Switched-capacitor filters

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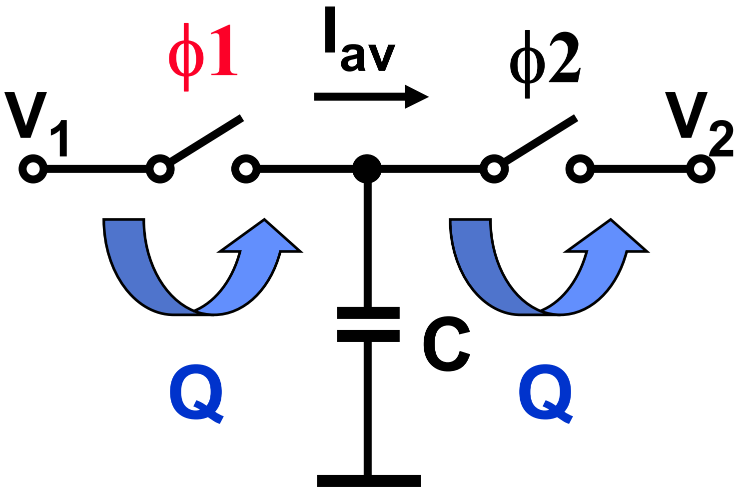
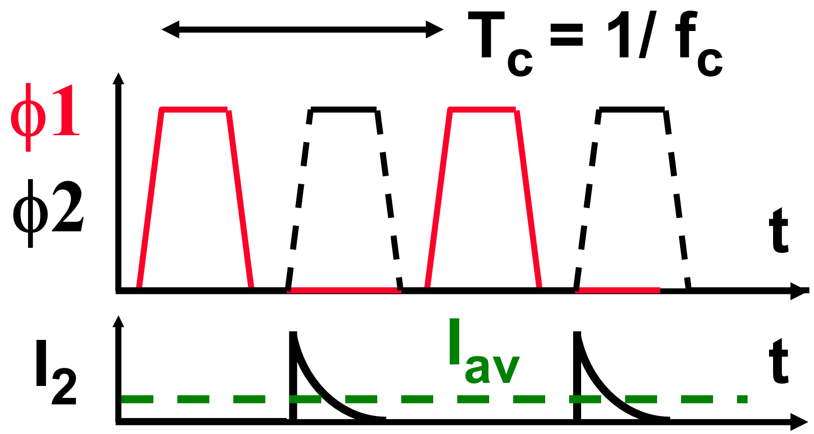
Switched-Capacitor Filters

- Introduction : principle
- Technology:
  - MOS capacitors
  - MOST switches
- SC Integrator
  - SC integrator : Exact transfer function
  - Stray insensitive integrator
  - Basic SC-integrator building blocks
- SC Filters : LC ladder / bi-quadratic section
- Opamp requirements
  - Charge transfer accuracy
  - Noise
- Switched-current filters

McCreary, JSSC Dec 75, 371-379
Gregorian, IEEE Proc. Aug 83, 941-986

Principle





- Non overlapping clocks
- Switches are MOSTs

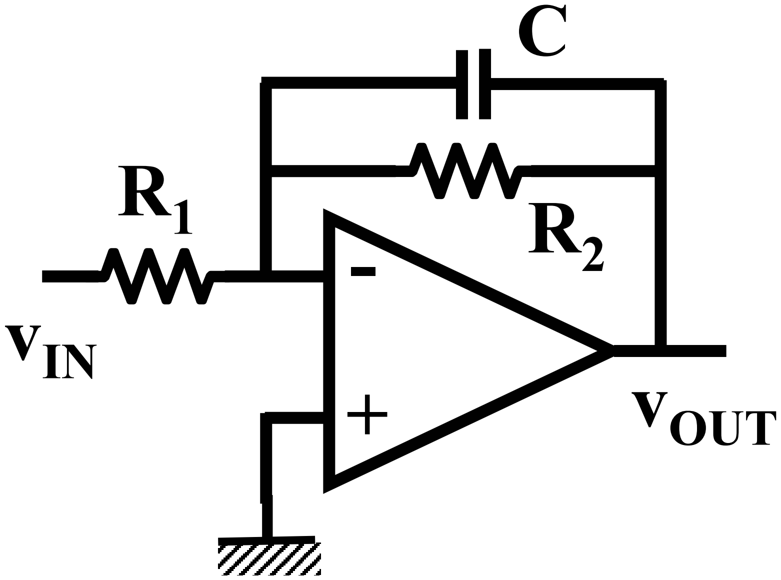
$$I_{av} = \frac{Q_{av}}{T_c} = \frac{C(V_1 - V_2)}{T_c}$$

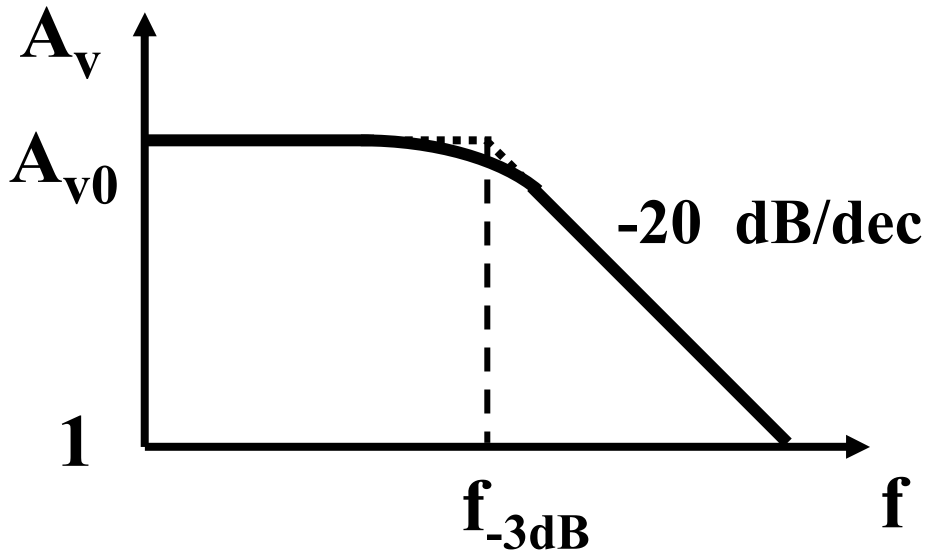
$$I_{av} = \frac{(V_1 - V_2)}{R}$$

$$R = \frac{T_c}{C} = \frac{1}{f_c C}$$

For C = 1 pF & f_c = 100 kHz   R = 10 MΩ

Low-Pass Filter with R's and C





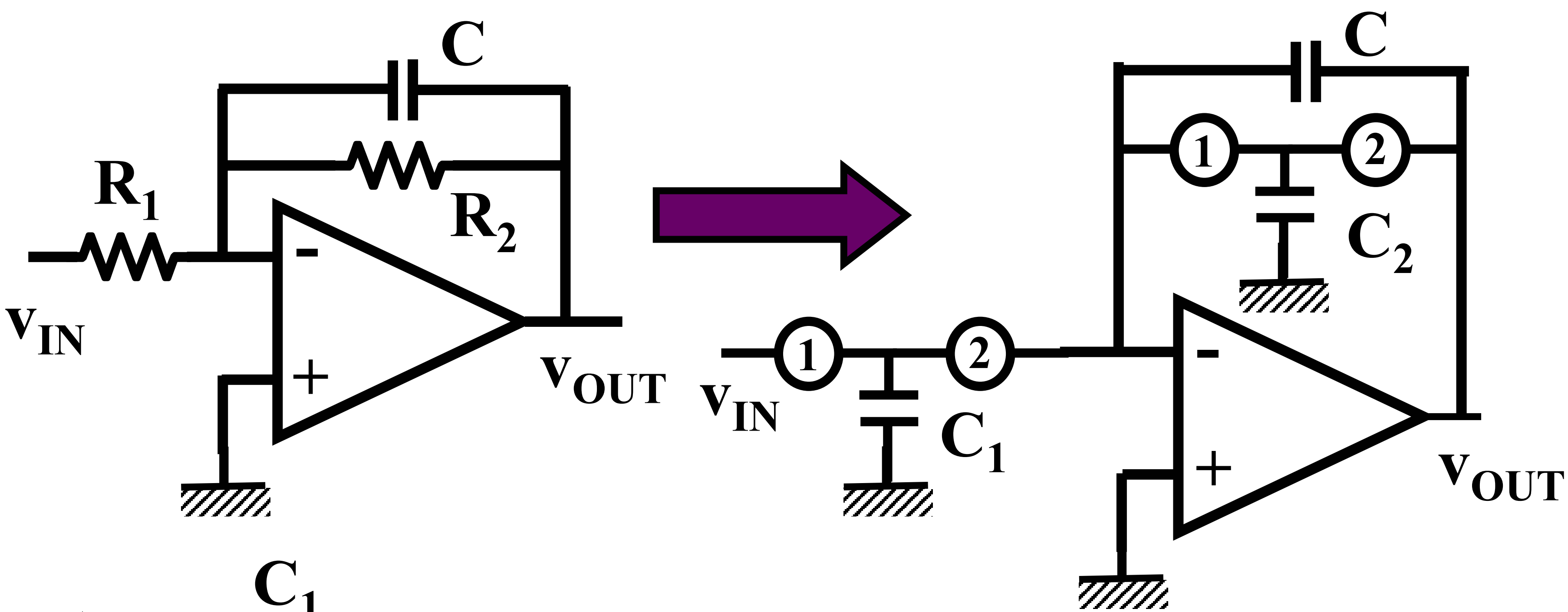
$$A_{v0} = \frac{R_2}{R_1}$$

Ratio's of R: 0.5% accuracy

$$f_{-3db} = \frac{1}{2\pi R_2 C}$$

Absolute value of RC : 20 % accuracy

Low-Pass Filter with switched C's



$$A_{v0} = \frac{C_1}{C_2}$$

$$f_{-3db} = \frac{f_c}{2\pi}\,\frac{C_2}{C}$$

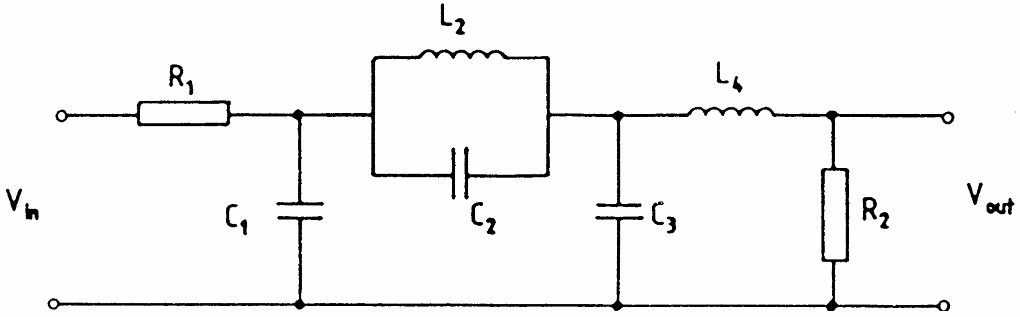
High accuracy: only ratio's of C: 0.2%

Only capacitors to drive : low power !

Tunable & easy to integrate !

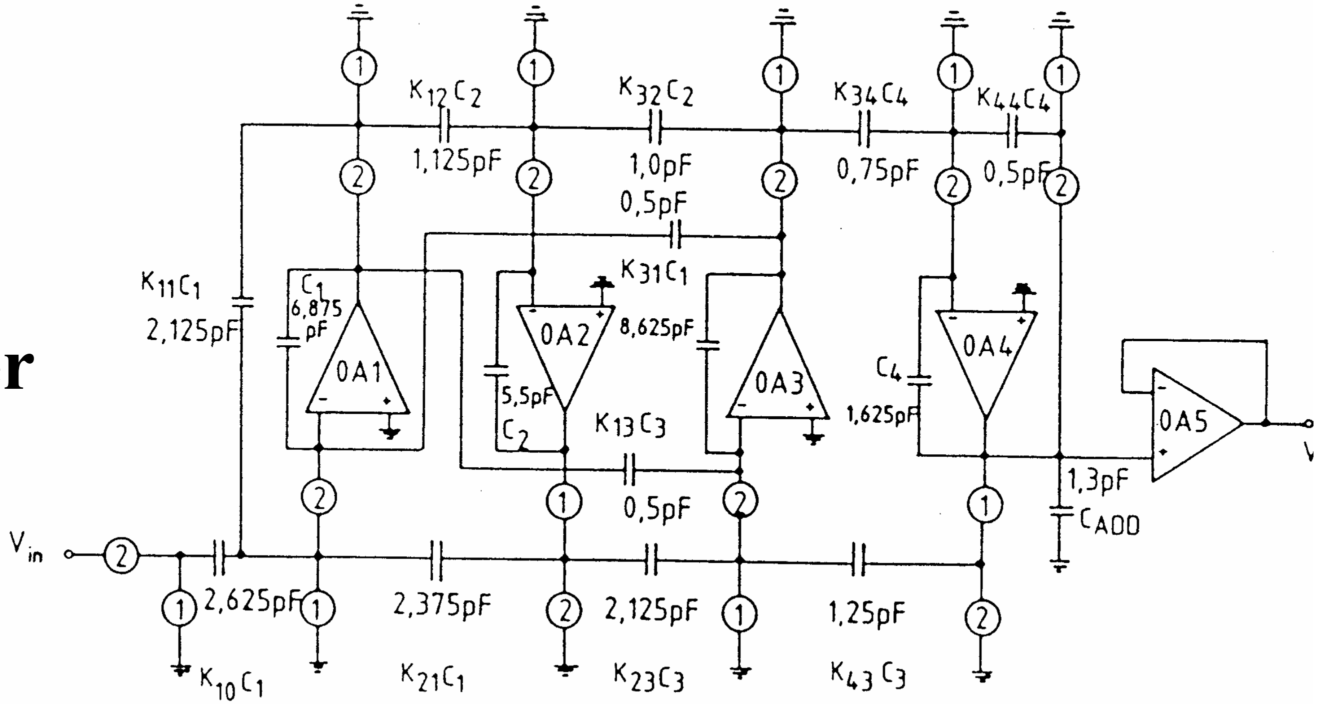
But : only for frequencies << f_c

Example of 4th-Order SC Low-Pass filter

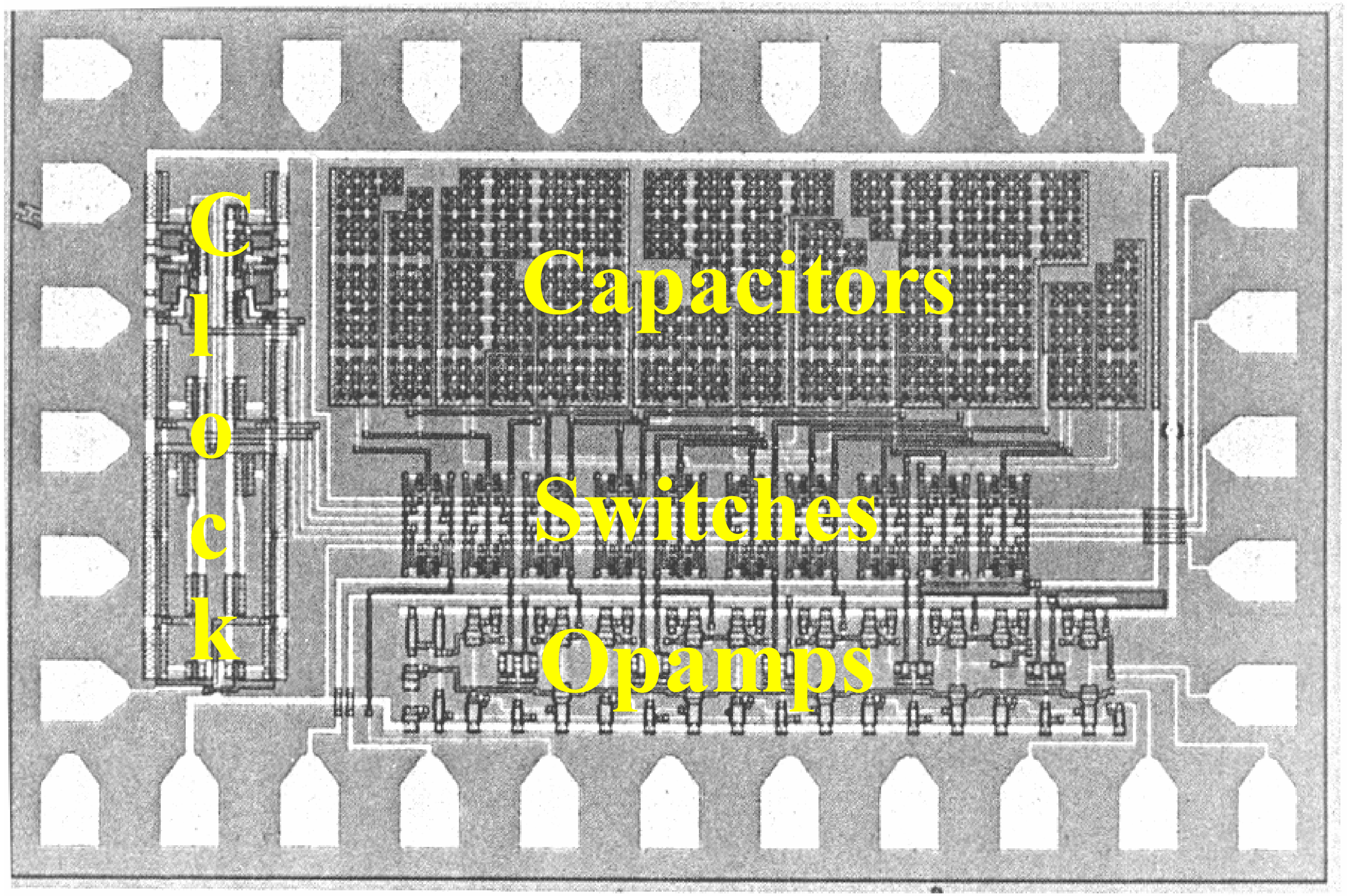


LC proto-type

SC ladder filter



4th-Order SC Low-Pass filter



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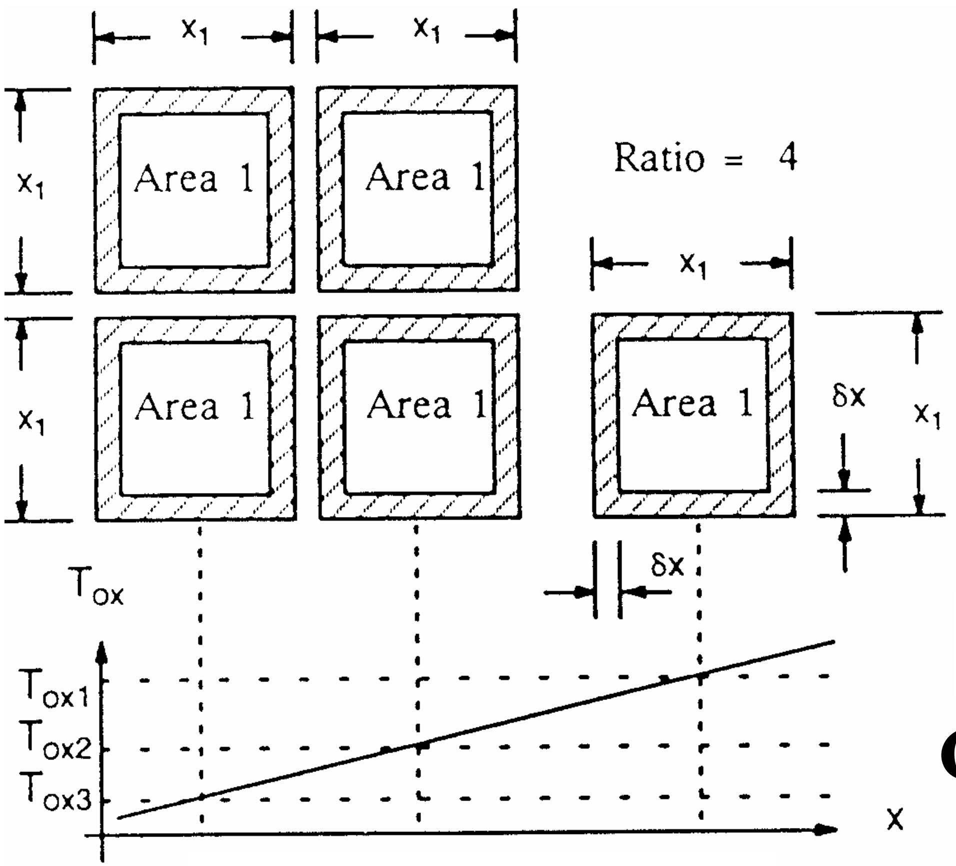
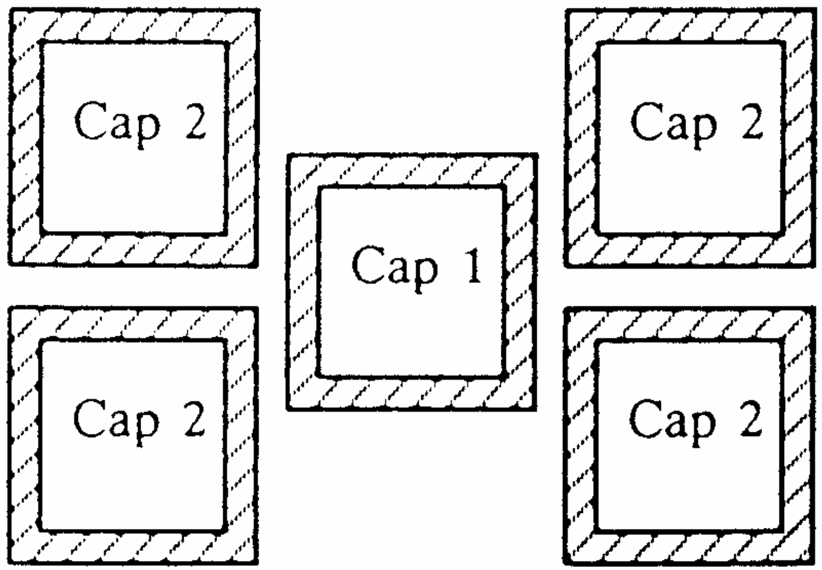
Capacitor Matching

