Mid-frequency AC equivalent circuit

$$
\begin{aligned}
A_{V_{mid}} &= \frac{V_o}{V_i} \\
&= \frac{(-g_m V_{GS})(R_L \parallel R_D \parallel r_d)}{V_{GS}} \\
&= -g_m(R_L \parallel R_D \parallel r_d) \\
&= -1.175mA/V(3.9k\Omega \parallel 3k\Omega \parallel 50k\Omega) \\
&= -1.175mA/V\left[3.9k\Omega \parallel \left(\frac{3k\Omega \times 50k\Omega}{3k\Omega + 50k\Omega}\right)\right] \\
&= -1.175mA/V[3.9k\Omega \parallel 2.83k\Omega] \\
&= -1.175mA/V\left(\frac{3k\Omega \times 50k\Omega}{3k\Omega + 50k\Omega}\right) \\
&= -1.175mA/V \times 1.63k\Omega = \mathbf{-1.92}
\end{aligned}
$$

Input impedance,
$Z_i = R_G = \mathbf{1M\Omega}$

$$
\begin{aligned}
A_{V_{S_{mid}}} &= \frac{V_o}{V_S} \\
&= \frac{V_o}{V_{in}} \times \frac{V_{in}}{V_S} \\
&= A_V \times \frac{V_{in}}{V_S} \\
&= A_V \times \frac{Z_i}{Z_i + R_{sig}} \\
&= A_V \times \frac{1M\Omega}{1M\Omega + 1k\Omega} \\
&= -1.92 \times 0.99 = \mathbf{-1.91}
\end{aligned}
$$

$\therefore A_{V_{S_{mid}}}$ in dB $= 20\log_{10}(1.91) = \mathbf{5.62dB}$

High frequency equivalent circuit:

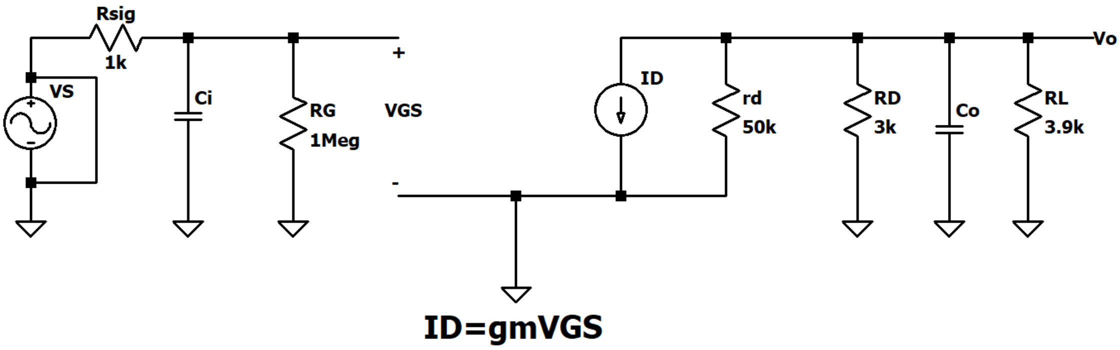

Figure 30: High frequency equivalent circuit

i) $C_i = C_{gs} + C_{mi} + C_{wi}$

$\therefore C_{mi} = C_{gd}(1 - A_{V_{S_{mid}}})$

$\therefore C_{mi} = 4pF(1 - (-1.91))$

$\therefore C_{mi} = 4pF(2.91) = \mathbf{11.64pF}$

$\therefore C_i = 6pF + 11.64pF + 3pF = \mathbf{20.64pF}$

i) $C_o = C_{wo} + C_{mo} + C_{ds}$

$$\therefore C_{mo} = C_{gd}\left(1 - \frac{1}{A_{V_{S_{mid}}}}\right)$$

$$\therefore C_{mo} = 4pF\left(1 - \frac{1}{(-1.91)}\right)$$

$$\therefore C_{mi} = 4pF\left(1 + \frac{1}{1.91}\right) = \mathbf{6.09pf}$$

$\therefore C_o = 5pF + 6.09pF + 1pF = \mathbf{12.09pF}$

For f_{H_i},

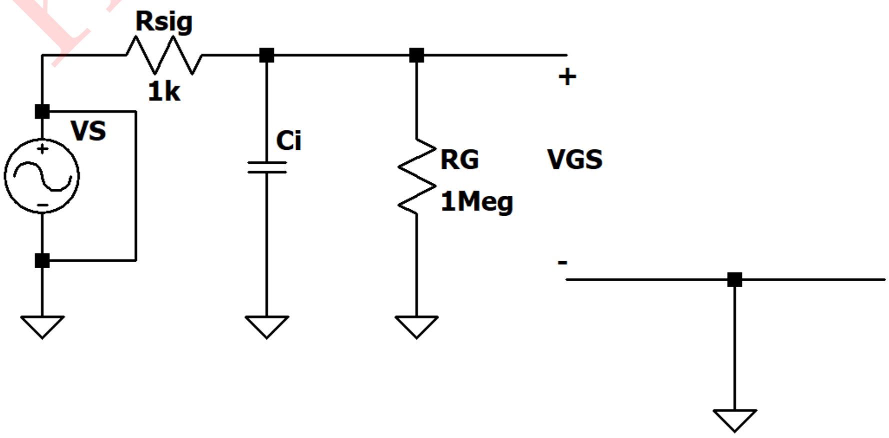

Figure 31: High frequency equivalent circuit for C_i

$$
\begin{aligned}
R_{eq} &= R_{sig} \parallel R_G \\
&= 1k\Omega \parallel 1M\Omega \\
&= \frac{1k\Omega \times 1M\Omega}{1k\Omega + 1M\Omega} = \mathbf{0.99k\Omega}
\end{aligned}
$$

$$
\begin{aligned}
f_{H_i} &= \frac{1}{2\pi R_{eq} C_i} \\
&= \frac{1}{2\pi \times 0.99k\Omega \times 20.64pF} = \mathbf{7.79MHz}
\end{aligned}
$$

For f_{H_o},

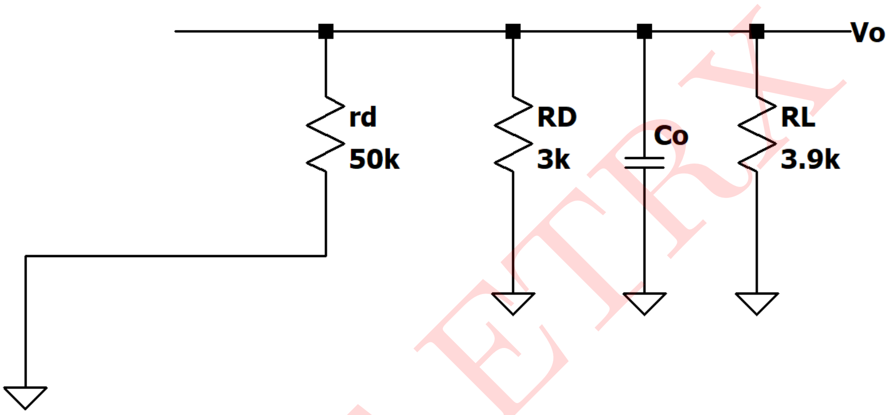

Figure 32: High frequency equivalent circuit for C_o

$$
\begin{aligned}
R_{eq} &= R_d \parallel R_D \parallel R_L \\
&= 50k\Omega \parallel 3k\Omega \parallel 3.9k\Omega \\
&= 50k\Omega \parallel \left(\frac{3k\Omega \times 3.9k\Omega}{3k\Omega + 3.9k\Omega}\right) \\
&= 50k\Omega \parallel 1.69k\Omega \\
&= \frac{50k\Omega \times 1.69k\Omega}{50k\Omega + 1.69k\Omega} = \mathbf{1.63k\Omega}
\end{aligned}
$$

$$
\begin{aligned}
f_{H_o} &= \frac{1}{2\pi R_{eq} C_o} \\
&= \frac{1}{2\pi \times 1.63k\Omega \times 12.09pF} = \mathbf{8.08MHz}
\end{aligned}
$$

$\because f_{H_i}$ is the lowest among f_{H_i} and f_{H_o}, the lower cut-off frequency of overall circuit(f_H) is 7.79Hz.

