

# Some

**Author:** Pannenets.F

**Date:** October 12, 2020

*Je reviendrai et je serai des millions. «Spartacus»*

## **0.1 MOSFET**



北京航空航天大學  
BEIHANG UNIVERSITY

| 微电子学院

# CMOS模拟集成电路设计

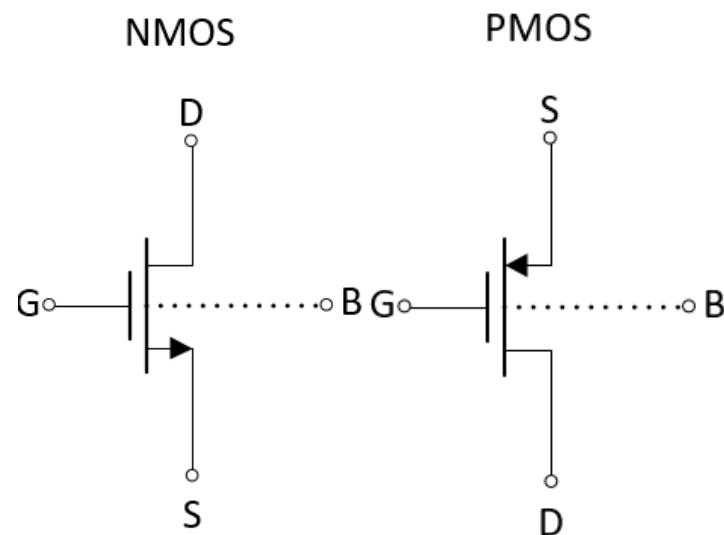
## 第一章：MOSFET

胡远奇

©2020

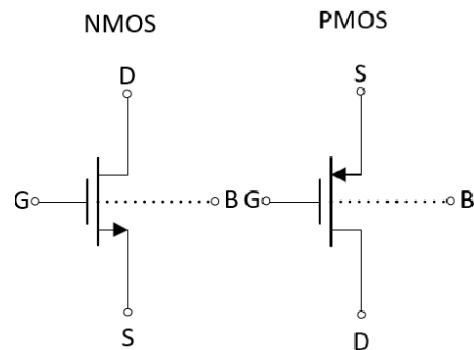
# »» MOSFET

---



**Id=?**

# 》》 MOSFET



线性区

vs

饱和区



如何选择?

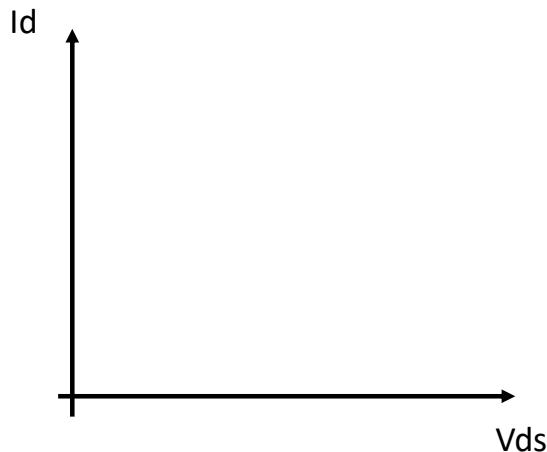
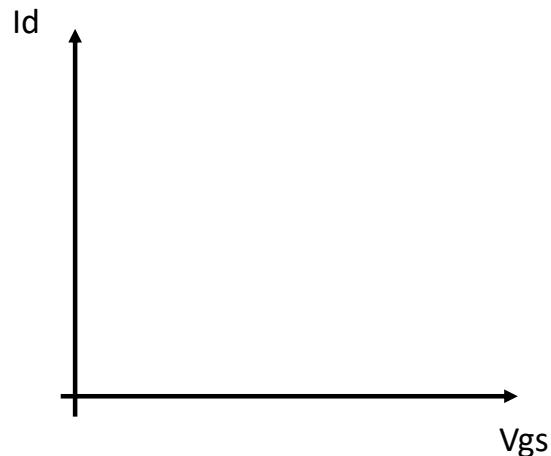
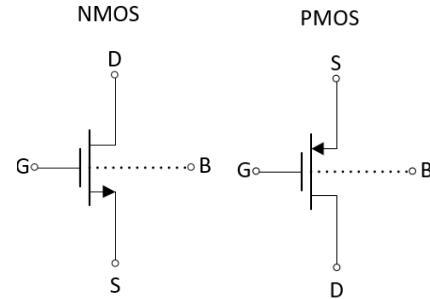


开关、电阻

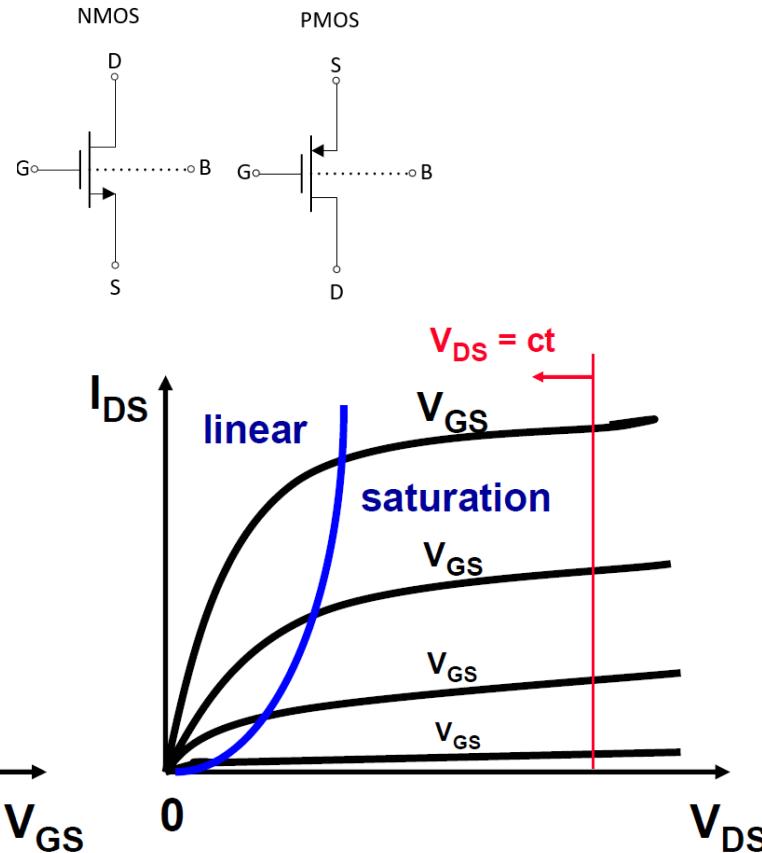
放大器



# MOSFET的I-V关系



# »» MOSFET的I-V关系





# MOSFET

---

- 晶体管工作在线性区：电阻
- 晶体管工作在饱和区：放大器
- 亚反型区/弱翻转区 (Weak Inversion)
- 速度饱和区 (Velocity Saturation)
- 特征频率

# »» MOSFET当作开关

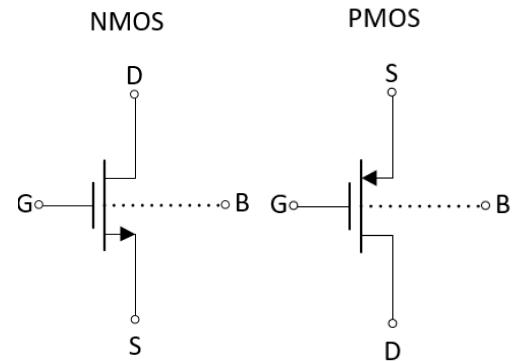
---

**线性区:**  $V_{DS} < V_{GS} - V_{TH}$

$$I_D =$$

- 小信号导通电阻:

$$R =$$



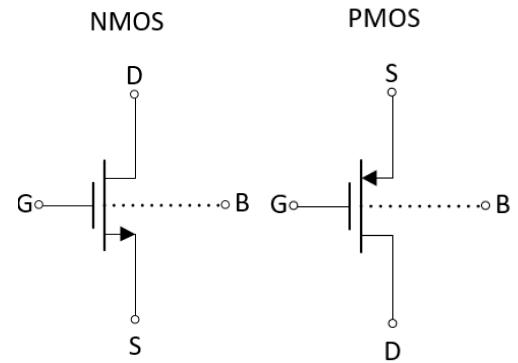
# 》》 MOSFET当作开关

**线性区:**  $V_{DS} < V_{GS} - V_{TH}$

$$I_D = \mu C_{ox} \frac{W}{L} \left( V_{GS} - V_{TH} - \frac{V_{DS}}{2} \right) V_{DS}$$

- 小信号导通电阻:

$$R \approx \frac{1}{\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})} \quad \text{for } V_{GS} - V_{TH} \gg V_{DS}$$



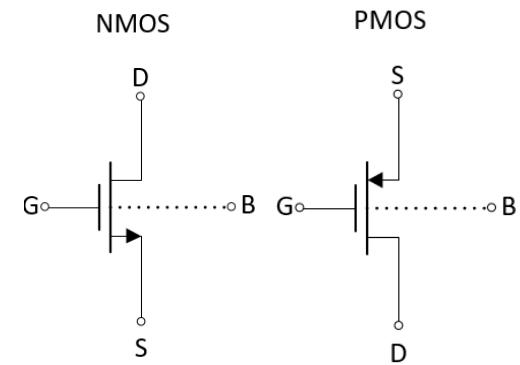
# »» MOSFET参数

$$R \approx \frac{1}{\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})}$$

- 电子迁移率 $\mu$ :  $\mu_p \approx 250 \text{ cm}^2/\text{Vs}$

$$\mu_n \approx 600 \text{ cm}^2/\text{Vs}$$

- 栅氧层电容 $C_{ox}$ :  $C_{ox} = \frac{\epsilon_{ox}}{t_{ox}}$

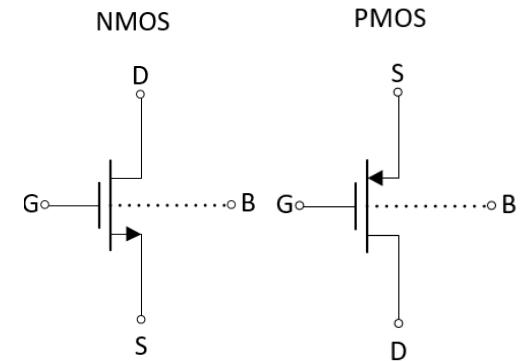


# »» MOSFET参数

**L<sub>min</sub> = 0.35 μm**

- 电子迁移率 $\mu$ :  $\mu_p \approx 250 \text{ cm}^2/\text{Vs}$   
 $\mu_n \approx 600 \text{ cm}^2/\text{Vs}$
- 棚氧层电容 $C_{ox}$ :  $C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} \approx 5 \cdot 10^{-7} \text{ F/cm}^2$

**KP<sub>n</sub> ≈ 300 μA/V<sup>2</sup>**





# MOSFET电阻快速估算

$$L_{min} = 0.35 \mu m \quad K_P n \approx 300 \mu A/V^2$$

$$R \approx \frac{1}{\mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})}$$

- 对于0.35um工艺方块NMOS晶体管，在 $V_{GS}-V_{TH}=1V$ 的驱动电压下，其电阻约为：

**3.4KΩ**



# MOSFET电阻快速估算

已知0.35um工艺方块NMOS晶体管，在 $V_{GS}-V_{TH}=1V$ 的驱动电压下，其电阻约为3.4Kohm，请估算电阻值：

1. 0.35um工艺方块PMOS晶体管， $V_{GS}-V_{TH}=1V$  **8.16KΩ**

2. 0.18um工艺方块NMOS晶体管， $V_{GS}-V_{TH}=1V$  **1.75KΩ**

$$\mu_p \approx 250 \text{ cm}^2/\text{Vs}$$

$$\mu_n \approx 600 \text{ cm}^2/\text{Vs}$$

$$t_{ox} = \frac{L_{min}}{50}$$



# MOSFET

---

- 晶体管工作在线性区：电阻
- **晶体管工作在饱和区：放大器**
- 亚反型区/弱翻转区 (Weak Inversion)
- 速度饱和区 (Velocity Saturation)
- 特征频率