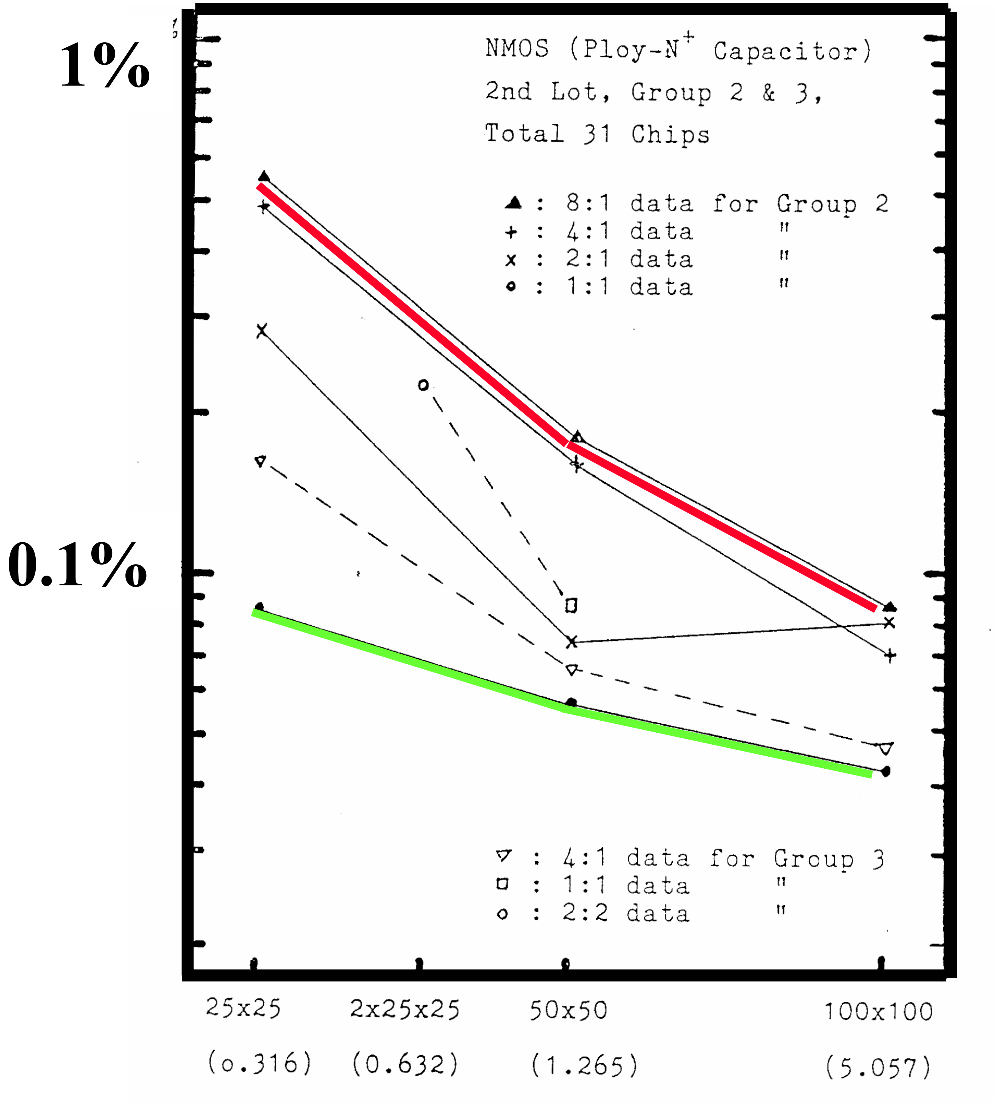


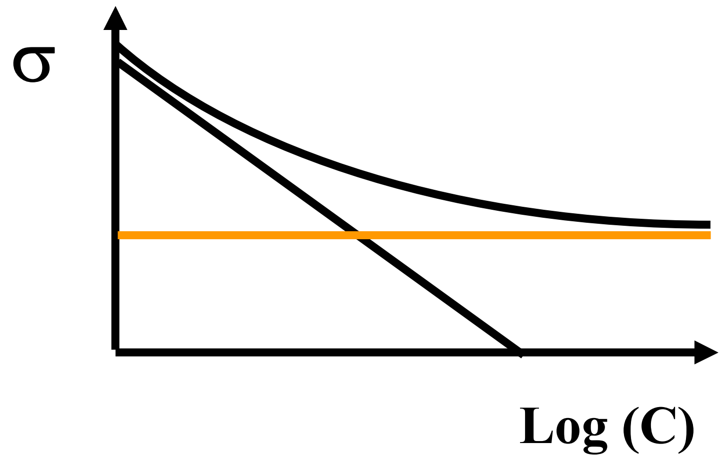


Constant Area / Perimeter !

Common centroide lay-out !

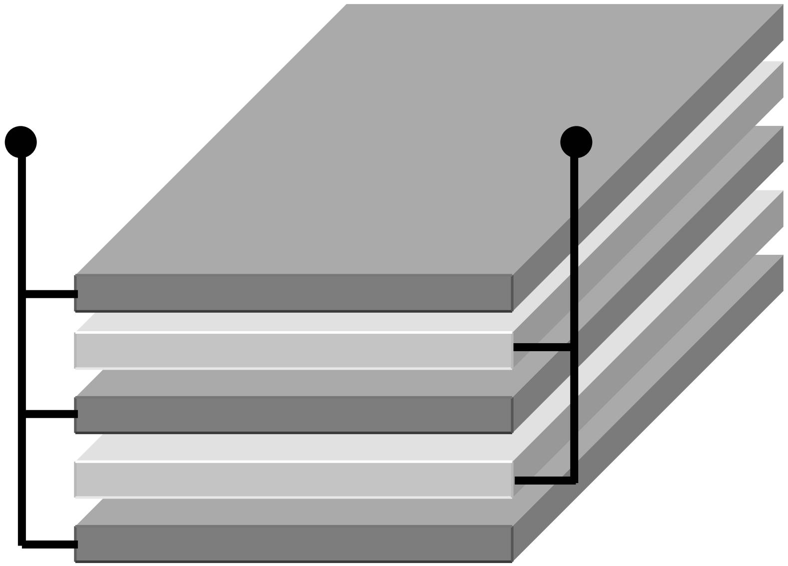
Random Error (σ)

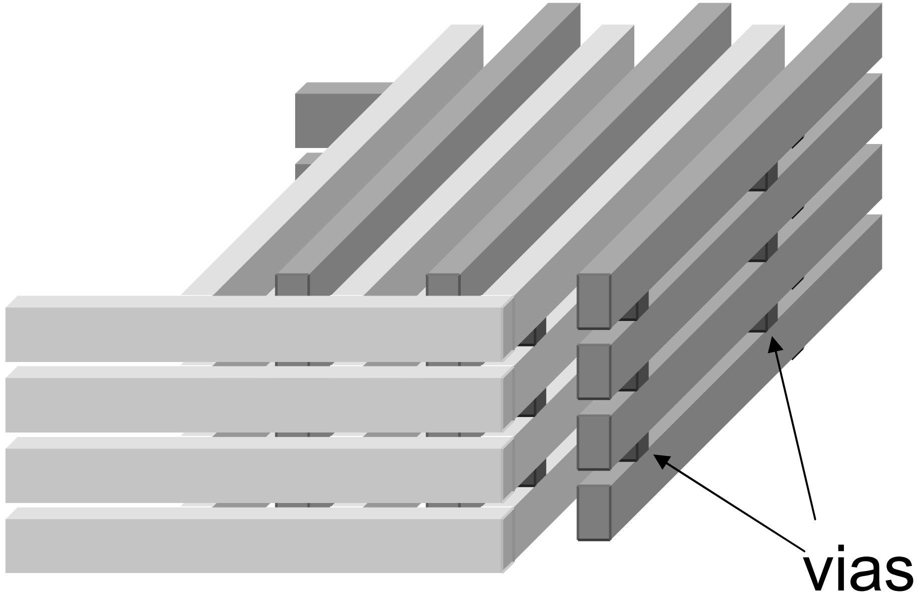




local vs global t_{ox} effects

Capacitances in nanometer CMOS





- MIM capacitors
- 5 metal layers, 0.35 fF/μ m^2
- Excellent matching

- Digital technology, no MIM cap.
- lateral metal-metal capacitance
- 8 metal layers, 1.7 fF/μ m^2
- Good matching

Aparicio, JSSC March 02, 384-393

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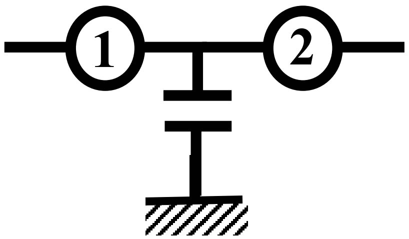
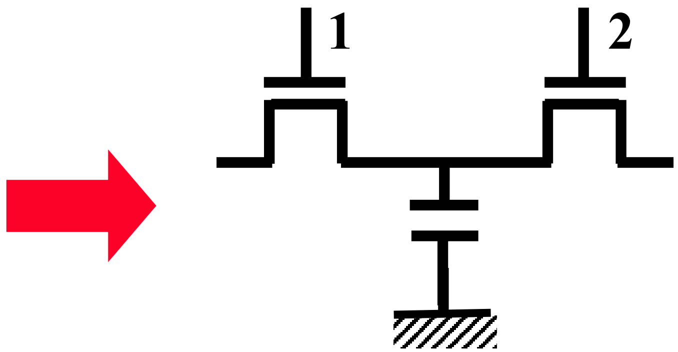
A MOST as a switch





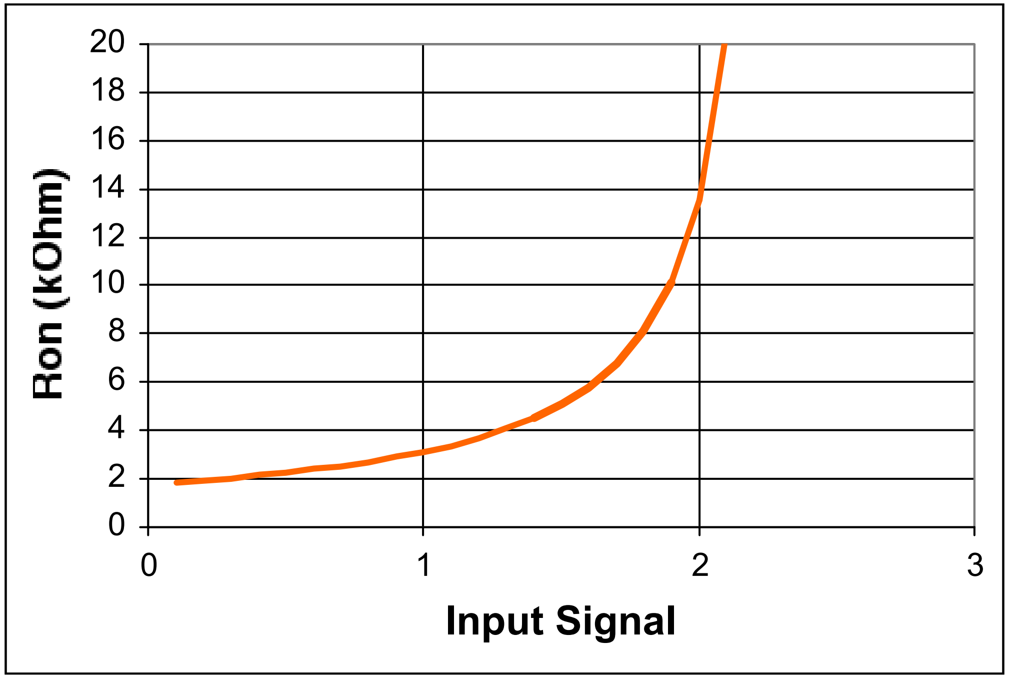
$$R_{on} = \frac{1}{KP_n \frac{W}{L} (V_h - V_T - V_{sign})}$$

$W = 2\ \mu m$ $L = 0.7\ \mu m$

$KP_n = 80\ \mu A/V^2$

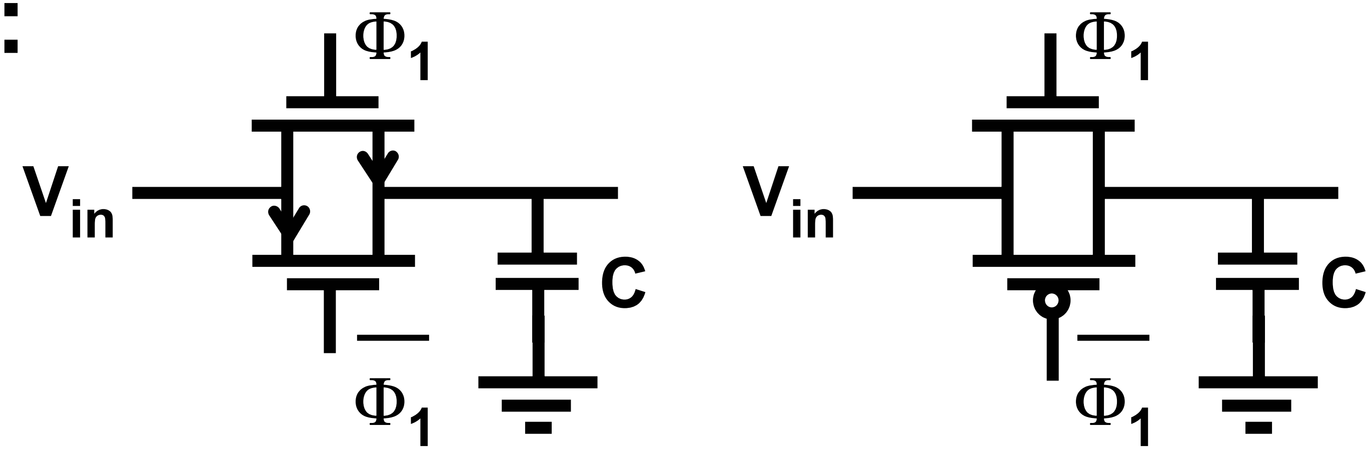
$V_T = 0.7\ V$

$V_h = 3\ V$



Double Switch or transmission gate

Switch:

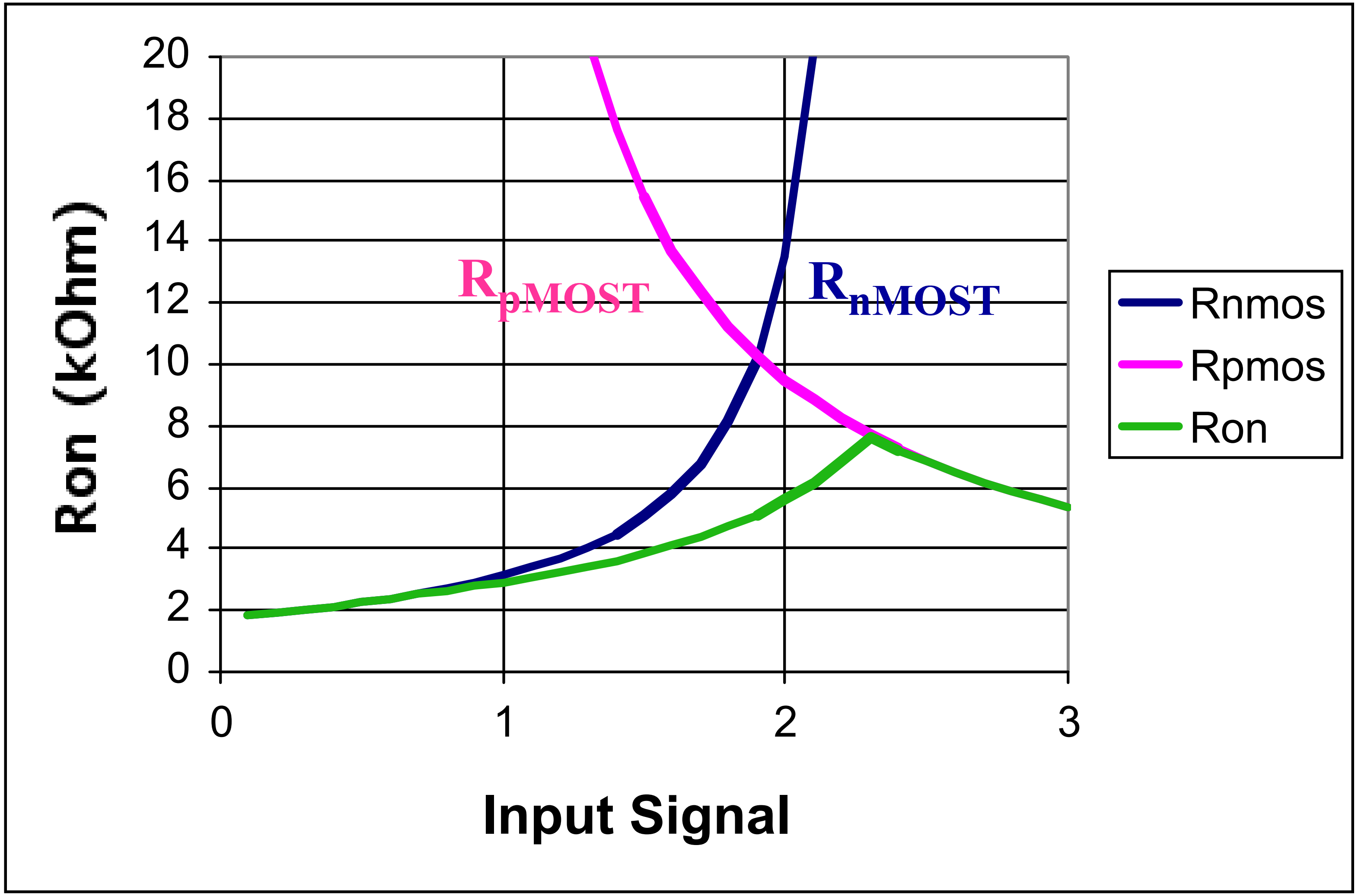


nMOST: V_{in} < V_{DD}-$V_{GS,n} \approx V_{DD}$ - 0.7 V

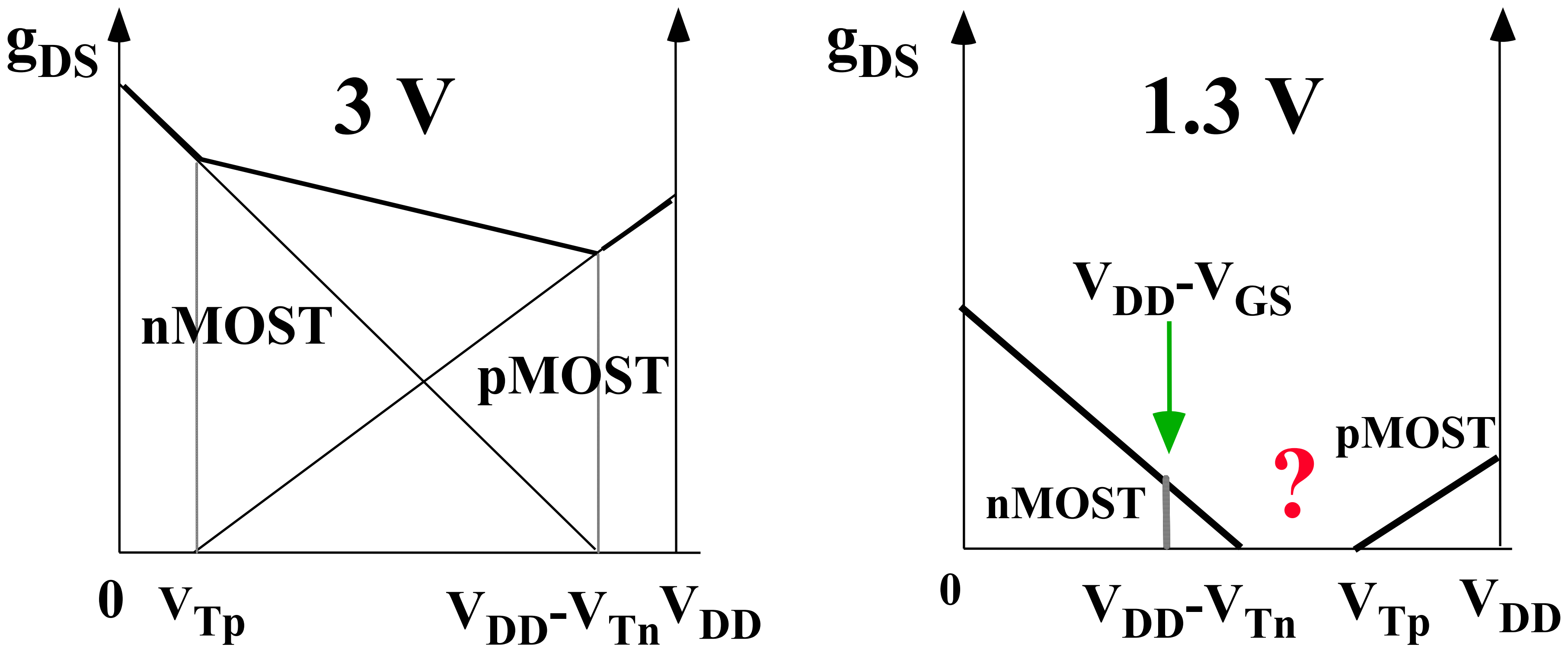
pMOST: V_{in} > $V_{GS,p} \approx$ 0.7 V

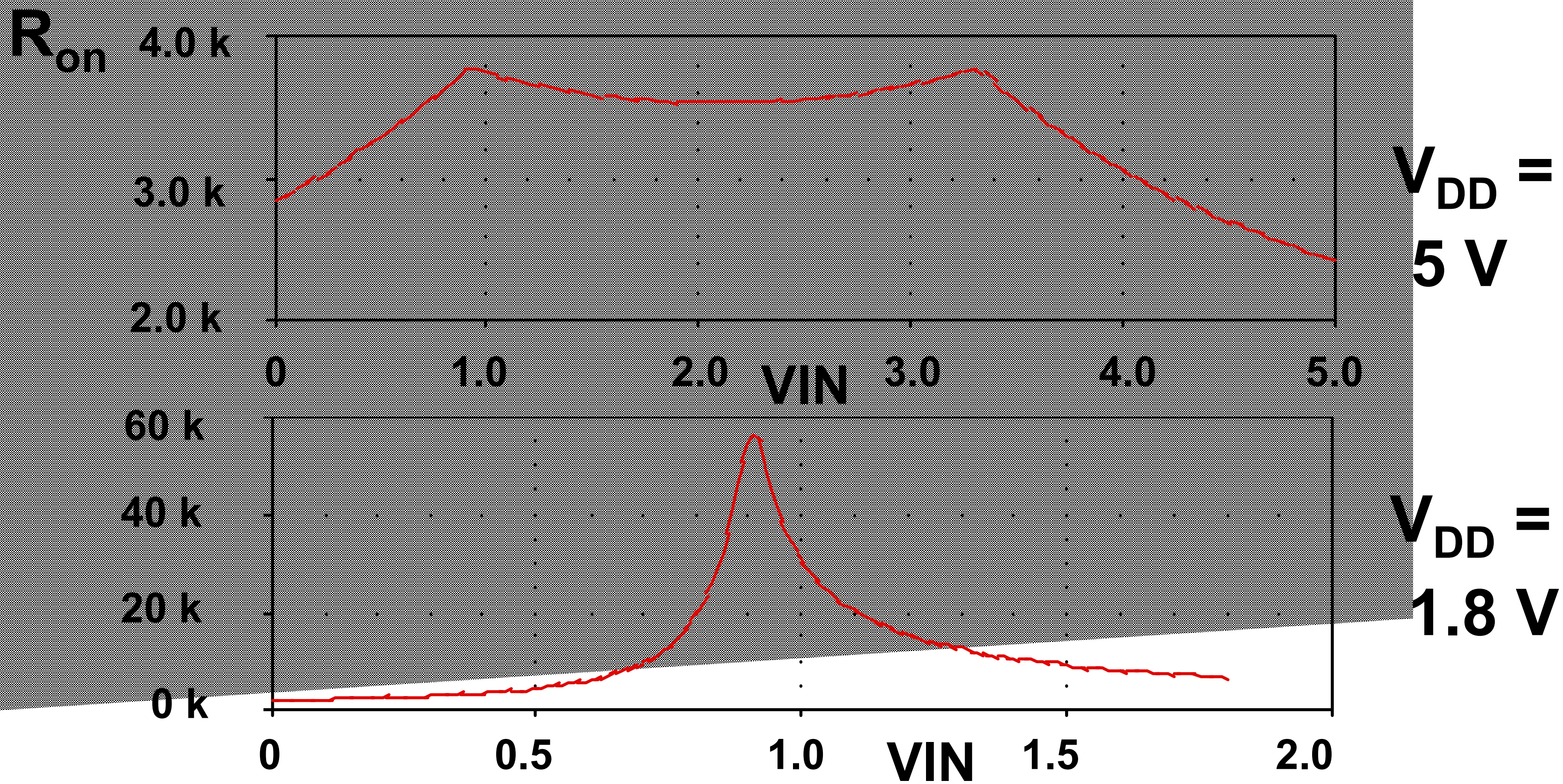
Minimum $V_h = V_{DD}$: V_{DD}-$V_{GS,n}$ = $V_{GS,p}$ => V_{DD} > 1.4 V

Double Switch



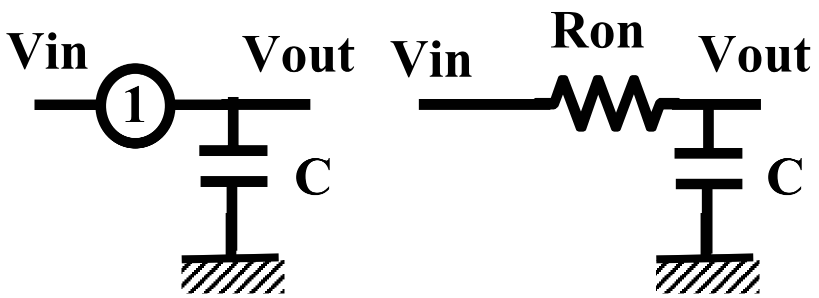
Low Voltage SC : MOST-Switch





Ron
4.0 k
3.0 k
2.0 k
0
1.0
2.0
VIN
3.0
4.0
5.0
VDD =
5 V
60 k
40 k
20 k
0 k
0
0.5
1.0
VIN
1.5
2.0
VDD =
1.8 V

Time constant of Ron

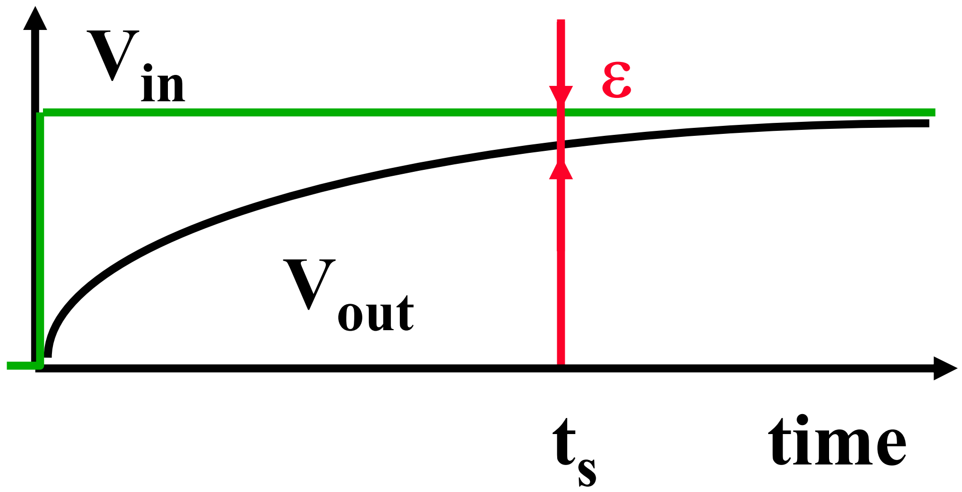


$$V_{out} = V_{in}\,(1\text{-}\exp(-\frac{t}{RC}))$$

⇩

$$t_s = RC\,\ln(1/\varepsilon)$$

$$t_s \approx 7\,RC \quad \text{for} \quad \varepsilon = 0.1\,\%$$



Speed ⇩ if

large C (low noise)

large R (small switch)

Maximum frequency of operation

For $W/L = 2$ and $V_{GS}\text{-}V_T \approx 1$ V

$R_{on} \approx 10$ kΩ

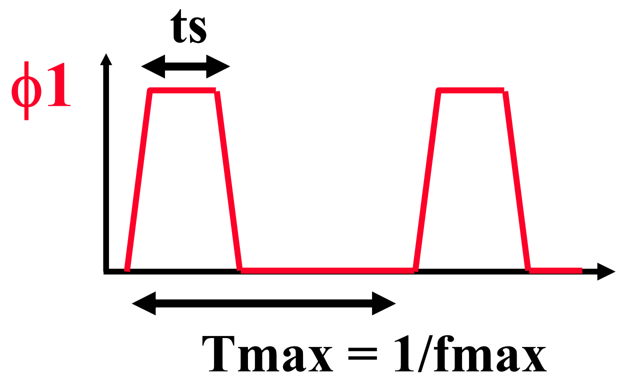
For $C \approx 1$ pF

For $\varepsilon \approx 0.1\%$

$t_s = 7\ RC \approx 70$ ns

$T_c = 140$ ns ⇨ $f_{max} \approx 7$ MHz

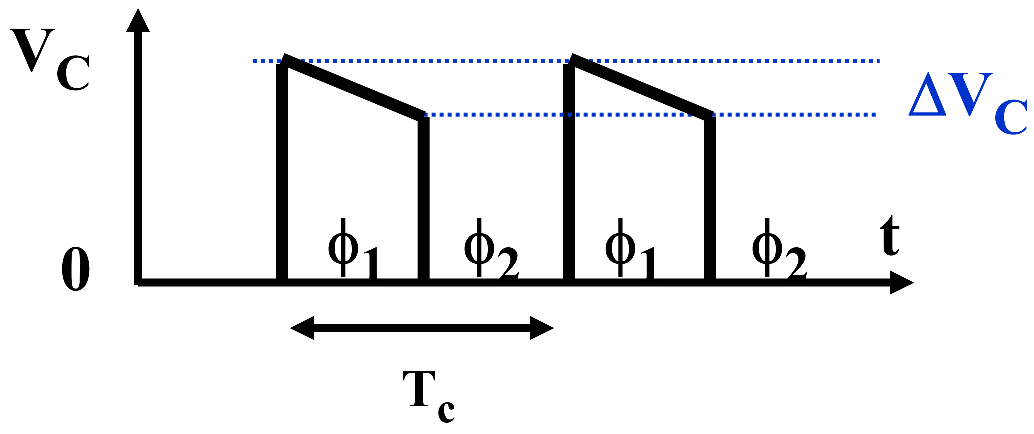
Due to only one switch



⇨ practical f_{max} : 1-10 MHz

$L \downarrow$ ⇨ $R_{on} \downarrow$

Minimum frequency of operation



$$\text{Leakage } i = C\,\frac{dV_C}{dt}$$

i is 10 nA/cm^2 at 25^o

is 10 μA/cm^2 at 125^o

For 10x1 μm: 2 fA (25^o)

or 2 pA (125^o)

For $C_{min} \approx 0.25$ pF (mismatch)

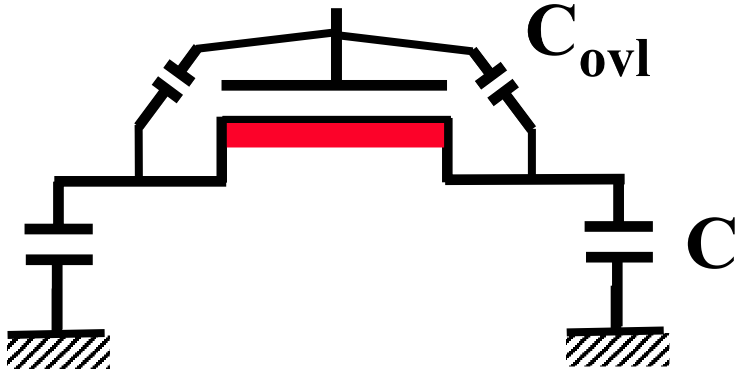
$\Delta V_c = 1\%$ of 0.1 V or $\Delta V_c = 1$ mV

$dt = T_c/2$ with $T_c = 1/f_{cmin}$

$$f_{cmin} = \frac{i}{2\,C_{min}\,\Delta V_c} = 4 \text{ Hz or } 4 \text{ kHz } (125^o)$$

Clock Feed-Through

Overlap Capacitors



$$C_{ovl} \approx W\, C_{ovlo}$$

$$W \uparrow \;\Rightarrow\; R \downarrow \text{ but } C_{ovl} \uparrow$$

Example : $W = 3\mu m \quad L = 0.7\mu m$

$$C_{ovlo} = 0.5 \text{ fF}/\mu m$$

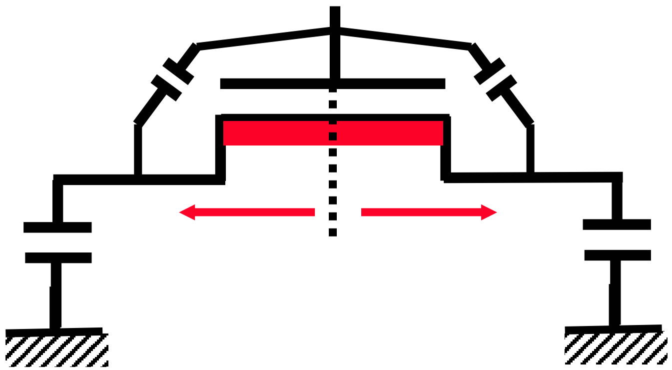
$$\Rightarrow C_{ovl} \approx 1 \text{ fF}$$

$$\Delta V\text{:}\;\; Q = C_{ovl}\,(V_h - V_l) \approx 1\text{fF} . 3\text{V} \approx 3\text{fC}$$

$$\Rightarrow \Delta V \approx \frac{Q}{C} \approx 3\text{fC}/1\text{pF} \approx 3 \text{ mV}$$

Charge redistribution

Inversion layer charge



$$Q_m \approx C_{ox}WL(V_h - V_{sign} - V_T)$$

Ex. $W = 3\mu m$ $L = 0.7\mu m$

$C_{ox} = 1.6\ fF/\mu m^2$

$V_T = 0.7V$ $V_{sign} = 1.5V$

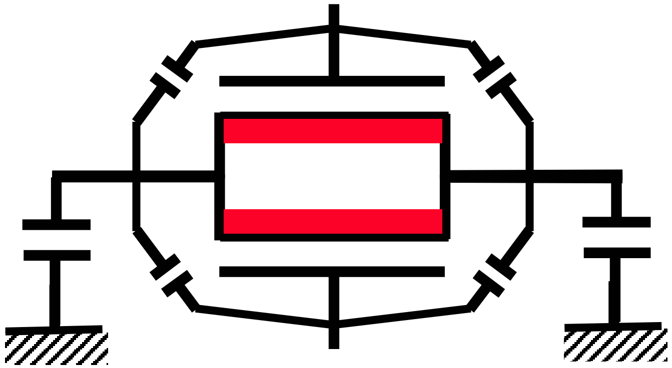
$\Rightarrow Q \approx 6\ fC$

ΔV: Half is stored in each cap

$\Rightarrow \Delta V \approx Q/2C \approx 3\ fC/1pF \approx 3\ mV$

$\boxed{\text{Total: } \Delta V \approx 10\ mV/pF}$ $C\uparrow \Rightarrow CD\downarrow$ Speed$\downarrow$ Power$\uparrow$

Clock injection & Charge redistribution



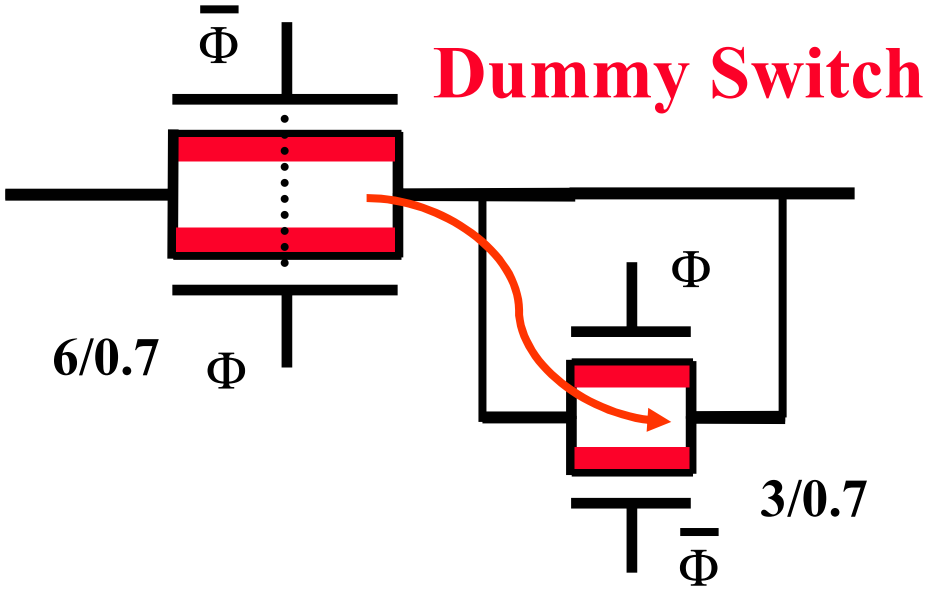
If

$C_{ovl,n} = C_{ovl,p}$

No Clock FT !

