K. J. SOMAIYA COLLEGE OF ENGINEERING
DEPARTMENT OF ELECTRONICS ENGINEERING
ELECTRONIC CIRCUITS
DC CIRCUITS

Numerical 1: For the circuit shown in figure 1:

a) Write the nodal equations and solve for the node voltages.

b) Find the value for current I

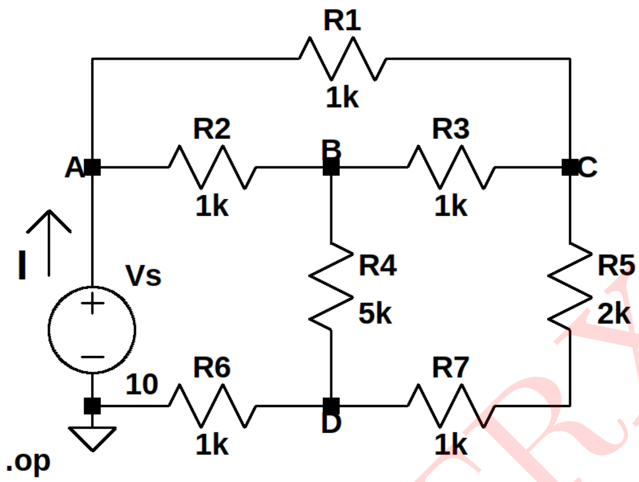

Figure 1: Circuit 1

SOLUTION:

We know that,

$V_A = V_s$

$\therefore V_A = \mathbf{10\ V}$

At Node B,

Applying KCL and Ohm's law we get,

$$\frac{V_A - 9}{1000} + \frac{V_B - V_D}{5000} + \frac{V_B - V_C}{1000} = 0$$

On solving, we get,

$11V_B - V_D$ - $5V_C = 50$ ...(1)

At Node C,

Applying KCL and Ohm's law we get,

$$\frac{V_C - 10}{1000} + \frac{V_C - V_D}{2000} + \frac{V_C - V_B}{1000} = 0$$

On solving, we get,

$5V_C - V_D - 2V_D = 20$ ...(2)

At Node D,

Applying KCL and Ohm's law we get,

$$\frac{V_D - V_B}{5000} + \frac{V_D - V_0}{1000} + \frac{V_D - V_C}{2000} = 0$$

On solving, we get,

$19V_D - 2V_B - 5V_C = 0$ ...(3)

On solving, we get,

$V_B = 8.4415584$V

$V_C = 7.9740259$V

$V_D = 2.9870129$V

At Node A,

Applying KCL,

$I = I_1 + I_2$

$$I = \frac{V_B - 10}{1000} + \frac{V_C - 10}{1000}$$

On solving, we get,

$I = -3.618181$mA

SIMULATED RESULTS:

The given circuit is simulated in LTSpice and the results obtained are as follows:

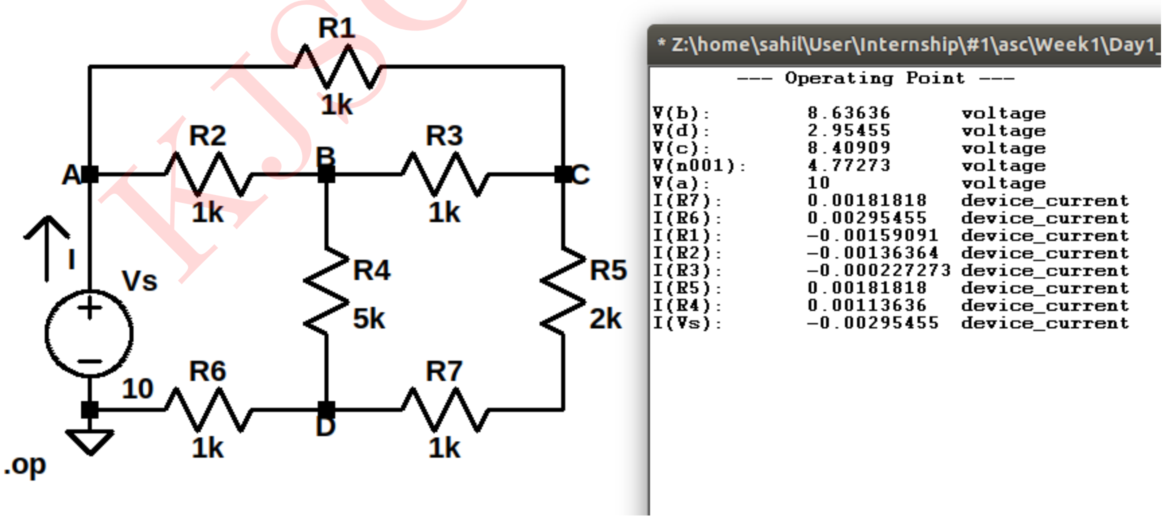

Figure 2: Circuit Schematic and Simulated Results

Comparison of theoretical and simulated values:

Parameters	Theoretical Values	Simulated Values
V_A	10V	10V
V_B	8.445584V	8.52941V
V_C	7.9740259V	8.08823V
V_D	2.9870129V	3.38235V
I	-3.61881mA	-3.3823529mA

Table 1: Numerical 1

Numerical 2: For the circuit shown in figure 3:

a) Write equilibrium equations for the network on nodal basis.

b) Obtain voltages V_1, V_2, V_3,

All resistors in the network are of 1Ω.

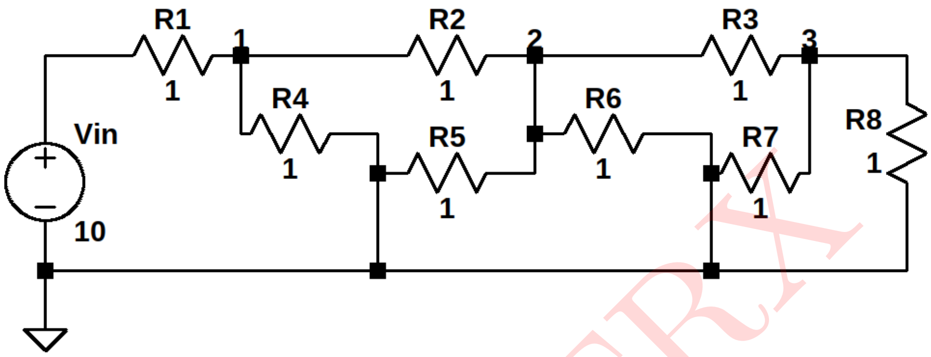

Figure 3: Circuit 2

Solution:

At Node 1,

Applying KCL and Ohm's law we get,

$$\frac{V_1 - 10}{1} + \frac{V_1 - V_2}{1} + \frac{V_1 - 0}{1} = 0$$

On solving, we get,

$3V_1 - V_2 = 10$ ...(1)

At Node 2,

Applying KCL and Ohm's law we get,

$$\frac{V_2 - V_1}{1} + \frac{V_2 - 0}{1} + \frac{V_2 - 0}{1} + \frac{V_2 - V_3}{1} = 0$$

On solving, we get,

$4V_2 - V_1 - V_3 = 0$ ...(2)

At Node 3,

Applying KCL and Ohm's law we get,

$$\frac{V_3 - V_2}{1} + \frac{V_3 - 0}{1} + \frac{V_3 - 0}{1} = 0$$

On solving, we get,

$3V_3 - V_2 = 0$ ...(3)

On solving, we get,

$V_1 = 3.66667\text{V}$

$V_2 = 1\text{V}$

$V_3 = 0.333333\text{V}$

SIMULATED RESULTS:

The given circuit is simulated in LTSpice and the results obtained are as follows:

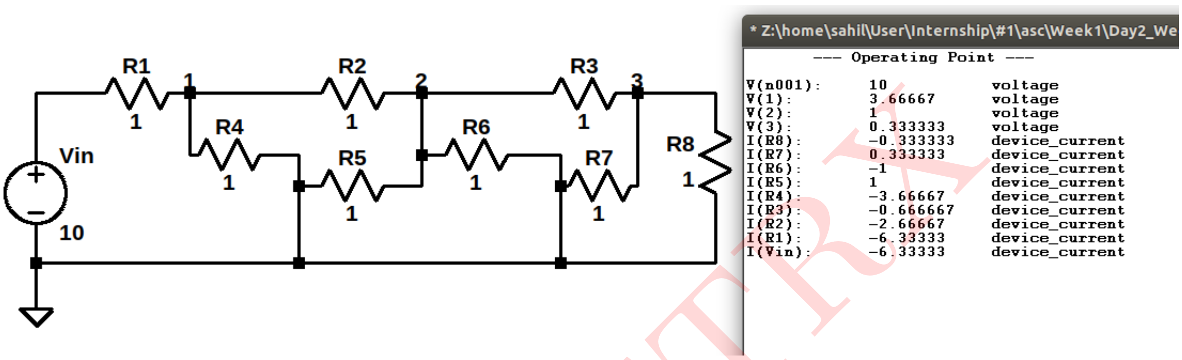

Figure 4: Circuit Schematic and Simulated Results

Comparison of theoretical and simulated values:

Parameters	Theoretical Values	Simulated Values
V_1	3.66667V	3.66667V
V_2	1V	1V
V_3	0.333333V	0.333333V

Table 2: Numerical 2

Numerical 3: By applying nodal method of network analysis, find current in the 15Ω resistor of the network shown.

