K. J. SOMAIYA COLLEGE OF ENGINEERING
DEPARTMENT OF ELECTRONICS ENGINEERING
ELECTRONIC CIRCUITS
AC CIRCUITS

Numerical 1: A series RLC circuit containing a resistance of 50Ω, an inductance of 0.2H and a capacitor of 120μF are connected in series across a 220V, 50Hz supply. Calculate:-
a) the current drawn by the circuit
b) V_R, V_L and V_C
c) Power factor
d) Draw the phasor digram

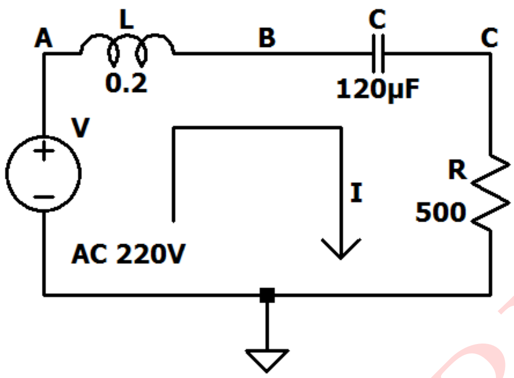


Figure 1: Circuit 1

Solution:

a) To calculate the current I we will first calculate the impedence Z

We have R = 50Ω, L = 0.2H, C = 120μF, V = 220 ∠ 0°V and f = 50Hz

$$\begin{aligned} X_L &= 2\pi f L \\ &= 2 \times \pi \times 50 \times 0.2 \\ &= 62.8318\Omega \end{aligned}$$

$$\begin{aligned} X_C &= \frac{1}{2\pi f C} \\ &= \frac{1}{2 \times \pi \times 50 \times 120 \times 10^{-6}} \\ &= 26.5258\Omega \end{aligned}$$

$$\begin{aligned} \therefore Z &= R + j(X_L - X_C) \\ &= 50 + j(62.8318 - 26.5258) \\ &= 50 + j36.306 \\ &= 61.791 \angle 35.9842^\circ \ \Omega \end{aligned}$$

$$\begin{aligned} \therefore I &= \frac{V}{Z} \\ &= \frac{220\angle 0^\circ}{61.791\angle 35.9842^\circ} \\ &= 3.5604 \angle -35.9842^\circ A \end{aligned}$$

$$\therefore \mathbf{I = 3.5604 \angle - 35.9842^\circ \ A}$$

∴The current is 3.5604A lagging by 35.9842° w.r.t voltage
b) Finding V_R, V_L and V_C

V_R is the voltage across the resistor.

$\therefore V_R = I \times R$

$= 3.5604 \angle -35.9842^\circ \times 50$

$= 178.019 \angle -35.9842^\circ V$

V_L is the voltage across the inductance.

$\therefore V_L = I \times X_L$

$= 3.5604 \angle -35.9842^\circ \times 62.8318 \angle 90^\circ$

$= 223.706 \angle 54.016^\circ V$

V_C is the voltage across the capacitor.

$\therefore V_C = I \times X_C$

$= 3.5604 \angle -35.9842^\circ \times 26.5258 \angle -90^\circ$

$= 94.4422 \angle -125.9842^\circ V$

$\therefore \mathbf{V_R = 178.019 \angle -35.9842^\circ\ V}$

$\therefore \mathbf{V_L = 223.706 \angle 54.016^\circ\ V}$

$\therefore \mathbf{V_C = 94.4422 \angle -125.9842^\circ\ V}$

c) Power Factor ($\cos\phi$)

$\phi = 35.9842^\circ$

$\therefore \cos\phi = \cos 35.9842 = 0.8092$ (lagging)

$\therefore$ **P.F = 0.8092**

d) Phasor Diagram

The phasor diagram for the circuit is as given figure 2:

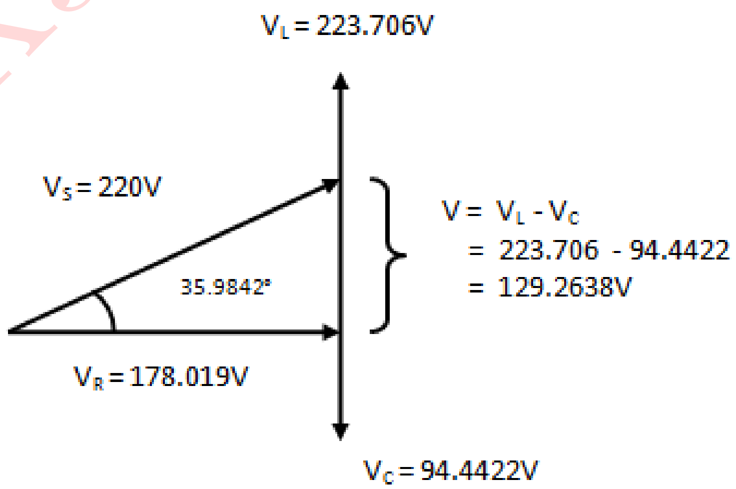


Figure 2: Phasor Diagram

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

I. Simulation of circuit to find V_R, I and p.f

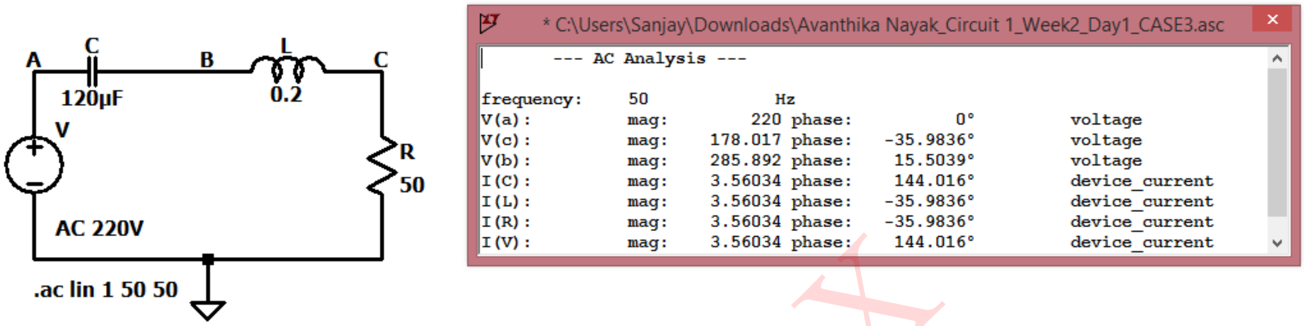


Figure 3: Circuit Schematic and Simulated Results for V_R, I and p.f

$V_R = 178.017 \angle -35.9836^{\circ}$V

$I(R) = 3.56034 \angle -35.9836^{\circ}$A

$P.F = \cos(-35.9386) = 0.80919$

II. Simulation of circuit to find V_L

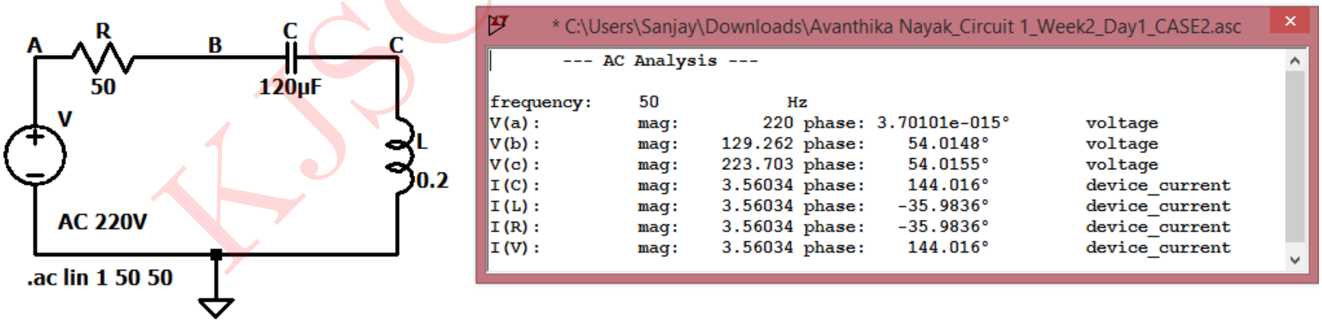
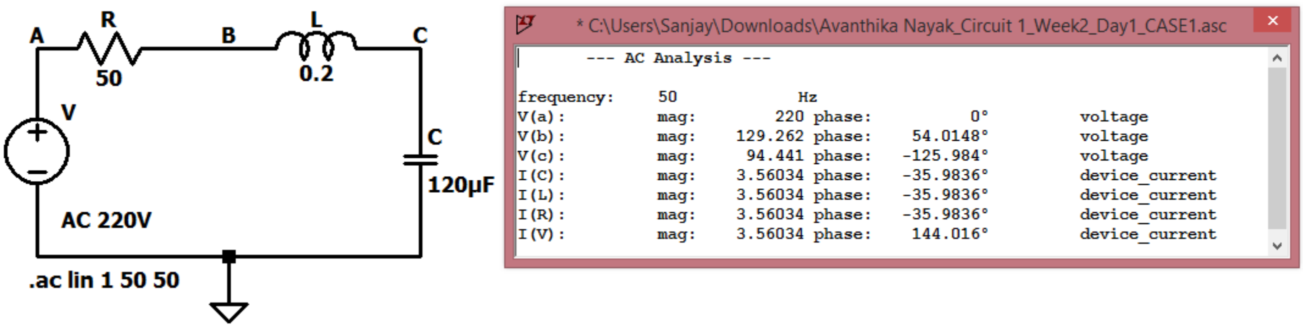


Figure 4: Circuit Schematic and Simulated Results for V_L

$V_L = 223.703 \angle 54.0155^{\circ}$V

III. Simulation of circuit to find V_C

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        --- AC Analysis ---

frequency:    50            Hz
V(a):         mag:       220 phase:          0°       voltage
V(b):         mag:   129.262 phase:    54.0148°       voltage
V(c):         mag:    94.441 phase:   -125.984°       voltage
I(C):         mag:   3.56034 phase:   -35.9836°       device_current
I(L):         mag:   3.56034 phase:   -35.9836°       device_current
I(R):         mag:   3.56034 phase:   -35.9836°       device_current
I(V):         mag:   3.56034 phase:    144.016°       device_current
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Figure 5: Circuit Schematic and Simulated Results for V_C

$V_C = 94.441 \angle -125.984^{\circ} V$

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
I	$3.5604 \angle -35.9842^{\circ}A$	$3.56034 \angle -35.9836^{\circ}A$
V_R	$178.019 \angle -35.9842^{\circ}V$	$178.017 \angle -35.9836^{\circ}V$
V_L	$223.706 \angle 54.016^{\circ}V$	$223.703 \angle 54.0155^{\circ}V$
V_C	$94.4422 \angle -125.9842^{\circ}V$	$94.441 \angle -125.984^{\circ}V$
P.F ($\cos\phi$)	0.8092	0.80919

Table 1: Numerical 1

Numerical 2: A 60 Hz sinusoidal voltage V = 141 sinωt is applied to a series R-L circuit. The values of the resistance and the inductance are 2Ω and 0.03 H respectively.
Calculate:
a) The RMS value of the current in the circuit and its phase angle w.r.t to the voltage.
b) The RMS value and the phase of the voltages appearing across the resistance and the inductance.
c) Power factor of the circuit

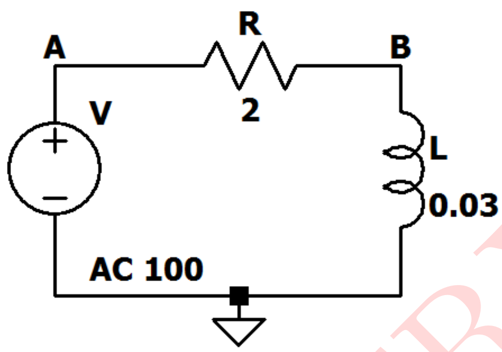


Figure 6: Circuit 2

Solution:

a) To calculate the current I we will first calculate the impedence Z.

