K. J. SOMAIYA COLLEGE OF ENGINEERING
DEPARTMENT OF ELECTRONICS ENGINEERING
ELECTRONIC CIRCUITS
Differential Amplifier Circuits

Numerical 1:

Determine the following for the circuit shown below in figure 1. Assume $\beta_1 = \beta_2 = 100$.
a) Name the circuit
b) Currents I_{C_1} and I_{C_2}
c) V_{C_1}, V_{C_2}, V_{CE_1}, V_{CE_2}
d) Differential voltage gain (A_d)
e) Common mode voltage gain (A_{cm})
f) CMRR in dB

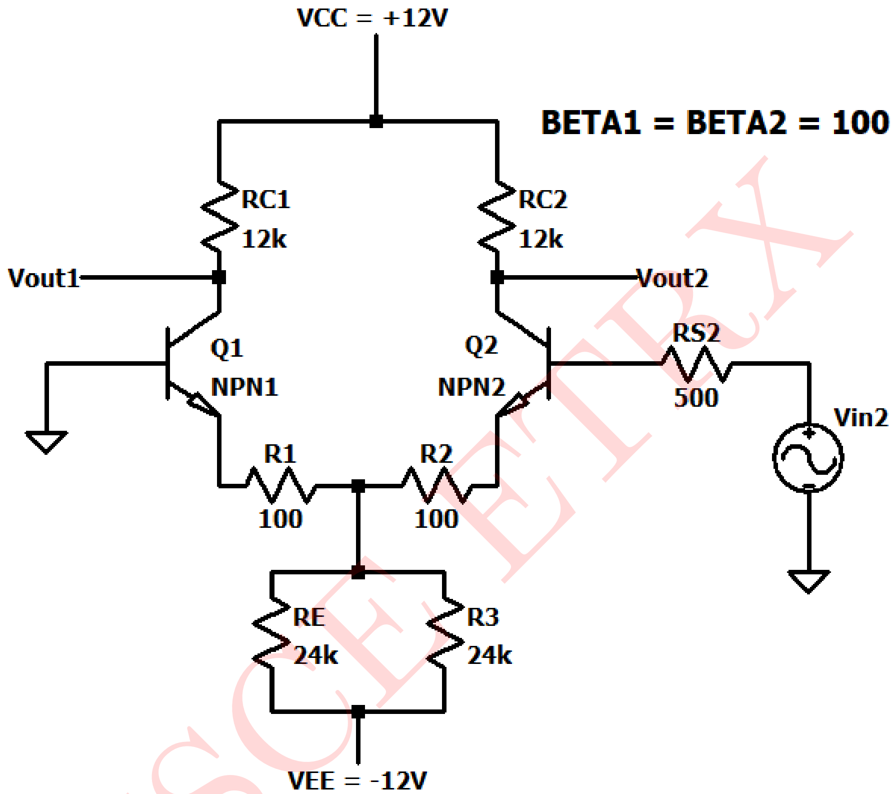

Figure 1: Circuit 1

Solution:

DC analysis for Q_1:

$$I_{C_1} = \beta_1 \left(\frac{V_{EE} - V_{BE}}{\beta_1 [2(R_E \parallel R_3) + R_1]} \right)$$

$$= 100 \left(\frac{12V - 0.7V}{100[2(24k\Omega \parallel 24k\Omega) + 100\Omega]} \right)$$

$$= 100 \left(\frac{11.3V}{100 \left[2 \times \frac{24k\Omega \times 24k\Omega}{24k\Omega + 24k\Omega} \right] + 100\Omega} \right)$$

$$= 100 \left(\frac{11.3V}{100 \times 24k\Omega + 100\Omega} \right)$$

$$= 100\left(\frac{11.3V}{2400.1k\Omega}\right)$$

$\therefore I_{C_1} = \mathbf{0.470mA}$

$\therefore I_{CQ_1} = \mathbf{0.470mA}$

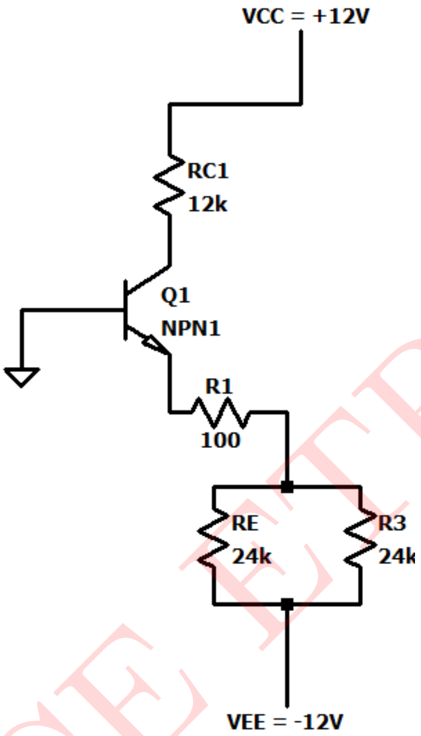

Figure 2: DC equivalent circuit for Q_1

$$\begin{aligned}
V_{CEQ_1} &= V_{CC} + V_{EE} - I_{CQ_1}[R_{C_1} + 2(R_E \parallel R_3) + R_1] \\
&= 12V + 12V - 0.470mA[12k\Omega + 2(24k\Omega \parallel 24k\Omega) + 100\Omega] \\
&= 12V + 12V - 0.470mA\left[12k\Omega + 2\left(\frac{24k\Omega \times 24k\Omega}{24k\Omega + 24k\Omega}\right) + 100\Omega\right] \\
&= 24V - 0.470mA\,[12k\Omega + 2(12k\Omega) + 100\Omega] \\
&= 24V - 0.470mA\,[12k\Omega + 24k\Omega + 100\Omega] \\
&= 24V - 0.470mA\,[36.1k\Omega] \\
&= 24V - 16.97V = \mathbf{7.033V}
\end{aligned}$$

