K. J. SOMAIYA COLLEGE OF ENGINEERING
DEPARTMENT OF ELECTRONICS ENGINEERING
ELECTRONIC CIRCUITS
LOW AND HIGH FREQUENCY RESPONSE OF SINGLE-STAGE AMPLIFIER

Numericals

1. For the circuit shown in Figure 1, determine
   a) r_π
   b) $A_{V_{mid}}$
   c) Z_i
   d) A_{VSmid}
   e) f_{LCC1}, f_{LCC2}, f_{LCE}
   f) lower cut-off frequency
   Given: $\beta = 120$, $V_T = 26$ mV

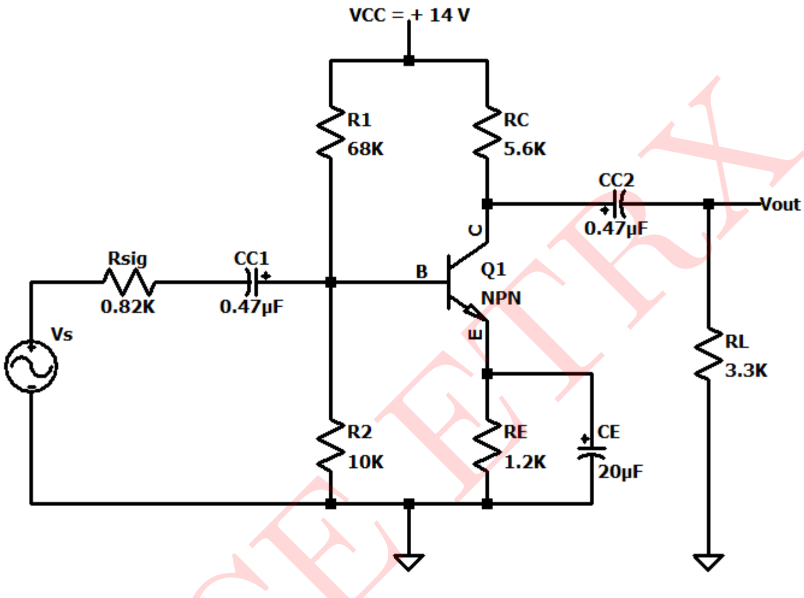

Figure 1: Circuit 1

Solution:

DC Analysis:

The capacitors act as open circuit. $f = 0, \therefore X_C = \dfrac{1}{2\pi f C} = \infty$

Applying Thevenin's equivalent at base,

$$V_{th} = \frac{R_2}{R_1 + R_2} \times V_{CC} = \frac{10k}{68k + 10k} \times 14$$

$\therefore V_{th} = 1.79487$ V

$R_{th} = R_1 || R_2 = 68k || 10k$

$\therefore R_{th} = 8.7179\ k\Omega$

Applying KVL to input base-emitter loop

$$I_B = \frac{V_{th} - V_{BE}}{R_{th} + (1+\beta)R_E} = \frac{1.7948 - 0.7}{8.7179k + (121 \times 1.2k)}$$

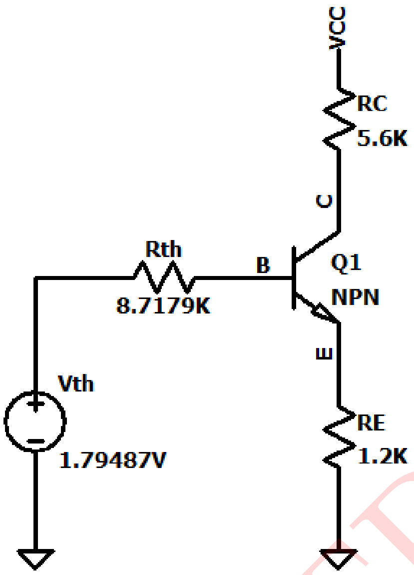

Figure 2: Thevenin's equivalent circuit

$\therefore I_B = 7.11335\ \mu$A

$I_{CQ} = \beta I_B = 120 \times 7.11335 \times 10^{-6}$

$\therefore I_{CQ} = 0.8536$ mA

Small-signal parameters

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{120 \times 26mV}{0.8536mA} = 3.655\ k\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.8536mA}{26mV} = 32.83077\frac{mA}{V}$$

$$r_o = \frac{V_A}{I_{CQ}}$$

Since V_A is not given, we assume r_o to be $\infty\Omega$

The low frequency equivalent circuit is shown below in Circuit 3.

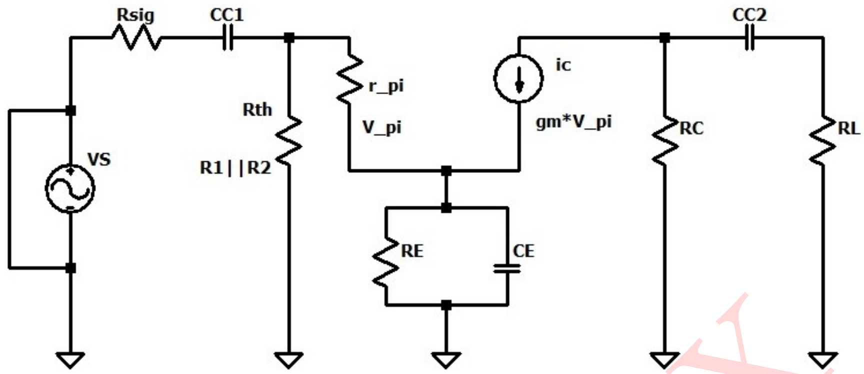

Figure 3: Small signal low frequency equivalent circuit

For $\mathbf{f_{L_{CC1}}}$:

As C_{C2}, C_E and V_S are short, Circuit 3 reduces to Figure 4

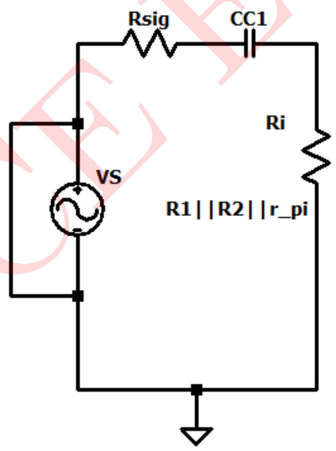

Figure 4: Small signal low frequency equivalent circuit for C_{C1}

$$f_{L_{CC1}} = \frac{1}{2\pi(R_{sig} + R_i) \times C_{C1}}$$

$R_{eq} = R_{sig} + R_i$

$R_i = R_1||R_2||r_\pi$

$\therefore R_i = 2.5753k\Omega$

$C_{C1} = 0.47\mu F,\ R_{sig} = 0.8k\Omega$

$$f_{L_{CC1}} = \frac{1}{2\pi(0.8k + 2.575k)(0.47\mu F)}$$

$\therefore f_{L_{CC1}} = 99.737$ Hz

For $\mathbf{f_{LCC2}}$:

The capacitors C_{C1} and C_E, and V_S are short, hence Circuit 3 reduces to Figure 5

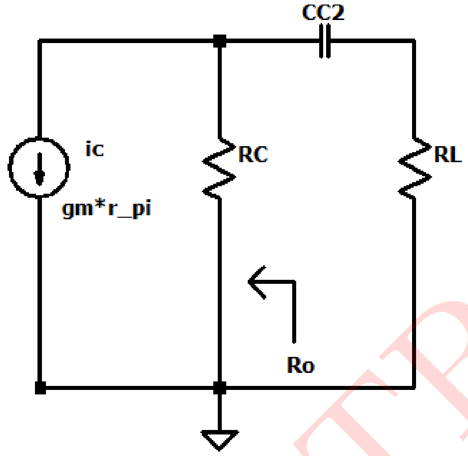

Figure 5: Small signal low frequency equivalent circuit for C_{C2}

$$f_{LCC2} = \frac{1}{2\pi(R_O + R_L)C_{C2}}$$

$R_O = R_C = 5.6k\Omega$

$C_{C2} = 0.47\mu\text{F}$, $R_L = 3.3k\Omega$

$$\therefore f_{LCC2} = \frac{1}{2\pi(5.6k + 3.3k) \times 0.47\mu F}$$

$\therefore f_{LCC2} = 38.048$ Hz

For $\mathbf{f_{LCE}}$:

C_{C1}, C_E and V_S are short, hence Circuit 3 reduces to Circuit 6

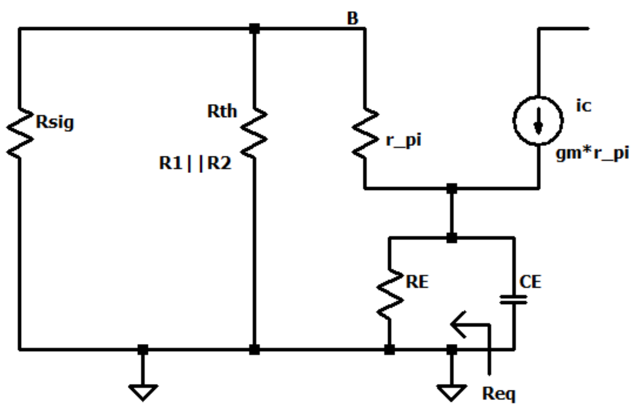

Figure 6: Small signal low frequency equivalent circuit for C_E

$$f_{LCE} = \frac{1}{2\pi R_{eq} C_E}$$

$$R_{eq} = R_E || \left(\frac{R_{sig}||R_{th} + r_\pi}{\beta} \right)$$

$$\therefore R_{eq} = 1.2k|| \left(\frac{0.82k||8.7179k + 3.655k}{120} \right) = 35.615\ k\Omega$$

$$\therefore f_{LCE} = \frac{1}{2\pi R - eqC_E} = \frac{1}{2\pi \times 35.615 \times 20 \times 10^{-6}}$$

$$\therefore f_{LCE} = 223.464 \text{ Hz}$$

Since $f_{LCE} = 223.464$ Hz is the largest among f_{LCC1} and f_{LCC2}, it is the lower cut-off frequency of the amplifier.
(Bypass capacitor C_E is determining the lower cut-off frequency of amplifier)

$\therefore f_L =$ **223.464 Hz**

$\mathbf{Av_{mid}}$ (Mid-band gain):

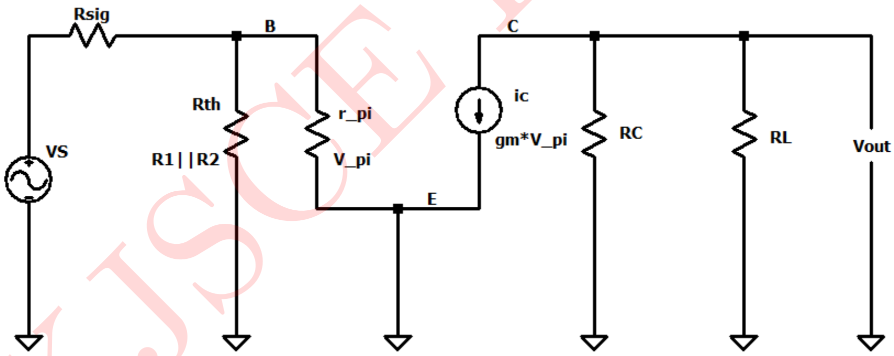

Figure 7: Mid frequency equivalent circuit

$Z_o = R_C||R_L = 2.0764\ k\Omega$

$$A_{V_{mid}} \text{ with } R_{sig} = A_{V_{Smid}} = \frac{V_o}{V_S} = \frac{V_o}{V_i} \times \frac{V_i}{V_S}$$

$$\frac{V_o}{V_i} = A_{V_{mid}} \text{ (without } R_{sig})$$

$$A_{V_{mid}} = \frac{V_o}{V_i} = -g_m \times (R_C||R_L) = -32.83077 \times 2.0764 \times 10^3 = -68.1699765$$

$$\frac{V_i}{V_S} = \frac{R_{th}||r_\pi}{R_{th}||r_\pi + R_{sig}} = \frac{2.5753k}{2.5753k + 0.82k}$$

$$\therefore \frac{V_i}{V_S} = 0.78549$$

$$\therefore A_{V_{Smid}} = 0.78549 \times (-68.1699) = -51.706133$$

$$|A_{V_{Smid}}| = 20log_{10}(-51.706133) = 34.27$$

SIMULATED RESULTS:
Above circuit is simulated using LTspice and the results are presented below:

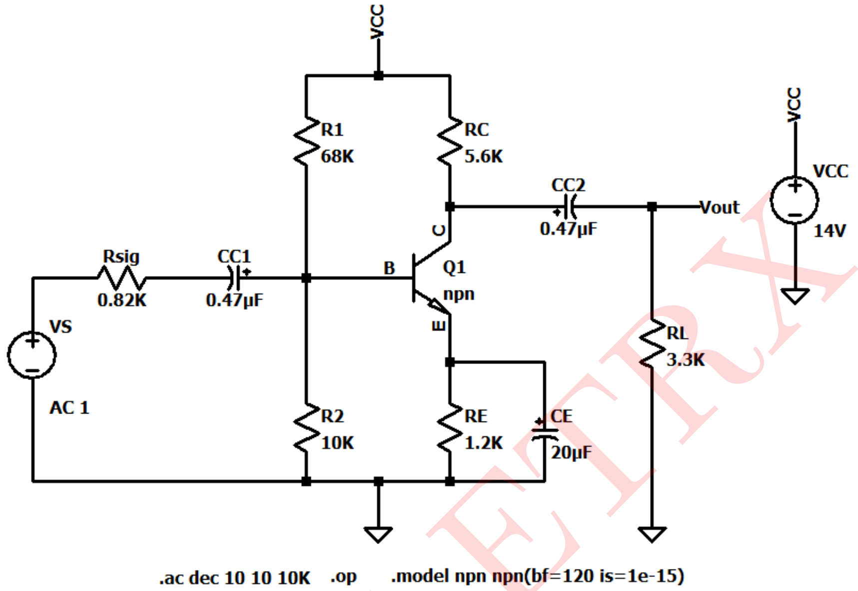

Figure 8: Circuit schematic

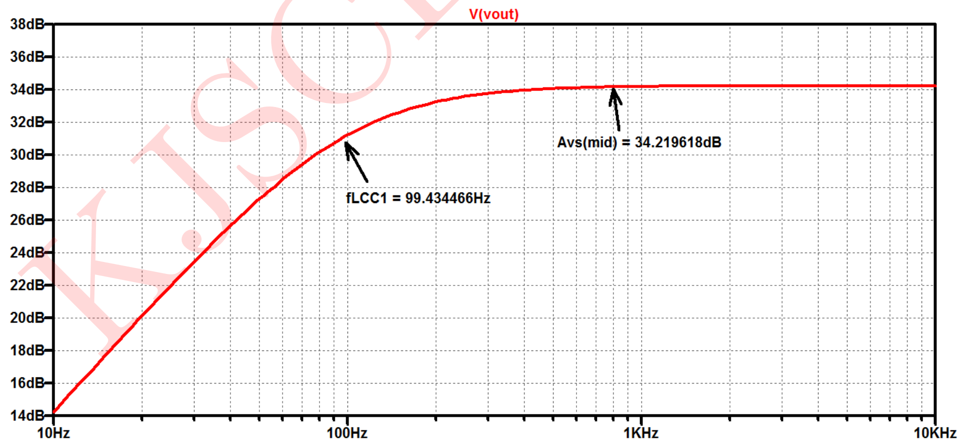

Figure 9: Low frequency response for C_{C1}

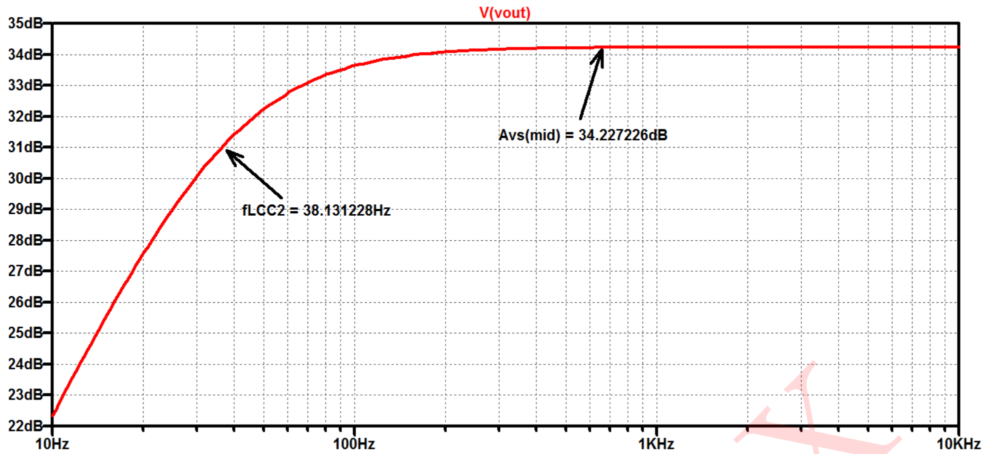

Figure 10: Low frequency response for C_{C2}

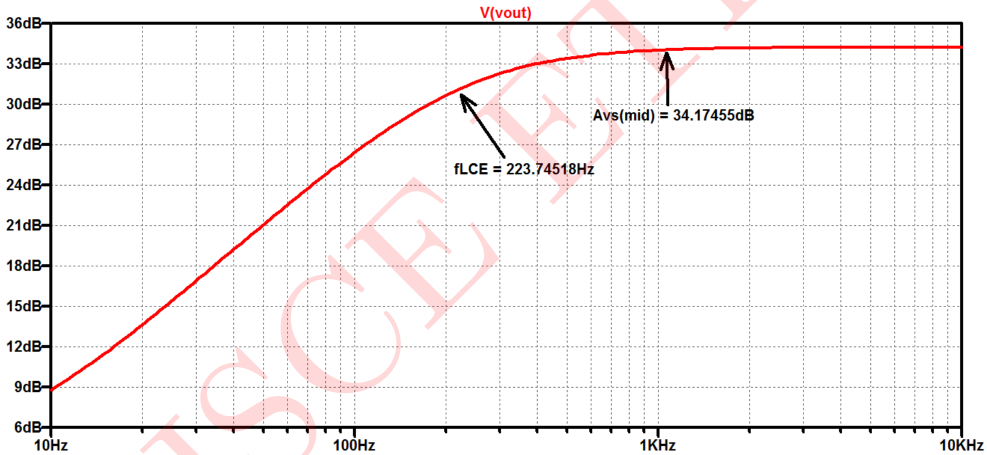

Figure 11: Low frequency response for C_E

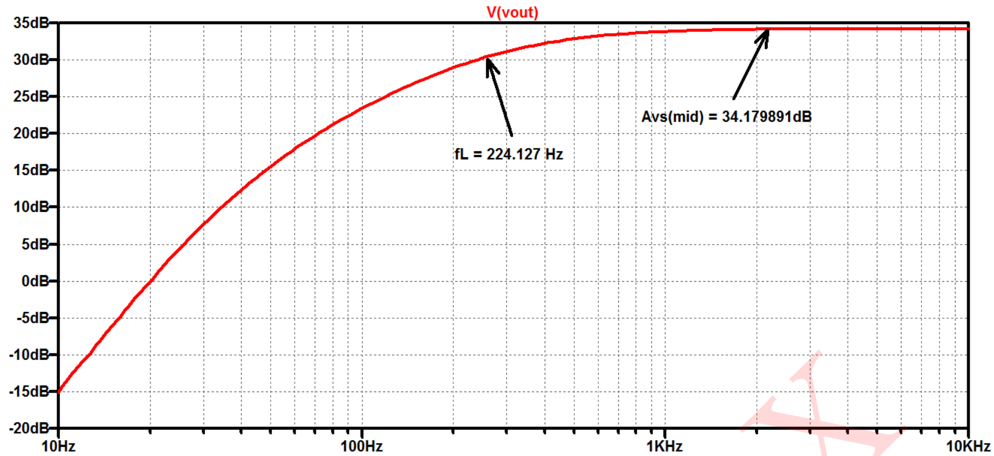

Figure 12: Low frequency response for the circuit

Comparison of theoretical and simulated values:

Parameters	Theoretical	Simulated
I_{CQ}	0.8536 mA	0.845559 mA
Lower cut-off frequency due to C_{C1}	99.737 Hz	99.43446 Hz
Lower cut-off frequency due to C_{C2}	38.048 Hz	38.13122 Hz
Lower cut-off frequency due to C_E	223.464 Hz	223.74518 Hz
Overall cut-off frequency (f_L)	223.464 Hz	224.127 Hz
Mid band voltage gain in dB	34.27 dB	34.1798 dB

Table 1: Numerical 1

2. For the circuit shown in Figure 13,
   a) Determine the mid-band voltage gain
   b) Find the Q-point
   c) Find the lower cut-off frequency of the circuit
   Given: $k_n = 1mA/V^2$, $V_{TN} = 2$V, $\lambda = 0$

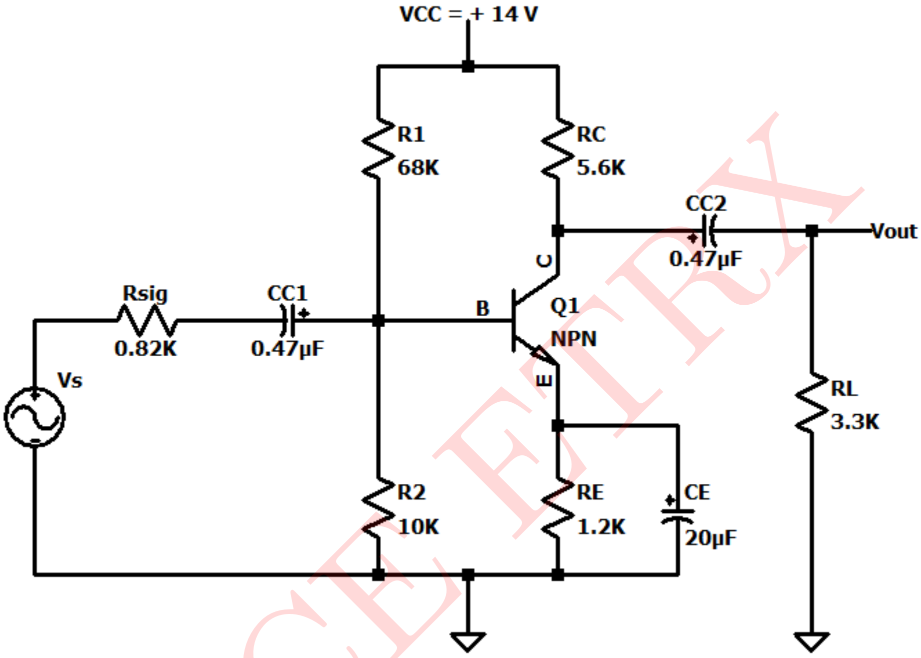

Figure 13: Circuit 2

Solution:

DC Analysis:

The capacitors act as open circuit. $f = 0, \therefore X_C = \dfrac{1}{2\pi f C} = \infty$