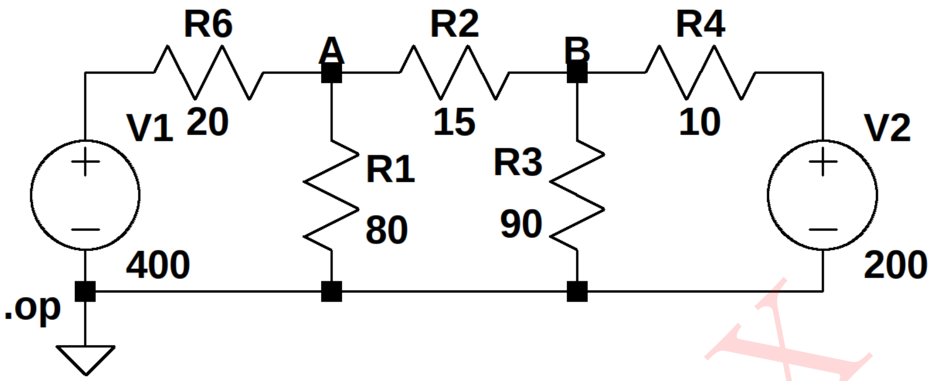

Figure 5: Circuit 3

Solution:

At Node A,

Applying KCL and Ohm's law we get,

$$\frac{V_A - 400}{20} + \frac{V_A - V_B}{15} + \frac{V_A - 0}{80} = 0$$

On solving, we get,

$$V_A\left(\frac{1}{15} + \frac{1}{80} + \frac{1}{90}\right) - \mathrm{V}_B\left(\frac{1}{15}\right) = 20...(1)$$

At Node B,

Applying KCL and Ohm's law we get,

$$\frac{V_B - 200}{10} + \frac{V_B - V_A}{15} + \frac{V_B - 0}{90} = 0$$

On solving, we get,

$$V_B\left(\frac{1}{10} + \frac{1}{15} + \frac{1}{90}\right) - \mathrm{V}_B\left(\frac{1}{15}\right) = 20...(2)$$

On solving, (1) and (2) we get,

$V_A = 264\text{V}$
$V_B = 211.5\text{V}$

For I_{15}, using Ohm's law,

$$I_{15} = \frac{V_A - V_B}{15} = \frac{264 - 211.5}{15} = 3.5mA$$

SIMULATED RESULTS:
The given circuit is simulated in LTSpice and the results obtained are as follows:

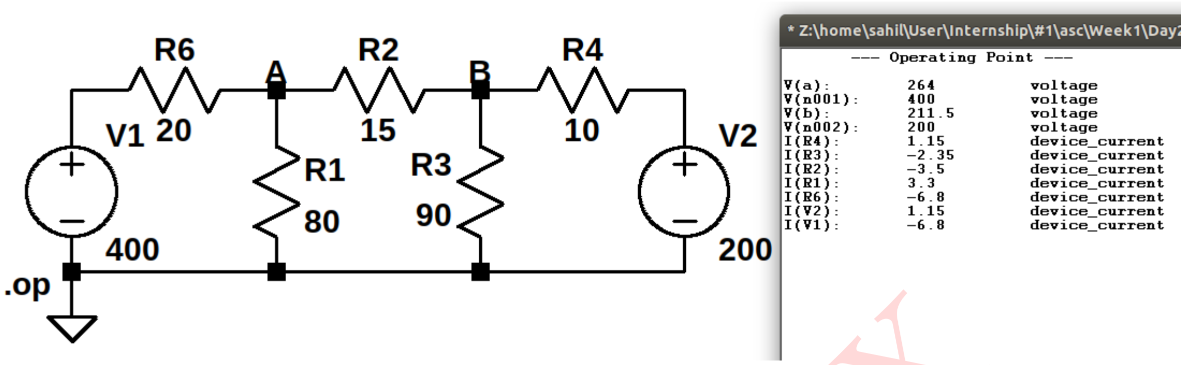

Figure 6: Circuit Schematic and Simulated Results

Comparison of theoretical and simulated values:

Parameters	Theoretical Values	Simulated Values
I_{15}	3.5mA	3.5mA

Table 3: Numerical 3

Numerical 4: The circuit given, is to control the speed of a motor such that the motor draws the current of 5A, 3A and 1A when the switch is set at High, Medium and Low respectively. The motor can be modelled as a load resistance of 20mΩ.
Determine the series dropping resistances R_1, R_2 and R_3.

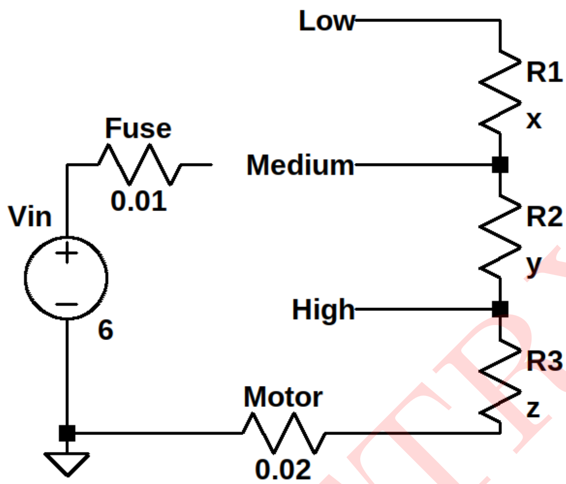

Figure 7: Circuit 4

Solution: According to the given circuit diagram, the fuse and the motor will put out a resistance in series at all positions.

Thus, motor and fuse resistance = $(0.02 + 0.01)\Omega$

At high position, motor draws 5A current,
Using ohms law,
$6 = (0.1 + 0.02 + R_3)\times 5$
$\therefore R_3 = 1.17\ \Omega$

At medium position, motor draws 3A current,
Using ohms law,
$6 = (0.1 + 0.02 + R_2 + R_3)\times 3$
$\therefore \mathrm{R}_2 = 0.8\ \Omega$

At low position, motor draws 1A current,
Using ohms law,
$6 = (0.1 + 0.02 + R_1 + R_2 + R_3)\times 1$
$\therefore R_1 = 4\ \Omega$

SIMULATED RESULTS:
The given circuit is simulated in LTSpice and the results obtained are as follows:

At high position,

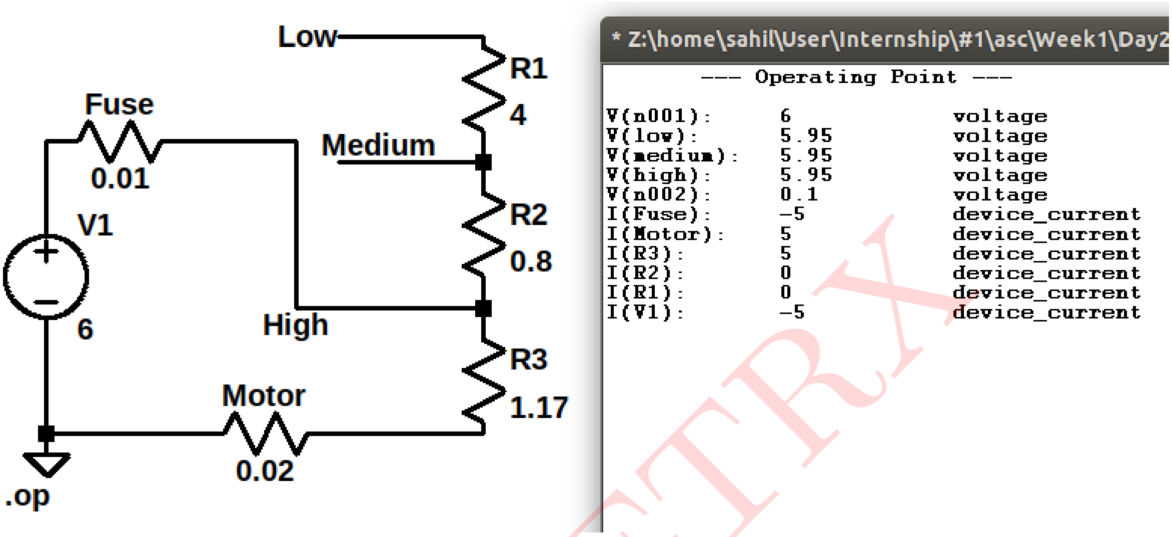

Figure 8: Circuit Schematic and Simulated Results at High Position

At medium position,

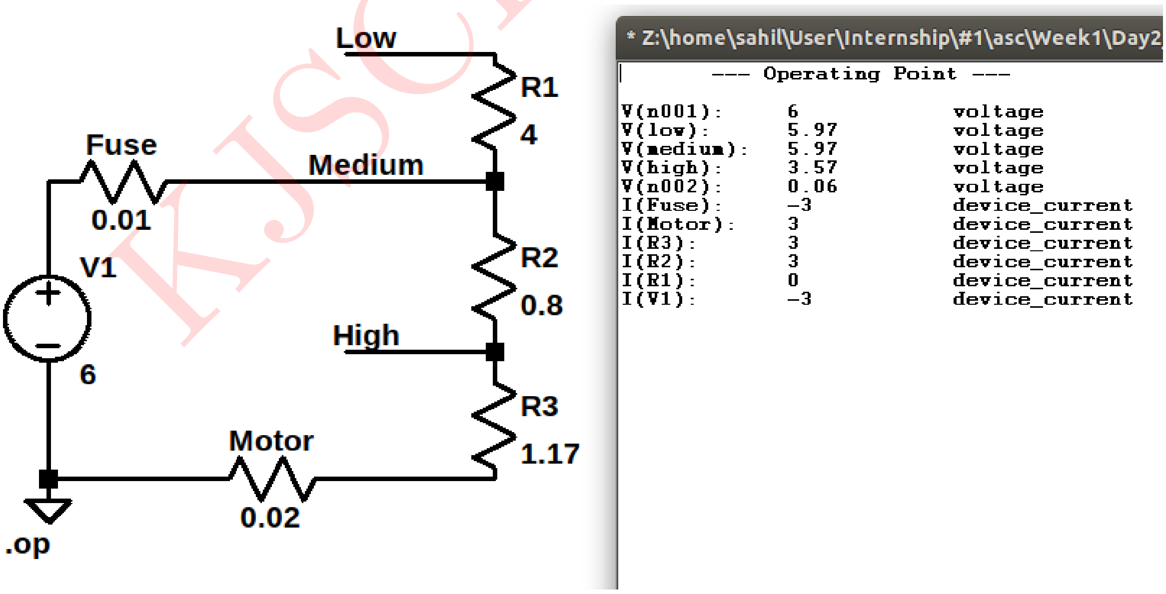

Figure 9: Circuit Schematic and Simulated Results at Medium Position

At low position,

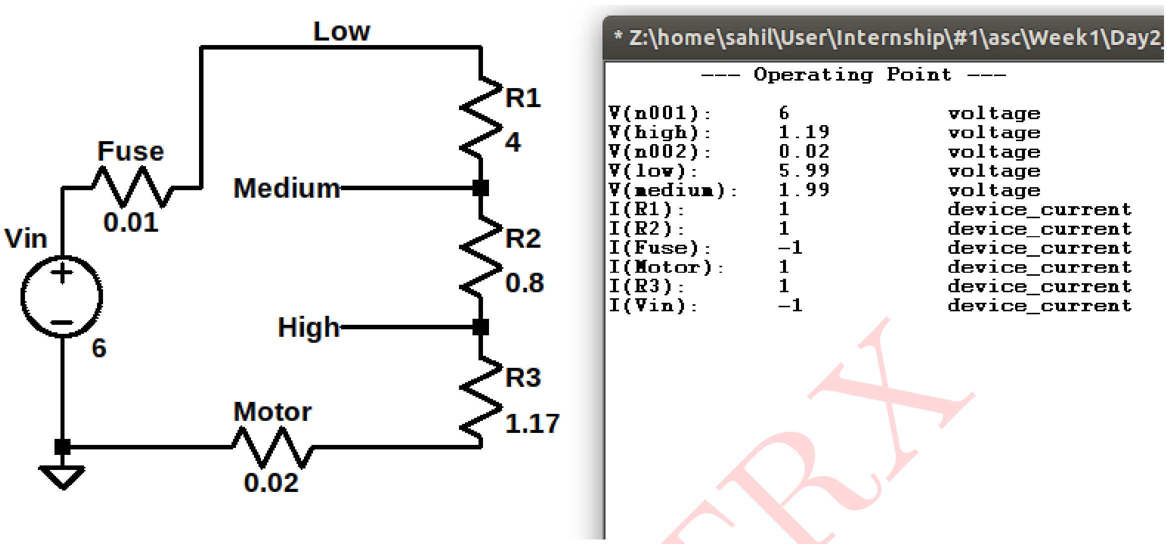

Figure 10: Circuit Schematic and Simulated Results at Low Position

Comparison of theoretical and simulated values:

Parameters	Theoretical Values	Simulated Values
I_{low}	1A	1A
I_{med}	3A	3A
I_{high}	5A	5A
R_1	4Ω	4Ω
R_2	0.8Ω	0.8Ω
R_3	1.17Ω	1.17Ω

Table 4: Numerical 4

Numerical 5: Find the current I flowing through shown in circuit 5, using star-delta transformation if the circuit is excited by 39V and the value of each resistor is 4Ω.

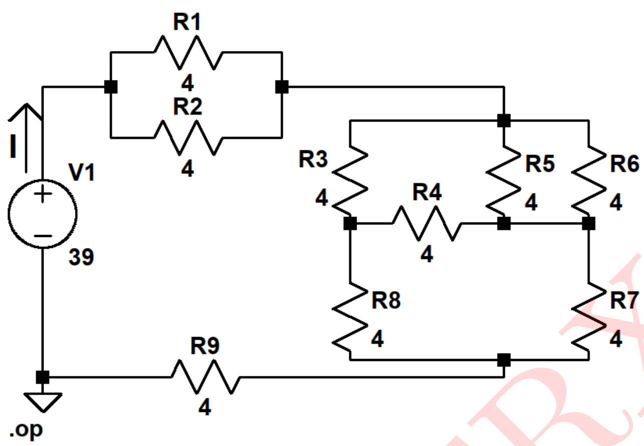

Figure 11: Circuit 5

SOLUTION:

In the following circuit, parallel resistors R_1, R_2 and R_4, R_5 can be replaced by a single resistor each of 2Ω

So the circuit becomes:

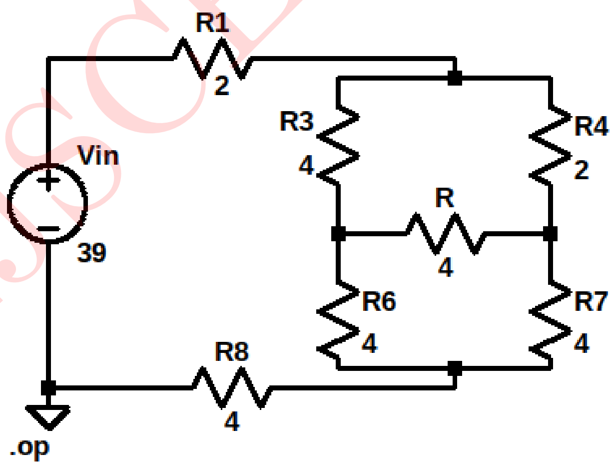

Figure 12: Circuit after replacing resistors in parallel connection

Using delta-star transformation for resistors R_3, R_4 and R

$$R_{34} = \frac{R_3 \times R_4}{R_3 + R_4 + R} = 0.8\Omega$$

$$R_3 = \frac{R_3 \times R}{R_3 + R_4 + R} = 1.6\Omega$$

$$R_4 = \frac{R_4 \times R}{R_3 + R_4 + R} = 0.8\Omega$$

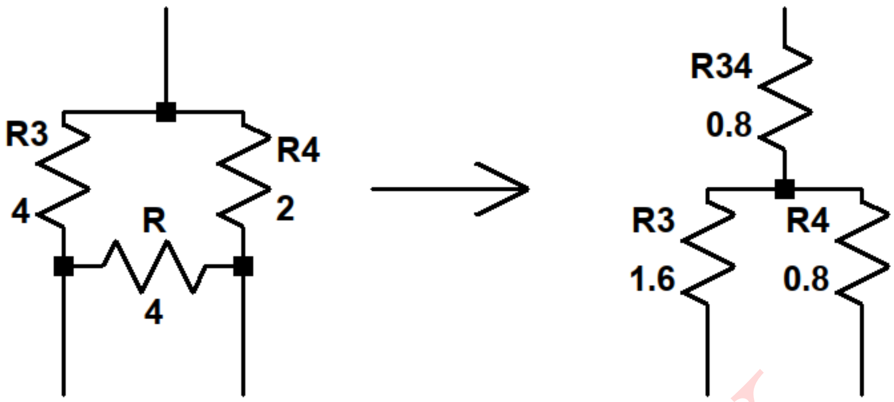

Figure 13: Delta-star transformation

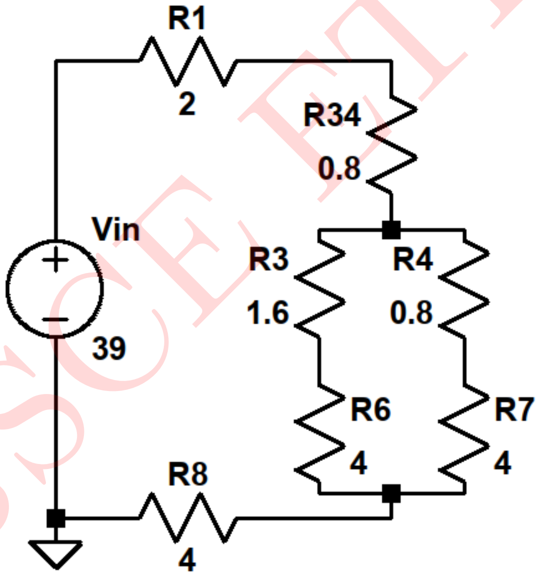

Figure 14: Circuit after delta-star transformation

Now, resistors R_1 and R_{34}, R_3 and R_6, resistors R_4 and R_7 can be replaced by a single resistor of 2.8Ω, 5.6Ω and 4.8Ω respectively.

