

K. J. SOMAIYA COLLEGE OF ENGINEERING
DEPARTMENT OF ELECTRONICS ENGINEERING
ELECTRONIC CIRCUITS
AC CIRCUITS

Numerical 1: A series R-L-C circuit containing a resistance of $20\ \Omega$, an inductance of 0.05H and a capacitor of $50\mu\text{F}$ are connected in series across a 230V , 50Hz supply.

Calculate:

- i) The current drawn by the circuit
- ii) V_R , V_L and V_C
- iii) Power factor of circuit
- iv) Draw the voltage phasor diagram

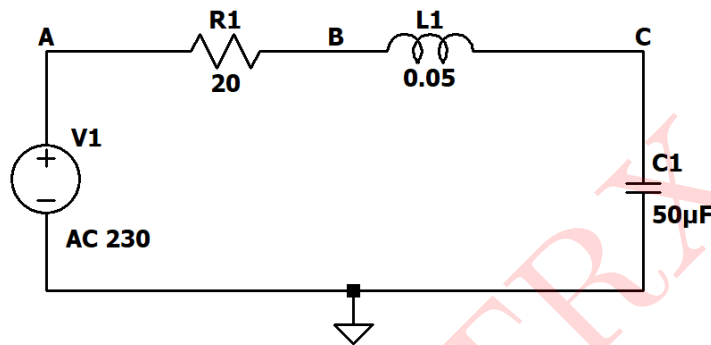


Figure 1: Circuit 1

Given:

$$R_1 = 20\ \Omega$$

$$L_1 = 0.05\text{H}$$

$$C_1 = 50\mu\text{F}$$

$$V_1 = 230\text{V}$$

$$f = 50\text{Hz}$$

To find:

- i) Total current: I
- ii) Individual voltage: V_R , V_C , V_L
- iii) Power factor: $\cos\phi$
- iv) Phasor diagram

Solution:

$$X_L = 2\pi fL_1$$

$$\therefore X_L = 2 \times 3.142 \times 50 \times 0.05$$

$$\therefore X_L = 15.71\ \Omega$$

$$X_C = \frac{1}{2\pi fC_1}$$

$$\therefore X_C = \frac{1}{2 \times 3.142 \times 50 \times 50 \times 10^{-6}}$$

$$\therefore X_C = 63.6537\ \Omega$$

Now,

$$X_L - X_C = 63.6537 - 15.71$$

$$\therefore X_L - X_C = 47.9437 \Omega$$

Impedance of the circuit is

$$Z_T = R_1 - j(X_C - X_L)$$

$$\therefore Z_T = 20 - j47.9437$$

$$\therefore Z_T = \mathbf{51.948 \angle -67.3562^\circ \Omega}$$

i) Total current:

$$I = \frac{V_1}{Z_T}$$

$$\therefore I = \frac{230 \angle 0^\circ}{51.948 \angle -67.3562^\circ}$$

$$\therefore I = 4.4275 \angle 67.3562^\circ$$

$$\therefore I = \mathbf{4.4275 \angle 67.3562^\circ A}$$

ii) Individual voltage:

$$V_R = I \times R_1$$

$$\therefore V_R = 4.4275 \times 20$$

$$\therefore V_R = 88.55 \angle 67.3562^\circ$$

$$\therefore V_R = \mathbf{88.55 \angle 67.3562^\circ V}$$

$$V_L = I \times X_L$$

$$\therefore V_L = 4.4275 \angle -67.3562^\circ \times 15.71 \angle 90^\circ$$

$$\therefore V_L = 69.556 \angle 157.3562^\circ$$

$$\therefore V_L = \mathbf{69.556 \angle 157.3562^\circ V}$$

$$V_C = I \times X_C$$

$$\therefore V_C = 4.4275 \angle -67.3562^\circ \times 63.6537 \angle -90^\circ$$

$$\therefore V_C = 281.8268 \angle -22.6437^\circ$$

$$\therefore V_C = \mathbf{281.8268 \angle -22.6437^\circ V}$$

iii) Power factor:

$$\cos\phi = \frac{R_1}{Z_T}$$

$$\therefore \cos\phi = \frac{20}{51.9480}$$

$$\therefore \cos\phi = 0.385$$

\therefore Power factor = 0.385 (Leading)

iv) Phasor diagram:

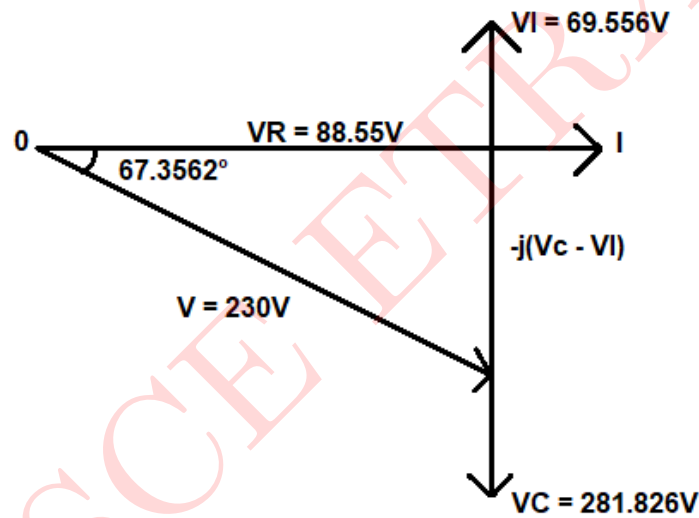


Figure 2: Phasor diagram

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

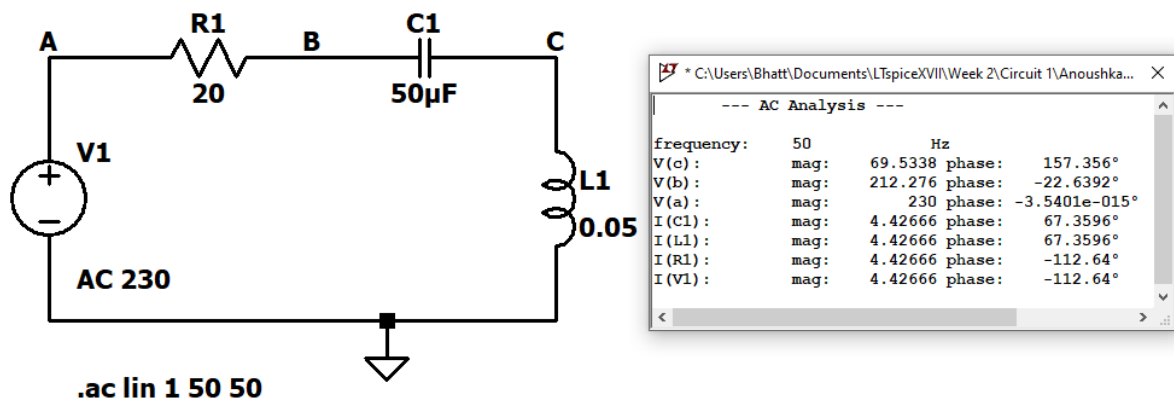


Figure 3: Circuit Schematic and Simulated Results: To determine V_L

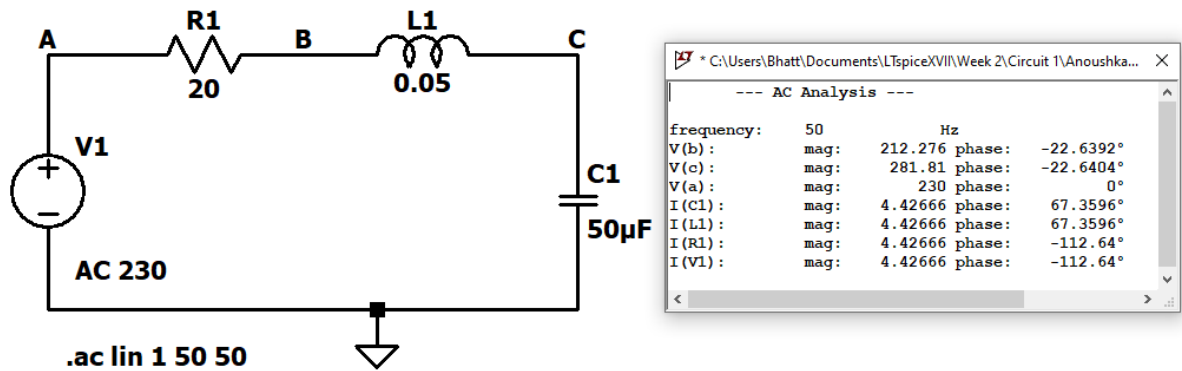


Figure 4: Circuit Schematic and Simulated Results: To determine V_C

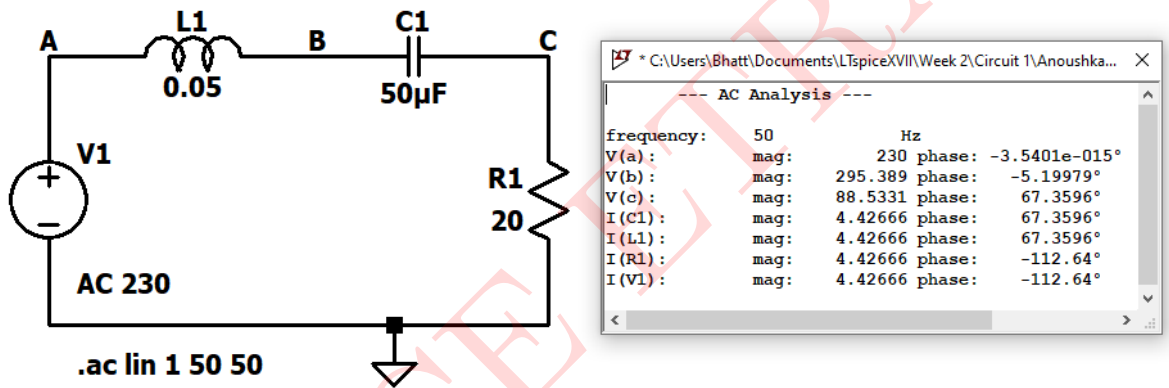


Figure 5: Circuit Schematic and Simulated Results: To determine V_R

Comparison of theoretical and simulated values:

| Parameters | Theoretical Values | Simulated Values |
|--------------|--|--|
| I | $4.4275 \angle 67.3562^\circ \text{ A}$ | $4.4266 \angle 67.3596^\circ \text{ A}$ |
| V_R | $88.55 \angle 67.3562^\circ \text{ V}$ | $88.5331 \angle 67.3596^\circ \text{ V}$ |
| V_C | $281.826 \angle -22.643^\circ \text{ V}$ | $281.81 \angle -22.6404^\circ \text{ V}$ |
| V_L | $69.556 \angle 157.356^\circ \text{ V}$ | $69.5338 \angle 157.356^\circ \text{ V}$ |
| Power factor | 0.385 | 0.384 |

