Stability of Operational amplifiers

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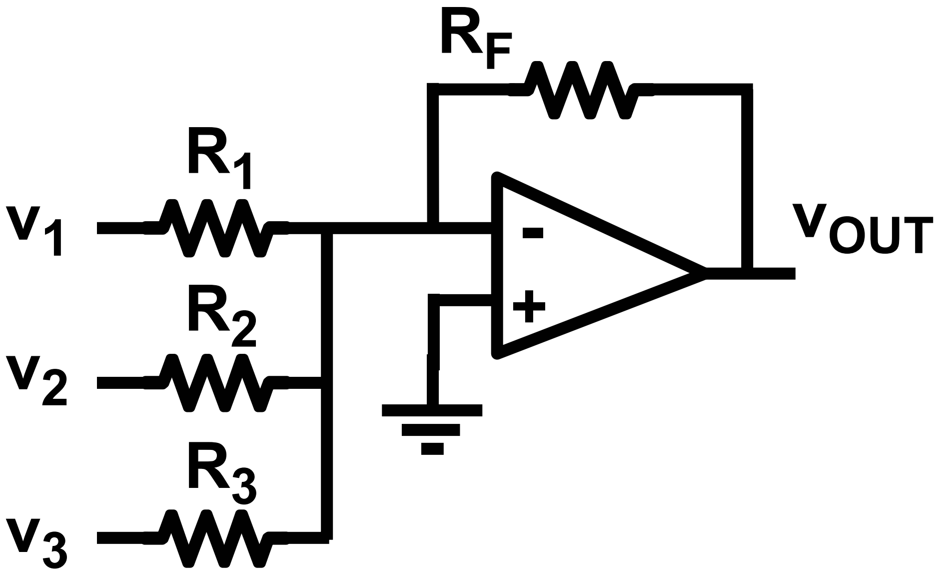
Operational amplifiers do operations

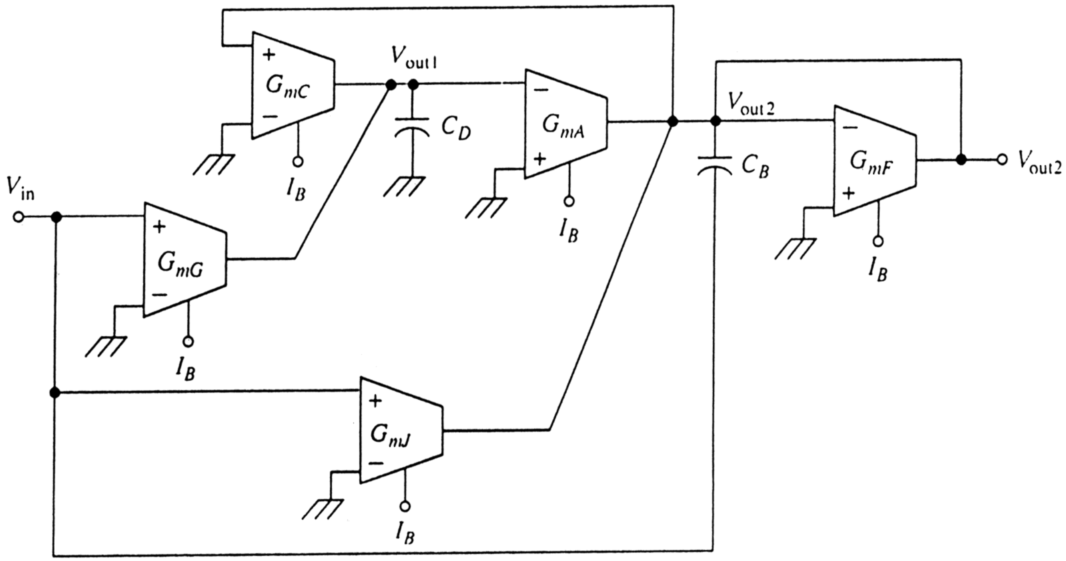


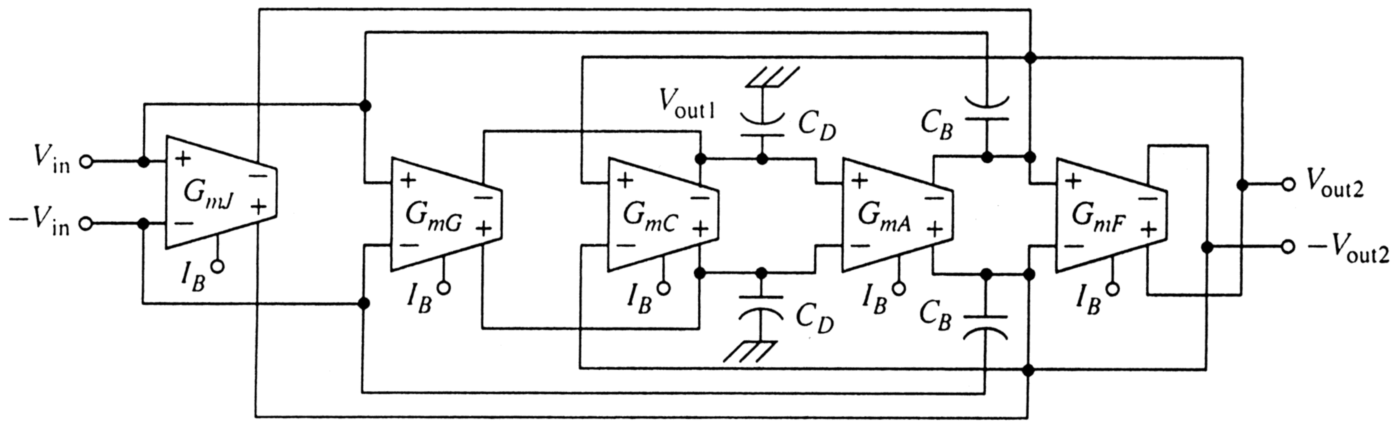
$$-\frac{v_{OUT}}{R_F} = \frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3}$$

Requires High gain
High speed
Low noise
Low power

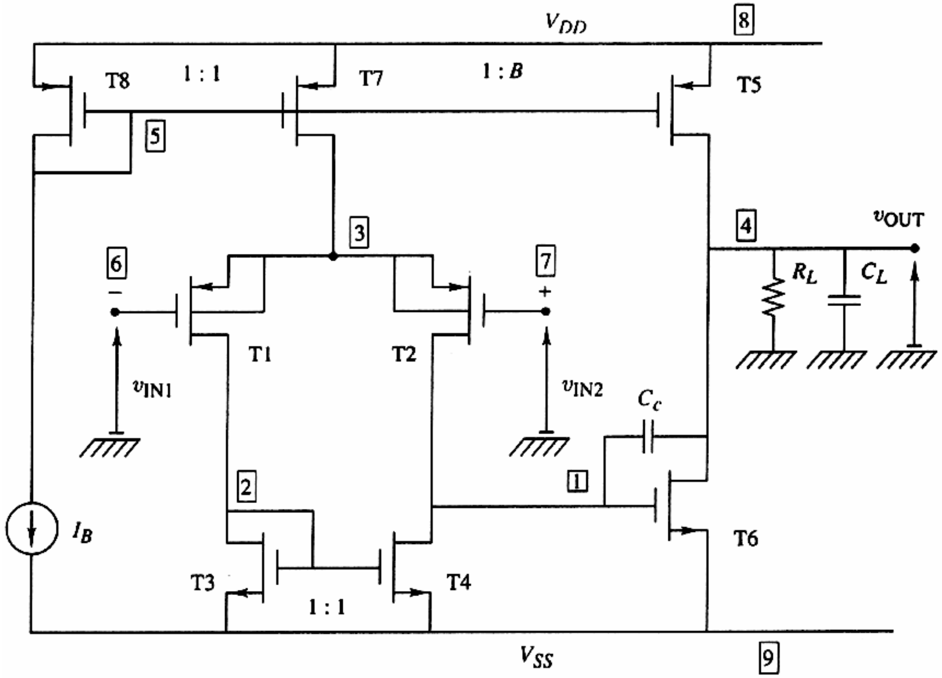
Opamp specs : Voltage gain is large
Differential input voltage ≈ 0
Input current = 0
Bandwidth is high
Gainbandwidth GBW is very, very high

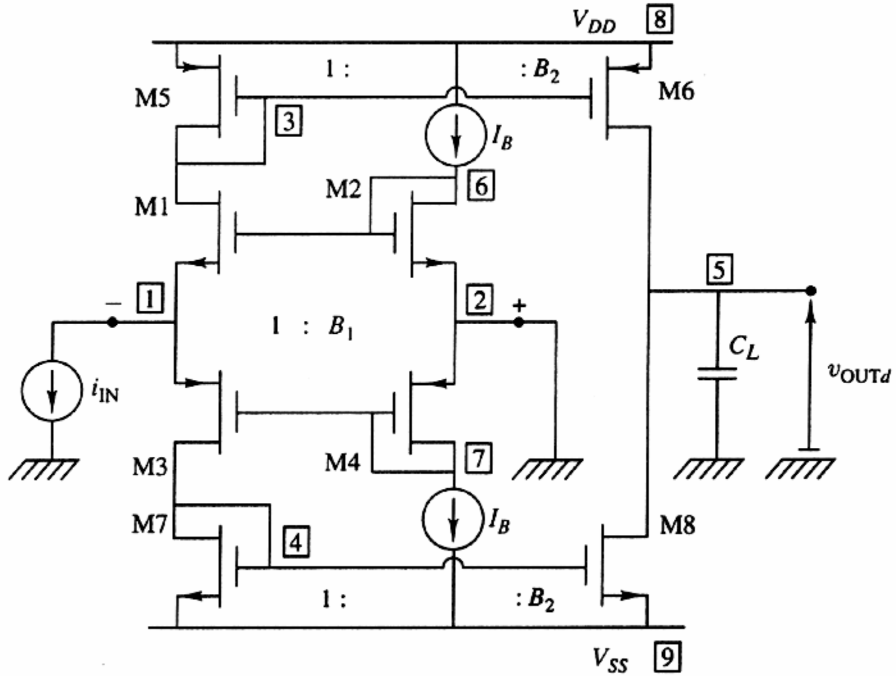
Single-ended or fully differential ?





Voltage input or current input ?