Problems: matching

$W_n = W_p$?

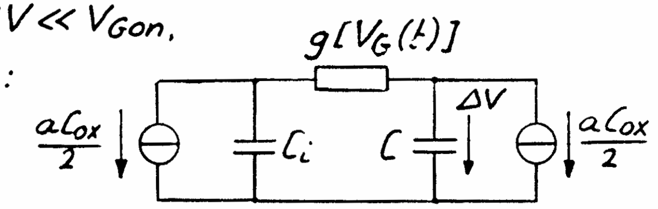


OK if Q is split equal 1/2

Problems: clock skew
rise/fall time
impedance

Quantitative Charge Redistribution

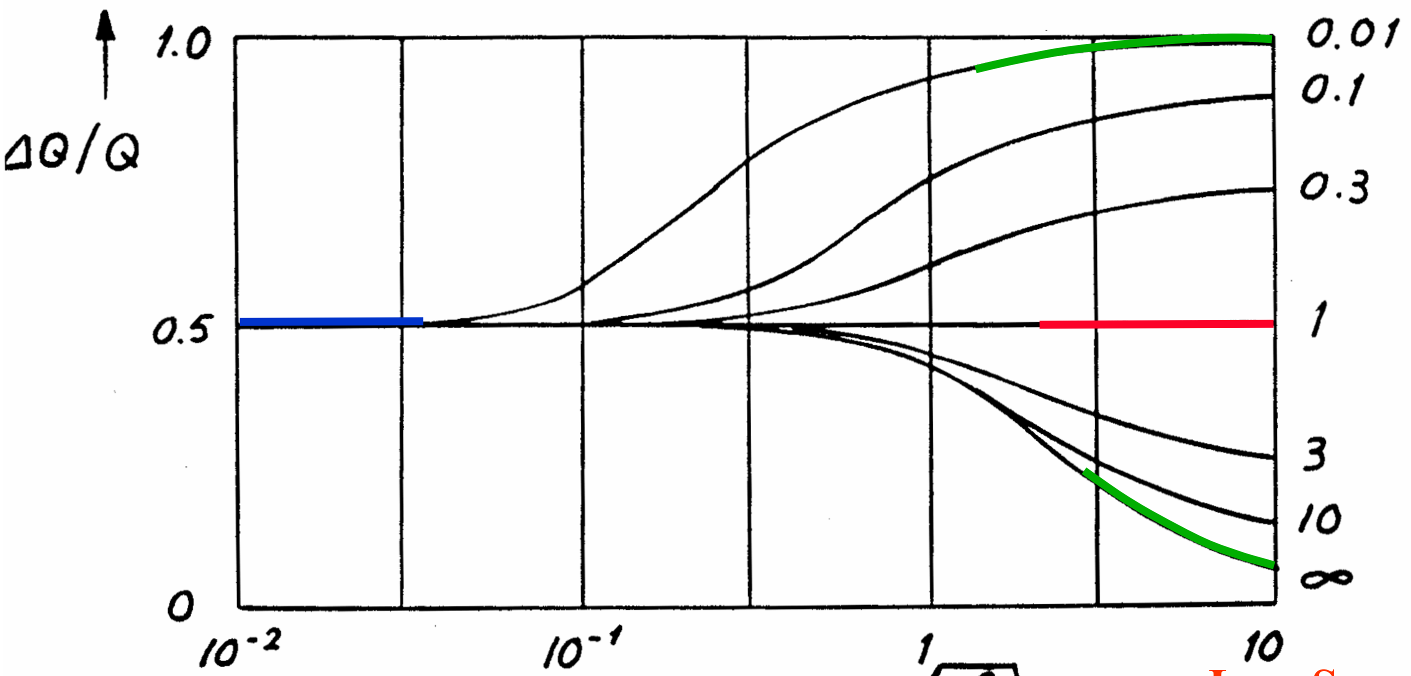
If $\Delta V \ll V_{Gon}$, then:



$\beta = \mu Cox.W/L$

$a = dV/dt$

V_{Te} = effective V_T (with bulk effect)



Ci << C, B >>1
ΔQ→Q
⇨ Dummy

Ci = C: ΔQ=Q/2
⇨ Dummy

Ci >> C, B >>1
ΔQ→0
NO dummy

High Speed Clocks

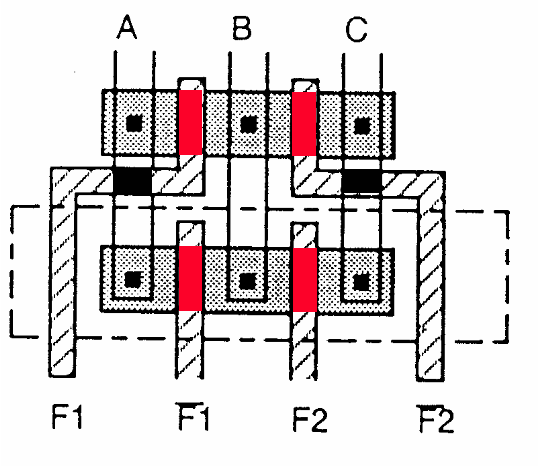
$$B = (V_{Gon} - V_{Te})\sqrt{\frac{\beta}{aC}}$$

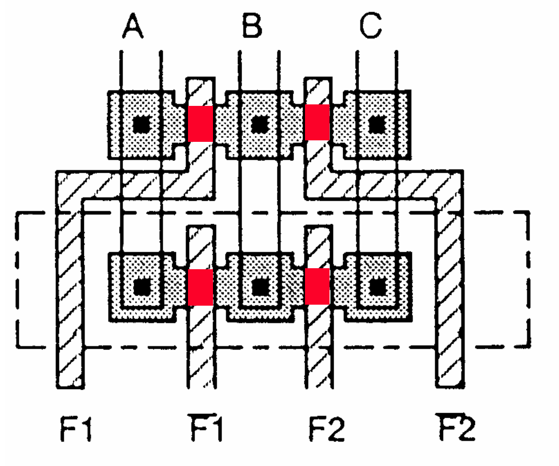
→ Low Speed Clocks

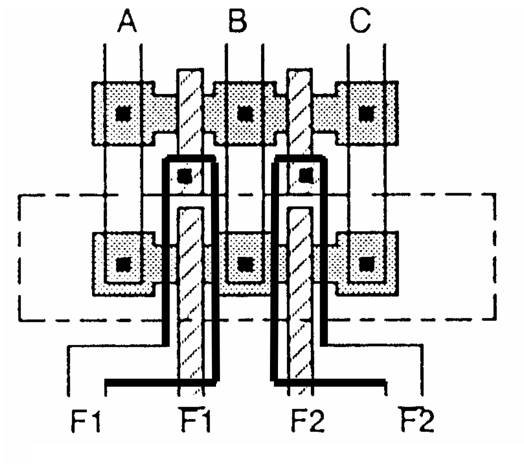
B<<1 : ΔQ=Q/2 ⇨ Dummy

Ref. Wegmann, Vittoz, JSSC Dec.87, 1091-1097

Layout considerations







Parasitic C

$\Downarrow$

CFT

Reduce C_{ox} area

Use metal to 'shield' clock lines

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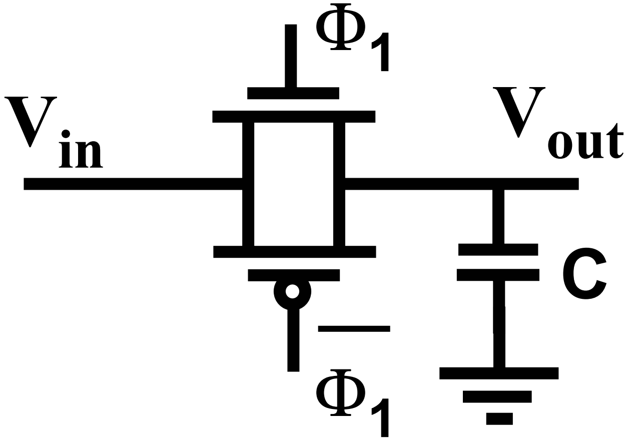
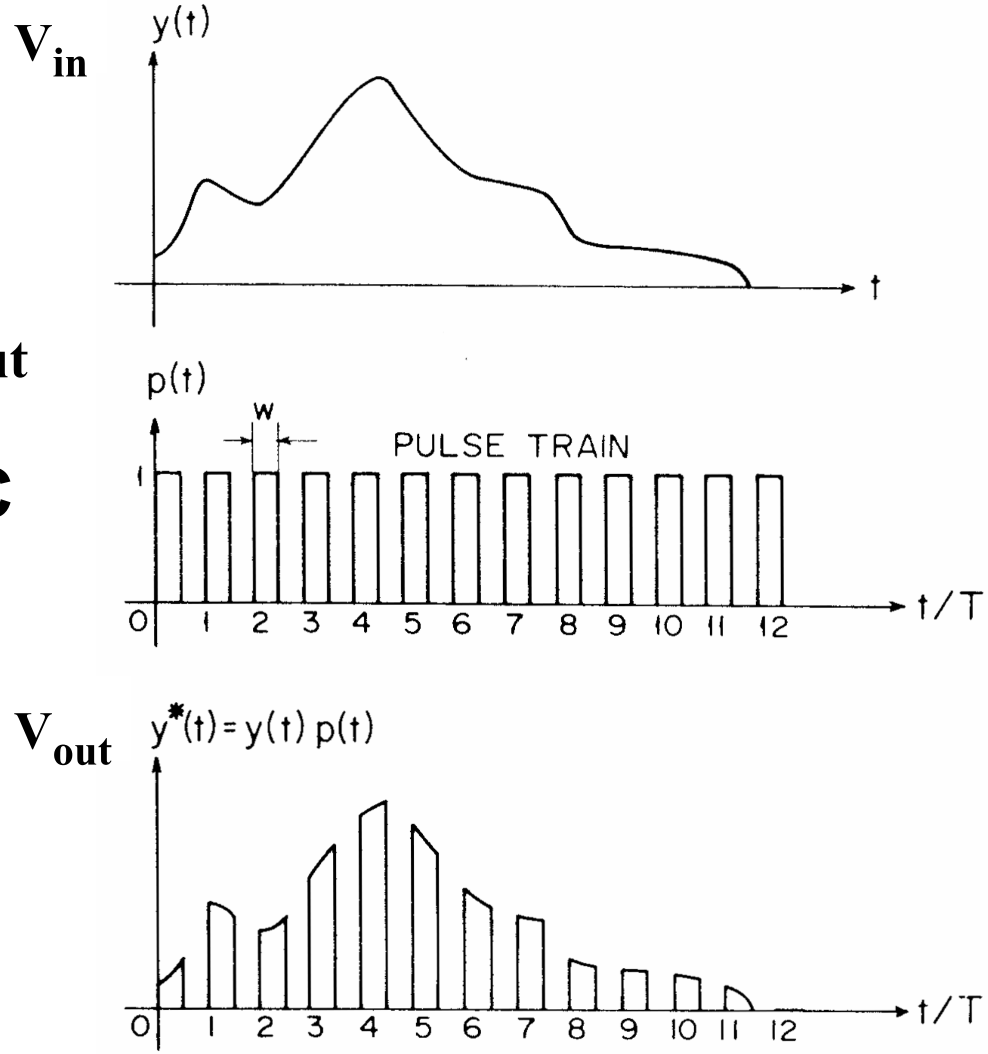
Sampling analog signals