Table 1: Numerical 1

Numerical 2: A 50Hz sinusoidal voltage $V = 141\sin\omega t$ is applied to a series R-L circuit. The values of the resistance and the inductance are $5\ \Omega$ and 0.02H respectively. Calculate:

- The RMS value of the current in the circuit and its phase angle with respect to the voltage
- The RMS value and the phase value of the voltages appearing across the resistance and the inductance
- Power factor of the circuit

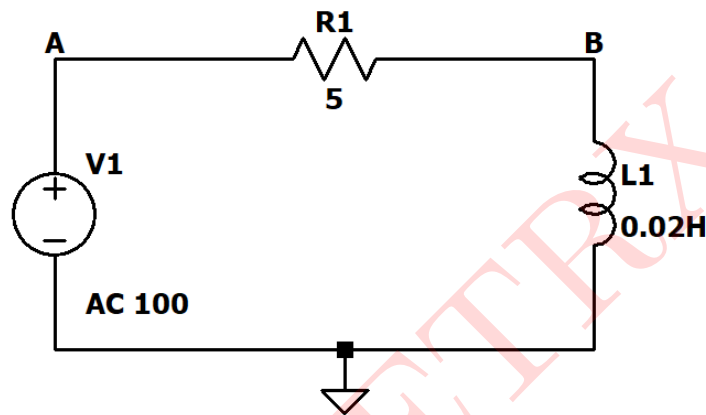


Figure 6: Circuit 2

Given:

$$R_1 = 5\ \Omega$$

$$L_1 = 0.02\text{H}$$

$$V_m = 141\text{V}$$

$$f = 50\text{Hz}$$

To find:

- The RMS value of the current and its phase angle with respect to the voltage
- The RMS value and the phase value of the voltages appearing across the resistance and the inductance
- Power factor

Solution:

$$V_1 = \frac{V_m}{\sqrt{2}}$$

$$\therefore V_1 = \frac{141}{\sqrt{2}}$$

$$\therefore V_1 = 100\text{V}$$

$$X_L = 2\pi fL_1$$

$$\therefore X_L = 2 \times 3.142 \times 50 \times 0.02$$

$$\therefore X_L = 6.284\ \Omega$$

Now, impedance of the circuit is

$$Z_T = R_1 + jX_L$$

$$\therefore Z_T = 5 + j6.284$$

$$\therefore Z_T = \mathbf{8.03048 \angle 51.4917^\circ \Omega}$$

i) RMS value and phase value of the current:

$$I = \frac{V_1}{Z_T}$$

$$\therefore I = \frac{100 \angle 0^\circ}{8.03048 \angle 51.4917^\circ}$$

$$\therefore I = 12.4526 \angle -51.4917^\circ$$

$$\therefore I = \mathbf{12.4526 A}$$

$$\therefore \phi = \mathbf{51.4917^\circ}$$

Since the phase is negative, current is lagging.

ii) RMS value and phase value of the current individual voltage:

$$V_R = I \times R_1$$

$$\therefore V_R = 12.4526 \angle -51.4917^\circ \times 5 \angle 0^\circ$$

$$\therefore V_R = 62.2628 \angle -51.4917^\circ$$

$$\therefore V_R = \mathbf{62.2628 V}$$

$$V_L = I \times X_L$$

$$\therefore V_L = 12.4526 \angle -51.4917^\circ \times 6.284 \angle 90^\circ$$

$$\therefore V_L = 78.2518 \angle 38.5083^\circ$$

$$\therefore V_L = \mathbf{78.2518 V}$$

iii) Power factor:

$$\cos \phi = \cos(51.4917^\circ)$$

$$\therefore \cos \phi = 0.6226$$

$$\therefore \mathbf{\text{Power factor} = 0.6226 \text{ (Lagging)}}$$

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

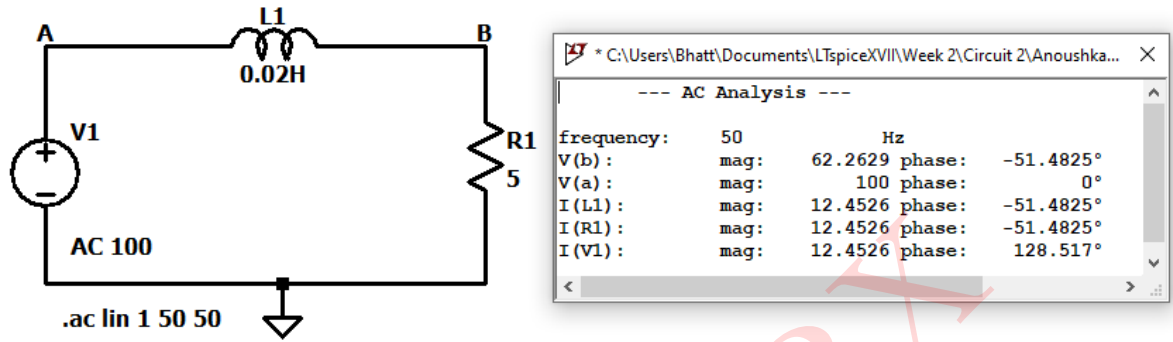


Figure 7: Circuit Schematic and Simulated Results: To determine V_R

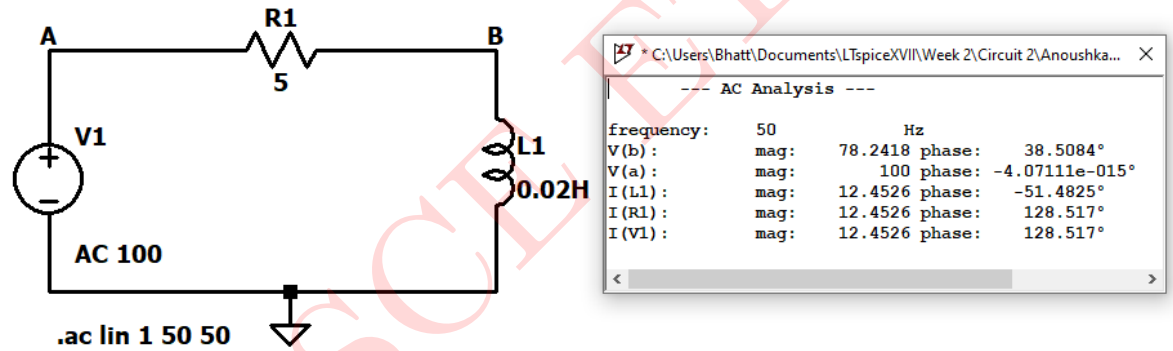


Figure 8: Circuit Schematic and Simulated Results: To determine V_L

Comparison of theoretical and simulated values:

| Parameters | Theoretical Values | Simulated Values |
|--------------|---|---|
| I | $12.4526 \angle -51.4917^\circ \text{ A}$ | $12.4526 \angle -51.4825^\circ \text{ A}$ |
| V_R | $62.2628 \angle -51.4917^\circ \text{ V}$ | $62.2629 \angle -51.4825^\circ \text{ V}$ |
| V_L | $78.2518 \angle 38.5083^\circ \text{ V}$ | $78.2418 \angle 38.5084^\circ \text{ V}$ |
| Power factor | 0.6226 | 0.6227 |

Table 2: Numerical 2

- Numerical 3:** A pure resistance of $40\ \Omega$ is in series with a pure capacitance of $47\mu\text{F}$. The series combination is connected across 220V, 50Hz supply. Find
- The impedance
 - Current
 - Power factor
 - Phase angle
 - Voltage across resistor
 - Voltage across capacitor

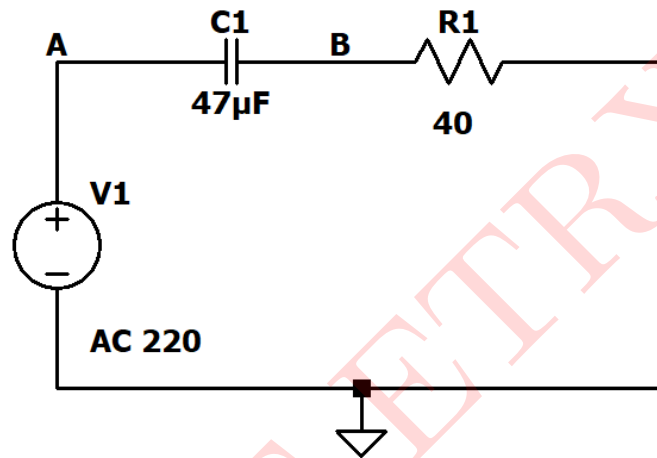


Figure 9: Circuit 3

Given:

$$R_1 = 40\ \Omega$$

$$C_1 = 47\mu\text{F}$$

$$V_1 = 220\text{V}$$

$$f = 50\text{Hz}$$

To find:

- The impedance: Z_T
- Current: I
- Power factor
- Phase angle: ϕ
- Voltage across resistor: V_R
- Voltage across capacitor: V_C

Solution:

$$X_C = \frac{1}{2\pi f C_1}$$

$$\therefore X_C = \frac{1}{2 \times 3.142 \times 50 \times 47 \times 10^{-6}}$$

$$\therefore X_C = 67.7167\ \Omega$$

i) Impedance of the circuit is

$$Z_T = \sqrt{R_1^2 + X_C^2}$$
$$\therefore Z_T = \sqrt{40^2 + 67.7167^2}$$
$$\therefore Z_T = \mathbf{78.6482 \, \Omega}$$

ii) Current:

$$I = \frac{V_1}{Z_T}$$
$$\therefore I = \frac{220}{78.6482}$$
$$\therefore I = \mathbf{2.7969A}$$

iii) Power factor:

$$\cos\phi = \frac{R_1}{Z_T}$$
$$\therefore \cos\phi = \frac{40}{78.64482}$$
$$\therefore \cos\phi = 0.508$$
$$\therefore \text{Power factor} = \mathbf{0.508 \text{ (Leading)}}$$

iv) Phase angle:

$$\phi = \cos^{-1}\left(\frac{R_1}{Z_T}\right)$$
$$\therefore \phi = \cos^{-1}\left(\frac{40}{78.6482}\right)$$
$$\therefore \phi = \cos^{-1}(0.508)$$
$$\therefore \phi = 59.4297^\circ$$
$$\therefore \text{Phase angle} = \mathbf{59.4297^\circ}$$

v) Voltage across resistor:

$$V_R = I \times R_1$$
$$\therefore V_R = 2.7973 \angle 59.4297^\circ \times 40 \angle 0^\circ$$
$$\therefore V_R = 111.89 \angle 59.4297^\circ$$
$$\therefore V_R = \mathbf{111.89V}$$

v) Voltage across capacitor:

$$V_C = I \times X_C$$

$$\therefore V_C = 2.7973 \angle 59.4297^\circ \times 67.7167 \angle -90^\circ$$

$$\therefore V_C = 189.4239 \angle -30.5703^\circ$$

$$\therefore V_C = 189.4239V$$

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

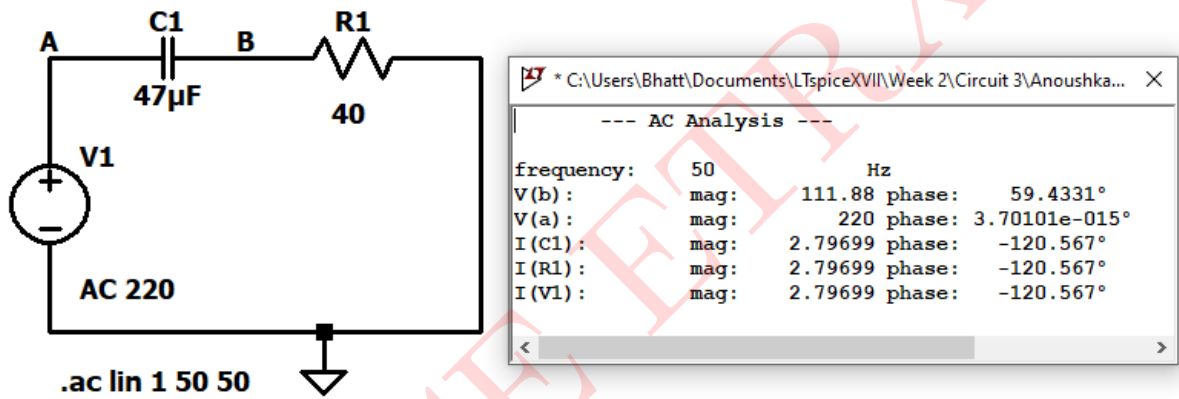


Figure 10: Circuit Schematic and Simulated Results: To determine V_R

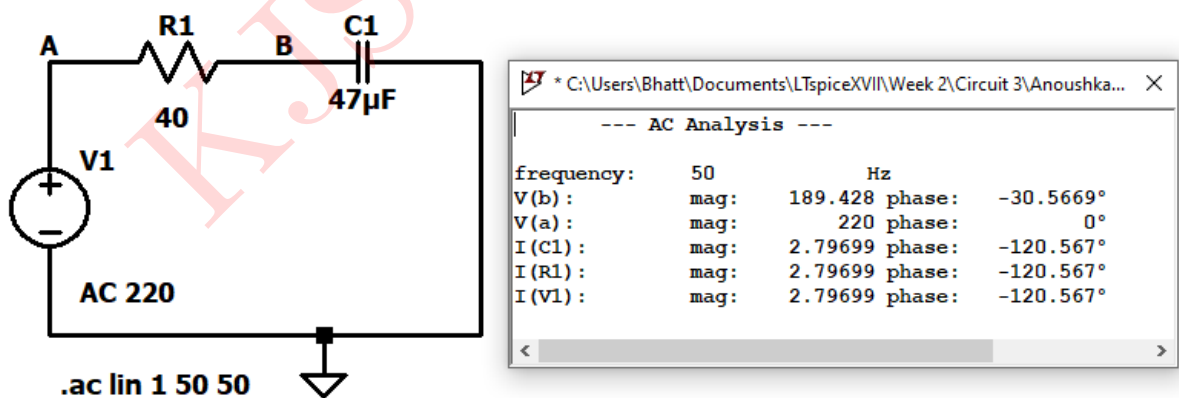


Figure 11: Circuit Schematic and Simulated Results: To determine V_C

Comparison of theoretical and simulated values:

| Parameters | Theoretical Values | Simulated Values |
|--------------|---|--|
| I | $2.7976\angle 59.4297^\circ \text{ A}$ | $2.7969\angle 59.4297^\circ \text{ A}$ |
| V_R | $111.89\angle 59.4297^\circ \text{ V}$ | $111.88\angle 59.4331^\circ \text{ V}$ |
| V_C | $189.4239\angle -30.5703^\circ \text{ V}$ | $189.428\angle -30.5669^\circ \text{ V}$ |
| Power factor | 0.508 | 0.508 |

Table 3: Numerical 3

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Numerical 4: The circuit shown in figure 12 consists of resistance of $15\ \Omega$, an inductance of 84mH and a capacitor of $60\mu\text{F}$ are connected in parallel across a 110V , 50Hz supply.

Calculate:

- Individual currents drawn by each element
- Total current drawn by the supply
- Overall power factor of circuit
- Draw phasor diagram

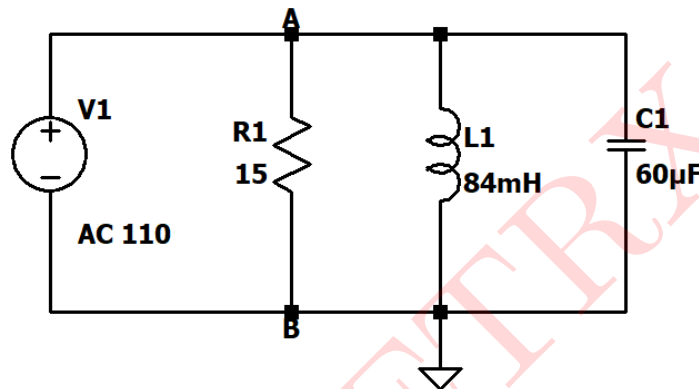


Figure 12: Circuit 4

Given:

$$R_1 = 15\ \Omega$$

$$L_1 = 84\text{mH}$$

$$C_1 = 60\mu\text{F}$$

$$V_1 = 110\text{V}$$

$$f = 50\text{Hz}$$

To find:

- Individual currents: I_R, I_C, I_L
- Total current: I
- Power factor: $\cos\phi$
- Phasor diagram

Solution:

$$X_L = 2\pi f L_1$$

$$\therefore X_L = 2 \times 3.142 \times 50 \times 84 \times 10^{-3}$$

$$\therefore X_L = 26.3928\ \Omega$$

$$X_C = \frac{1}{2\pi f C_1}$$

$$\therefore X_C = \frac{1}{2 \times 3.142 \times 50 \times 60 \times 10^{-6}}$$

$$\therefore X_C = 53.0447\ \Omega$$

i) Individual currents:

$$I_R = \frac{V_1}{R_1}$$

$$\therefore I_R = \frac{110\angle 0^\circ}{15\angle 0^\circ}$$

$$\therefore I_R = 7.3333\angle 0^\circ$$

$$\therefore I_R = \mathbf{7.3333A}$$

$$I_C = \frac{V_1}{X_C}$$

$$\therefore I_C = \frac{110\angle 0^\circ}{53.0447\angle -90^\circ}$$

$$\therefore I_C = 2.0737\angle 90^\circ$$

$$\therefore I_C = \mathbf{2.0737A}$$

$$I_L = \frac{V_1}{X_L}$$

$$\therefore I_L = \frac{110\angle 0^\circ}{26.3928\angle 90^\circ}$$

$$\therefore I_L = 4.1683\angle -90^\circ$$

$$\therefore I_L = \mathbf{4.1683A}$$

Now, impedance of the circuit is

$$\frac{1}{Z} = \sqrt{\left(\frac{1}{R_1}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

$$\therefore \frac{1}{Z} = \sqrt{\left(\frac{1}{15}\right)^2 + \left(\frac{1}{4.1683} - \frac{1}{53.04}\right)^2}$$

$$\therefore \frac{1}{Z} = 0.0693\angle -15.9369^\circ$$

$$\therefore Z_T = \frac{1}{Z} = \mathbf{14.4234\angle 15.9369^\circ \Omega}$$

ii) Total current:

$$I = \frac{V_1}{Z_T}$$

$$\therefore I = \frac{110\angle 0^\circ}{14.4234\angle 15.9369^\circ}$$

$$\therefore I = 7.6268\angle -15.9369^\circ$$

$$\therefore I = 7.6268A \text{ and } \cos\phi = 15.9369^\circ$$

iii) Power factor:

$$\therefore \cos\phi = \cos(15.9369^\circ)$$

$$\therefore \cos\phi = 0.9615$$

$$\therefore \text{Power factor} = 0.9615$$

iv) Phasor diagram:

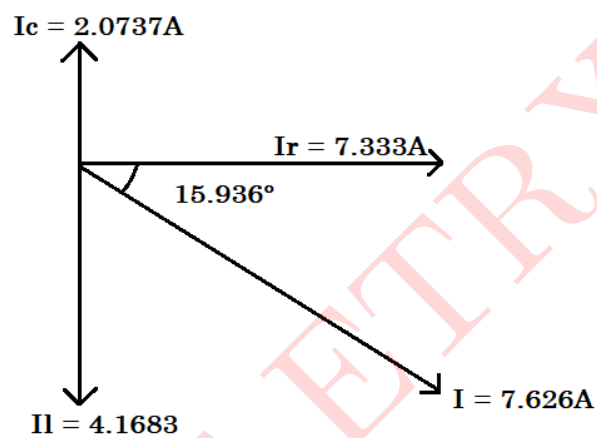


Figure 13: Phasor diagram

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

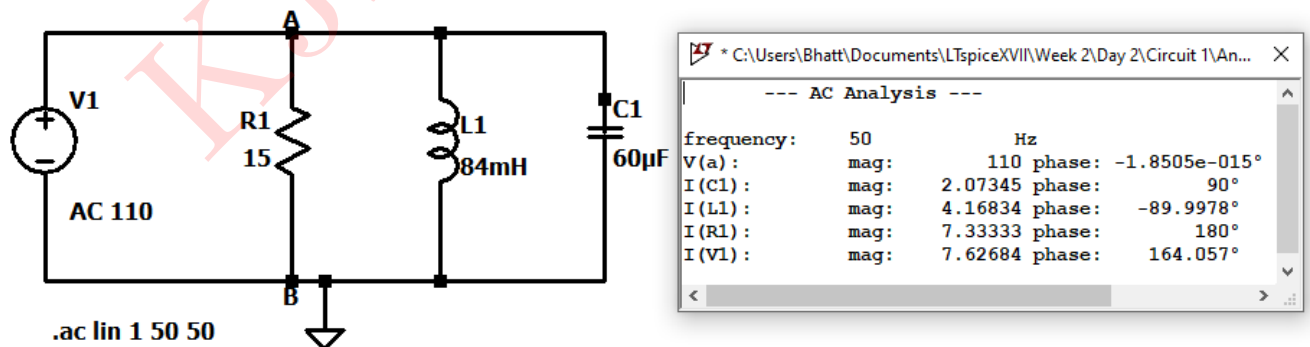


Figure 14: Circuit Schematic and Simulated Results

Comparison of theoretical and simulated values:

| Parameters | Theoretical Values | Simulated Values |
|--------------|---|---|
| I_R | $7.3333\angle 0^\circ \text{ A}$ | $7.3333\angle 0^\circ \text{ A}$ |
| I_L | $4.1683\angle -90^\circ \text{ A}$ | $4.1683\angle -90^\circ \text{ A}$ |
| I_C | $2.0737\angle 90^\circ \text{ A}$ | $2.07345\angle 90^\circ \text{ A}$ |
| I | $7.6268\angle -15.9369^\circ \text{ A}$ | $7.6268\angle -15.9369^\circ \text{ A}$ |
| Power factor | 0.9615 | 0.9615 |

Table 4: Numerical 4

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Numerical 5: A coil having a resistance of $R_1 = 3 \Omega$ and an inductance of $L_1 = 0.015\text{H}$ is arranged in parallel with another coil having a resistance of $R_2 = 0.5 \Omega$ and an inductance of $L_2 = 0.06\text{H}$. Calculate current I , I_1 and I_2 when voltage of $V_1 = 100\text{V}$ at 50Hz is applied. Also calculate the power factor.