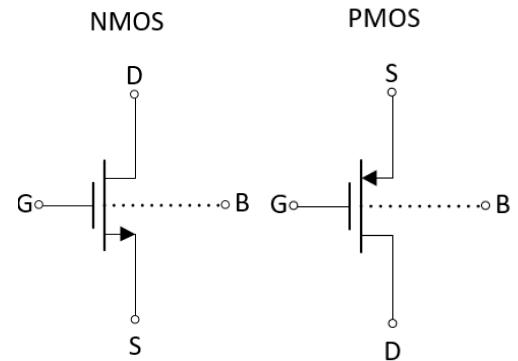
# MOSFET当作放大器

**饱和区：**  $V_{DS} > V_{GS} - V_{TH}$

$$I_D =$$

- 小信号跨导：

$$g_m \approx$$





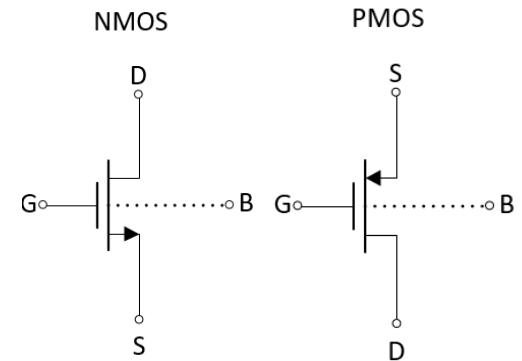
# MOSFET当作放大器

**饱和区:**  $V_{DS} > V_{GS} - V_{TH}$

$$I_D = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})$$

- 小信号跨导:

$$g_m \approx \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_{DS}} = \frac{2I_{DS}}{V_{GS} - V_{TH}}$$



## 》》 晶体管跨导gm

---

$$g_m \approx \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH}) = \sqrt{2\mu_n C_{ox} \frac{W}{L} I_{DS}} = \frac{2I_{DS}}{V_{GS} - V_{TH}}$$

问：小信号跨导 $g_m$ 与 $I_{DS}$ 的关系？

$$g_m \propto \sqrt{I_{DS}}$$



测试中尺寸固定

VS

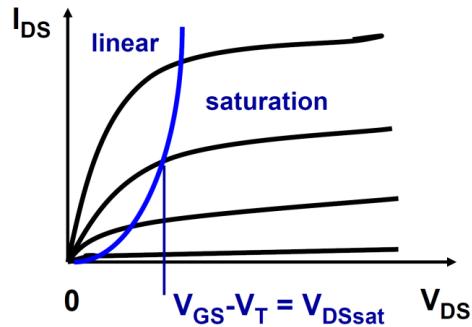
$$g_m \propto I_{DS}$$



设计中偏置固定

## 》》 输出电阻 $r_0$

---



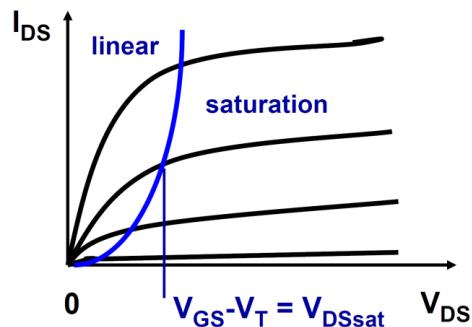
$$I_{DS} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})$$

- 输出电阻：

$$r_0 =$$

## 》》 输出电阻 $r_0$

---



$$I_{DS} = \frac{1}{2} \mu_n C_{ox} \frac{W}{L} (V_{GS} - V_{TH})^2 (1 + \lambda V_{DS})$$

- 输出电阻：

$$r_0 \approx \frac{1}{\lambda I_{DS}} = \frac{V_E \cdot L}{I_{DS}}$$



$$V_{En} = 4 \text{ V}/\mu\text{mL}$$

$$L = 1 \text{ } \mu\text{m}$$

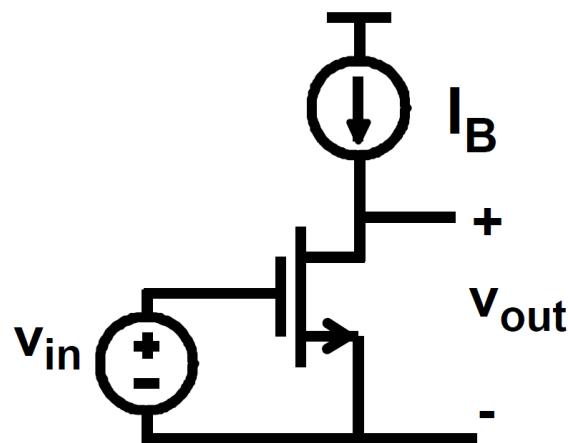
$$I_{DS} = 100 \text{ } \mu\text{A}$$

$$r_0 = 40 \text{ k}\Omega$$



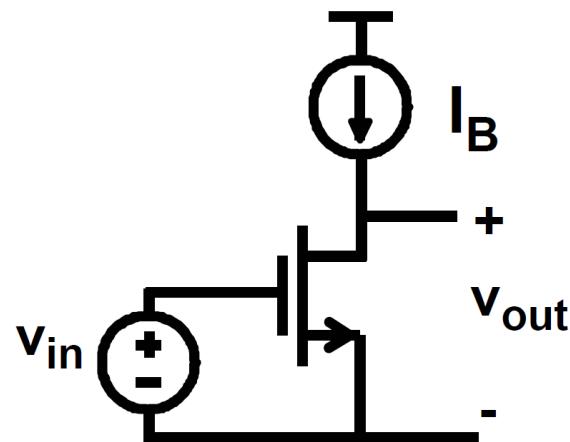
# 单晶体管放大器

---



## 》》 单晶体管放大器

---



$$A_v = g_m r_{DS} = \frac{2 V_E L}{V_{GS} - V_T}$$

$$A_v \approx 100$$

If  $V_E L \approx 10 \text{ V}$   
and  $V_{GS} - V_T \approx 0.2 \text{ V}$

# 》》运放设计Trade-off

---

$$g_m \approx \frac{2I_{DS}}{V_{GS} - V_{TH}}$$

$$A = \frac{2 \cdot V_E \cdot L}{V_{GS} - V_{TH}}$$

	高增益	高速
$V_{GS} - V_{TH}$		
L		

# 》》运放设计Trade-off

---

$$g_m \approx \frac{2I_{DS}}{V_{GS} - V_{TH}}$$

$$A = \frac{2 \cdot V_E \cdot L}{V_{GS} - V_{TH}}$$

	高增益	高速
$V_{GS} - V_{TH}$	↓	↑
L	↑	↓



# MOSFET

---

- 晶体管工作在线性区：电阻
- 晶体管工作在饱和区：放大器
- **亚反型区/弱翻转区 (Weak Inversion)**
- 速度饱和去 (Velocity Saturation)
- 特征频率

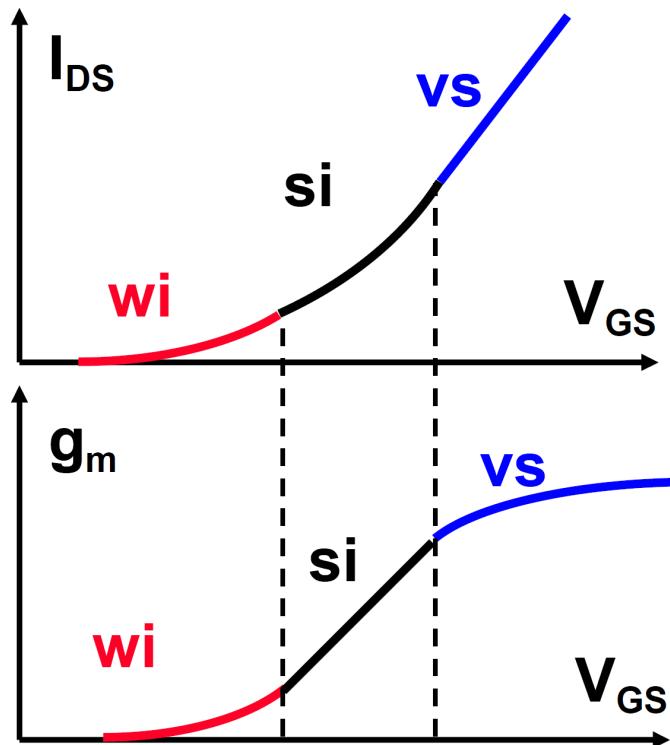
## 》》弱反型区和速度饱和区

---

- 假设晶体管始终工作在饱和区： $V_{DS} > V_{GS} - V_{TH}$



## 》》 弱反型区 (weak inversion)



- 弱反型区:

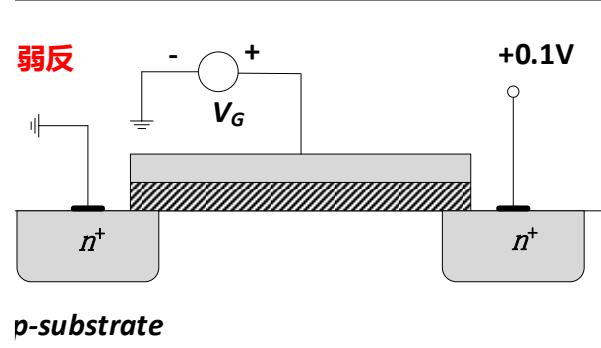
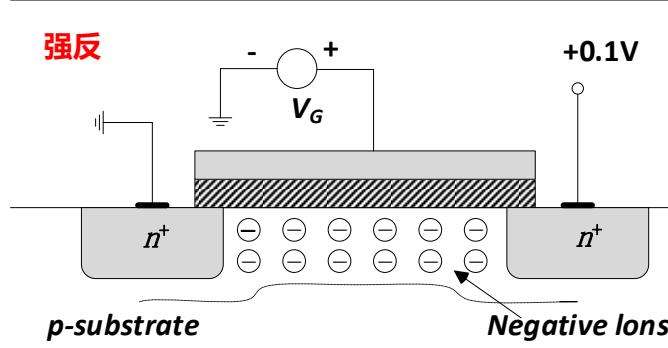
$$I_{D,wi} = I_{D0} \frac{W}{L} e^{\frac{V_{GS}}{nkT/q}}$$

$$g_{m,wi} = \frac{I_{D,wi}}{nkT/q}$$



$n > 1$ , 取决于偏置电压

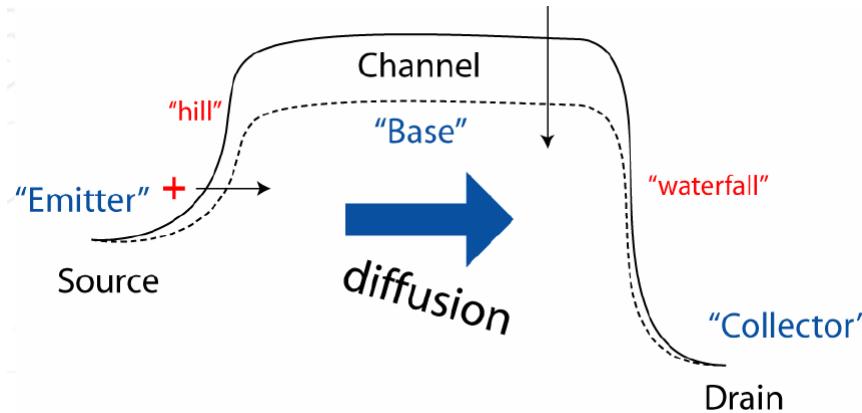
# 》》弱反型区



- 场效应管实际是一个水平的BJT三极管
- 漏极的反向偏置形成二极管，增加栅极电压将线性降低二极管的电势壁垒 (Potential Barrier)
- 主要的电流是扩散电流(diffusion)而非漂移电流(drift)
- 通道中电势几乎不变，离子浓度线性变化