We have R = 2Ω, L = 0.03H, V = 141 sinωt and f = 60Hz

$$\begin{aligned} X_L &= 2\pi f L \\ &= 2 \times \pi \times 60 \times 0.03 \\ &= 11.3097\Omega \end{aligned}$$

$$\begin{aligned} V_{RMS} &= \frac{V_{peak}}{\sqrt{2}} \\ &= \frac{141}{\sqrt{2}} \\ &= 100V \end{aligned}$$

$$\begin{aligned} \therefore Z &= R + jX_L \\ &= 2 + j11.3097 \\ &= 11.4852 \angle 79.9715^\circ \ \Omega \end{aligned}$$

$$\begin{aligned} \therefore I &= \frac{V}{Z} \\ &= \frac{100\angle 0^\circ}{11.4852\angle 79.9715^\circ} \\ &= 8.7069 \angle -79.9715^\circ A \end{aligned}$$

$\therefore$ $\mathbf{I = 8.7069 \angle - 79.9715^\circ\ A}$

$\therefore$ The current is 8.7069A lagging by 79.9715° w.r.t voltage

b) Finding V_R and V_L

V_R is the voltage across the resistor.

$\therefore V_R = I \times R$

$= 8.7069 \angle -79.9715^{\circ} \times 2$

$= 17.4137 \angle -79.9715^{\circ} V$

V_L is the voltage across the inductance.

$\therefore V_L = I \times X_L$

$= 8.7069 \angle -79.9715^{\circ} \times 11.3097 \angle 90^{\circ}$

$= 98.4724 \angle 10.0285^{\circ} V$

$\therefore \mathbf{V_R = 17.4137 \angle -79.9715^{\circ}\ V}$

$\therefore \mathbf{V_L = 98.4724 \angle 10.0285^{\circ}\ V}$

c) Power Factor ($\cos\phi$)

$\phi = 79.9715^{\circ}$

$\therefore \cos\phi = \cos 79.9715 = 0.17414$ (lagging)

$\therefore \mathbf{P.F = 0.17414}$

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

I. Simulation of circuit to find V_R, I and p.f

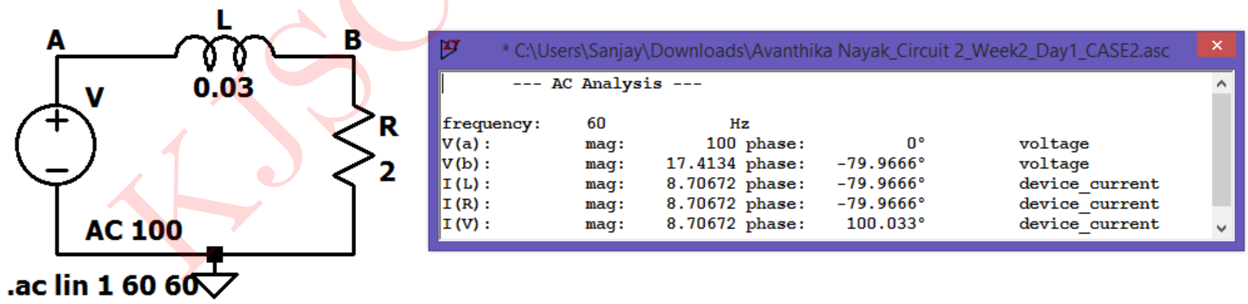


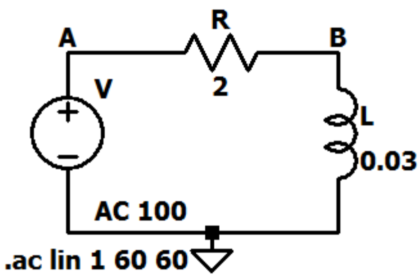
Figure 7: Circuit Schematic and Simulated Results for V_R, I and p.f

$V_R = 17.4134 \angle -79.9666^{\circ} V$

$I(R) = 8.70672 \angle -79.9666^{\circ} A$

$P.F = \cos(-79.9666) = 0.17422$

II. Simulation of circuit to find V_L



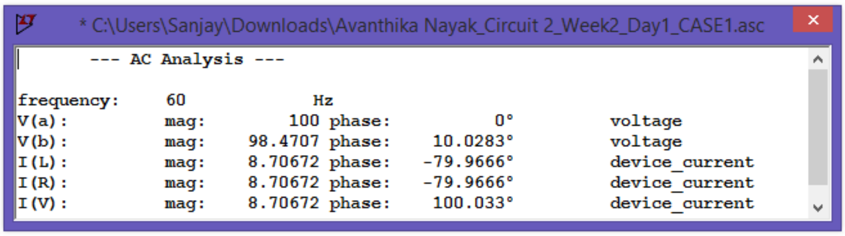


Figure 8: Circuit Schematic and Simulated Results for V_L

$V_L = 98.4707 \angle 10.0283^\circ V$

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
I	$8.7069 \angle -79.9715^\circ A$	$8.70672 \angle -79.9666^\circ A$
V_R	$17.4137 \angle -79.9715^\circ V$	$17.4134 \angle -79.9666^\circ V$
V_L	$98.4724 \angle 10.0285^\circ V$	$98.4702 \angle 10.0283^\circ V$
P.F ($\cos\phi$)	0.17414	0.17422

Table 2: Numerical 2

Numerical 3: A pure resistance of 35 ohms is in series with a pure capacitance of 80uF. The series combination is connected across 100V, 60 Hz supply. Find :-
(a) the impedance
(b) current
(c) phase angle
(d) power factor
(e) voltage across resistor
(f) voltage across capacitor.

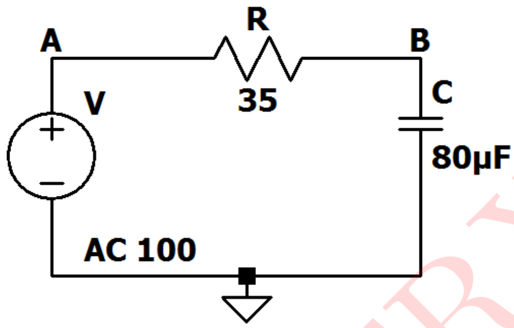


Figure 9: Circuit 3

Solution:

a) Finding impedence Z

We have R = 35Ω, C = 80μF, V = 100V and f = 60Hz

$$X_C = \frac{1}{2\pi f C}$$
$$= \frac{1}{2 \times \pi \times 60 \times 80 \times 10^{-6}}$$
$$= 33.1573\Omega$$

$$\therefore Z = R - jX_C$$
$$= 35 - j33.1573$$
$$= 48.2121\ \angle -43.4513^\circ\ \Omega$$

$\therefore$ **Z = 48.2121$\angle -$43.4513°Ω**

b) Finding Current I

$$\therefore I = \frac{V}{Z}$$
$$= \frac{100\angle 0^\circ}{48.2121\angle -43.4513^\circ}$$
$$= 2.0742\ \angle\ 43.4513^\circ A$$

$\therefore$ **I = 2.0742 $\angle$43.4513° A**

$\therefore$ The current is 2.0742A leading by 43.4513° w.r.t voltage

c) Phase Angle (ϕ)

ϕ = 45.4513°

$\therefore$ **Phase Angle = 45.4513°**

d) Power Factor ($\cos\phi$)

$\phi = 45.4513^{\circ}$

$\therefore \phi = \cos 45.4513 = 0.72596$ (leading)

$\therefore$ **P.F = 0.72596**

e) Finding V_R

V_R is the voltage across the resistor.

$\therefore V_R = I \times R$

$= 2.0742 \angle 43.4513^{\circ} \times 35$

$= 72.597 \angle -43.4513^{\circ}$V

$\therefore \mathbf{V_R = 72.597 \angle 43.4513^{\circ}}$ **V**

f) Finding V_C

V_C is the voltage across the capacitance.

$\therefore V_L = I \times X_C$

$= 2.0742 \angle 43.4513^{\circ} \times 33.157 \angle -90^{\circ}$

$= 68.7749 \angle -46.5487^{\circ}$V

$\therefore \mathbf{V_C = 68.7749 \angle -46.5487^{\circ}}$ **V**

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

I. Simulation of circuit to find V_R, I, Z, phase angle and p.f

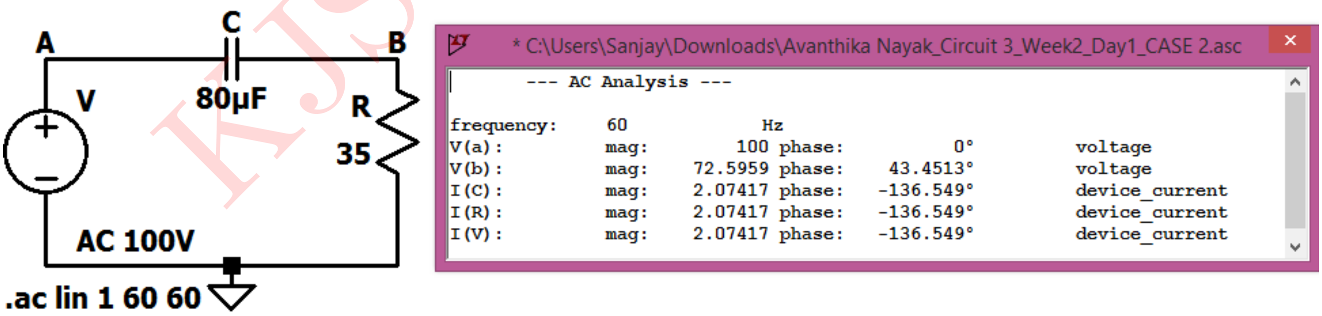


Figure 10: Circuit Schematic and Simulated Results for V_R, I, Z, phase angle and p.f

$V_R = 72.5959 \angle 43.4513^{\circ}$V

$I(R) = 2.07417 \angle -136.549^{\circ}$A

$$Z = \frac{V}{I(V)} = \frac{100\angle 0^{\circ}}{2.07417\angle -136.549^{\circ}} = 48.21206 \angle 136.549^{\circ}$$

$\phi = 43.4513^{\circ}$

$P.F = \cos(43.4513) = 0.72596$

II. Simulation of circuit to find V_C

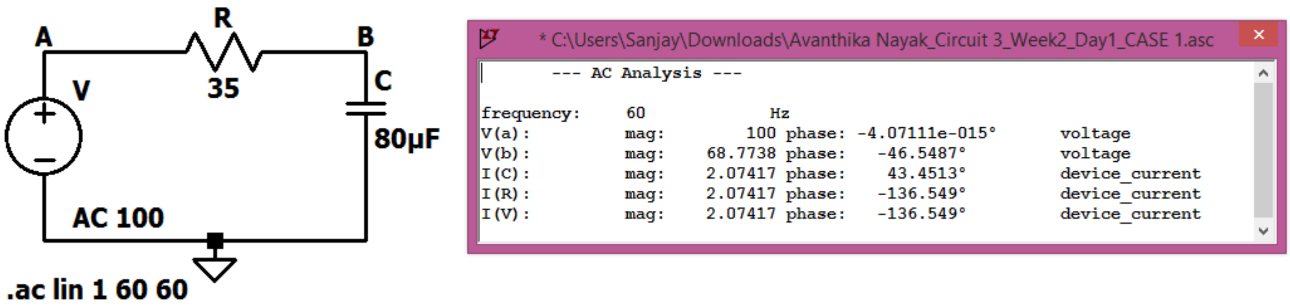


Figure 11: Circuit Schematic and Simulated Results for V_C

$V_C = 68.7738 \angle -46.5487^{\circ}V$

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
Z	$48.2121 \angle -43.4513^{\circ}\Omega$	$48.21206 \angle 136.549^{\circ}\Omega$
I	$2.0742 \angle 43.4513^{\circ}A$	$2.07417 \angle -136.549^{\circ}A$
V_R	$72.597 \angle 43.4513^{\circ}V$	$72.5959 \angle 43.4513^{\circ}V$
V_C	$68.7749 \angle -46.5487^{\circ}V$	$68.7738 \angle -46.5487^{\circ}V$
ϕ	43.4513°	43.4513°
P.F $(\cos\phi)$	0.72596	0.72596

Table 3: Numerical 3

Numerical 4: A circuit consists of resistance of 45Ω, an inductance of 34mH and a capacitor of 50μF are connected in parallel across a 110V, 50Hz supply.
Calculate:
a) Individual currents drawn by each element.
b) Total current drawn from the supply.
c) Overall power factor of the circuit.
d) Draw the phasor diagram.

