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DEPARTMENT OF ELECTRONICS ENGINEERING
ELEMENTS OF ELECTRICAL AND ELECTRONICS ENGINEERING
AC CIRCUITS

Numerical 1: A series RLC circuit containing a resistor of 10Ω , an inductance of 0.4H and a capacitor of $20\mu\text{F}$ are connected in series across a 220V 60Hz supply as shown in figure 1. Calculate:

- The current drawn by the circuit.
- V_R , V_L and V_C .
- Power factor.
- Draw the voltage phasor diagram.

Solution:

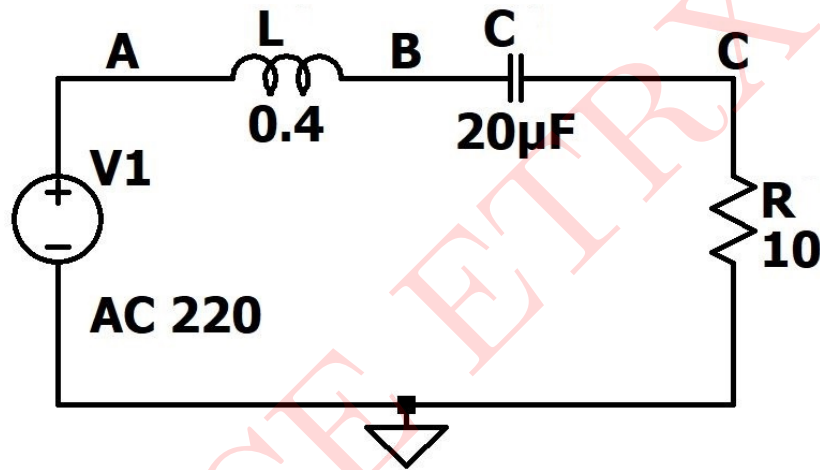


Figure 1: Circuit 1

i] Finding Current,

$$X_L = \omega L = 2\pi \times 60 \times 0.4 = 150.7964\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 60 \times 20 \times 10^{-6}} = 132.629\Omega$$

$$Z = R + (X_L - X_C)j$$

$$Z = 10 + 18.1674j$$

$$Z = 20.7377\angle 61.17^\circ$$

$$I = \frac{V_1}{Z} = \frac{200}{29.7377} = 19.6086\text{A}$$

$$\mathbf{I = 19.6086A}$$

ii] Finding V_R , V_L and V_C ,

$$V_R = I \times R = 19.6086 \times 10 = 196.086\text{V}$$

$$V_C = I \times X_C = 19.6086 \times 132.629 = 2600.07\text{V}$$

$$V_L = I \times X_L = 19.6086 \times 150.7964 = 2956.738\text{V}$$

iii] Power factor = $\frac{V_R}{V_1} = \frac{106.085}{220}$

∴ Power factor = 0.4822(lagging)

iv] Phasor diagram,

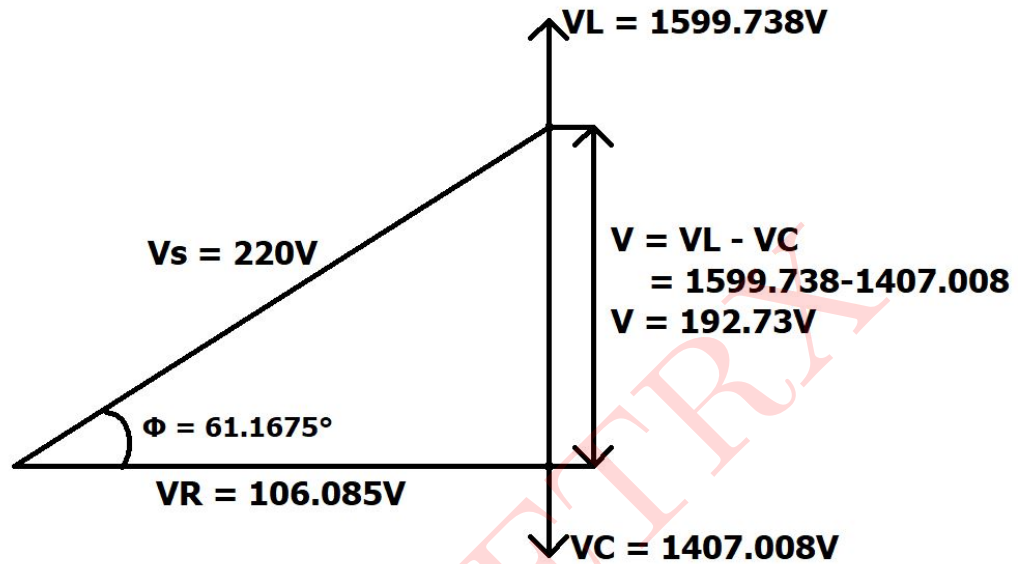


Figure 2: Phasor diagram for circuit 1

SIMULATED RESULTS:

The above circuit is simulated in LTspice. The results are presented below.

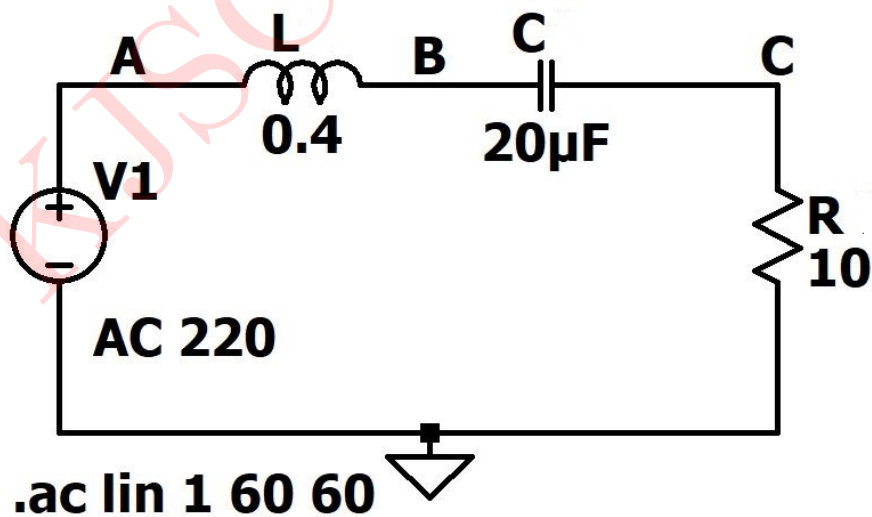


Figure 3: Circuit schematic

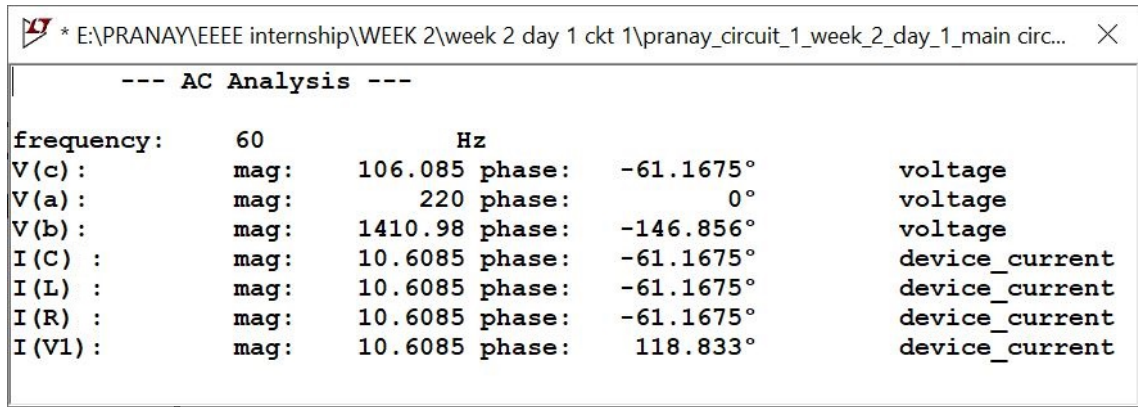


Figure 4: Simulated results

Here,

$$I = I_{V1} = 10.6085A$$

$$V_L = V_A - V_B = 200\angle 0^\circ - 1410.98\angle -146.856^\circ = 1599.72\angle 28.8321^\circ$$

$$V_R = V_C = 106.085\angle -61.1675^\circ$$

$$V_C = V_B - V_C = 1410.98\angle -146.856^\circ - 106.085\angle -61.1675^\circ = 1406.99\angle -151.167^\circ$$

$$\Phi = 61.1675^\circ$$

$$Power\ factor = \frac{V_R}{V_1} = \frac{106.085}{220} = 0.4822(lagging)$$

Comparison table between theoretical and simulated values:

Parameter	Theoretical value	Simulated values
I	10.6086A	10.6087A
V_R	106.086V	106.085V
V_C	1407.008V	1406.99V
V_L	1599.738V	1599.72V
Power factor	0.4822	0.4822

Table 1: Numerical 1

Numerical 2: A voltage $V_1 = 200 \sin(314t)$ is applied to a circuit consisting a 10Ω resistor and a $120\mu\text{F}$ capacitor in series.

Determine:

- an expression of current flowing a any instant.
- V_R and V_C .
- Power factor.

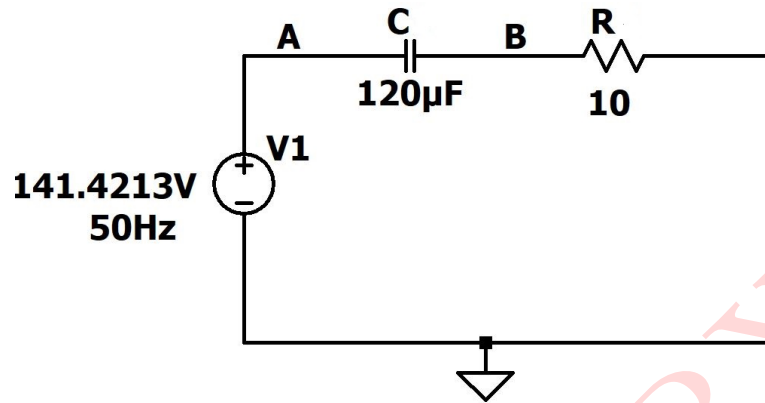


Figure 5: Circuit 2

Solution:

i) $V_1 = 200 \sin(314t)$

$\therefore n = 50\text{Hz}$

$$V_{rms} = \frac{200}{\sqrt{2}} = 141.42\text{V}$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 50 \times 120 \times 10^{-6}}$$

$\therefore X_C = 26.5258\Omega$

$$Z = R + (X_L - X_C)j$$

$\therefore Z = 10 - 26.5258j$

$\therefore Z = 28.3481 \angle -69.3439^\circ$

$$I_{rms} = \frac{V_{rms}}{Z}$$

$\therefore I_{rms} = \frac{141.42}{28.3481}$

$\therefore I_{rms} = 4.98869\text{A}$

$$I_m = I_{rms} \times \sqrt{2}$$

$\therefore I_m = 4.98869 \times \sqrt{2}$

$\therefore I_m = 7.05507\text{A}$

$$I_m = 7.05507 \sin(\omega t + 69.3439) \text{A}$$

ii] $V_C = I \times X_C$

$$\therefore V_C = 4.98869 \times 26.5258$$

$$\therefore V_C = 132.325\text{V}$$

$$V_R = I \times R$$

$$\therefore V_R = 4.98869 \times 10$$

$$V_R = 49.8869\text{V}$$

$$\text{iii] Power factor} = \frac{V_R}{V_1} = \frac{49.8869}{141.4213}$$

$$\therefore \text{Powerfactor} = 0.35275$$

SIMULATED RESULTS:

The above circuit is simulated in LTspice. The results are presented below.