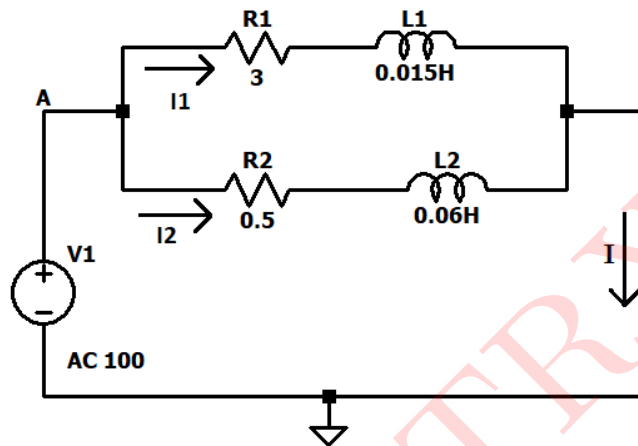


Figure 15: Circuit 5

Given:

$$\begin{aligned} R_1 &= 3 \Omega \\ L_1 &= 0.015\text{H} \\ R_2 &= 0.5 \Omega \\ L_2 &= 0.06\text{H} \\ V_1 &= 100\text{V} \\ f &= 50\text{Hz} \end{aligned}$$

To find:

- i) Total current: I
- ii) Individual currents: I_1, I_2
- iii) Power factor: $\cos\phi$

Solution:

$$\begin{aligned} X_{L1} &= 2\pi f L_1 \\ \therefore X_L &= 2 \times 3.142 \times 50 \times 0.015 \\ \therefore X_L &= 4.713 \Omega \end{aligned}$$

$$\begin{aligned} X_{L2} &= 2\pi f L_2 \\ \therefore X_L &= 2 \times 3.142 \times 50 \times 0.06 \\ \therefore X_L &= 18.852 \Omega \end{aligned}$$

$$Z_1 = R_1 + jX_{L1}$$

$$\therefore Z_1 = 3 + j4.731$$

$$\therefore Z_1 = 5.5868 \angle -57.5217^\circ \Omega$$

$$Z_2 = R_2 + jX_{L2}$$

$$\therefore Z_2 = 0.5 + j18.852$$

$$\therefore Z_2 = 18.8586 \angle -88.4807^\circ \Omega$$

Now,

$$Y = \frac{Z_1 + Z_2}{Z_1 \times Z_2}$$

$$\therefore Y = \frac{(3 + j4.731) + (0.5 + j18.852)}{(3 + j4.731) \times (0.5 + j18.852)}$$

$$\therefore Y = \frac{3.5 + j23.565}{-87.349 + j58.9125}$$

$$\therefore Y = \mathbf{0.2261 \angle -64.45^\circ \Omega}$$

i) Total current:

$$I = V_1 \times Y$$

$$\therefore I = 100 \angle 0^\circ \times 0.2261 \angle -64.45^\circ$$

$$\therefore I = \mathbf{22.6131 \angle -64.45^\circ A}$$

ii) Individual currents:

$$I_1 = \frac{V_1}{Z_1}$$

$$\therefore I_1 = \frac{100 \angle 0^\circ}{5.5868 \angle 57.5217^\circ}$$

$$\therefore I_1 = 17.8993 \angle -57.5217^\circ$$

$$\therefore I_1 = \mathbf{17.8993 \angle -57.5217^\circ A}$$

$$I_2 = \frac{V_1}{Z_2}$$

$$\therefore I_2 = \frac{100 \angle 0^\circ}{18.8586 \angle 88.4807^\circ}$$

$$\therefore I_2 = 5.3026 \angle -88.4807^\circ$$

$$\therefore I_2 = \mathbf{5.3026 \angle -88.4807^\circ A}$$

iii) Power factor:

$$\cos\phi = \cos(64.45^\circ)$$

$$\therefore \cos\phi = 0.4312$$

$$\therefore \text{Power factor} = 0.4312$$

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

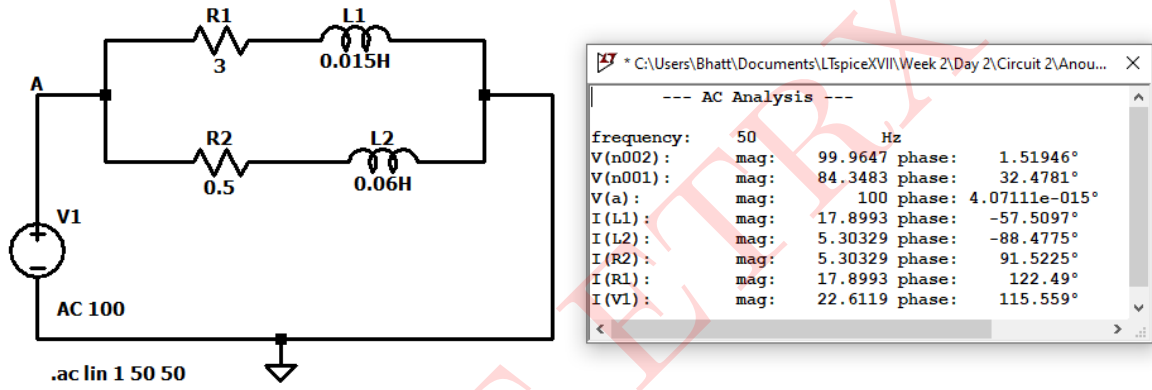


Figure 16: Circuit Schematic and Simulated Results

Comparison of theoretical and simulated values:

| Parameters | Theoretical Values | Simulated Values |
|--------------|--|--|
| I | $22.6131 \angle -64.45^\circ \text{ A}$ | $22.6119 \angle -64.441^\circ \text{ A}$ |
| I_1 | $17.8993 \angle -57.521^\circ \text{ A}$ | $17.8993 \angle -57.5^\circ \text{ A}$ |
| I_2 | $5.3026 \angle -88.4807^\circ \text{ A}$ | $5.303 \angle -88.477^\circ \text{ A}$ |
| Power factor | 0.4312 | 0.4312 |

Table 5: Numerical 5

Numerical 6: Find I , I_1 and I_2 and voltage drop in each branch in the circuit shown in figure 17 if $R_1 = 6\Omega$, $L_1 = j4\Omega$, $R_2 = 20\Omega$, $L_2 = j8\Omega$, $R_3 = 9\Omega$, $C_1 = -j6\Omega$ and $V = 100V$, frequency = 50Hz.

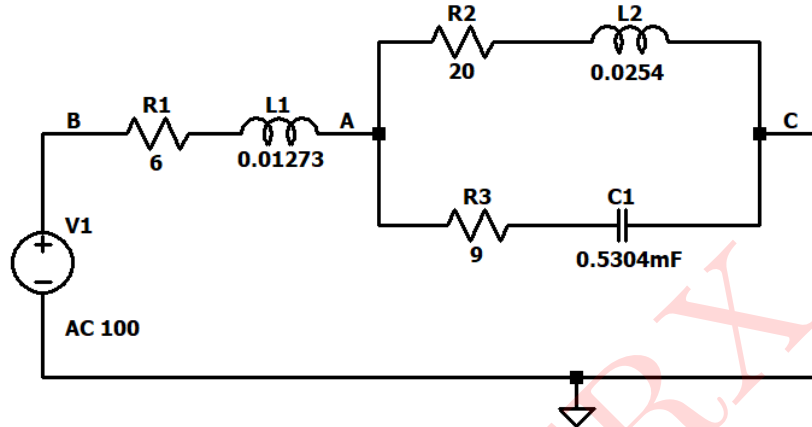


Figure 17: Circuit 6

Given:

$$\begin{aligned} R_1 &= 6\Omega \\ L_1 &= j4\Omega \\ R_2 &= 20\Omega \\ L_2 &= j8\Omega \\ R_3 &= 9\Omega \\ C_1 &= -j6\Omega \\ V_1 &= 100V \\ f &= 50\text{Hz} \end{aligned}$$

To find:

- Total current: I
- Individual voltages: V_{AB} , V_{BC}
- Individual currents: I_1 , I_2

Solution:

$$\begin{aligned} Z_1 &= R_1 + jL_1 \\ \therefore Z_1 &= 6 + j4 \\ \therefore Z_1 &= 7.2111\angle 33.69^\circ \Omega \end{aligned}$$

$$\begin{aligned} Z_2 &= R_2 + jL_2 \\ \therefore Z_2 &= 20 + j8 \\ \therefore Z_2 &= 21.54\angle 21.801^\circ \Omega \end{aligned}$$

$$Z_3 = R_3 + jC_1$$

$$\therefore Z_3 = 9 - j6$$

$$\therefore Z_3 = 10.8166 \angle -33.697^\circ \Omega$$

Impedance of the circuit is

$$Z = Z_1 + \frac{Z_2 \times Z_3}{Z_2 + Z_3}$$

$$\therefore Z = 6 + j4 + \frac{(20 + j8) \times (9 - j6)}{20 + j8 + 9 - j6}$$

$$\therefore Z = 6 + j4 + \frac{(20 + j8) \times (9 - j6)}{29 + j2}$$

$$\therefore Z = 13.83 \angle 7.533^\circ \Omega$$

i) **Total current:**

$$I = \frac{V_1}{Z}$$

$$\therefore I = \frac{100 \angle 0^\circ}{1.83 \angle 7.533^\circ}$$

$$\therefore I = 7.2306 \angle -7.533^\circ$$

$$\therefore I = \mathbf{7.2306A}$$

ii) **Individual voltages:**

$$V_{AB} = I \times Z_1$$

$$\therefore V_{AB} = 7.2306 \angle -7.533^\circ \times 7.2111 \angle 33.69^\circ$$

$$\therefore V_{AB} = 52.1405 \angle 26.157^\circ$$

$$\therefore V_{AB} = \mathbf{52.1405V}$$

$$V_{BC} = I \times Z_2$$

$$\therefore V_{BC} = 7.2306 \angle -7.533^\circ \times 8.0151 \angle -15.834^\circ$$

$$\therefore V_{BC} = 57.9539 \angle -23.3674^\circ V$$

$$\therefore V_{BC} = \mathbf{57.9539V}$$

iii) **Individual currents:**

$$I_1 = I$$

$$\therefore I_1 = \mathbf{7.2306 \angle -7.533^\circ A}$$

$$I_2 = \frac{V_{BC}}{Z_2}$$

$$\therefore I_2 = \frac{57.5939 \angle -23.3674^\circ}{21.54 \angle 21.801^\circ}$$