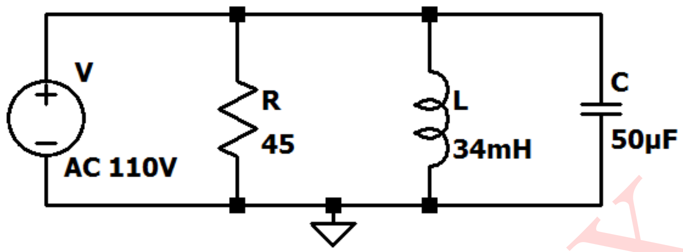


Figure 12: Circuit 4

Solution:

a) To calculate the individual currents I_R, I_L and I_C, we will first calculate the impedences Z_R, Z_L and Z_C across R1, L1 and C1 respectively.

We have R = 45Ω, L = 34mH, C = 50μF, V = 110 ∠ 0°V and f = 50Hz

$$\begin{aligned} X_L &= 2\pi f L \\ &= 2 \times \pi \times 50 \times 34 \times 10^{-3} \\ &= 10.6814\Omega \end{aligned}$$

$$\begin{aligned} X_C &= \frac{1}{2\pi f C} \\ &= \frac{1}{2 \times \pi \times 50 \times 50 \times 10^{-6}} \\ &= 63.662\Omega \end{aligned}$$

$$\therefore Z_R = R = 45\Omega$$

$$\therefore Z_L = jX_L = j10.6814 = 10.6814 \angle 90^\circ \ \Omega$$

$$\therefore Z_C = -jX_C = -j63.662 = 63.662 \angle{-90^\circ} \ \Omega$$

$$\begin{aligned} \therefore I_R &= \frac{V}{Z_R} \\ &= \frac{110\angle 0^\circ}{45\angle 0^\circ} \\ &= 2.4444 \angle 0^\circ A \end{aligned}$$

$$\begin{aligned} \therefore I_L &= \frac{V}{Z_L} \\ &= \frac{110\angle 0^\circ}{10.6814\angle 90^\circ} \\ &= 10.2983 \angle -90^\circ A \end{aligned}$$

$$\therefore \mathrm{I_C} = \frac{V}{Z_C}$$
$$= \frac{110\angle 0^\circ}{63.662\angle -90^\circ}$$
$$= 1.7279 \angle 90^\circ \mathrm{A}$$

$\therefore \mathbf{I_R = 2.4444 \angle 0^\circ \ A}$

The current across the resistor is 2.4444A in phase with the voltage.

$\therefore \mathbf{I_L = 10.2983 \angle -90^\circ \ A}$

The current across the inductor is 10.2983A lagging by $\angle -90^\circ$ w.r.t voltage.

$\therefore \mathbf{I_C = 1.7279 \angle 90^\circ \ A}$

The current across the capacitance is 1.7279A leading by $\angle$ 90° w.r.t voltage.

b) Finding the total current I

$\mathrm{I = I_R + I_L + I_C}$

$= 2.4444 \angle 0^\circ + 10.2983 \angle -90^\circ + 1.7279 \angle 90^\circ$

$= 2.4444 + j0 + 0 - j10.2983 + 0 + j1.7279$

$= 2.4444 - j8.5704$

$= 8.9122 \angle -74.081^\circ$

$\therefore \mathbf{I = 8.9122 \angle -74.081^\circ \ A}$

$\therefore$The total current is 8.9122A lagging by 74.081° w.r.t voltage.

c) Power Factor ($\cos\phi$)

$\phi = 74.081^\circ$

$\therefore \cos\phi = \cos(74.081) = 0.2743$ (lagging)

$\therefore$ **P.F = 0.2743**

d) Phasor Diagram

The phasor diagram for the circuit is as given figure 13:

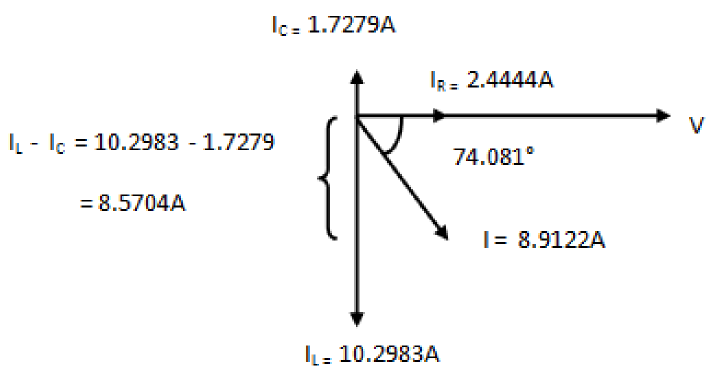


Figure 13: Phasor Diagram

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

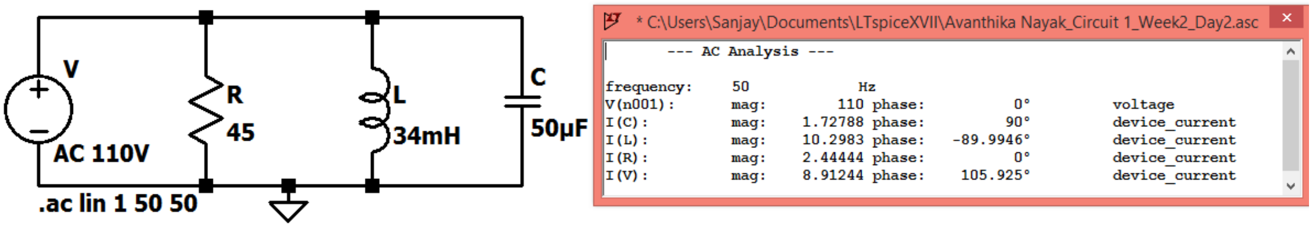


Figure 14: Circuit Schematic and Simulated Results

I(R) $= 2.4444 \angle 0^{\circ}$A

I(L) $= 10.2983 \angle -89.9946^{\circ}$A

I(C) $= 1.72788 \angle 90^{\circ}$A

I(V) $= 8.91244 \angle 105.925^{\circ} = 8.91244 \angle 105.925 - 180^{\circ} = 8.91244 \angle -74.075^{\circ}$A

P.F $= \cos(74.075) = 0.2744$

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
I	$8.9122 \angle -74.081^{\circ}$A	$8.91244 \angle -74.075^{\circ}$A
I_R	$2.4444 \angle 0^{\circ}$A	$2.4444 \angle 0^{\circ}$A
I_L	$10.2983 \angle -90^{\circ}$A	$10.2983 \angle -89.9946^{\circ}$A
I_C	$1.7279 \angle 90^{\circ}$A	$1.72788 \angle 90^{\circ}$A
P.F ($\cos\phi$)	0. 2743	0.2744

Table 4: Numerical 4

Numerical 5: A coil having a resistance of R1 = 4Ω and an inductance of L1 = 0.03 H is arranged in parallel with another coil having a resistance of R2 = 3Ω and an inductance of L2 = 0.09 H. Calculate the currents I, I1 and I2 when a voltage of V1 = 100 V at 50 Hz is applied. Also calculate the power factor of the circuit.

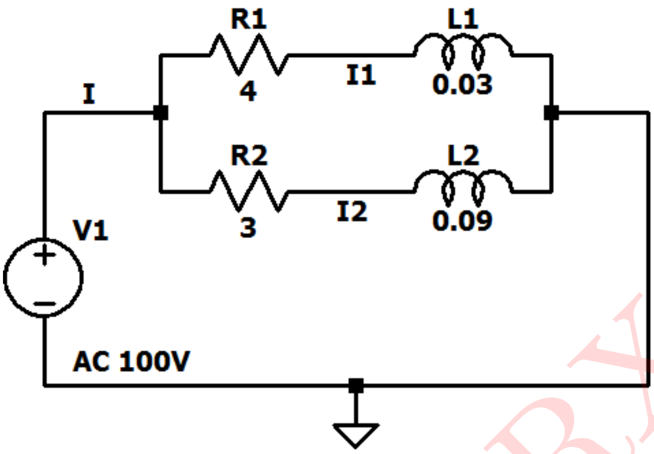


Figure 15: Circuit 5

Solution:

To calculate the currents, we will first calculate the impedences.

We have $R_1 = 4\Omega$, $R_2 = 3\Omega$, $L_1 = 0.03H$, $L_2 = 0.09H$, $V = 100V$ and $f = 50Hz$.

$$\begin{aligned}X_{L_1} &= 2\pi f L_1\\ &= 2 \times \pi \times 50 \times 0.03\\ &= 9.4248\Omega\end{aligned}$$

$$\begin{aligned}X_{L_2} &= 2\pi f L_2\\ &= 2 \times \pi \times 50 \times 0.09\\ &= 28.2743\Omega\end{aligned}$$

$$\begin{aligned}\therefore Z_1 &= R + jX_{L_1}\\ &= 4 + j9.4248\\ &= 10.2385 \angle 67^\circ \ \Omega\end{aligned}$$

$$\begin{aligned}\therefore Z_2 &= R + jX_{L_2}\\ &= 3 + j28.2743\\ &= 28.433 \angle 83.9433^\circ \ \Omega\end{aligned}$$

$$\begin{aligned}\therefore I_1 &= \frac{V}{Z_1}\\ &= \frac{100\angle 0^\circ}{10.2385\angle 67^\circ}\\ &= 9.7671 \angle -67^\circ A\end{aligned}$$

$$\therefore \mathrm{I}_2 = \frac{V}{Z_2}$$
$$= \frac{100\angle 0^\circ}{28.433\angle 83.9433^\circ}$$
$$= 3.51704\ \angle -83.9433^\circ \mathrm{A}$$

$$\mathrm{I} = \mathrm{I}_1 + \mathrm{I}_2$$
$$= 9.7671\ \angle -67^\circ + 3.51704\ \angle -83.9433^\circ$$
$$= 3.8163 - j8.9907 + 0.3711 - j3.4974$$
$$= 4.1874 - j12.4881$$
$$= 13.1714\ \angle -71.463^\circ$$

$\therefore$ $\mathbf{I_1 = 9.7671\angle -67^\circ\ A}$

The current across Z_1 is 9.7671A lagging by 67° w.r.t voltage

$\therefore$ $\mathbf{I_2 = 3.51704\angle -83.9433^\circ\ A}$

The current across Z_2 is 3.517041A lagging by 83.9433° w.r.t voltage

$\therefore$ $\mathbf{I = 13.1714\angle -71.463^\circ\ A}$

The total current is 13.1714A lagging by 71.463° w.r.t voltage

c) Power Factor ($\cos\phi$)

$\phi = 71.463^\circ$

$\therefore \cos\phi = \cos 71.463 = 0.31792$ (lagging)

$\therefore$ **P.F = 0.31792**

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

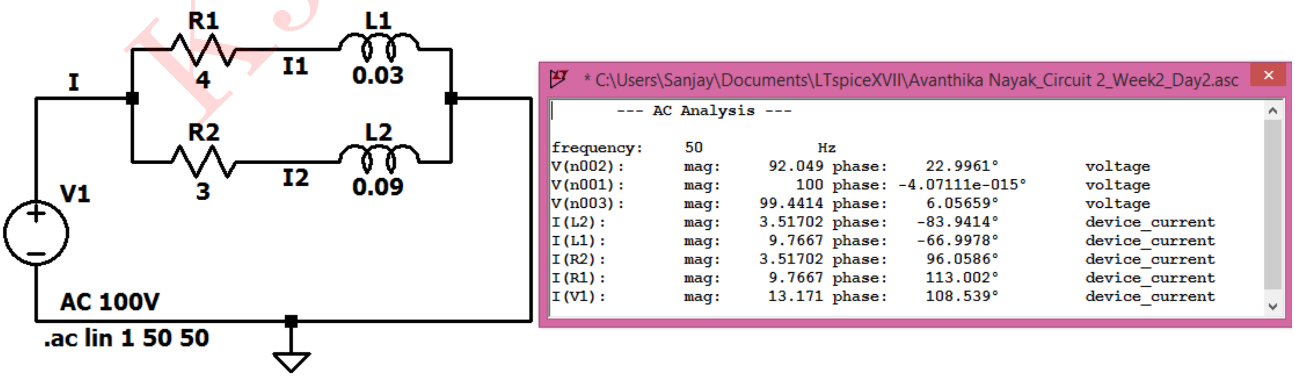


Figure 16: Circuit Schematic and Simulated Results

I_1 = I(L1) = 9.7667 $\angle -$ 66.9978°A

I_2 = I(L2) = 3.51702 $\angle -$ 83.9414°A

I = 13.171 $\angle$ 108.539°= 13.171 $\angle$ 108.539°- 180°= 13.171 $\angle -$ 71.461°A

