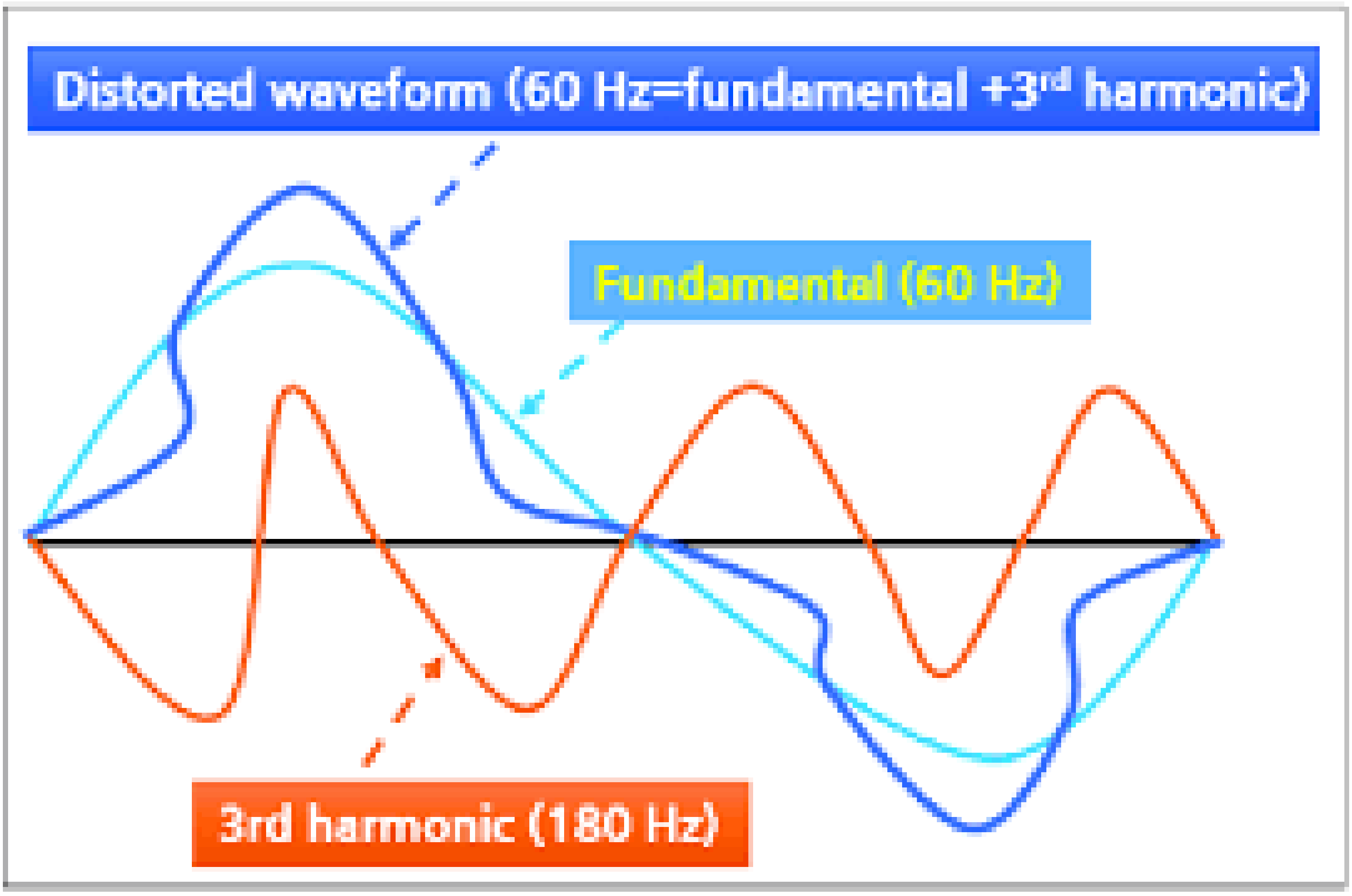
Waveform Distortion

Waveform distortion occurs when the received signal is not identical in shape to the transmitted signal. This is due to the non-ideal behavior of the transmission line parameters (R, L, G, C), which are functions of frequency.

There are two primary types of distortion:

1. **Frequency Distortion (or Attenuation Distortion):**
   - **Cause: The attenuation constant (α) is a function of frequency (ω).**
   - **Effect: Different frequency components of the signal are attenuated (reduced in amplitude) by different amounts, altering the overall waveform shape.**
2. **Phase Distortion (or Delay Distortion):**
   - **Cause: The phase velocity ($v_p = \omega/\beta$) is a function of frequency.**
   - **Effect: Different frequency components travel at different speeds, arriving at the receiver at different times (non-uniform delay), which smears the composite signal and changes its shape.**



The Distortion-less Line

A line is called distortion-less if the shape of the transmitted signal remains unchanged during propagation. This requires that both frequency distortion and phase distortion are eliminated simultaneously.