## 》》弱反型区

---



$$I_{D,wi} = I_{D0} \frac{W}{L} \exp\left(\frac{V_{GS}}{nU_T}\right) \cdot \left[ 1 - \exp\left(-\frac{V_{DS}}{U_T}\right) \right]$$

$$I_{D,wi} = I_{D0} \frac{W}{L} \exp\left(\frac{V_{GS}}{nU_T}\right) \quad \text{for } V_{DS} > 4U_T \text{ (饱和)}$$

# 》》弱反型与强反型

---

弱反型区 (Weak Inversion)

$$g_{m,wi} = \frac{I_{DS}}{nkT/q}$$



$$\frac{g_{m,wi}}{I_{DS}} = \frac{1}{nkT/q}$$

强反型区 (Strong Inversion)

$$g_{m,si} = \frac{2I_{DS}}{V_{GS} - V_{TH}}$$



$$\frac{g_{m,si}}{I_{DS}} = \frac{2}{V_{GS} - V_{TH}}$$

$$V_{GS} - V_{TH} = 2n \frac{kT}{q} \approx 70mV$$



## 弱-强反型区转换点

---

$$V_{GS} - V_{TH} = 2n \frac{kT}{q} \approx 70mV$$

- 强弱的转换点独立于工艺的尺寸
- 保证  $V_{GS} - V_{TH} = 0.2V$  可以在不同工艺中均保证晶体管工作在强反型区
- 转换点电流:  $I_{DSt} = K'_n \frac{W}{L} (V_{GS} - V_{TH})^2$

$$I_{DSt} = \frac{W}{L} \cdot \textcolor{red}{0.} \textcolor{red}{x} \mu A$$

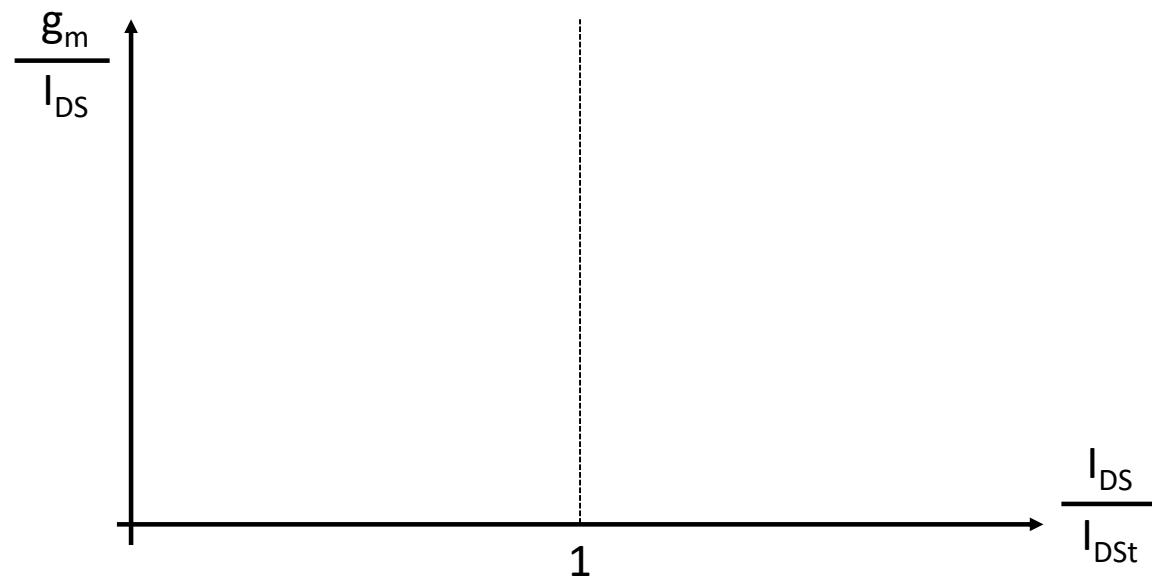
# 》》弱反型与强反型

弱反型区 (Weak Inversion)

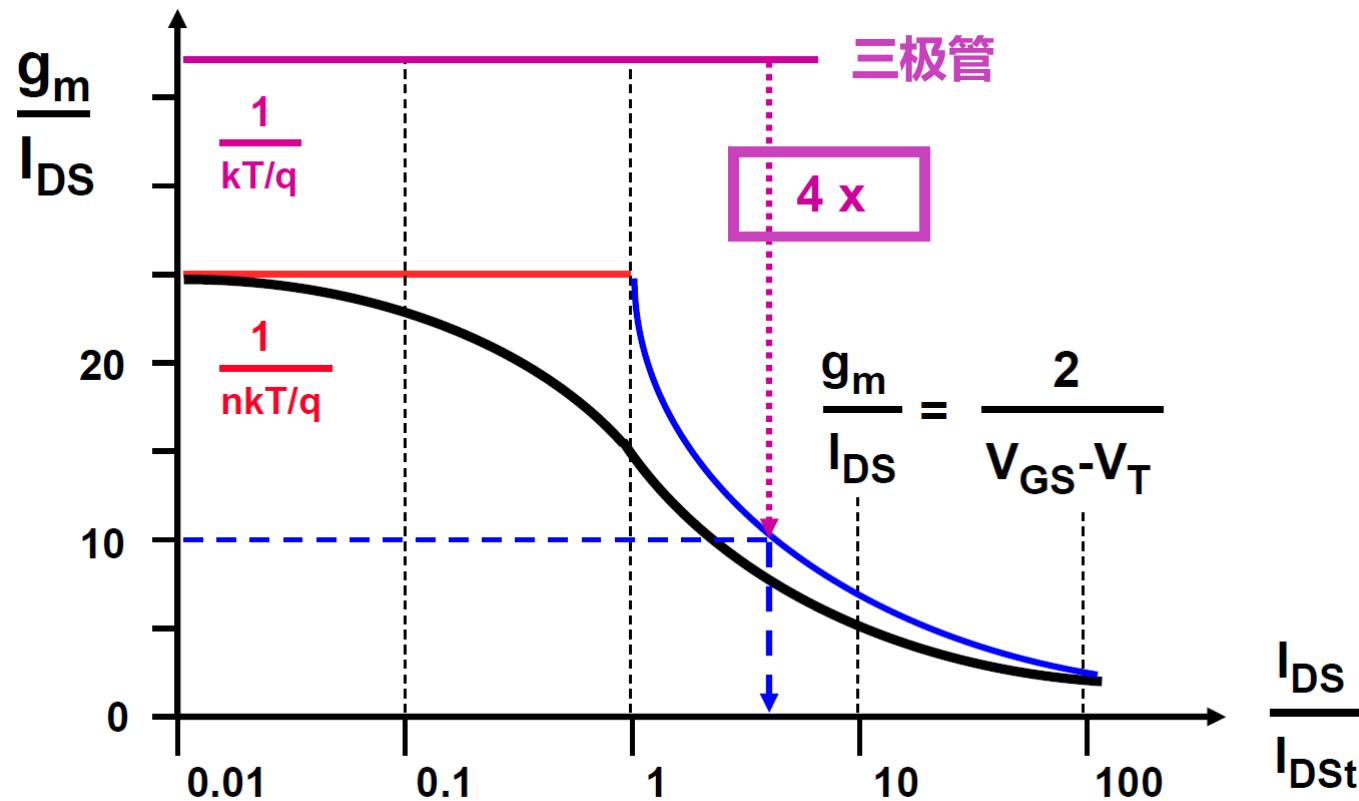
$$\frac{g_{m,wi}}{I_{DS}} = \frac{1}{nkT/q}$$

强反型区 (Strong Inversion)

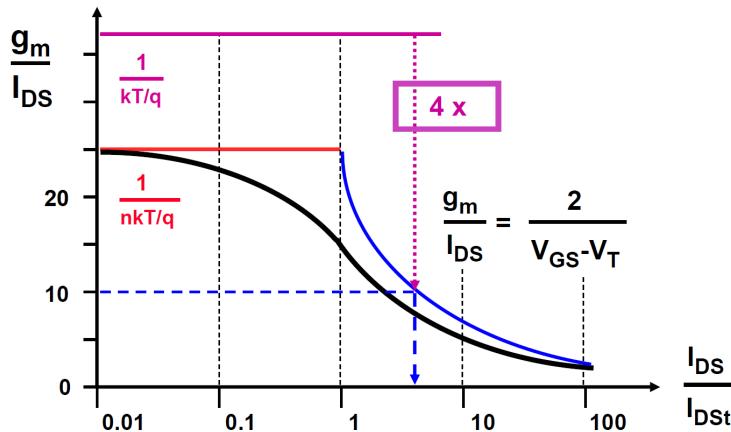
$$\frac{g_{m,si}}{I_{DS}} = \frac{2}{V_{GS} - V_{TH}}$$



## 》》弱反型与强反型



# 弱反型与强反型：EKV模型

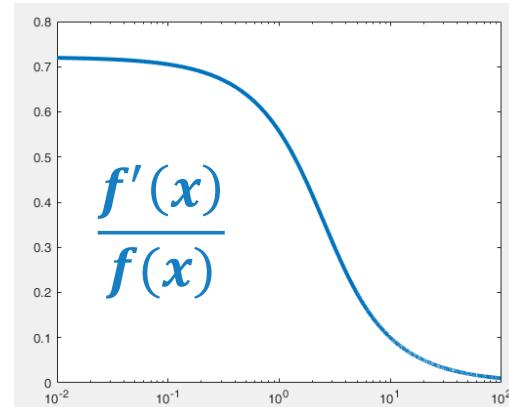


$$I_{DS} = K' \frac{W}{L} V_{GSTt}^2 \cdot \ln^2(1 + e^\nu)$$

$$\nu = \frac{V_{GST}}{V_{GSTt}}$$

- 能否通过一个公式同时表述两种反型区？

$$\ln^2(1 + e^\nu)$$



## 》》弱反型与强反型：EKV模型

---

$$I_{DS} = K' \frac{W}{L} V_{GSTt}^2 \cdot \ln^2(1 + e^\nu), \quad \nu = \frac{V_{GST}}{V_{GSTt}}, \quad V_{GSTt} = (V_{GS} - V_T)_t = 2n \frac{kT}{q}$$

- 弱反型区中:
- 强反型区中:

## 》》弱反型与强反型：EKV模型

---

$$I_{DS} = K' \frac{W}{L} V_{GSTt}^2 \cdot \ln^2(1 + e^\nu), \quad \nu = \frac{V_{GST}}{V_{GSTt}}, \quad V_{GSTt} = (V_{GS} - V_T)_t = 2n \frac{kT}{q}$$

- 弱反型区中:  $\ln(1 + e^\nu) \approx e^\nu$

$$I_{DS} = K' \frac{W}{L} V_{GSTt}^2 \cdot e^{2\nu} = I_{DSt} \cdot \exp\left(\frac{V_{GS} - V_T}{nkT/q}\right)$$

- 强反型区中:  $\ln(1 + e^\nu) \approx \nu$

$$I_{DS} = K' \frac{W}{L} V_{GSTt}^2 \cdot \nu^2 = K' \frac{W}{L} (V_{GS} - V_T)^2$$

## 》》弱反型与强反型：反型系数

---

$$I_{DS} = K' \frac{W}{L} V_{GSTt}^2 \cdot \ln^2(1 + e^\nu), \quad \nu = \frac{V_{GST}}{V_{GSTt}}, \quad V_{GSTt} = (V_{GS} - V_T)_t = 2n \frac{kT}{q}$$

