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DEPARTMENT OF ELECTRONICS ENGINEERING
ELECTRONIC CIRCUITS
Low and High-frequency response of single-stage amplifier

Numerical 1: For the network shown in figure 1 determine:

- a. r_{π} b. Z_i c. $A_{V_{mid}} = \frac{V_{out}}{V_{in}}$ d. $A_{V_{smd}} = \frac{V_{out}}{V_s}$ e. $f_{LCE}, f_{LCC1}, f_{LCC2}$ f. f_L

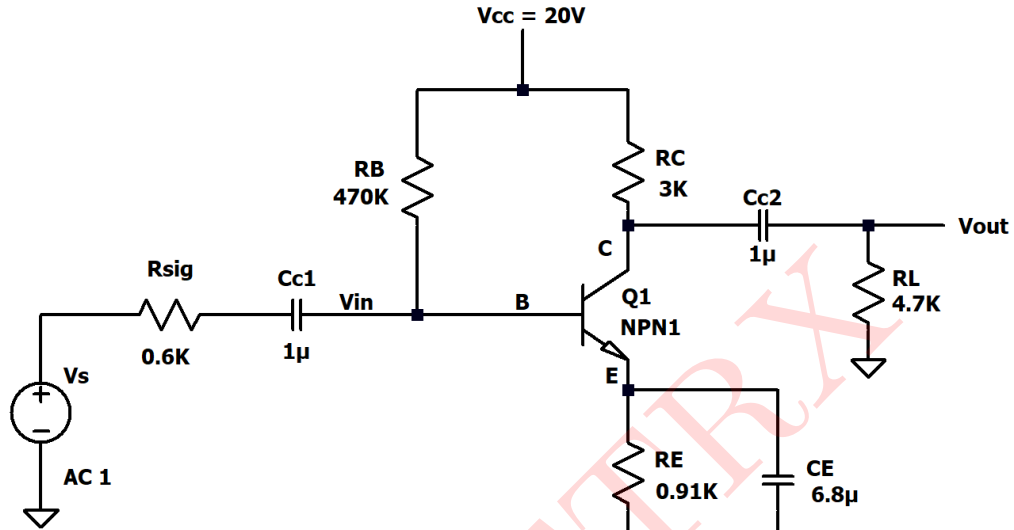


Figure 1: Circuit 1

Solution:

DC ANALYSIS:

$f = 0$, thus $X_C = \infty$, so we replace each capacitor with short circuit.

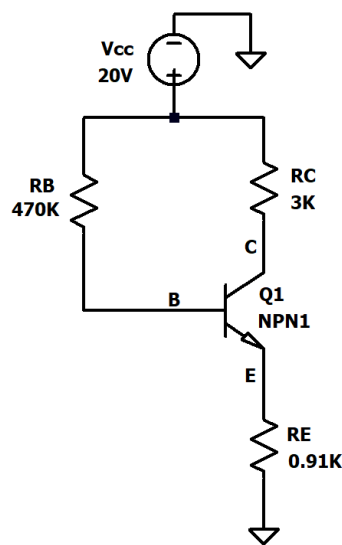


Figure 2: DC Equivalent Circuit

Applying KVL to the Base - emitter loop;

$$V_{CC} - I_B R_B - V_{BE} - I_E R_E = 0$$

$$\text{But, } I_E = (\beta + 1)I_B \text{ and } V_{BE} = 0.7V$$

$$V_{CC} - I_B R_B - V_{BE} - (\beta + 1)I_B R_E = 0$$

$$\therefore V_{CC} - V_{BE} = I_B R_B + (\beta + 1)I_B R_E = I_B (R_B + (\beta + 1)R_E)$$

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + (\beta + 1)R_E} = \frac{20 - 0.7}{470k + (101)(0.91 \times 10^3)} = \frac{19.3}{56190} = \mathbf{34.34\mu A}$$

$$I_{BQ} = \mathbf{34.343\mu A}$$

$$\text{Now, } I_C = \beta I_B = 100 \times 34.34 \times 10^{-6}$$

$$I_{CQ} = \mathbf{3.435mA}$$

Small Signal parameters:

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{100 \times 26mV}{3.435mA} = \mathbf{756.914\Omega}$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{3.435mA}{26mV} = \mathbf{132.115mA/V}$$

AC (mid frequency) equivalent circuit:

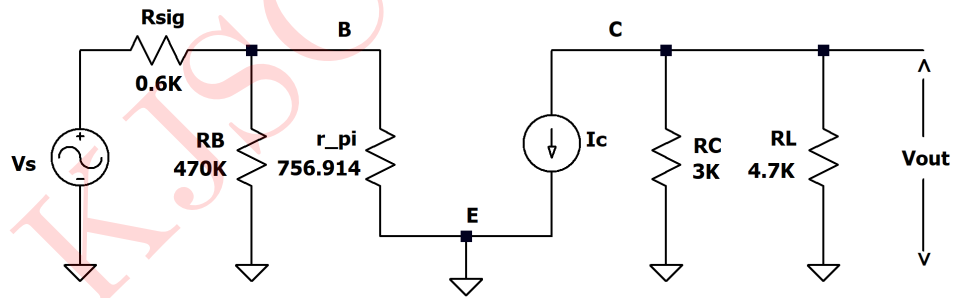


Figure 3: AC (mid frequency) equivalent circuit

$$Z_i = R_B \parallel r_\pi = 470k \parallel 756.914 = \mathbf{755.696\Omega}$$

.....1

$A_{V_{mid}}$ (Mid frequency gain):

$$A_{V_{mid}} = \frac{V_{out}}{V_s} = \frac{V_{out}}{V_{in}} \times \frac{V_{in}}{V_s}$$

$$\frac{V_{out}}{V_{in}} = \frac{-g_m V_\pi (R_C \parallel R_L)}{V_\pi} \quad (\text{As } V_{in} = V_\pi)$$

$$\frac{V_{out}}{V_{in}} = -g_m (R_C \parallel R_L) = -(132.115 \times 10^{-3})(3k \parallel 4.7k) = \mathbf{-241.925}$$

.....2

$A_{V_{mid}}$ with R_{sig} :

$$A_{V_{s(mid)}} = \frac{V_{out}}{V_s} = \frac{V_{out}}{V_{in}} \times \frac{V_{in}}{V_s}$$

Here, $\frac{V_{out}}{V_{in}} = -241.925$

$$V_{in} = \frac{(Z_i)V_s}{R_{sig} + Z_i} = \frac{(R_B \parallel r_\pi) \times V_s}{R_{sig} + (R_B \parallel r_\pi)} \quad (\text{As } Z_i = R_B \parallel r_\pi)$$

$$\frac{V_{in}}{V_s} = \frac{R_B \parallel r_\pi}{R_{sig} + (R_B \parallel r_\pi)} = \frac{470k \parallel 756.914}{0.6k + (470k \parallel 756.914)} = \frac{755.696}{0.6k + 755.696} = \mathbf{0.5574}$$

$$A_{V_{s(mid)}} = \frac{V_{out}}{V_{in}} \times \frac{V_{in}}{V_s} = -241.925 \times 0.5574 = \mathbf{-134.8}$$

$$A_{V_{s(mid)}}(indB) = 20\log_{10}(134.8) = \mathbf{42.5dB}$$

Low frequency equivalent circuit:

We short circuit the AC source V_s

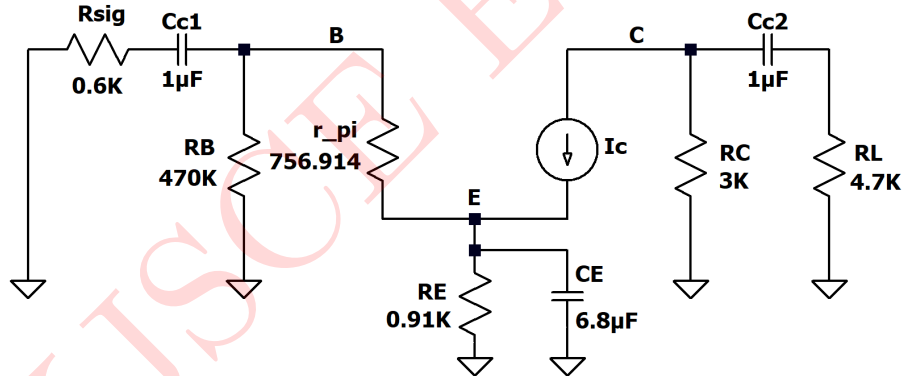


Figure 4: Small signal low frequency equivalent circuit

Low frequency AC equivalent circuit due to C_{C1} alone:

We short circuit other two capacitors C_E and C_{C2} and also short AC source V_s

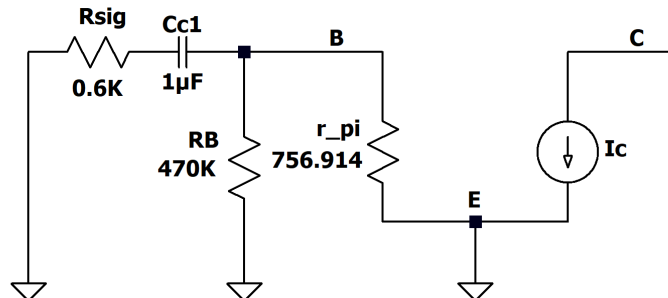


Figure 5: Small signal low frequency equivalent circuit for C_{C1}

$$f_{LCC1} = \frac{1}{2\pi(R_i + R_{sig})C_{C1}} \quad (\text{Where, } C_{C1} = 1\mu F)$$