$$\therefore I_2 = 2.6905 \angle -45.1684^\circ \text{ A}$$

$$I_3 = \frac{V_{BC}}{Z_3}$$

$$\therefore I_3 = \frac{57.5939 \angle -23.3674^\circ}{10.8166 \angle -33.69^\circ}$$

$$\therefore I_3 = 5.3578 \angle 10.3226^\circ \text{ A}$$

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

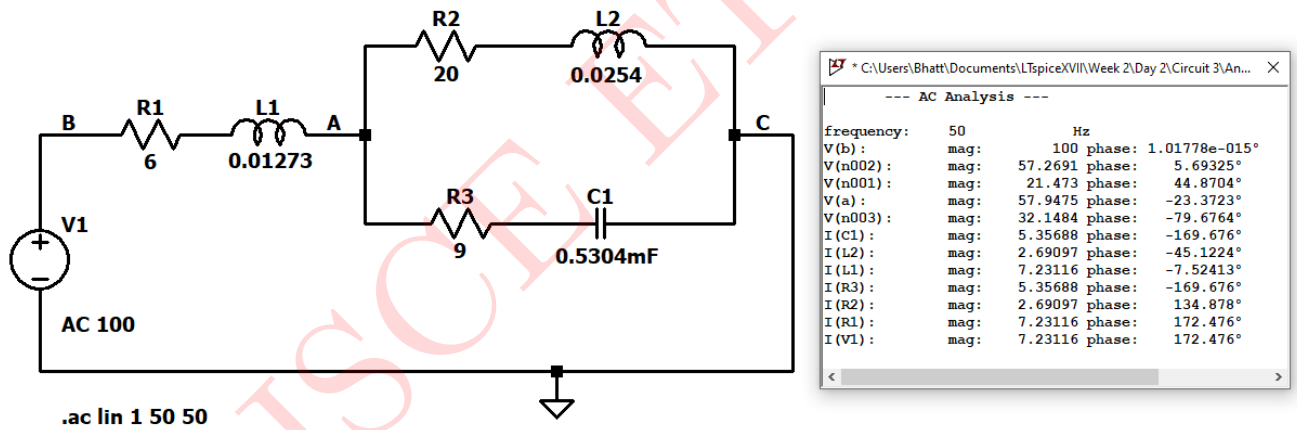


Figure 18: Circuit Schematic and Simulated Results

Calculation:

Voltage drop across branch AB

$$V_{AB} = V(a) - V(b)$$

$$\therefore V_{AB} = 100 - 57.9618$$

$$\therefore V_{AB} = 52.110V$$

Comparison of theoretical and simulated values:

| Parameters | Theoretical Values | Simulated Values |
|------------|--|--|
| I | $7.2306\angle 172.467^\circ \text{ A}$ | $7.2311\angle 172.467^\circ \text{ A}$ |
| I_1 | $7.2306\angle 172.467^\circ \text{ A}$ | $7.23116\angle 172.467^\circ \text{ A}$ |
| I_2 | $2.6905\angle -45.1684^\circ \text{ A}$ | $2.6909\angle -45.1224^\circ \text{ A}$ |
| I_3 | $5.3578\angle 169.677^\circ \text{ A}$ | $5.3568\angle 169.676^\circ \text{ A}$ |
| V_{AB} | $52.1405\angle 26.157^\circ \text{ V}$ | $52.11\angle 26.142^\circ \text{ V}$ |
| V_{BC} | $57.9539\angle -23.3674^\circ \text{ V}$ | $57.9618\angle -23.3358^\circ \text{ V}$ |

Table 6: Numerical 6

Numerical 7: A 50Hz sinusoidal voltage $V = 141\sin\omega t$ is applied to a series R-L circuit shown in figure 19. The values of the resistance and the inductance are $5\ \Omega$ and 0.02H respectively.

Determine the following:

- Calculate the peak voltage across resistor and inductor and also find the peak value of source current in LTspice.
- Plot input source voltage $V_S(t)$ Vs input source current $I_S(t)$ in LTspice.
- Measure the phase difference between $V_S(t)$ Vs $I_S(t)$ in time and degrees.
- Plot input source voltage $V_S(t)$ Vs input source voltage across resistor $V_R(t)$ in LTspice.
- Measure the phase difference between $V_S(t)$ Vs $V_R(t)$ in time and degrees.
- Plot input source voltage $V_S(t)$ Vs input source voltage across inductor $V_L(t)$ in LTspice.
- Measure the phase difference between $V_S(t)$ Vs $V_L(t)$ in time and degrees.
- Calculate power factor of the circuit

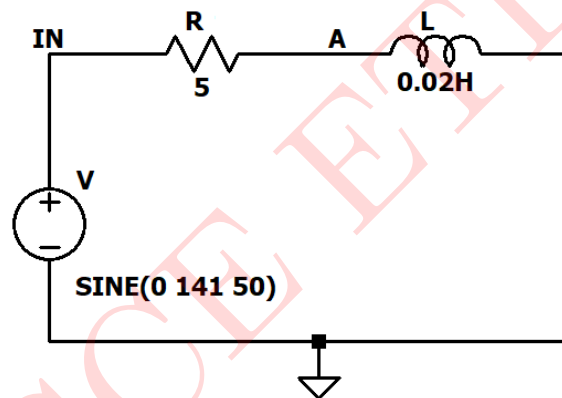


Figure 19: Circuit 7

Given:

$$R = 5\ \Omega$$

$$L = 0.02\text{H}$$

$$V_m = 141\text{V}$$

$$f = 50\text{Hz}$$

Solution:

$$V = \frac{V_m}{\sqrt{2}}$$

$$\therefore V = \frac{141}{\sqrt{2}}$$

$$\therefore V = 100\text{V}$$

$$X_L = 2\pi fL$$

$$\therefore X_L = 2 \times 3.142 \times 50 \times 0.02$$

$$\therefore X_L = 6.284 \Omega$$

$$Z = R + jX_L$$

$$\therefore Z = 5 + j6.284$$

$$\therefore Z = 8.03048 \angle 51.4917^\circ \Omega$$

$$I = \frac{V}{Z}$$

$$\therefore I = \frac{100 \angle 0^\circ}{8.03048 \angle 51.4917^\circ}$$

$$\therefore I = 12.4525 \angle -51.4917^\circ$$

$$\therefore I = \mathbf{12.4525A}$$

Now,

$$I_m = I \times \sqrt{2}$$

$$\therefore I_m = 12.4525 \times \sqrt{2}$$

$$\therefore I_m = \mathbf{17.61027A}$$

\therefore Peak value of source current is 17.61057A and the current lags behind the voltage by 51.4917°

$$V_R = I \times R$$

$$\therefore V_R = 12.4525 \angle -51.4917^\circ \times 5 \angle 0^\circ$$

$$\therefore V_R = \mathbf{62.2627 \angle -51.4917^\circ V}$$

Now,

$$V_{Rm} = V_R \times \sqrt{2}$$

$$\therefore V_{Rm} = 62.2627 \times \sqrt{2}$$

$$\therefore V_{Rm} = \mathbf{88.05286V}$$

\therefore Phase difference between input source voltage and voltage across resistor is 51.4917°

$$V_L = I \times X_L$$

$$\therefore V_L = 12.4525 \angle -51.4917^\circ \times 6.284 \angle 90^\circ$$

$$\therefore V_L = \mathbf{78.25186 \angle 38.5083^\circ V}$$

Now,

$$V_{Lm} = V_L \times \sqrt{2}$$

$$\therefore V_{Lm} = 78.25186 \times \sqrt{2}$$

$$\therefore V_{Lm} = \mathbf{110.6648 V}$$

\therefore Phase difference between input source voltage and voltage across inductor is 51.4917°

Power factor is

$$\cos \phi = \cos(51.4917^\circ)$$

$$\therefore \cos \phi = 0.6226$$

$$\therefore \mathbf{\text{Power factor} = 0.6226}$$

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

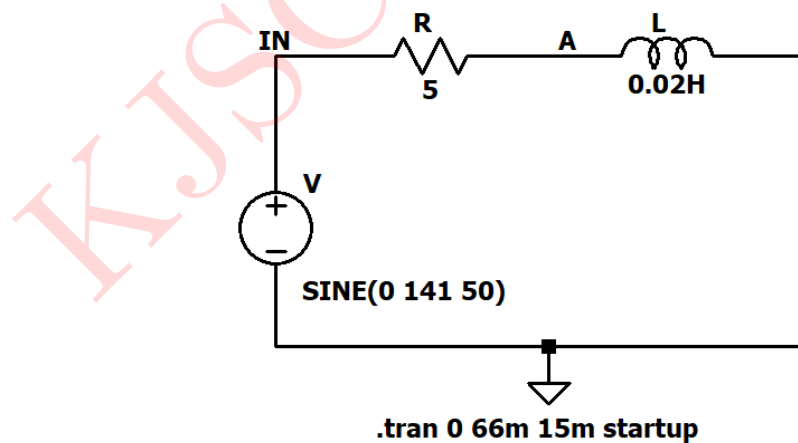


Figure 20: Circuit Schematic and Simulated Results

The input voltage and current waveforms are shown in figure 21

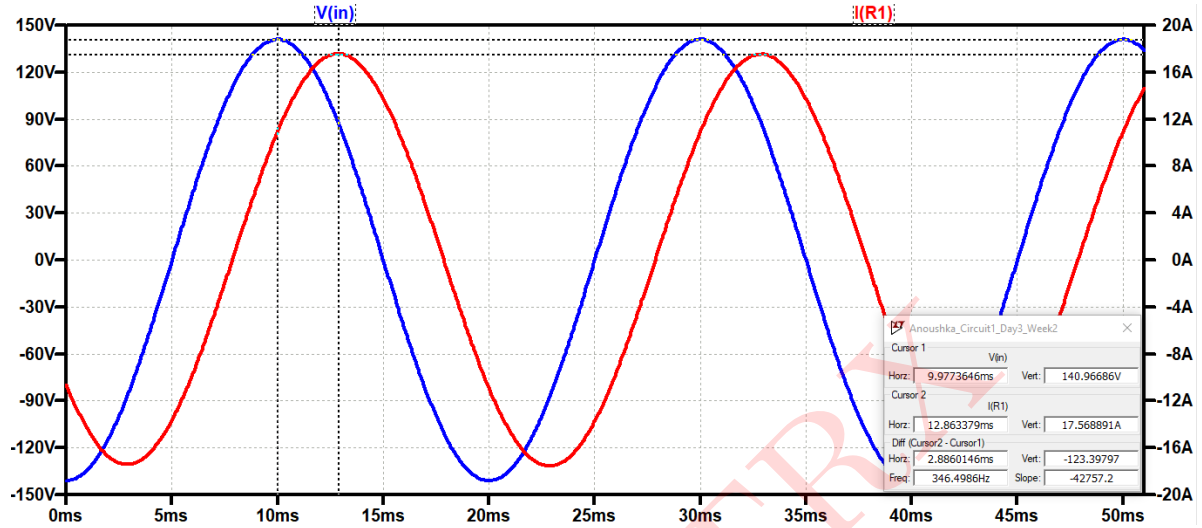


Figure 21: Input source voltage $V_S(t)$ Vs Input source current $I_S(t)$

Calculation:

Phase difference between $V_S(t)$ Vs $I_S(t)$

In time: $12.8634 - 9.9773 = 2.88601\text{ms}$

In degrees:

$$\phi = \frac{2.88601 \times 10^{-3} \times 360}{0.02}$$

$$\therefore \phi = 51.5117^\circ$$

The waveforms of input voltage and voltage across resistor are shown in figure 22

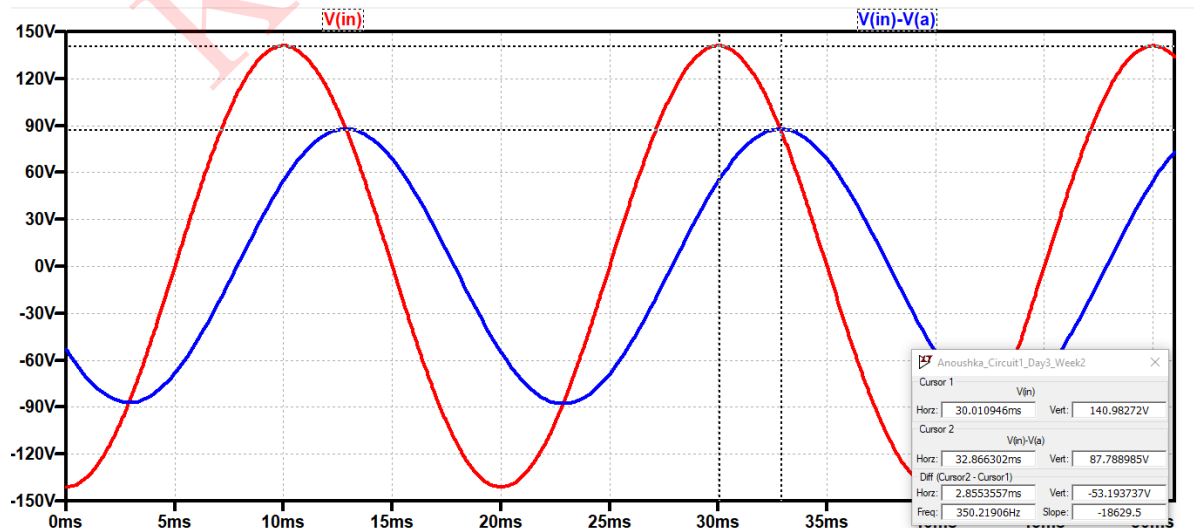


Figure 22: Input source voltage $V_S(t)$ Vs Voltage across resistor $V_R(t)$

Calculation:

Phase difference between $V_S(t)$ Vs $V_R(t)$

In time: $12.8634 - 9.9773 = 2.8553\text{ms}$

In degrees:

$$\phi = \frac{2.8553 \times 10^{-3} \times 360}{0.02}$$

$$\therefore \phi = 51.3964^\circ$$

The waveforms of input voltage and voltage across inductor are shown in figure 23

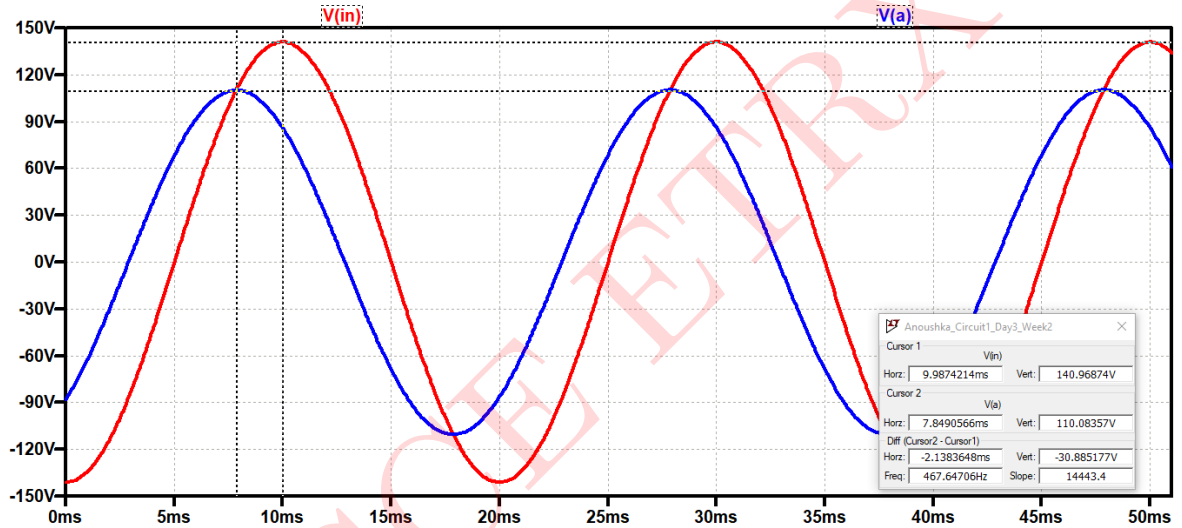


Figure 23: Input source voltage $V_S(t)$ Vs Voltage across inductor $V_L(t)$

Calculation:

Phase difference between $V_S(t)$ Vs $V_L(t)$

In time: $12.8634 - 9.9773 = 2.1383\text{ms}$

In degrees:

$$\phi = \frac{2.1383 \times 10^{-3} \times 360}{0.02}$$

$$\therefore \phi = 38.4905^\circ$$

Comparison of theoretical and simulated values:

| Parameters | Theoretical Values | Simulated Values |
|---|--------------------|------------------|
| Peak value of source current | 17.6105A | 17.5667A |
| Peak voltage across resistor | 88.05268V | 87.7889A |
| Peak voltage across inductor | 110.6648V | 110.0835V |
| Phase difference between $V_S(t)$ Vs $I_S(t)$ | 51.4917° | 51.5117° |
| Phase difference between $V_S(t)$ Vs $V_R(t)$ | 51.4917° | 51.3964° |
| Phase difference between $V_S(t)$ Vs $V_L(t)$ | 38.5083° | 38.4905° |
| Power factor | 0.6226 | 0.6226 |

Table 7: Numerical 7

Numerical 8: A pure resistance of $40\ \Omega$ is in series with a pure capacitance of $47\mu\text{F}$. The series combination is connected across 220V , 50Hz supply.

Determine the following:

- Calculate the peak voltage across resistor, capacitor and also find the peak value of source current in LTspice.
- Plot input source voltage $V_S(t)$ Vs input source current $I_S(t)$ in LTspice.
- Measure the phase difference between $V_S(t)$ Vs $I_S(t)$ in time and degrees.
- Plot input source voltage $V_S(t)$ Vs input source voltage across resistor $V_R(t)$ in LTspice.
- Measure the phase difference between $V_S(t)$ Vs $V_R(t)$ in time and degrees.
- Plot input source voltage $V_S(t)$ Vs input source voltage across capacitor $V_C(t)$ in LTspice.
- Measure the phase difference between $V_S(t)$ Vs $V_C(t)$ in time and degrees.
- Calculate power factor of the circuit.

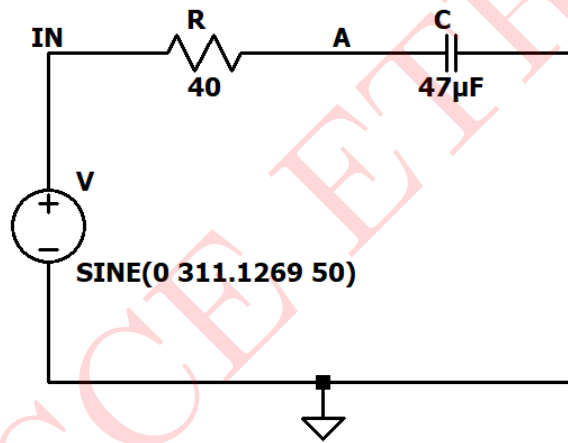


Figure 24: Circuit 8

Given:

$$R = 5\ \Omega$$

$$C = 47\mu\text{F}$$

$$V = 220\text{V}$$

$$f = 50\text{Hz}$$

Solution:

$$V_m = V \times \sqrt{2}$$

$$\therefore V_m = 220 \times \sqrt{2}$$

$$\therefore V_m = 311.1269\text{V}$$

$$X_C = \frac{1}{2\pi fC}$$

$$\therefore X_C = \frac{1}{2 \times 3.142 \times 47 \times 10^{-6}}$$

$$\therefore X_C = 67.7167\ \Omega$$

$$Z = R - jX_C$$

$$\therefore Z = 40 - j67.7167$$

$$\therefore Z = 78.6483 \angle -59.4298^\circ \Omega$$

$$I = \frac{V}{Z}$$

$$\therefore I = \frac{220 \angle 0^\circ}{78.6483 \angle -59.4298^\circ}$$

$$\therefore I = 2.7972 \angle 59.4298^\circ$$

$$\therefore I = \mathbf{2.7972A}$$

Now,

$$I_m = I \times \sqrt{2}$$

$$\therefore I_m = 2.7972 \times \sqrt{2}$$

$$\therefore I_m = \mathbf{3.9559A}$$

\therefore Peak value of source current is 3.9559A and the current leads the voltage by 59.4298°

$$V_R = I \times R$$

$$\therefore V_R = 2.7972 \angle 59.4298^\circ \times 40 \angle 0^\circ$$

$$\therefore V_R = \mathbf{111.888 \angle 59.4298^\circ V}$$

Now,

$$V_{Rm} = V_R \times \sqrt{2}$$

$$\therefore V_{Rm} = 111.888 \times \sqrt{2}$$

$$\therefore V_{Rm} = \mathbf{158.2335V}$$

\therefore Phase difference between input source voltage and voltage across resistor is 59.4298°

$$V_L = I \times X_C$$

$$\therefore V_C = 2.7972 \angle 59.4298^\circ \times 67.284 \angle -90^\circ$$

$$\therefore V_C = \mathbf{189.4171 \angle -30.5702^\circ V}$$

Now,

$$V_{Cm} = V_C \times \sqrt{2}$$

$$\therefore V_{Cm} = 189.4171 \times \sqrt{2}$$

$$\therefore V_{Cm} = \mathbf{267.8763V}$$

\therefore Phase difference between input source voltage and voltage across capacitor is 30.5702°