The DC equivalent circuit is shown in Figure 14

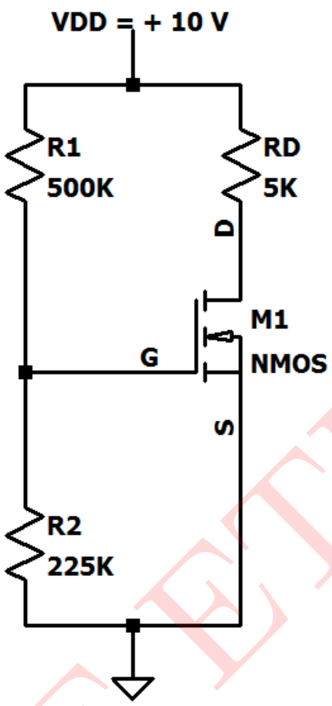

Figure 14: DC equivalent circuit

$$k_n = \frac{I_D}{[V_{GS} - V_{TN}]^2}$$

$$\therefore 1 \times 10^{-3} = \frac{I_D}{[V_{GS} - 2]^2}$$

$$\therefore I_D = 10^{-3} \times [V_{GS} - 2]^2 \qquad ....(1)$$

$$V_G = \frac{R_2}{R_1 + R_2} \times V_{DD} = \frac{225k}{500k + 225k} \times 10$$

$$\therefore V_G = 3.103448 \text{ V}$$

$$V_S = I_D R_S$$

$$R_S = 0, \therefore V_S = 0 \text{ V}$$

$$V_{GS} = V_G - V_S = V_G - 0 = 3.103448 \text{ V}$$

$$\therefore V_{GSQ} = 3.103448 \text{ V}$$

From equation (1), $I_D = 10^{-3} \times [3.103448 - 2]^2$

$$\therefore I_{DQ} = 1.217597mA$$

Small-signal parameters:

$r_d = \dfrac{1}{\lambda I_{DQ}}$

$\lambda = 0, \therefore r_d = \infty \Omega$

$g_m = 2k_n(V_{GSQ} - V_{TN}) = 2 \times 10^{-3} \times (3.103448 - 2)$

$\therefore g_m = 2.206896 \dfrac{mA}{V}$

The mid-frequency AC equivalent circuit is shown in Figure 15

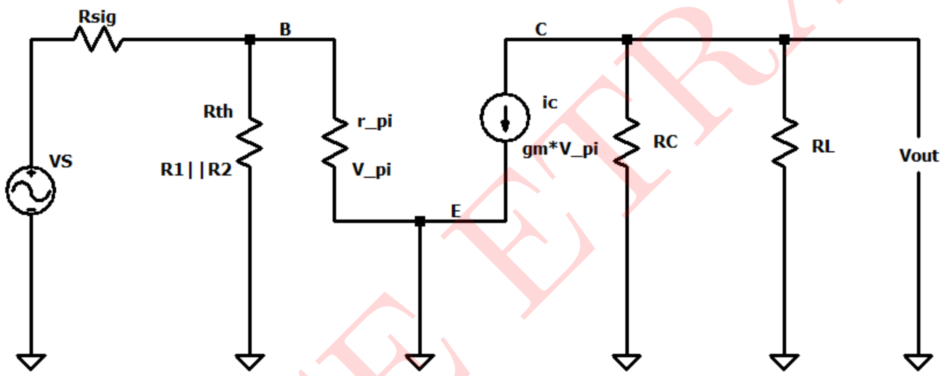

Figure 15: Mid-frequency equivalent circuit

$A_{V_{mid}} = \dfrac{V_o}{V_i} = \dfrac{-g_m V_{gs} R_D}{V_{gs}}$

$\therefore A_{V_{mid}} = -g_m R_D = -2.206896 \times 10^{-3} \times 5 \times 10^{-3}$

$\therefore \dfrac{V_o}{V_i} = -11.03445$

$A_{V_{Smid}} = \dfrac{V_o}{V_S} = \dfrac{V_o}{V_i} \times \dfrac{V_i}{V_S}$

$\dfrac{V_i}{V_S} = \dfrac{R_i}{R_{sig}}$

$R_i = R_1 || R_2 = 500k || 225k = 155.17 \text{ k}\Omega$

$\therefore \dfrac{V_i}{V_S} = \dfrac{155.17k}{155.17k + 1k} = 0.9935967$

$A_{V_{Smid}} = -11.03445 \times 0.9935967 = -10.96379311$

$|A_{V_{Smid}}|$ (in dB) $= 20log_{10}(10.96379) = 20.7921633dB$

The low frequency AC equivalent circuit is shown in Figure 16

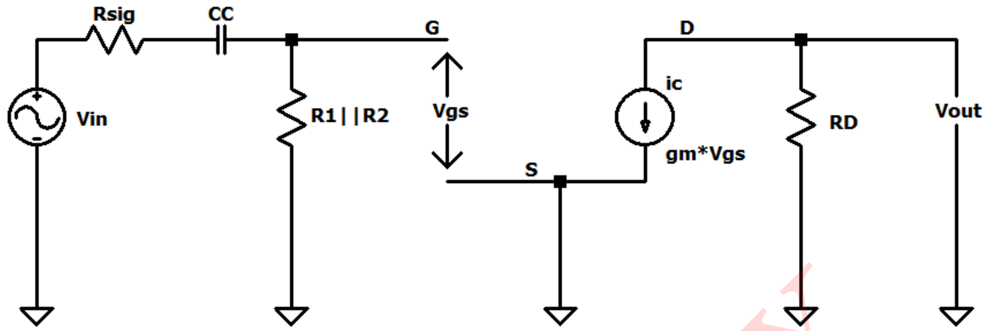

Figure 16: Low-frequency equivalent circuit

For f_{LCC}, we short V_{in}, hence the circuit reduces to Figure 17

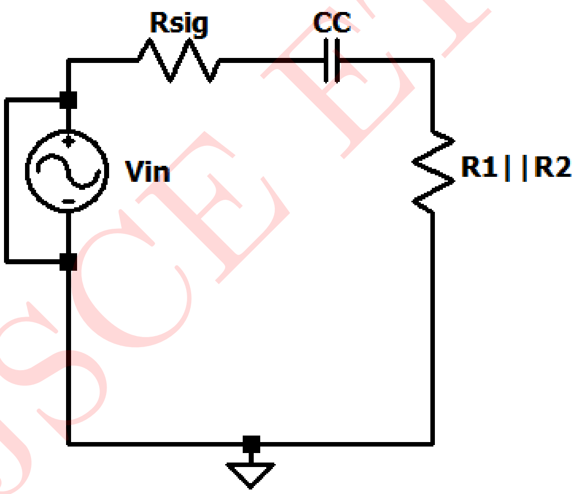

Figure 17: Low frequency equivalent circuit for C_C

$$f_{LCC} = \frac{1}{2\pi(R_{sig} + R_i)C_{C1}}$$

$$R_{sig} + R_i = 1k + 155.17k = 156.17 \text{ k}\Omega$$

$$\therefore f_{LCC} = \frac{1}{2\pi \times 156.17k \times 10 \times 10^{-6}}$$

$$\therefore f_{LCC} = 0.101911 \text{ Hz}$$

Here C_C is the only capacitor, hence f_{LCC} is considered to be the lower cut-off frequency.