So the circuit thus becomes,

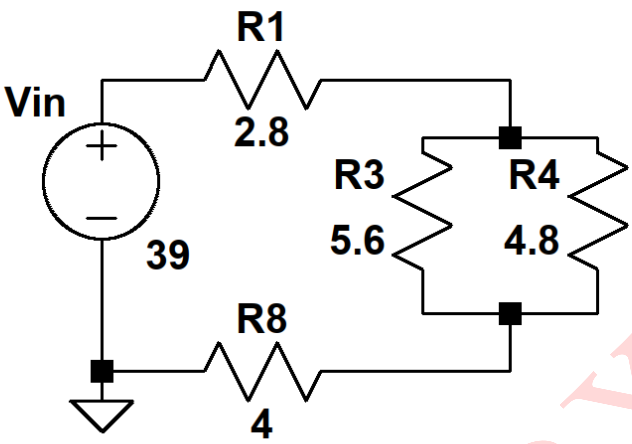

Figure 15: Circuit after replacing resistors in series connection

Now, all the resistors of the circuit can be replaced by a singe resistor of value as

$$\mathrm{R} = 4 + 0.8\left(\frac{1}{\frac{1}{5.6} - \frac{1}{4.8}}\right) + 4$$

$\therefore R = 9.3846\Omega$

This is the total resistance in the circuit.

Now, Current $(I) = \dfrac{Voltage}{Resistance}$

$\therefore$ Current $(I) = \dfrac{39}{9.3846}$

$\therefore$ Current $(I) = 4.15574\mathrm{A}$

SIMULATED RESULTS:
The given circuit is simulated in LTSpice and the results obtained are as follows:

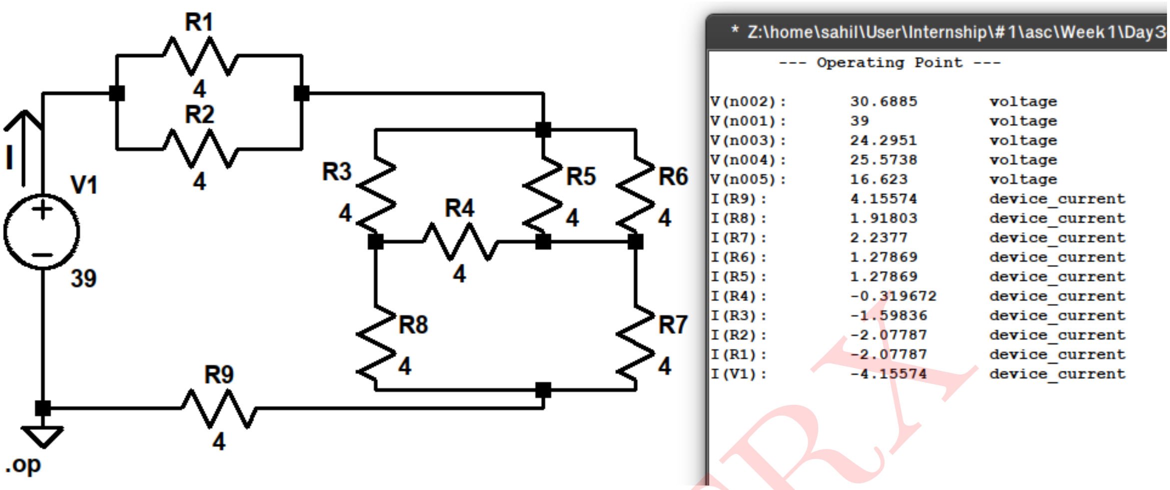

Figure 16: Circuit Schematic and Simulated Results

Comparison of theoretical and simulated values:

Parameters	Theoretical Values	Simulated Values
I	4.15574A	4.15574A

Table 5: Numerical 5

Numerical 6: Find the node voltages V_1, V_2, V_3, V_4 in the shown circuit 6.

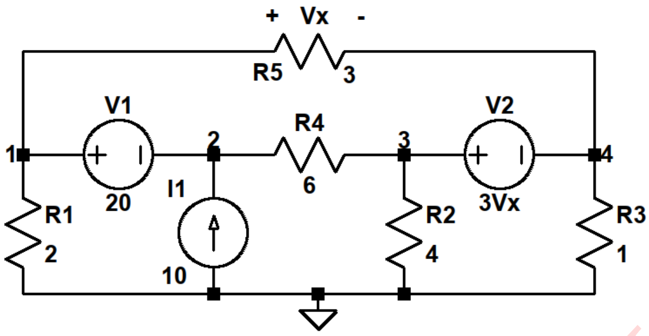

Figure 17: Circuit 6

SOLUTION:

Nodes 1 and 2 form a super-node, so do nodes 3 and 4.

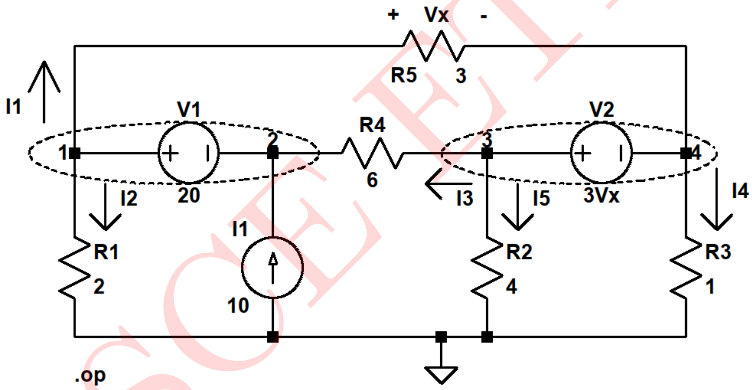

Figure 18: Modified Circuit

So we apply KCL to them,

At super-node 1-2,

$I_3 + 10 = I_1 + I_2$

Using Ohm's law,

$$\frac{V_3 - V_2}{6} + 10 = \frac{V_1 - V_4}{3} + \frac{V_1}{2}$$

i.e. $5V_1 + V_2 - V_3 - 2V_4 = 60$ ...(1)

At super-node 3-4,

$I_1 = I_3 + I_4 + I_5$

Using Ohm's law,

$$\frac{V_1 - V_4}{3} = \frac{V_3 - V_2}{6} + \frac{V_4}{1} + \frac{V_3}{4}$$

i.e. $4V_1 + 2V_2 - 5V_3 - 16V_4 = 0$ ...(2)

We now apply KVL to the branches involving the voltage sources as in the figure below,

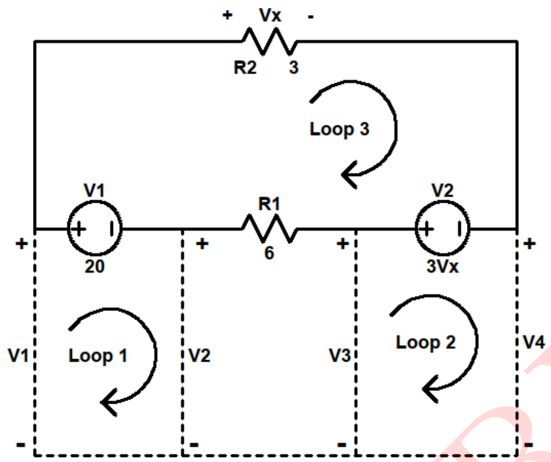

Figure 19: Modified Circuit

For loop 1,
$-V_1 + 20 + V_2 = 0$
i.e. $V_1 - V_2 = 20$ ...(3)

For loop 2,
$V_3 + 3V_x + V_4 = 0$ ...(4)

For loop 3,
$V_x - 3V_x + 6I_3 - 20 = 0$
But, $6I_3 = V_3 - V_2$ and $V_x = V_1 - V_4$
$\therefore -2\text{V}_1 - V_2 + V_3 + 2V_4 = 20$ ...(5)

Solving for equations 1 to 4, we get,
$V_1 = 26.66666667\text{V}$
$V_2 = 6.666667\text{V}$
$V_3 = 173.33333\text{V}$
$V_4 = -46.6666667\text{V}$

Equation 5 can be used for cross verifying the results.