Power factor is

$$\cos\phi = \cos(59.4298^\circ)$$

$$\therefore \cos\phi = 0.5086$$

$$\therefore \text{Power factor} = 0.5086$$

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

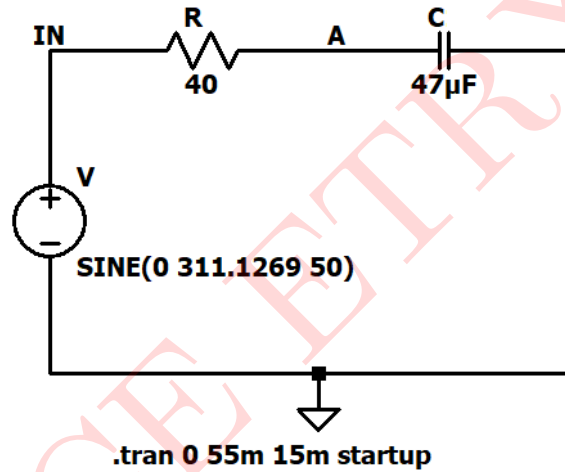


Figure 25: Circuit Schematic and Simulated Results

The input voltage and current waveforms are shown in figure 26

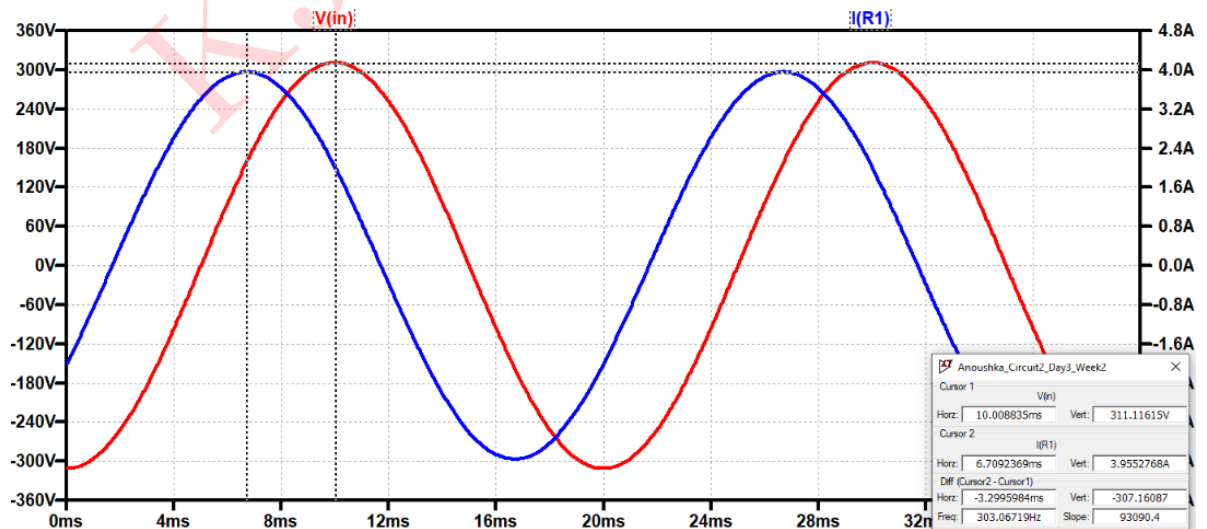


Figure 26: Input source voltage $V_S(t)$ Vs Input source current $I_S(t)$

Calculation:

Phase difference between $V_S(t)$ Vs $I_S(t)$

In time: $12.8634 - 9.9773 = 3.2995\text{ms}$

In degrees:

$$\phi = \frac{3.2995 \times 10^{-3} \times 360}{0.02}$$

$$\therefore \phi = 59.3927^\circ$$

The waveforms of input voltage and voltage across resistor are shown in figure 27

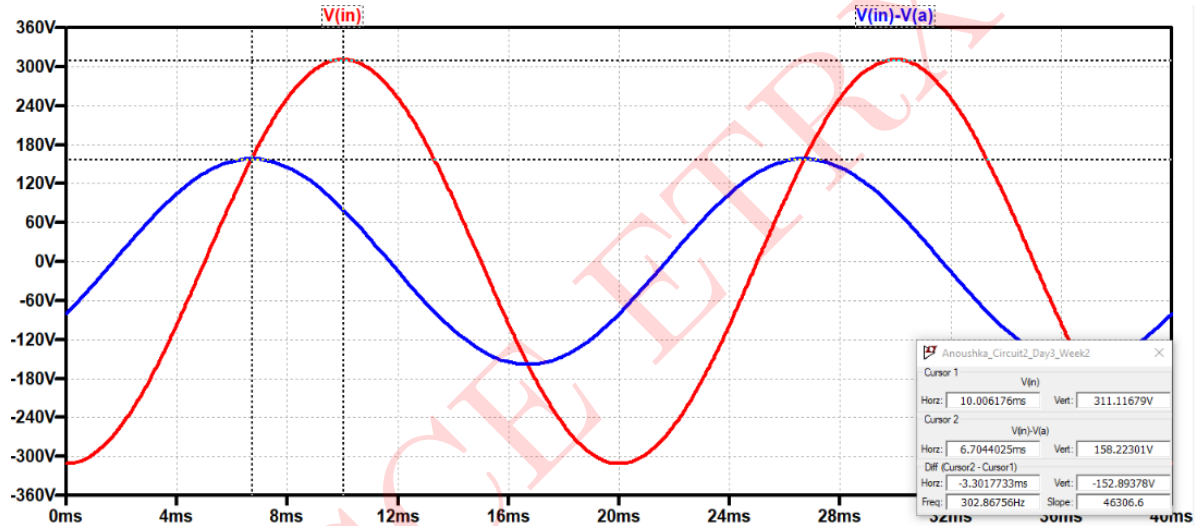


Figure 27: Input source voltage $V_S(t)$ Vs Voltage across resistor $V_R(t)$

Calculation:

Phase difference between $V_S(t)$ Vs $V_R(t)$

In time: $12.8634 - 9.9773 = 3.3017\text{ms}$

In degrees:

$$\phi = \frac{3.3017 \times 10^{-3} \times 360}{0.02}$$

$$\therefore \phi = 59.4319^\circ$$

The waveforms of input voltage and voltage across capacitor are shown in figure 28

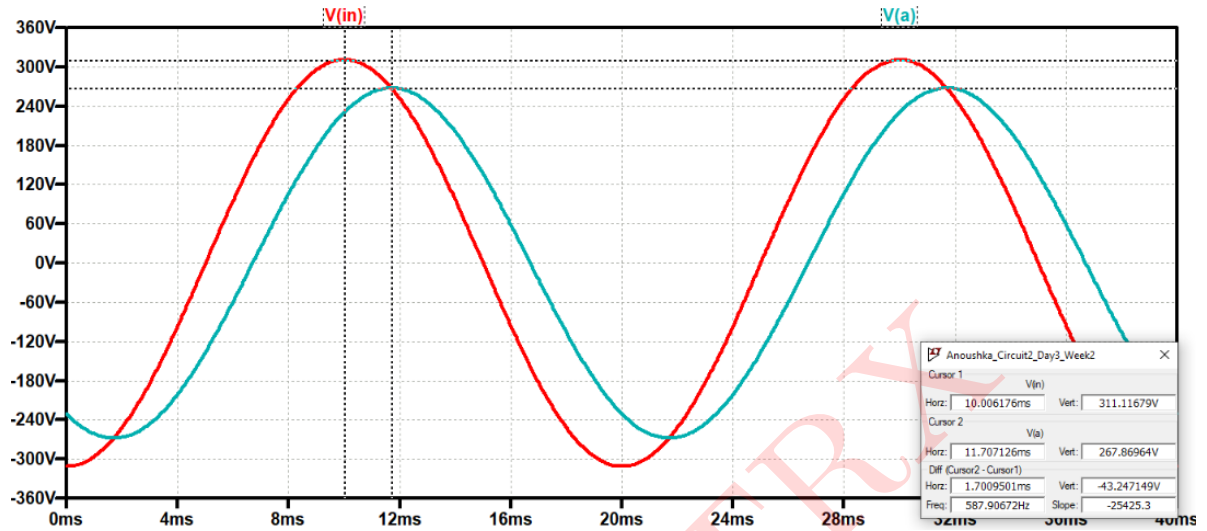


Figure 28: Input source voltage $V_S(t)$ Vs Voltage across capacitor $V_C(t)$

Calculation:

Phase difference between $V_S(t)$ Vs $V_C(t)$

In time: $12.8634 - 9.9773 = 1.7009\text{ms}$

In degrees:

$$\phi = \frac{1.7009 \times 10^{-3} \times 360}{0.02}$$

$$\therefore \phi = 30.6171^\circ$$

Comparison of theoretical and simulated values:

| Parameters | Theoretical Values | Simulated Values |
|---|--------------------|------------------|
| Peak value of source current | 3.9559A | 3.95522A |
| Peak voltage across resistor | 158.2335V | 158.22303A |
| Peak voltage across capacitor | 267.8763V | 267.8690V |
| Phase difference between $V_S(t)$ Vs $I_S(t)$ | 59.4289° | 59.3927° |
| Phase difference between $V_S(t)$ Vs $V_R(t)$ | 59.4289° | 59.4319° |
| Phase difference between $V_S(t)$ Vs $V_C(t)$ | 30.5702° | 30.6171° |
| Power factor | 0.5086 | 0.5091 |

Table 8: Numerical 8

Numerical 9: A series resonance network consisting of a resistor $30\ \Omega$, a capacitor of $1\mu\text{F}$ and an inductor of 30mH is connected across a sinusoidal supply voltage which has a constant supply output of AC 9V at all frequencies. Calculate the resonant frequency, the current at resonance, the voltage across inductor and capacitor at resonance, the quality factor and the bandwidth of the circuit. Plot the resonance curve, the current at resonance, the voltage across the inductor and capacitor at resonance in LTspice.

