K. J. SOMAIYA COLLEGE OF ENGINEERING DEPARTMENT OF ELECTRONICS ENGINEERING ELECTRONIC CIRCUITS DC CIRCUITS

Numerical 1: For the circuit shown in Figure 1,

- a) Write the nodal equations and solve for the node voltages. Then, find the value of I_1 .
- b) Could this problem be solved in another (simpler) way?

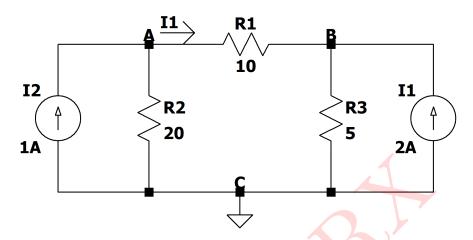


Figure 1: Circuit 1

Solution:

a) Since C is the reference node,

$$V_C = 0$$

Applying Kirchhoff's Current Law(KCL) and Ohm's Law at A:

$$\frac{V_A - V_B}{10} + \frac{V_A}{20} - 1 = 0$$

$$3V_A - 2V_B = 20$$
 ...(1)

Applying Kirchhoff's Current Law(KCL) and Ohm's Law at B:

$$\frac{V_B - V_A}{10} + \frac{V_B}{5} - 2 = 0$$

$$-V_A + 3V_B = 20 \qquad ...(2)$$

From (1) and (2) we get:

$$V_A = 14.2857 V$$

$$V_B = 11.42857 V$$

From Ohm's Law:

$$I_1 = \frac{V_A - V_B}{10}$$

$$I_1 = \mathbf{0.285713A}$$

SIMULATED RESULTS:

The following circuit has been simulated in LTspice and the readings obtained are as follows:

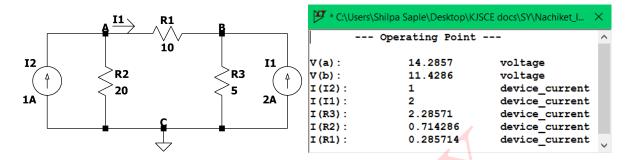


Figure 2: Circuit Schematic and Simulated Results

Comparison of theoretical and simulated values:

Quantity	Calculated Value	Simulated Value
V_A	14.2857V	14.286V
V_B	11.42857V	11.429V
I_1	0.285713A	0.28571A

Table 1: Comparison of calculated and simulated results

b) Yes, the problem can be solved using Source Transformation in a simpler way.

Numerical 2: By applying nodal analysis to the below circuit in Figure 3, find I_{ab} , I_{bd} and I_{bc} . All resistance values are in ohms.

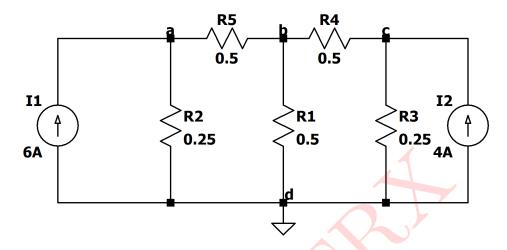


Figure 3: Circuit 2

Solution:

Since d is the reference node,

$$V_d = 0$$

Applying Kirchoff's Current Law(KCL) and Ohm's Law at a:

$$\frac{V_a - V_b}{0.5} + \frac{V_a}{0.25} - 6 = 0$$

$$6V_a - 2V_b = 6$$
(1)

Applying Kirchoff's Current Law(KCL) and Ohm's Law at b:

$$\frac{V_b - V_a}{0.5} + \frac{V_b - V_c}{0.5} + \frac{V_b}{0.5} = 0$$
$$-V_a + 3V_b - V_c = 0 \qquad(2)$$

Applying Kirchoff's Current Law(KCL) and Ohm's Law at c:

$$\frac{V_c - V_b}{0.5} + \frac{V_c}{0.25} - 4 = 0$$

$$-2V_b + 6V_c = 4 \qquad \dots(3)$$

From (1), (2) and (3) we get:

 $V_a = 1.23809 V$

 $V_b = \mathbf{0.71428V}$

 $V_c = \mathbf{0.90476V}$

From Ohm's Law:

$$I_{ab} = \frac{V_a - V_b}{0.5}$$

$$I_{bd} = \frac{V_b - V_d}{0.5}$$

$$I_{bc} = \frac{V_b - V_c}{0.5}$$

 $I_{ab}=\mathbf{1.04762A}$

 $I_{bd} = 1.42857 A$

 $I_{bc} = -0.38095 A$

SIMULATED RESULTS:

The following circuit has been simulated in LTspice and the readings obtained are as follows:

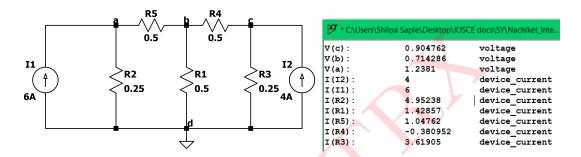


Figure 4: Circuit Schematic and Simulated Results

Quantity	Calculated Value	Simulated Value
I_{ab}	1.04762A	1.04762A
I_{bd}	1.42857A	1.42857A
I_{bc}	-0.38095A	-0.380952A

Table 2: Comparison of calculated and simulated results

Numerical 3: Using Nodal voltage method, compute the power dissipated in the 9Ω resistor of the circuit given in Figure 5.