Here, $R_i = Z_i = R_B \parallel r_\pi$

$R_i = 755.696\Omega$ (from 1)

$$\therefore f_{LCC1} = \frac{1}{2\pi(0.6k + 755.696)(1 \times 10^{-6})} = \mathbf{117.4Hz}$$

Low frequency AC equivalent circuit due to C_{C2} alone:

We short the other capacitors C_E and C_{C1} and also the AC source V_s

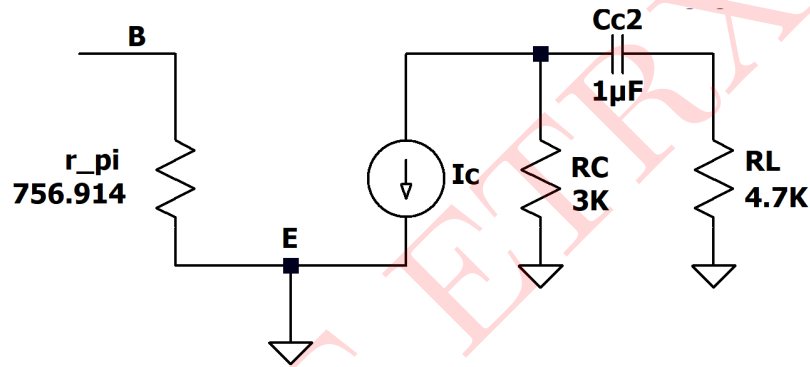


Figure 6: Small signal low frequency equivalent circuit for C_{C2}

$$f_{LCC2} = \frac{1}{2\pi(R_{eq}C_{C2})} \quad (\text{Here } C_{C2} = 1\mu F)$$

Here, $R_{eq} = R_C + R_L = 3k + 4.7k = \mathbf{7.7k}$

$$f_{LCC2} = \frac{1}{2\pi(7.7 \times 10^3 \times 1 \times 10^{-6})} = \mathbf{20.66Hz}$$

Low frequency AC equivalent circuit due to C_E alone:

We short circuit other two capacitors C_{C1} and C_{C2} and also short AC source V_s

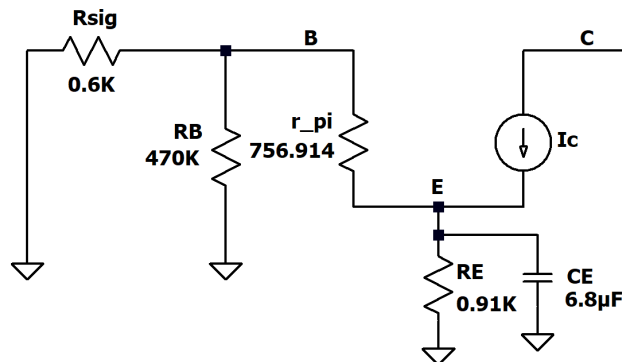


Figure 7: Small signal low frequency equivalent circuit for C_E

$$f_{LCE} = \frac{1}{2\pi(R_{eq1})C_E} \quad (\text{where, } C_E = 20\mu F)$$

$$\text{Here, } R_{eq1} = R_E \parallel \left(\frac{R_{sig} \parallel R_B + r_\pi}{\beta} \right) = 0.91k \parallel \left(\frac{(0.6k \parallel 470k) + 765.914}{100} \right)$$

$$= 0.91k \parallel 13.6514 = \mathbf{13.449\Omega}$$

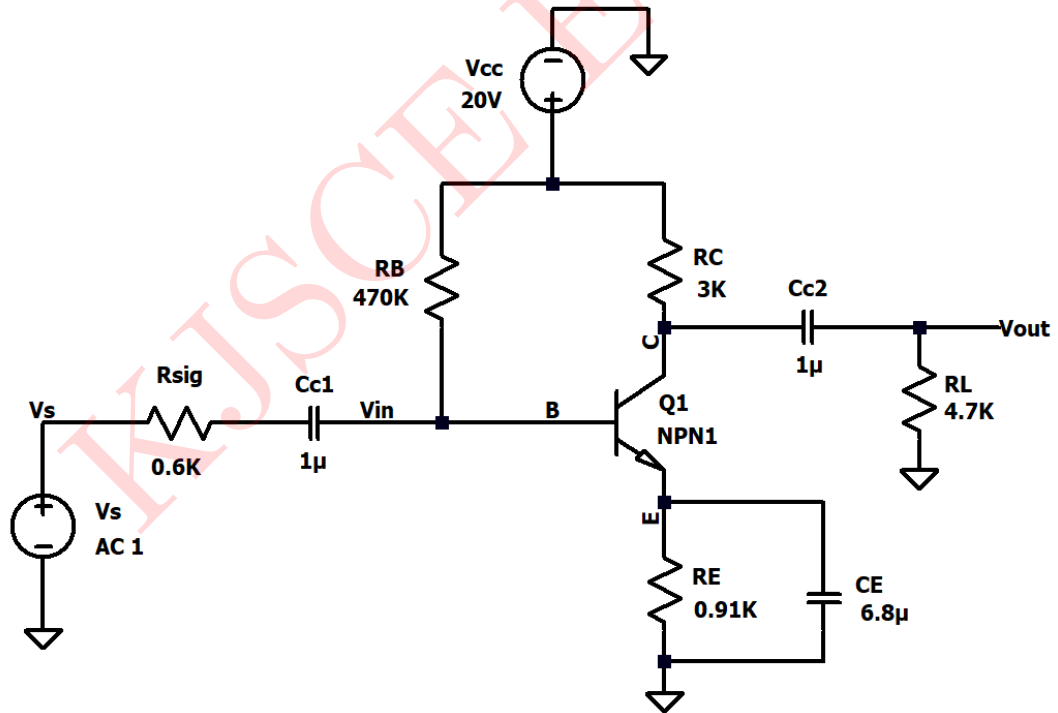
$$f_{LCE} = \frac{1}{2\pi(13.449 \times 6.8 \times 10^{-6})} = \mathbf{1.74kHz}$$

Since, $f_{LCE} = 1.75kHz$ is largest as compared to f_{LCC1} , f_{LCC2} , it is the lower cut-off frequency of the amplifier

$$f_L = \mathbf{1.74kHz}$$

SIMULATED RESULTS:

Above circuit is simulated in LTspice and the result is as follows:



.model NPN1 npn(bf=100)
.ac dec 10 10 10K

Figure 8: Circuit Schematic

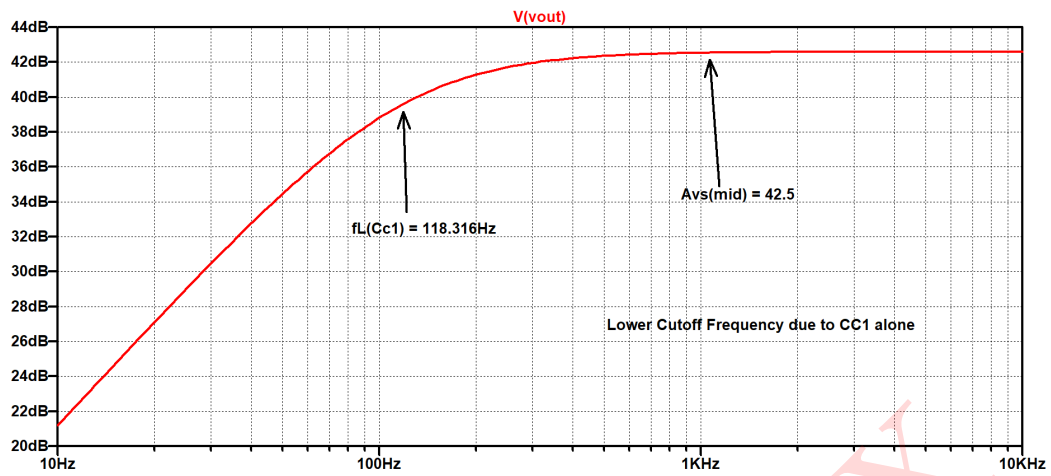


Figure 9: Low frequency response of C_{C1}

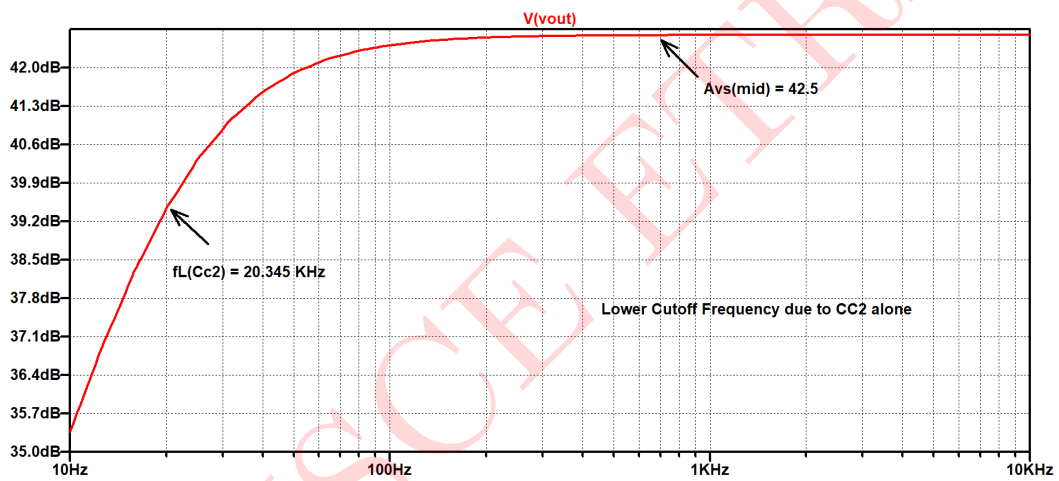


Figure 10: Low frequency response of C_{C2}

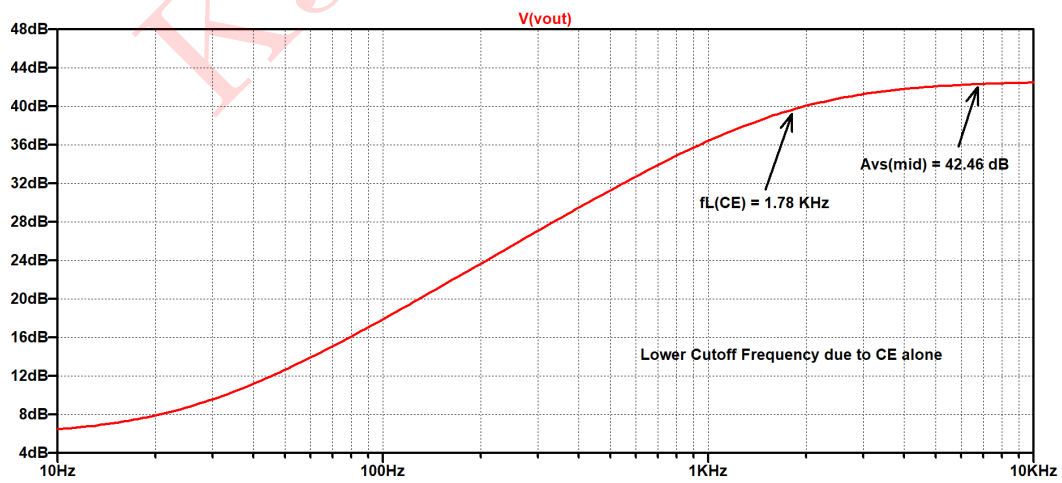


Figure 11: Low frequency response of C_E

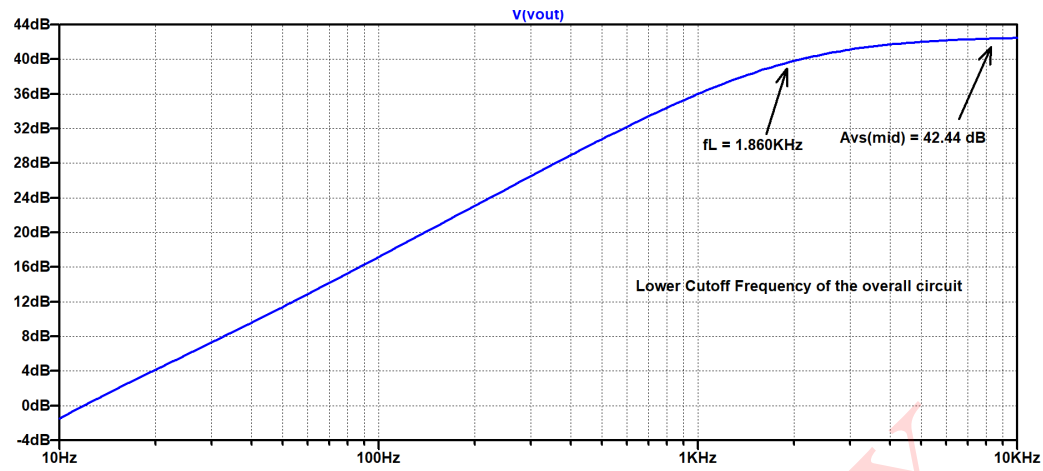


Figure 12: Low frequency response of the circuit

Comparison between Theoretical and Simulated values :-

Parameters	Simulated	Theoretical
I_{CQ}	3.435mA	3.41mA
Lower cur-off frequency due to C_{C1}	117.4Hz	118.31Hz
Lower cut-off frequency due to C_{C2}	20.66Hz	20.345Hz
Lower cut-off frequency due to C_E	1.74kHz	1.78kHz
overall cut-off frequency	1.74kHz	1.86kHz
Mid band voltage gain (in dB)	42.5	42.44

Table 1: Numerical 1

Numerical 2: For the network shown in figure 13 the parameters are $k_p = 1\text{mA}/V^2$, $V_{TP} = -1.5V$ and $\lambda = 0$

- Determine quiescent and small signal parameters of the transistor
- Find lower cut-off frequency

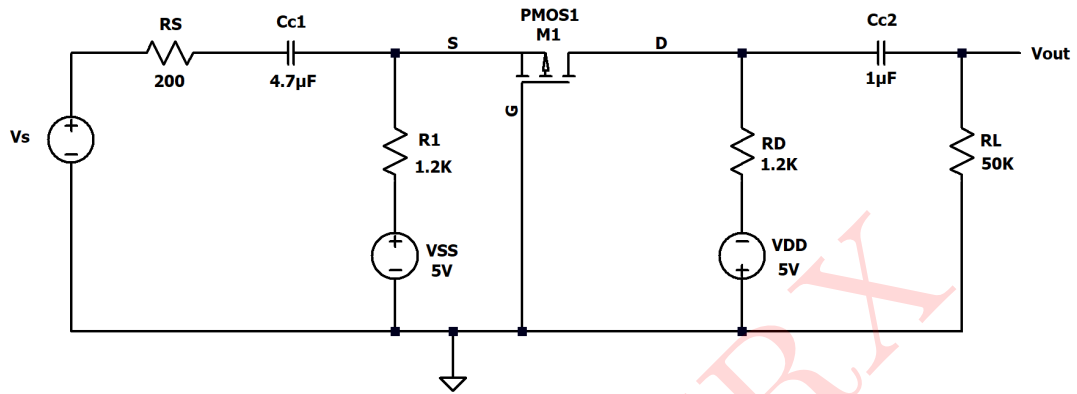


Figure 13: Circuit 2

Solution:

The above circuit is a common gate amplifier employing pmosfet.

DC equivalent circuit:

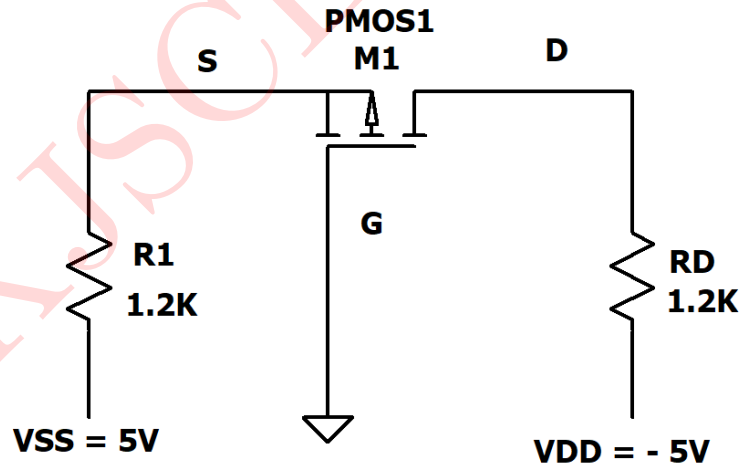


Figure 14: DC Equivalent Circuit

Applying KVL to the Source - Gate loop;

$$V_{SS} - I_S R_1 - V_{SG} = 0$$

But, $I_D = I_S$

$$V_{SS} - I_D R_1 - V_{SG} = 0$$

$$\therefore I_D = \frac{V_{SS} - V_{SG}}{R_1} = \frac{5 - V_{SG}}{1.2 \times 10^3} \quad \dots 1$$

Also we know that for pmosfet in saturation region,

$$I_D = k_p (V_{SG} + V_{TP})^2$$

here, $k_p = 1 \text{mA/V}^2$ and $V_{TP} = -1.5 \text{V}$

$$I_D = 1 \times 10^{-3} (V_{GS} - 1.5)^2 \quad \dots 2$$

From 1 and 2, we get;

$$\frac{5 - V_{SG}}{1.2 \times 10^3} = 1 \times 10^{-3} (V_{GS} - 1.5)^2$$

$$5 - V_{SG} = 1.2 (V_{SG}^2 - 3V_{SG} + 2.25)$$

$$1.2V_{SG}^2 - 3.6V_{SG} + 2.7 + V_{SG} - 5 = 0$$

$$1.2V_{SG}^2 - 2.6V_{SG} - 2.3 = 0$$

On solving we get, $V_{SG} = 2.841 \text{V}$ or -0.6748V

$$V_{GS} = -V_{SG}$$

$$\therefore V_{GS} = 0.67 \text{V} \text{ or } -2.841 \text{V}$$

But as $|V_{GS}| > V_{TP}$

$$\therefore V_{GSQ} = -2.841 \text{V}$$

So, $V_{SGQ} = \mathbf{2.841 \text{V}}$

From 1;

$$I_D = \frac{5 - 2.841}{1.2 \times 10^3} = \mathbf{1.799 \text{mA}}$$

Small Signal parameters:

$$g_m = 2k_p (V_{SG} + V_{TP})$$

here, $V_{TP} = -1.5$

$$\therefore g_m = 2 \times 1 \times 10^{-3} (2.841 - 1.5) = \mathbf{2.682 \text{mA/V}^2}$$