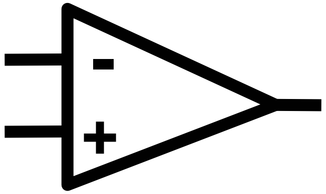
Voltage input
Current output

Current input
Current output

Classification

Opamp

Operational amplifier



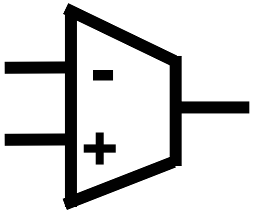
$$A_v = \frac{v_{OUT}}{v_{IN}}$$

$$A_v =$$

GBW

OTA

Operational Transconduct. amplifier

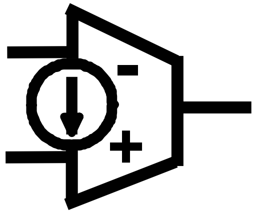


$$A_g = \frac{i_{OUT}}{v_{IN}}$$

$$= A_g \, \color{red}{R_L}$$

OCA

Operational Current amplifier

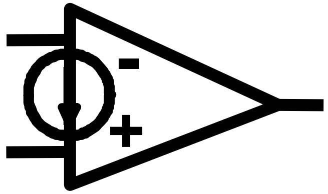


$$A_i = \frac{i_{OUT}}{i_{IN}}$$

$$= A_i \frac{\color{red}{R_L}}{\color{blue}{R_S}}$$

CM amp

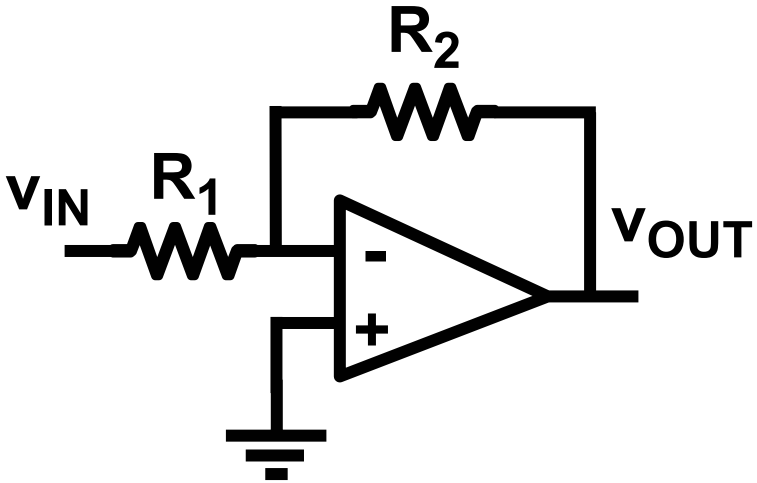
Current Mode amplifier

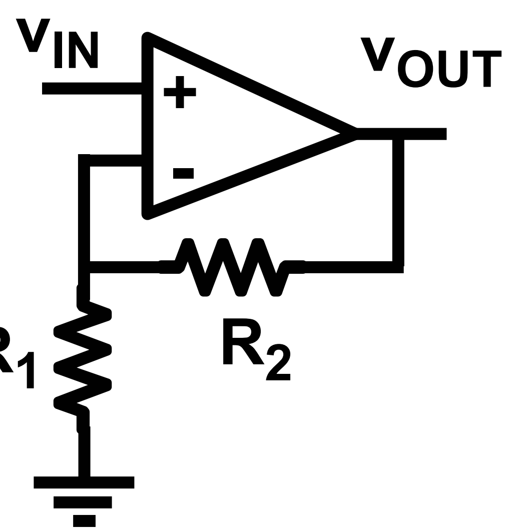


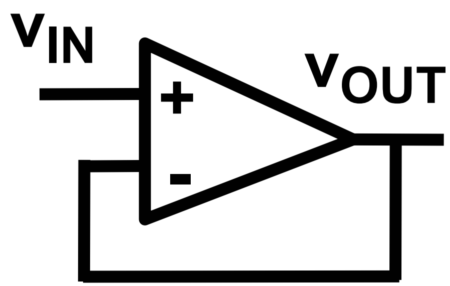
$$A_r = \frac{v_{OUT}}{i_{IN}}$$

$$= A_r \frac{1}{\color{blue}{R_S}}$$

Feedback configurations







$$A_v = -\frac{R_2}{R_1}$$

$$A_v = 1 + \frac{R_2}{R_1}$$

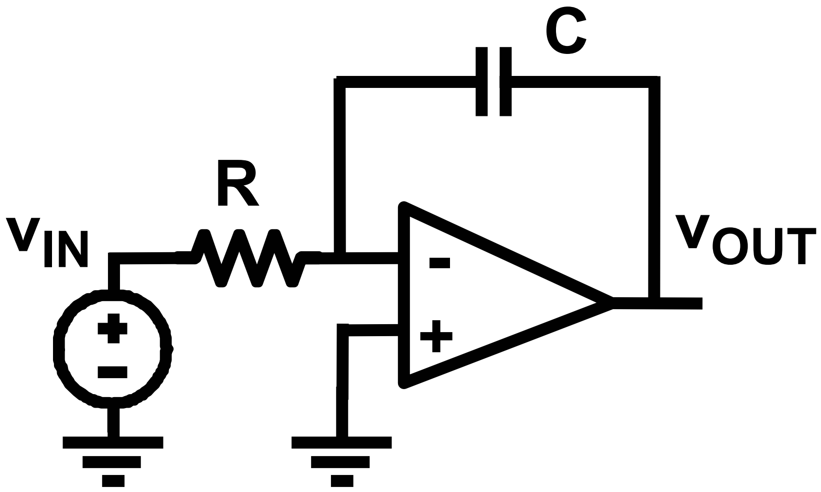
$$A_v = 1$$

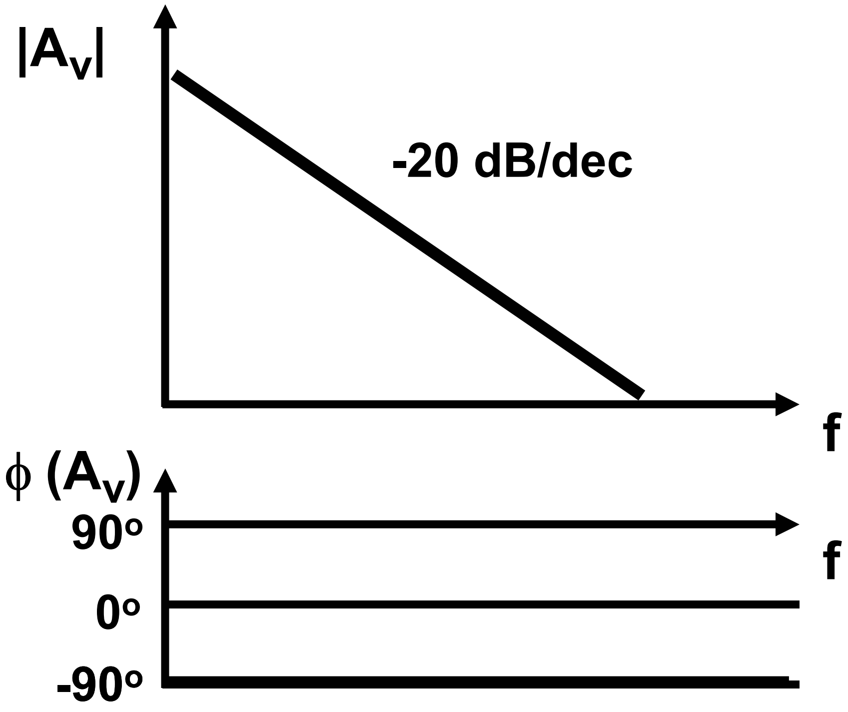
$$R_{IN} = R_1$$

$$R_{IN} = \infty$$

$$R_{IN} = \infty$$

Integrator

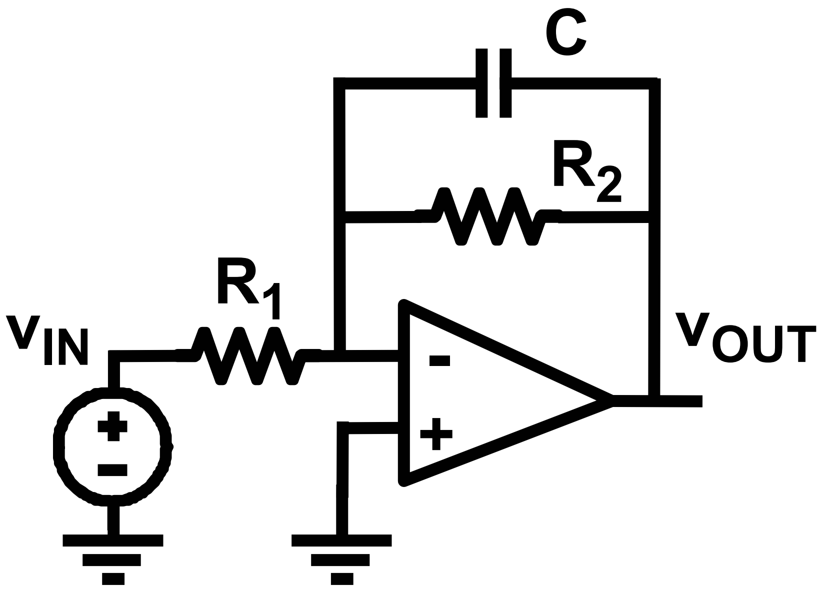


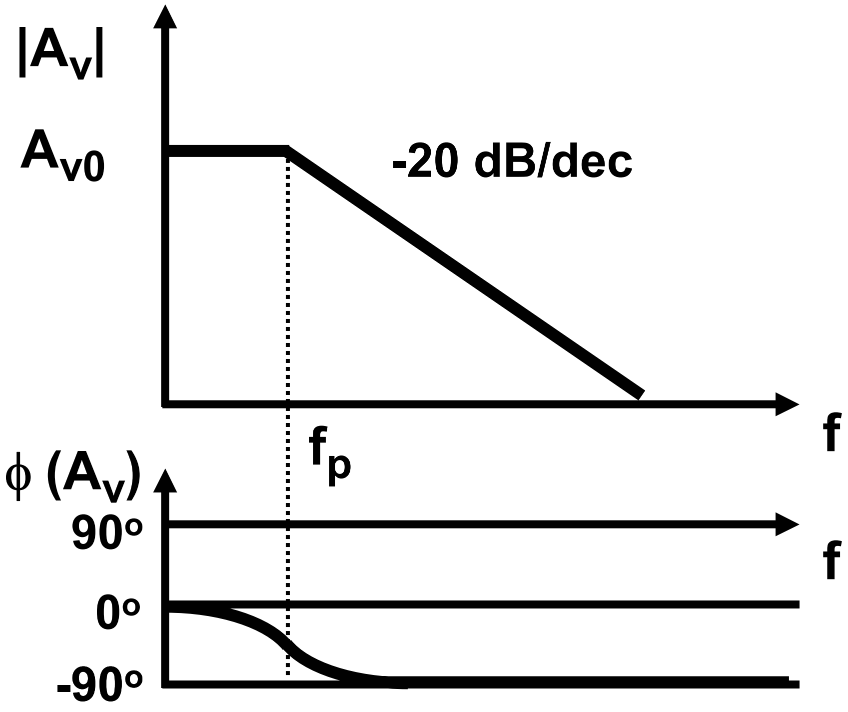


$$A_v = \frac{1}{j\,\frac{f}{f_p}}$$

$$f_p = \frac{1}{2\pi\,RC}$$

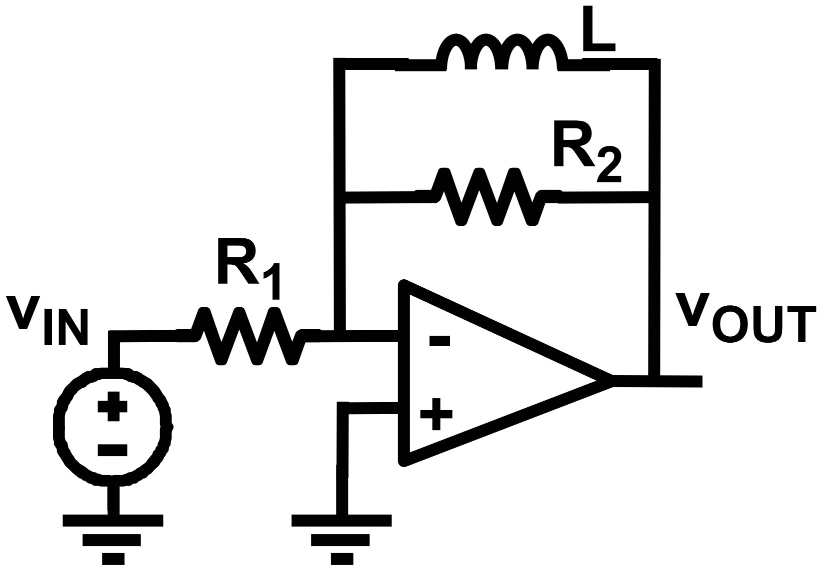
Low-pass filter

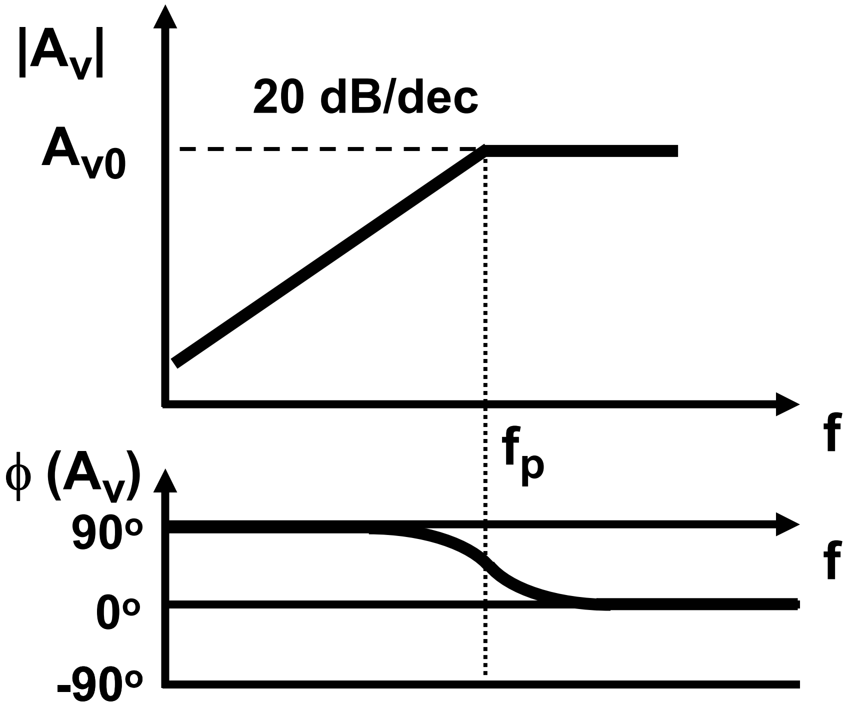




$$A_{v0} = -\frac{R_2}{R_1} \qquad A_v = \frac{A_{v0}}{\left(1 + j\dfrac{f}{f_p}\right)} \qquad f_p = \frac{1}{2\pi R_2 C}$$

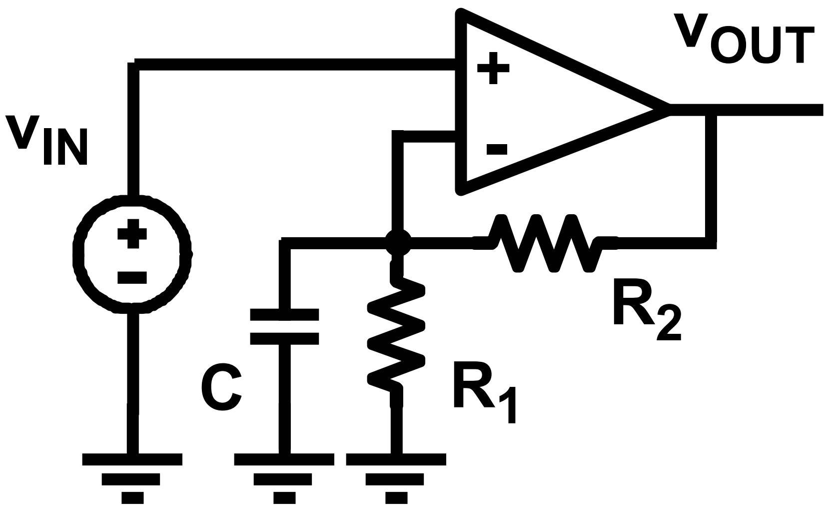
High-pass filter

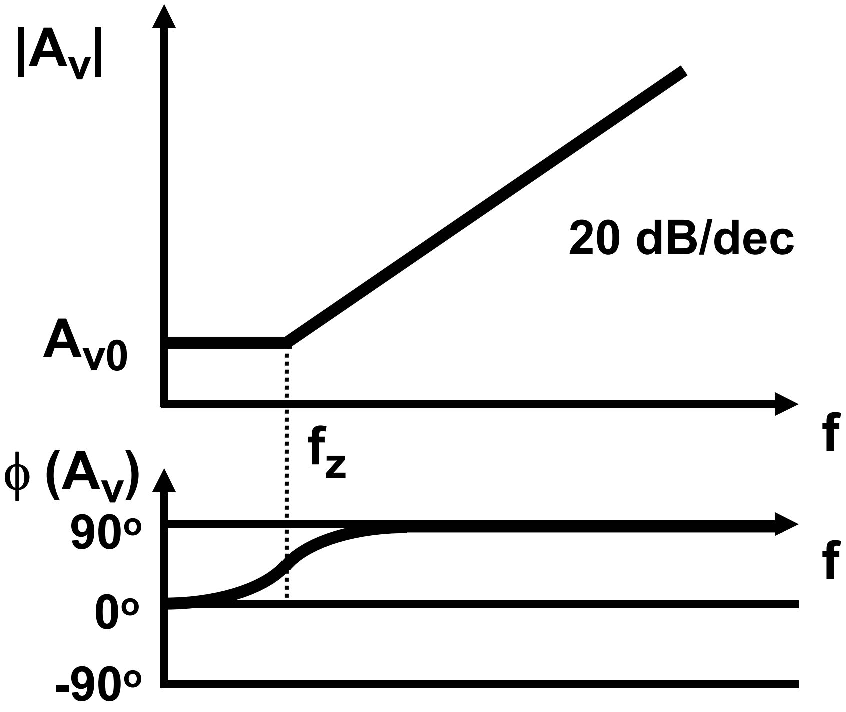




$$A_{v0} = -\frac{R_2}{R_1} \qquad A_v = A_{v0}\frac{j\frac{f}{f_p}}{\left(1 + j\frac{f}{f_p}\right)} \qquad f_p = \frac{R_2}{2\pi L}$$

High-pass filter





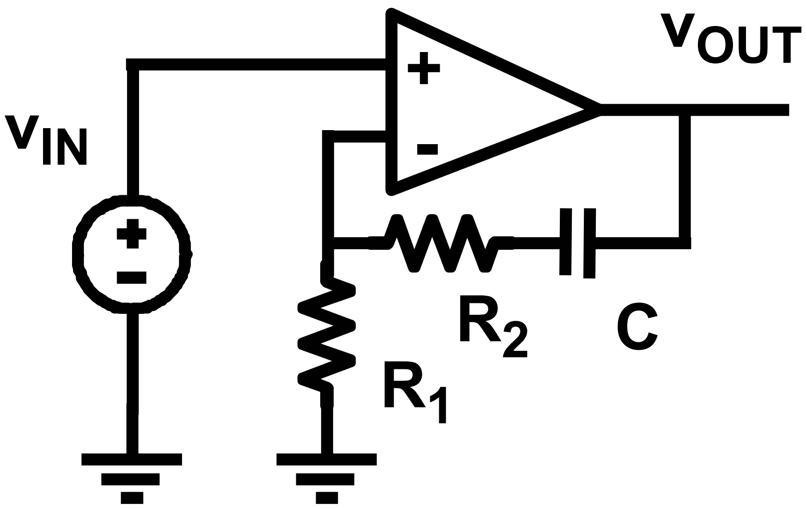
$$A_{v0} = 1 + \frac{R_2}{R_1}$$

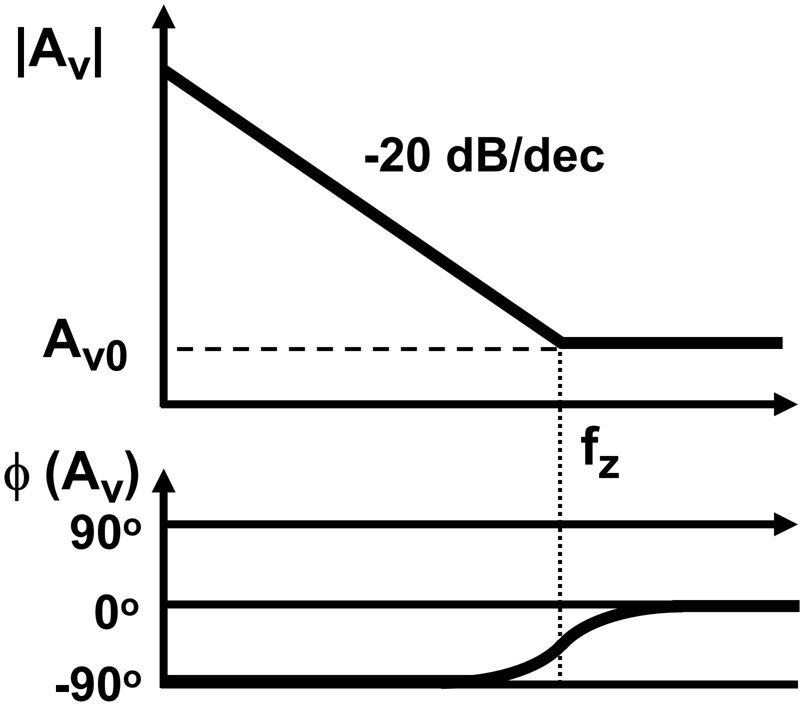
$$A_v = A_{V0}\,(1 + j\,\frac{f}{f_z})$$

$$f_z = \frac{1}{2\pi\,RC}$$

$$R = R_1//R_2 = \frac{R_1R_2}{R_1+R_2}$$

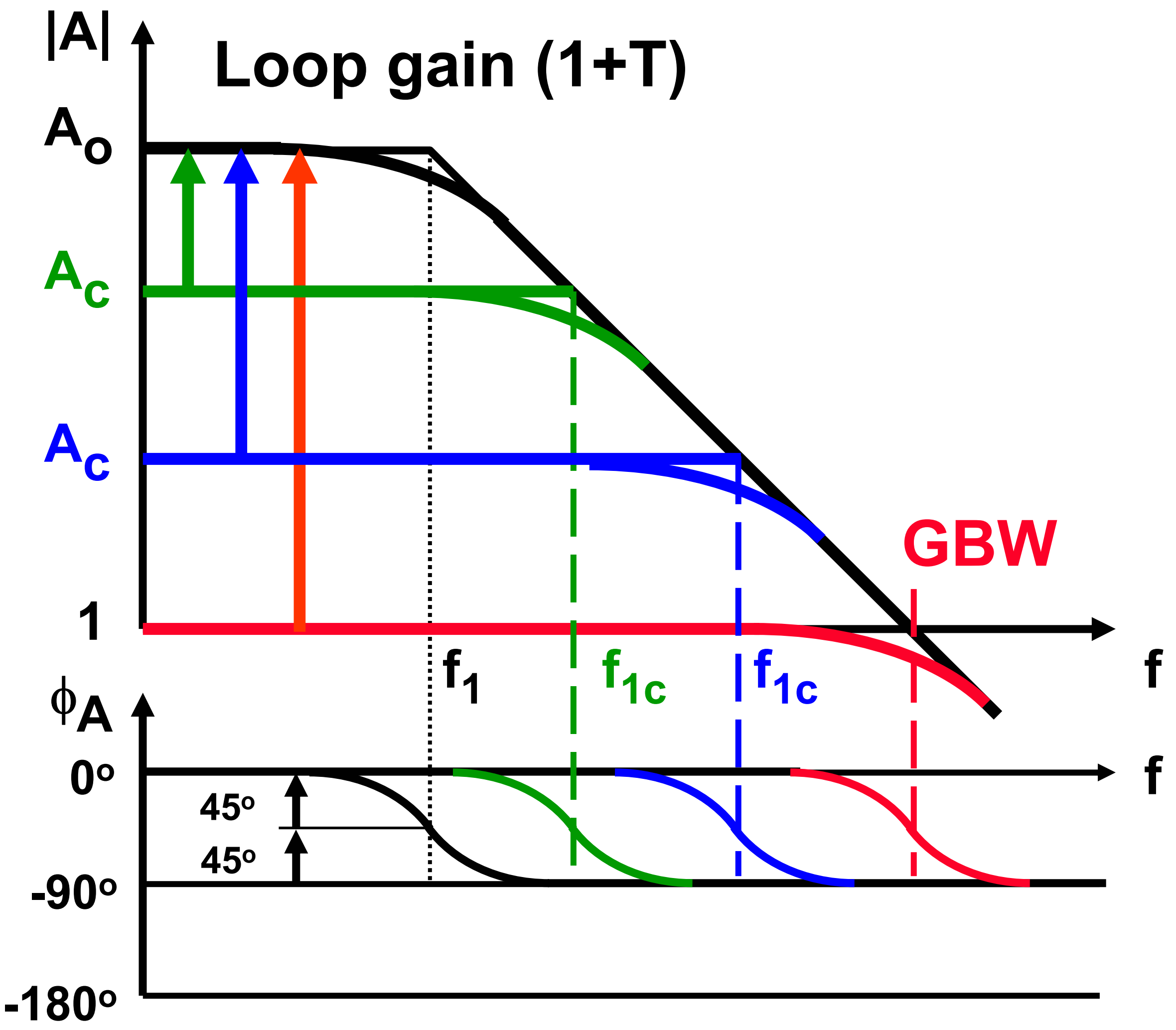
Low-pass filter with finite attenuation





$$A_{v0} = 1 + \frac{R_2}{R_1} \qquad A_v = A_{v0}\,\frac{\left(1 + j\,\frac{f}{f_z}\right)}{j\,\frac{f}{f_z}} \qquad f_z = \frac{1}{2\pi\, RC} \qquad R = R_1 + R_2$$