反型系数 $i$ :  $i = \frac{I_{DS}}{I_{DSt}} = \ln^2(1 + e^\nu)$



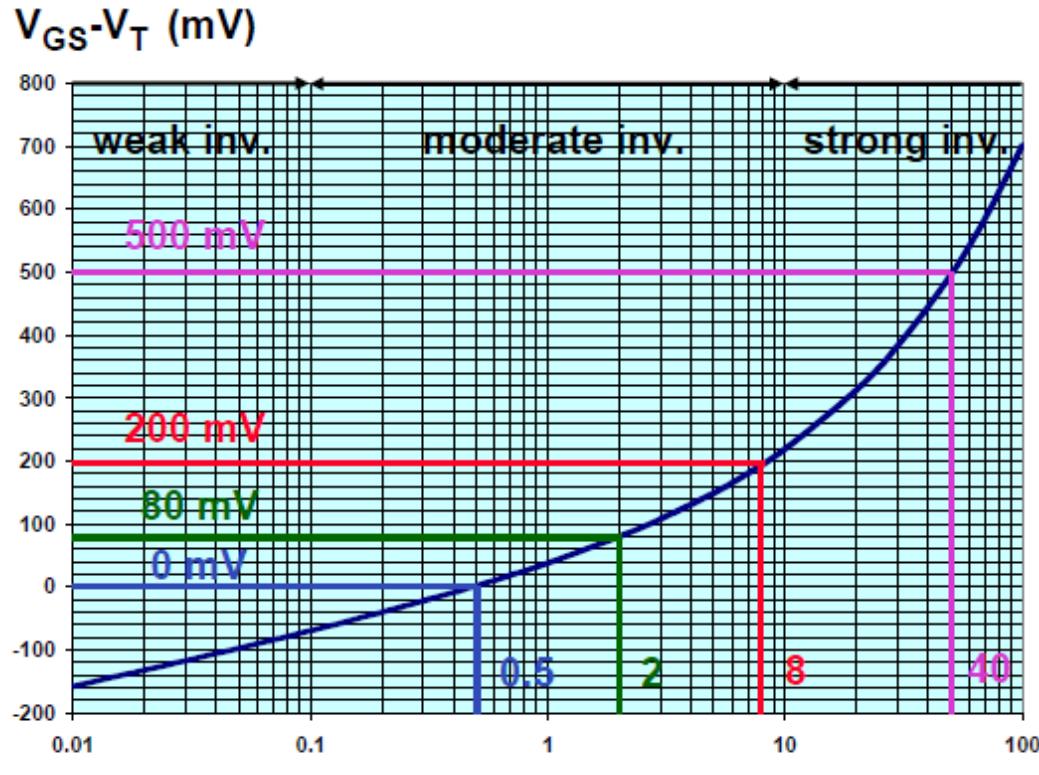
$$\nu = \ln(e^{\sqrt{i}} - 1)$$



$$V_{GST} = V_{GSTt} \cdot \ln(e^{\sqrt{i}} - 1)$$

$$V_{GSTt} = 2n \frac{kT}{q} \approx 70mV$$

## 》》弱反型与强反型：反型系数



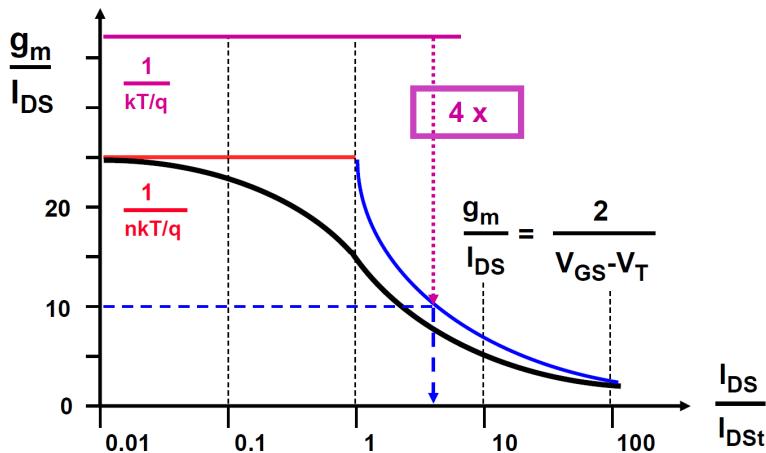
$$V_{GST} = V_{GSTt} \cdot \ln(e^{\sqrt{i}} - 1)$$

$$V_{GSTt} = 2n \frac{kT}{q} \approx 70mV$$

$$i = \frac{I_{DS}}{I_{DSt}}$$



## 弱反型与强反型：反型系数与 $g_m/I_{DS}$

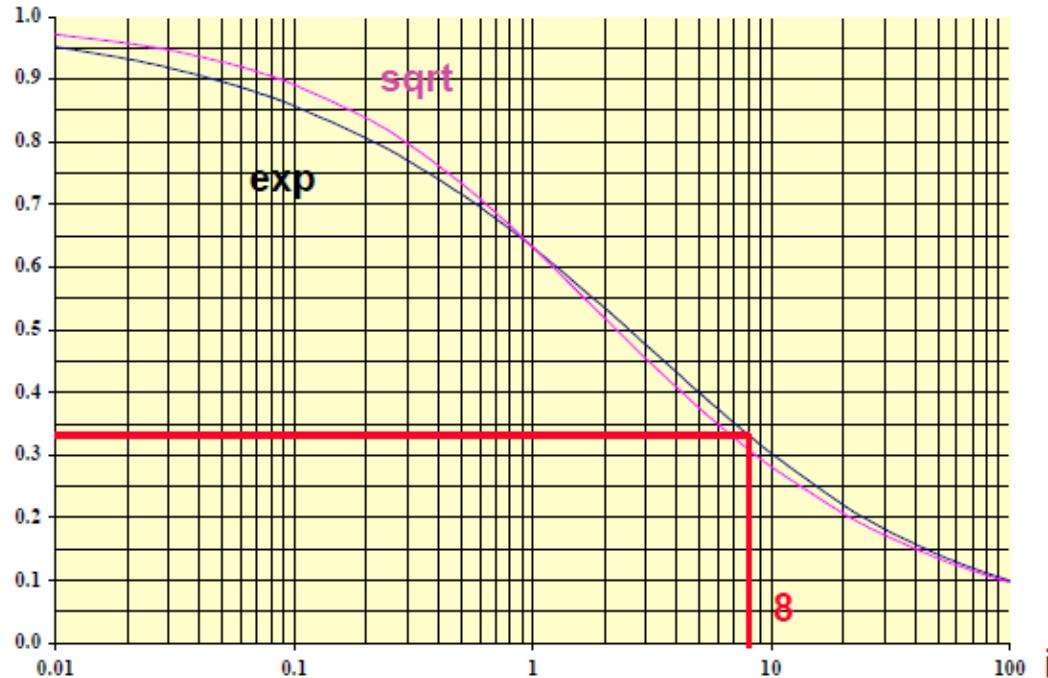


晶体管最大的  $g_m/I_{DS}$  出现在弱饱和区，且随着反型系数增加而下降

归一化效率 GM/ID:  $GM/ID = \frac{g_m/I_{DS}}{(g_m/I_{DS})_{max}}$

→  $GM/ID = \frac{1 - e^{-\sqrt{i}}}{\sqrt{i}}$

## 弱反型与强反型：反型系数与 $g_m/I_{DS}$



$$GM/ID = \frac{1 - e^{-\sqrt{i}}}{\sqrt{i}}$$

- 反型系数为8，即 $V_{GS} - V_T = 200\text{mV}$ 时，晶体管在跨导值和跨导效率中取得一个较好的平衡。

## » 弱反型与强反型

- 晶体管不会立即从弱反型区的指数行为切换为强反型区的二次项行为。在这两个极端之间有一个平滑的过渡，其中漂移和扩散均会产生电流。对该区域进行建模非常困难，通常将其行为理解为弱反型和强反型行为的混合体。

$$V_{GS} > V_T + 100\text{mV}$$

strong inversion

$$V_T + 100\text{mV} > V_{GS} > V_T - 100\text{mV}$$

moderate inversion

$$V_{GS} < V_T - 100\text{mV}$$

weak inversion

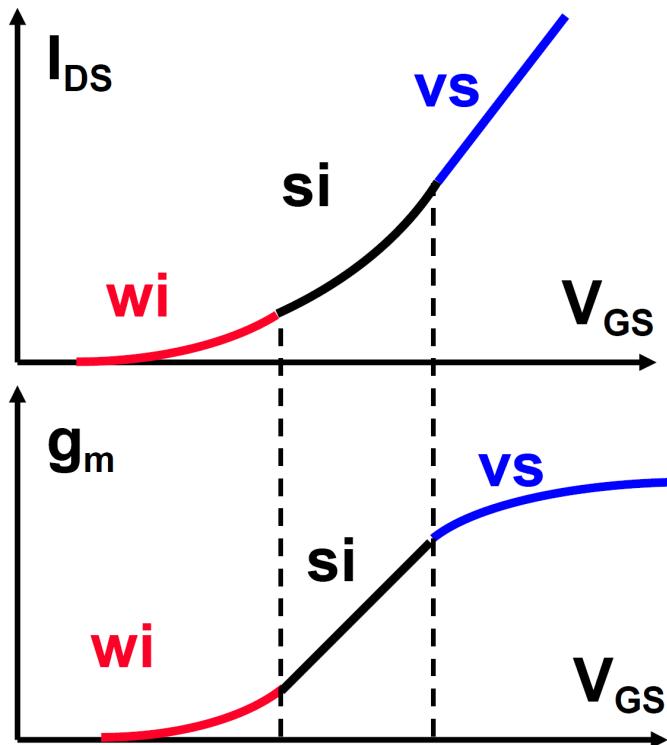


# MOSFET

---

- 晶体管工作在线性区：电阻
- 晶体管工作在饱和区：放大器
- 亚反型区/弱翻转区 (Weak Inversion)
- **速度饱和区 (Velocity Saturation)**
- 特征频率

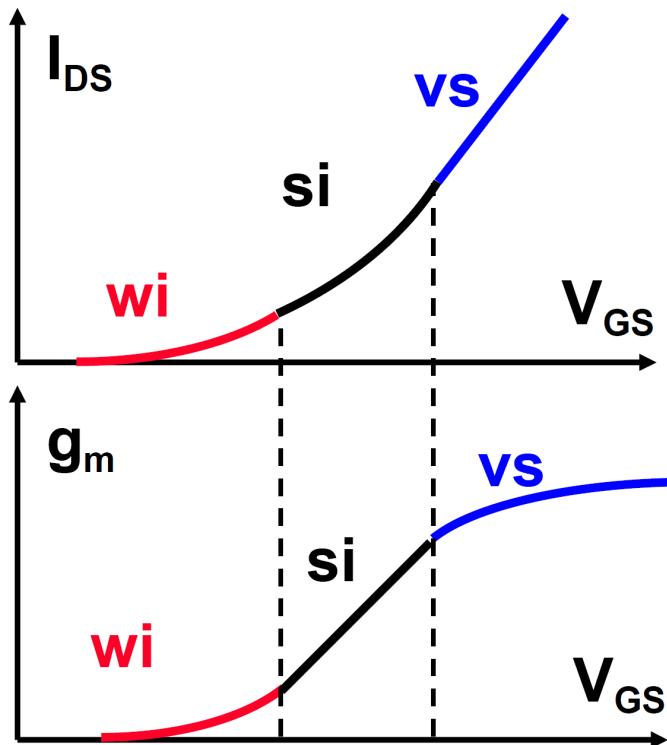
# 》》速度饱和区 (Velocity Saturation)



□ 如何理解饱和区电流中 $V_{GS}$ 的平方项?