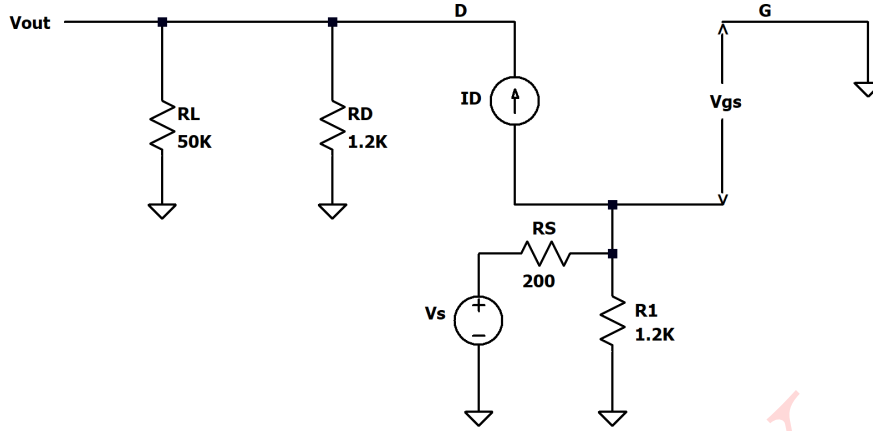


Figure 15: AC (mid frequency) equivalent circuit

A_V (mid frequency gain):

$$A_V = \frac{V_{out}}{V_{in}} = g_m(R_D \parallel R_L) = 2.682 \times 10^{-3}(1.2k \parallel 50k) = \mathbf{3.143}$$

A_{VS} (mid band gain with R_S):

$$A_{VS} = \frac{V_{out}}{V_{in}} \times \frac{V_{in}}{V_S}$$

$$\frac{V_{out}}{V_{in}} = A_V = 3.143$$

$$\frac{V_{in}}{V_S} = \frac{\frac{1}{g_m} \parallel R_1}{\left(\frac{1}{g_m} \parallel R_1\right) + R_S}$$

$$\frac{1}{g_m} \parallel R_1 = \frac{1}{2.682 \times 10^{-3}} \parallel 1.2k = 372.856 \parallel 1.2k = \mathbf{284.46}$$

$$\frac{V_{in}}{V_S} = \frac{284.46}{284.46 + 200} = \mathbf{0.5871}$$

$$A_{VS(mid)} = 3.143 \times 0.5871 = \mathbf{1.845}$$

$$A_{VS(mid)}(indB) = 20\log(1.845) = \mathbf{5.322dB}$$

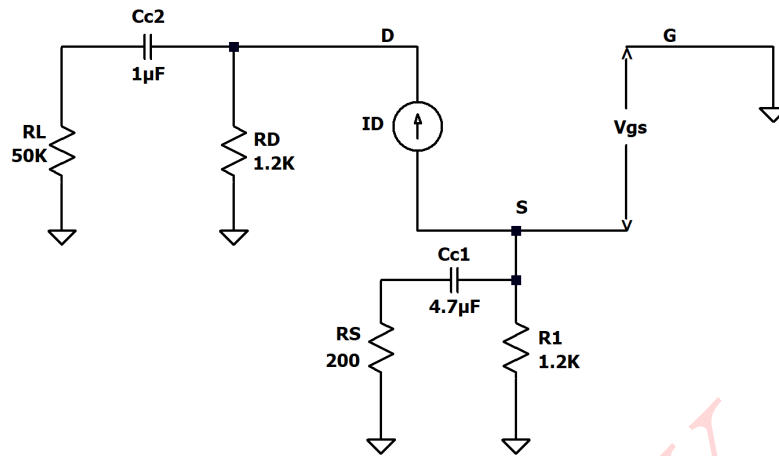


Figure 16: AC low frequency equivalent circuit

Low frequency AC equivalent circuit due to C_{C1} alone:

We short circuit C_{C2} and also short AC source V_S

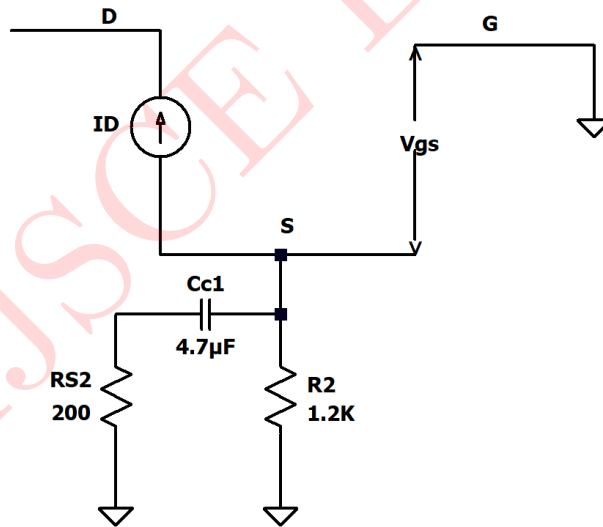


Figure 17: Small signal low frequency equivalent circuit for C_{C1}

$$f_{LCC1} = \frac{1}{2\pi R_{eq} C_{C1}}$$

$$\text{Here, } R_{eq} = R_S + R_1 \parallel \frac{1}{g_m} = 200 + (1.2k \parallel \frac{1}{2.68 \times 10^{-3}}) = \mathbf{484.63}$$

$$f_{LCC1} = \frac{1}{2\pi 484.63 \times 4.7 \times 10^{-6}} = \mathbf{69.87Hz}$$

Low frequency AC equivalent circuit due to C_{C2} alone:

We short C_{C1} and also the AC source V_s

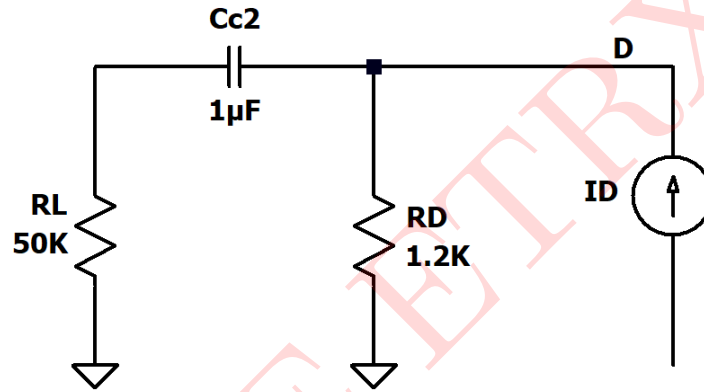


Figure 18: Small signal low frequency equivalent circuit for C_{C2}

$$f_{LCC2} = \frac{1}{2\pi R_{eq} C_{C2}}$$

$$\text{Here, } R_{eq2} = R_L + R_D = 50k + 1.2k = \mathbf{51.2k}$$

$$f_{LCC2} = \frac{1}{2\pi (51.2 \times 10^3 \times 1 \times 10^{-6})} = \mathbf{3.108Hz}$$

Since, f_{LCC1} is greater than f_{LCC2} we choose the lower cut- off frequency of the circuit as $69.87Hz$

$$f_L = \mathbf{69.87Hz}$$

SIMULATED RESULTS:

Above circuit is simulated in LTspice and the result is as follows:

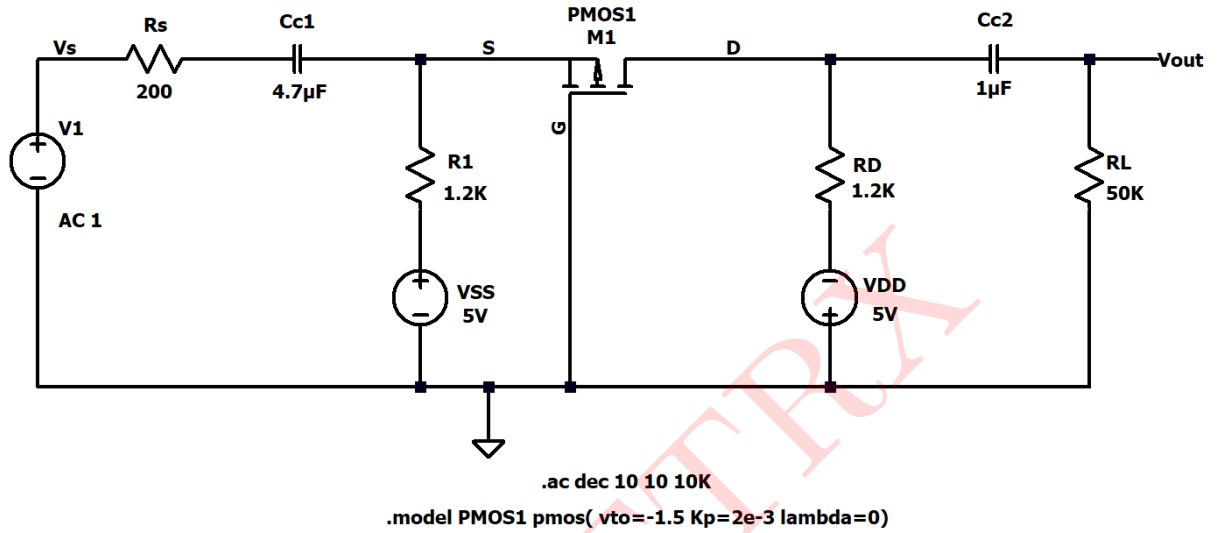


Figure 19: Circuit Schematic

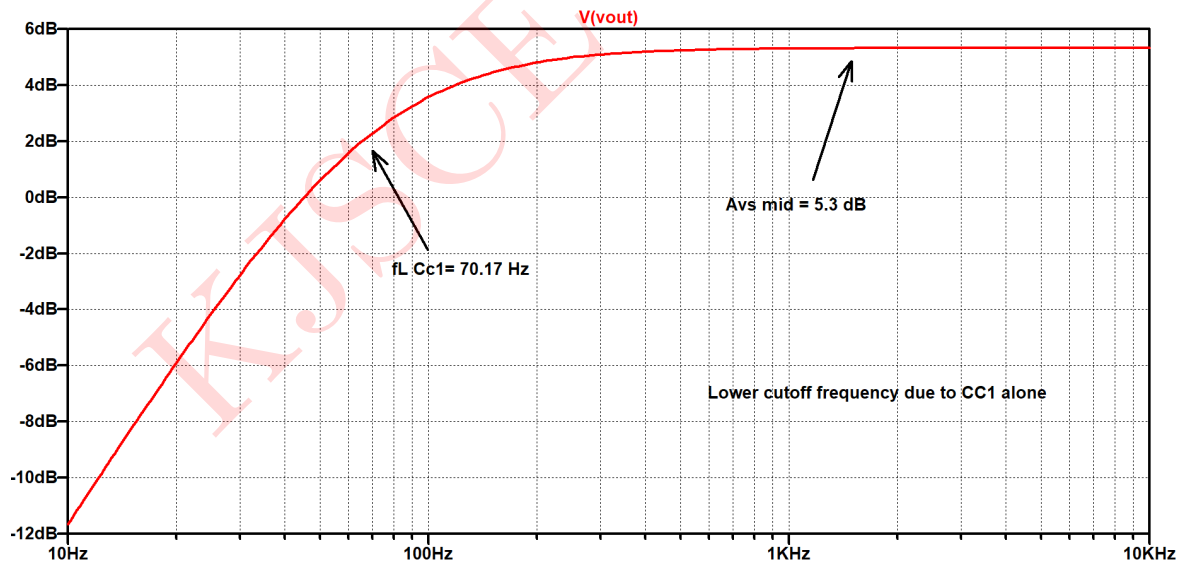


Figure 20: Low frequency response of C_{C1}

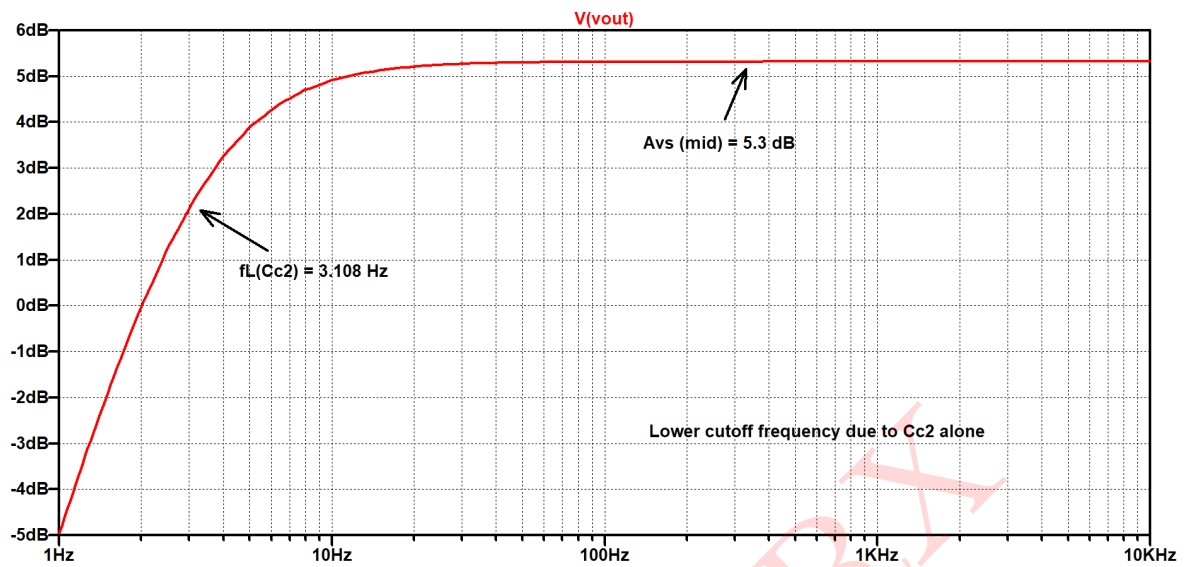


Figure 21: Low frequency response of C_{C2}

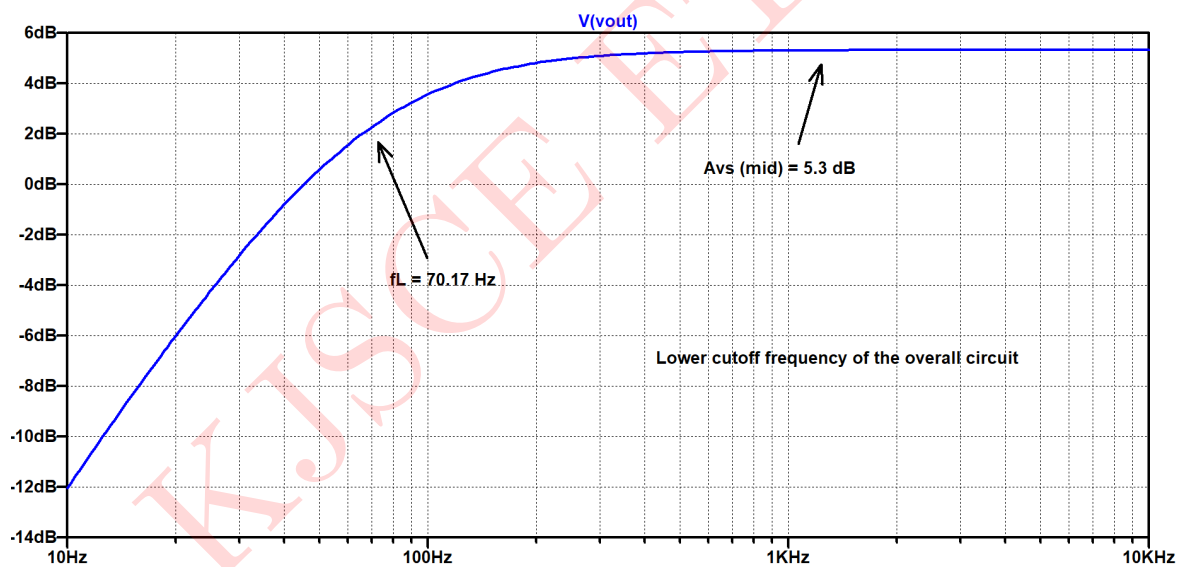


Figure 22: Low frequency response of the circuit

Comparison between Theoretical and Simulated values :-

Parameters	Simulated	Theoretical
I_{CQ}	1.799mA	1.798mA
V_{SGQ}	2.841V	2.84125V
Lower cur-off frequency due to C_{C1}	69.87Hz	70.17Hz
Lower cut-off frequency due to C_{C2}	3.108Hz	3.108Hz
Lower cut-off frequency f_L	69.87Hz	70.17Hz
Mid band voltage gain (in dB)	5.322dB	5.3dB

Table 2: Numerical 2

Numerical 3: For the network shown in figure 23 determine:

- a. r_π b. Z_i c. $A_{V(mid)} = \frac{V_{out}}{V_\pi}$ d. $A_{VS(mid)} = \frac{V_{out}}{V_S}$ e. $f_{LCE}, f_{LCC1}, f_{LCC2}$
 f. Lower cut-off frequency g. Higher cut-off frequency
 where, $C_{wi} = 7pF$, $C_{bc} = 6pF$, $C_{wo} = 11pF$, $C_{be} = 2pF$, $C_{ce} = 10pF$

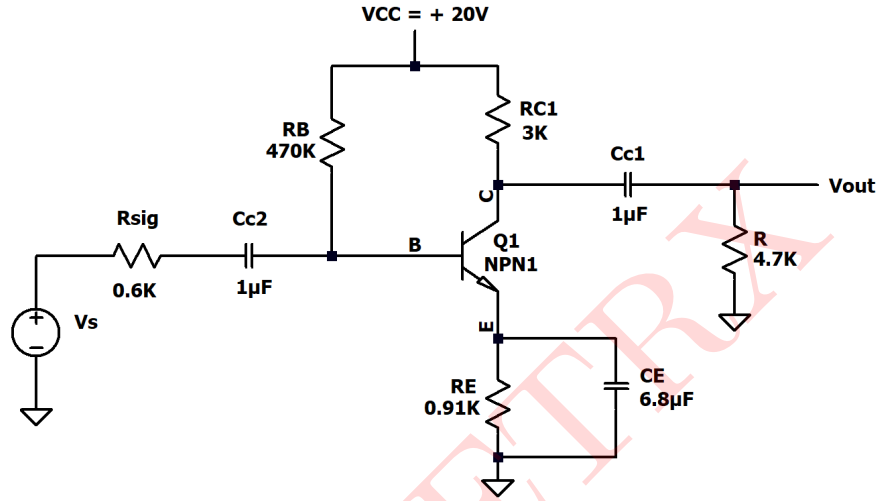


Figure 23: Circuit 3

Solution:

DC ANALYSIS:

$f = 0$, thus $X_C = \infty$, So we replace each capacitor with short circuit,

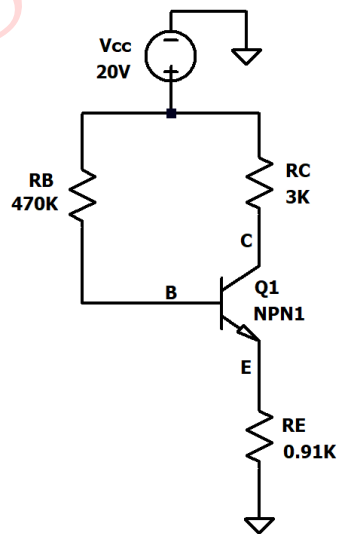


Figure 24: DC Equivalent Circuit

Applying KVL to the Base - emitter loop;

$$V_{CC} - I_B R_B - V_{BE} - I_E R_E = 0$$

But, $I_E = (\beta + 1)I_B$ and $V_{BE} = 0.7$

$$V_{CC} - I_B R_B - V_{BE} - (\beta + 1)I_B R_E = 0$$

$$V_{CC} - V_{BE} = I_B R_B + (\beta + 1)I_B R_E = I_B (R_B + (\beta + 1)R_E)$$

$$I_B = \frac{V_{CC} - V_{BE}}{R_B + (\beta + 1)R_E} = \frac{20 - 0.7}{470k + (101)(0.91 \times 10^3)} = \frac{19.3}{561910} = \mathbf{34.34\mu A}$$

$$\text{Now, } I_C = \beta I_B = 100 \times 34.34 \times 10^{-6} = \mathbf{3.434mA}$$

Small Signal Parameters:

$$r_\pi = \frac{\beta V_T}{I_{CQ}} = \frac{100 \times 26mV}{3.435mA} = \mathbf{756.914\Omega}$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{3.435mA}{26mV} = \mathbf{132.115mA/V}$$

AC (mid frequency) equivalent circuit:

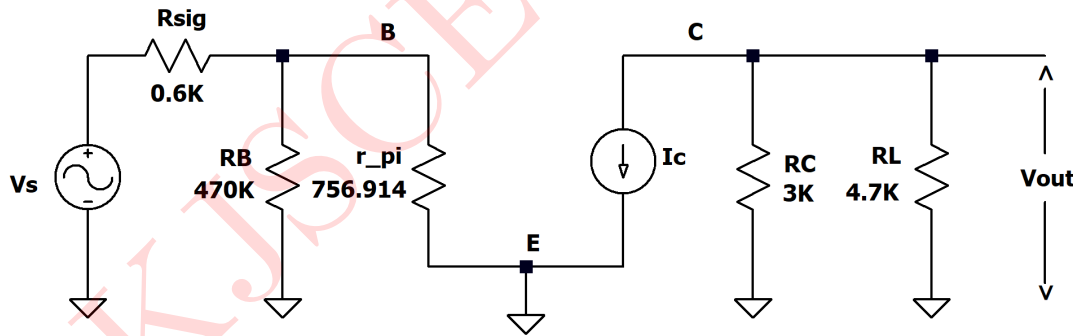


Figure 25: AC (mid frequency) equivalent circuit

$$Z_i = R_B \parallel r_\pi = 470k \parallel 756.914 = \mathbf{755.696\Omega}$$

.....1

$A_{V_{mid}}$ (Mid frequency gain):

$$A_{V_{mid}} = \frac{V_{out}}{V_S} = \frac{V_{out}}{V_{in}} \times \frac{V_{in}}{V_S}$$

$$\frac{V_{out}}{V_{in}} = \frac{-g_m V_\pi (R_C \parallel R_L)}{V_\pi} \quad (\text{As } V_{in} = V_\pi)$$

$$\frac{V_{out}}{V_{in}} = -g_m (R_C \parallel R_L) = -(132.115 \times 10^{-3})(3k \parallel 4.7k) = \mathbf{-241.925}$$

.....2

$A_{V_{mid}}$ with R_{sig} :

$$A_{V_{s(mid)}} = \frac{V_{out}}{V_S} = \frac{V_{out}}{V_{in}} \times \frac{V_{in}}{V_S}$$

Here, $\frac{V_{out}}{V_{in}} = -241.925$

$$V_{in} = \frac{(Z_i)V_S}{R_{sig} + Z_i} = \frac{(R_B \parallel r_\pi) \times V_S}{R_{sig} + (R_B \parallel r_\pi)} \quad (\text{As } Z_i = R_B \parallel r_\pi)$$

$$\frac{V_{in}}{V_S} = \frac{R_B \parallel r_\pi}{R_{sig} + (R_B \parallel r_\pi)} = \frac{470k \parallel 756.914}{0.6k + (470k \parallel 756.914)} = \frac{755.696}{0.6k + 755.696} = \mathbf{0.5574}$$

$$A_{V_{s(mid)}} = \frac{V_{out}}{V_{in}} \times \frac{V_{in}}{V_S} = -241.925 \times 0.5574 = \mathbf{-134.8}$$

$$A_{V_{s(mid)}}(\text{indB}) = 20\log(134.8) = \mathbf{42.5dB}$$

Low frequency equivalent circuit:

We short circuit the AC source V_S

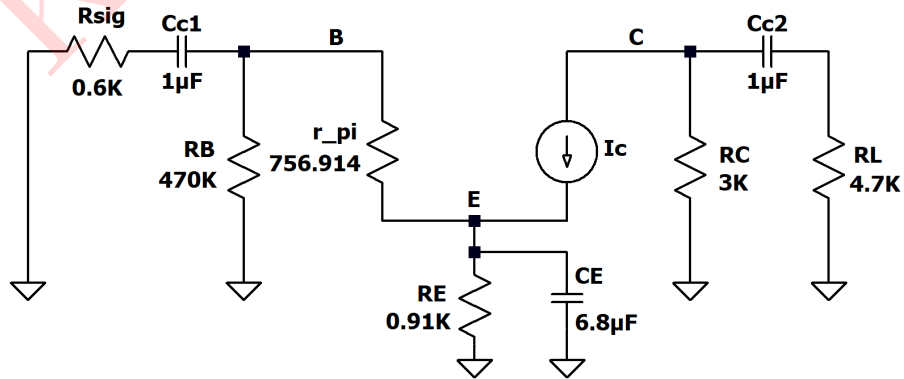


Figure 26: AC low frequency equivalent circuit

Low frequency AC equivalent circuit due to C_{C1} alone:

We short circuit C_{C2} and also short AC source V_S

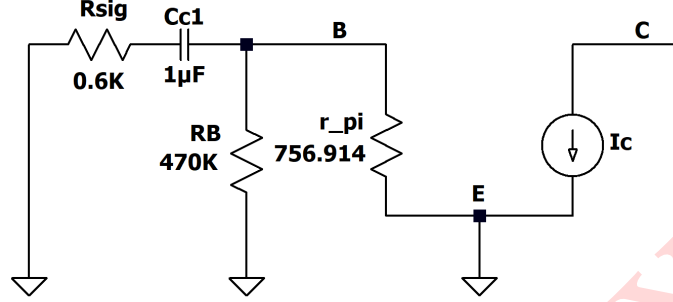


Figure 27: Small signal low frequency equivalent circuit for C_{C1}

$$f_{LCC1} = \frac{1}{2\pi(R_i + R_{sig})C_{C1}} \quad (\text{Where, } C_{C1} = 1\mu F)$$

Here, $R_i = Z_i = R_B \parallel r_\pi$

$R_i = 755.696\Omega$ (from 1)

$$\therefore f_{LCC1} = \frac{1}{2\pi(0.6k + 755.696)(1 \times 10^{-6})} = \mathbf{117.4Hz}$$

Low frequency AC equivalent circuit due to C_{C2} alone:

We short the other capacitors C_E and C_{C1} and also the AC source V_S

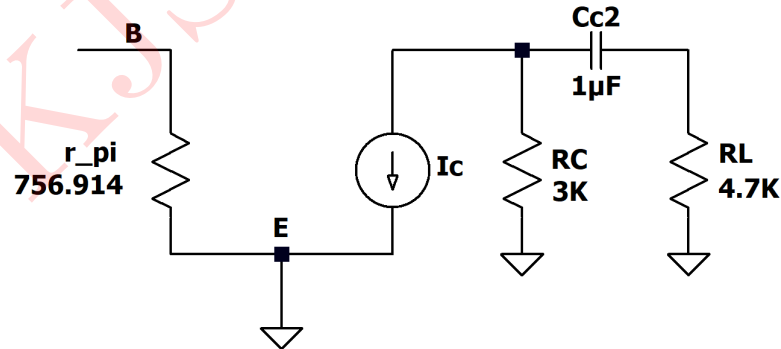


Figure 28: Small signal low frequency equivalent circuit for C_{C2}

$$f_{LCC2} = \frac{1}{2\pi(R_{eq}C_{C2})} \quad (\text{Here } C_{C2} = 1\mu F)$$

Here, $R_{eq} = R_C + R_L = 3k + 4.7k = \mathbf{7.7k}$

$$f_{LCC2} = \frac{1}{2\pi(7.7 \times 10^3 \times 1 \times 10^{-6})} = \mathbf{20.66Hz}$$

