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DEPARTMENT OF ELECTRONICS ENGINEERING
ELECTRONIC CIRCUITS
Multi-transistor circuits

Numerical 1:

The parameters of each transistor in the circuit given below in figure 1 are

$\beta_1 = \beta_2 = 100$, $V_A = \infty$ and $V_{BE1} = V_{BE2} = 0.7V$

- Determine the small-signal parameters g_m , r_π and r_o for both transistors
- Determine the small-signal voltage gain $A_{V1} = V_{o1}/V_S$, assuming V_{o1} is connected to an open circuit and determine the gain $A_{V2} = V_{out}/V_{o1}$
- Determine the overall small-signal voltage gain $A_{VT} = V_{out}/V_S$

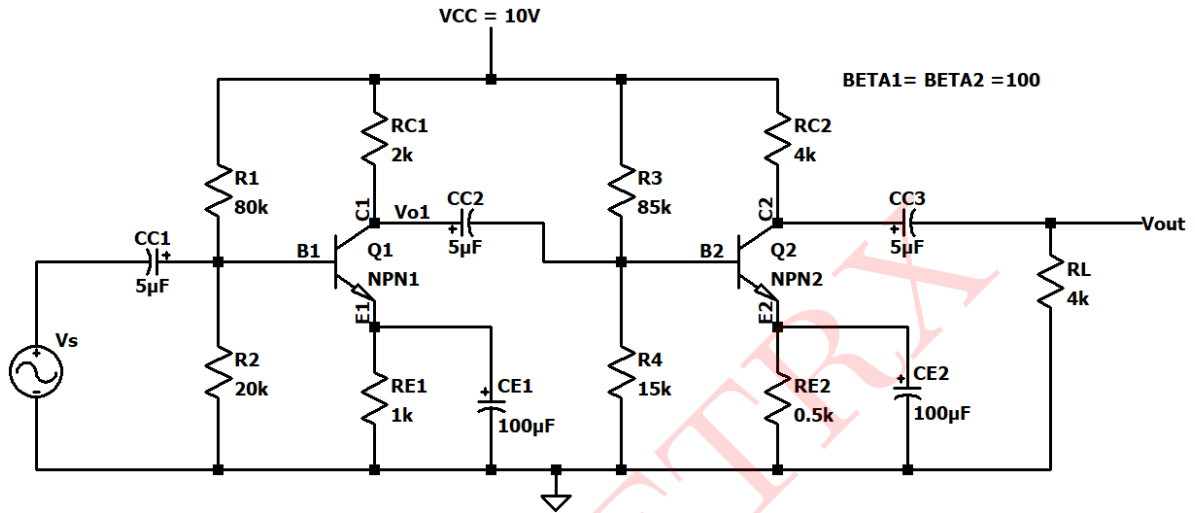


Figure 1: Circuit 1

Solution:

The circuit given above is a 2-stage RC-coupled CE-CE amplifier.

DC analysis:

Due to R-C coupling, both the stage's Q-point are isolated.

Since R-C coupling is employed, DC analysis of both stages can be performed individually.

DC analysis of stage 1 CE amplifier:

$$V_{TH1} = \frac{R_2}{R_1 + R_2} \times V_{CC} = \frac{20k\Omega}{80k\Omega + 20k\Omega} \times 10V = 2V$$

$$R_{B1} = \frac{R_1 R_2}{R_1 + R_2} = \frac{80k\Omega \times 20k\Omega}{80k\Omega + 20k\Omega} = 16k\Omega$$

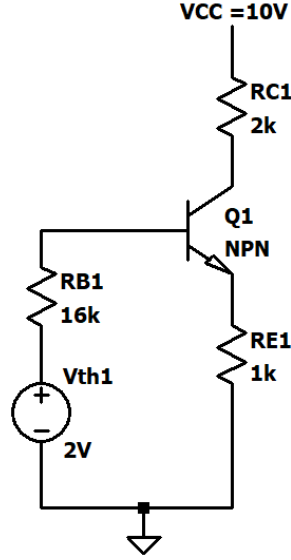


Figure 2: Thevenin's equivalent circuit of stage 1

Applying KVL to base-emitter loop,

$$V_{th1} - I_{B1}R_{B1} - V_{BE1} - I_{E1}R_{E1} = 0$$

$$\text{i.e } V_{th1} - I_{B1}R_{B1} - V_{BE1} - (\beta_1 + 1)I_{B1}R_{E1} = 0 \quad (\because I_E = (\beta + 1)I_B)$$

$$\therefore 2V - I_{B1}(16k\Omega) - 0.7V - (101)I_{B1}(1k\Omega) = 0$$

$$\therefore 1.3V - I_{B1}(16k\Omega + 101(1k\Omega)) = 0$$

$$I_{B1} = \frac{1.3V}{16k\Omega + 101k\Omega} = \mathbf{11.111\mu A}$$

$$\therefore I_{C1} = \beta_1 I_{B1} = (100)(11.111\mu A) = \mathbf{1.111mA}$$

$$\therefore I_{E1} = (\beta_1 + 1)I_{B1} = (100 + 1)(11.111\mu A) = \mathbf{1.122mA}$$

Applying KVL to collector emitter loop,

$$V_{CC} - I_{C1}R_{C1} - V_{CE1} - I_{E1}R_{E1} = 0$$

$$10V - (1.111mA)(2k\Omega) - V_{CE1} - (1.122mA)(1k\Omega) = 0$$

$$10V - 2.22V - V_{CE1} - 1.122V = 0$$

$$\therefore V_{CE1} = 10 - 3.342 = \mathbf{6.658V}$$

DC analysis of stage 2 CE amplifier:

$$V_{TH2} = \frac{R_4}{R_3 + R_4} \times V_{CC} = \frac{15k\Omega}{85k\Omega + 15k\Omega} \times 10V = \mathbf{1.5V}$$

$$R_{B2} = \frac{R_3 R_4}{R_3 + R_4} = \frac{15k\Omega \times 85k\Omega}{15k\Omega + 85k\Omega} = \mathbf{12.75k\Omega}$$

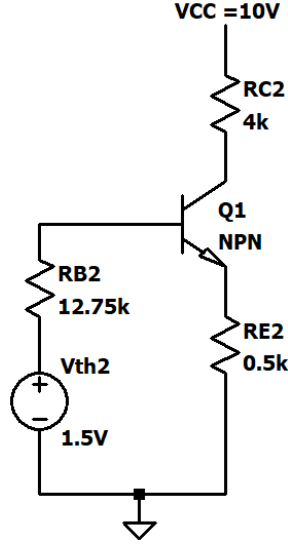


Figure 3: Thevenin's equivalent circuit of stage 2

Applying KVL to base-emitter loop,

$$V_{th2} - I_{B2}R_{B2} - V_{BE2} - I_{E2}R_{E2} = 0$$

$$\text{i.e } V_{th2} - I_{B2}R_{B2} - V_{BE2} - (\beta_2 + 1)I_{B2}R_{E2} = 0 \quad (\because I_E = (\beta + 1)I_B)$$

$$\therefore 1.5V - I_{B2}(12.75k\Omega) - 0.7V - (101)I_{B2}(0.5k\Omega) = 0$$

$$\therefore 0.8V - I_{B2}(12.75k\Omega + 101(.05k\Omega)) = 0$$

$$I_{B2} = \frac{0.8V}{12.75k\Omega + 50.5k\Omega} = \mathbf{12.648\mu A}$$

$$\therefore I_{C2} = \beta_2 I_{B2} = (100)(12.64\mu A) = \mathbf{1.264mA}$$

$$\therefore I_{E2} = (\beta_2 + 1)I_{B2} = (100 + 1)(12.648\mu A) = \mathbf{1.277mA}$$