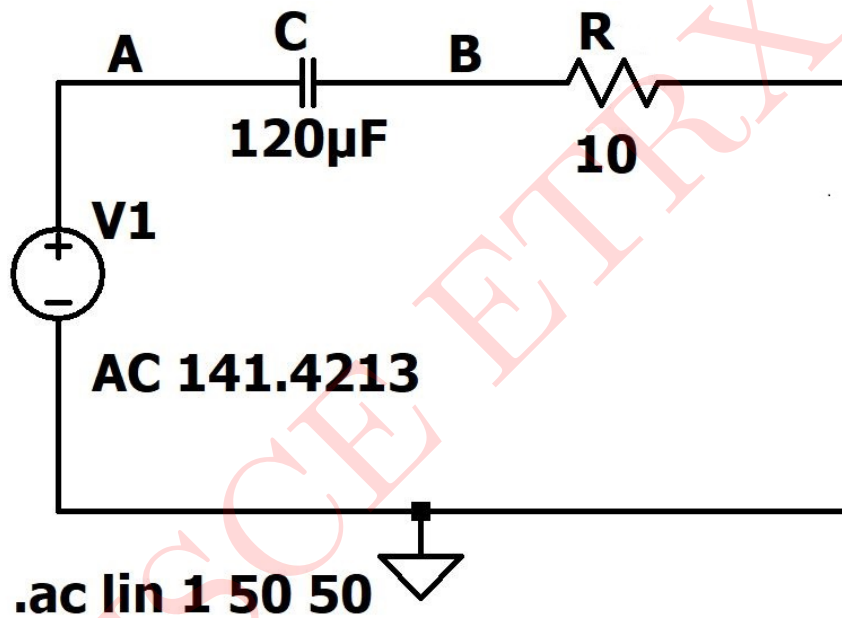


Figure 6: Circuit schematic

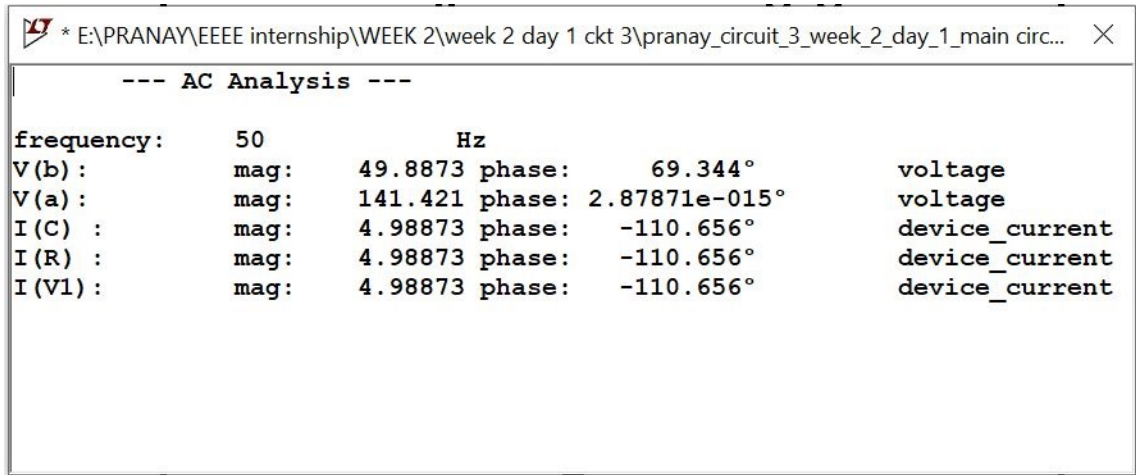


Figure 7: Simulated results

Here,

$$\phi = 69.3439^\circ$$

$$I_{rms} = 4.9883A$$

$$I_m = I_{rms} \times \sqrt{2} = 4.9883 \times \sqrt{2} = 7.05512A$$

$$\therefore I_m = 7.05507 \sin(\omega t + 69.3439)A$$

$$V_R = V_B = 49.8873 \angle 69.344^\circ$$

$$V_C = V_A - V_B = 141.421 \angle 0^\circ - 49.8873 \angle 69.344^\circ$$

$$V_C = 132.33 \angle -20.656^\circ V$$

$$Power\ factor = \cos(69.344^\circ)$$

$$Powerfactor = 0.35275(\text{leading})$$

Comparison table between theoretical and simulated values:

Parameter	Theoretical value	Simulated values
I	$7.05507 \sin(\omega t + 69.3439)$	$7.05512 \sin(\omega t + 69.344)$
V_C	$132.325 \angle -20.656^\circ V$	$132.33 \angle -20.656^\circ V$
V_R	$49.8869 \angle 69.344^\circ V$	$49.887349.8869 \angle 69.344^\circ V$
Power factor	0.35275	0.35275

Table 2: Numerical 2

Numerical 3: A circuit consists of resistance of 55Ω and inductor of 74mH and a capacitor of $90\mu\text{F}$ are connected in parallel across a 110V , 50Hz supply.

Calculate:

- individual currents drawn by each element.
- total current drawn from the supply.
- overall power factor of the circuit.
- draw the phasor diagram.

Solution:

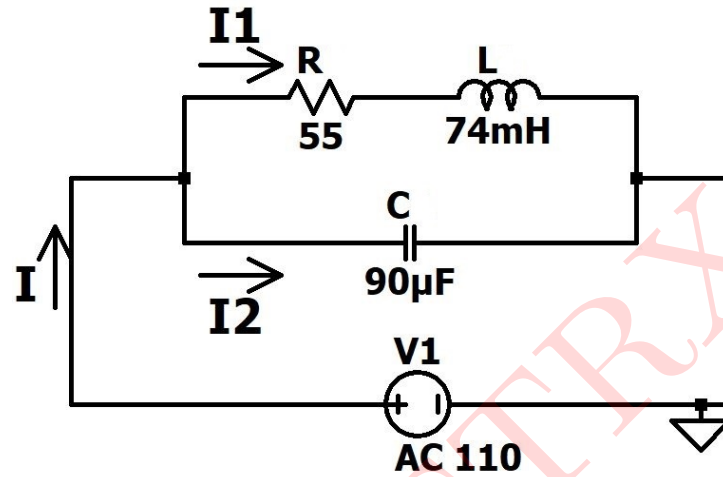


Figure 8: Circuit 3

$$\text{a] } X_L = \omega L = 2 \times \pi \times 50 \times 74 \times 10^{-3} = 23.2477\Omega$$

$$\therefore Z_1 = R + j(X_L - X_C) = 55 + 23.2477j = 59.7114\angle 22.91^\circ\Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2 \times \pi \times 50 \times 90 \times 10^{-6}} = 35.3677\Omega$$

$$\therefore Z_2 = R + j(X_L - X_C) = -35.3677j = 35.3677\angle -90^\circ\Omega$$

$$\therefore I_1 = \frac{V_1}{Z_1} = \frac{110}{59.7114\angle 22.91^\circ} = 1.84219\angle -22.91^\circ\text{A}$$

$$\therefore I_2 = \frac{V_1}{Z_2} = \frac{110}{35.3677\angle 90^\circ} = 3.1101\angle 90^\circ\text{A}$$

$$\text{b] } I = I_1 + I_2$$

$$\therefore I = 1.84219\angle -22.91^\circ + 3.1101\angle 54.6591^\circ$$

$$\therefore I = 2.9335\angle 54.6591^\circ\text{A}$$

$$\text{c] } \phi = 54.6591^\circ$$

$$\therefore \text{Power factor} = \cos(\phi) = \cos(54.6591^\circ)$$

$$\therefore \text{Power factor} = 0.57844(\text{lagging})$$

d] Phasor diagram:

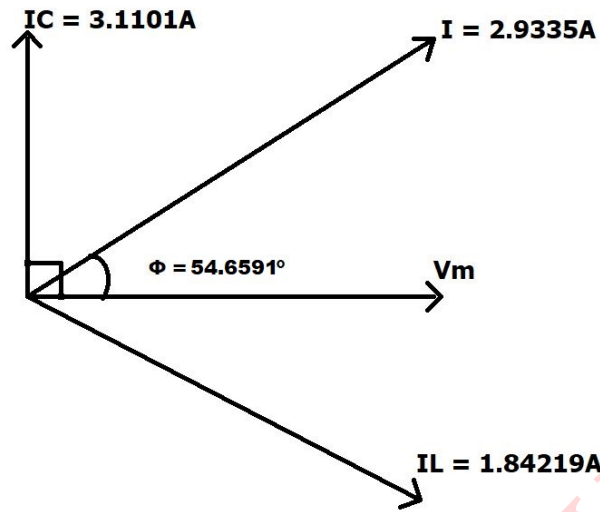


Figure 9: Phasor diagram for circuit 8

SIMULATED RESULTS:

The above circuit is simulated in LTspice. The results are presented below.