Low frequency AC equivalent circuit due to C_E alone:

We short circuit other two capacitors C_{C1} and C_{C2} and also short AC source V_S

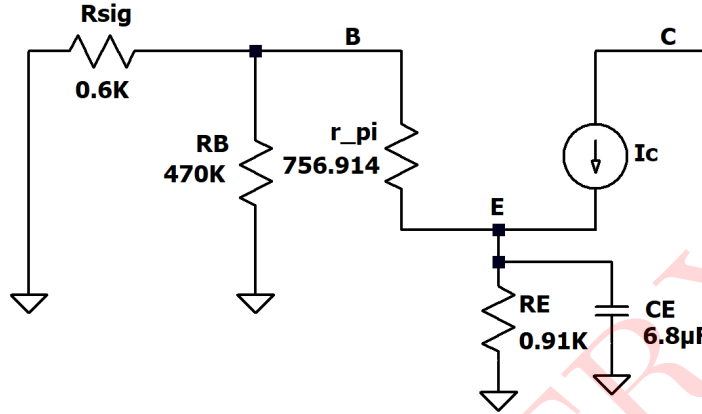


Figure 29: Small signal low frequency equivalent circuit for C_E

$$f_{LCE} = \frac{1}{2\pi(R_{eq1})C_E} \quad (\text{where, } C_E = 20\mu F)$$

$$\begin{aligned} \text{Here, } R_{eq1} &= R_E \parallel \left(\frac{R_{sig} \parallel R_B + r_\pi}{\beta} \right) = 0.91k \parallel \left(\frac{(0.6k \parallel 470k) + 765.914}{100} \right) \\ &= 0.91k \parallel 13.6514 = \mathbf{13.449\Omega} \end{aligned}$$

$$f_{LCE} = \frac{1}{2\pi(13.449 \times 6.8 \times 10^{-6})} = \mathbf{1.74kHz}$$

Since, $f_{LCE} = 1.75kHz$ is largest as compared to f_{LCC1} , f_{LCC2} , it is the lower cut-off frequency of the amplifier

$$f_L = \mathbf{1.74kHz}$$

High frequency equivalent circuit:

Here we short circuit capacitors C_{C1} , C_{C2} , C_{CE} Here, $C_i = C_{wi} + C_{mi} + C_{be}$

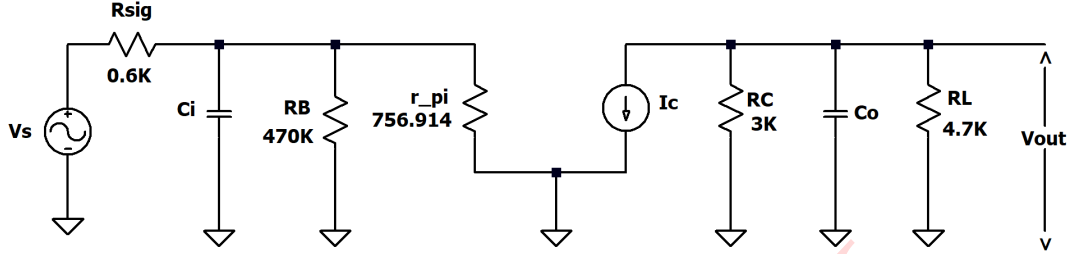


Figure 30: High frequency equivalent circuit

$$C_{mi} = C_{bc}(1 - A_{V(mid)})$$

$$\text{Here } A_{V(Mid)} = -134.8$$

$$\therefore C_{mi} = 6 \times 10^{-12}(1 - (-134.8)) = 814.8 \text{ pF}$$

$$C_i = 814.8 \text{ pF} + 7 \text{ pF} + 20 \text{ pF} = 841.8 \text{ pF}$$

$$\text{Also, } C_o = C_{wo} + C_{mo} + C_{cb} + C_{ce}$$

$$C_{mo} = C_{bc} \left(1 - \frac{1}{A_{V(mid)}} \right) = 4 \times 10^{-12} \left(1 - \frac{1}{-134.8} \right) = 4.0296 \text{ pF}$$

$$C_o = 4.0296 \text{ pF} + 11 \text{ pF} + 10 \text{ pF} = 25.0296 \text{ pF}$$

$$f_{Hi} = \frac{1}{2\pi R_{eq} C_i}$$

$$\text{Here, } R_{eq} = R_{sig} \parallel R_B \parallel r_{\pi} = 0.6k \parallel 470k \parallel 756.914 = 334.45 \Omega$$

$$f_{Hi} = \frac{1}{2\pi(334.45 \times 841.8 \times 10^{-12})} = 565.3 \text{ kHz}$$

$$\text{Also, } f_{Ho} = \frac{1}{2\pi(R_{eq2})C_o}$$

$$\text{Here, } R_{eq2} = R_C \parallel R_L = 3k \parallel 4.7K = 1831.16 \Omega$$

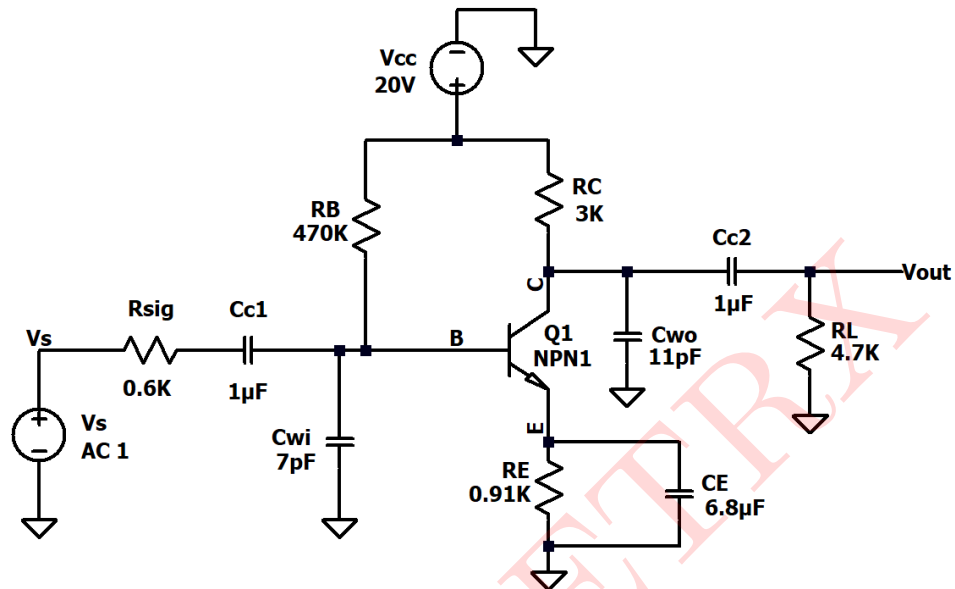
$$f_{Ho} = \frac{1}{2\pi(181.16)(25.0296)} = 3.472 \text{ MHz}$$

Since f_{Hi} is lowest among f_{Ho} and f_{Hi} , we select higher cut-off frequency as $f_H = f_{Hi}$

$$f_H = 565.3 \text{ kHz}$$

SIMULATED RESULTS:

Above circuit is simulated in LTspice and the result is as follows:



```
.model NPN1 npn(bf=100 cje=20pF cjc=6pF)
.ac dec 10 10 10MEG
```