Applying KVL to collector emitter loop,

$$V_{CC} - I_{C2}R_{C2} - V_{CE2} - I_{E2}R_{E2} = 0$$

$$10V - (1.264mA)(4k\Omega) - V_{CE2} - (1.277mA)(0.5k\Omega) = 0$$

$$10V - 5.056V - V_{CE2} - 0.6385 = 0$$

$$\therefore V_{CE2} = 10 - 5.6885 = \mathbf{4.3115V}$$

Small signal parameters of stage 1:

$$\text{i) } r_{\pi_1} = \beta_1 \frac{V_T}{I_{C1}} = 100 \times \frac{26mV}{1.111mA} = \mathbf{2.340k\Omega}$$

$$\text{ii) } g_{m1} = \frac{I_{C1}}{V_T} = \frac{1.111mA}{26mV} = 42.730mA/V$$

$$\text{iii) } r_{o1} = \infty$$

Small signal parameters of stage 2:

$$\text{i) } r_{\pi_2} = \beta_2 \frac{V_T}{I_{C_2}} = 100 \times \frac{26mV}{1.264mA} = \mathbf{2.056k\Omega}$$

$$\text{ii) } g_{m_2} = \frac{I_{C_2}}{V_T} = \frac{1.264mA}{26mV} = \mathbf{48.61mA/V}$$

$$\text{iii) } r_{o_2} = \infty$$

Mid-band AC equivalent circuit:

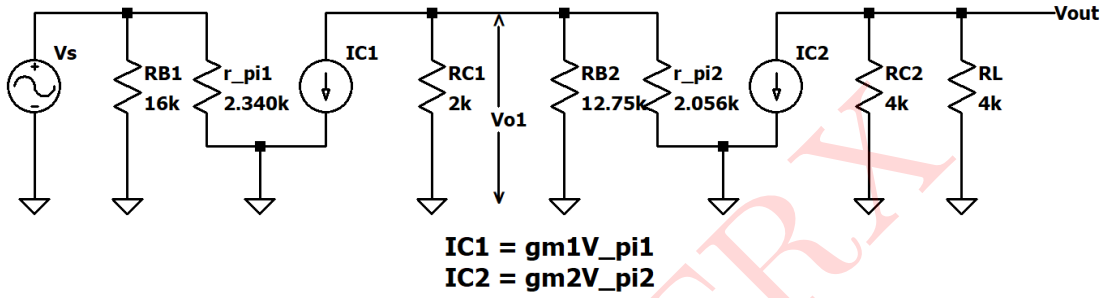


Figure 4: Mid-band AC equivalent circuit

Input impedance of stage 1

$$\begin{aligned} Z_i &= R_{B1} \parallel r_{\pi_1} \\ &= 16k\Omega \parallel 2.340k\Omega \\ &= \frac{16k\Omega \times 2.340k\Omega}{16k\Omega + 2.340k\Omega} = \mathbf{2.041k\Omega} \end{aligned}$$

Output impedance of stage 2

$$\begin{aligned} Z_o &= R_{C2} \parallel R_L \\ &= 4k\Omega \parallel 4k\Omega \\ &= \frac{4k\Omega \times 4k\Omega}{4k\Omega + 4k\Omega} = \mathbf{2k\Omega} \end{aligned}$$

$$\begin{aligned} A_{V_1} &= \frac{V_{o1}}{V_S} \\ &= \frac{(-g_{m1}V_{\pi_1})(R_{C1} \parallel R_{B2} \parallel r_{\pi_2})}{V_{\pi_1}} \\ &= -g_{m1}(R_{C1} \parallel R_{B2} \parallel r_{\pi_2}) \\ &= -42.730mA/V(2k\Omega \parallel 12.75k\Omega \parallel 2.065k\Omega) \\ &= -42.730mA/V \left[2k\Omega \parallel \left(\frac{12.75k\Omega \times 2.056k\Omega}{12.75k\Omega + 2.056k\Omega} \right) \right] \\ &= -42.730mA/V[2k\Omega \parallel 1.77k\Omega] \\ &= -42.730mA/V \left(\frac{2k\Omega \times 1.77k\Omega}{2k\Omega + 1.77k\Omega} \right) \\ &= -42.730mA/V \times 0.938k\Omega = \mathbf{-40.123} \end{aligned}$$

$$\begin{aligned}
A_{V_2} &= \frac{V_{out}}{V_{o1}} \\
&= \frac{(-g_{m_2} V_{\pi_2})(R_{C_2} \parallel R_L)}{V_{\pi_1}} \\
&= -g_{m_2}(R_{C_2} \parallel R_L) \\
&= -48.61 \text{mA/V}(4 \text{k}\Omega \parallel 4 \text{k}\Omega) \\
&= -48.61 \text{mA/V} \left(\frac{4 \text{k}\Omega \times 4 \text{k}\Omega}{4 \text{k}\Omega + 4 \text{k}\Omega} \right) \\
&= -48.61 \text{mA/V} \times 2 \text{k}\Omega = \mathbf{-97.22}
\end{aligned}$$

Overall small signal voltage gain,

$$\begin{aligned}
A_{V_T} &= \frac{V_{out}}{V_S} \\
&= \frac{V_{out}}{V_{o1}} \times \frac{V_{o1}}{V_S} \\
&= A_{V_2} \times A_{V_1} \\
&= -97.22 \times -40.123 = \mathbf{3900.7580}
\end{aligned}$$

Overall voltage gain in dB,

$$|A_{V_T}|_{dB} = 20 \log_{10}(3900.7580) = \mathbf{71.82 \text{dB}}$$

Output voltage,

$$\begin{aligned}
V_{out} &= A_{V_T} \times V_S \\
&= 3900.7580 \times 0.4 \text{mV} = \mathbf{1.560 \text{V}}
\end{aligned}$$

SIMULATED RESULTS:

Above circuit is simulated in LTspice and results are as follows:

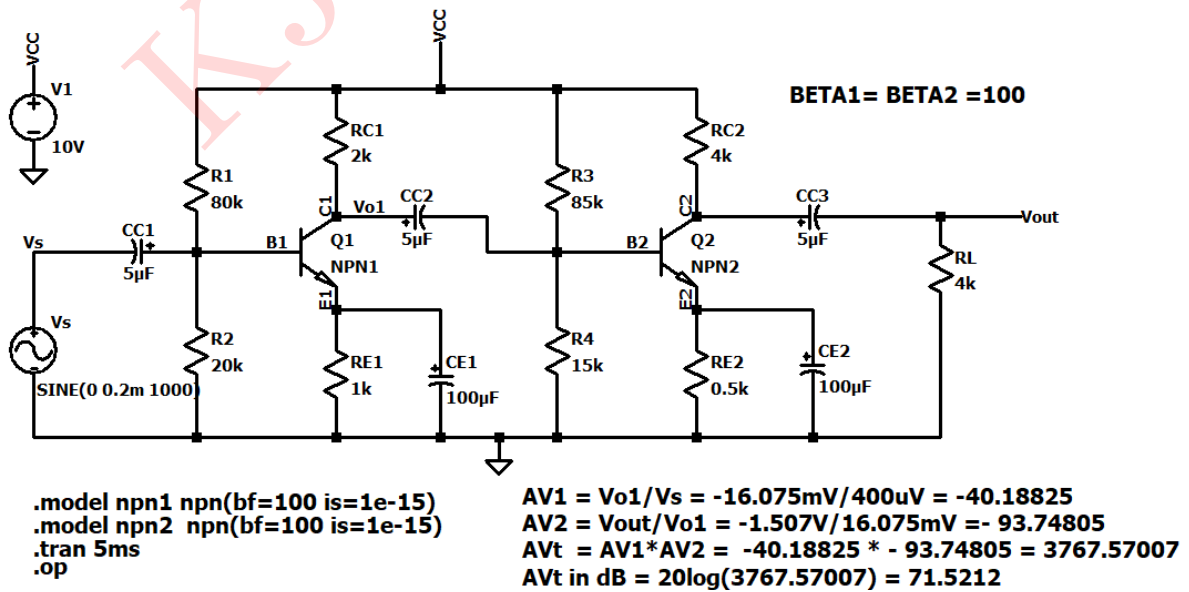


Figure 5: Circuit Schematic

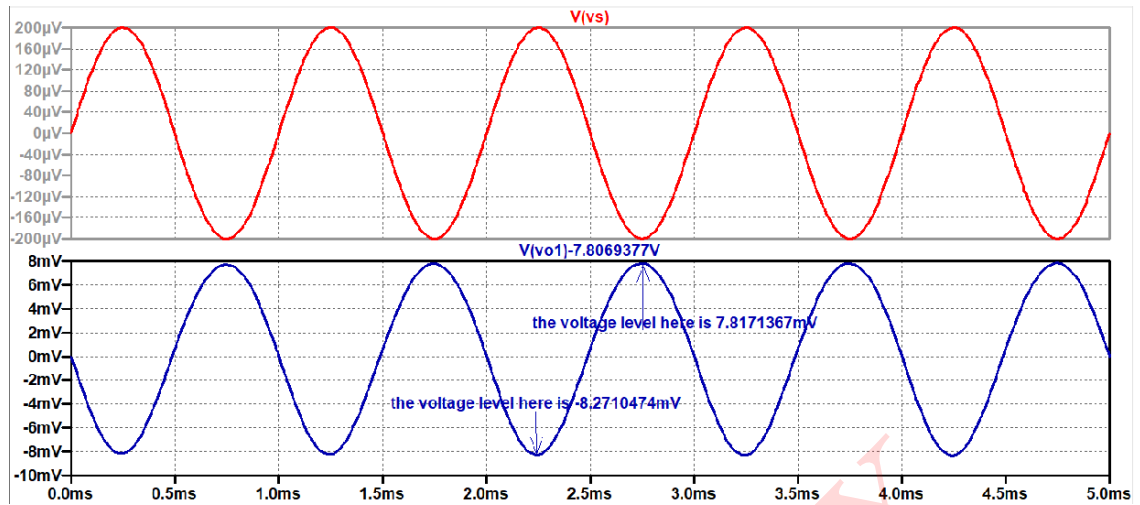


Figure 6: Input and ouput waveforms for Stage 1 voltage gain

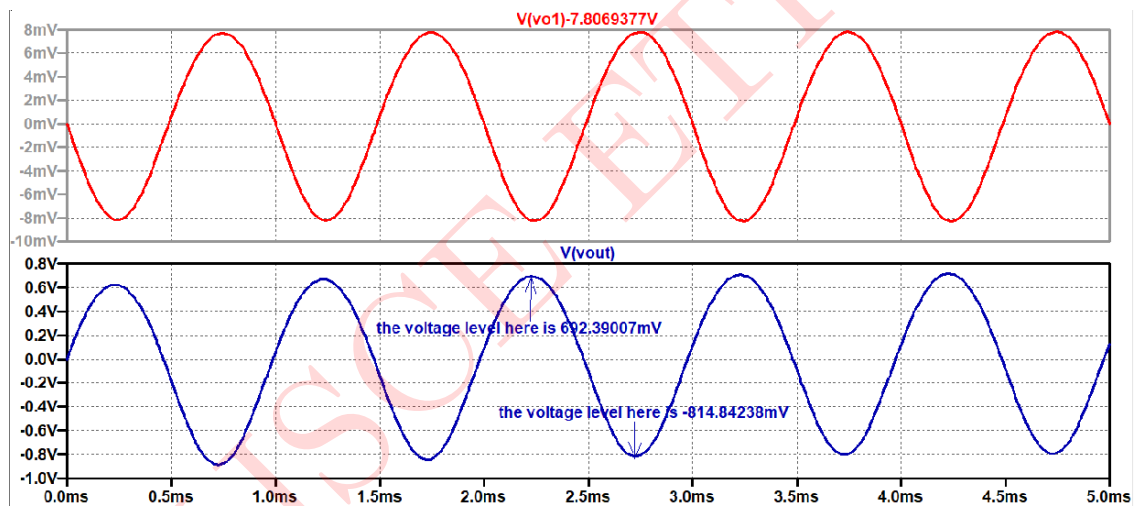


Figure 7: Input and ouput waveforms for Stage 2 voltage gain

Comparison between theoretical and simulated values:

Parameters	Theoretical values	Simulated values
1 st stage Q-point I_{C_1Q} , V_{CE_1Q}	1.111mA, 6.658V	1.0965mA, 6.6994V
2 nd stage Q-point I_{C_2Q} , V_{CE_2Q}	1.264mA, 4.3115V	1.23305mA, 4.4450V
Voltage gain of 1st stage: A_{V1}	-40.123	-40.18825
Voltage gain of 2nd stage: A_{V2}	-97.22	-98.7480
Overall voltage gain: A_{VT} in dB	71.82dB	71.52dB
Input impedance of 1st stage (Z_i)	2.041k Ω	-
Output impedance of 2nd stage (Z_o)	2k Ω	-
Output voltage (V_o)	1.560V	1.5078V

Table 1: Numerical 1

Numerical 2:

A two stage circuit is shown in figure 8 below, BJT parameters are $V_{BE} = 0.7V$, $\beta = 200$, JFET parameters are $I_{DSS} = 10mA$, $V_P = -4V$

- Determine the Q point for both stages
- Draw mid-frequency equivalent circuit
- Calculate A_{V_1} and A_{V_2}
- Calculate A_{V_T} in dB
- Calculate V_{out} if $V_S = 2mV_{p-p}$
- Calculate Z_i and Z_o

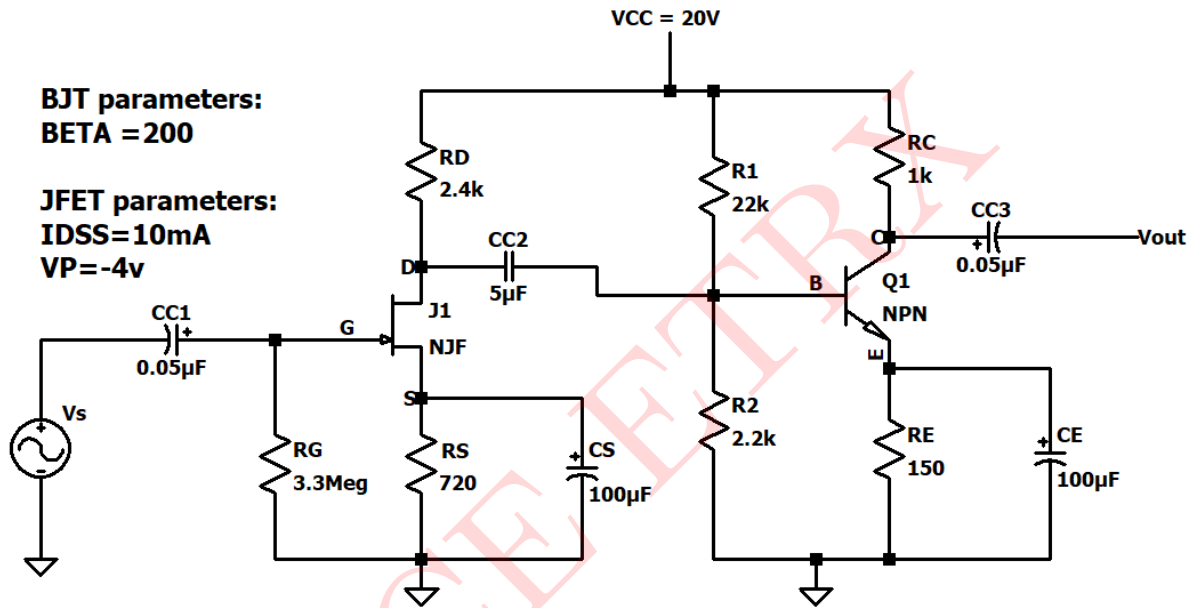


Figure 8: Circuit 2

Solution:

Since RC coupling is employed, DC analysis of both stages can be performed individually.