Exchange of gain and bandwidth



A_o open loop gain

A_c closed loop gain

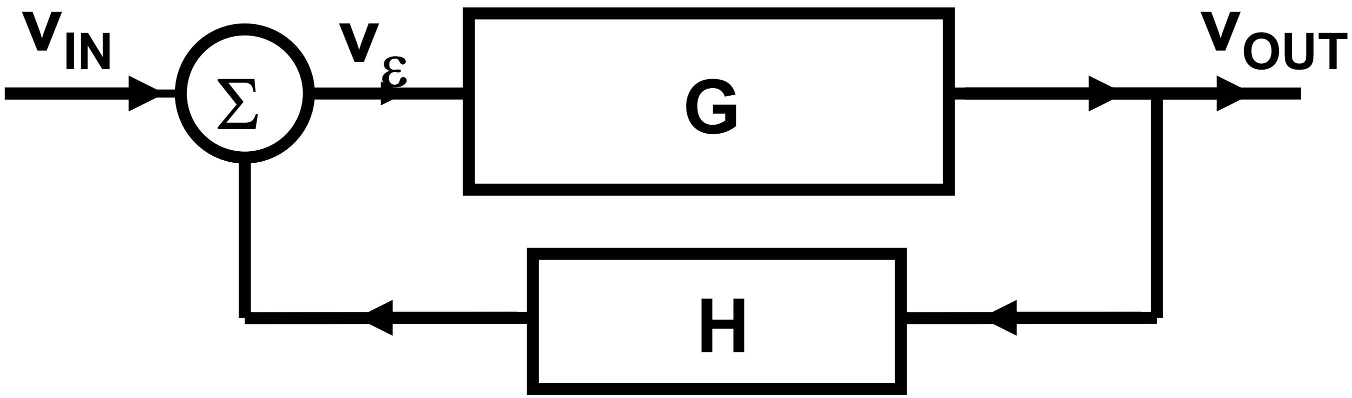
$A_o\, f_1 =$

$A_c\, f_{1c} =$

$A_c\, f_{1c} =$

GBW

Open- and closed-loop gain

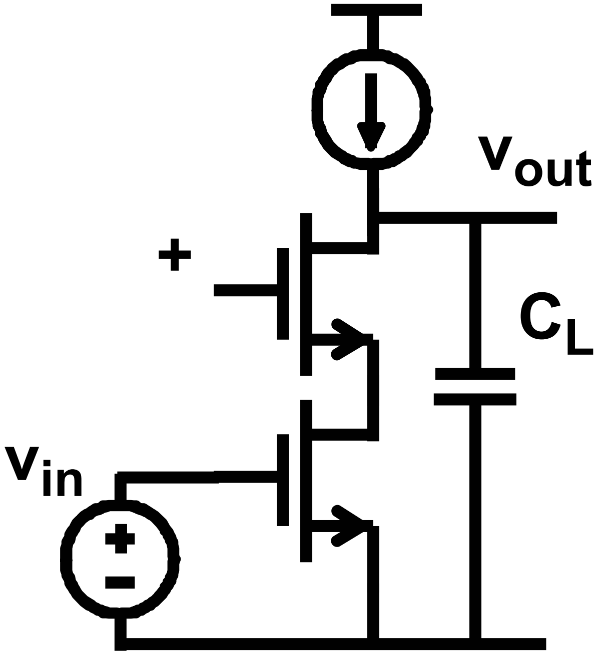


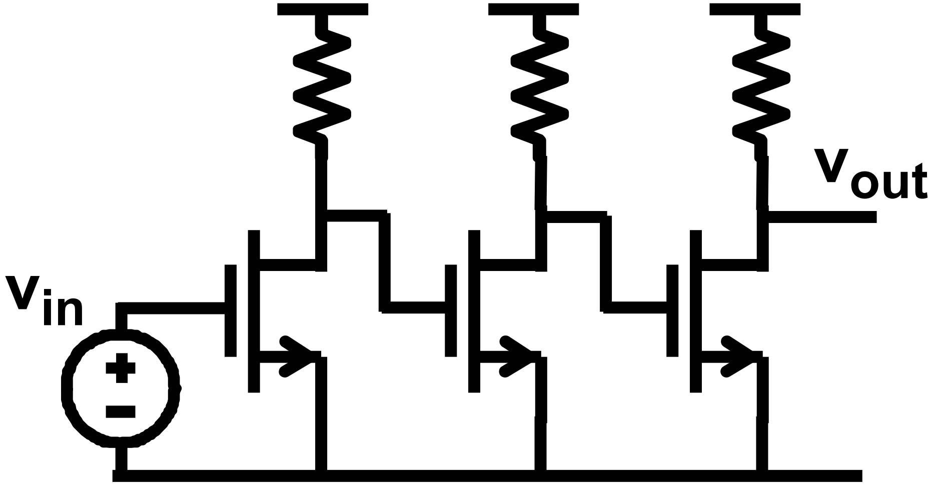
$$\left.\begin{array}{l} v_\varepsilon = v_{IN} - H\, v_{OUT} \\ v_{OUT} = G\, v_\varepsilon \end{array}\right\} \quad A_c = \frac{v_{OUT}}{v_{IN}} = \frac{G}{1 + GH} \approx \frac{1}{H}$$

if the loop gain GH = T >> 1

P. Gray, P.Hurst, S.Lewis, R. Meyer: Design of analog integrated circuits, 4th ed., Wiley 2001

What makes an opamp an opamp ?

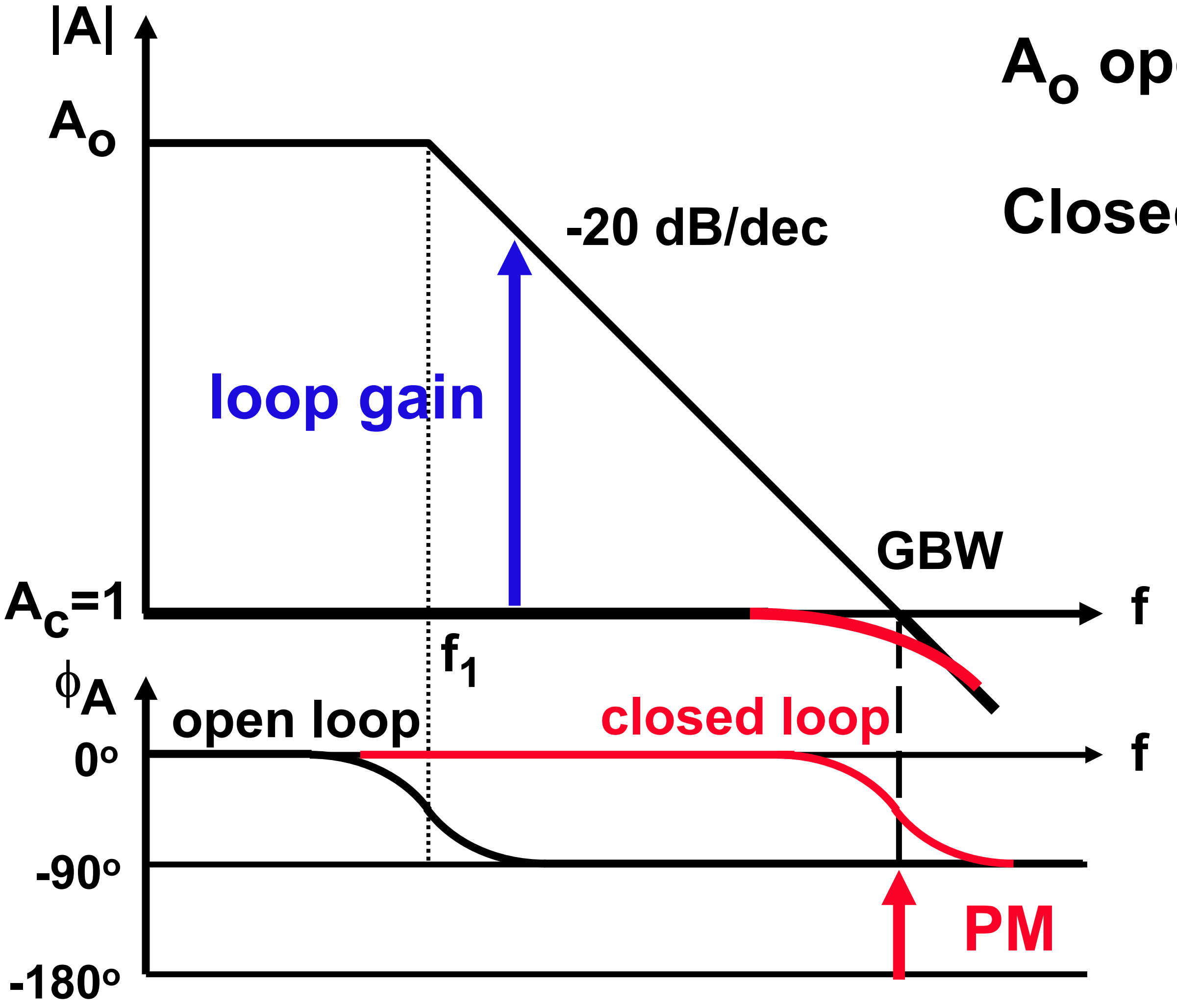




Operational amplifier :
Single-pole amplifier
High impedance = high gain
Exchange Gain-Bandwidth
Stable for all gain values

Wideband amplifier :
Multiple-pole amplifier
Low impedances at nodes
Wide Bandwidth
Stable for one gain only