□ 如何理解饱和区跨导与 $V_{GS}$ 的线性关系

# 》》速度饱和区 (Velocity Saturation)

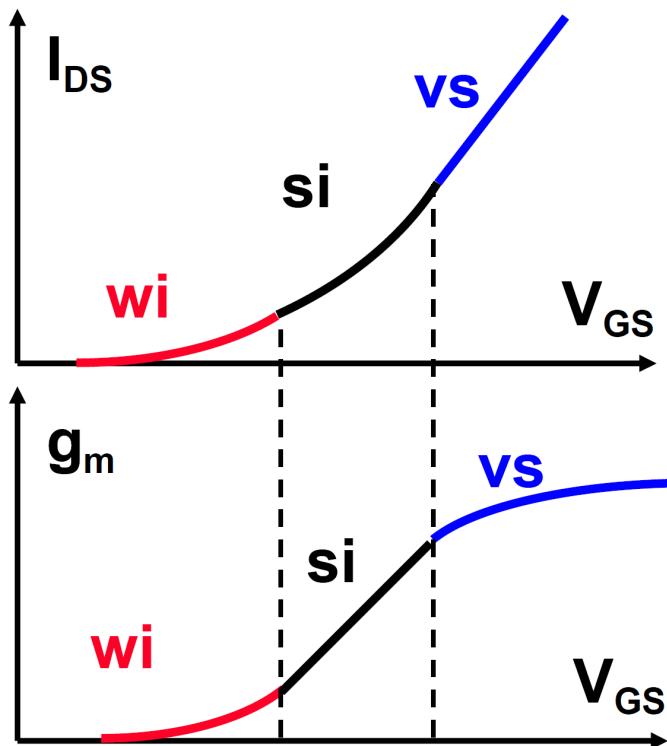


- **速度饱和区:**

- ✓ 电子以最大速度流过沟道
- ✓ 电流随着驱动电压线性增加

$$I_{DSvs} = ?$$

## 》》速度饱和区 (Velocity Saturation)



- 速度饱和区:

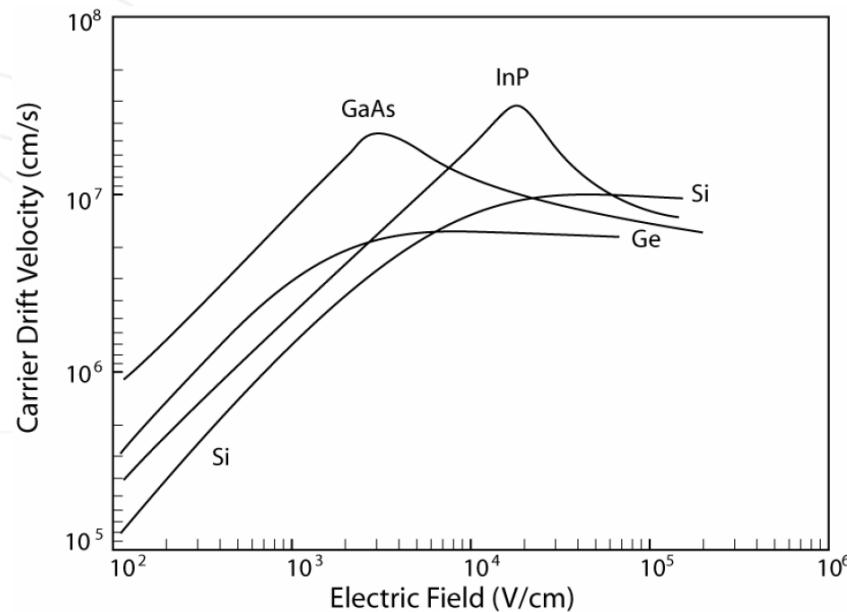
$$I_{DSvs} = WC_{ox}v_{sat} (V_{GS} - V_T)$$

$$v_{sat} \approx 10^7 \text{ cm/s}$$

$$g_{msat} = WC_{ox}v_{sat}$$

跨导到达最大值!

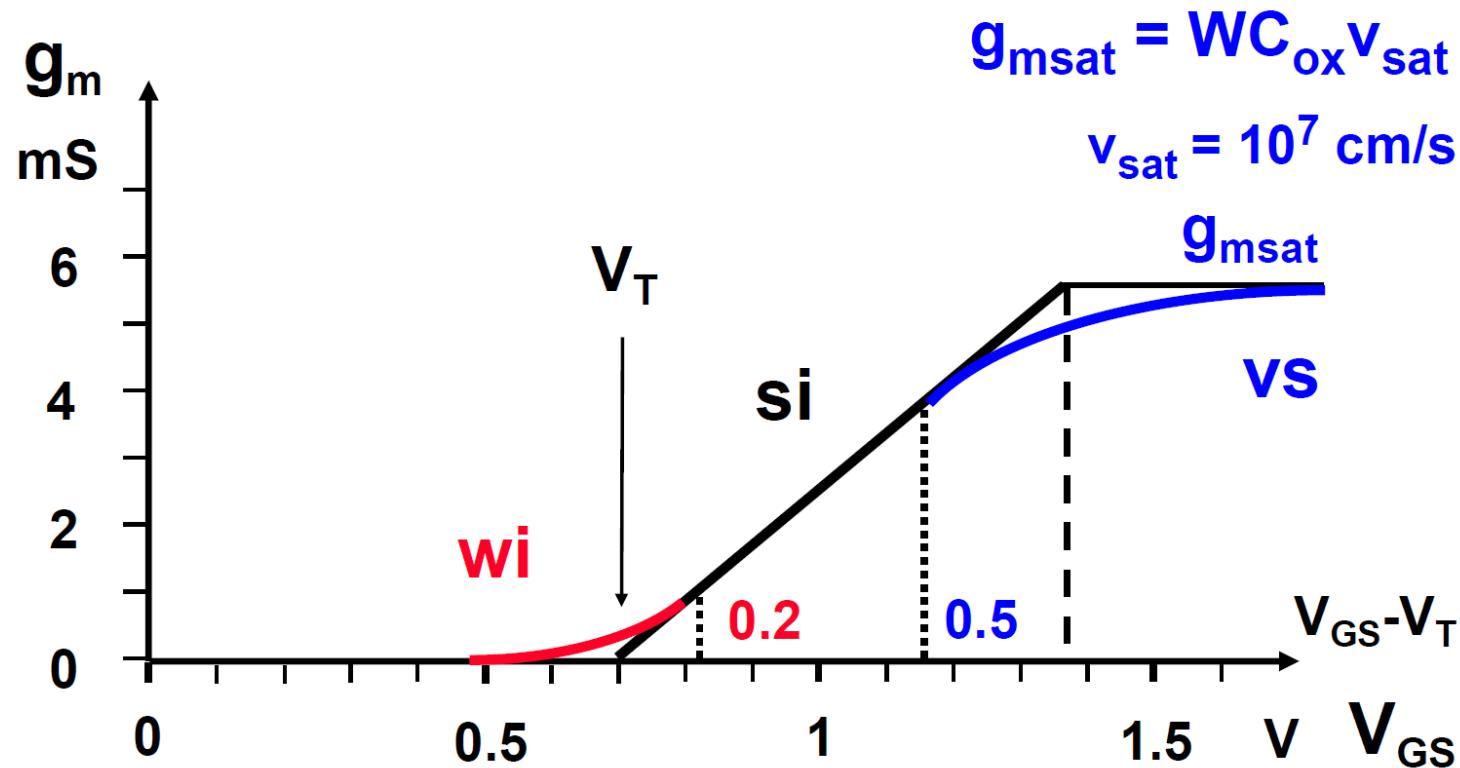
# 》》速度饱和区 (Velocity Saturation)



$$g_{msat} = WC_{ox}v_{sat}$$

- $g_{msat}/W$ 只与工艺参数 $C_{OX}$ 和物理参数 $v_{sat}$ 有关
- 模拟电路中一般不使用该区域

## 》》速度饱和区



## 》》速度饱和区

强反型区(Strong Inversion)

$$g_{m,si} \approx \mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$

速度饱和区(Velocity Saturation)

$$g_{m,sat} = WC_{OX}v_{sat}$$



$$(V_{GS} - V_{TH})_{vs} = \frac{L \cdot v_{sat}}{\mu}$$

$$v_{sat} \approx 10^7 \text{ cm/s}$$

$$\mu_p \approx 250 \text{ cm}^2/\text{Vs}$$

$$\mu_n \approx 600 \text{ cm}^2/\text{Vs}$$



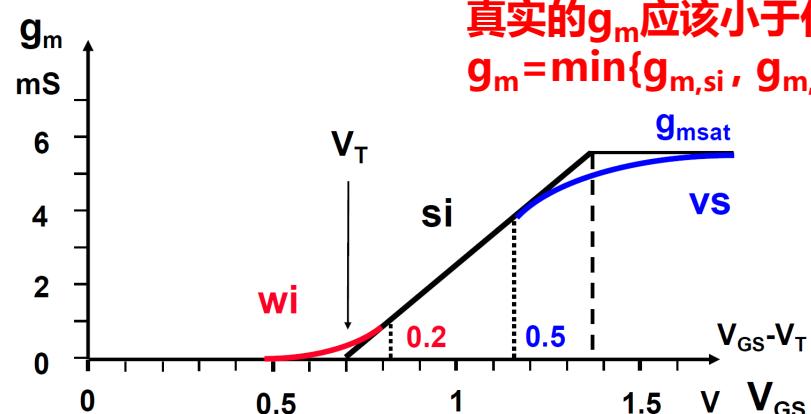
$$(V_{GS} - V_{TH})_{vs} = 0.58V$$

$$L_{min} = 0.35 \mu\text{m}$$

# 》》速度饱和区

强反型区(Strong Inversion)

$$g_{m,si} \approx \mu C_{ox} \frac{W}{L} (V_{GS} - V_{TH})$$



速度饱和区(Velocity Saturation)

$$g_{m,sat} = W C_{OX} v_{sat}$$

真实的 $g_m$ 应该小于任意一个表达式

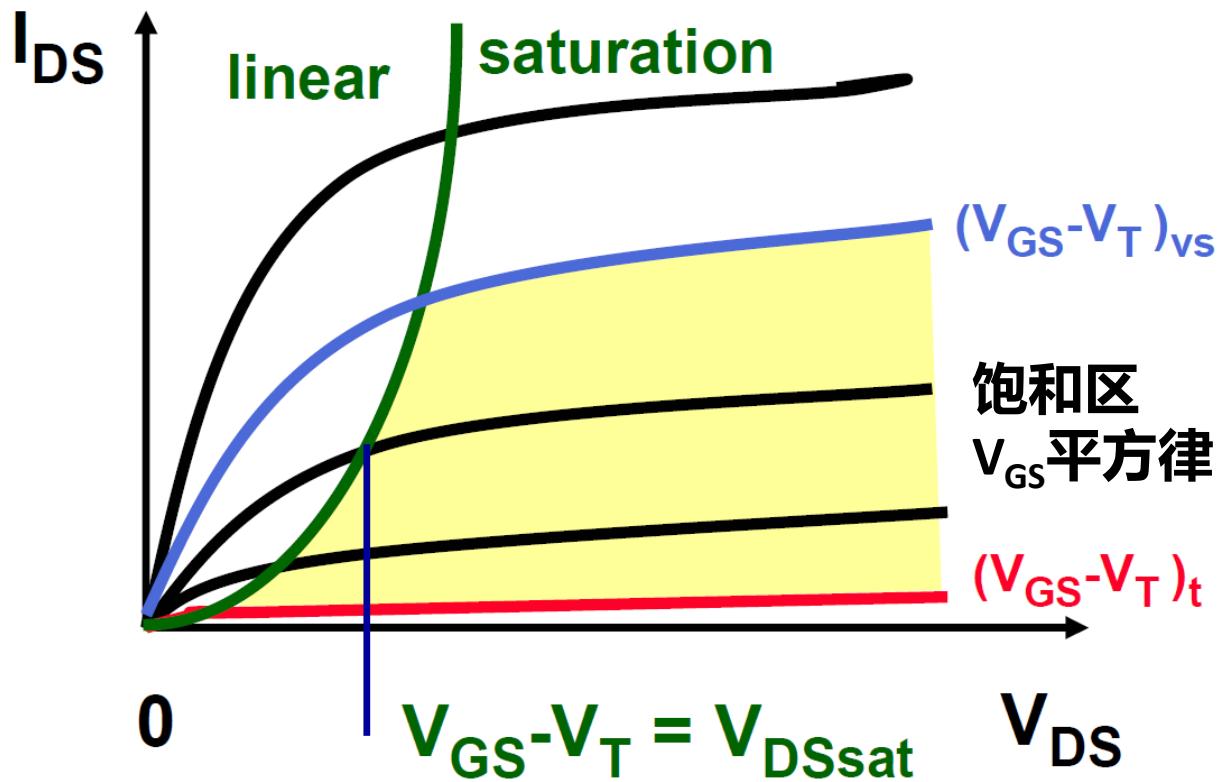
$$g_m = \min\{g_{m,si}, g_{m,sat}\}$$

$$\frac{1}{g_m} = \frac{1}{g_{m,si}} + \frac{1}{g_{m,sat}}$$

$$g_m \approx \frac{W}{L} \frac{17 \cdot 10^{-5}}{1 + 2.8 \cdot 10^4 L / V_{GST}}$$