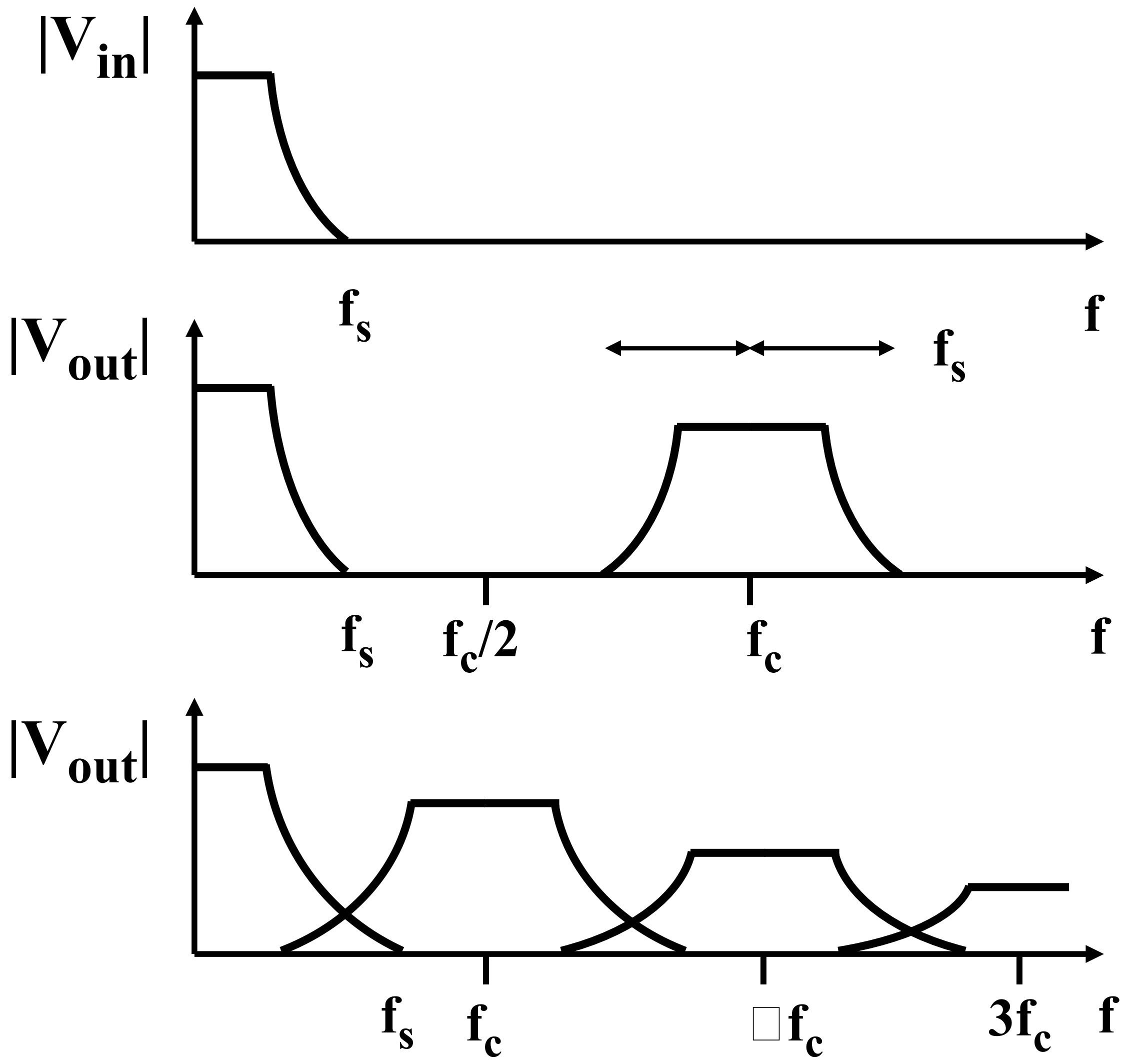


Spectra

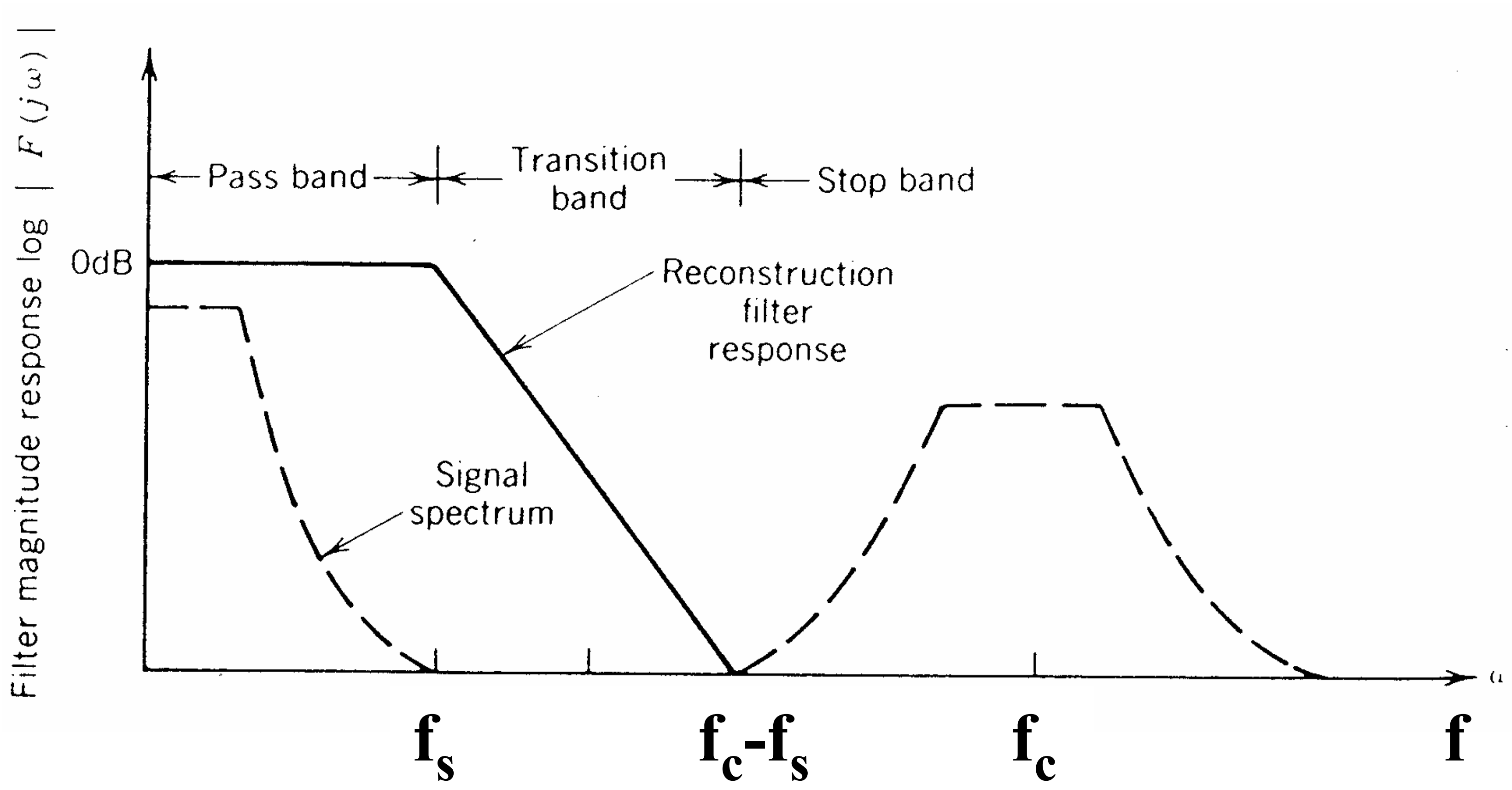
Input signal v_{in}

Sampled signal
$f_c/2 >> f_{signal}$

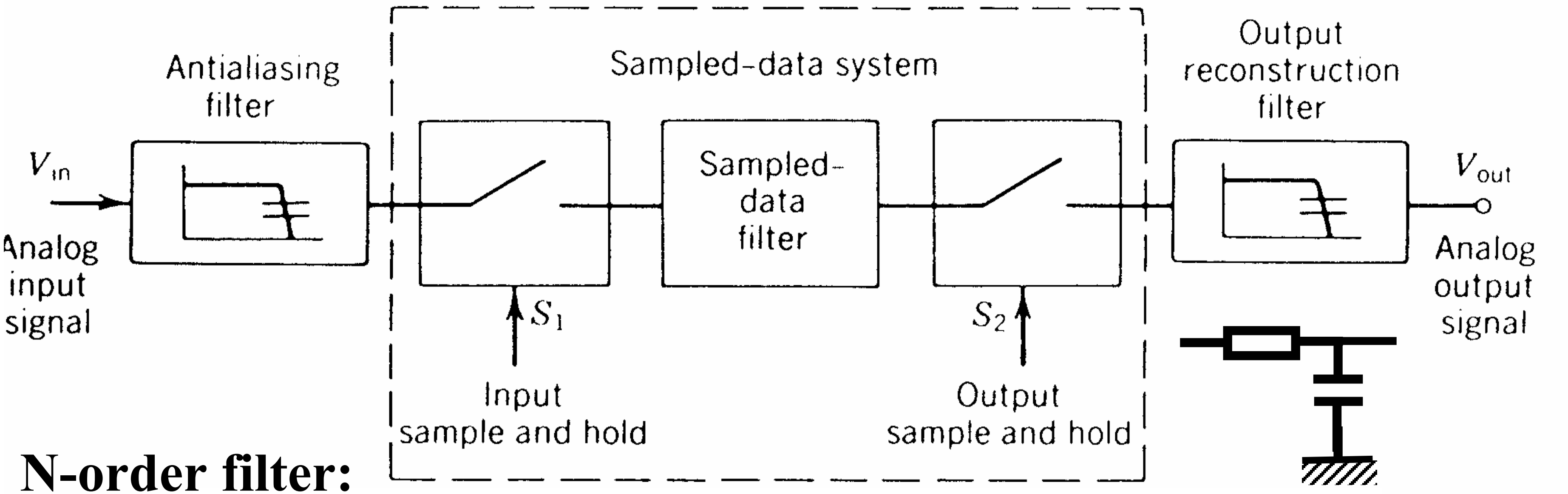
Sampled signal
$f_c/2 < f_{signal}$
Nyquist !



Anti-Aliasing filter



Anti-aliasing / Reconstruction



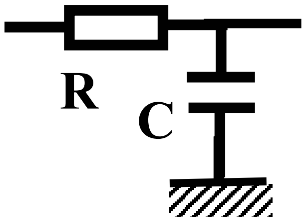
N-order filter:

$$\left[\frac{fs}{fc-fs}\right]^N = 10^{\frac{-Attenuation}{20}} \qquad fc = fs.10^{\frac{Attenuation}{20.N}}$$

Ex. Attenuation = 40 dB; fs = 10 kHz ; N = 1 ⇨ fc = 1 MHz

Sampled Data Basics : z-transform

Analog System: $s = j\omega$



$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + sRC}$$

Sampled data: z-transforms

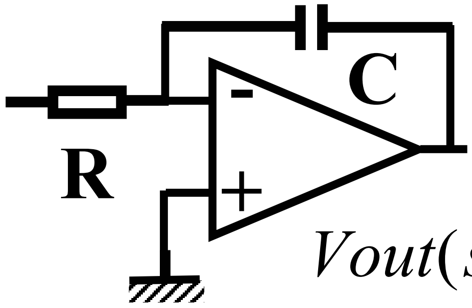
1 delay is z^{-1}

$$z = e^{j\omega T_c} = e^{j\frac{2\pi f}{f_c}}$$

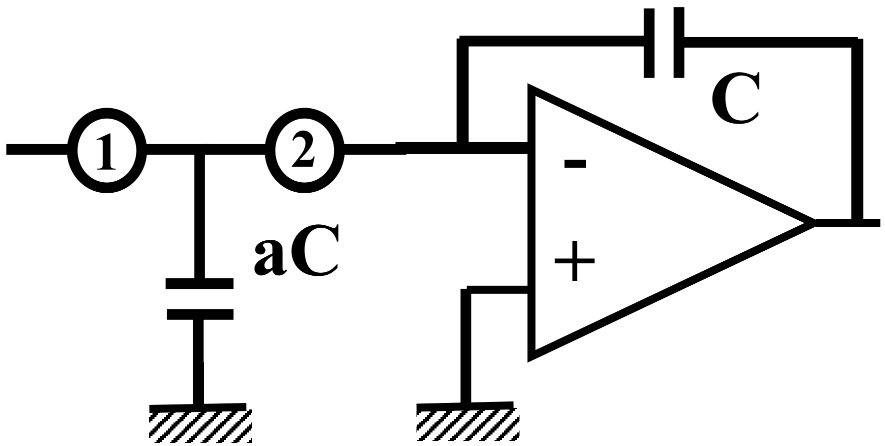
$$e^{j\omega T_c} = 1 + j\omega T_c + \frac{(j\omega T_c)^2}{2} + \ldots\ldots \quad \text{if } \omega T_c << 1$$

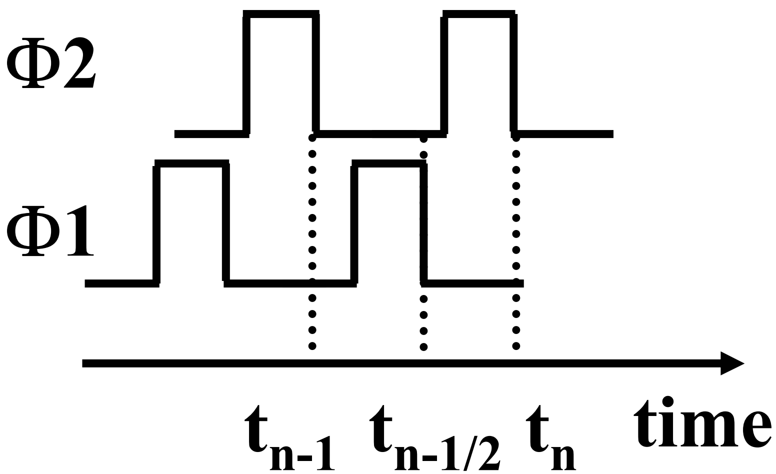
z-TRANSFORM	SEQUENCE
$a\,X(z) + b\,V(z)$	$ax(n) + bv(n)$
$\boxed{z^{-n_1}\,Y(z)}$	$\boxed{y(n - n_1)}$
$Y(z/b)$	$b^n\,y(n)$
$-z\dfrac{dY(z)}{dz}$	$n\,y(n)$
$Y(z^{-1})$	$y(-n)$
$X(z)\,V(z)$	$x(n) * v(n)$

SC-Integrator in phase 1



$$Vout(s) = \frac{-Vin(s)}{sRC}$$



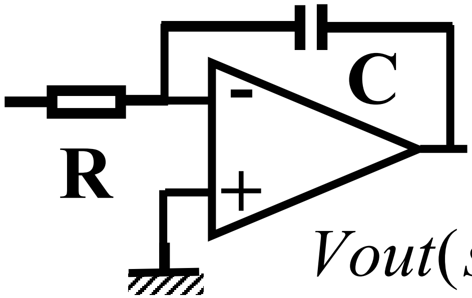


$\Phi 1$ $\quad Q_{aC1} = aC\ V_{in}(n\text{-}1/2)$

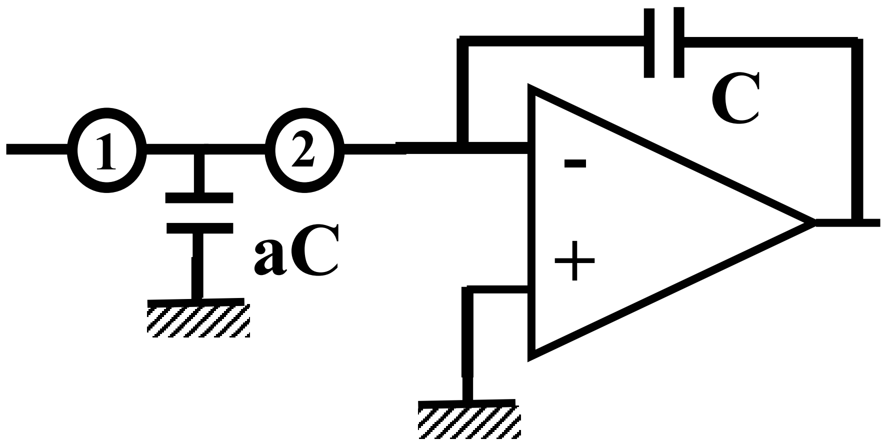
$Q_{C1} = -\ C\ V_{out}\ (n\text{-}1)$

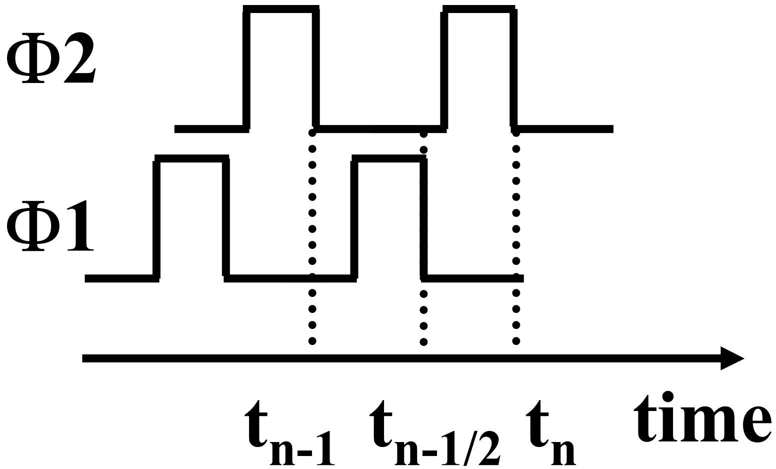
$V_{out}(n\text{-}1/2) = V_{out}(n\text{-}1)$

SC-Integrator in phase 2 : charge conservation



$$Vout(s) = \frac{-Vin(s)}{sRC}$$





$\Phi 2 \quad Q_{aC2} = 0$

$Q_{C2} = - C\, V_{out}\,(n)$

$$Q_{aC2} + Q_{C2} = Q_{aC1} + Q_{C1}$$

$$- C\, V_{out}\,(n) = aC\, V_{in}\,(n\text{-}1/2) - C\, V_{out}\,(n\text{-}1)$$

SC-Integrator : approximate transfer function

$$- C\,V_{out}(n) = aC\,V_{in}(n-1/2) - C\,V_{out}(n-1)$$

$$V_{out}(n-1) = z^{-1}\,V_{out}$$

$$\Rightarrow \quad C.V_{out} = z^{-1}\,C\,V_{out} - z^{-1/2}\,aC\,V_{in}$$

$$\frac{V_{out}}{V_{in}} = -a\,\frac{z^{-1/2}}{1-z^{-1}} \qquad z^{-1} = e^{-j\omega T_c} \approx 1 - j\omega T_c$$

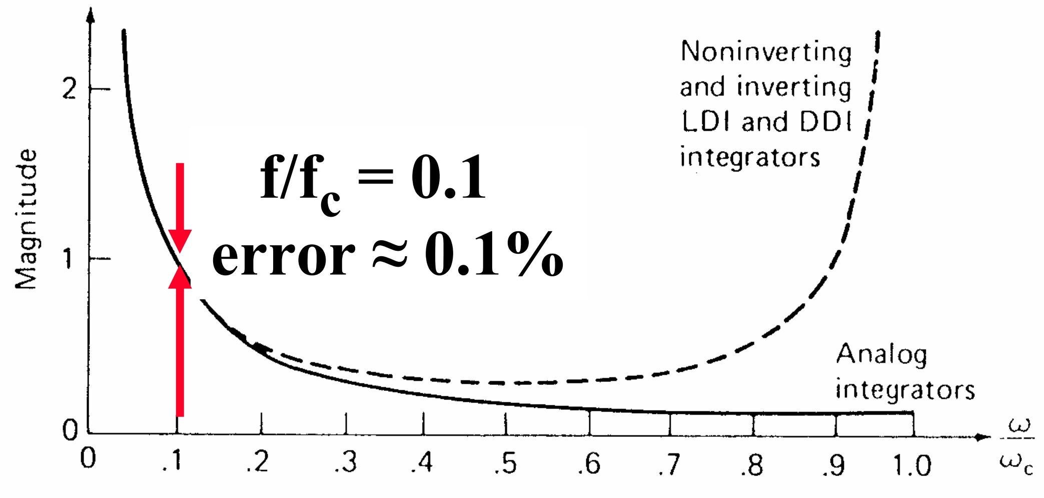
$$\Rightarrow \frac{Vout}{Vin} \approx -\frac{a(1-\cancel{j\omega Tc/2})}{j\omega Tc} \approx -\frac{a}{j\omega Tc}$$

Integrator

$$RC = \frac{T_c}{a}$$

Exact Transfer function

$$H(z) = -\frac{az^{-1/2}}{1 - z^{-1}}$$



$$H(e^{j\omega T_c}) = -\frac{ae^{-j\omega T_c/2}}{1 - e^{-j\omega T_c}}$$

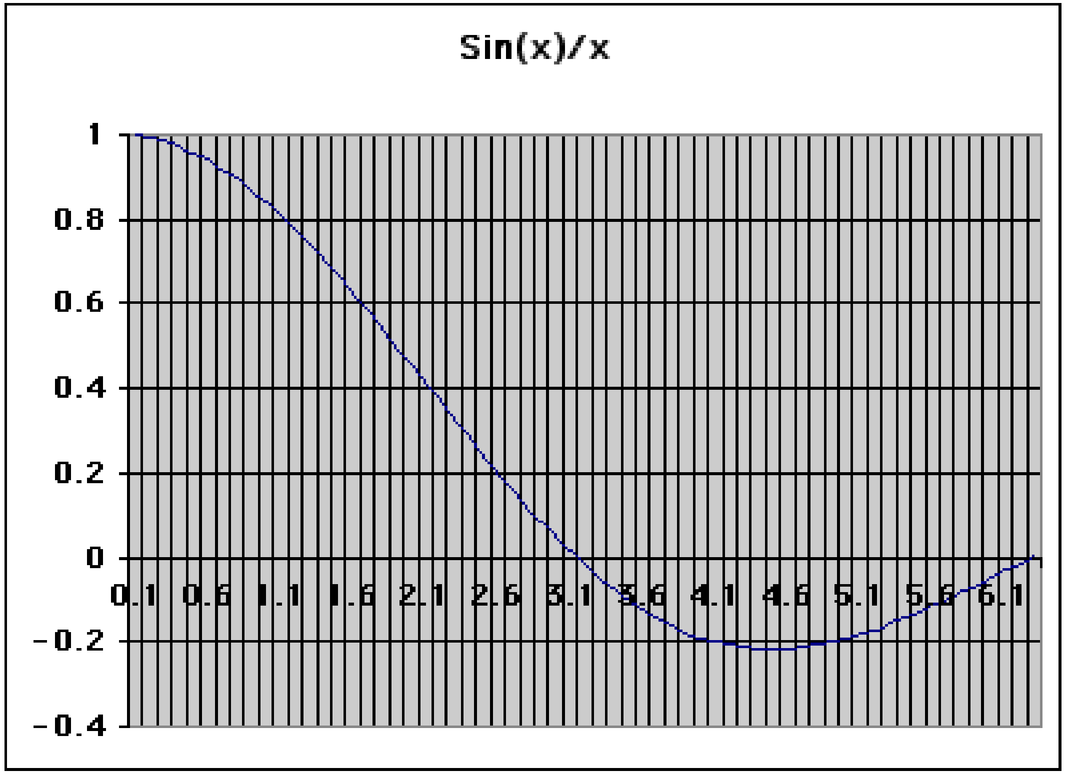
$$H(e^{j\omega T_c}) = -\frac{a}{e^{j\omega T_c/2} - e^{-j\omega T_c/2}}$$

Euler's relationship:

$$\sin(x) = \frac{e^{+jx} - e^{-jx}}{2j}$$

$$H(e^{j\omega T_c}) = -\frac{a}{j\omega T_c}\,\frac{\omega T_c/2}{\sin(\omega T_c/2)}$$

The sin(x)/x function



$$\sin(x) \approx x - \frac{x^3}{3} + ..$$

$$\frac{\sin(x)}{x} \approx 1 - \frac{x^2}{3} + ..$$

For $x = 0.1$

$\sin(x)/x \approx 1 - 0.003$

For $x = 0.05$

$\sin(x)/x \approx 1 - 0.0008$

$\approx 1 - 0.001$

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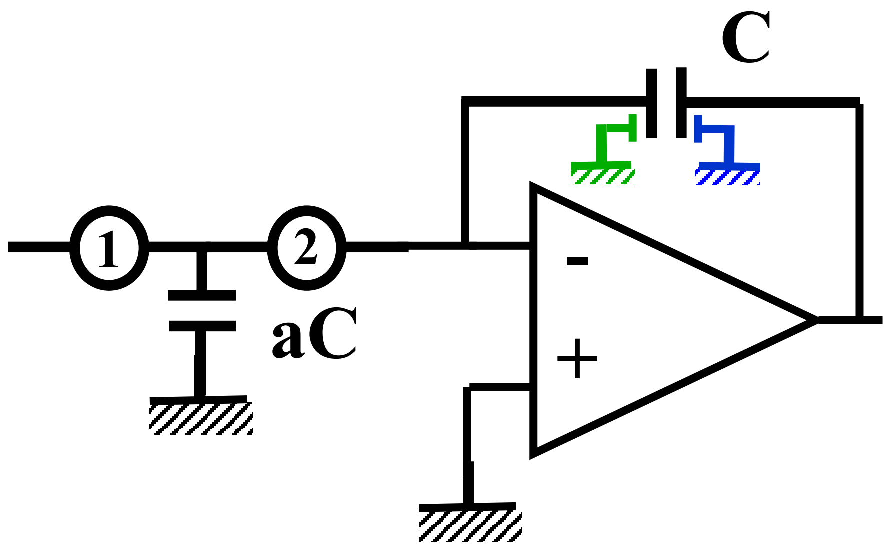
McCreary, JSSC Dec 75, 371-379
Gregorian, IEEE Proc. Aug 83, 941-986