Single-pole system

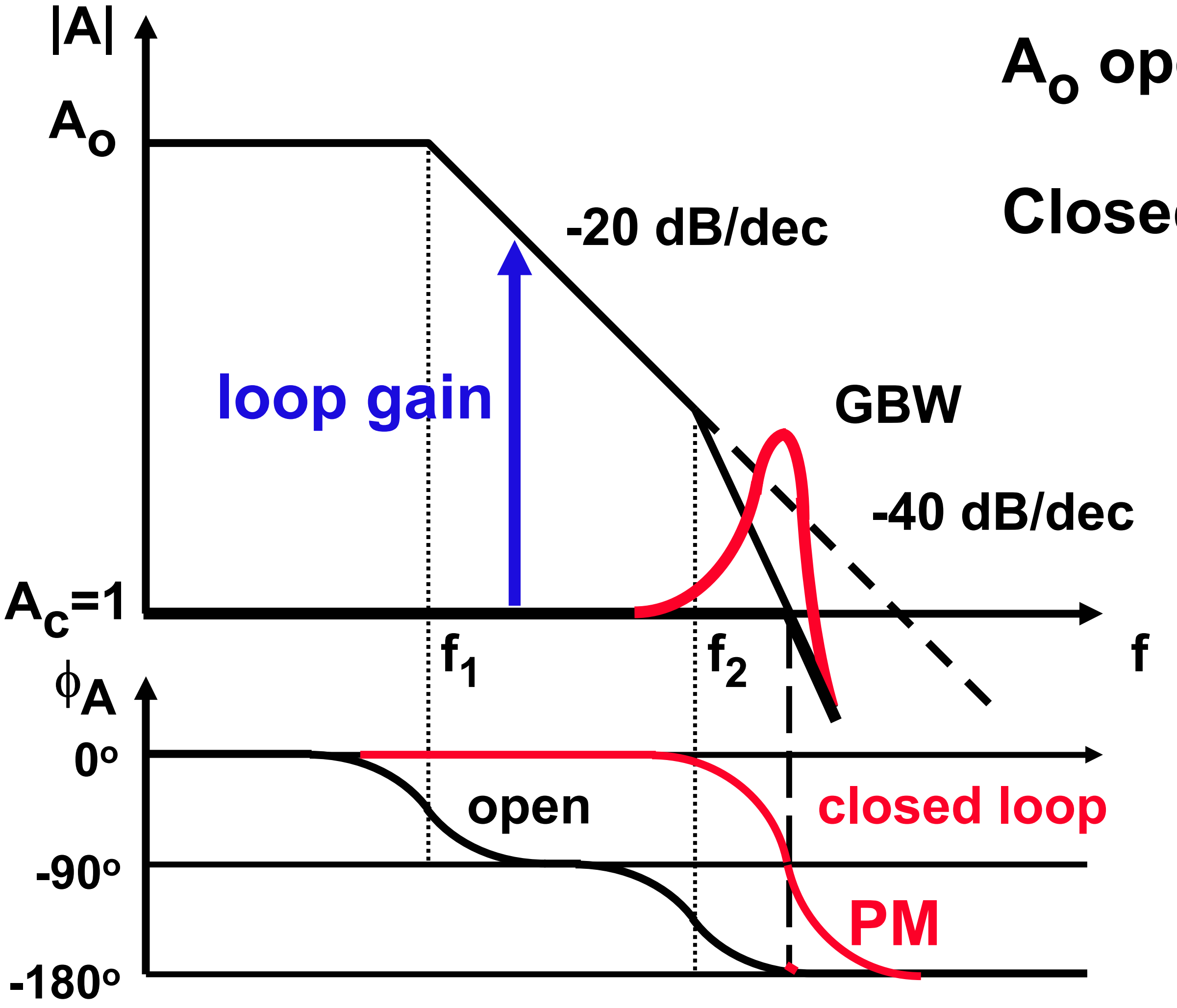


A_o open loop gain

Closed loop gain $A_c = 1$

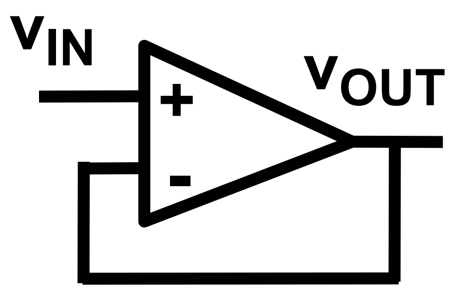
PM phase margin

Two-pole system



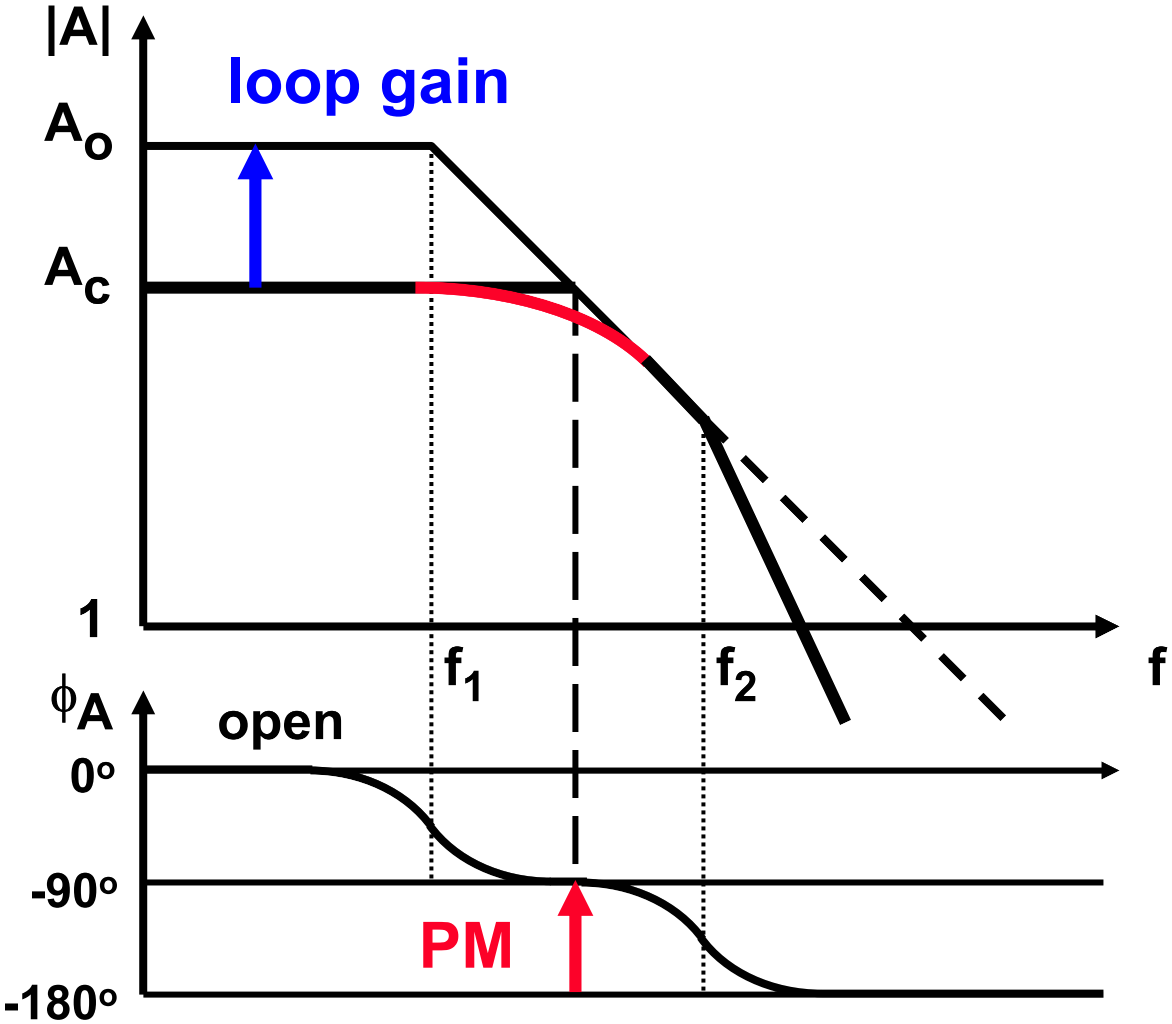
A_o open loop gain

Closed loop gain $A_c = 1$



PM phase margin

Higher loop gain gives less PM

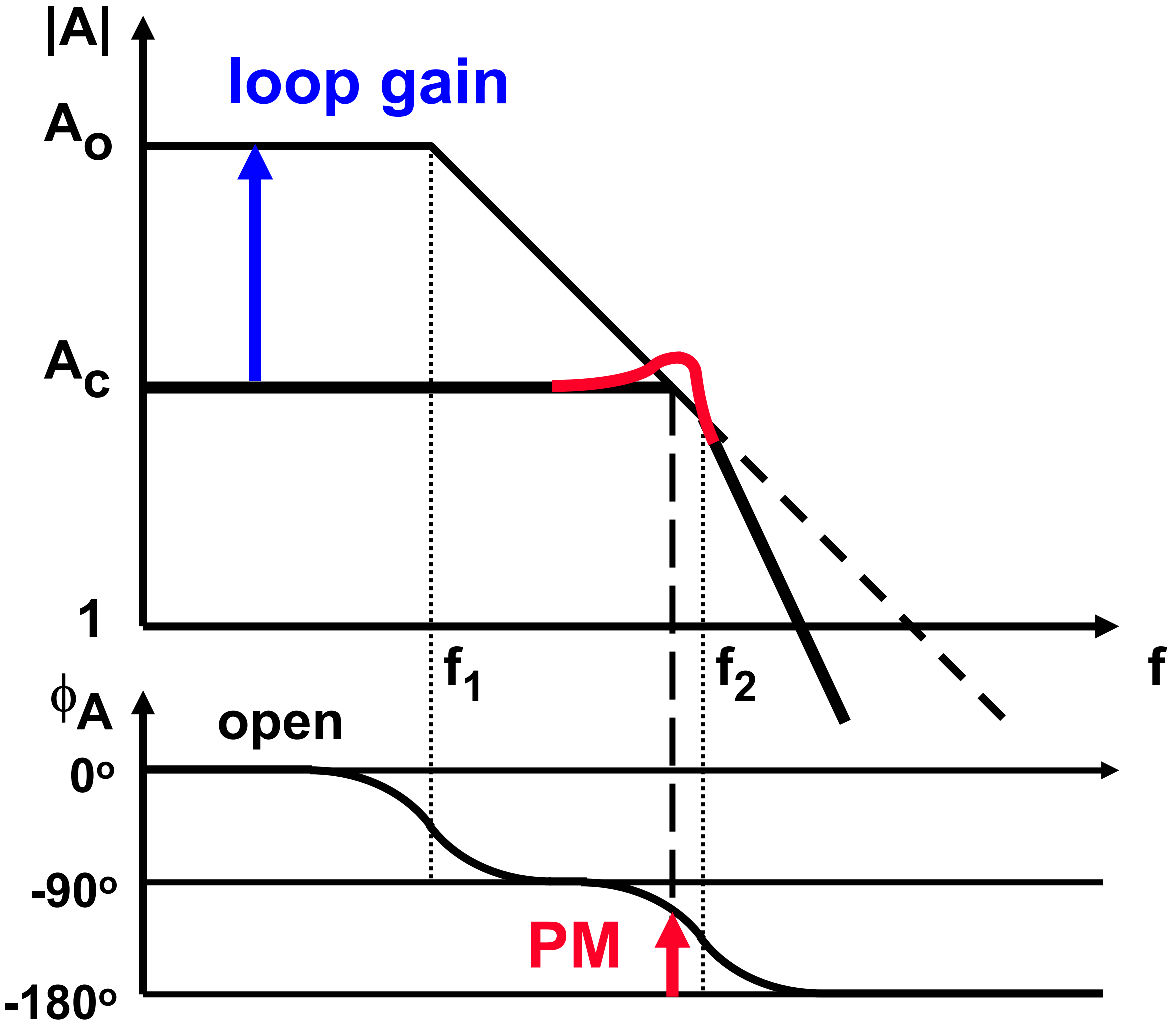


A_o open loop gain

A_c closed loop gain

PM phase margin

Higher loop gain gives less PM

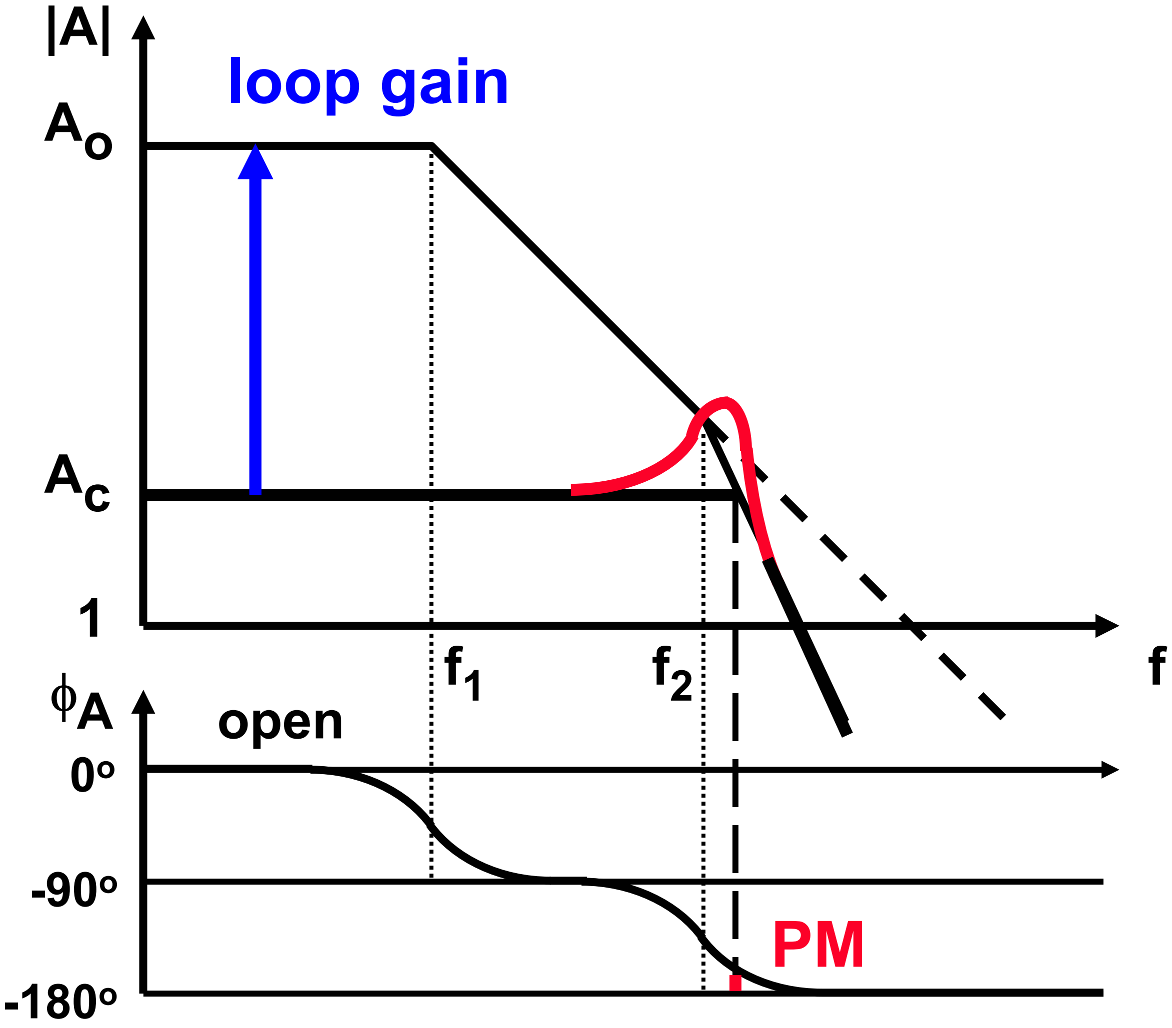


A_o open loop gain

A_c closed loop gain

PM phase margin

Higher loop gain gives less PM

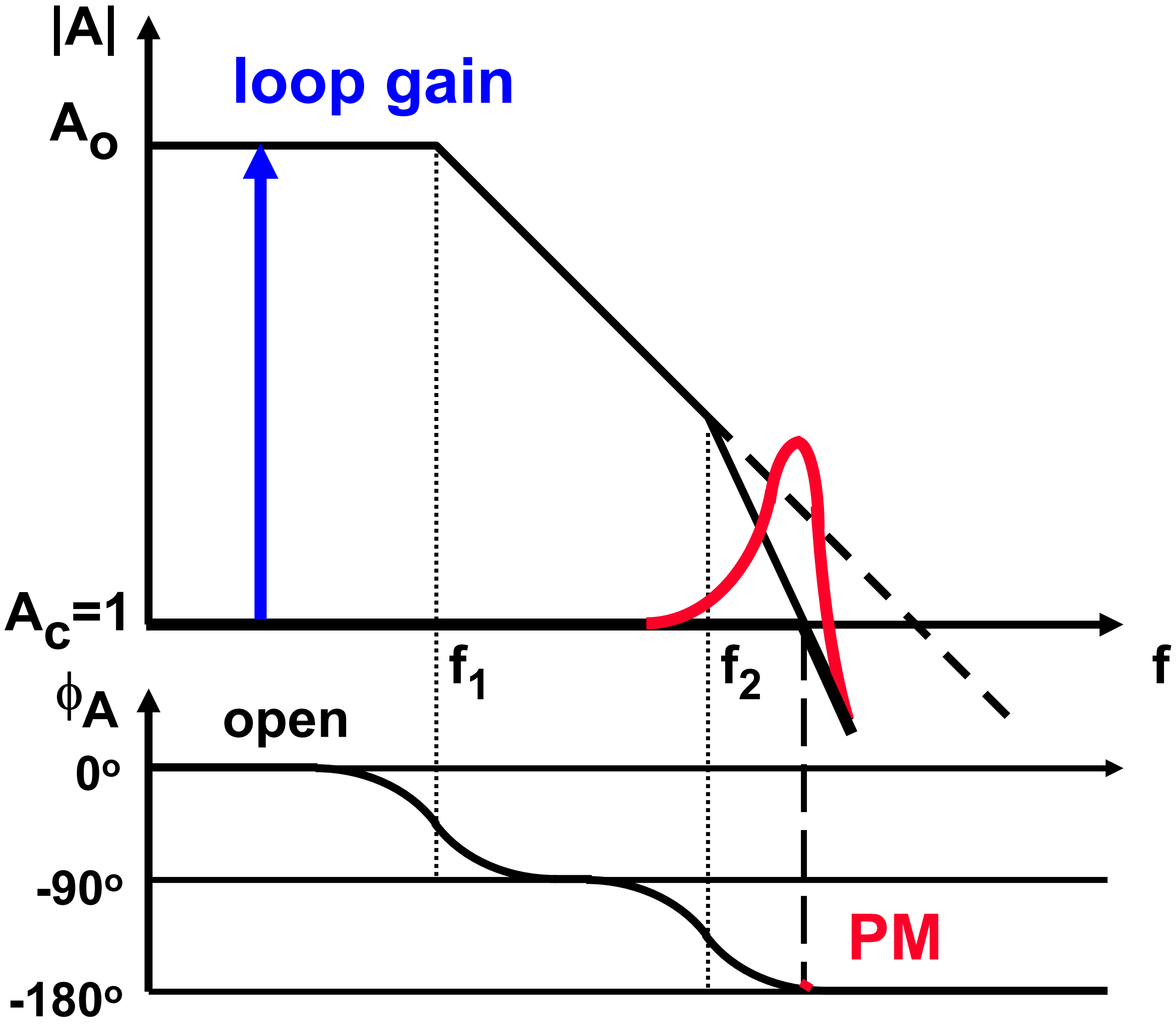


A_o open loop gain

A_c closed loop gain

PM phase margin

Higher loop gain gives less PM



A_o open loop gain

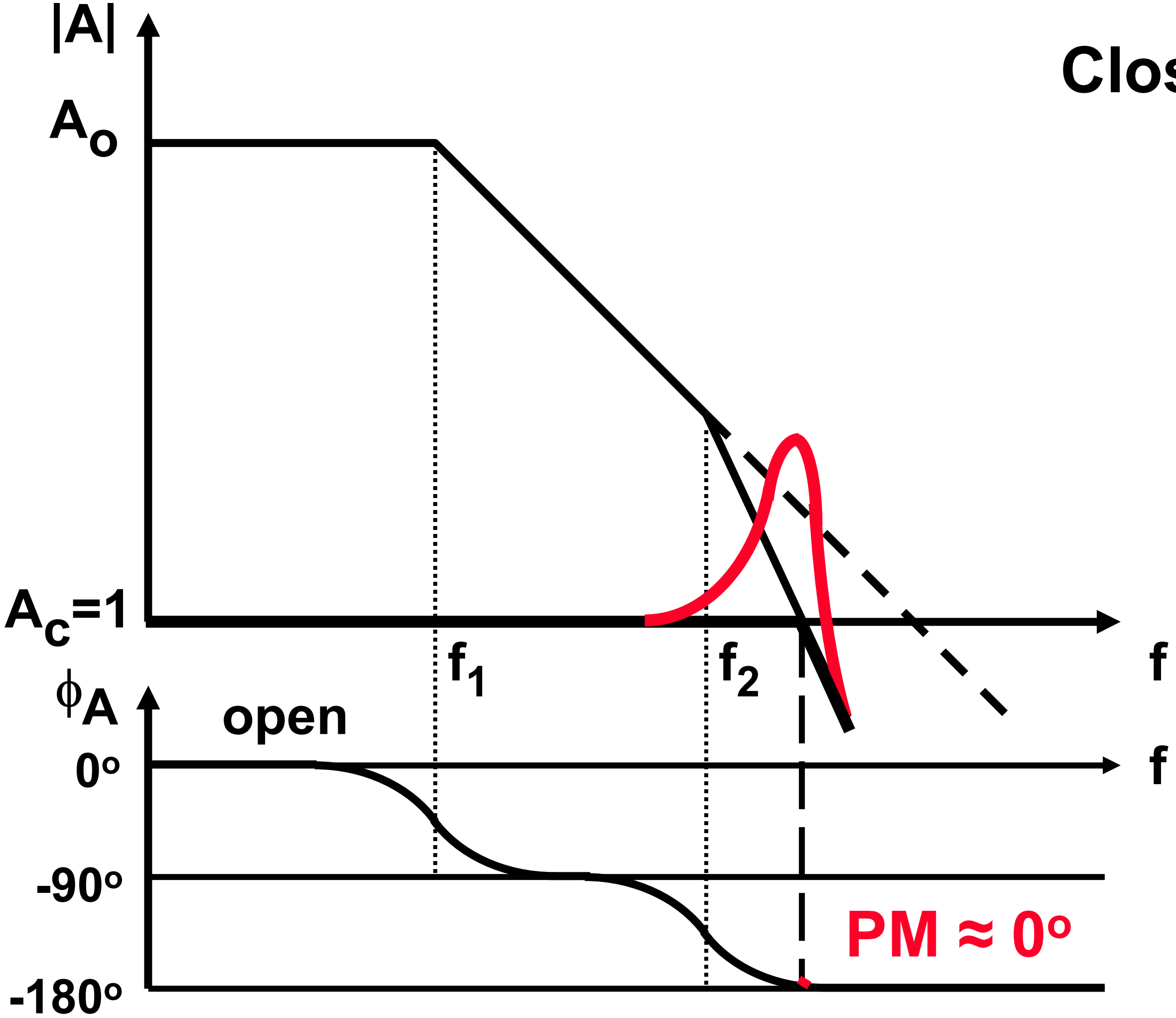
A_c closed loop gain

Worst case for $A_c = 1$

PM phase margin

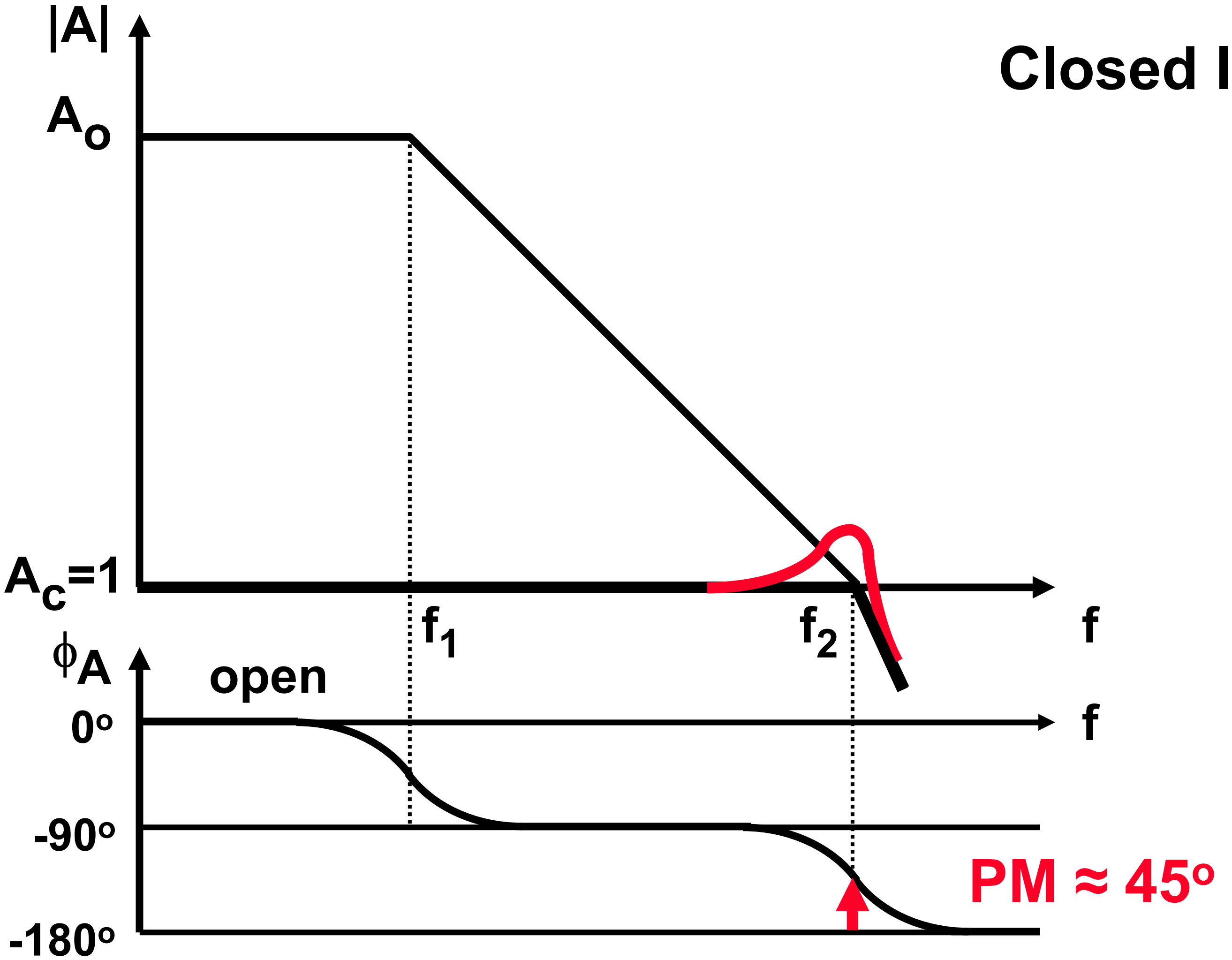
Increase PM by increasing f_2 : low f_2

Closed loop gain $A_c = 1$

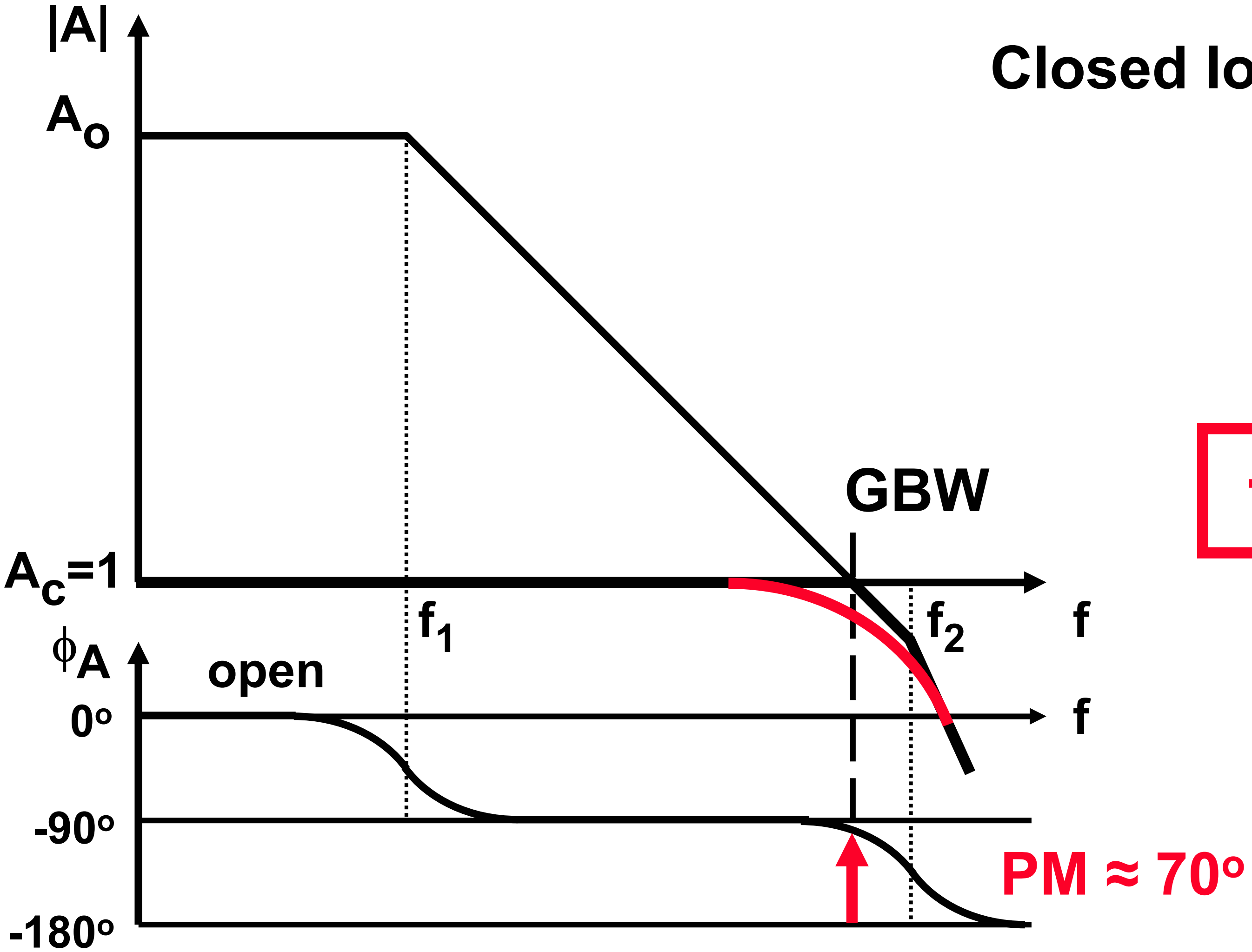


Increase PM by increasing f_2

Closed loop gain $A_c = 1$



Set PM by setting $f_2 \approx 3$ GBW

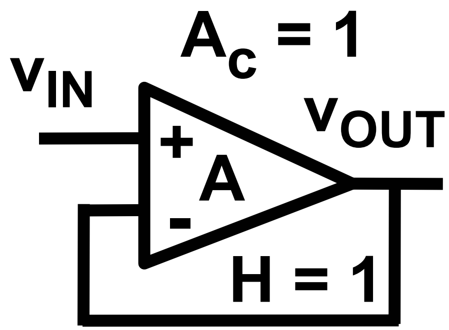


Closed loop gain $A_c = 1$

$f_2 \approx 3$ GBW

Calculate PM for $f_2 \approx 3$ GBW

Open loop gain $A = \dfrac{A_o}{(1 + j\dfrac{f}{f_1})(1 + j\dfrac{f}{f_2})}$



Closed loop gain $A_c = \dfrac{A}{1+A} \approx \dfrac{1}{1 + j\dfrac{f}{GBW} + j^2\dfrac{f^2}{GBW\, f_2}}$

$$\approx \frac{1}{1 + j\,2\zeta\dfrac{f}{f_r} + j^2\dfrac{f^2}{f_r^2}}$$

ζ is the damping (=1/2Q)

