K. J. SOMAIYA COLLEGE OF ENGINEERING
DEPARTMENT OF ELECTRONICS ENGINEERING
ELECTRONIC CIRCUITS
Low & High Frequency Response of Single-Stage Amplifier

Numerical 1:
For the circuit shown in figure 1, The transistor parameters are $\beta = 100$, $V_{BE_{ON}} = 0.7V$.
a) Calculate Lower corner frequency b) Determine mid-band voltage gain.

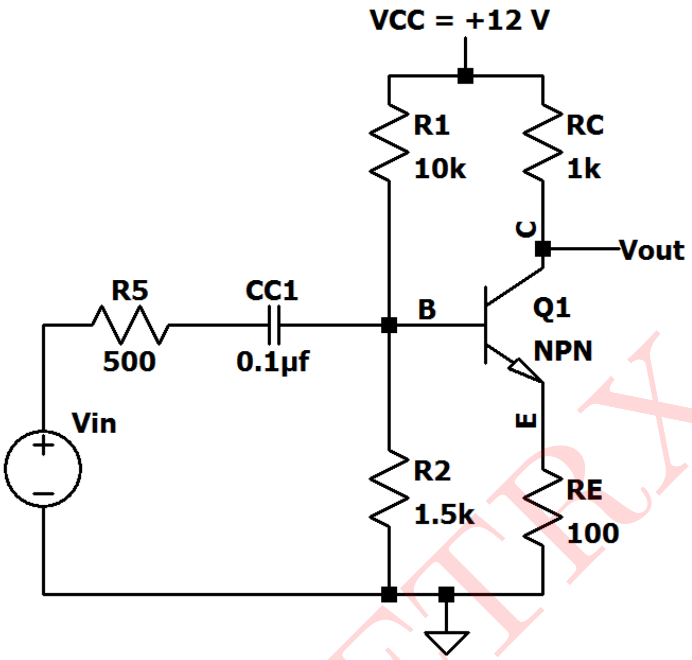

Figure 1: Circuit 1

Solution:

Above circuit 1 is a common-emitter BJT amplifier

DC Analysis:-

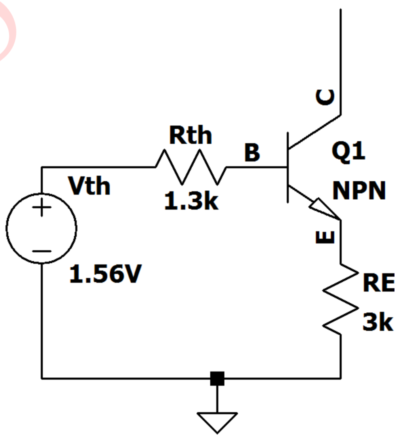

Figure 2: DC Equivalent circuit

$$V_B = V_{th} = \frac{R_2}{R_1 + R_2} \times V_{CC} = \frac{1.5k\Omega}{10k\Omega + 1.5k\Omega} \times 12 = 1.56V$$

$\boldsymbol{V_{th} = 1.56V}$

$R_{th} = R1 \mid\mid R2 = 1.30k\Omega$

$\boldsymbol{R_{th} = 1.30k\Omega}$

Applying KVL to the base-emitter loop:-

$V_{th} - I_B R_{th} - V_{BE(ON)} - I_E R_E = 0$

$I_E = I_C + I_B = (\beta + 1) I_B$

Assume $V_{BE(ON)} = 0.7V$

$V_{th} - I_B R_{th} - 0.7V - (\beta + 1) I_B R_E = 0$

$$I_B = \frac{V_{th} - 0.7V}{R_{th} + ((\beta + 1) R_E)} = \frac{1.56 - 0.7}{1.3k\Omega + (101 \times 0.1k\Omega)} = 75.43\mu A$$

$\boldsymbol{I_B = 75.43\mu A}$

$I_C = \beta I_B = 100 \times 75.43\mu A = 7.5mA$

$\boldsymbol{I_C = 7.5mA}$

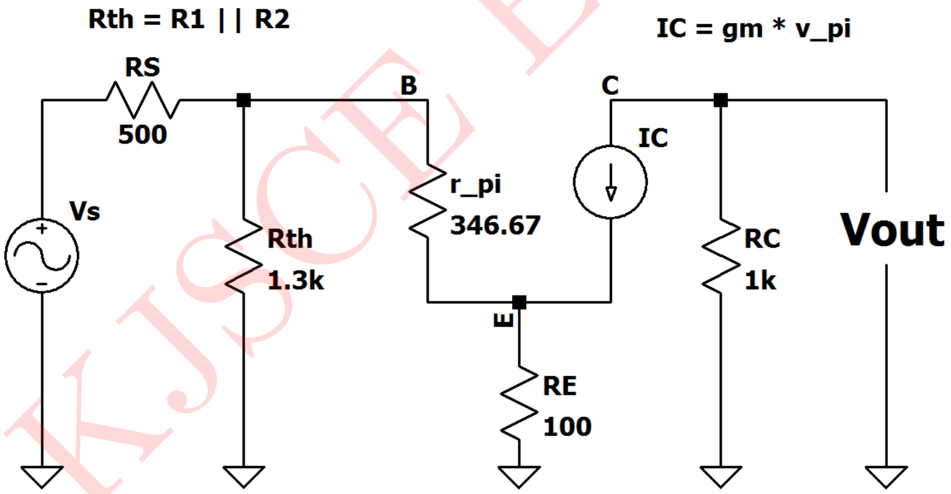

Figure 3: Small Signal Equivalent Circuit

Small-Signal parameters:-

$$g_m = \frac{I_C}{V_T} = \frac{7.5mA}{26mV} = 288.46\frac{mA}{V}$$

$\boldsymbol{g_m = 288.46\frac{mA}{V}}$

$$r_\pi = \frac{V_T}{I_B} = \frac{26mV}{75.43\mu A} = 346.67\Omega$$

$\boldsymbol{r_\pi = 346.67\Omega}$

Applying KCL at the collector terminal:-

$$g_m V_\pi + \frac{V_{out}}{R_C} = 0$$

$$V_\pi = -\frac{V_{out}}{g_m R_C} \qquad \text{.........(1)}$$

Applying KCL at emitter node,

$$\left(\frac{V_\pi}{r_\pi} + g_m V_\pi\right) R_E = \text{Voltage drop on } R_E$$

$$\left[\frac{-V_{out}}{g_m R_C} \times \frac{1}{r_\pi} + g_m\left(\frac{-V_{out}}{g_m R_C}\right)\right] R_E = \text{Voltage drop on } R_E \qquad \text{.........(2)}$$

Applying KVL at the base-emitter loop:-

$V_{in} = V_\pi + $ Voltage drop on R_E

From (1) & (2)

$$V_{in} = \frac{-V_{out}}{g_m R_C} - \frac{V_{out}}{g_m r_\pi R_C} R_E - \frac{V_{out}}{R_C} R_E$$

$$V_{in} = \frac{-V_{out}}{g_m R_C} - \frac{V_{out}}{\beta R_C} R_E - \frac{V_{out}}{R_C} R_E \qquad [\because g_m R_\pi = \beta]$$

$$V_{in} = \frac{-V_{out}\beta + g_m R_E V_{out} + \beta g_m R_E V_{out}}{\beta (g_m R_C)}$$

$$\frac{V_{out}}{V_{in}} = \frac{-\beta g_m R_C}{\beta + (\beta + 1) g_m R_E}$$

$$A_V = \frac{V_{out}}{V_{in}} = \frac{-R_C}{\frac{1}{g_m} + \left(\frac{\beta + 1}{\beta}\right) R_E}$$

$$A_V = \frac{V_{out}}{V_{in}} = \frac{-1k\Omega}{\frac{1}{0.28846} + \left(\frac{101}{100}\right) \times 100} = -9.57$$

$\boldsymbol{A_V = -9.57}$

$$A_{V_{mid}} \text{ with } R_S = \frac{V_{out}}{V_S} = \frac{V_{out}}{V_{in}} \times \frac{V_{in}}{V_S}$$

Current through base $= I_B$, Current through emitter $= I_E = (\beta + 1) I_B$

$$Z_i = R_{th} \mid\mid [r_\pi + (\beta + 1) R_E]$$

$$\frac{V_{in}}{V_S} = \frac{Z_i}{Z_i + R_S}$$

$$Z_i = 1300 \mid\mid [346.67 + (10100)] = 1300 \mid\mid 10446.67 = 1.16k\Omega$$