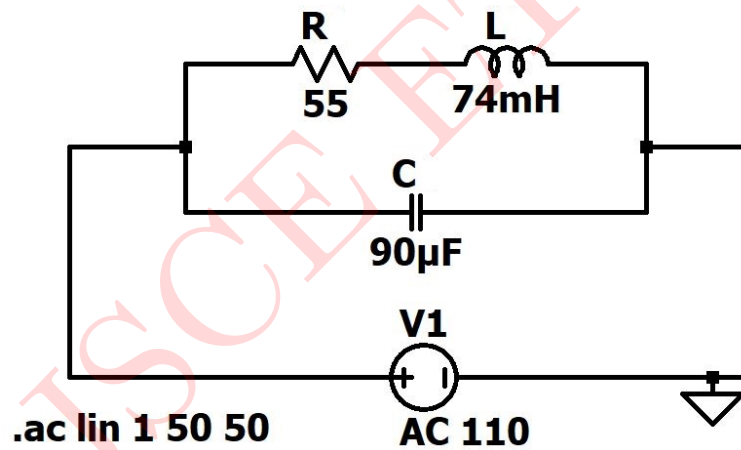


Figure 10: Circuit schematic

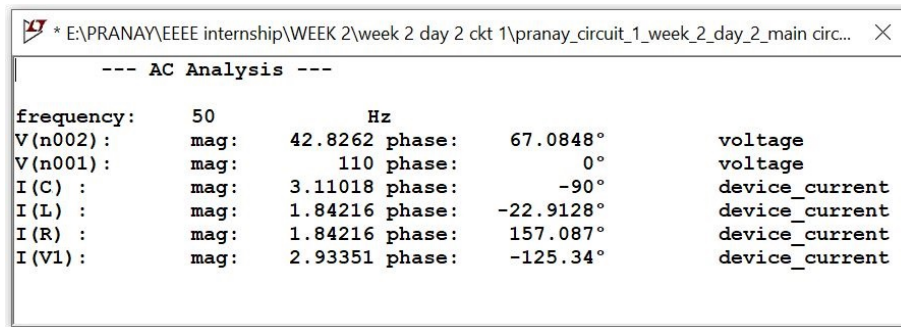


Figure 11: Simulated results

Here,

$$I = I_{V_1} = 2.93351 \angle 54.66^\circ A$$

$$I_1 = I_L = 1.8216 \angle -22.9128^\circ A$$

$$I_2 = I_C = 3.11018 \angle 90^\circ A$$

$$\phi = 180 - 125.34 = 54.66^\circ$$

$$\therefore \text{Power factor} = \cos(\phi) = \cos(54.66^\circ)$$

$$\therefore \text{Power factor} = 0.5784$$

Comparison table between theoretical and simulated values:

Parameter	Theoretical value	Simulated values
I	$2.9335 \angle 54.6591^\circ A$	$2.93351 \angle 54.66^\circ A$
I_1	$1.84219 \angle -22.91^\circ A$	$1.84216 \angle -22.9128^\circ A$
I_2	$3.1101 \angle 90^\circ A$	$3.11018 \angle 90^\circ A$
Power factor	0.57844	0.5784

Table 3: Numerical 3

Numerical 4: A coil having a resistance of $R_1 = 3.5\Omega$ and an inductance of $L_1 = 0.025H$ is arranged in parallel with another coil having a resistance of $R_2 = 1\Omega$ and an inductance of $L_2 = 0.065H$. Calculate the current I , I_1 , I_2 when a voltage $V_1 = 110V$ at $60Hz$ is applied. Also calculate power factor of the circuit.

Solution:

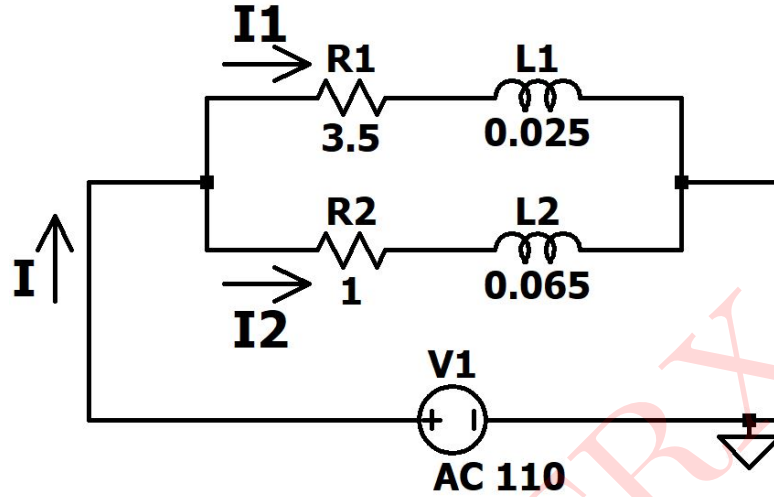


Figure 12: Circuit 4

$$\begin{aligned} \text{a] } X_{L_1} &= \omega L = 2 \times \pi 60 \times 0.025 = 9.4247\Omega \\ \therefore Z_1 &= R + j(X_L - X_C) = 3.5 + 9.4247j = 10.0536\angle 69.6267^\circ \Omega \end{aligned}$$

$$\begin{aligned} X_{L_2} &= \omega L = 2 \times \pi 60 \times 0.065 = 24.5044\Omega \\ \therefore Z_1 &= R + j(X_L - X_C) = 1 + 24.5044j = 24.5247\angle 87.6631^\circ \Omega \end{aligned}$$

$$\begin{aligned} Z &= \frac{Z_1 \times Z_2}{Z_1 + Z_2} \\ Z &= \frac{10.0536\angle 69.6267^\circ \times 24.5247\angle -87.6631^\circ}{10.0536\angle 69.6267^\circ + 24.5247\angle -87.6631^\circ} \\ Z &= 7.2039\angle -74.8448^\circ \Omega \end{aligned}$$

$$\begin{aligned} I &= \frac{V_1}{Z} = \frac{110}{7.2039\angle -74.8448^\circ} \\ I &= 15.2695\angle -74.8448^\circ A \end{aligned}$$

$$\therefore I_1 = \frac{V_1}{Z_1} = \frac{110}{7.2039\angle -74.8448^\circ} = 10.0536\angle 69.6267^\circ A$$

$$\therefore I_2 = \frac{V_1}{Z_2} = \frac{110}{24.5247\angle 87.6631^\circ} = 4.4852\angle -87.6631^\circ A$$

$$\begin{aligned} \text{c] } \phi &= 74.8448^\circ \\ \therefore \text{Power factor} &= \cos(\phi) = \cos(74.8448^\circ) \\ \therefore \text{Power factor} &= 0.2614(\text{leading}) \end{aligned}$$

SIMULATED RESULTS:

The above circuit is simulated in LTspice. The results are presented below.

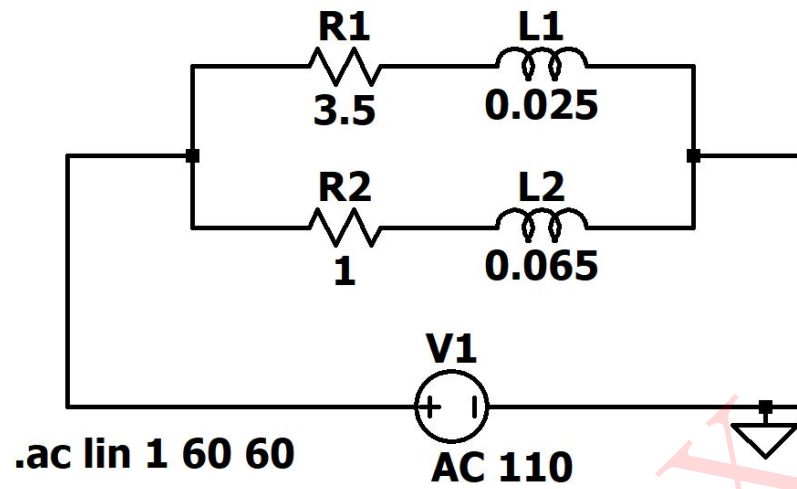


Figure 13: Circuit schematic

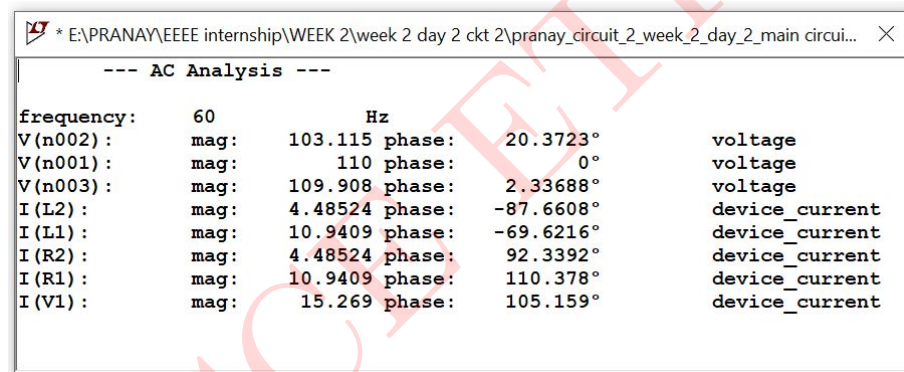


Figure 14: Simulated results

Here,

$$I = I_{V_1} = 15.2695 \angle -74.841^\circ \text{ A}$$

$$I_1 = I_{L_1} = 10.9409 \angle -69.6216^\circ \text{ A}$$

$$I_2 = I_{L_2} = 4.48524 \angle -87.6608^\circ \text{ A}$$

$$\phi = 180 - 105.159 = 74.84^\circ$$

$$\therefore \text{Power factor} = \cos(\phi) = \cos(74.84^\circ)$$

$$\therefore \text{Power factor} = 0.2615$$

Comparison table between theoretical and simulated values:

Parameter	Theoretical value	Simulated values
I	$15.2695\angle -74.8448^\circ \text{ A}$	$15.269\angle -75.841^\circ \text{ A}$
I_1	$10.9413\angle -69.6267^\circ \text{ A}$	$10.9409\angle -69.6216^\circ \text{ A}$
I_2	$4.4852\angle -87.6631^\circ \text{ A}$	$4.48524\angle -87.6608^\circ \text{ A}$
Power factor	0.2614	0.2615

Table 4: Numerical 4

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Numerical 5: Find I , I_1 , I_2 and voltage drop in each branch in the following figure, If $R_1 = 15\Omega$, $L_1 = j10\Omega$ and $R_1 = 15\Omega$, $L_1 = j10\Omega$ and $R_1 = 15\Omega$, $L_1 = j10\Omega$ with supply $V_1 = 100V$, $50Hz$.

