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DEPARTMENT OF ELECTRONICS ENGINEERING
ELECTRONIC CIRCUITS
Darlington Amplifier

Numerical 1:

For each transistor given below in the figure 1, the parameters are $\beta = 100$ and $V_A = \infty$

a) Determine DC parameters of both the stages

b) Determine overall voltage gain A_{V_T}

c) Determine overall current gain A_{i_T}

d) Determine input impedance of 1st stage

e) Determine output impedance of 2nd stage

f) Determine the output voltage

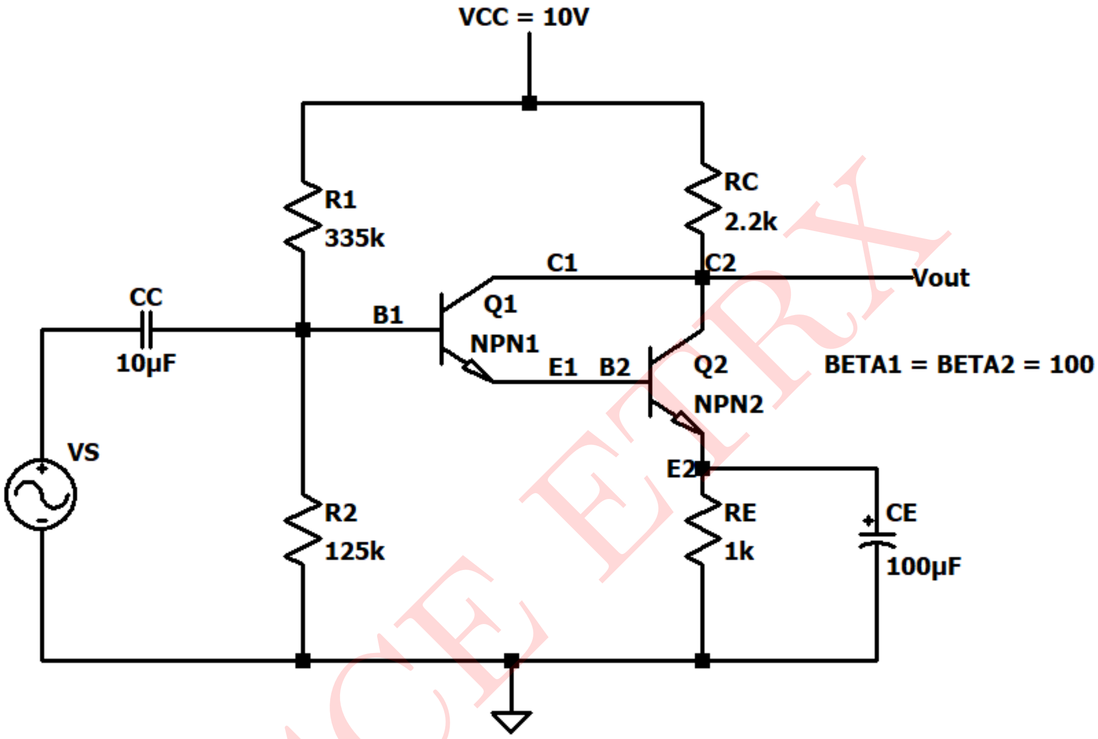

Figure 1: Circuit 1

Solution:

DC analysis:

$$V_{TH} = \frac{R_2}{R_1 + R_2} \times V_{CC} = \frac{125k\Omega}{125k\Omega + 335k\Omega} \times 10V = \mathbf{2.71V}$$

$$R_B = \frac{R_1 R_2}{R_1 + R_2} = \frac{335k\Omega \times 125k\Omega}{335k\Omega + 125k\Omega} = \mathbf{91.03k\Omega}$$