SIMULATED RESULTS:
The given circuit is simulated in LTSpice and the results obtained are as follows:

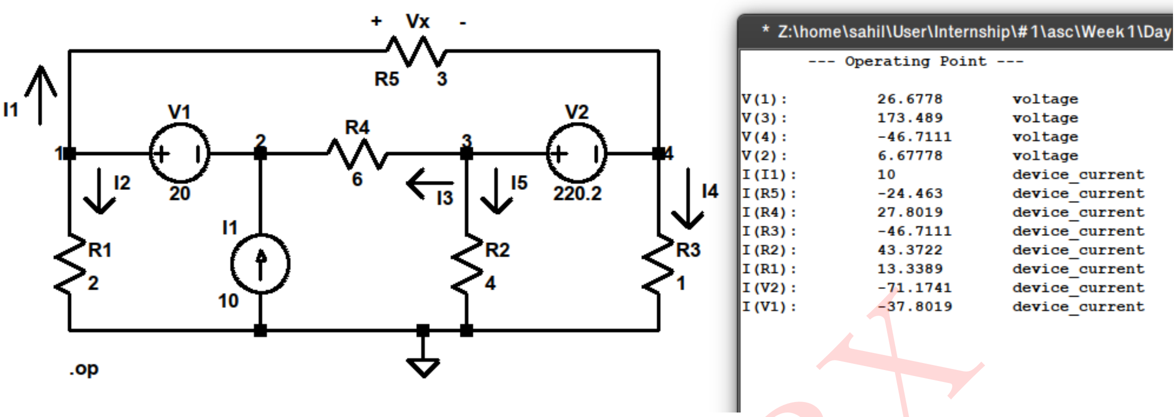

Figure 20: Circuit Schematic and Simulated Results

Comparison of theoretical and simulated values:

Parameters	Theoretical Values	Simulated Values
V_1	26.66666667V	26.6778V
V_2	6.666667V	6.67778V
V_3	173.33333V	173.489V
V_4	-46.6666667V	-46.7111V

Table 6: Numerical 6

Numerical 7: Determine the Norton's equivalent for the circuit 7.
Express your result in terms of I and R.

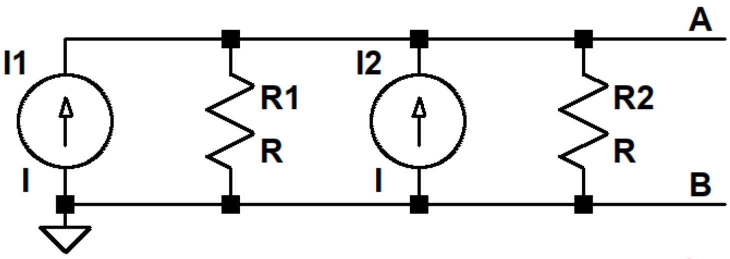

Figure 21: Circuit 7

SOLUTION:

Here, the two current sources can be represented by a single source of $2I$.
Also, the resistors can be replaced by a single resistor of $0.5R$.
The circuit thus becomes:

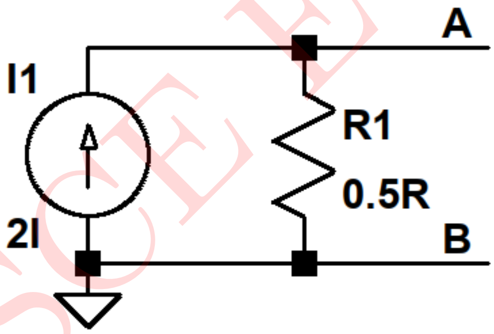

Figure 22: Circuit after replacing resistors and current sources

Now, the $2I$ current source and the $0.5R$ resistor in parallel can be replaced by a voltage source and a resistor in series as follows,
Using, Ohm's law,
$V = I\ \times$R

The circuit thus becomes:

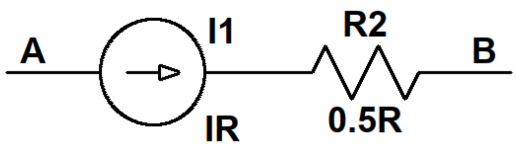

Figure 23: Circuit after replacing resistor and current source

Now, current across AB can be calculated as:

$I_{SC} = \dfrac{V}{R} = \dfrac{I \times R}{0.5R} = 2\text{I}$

For R_{TH},
We short all current sources and the circuit becomes:

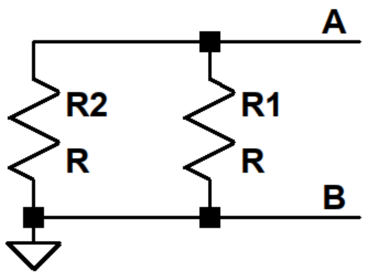

Figure 24: Calculating R_{TH}

Thus,

$R_{TH} = R \mid\mid R = \dfrac{R}{2} = 0.5\text{R}$

Thus, the Norton's Equivalent circuit is:

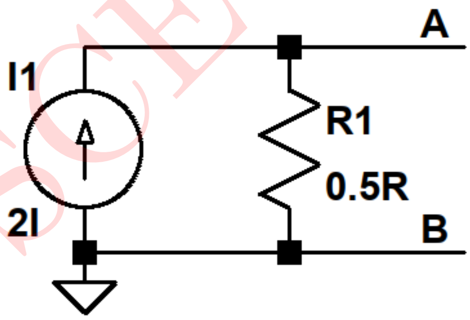

Figure 25: Norton's Equivalent

To cross verify the values we got for I and R, we take I as 5A and R as 10Ω in the given circuit for LTSpice simulations,
So, I_{SC} = 2I = 2 ×5 = 10A

R_{TH} = 0.5R = 0.5 ×10 = 5Ω

V_{AB} = 50V

SIMULATED RESULTS:

The given circuit is simulated in LTSpice and the results obtained are as follows:

For I_{SC},

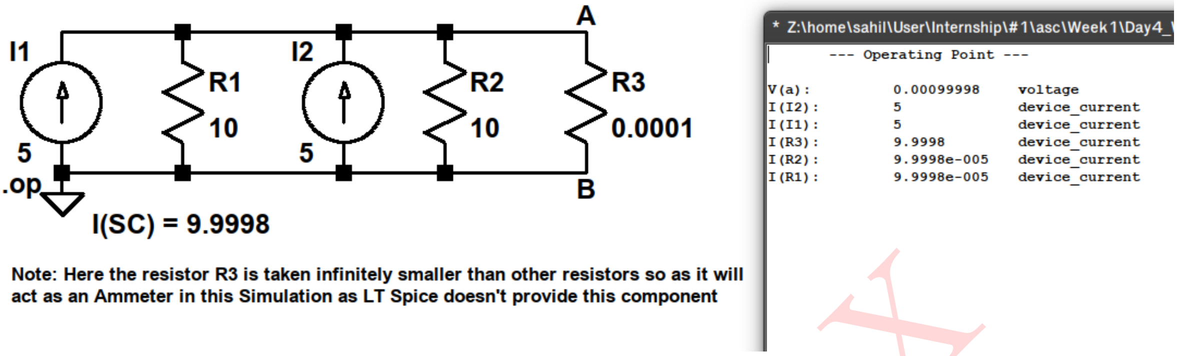

Figure 26: Circuit Schematic for verifying I_{SC} and Simulated Results

For V_{AB},

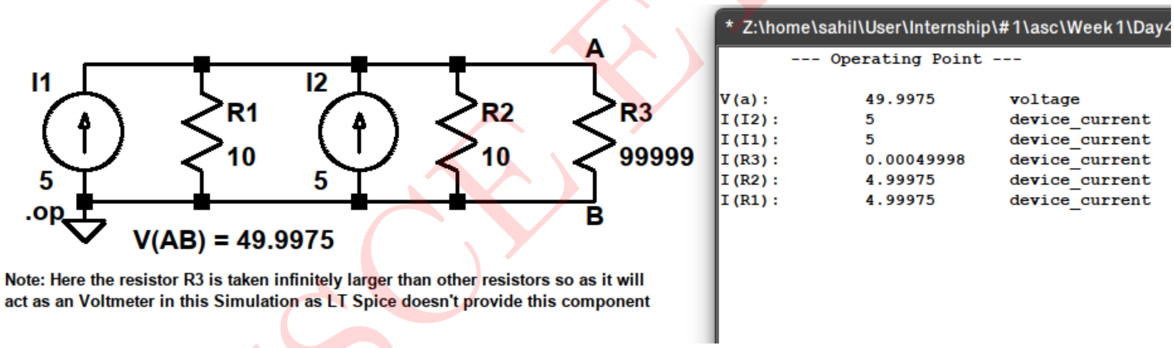

Figure 27: Circuit Schematic for verifying V_{AB} and Simulated Results

Comparison of theoretical and simulated values:

Parameters	Theoretical Values	Simulated Values
I_{SC}	10A	9.998A
R_{TH}	5Ω	5Ω
V_{AB}	50V	49.9975V

Table 7: Numerical 7

Numerical 8: Use Norton's theorem to find V_0 in the circuit 8.

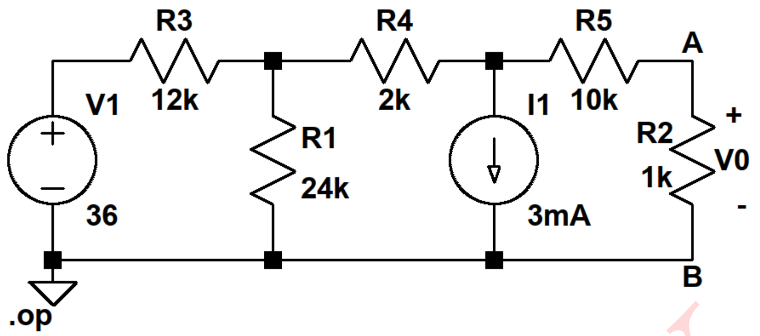

Figure 28: Circuit 8

In the figure, the resistor across V_0 acts as the load resistance and to find I_{SC} we short circuit it.
The short circit current I_{SC} is given by,
$I_{SC} = I_1 + I_2$ ...(1)

where, I_1 and I_2 are short circuit currents due to the 36V source and 3mA source respectively.