DC analysis of JFET:

Applying KVL to gate-source loop,

$$-I_g R_G - V_{GS} - I_D R_S = 0$$

$$\text{i.e } 0 - V_{GS} - I_D R_S = 0 \quad (\because I_g = 0, I_g R_G = 0)$$

$$\therefore V_{GS} = -I_D R_S$$

$$\therefore V_{GS} = -I_D (720\Omega)$$

$$V_{GS} = -720 I_D \quad \dots(1)$$

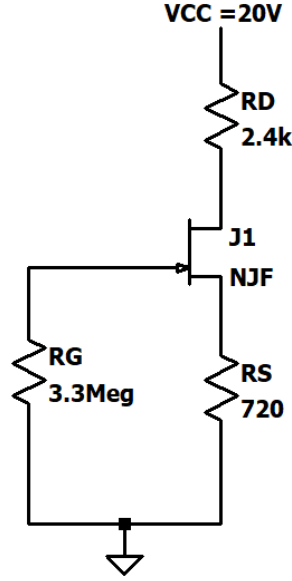


Figure 9: DC equivalent circuit of stage 1

In JFET,

$$I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P} \right)^2$$

$$\therefore I_D = 10mA \left(1 - \frac{V_{GS}}{(-4V)} \right)^2$$

$$\therefore I_D = 10mA \left(1 + \frac{V_{GS}}{4V} \right)^2 \quad \dots (2)$$

Put (2) in (1), we get

$$V_{GS} = -720 \times \left[10mA \left(1 + \frac{V_{GS}}{4} \right)^2 \right]$$

$$V_{GS} = -720 \times 10mA \left(1 + \frac{2V_{GS}}{4} + \frac{V_{GS}^2}{16} \right)$$

$$V_{GS} = -7.2 \left(1 + \frac{V_{GS}}{2} + \frac{V_{GS}^2}{16} \right)$$

$$V_{GS} = -7.2 - 3.6V_{GS} - 0.45V_{GS}^2$$

$$0 = -7.2 - 4.6V_{GS} - 0.45V_{GS}^2$$

$$\therefore V_{GS} = -1.929V \text{ or } V_{GS} = -8.292V$$

We reject the value $V_{GS} = -8.292V$ because $V_{GSQ} > V_P$

$$\therefore V_{GS} = -1.929V$$

$$\therefore I_{DQ} = I_{DSS} \left(1 - \frac{V_{GSQ}}{V_P} \right)^2$$

$$\therefore I_{DQ} = 10mA \left(1 - \frac{(-1.929V)}{(-4V)} \right)^2$$

$$\therefore I_{DQ} = 10mA \left(1 - \frac{1.929V}{4V} \right)^2$$

$$\therefore I_{DQ} = 10mA (1 - 0.48225)^2$$

$$\therefore I_{DQ} = 10mA (0.51775)^2 = \mathbf{2.678mA}$$

Small signal parameters of JFET:

$$\begin{aligned} \text{i) } g_{m1} &= \frac{2I_{DSS}}{|V_P|} \left(1 - \frac{V_{GSQ}}{V_P}\right) \\ &= \frac{2 \times 10mA}{4V} \left(1 - \frac{(-1.929)}{(-4)}\right) \\ &= 5mA/V (1 - 0.48225) \\ &= 5mA/V (0.51775) = \mathbf{2.588mA/V} \end{aligned}$$

$$r_d = \infty$$

DC analysis of BJT:

$$V_{TH} = \frac{R_2}{R_1 + R_2} \times V_{CC} = \frac{2.2k\Omega}{22k\Omega + 2.2k\Omega} \times 20V = \mathbf{1.8V}$$

$$R_B = \frac{R_1 R_2}{R_1 + R_2} = \frac{22k\Omega \times 2.2k\Omega}{22k\Omega + 2.2k\Omega} = \mathbf{2k\Omega}$$

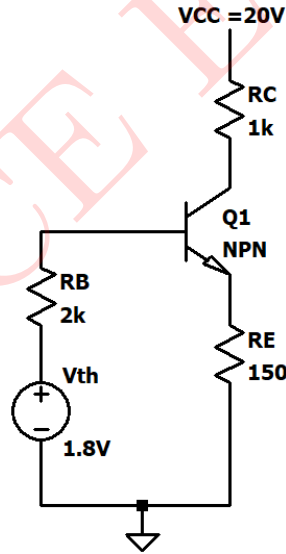


Figure 10: Thevenin's equivalent circuit of stage 2

Applying KVL to base-emitter loop,

$$V_{th} - I_B R_B - V_{BE} - I_E R_E = 0$$

$$\text{i.e } V_{th} - I_B R_B - V_{BE} - (\beta + 1)I_B R_E = 0$$

$$\therefore 1.8V - I_B(2k\Omega) - 0.7V - (201)I_B(150\Omega) = 0$$

$$\therefore 1.1V - I_B(2k\Omega + 201(150\Omega)) = 0$$

$$I_B = \frac{1.1V}{2k\Omega + 30.15k\Omega} = \mathbf{34.214\mu A}$$

$$\therefore I_C = \beta I_B = (200)(34.214\mu A) = \mathbf{6.842mA}$$

$$\therefore I_E = (\beta + 1)I_B = (200 + 1)(34.214\mu A) = \mathbf{6.877mA}$$

Applying KVL to collector emitter loop,

$$V_{CC} - I_C R_C - V_{CE} - I_E R_E = 0$$

$$20V - (6.842mA)(1k\Omega) - V_{CE} - (6.877mA)(150\Omega) = 0$$

$$20V - 6.842V - V_{CE} - 1.031V = 0$$

$$\therefore V_{CEQ} = 20 - 7.873 = \mathbf{12.126V}$$

Small signal parameters of BJT:

$$\text{i) } r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{200 \times 26mV}{6.842mA} = \mathbf{0.76k\Omega}$$

$$\text{ii) } g_{m2} = \frac{I_{CQ}}{V_T} = \frac{6.842mA}{26mV} = \mathbf{263.15mA/V}$$

$$\text{iii) } r_o = \infty$$

Mid-band AC equivalent circuit:

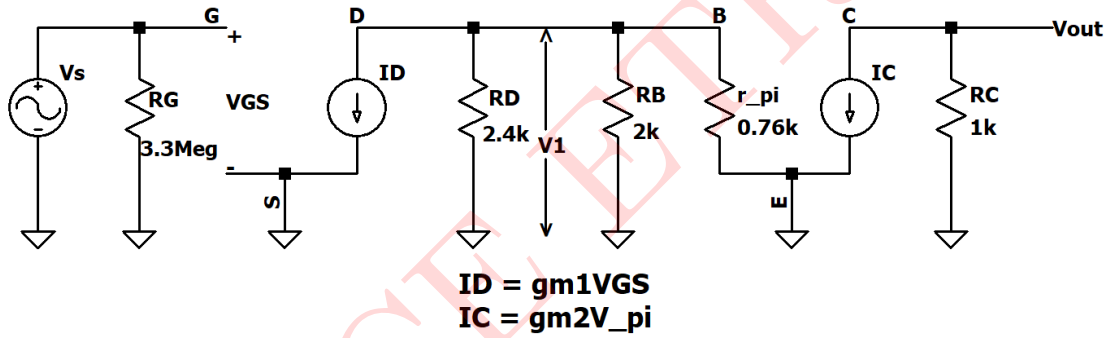


Figure 11: Mid-band AC equivalent circuit

Input impedance of 1st stage

$$Z_i = R_G = \mathbf{3.3M\Omega}$$

Output impedance of 2nd stage

$$Z_o = R_C = \mathbf{1k\Omega}$$

$$\begin{aligned} A_{V_1} &= \frac{V_1}{V_S} \\ &= \frac{(-g_{m1} V_{gs})(R_D \parallel R_B \parallel r_\pi)}{V_{gs}} \\ &= -g_{m1}(R_D \parallel R_B \parallel r_\pi) \\ &= -2.588mA/V(2.4k\Omega \parallel 2k\Omega \parallel 0.76k\Omega) \\ &= -2.588mA/V \left[2.4k\Omega \parallel \left(\frac{2k\Omega \times 0.76k\Omega}{2k\Omega + 0.76k\Omega} \right) \right] \\ &= -2.588mA/V[2.4k\Omega \parallel 0.55k\Omega] \\ &= -2.588mA/V \times 0.44k\Omega = \mathbf{-1.158} \end{aligned}$$

$$\begin{aligned}
A_{V_2} &= \frac{V_{out}}{V_1} \\
&= \frac{-g_{m_2} V_{\pi} R_C}{V_{\pi}} \\
&= -g_{m_2} R_C \\
&= -263.15 \text{ mA/V} \times 1 \text{ k}\Omega = \mathbf{-263.15}
\end{aligned}$$

Overall small signal voltage gain,

$$\begin{aligned}
A_{V_T} &= \frac{V_{out}}{V_S} \\
&= \frac{V_{out}}{V_1} \times \frac{V_1}{V_S} \\
&= A_{V_2} \times A_{V_1} \\
&= -263.15 \times -1.158 = \mathbf{304.733}
\end{aligned}$$

Overall voltage gain in dB,

$$|A_{V_T}|_{dB} = 20 \log_{10}(304.733) = \mathbf{49.678 \text{ dB}}$$

$$\therefore A_{V_T} = \frac{V_{out}}{V_S}$$

$$\therefore V_{out} = A_{V_T} \times V_S = 304.733 \times 2 \text{ mV} = \mathbf{0.60 \text{ V}}$$

i.e $V_{out} = 0.60 \text{ V}$ for $V_S = 2 \text{ mV}_{P-P}$

SIMULATED RESULTS:

Above circuit is simulated in LTspice and results are as follows:

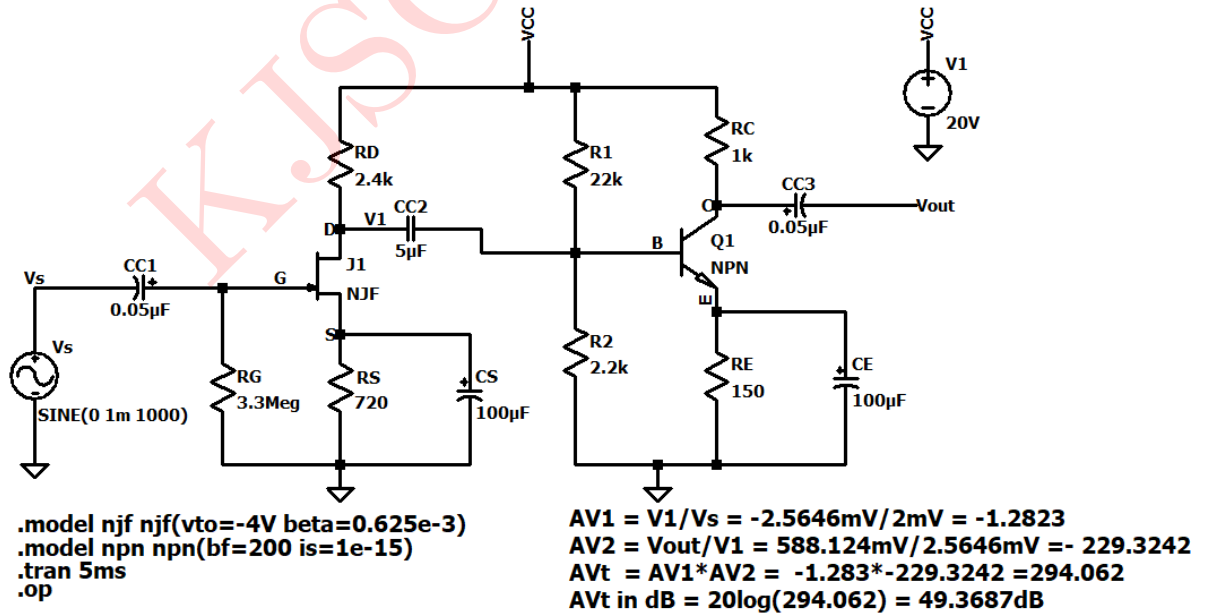


Figure 12: Circuit Schematic

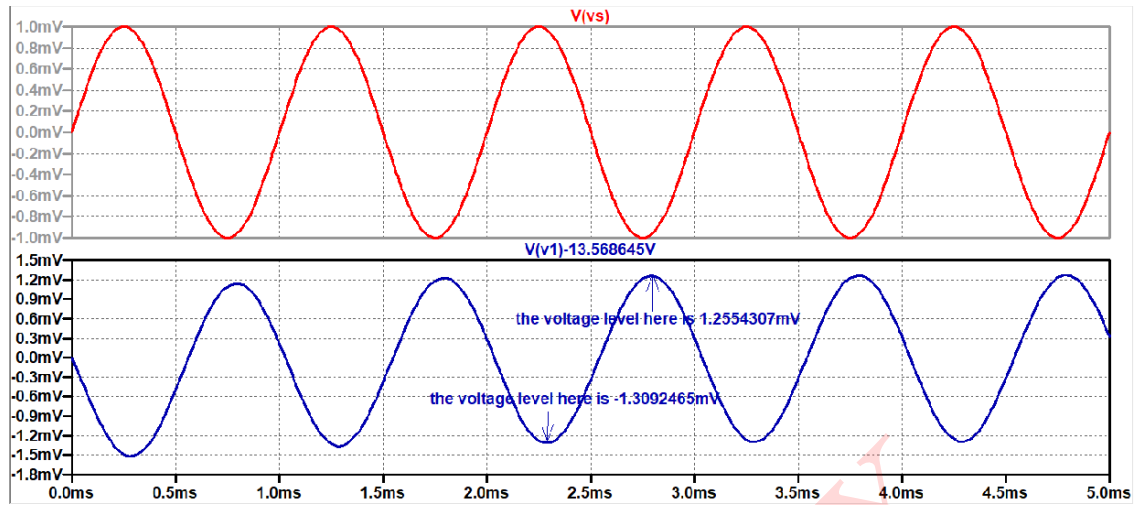


Figure 13: Input and output waveforms for Stage 1 voltage gain

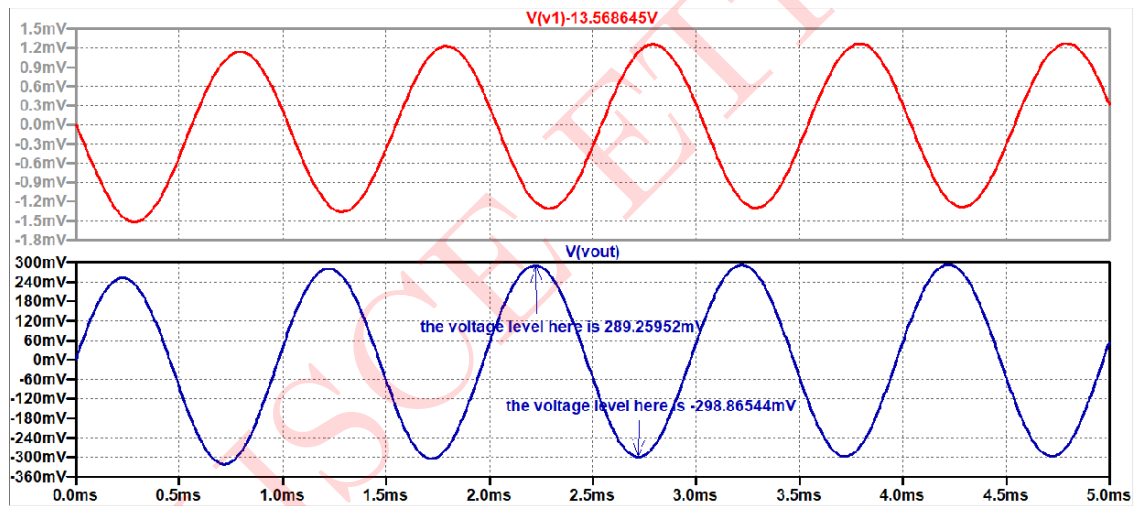


Figure 14: Input and output waveforms for Stage 2 voltage gain

Comparison between theoretical and simulated values:

Parameters	Theoretical values	Simulated values
1st stage Q-point I_{DQ}, V_{GSQ}	2.678mA, -1.929V	2.6797mA, -1.9294V
2nd stage Q-point I_{CQ}, V_{CEQ}	6.842mA, 12.126V	6.5620mA, 12.4487V
Voltage gain of 1st stage: A_{V1}	-1.158	-1.2823
Voltage gain of 2nd stage: A_{V2}	-263.15	-229.3242
Overall voltage gain: A_{VT} in dB	49.678dB	49.3687dB
Input impedance of 1st stage (Z_i)	3.3M Ω	—
Output impedance of 2nd stage (Z_o)	1k Ω	—
Output voltage (V_o)	0.60V	0.5882V

Table 2: Numerical 2

Numerical 3:

A two stage circuit is shown in figure 15 below, BJT parameters are $V_{BE1} = V_{BE2} = 0.6V$, $\beta_1 = \beta_2 = 100$

- Determine all node voltages and terminal currents under DC analysis
- Determine overall voltage gain of the circuit

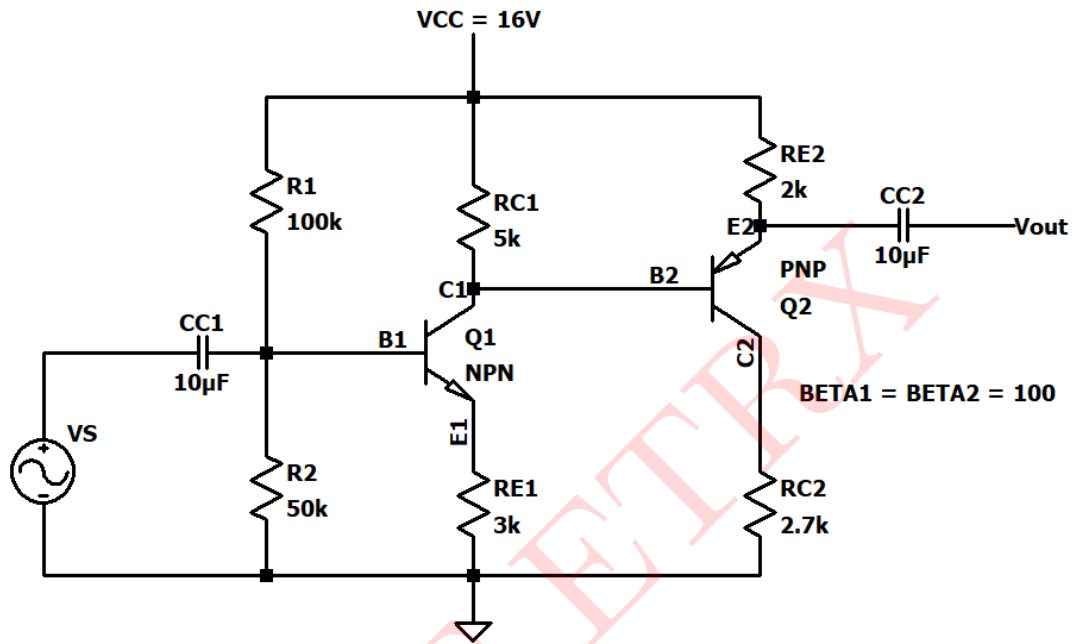


Figure 15: Circuit 3

Solution:

DC analysis:

Considering the thevenin's equivalent of base circuit of transistor Q_1 ,

$$V_{TH} = \frac{R_2}{R_1 + R_2} \times V_{CC} = \frac{50k\Omega}{100k\Omega + 50k\Omega} \times 16V = \mathbf{5.33V}$$

$$R_{B1} = \frac{R_1 R_2}{R_1 + R_2} = \frac{50k\Omega \times 100k\Omega}{50k\Omega + 100k\Omega} = \mathbf{33.33k\Omega}$$

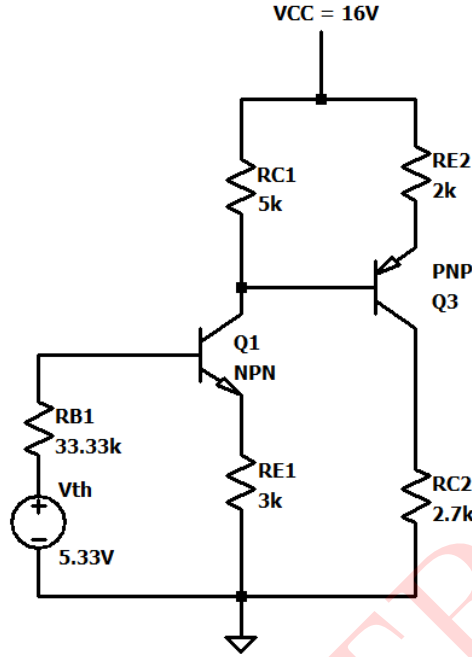


Figure 16: DC equivalent circuit

Applying KVL to base-emitter loop of transistor Q_1 ,

$$V_{th1} - I_{B1}R_{B1} - V_{BE1} - I_{E1}R_{E1} = 0$$

$$\text{i.e } V_{th1} - I_{B1}R_{B1} - V_{BE1} - (\beta_1 + 1)I_{B1}R_{E1} = 0$$

$$\therefore 5.33V - I_{B1}(33.33k\Omega) - 0.6V - (101)I_{B1}(3k\Omega) = 0$$

$$\therefore 4.73V - I_{B1}(33.33k\Omega + 101(3k\Omega)) = 0$$

$$I_{B1} = \frac{4.73V}{33.33k\Omega + 303k\Omega} = \mathbf{14.063\mu A}$$

$$\therefore I_{C1} = \beta I_{B1} = (100)(14.063\mu A) = \mathbf{1.4063mA}$$

$$\therefore I_{E1} = (\beta + 1)I_{B1} = (100 + 1)(14.063\mu A) = \mathbf{1.420mA}$$

$$V_{C1} = V_{CC} - I_{C1}R_{C1} \text{ (Ignoring } I_{B2} \text{ and } I_{RC} \approx I_{C1}) \quad \dots(1)$$