$$\begin{aligned}
V_{C_1} &= V_{CC} - I_{C_1}R_{C_1} \\
&= 12V - (0.470mA)(12k\Omega) \\
&= 12V - 5.6V = \mathbf{6.36V}
\end{aligned}$$

DC analysis for Q_2:

$$I_{C_2} = \beta_2\left(\frac{V_{EE}-V_{BE}}{R_S+\beta_2[2(R_E \parallel R_3)+R_2]}\right)$$

$$= 100\left(\frac{12V-0.7V}{500\Omega+100[2(24k\Omega \parallel 24k\Omega)+100\Omega]}\right)$$

$$= 100\left(\frac{11.3V}{500\Omega+100\left[2\times\frac{24k\Omega\times 24k\Omega}{24k\Omega+24k\Omega}\right]+100\Omega}\right)$$

$$= 100\left(\frac{11.3V}{500\Omega+100\times 24k\Omega+100\Omega}\right)$$

$$= 100\left(\frac{11.3V}{2400.6k\Omega}\right)$$

$\therefore I_{C_2} = \mathbf{0.470mA}$

$\therefore I_{CQ_2} = \mathbf{0.470mA}$

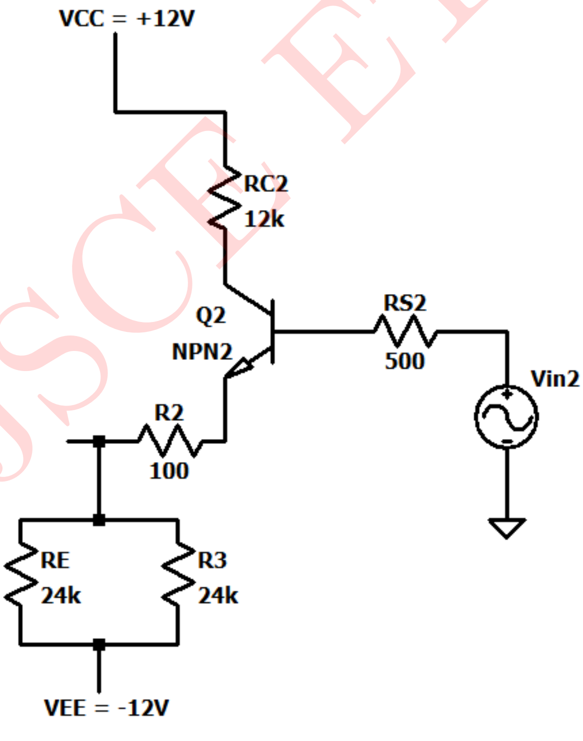

Figure 3: DC equivalent circuit for Q_2

$$V_{CEQ_2} = V_{CC}+V_{EE}-I_{CQ_2}[R_{C_2}+2(R_E \parallel R_3)+R_2]$$

$$= 12V+12V-0.470mA[12k\Omega+2(24k\Omega \parallel 24k\Omega)+100\Omega]$$

$$= 12V+12V-0.470mA\left[12k\Omega+2\left(\frac{24k\Omega\times 24k\Omega}{24k\Omega+24k\Omega}\right)+100\Omega\right]$$

$$= 24V-0.470mA\,[12k\Omega+2(12k\Omega)+100\Omega]$$

$$= 24V-0.470mA\,[12k\Omega+24k\Omega+100\Omega]$$

$$= 24V - 0.470mA\,[36.1k\Omega]$$

$$= 24V - 16.97V = \mathbf{7.033V}$$

$$V_{C_2} = V_{CC} - I_{C_2}R_{C_2}$$

$$= 12V - (0.470mA)(12k\Omega)$$

$$= 12V - 5.64V = \mathbf{6.36V}$$

$$r_\pi = \frac{\beta V_T}{I_{CQ}}$$

$$= \frac{100 \times 26mV}{0.470mA} \qquad (\because \beta_1 = \beta_2 = \beta = 100, I_{CQ_1} = I_{CQ_2} = I_{CQ} = 0.470mA)$$

$$= 100 \times 55.31 = \mathbf{5.531k\Omega}$$

Differential voltage gain(A_d):

$$A_d = \frac{\beta R_C}{R_S + r_\pi + \beta R_{Eswamp}}$$

$$= \frac{100 \times 12k\Omega}{500\Omega + 5.531k\Omega + 100 \times 100\Omega} (\because R_{C_1} = R_{C_2} = R_C = 12k\Omega, R_1 = R_2 = R_{Eswamp})$$

$$= \frac{1200k\Omega}{500\Omega + 5.531k\Omega + 10000\Omega} = \mathbf{74.85}$$

