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DEPARTMENT OF ELECTRONICS ENGINEERING
ELECTRONIC CIRCUITS
Low & High-frequency response of single-stage amplifier

Numerical 1

For the BJT network shown in figure 1.

- Calculate r_π
- Find $A_{V(mid)} = \frac{V_o}{V_i}$
- Calculate Z_i
- Find $A_{VS(mid)} = \frac{V_o}{V_s}$
- Determine f_{LCE} , f_{LCC1} & f_{LCC2}
- Determine the low cutoff frequency

Given: $\beta = 135$, $r_o = 50k\Omega$

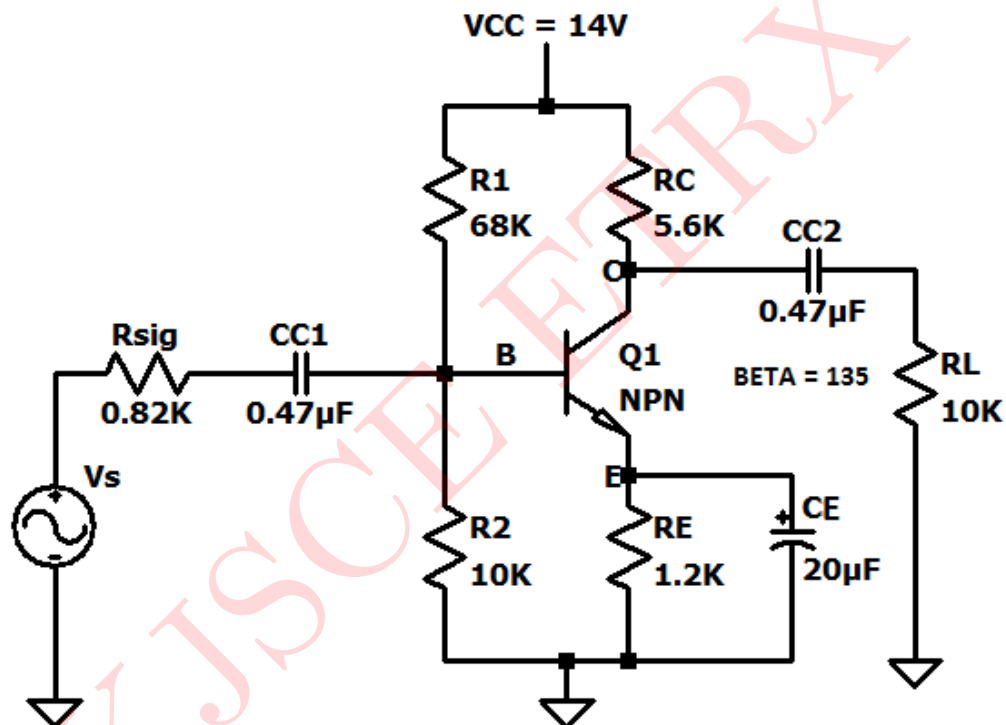


Figure 1: Circuit for Numerical 1

Solution:

DC Analysis: During DC analysis, capacitors become open circuit.

From figure 1 we get,

$$R_{th} = R_1 \parallel R_2 = 68k \parallel 10k$$

$$\therefore R_{th} = 8.718k\Omega$$

$$\text{We know that } V_{th} = \frac{R_2}{R_1 + R_2} \times V_{CC} = \frac{10k}{10k + 68k} \times 14$$

$$\therefore V_{th} = 1.79V$$

Therefore the Thevenin's equivalent circuit is shown in figure 2

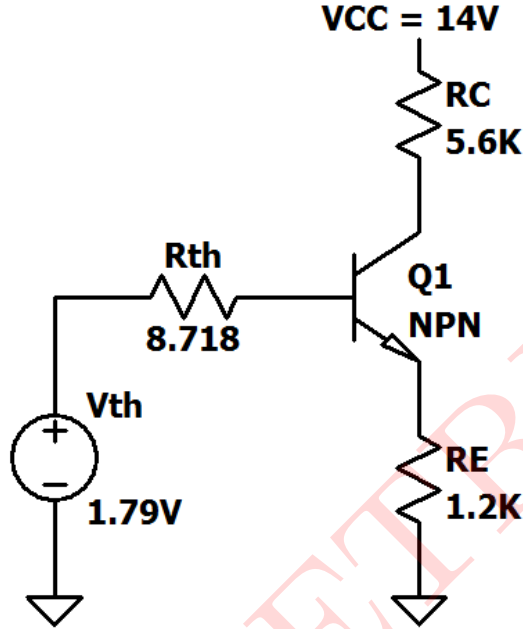


Figure 2: Thevenin's Equivalent Circuit

Applying KVL to B-E loop of figure 2, we get

$$V_{th} - I_{BQ}R_{th} - V_{BE} - I_E R_E = 0$$

$$V_{th} - I_{BQ}R_{th} - V_{BE} - (\beta + 1)I_{BQ}R_E = 0$$

$$I_{BQ} = \frac{V_{th} - V_{BE}}{R_{th} + (\beta + 1)R_E} = \frac{1.79V - 0.7V}{8.71k + (1 + 135) \times 1.2k}$$

$$\therefore I_{BQ} = 6.34\mu A$$

$$I_{CQ} = \beta \times I_{BQ} = 135 \times 6.34\mu A$$

$$I_{CQ} = 0.856mA$$

AC Analysis: During AC analysis, capacitors become short circuit.

Calculation of small signal parameters is shown below

$$r_o = 50k\Omega \quad (\text{Given})$$

$$r_\pi = \frac{\beta \times V_T}{I_{CQ}} = \frac{135 \times 26mV}{0.856mA}$$

$$\therefore r_\pi = 4.100k\Omega$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{0.856mA}{26mV}$$

$$\therefore g_m = 32.923mA/V$$

The low frequency small signal equivalent circuit is shown in figure 3

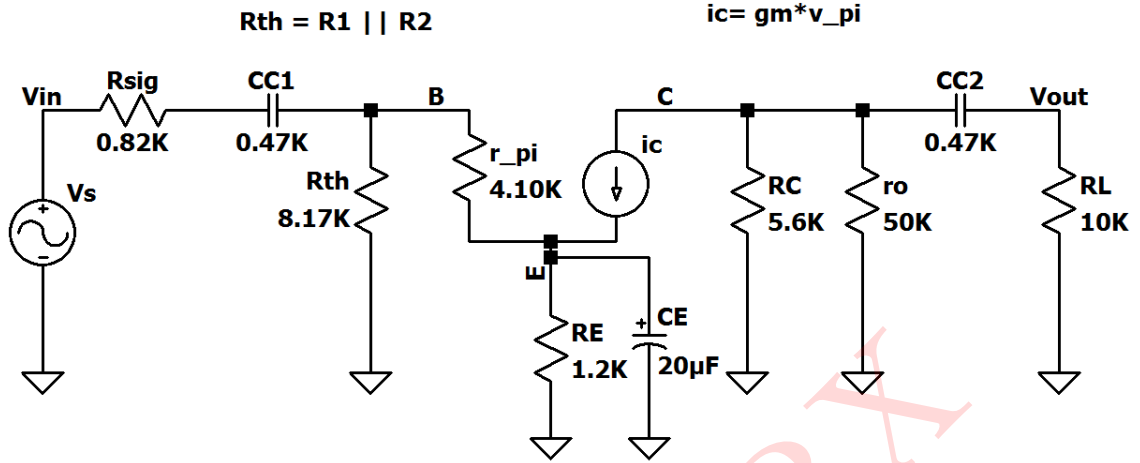


Figure 3: Low Frequency Small Signal Equivalent Circuit

Calculation of $f_{L_{CC1}}$:

$$f_{L_{CC1}} = \frac{1}{2\pi \times R_{eq} \times C_{C1}}$$

$$R_{eq} = R_{sig} + (R_1 \parallel R_2 \parallel r_{\pi}) = 0.82k + (10k \parallel 68k \parallel 4.10k)$$

$$\therefore R_{eq} = 3.608k\Omega$$

$$C_{C1} = 0.47\mu F$$

$$f_{L_{CC1}} = \frac{1}{2\pi \times 3.608k \times 0.47\mu F}$$

$$f_{L_{CC1}} = 93.85\text{Hz}$$

Small signal low frequency equivalent circuit for C_{C1} is shown in figure 5

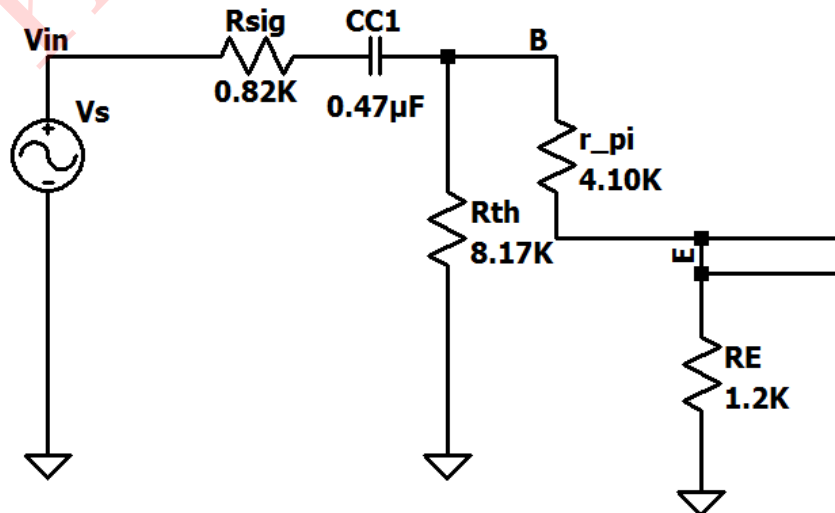


Figure 4: Small signal low frequency equivalent circuit for C_{C1}

Calculation of $f_{L_{CC2}}$:

$$f_{L_{CC2}} = \frac{1}{2\pi \times R_{eq} \times C_{C2}}$$

$$R_{eq} = (R_C \parallel r_o) + R_L = (5.6k \parallel 4.1k) + 10k$$

$$\therefore R_{eq} = 12.367k\Omega$$

$$C_{C2} = 0.47\mu F$$

$$f_{L_{CC2}} = \frac{1}{2\pi \times 12.367k \times 0.47\mu F}$$

$$f_{L_{CC2}} = 27.38Hz$$

Small signal low frequency equivalent circuit for C_{C2} is shown in figure 5

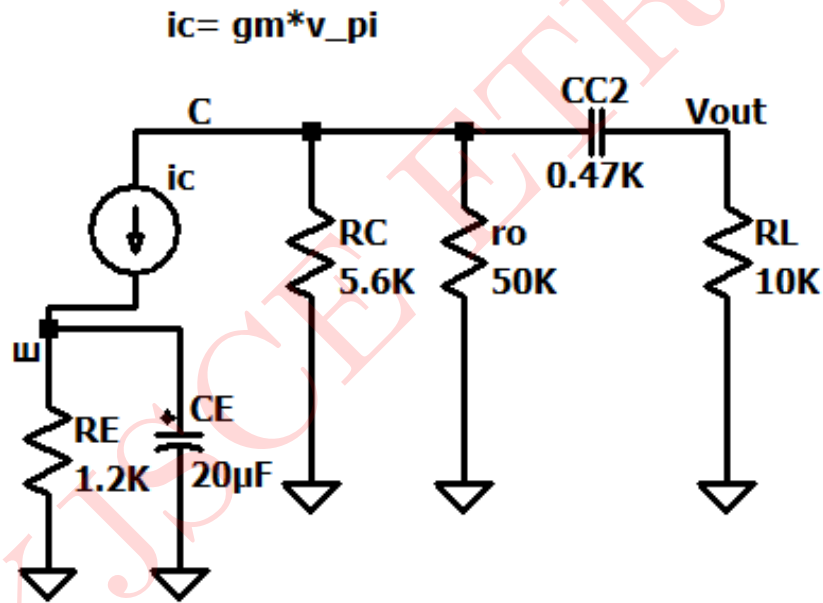


Figure 5: Small signal low frequency equivalent circuit for C_{C2}

Calculation of $f_{L_{CE}}$:

$$f_{L_{CE}} = \frac{1}{2\pi \times R_{eq} \times C_E}$$

$$R_{eq} = \left(\frac{R_{sig} \parallel R_1 \parallel R_2 + r_\pi}{\beta} \right) \parallel R_E = \left(\frac{0.82k \parallel 68k \parallel 10k + 4.10k}{135} \right) \parallel 1.2k$$

$$\therefore R_{eq} = 34.874\Omega$$

$$C_E = 20\mu F$$

$$f_{L_{CE}} = \frac{1}{2\pi \times 34.874 \times 20\mu F}$$

$$f_{L_{CE}} = 228.185Hz$$

Since, $f_{L_{CE}} = 228.185\text{Hz}$ is the largest among $f_{L_{CC1}}$ & $f_{L_{CC2}}$, it is the lower cutoff frequency of the amplifier.