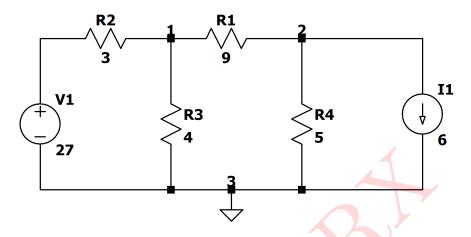


Figure 5: Circuit 3

Solution:

Since 3 is the reference node,

$$V_3 = 0$$

Applying Kirchoff's Current Law(KCL) and Ohm's Law at 1:

$$\frac{V_1 - 27}{3} + \frac{V_1}{4} - \frac{V_1 - V_2}{9} = 0$$

$$\frac{25V_1}{36} - \frac{V_2}{9} = 9 \qquad \dots (1)$$

Applying Kirchoff's Current Law(KCL) and Ohm's Law at 2:

$$\frac{V_2 - V_1}{9} + \frac{V_2}{5} + 6 = 0$$

$$\frac{-V_1}{9} + \frac{14V_2}{45} = -6 \qquad \dots (2)$$

From (1) and (2) we get:

$$V_1 = 10.47273V$$

$$V_2 = -15.54545V$$

From Ohm's Law, Current through 9Ω resistor:

$$I_{9\Omega} = \frac{V_1 - V_2}{9}$$

$$I_{9\Omega} = 2.89091A$$

Power dissipated in a resistor:

$$P = VI = I^2R$$

Power dissipated in 9Ω resistor:

$$P_{9\Omega} = 9 \times I_{9\Omega}^2 = 75.21621W$$

SIMULATED RESULTS:

The following circuit has been simulated in LTspice and the readings obtained are as follows:

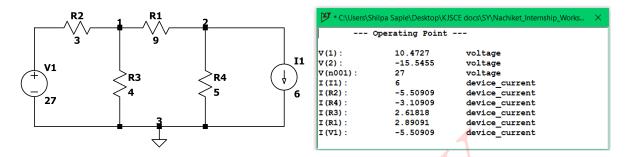


Figure 6: Circuit Schematic and Simulated Results

From the simulated results, $V_{12} = V_1 - V_2 = 10.4727 + 15.5455 = 26.0182V$

Quantity	Calculated Value	Simulated Value
V_{12}	26.0182V	26.0182V
$I_{9\Omega}$	2.89091A	2.89091A
$P_{9\Omega}$	75.21621W	75.21627W

Table 3: Comparison of calculated and simulated results

Numerical 4: For the circuit shown in Figure 7, find the equivalent resistance. All resistors are of 60Ω .

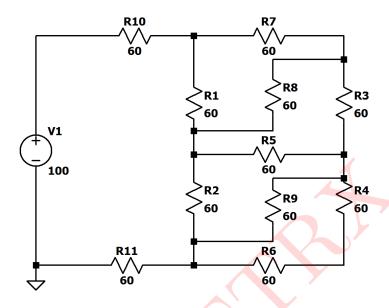


Figure 7: Circuit 4

Solution:

Converting Delta configuration of resistor to Star for Resistors R_1, R_7 and R_8 .

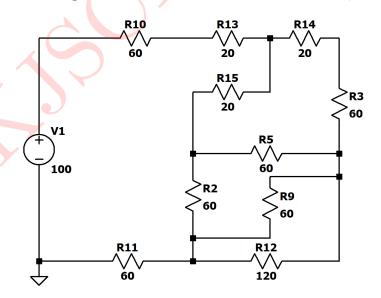


Figure 8: Modified circuit after delta to star conversion

Adding resistances R_{10} and R_{13} , R_{14} and R_3 in series.

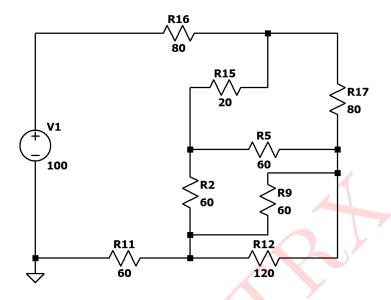


Figure 9: Circuit after adding the series resistances

Applying Delta to Star Conversion on R_5 , R_{15} and R_{17} .

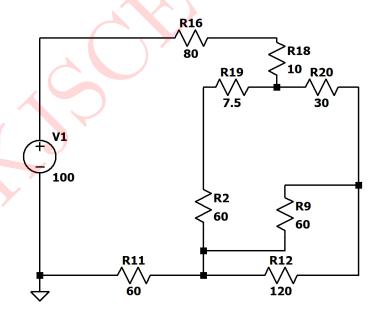


Figure 10: Modified circuit after delta to star conversion

Adding resistances R_{16} and R_{18} , R_{2} and R_{19} in series, R_{9} and R_{12} in parallel.

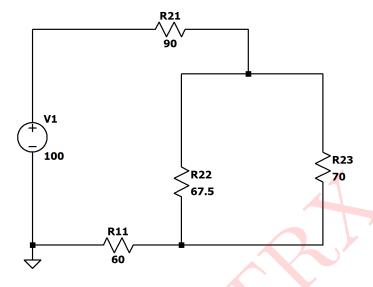


Figure 11: Modified circuit after series and parallel combination

Finally, after solving the last series and parallel resistances from Figure 9, we get:

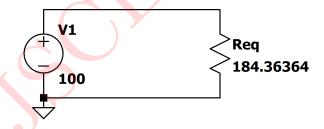


Figure 12: Equivalent Resistance

Hence, Equivalent Resistance $R_{eq}=184.36364\Omega$

SIMULATED RESULTS:

The following circuit has been simulated in LTspice and the readings obtained are as follows:

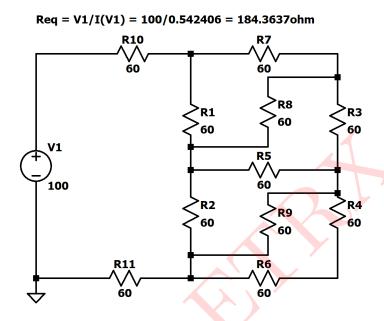


Figure 13: Circuit Schematic and Simulated Results

Quantity	Calculated Value	Simulated Value
R_{eq}	184.3636Ω	184.3637Ω

Table 4: Comparison of calculated and simulated results

Numerical 5: Find the current in the 11Ω resistor in Figure 14 by using star delta conversion. All resistances are in ohm.