Solution:

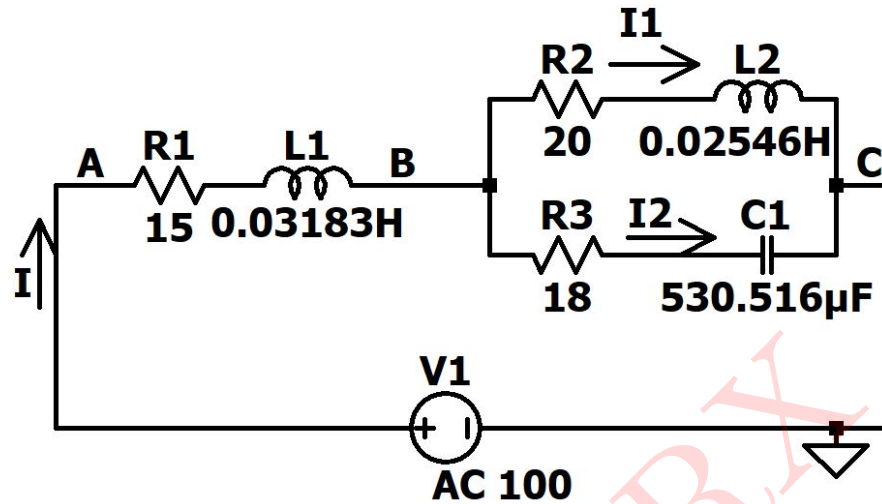


Figure 15: Circuit 5

a] $Z_1 = 15 + 10j$

$X_{L1} = 10\Omega$

But,

$$L_1 = \frac{X_{L1}}{2\pi f}$$

$$\therefore L_1 = \frac{10}{2 \times \pi \times 50} = 0.03183H$$

$Z_2 = 20 + 8j$

$X_{L2} = 8\Omega$

But,

$$L_2 = \frac{X_{L2}}{2\pi f}$$

$$\therefore L_2 = \frac{8}{2 \times \pi \times 50} = 0.02546H$$

$Z_3 = 18 - 6j$

$X_{C1} = 6\Omega$

But,

$$C_1 = \frac{1}{X_{C1} \times 2\pi f}$$

$$\therefore C_1 = \frac{1}{6 \times 2 \times \pi \times 50} = 530.516\mu F$$

$$\begin{aligned} \text{b) } Z &= Z_1 + \frac{Z_2 \times Z_3}{Z_2 + Z_3} \\ Z &= 15 + 10j + \frac{(20 + 8j)(18 - 6j)}{(20 + 8j) + (18 - 6j)} \\ Z &= 27.6386 \angle 21.3590^\circ \Omega \end{aligned}$$

$$\begin{aligned} I &= \frac{V_1}{Z} \\ I &= \frac{100}{27.6386 \angle 21.359^\circ} \\ I &= 3.6181 \angle -21.359^\circ A \end{aligned}$$

$$\begin{aligned} I_1 &= \frac{I \times Z_3}{Z_2 + Z_3} \\ I_1 &= \frac{(3.369 - 1.3177j) \times (18 - 6j)}{18 - 6j + 20 + 8j} \\ I_1 &= 1.3235 - 1.2258j A \end{aligned}$$

$$I_1 = 1.8040 \angle -42.8067^\circ A$$

$$\begin{aligned} I_2 &= \frac{I \times Z_2}{Z_2 + Z_3} \\ I_2 &= \frac{(3.369 - 1.3177j) \times (20 + 8j)}{18 - 6j + 20 + 8j} \\ I_2 &= 2.0461 - 0.0918j A \end{aligned}$$

$$I_2 = 2.048119 \angle -2.5703^\circ A$$

$$\begin{aligned} \text{c) } V_{AB} &= I \times Z_{AB} = (3.369 - 1.3177j) \times (15 + 10j) \\ V_{AB} &= 65.2262 \angle -21.359^\circ V \end{aligned}$$

$$V_{AB} = 65.2262 \angle -21.359^\circ V$$

$$\begin{aligned} Z_{BC} &= (20 + 8j) \parallel (18 - 6j) \\ Z_{BC} &= \frac{(20 + 8j)(18 - 6j)}{(20 + 8j) + (18 - 6j)} \\ Z_{BC} &= 10.7402 + 0.0662j \Omega \\ V_{BC} &= I_2 \times Z_{BC} = (3.369 - 1.3177j) \times (10.7402 + 0.0662j) \\ V_{BC} &= 36.2709 - 13.9293j V \\ V_{BC} &= 38.8536 \angle -21.0085^\circ V \end{aligned}$$

SIMULATED RESULTS:

The above circuit is simulated in LTspice. The results are presented below.

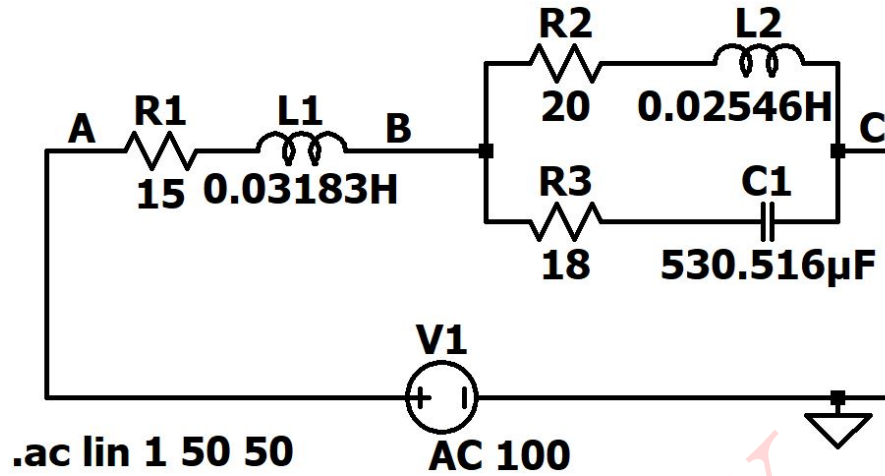


Figure 16: Circuit schematic

--- AC Analysis ---					
frequency:	50	Hz			
V(n001):	mag:	14.4291	phase:	47.1906°	voltage
V(b):	mag:	38.8595	phase:	-21.0056°	voltage
V(n002):	mag:	53.259	phase:	21.7831°	voltage
V(n003):	mag:	12.2885	phase:	-92.5706°	voltage
V(a):	mag:	100	phase:	0°	voltage
I(C1):	mag:	2.04807	phase:	177.429°	device_current
I(L2):	mag:	1.80397	phase:	-42.8023°	device_current
I(L1):	mag:	3.61804	phase:	-21.3569°	device_current
I(R1):	mag:	3.61804	phase:	158.643°	device_current
I(R3):	mag:	2.04807	phase:	-2.5706°	device_current
I(R2):	mag:	1.80397	phase:	137.198°	device_current
I(V1):	mag:	3.61804	phase:	158.643°	device_current

Figure 17: Simulated results

Here,

$$I = I_{V_1} = 3.6180 \angle -21.357^\circ A$$

$$I_1 = I_{L_1} = 1.80397 \angle -42.8023^\circ A$$

$$I_2 = I_{L_2} = 2.0481 \angle -2.571^\circ A$$

$$V_A = 100V$$

$$V_B = 38.8595 \angle -21.0056^\circ V$$

$$V_C = 0V$$

$$V_{AB} = 100 - 38.8595 \angle -21.0056^\circ$$

$$V_{AB} = 65.2275 \angle 12.3306^\circ V$$

$$V_{BC} = V_B - V_C$$

$$V_{BC} = 38.8595 \angle -21.0056^\circ V$$

Comparison table between theoretical and simulated values:

Parameter	Theoretical value	Simulated values
I	$3.6181\angle -21.359^\circ \text{A}$	$3.6180\angle -21.357^\circ \text{A}$
I_1	$1.8040\angle -42.8067^\circ \text{A}$	$1.80397\angle -42.8023^\circ \text{A}$
I_2	$2.0481\angle -2.5703^\circ \text{A}$	$2.0481\angle -2.571^\circ \text{A}$
V_{AB}	$65.2262\angle 12.3310^\circ \text{V}$	$65.2275\angle 12.3301^\circ \text{V}$
V_{BC}	$38.8593\angle -21.0053^\circ \text{V}$	$38.8595\angle -21.0056^\circ \text{V}$

Table 5: Numerical 5

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Numerical 6: A 50 Hz sinusoidal voltage $V_1 = 141\sin\omega t$ is applied to a series RL circuit. The values of the resistance and the inductor are 4.2Ω and 0.017H respectively.

Determine the following:

- Calculate peak voltage across resistor and inductor and also find the peak value of source current in LTspice.
- Plot input source voltage $V_S(t)$ input source current $I_S(t)$ in LT spice.
- Measure phase difference between $V_S(t)$ Vs $I_S(t)$.
- Plot input voltage source $V_S(t)$ voltage across resistor $V_R(t)$ in LT spice.
- Measure the phase difference between $V_S(t)$ and $V_R(t)$.
- Plot input voltage source $V_S(t)$ voltage across inductor $V_L(t)$ in LT spice.
- Measure the phase difference between $V_S(t)$ and $V_L(t)$ in time and degrees.
- Calculate power factor of the circuit.

Solution:

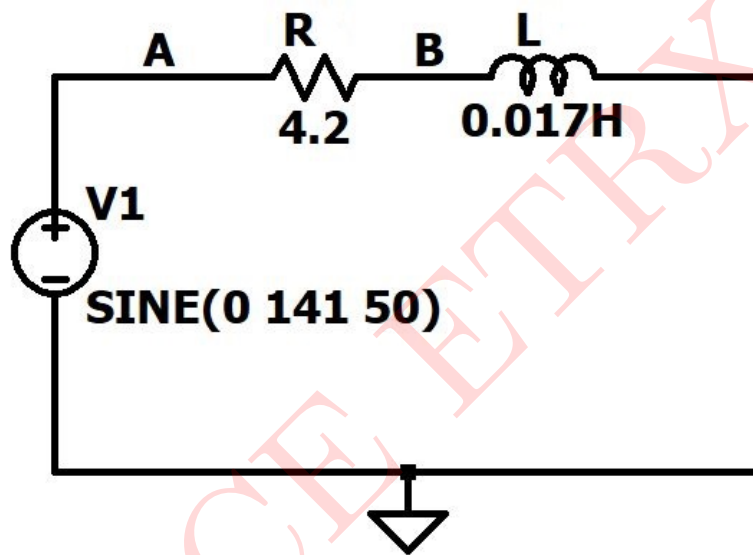


Figure 18: Circuit 6

$$\text{a) } V_1 = V_p = 141\text{V}$$

$$V_{rms} = \frac{141}{\sqrt{2}}$$

$$\therefore V_{rms} = 99.702\text{V}$$

$$\text{b) } X_L = 2 \times \pi \times 50 \times 0.017$$

$$\therefore X_L = 5.340\Omega$$

$$Z = 4.2 + 5.340j$$

$$\therefore Z = 6.7938\angle 51.814^\circ\Omega$$

$$\text{c) } I = \frac{V_{rms}}{Z} = \frac{99.702}{6.7938\angle 51.814^\circ}$$

$$\therefore I = 14.6754\angle -51.814^\circ\text{A}$$

$$I_p = 14.6754\angle -51.814^\circ \times \sqrt{2}$$

$$\therefore I_p = 20.7541 \angle -51.814^\circ \text{ A}$$

$$d) V_{R_p} = I_p \times R = 20.7541 \angle -51.814^\circ \times 4.2$$

$$\therefore V_{R_p} = 87.1674 \angle -51.814^\circ \text{ V}$$

$$V_{L_p} = I_p \times X_L = 20.7541 \angle -51.814^\circ \times 5.340$$

$$\therefore V_{L_p} = 110.8268 \angle -51.814^\circ \text{ V}$$

$$\phi = 51.814^\circ \text{ Power factor} = \cos(\phi) = \cos(51.814^\circ)$$

$$\text{Power factor} = 0.6182$$

SIMULATED RESULTS:

The above circuit is simulated in LTspice. The results are presented below.