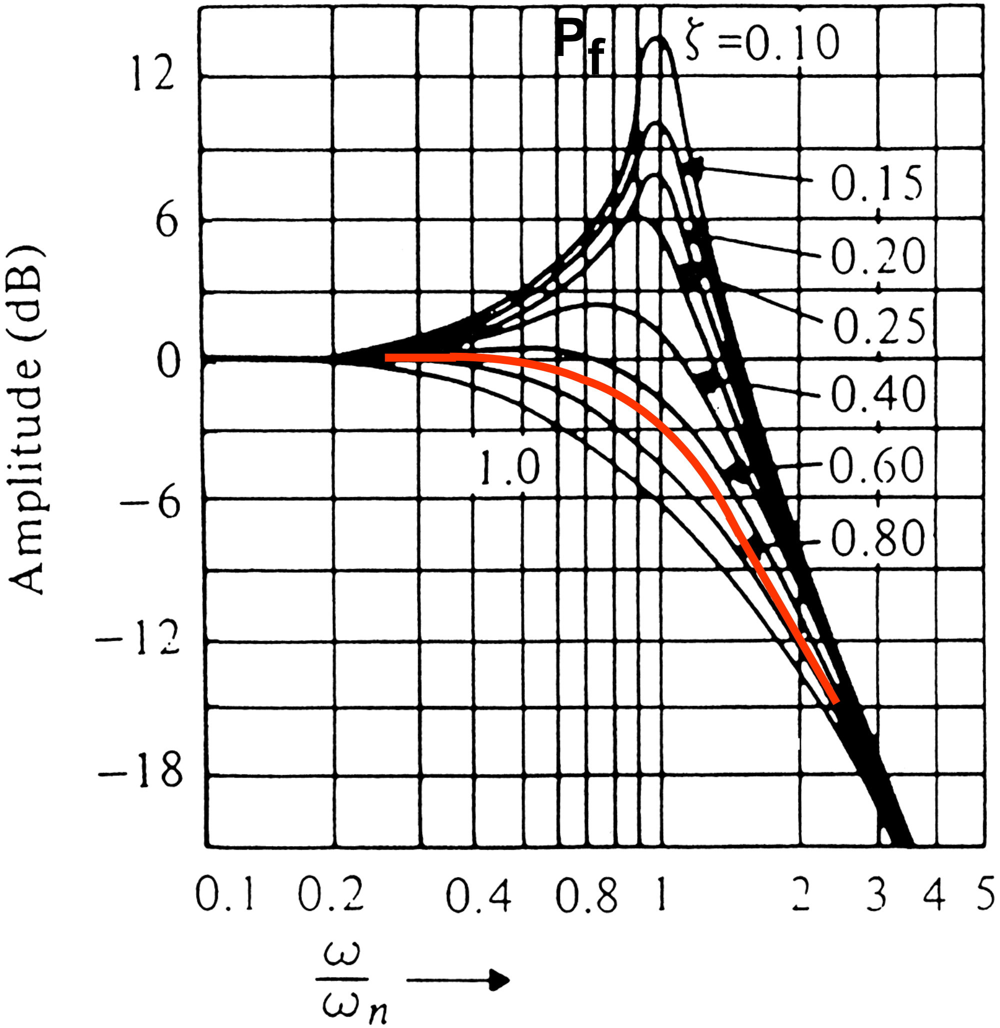
f_r is the resonant frequency

Relation PM, damping and f_2/GBW

$$f_r = \sqrt{GBW\, f_2} \qquad PM\,(^\circ) = 90^\circ - \arctan\frac{GBW}{f_2} = \arctan\frac{f_2}{GBW}$$

$\frac{f_2}{GBW}$	PM (°)	$\zeta = \frac{1}{2}\sqrt{\frac{f_2}{GBW}}$	P_f (dB)	P_t (dB)
0.5	27	0.35	3.6	2.3
1	45	0.5	1.25	1.3
1.5	56	0.61	0.28	0.73
2	63	0.71	0	0.37
3	72	0.87	0	0.04

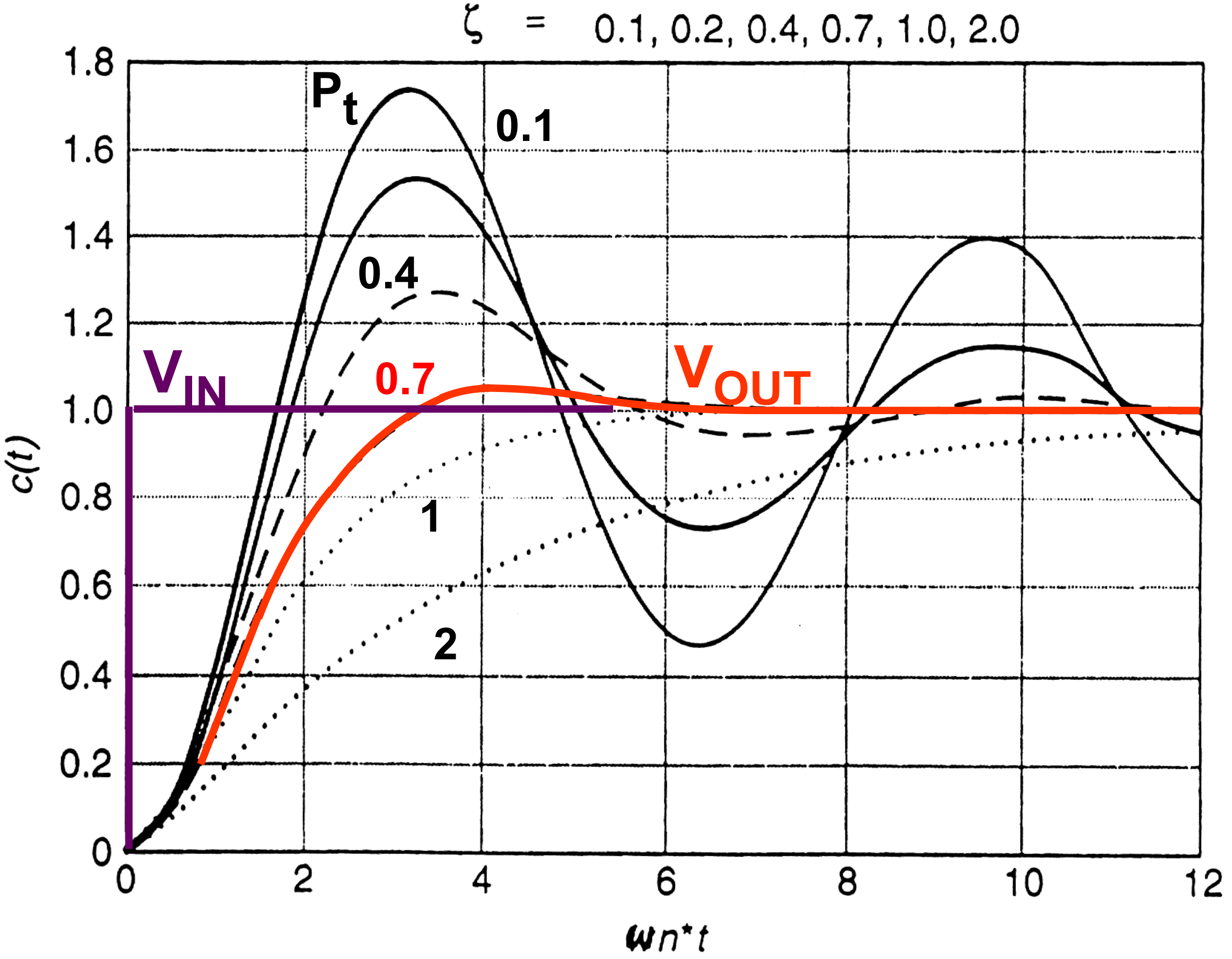
Amplitude response vs frequency



ζ = Q = 0.7

$$P_f = \frac{1}{2\zeta\sqrt{1-\zeta^2}}$$

Amplitude response vs time



$\zeta = Q = 0.7$

$$P_t = 1 + e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$$

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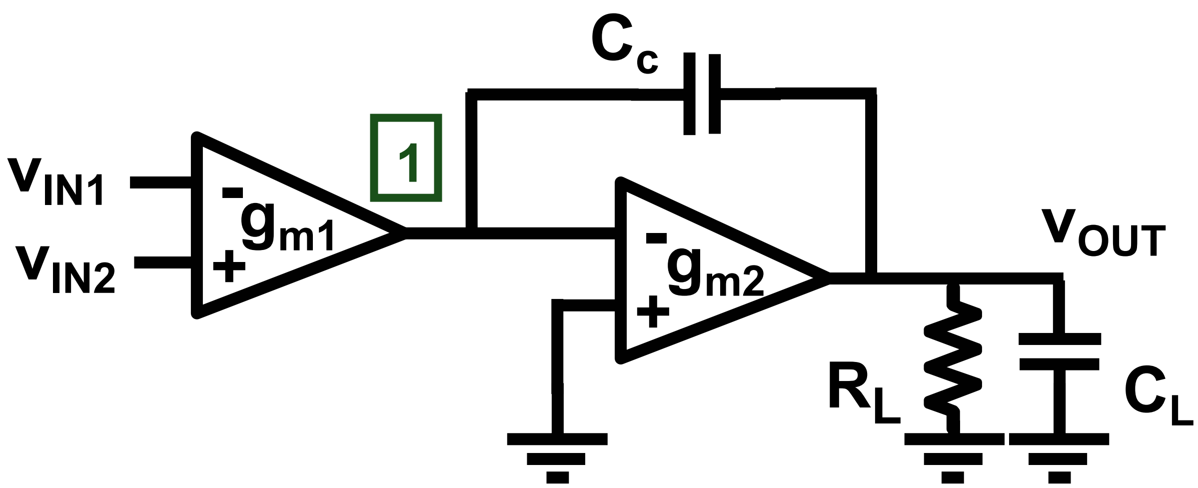
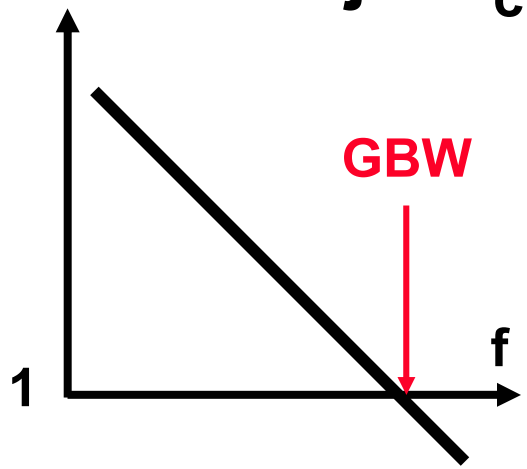
Generic 2-stage opamp



$$A_v = g_{m1} \frac{1}{j\omega C_c}$$

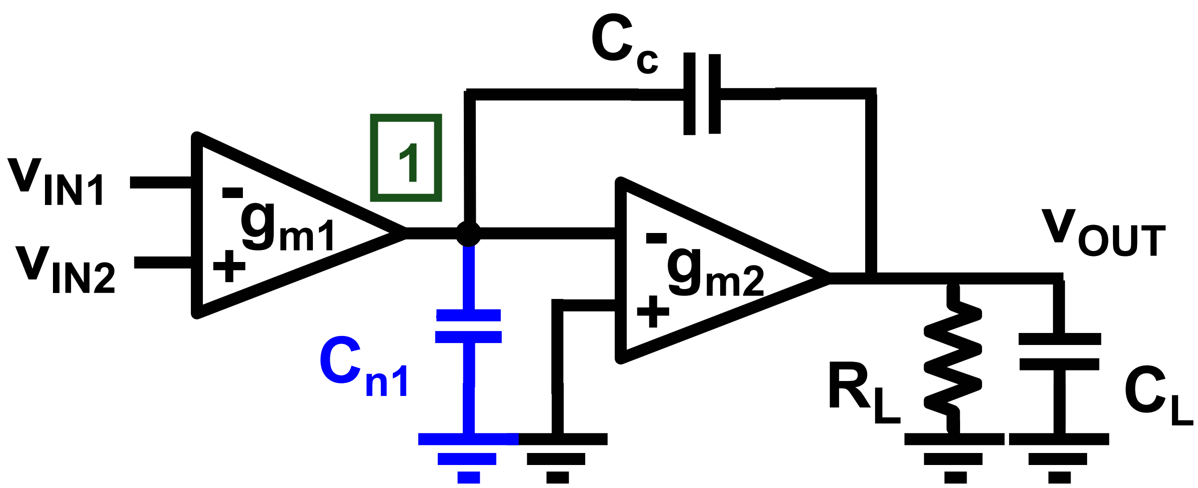


$|A_v| = 1 \Rightarrow$

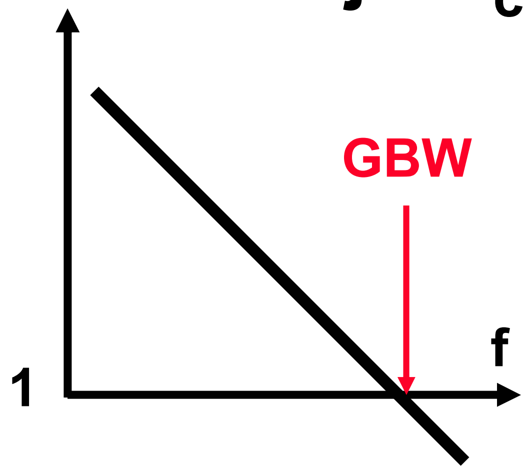
$$\boxed{GBW = \frac{g_{m1}}{2\pi C_c}}$$

$$\boxed{f_{nd} = \frac{g_{m2}}{2\pi C_L}}$$

Generic 2-stage opamp



$$A_v = g_{m1} \frac{1}{j\omega C_c}$$



$|A_v| = 1 \Rightarrow$

$$\boxed{GBW = \frac{g_{m1}}{2\pi C_c}}$$

$$\boxed{f_{nd} = \frac{g_{m2}}{2\pi C_L} \frac{1}{1 + \frac{C_{n1}}{C_c}}}$$

Elementary design of 2-stage opamp

$$GBW = \frac{g_{m1}}{2\pi C_c} \qquad f_{nd} = 3\ GBW = \frac{g_{m2}}{2\pi C_L} \frac{1}{1 + \underbrace{\frac{C_{n1}}{C_c}}_{\approx 0.3}}$$

$$\boxed{\frac{g_{m2}}{g_{m1}} \approx 4 \frac{C_L}{C_c}}$$

Larger current in 2nd stage !

GBW = 100 MHz for C_L = 2 pF

Solution: choose C_c = 1 pF

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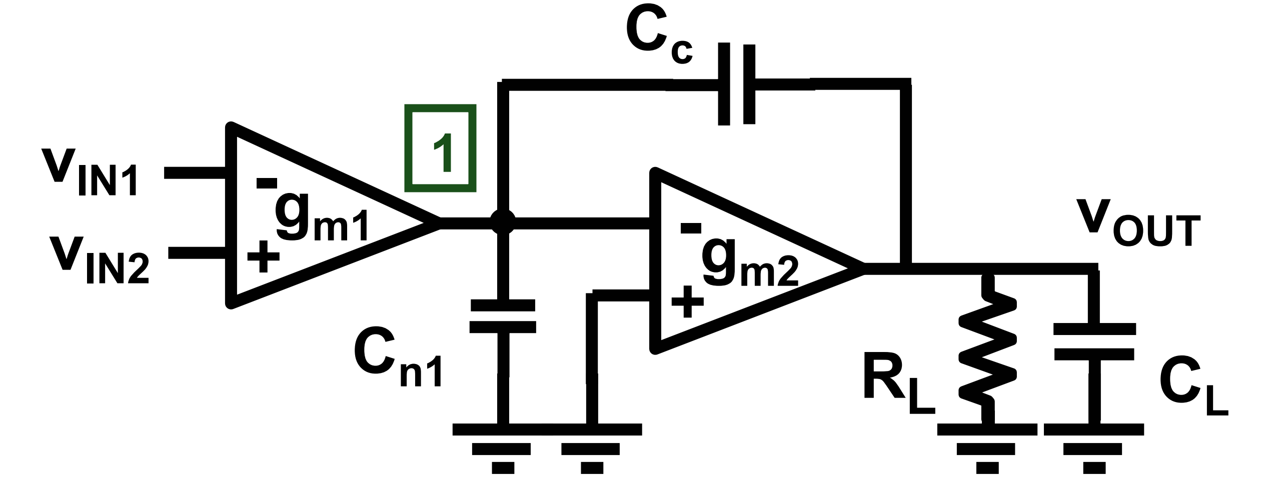
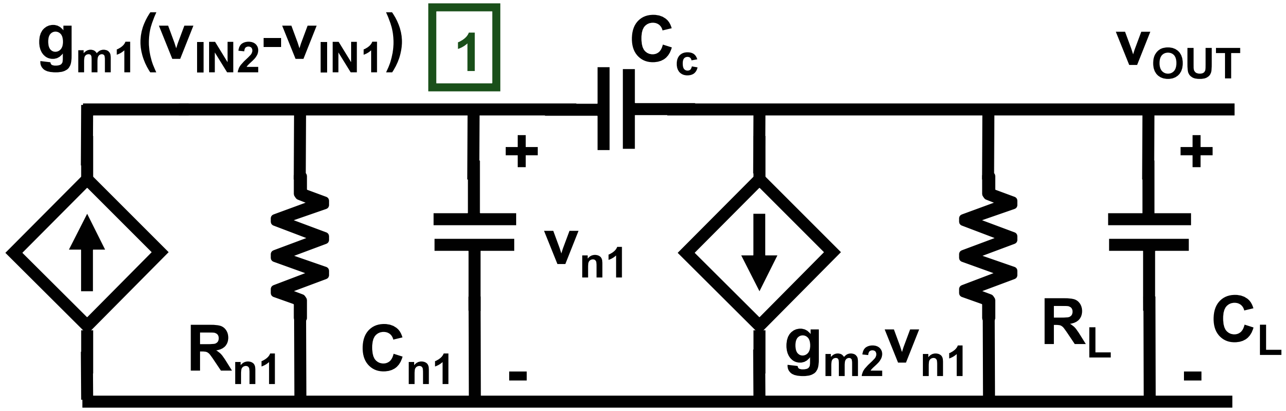
Generic 2-stage opamp : Miller OTA