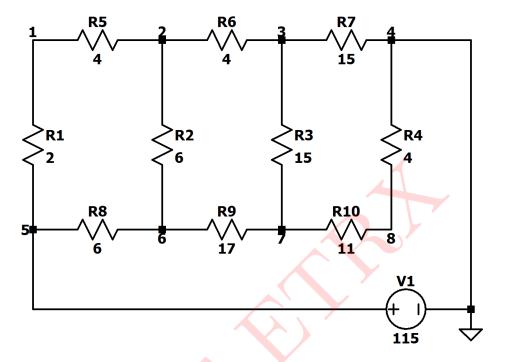


Figure 14: Circuit 5

Solution:

Adding resistances R_1 and R_5

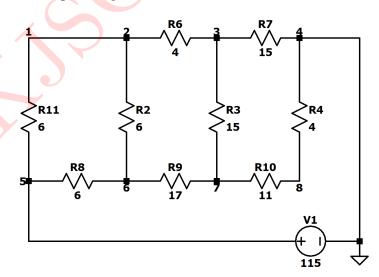


Figure 15: Modified circuit after series addition of resistances

Delta to Star conversion of resistors: R_2 , R_8 and R_{11}

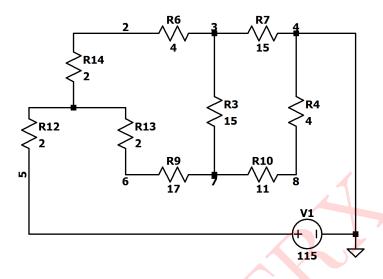


Figure 16: Modified circuit after Delta to Star conversion

Adding series combination of resistances: R_6 and R_{14} , R_{13} and R_9

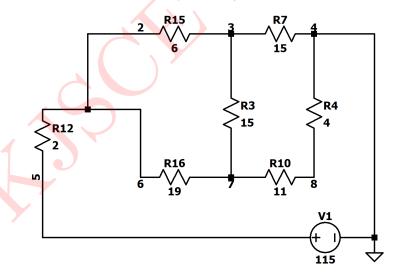


Figure 17: Modified circuit after series addition of resistances

Delta to Star conversion of resistors: R_{15} , R_{16} and R_{3}

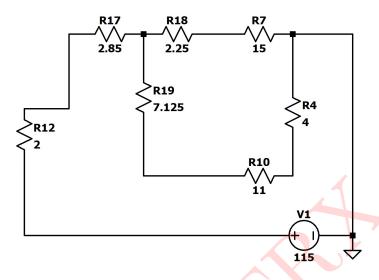


Figure 18: Modified circuit after Delta to Star conversion

Adding series combination of resistances: R_{12} and R_{17} , R_{18} and R_{7} , R_{4} and R_{19}

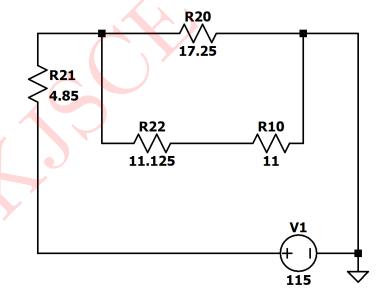


Figure 19: Modified circuit after series addition of resistances

Now, equivalent resistance of the parallel combination is:

$$\frac{17.25 \times (11.125 + 11)}{17.25 + 11.125 + 11} = 9.69286\Omega$$

Total current supplied by the voltage source = I:

$$I = \frac{115}{4.85 + 9.69286} = 7.90766A$$

Finally, current through 11Ω branch is:

$$I_{11\Omega} = \frac{17.25}{39.375} \times I = \frac{17.25}{39.375} \times 7.90766$$

$$I_{11\Omega} = 3.46431A$$

SIMULATED RESULTS:

The following circuit has been simulated in LTspice and the readings obtained are as follows:

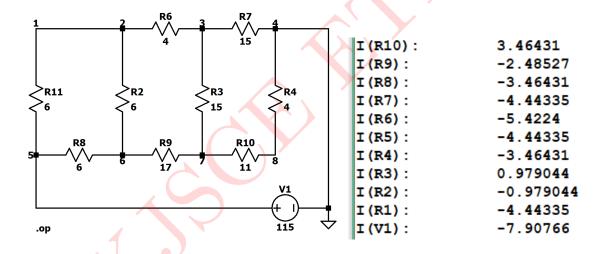


Figure 20: Circuit Schematic and Simulated Results

Quantity	Calculated Value	Simulated Value
I_{V_1}	7.90766A	7.90766A
$I_{11\Omega}$	3.46431A	3.46431A

Table 5: Comparison of calculated and simulated results

Numerical 6: Determine the voltages at the nodes V_1 , V_2 and V_3 given in Figure 21.