P.F = cos(71.461) = 0.31795

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
I_1	9.7671 $\angle -$ 67°A	9.7667 $\angle -$ 66.9978°A
I_2	3.51704 $\angle -$ 83.9433°A	3.51702 $\angle -$ 83.9414°A
I	13.1714 $\angle -$ 71.463°A	13.171 $\angle -$ 71.461°A
P.F ($\cos\phi$)	0. 31792	0.31795

Table 5: Numerical 5

Numerical 6: Find I, I_1 and I_2 and voltage drops in each branch in the circuit shown in figure 6, if $R_1 = 2\Omega$, $L_1 = j8\Omega$, $R_2 = 15\Omega$, $L_2 = j10\Omega$, $R_3 = 12\Omega$, $C_1 = -j2\Omega$, V1 = 100V and frequency = 50Hz

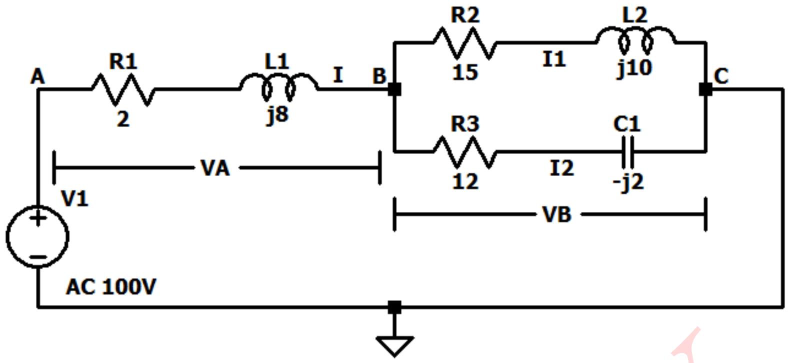


Figure 17: Circuit 6

Solution:

To calculate the currents and voltage drops we will first calculate the impedences.

We have $R_1 = 2\Omega$, $L_1 = j8\Omega$, $R_2 = 15\Omega$, $L_2 = j10\Omega$, $R_3 = 12\Omega$, $C_1 = -j2\Omega$, V1 = 100V and $f = 50$Hz

$$X_{L_1} = 2\pi f L_1$$
$$\therefore 8 = 2\pi f L_1$$
$$\therefore L_1 = \frac{8}{2\pi \times 50}$$
$$= 0.02546\text{H}$$

$$X_{L_2} = 2\pi f L_2$$
$$\therefore 10 = 2\pi f L_2$$
$$\therefore L_1 = \frac{10}{2\pi \times 50}$$
$$= 0.031831\text{H}$$

$$X_{C_1} = \frac{1}{2\pi f C_1}$$
$$\therefore 2 = \frac{1}{2\pi f C_1}$$
$$\therefore C_1 = \frac{1}{2 \times \pi \times 50 \times 2}$$
$$= 1.59155\text{mF}$$

$$\therefore Z_1 = R + jX_{L_1}$$
$$= 2 + j8$$
$$= 8.2462 \angle 75.9638^\circ \ \Omega$$

$$\therefore Z_2 = R + jX_{L_2}$$
$$= 15 + j10$$
$$= 18.0278 \angle 33.69^\circ \ \Omega$$

$\therefore Z_3 = R = jX_{C_1}$

$= 12 - j2$

$= 12.1655 \angle -9.4623^\circ \ \Omega$

Impedences Z_2 and Z_3 are connected in parallel and this combination in turn, is connected in series with Z_1.

So the effective impedence Z_{eff} is :-

$$\therefore Z_{eff} = Z_1 + (Z_2 \mid\mid Z_3)$$

$$= Z_1 + \frac{Z_2 \times Z_3}{Z_2 + Z_3}$$

$$= 8.2462 \angle 75.9638^\circ + \frac{18.0278\angle 33.69^\circ \times 12.1655\angle -9.4623}{18.0278\angle 33.69^\circ + 12.1655\angle -9.4623}$$

$$= 2 + j8 + \frac{219.372\angle 24.2277^\circ}{15 + j10 + 12 - j2}$$

$$= 2 + j8 + \frac{219.372\angle 24.2277^\circ}{28.1603\angle 16.5044^\circ}$$

$$= 2 + j8 + 7.7882 \angle 7.7233^\circ$$

$$= 2 + j8 + 7.7176 + j1.0466$$

$$= 9.7176 + j9.0466$$

$$= 13.2768 \angle 42.952^\circ$$

$$\therefore I = \frac{V}{Z_{eff}}$$

$$= \frac{100\angle 0^\circ}{13.2768\angle 42.952^\circ}$$

$$= 7.53193 \angle -42.952^\circ A$$

By Current Division Rule, we have

$$I_1 = I \times \frac{Z_3}{Z_2 + Z_3}$$

$$= 7.53193 \angle -42.952^\circ \times \frac{12.1655\angle -9.4623^\circ}{15 + j10 + 12 - j2}$$

$$= \frac{91.6297\angle -52.4143^\circ}{27 + j8}$$

$$= \frac{91.6297\angle -52.4143^\circ}{28.1603\angle 16.5044^\circ}$$

$$= 3.25386 \angle -68.919^\circ A$$

$$\therefore I_2 = I - I_1$$

$$= 7.53193 \angle -42.952 - 3.25386 \angle -68.919$$

$$= 5.5128 - j5.1321 - 1.17037 + j3.0361$$

$$= 4.34243 - j2.096$$

$$= 4.82182 \angle -25.7656^\circ A$$

$\therefore \mathbf{I = 7.53193 \angle -42.952^\circ \ A}$

The total current is 7.53193A lagging by 42.952° w.r.t voltage

$\therefore \mathbf{I_1 = 3.25386 \angle -68.919^\circ \ A}$

The current through Z_1 is 3.25386A lagging by 68.919° w.r.t voltage

$\therefore \mathbf{I_2 = 4.82182 \angle -25.7656^\circ \ A}$

The current through Z_2 is 4.82182A lagging by 25.7656° w.r.t voltage

The branch voltages are V_A and V_B.

$\therefore V_B = I_1 \ Z_2$

$$= 3.25386 \angle -68.919^\circ \times 18.0278 \angle 33.69^\circ$$

$$= 58.6599 \angle -35.229^\circ \mathrm{V}$$

$\therefore V_A = I \ Z_1$

$$= 7.53193 \angle -42.952^\circ \times 8.2462 \angle 75.9638^\circ$$

$$= 62.1098 \angle 33.0118^\circ \mathrm{V}$$

$\therefore \mathbf{V_A = 62.1098 \angle 33.0118^\circ \ V}$

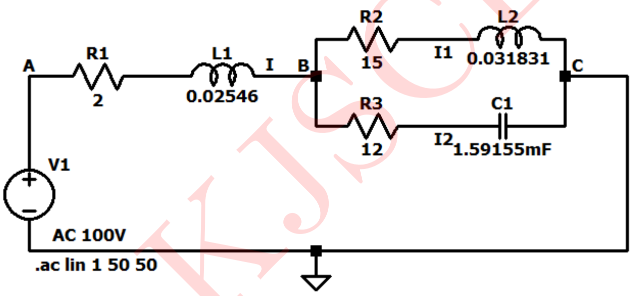
The branch voltage through Z_1 is 62.1098V leading by 33.0118° w.r.t supply voltage

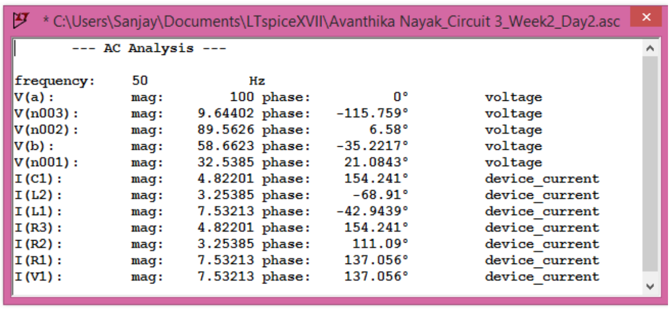
$\therefore \mathbf{V_B = 58.6599 \angle -35.229^\circ \ V}$

The branch voltage through Z_2 and Z_2 is 58.6599V lagging by 35.229° w.r.t supply voltage

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:





frequency:	50	Hz			
V(a):	mag:	100	phase:	0°	voltage
V(n003):	mag:	9.64402	phase:	-115.759°	voltage
V(n002):	mag:	89.5626	phase:	6.58°	voltage
V(b):	mag:	58.6623	phase:	-35.2217°	voltage
V(n001):	mag:	32.5385	phase:	21.0843°	voltage
I(C1):	mag:	4.82201	phase:	154.241°	device_current
I(L2):	mag:	3.25385	phase:	-68.91°	device_current
I(L1):	mag:	7.53213	phase:	-42.9439°	device_current
I(R3):	mag:	4.82201	phase:	154.241°	device_current
I(R2):	mag:	3.25385	phase:	111.09°	device_current
I(R1):	mag:	7.53213	phase:	137.056°	device_current
I(V1):	mag:	7.53213	phase:	137.056°	device_current

Figure 18: Circuit Schematic and Simulated Results

$I_1 = I(L2) = 3.25385 \angle -68.91^\circ \mathrm{A}$

$I_2 = I(C1) = 4.82201 \angle 154.241^\circ = 4.82201 \angle 154.241^\circ - 180^\circ = 4.82201 \angle -25.759^\circ \mathrm{A}$

$I = 7.53213 \angle 137.056^\circ = 7.53213 \angle 137.056^\circ - 180^\circ = 7.53213 \angle -42.944^\circ \mathrm{A}$

$V_B = V(b) = 58.6623 \angle -35.2217^\circ \mathrm{V}$

$V_A = V(a) - V(b) = 100 \angle 0^\circ - 58.6623 \angle -35.2217^\circ = 62.1024 \angle 33.0106^\circ \mathrm{V}$

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
I_1	$3.25386 \angle -68.919^{\circ}$A	$3.25385 \angle -68.91^{\circ}$A
I_2	$4.82182 \angle -25.7656^{\circ}$A	$4.82201 \angle -25.759^{\circ}$A
I	$7.53193 \angle -42.952^{\circ}$A	$7.53213 \angle -42.944^{\circ}$A
V_A	$62.1098 \angle 33.0118^{\circ}$A	$62.1024 \angle 33.0106^{\circ}$V
V_B	$58.6599 \angle -35.229^{\circ}$A	$58.6623 \angle -35.2217^{\circ}$V

Table 6: Numerical 6

Numerical 7: A 60 Hz sinusoidal voltage V = 141 sinωt is applied to a series R-L circuit. The values of the resistance and the inductance are 2Ω and 0.03 H respectively.
Determine the following:
a) Calculate the peak voltage across resistor and inductor and also find the peak value of source current in LTspice
b) Plot input source voltage $V_S(t)$ vs input source current $I_S(t)$ in LTspice
c) Measure the phase delay/difference between $V_S(t)$ vs $I_S(t)$ in time and degrees
d) Plot input source voltage $V_S(t)$ vs voltage across resistor $V_R(t)$ in LTspice
e) Measure the phase delay/difference between $V_S(t)$ vs $V_R(t)$ in time and degrees
f) Plot input source voltage $V_S(t)$ vs voltage across inductor $V_L(t)$ in LTspice
g) Measure the phase delay/difference between $V_S(t)$ vs $V_L(t)$ in time and degrees
h) Calculate the power factor of the circuit.