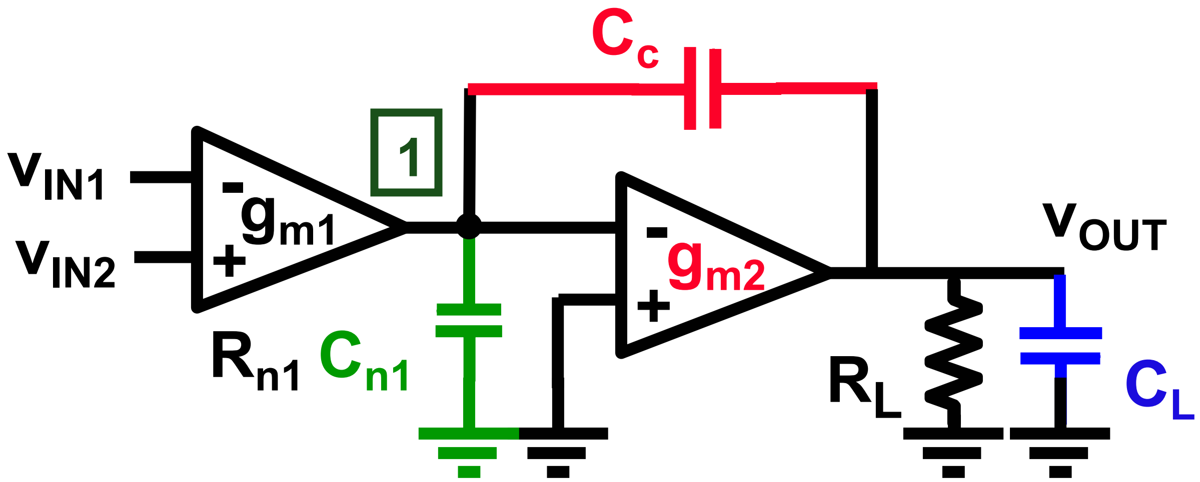


$$A_{v0} = - A_{v1}A_{v2}$$

$$A_{v1} = g_{m1}R_{n1}$$

$$A_{v2} = - g_{m2}R_L$$

Generic two-stage opamp



$$A_{v0} = - A_{v1}A_{v2}$$

$$A_{v1} = g_{m1}R_{n1}$$

$$A_{v2} = g_{m2}R_L$$

$$A_v = A_{v0} \frac{1 - \frac{C_c}{g_{m2}} s}{1 + (R_{n1}C_{n1} + A_{v2}R_{n1}C_c + R_LC_L)s + R_{n1}R_LCCs^2}$$

$$CC = C_{n1}C_c + C_{n1}C_L + C_cC_L$$

Approximate poles and zeros

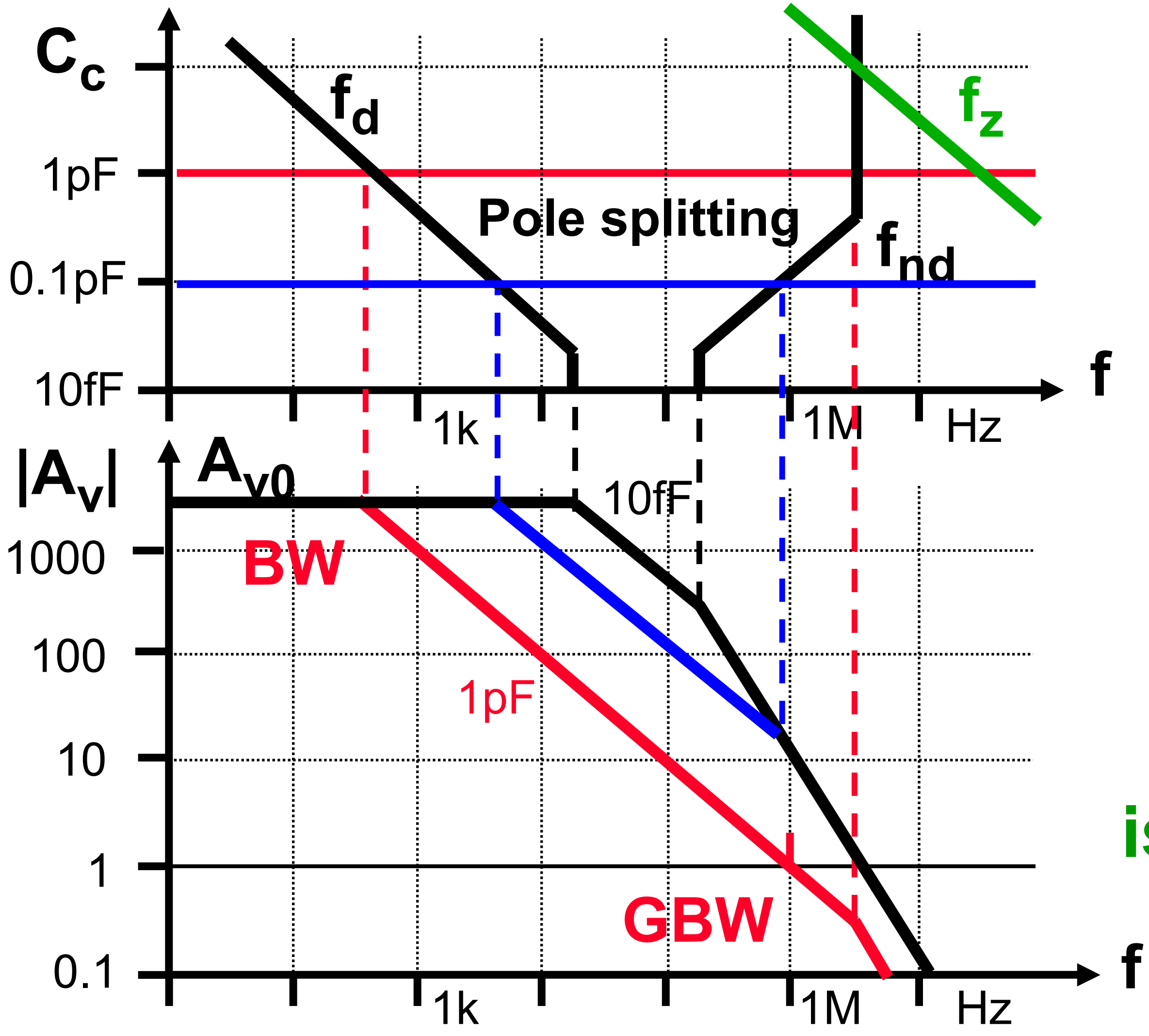
$$A = A_0 \frac{1 - cs}{1 + a\,s + b\,s^2}$$

Zero $s = \frac{1}{c}$

Pole $s_1 = -\frac{1}{a}$

$s_2 = -\frac{a}{b}$ if $s_2 >> s_1$

Miller OTA : pole splitting with C_c



Pole splitting for high C_c :

$$f_d = \frac{1}{2\pi\, A_{v2} R_{n1} C_c}$$

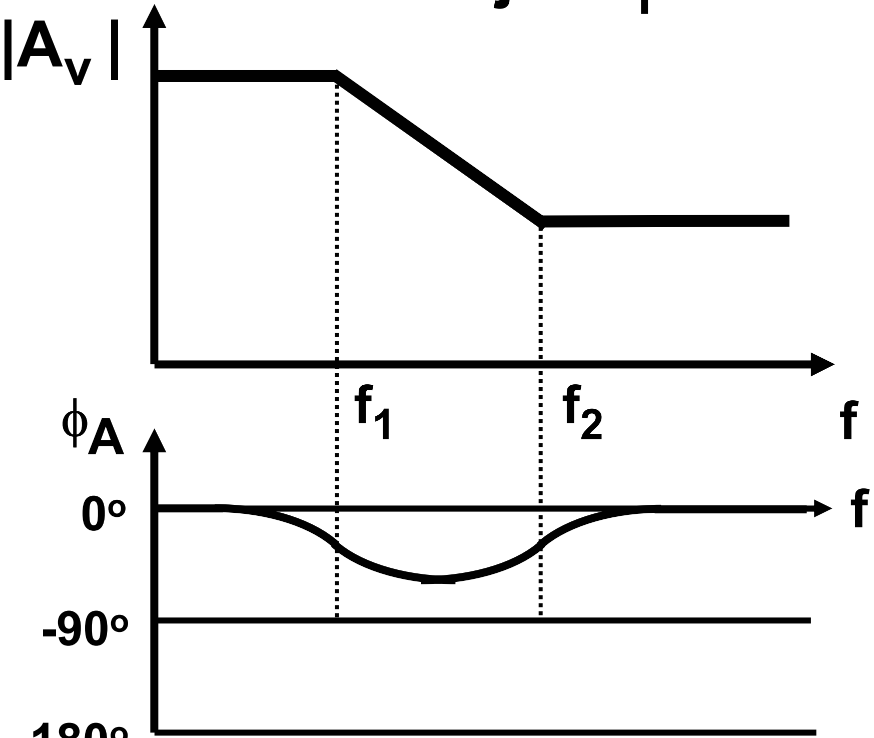
$$f_z = \frac{g_{m2}}{2\pi\, C_c}$$

is a positive zero !

Effect of positive zero

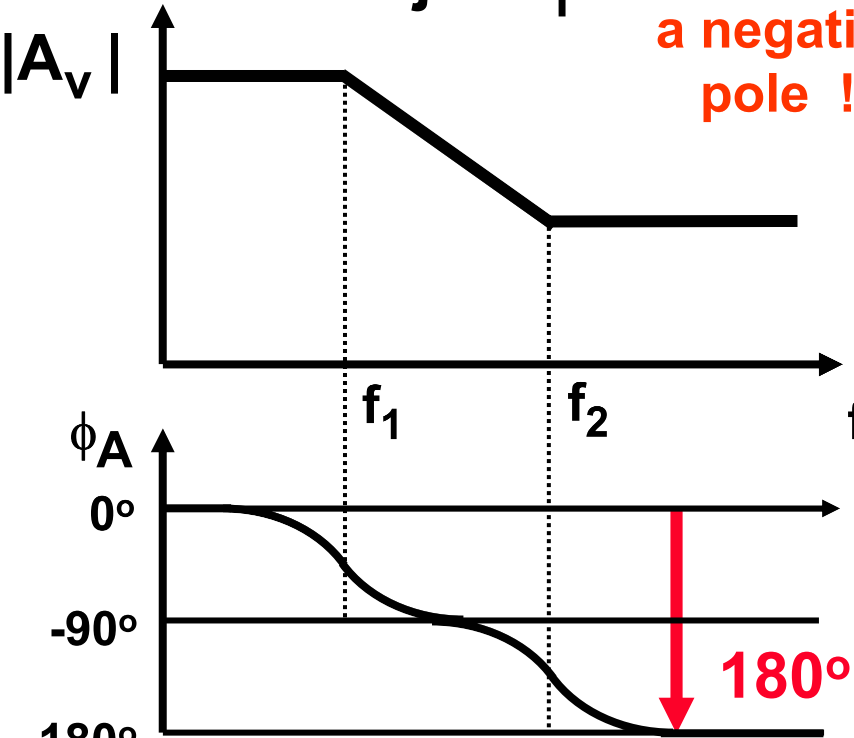
Negative zero

$$A_v = A_{v0} \frac{1 + j f / f_2}{1 + j f / f_1}$$



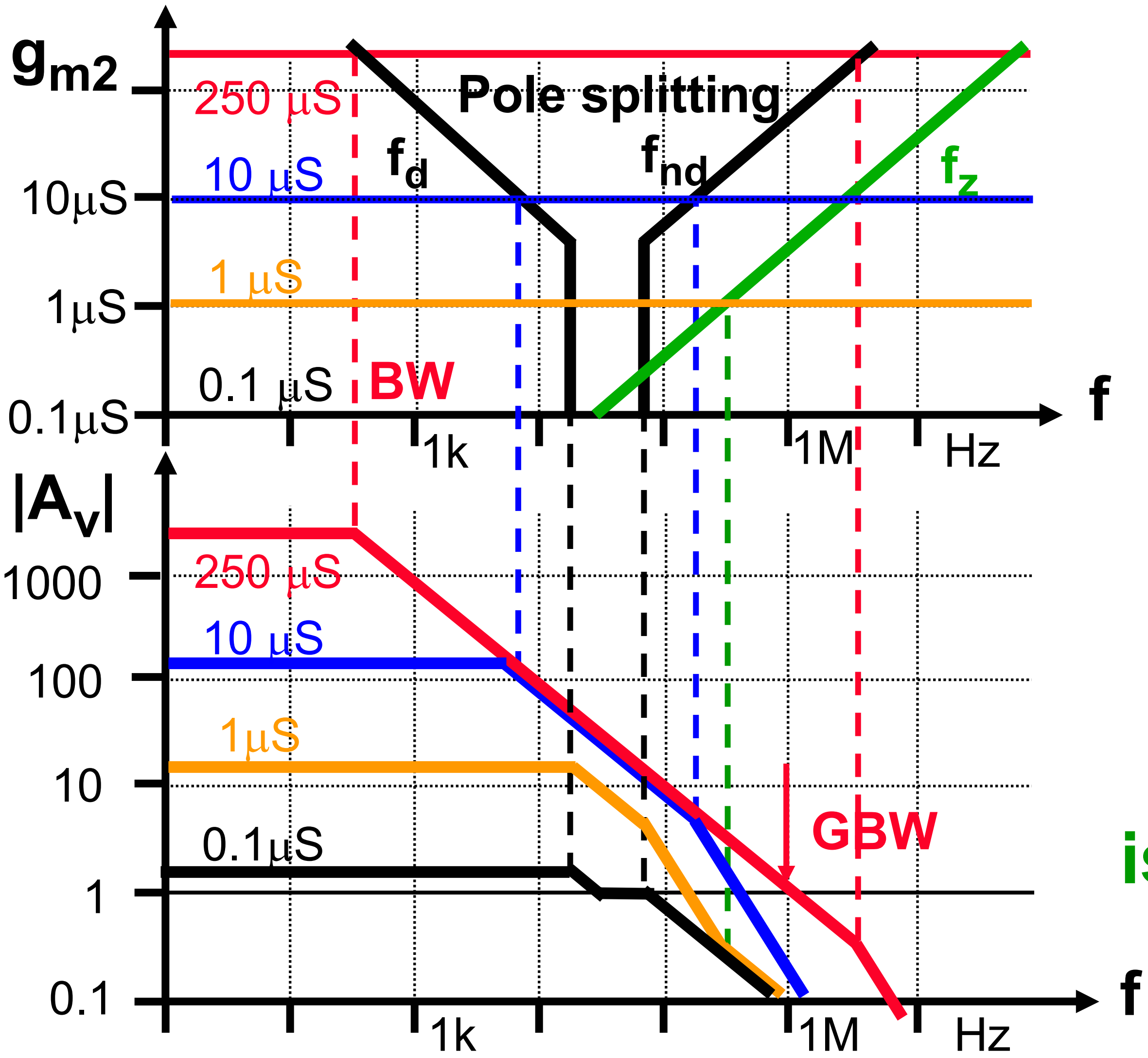
Positive zero

$$A_v = A_{v0} \frac{1 - j f / f_2}{1 + j f / f_1}$$



For phase, a positive zero is like a negative pole !!!

Miller OTA : pole splitting with g_{m2}



Pole splitting for high g_{m2} :

$$f_d = \frac{1}{2\pi\, A_{v2} R_{n1} C_c}$$

$$f_z = \frac{g_{m2}}{2\pi\, C_c}$$

is a positive zero !

Pole splitting by ...

$$\boxed{\frac{g_{m2}}{g_{m1}} \approx 4\ \frac{C_L}{C_c}}$$

or $g_{m2}\, C_c \approx 4\ g_{m1}\, C_L$

$$\boxed{\text{both}\quad g_{m2}\, C_c}$$

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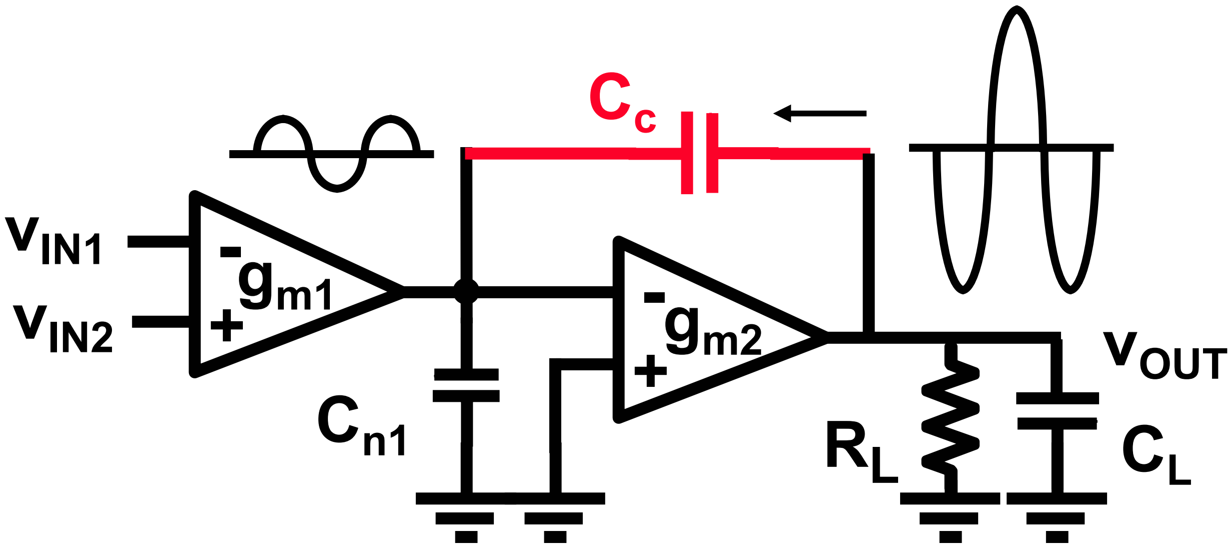
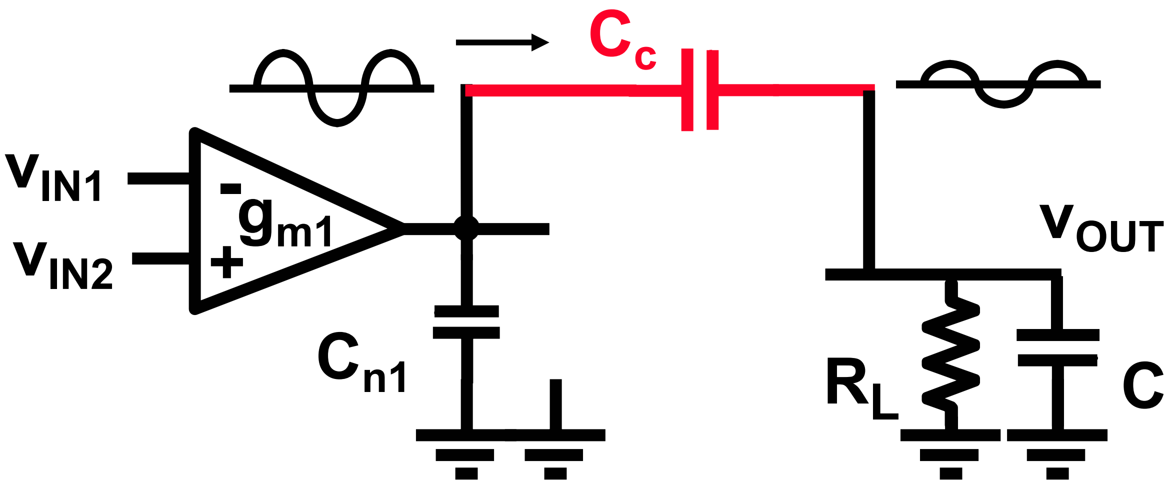
Positive zero because feedforward