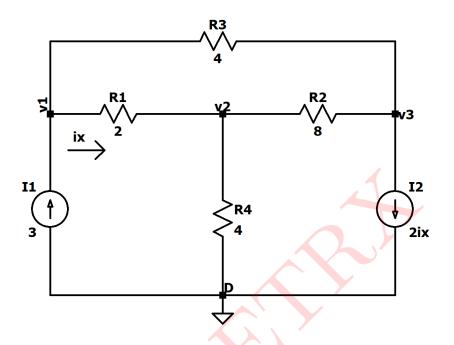


Figure 21: Circuit 6

Solution:

From Ohm's Law:

$$I_x = \frac{V_1 - V_2}{2} \qquad(1)$$

Applying Kirchoff's Current Law(KCL) and Ohm's Law at 1:

$$\frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{4} - 3 = 0$$

$$3V_1 - 2V_2 - V_3 = 12$$
(2)

Applying Kirchoff's Current Law(KCL) and Ohm's Law at 2:

$$\frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{8} + \frac{V_2}{4} = 0$$

$$-4V_1 + 7V_2 - V_3 = 0 \qquad \dots (3)$$

Applying Kirchoff's Current Law(KCL) and Ohm's Law at node 3:

$$\frac{V_3 - V_1}{4} + \frac{V_3 - V_2}{8} + 2I_x = 0$$

Substituting the expression for I_x from (1), we get

From (2), (3) and (4) we get:

 $V_1 = 4.8 \mathbf{V}$

 $V_2 = \mathbf{2.4V}$

 $V_3 = -2.4 \mathrm{V}$

SIMULATED RESULTS:

The following circuit has been simulated in LTspice and the readings obtained are as follows:

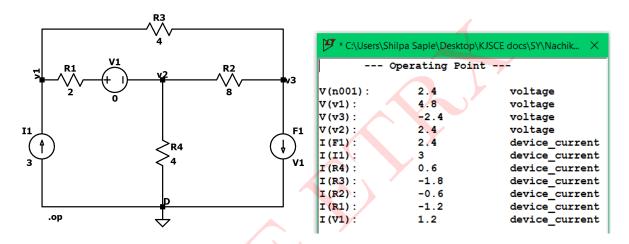


Figure 22: Circuit Schematic and Simulated Results

Quantity	Calculated Value	Simulated Value
V_1	4.8V	4.8V
V_2	2.4V	2.4V
V_3	-2.4V	-2.4V

Table 6: Comparison of calculated and simulated results

Numerical 7: Find the voltages at the three non-reference nodes 1, 2 and 3 in the circuit of Figure 23.

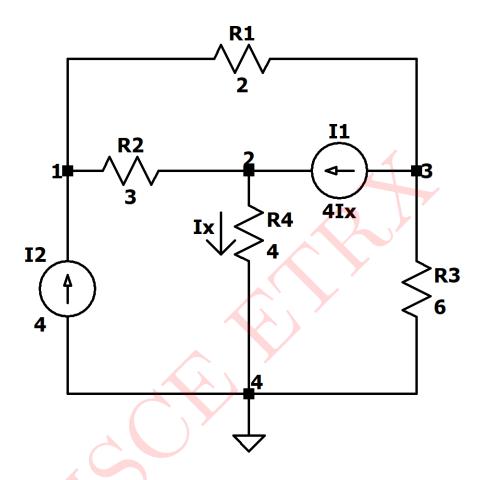


Figure 23: Circuit 7

Solution:

From Ohm's Law:

$$I_x = \frac{V_2}{4}$$

Applying Kirchoff's Current Law(KCL) at 1:

$$\frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{2} - 4 = 0$$

$$\frac{5V_1}{6} - \frac{V_2}{3} - \frac{V_3}{2} = 4$$
(1)

Applying Kirchoff's Current $\mathrm{Law}(KCL)$ at 2:

$$\frac{V_2 - V_1}{3} + \frac{V_2}{4} - 4I_x = 0$$

$$\frac{V_1}{3} + \frac{5V_2}{12} = 0$$
(2)

Applying Kirchoff's Current Law(KCL) at node 3:

$$\frac{V_3}{6} + \frac{V_3 - V_1}{2} + 4I_x = 0$$

$$\frac{-V_1}{2} + V_2 + \frac{2V_3}{3} = 0$$
(3)

From (2), (3) and (4) we get:

$$V_1 = \mathbf{32V}$$

$$V_2 = -\mathbf{25.6V}$$

$V_3 = 62.4$

SIMULATED RESULTS:

The following circuit has been simulated in LTspice and the readings obtained are as follows:

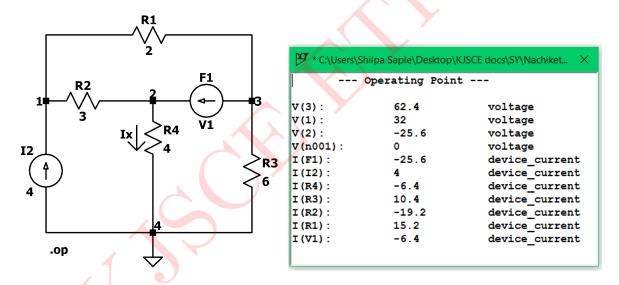


Figure 24: Circuit Schematic and Simulated Results

Quantity	Calculated Value	Simulated Value
V_1	32V	32V
V_2	-25.6V	-25.6V
V_3	62.4V	62.4V

Table 7: Comparison of calculated and simulated results

Numerical 8: A two-terminal network is shown in the Figure 25. It is a four bit ladder network used for digital-to-analog conversion. In terms of four (digital) voltages D_0 , D_1 , D_2 and D_3 and resistance R, express:

- 1. Thévenin (or equivalent) voltage
- 2. Thévenin (or equivalent) resistance

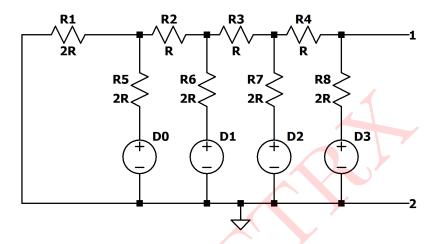


Figure 25: Circuit 8

Solution:

Applying Source Transformation on resistance R_5 and voltage source D_0

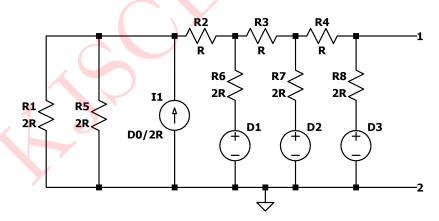


Figure 26: Modified circuit after source transformation

Adding the parallel combinations of resistances \mathbb{R}_1 and \mathbb{R}_5

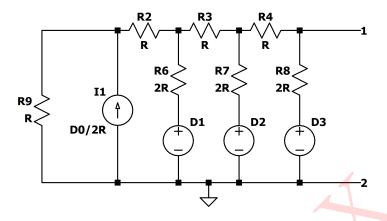


Figure 27: Modified circuit after addition of resistors

Applying source transformation on resistance R_9 and current source I_1

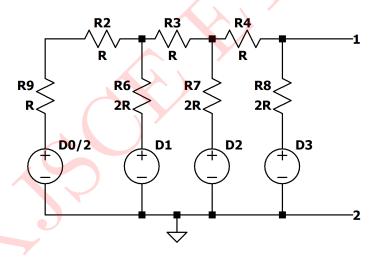


Figure 28: Modified circuit after source transformation

Adding series resistances: R_2 and R_9

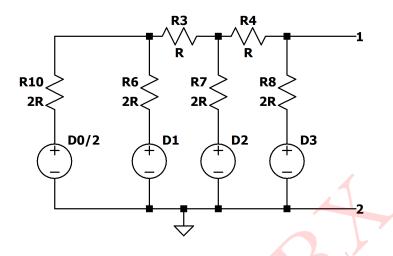


Figure 29: Modified circuit after series addition of resistances

Applying source transformation on: resistance R_{10} and voltage source $\frac{D_0}{2}$ and resistance R_6 and voltage source D_1

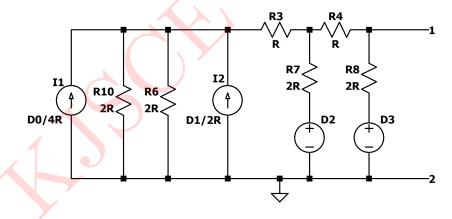


Figure 30: Modified circuit after source transformations

Adding parallel resistances: R_{10}, R_{6} and current sources: I_{1}, I_{2}

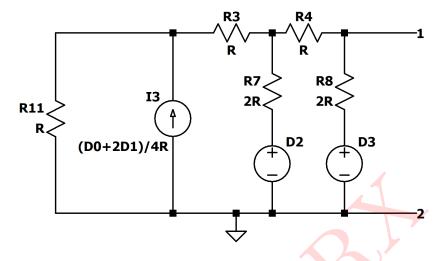


Figure 31: Modified circuit after combination of resistances and current sources

Applying source transformation on: resistance R_{11} and current source I_3

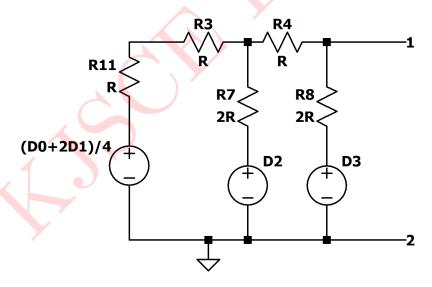


Figure 32: Modified circuit after source transformations

Adding series resistances: R_3 and R_{11}

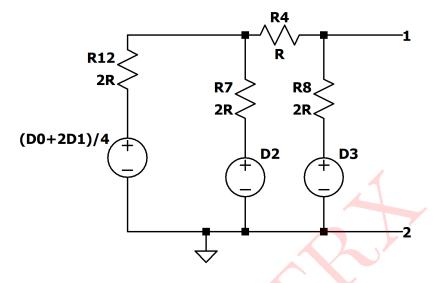


Figure 33: Modified circuit after series addition of resistances

Applying source transformation on: resistance R_{12} and voltage source $\frac{D_0+2D_1}{4}$ and resistance R_7 and voltage source D_2

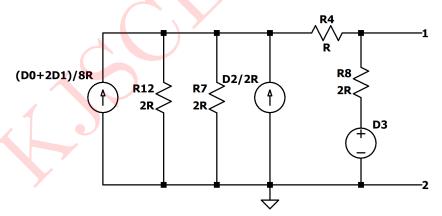


Figure 34: Modified circuit after source transformations

Adding parallel resistances: R_7 and R_{12} and current sources: $\frac{D_0+2D_1}{8R}$ and $\frac{D_2}{2R}$

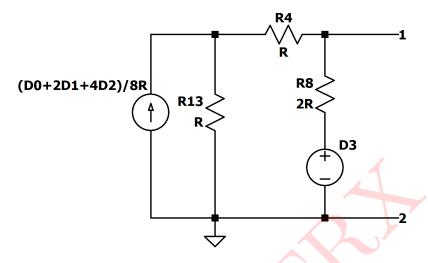


Figure 35: Modified circuit after combination of resistances and current sources

Applying source transformation on: R_{12} and current source $\frac{(D_0 + 2D_1 + 4D_2)}{8R}$