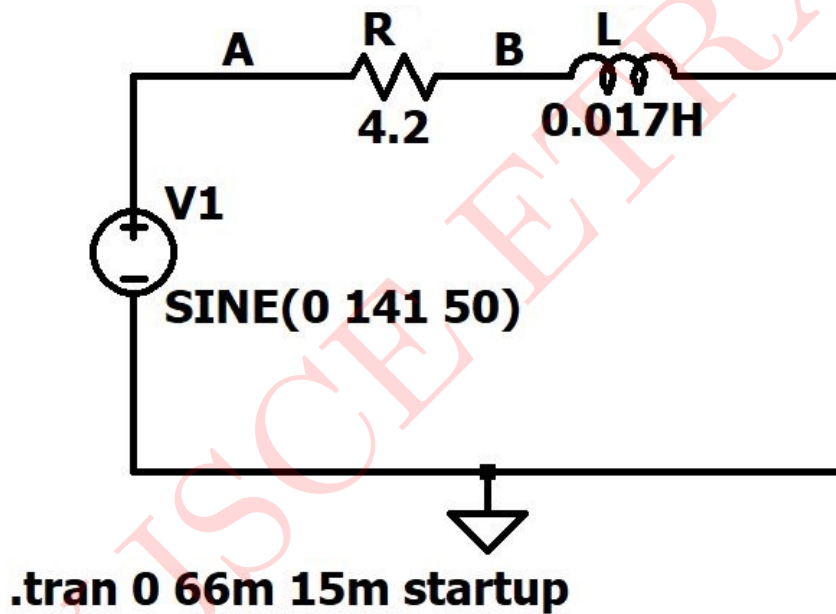


Figure 19: Circuit schematic

a) For peak voltage across resistor and inductor.

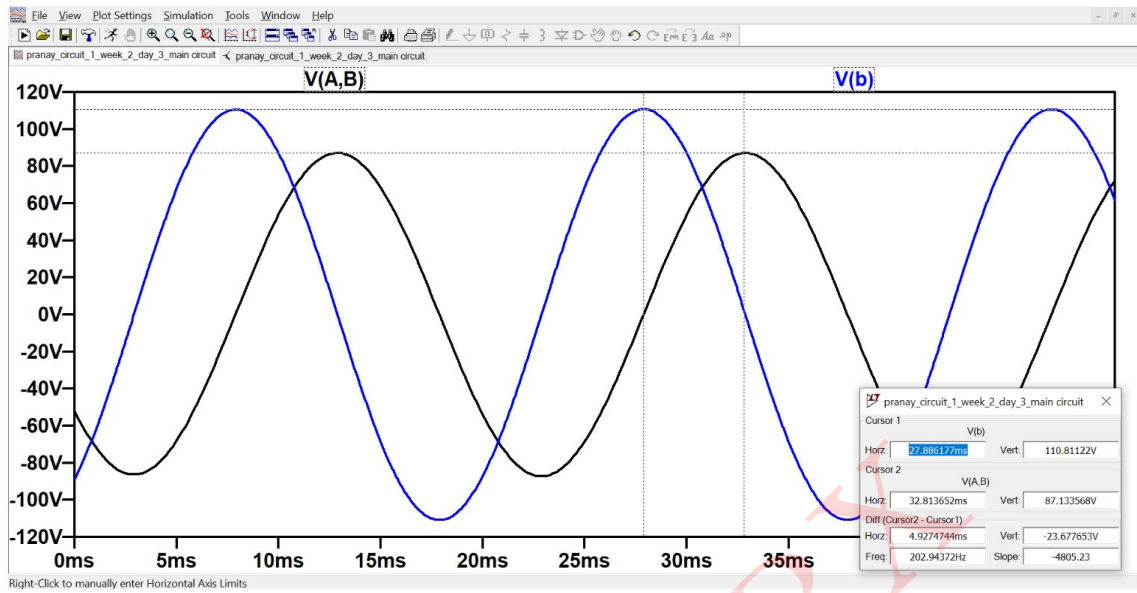


Figure 20: Graph for V_{R_p} vs V_{L_p}

From graph,

$$V_{R_p} = 87.133V$$

$$V_{L_p} = 110.8112V$$

b) For input source voltage $V_S(t)$ vs input source current $I_S(t)$.

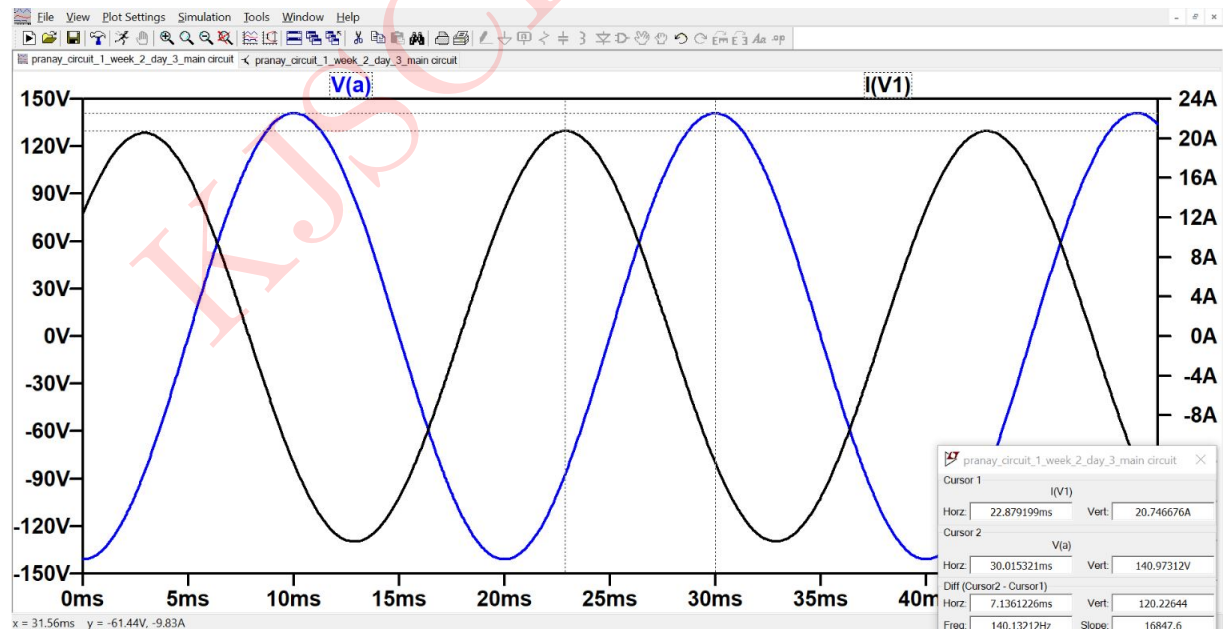


Figure 21: Graph for $V_S(t)$ vs $I_S(t)$

$$\Delta t = 7.0748ms$$

$$\phi = \frac{\Delta t}{T} \times 360$$

$$\phi = \frac{7.0748}{20} \times 360 = 127.396^\circ$$

$$\phi = 180 - 127.346 = 52.6536^\circ$$

c) For input source voltage $V_S(t)$ vs voltage across resistor $V_R(t)$.

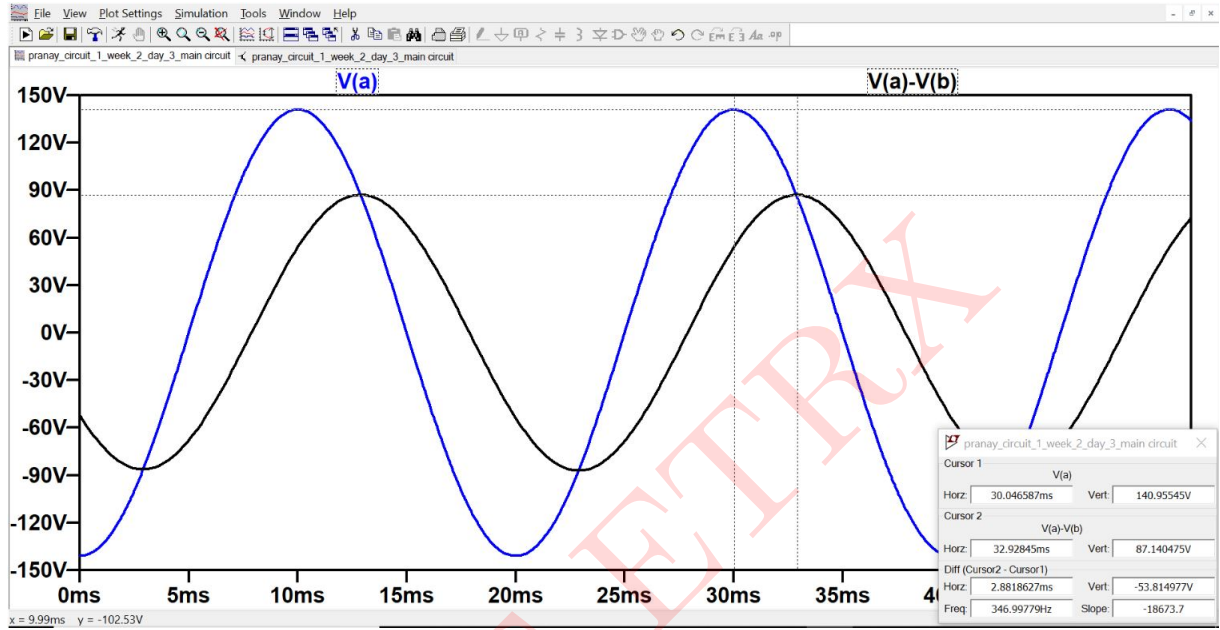


Figure 22: Graph for $V_S(t)$ vs $V_R(t)$

$$\Delta t = 2.8767ms$$

$$\phi = \frac{\Delta t}{T} \times 360$$

$$\phi = \frac{2.8767}{20} \times 360$$

$$\phi = 51.6186^\circ$$

d) For input source voltage $V_S(t)$ vs voltage across inductor $V_L(t)$.