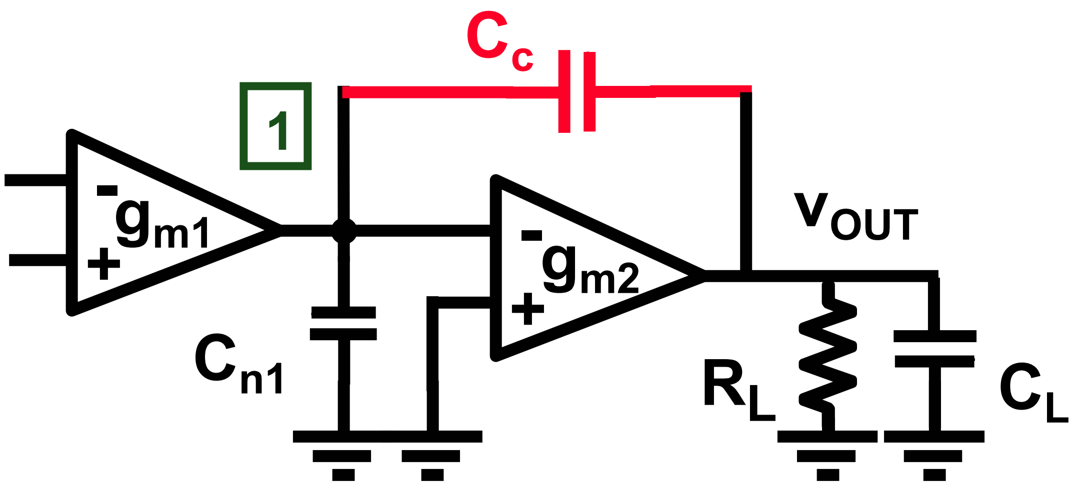
Miller effect
Is feedback

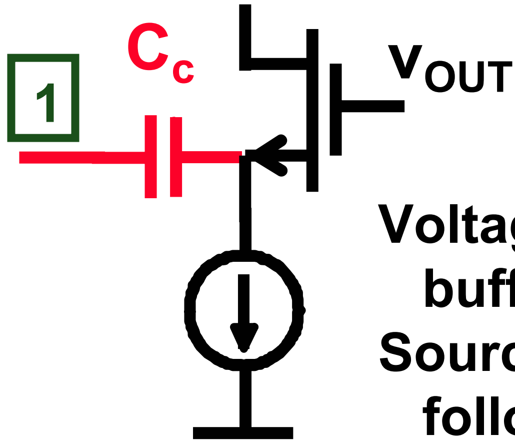


Feedforward

Cut !

Cut feedforward through C_c - 1

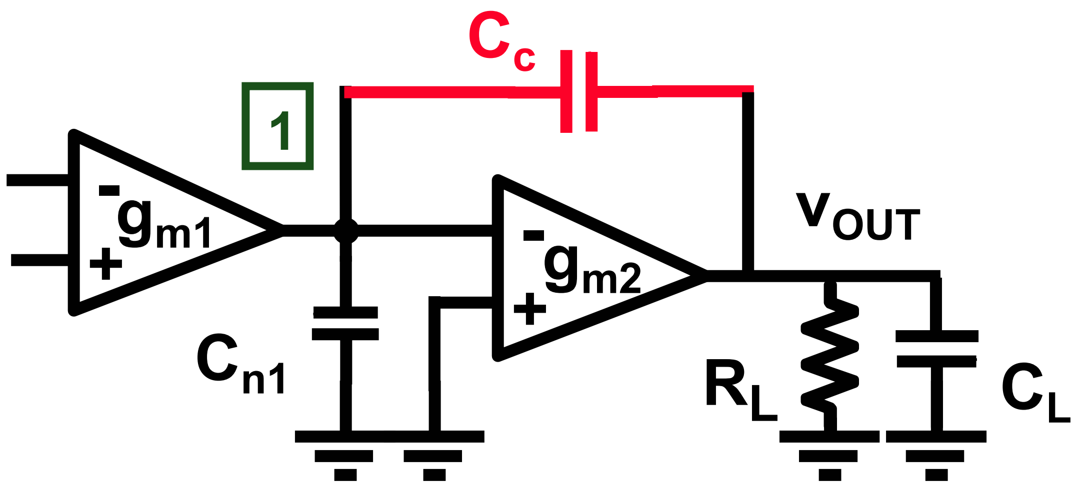


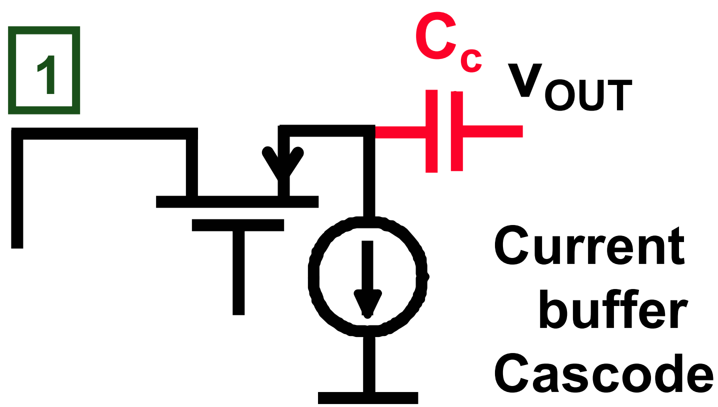


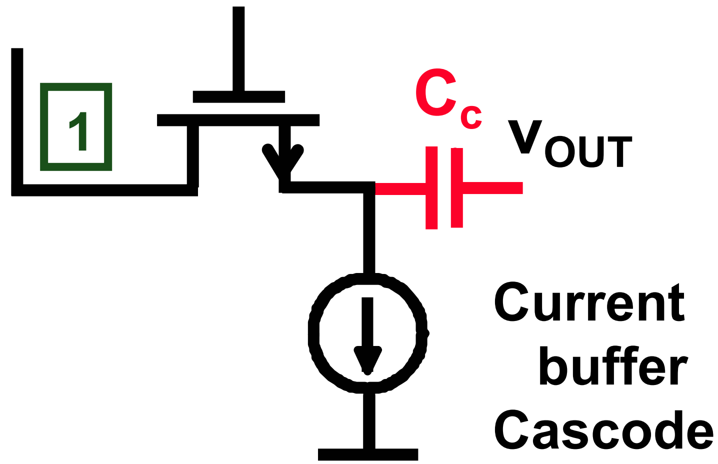
Voltage buffer
Source follower

Ref. Tsividis, JSSC Dec.76, 748-753

Cut feedforward through C_c - 2

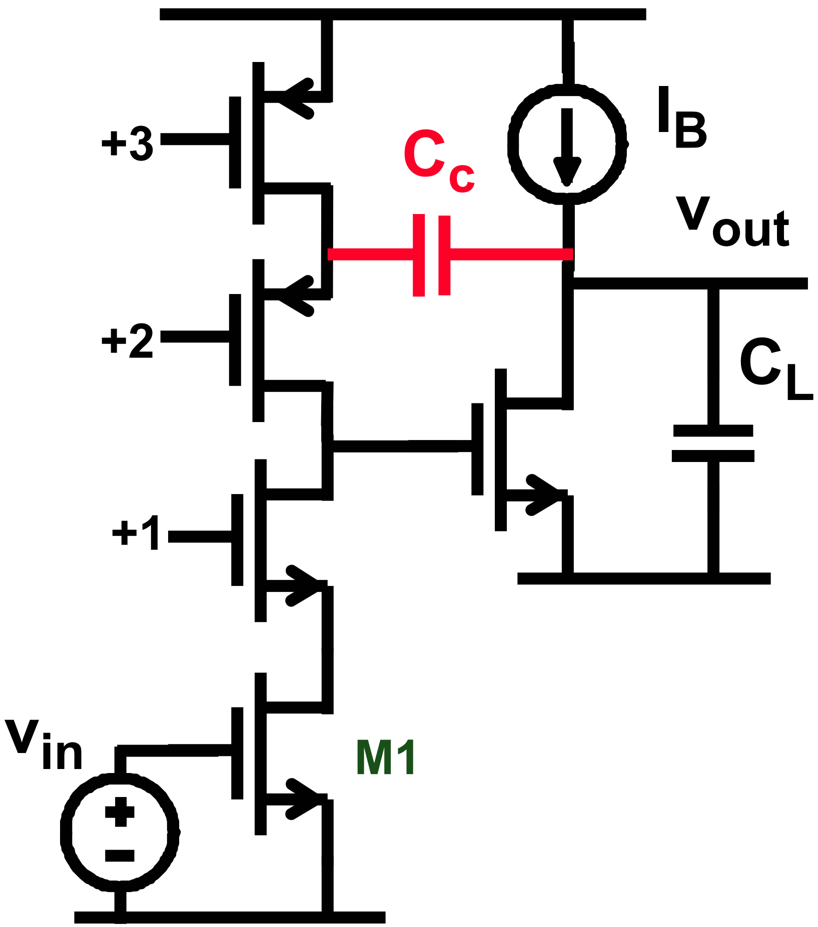


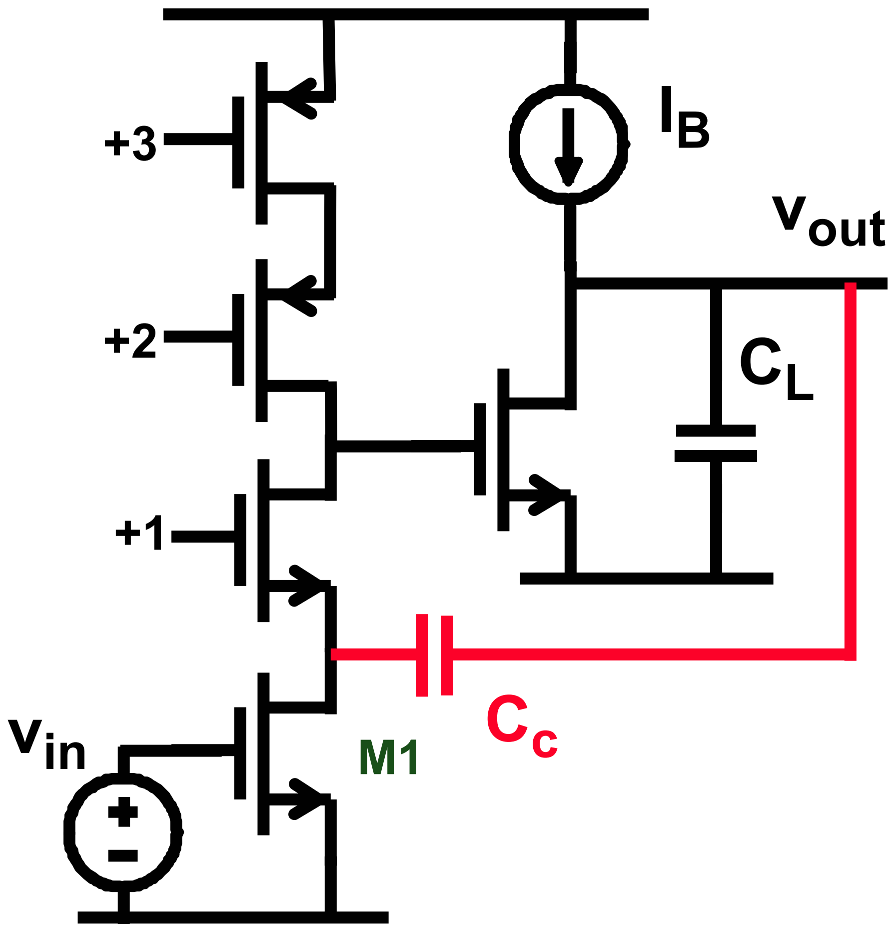




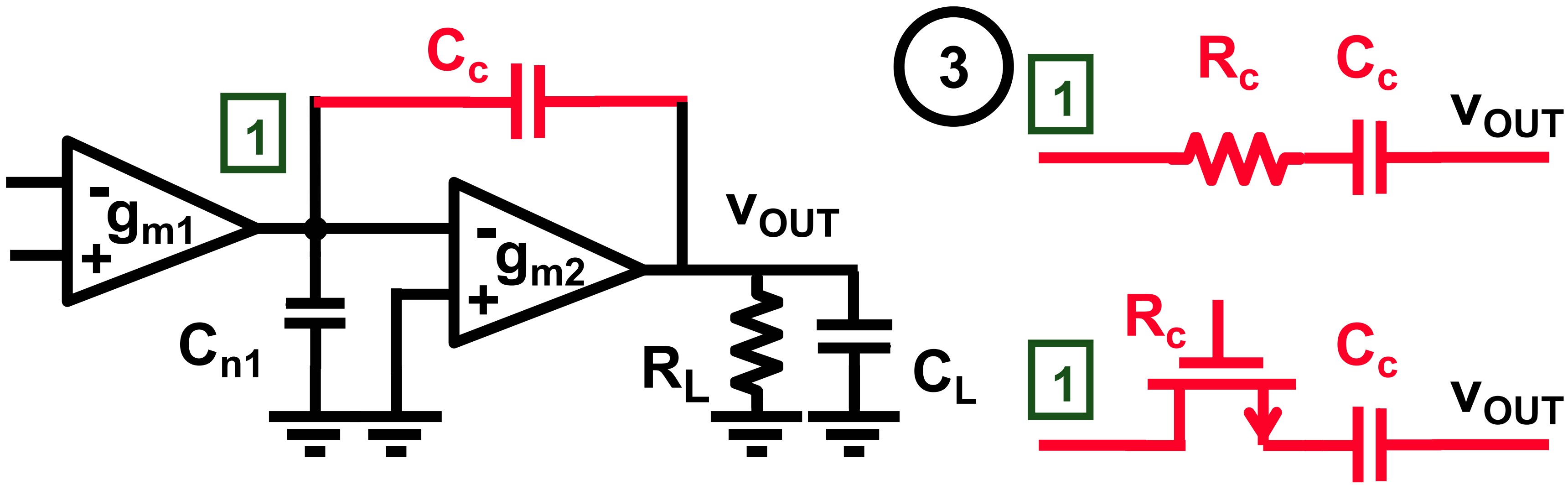
Ref. Ahuja, JSSC Dec 83, 629-633

Compensation with cascodes





Cut feedforward through C_c - 3



$$f_z = \frac{1}{2\pi\, C_c\, (1/g_{m2} - R_c)}$$

$R_c = 1/g_{m2}$ No zero

$R_c > 1/g_{m2}$ Negative zero

Ref. Senderovics, JSSC Dec 78, 760-766

Negative zero compensation

$R_c >> 1/g_{m2}$ $\Rightarrow$ $f_z = -\dfrac{1}{2\pi C_c R_c}$

$f_z = 3\ GBW$ $\Rightarrow$ $R_c = \dfrac{1}{3\ g_{m1}}$

Final choice :

$$\boxed{\frac{1}{g_{m2}} < R_c < \frac{1}{3g_{m1}}}$$

Exercise of 2-stage opamp

GBW = 50 MHz for C_L = 2 pF

Find I_{DS1}; I_{DS2} ; C_c and R_c !

Choose C_c = 1 pF > $g_{m1} = 2\pi\, C_c GBW = 315\ \mu S$

$I_{DS1} = 31.5\ \mu A$ & $1/g_{m1} \approx 3.2\ k\Omega$

f_{nd} = 150 MHz > $g_{m2} = 2\pi\, C_L 4GBW = 8g_{m1} = 2520\ \mu S$

$I_{DS2} = 252\ \mu A$ & $1/g_{m2} \approx 400\ \Omega$

$400\ \Omega < R_c < 1\ k\Omega$: $R_c = 1/\sqrt{2.5} \approx 400\sqrt{2.5} \approx 640\ \Omega \pm 60\%$

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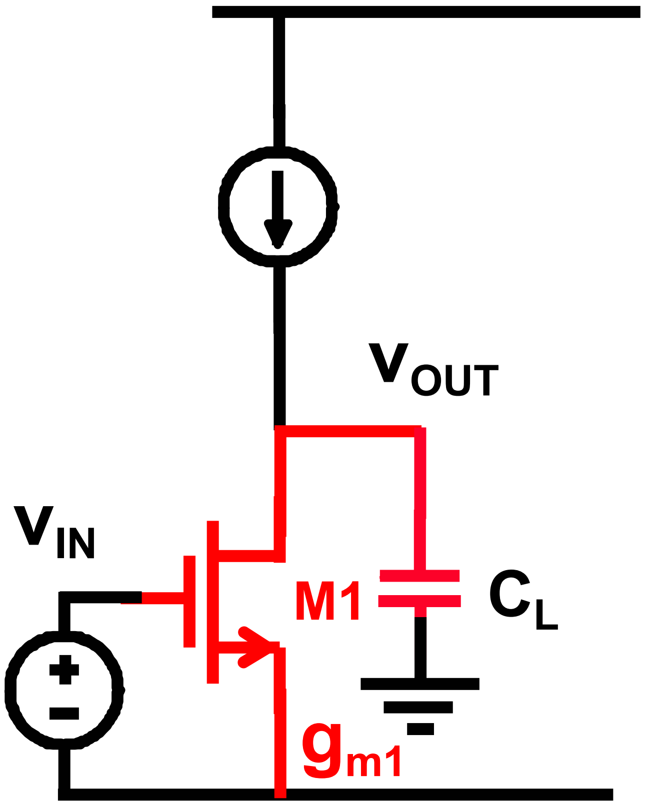
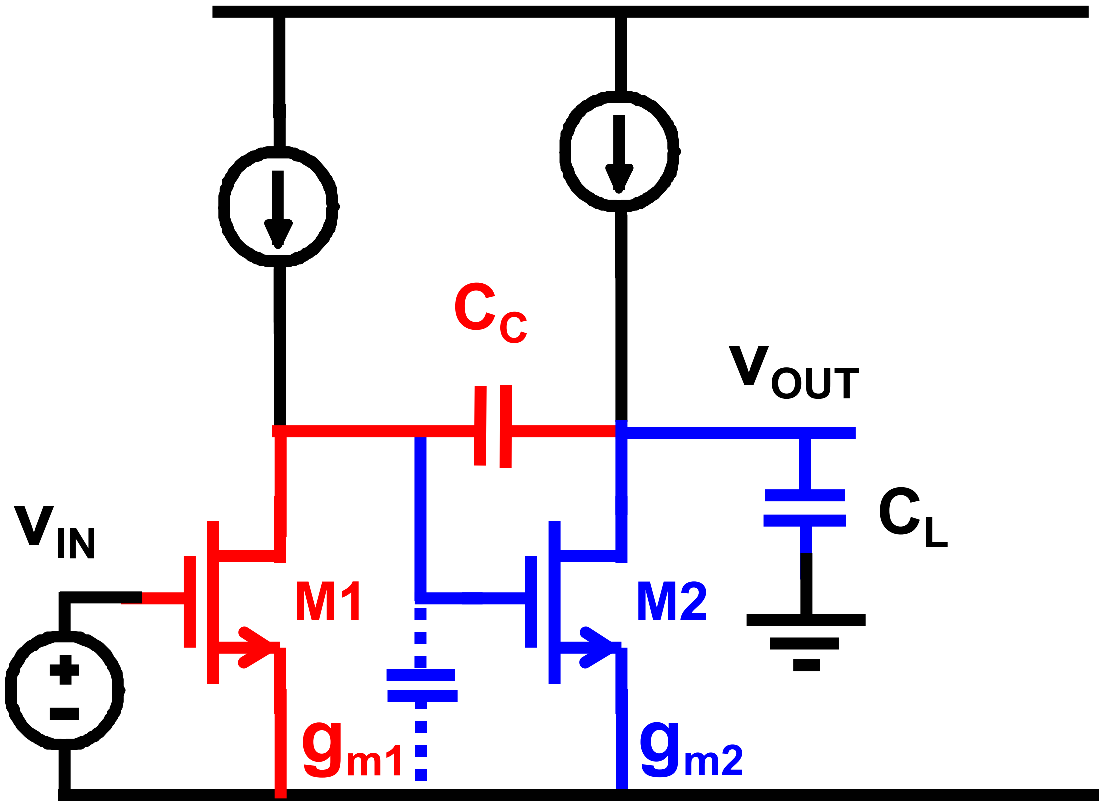
1-stage CMOS OTA