$\therefore f_H = \mathbf{7.79MHz}$

SIMULATED RESULTS:
Above circuit is simulated in LTspice and results are as follows:

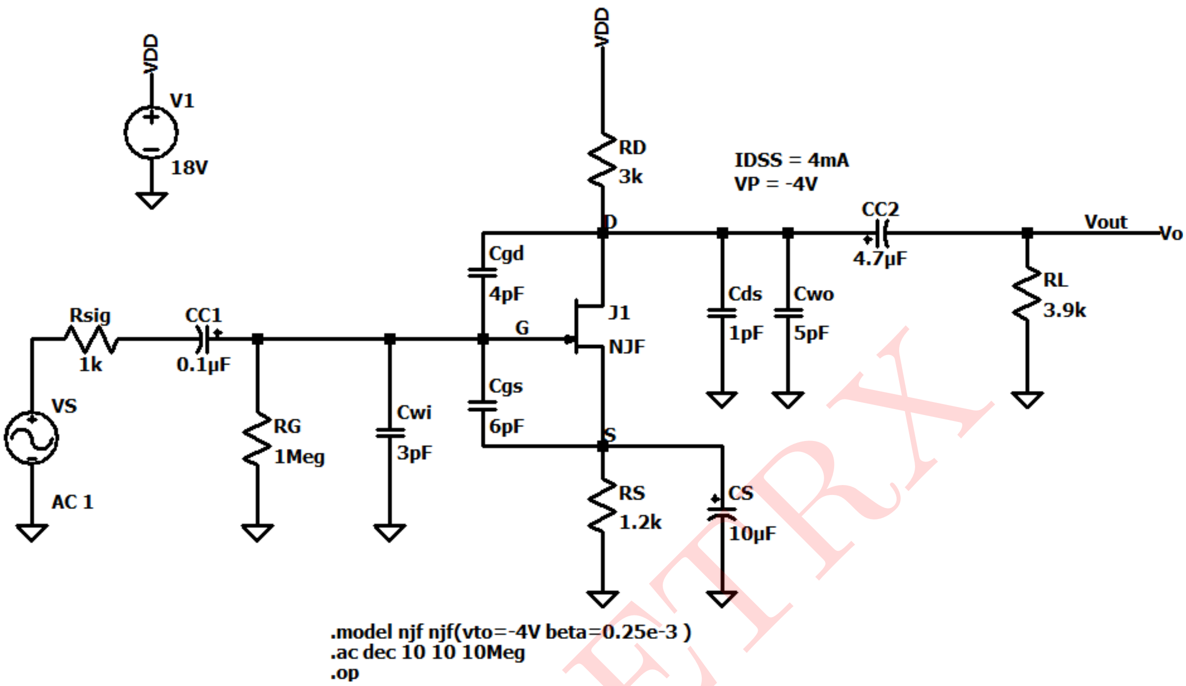

Figure 33: Circuit Schematic

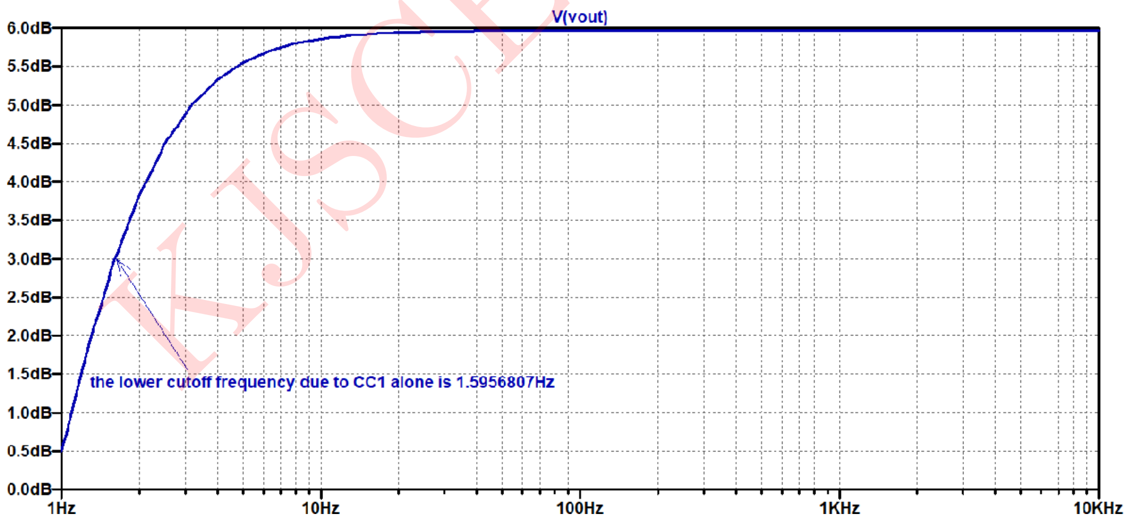

Figure 34: Lower frequency response for C_{C1}

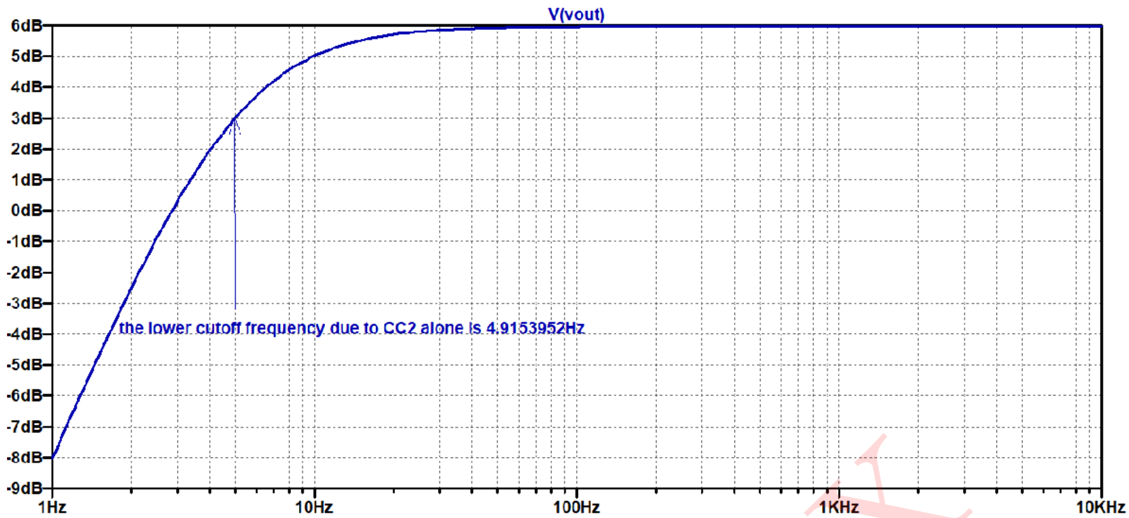

Figure 35: Lower frequency response for C_{C2}