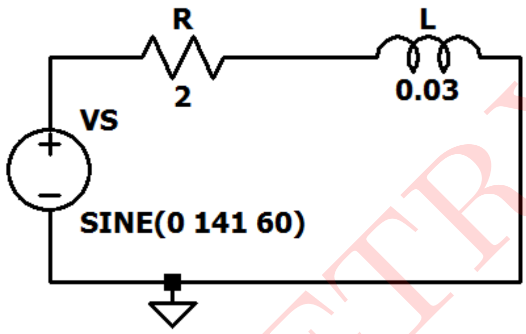


Figure 19: Circuit 7

Solution:

a) Finding peak voltages across resistor and inductor($V_{R_{(peak)}}$ and $V_{L_{(peak)}}$) and peak current($I_{(peak)}$)

We have R = 2Ω, L = 0.03H, V = 141sinωt and f = 60Hz

$$\begin{aligned} V_{RMS} &= \frac{V_{peak}}{\sqrt{2}} \\ &= \frac{141}{\sqrt{2}} \\ &= 100V \end{aligned}$$

$$\begin{aligned} X_L &= 2\pi f L \\ &= 2 \times \pi \times 60 \times 0.03 \\ &= 11.3097\Omega \end{aligned}$$

$$\begin{aligned} \therefore Z &= R + jX_L \\ &= 2 + j11.3097 \\ &= 11.4852 \angle 79.9715^\circ \ \Omega \end{aligned}$$

$$\begin{aligned} \therefore I_{RMS} &= \frac{V_{RMS}}{Z} \\ &= \frac{100\angle 0^\circ}{11.4852\angle 79.9715^\circ} \\ &= 8.7069 \angle -79.9715^\circ A \end{aligned}$$

$$\therefore I_{(peak)} = I_{RMS} \times \sqrt{2}$$

$$= 8.7069 \angle -79.9715^{\circ} \times \sqrt{2}$$

$$= 12.3133 \angle -79.9715^{\circ}\mathrm{A}$$

$$V_R = I_{RMS} \times R$$

$$= 8.7069 \angle -79.9715 \times 2$$

$$= 17.41372 \angle -79.9715^{\circ}\mathrm{V}$$

$$\therefore V_{R_{(peak)}} = V_R \times \sqrt{2}$$

$$= 17.41372 \angle -79.9715^{\circ} \times \sqrt{2}$$

$$= 24.6267 \angle -79.9715^{\circ}\mathrm{V}$$

$$V_L = I_{RMS} \times X_L$$

$$= 8.7069 \angle -79.9715^{\circ} \times 11.3097 \angle 90^{\circ}$$

$$= 98.4719 \angle 10.0285^{\circ}\mathrm{V}$$

$$\therefore V_{L_{(peak)}} = V_L \times \sqrt{2}$$

$$= 98.4719 \angle 10.02855^{\circ} \times \sqrt{2}$$

$$= 139.26047 \angle 10.0285^{\circ}\mathrm{V}$$

$$\therefore \mathbf{I_{(peak)}} = \mathbf{12.3133 \angle -79.9715^{\circ}\ A}$$

$$\therefore \mathbf{V_{R_{(peak)}}} = \mathbf{24.6267 \angle -79.9715^{\circ}\ V}$$

$$\therefore \mathbf{V_{L_{(peak)}}} = \mathbf{139.2604 \angle 10.0285^{\circ}\ V}$$

b) Graph of input source voltage $V_S(t)$ vs input source current $I_S(t)$

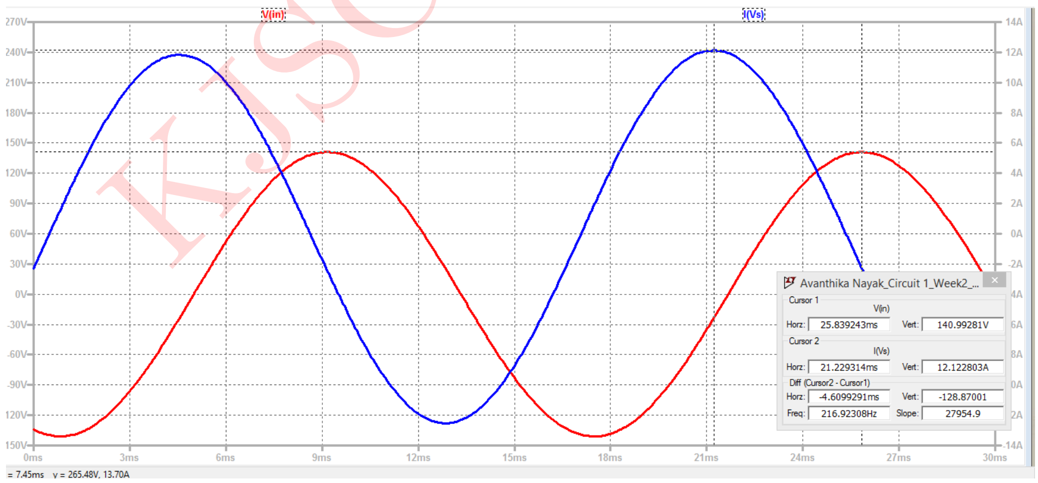


Figure 20: $V_S(t)$ vs $I_S(t)$

c) Phase delay/difference between $V_S(t)$ vs $I_S(t)$ in time and degrees.

f = 60Hz

$$\therefore \mathrm{T} = \frac{1}{f} = \frac{1}{50} = 16.67\mathrm{ms}$$

We have $V = 100 \angle 0^\circ,\ I = 8.7069 \angle -79.9715^\circ A$

Phase difference = $\Delta\theta$ = 180°- 79.9715°= 100.0285°

$$\text{Phase delay} = \Delta t = \frac{\Delta\theta \times T}{360}$$
$$= \frac{100.0285 \times 16.67}{360}$$
$$= 4.63187\mathrm{ms}$$

$\therefore$ **$\Delta\theta$ = 100.0285°**

$\therefore$ **Δt = 4.63187 ms**

d) Graph of input source voltage $V_S(t)$ vs voltage across resistor $V_R(t)$

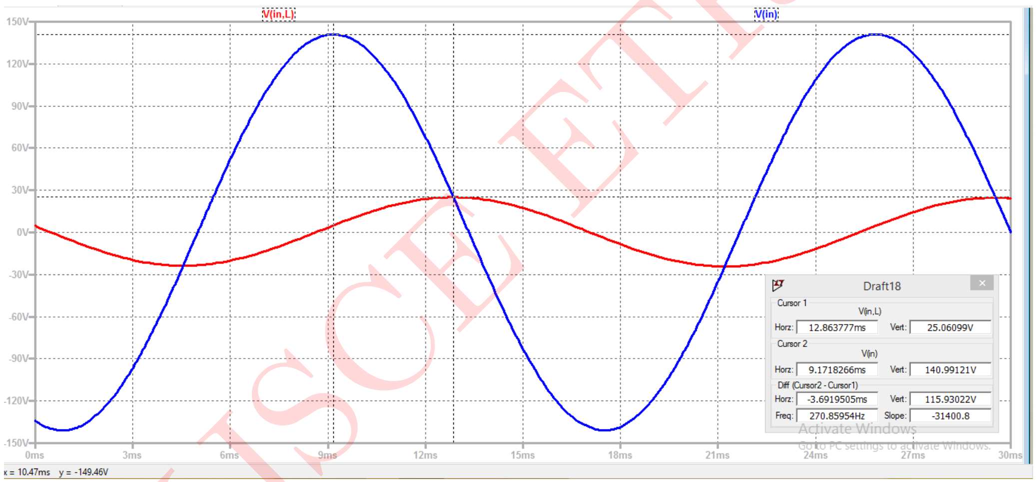


Figure 21: $V_S(t)$ vs $V_R(t)$

e) Phase delay/difference between $V_S(t)$ vs $V_R(t)$ in time and degrees.

We have $V = 100 \angle 0^\circ,\ V_R = 17.41372 \angle -79.9715^\circ V$

Phase difference = $\Delta\theta$ = 79.9715°

$$\text{Phase delay} = \Delta t = \frac{\Delta\theta \times T}{360}$$
$$= \frac{79.9715 \times 16.67}{360}$$
$$= 3.703124\mathrm{ms}$$

$\therefore$ **$\Delta\theta$ = 79.9715°**

$\therefore$ **Δt = 3.703124 ms**

f) Graph of input source voltage $V_S(t)$ vs voltage across inductor $V_L(t)$

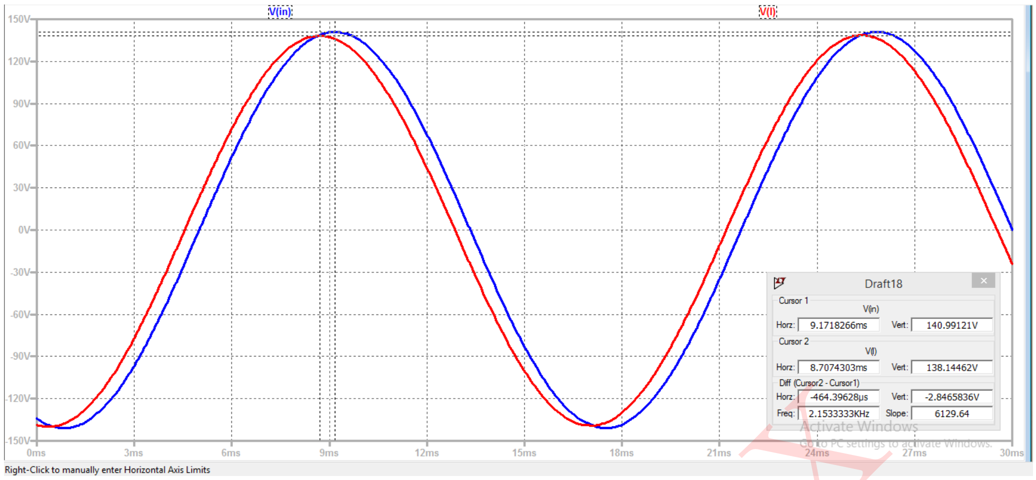


Figure 22: $V_S(t)$ vs $V_L(t)$

g) Phase delay/difference between $V_S(t)$ vs $V_L(t)$ in time and degrees.

We have $V = 100 \angle 0^\circ$, $V_L = 98.4719 \angle 10.0285^\circ V$

Phase difference $= \Delta\theta = 10.0285^\circ$

$$\text{Phase delay} = \Delta t = \frac{\Delta\theta \times T}{360}$$
$$= \frac{10.0285 \times 16.67}{360}$$
$$= 0.464375\text{ms}$$

$\therefore$ $\mathbf{\Delta\theta = 10.0285^\circ}$

$\therefore$ $\mathbf{\Delta t = 0.464375\ ms}$

h) Power Factor ($\cos\phi$)

$\phi = 79.9715^\circ$

$\therefore \cos\phi = \cos\,(79.9715) = 0.174138$ (lagging)

$\therefore$ **P.F = 0.174138**

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

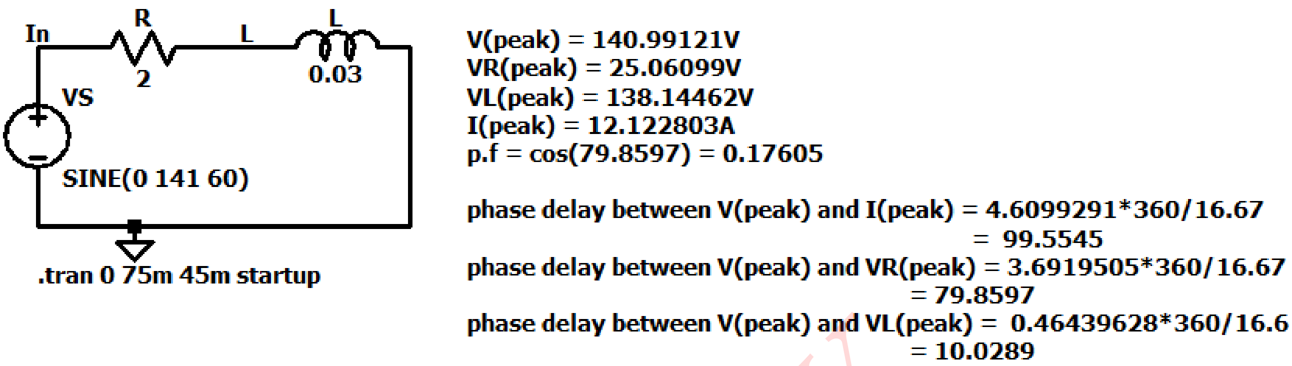


Figure 23: Circuit Schematic and Simulated Results

Graph of input source voltage $V_S(t)$ vs input source current $I_S(t)$ is shown in figure 24