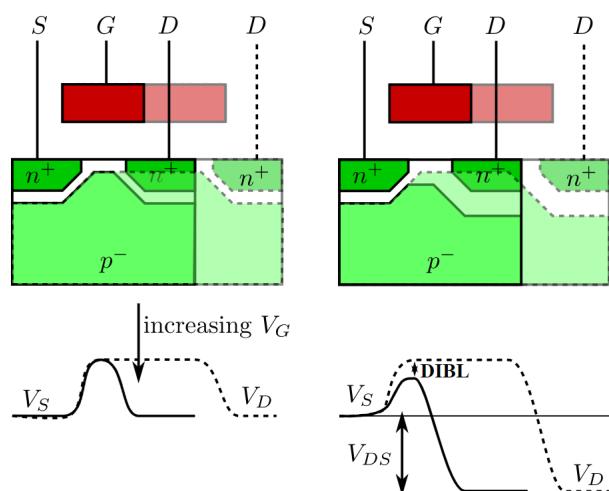
L  
in cm

## » 饱和区VS速度饱和区

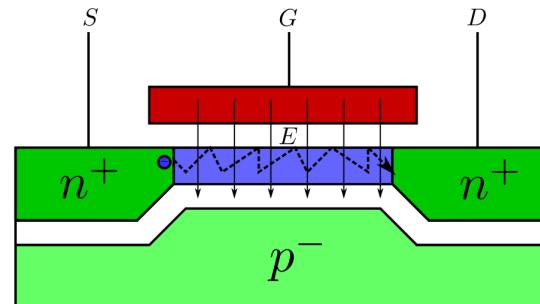


# 》》 其他因素

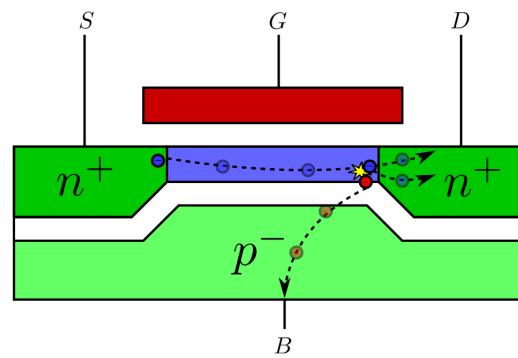
## Drain-Induced Barrier Lowering (DIBL)



## Surface Scattering



## Impact ionization



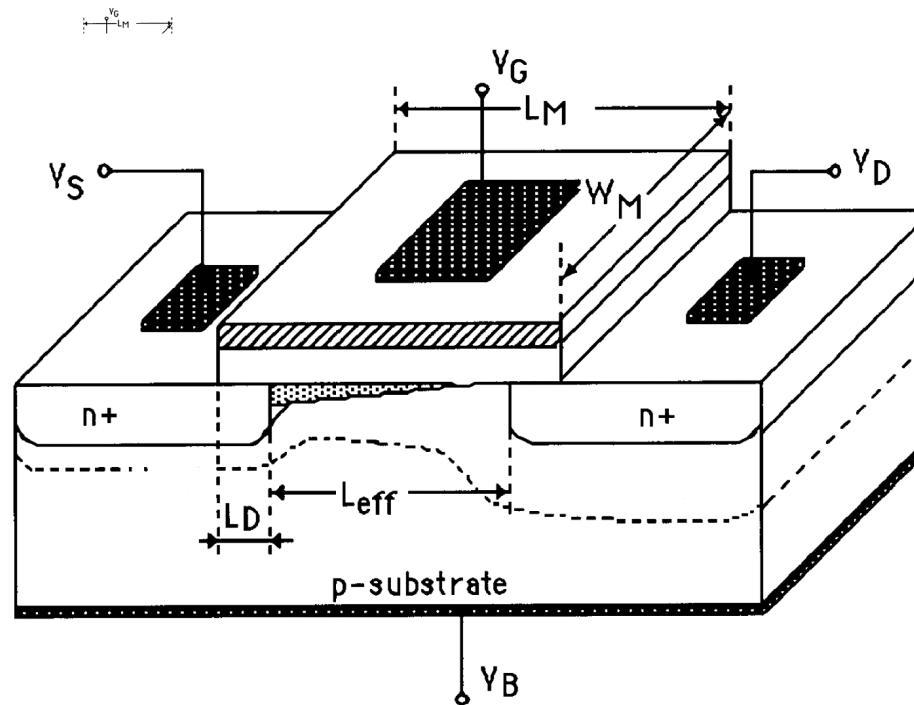


# MOSFET

---

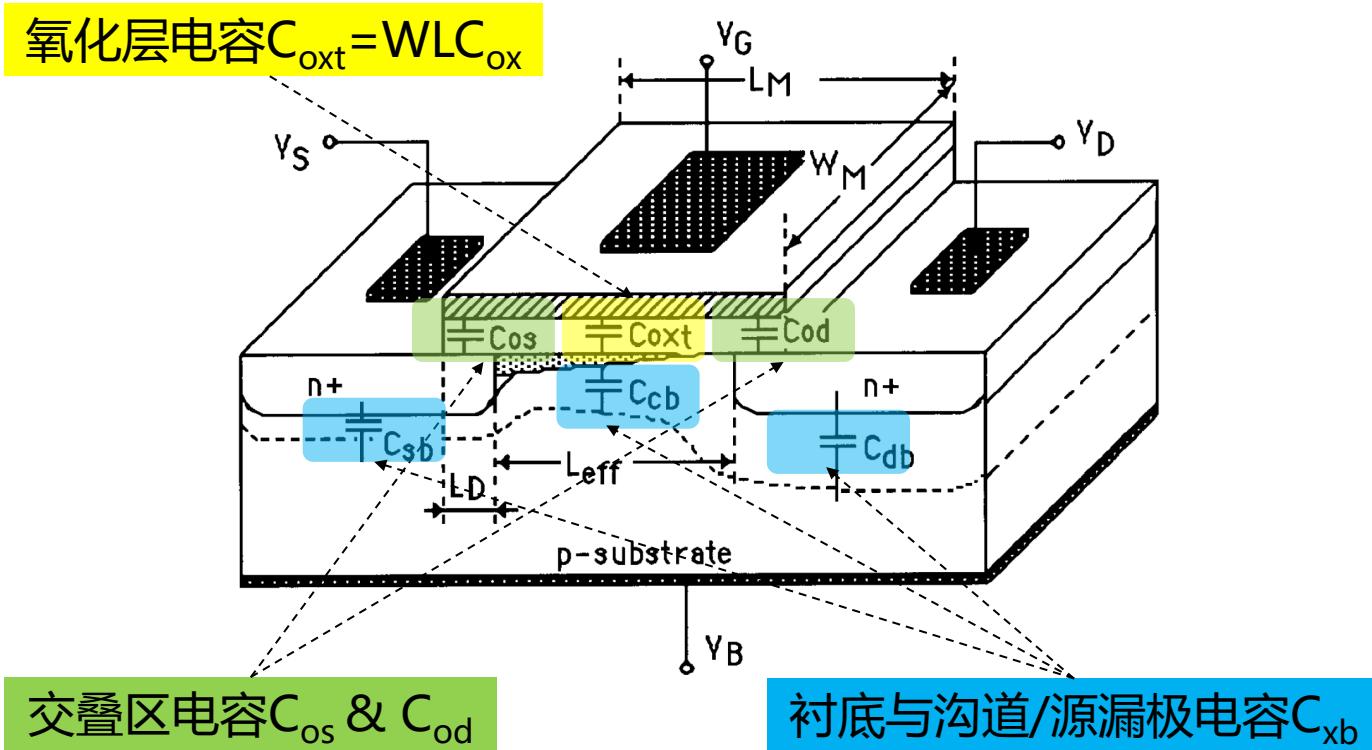
- 晶体管工作在线性区：电阻
- 晶体管工作在饱和区：放大器
- 亚反型区/弱翻转区 (Weak Inversion)
- 速度饱和区 (Velocity Saturation)
- **特征频率**