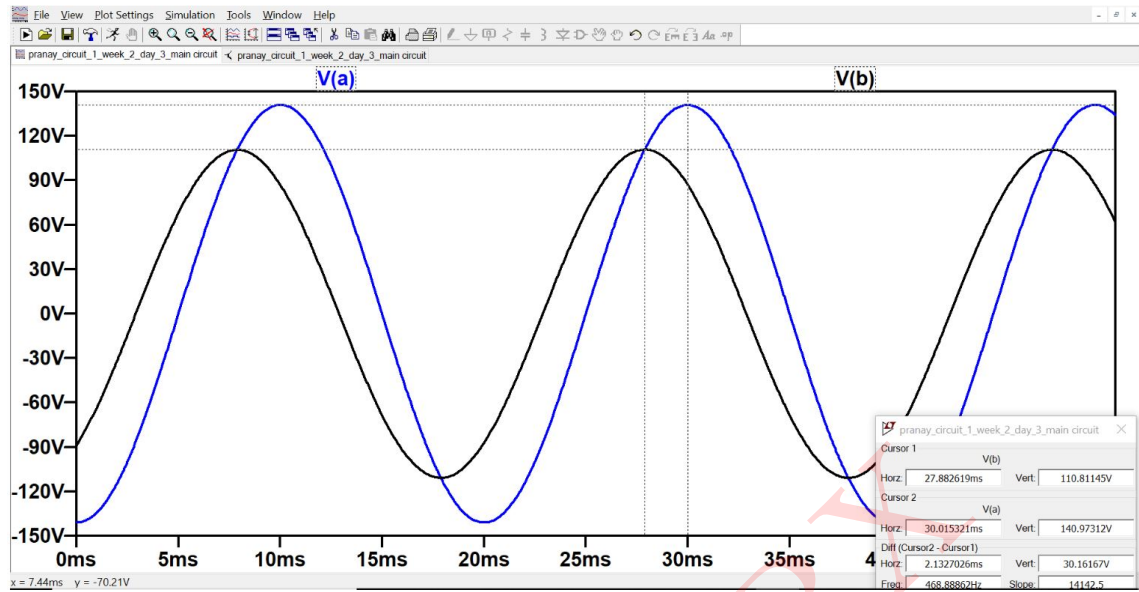


Figure 23: Graph for $V_S(t)$ vs $V_L(t)$

$$\Delta t = 2.1226ms$$

$$\phi = \frac{\Delta t}{T} \times 360$$

$$\phi = \frac{2.1226}{20} \times 360 = 38.196^\circ$$

$$\phi = 90 - 38.196 = 51.804^\circ$$

$$\phi = 51.804^\circ \text{ Power factor} = \cos(\phi) = \cos(51.804^\circ)$$

$$\text{Power factor} = 0.6183$$

Comparison table between theoretical and simulated values:

Parameter	Theoretical value	Simulated values
V_{R_p}	$87.1674\angle -51.814^\circ \text{A}$	$87.1357\angle -51.814^\circ \text{A}$
V_{L_p}	$110.8268\angle -51.814^\circ \text{A}$	$110.81\angle -51.804^\circ \text{A}$
I_{S_p}	20.754A	20.7466A
Phase difference between $V_S(t)$ and $I_S(t)$ in time and degrees	7.0748ms and -52.6536°	-51.814°
Phase difference between $V_S(t)$ and $V_R(t)$ in time and degrees	2.8768ms and -51.6186°	51.814°
Phase difference between $V_S(t)$ and $V_L(t)$ in time and degrees	2.1226ms and -51.804°	51.814°
Power factor	0.6182	0.6183

Table 6: Numerical 6

Numerical 7: A pure resistance of 55Ω is in series with a pure capacitor of $110\mu\text{F}$. The series combination is connected across 110V , 60Hz supply.

Determine the following:

- Calculate peak voltage across resistor and capacitor and also find the peak value of source current in LTspice.
- Plot input source voltage $V_S(t)$ input source current $I_S(t)$ in LT spice.
- Measure phase difference between $V_S(t)$ Vs $I_S(t)$.
- Plot input voltage source $V_S(t)$ voltage across resistor $V_R(t)$ in LT spice.
- Measure the phase difference between $V_S(t)$ and $V_R(t)$.
- Plot input voltage source $V_S(t)$ voltage across capacitor $V_C(t)$ in LT spice.
- Measure the phase difference between $V_S(t)$ and $V_C(t)$ in time and degrees.
- Calculate power factor of the circuit.

Solution:

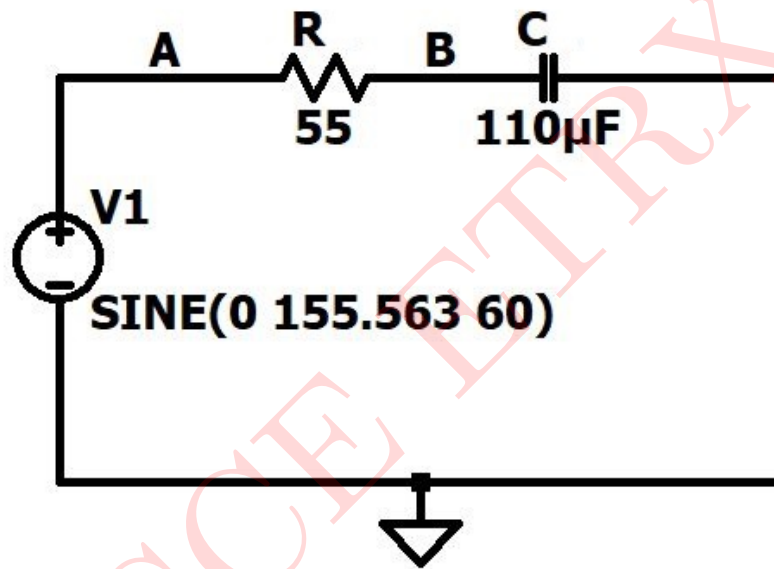


Figure 24: Circuit 7

$$\text{a) } X_C = \frac{1}{\omega C} = \frac{1}{2 \times \pi \times 60 \times 110 \times 10^{-6}}$$

$$\therefore X_C = 24.114\Omega$$

$$\text{b) } Z = R + (X_L - X_C)j$$

$$\therefore Z = 55 + 24.114j\Omega$$

$$\therefore Z = 60.054\angle 23.674^\circ$$

$$\text{c) } I = \frac{V_1}{Z} = \frac{110}{60.054\angle -23.674}$$

$$\therefore I = 1.8316\angle -23.674^\circ \text{ A}$$

$$\therefore I_p = 1.8316\angle -23.674^\circ \times \sqrt{2}$$

$$\therefore I_p = 2.5903\angle -23.674^\circ \text{ A}$$

$$d) V_{R_p} = I_p \times R = 2.5903 \angle -23.674^\circ \times 55$$

$$\therefore V_{R_p} = 142.44665 \angle -23.674^\circ V$$

$$V_{C_p} = I_p \times X_C = 2.5903 \angle -23.674^\circ \times 24.114$$

$$\therefore V_{C_p} = 62.4625 \angle -23.674^\circ V$$

$$\phi = 23.674$$

$$\text{Power factor} = \cos(\phi) = \cos(23.674^\circ)$$

$$\text{Power factor} = 0.91584$$

SIMULATED RESULTS:

The above circuit is simulated in LTspice. The results are presented below.

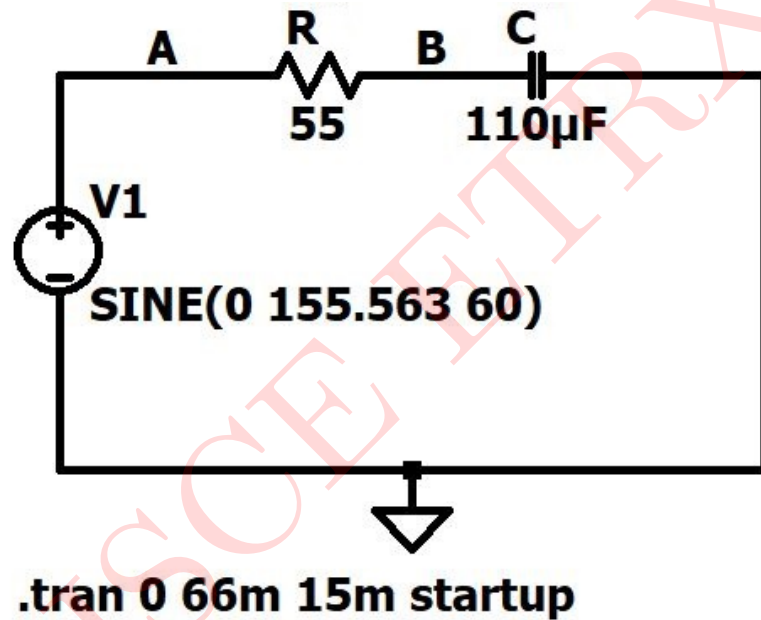


Figure 25: Circuit schematic

a) For peak voltage across resistor and inductor.

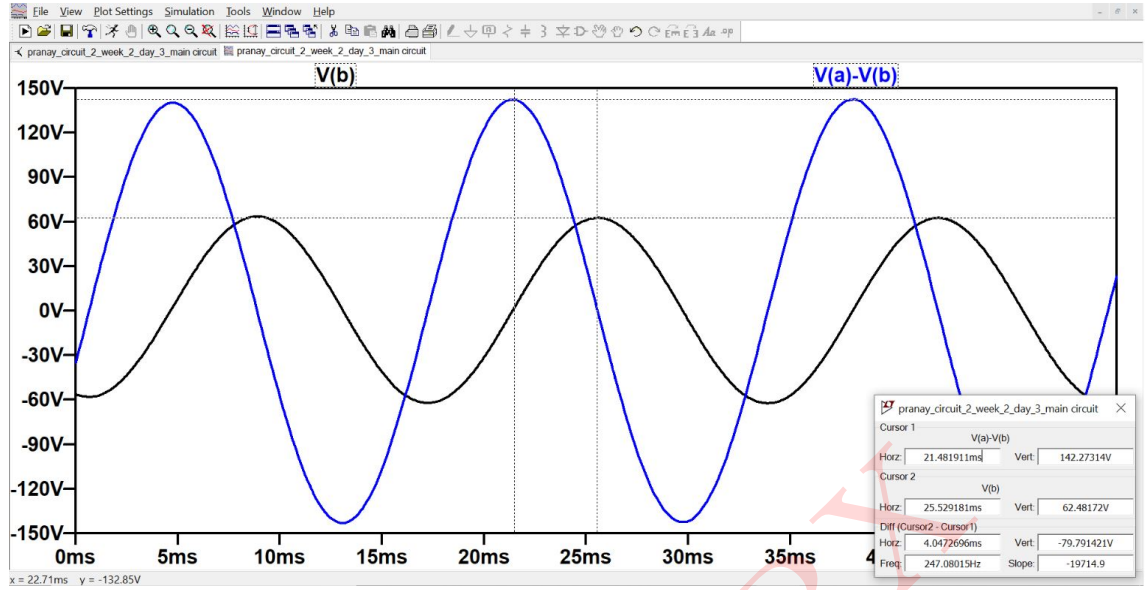


Figure 26: Graph for V_{R_p} vs V_{C_p}

From graph,

$$V_{R_p} = 142.2731V$$

$$V_{C_p} = 62.4806V$$

b) For input source voltage $V_S(t)$ vs input source current $I_S(t)$.