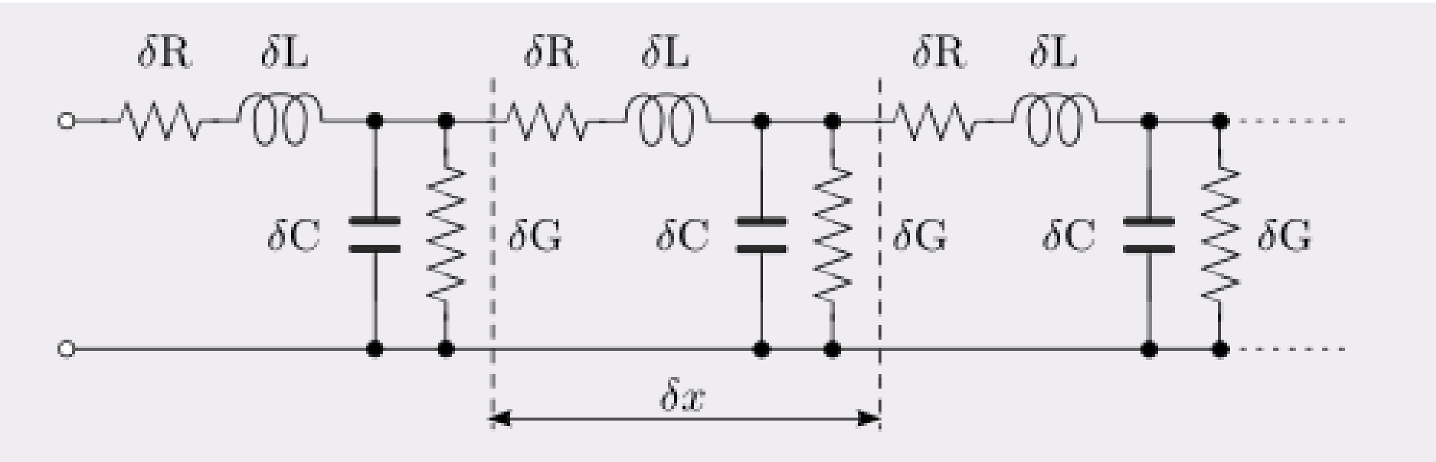
The two necessary conditions for a distortion-less line are:

1. The Attenuation Constant (α) must be independent of frequency (ω).
2. The Phase Constant (β) must be linearly proportional to frequency (ω).

Practical Significance (Heaviside's Condition)

The distortion-less condition was historically significant. Oliver Heaviside (19th century) showed that this condition would make long-distance telephony and telegraphy practical.

- In typical low-frequency cables, the inductance (L) is small, and the condition $R/L = G/C$ is rarely met (usually $R/L \gg G/C$).
- The practical solution to achieve the distortion-less condition is by increasing the inductance (L) of the line. This is known as loading, often implemented by inserting lumped loading coils periodically along the line.



Derivation and Problem Solving

A. Derivation of the Distortion-less Condition

The propagation constant (γ) of a transmission line is given by:

$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Step 1: Expand the Expression

Factor out L and C from the respective terms:

$$\gamma = \sqrt{LC \left(\frac{R}{L} + j\omega\right) \left(\frac{G}{C} + j\omega\right)}$$

Step 2: Apply the Condition for Distortion-less Line

For the line to be distortion-less, the phase constant (β) must be linear with frequency (ω), and the attenuation constant (α) must be independent of ω. This is achieved if the ratios of R/L and G/C are equal:

$$\mathbf{\frac{R}{L} = \frac{G}{C} \quad \text{or} \quad RC = LG}$$

Step 3: Substitute the Condition into γ

Let $\frac{R}{L} = \frac{G}{C} = A$ (where A is a constant).

$$\gamma = \sqrt{LC (A + j\omega)(A + j\omega)}$$

$$\gamma = \sqrt{LC (A + j\omega)^2}$$

$$\gamma = (A + j\omega)\sqrt{LC}$$

$$\gamma = A\sqrt{LC} + j\omega\sqrt{LC}$$

Step 4: Equate Real and Imaginary Parts

Since $\gamma = \alpha + j\beta$, we equate the real and imaginary parts to find the resulting parameters:

Parameter	Resulting Expression	Frequency Dependence
Attenuation Constant (α)	$\alpha = A\sqrt{LC} = \mathbf{\frac{R}{L}\sqrt{LC} = \sqrt{RG}}$	Independent of frequency
Phase Constant (β)	$\beta = \mathbf{\omega\sqrt{LC}}$	Linear function of frequency

The condition $\mathbf{RC = LG}$ satisfies the requirements for a distortion-less line.

B. Resulting Secondary Parameters

When the distortion-less condition ($R/L = G/C$) is satisfied, the other key parameters also simplify:

1. **Phase Velocity (v_p):**
2. $$v_p = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \mathbf{\frac{1}{\sqrt{LC}}}$$
    - **Conclusion: The phase velocity is independent of frequency, ensuring all components arrive at the same time (no phase distortion).**
3. **Characteristic Impedance (Z_0):**
4. $$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$
5. **Substitute $R = G \cdot (L/C)$:**
6. $$Z_0 = \sqrt{\frac{G(L/C) + j\omega L}{G + j\omega C}} = \sqrt{\frac{L(G/C + j\omega)}{C(G + j\omega C)}}$$
7. $$Z_0 = \sqrt{\frac{L(G/C + j\omega)}{C(G/C + j\omega)}} = \mathbf{\sqrt{\frac{L}{C}} = \sqrt{\frac{R}{G}}}$$
    - **Conclusion: The characteristic impedance is a purely real resistance and independent of frequency.**