$\boldsymbol{Z_i = 1.16k\Omega}$

$$\frac{V_{in}}{V_S} = \frac{1160}{1160 + 500} = 0.6987$$

$$\boldsymbol{\frac{V_{in}}{V_S} = 0.6987}$$

$$A_{V_{mid}} \text{ with } R_S = \frac{V_{out}}{V_{in}} \times \frac{V_{in}}{V_S} = -9.57 \times 0.6987 = -6.686$$

$$\boldsymbol{A_{V_{mid}} \text{ with } R_S = -6.686}$$

$$A_{V_{mid}} \text{ in dB} = 20log_{10}(6.686) = 16.5dB$$

$$\boldsymbol{A_{V_{mid}} \text{ in dB} = 16.5dB}$$

Lower cut-off frequency analysis:-

Due to C_{C_1} alone:-

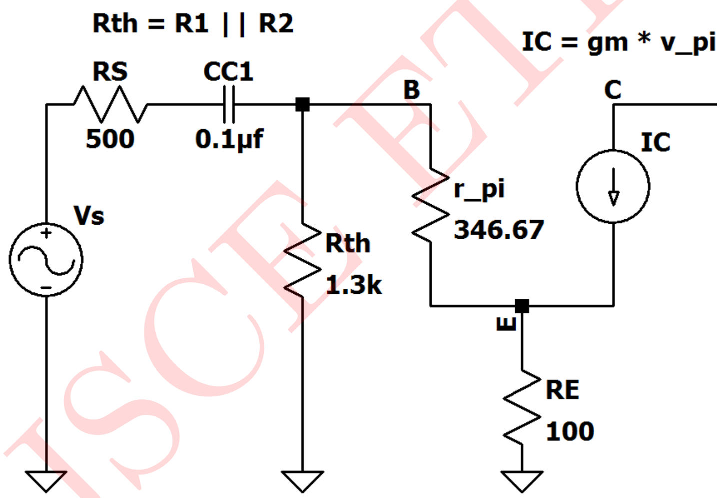

Figure 4: Small Signal low frequency equivalent circuit for C_{C_1}

To find R_{eq}

$R_{eq} = R_S + R_i$

$R_i = R_{th}||(r_\pi + (\beta + 1)R_E)$

$R_i = R_{eq} = 1.16k\Omega$

$$f_{LCC1} = \frac{1}{2\pi \times C_{C_1} \times R_{eq}} = \frac{1}{2\pi \times 0.1 \times 10^{-6} \times 1.66 \times 10^3} = 958.76Hz$$

Since we have only one capacitor, Overall cut-off frequency f_L will be:-

$$\boldsymbol{f_L = f_{LCC1} = 958.76Hz}$$

SIMULATED RESULTS:

Above circuit was simulated in LTSpice and results are presented below:

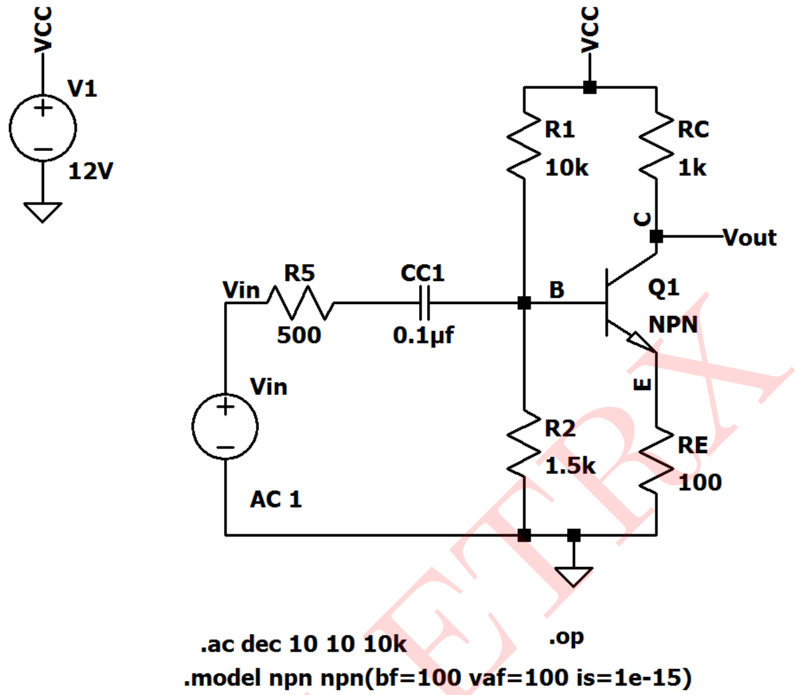

Figure 5: Circuit Schematic 1

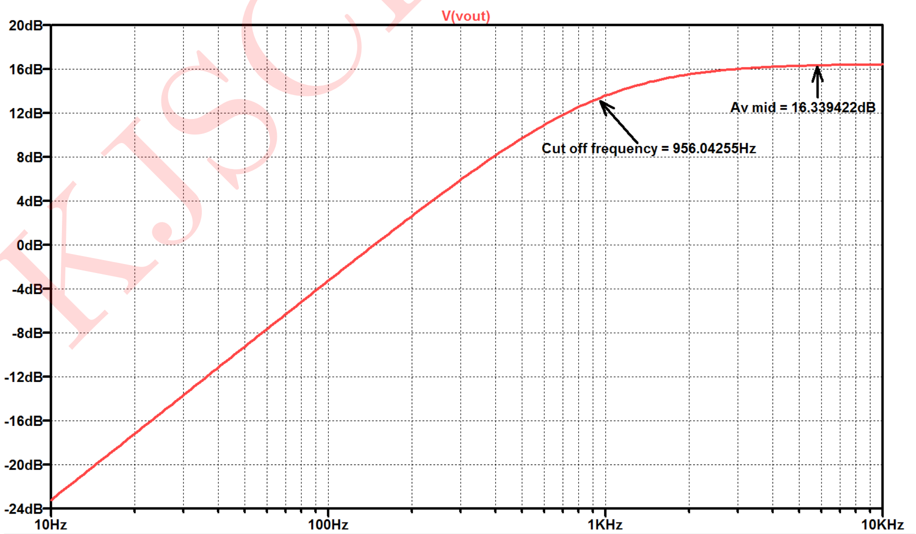

Figure 6: Low frequency response

Comparison of Theoretical and Simulated Values:

Parameters	Theoretical	Simulated
I_C	$7.5mA$	$7.1mA$
$A_{V_{mid}}(dB)$	16.5	16.34
Overall Cut-off frequency f_L	$958.76Hz$	$956.04Hz$

Table 1: Numerical 1

Numerical 2:

For the circuit shown in figure 7, a) Determine V_{GS}, I_D b) Find g_{m_o}, g_m c) Calculate mid-band gain(A_V) d) Determine Z_i e) Calculate A_{V_S} f) Determine f_{LCC1}, f_{LCC2}, f_{L_S} g) Determine cut-off frequency

Given: $I_{DSS} = 6mA$, $V_P = -6V$, $r_d = 100k\Omega$

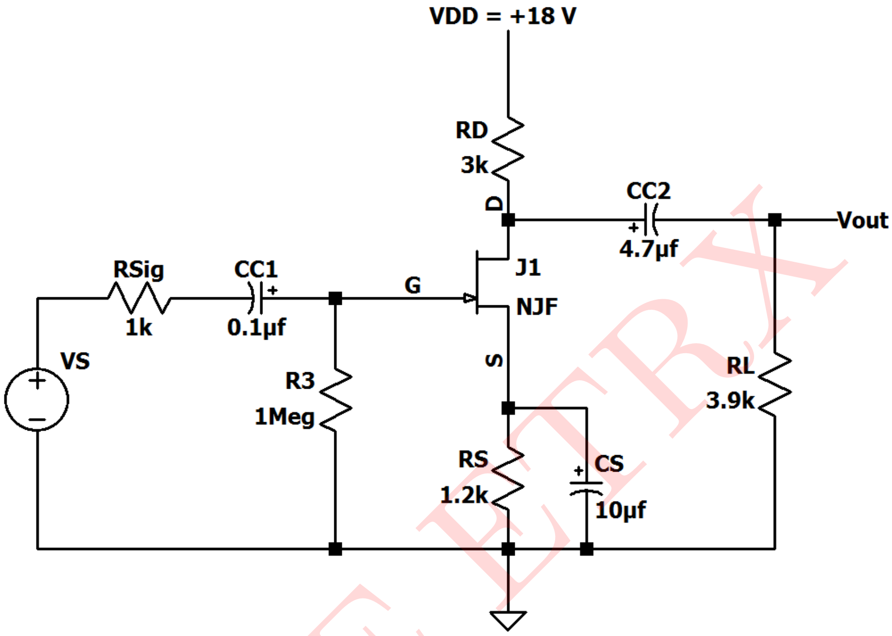

Figure 7: Circuit 2

Solution:

DC Analysis:-

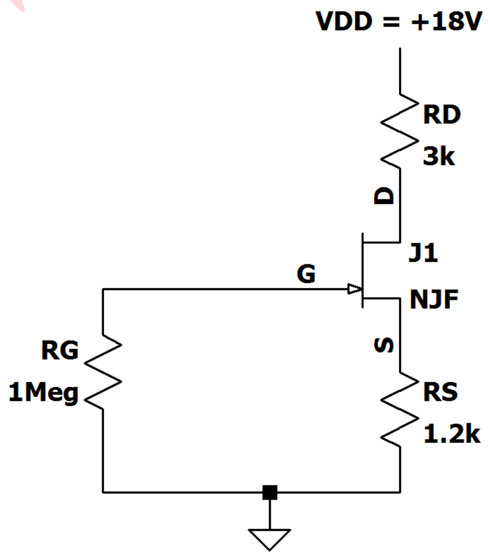

Figure 8: DC Equivalent circuit

Applying KVL to the Gate-Source loop:-

$V_{GS} = -I_D R_S$

$I_D = I_{DSS}\left(1 - \dfrac{V_{GS}}{V_P}\right)^2$

$V_{GS} = -6mA\left(1 + \dfrac{V_{GS}}{6}\right)^2 \times 1.2k\Omega$

$V_{GS} = -7.2\left(1 + \dfrac{V_{GS}}{3} + \dfrac{{V_{GS}}^2}{36}\right)^2$

$V_{GS} = -7.2 - 0.2{V_{GS}}^2 - 2.4V_{GS}$

$0.2{V_{GS}}^2 + 3.4V_{GS} + 7.2 = 0$

Solving above quadratic equation, we get

$V_{GS} = -2.479V$

or

$V_{GS} = -14.52V$, We reject this value, as $(V_{GS} > V_P)$

$\therefore \boldsymbol{V_{GS} = -2.479V}$

$I_D = -\dfrac{V_{GS}}{R_S} = \dfrac{2.479V}{1.2k\Omega} = 2.065mA$

$\boldsymbol{I_D = 2.065mA}$

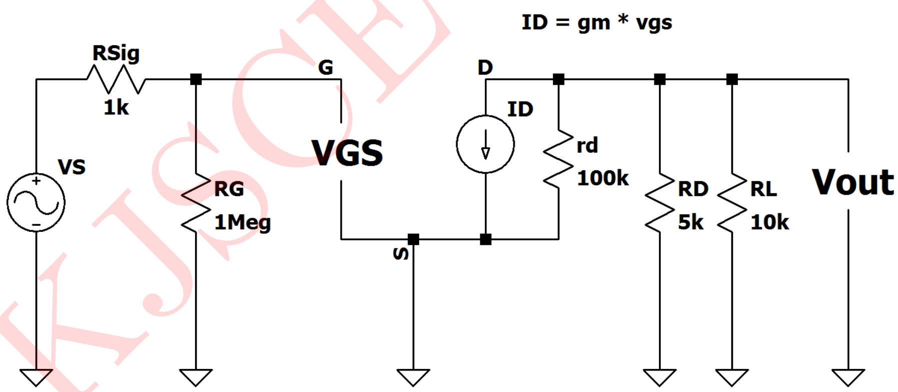

Figure 9: Small Signal Equivalent Circuit

Small-Signal parameters:-

$g_{m_o} = \left|\dfrac{2I_{DSS}}{V_P}\right| = \dfrac{2 \times 6}{6} = 2mA/V$

$g_m = g_{m_o}\left(1 - \dfrac{V_{GS}}{V_P}\right)^2 = 2 \times \left(1 - \dfrac{2.479}{6}\right) = 1.1736\ mA/V$

$\boldsymbol{g_m = 1.1736\ mA/V}$

$\boldsymbol{r_d = 100k\Omega}$ (Given)

Applying KCL at the drain terminal:-

$$g_m V_{GS} + \frac{V_{out}}{r_d} + \frac{V_{out}}{R_D} + \frac{V_{out}}{R_L} = 0$$

$$V_{GS} = -\frac{V_{out}}{g_m}\left(\frac{1}{r_d} + \frac{1}{R_D} + \frac{1}{R_L}\right)$$

$$V_{GS} = V_{in}$$

$$A_{V_{mid}} = \frac{V_{out}}{V_{in}} = -g_m(r_d \mid\mid R_D \mid\mid R_L\ )$$

$$A_{V_{mid}} = -1.1736mA/V(100k \mid\mid 3k \mid\mid 3.9k\ ) = -1.956$$

$$\boldsymbol{A_{V_{mid}} = -1.956}$$

Input Impedance:-

$$Z_i = R_{sig} + R_G$$

$$Z_i = 1k\Omega + 1M\Omega = 1001k\Omega$$

$$\boldsymbol{Z_i = 1001k\Omega}$$

$$A_{V_{mid}} \text{ with } R_{sig} = \frac{V_{out}}{V_S} = \frac{V_{out}}{V_{in}} \times \frac{V_{in}}{V_S}$$

$$\frac{V_{in}}{V_S} = \frac{R_G}{R_G + R_{sig}} = \frac{1M\Omega}{1001k\Omega} = 0.999$$

$$A_{V_{mid}} \text{ with } R_{sig} = 0.99 \times -1.956 = -1.954$$

$$\boldsymbol{A_{V_{mid}} \text{ with } R_{sig} = -1.954}$$

$$A_{V_{mid}} \text{ in } dB = 20log_{10}(1.954) = 5.818dB$$

$$\boldsymbol{A_{V_{mid}} \text{ in } dB = 5.818dB}$$

Due to C_{C_1} alone:-

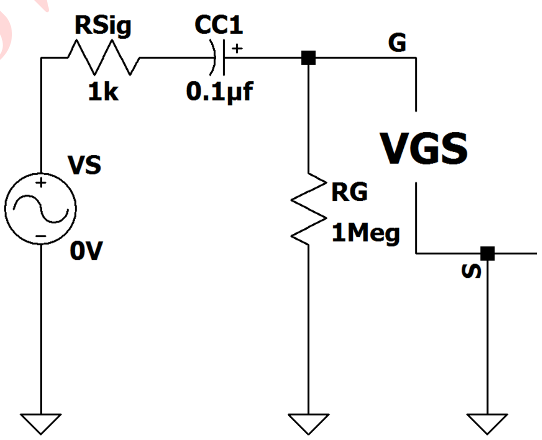

Figure 10: Small Signal low frequency equivalent circuit for C_{C_1}

$R_{eq} = R_{sig} + R_G = 1k\Omega + 1M\Omega = 1001k\Omega$

$$f_{LCC1} = \frac{1}{2\pi \times C_{C_1} \times R_{eq}} = \frac{1}{2\pi \times 1001 \times 1000 \times 0.1} = 1.589Hz$$

$\boldsymbol{f_{LCC1} = 1.589Hz}$

Due to C_{C_2} alone:-

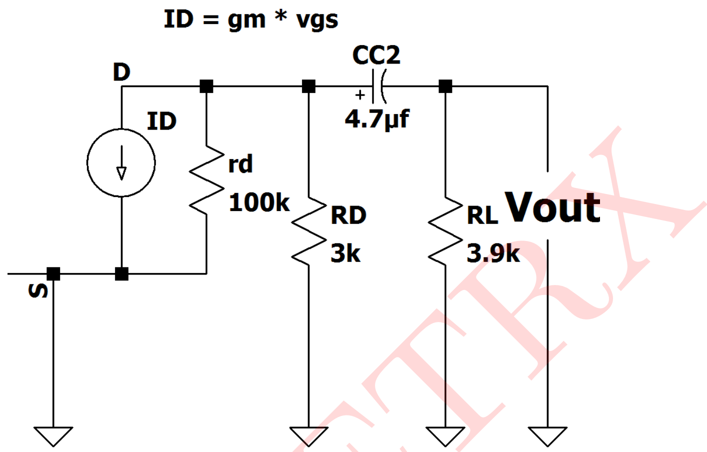

Figure 11: Small Signal low frequency equivalent circuit for C_{C_2}

$R_{eq} = R_D + R_L = (r_d \mid\mid R_D) + R_L = (100k \mid\mid 3k) + 3.9k = 6.813k\Omega$

$$f_{LCC2} = \frac{1}{2\pi \times C_{C_2} \times R_{eq}} = \frac{1}{2\pi \times 6.813k \times 4.7} = 4.97Hz$$