(Bypass capacitor C_E is determining the lower cutoff frequency of the amplifier)

$$f_L = 228.185\text{Hz}$$

Calculation of mid frequency voltage gain:

The mid frequency small signal equivalent circuit is shown in figure 6

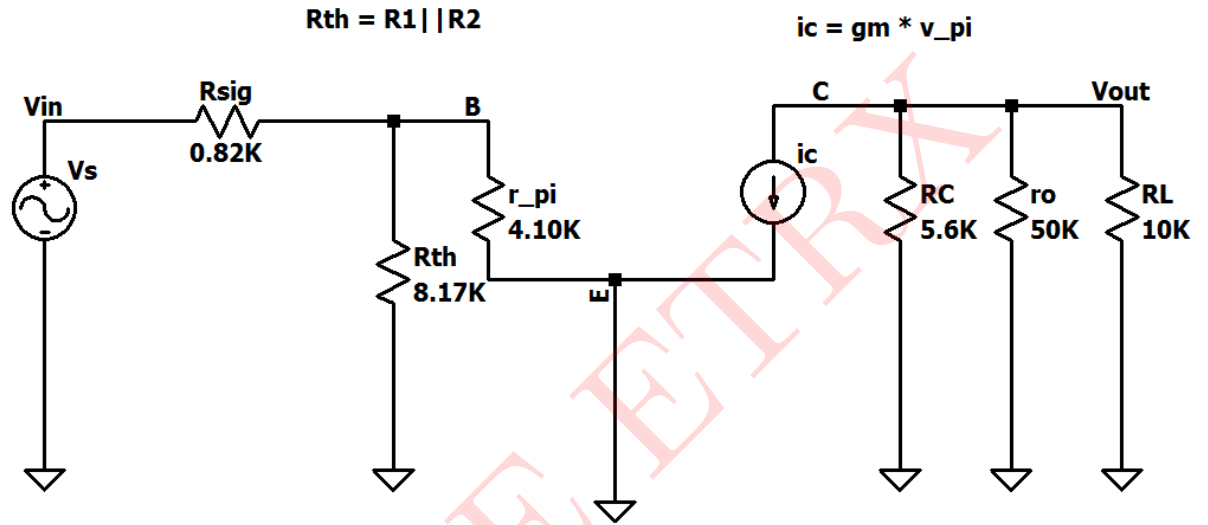


Figure 6: Mid Frequency Small Signal Equivalent Circuit

$$A_V = \frac{V_{out}}{V_{in}} = -g_m(R_C \parallel R_L \parallel r_o) = 32.923 \times 10^{-3}(5.6k \parallel 10k \parallel 50k)$$

$$A_{V(\text{mid})} = -110.25$$

$$A_{VS(\text{mid})} = \frac{V_{out}}{V_s} = \frac{V_{out}}{V_{in}} \times \frac{V_{in}}{V_s} = A_V \times \frac{V_{in}}{V_s}$$

$$\frac{V_{in}}{V_s} = \frac{R_1 \parallel R_2 \parallel r_\pi}{R_1 \parallel R_2 \parallel r_\pi + R_{sig}} = \frac{68k \parallel 10k \parallel 4.10k}{68k \parallel 10k \parallel 4.10k + 0.82k}$$

$$\frac{V_{in}}{V_s} = 0.773$$

$$A_{VS(\text{mid})} = -110.25 \times 0.773$$

$$A_{VS(\text{mid})} = -85.223$$

$$A_{VS(\text{dB})} = 38.611\text{dB}$$

Calculation of Z_i :

$$Z_i = R_{sig} + R_1 \parallel R_2 \parallel r_\pi = 0.82k + 68k \parallel 10k \parallel 4.10k$$

$$Z_i = 3.608k\Omega$$

SIMULATED RESULTS:

Above circuit is simulated in LTspice and results are as follows:

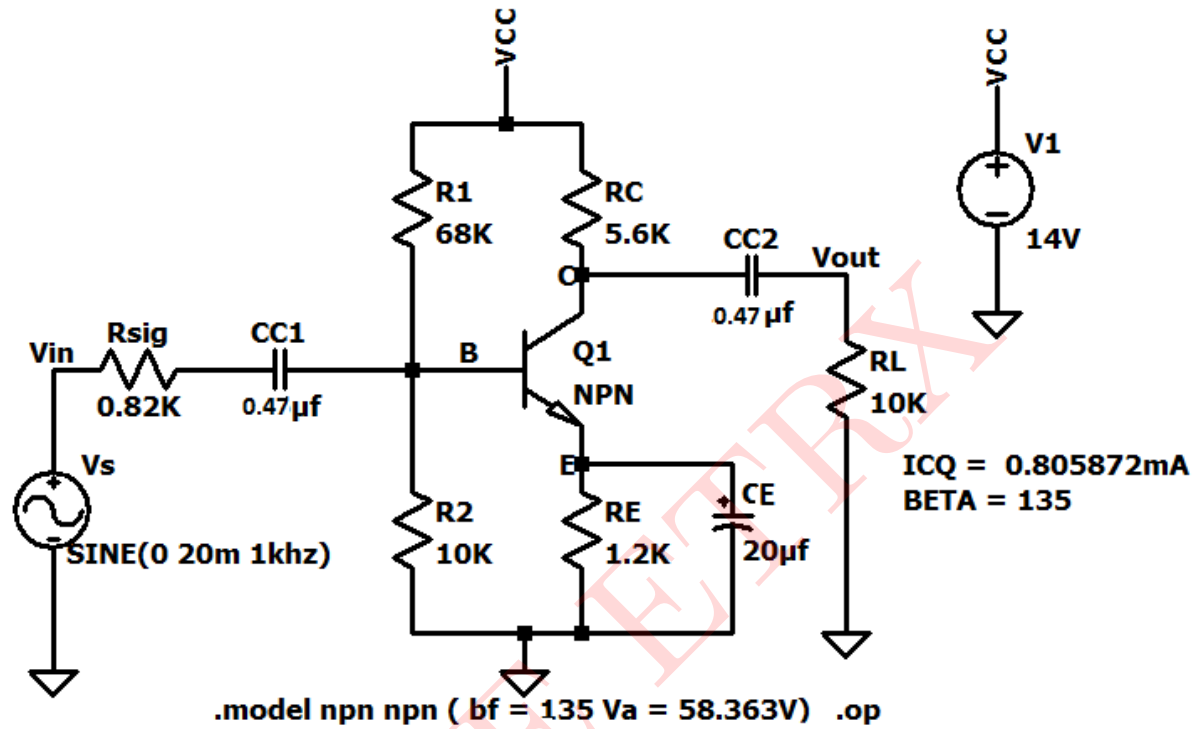


Figure 7: Circuit Schematic: Results

The output bode plots are shown below

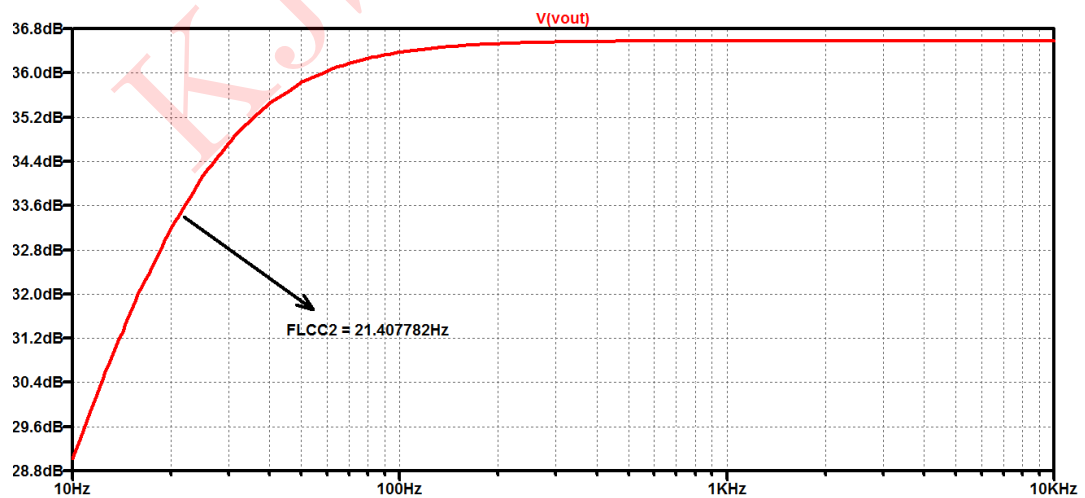


Figure 8: Low frequency response for C_{C2}

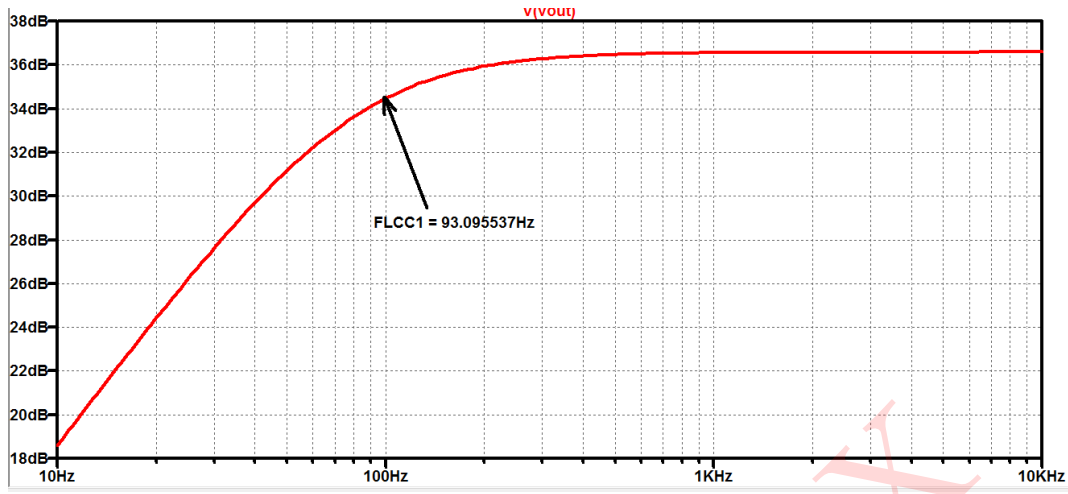


Figure 9: Low frequency response for C_{C1}

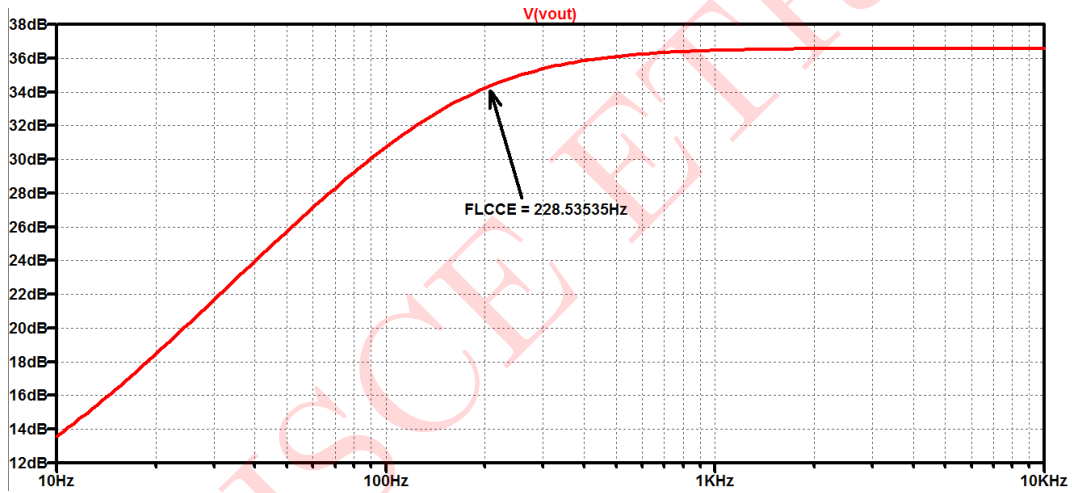


Figure 10: Low frequency response for C_E

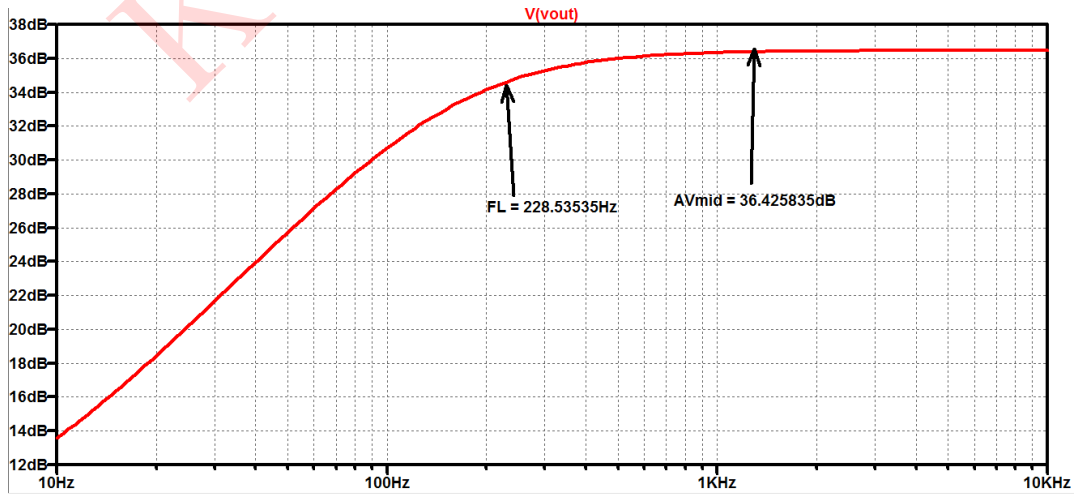


Figure 11: Low frequency response for the circuit

Comparison between theoretical and simulated values is given below:

Parameters	Simulated Values	Theoretical Values
I_{CQ}	$0.815mA$	$0.856mA$
Lower cutoff frequency due to C_{C1}	$93.09Hz$	$93.85Hz$
Lower cutoff frequency due to C_{C2}	$21.0477Hz$	$27.38Hz$
Lower cutoff frequency due to C_E	$228.53Hz$	$228.185Hz$
Overall cutoff frequency	$228.6Hz$	$228.185Hz$
Mid band voltage gain A_{VS} in dB	$36.5dB$	$38.61dB$

Table 1: Numerical 1

Numerical 2

For the N-JFET network shown in figure 12.

- Determine V_{GSQ} & I_{DQ}
- Determine g_m & g_{m_o}
- Find $A_{V(mid)} = \frac{V_o}{V_i}$
- Calculate Z_i
- Find $A_{VS(mid)} = \frac{V_o}{V_s}$
- Determine f_{LCC1} , f_{LCC2} & f_{LCS}
- Determine the low cutoff frequency

Given: $I_{DSS} = 6mA$, $V_P = -6V$

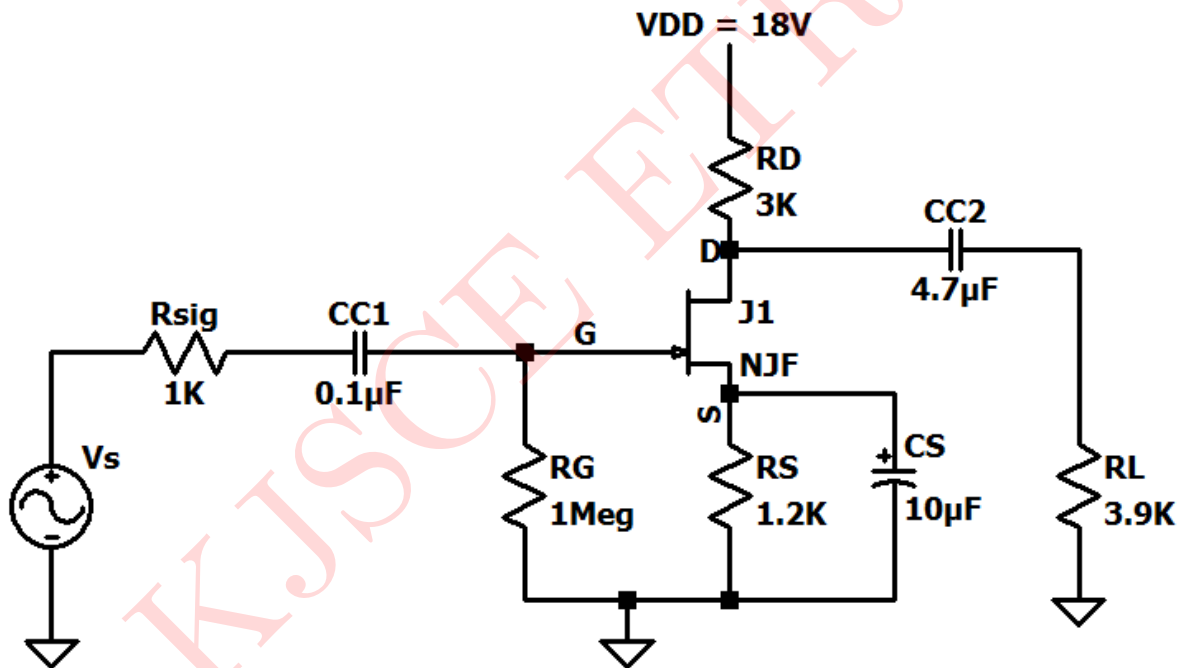


Figure 12: Circuit for Numerical 2

Solution:

DC Analysis: During DC analysis, capacitors become open circuit.

The thevenin's equivalent DC circuit is shown in figure 13

Applying KVL to D-S loop of figure 13, we get

$$V_{GS} = -I_D R_S \quad \text{.....(1)}$$

$$\text{We know that } I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2 \quad \text{.....(2)}$$

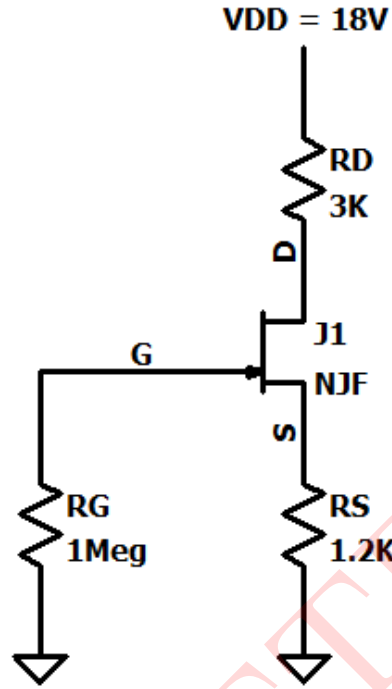


Figure 13: DC Equivalent Circuit

Substituting (2) in (1) we get,

$$V_{GS} = -I_{DSS}R_S \left(1 - \frac{V_{GS}}{V_P}\right)^2 = -6mA \times (1.2k) \left(1 + \frac{V_{GS}}{6}\right)^2$$

$$V_{GS} = -7.2 - 0.2V_{GS}^2 - 2.4V_{GS}$$

$$0.2V_{GS}^2 + 3.4V_{GS} + 7.2 = 0$$

Solving the quadratic equation we get,

$$V_{GS} = -2.479V \text{ or } V_{GS} = -14.521V$$

$$\therefore V_{GS} = -2.479V \quad (\because V_{GS} > V_P)$$

$$\therefore I_D = 6mA \times \left[1 - \frac{(-2.479)}{(-6)}\right]^2$$

$$\therefore I_D = 2.066mA$$

AC Analysis: During AC analysis, capacitors become short circuit.

Calculation of small signal parameters is shown below

$$g_m = \frac{2I_{DSS}}{|V_P|} \left(1 - \frac{V_{GS}}{V_P} \right) = \frac{2 \times 6mA}{|-6|} \left[1 - \frac{(-2.479V)}{(-6V)} \right]$$

$$\therefore g_m = 1.174mA/V$$

$$g_{m_o} = \frac{2I_{DSS}}{|V_P|} = \frac{2 \times 6mA}{|-6|}$$

$$\therefore g_{m_o} = 2mA/V$$

Calculation of voltage gain:

The mid frequency small signal equivalent circuit is shown in figure 14

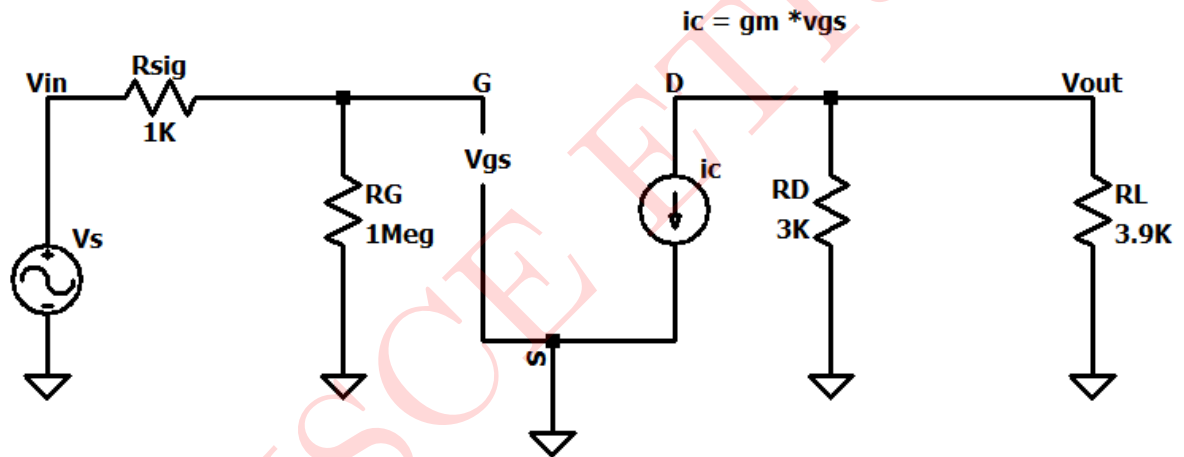


Figure 14: Mid Frequency Small Signal Equivalent Circuit

$$A_V = \frac{V_{out}}{V_{in}} = -g_m(R_D \parallel R_L) = 1.174 \times 10^{-3}(3k \parallel 3.9k)$$