## 》》 特征频率 $f_T$

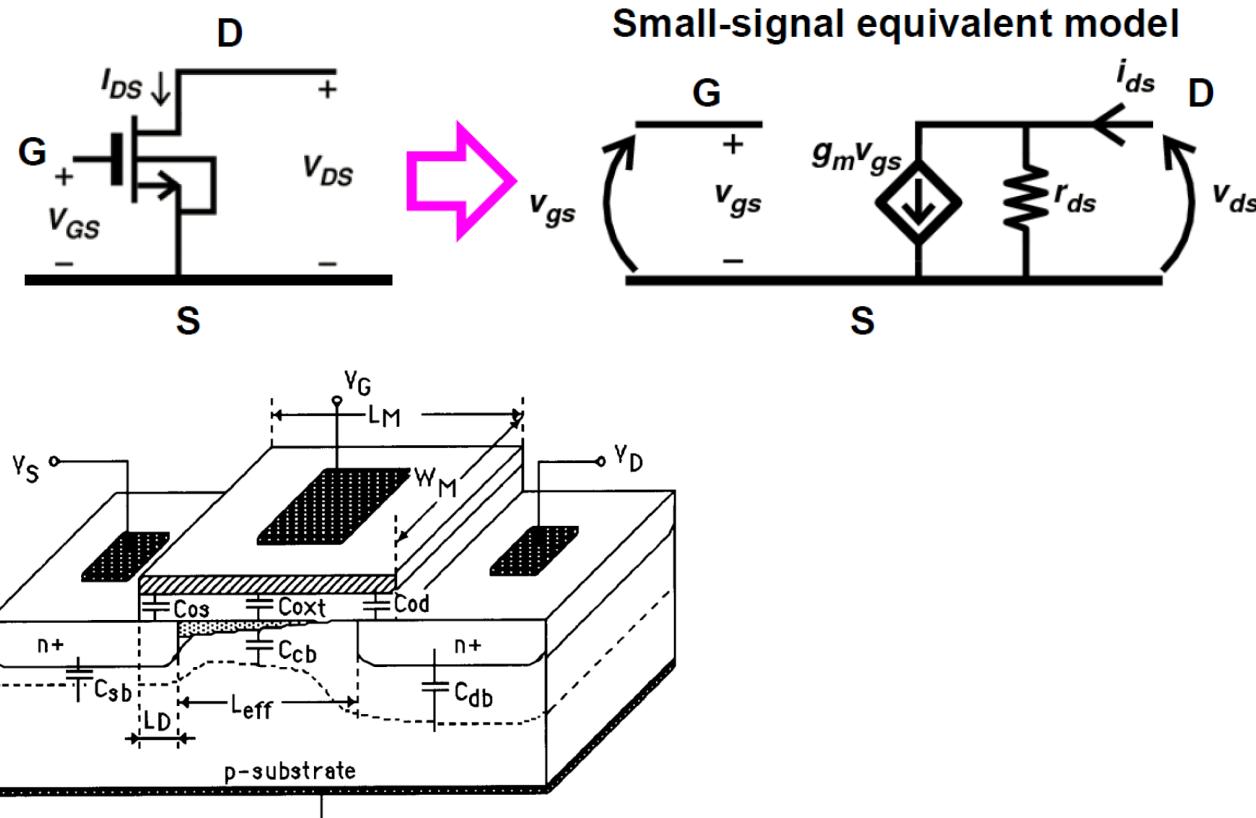


- 特征频率是表征晶体管在高频时放大能力的一个基本参量，主要由晶体管的**跨导**和**寄生电容**决定。

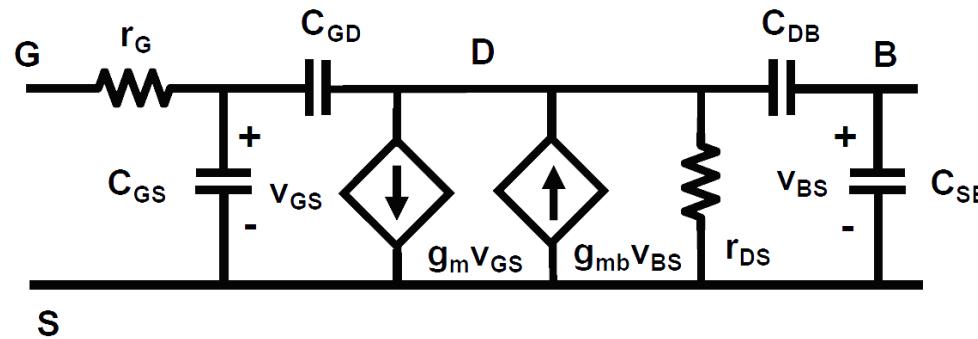
# 》》 特征频率 $f_T$



# 》》高频小信号模型



# 》》高频小信号模型



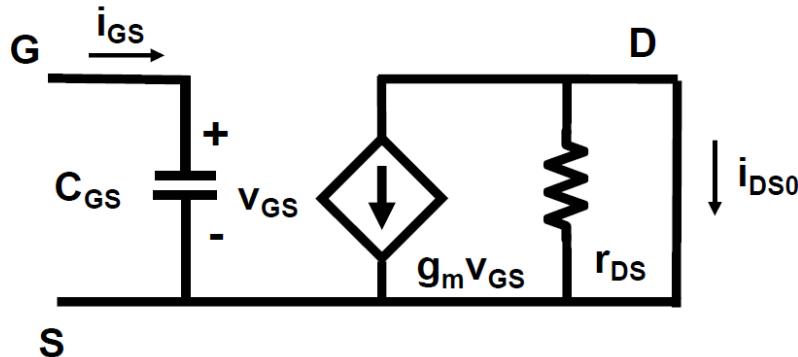
$$C_{GS} \approx \frac{2}{3} WLC_{ox} \approx 2W \text{ fF}/\mu\text{m for } L_{min}$$

$$L_{min} C_{ox} \approx L_{min} \frac{\epsilon_{ox}}{t_{ox}} \approx 50 \epsilon_{ox} \approx 2 \text{ fF}/\mu\text{m}$$

$$C_{GD} = W C_{gdo}$$

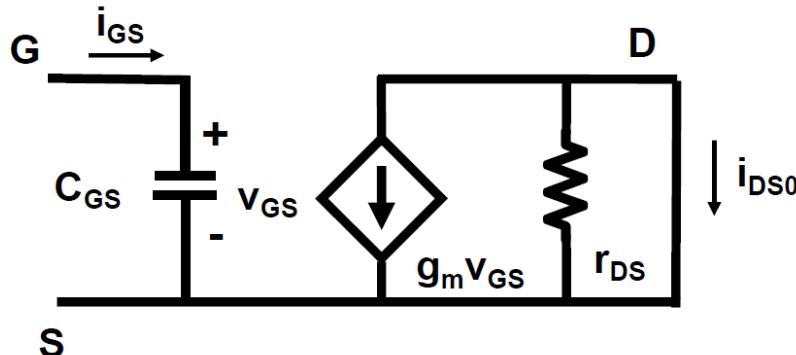
## 》》 特征频率 $f_T$ , $i_{DS} = i_{GS}$

---



$$i_{GS} =$$
$$i_{DS0} =$$

## 》》 特征频率 $f_T$ , $i_{DS} = i_{GS}$



$$i_{GS} = v_{GS} C_{GS} s$$

$$i_{DS} = g_m v_{GS}$$

$$C_{GS} = \frac{2}{3} WLC_{ox} \quad g_m = 2K' \frac{W}{L} (V_{GS} - V_T) \quad K' = \frac{\mu C_{ox}}{2n}$$

$$f_T = \frac{g_m}{2\pi C_{GS}} = \frac{1}{2\pi} \frac{3}{2n} \frac{\mu}{L^2} (V_{GS} - V_T)$$

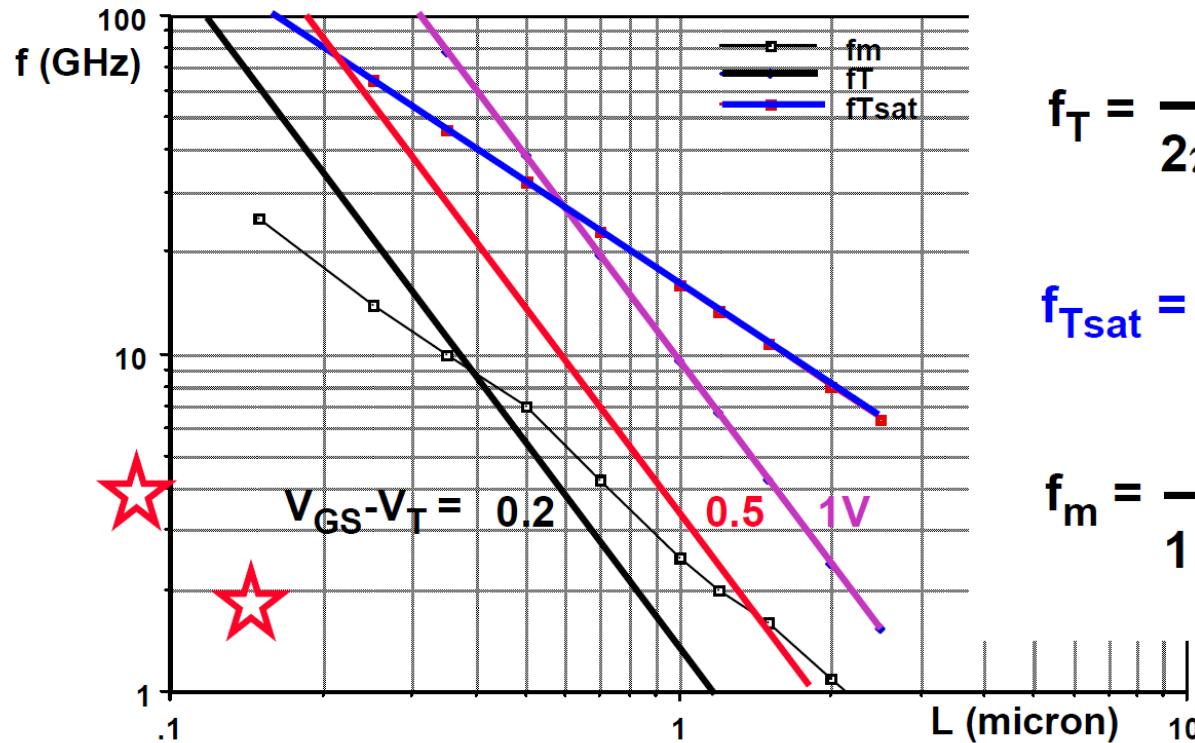
↑  
反型区

or

$$\approx \frac{v_{sat}}{2\pi L}$$

↑  
速度饱和区

# 》》 f<sub>T</sub>与沟道长度L的关系



Processors

$$f_T = \frac{\mu}{2\pi L^2} \underbrace{(V_{GS} - V_T)}_{0.2 \dots 1 \text{ V}}$$

$$f_{Tsat} = \frac{v_{sat}}{2\pi L}$$

$$f_m = \frac{f_T}{1 + \alpha_{BD}}$$

$$\alpha_{BD} \approx \frac{C_{BD}}{C_{ox}}$$



## $f_T$ 与沟道长度L的关系

---

$$f_T = \frac{1}{L} \frac{13.5}{1 + 2.8 L / V_{GST}} \text{ GHz}$$

$L$  in  $\mu\text{m}$

If  $V_{GST} = 0.2 \text{ V}$ ,  $v_{sat}$  takes over for  $L < 65 \text{ nm}$

If  $V_{GST} = 0.5 \text{ V}$  for  $L < 0.15 \mu\text{m}$



## f<sub>T</sub>在强反型区和弱反型区中模型

---

$$\left\{ \begin{array}{l} GM/ID = \frac{g_m/I_{DS}}{(g_m/I_{DS})_{max}} = \frac{1 - e^{-\sqrt{i}}}{\sqrt{i}} \\ i = \frac{I_{DS}}{I_{DSt}} \end{array} \right.$$

$$f_T =$$

## 》》 $f_T$ 在强反型区和弱反型区中模型

$$\left\{ \begin{array}{l} GM/ID = \frac{g_m/I_{DS}}{(g_m/I_{DS})_{max}} = \frac{1 - e^{-\sqrt{i}}}{\sqrt{i}} \\ i = \frac{I_{DS}}{I_{DSt}} \end{array} \right.$$

$$f_T = \frac{g_m}{2\pi C_{GS}} = \frac{1}{2\pi C_{GS}} \frac{I_{DSt}}{nkT/q} \sqrt{i}(1 - e^{\sqrt{i}})$$

$$= \frac{2\mu kT/q}{2\pi L^2} \cdot \sqrt{i}(1 - e^{\sqrt{i}})$$

**尺寸设计    偏置设计**



## 设计思路总结

---

1. 手工计算用来估算尺寸，精确设计依赖仿真结果。
2. 时刻牢记 $g_m/I_D$ 的曲线及大致数值。
3. 低功耗电路取 $V_{GS} - V_{TH} < -0.1V$ ；高增益电路取 $V_{GS} - V_{TH} = 0.2V$ ；  
高速电路取 $V_{GS} - V_{TH} = 0.5V$ 。
4. 通过 $f_T$ 的公式来估算设计的特征频率。

》》 课程群（钉钉）

---



扫一扫二维码，加入班级

## 》》课后作业

---

1. 分别对于 $W/L=1\text{um}/1\text{um}$ 的NMOS和PMOS，在 $0-1.8\text{V}$ 的区域内，扫描 $V_{GS}$ ，同时令 $V_{DS}=1.8\text{V}$ ，画出 $I_{DS}$ ,  $g_m$ 和 $g_m/I_{DS}$ 的曲线。（提示：改变坐标的刻度表现形式，以得到更直观的展示）
2. 估算 $V_{GS}=0.5\text{V}$ ,  $0.8\text{V}$ 和 $1.1\text{V}$ 且 $V_{DS}=1.8\text{V}$ 时， $W/L=1\text{um}/0.18\text{um}$ 的NMOS的 $g_m$ 和 $r_0$ 。
3. 估算 $V_{GS}=0.5\text{V}$ ,  $0.8\text{V}$ 和 $1.1\text{V}$ 且 $V_{DS}=1.8\text{V}$ 时， $W/L=1\text{um}/0.18\text{um}$ 的NMOS的特征频率 $f_T$ ，并通过AC仿真得到所求的特征频率。