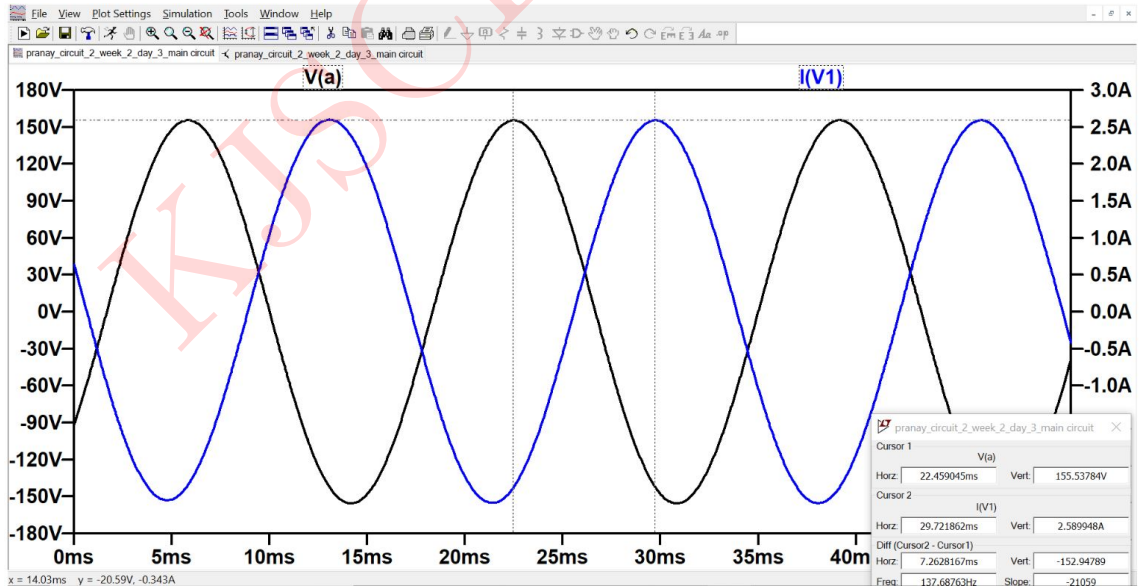


Figure 27: Graph for $V_S(t)$ vs $I_S(t)$

$$\Delta t = 7.2285ms$$

$$\phi = \frac{\Delta t}{T} \times 360$$

$$\phi = \frac{7.2285}{16.67} \times 360 = 156.1356^\circ$$

$$\phi = 180 - 156.1356 = 23.8644^\circ$$

c) For input source voltage $V_S(t)$ vs voltage across resistor $V_R(t)$.

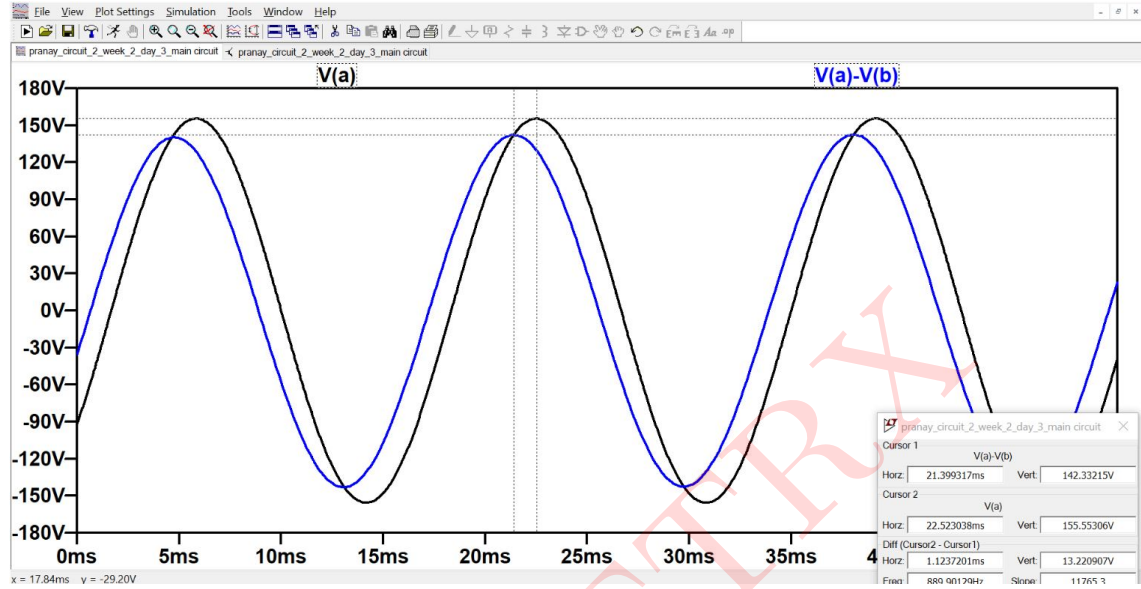


Figure 28: Graph for $V_S(t)$ vs $V_R(t)$

$$\Delta t = 1.0892ms$$

$$\phi = \frac{\Delta t}{T} \times 360$$

$$\phi = \frac{1.0892}{16.67} \times 360$$

$$\phi = 23.526^\circ$$

d) For input source voltage $V_S(t)$ vs voltage across capacitor $V_C(t)$.

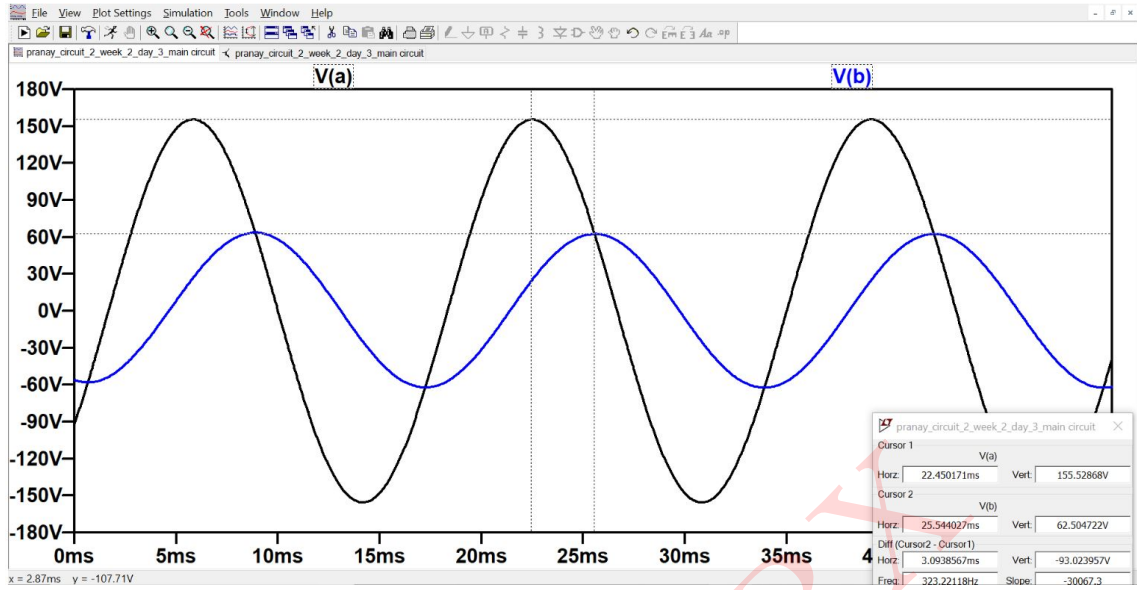


Figure 29: Graph for $V_S(t)$ vs $V_C(t)$

$$\Delta t = 3.01642ms$$

$$\phi = \frac{\Delta t}{T} \times 360$$

$$\phi = \frac{3.01642}{16.67} \times 360 = 65.1546^\circ$$

$$\phi = 90 - 65.1546 = 24.8453^\circ$$

$$\phi = 23.8644^\circ \text{ Power factor} = \cos(\phi) = \cos(23.8644^\circ)$$

$$\text{Power factor} = 0.9145$$

Comparison table between theoretical and simulated values:

Parameter	Theoretical value	Simulated values
V_{R_p}	$142.4665\angle -23.674^\circ\text{V}$	$142.2789\angle -23.8644^\circ\text{V}$
V_{L_p}	$62.4625\angle -23.674^\circ\text{V}$	$62.4806\angle -23.8644^\circ\text{V}$
I_{S_p}	2.5903A	2.5989A
Phase difference between $V_S(t)$ and $I_S(t)$ in time and degrees	77.2285ms and -23.8644°	-23.674°
Phase difference between $V_S(t)$ and $V_R(t)$ in time and degrees	1.0892ms and -23.526°	-23.674°
Phase difference between $V_S(t)$ and $V_C(t)$ in time and degrees	3.01642ms and -24.8453°	-23.674°
Power factor	0.91584	0.9145

Table 7: Numerical 7

Numerical 8: A series resonance network consisting of a resistor 27Ω , capacitor of $2.5\mu\text{F}$ is connected across a sinusoidal supply voltage which has a constant output of AC 9 volts at all frequencies. Calculate the resonant frequency, the current at resonance, the voltage across the inductor and capacitor at resonance, the quality factor and bandwidth of the circuit. Plot the resonance curve, the voltage across the inductor and capacitor at resonance in LTspice.