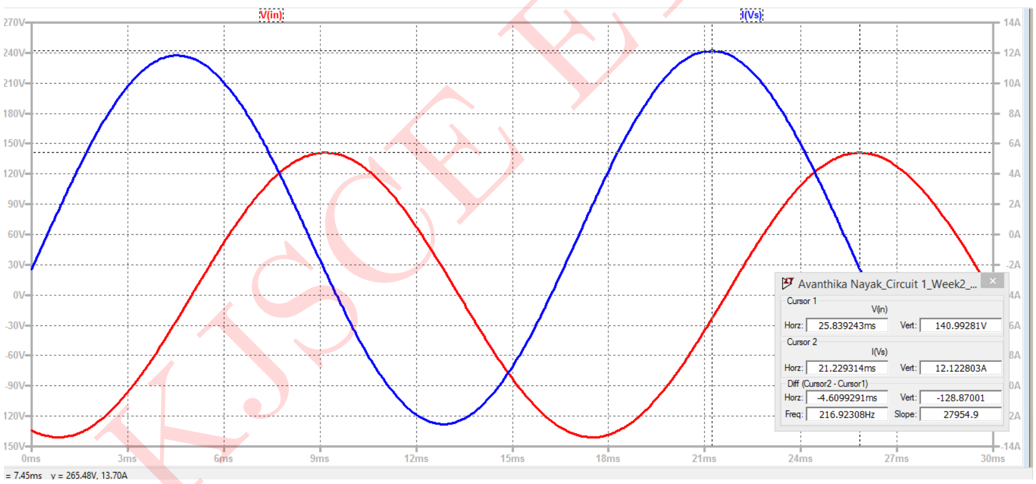


Figure 24: $V_S(t)$ vs $I_S(t)$

Graph of input source voltage $V_S(t)$ vs voltage across resistor $V_R(t)$ is shown in figure 25

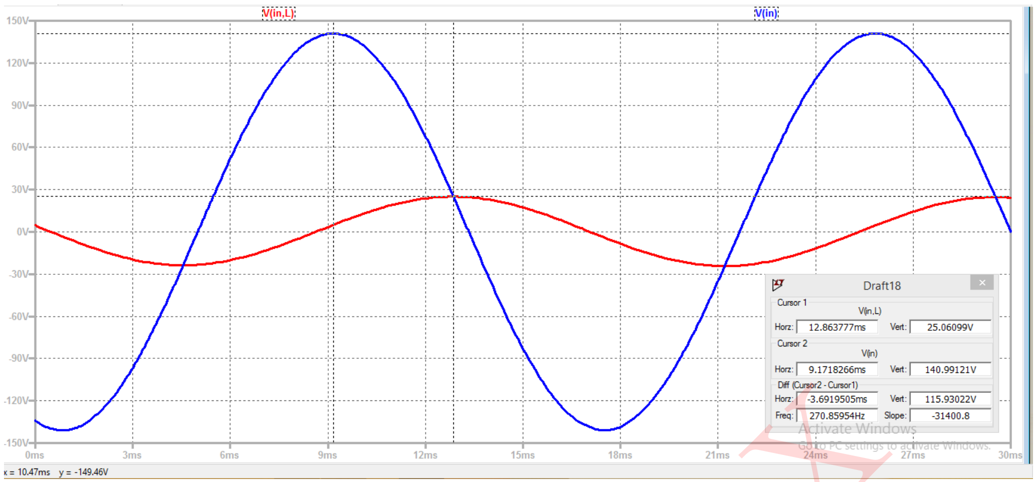


Figure 25: $V_S(t)$ vs $V_R(t)$

Graph of input source voltage $V_S(t)$ vs voltage across inductor $V_L(t)$ is shown in figure 26

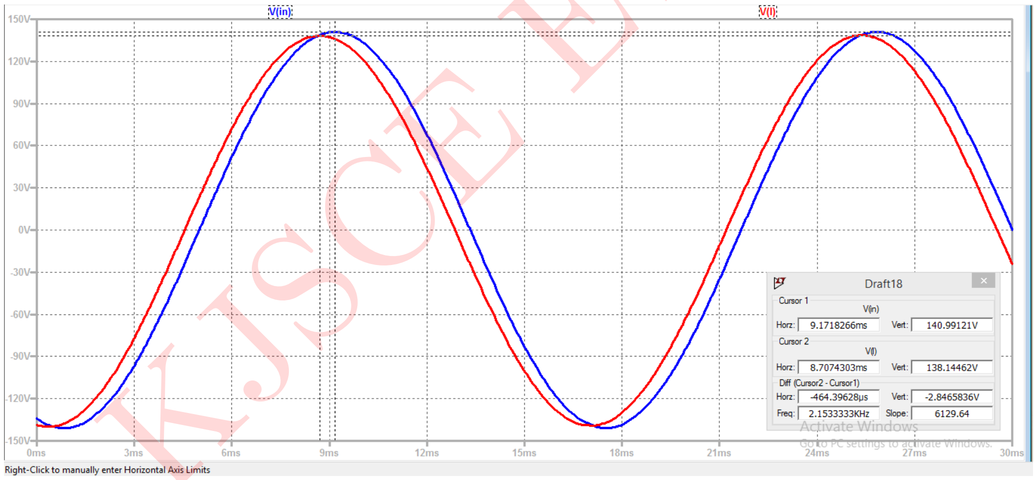


Figure 26: $V_S(t)$ vs $V_L(t)$

Comparison of Theoretical and Simulated values:-

Parameters	Theoretical values	Simulated values
$V_{R(peak)}$	24.6767V	25.06099V
$V_{L(peak)}$	139.26047V	138.14462V
I_{peak}	12.3133A	12.122803A
Phase delay between $V_S(t)$ vs $I_S(t)$	4.63187ms 100.0285°	4.6099291ms 99.5545°
Phase delay between $V_S(t)$ vs $V_R(t)$	3.70312ms 79.9715°	3.6919505ms 79.8597°
Phase delay between $V_S(t)$ vs $V_L(t)$	0.464375ms 10.0285°	0.46439628ms 10.0289°
P.F ($\cos\phi$)	0.174138	0.17605

Table 7: Numerical 7

Numerical 8: A pure resistance of 35Ω is in series with a pure capacitance of 80μF. The series combination is connected across 100V, 60 Hz supply.
Determine the following:
a) Calculate the peak voltage across resistor and capacitor and also find the peak value of source current in LTspice
b) Plot input source voltage $V_S(t)$ vs input source current $I_S(t)$ in LTspice
c) Measure the phase delay/difference between $V_S(t)$ vs $I_S(t)$ in time and degrees
d) Plot input source voltage $V_S(t)$ vs voltage across resistor $V_R(t)$ in LTspice
e) Measure the phase delay/difference between $V_S(t)$ vs $V_R(t)$ in time and degrees
f) Plot input source voltage $V_S(t)$ vs voltage across capacitor $V_L(t)$ in LTspice
g) Measure the phase delay/difference between $V_S(t)$ vs $V_L(t)$ in time and degrees
h) Calculate the power factor of the circuit.

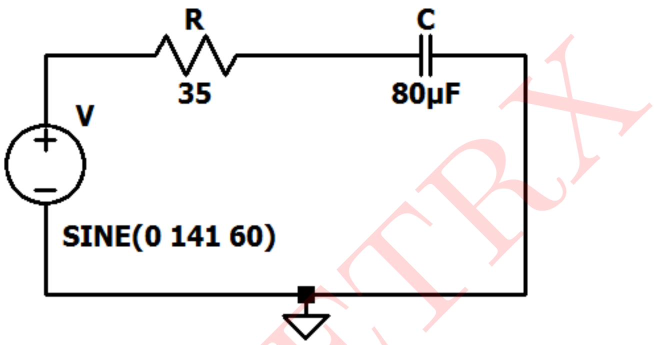


Figure 27: Circuit 8

Solution:

a) Finding peak voltages across resistor and capacitor($V_{R_{(peak)}}$ and $V_{C_{(peak)}}$) and peak current($I_{(peak)}$)

We have R = 35Ω, C = 80μF, V = 100V and f = 60Hz

$$\begin{aligned} V_{(peak)} &= V_{RMS}\ \sqrt{2} \\ &= 100 \times \sqrt{2} \\ &= 141\text{V} \end{aligned}$$

$$\begin{aligned} X_C &= \frac{1}{2\pi f C} \\ &= \frac{1}{2 \times \pi \times 60 \times 80 \times 10^{-6}} \\ &= 33.1573\Omega \end{aligned}$$

$$\begin{aligned} \therefore Z &= R - jX_C \\ &= 2 - j33.1573 \\ &= 48.2121\ \angle -\ 43.4513^\circ\ \Omega \end{aligned}$$

$$\begin{aligned} \therefore I_{RMS} &= \frac{V_{RMS}}{Z} \\ &= \frac{100\angle 0^\circ}{48.2121\angle - 43.4513^\circ} \\ &= 2.0741\ \angle\ 43.4513\ ^\circ\text{A} \end{aligned}$$

$$\therefore \mathrm{I}_{(peak)} = \mathrm{I}_{RMS} \times \sqrt{2}$$
$$= 2.0741 \angle 43.4513^\circ \times \sqrt{2}$$
$$= 2.9333 \angle 43.4513^\circ \mathrm{A}$$

$$\mathrm{V}_{R} = \mathrm{I}_{RMS} \times \mathrm{R}$$
$$= 2.0741 \angle 43.4513^\circ \times 35$$
$$= 72.5935 \angle 43.4513^\circ \mathrm{V}$$

$$\therefore \mathrm{V}_{R_{(peak)}} = \mathrm{V}_{R} \times \sqrt{2}$$
$$= 72.5935 \angle 43.4513^\circ \times \sqrt{2}$$
$$= 102.6627 \angle 43.4513^\circ \mathrm{V}$$

$$\mathrm{V}_{C} = \mathrm{I}_{RMS} \times \mathrm{X}_{C}$$
$$= 2.0741 \angle 43.4513^\circ \times 33.1573 \angle -90^\circ$$
$$= 68.7715 \angle -46.5487^\circ \mathrm{V}$$

$$\therefore \mathrm{V}_{C_{(peak)}} = \mathrm{V}_{C} \times \sqrt{2}$$
$$= 68.7715 \angle -46.5487 \times \sqrt{2}$$
$$= 97.2577 \angle -46.5487^\circ \mathrm{V}$$

$$\therefore \mathbf{I_{(peak)} = 2.9333 \angle 43.4513^\circ \ A}$$
$$\therefore \mathbf{V_{R_{(peak)}} = 102.6627 \angle 43.4513^\circ \ V}$$
$$\therefore \mathbf{V_{C_{(peak)}} = 97.2577 \angle -46.5487^\circ \ V}$$

b) Graph of input source voltage $V_S(t)$ vs input source current $I_S(t)$

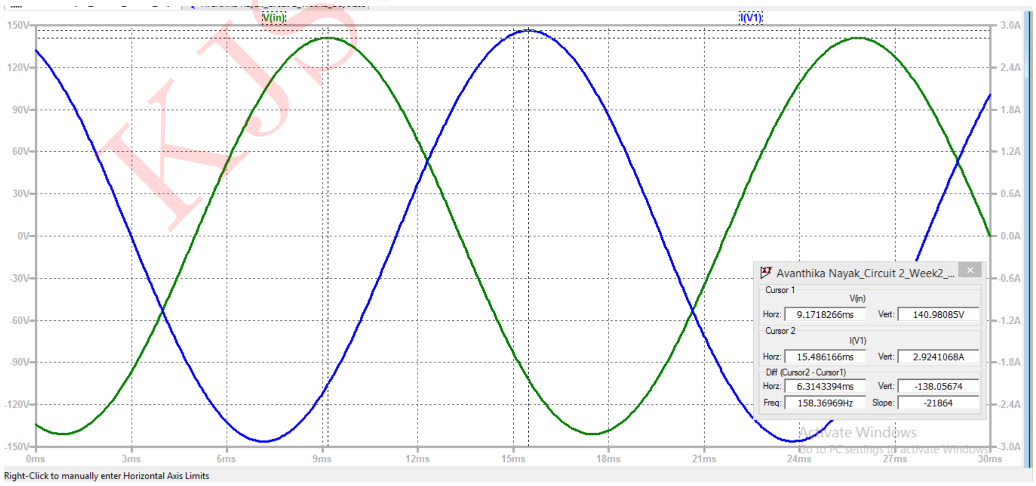


Figure 28: $V_S(t)$ vs $I_S(t)$

c) Phase delay/difference between $V_S(t)$ vs $I_S(t)$ in time and degrees.