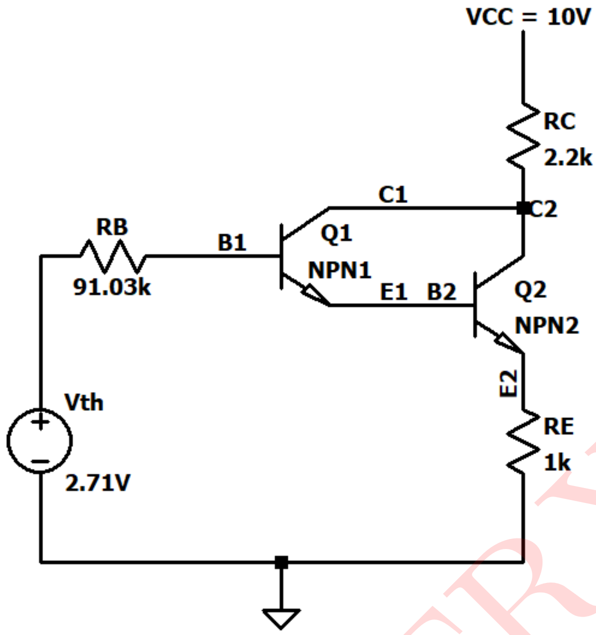

Figure 2: Thevenin's equivalent circuit

Applying KVL to base-emitter loop,

$V_{th} - I_B R_B - V_{BE} - I_E R_E = 0$

i.e $V_{th} - I_B R_B - V_{BE} - (\beta + 1) I_B R_E = 0$

$\because V_{BE} = V_{BE_1} + V_{BE_2}$

$\quad = 0.7V + 0.7V = \mathbf{1.4V}$

Applying KVL to base-emitter loop,

$V_{th} - I_{B_1} R_B - V_{BE} - I_{E_1} R_E = 0$

i.e $V_{th} - I_{B_1} R_B - V_{BE} - (\beta_1 + 1)(\beta_2 + 1) I_{B_1} R_E = 0$

i.e $V_{th} - I_{B_1} R_B - V_{BE} - (\beta_1 + 1)^2 I_{B_1} R_E = 0$

$\therefore 2.71V - I_{B_1}(91.03k\Omega) - 1.4V - (101)^2 I_{B_1}(1k\Omega) = 0$

$\therefore 2.71V - 1.4V - I_{B_1}(91.03k\Omega + 10201k\Omega) = 0$

$$I_{B_1} = \frac{1.3V}{91.03k\Omega + 10201k\Omega} = \mathbf{0.1263\mu A}$$

$\therefore I_{C_1} = \beta_1 I_{B_1}$

$\quad = (100)(0.1263\mu A) = \mathbf{0.01263mA}$

$\therefore I_{E_1} = I_{C_1} + I_{B_1}$

$\quad = 0.01263mA + 0.01263\mu A = \mathbf{0.01275mA}$

$\because I_{E_1} = I_{B_2} = \mathbf{0.01275mA}$

$\therefore I_{C_2} = \beta_2 I_{B_2}$

$\quad = (100)(0.01275\mu A) = \mathbf{1.275mA}$

$$\therefore I_{E_2} = I_{C_2} + I_{B_2}$$
$$= 1.275mA + 0.01275\mu A = \mathbf{1.2877mA}$$

$$\because V_{E_2} = I_{E_2} R_E$$
$$= 1.2877mA \times 1k\Omega = \mathbf{1.2688V}$$

$$\because V_{C_2} = V_{CC} - I_{C_2} R_C$$
$$= 10V - (1.275mA)(2.2k\Omega) = \mathbf{7.195V}$$

$$\because V_{CE_2} = V_{C_2} - V_{E_2}$$
$$= 7.195V - 1.2877V = \mathbf{5.9073V}$$

Small signal parameters:

$$r_{\pi_1} = \frac{\beta_1 V_T}{I_{C_1}}$$
$$= \frac{100 \times 26mV}{0.01263mA} = \mathbf{205.859k\Omega}$$
$$r_{\pi_2} = \frac{\beta_2 V_T}{I_{C_2}}$$
$$= \frac{100 \times 26mV}{1.275mA} = \mathbf{2.039k\Omega}$$
$$r_{o_1} = \infty$$
$$r_{o_2} = \infty$$
$$g_{m_1} = \frac{I_{C_1}}{V_T}$$
$$= \frac{0.01263mA}{26mV} = \mathbf{0.048mA/V}$$
$$g_{m_2} = \frac{I_{C_2}}{V_T}$$
$$= \frac{1.275mA}{26mV} = \mathbf{49.03mA/V}$$