Stray Capacitances



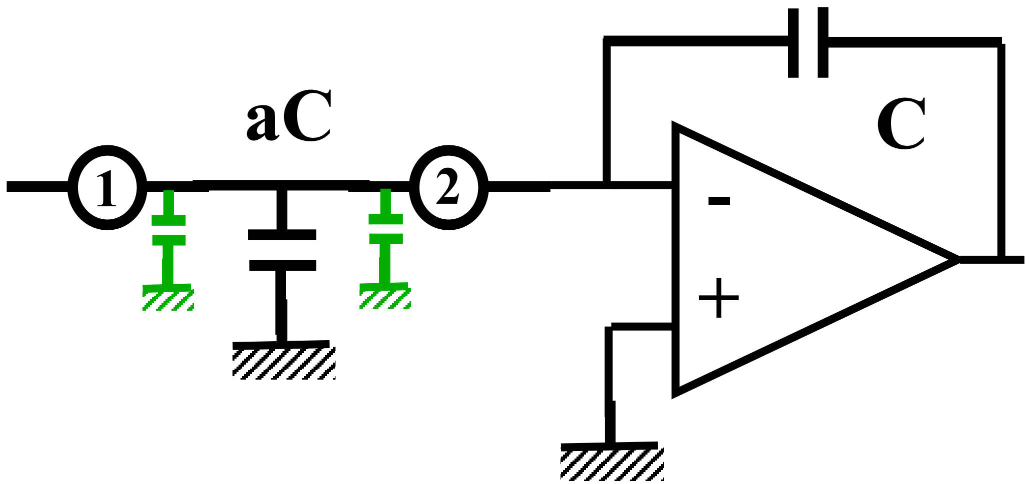
Stray Cap at input:

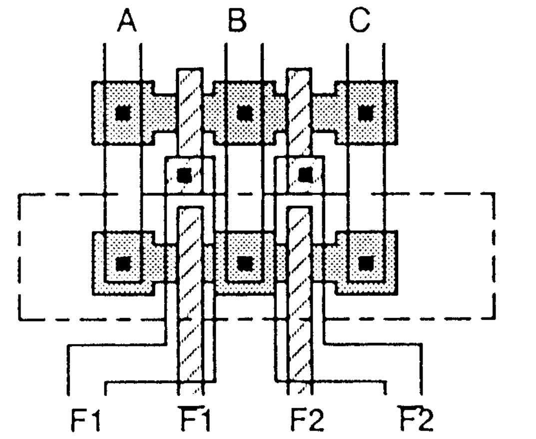
Substrate coupling
Continuous time
PSRR very bad

Stray Cap at output:

Cp is extra load
for opamp

Stray Capacitances



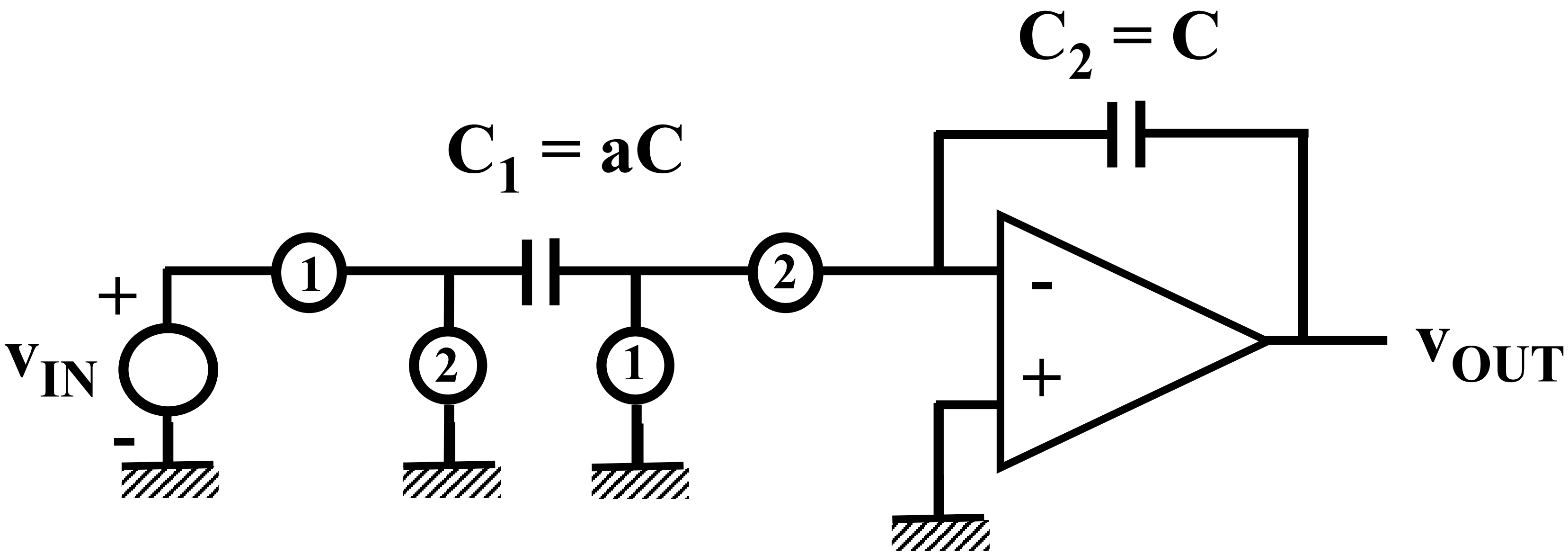


$$Cp \approx 2.C_{jS}.Area \approx 20\ fF$$

$$Gain = \frac{aC + 2Cp}{C}$$

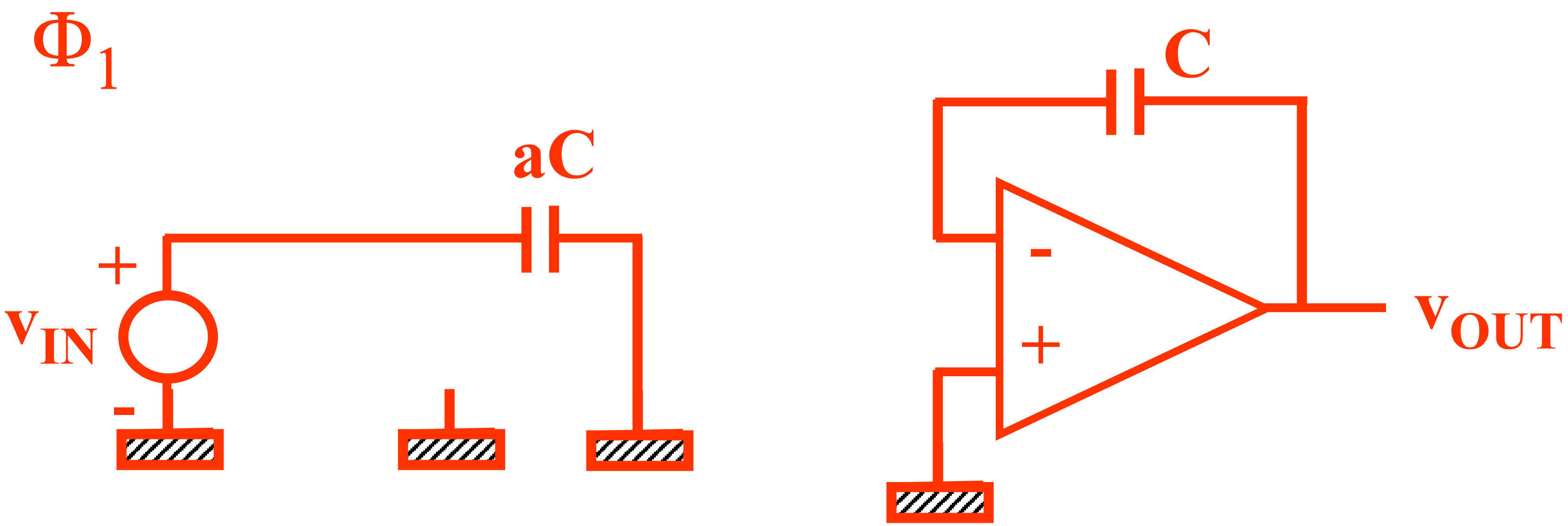
$$error \approx \frac{2Cp}{aC} \approx 5 - 10\%$$

Stray Insensitive SC integrator



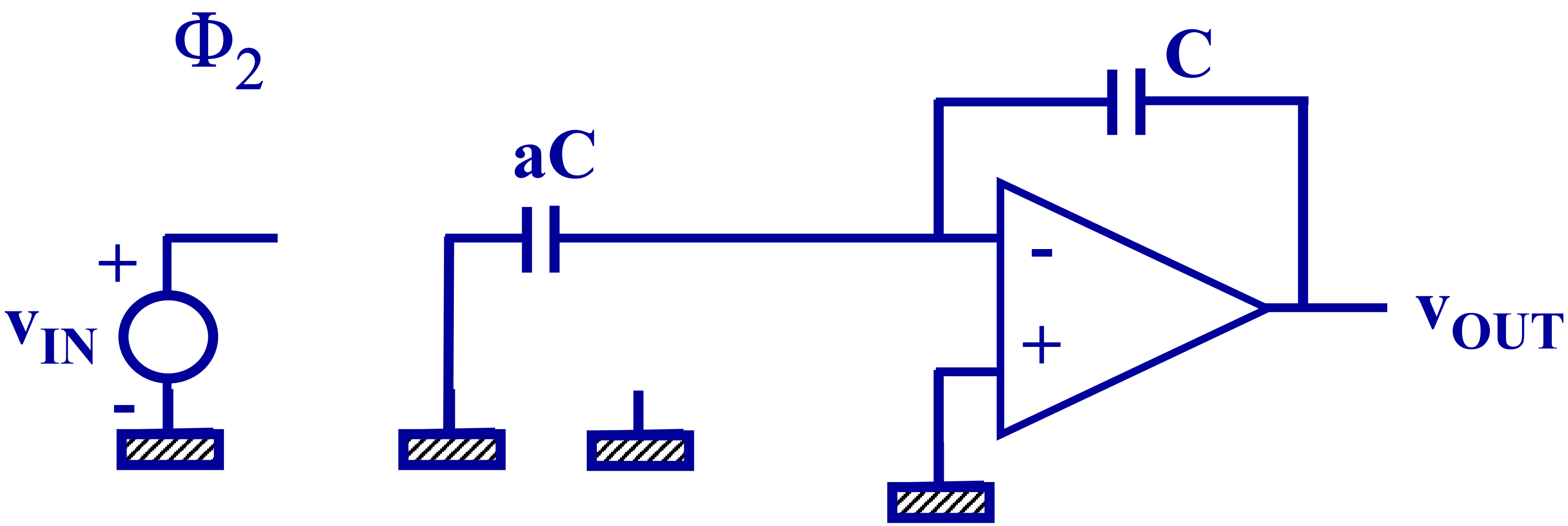
$$A_v = \frac{C_1}{C_2} = a$$

Stray Insensitive SC integrator



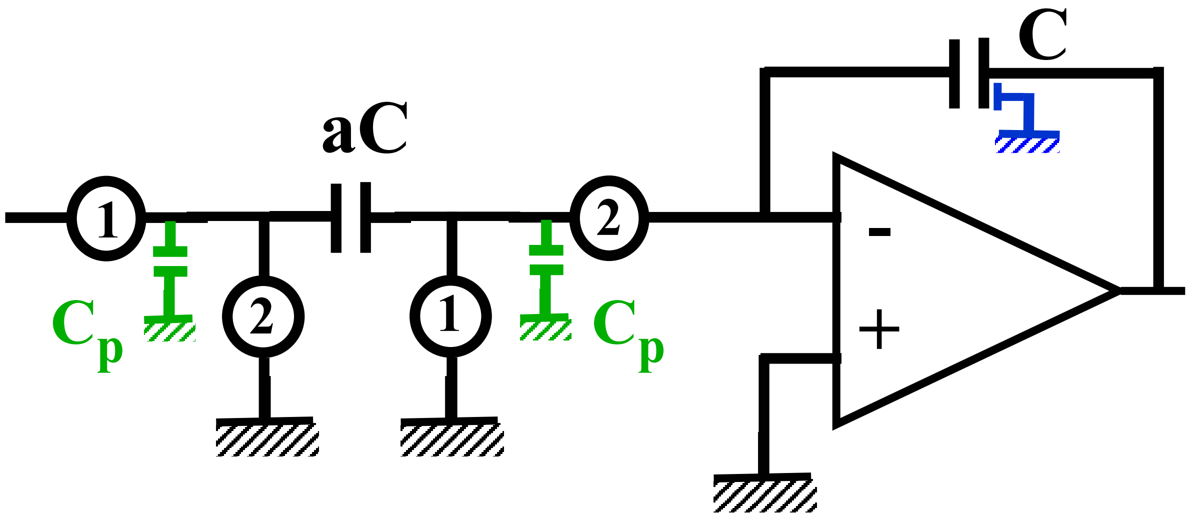
$$A_v = \frac{C_1}{C_2} = a$$

Stray Insensitive SC integrator



$$A_v = \frac{C_1}{C_2} = a$$

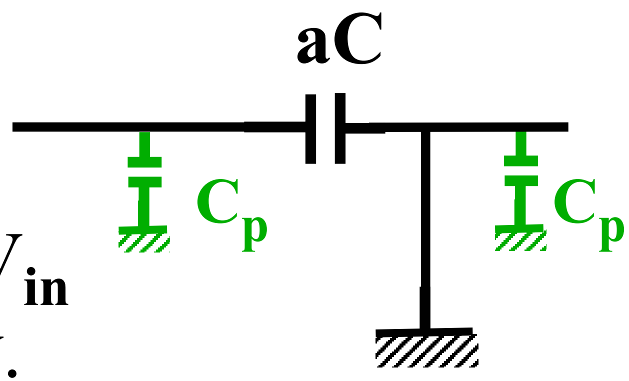
Stray Insensitive Integrator during phase 1



$$A_v = \frac{C_1}{C_2} = a$$

$$\tau = \frac{1}{af_c}$$

$\Phi 1$

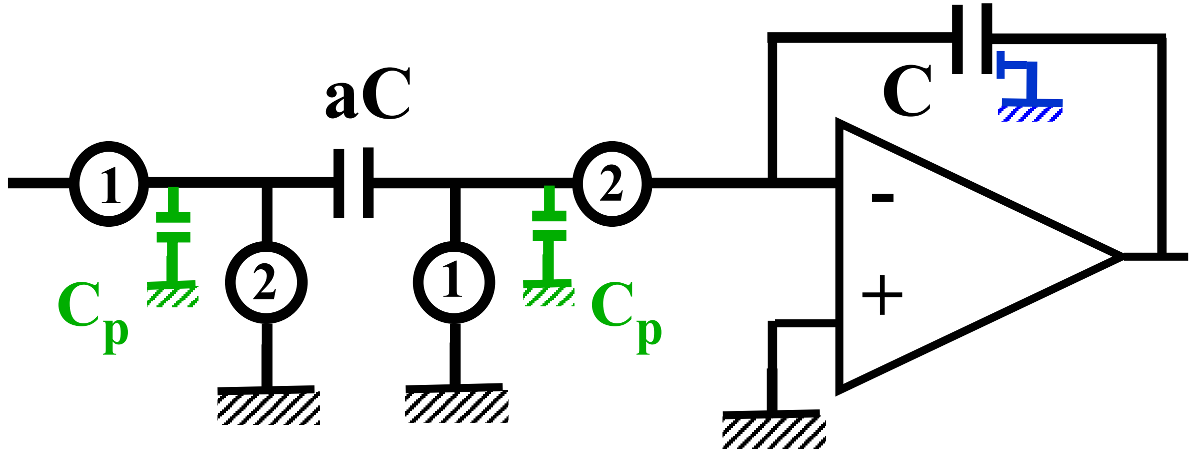


$Q_{aC} = aC\ V_{in}$

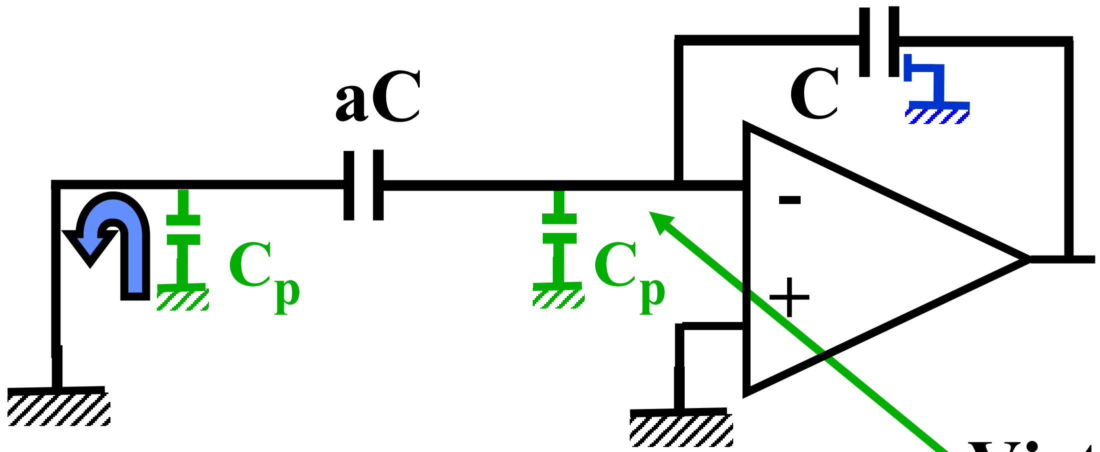
$Q_{cp} = C_p\ V_{in}$

$Q = 0 \Rightarrow$ no effect

Stray Insensitive Integrator during phase 2



$\Phi 2$



Q_{cp} is discharged to gnd

Only Q_{aC} is transferred

Virtual ground
Q_{cp} remains 0

Switched-Capacitor Filters

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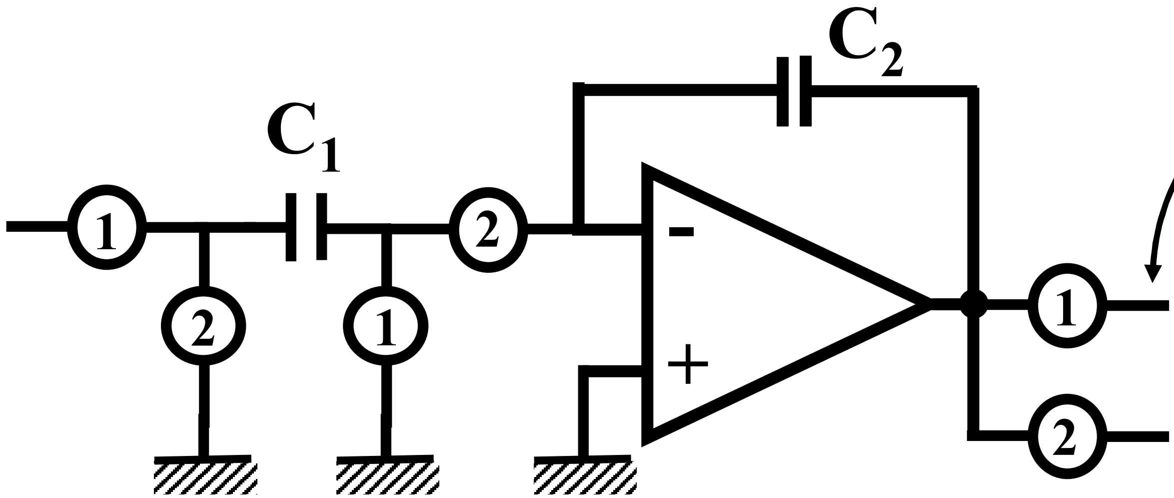
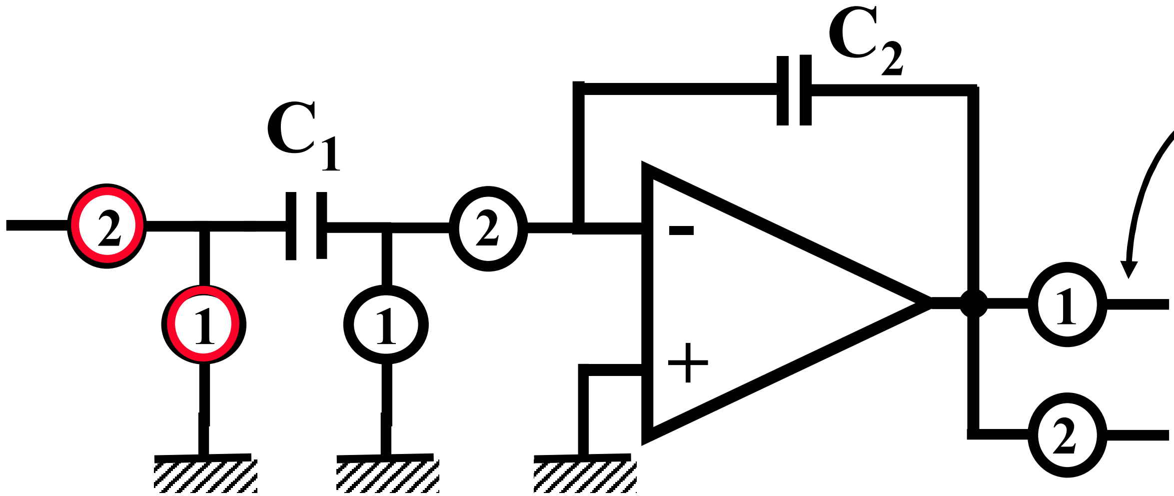
McCreary, JSSC Dec 75, 371-379
Gregorian, IEEE Proc. Aug 83, 941-986