f = 60Hz

$\therefore \mathrm{T} = \frac{1}{f} = \frac{1}{50} = 16.67\mathrm{ms}$

We have V = 100 $\angle$ 0°, I = 2.0741 $\angle$ 43.4513°A

Phase difference = $\Delta\theta$ = 180°- 43.4513°= 136.5487°

$$
\begin{aligned}
\text{Phase delay} = \Delta t &= \frac{\Delta\theta \times T}{360} \\
&= \frac{136.54875 \times 16.67}{360} \\
&= 6.3229\mathrm{ms}
\end{aligned}
$$

$\therefore \boldsymbol{\Delta\theta = 136.54875°}$

$\therefore \boldsymbol{\Delta t = 6.3229\ \mathrm{ms}}$

d) Graph of input source voltage $V_S(t)$ vs voltage across resistor $V_R(t)$

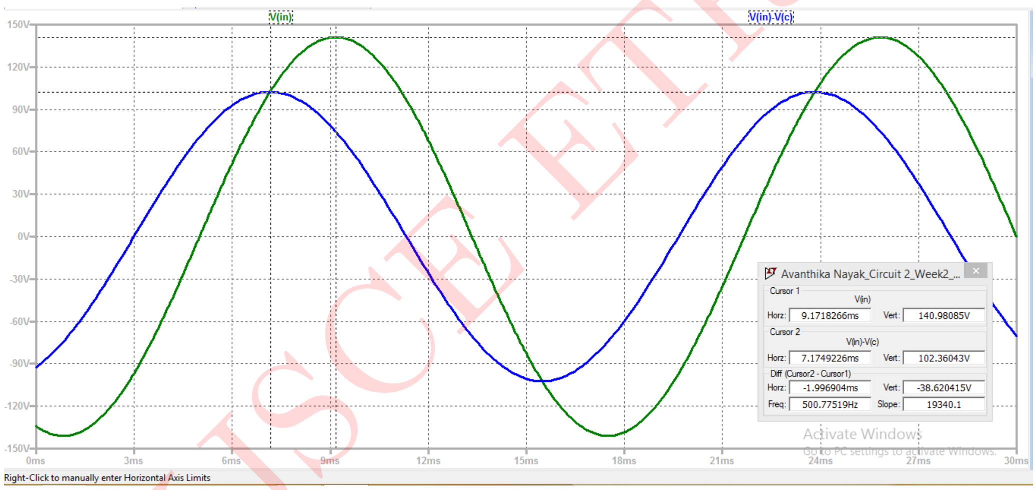


Figure 29: $V_S(t)$ vs $V_R(t)$

e) Phase delay/difference between $V_S(t)$ vs $V_R(t)$ in time and degrees.

We have V = 100 $\angle$ 0°, V_R = 72.5935 $\angle$ 43.4513°V

Phase difference = $\Delta\theta$ = 43.4513°

$$
\begin{aligned}
\text{Phase delay} = \Delta t &= \frac{\Delta\theta \times T}{360} \\
&= \frac{43.4513 \times 16.67}{360} \\
&= 2.01204\mathrm{ms}
\end{aligned}
$$

$\therefore \boldsymbol{\Delta\theta = 43.4513°}$

$\therefore \boldsymbol{\Delta t = 2.01204\ \mathrm{ms}}$

f) Graph of input source voltage $V_S(t)$ vs voltage across capacitor $V_C(t)$

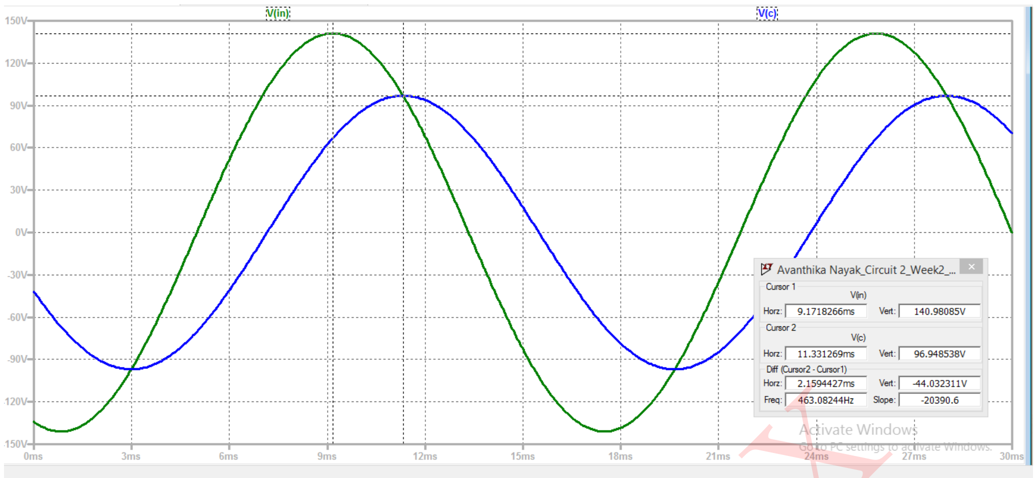


Figure 30: $V_S(t)$ vs $V_C(t)$

g) Phase delay/difference between $V_S(t)$ vs $V_C(t)$ in time and degrees.

We have $V = 100 \angle 0^\circ$, $V_C = 68.7715 \angle -46.5487^\circ V$

Phase difference $= \Delta\theta = 46.5487^\circ$

$$\text{Phase delay} = \Delta t = \frac{\Delta\theta \times T}{360} = \frac{46.5487 \times 16.67}{360} = 2.1555\text{ms}$$

$\therefore$ **$\Delta\theta = 46.5487^\circ$**

$\therefore$ **$\Delta t = 2.1555$ ms**

h) Power Factor ($\cos\phi$)

$\phi = 43.4513^\circ$

$\therefore \cos\phi = \cos(43.4513) = 0.7259$ (leading)

$\therefore$ **P.F $= 0.7259$**

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

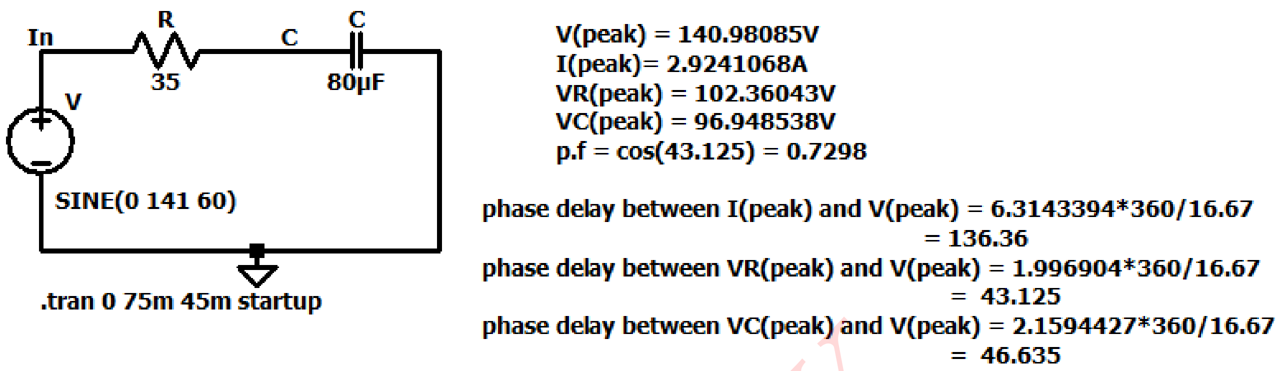


Figure 31: Circuit Schematic and Simulated Results

Graph of input source voltage $V_S(t)$ vs input source current $I_S(t)$ is shown in figure 32

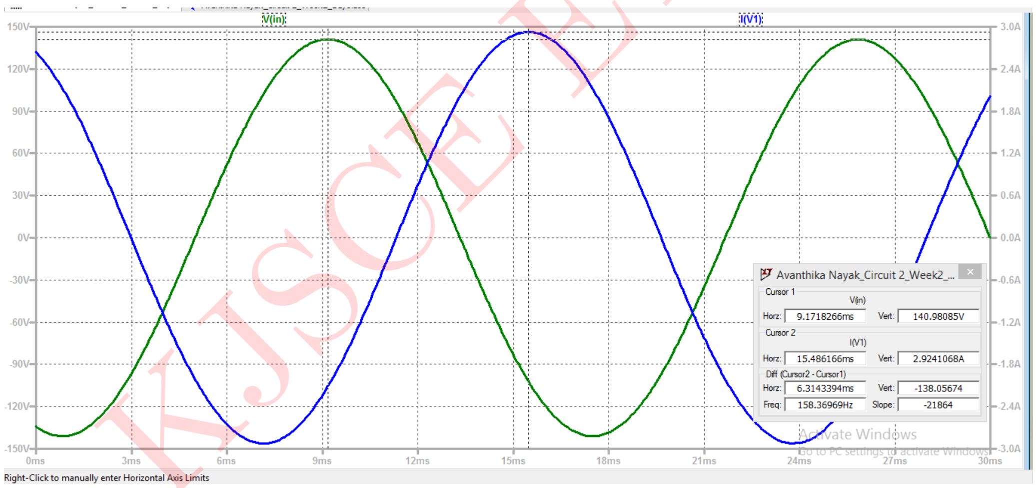


Figure 32: $V_S(t)$ vs $I_S(t)$

Graph of input source voltage $V_S(t)$ vs voltage across resistor $V_R(t)$ is shown in figure 33

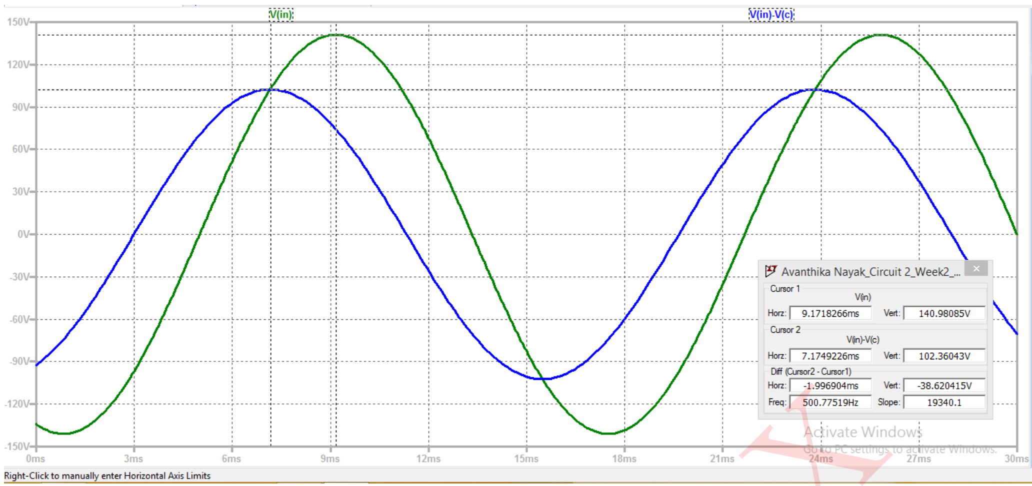


Figure 33: $V_S(t)$ vs $V_R(t)$

Graph of input source voltage $V_S(t)$ vs voltage across capacitor $V_C(t)$ is shown in figure 34

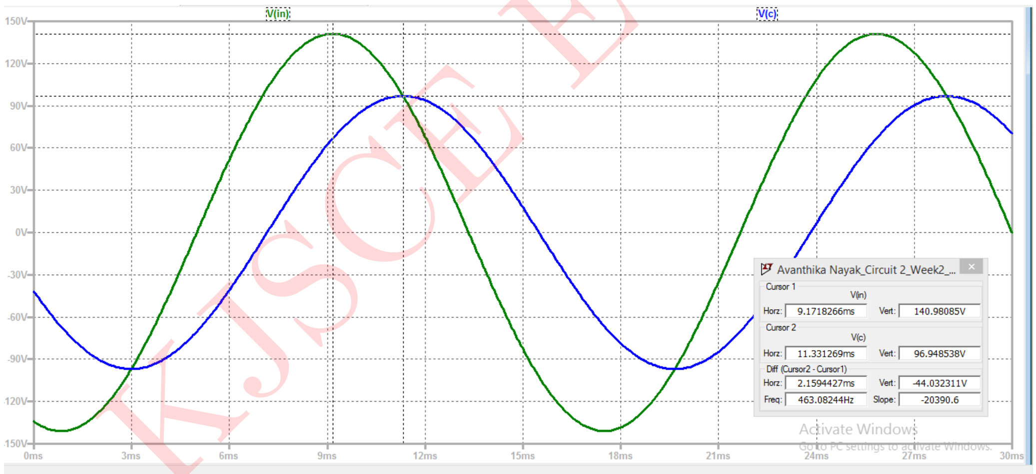


Figure 34: $V_S(t)$ vs $V_C(t)$

Comparison of Theoretical and Simulated values:-

Parameters	**Theoretical values**	**Simulated values**
$V_{R(peak)}$	102.6627V	102.36043V
$V_{C(peak)}$	97.2577V	96.948538V
I_{peak}	2.9333A	2.92481068A
Phase delay between $V_S(t)$ vs $I_S(t)$	6.32297ms 136.54875°	6.3143394ms 136.36°
Phase delay between $V_S(t)$ vs $V_R(t)$	2.01204ms 43.4513°	1.996904ms 43.125°
Phase delay between $V_S(t)$ vs $V_C(t)$	2.1555ms 46.5487°	2.1594427ms 46.635°
P.F ($\cos\phi$)	0.7259	0.7298

Table 8: Numerical 8

Numerical 9: A series resonance network consisting of a resistor of 27Ω, a capacitor of 2μF and an inductor of 25mH is connected across a sinusoidal supply voltage which has a constant output of AC 9 volts at all frequencies. Calculate, the resonant frequency, the current at resonance, the voltage across the inductor and capacitor at resonance, the quality factor and the bandwidth of the circuit. Plot the resonance curve, the current at resonance, the voltage across the inductor and capacitor at resonance in LTspice.