Mid-frequency AC equivalent circuit:

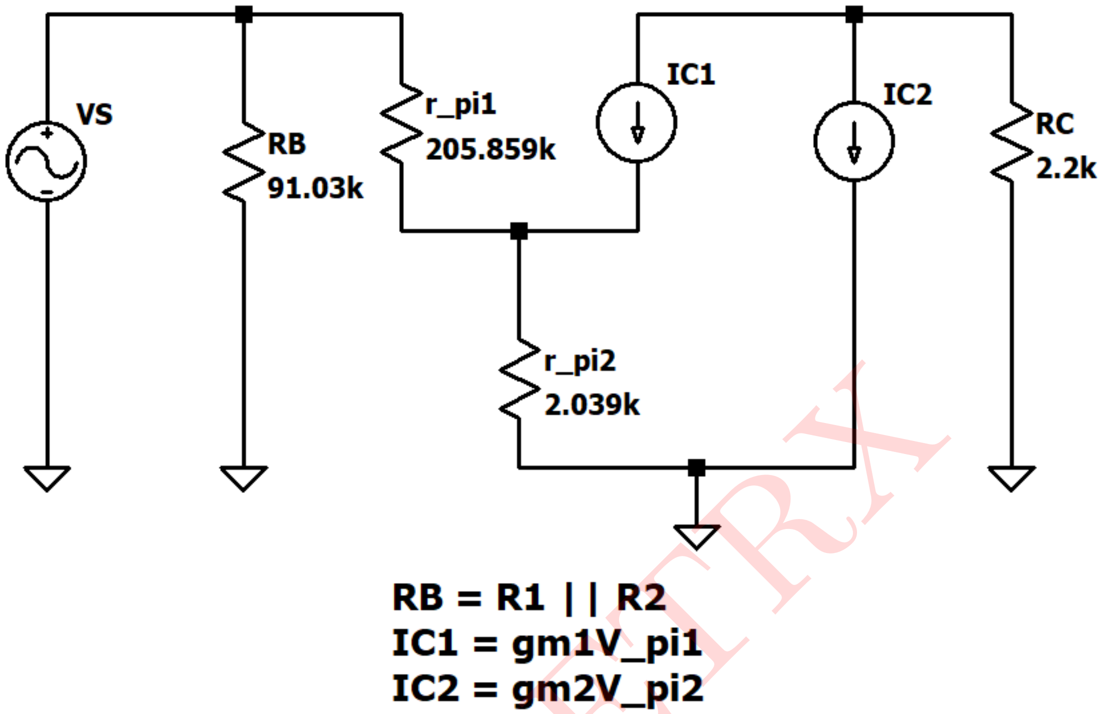

Figure 3: Mid-frequency AC equivalent circuit

$$
\begin{aligned}
Z_{i_1} &= r_{\pi_1} + (1+\beta_1)r_{\pi_2} \\
&= 205.859k\Omega + (1+100)(2.039k\Omega) \\
&= 205.859k\Omega + 205.939k\Omega = \mathbf{411.798k\Omega}
\end{aligned}
$$

Input impedance of 1st stage,

$$
\begin{aligned}
Z_i &= R_B \parallel Z_{i_1} \\
&= 91.03k\Omega \parallel 411.798k\Omega \\
&= \frac{91.03k\Omega \times 411.798k\Omega}{91.03k\Omega + 411.798k\Omega} = \mathbf{74.550k\Omega}
\end{aligned}
$$

$$
\begin{aligned}
Z_{o_1} &= \frac{R_B + r_{\pi_1}}{1+\beta_1} \\
&= \frac{91.03k\Omega + 205.859k\Omega}{1+100} = \mathbf{2.93k\Omega}
\end{aligned}
$$

$$
\begin{aligned}
Z_{o_2} &= \frac{Z_{o_1} + r_{\pi_2}}{1+\beta_2} \\
&= \frac{2.93k\Omega + 2.039k\Omega}{1+100} = \mathbf{0.04k\Omega}
\end{aligned}
$$