For I_1 due to 36V source acting alone,
Consider the circuit diagram below,

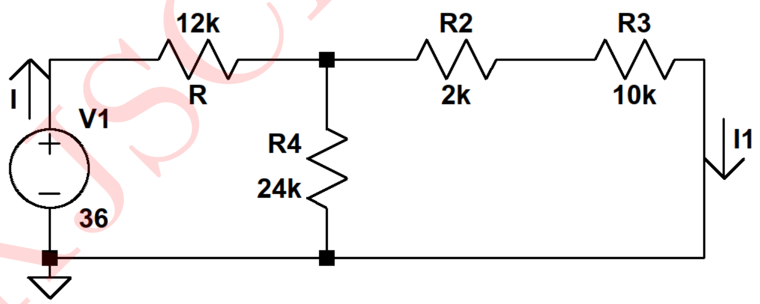

Figure 29: Circuit for 36V source acting alone

The total resistance is given by,
$R = 12\text{k}\Omega + 24k\Omega \,||\, (2k\Omega + 10k\Omega)$

$$\therefore \text{R} = \left(12 + \frac{24 \times 12}{24 + 12}\right) \times 10^3 \ \Omega$$

$R = 20\text{k}\Omega$ ...(2)

Total current, $I = \dfrac{V}{R} = \dfrac{24 + 12}{20 \times 10^3} = 1.8mA$

Now, $I_1 = \dfrac{24}{36} \times 1.8 \times 10^{-3}$ ...(By current division)

$\therefore \text{I}_1 = 1.2\text{mA}$

For I_2 due to 3mA source acting alone,
Consider the circuit diagram below,

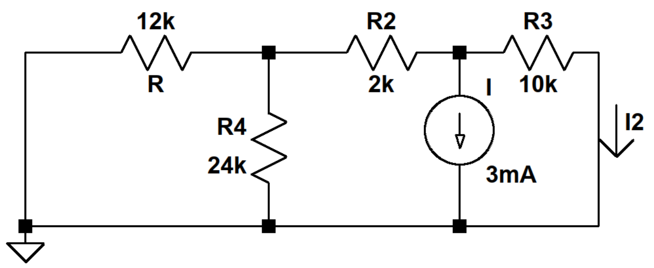

Figure 30: Circuit for 3mA source acting alone

The total resistance is given by,
$R = (12\text{k}\Omega \,||\, 24k\Omega) + 2k\Omega$

$\therefore \text{R} = 10k\Omega$

Now, $I_2 = \dfrac{10}{10+10} \times -3 \times 10^{-3}$ ...(By current division)

$\therefore \text{I}_2 = -1.5\text{mA}$

The circuit diagram is thus modified as follows,

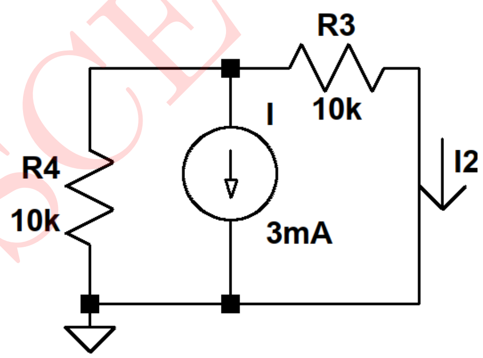

Figure 31: Modified Circuit

$\therefore \text{I}_{SC} = I_1 + I_2 = (1.2 - 1.5) \times 10^{-3} = -0.3\text{mA}$

Also, our R from (2) is our Norton's Resistance, R_{TH}

Thus, the Norton's equivalent circuit is:

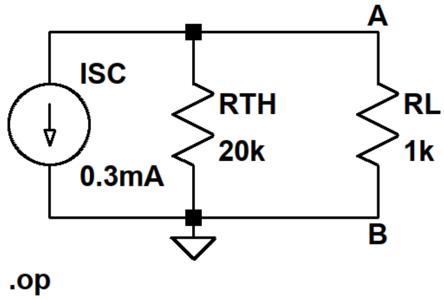

Figure 32: Norton's Equivalent Circuit

The voltage V_0 can be calculated by:

$$V_0 = \frac{1}{20+1} \times 0.3 \times 10^{-3}$$

$$\therefore \mathrm{V}_0 = -0.2857\mathrm{V}$$

SIMULATED RESULTS:
The given circuit is simulated in LTSpice and the results obtained are as follows:

For I_{SC},

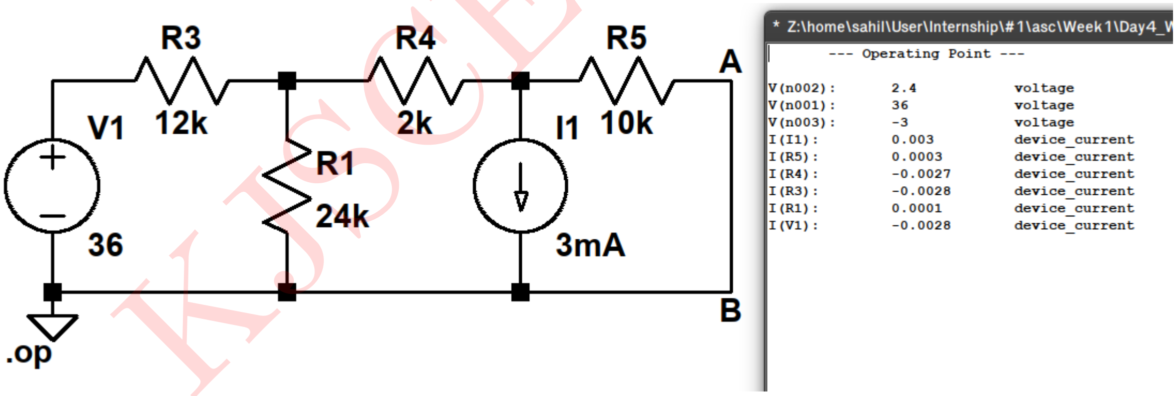

Figure 33: Circuit Schematic for verifying I_{SC} and Simulated Results

For V_{AB},

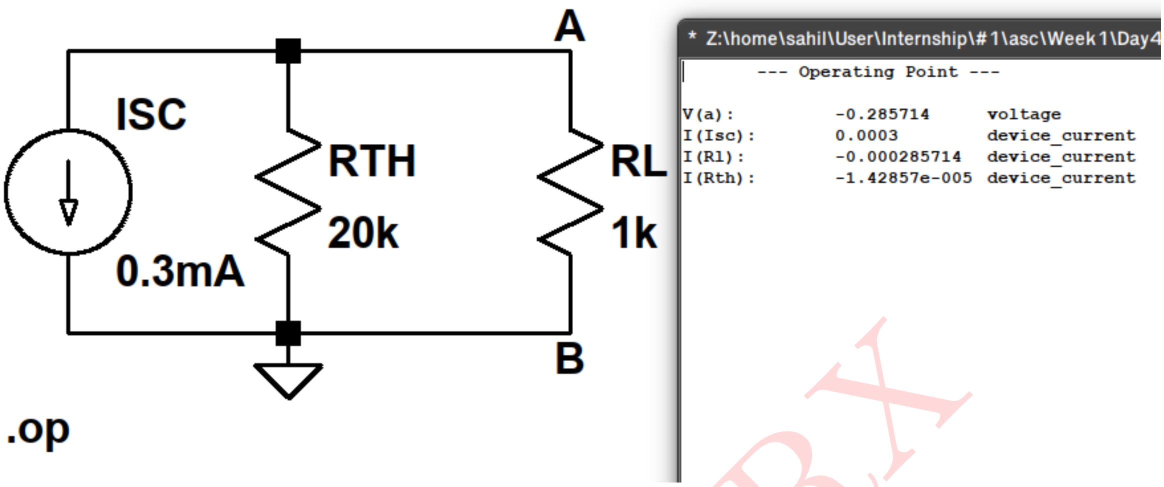

Figure 34: Circuit Schematic for verifying V_{AB} and Simulated Results

Comparison of theoretical and simulated values:

Parameters	Theoretical Values	Simulated Values
I_{SC}	-0.3mA	-0.3mA
R_{TH}	20kΩ	20kΩ
V_{AB}	-0.2857V	-0.285714V

Table 8: Numerical 8

Numerical 9: Determine the value of R_L that will draw the maximum power from the circuit 9. Calculate the maximum power.