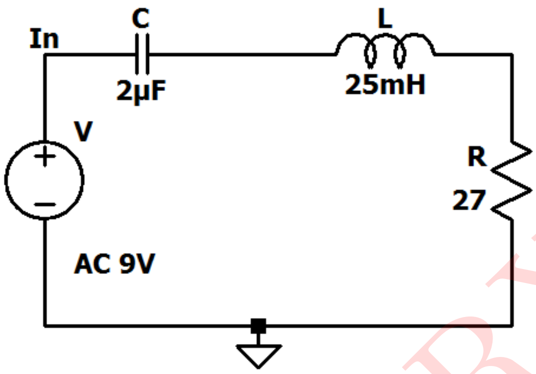


Figure 35: Circuit 9

Solution:

a) Calculating Resonant Frequency (f_R)

We have R = 27Ω, C = 2μF, L = 25mH and V = 9V.

$$\begin{aligned} V_{(peak)} &= V_{RMS}\ \sqrt{2} \\ &= 9 \times \sqrt{2} \\ &= 12.727V \end{aligned}$$

$$\begin{aligned} \therefore f_R &= \frac{1}{2\pi\sqrt{LC}} \\ &= \frac{1}{2\pi\sqrt{25 \times 10^{-3} \times 2 \times 10^{-6}}} \\ &= 711.76\text{Hz} \end{aligned}$$

$\therefore$ **Resonant Frequency = $\mathbf{f_R}$ = 711.76 Hz**

b) Calculating the current at resonance

$$\begin{aligned} \therefore I_{RMS} &= \frac{V}{R} \\ &= \frac{9}{27} \\ &= 333.333\text{mA} \end{aligned}$$

$$\begin{aligned} \therefore I_{(peak)} &= I_{RMS} \times \sqrt{2} \\ &= 333.333 \times \sqrt{2} \\ &= 471.404\text{mA} \end{aligned}$$

$\therefore$ **$\mathbf{I_{(peak)}}$ = 2.9333 ∠43.4513° A**

c) Calculating the voltage across inductor and capacitor at resonance

Inductive reactance $X_L = 2\pi f_R \, L$

$$= 2 \times \pi \times 711.76 \times 25 \times 10^{-3}$$

$$= 111.8037\Omega$$

At resonance $X_L = X_C$

$\therefore X_C = 111.8037\Omega$

At resonance,

$$V_L = V_C = I_{RMS} \times X_L$$

$$= 0.3333 \times 111.8037$$

$$= 37.2641V$$

$$\therefore V_{L_{(peak)}} = V_{C_{(peak)}} = V_L \times \sqrt{2}$$

$$= 37.2641 \times \sqrt{2}$$

$$= 52.6995V$$

$\therefore \mathbf{V_{L_{(peak)}} = V_{C_{(peak)}} = 52.6995\ V}$

d) Calculating the Quality Factor (Q)

$$Q = \frac{X_L}{R}$$

$$= \frac{111.8037}{27}$$

$$= 4.14088$$

$\therefore \mathbf{Q = 4.14088}$

d) Calculating the Bandwidth (BW)

$$BW = \frac{f_R}{Q}$$

$$= \frac{711.76}{4.14088}$$

$$= 171.8862Hz$$

$\therefore \mathbf{BW = 171.8862\ Hz}$

SIMULATED RESULTS:

The given circuit is simulated in LTspice and the results obtained are as follows:

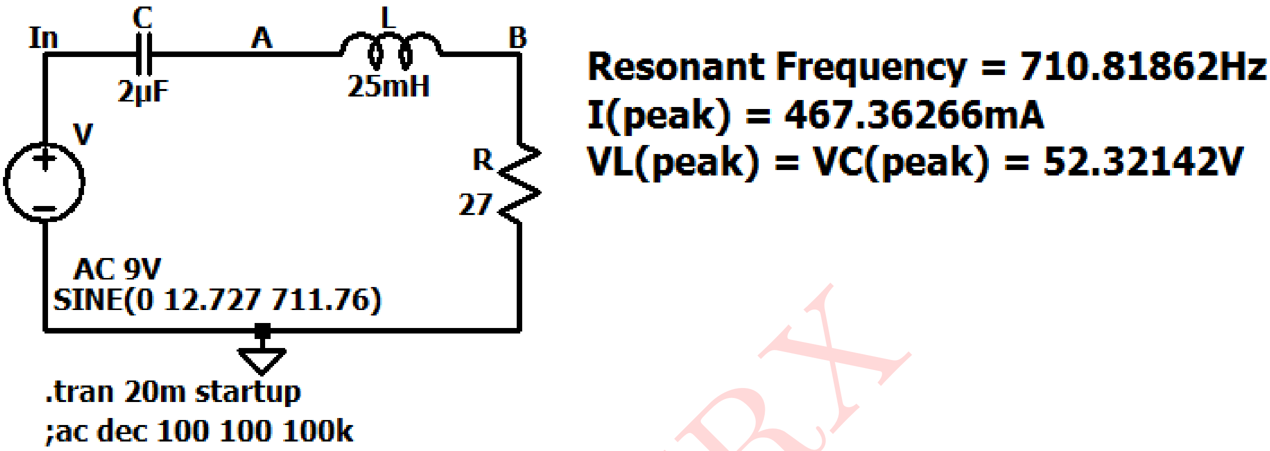


Figure 36: Circuit Schematic and Simulated Results

Plot of Resonant Frequency is shown in figure 37

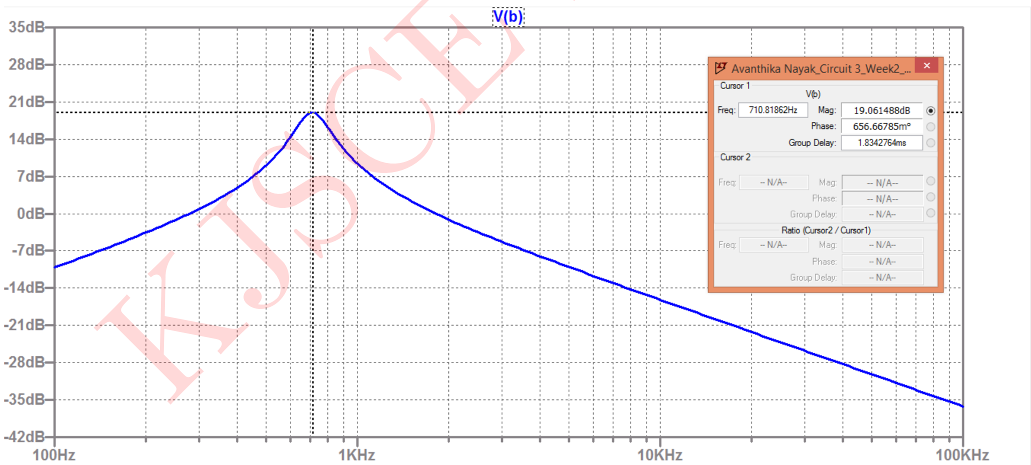


Figure 37: Resonant Frequency

Plot of V_L and V_C at resonance is shown in figure 38

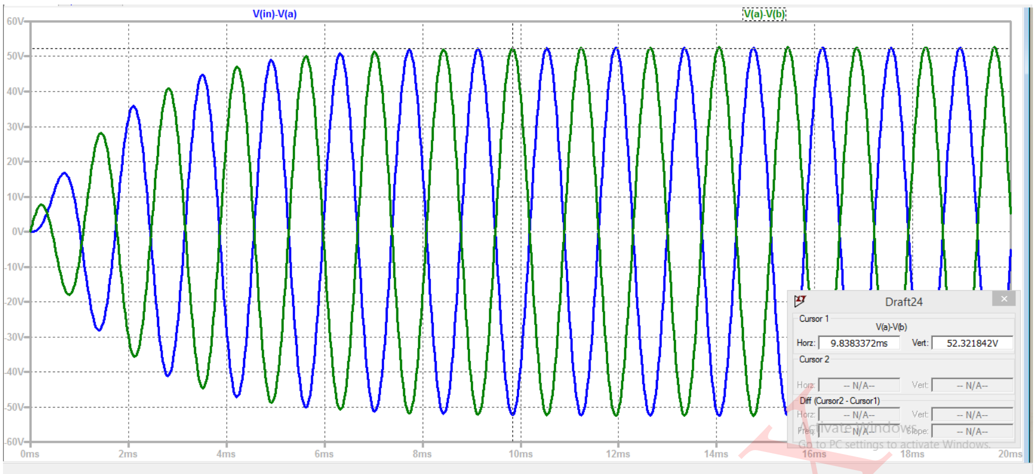


Figure 38: V_L and V_C at resonance

Plot of I at resonance is shown in figure 39

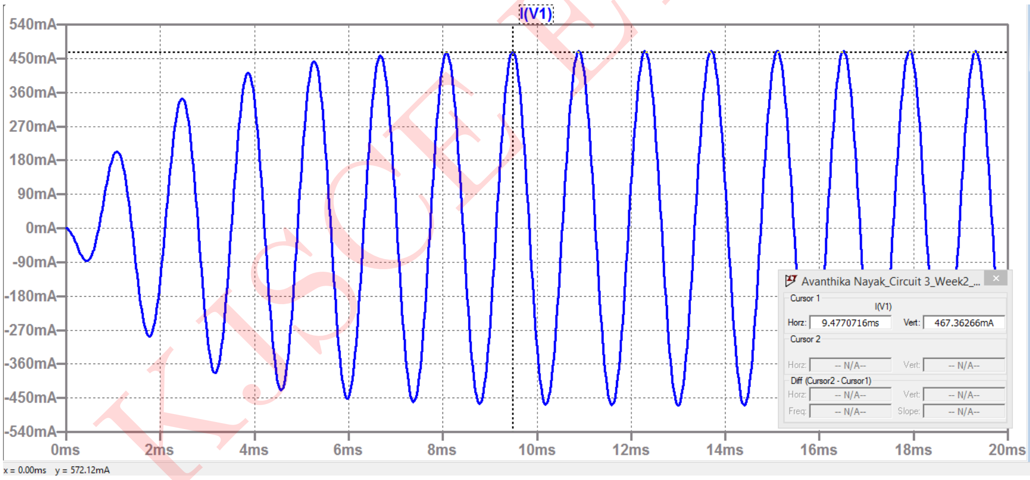


Figure 39: I at resonance

Comparison of Theoretical and Simulated values:-

Parameters	**Theoretical values**	**Simulated values**
$V_{R(peak)}$	52.6995V	52.32142V
$V_{C(peak)}$	52.6995V	52.32142V
I_{peak}	471.404mA	467.36266mA
Resonant Frequency	711.76Hz	710.81826Hz

Table 9: Numerical 9