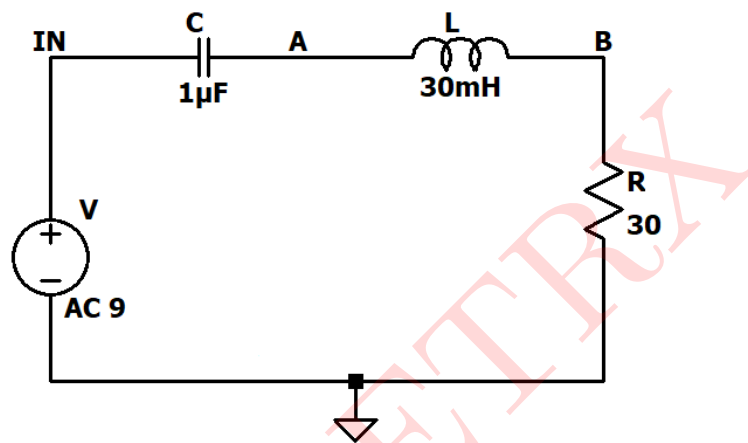


Figure 29: Circuit 9

Given:

$$\begin{aligned} R &= 30\ \Omega \\ L &= 30\text{mH} \\ C &= 1\mu\text{F} \\ V &= 9\text{V} \end{aligned}$$

To find:

- i) Resonant frequency: f_r
- ii) Current at resonance: I
- iii) Voltage across inductor and capacitor at resonance
- iv) Quality factor: Q
- v) Bandwidth: BW

Solution:

i) Resonant frequency

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore f_r = \frac{1}{2 \times 3.142\sqrt{30 \times 1 \times 10^{-9}}}$$

$$\therefore f_r = \mathbf{919.3475\text{Hz}}$$

ii) **Current at resonance:**

$$I = \frac{V}{R}$$

$$\therefore I = \frac{9}{30}$$

$$\therefore I = 0.3\text{A}$$

Now,

$$I_m = I \times \sqrt{2}$$

$$\therefore I_m = 0.3 \times \sqrt{2}$$

$$\therefore I_m = \mathbf{424.264\text{mA}}$$

$$X_L = 2\pi fL$$

$$X_L = 2 \times 3.142 \times 919.34 \times 30 \times 10^{-3}$$

$$X_L = 173.31 \Omega$$

iii) **Voltage across inductor and capacitor at resonance:**

$$V_L = V_C$$

$$V_L = I \times X_L$$

$$V_L = 0.3 \times 173.31$$

$$V_L = 51.9615\text{V}$$

Now,

$$\therefore V_{Lm} = V_L \times \sqrt{2}$$

$$\therefore V_{Lm} = 51.9615 \times \sqrt{2}$$

$$\therefore V_{Lm} = \mathbf{73.4846\text{V}}$$

$$\text{Also, } V_{Lm} = V_{Cm}$$

$$\therefore V_{Lm} = V_{Cm} = \mathbf{73.4846\text{V}}$$

iv) **Quality factor:**

$$Q = \frac{X_L}{R}$$

$$Q = \frac{173.31}{30}$$

$$Q = \mathbf{5.7664}$$

v) Bandwidth:

$$BW = \frac{f_r}{Q}$$

$$BW = \frac{919.3475}{5.7664}$$

$$BW = 159.1549\text{Hz}$$

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

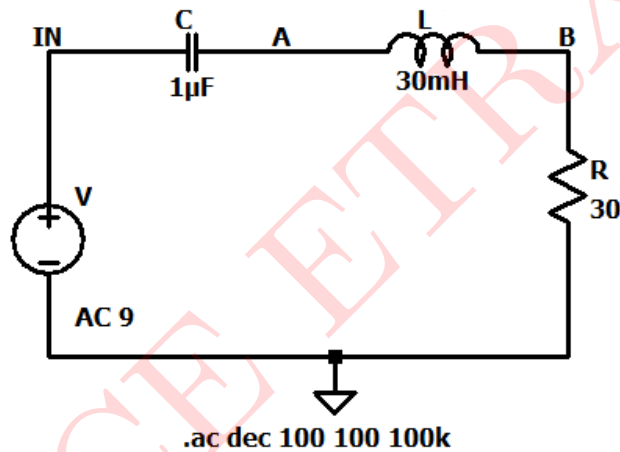


Figure 30: Circuit Schematic and Simulated Results: To determine f_r

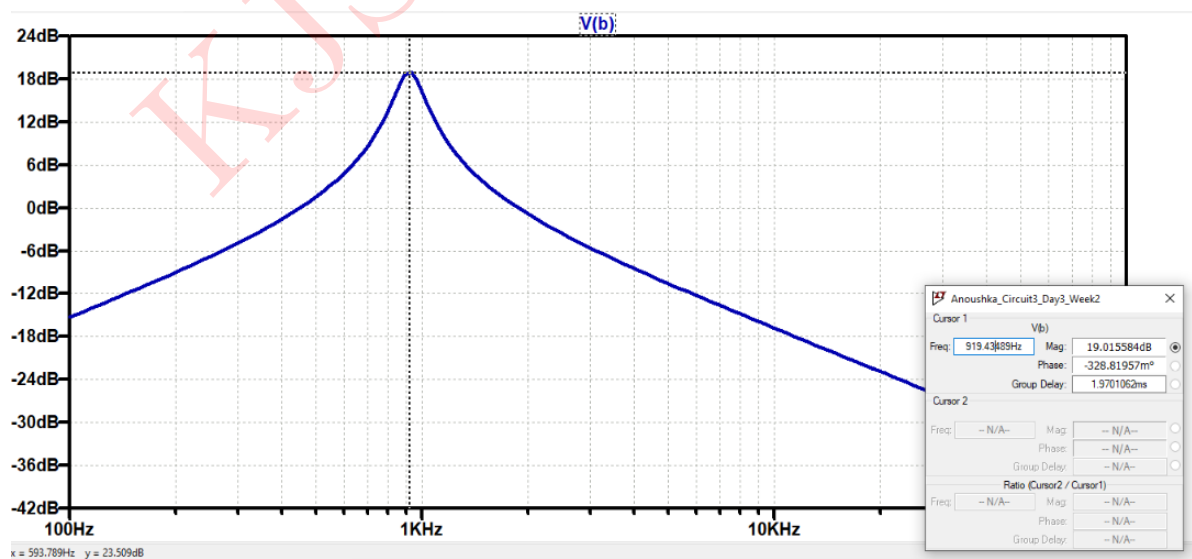


Figure 31: Resonance curve

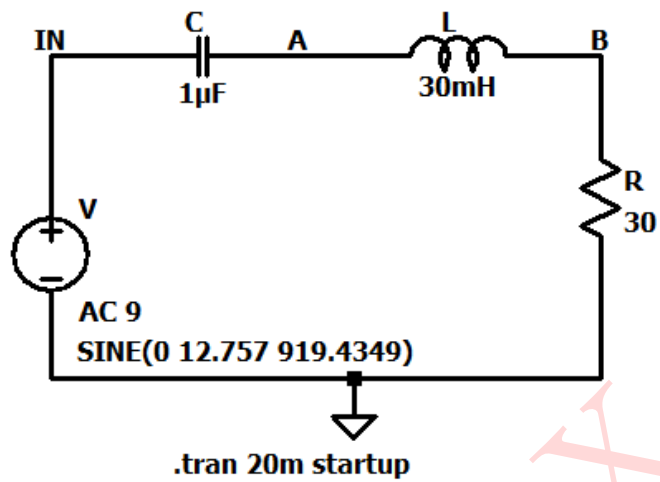


Figure 32: Circuit Schematic and Simulated Results

The waveform of current at resonance is shown in figure 33

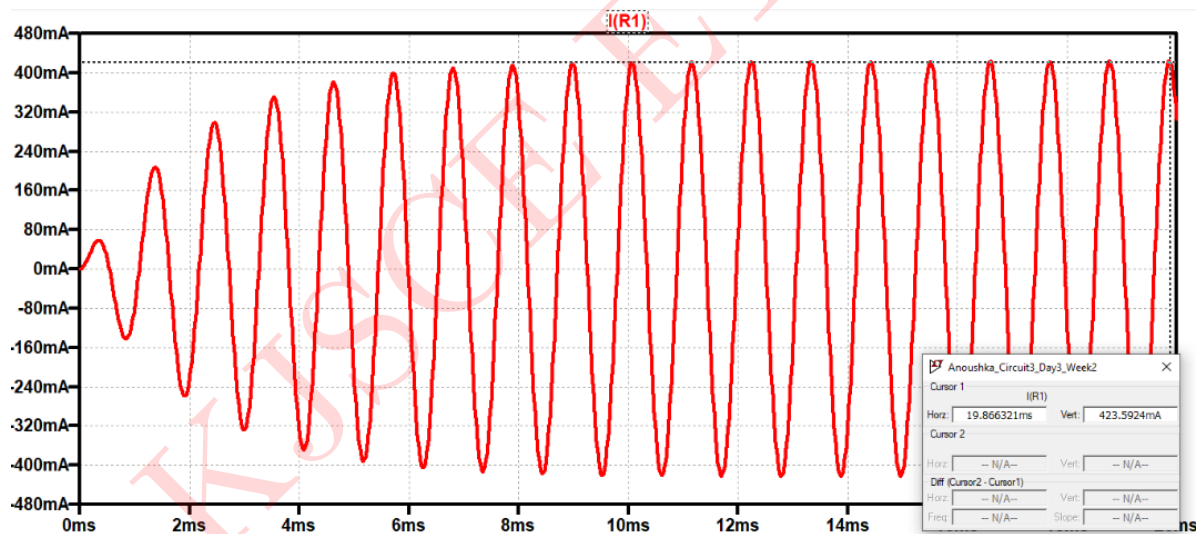


Figure 33: Current at resonance

The waveforms of voltage across inductor and capacitor are shown in figure 34

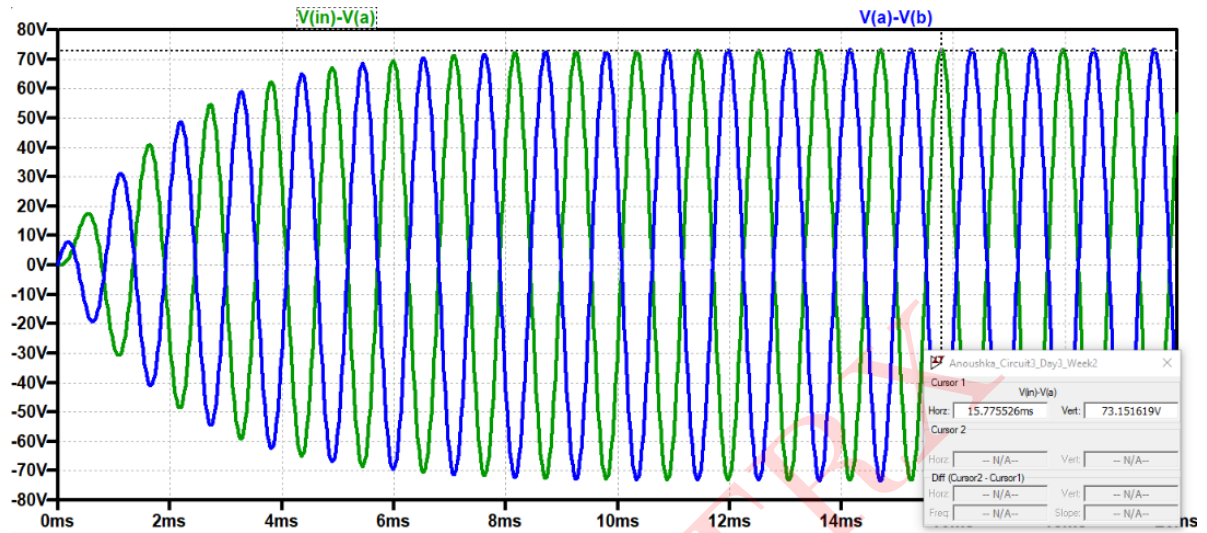


Figure 34: Voltage across inductor and capacitor at resonance

Comparison of theoretical and simulated values:

| Parameters | Theoretical Values | Simulated Values |
|---------------------------------------|--------------------|------------------|
| Resonant frequency | 919.3475Hz | 919.4349Hz |
| Current at resonance | 424.264mA | 423.5942mA |
| Voltage across inductor and capacitor | 73.4846V | 73.1515V |

Table 9: Numerical 9