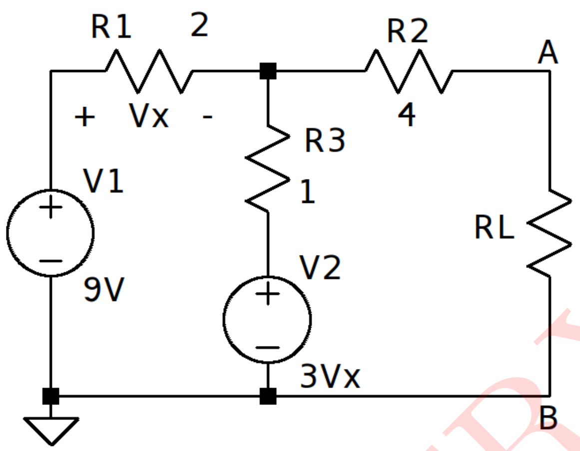

Figure 35: Circuit 9

SOLUTION:

Here we disconnect the load resistance to calculate open circuit voltage,

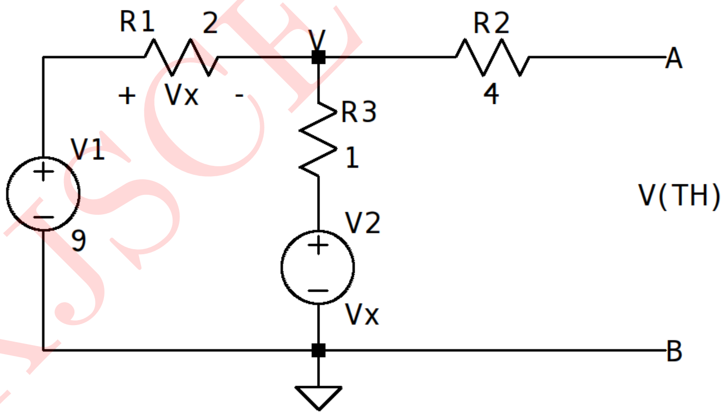

Figure 36: Circuit after disconnecting load resistance

For voltage V_x,
$V_x = 9 - V_A$ ...(1)

Applying KCL to node A,

$$\frac{V_A - 9}{2} + \frac{V_A - 3V_x}{1} = 0$$

i.e. $V_A - 9 + 2V_A - 6V_x = 0$

Using (1),
$3V_A - 9 - 6\,[9 - V_A] = 0$

On solving, we get: $V_A = 7$V
$\therefore$ V$_x$ = 7V

Here, $V_x = V_{TH}$
$\therefore$ V$_{TH}$ = 7V

Now, we short all voltage sources to find R_{TH},

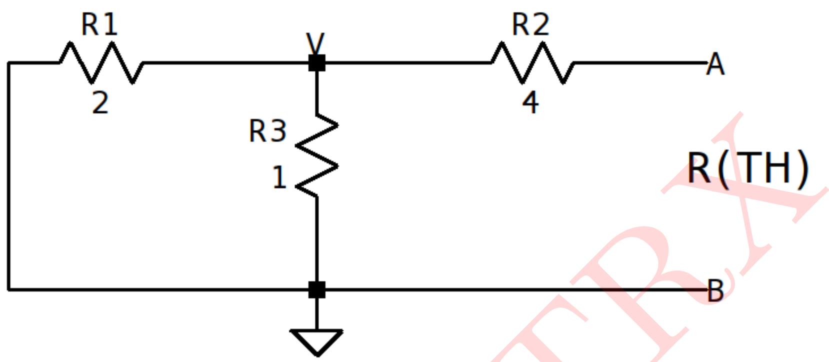

Figure 37: Modified Circuit for finding R_{TH}

From the figure, we get: $R_{TH} = 4 + (2 \mid\mid 1)$
$\therefore$ R$_{TH}$ = 4.6666Ω

Now, Max power absorbed by the load resistance is given by:

$$P_{max} = \frac{(V_{TH})^2}{R_{TH}}$$

Substituting the values for R_{TH} and V_{TH}, we get:
$P_{max} = 2.625$W

SIMULATED RESULTS:
The given circuit is simulated in LTSpice and the results obtained are as follows:

For V_{TH}:

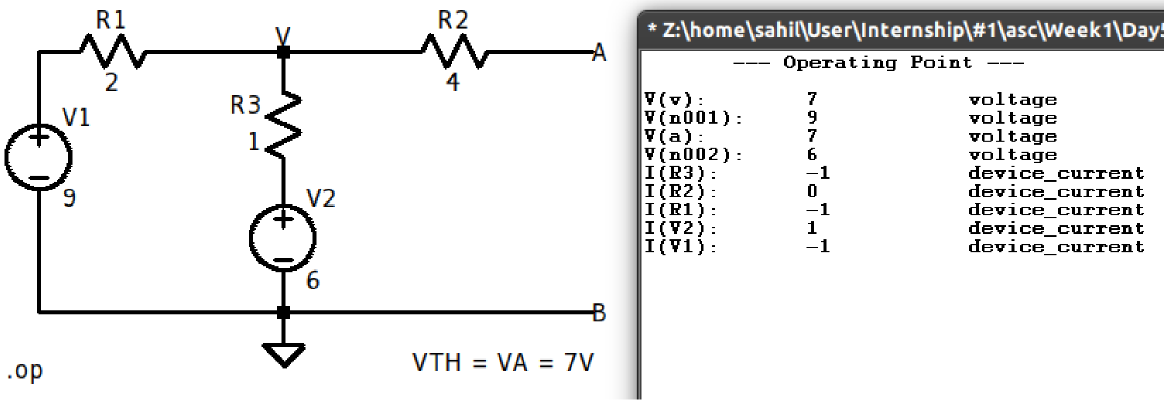

Figure 38: Circuit Schematic and Simulated Results to find V_{TH}

For R_{TH}:

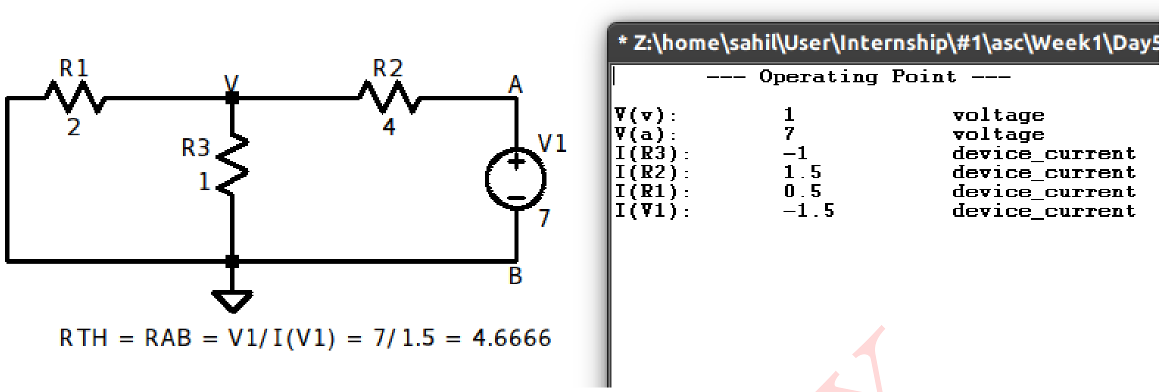

Figure 39: Circuit Schematic and Simulated Results to find R_{TH}

Thevenin's Equivalent Circuit:

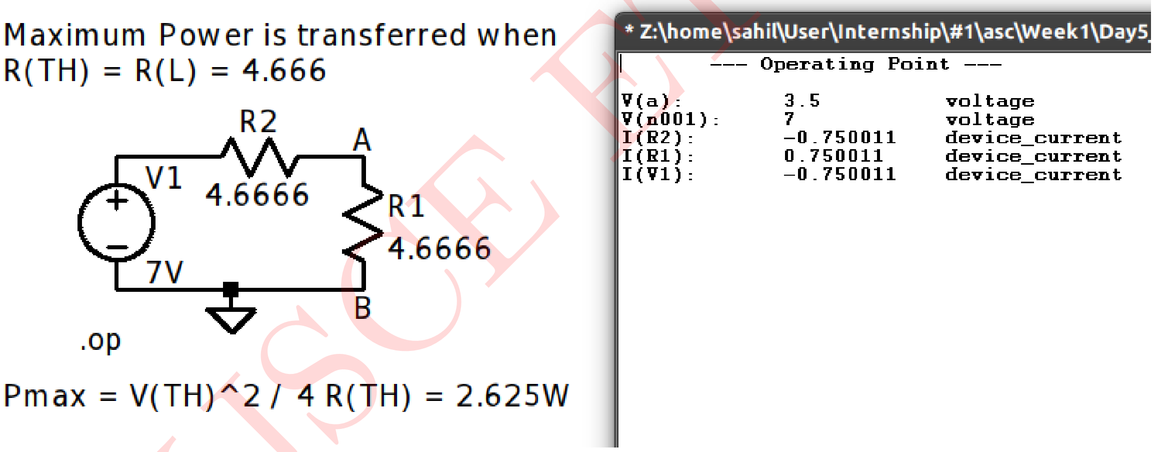

Figure 40: Circuit Schematic and Simulated Results for Thevenin's Equivalent Circuit

Comparison of theoretical and simulated values:

Parameters	Theoretical Values	Simulated Values
R_{TH}	4.6666Ω	4.6666Ω
V_{TH}	7V	7V
P_{max}	2.625W	2.625W

Table 9: Numerical 9

Numerical 10: Calculate the value of R that will absorb maximum power from the circuit 10. Also compute the maximum power.