$\therefore f_L =$ **0.101911 Hz**

SIMULATED RESULTS:
Above circuit is simulated using LTspice and the results are presented below:

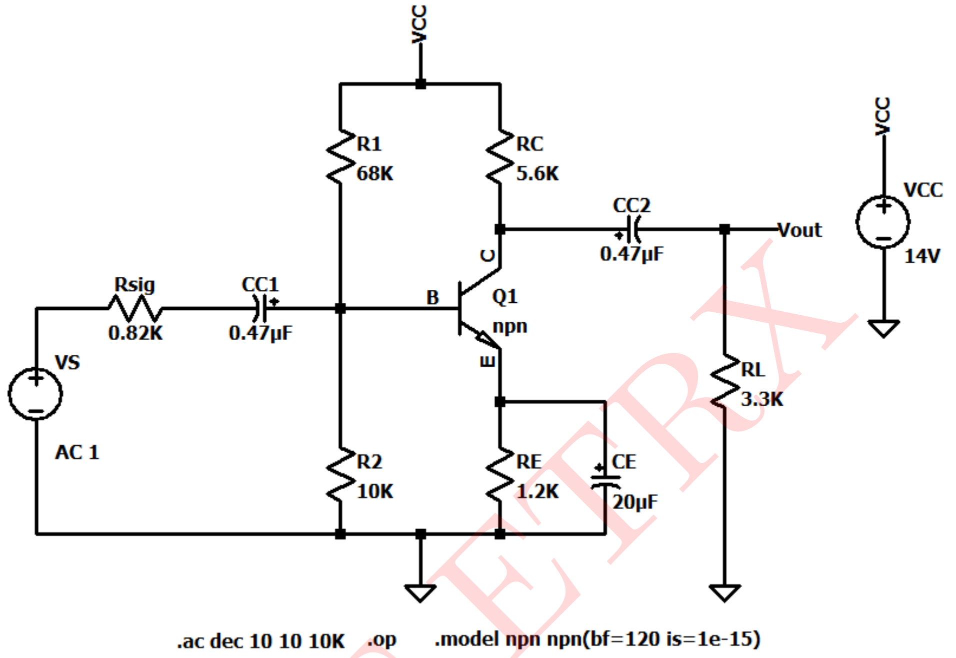

Figure 18: Circuit schematic

The frequency response for the circuit is shown in Figure 19

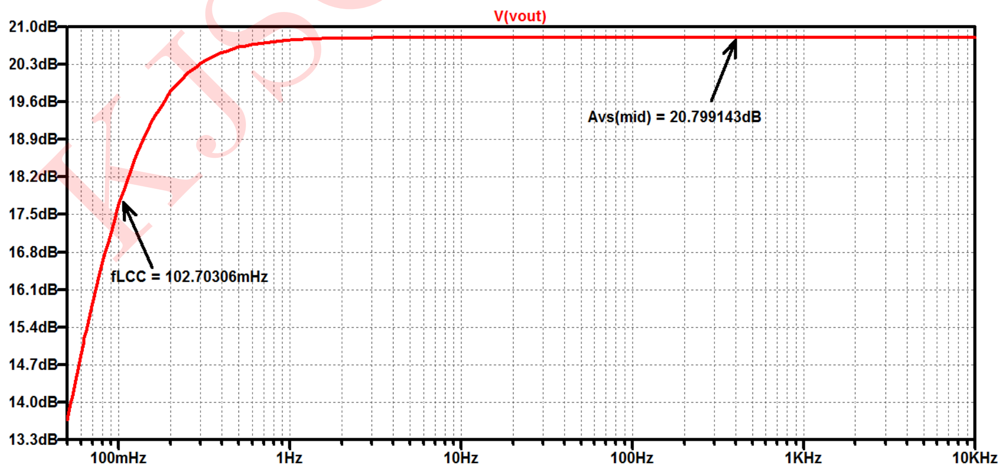

Figure 19: Low frequency response for the circuit

Comparison of theoretical and simulated values:

Parameters	**Theoretical**	**Simulated**
I_{DQ}	1.217597 mA	1.2176 mA
V_{GSQ}	3.103448 V	3.10345 V
Lower cut-off frequency f_L	0.1019911 Hz	0.10270306 Hz
Mid band voltage gain in dB	20.79921663 dB	20.799143 dB

Table 2: Numerical 2

3. For the circuit shown in Figure 20, determine
   a) r_π
   b) $A_{V_{mid}}$
   c) Z_i
   d) A_{VSmid}
   e) f_{LCC1}, f_{LCC2}, f_{LCE}
   f) lower cut-off frequency
   g) higher cut-off frequency
   Given: $\beta = 120$, $V_T = 26$ mV, $C_{\omega o} = 8$ pF, $C_{\omega i} = 5$ pF, $C_{bc} = 12$ pF, $C_{be} = 40$ pF, $C_{ce} = 8$ pF

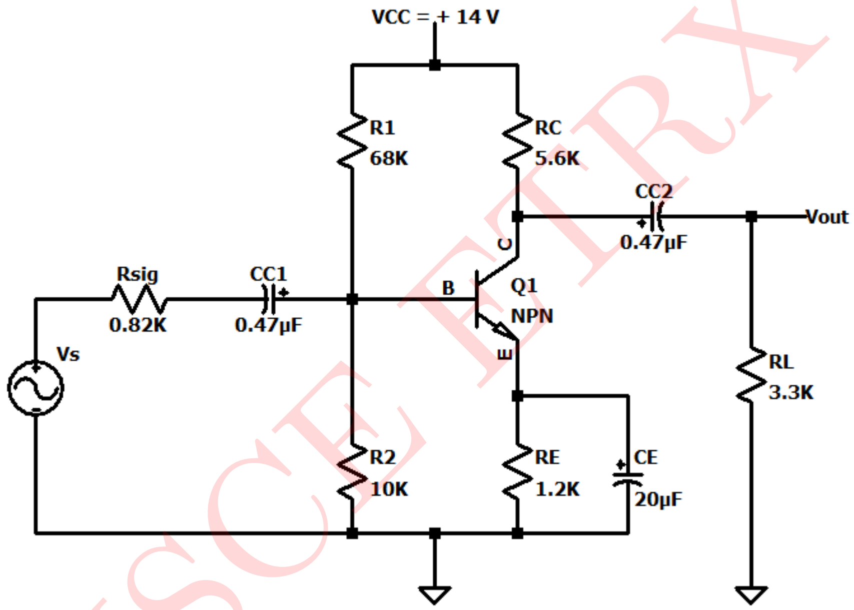

Figure 20: Circuit 3

Solution:

DC Analysis:

The capacitors act as open circuit. $f = 0, \therefore X_C = \dfrac{1}{2\pi f C} = \infty$

Applying Thevenin's equivalent at base,

$$V_{th} = \frac{R_2}{R_1 + R_2} \times V_{CC} = \frac{10k}{68k + 10k} \times 14$$

$\therefore V_{th} = 1.79487$ V

$R_{th} = R_1 || R_2 = 68k || 10k$

$\therefore R_{th} = 8.7179\ k\Omega$

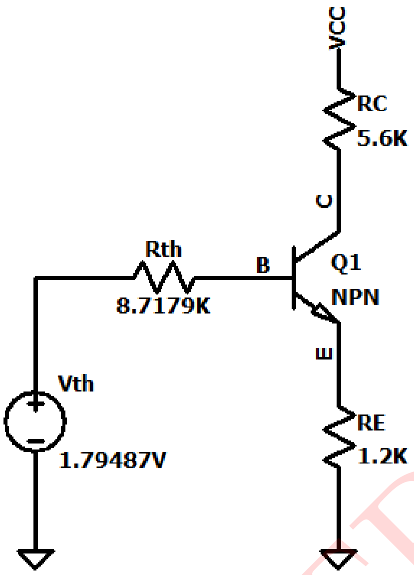

Figure 21: Thevenin's equivalent circuit

Applying KVL to input base-emitter loop

$$I_B = \frac{V_{th} - V_{BE}}{R_{th} + (1+\beta)R_E} = \frac{1.7948 - 0.7}{8.7179k + (121 \times 1.2k)}$$

$\therefore I_B = 7.11335\ \mu\text{A}$

$I_{CQ} = \beta I_B = 120 \times 7.11335 \times 10^{-6}$

$\therefore I_{CQ} = 0.8536\ \text{mA}$

Small-signal parameters

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{120 \times 26mV}{0.8536mA} = 3.655\ k\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.8536mA}{26mV} = 32.83077\frac{mA}{V}$$

$$r_o = \frac{V_A}{I_{CQ}}$$

Since V_A is not given, we assume r_o to be $\infty\Omega$

The low frequency equivalent circuit is shown below in Figure 22.