Solution:

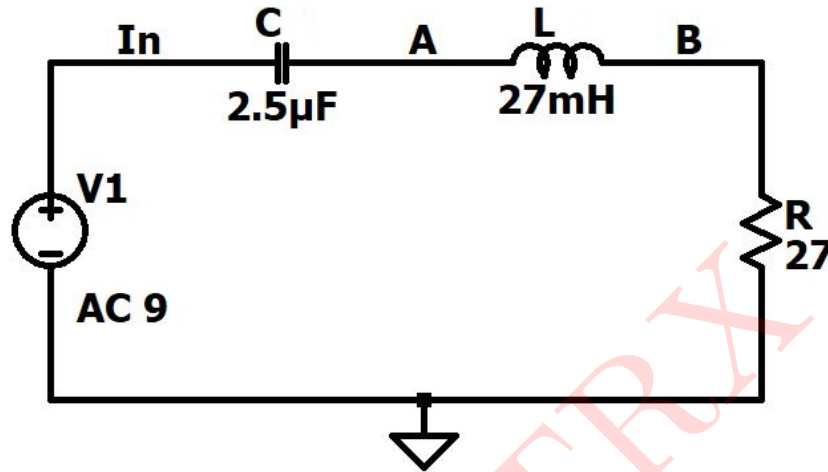


Figure 30: Circuit 8

$$\text{a) Resonance Frequency} = \frac{1}{2\pi\sqrt{LC}}$$

$$\therefore \text{Resonance Frequency} = \frac{1}{2\pi\sqrt{27 \times 10^{-3} \times 2.5 \times 10^{-6}}}$$

$$\therefore \text{Resonance Frequency} = \mathbf{612.587\text{Hz}}$$

$$\text{b) } I_{rms} = \frac{V_1}{Z} = \frac{9}{27}$$

$$\therefore I_{rms} = 0.3333\text{A}$$

$$\therefore I_p = I_{rms} \times \sqrt{2} = 0.3333 \times \sqrt{2}$$

$$\therefore I_p = \mathbf{471.404\text{mA}}$$

c) At resonance,

$$X_L = X_C = \omega L = 103.9229\Omega$$

$$\therefore V_{L_p} = V_{C_p} = I_p \times X_L$$

$$\therefore V_{L_p} = V_{C_p} = 471.404 \times 10^{-3} \times 103.9229$$

$$\therefore V_{L_p} = V_{C_p} = \mathbf{48.989\text{V}}$$

$$\text{d) Quality factor} = \frac{X_L/X_C}{R} = \frac{103.9229}{27}$$

$$\therefore \text{Quality factor} = \mathbf{3.848}$$

$$e) \text{ Bandwidth} = \frac{R}{2\pi\sqrt{LC} \times X_L}$$

$$\therefore \text{ Bandwidth} = \frac{27}{2\pi\sqrt{27 \times 10^{-3} \times 2.5 \times 10^{-6} \times 103.9229}}$$

$$\text{Bandwidth} = 159.1962\text{Hz}$$

SIMULATED RESULTS:

The above circuit is simulated in LTspice. The results are presented below.

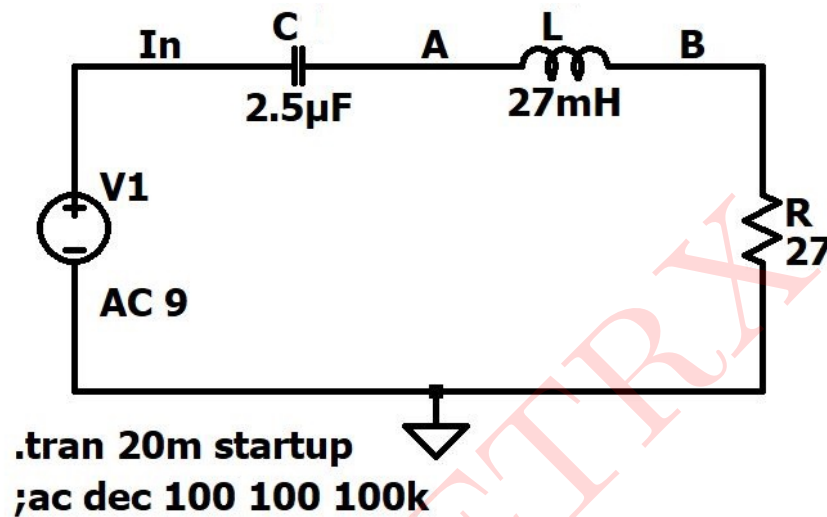


Figure 31: Circuit schematic

a) Resonant frequency curve.

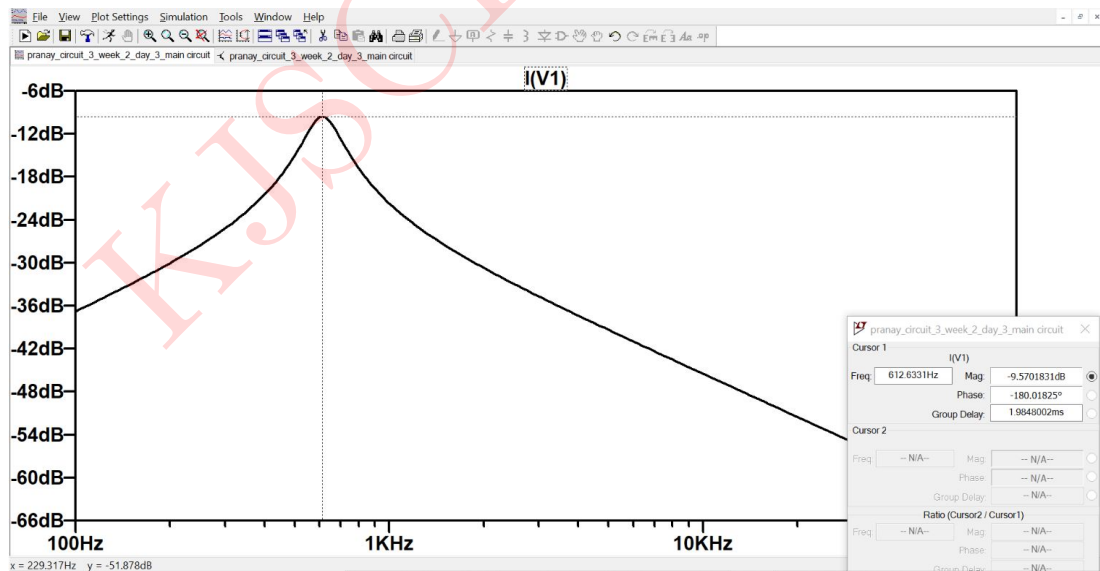


Figure 32: Resonance curve

From graph,

Resonance frequency = 612.6767Hz

b) Graph for current at resonance.

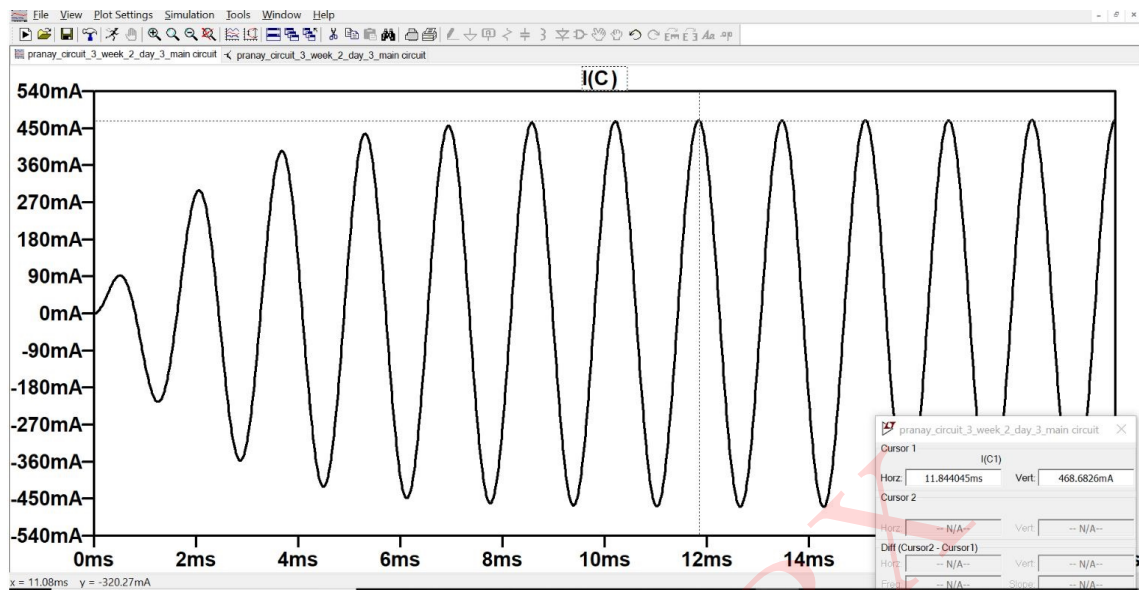


Figure 33: For current at resonance

From graph,

$$I_{V_1} = 468.683mA$$

c) Graph for voltage across inductor and capacitor at resonance.

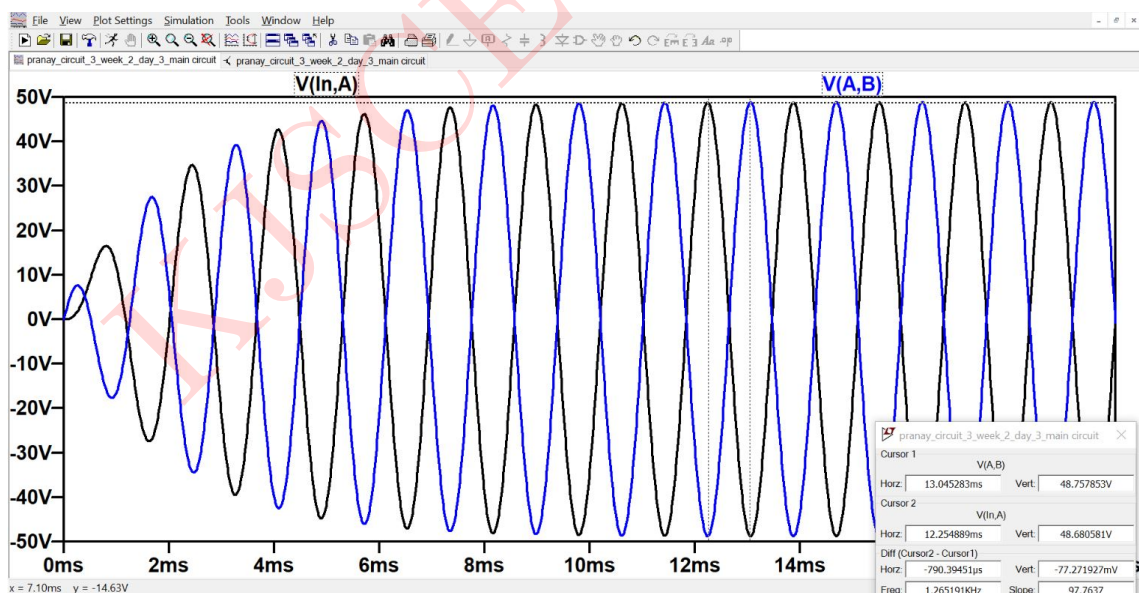


Figure 34: For voltage at resonance

From graph,

$$V_L = 48.7540V$$

$$V_C = 48.7683V$$

Comparison table between theoretical and simulated values:

Parameter	Theoretical value	Simulated values
Resonance frequency	612.587Hz	612.6767Hz
I_p	471.404mA	469.926mA
$V_L = V_C$	48.989V	48.7682V
Bandwidht	159.1962	159.2195

Table 8: Numerical 8

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