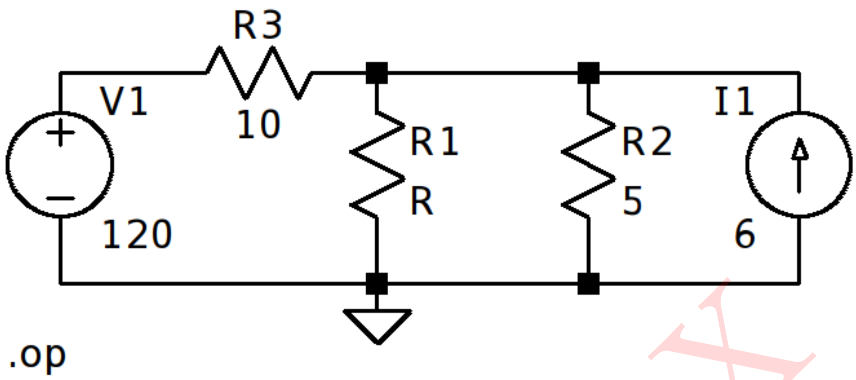

Figure 41: Circuit 10

SOLUTION:

Here we disconnect the load resistance to calculate open circuit voltage,

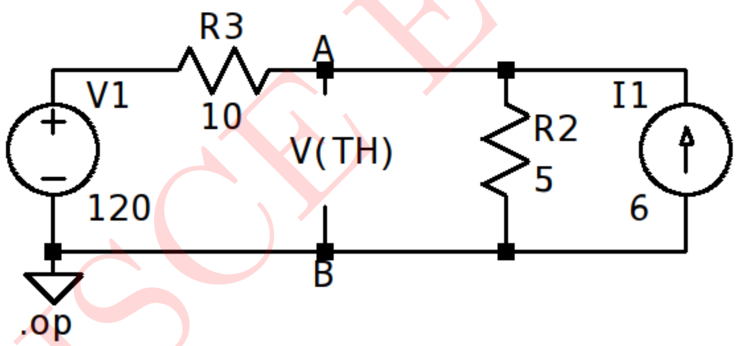

Figure 42: Circuit after disconnecting load resistance

Applying KCL to node A,

$$\frac{V_A - 120}{10} + \frac{V_A - 0}{5} - 6 = 0$$

i.e. $V_A = 60$V

Here, $V_A = V_{TH} = 60$V

Now, we short all voltage and current sources to find R_{TH},

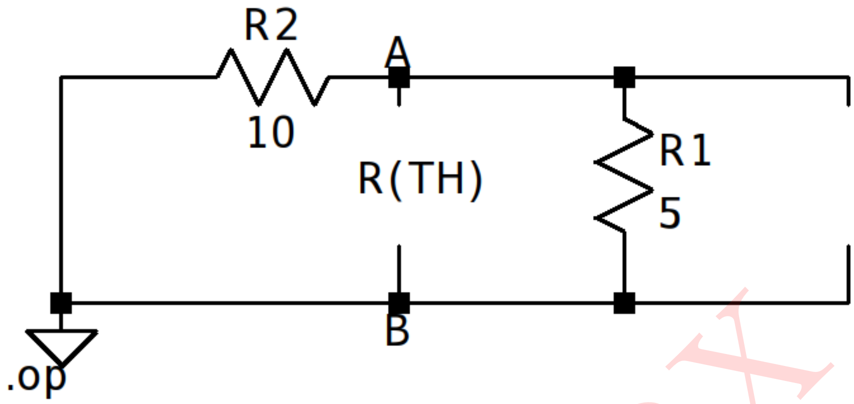

Figure 43: Modified Circuit for finding R_{TH}

From the figure, we get: $R_{TH} = 10 \mid\mid 5$

$\therefore \mathrm{R}_{TH} = 3.3333\Omega$

Now, Max power absorbed by the load resistance is given by:

$$P_{max} = \frac{(V_{TH})^2}{R_{TH}}$$

Substituting the values for R_{TH} and V_{TH}, we get:
$P_{max} = 270.0027\mathrm{W}$

SIMULATED RESULTS:
The given circuit is simulated in LTSpice and the results obtained are as follows:

For V_{TH}:

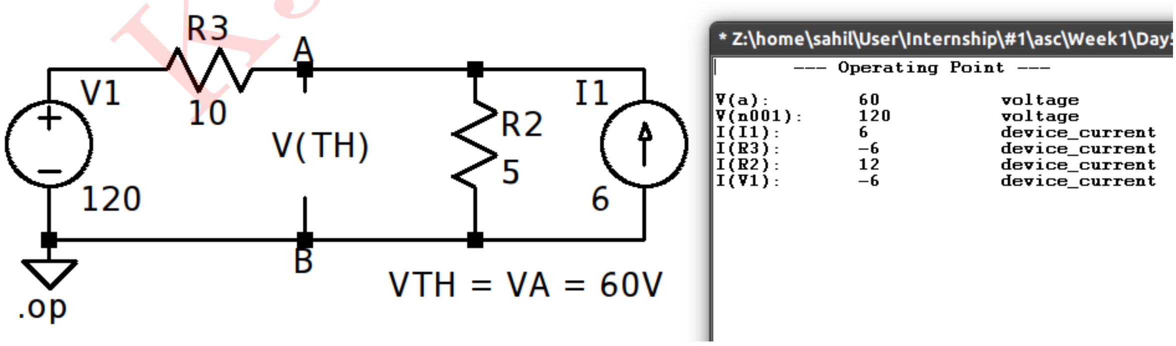

Figure 44: Circuit Schematic and Simulated Results to find V_{TH}

For R_{TH}:

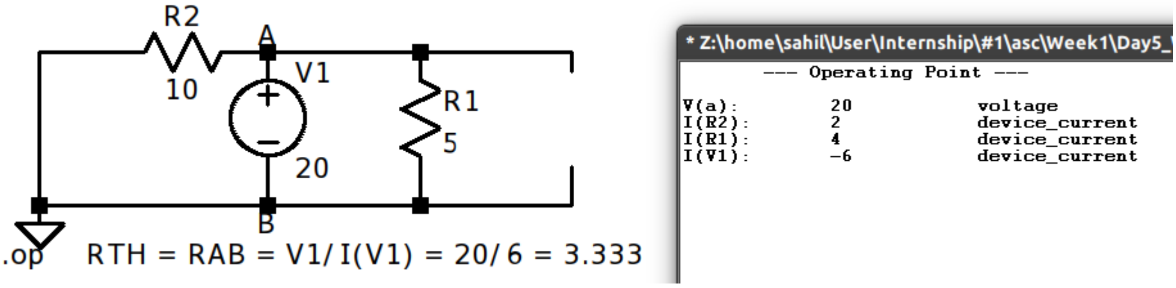

Figure 45: Circuit Schematic and Simulated Results to find R_{TH}

Thevenin's Equivalent Circuit:

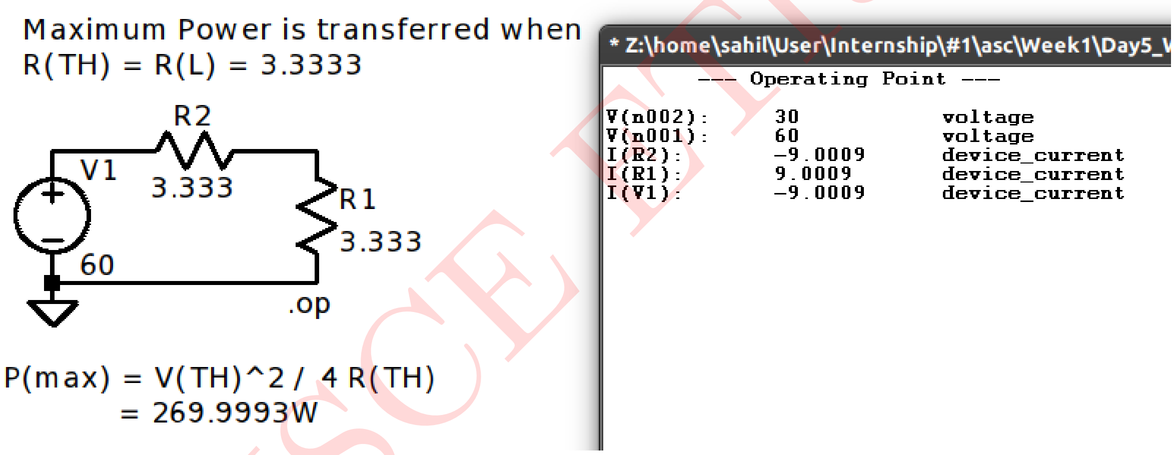

Figure 46: Circuit Schematic and Simulated Results for Thevenin's Equivalent Circuit

Comparison of theoretical and simulated values:

Parameters	Theoretical Values	Simulated Values
R_{TH}	3.3333Ω	3.3333Ω
V_{TH}	60V	60V
P_{max}	270.0027W	269.9993W

Table 10: Numerical 10