Figure 31: Circuit Schematic

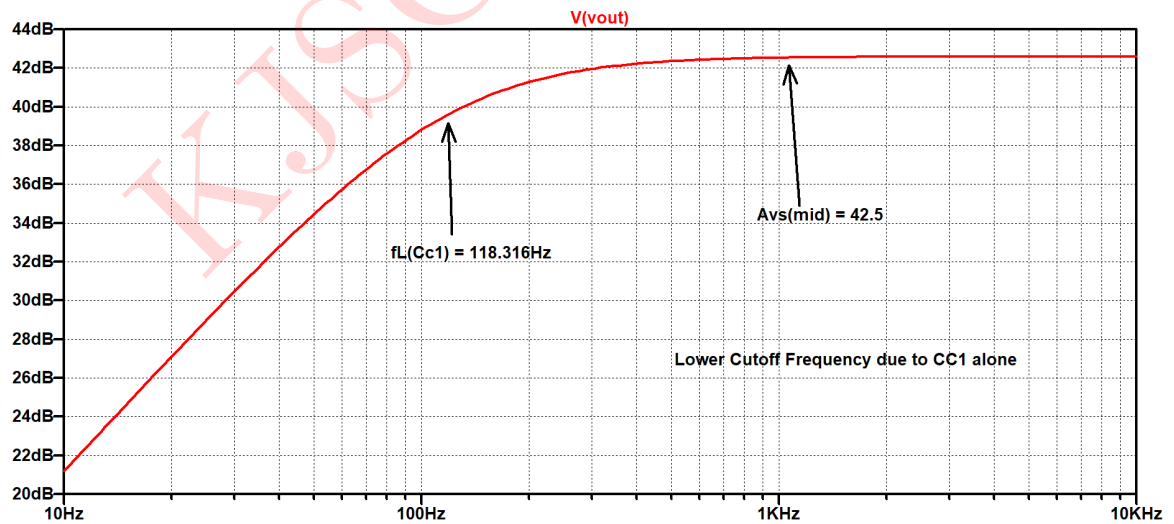


Figure 32: Low frequency response of C_{C1}

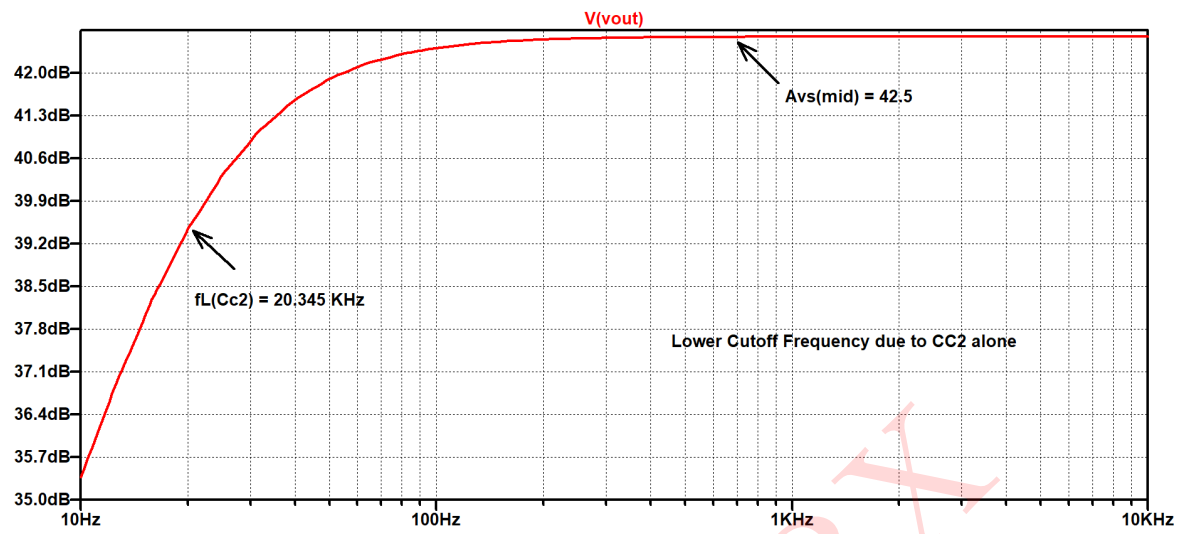


Figure 33: Low frequency response of C_{C2}

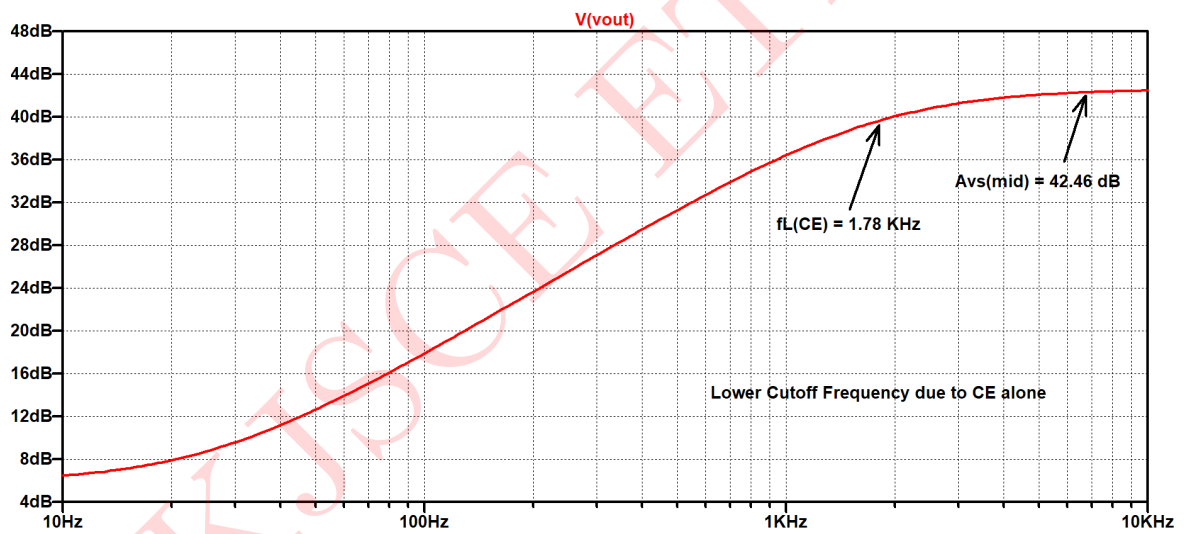


Figure 34: Low frequency response of C_E

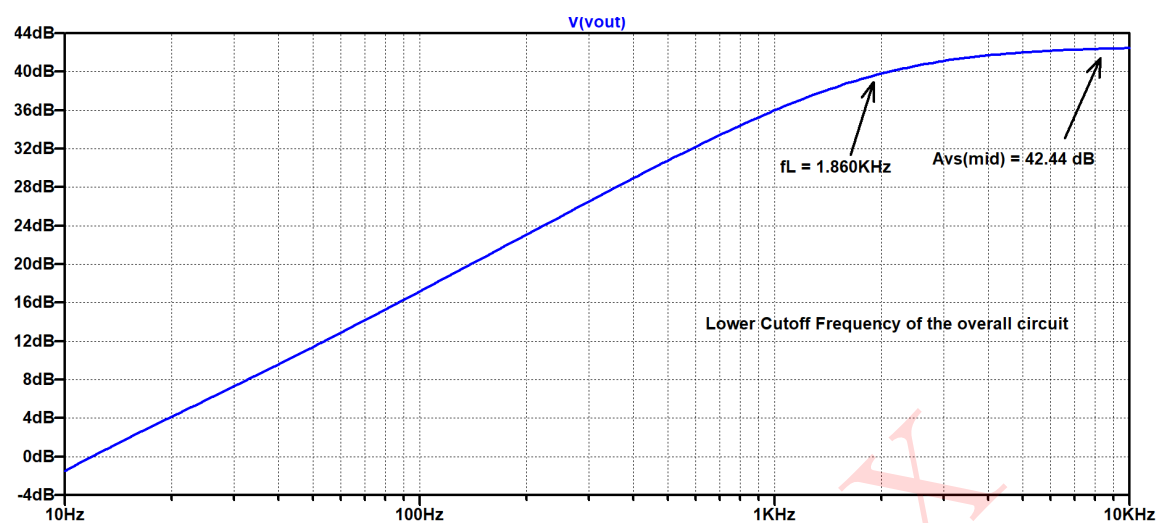


Figure 35: Low frequency response of the circuit

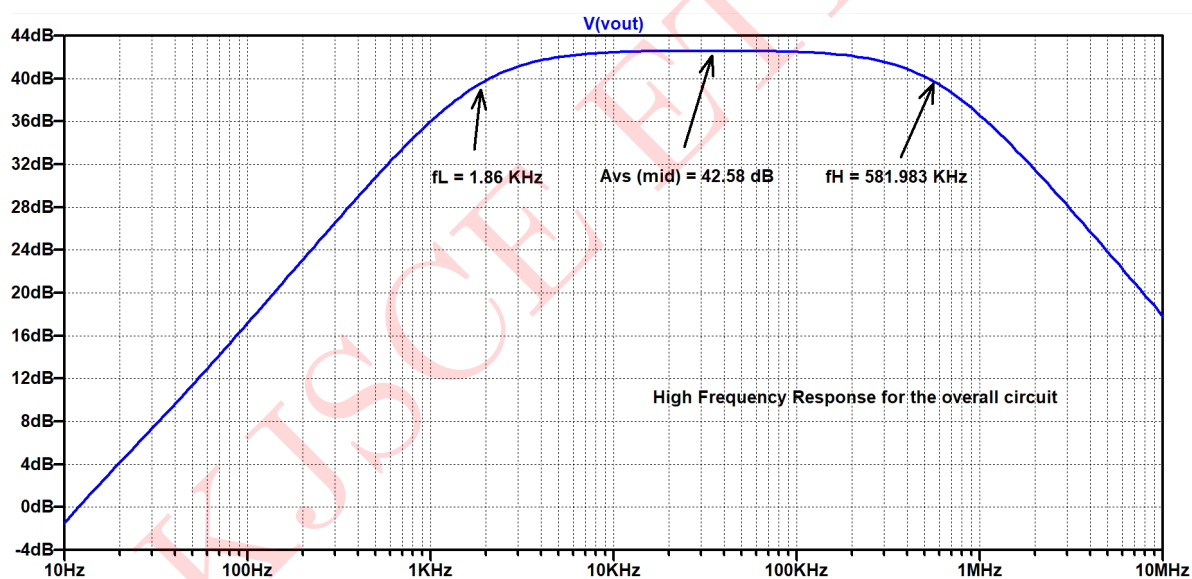


Figure 36: High frequency response of the circuit

Comparison between Theoretical and Simulated values :-

Parameters	Simulated	Theoretical
I_{CQ}	3.435mA	3.41mA
Lower cut-off frequency due to C_{C1}	117.4Hz	118.31Hz
Lower cut-off frequency due to C_{C2}	20.66Hz	20.345Hz
Lower cut-off frequency due to C_{CE}	1.74kHz	1.78kHz
Overall cut-off frequency f_L	1.74kHz	1.86Hz
Mid band voltage gain (in dB)	42.5dB	42.44dB
Overall cut-off frequency f_H	565.3kHz	581.983kHz

Table 3: Numerical 1