Common mode voltage gain(A_{cm}):

$$A_{cm} = \frac{R_C}{2(R_E \parallel R_3)}$$

$$= \frac{12k\Omega}{2(24k\Omega \parallel 24k\Omega)}$$

$$= \frac{12k\Omega}{2\left(\frac{24k\Omega \times 24k\Omega}{24k\Omega + 24k\Omega}\right)}$$

$$= \frac{12k\Omega}{2 \times 12k\Omega} = \mathbf{0.5}$$

$$\text{CMRR} = \left|\frac{A_d}{A_{cm}}\right|$$

$$= \frac{74.85}{0.5} = \mathbf{149.70}$$

CMRR in dB $= 20\log_{10} 149.70 = \mathbf{43.50dB}$

SIMULATED RESULTS:

Above circuit is simulated in LTspice and results are as follows:

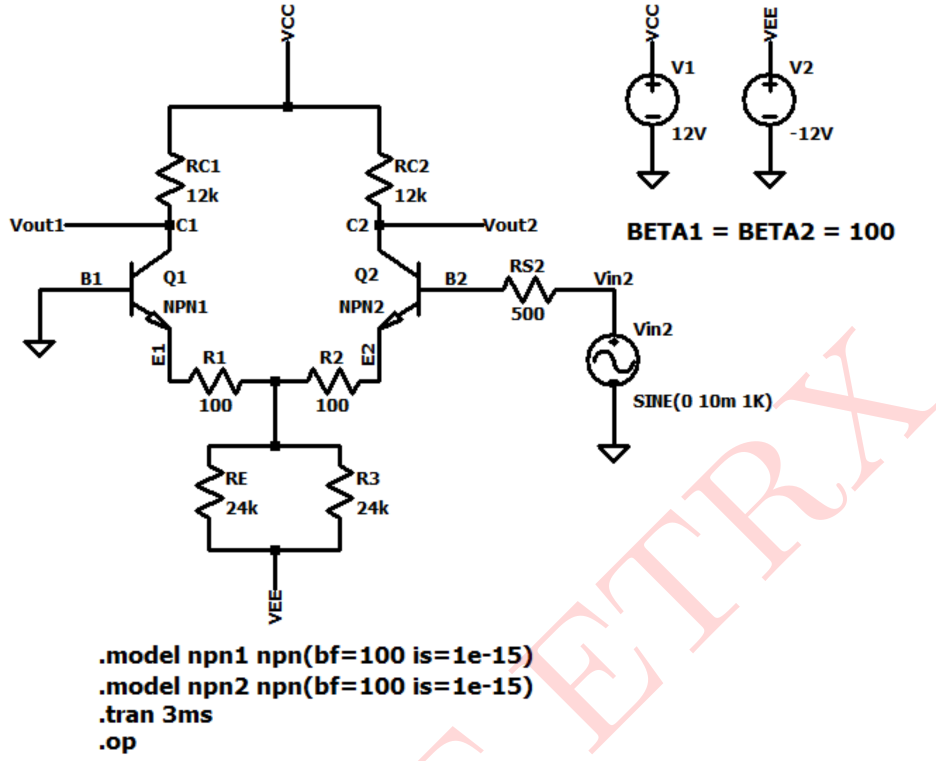

Figure 4: Circuit Schematic

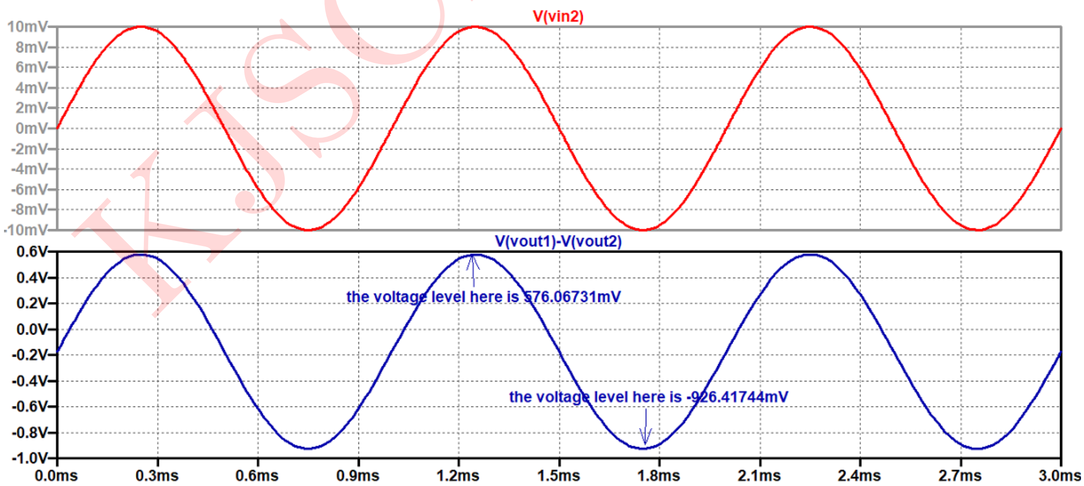

Figure 5: Input and output voltage waveform

Comparison between theoretical and simulated values:

Parameters	Theoretical values	Simulated values
I_{C_1}, I_{C_2}	0.470mA, 0.470mA	0.4716mA, 0.4751mA
V_{C_1}, V_{C_2}	6.36V, 6.36V	6.3396V, 6.5146V
V_{CE_1}, V_{CE_2}	7.033V, 7.033V	7.0348V, 7.211V
Differential voltage gain(A_d)	74.85	75.1242
Common mode voltage gain(A_{cm})	0.5	–
CMRR in dB	43.50dB	–

Table 1: Numerical 1

Numerical 2:
Consider the differential amplifier given below in figure 6. The transistor parameters are $k_{n_1} = k_{n_2} = 55\mu A/V^2$, $\lambda_1 = \lambda_2 = 0.02V^{-1}$ and $V_{TN_1} = V_{TN_2} = 1V$.
a) Determine I_S, I_{D_1}, I_{D_2} and V_{o_2} for $V_1 = V_2 = 0$
b) Determine the differential mode voltage gain A_d, common mode voltage gain A_{cm} and the CMRR in dB

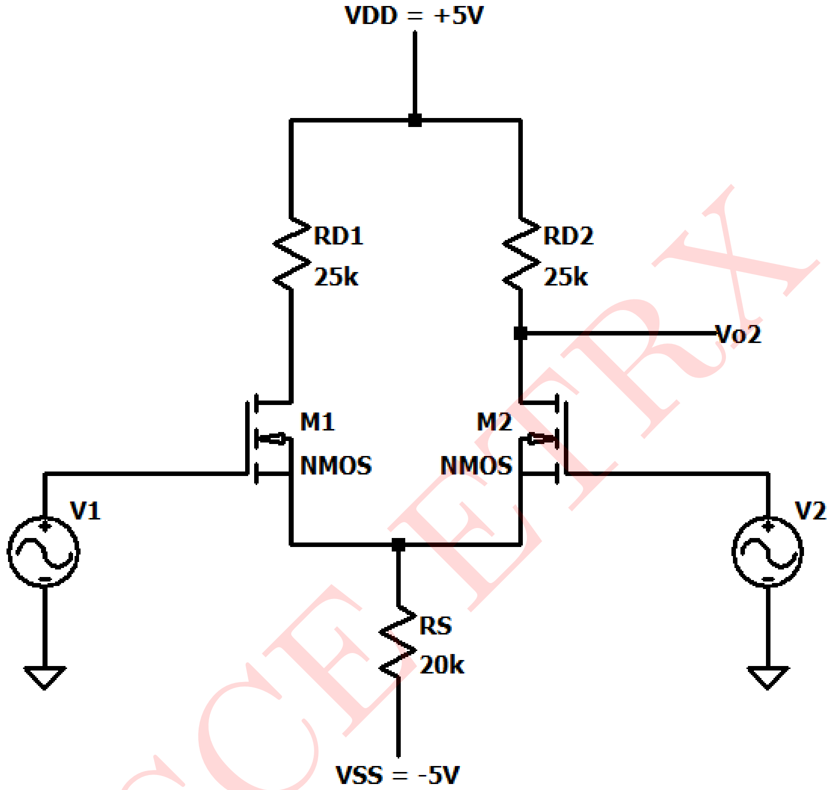

Figure 6: Circuit 2

Solution:
DC analysis:

Applying KVL to gate-source loop,

$-V_{GS_1} - 2I_{D_1}R_S - V_{SS} = 0$

$\therefore V_{GS_1} = -V_{SS} - 2I_{D_1}R_S$

$= 5V - 2I_{D_1}(20k\Omega)$

$= 5V - I_{D_1}(40k\Omega) \qquad ....(1)$

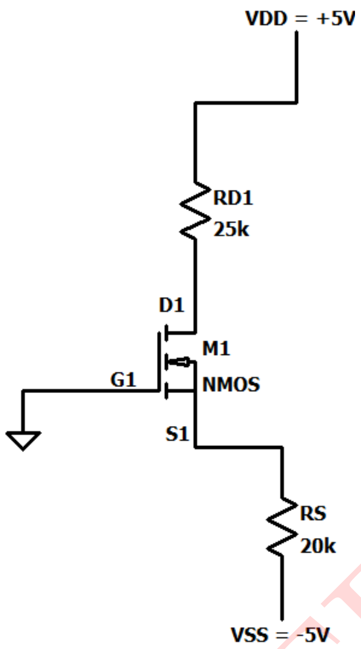

Figure 7: DC equivalent circuit

Applying KVL at drain-source loop,

$V_{DD} - I_{D_1}R_{D_1} - V_{DS_1} - 2I_{D_1}R_S - V_{SS} = 0$

$V_{DS_1} = V_{DD} - I_{D_1}R_{D_1} - 2I_{D_1}R_S - V_{SS}$

$V_{DS_1} = 5 - I_{D_1}(25k\Omega) - I_{D_1}(40k\Omega) + 5$

$V_{DS_1} = 10 - I_{D_1}(65k\Omega) \qquad ....(2)$

In MOSFET,

$I_{D_1} = k_{n_1}(V_{GS_1} - V_{TN_1})^2(1 + \lambda V_{DS_1})$

$I_{D_1} = 55 \times 10^{-6}[5 - I_{D_1}(40k\Omega) - 1V]^2 \times [1 + 0.02(10 - I_{D_1}(65k\Omega))] \qquad [from(1)and(2)]$