$$\therefore V_{C1} = 16 - 1.4063mA(5k\Omega)$$

$$\therefore V_{C1} = 16 - 7.0315 = \mathbf{8.968V}$$

$$V_{E2} = V_{B2} + V_{EB2(ON)}$$

$$\therefore V_{E2} = V_{C1} + V_{EB2(ON)} \quad (\because I_{B2} \text{ and } I_{RC} \approx I_{C1})$$

$$\therefore V_{E2} = 8.968V + 0.6V = \mathbf{9.568V}$$

According to KCL at emitter of transistor Q_2 ,

$$I_{E2} = \frac{V_{CC} - V_{E2}}{R_{E2}}$$

$$\therefore I_{E2} = \frac{16V - 9.568V}{2k\Omega}$$

$$\therefore I_{E2} = \frac{6.432V}{2k\Omega} = \mathbf{3.216mA}$$

$$\begin{aligned}\therefore I_{C_2} &= \frac{\beta_2}{1 + \beta_2} \times I_{E_2} \\ &= \frac{100}{101} \times 3.216mA = \mathbf{3.184mA}\end{aligned}$$

$$\begin{aligned}\therefore I_{B_2} &= \frac{I_{E_2}}{1 + \beta_2} \\ &= \frac{3.216mA}{101} = \mathbf{31.814\mu A}\end{aligned}$$

Now, rewriting exact expression for equation (1)

$$V_{C_1} = V_{CC} - I_{RC_1} R_{C_1}$$

$$\therefore I_{RC_1} = I_{C_1} - I_{B_2}$$

$$\therefore I_{RC_1} = 1.4063mA - 31.841\mu A = \mathbf{1.374mA}$$

$$\therefore V_{C_1new} = 16 - (1.374mA)(5k\Omega) = \mathbf{9.13V}$$

$$\therefore V_{E_2new} = V_{C_1new} + V_{EB_2(ON)}$$

$$\therefore V_{E_2new} = 9.13V + 0.6V = \mathbf{9.73V}$$

$$\begin{aligned}\therefore I_{E_2new} &= \frac{V_{CC} - V_{E_2new}}{R_{E_2}} \\ &= \frac{16V - 9.73V}{2k\Omega} = \mathbf{3.135mA}\end{aligned}$$

$$\begin{aligned}\therefore I_{C_2new} &= \frac{\beta_2}{1 + \beta_2} \times I_{E_2new} \\ &= \frac{100}{101} \times 3.135mA = \mathbf{3.103mA}\end{aligned}$$

$$\begin{aligned}\therefore I_{B_2new} &= \frac{I_{E_2new}}{1 + \beta_2} \\ &= \frac{3.135mA}{101} = \mathbf{31.039\mu A}\end{aligned}$$

$$\begin{aligned}\therefore V_{E_1} &= I_{E_1} R_{E_1} \\ &= (1.420mA)(3k\Omega) = \mathbf{4.26V}\end{aligned}$$

$$\begin{aligned}\therefore V_{C_2} &= I_{C_2} R_{C_2} \\ &= (3.184mA)(2.7k\Omega) = \mathbf{8.596V}\end{aligned}$$

$$\begin{aligned}\therefore V_{B_1} &= V_{BE_1(ON)} + V_{E_1} \\ &= 0.6V + 4.26V = \mathbf{4.86V}\end{aligned}$$

Node voltages:

$$V_{B_1} = 4.86V$$

$$V_{C_1} = 9.13V$$

$$V_{E_1} = 4.26V$$

$$V_{C_2} = 8.596V$$

$$V_{E_2} = 9.73V$$

$$V_{B_2} = 9.13V$$

Terminal currents:

$$I_{B_1} = 14.063\mu A$$

$$I_{C_1} = 1.4063mA$$

$$I_{E_1} = 1.420mA$$

$$I_{B_2} = 31.039\mu A$$

$$I_{C_2} = 3.103mA$$

$$I_{E_2} = 3.135mA$$

Small signal parameters:

$$\begin{aligned} r_{\pi_1} &= \frac{\beta_1 V_T}{I_{CQ_1}} \\ &= \frac{100 \times 26mV}{1.4063mA} = \mathbf{1.848k\Omega} \\ r_{\pi_2} &= \frac{\beta_2 V_T}{I_{CQ_2}} \\ &= \frac{100 \times 26mV}{3.103mA} = \mathbf{837.89\Omega} \end{aligned}$$

Mid-frequency AC equivalent circuit:

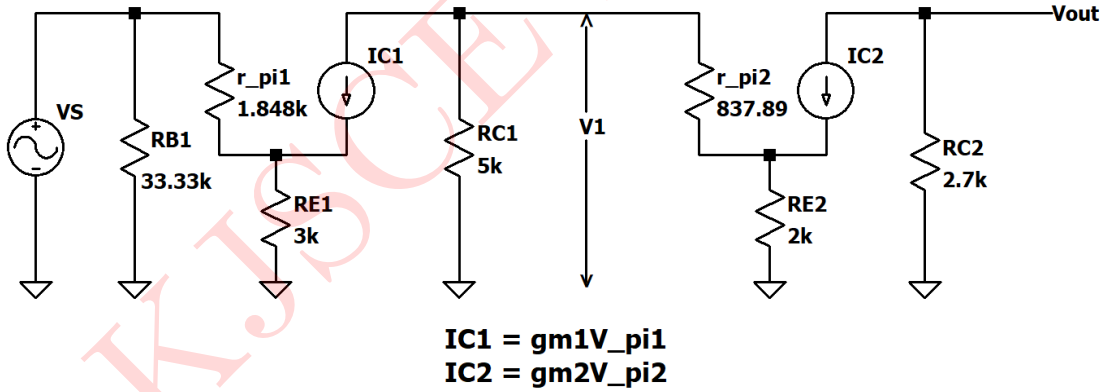


Figure 17: Mid-frequency AC equivalent circuit

$$\begin{aligned} A_{V_1} &= \frac{V_1}{V_S} \\ &= \frac{(-g_{m_1} V_{\pi_1}) R_{C_1}}{I_{B_1} [r_{\pi_1} + (1 + \beta_1) R_{E_1}]} \\ &= \frac{-(\beta_1 I_{B_1}) R_{C_1}}{I_{B_1} [r_{\pi_1} + (1 + \beta_1) R_{E_1}]} \\ &= \frac{-\beta_1 R_{C_1}}{r_{\pi_1} + (1 + \beta_1) R_{E_1}} \\ &= \frac{-100 \times 5k\Omega}{1.848k\Omega + (1 + 100)3k\Omega} = \mathbf{-1.640} \end{aligned}$$

$$\begin{aligned}
A_{V_2} &= \frac{V_o}{V_1} \\
&= \frac{(g_{m_2} V_{\pi_2}) R_{E_2}}{I_{B_2} [r_{\pi_2} + (1 + \beta_2) R_{E_2}]} \\
&= \frac{(\beta_2 I_{B_2}) R_{E_2}}{I_{B_2} [r_{\pi_2} + (1 + \beta_2) R_{E_2}]} \\
&= \frac{\beta_2 R_{E_2}}{r_{\pi_2} + (1 + \beta_2) R_{E_2}} \\
&= \frac{100 \times 2k\Omega}{837.89\Omega + (1 + 100)2k\Omega} = \mathbf{0.986}
\end{aligned}$$

Overall voltage gain,

$$\begin{aligned}
\therefore V_{out} &= A_{V_T} \times A_{V_2} \\
&= -1.640 \times 0.986 = \mathbf{-1.617} \\
\therefore A_{V_T} &= \frac{V_{out}}{V_S} \\
\therefore V_{out} &= A_{V_T} \times V_S \\
&= -1.617 \times 40mV = \mathbf{-64.68V}
\end{aligned}$$

SIMULATED RESULTS:

Above circuit is simulated in LTspice and results are as follows:

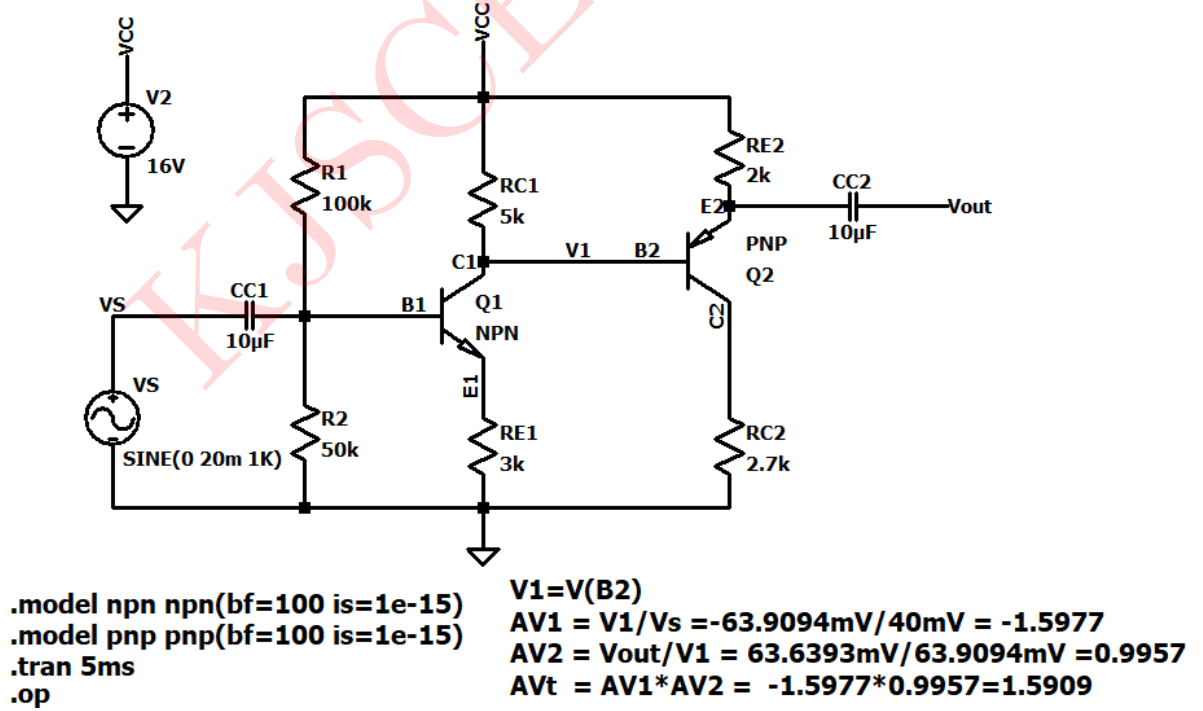


Figure 18: Circuit Schematic

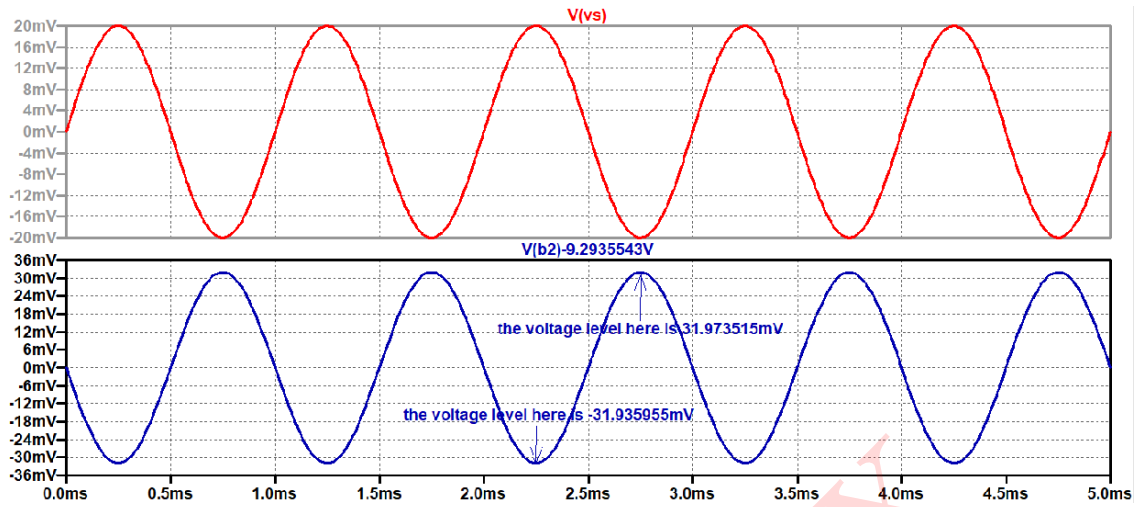


Figure 19: Input and output waveforms for Stage 1 voltage gain

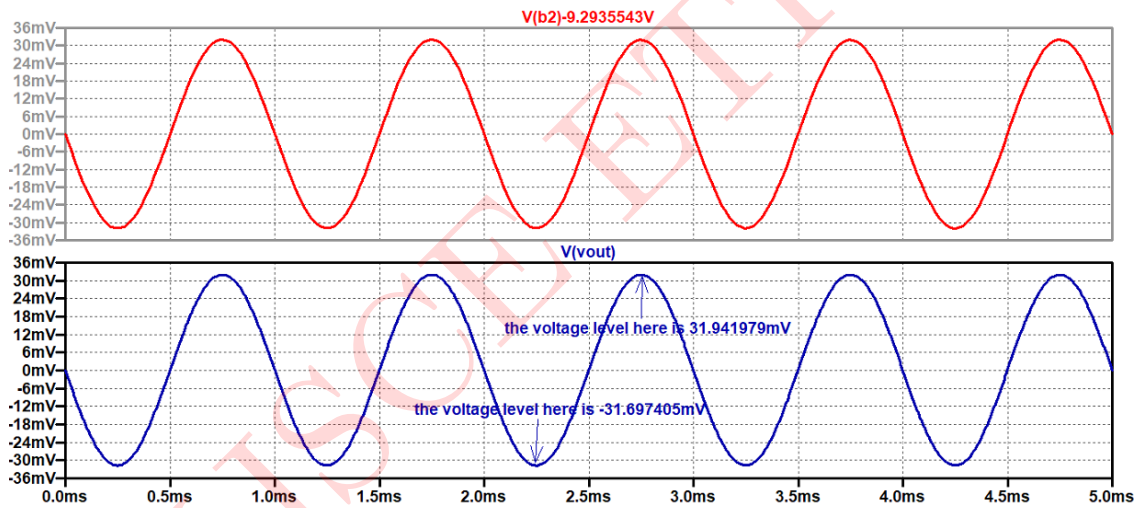


Figure 20: Input and output waveforms for Stage 2 voltage gain

Comparison between theoretical and simulated values:

Parameters	Theoretical values	Simulated values
I_{B_1}	14.063 μ A	13.7081 μ A
I_{C_1}, I_{E_1}	1.4063mA, 1.420mA	1.3708mA, 1.3845mA
I_{B_2}	31.039 μ A	29.5236 μ A
I_{C_2}, I_{E_2}	3.103mA, 3.135mA	2.9523mA, 2.98188mA
V_{C_1}	9.13V	9.2935V
V_{C_2}	8.596V	7.9718V
V_{E_1}	4.26V	4.1535V
V_{E_2}	9.73V	10.0362V
V_{B_1}	4.86V	4.8764V
V_{B_2}	9.13V	9.2935V
A_{V_T}	-1.617	-1.5909
V_{out}	-64.68mV	-63.6393mV

Table 3: Numerical 3