Loss-less Integrators



$$H(z) = \frac{C_1}{C_2} \frac{z^{-1}}{1 - z^{-1}}$$

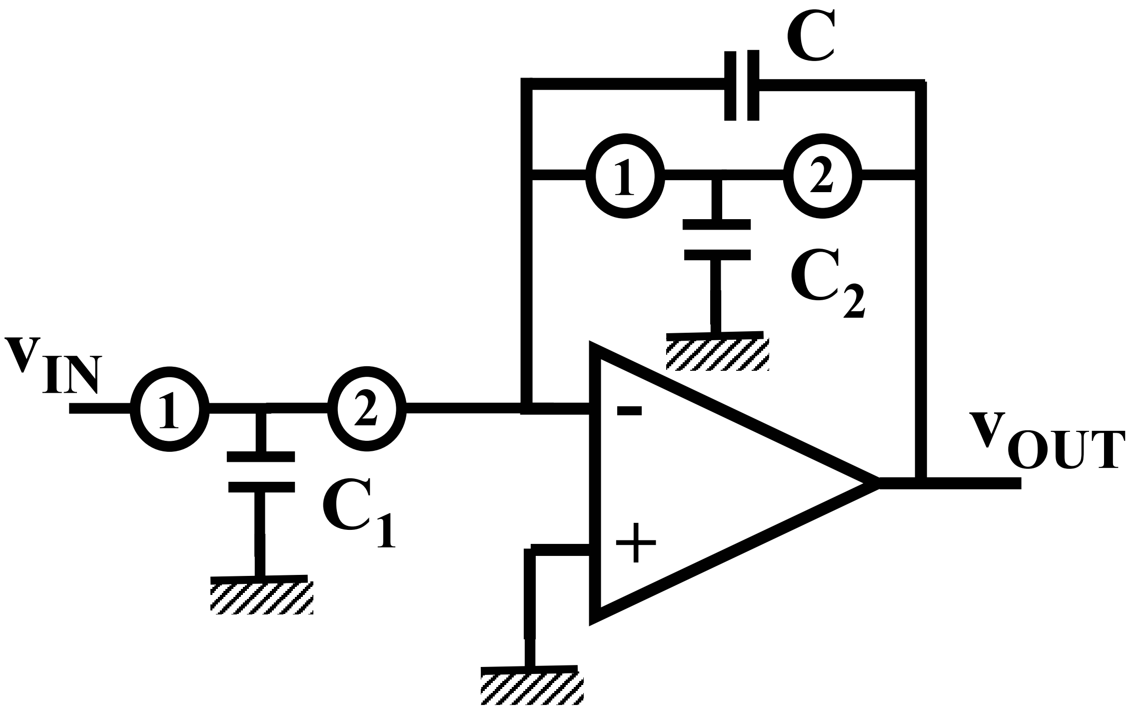
$$H(z) = \frac{C_1}{C_2} \frac{z^{-1/2}}{1 - z^{-1}}$$



$$H(z) = -\frac{C_1}{C_2} \frac{z^{-1/2}}{1 - z^{-1}}$$

$$H(z) = -\frac{C_1}{C_2} \frac{1}{1 - z^{-1}}$$

Low-pass filter of 1st order

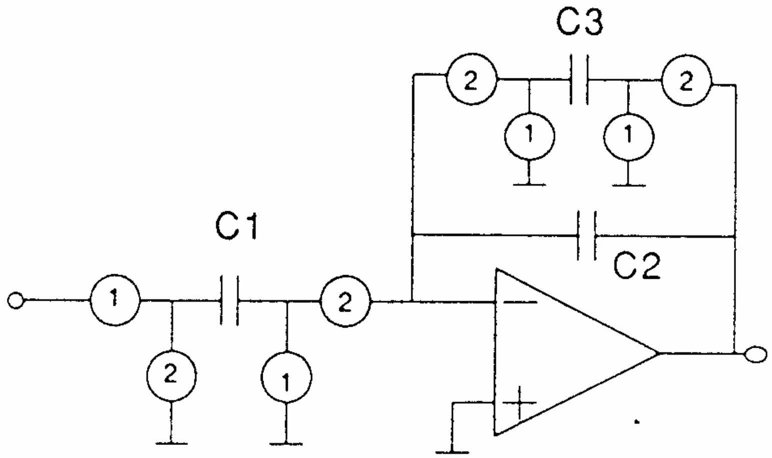


$$A_v = \frac{C_1}{C_2}$$

$$BW = \frac{f_c}{2\pi}\,\frac{C_2}{C}$$

Damped because of R//C !

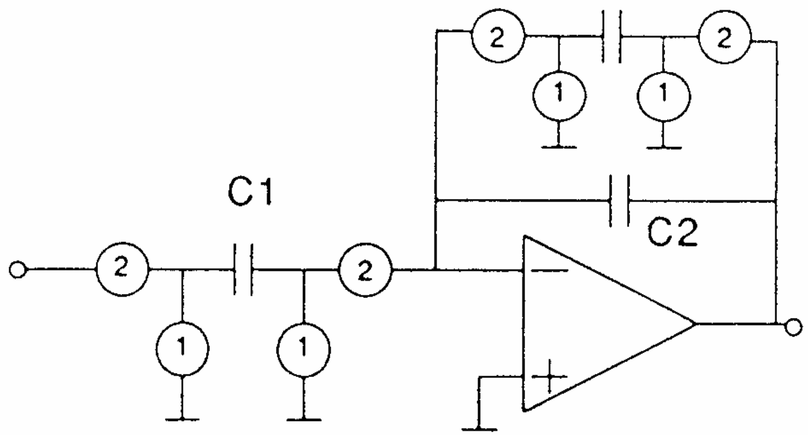
Damped integrators



(c)

Non-inverting damped integrator

$$H(z) = \frac{z^{1/2} C_1/C_2}{1 + C_3/C_2 - z^{1}}$$

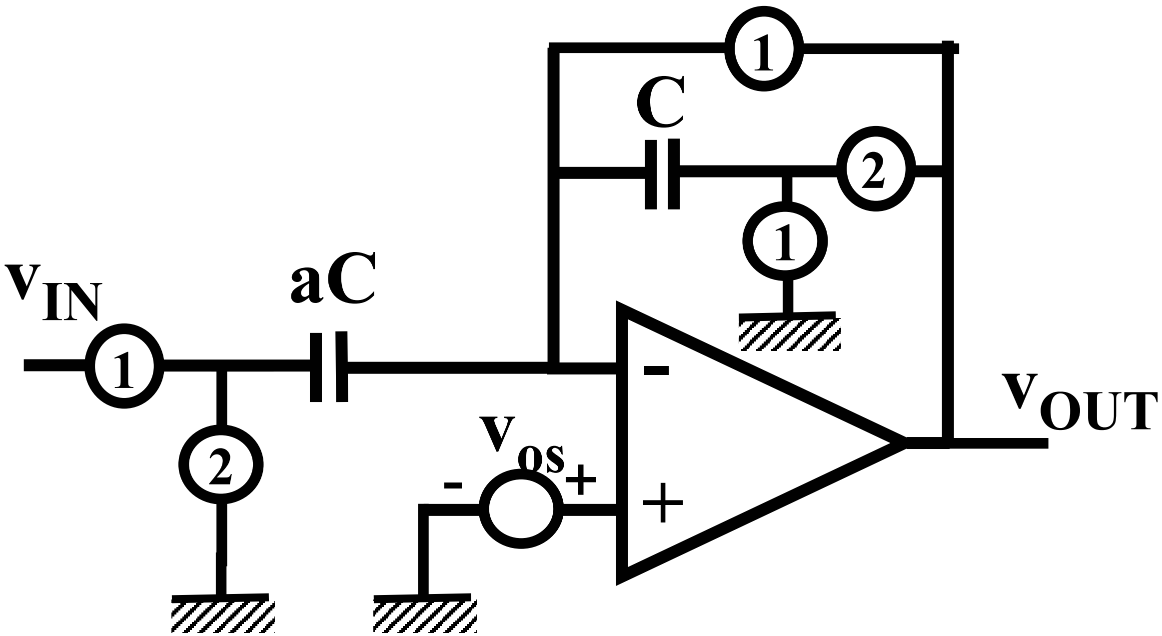


(d)

Inverting Damped integrator

$$H(z) = - \frac{z^{1/2} C_1/C_2}{1 + C_3/C_2 - z^{1}}$$

Offset compensation

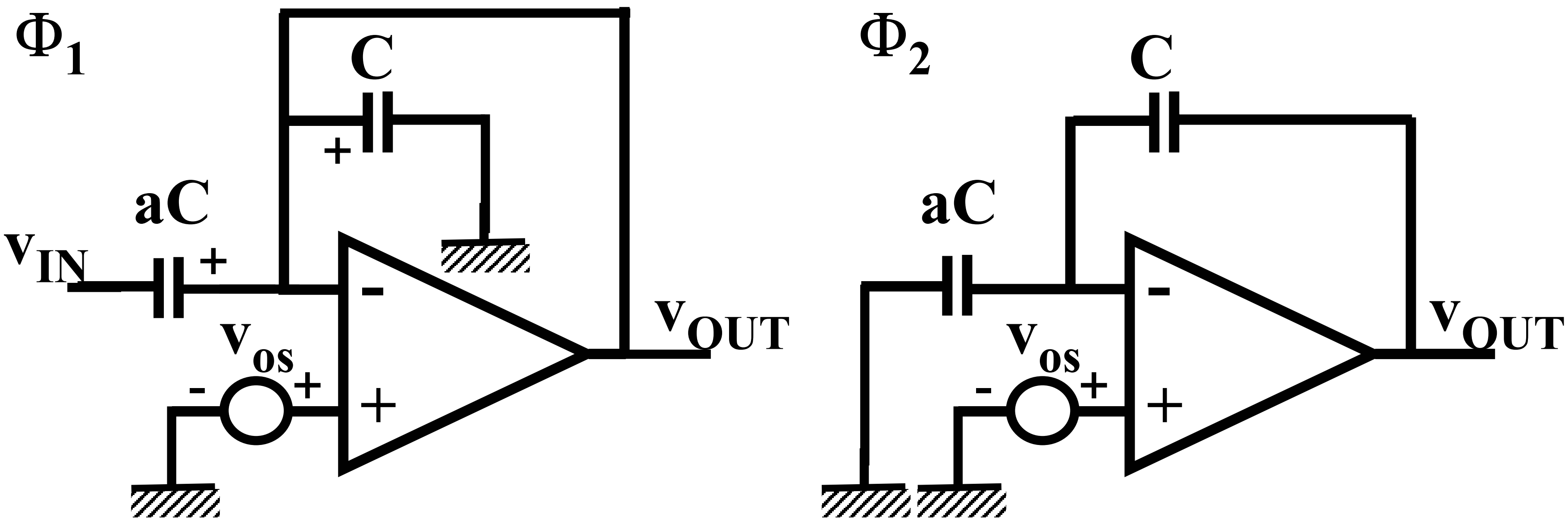


$A_v = a\ z^{-1/2}$

independent of v_{os}

Gregorian, IEEE Proc. Aug 83, 941-986

Offset compensation



$Q_{aC1} = aC\,(v_{os} - v_{IN}(n\text{-}1/2))$

$Q_{C1} = C\,v_{os}$

$Q_{aC2} = aC\,v_{os}$

$Q_{C2} = C\,(v_{os} - v_{OUT}(n))$

$Q_{aC1} + Q_{C1} = Q_{aC2} + Q_{C2} \Rightarrow A_v = a\,z^{-1/2}$

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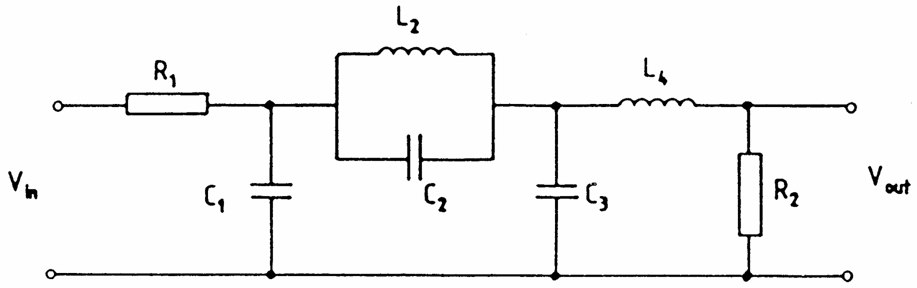
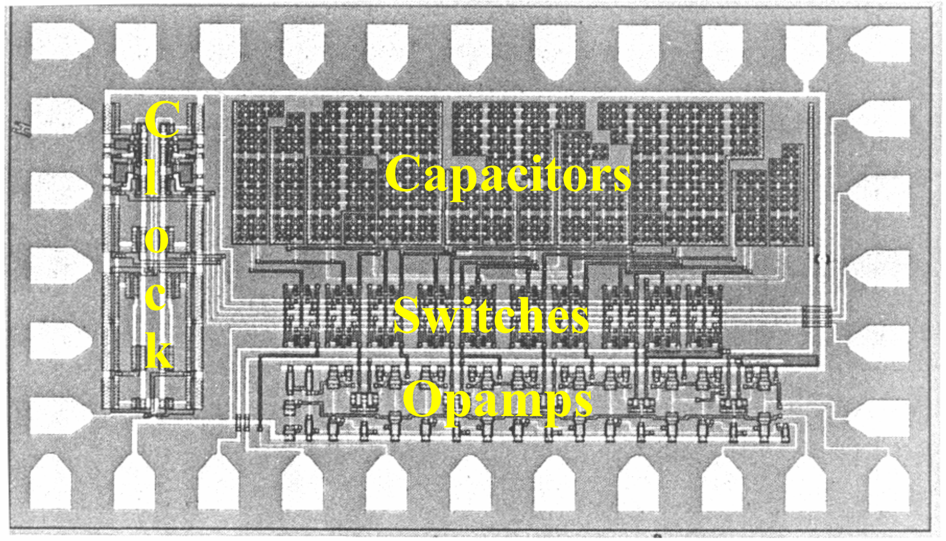
Gregorian, Temes, Analog MOS Integrated Circuits for Signal Processing, Wiley, 1986

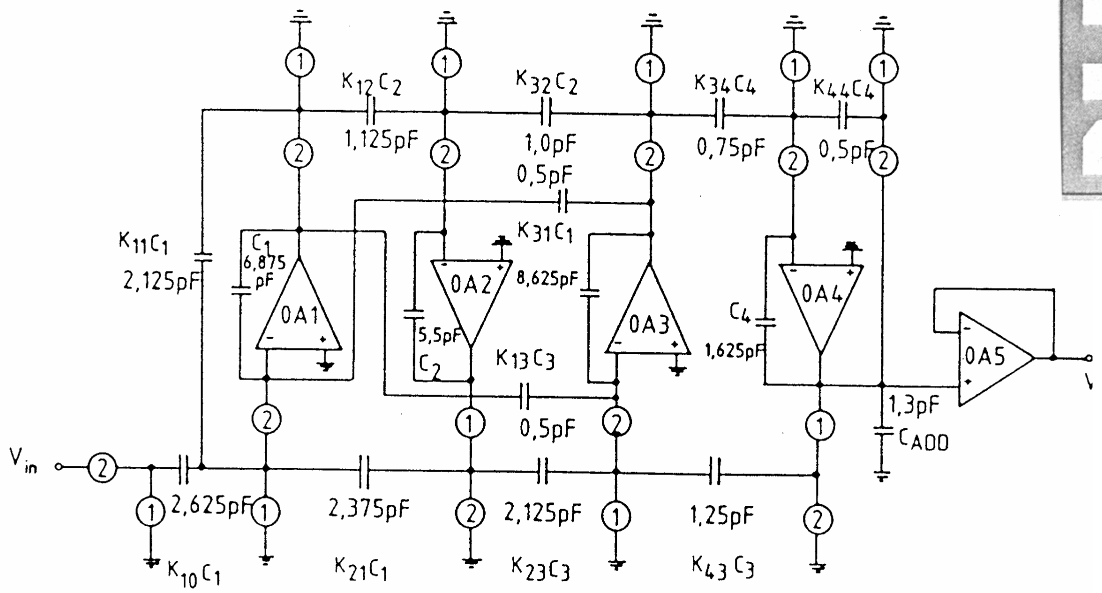
Laker, Sansen, Design of Analog Integrated Circuits and Systems, McGrawHill, 1994