$I_{D_1} = 55 \times 10^{-6}[4 - I_{D_1}(40k\Omega)]^2 \times [1 + 0.2 - I_{D_1}(1.3k\Omega)]$

$I_{D_1} = 55 \times 10^{-6}[4 - I_{D_1}(40k\Omega)]^2 \times [1.2 - I_{D_1}(1.3k\Omega)]$

$I_{D_1} = 55 \times 10^{-6}[16 - I_{D_1}(320 \times 10^3) + {I_{D_1}}^2(1600 \times 10^6)] \times [1.2 - I_{D_1}(1.3k\Omega)]$

$I_{D_1} = 55 \times 10^{-6}[19.2 - I_{D_1}(304.8 \times 10^3) + {I_{D_1}}^2(2336 \times 10^6) - {I_{D_1}}^3(2080 \times 10^9)]$

$I_{D_1} = 1.056 \times 10^{-3} - I_{D_1}(22.264) + {I_{D_1}}^2(128.48 \times 10^3) - {I_{D_1}}^3(114.4 \times 10^6)$

${I_{D_1}}^3(114.4 \times 10^6) - {I_{D_1}}^2(128.48 \times 10^3) + I_{D_1}(23.264) - 1.056 \times 10^{-3} = 0$

$I_{D_1} = 0.9109mA$ or $I_{D_1} = 0.1394mA$ or $I_{D_1} = 0.0726mA$

Let $I_{D_1} = 0.9109mA$

$\therefore V_{GS_1} = 5 - (0.9109mA)(40k\Omega) = \mathbf{-31.436V}$

Let $I_{D_1} = 0.1394mA$

$\therefore V_{GS_1} = 5 - (0.1394mA)(40k\Omega) = \mathbf{-0.56V}$

Let $I_{D_1} = 0.0726mA$

$\therefore V_{GS_1} = 5 - (0.0726mA)(40k\Omega) = \mathbf{2.096V}$

We reject both negative values of V_{GS_1} because $V_{GS_1} > V_{TN_1}$

$\therefore V_{GS_1} = \mathbf{2.096V}$

$$\begin{aligned} V_{DS_1} &= 10 - I_{D_1}(65k\Omega) \\ &= 10 - (0.0726mA)(65k\Omega) = \mathbf{5.281V} \end{aligned}$$

Now,

$$\begin{aligned} V_{D_1} &= V_{DD} - I_{D_1}R_{D_1} \\ &= 5V - (0.0726mA)(25k\Omega) = \mathbf{3.185V} \end{aligned}$$

$\because$ Both the transistor are identical,

$\therefore I_{D_1} = I_{D_2} = \mathbf{0.0726mA}$

$V_{DS_1} = V_{DS_2} = \mathbf{5.281V}$

$V_{D_1} = V_{D_2} = \mathbf{3.185V}$

$$\begin{aligned} \because I_S &= I_{D_1} + I_{D_2} \\ &= 0.0726mA + 0.0726mA = \mathbf{0.1452mA} \end{aligned}$$

Applying KVL at drain-source loop of M_2,

$V_{DD} - V_{O_2} - V_{DS_2} - I_S R_S + V_{SS} = 0$

$5V - V_{O_2} - 2.096V - (0.15 = 452mA)(20k\Omega) + 5V = 0$

$10V - 2.096V - 2.904V + 5V = V_{O_2}$

$V_{O_2} = 10V - 5V = \mathbf{5V}$

Small signal parameters:

$$\begin{aligned} g_{m_1} &= 2k_n(V_{GS_1} - V_{TN_1})(1 + \lambda V_{DS_1}) \\ &= 2 \times 55 \times 10^{-6}(2.096V - 1V)[1 + (0.02)(5.281V)] \\ &= 110 \times 10^{-6}(1.096V)[1.105] = \mathbf{0.133mA/V} \end{aligned}$$

$\therefore g_{m_1} = g_{m_2} = g_m = \mathbf{0.133mA/V}$

$$\begin{aligned} r_{d_1} &= \frac{1}{\lambda_1 I_{D_1}} \\ &= \frac{1}{0.02 \times 0.0726mA} = \mathbf{688.70k\Omega} \end{aligned}$$