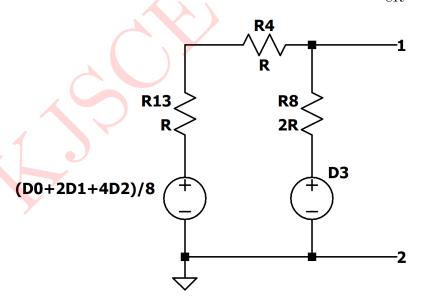


Figure 36: Modified circuit after source transformations

Adding series resistances: R_4 and R_{13}

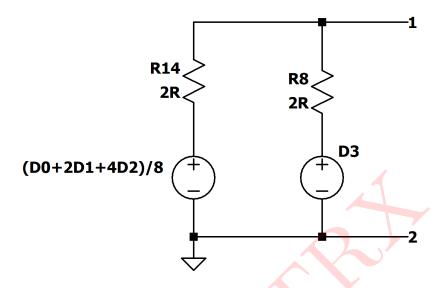


Figure 37: Modified circuit after series addition of resistances

Applying source transformation and combining the resistance and current sources

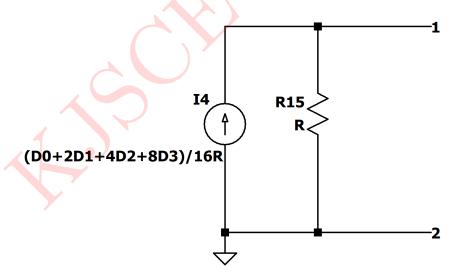


Figure 38: Modified circuit after source transformations

Now, after applying source transformation we get the venin's equivalent voltage V_{th} and the venin's resistance R_{th}

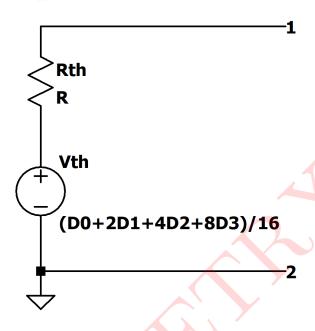


Figure 39: Thevenin's equivalent circuit

... Thevenin's equivalent voltage

$$V_{th} = \frac{D_0 + 2D_1 + 4D_2 + 8D_3}{16}$$

Thevenin's equivalent resistance

$$R_{th} = R$$

SIMULATED RESULTS:

follows:

To verify V_{th} in the simulated results, take $R = 10\Omega$, $D_0 = D_1 = D_2 = D_3 = 10V$ The following circuit has been simulated in LTspice and the readings obtained are as

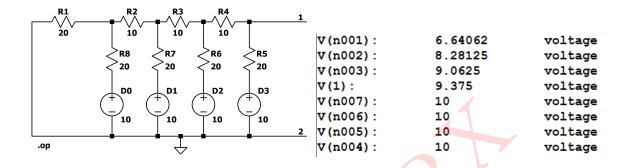


Figure 40: Circuit Schematic and Simulated Results

Hence, simulated value of V_{th} for given resistance and voltage values = 9.375V

From calculations:
$$V_{th} = \frac{D_0 + 2D_1 + 4D_2 + 8D_3}{16} = \frac{150}{16} = 9.375V$$

Connect a small resistance between 1 and 2 and find the short circuit current I_{sc} The following circuit has been simulated in LTspice and the readings obtained are as follows:

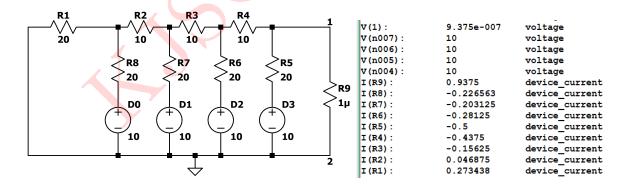


Figure 41: Circuit Schematic and Simulated Results

From the simulated readings, $I_{sc} = I_9 = 0.9375A$

Hence,
$$R_{th} = \frac{V_{12}}{I_{sc}} = \frac{9.375V}{0.9375A} = 10\Omega$$

Quantity	Calculated Value	Simulated Value
V_{th}	9.375V	9.375V
R_{th}	10Ω	10Ω

Table 8: Comparison of calculated and simulated results



Numerical 9: Obtain the Norton's equivalent of the circuit in Figure 42 to the left of terminals a-b. Use the results to find current i.

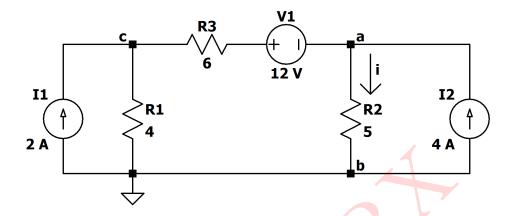


Figure 42: Circuit 9

Solution:

Applying source transformation on current source I_1 and resistance R_1 $V_2 = I_1 \times R_1 = 2 \times 4 = 8V$

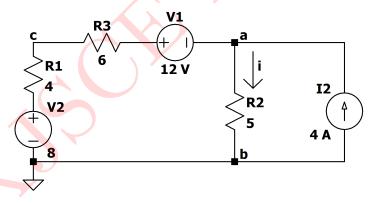


Figure 43: Modified circuit after source transformation

Adding series resistances $R_1,\,R_3$ and voltage sources $V_2,\,V_1$

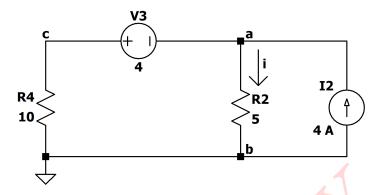


Figure 44: Modified circuit after combination of resistances and voltage sources

Applying source transformation on voltage source V_3 and resistance R_4

$$I_3 = \frac{V_3}{R_4} = \frac{4}{10} = 0.4A$$

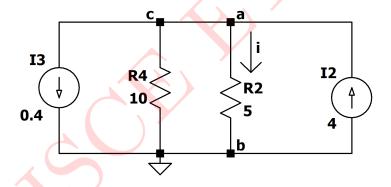


Figure 45: Modified circuit after source transformation

Subtracting current sources I_2 , I_3 as they are in opposite directions

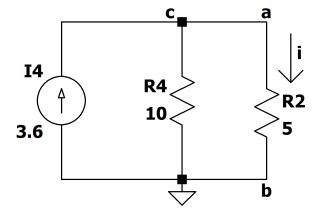


Figure 46: Modified circuit after combining the current sources

Hence the Norton's equivalent circuit is:

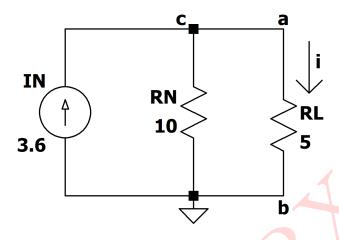


Figure 47: Norton's Equivalent Circuit

∴ Norton's current: $I_N = 3.6A$ ∴ Norton's resistance: $R_N = 10\Omega$

Load resistance: $R_L = 5\Omega$

Using current division rule to find current through load resistance R_L

$$I_{R_L} = i = \frac{R_N \times I_N}{R_N + R_L} = \frac{10 \times 3.6}{10 + 5} = 2.4A$$

 $\therefore i = 3.6A$

SIMULATED RESULTS:

The following circuit has been simulated in LTspice and the readings obtained are as follows:

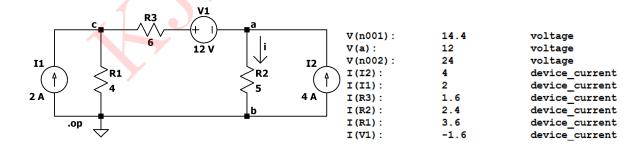


Figure 48: Circuit Schematic and Simulated Results

Quantity	Calculated Value	Simulated Value
i	2.4A	2.4A

Table 9: Comparison of calculated and simulated results

Numerical 10: In Figure 49 what value of resistor R_3 will allow maximum power transfer to the load? Also calculate the maximum total load power. All resistances are in ohms.

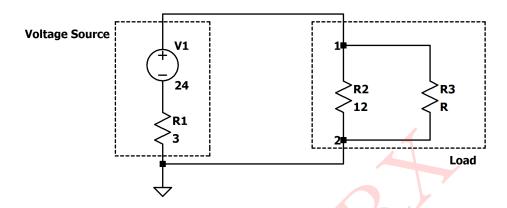


Figure 49: Circuit 10

Solution:

For maximum power transfer to the load, internal resistance of the source must be equal to the load resistance

 $R_{internal} = R_{load}$

$$R_1 = R_2 || R_3$$

$$3 = (12||R)$$

$$3 = \frac{12 \times R}{12 + R}$$

$$36 + 3R = 12R$$

$$9R = 36$$

$$R = 4\Omega$$

Maximum total load power = P_{max}

$$P_{max} = \frac{V_s^2}{4R_s}$$

$$P_{max} = \frac{24^2}{4 \times 3}$$

$$P_{max} = 48W$$

.:. maximum total load power = 48W

SIMULATED RESULTS:

The following circuit has been simulated in LTspice and the readings obtained are as follows:

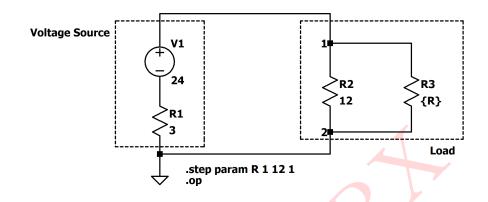


Figure 50: Circuit Schematic



Figure 51: Simulated Results

The graph shows various values of power dissipated across load (y-axis) for values of the resistance R_3 from 1Ω to 12Ω (x-axis).

From the graph, we can see that the power in the load is maximum when $R_3 = 4\Omega$ and the value of maximum power is 48W.

Quantity	Calculated Value	Simulated Value
R_3	4Ω	4Ω
P_{max}	48W	48W

Table 10: Comparison of calculated and simulated results

Numerical 11: In Figure 52 find the value of load resistance R_L for which maximum power will be dissipated in it. Also find the value of maximum power.

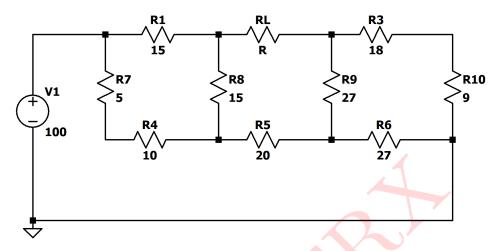


Figure 52: Circuit 11

Solution:

Adding the series resistances: R_7 , R_4 and R_3 , R_{10}

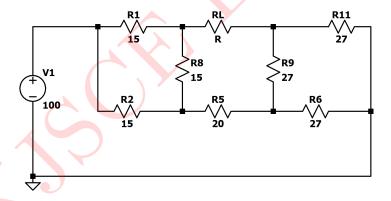


Figure 53: Modified circuit after addition of series resistances

Applying delta to star conversion on resistors: R_1 , R_2 , R_8 and R_6 , R_9 , R_{11}