Output impedance of 2nd stage,

$$Z_o = Z_{o_2} \parallel R_C$$

$$
\begin{aligned}
Z_o &= 0.04k\Omega \parallel 2.2k\Omega \\
&= \frac{0.04k\Omega \times 2.2k\Omega}{0.04k\Omega + 2.2k\Omega} = \mathbf{0.039k\Omega}
\end{aligned}
$$

Current gain of 1st stage,

$$A_{i_1} = \frac{I_{C_1}}{I_{B_1}} = \frac{\beta_1 I_{B_1}}{I_{B_1}} = \beta_1 = \mathbf{100}$$

Current gain of 1st stage,

$$A_{i_1} = \frac{I_{C_2}}{I_{B_2}} = \frac{\beta_2 I_{B_2}}{I_{B_2}} = \beta_2 = \mathbf{100}$$

$$\therefore A_{i_t} = A_{i_1} \times A_{i_2} = 100 \times 100 = \mathbf{10000}$$

Overall current gain,

$$\begin{aligned} A_{i_S} &= \frac{I_o}{I_i} \\ &= \frac{I_o}{I_{B_2}} \times \frac{I_{B_2}}{I_{B_1}} \times \frac{I_{B_1}}{I_i} \\ &= A_{i_2} \times A_{i_1} \times \frac{I_{B_1}}{I_i} \qquad (\because I_{B_2} = I_{E_1}, I_{E_1} \approx I_{C_1}) \\ &= A_{i_2} \times A_{i_1} \times \frac{R_B}{R_B + Z_{i_2}} \qquad \left(\because \frac{I_{B_1}}{I_i} = \frac{R_B}{R_B + Z_{i_2}}\right) \\ &= 100 \times 100 \times \frac{91.03k\Omega}{91.03k\Omega + 411.798k\Omega} = \mathbf{1810.036} \end{aligned}$$

$$\begin{aligned} A_{V_1} &= \frac{V_1}{V_S} \\ &= \frac{I_{E_1}}{I_{B_1}} \times \frac{r_{\pi 2}}{Z_{i_1}} \\ &= \frac{I_{C_1}}{I_{B_1}} \times \frac{r_{\pi 2}}{Z_{i_1}} \qquad (\because I_{E_1} \approx I_{C_1}) \\ &= A_{i_1} \times \frac{r_{\pi 2}}{Z_{i_1}} \\ &= 100 \times \frac{2.039k\Omega}{411.798k\Omega} = \mathbf{10.49} \end{aligned}$$

$$\begin{aligned} A_{V_2} &= \frac{V_o}{V_1} \\ &= \frac{I_o}{I_{B_2}} \times \frac{R_C}{r_{\pi 2}} \\ &= \frac{I_{C_2}}{I_{B_2}} \times \frac{R_C}{r_{\pi 2}} \end{aligned}$$

$$= A_{i_2} \times \frac{R_C}{r_{\pi_2}}$$

$$= 100 \times \frac{2.2k\Omega}{20 = .039k\Omega} = \mathbf{107.89}$$

$$A_{V_T} = A_{V_1} \times A_{V_2}$$

$$= 0.49 \times 107.89 = \mathbf{52.86}$$

$$\text{Also, } A_{V_T} = \frac{V_{out}}{V_S}$$

$$\therefore V_{out} = A_{V_T} \times V_S$$

$$= 52.86 \times 20mV = \mathbf{1.057V}$$

SIMULATED RESULTS:

Above circuit is simulated in LTspice and results are as follows:

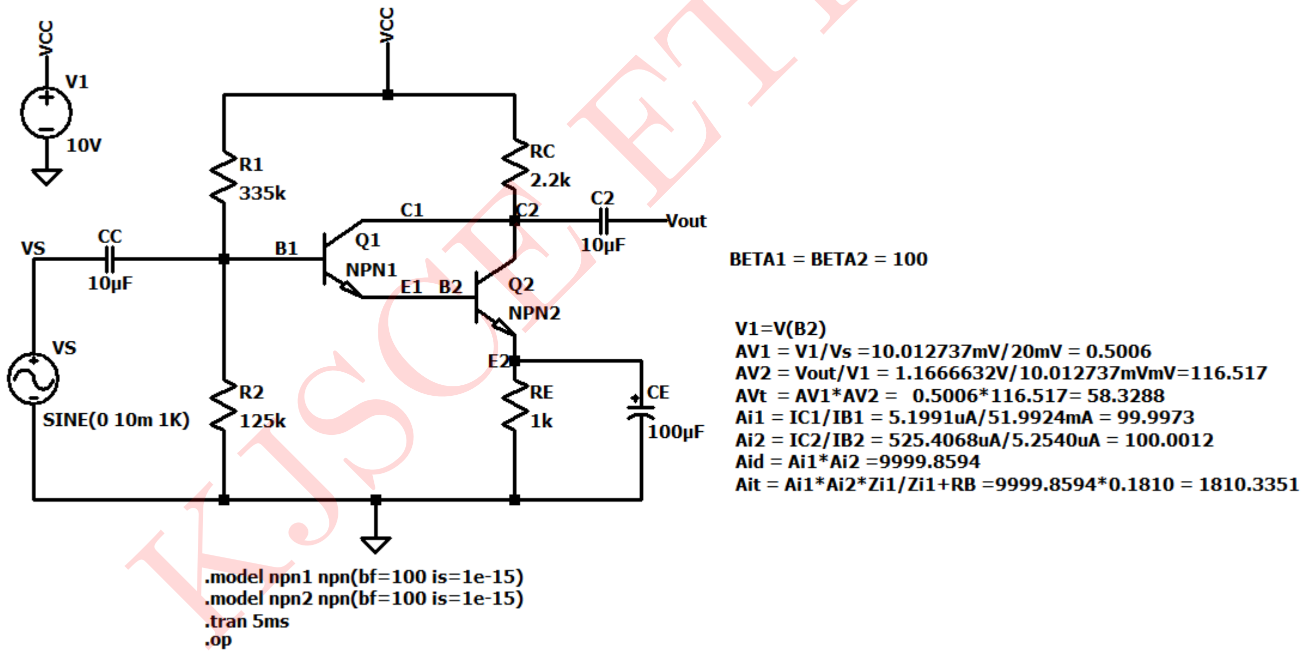

Figure 4: Circuit Schematic 1

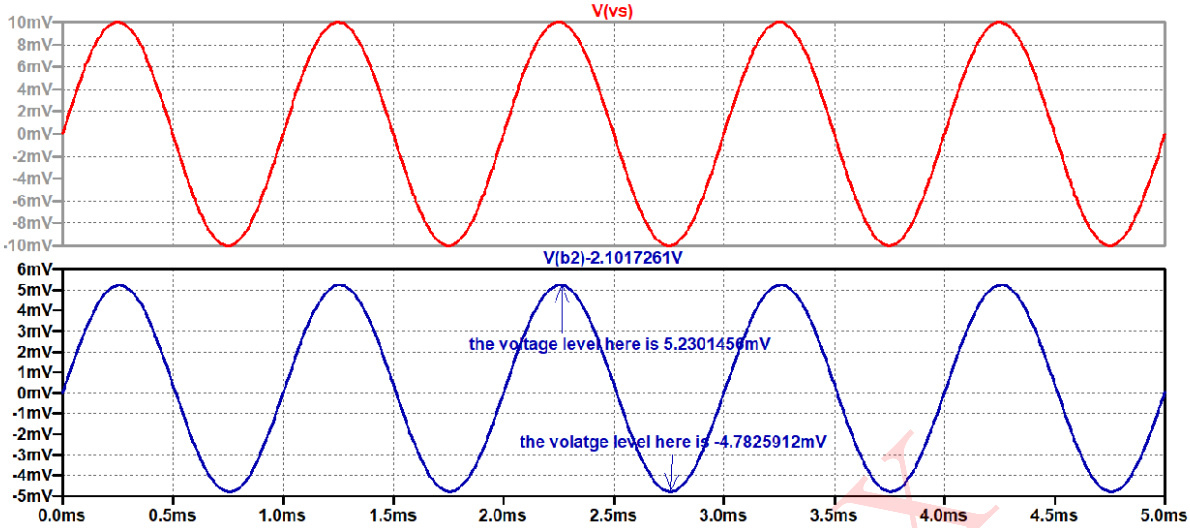

Figure 5: Input and ouput waveforms for Stage 1 voltage gain

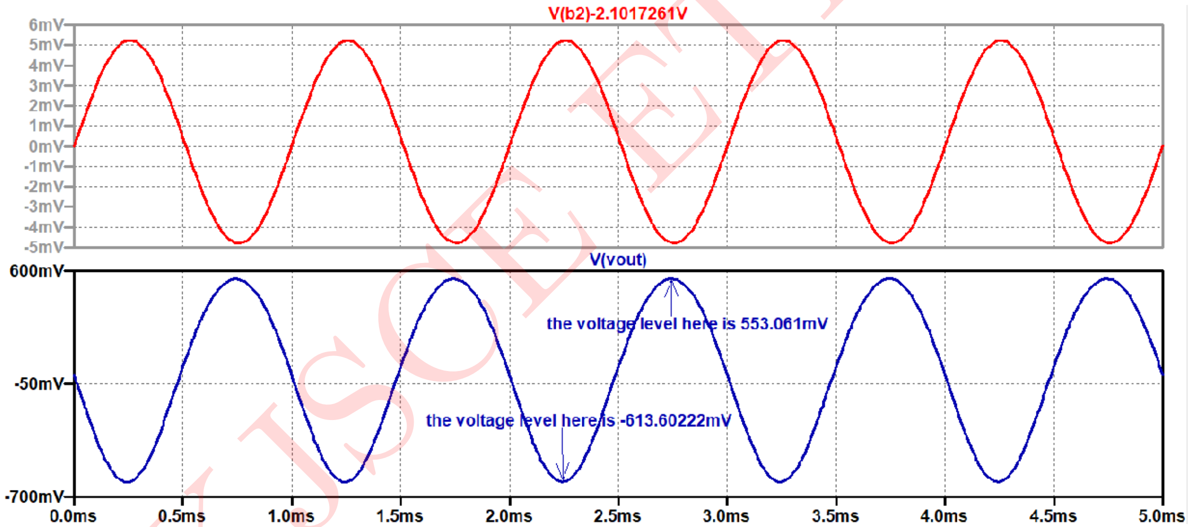

Figure 6: Input and ouput waveforms for Stage 2 voltage gain

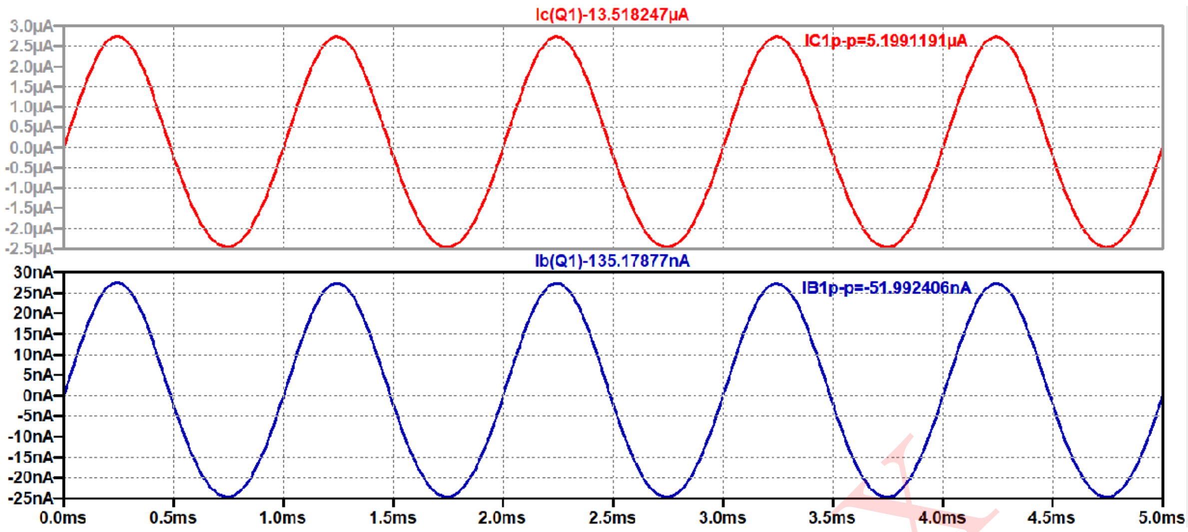

Figure 7: Input and ouput waveforms for Stage 1 current gain

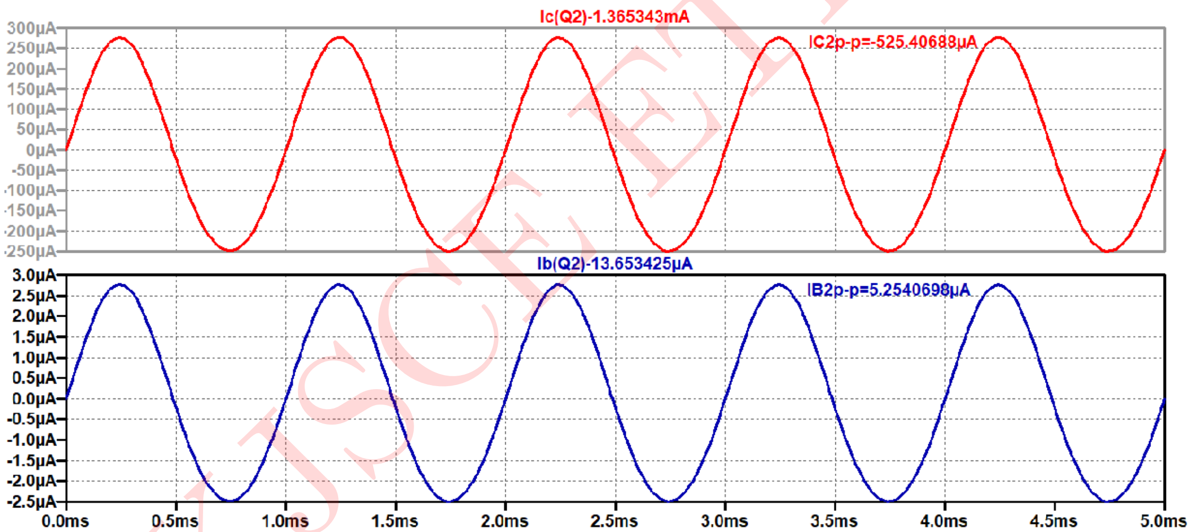

Figure 8: Input and ouput waveforms for Stage 2 current gain

Comparison between theoretical and simulated values:

Parameters	Theoretical values	Simulated values
1st stage DC parameters: I_{B_1}, I_{C_1}, I_{E_1}	0.1263μA, 0.1263mA, 0.1275mA	0.1351μA, 0.1351mA, 0.1365mA
2nd stage DC parameters: I_{B_2}, I_{E_2}, V_{E_2}, V_{C_2}	0.1275mA, 1.2877mA, 1.2877V, 7.195V	0.1365mA, 1.379mA, 1.379V, 6.9665V
Voltage gain of 1st stage A_{V_1}	0.49	0.5006
Voltage gain of 2nd stage A_{V_2}	107.89	116.517
Overall voltage gain A_{V_T}	52.86	58.3288
Current gain of 1st stage A_{i_1}	100	99.9973
Current gain of 2nd stage A_{i_2}	100	100.0012
Overall current gain A_{i_T}	10000	9999.8594
Input impedance of 1st stage	75.550kΩ	–
Output impedance of 2nd stage	0.039kΩ	–
Output voltage	1.057V	1.1666632V

Table 1: Numerical 1

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