$\therefore r_{d_1} = r_{d_2} = \mathbf{688.70k\Omega}$

Differential voltage gain(A_d):

$$A_d = \frac{g_m(R_D \parallel r_d)}{2}$$

$$= \frac{0.133mA/V(25k\Omega \parallel 688.70k\Omega)}{2}$$

$$= \frac{0.133mA/V\left(\frac{25k\Omega \times 688.70k\Omega}{25k\Omega + 688.70k\Omega}\right)}{2}$$

$$= \frac{0.133mA/V \times 24.12k\Omega}{2}$$

$$= \frac{3.208}{2} = \mathbf{1.60}$$

Common mode voltage gain(A_{cm}):

$$A_{cm} = \frac{g_m(r_d \parallel R_D)}{1 + 2g_m R_S}$$

$$= \frac{0.133mA/V(25k\Omega \parallel 688.70k\Omega)}{1 + 2(0.133mA/V)(20k\Omega)}$$

$$= \frac{0.133mA \times 24.12k\Omega}{1 + 5.32}$$

$$= \frac{3.207}{6.32} = \mathbf{0.50}$$

$$\text{CMRR} = \left|\frac{A_d}{A_{cm}}\right|$$

$$= \frac{1.60}{0.50} = \mathbf{3.2}$$

CMRR in dB $= 20\log_{10} 3.2 = \mathbf{10.10dB}$

SIMULATED RESULTS:
Above circuit is simulated in LTspice and results are as follows:

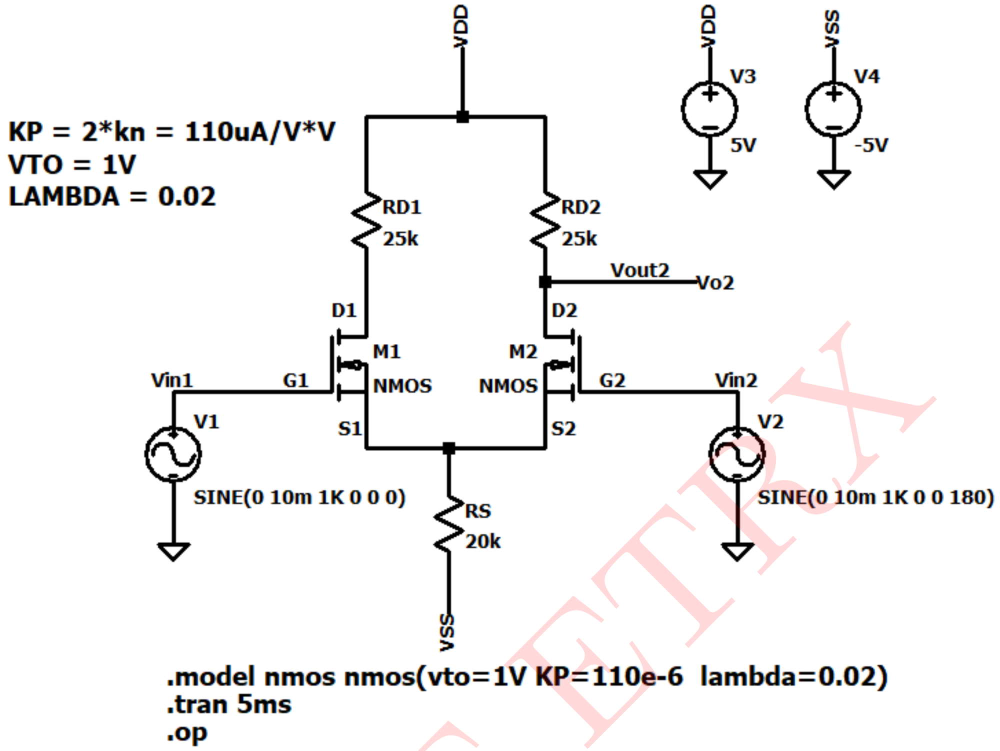

Figure 8: Circuit Schematic

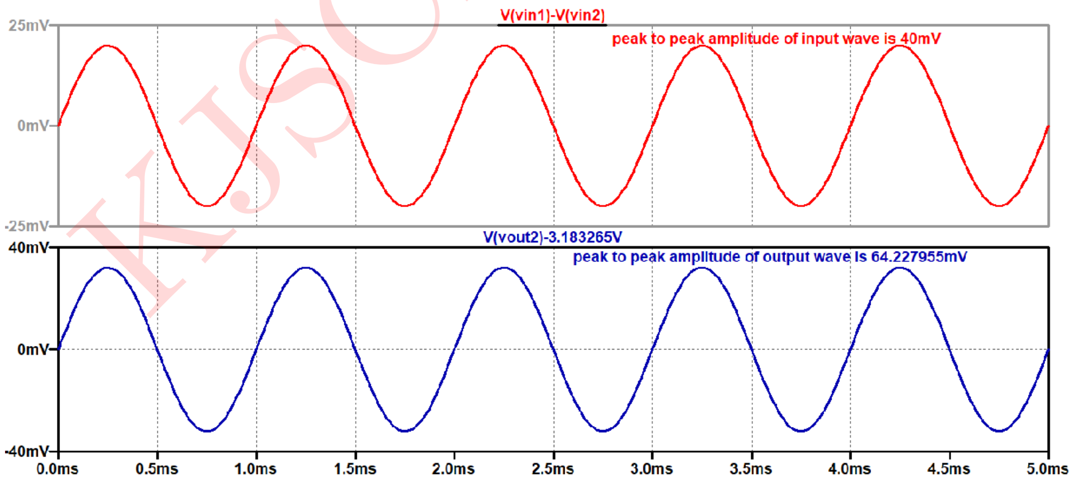

Figure 9: Input and output voltage waveform

Comparison between theoretical and simulated values:

Parameters	Theoretical values	Simulated values
I_S, I_{D_1}, I_{D_2}	0.1452mA, 0.0726mA, 0.0726mA	0.1453mA, 0.0726mA, 0.0726mA
V_{D_1}, V_{D_2}	3.185V, 3.185	3.1832V, 3.1832V
V_{DS_1}, V_{DS_2}	5.281V, 5.281V	5.2764V, 5.2764V
Differential voltage gain(A_d)	1.60	1.6056
Common mode voltage gain(A_{cm})	0.50	–
CMRR in dB	10.10dB	–

Table 2: Numerical 2

Numerical 3:
For the amplifier shown below, JFET parameters are $I_{DSS_1} = I_{DSS_2} = 12mA$ and $V_{P_1} = V_{P_2} = -2.5V$. Find
a) I_{D_1}, I_{D_2}, V_{D_1}, V_{D_2}
b) DC values of V_{o_1} and V_{o_2}
c) Double ended gain$\left(\dfrac{V_{o_1} - V_{o_2}}{V_1 - V_2}\right)$

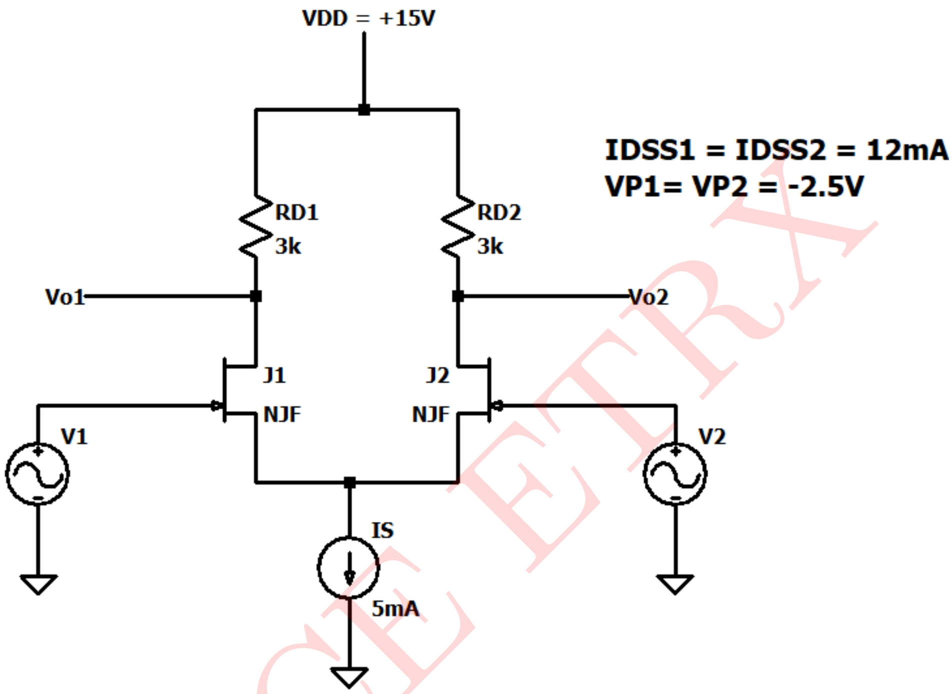

Figure 10: Circuit 3

Solution:
Above circuit is dual input balanced output(DIBO) differential amplifier consisting of JFETs. The circuit consists of identical transistors and circuit is vertically symmetric.

$$I_{D_1} = I_{D_2} = \frac{I_S}{2}$$

$$I_{D_1} = I_{D_2} = \frac{5mA}{2}$$

$$I_{D_1} = I_{D_2} = \mathbf{2.5mA}$$

Now,

$$\begin{aligned} \text{DC value of } V_{o_1} = V_{o_2} &= V_{DD} - I_D R_D \\ &= 15V - (2.5mA)(3k\Omega) \qquad (I_{D_1} = I_{D_2} = I_D = 2.5mA) \\ &= 15V - 7.5V = \mathbf{7.5V} \end{aligned}$$

$\because V_{D_1} = \text{DC value of } V_{o_1}$ and $V_{D_2} = \text{DC value of } V_{o_2}$

$\therefore V_{D_1} = V_{D_2} = \mathbf{7.5V}$

Small signal parameters:

In JFET,

$$I_D = I_{DSS}\left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$\therefore \frac{I_D}{I_{DSS}} = \left(1 - \frac{V_{GS}}{V_P}\right)^2$$

$$\therefore \sqrt{\frac{I_D}{I_{DSS}}} = 1 - \frac{V_{GS}}{V_P} \qquad ....(1)$$

$$\because g_m = \frac{2I_{DSS}}{|V_P|} \times \left(1 - \frac{V_{GS}}{V_P}\right)$$

$$\therefore g_m = \frac{2I_{DSS}}{|V_P|} \times \sqrt{\frac{I_D}{I_{DSS}}} \qquad \text{(from (1))}$$

$$\therefore g_m = \frac{2 \times 12mA}{2.5V} \times \sqrt{\frac{2.5mA}{12mA}}$$

$$\therefore g_m = 2 \times 4.8mA/V \times 0.45 = \mathbf{4.38mA/V}$$

$$\therefore g_{m_1} = g_{m_2} = g_m = \mathbf{4.38mA/V}$$

Differential ended gain(A_d):

$$A_d = \frac{V_{o_1} - V_{o_2}}{V_1 - V_2} \qquad \text{(assuming } V_{o_2} > V_{o_1}\text{)}$$

$$\therefore A_d = -g_m R_D$$

$$= -4.38mA/V \times 3k\Omega = \mathbf{-13.14}$$

SIMULATED RESULTS:

Above circuit is simulated in LTspice and results are as follows:

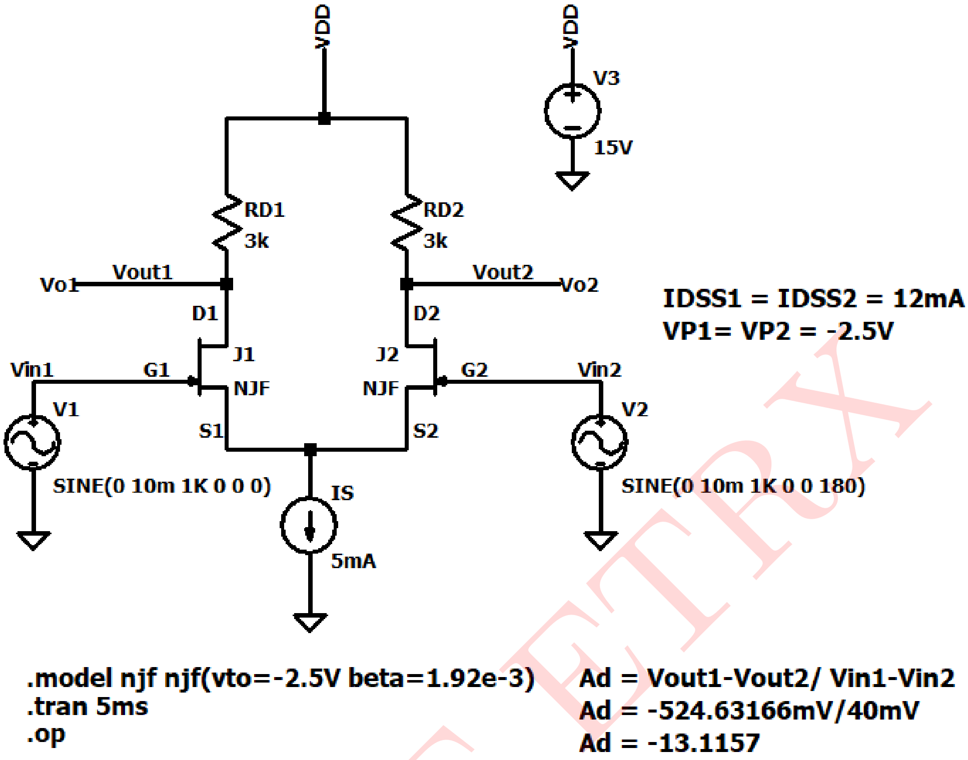

Figure 11: Circuit Schematic

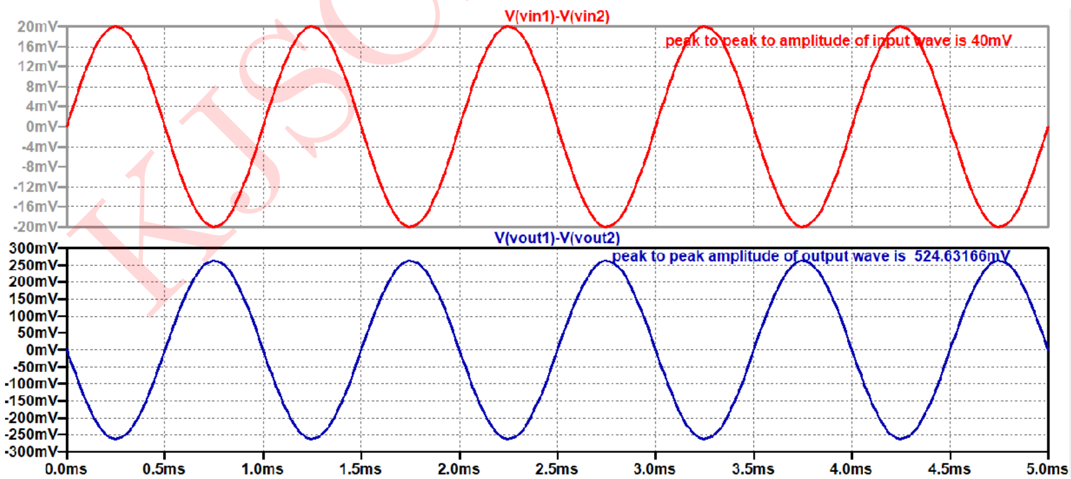

Figure 12: Input and output voltage waveform

Comparison between theoretical and simulated values:

Parameters	Theoretical values	Simulated values
I_{D_1}, I_{D_2}	2.5mA, 2.5mA	2.500mA, 2.500mA,
V_{D_1}, V_{D_2}	7.5V, 7.5V	7.5V, 7.5V
Differential voltage gain(A_d)	-13.14	-13.1157

Table 3: Numerical 3

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