Johns, Martin, Analog Integrated Circuit Design, Wiley 1997

4th Order SC low-pass ladder filter

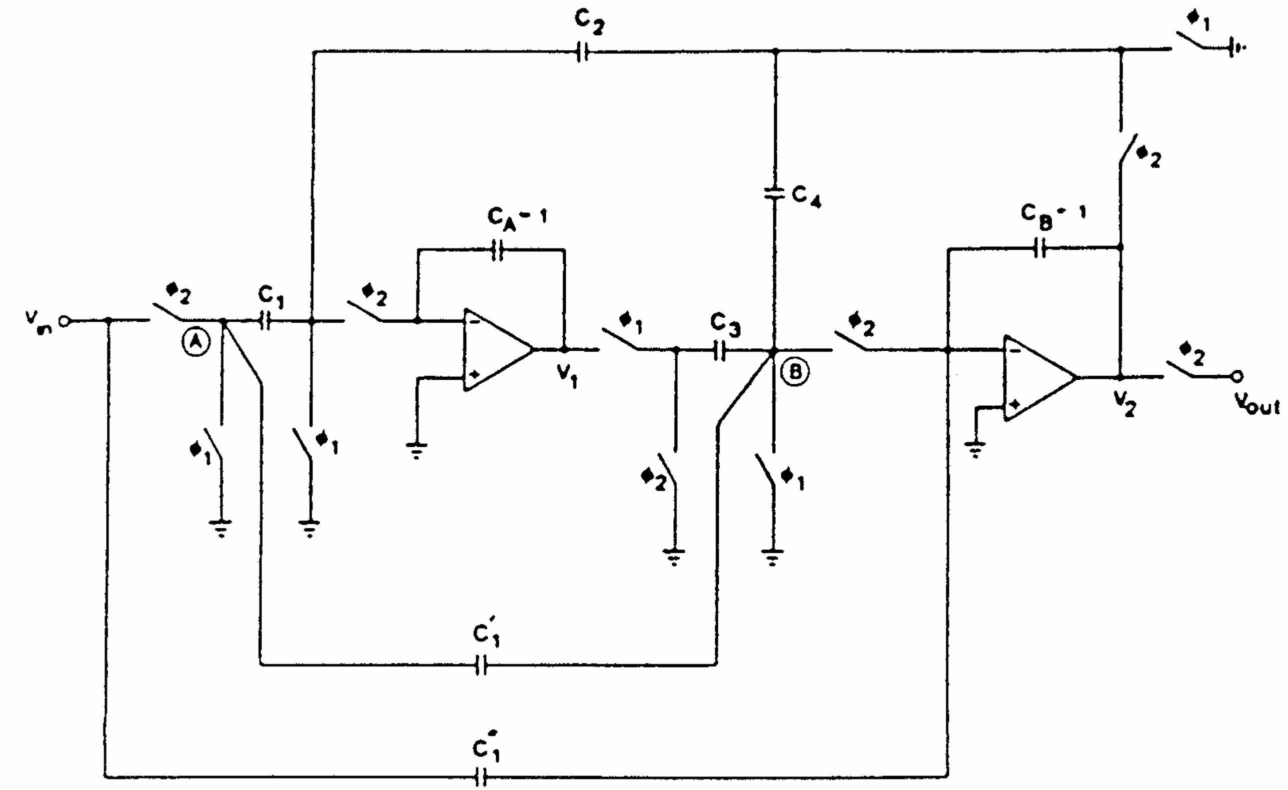






Clock freq	**100 kHz**
Cut-off	**5 kHz**
Pass ripple	**0.25dB**
Stop reject	**>45 dB**
Power	**190µW (± 2.5V)**
S/N	**75 dB**
Harm dist	**0.25%**
Area	**0.9 mm²**

Biquadratic filter



$$H(z) = -\frac{a_2z^2 + a_1z + a_o}{b_2z^2 + b_1z + b_o} = -\frac{(C_1' + C_1'')z^2 + (C_1C_3 - C_1' - 2C_1'')z + C_1''}{(1 + C_4)z^2 + (C_2C_3 - C_4 - 2)z + 1}$$

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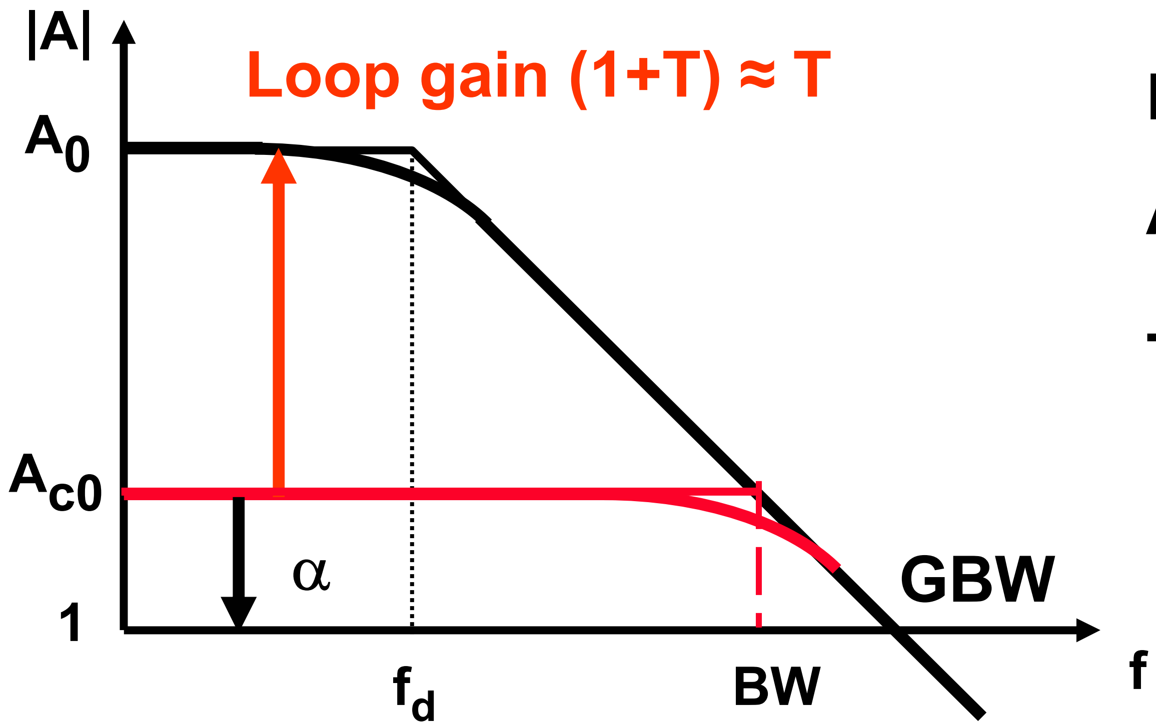
McCreary, JSSC Dec 75, 371-379
Gregorian, IEEE Proc. Aug 83, 941-986

Opamp parameters



Feedback factor α

$$A_{c0} = 1/\,\alpha$$

$$T = A_0 \,/\, A_{c0} = \alpha\, A_0$$

$$GBW = \frac{g_m}{2\pi\, C_{eff}}$$

$$BW = \alpha\ GBW$$

Static error

$$V_{out,\, t\,=\,\infty} = -\frac{Ao.Vstep}{1+\alpha.Ao}$$

$$\varepsilon_S = \frac{Vstep/\alpha - Vout}{Vstep/\alpha} = 1 - \frac{Ao}{1+Ao.\alpha} \approx \frac{1}{\alpha.Ao}$$

Minimum Gain

$$\boxed{Ao > \frac{1}{\alpha.\varepsilon_S}}$$

ε = 0.05%

⇩

$A_0 \approx$ 1-10k

$\approx$ 60-80 dB

Dynamic error

$$\mathcal{E}_D = EXP\left(-\frac{\alpha . gm . t_S}{C_{L,ef}}\right)$$

$$GBW = \frac{gm}{2\pi C_{L,ef}}$$

$$\mathcal{E}_D = EXP(-\alpha . 2\pi . GBW . t_S)$$

$$t_S = \frac{1}{2 f_c}$$

$$GBW = \frac{1}{\alpha . 2\pi . t_S} \ln\left(\frac{1}{\mathcal{E}_D}\right) = \frac{2 f_c}{2\pi . \alpha} \ln\left(\frac{1}{\mathcal{E}_D}\right)$$

Minimum GBW: $\boxed{GBW > \frac{f_c}{\pi . \alpha} \ln\left(\frac{1}{\mathcal{E}_D}\right)}$

ε = 0.05%

⇩

GBW ≈ 2-3*f_c

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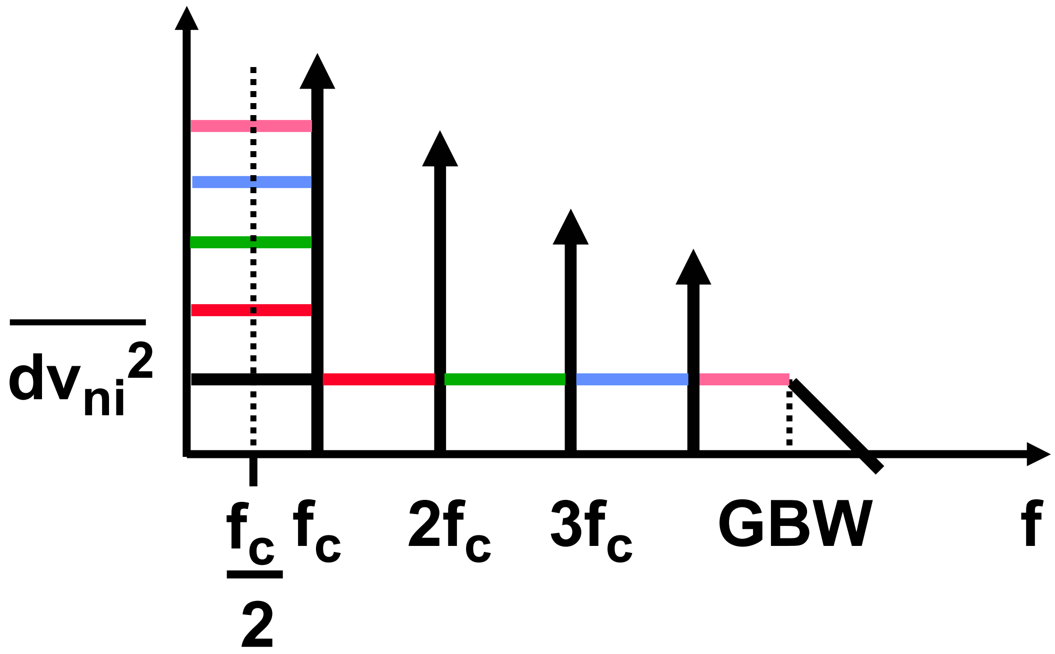
kT/C versus kTR noise

Narrow-band noise >> noise density : $\overline{dv_{ni}^2} = 4kT\ R\ df$

Wide-band noise >> integrated noise : $\overline{v_{ni}^2} = \dfrac{kT}{C}$



$$\overline{v_{ni}^2} = \frac{kT}{C}\ \frac{GBW}{f_c/2}$$

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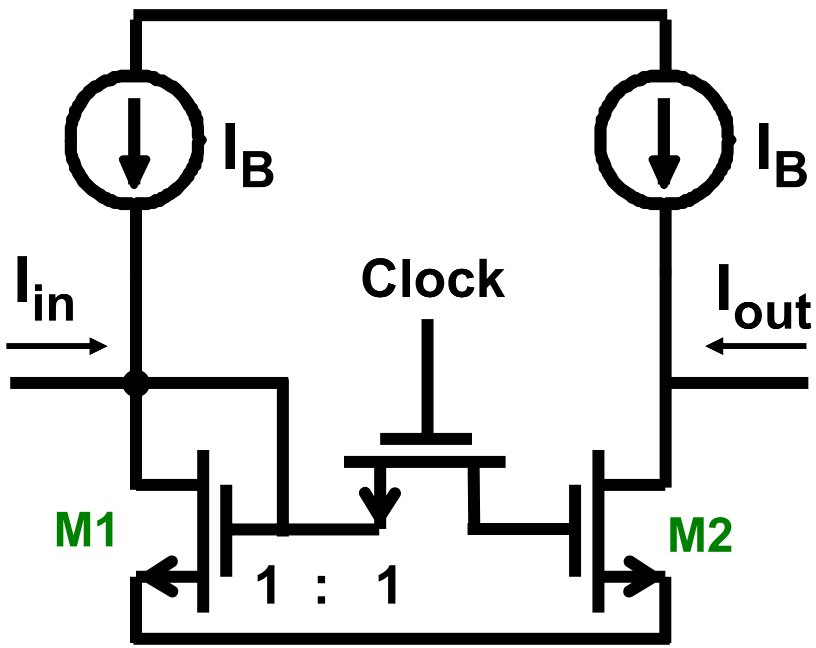
Switched-current delay block



Switch closed : track V_{GS}

$$I_{out} = I_{in}$$

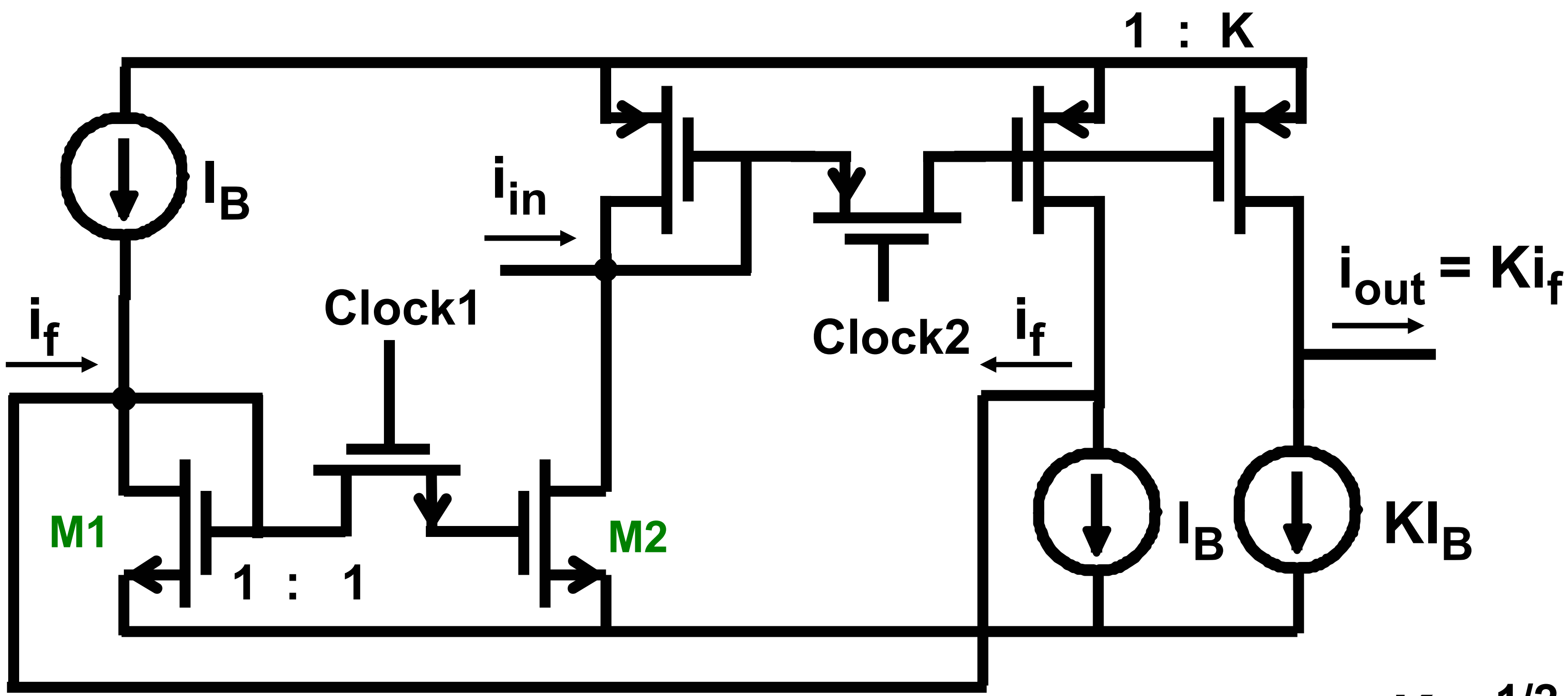
Switch open : hold V_{GS}

$$I_{out} = I_{in}\,(\Delta T_c)$$

$$I_{out} = I_{in}\,z^{-1/2}$$

Ref. Zele JSSC Feb. 96, 157- 168

Switched-current low-pass filter

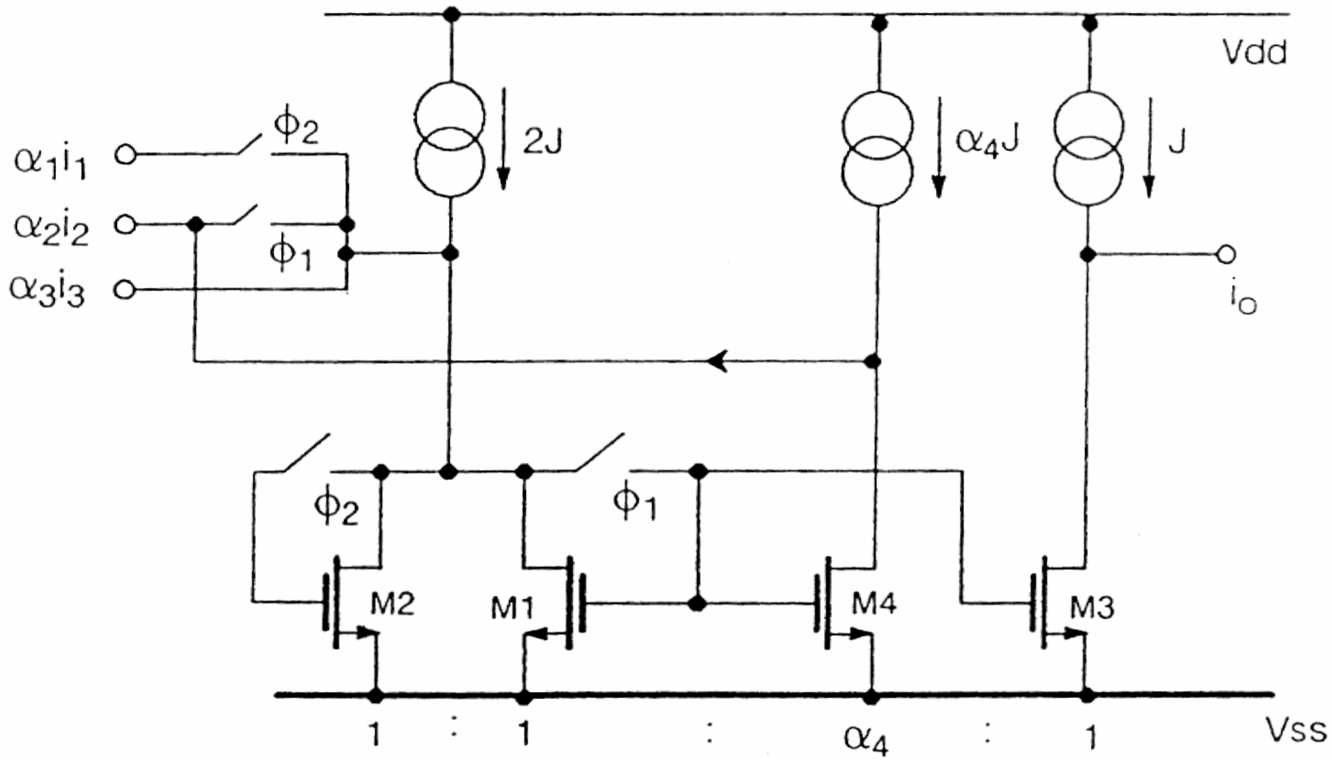


$$i_f = i_f\ z^{-1} - i_{in}\ z^{-1/2}$$

$$\frac{i_{out}}{i_{in}} = \frac{K\, z^{-1/2}}{1 - z^{-1}}$$

Ref. Zele JSSC Feb. 96, 157- 168

2nd-generation switched-current filter



$$A_1 = \frac{\alpha_1}{1 + \alpha_4}$$

$$A_2 = \frac{\alpha_2}{1 + \alpha_4}$$

$$A_3 = \frac{\alpha_3}{1 + \alpha_4}$$

$$B = \frac{1}{1 + \alpha_4}$$

$$i_o(z) = \frac{A_1 z^{-1}}{1 - Bz^{-1}} i_1(z) - \frac{A_2 z^{-1}}{1 - Bz^{-1}} i_2(z) - \frac{A_3(1 - z^{-1})}{1 - Bz^{-1}} i_3(z)$$

Comparison SC - SI

	SC	SI
Signal :	Voltage	Current
	Charge on linear C	Charge on MOST C_{GS}
	$Q = C\ V$	$Q = I\ t$
Accuracy :	Capacitor ratio	MOST area ratio
	0.2 %	2 %
Amps :	Opamps	Current mirrors
S/N+D	70 dB	50 dB

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- **Opamp requirements**
  - **Charge transfer accuracy**
  - **Noise**
- **Switched-current filters**

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Gregorian, IEEE Proc. Aug 83, 941-986