$$A_{V(mid)} = -1.989$$

$$A_{VS(mid)} = \frac{V_{out}}{V_s} = \frac{V_{out}}{V_{in}} \times \frac{V_{in}}{V_s} = A_V \times \frac{V_{in}}{V_s}$$

$$\frac{V_{in}}{V_s} \approx 1 \quad (\because R_G \text{ is in Mega Ohms})$$

$$A_{VS(mid)} = -1.989 \times 1 = -1.989$$

$$A_{VS(dB)} = 5.973dB$$

Calculation of Zi:

$$Z_i = R_{sig} + R_G \approx 1M\Omega$$

$$Z_i = 1M\Omega$$

Low frequency small signal equivalent circuit is shown in figure 15

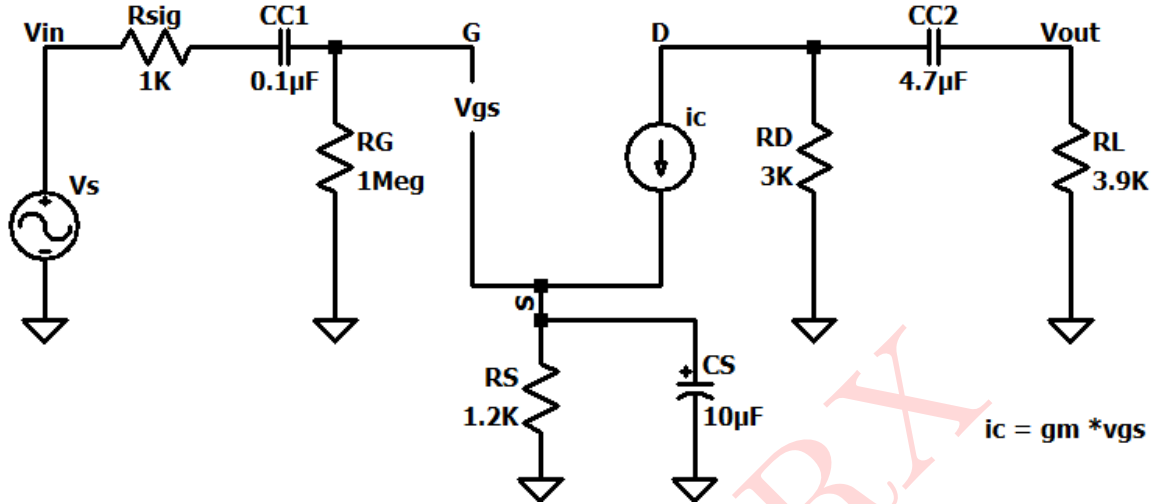


Figure 15: Low Frequency Small Signal Equivalent Circuit

Calculation of $f_{L_{CC1}}$:

$$C_{C1} = 0.1\mu F$$

$$R_{eq} = R_{sig} + R_G \approx 1M\Omega \quad (\because R_G \text{ is in Mega Ohms})$$

$$\therefore R_{eq} = 1M\Omega$$

$$f_{L_{CC1}} = \frac{1}{2\pi \times R_{eq} \times C_{C1}} = \frac{1}{2\pi \times 1M \times 0.1\mu F}$$

$$f_{L_{CC1}} = 1.591\text{Hz}$$

Small signal low frequency equivalent circuit for C_{C1} is shown in figure 16

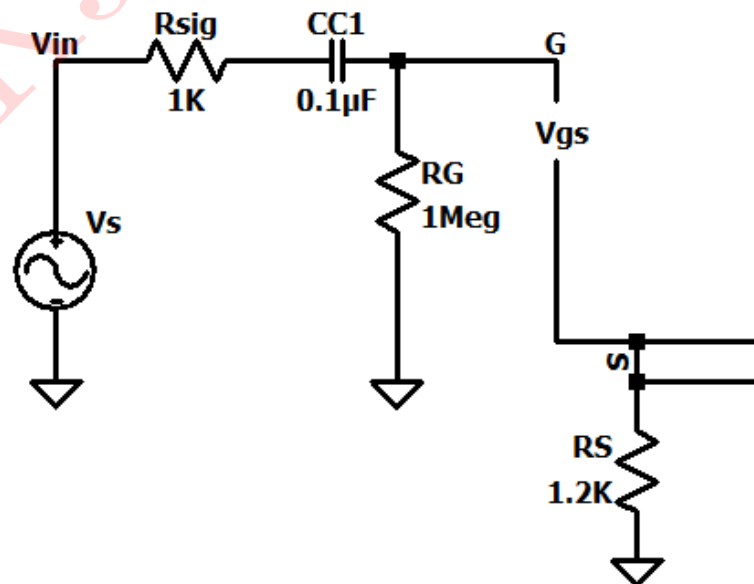


Figure 16: Small signal low frequency equivalent circuit for C_{C1}

Calculation of $f_{L_{CC2}}$:

$$C_{C2} = 4.7\mu F$$

$$R_{eq} = R_D + R_L = 3k \parallel 3.9k$$

$$\therefore R_{eq} = 6.9k\Omega$$

$$f_{L_{CC2}} = \frac{1}{2\pi \times R_{eq} \times C_{C2}} = \frac{1}{2\pi \times 6.9k \times 4.7\mu F}$$

$$f_{L_{CC2}} = 4.908Hz$$

Small signal low frequency equivalent circuit for C_{C2} is shown in figure 17

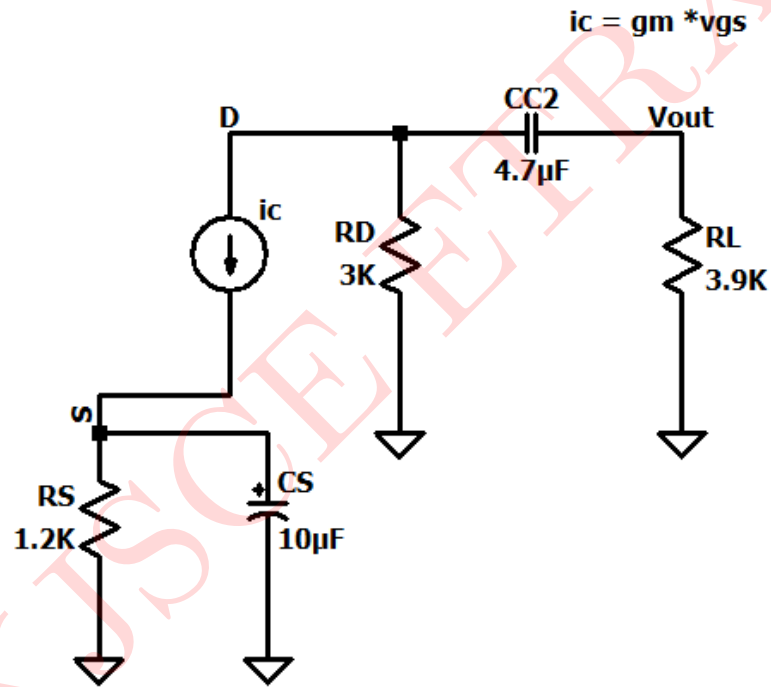


Figure 17: Small signal low frequency equivalent circuit for C_{C2}

Calculation of $f_{L_{CS}}$:

$$C_S = 10\mu F$$

$$R_{eq} = R_S \parallel \frac{1}{g_m} = 1.2k \parallel \frac{1}{1.174mA/V}$$

$$\therefore R_{eq} = 0.498k\Omega$$

$$f_{L_{CS}} = \frac{1}{2\pi \times R_{eq} \times C_S} = \frac{1}{2\pi \times 0.498k \times 10\mu F}$$

$$f_{L_{CS}} = 31.959Hz$$

Small signal low frequency equivalent circuit for C_S is shown in figure 18

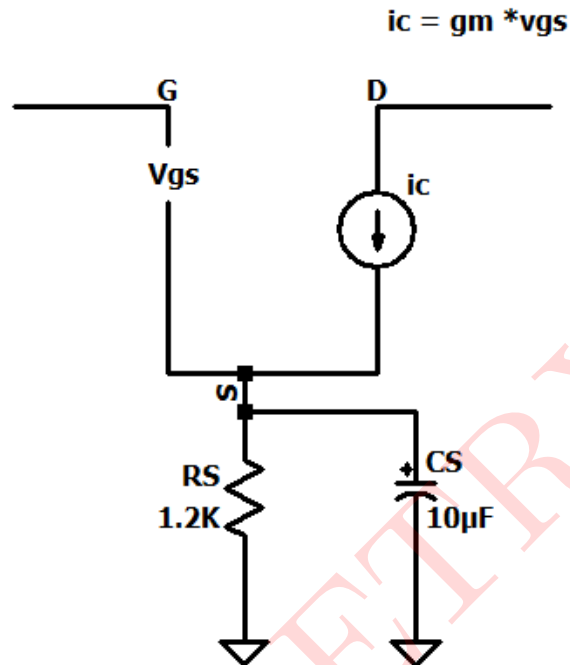


Figure 18: Small signal low frequency equivalent circuit for C_S

Lower Cutoff Frequency f_L :

Since, $f_{L_{C_S}}$ is the largest among $f_{L_{C_{C1}}}$ & $f_{L_{C_{C2}}}$, it is the lower cutoff frequency of the amplifier.

(Bypass capacitor C_S is determining the lower cutoff frequency of the amplifier)

$$f_L = 31.959\text{Hz}$$

SIMULATED RESULTS:

Above circuit is simulated in LTspice and results are as follows:

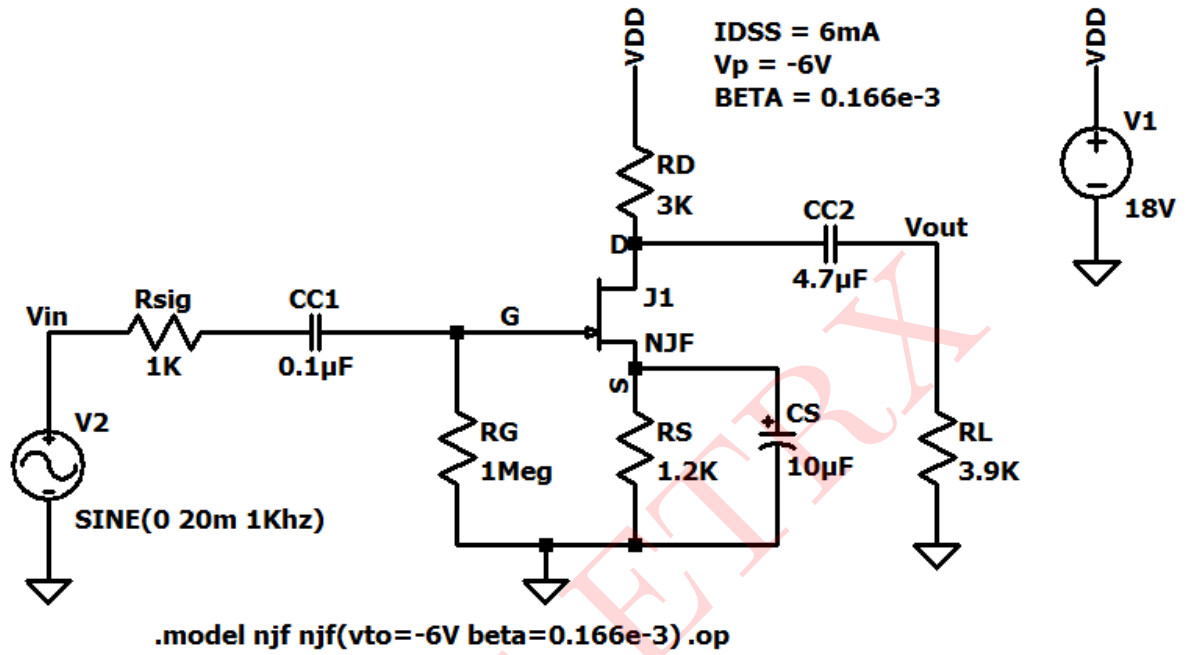


Figure 19: Circuit Schematic: Results

Output Waveforms:

The output bode plots are shown below

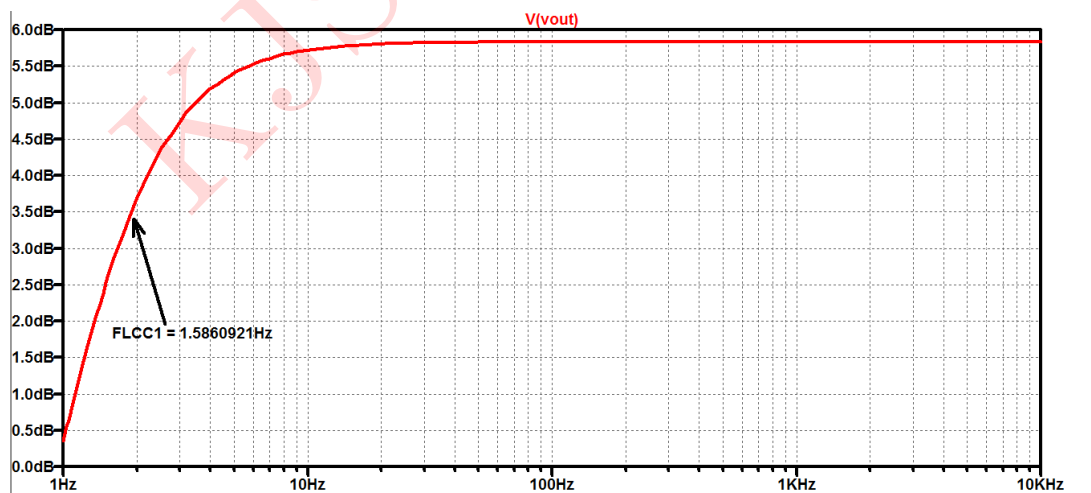


Figure 20: Low frequency response for C_{C1}

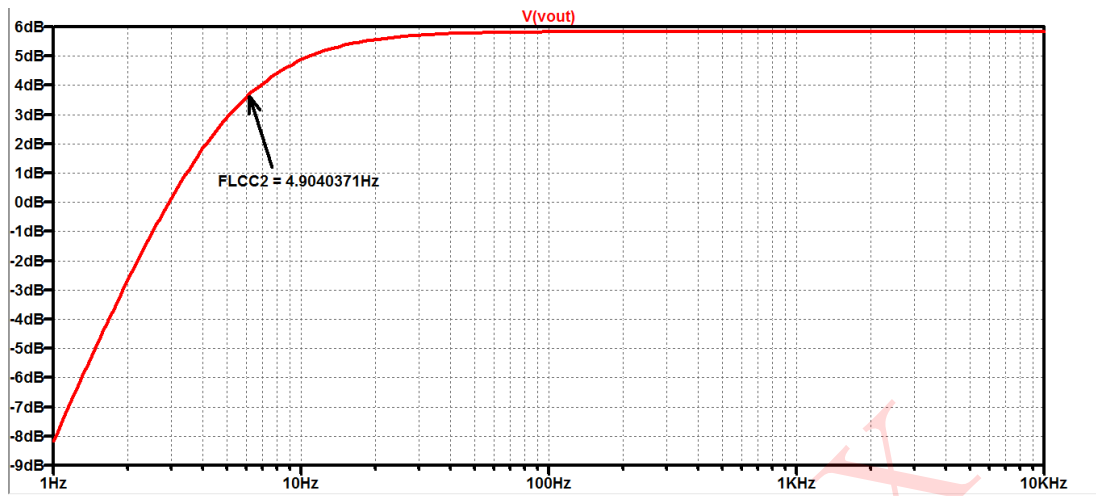


Figure 21: Low frequency response for C_{C2}

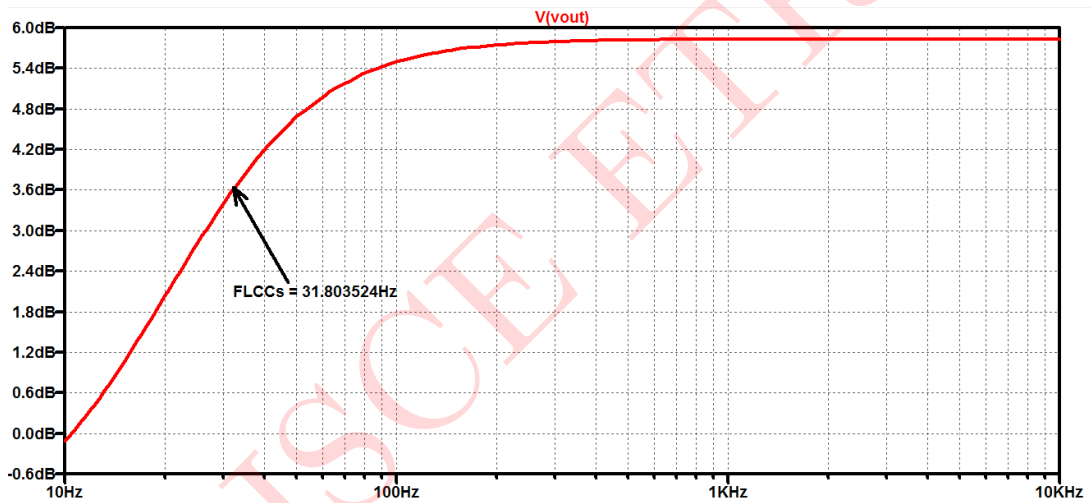


Figure 22: Low frequency response for C_S

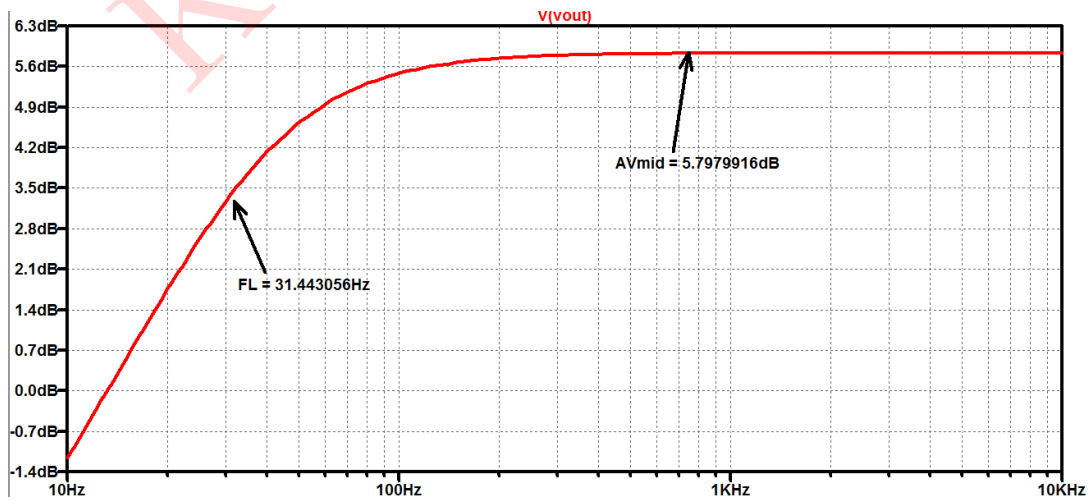


Figure 23: Low frequency response for the circuit

Comparison between theoretical and simulated values is given below:

Parameters	Simulated Values	Theoretical Values
I_{DQ}	$2.06mA$	$2.066mA$
Lower cutoff frequency due to C_{C1}	$1.586Hz$	$1.591Hz$
Lower cutoff frequency due to C_{C2}	$4.904Hz$	$4.908Hz$
Lower cutoff frequency due to C_S	$31.80Hz$	$31.959Hz$
Overall cutoff frequency	$31.44Hz$	$31.959Hz$
Mid band voltage gain A_{VS} in dB	$5.79dB$	$5.793dB$

Table 2: Numerical 2

Numerical 3

For the N-JFET network shown in figure 24.

a) Determine V_{GSQ} & I_{DQ}

b) Determine g_m & g_{m_o}

c) Find $A_{V(mid)} = \frac{V_o}{V_i}$

d) Calculate Z_i

e) Find $A_{VS(mid)} = \frac{V_o}{V_s}$

f) Determine f_{LCC1} , f_{LCC2} & f_{LCS}

g) Determine the lower cutoff frequency

h) Determine the higher cutoff frequency

Given: $I_{DSS} = 6mA$, $V_P = -6V$, $C_{wi} = 3pF$, $C_{wo} = 8pF$, $C_{gd} = 4pF$, $C_{gs} = 8pF$, $C_{ds} = 2pF$

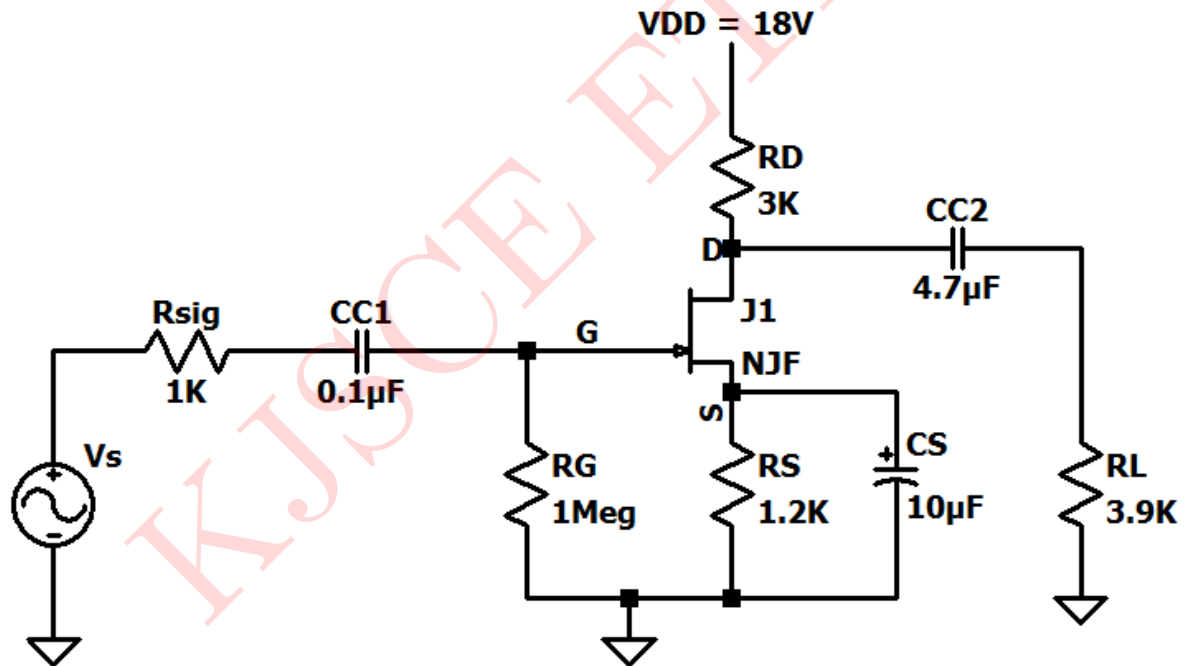


Figure 24: Circuit for Numerical 3

Solution:

DC Anaylsis: During DC analysis, capacitors become open circuit.