$$GBW = \frac{g_{m1}}{2\pi\, C_L}$$

2-stage Miller CMOS OTA

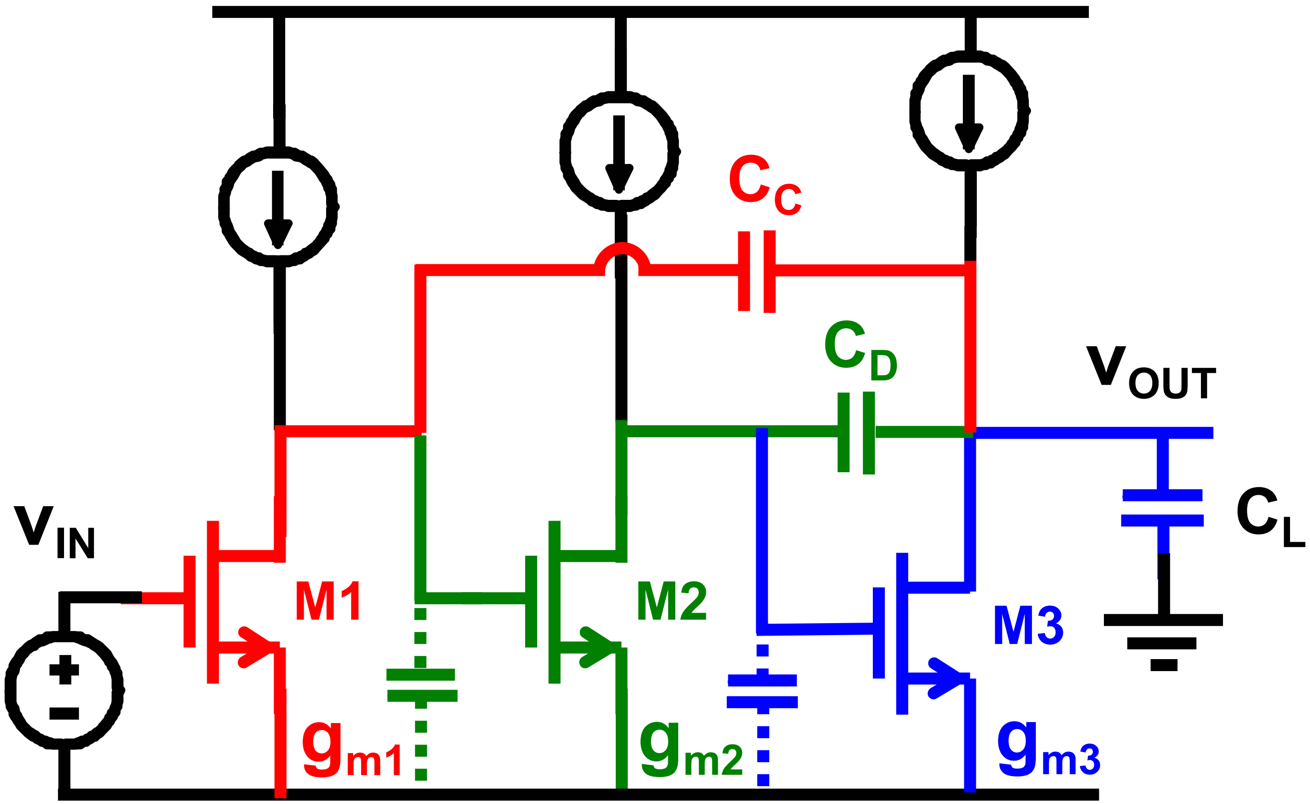


$$GBW = \frac{g_{m1}}{2\pi C_C}$$

$$f_{nd1} = \frac{g_{m2}}{2\pi C_L}$$

$$f_{nd1} = 3\ GBW$$

3-stage Nested Miller CMOS OTA



$$GBW = \frac{g_{m1}}{2\pi C_C}$$

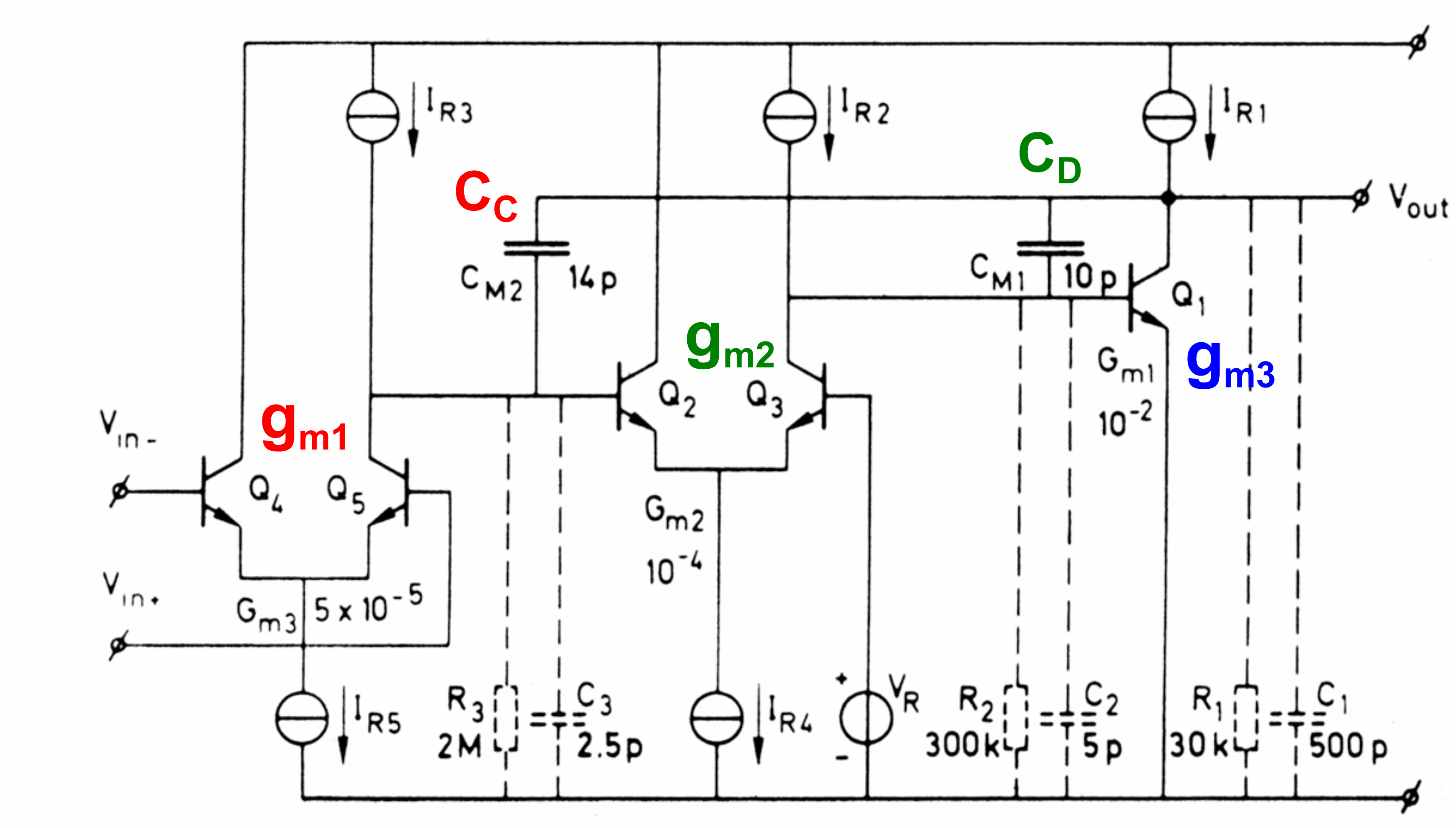
$$f_{nd1} = \frac{g_{m2}}{2\pi C_D}$$

$$f_{nd2} = \frac{g_{m3}}{2\pi C_L}$$

$f_{nd1} = 3\ GBW$

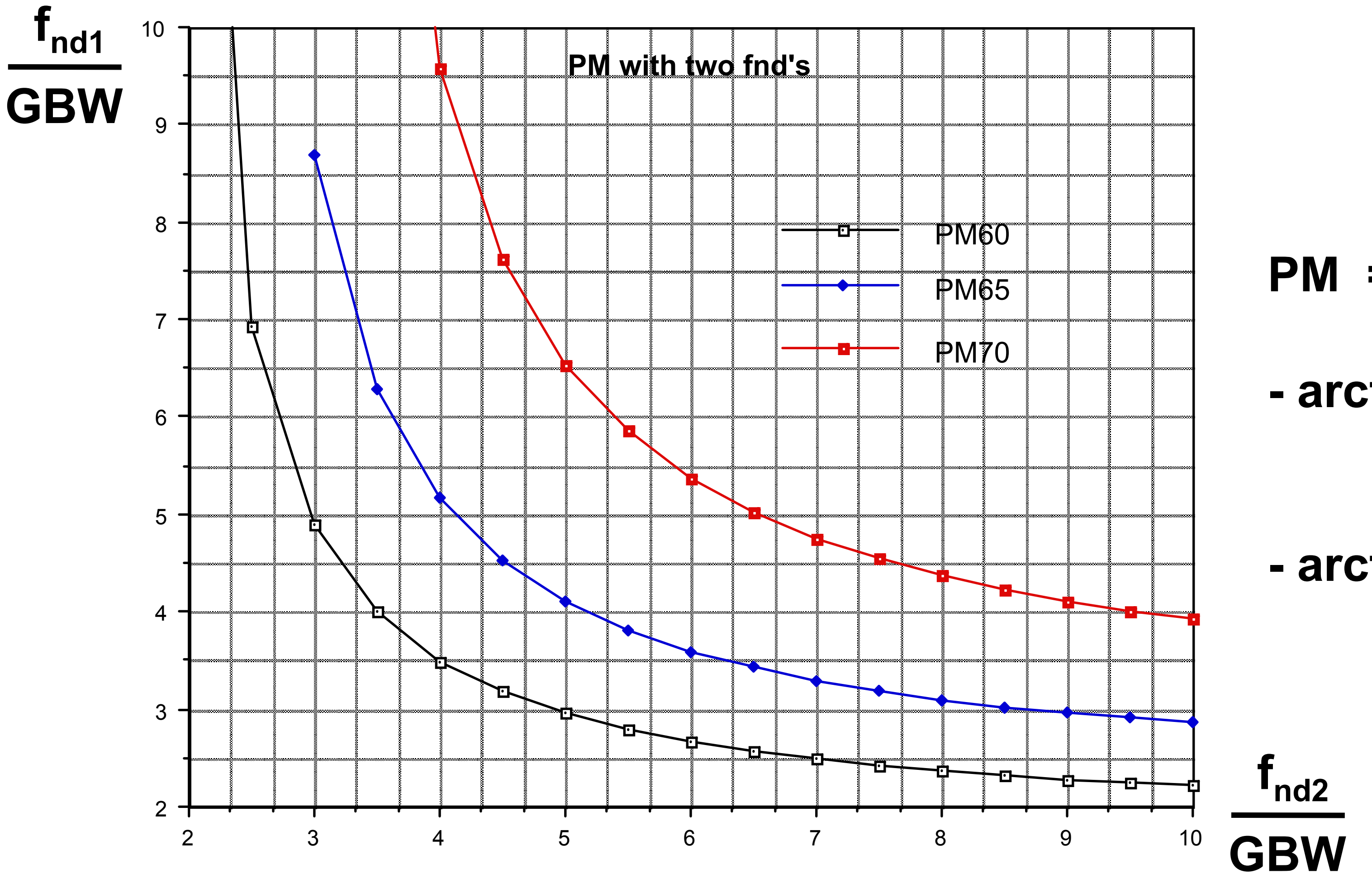
$f_{nd2} = 5\ GBW$

Nested Miller with differential pair



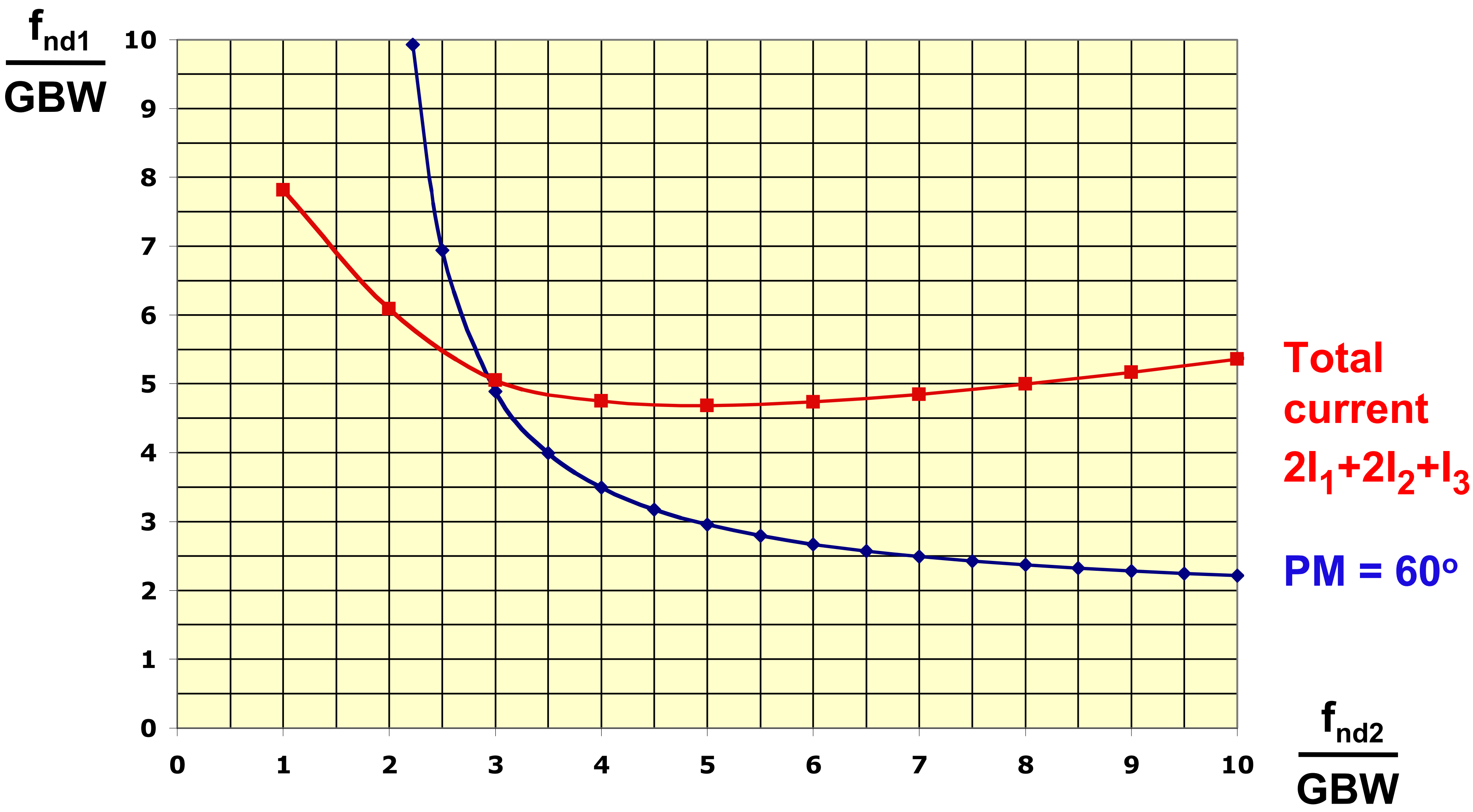
Huijsing, JSSC Dec.85, pp.1144-1150

Relation between the f_{nd}'s



$$PM = 90^o - \arctan\left(\frac{GBW}{f_{nd1}}\right) - \arctan\left(\frac{GBW}{f_{nd2}}\right)$$

Relation f_{nd}'s and power



Elementary design of 3-stage opamp

$$GBW = \frac{g_{m1}}{2\pi\, C_C}$$

$$f_{nd1} = 3\ GBW = \frac{g_{m2}}{2\pi\, C_D}$$

$$f_{nd2} = 5\ GBW = \frac{g_{m3}}{2\pi\, C_L}$$

Choose $C_D \approx C_C$!

$$\frac{g_{m2}}{g_{m1}} \approx 3 \qquad \frac{g_{m3}}{g_{m1}} \approx 5\ \frac{C_L}{C_C}$$

Even larger current in output stage !

Exercise of 3-stage opamp

GBW = 50 MHz for C_L = 2 pF

Find I_{DS1}; I_{DS2} ; I_{DS3} ; C_C and C_D !

Choose $C_C = C_D = 1$ pF > $g_{m1} = 2\pi\, C_C GBW = 315\ \mu S$

$I_{DS1} = 31\ \mu A$

$f_{nd1} = 150$ MHz > $g_{m2} = 2\pi\, C_D 3GBW = 3g_{m1} = 945\ \mu S$

$I_{DS2} = 95\ \mu A$

$f_{nd2} = 250$ MHz > $g_{m3} = 2\pi\, C_L 5GBW = 10g_{m1} = 3150\ \mu S$

$I_{DS3} = 315\ \mu A$

Comparison 1, 2 & 3 stage designs

GBW = 50 MHz for C_L = 2 pF

Single stage : $I_{DS1} = 31\ \mu A$ $I_{TOT} = 2I_{DS1} = 62\ \mu A$

Two stages : Choose $C_C = 1$ pF

$I_{DS1} = 31\ \mu A$ $I_{DS2} = 252\ \mu A$ $I_{TOT} = 2I_{DS1} + I_{DS2} = 314\ \mu A$

Three stages : Choose $C_C = C_D = 1$ pF

$I_{DS1} = 31\ \mu A$ $I_{DS2} = 95\ \mu A$ $I_{DS3} = 315\ \mu A$

$I_{TOT} = 2I_{DS1} + 2I_{DS2} + I_{DS3} = 567\ \mu A$

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