$\boldsymbol{f_{LCC2} = 4.97Hz}$

Due to C_S alone:-

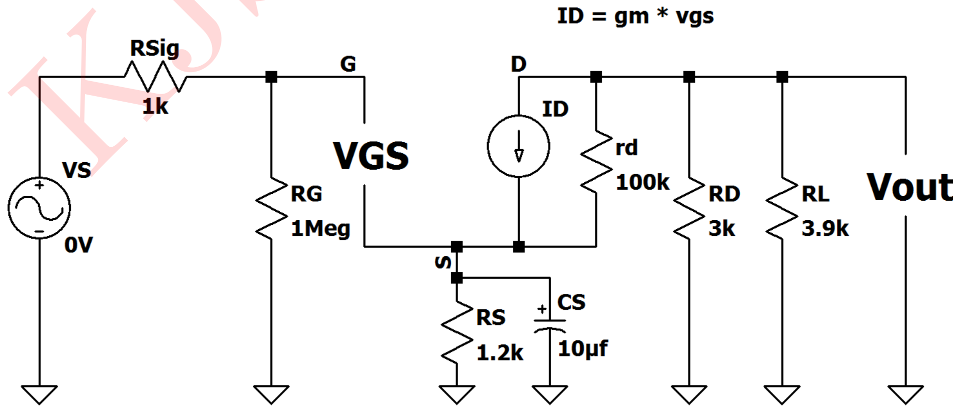

Figure 12: Small Signal low frequency equivalent circuit for C_S

Since $V_S = 0$; and also gate and source are open circuit $\rightarrow \therefore R_{sig}$ & R_G are ignored

$$R_{eq} = R_S \,||\, \left[\frac{r_d + (R_D \,||\, R_L)}{1 + g_m r_d}\right] = 1.2k \,||\, \left[\frac{100k + 1.695k}{1 + 117.36}\right] = 1.2k \,||\, 0.8592k = 500.7\Omega$$

$$f_{LCS} = \frac{1}{2\pi \times C_S \times R_{eq}} = \frac{1}{2\pi \times 10 \times 500.7} = 29.68Hz$$

$\boldsymbol{f_{Ls} = 29.68Hz}$

Since, $f_{LCS} > f_{LCC2} > f_{LCC1}$

$\therefore$ **Lower cut-off frequency** $= \boldsymbol{f_{LCS} = 29.68Hz}$

SIMULATED RESULTS:
Above circuit was simulated in LTSpice and results are presented below:

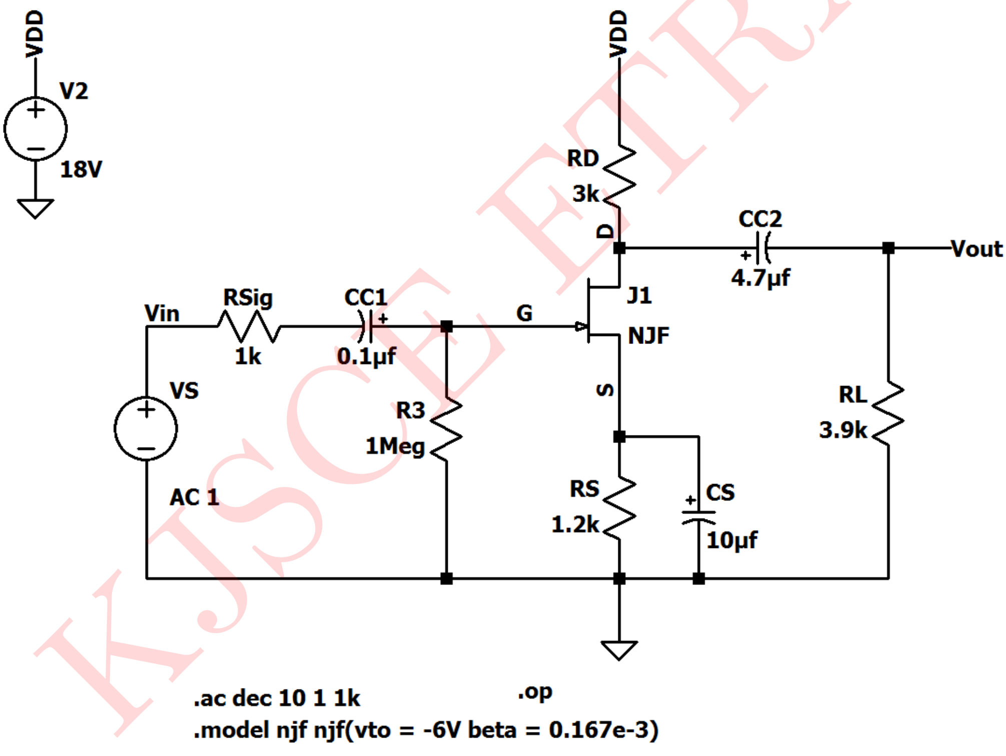

Figure 13: Circuit Schematic 2

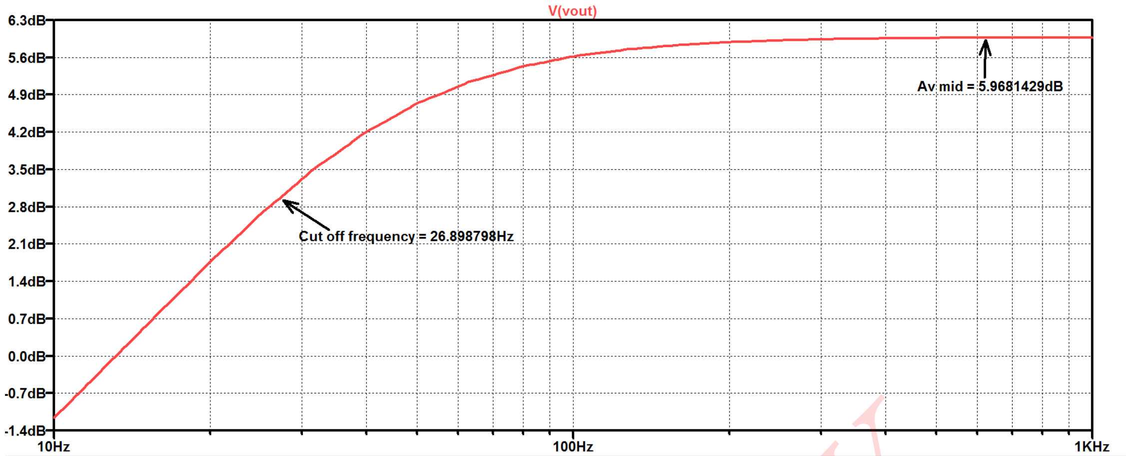

Figure 14: Low frequency response of the circuit

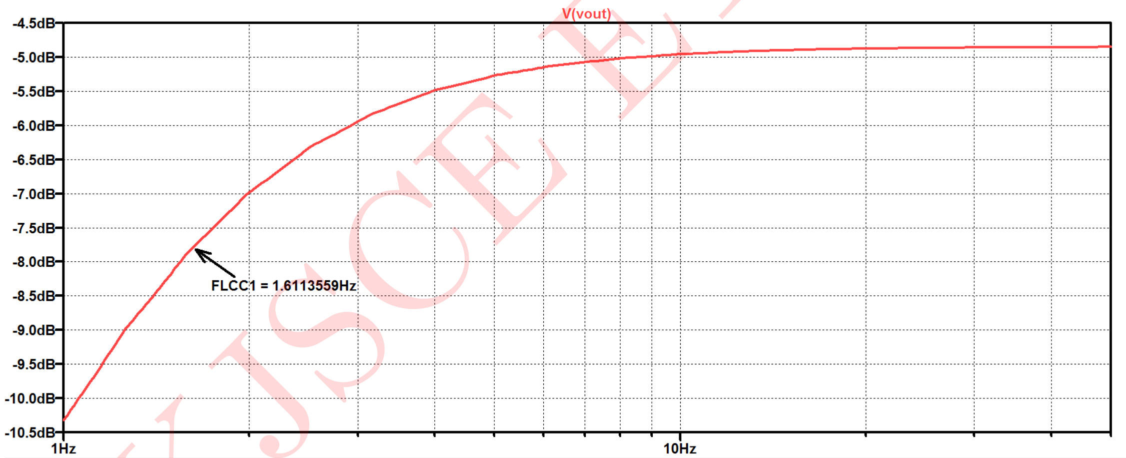

Figure 15: Low frequency response for C_{C_1}

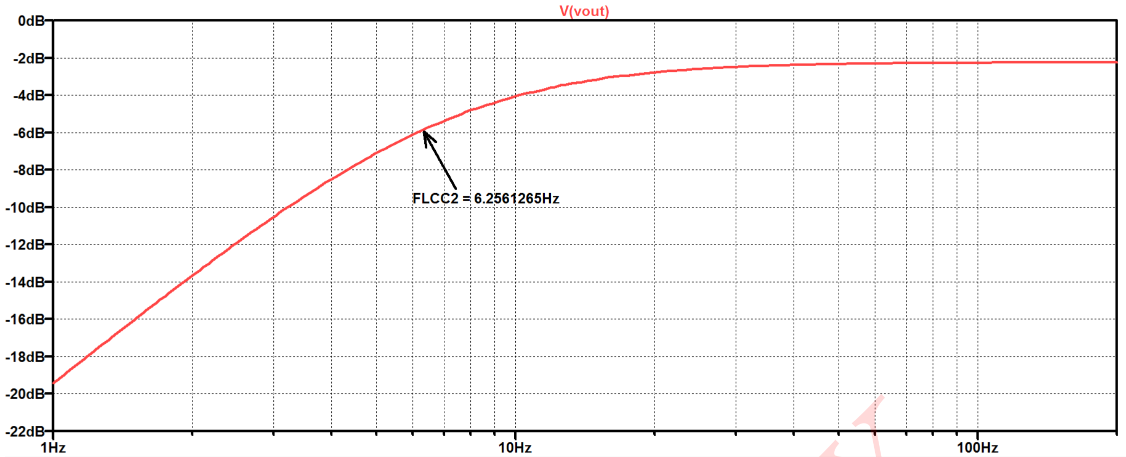

Figure 16: Low frequency response for C_{C_2}

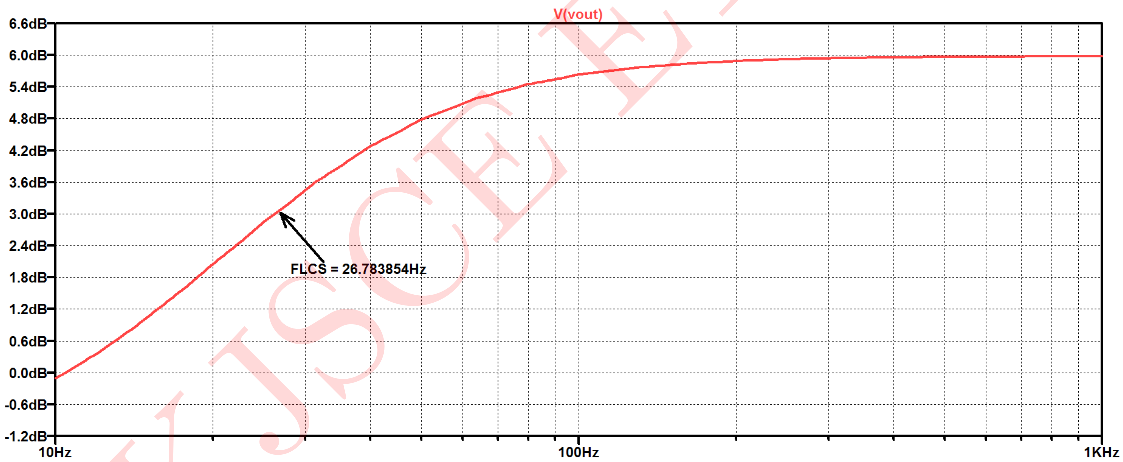

Figure 17: Low frequency response for C_S

Comparison of Theoretical and Simulated Values:

Parameters	Theoretical	Simulated
I_D, V_{GS}	$2.065mA$, $-2.479V$	$2.067mA$, $-2.481V$
Lower cut-off frequency due to C_{C_1}	$1.589Hz$	$1.611Hz$
Lower cut-off frequency due to C_{C_2}	$4.97Hz$	$6.2Hz$
Lower cut-off frequency due to C_S	$29.68Hz$	$26.78Hz$
Overall cut-off frequency f_L	$29.68Hz$	$26.86Hz$
Mid-band Voltage gain A_V in dB	$5.82dB$	$5.96dB$

Table 2: Numerical 2

Numerical 3:

For the circuit shown in figure 18, a) Determine V_{GS}, I_D b) Find g_{m_o}, g_m c) Calculate mid-band gain(A_V) d) Determine Z_i e) Calculate A_{V_S} f) Determine f_{LCC1}, f_{LCC2}, f_{L_S} g) Determine low cut-off frequency h) Determine high cut-off frequency

Given: $I_{DSS} = 6mA$, $V_P = -6V$, $r_d = 100k\Omega$, $C_{wi} = 3pF$, $C_{wo} = 5pF$, $C_{gd} = 4pF$, $C_{gs} = 6pF$ and $C_{ds} = 1pF$

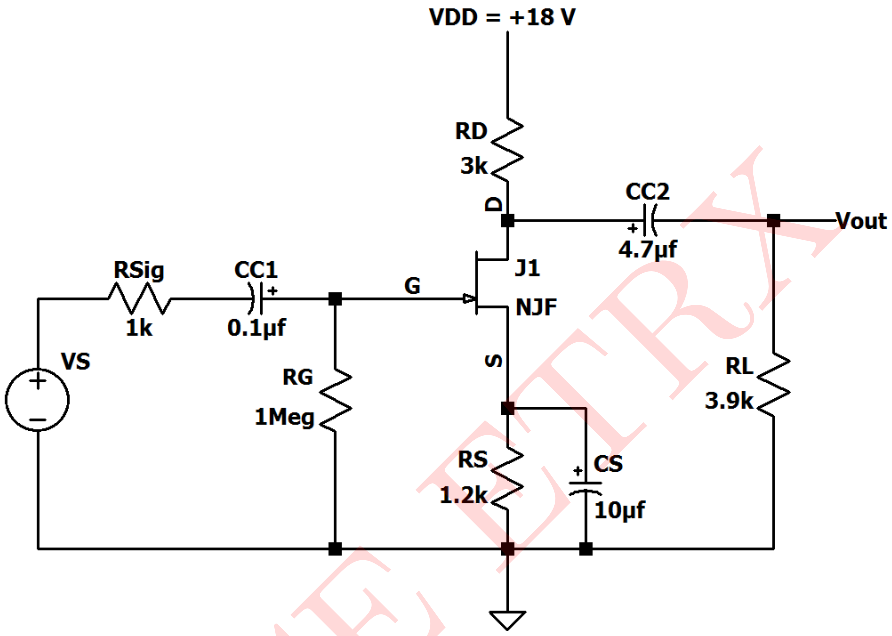

Figure 18: Circuit 3

Solution:

DC Analysis:-

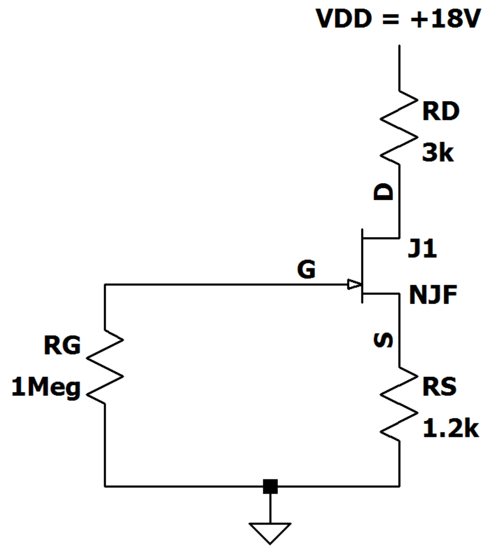

Figure 19: DC Equivalent circuit

Applying KVL to the Gate-Source loop:-

$V_{GS} = -I_D R_S$

$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$

$V_{GS} = -6mA\left(1 + \frac{V_{GS}}{6}\right)^2 \times 1.2k\Omega$

$V_{GS} = -7.2\left(1 + \frac{V_{GS}}{3} + \frac{V_{GS}{}^2}{36}\right)^2$

$V_{GS} = -7.2 - 0.2V_{GS}{}^2 - 2.4V_{GS}$

$0.2V_{GS}{}^2 + 3.4V_{GS} + 7.2 = 0$

Solving above quadratic equation, we get

$V_{GS} = -2.479V$

or

$V_{GS} = -14.52V$, We reject this value, as $(V_{GS} > V_P)$

$\therefore \boldsymbol{V_{GS} = -2.479V}$

$I_D = -\frac{V_{GS}}{R_S} = \frac{2.479V}{1.2k\Omega} = 2.065mA$

$\boldsymbol{I_D = 2.065mA}$

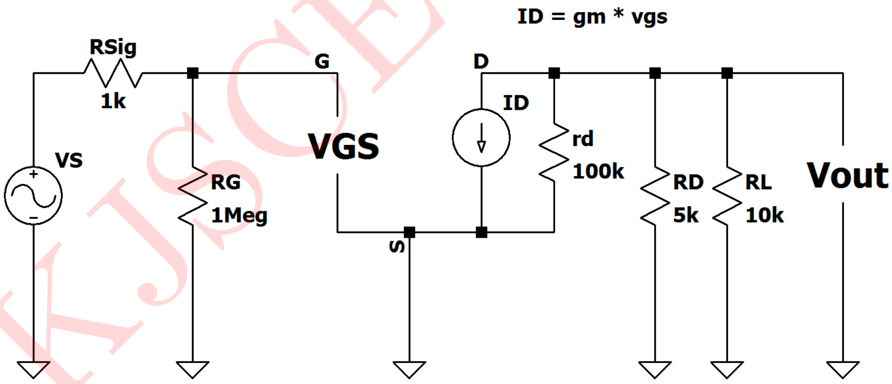

Figure 20: Small Signal Equivalent Circuit

Small-Signal parameters:-

$g_{m_o} = \left|\frac{2I_{DSS}}{V_P}\right| = \frac{2 \times 6}{6} = 2mA/V$

$g_m = g_{m_o}\left(1 - \frac{V_{GS}}{V_P}\right)^2 = 2 \times \left(1 - \frac{2.479}{6}\right) = 1.1736\ mA/V$

$\boldsymbol{g_m = 1.1736\ mA/V}$

$\boldsymbol{r_d = 100k\Omega}$ (Given)

Applying KCL at the drain terminal:-

$$g_m V_{GS} + \frac{V_{out}}{r_d} + \frac{V_{out}}{R_D} + \frac{V_{out}}{R_L} = 0$$

$$V_{GS} = -\frac{V_{out}}{g_m}\left(\frac{1}{r_d} + \frac{1}{R_D} + \frac{1}{R_L}\right)$$

$$V_{GS} = V_{in}$$

$$A_{V_{mid}} = \frac{V_{out}}{V_{in}} = -g_m(r_d \ || \ R_D \ || \ R_L \ )$$

$$A_{V_{mid}} = -1.1736mA/V(100k \ || \ 3k \ || \ 3.9k \ ) = -1.956$$

$$\boldsymbol{A_{V_{mid}} = -1.956}$$

Input Impedance:-

$$Z_i = R_{sig} + R_G$$

$$Z_i = 1k\Omega + 1M\Omega = 1001k\Omega$$

$$\boldsymbol{Z_i = 1001k\Omega}$$

$$A_{V_{mid}} \text{ with } R_{sig} = \frac{V_{out}}{V_S} = \frac{V_{out}}{V_{in}} \times \frac{V_{in}}{V_S}$$

$$\frac{V_{in}}{V_S} = \frac{R_G}{R_G + R_{sig}} = \frac{1M\Omega}{1001k\Omega} = 0.999$$

$$A_{V_{mid}} \text{ with } R_{sig} = 0.99 \times -1.956 = -1.954$$

$$\boldsymbol{A_{V_{mid}} \textbf{ with } R_{sig} = -1.954}$$

$$A_{V_{mid}} \text{ in } dB = 20log_{10}(1.954) = 5.818dB$$

$$\boldsymbol{A_{V_{mid}} \textbf{ in } dB = 5.818dB}$$

Due to C_{C_1} alone:-

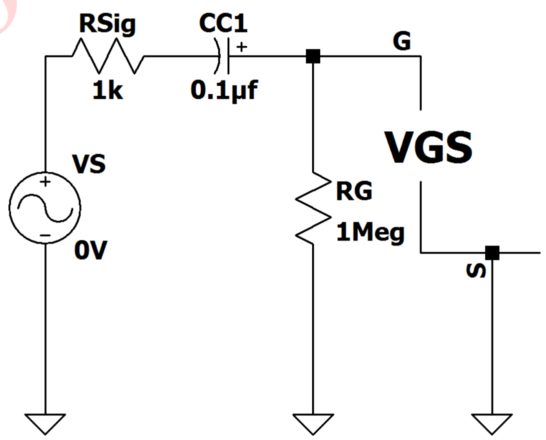

Figure 21: Small Signal low frequency equivalent circuit for C_{C_1}

$R_{eq} = R_{sig} + R_G = 1k\Omega + 1M\Omega = 1001k\Omega$

$$f_{LCC1} = \frac{1}{2\pi \times C_{C_1} \times R_{eq}} = \frac{1}{2\pi \times 1001 \times 1000 \times 0.1} = 1.589Hz$$

$\boldsymbol{f_{LCC1} = 1.589Hz}$

Due to C_{C_2} alone:-

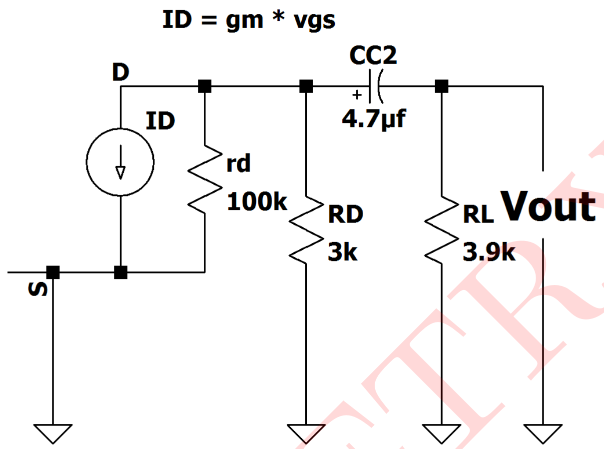

Figure 22: Small Signal low frequency equivalent circuit for C_{C_2}

$R_{eq} = R_D + R_L = (r_d \parallel R_D) + R_L = (100k \parallel 3k) + 3.9k = 6.813k\Omega$

$$f_{LCC2} = \frac{1}{2\pi \times C_{C_2} \times R_{eq}} = \frac{1}{2\pi \times 6.813k \times 4.7} = 4.97Hz$$

$\boldsymbol{f_{LCC2} = 4.97Hz}$

Due to C_S alone:-

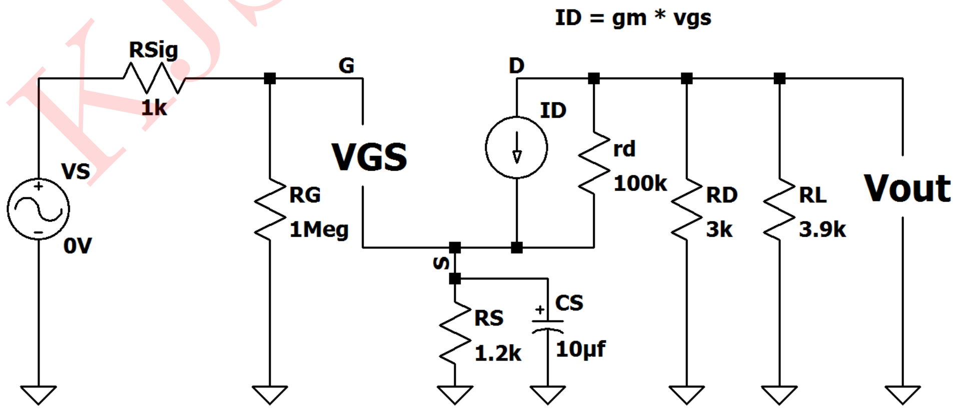

Figure 23: Small Signal low frequency equivalent circuit for C_S

Since $V_S = 0$; and also gate and source are open circuit $\rightarrow \therefore R_{sig}$ & R_G are ignored

$$R_{eq} = R_S \,||\, \left[\frac{r_d + (R_D \,||\, R_L)}{1 + g_m r_d}\right] = 1.2k \,||\, \left[\frac{100k + 1.695k}{1 + 117.36}\right] = 1.2k \,||\, 0.8592k = 500.7\Omega$$

$$f_{L_{CS}} = \frac{1}{2\pi \times C_S \times R_{eq}} = \frac{1}{2\pi \times 10 \times 500.7} = 29.68Hz$$