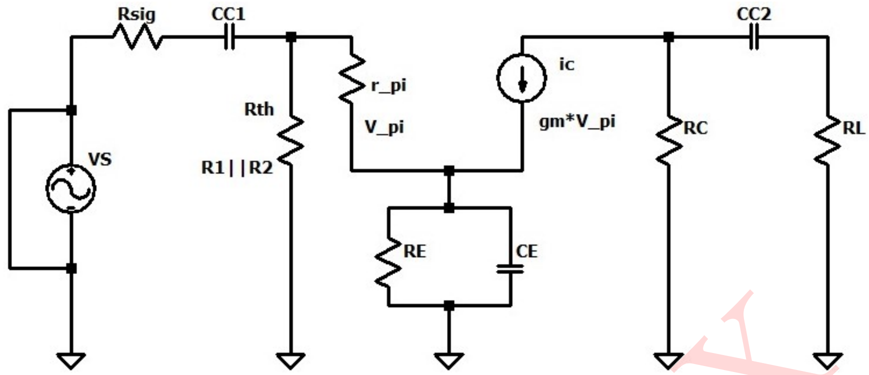

Figure 22: Small signal low frequency equivalent circuit

For $\mathbf{f_{LCC1}}$:

As C_{C2}, C_E and V_S are short, Figure 3 reduces to Figure 23

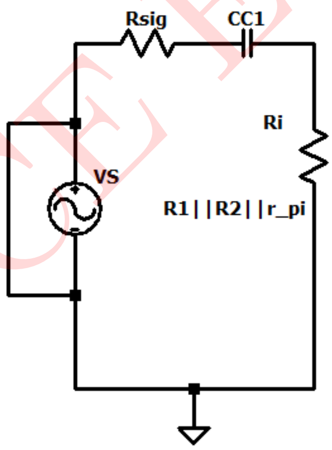

Figure 23: Small signal low frequency equivalent circuit for C_{C1}

$$f_{LCC1} = \frac{1}{2\pi(R_{sig} + R_i) \times C_{C1}}$$

$R_{eq} = R_{sig} + R_i$

$R_i = R_1||R_2||r_\pi$

$\therefore R_i = 2.5753k\Omega$

$C_{C1} = 0.47\mu F$, $R_{sig} = 0.8k\Omega$

$$f_{LCC1} = \frac{1}{2\pi(0.8k + 2.575k)(0.47\mu F)}$$

$\therefore f_{LCC1} = 99.737$ Hz

For $\mathbf{f_{LCC2}}$:

The capacitors C_{C1} and C_E, and V_S are short, hence Figure 22 reduces to Figure 24

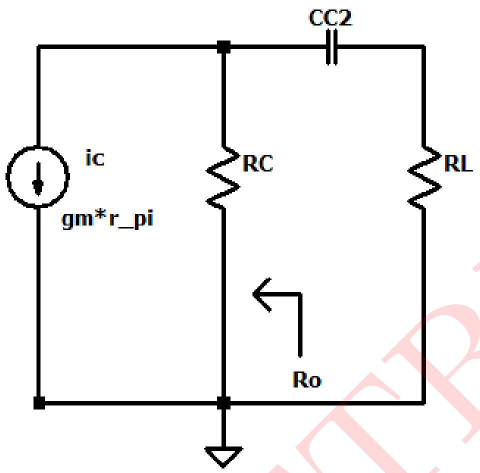

Figure 24: Small signal low frequency equivalent circuit for C_{C2}

$$f_{LCC2} = \frac{1}{2\pi(R_O + R_L)C_{C2}}$$

$R_O = R_C = 5.6k\Omega$

$C_{C2} = 0.47\mu\text{F}$, $R_L = 3.3k\Omega$

$$\therefore f_{LCC2} = \frac{1}{2\pi(5.6k + 3.3k) \times 0.47\mu F}$$

$\therefore f_{LCC2} = 38.048$ Hz

For $\mathbf{f_{LCE}}$:

C_{C1}, C_E and V_S are short, hence Circuit 22 reduces to Circuit 25

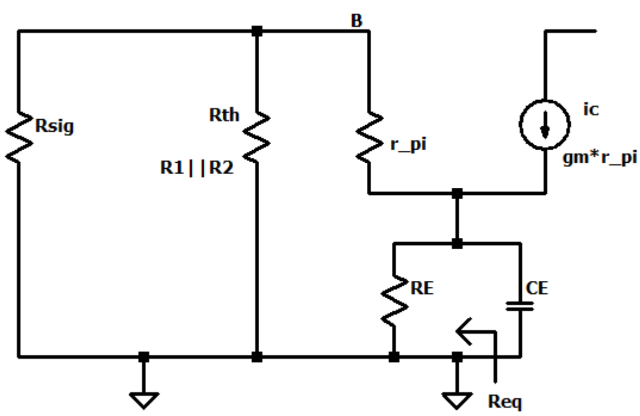

Figure 25: Small signal low frequency equivalent circuit for C_E

$$f_{LCE} = \frac{1}{2\pi R_{eq} C_E}$$

$$R_{eq} = R_E || \left(\frac{R_{sig}||R_{th} + r_\pi}{\beta}\right)$$

$$\therefore R_{eq} = 1.2k|| \left(\frac{0.82k||8.7179k + 3.655k}{120}\right) = 35.615\ k\Omega$$

$$\therefore f_{LCE} = \frac{1}{2\pi R_{eq} C_E} = \frac{1}{2\pi \times 35.615 \times 20 \times 10^{-6}}$$

$$\therefore f_{LCE} = 223.464 \text{ Hz}$$

Since $f_{LCE} = 223.464$ Hz is the largest among f_{LCC1} and f_{LCC2}, it is the lower cut-off frequency of the amplifier.
(Bypass capacitor C_E is determining the lower cut-off frequency of amplifier)

$\therefore f_L =$ **223.464 Hz**

$A_{V_{mid}}$ (Mid-band gain):

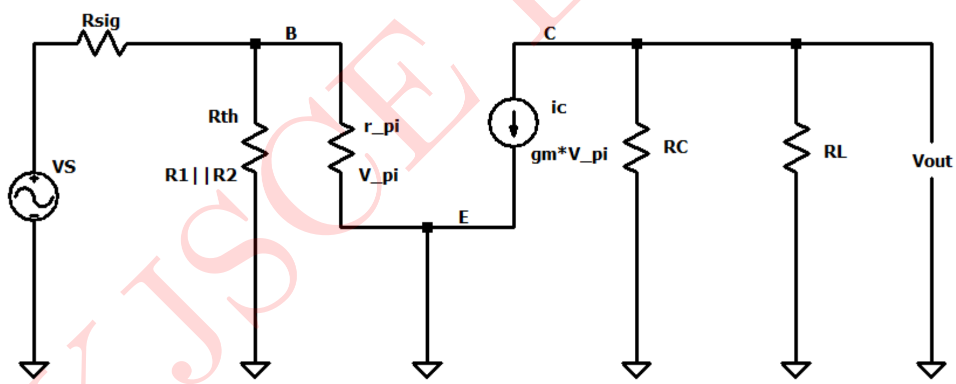

Figure 26: Mid frequency equivalent circuit

$$Z_o = R_C||R_L = 2.0764\ k\Omega$$

$$A_{V_{mid}} \text{ with } R_{sig} = A_{V_{Smid}} = \frac{V_o}{V_S} = \frac{V_o}{V_i} \times \frac{V_i}{V_S}$$

$$\frac{V_o}{V_i} = A_{V_{mid}} \text{ (without } R_{sig})$$

$$A_{V_{mid}} = \frac{V_o}{V_i} = -g_m \times (R_C||R_L) = -32.83077 \times 2.0764 \times 10^3$$

$$\therefore A_{V_{mid}} = -68.1699765$$

$$\frac{V_i}{V_S} = \frac{R_{th}||r_\pi}{R_{th}||r_\pi + R_{sig}} = \frac{2.5753k}{2.5753k + 0.82k}$$

$$\therefore \frac{V_i}{V_S} = 0.78549$$

$$\therefore A_{V_{Smid}} = 0.78549 \times (-68.1699) = -51.706133$$

$|A_{V_{Smid}}| = 20log_{10}(-51.706133) = 34.27$ dB

The high frequency equivalent circuit is shown below in Figure 27.