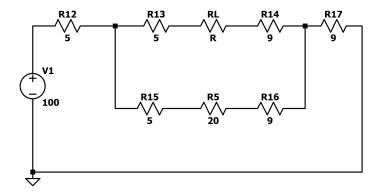


Figure 54: Modified circuit after delta to star conversion

Adding series resistances: R_{13} , R_{14} and R_{15} , R_5 , R_{16}

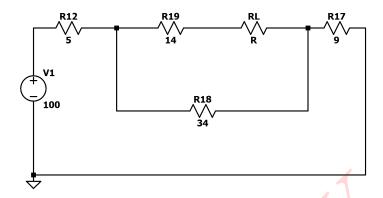


Figure 55: Modified circuit after delta to star conversion

Adding series resistances: R_{12} , R_{17}

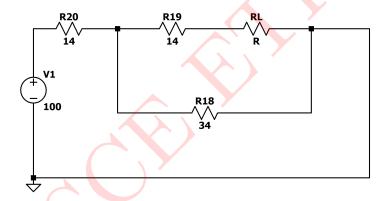


Figure 56: Modified circuit after series combination

Let the resistance of the parallel combination be R_x then,

$$R_x = (14 + R_L)||34$$

$$R_x = \frac{(14 + R_L) \times (34)}{48 + R_L}$$

$$R_x = \frac{34R_L + 476}{48 + R_L} \qquad \dots (1)$$

Total current supplied by V_1 :

$$I_{V_1} = \frac{100}{14 + R_x} \qquad \dots (2)$$

From current division rule, current through load resistance R_L :

$$I_{R_L} = \frac{34 \times I_{V_1}}{48 + R_L}$$

Substituting the value of I_{V_1} from (2) we get,

$$I_{R_L} = \frac{34}{48 + R_L} \times \frac{100}{14 + R_x}$$

Substituting the value of R_x from (1) we get,

$$I_{R_L} = \frac{34}{48 + R_L} \times \frac{100}{14 + \frac{34R_L + 476}{48 + R_L}}$$

$$I_{R_L} = \frac{34}{48 + R_L} \times \frac{100 \times (48 + R_L)}{1148 + 48R_L}$$

$$I_{R_L} = \frac{3400}{1148 + 48R_L}$$

$$I_{R_L} = \frac{850}{287 + 12R_L} \qquad(3)$$

Power dissipated across load resistance:

$$P_{R_L} = I_{R_L}^2 \times R_L$$

Substituting the value of I_{R_L} from (3)

$$P_{R_L} = \left(\frac{850}{287 + 12R_L}\right)^2 \times R_L \qquad(4)$$

For maximum power dissipation:

$$\frac{dP_{R_L}}{dR_L} = 0 \dots (5)$$

Differentiating (4) with respect to R_L

$$\frac{dP_{R_L}}{dR_L} = \frac{d}{dR_L} \left[\left(\frac{850}{287 + 12R_L} \right)^2 \times R_L \right]$$

$$\frac{dP_{R_L}}{dR_L} = 850^2 \times \frac{d}{dR_L} \left[\frac{R_L}{(287 + 12R_L)^2} \right]$$

$$\frac{dP_{R_L}}{dR_L} = 850^2 \times \left[\frac{(1)(287 + 12R_L)^2 - (R_L)(2)(287 + 12R_L)(12)}{(287 + 12R_L)^4} \right]$$

Dividing numerator and denominator by $(287 + 12R_L)$

$$\frac{dP_{R_L}}{dR_L} = 850^2 \times \left[\frac{(287 + 12R_L) - (R_L)(2)(12)}{(287 + 12R_L)^3} \right]$$

$$\frac{dP_{R_L}}{dR_L} = 850^2 \times \left[\frac{287 - 12R_L}{(287 + 12R_L)^3} \right] \qquad \dots (6)$$

From (5) and (6):

$$850^2 \times \left[\frac{287 - 12R_L}{\left(287 + 12R_L\right)^3} \right] = 0$$

$$287 - 12R_L = 0$$

$$12R_L = 287$$

$$R_L = \frac{287}{12}$$

$$R_L = 23.9167\Omega$$

 \therefore we will get maximum power dissipation in load resistance R_L for:

$$R_L = 23.9167\Omega$$

From (4) we can find the value of maximum power by substituting the value of R_L

$$P_{R_L} = \left(\frac{850}{287 + 12R_L}\right)^2 \times R_L$$

$$P_{R_L} = \left(\frac{850}{287 + 12(23.9167)}\right)^2 \times 23.9167$$

 \therefore the maximum power dissipated in load resistor:

$$P_{R_L max} = 52.45W$$

SIMULATED RESULTS:

The following circuit has been simulated in LTspice and the graph obtained is as follows:

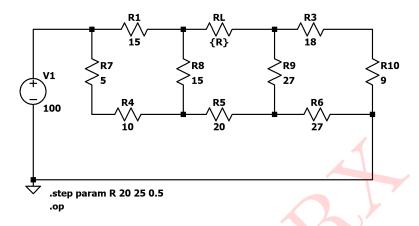


Figure 57: Circuit Schematic

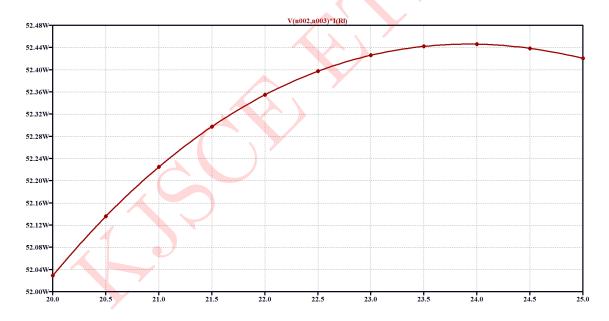


Figure 58: Simulated Result

Quantity	Calculated Value	Simulated Value
R_L	23.9167Ω	23.95Ω
$P_{R_L} max$	52.45W	52.44W

Table 11: Comparison of calculated and simulated results