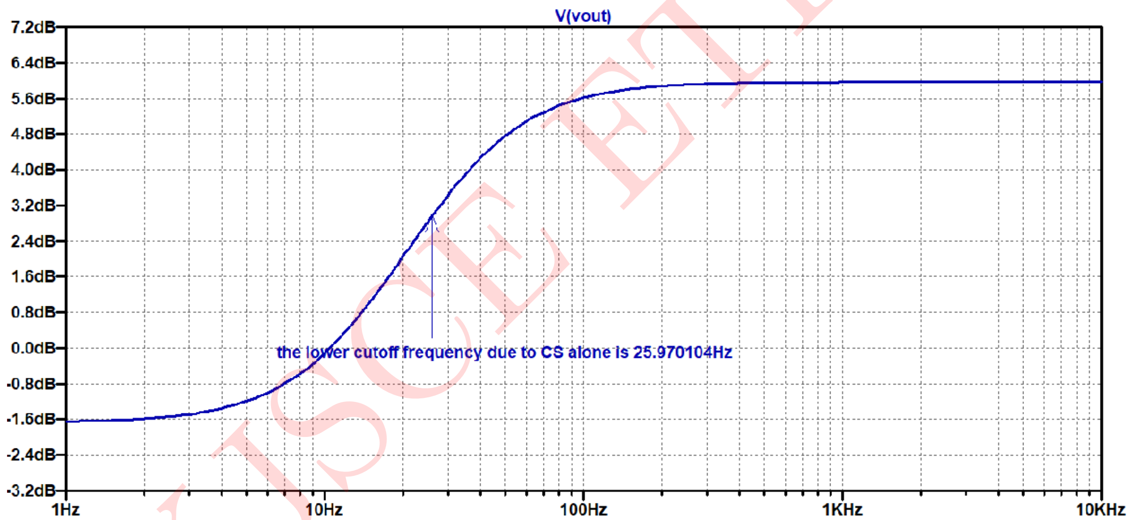

Figure 36: Lower frequency response for C_S

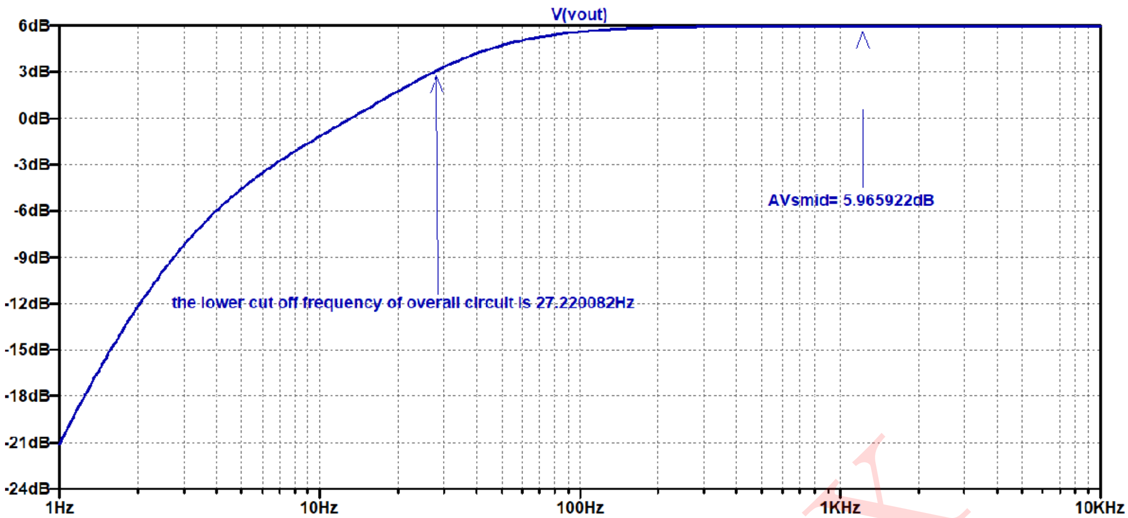

Figure 37: Lower frequency response for overall circuit

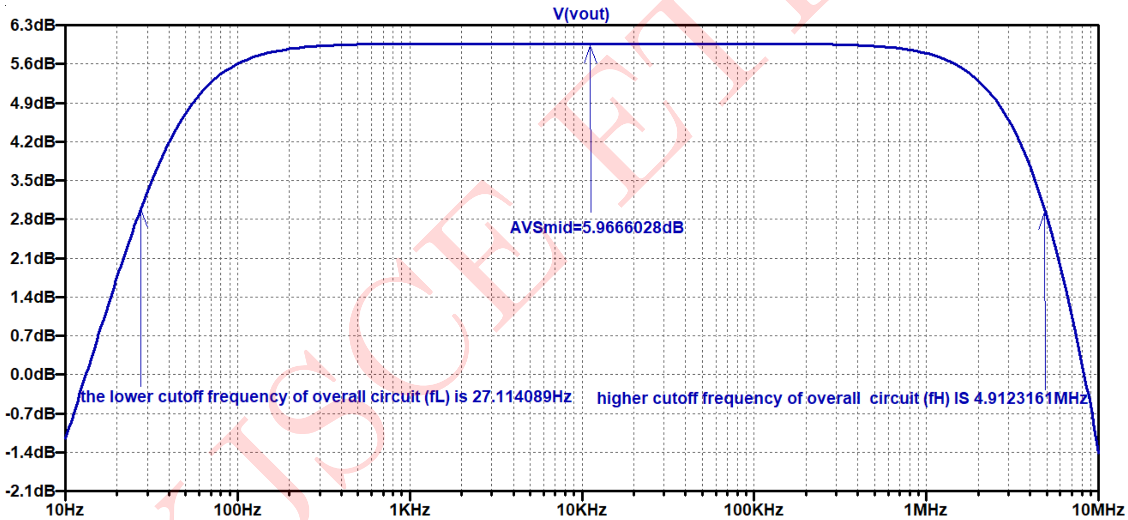

Figure 38: Higher frequency response for overall circuit

Comparison between theoretical and simulated values:

Parameters	Theoretical values	Simulated values
I_{DQ}	1.38mA	1.377mA
V_{GSQ}	-1.65V	-1.65281V
Lower cut-off frequency due to C_{C1}	1.59Hz	1.59568Hz
Lower cut-off frequency due to C_{C2}	5.03Hz	4.9153Hz
Overall cut-off frequency f_L	31.59Hz	27.2200Hz
Midband voltage gain $A_{V_{S_{mid}}}$ in dB	5.62dB	5.9659dB
Overall cut-off frequency f_H	7.79MHz	4.912MHz

Table 3: Numerical 3

****