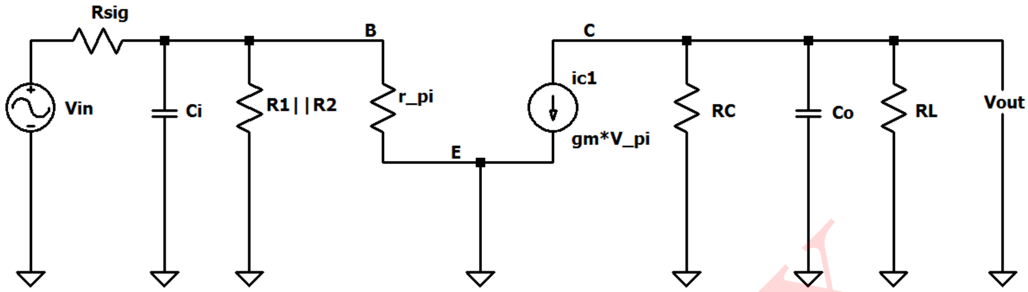

Figure 27: Small signal high frequency equivalent circuit

$C_i = C_{\omega i} + C_{mi} + C_{be}$

$C_{mi} = C_{bc} \times (1 - A_{V_{Smid}}) = 12 \times 10^{-12} \times [1 - (-51.705133)]$

$\therefore C_{mi} = 632.473596$ pF

$C_i = 5$ pF $+ 632.473596$ pF $+ 40$ pF

$\therefore C_i = 677.473596$ pF

$C_o = C_{\omega o} + C_{mo} + C_{CB} + C_{CE}$

$$C_{mo} = C_{bc} \times \left(1 - \frac{1}{A_{V_{Smid}}}\right)$$

$$\therefore C_{mo} = 12 \times 10^{-12} \times \left(1 - \frac{1}{-51.706133}\right) = 12.232 \text{ pF}$$

$C_o = 8$ pF $+ 12.232$ pF $+ 0$ pF $+ 8$ pF $= 28.232$ pF

The circuit for f_{Hi} is shown in Figure 28

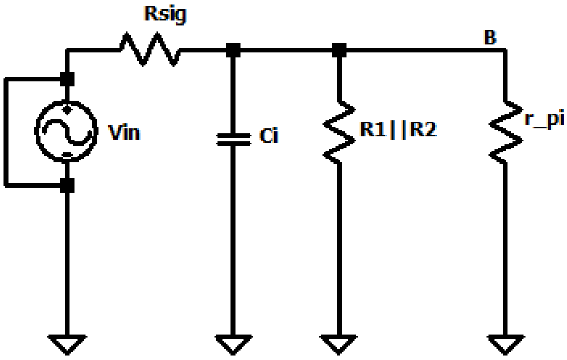

Figure 28: Small signal high frequency equivalent circuit for C_i

$f_{Hi} = \dfrac{1}{2\pi R_{eq} C_i}$

$R_{eq} = R_{sig}||R_{th}||r_{\pi} = 0.82k||8.7179k||3.655k$

$\therefore R_{eq} = 622.66386$ Hz

$\therefore f_{Hi} = \dfrac{1}{2\pi \times 622.66836\ times677.473596 \times 10^{-12}}$

$\therefore f_{Hi} = 377.2862348$ kHz

For f_{Ho}, we short V_S and C_i, hence Figure 8 reduces to figure 29

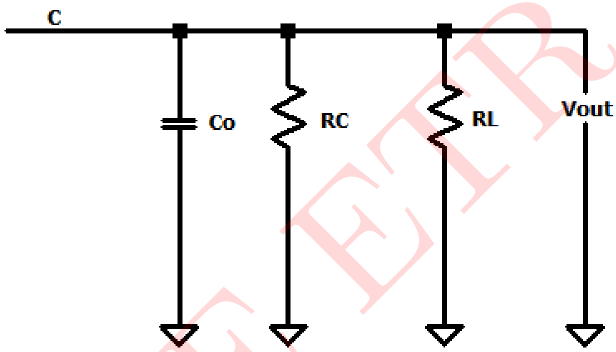

Figure 29: Small signal high frequency equivalent circuit for C_o

$f_{Ho} = \dfrac{1}{2\pi R_{eq} C_o}$

$R_{eq} = R_C||R_L = 5.6k||3.3k = 2.0764$ Hz

$\therefore f_{Ho} = \dfrac{1}{2\pi \times 2.0764 \times 28.232 \times 10^{-12}}$

$\therefore f_{Ho} = 2.71498$ MHz

We select the lowest of f_{Ho} and f_{Hi} as the higher cut-off frequency of BJT amplifier

$\therefore f_H = f_{Hi} = 377.286$ kHz

SIMULATED RESULTS:
Above circuit is simulated using LTspice and the results are presented below:

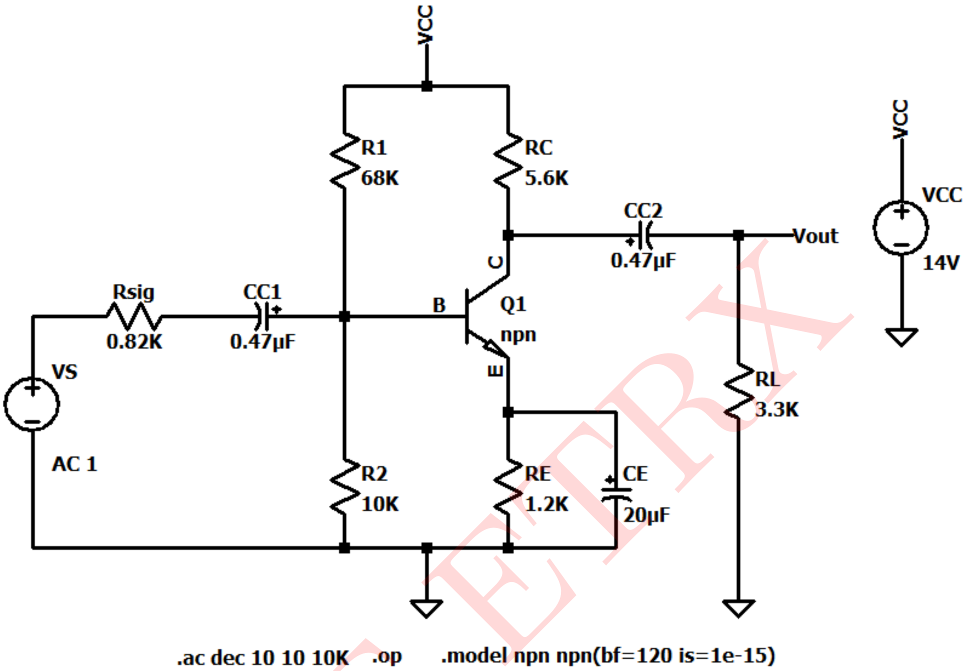

Figure 30: Circuit schematic

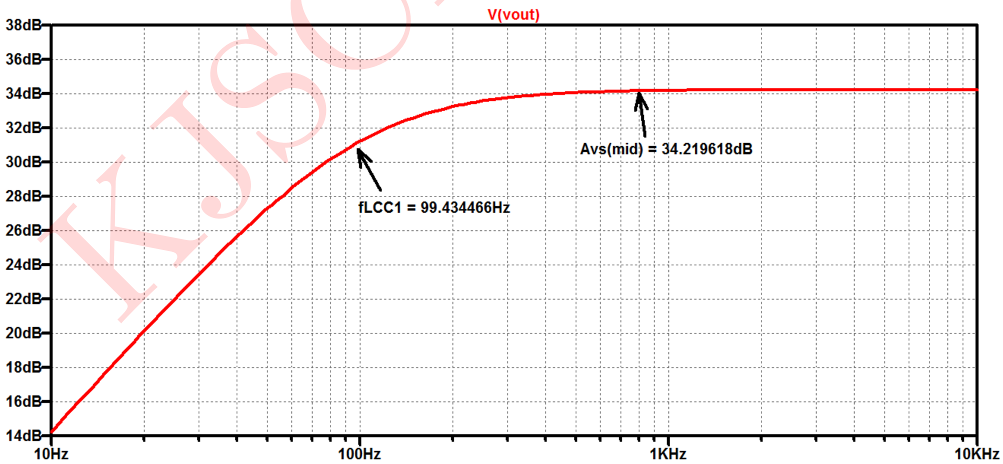

Figure 31: Low frequency response for C_{C1}

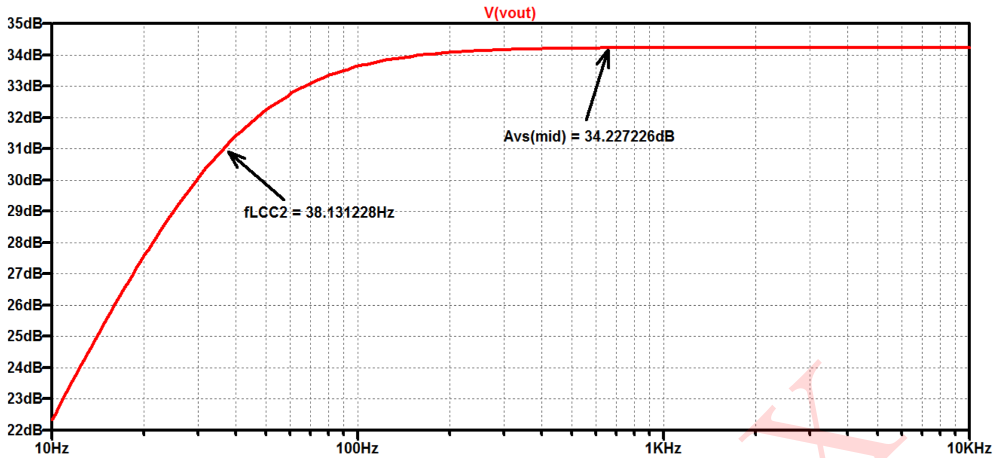

Figure 32: Low frequency response for C_{C2}

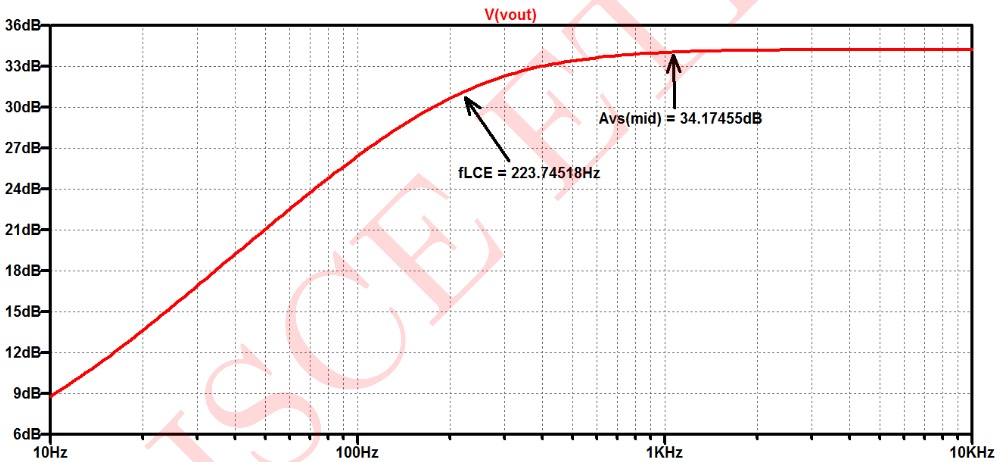

Figure 33: Low frequency response for C_E

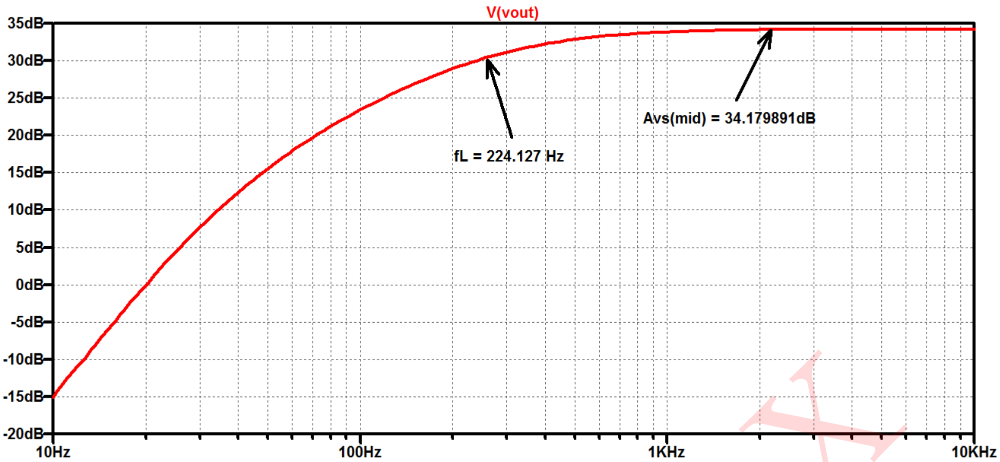

Figure 34: Low frequency response for the circuit

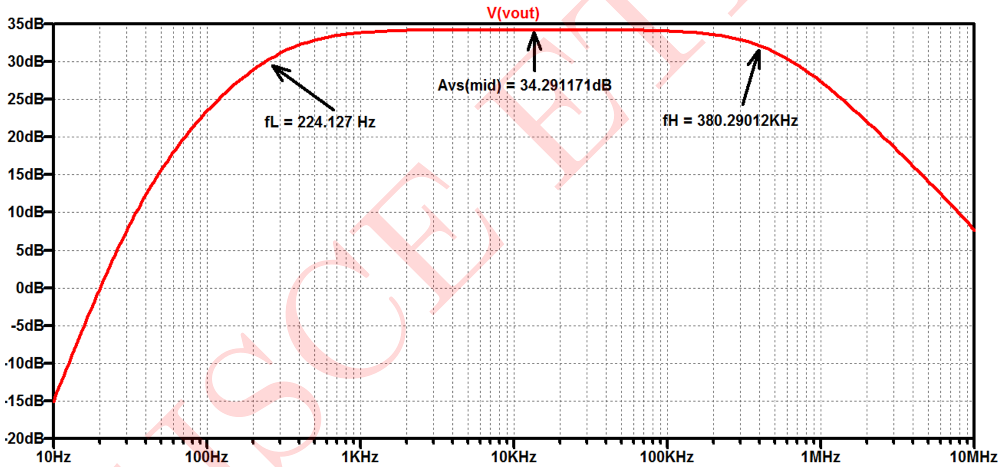

Figure 35: High frequency response for the circuit

Comparison of theoretical and simulated values:

Parameters	**Theoretical**	**Simulated**
I_{CQ}	0.8536 mA	0.845559 mA
V_{CEQ}	8.1869 V	8.24185 V
Lower cut-off frequency due to C_{C1}	99.737 Hz	99.43446 Hz
Lower cut-off frequency due to C_{C2}	38.048 Hz	38.13122 Hz
Lower cut-off frequency due to C_E	223.464 Hz	223.74518 Hz
Overall cut-off frequency f_L	223.464 Hz	224.127 Hz
Overall cut-off frequency f_H	377.286 kHz	380.2901 kHz
Mid band voltage gain in dB	34.27 dB	34.1798 dB

Table 3: Numerical 3