The equivalent DC circuit is shown in figure 25

Applying KVL to D-S loop of figure 25, we get

$$V_{GS} = -I_D R_S \quad \text{.....(1)}$$

$$\text{We know that } I_D = I_{DSS} \left(1 - \frac{V_{GS}}{V_P}\right)^2 \quad \text{.....(2)}$$

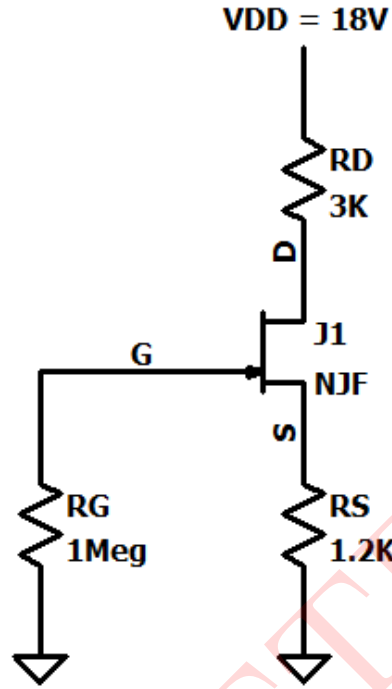


Figure 25: DC Equivalent Circuit

Substituting (2) in (1) we get,

$$V_{GS} = -I_{DSS}R_S \left(1 - \frac{V_{GS}}{V_P}\right)^2 = -6mA \times (1.2k) \left(1 + \frac{V_{GS}}{6}\right)^2$$

$$V_{GS} = -7.2 - 0.2V_{GS}^2 - 2.4V_{GS}$$

$$0.2V_{GS}^2 + 3.4V_{GS} + 7.2 = 0$$

Solving the quadratic equation we get,

$$V_{GS} = -2.479V \text{ or } V_{GS} = -14.521V$$

$$\therefore V_{GS} = -2.479V \quad (\because V_{GS} > V_P)$$

$$\therefore I_D = 6mA \times \left[1 - \frac{(-2.479)}{(-6)}\right]^2$$

$$\therefore I_D = 2.066mA$$

AC Analysis: During AC analysis, capacitors become short circuit.

Calculation of small signal parameters is shown below

$$g_m = \frac{2I_{DSS}}{|V_P|} \left(1 - \frac{V_{GS}}{V_P} \right) = \frac{2 \times 6mA}{|-6|} \left[1 - \frac{(-2.479V)}{(-6V)} \right]$$

$$\therefore g_m = 1.174mA/V$$

$$g_{m_o} = \frac{2I_{DSS}}{|V_P|} = \frac{2 \times 6mA}{|-6|}$$

$$\therefore g_{m_o} = 2mA/V$$

Calculation of voltage gain:

The mid frequency small signal equivalent circuit is shown in figure 26

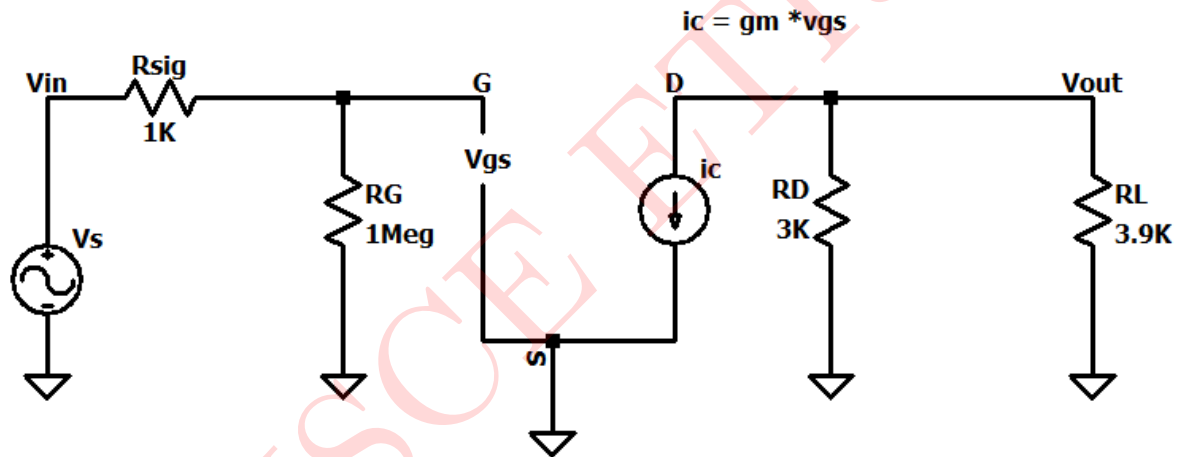


Figure 26: Mid Frequency Small Signal Equivalent Circuit

$$A_V = \frac{V_{out}}{V_{in}} = -g_m(R_D \parallel R_L) = 1.174 \times 10^{-3}(3k \parallel 3.9k)$$

$$A_{V(mid)} = -1.989$$

$$A_{VS(mid)} = \frac{V_{out}}{V_s} = \frac{V_{out}}{V_{in}} \times \frac{V_{in}}{V_s} = A_V \times \frac{V_{in}}{V_s}$$

$$\frac{V_{in}}{V_s} \approx 1 \quad (\because R_G \text{ is in Mega Ohms})$$

$$A_{VS(mid)} = -1.989 \times 1 = -1.989$$

$$A_{VS(dB)} = 5.973dB$$

Calculation of Zi:

$$Z_i = R_{sig} + R_G \approx 1M\Omega$$

$$Z_i = 1M\Omega$$

Low frequency small signal equivalent circuit is shown in figure 27

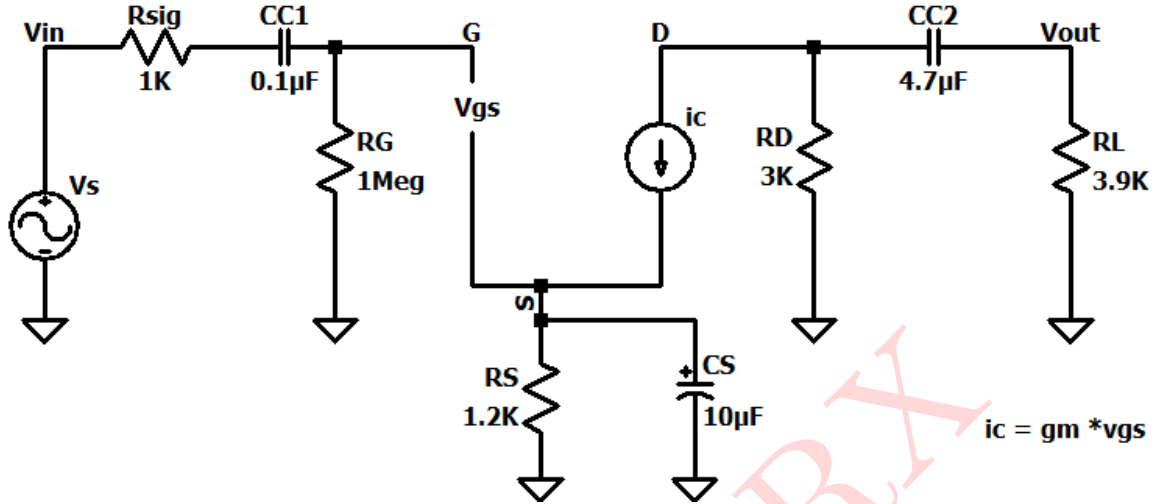


Figure 27: Low Frequency Small Signal Equivalent Circuit

Calculation of $f_{L_{CC1}}$:

$$C_{C1} = 0.1\mu F$$

$$R_{eq} = R_{sig} + R_G \approx 1M\Omega \quad (\because R_G \text{ is in Mega Ohms})$$

$$\therefore R_{eq} = 1M\Omega$$

$$f_{L_{CC1}} = \frac{1}{2\pi \times R_{eq} \times C_{C1}} = \frac{1}{2\pi \times 1M \times 0.1\mu F}$$

$$f_{L_{CC1}} = 1.591\text{Hz}$$

Small signal low frequency equivalent circuit for C_{C1} is shown in figure 28

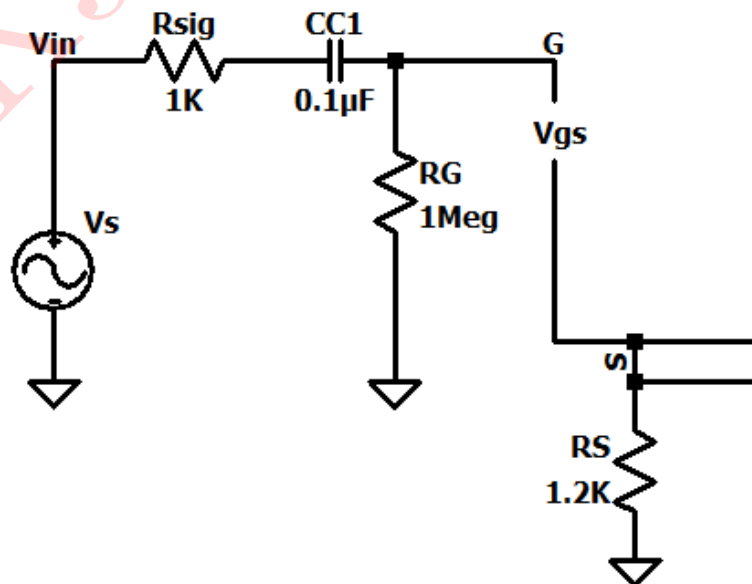


Figure 28: Small signal low frequency equivalent circuit for C_{C1}

Calculation of $f_{L_{CC2}}$:

$$C_{C2} = 4.7\mu F$$

$$R_{eq} = R_D + R_L = 3k \parallel 3.9k$$

$$\therefore R_{eq} = 6.9k\Omega$$

$$f_{L_{CC2}} = \frac{1}{2\pi \times R_{eq} \times C_{C2}} = \frac{1}{2\pi \times 6.9k \times 4.7\mu F}$$

$$f_{L_{CC2}} = 4.908Hz$$

Small signal low frequency equivalent circuit for C_{C2} is shown in figure 29

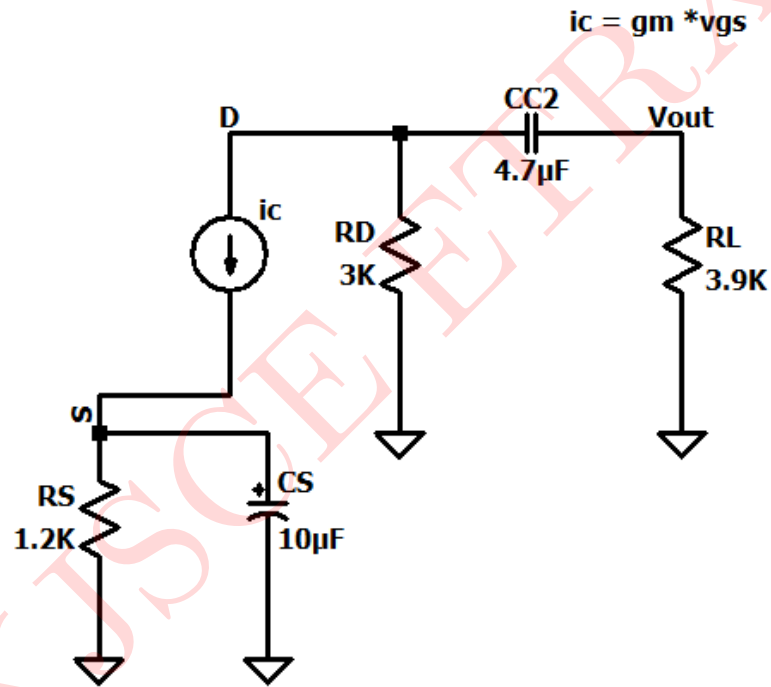


Figure 29: Small signal low frequency equivalent circuit for C_{C2}

Calculation of $f_{L_{CS}}$:

$$C_S = 10\mu F$$

$$R_{eq} = R_S \parallel \frac{1}{g_m} = 1.2k \parallel \frac{1}{1.174mA/V}$$

$$\therefore R_{eq} = 0.498k\Omega$$

$$f_{L_{CS}} = \frac{1}{2\pi \times R_{eq} \times C_S} = \frac{1}{2\pi \times 0.498k \times 10\mu F}$$

$$f_{L_{CS}} = 31.959Hz$$

Small signal low frequency equivalent circuit for C_S is shown in figure 30

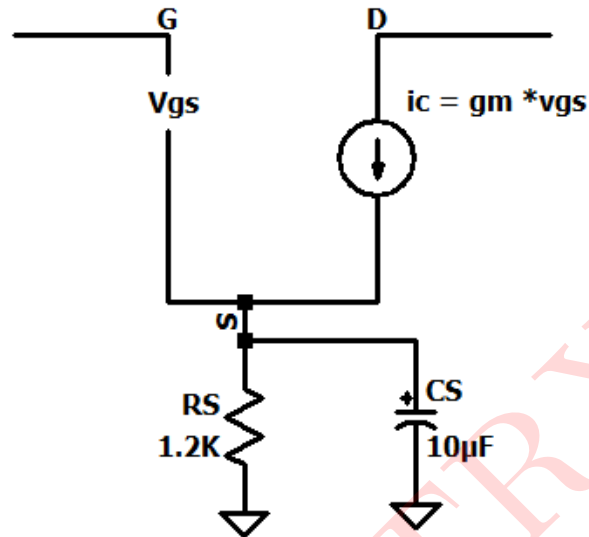


Figure 30: Small signal low frequency equivalent circuit for C_S

Lower Cutoff Frequency f_L :

Since, $f_{L_{CS}}$ is the largest among $f_{L_{CC1}}$ & $f_{L_{CC2}}$, it is the lower cutoff frequency of the amplifier.

(Bypass capacitor C_S is determining the lower cutoff frequency of the amplifier)

$$f_L = 31.959\text{Hz}$$

High Frequency Analysis

High frequency small signal equivalent circuit is shown in figure 31

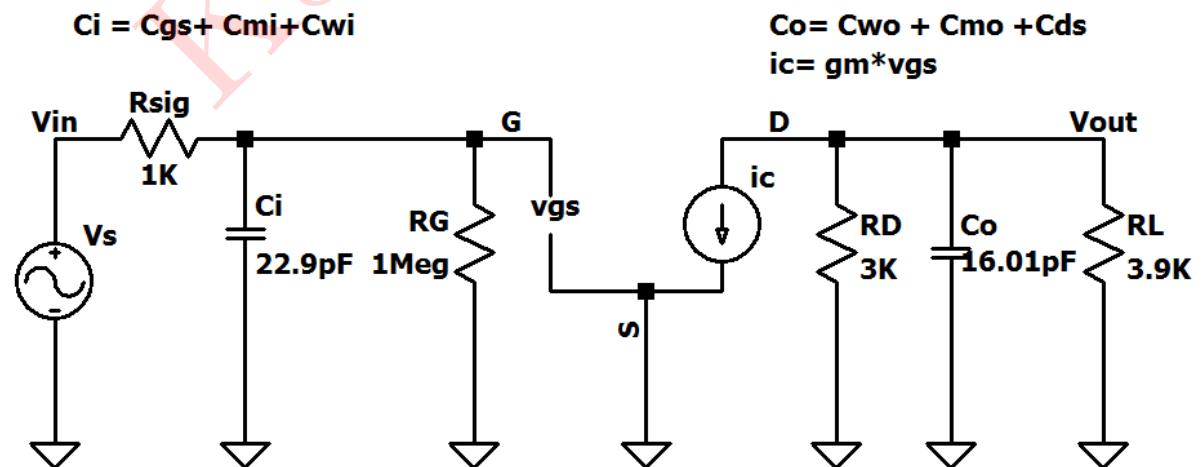


Figure 31: High Frequency Small Signal Equivalent Circuit

Calculation of f_{Hi} :

$$C_{mi} = C_{gd}[1 - A_{V(mid)}] = 4pF[1 - (-1.989)]$$

$$\therefore C_{mi} = 11.956pF$$

$$C_i = C_{gs} + C_{mi} + C_{wi} = 8pF + 11.956pF + 3pF$$

$$\therefore C_i = 22.956pF$$

$$R_{eq} = R_{sig} \parallel R_G = 1k \parallel 1M$$

$$R_{eq} = 0.999k\Omega$$

$$f_{Hi} = \frac{1}{2\pi \times R_{eq} \times C_i} = \frac{1}{2\pi \times 0.999k \times 22.956pF}$$

$$\therefore f_{Hi} = 6.939MHz$$

$$C_i = C_{gs} + C_{mi} + C_{wi}$$

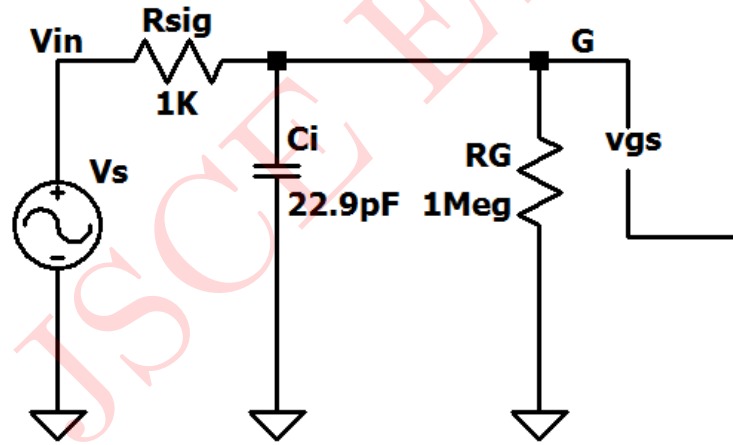


Figure 32: High Frequency Small Signal Equivalent Circuit for C_i

Calculation of f_{H_o} :

$$C_{mo} = C_{gd} \left[1 - \frac{1}{A_{V(mid)}} \right] = 4pF \left[1 - \frac{1}{(-1.989)} \right]$$

$$\therefore C_{mo} = 6.011pF$$

$$C_o = C_{ds} + C_{mo} + C_{wo} = 2pF + 6.011pF + 8pF$$

$$\therefore C_o = 16.011pF$$

$$R_{eq} = R_D \parallel R_L = 3k \parallel 3.9k$$

$$R_{eq} = 1.696k\Omega$$

$$f_{H_o} = \frac{1}{2\pi \times R_{eq} \times C_o} = \frac{1}{2\pi \times 1.696k \times 16.011pF}$$

$$\therefore f_{H_o} = 5.861MHz$$

$$C_o = C_{wo} + C_{mo} + C_{ds}$$

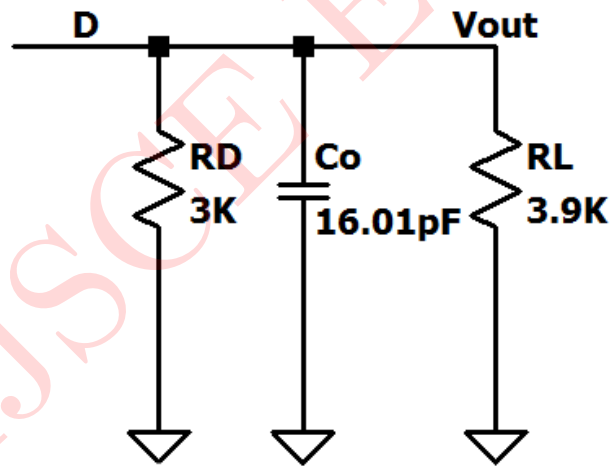


Figure 33: High Frequency Small Signal Equivalent Circuit for C_o

Higher Cutoff Frequency f_H :

Since, f_{H_o} is the lowest frequency among f_{H_i} and f_{H_o} , it is the higher cutoff frequency of the amplifier.

$$f_H = 5.861MHz$$

SIMULATED RESULTS:

Above circuit is simulated in LTspice and results are as follows:

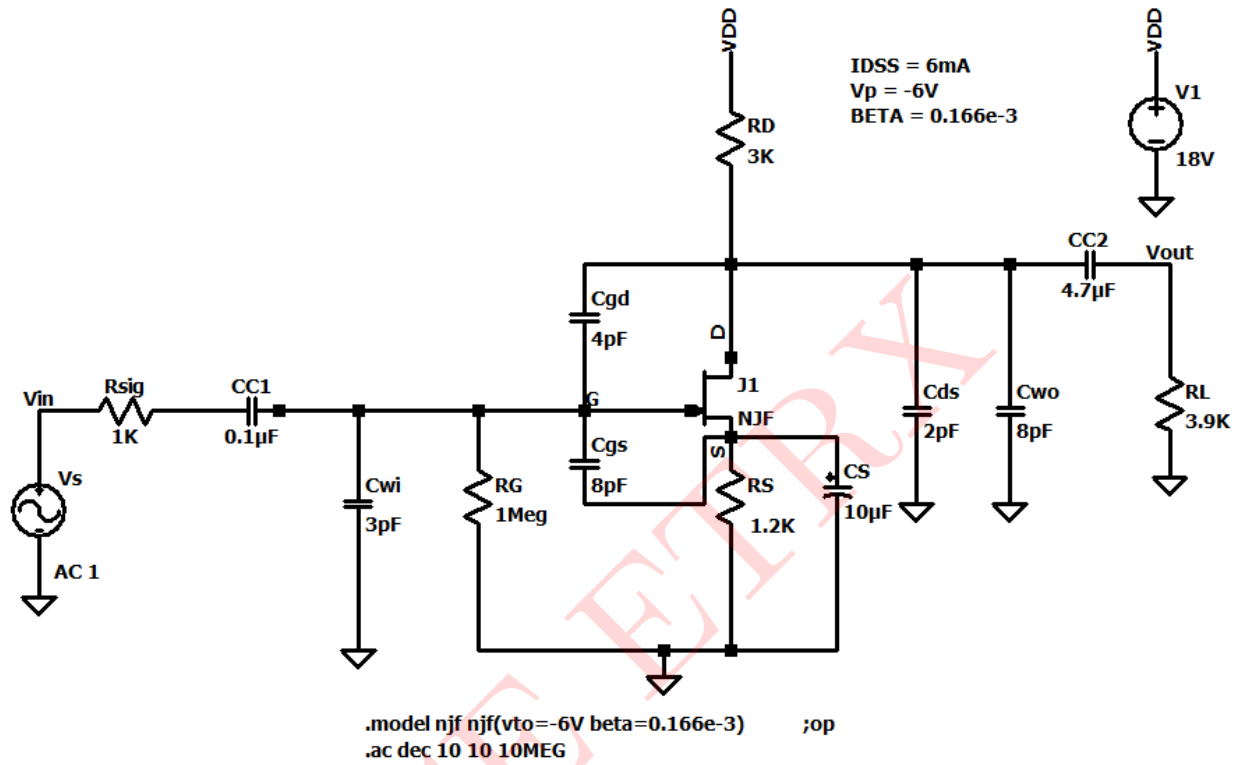


Figure 34: Circuit Schematic: Results

Output Waveforms:

The output bode plots are shown below

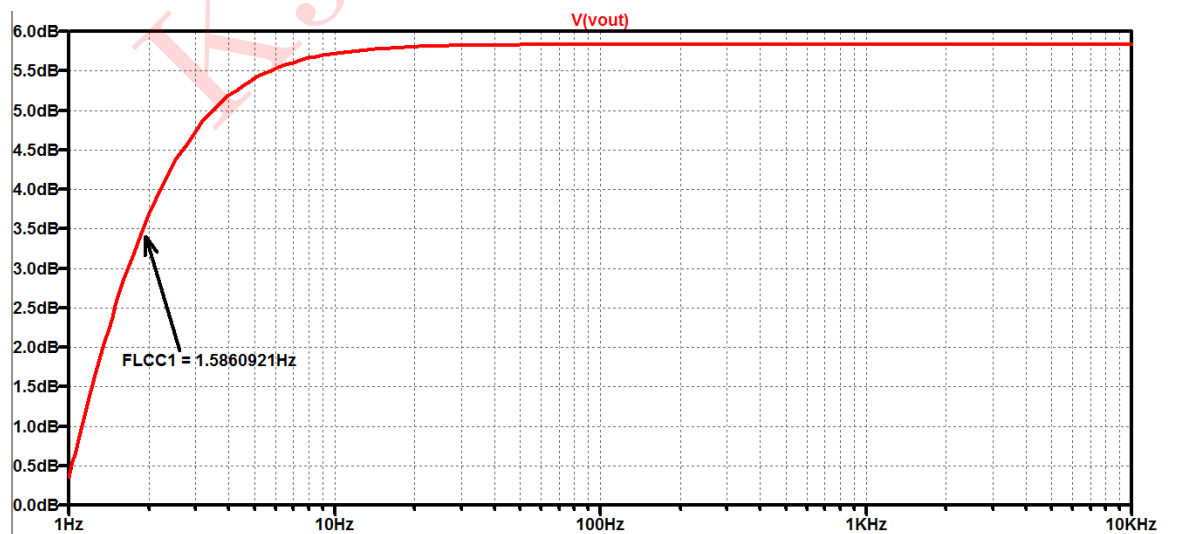


Figure 35: Low frequency response for C_{C1}

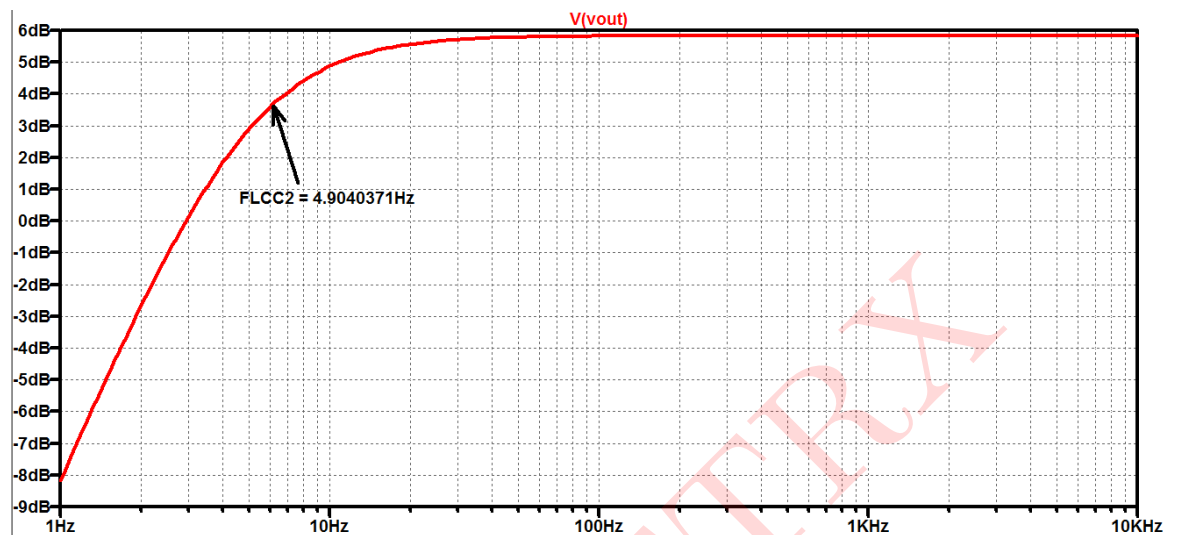


Figure 36: Low frequency response for C_{C2}

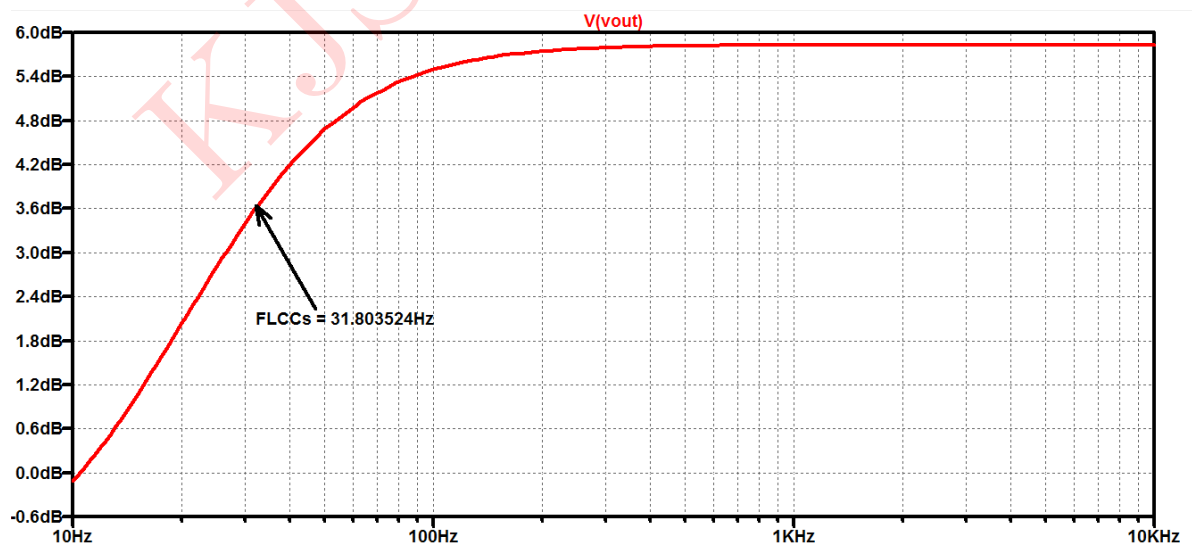


Figure 37: Low frequency response for C_S

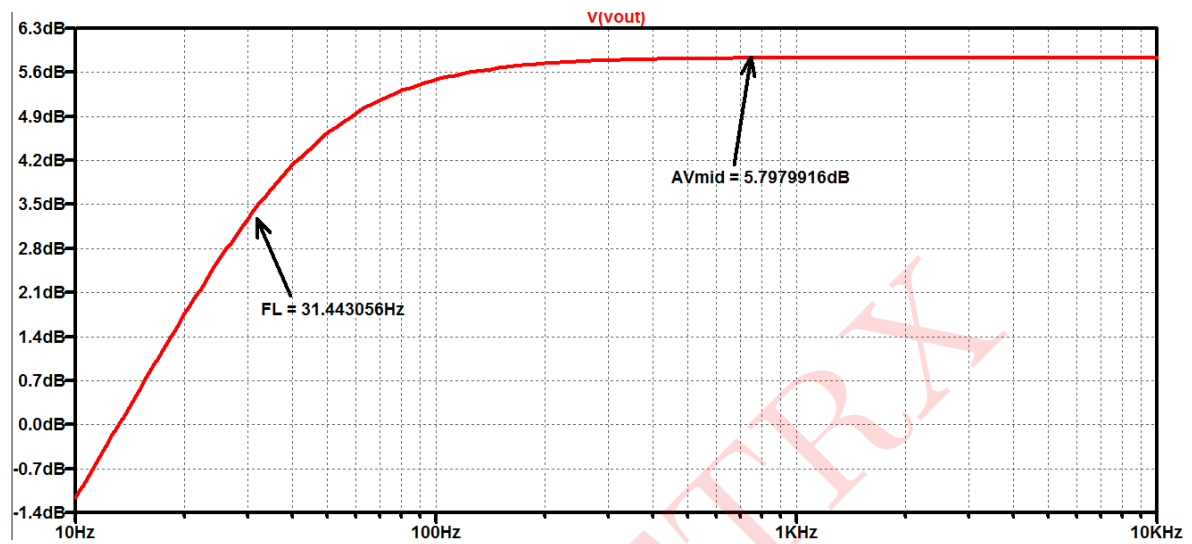


Figure 38: Low frequency response for the circuit

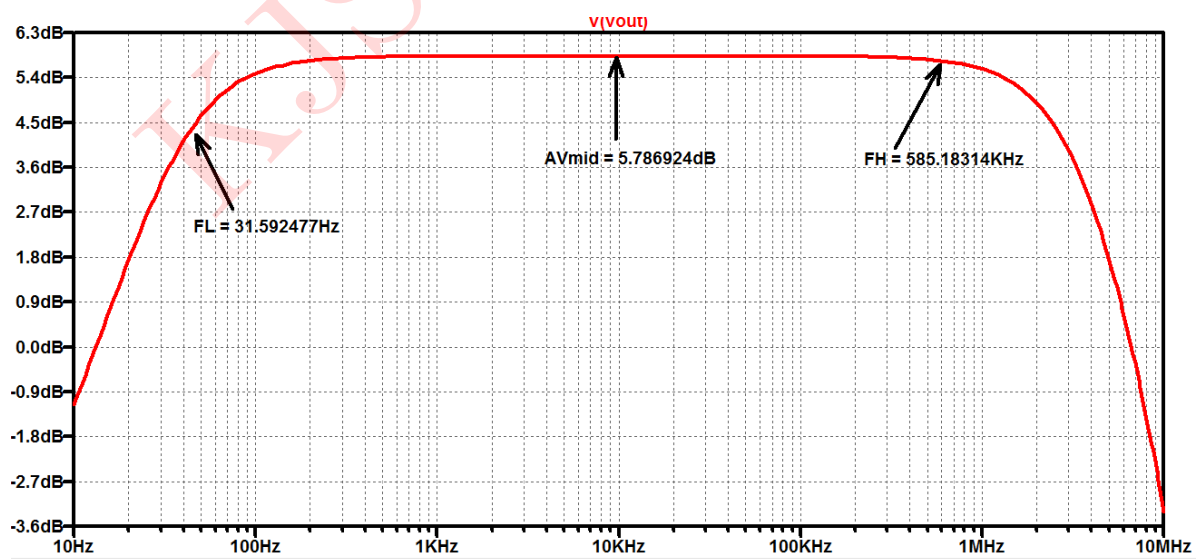


Figure 39: Frequency response for the circuit

Comparison between theoretical and simulated values is given below:

Parameters	Simulated Values	Theoretical Values
I_{DQ}	$2.06mA$	$2.066mA$
Lower cutoff frequency due to C_{C1}	$1.586Hz$	$1.591Hz$
Lower cutoff frequency due to C_{C2}	$4.904Hz$	$4.908Hz$
Lower cutoff frequency due to C_S	$31.80Hz$	$31.959Hz$
Lower cutoff frequency	$31.44Hz$	$31.959Hz$
Higher cutoff frequency	$5.851MHz$	$5.861MHz$
Mid band voltage gain A_{VS} in dB	$5.79dB$	$5.793dB$

Table 3: Numerical 3