$\boldsymbol{f_{L_S} = 29.68Hz}$

Since, $f_{L_{CS}} > f_{L_{CC2}} > f_{L_{CC1}}$

$\therefore$ **Lower cut-off frequency** $= \boldsymbol{f_{L_{CS}} = 29.68Hz}$

High frequency equivalent circuit:-

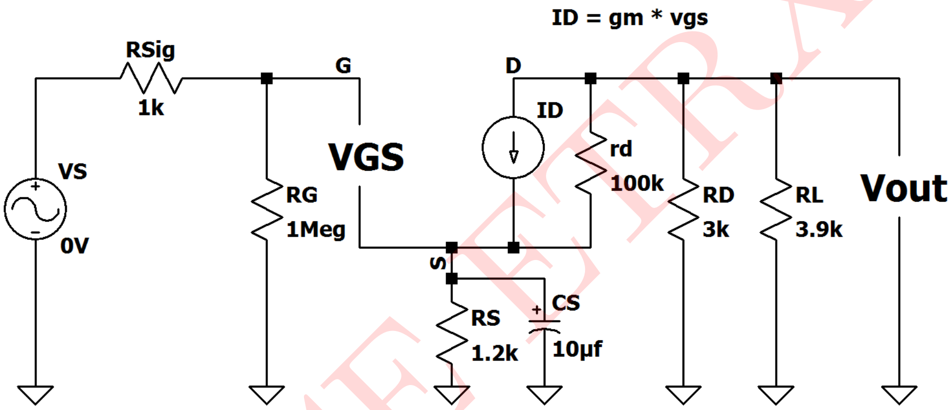

Figure 24: Small Signal high frequency equivalent circuit

$C_i = C_{gs} + C_{mi} + C_{wi}$

$C_{mi} = C_{gd}(1 - A_{V_{mid}}) = 4(1 + 5.82) = 27.28pF$

$C_i = 6pF + 27.28pF + 3pF = 36.28pF$

$C_o = C_{wo} + C_{mo} + C_{ds}$

$$C_{mo} = C_{gd}\left(1 - \frac{1}{A_{V_{mid}}}\right) = 4\left(1 - \frac{1}{5.82}\right) = 4.687pF$$

$C_o = 5pF + 4.687pF + 1pF = 10.687pF$

For f_{H_i}

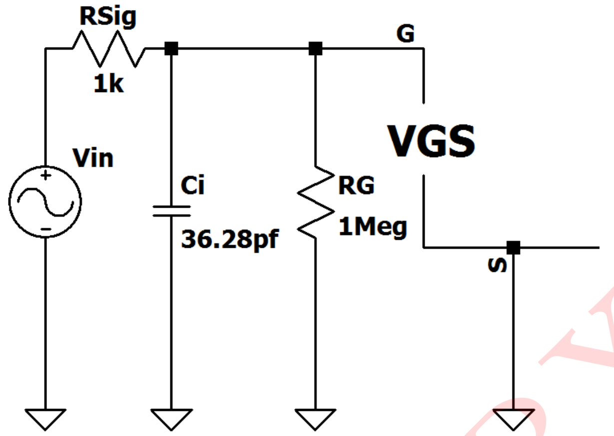

Figure 25: Small Signal equivalent circuit for f_{H_i}

$R_{eq} = R_{sig} \mid\mid R_G$

$R_{eq} = 1k\Omega \mid\mid 1M\Omega$

$R_{eq} = 999\Omega$

$$f_{H_i} = \frac{1}{2\pi R_{eq} C_i} = \frac{10^{12}}{2\pi \times 999 \times 36.28} = 4.39MHz$$

$\boldsymbol{f_{H_i} = 4.39MHz}$

For f_{H_o}

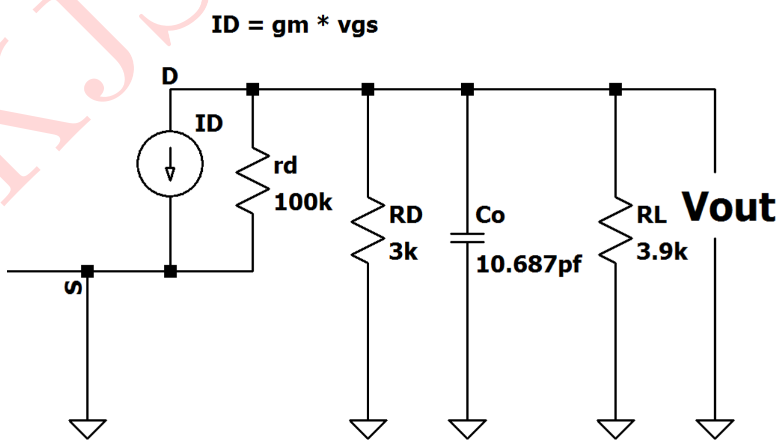

Figure 26: Small Signal equivalent circuit for f_{H_o}

$R_{eq} = r_d \;||\; R_D \;||\; R_L$

$R_{eq} = 100k\Omega \;||\; 3k\Omega \;||\; 3.9k\Omega$

$R_{eq} = 1.667k\Omega$

$$f_{H_o} = \frac{1}{2\pi R_{eq} C_o} = \frac{10^{12}}{2\pi \times 1.667 \times 10^3 \times 10.687} = 8.93MHz$$

$\boldsymbol{f_{H_o} = 8.93MHz}$

Since, $f_{H_i} < f_{H_o}$

$\therefore$ **Higher cut-off frequeny** $\boldsymbol{f_H = f_{H_i} = 4.39MHz}$

SIMULATED RESULTS:

Above circuit was simulated in LTSpice and results are presented below:

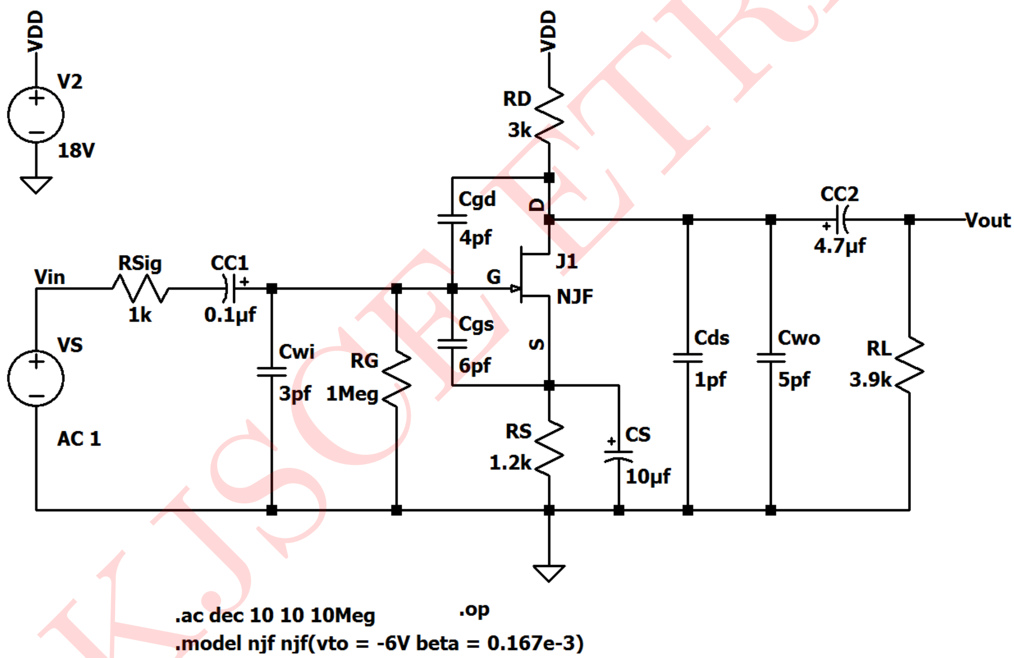

Figure 27: Circuit Schematic 3

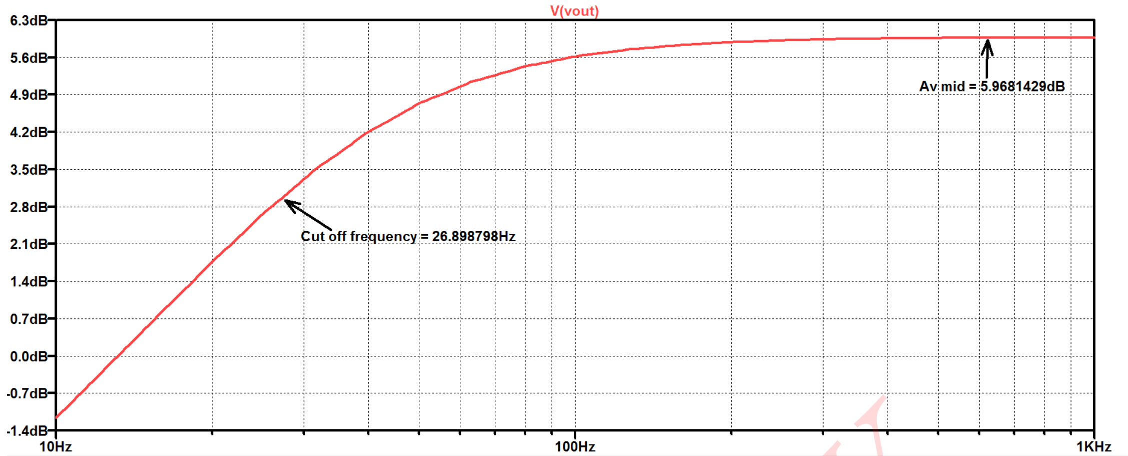

Figure 28: Low frequency response of the circuit

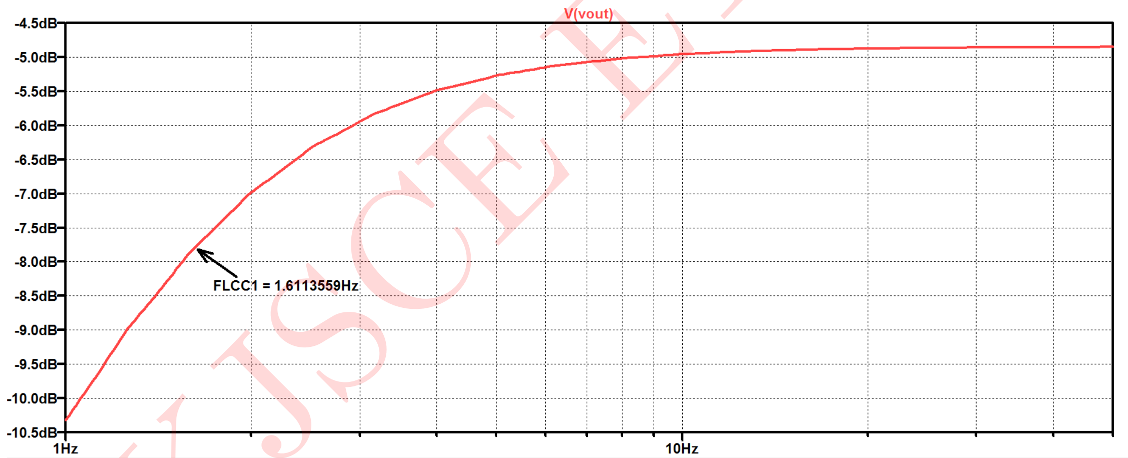

Figure 29: Low frequency response for C_{C_1}

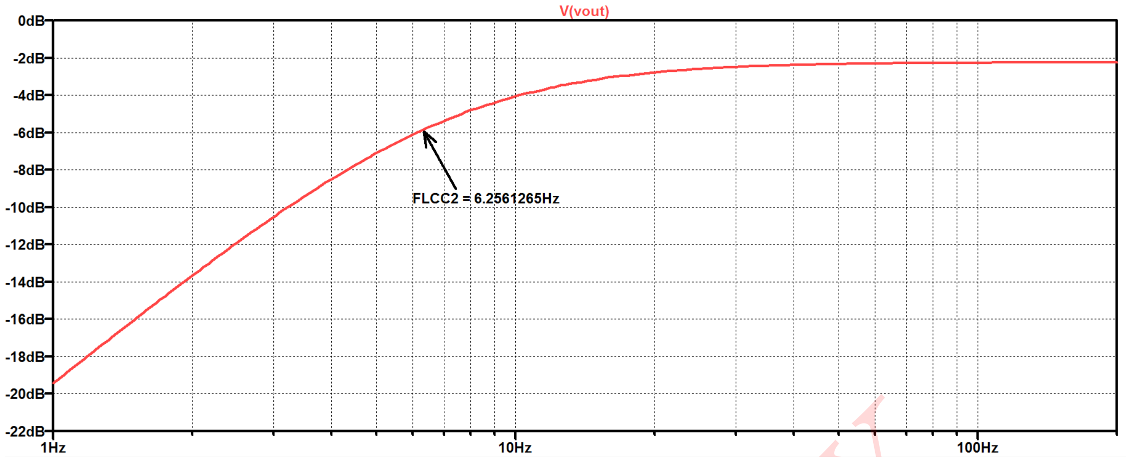

Figure 30: Low frequency response for C_{C_2}

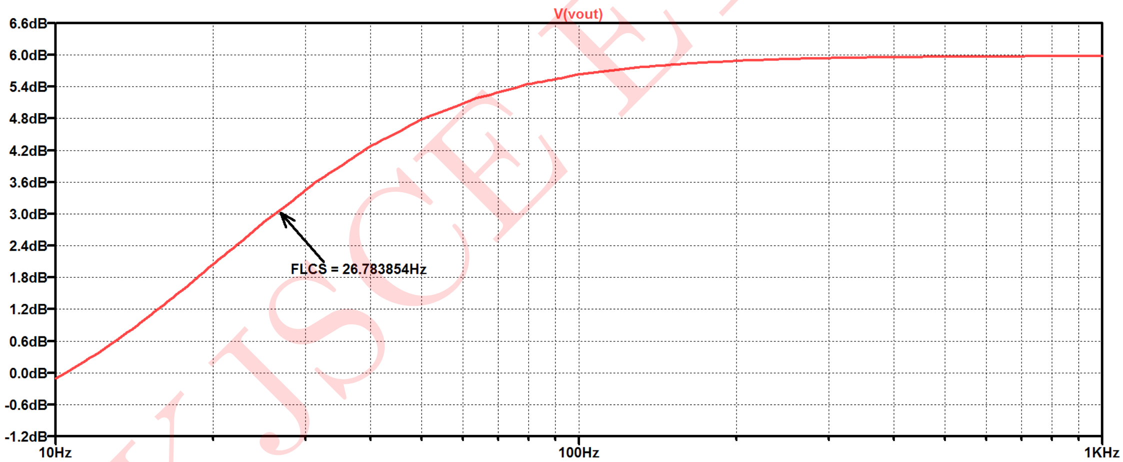

Figure 31: Low frequency response for C_S

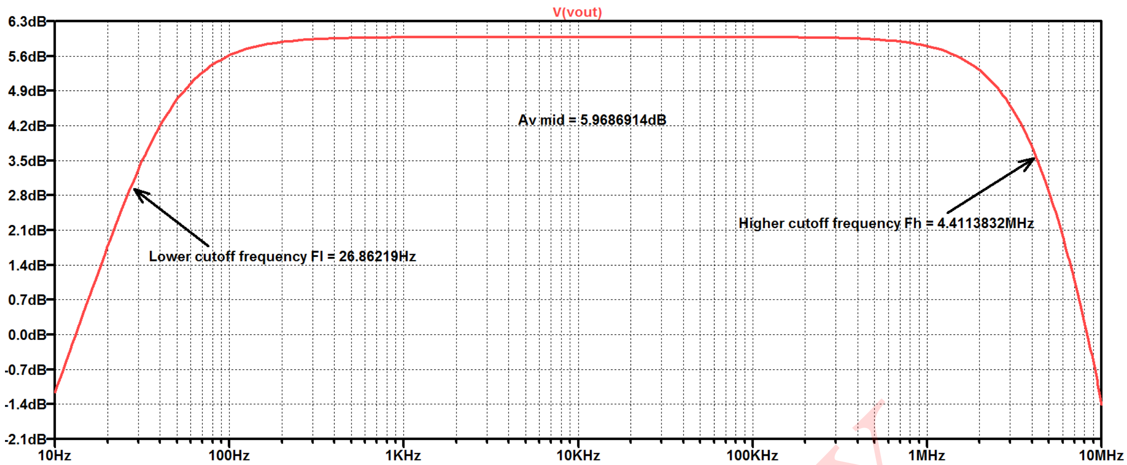

Figure 32: Complete frequency Response

Comparison of Theoretical and Simulated Values:

Parameters	Theoretical	Simulated
I_D, V_{GS}	$2.065mA$, $-2.479V$	$2.067mA$, $-2.481V$
Lower cut-off frequency due to C_{C_1}	$1.589Hz$	$1.611Hz$
Lower cut-off frequency due to C_{C_2}	$4.97Hz$	$6.2Hz$
Lower cut-off frequency due to C_S	$29.68Hz$	$26.78Hz$
Overall cut-off frequency f_L	$29.68Hz$	$26.86Hz$
Mid-band Voltage gain A_V in dB	$5.82dB$	$5.96dB$
Overall cut-off frequency f_H	$4.39MHz$	$4.41MHz$

Table 3: Numerical 3

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