**0.2**



北京航空航天大學  
BEIHANG UNIVERSITY

| 微电子学院

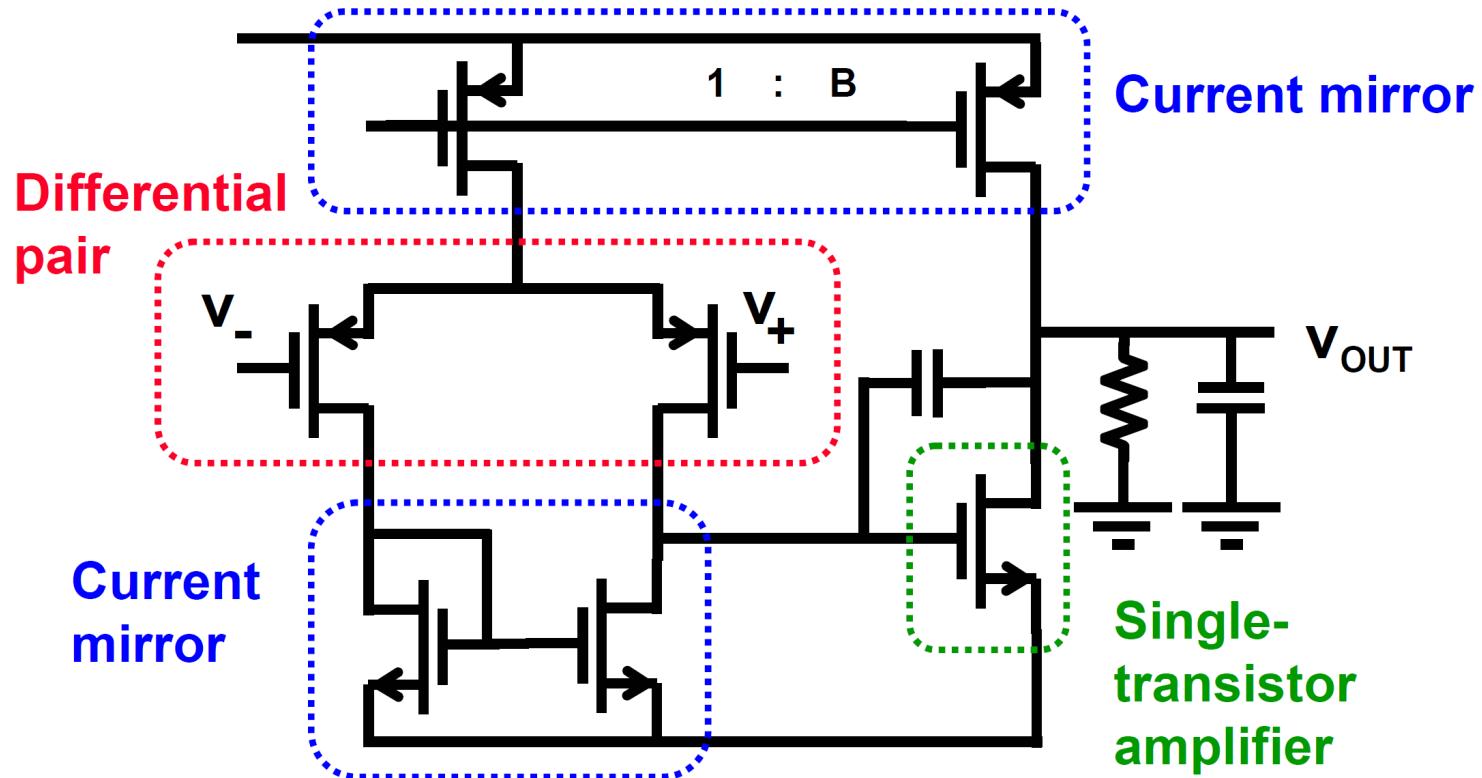
# CMOS模拟集成电路设计

## 第二章：模拟电路的基本构成

胡远奇

©2020

# 》》运算放大器





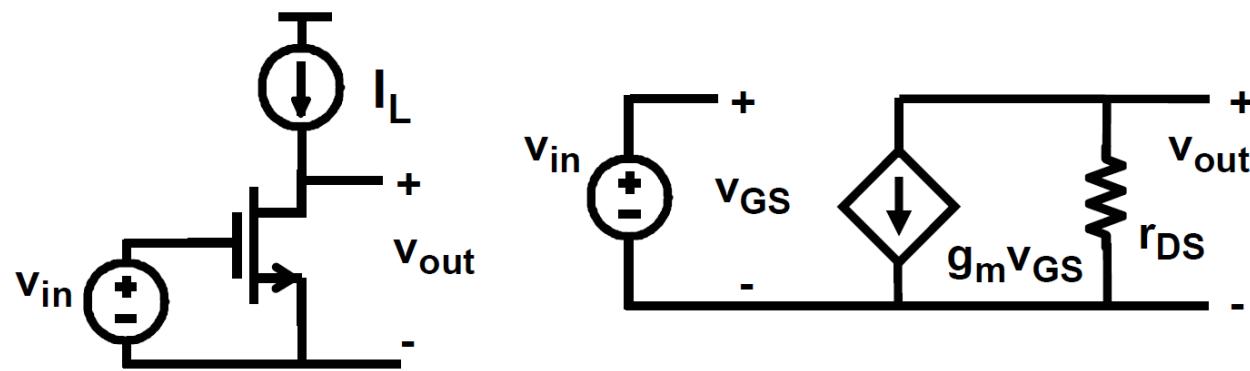
# 模拟电路的基本结构

---

1. 单晶体管放大器
2. 源极跟随器
3. Cascode (共源共栅极)
4. 电流镜
5. 差分对

# 》》单晶体管放大器

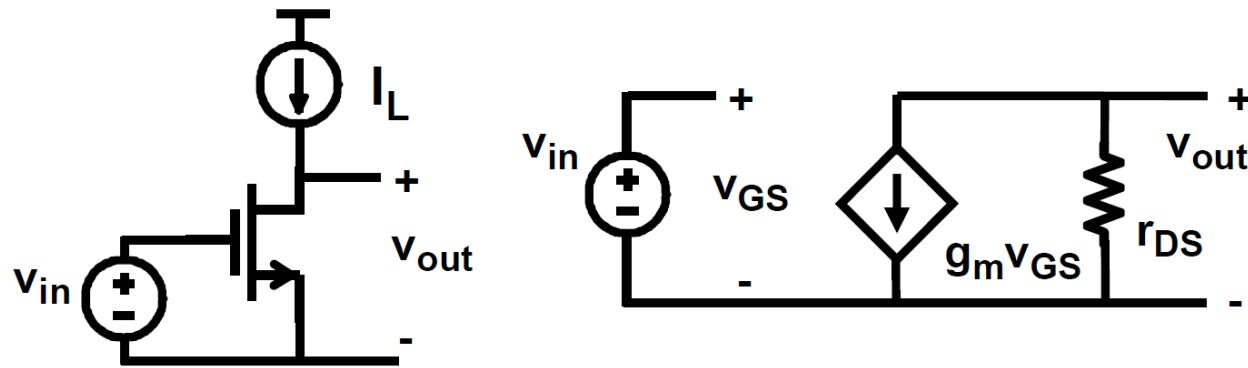
---



$$A_v =$$

# 》》单晶体管放大器

---



$$A_v = g_m r_{DS} = \frac{2 I_{DS}}{V_{GS} - V_T} \frac{V_E L}{I_{DS}} = \frac{2 V_E L}{V_{GS} - V_T}$$

$$A_v \approx 100 \quad \text{if } V_E L \approx 10 \text{ V and } V_{GS} - V_T \approx 0.2 \text{ V}$$

# 》》单晶体管放大器

如何获得高增益?

降低 $V_{GS} - V_T$

增加L

0.15-0.2V

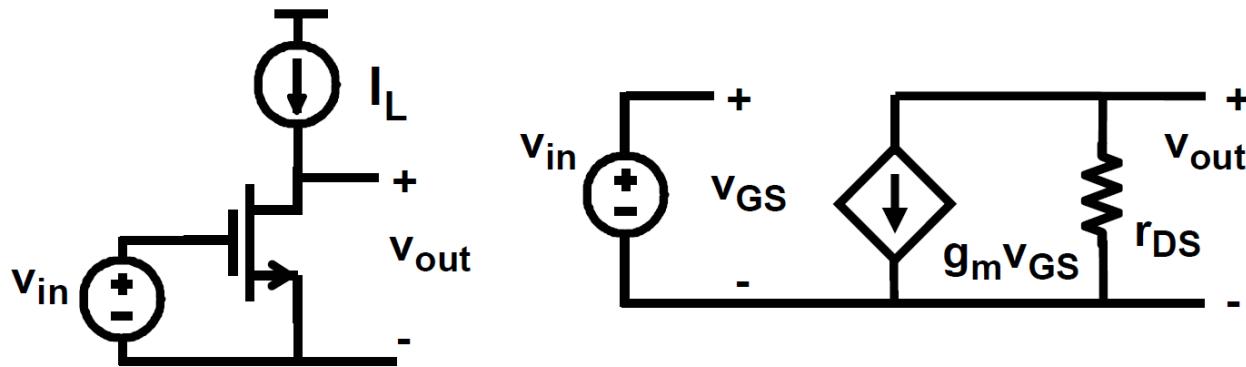
4-5x minL

进一步减小会牺牲信噪  
比SNR和跨导gm

进一步增大会牺牲速度  
和面积

## 》》单晶体管放大器

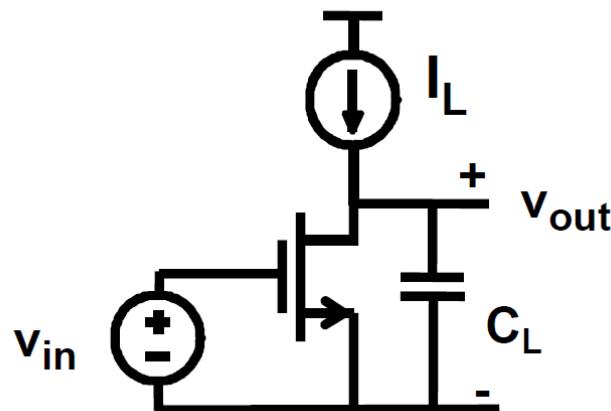
---



□问：对于一个55nm工艺的单晶体管放大器，其典型的放大倍数是多少？( $V_E=4V/\mu m$ )

## 》》 单晶体管放大器的频率特性

- 如果只有大的负载电容



$$A_{v0} = g_m r_{DS}$$

$$BW = \frac{1}{2\pi r_{DS} C_L}$$

$$GBW = \frac{g_m}{2\pi C_L}$$

For all single-stage  
Operational amplifiers

## 》》单晶体管放大器的频率特性

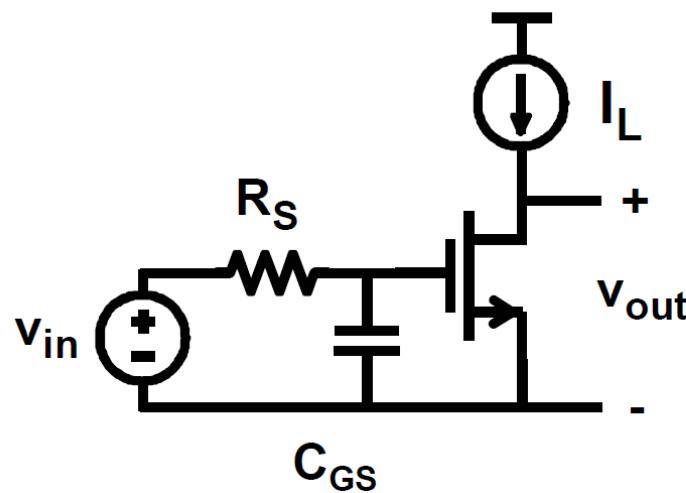
---

□设计指标GBW=100M，负载电容2pF的NMOS单晶体放大器，  
根据工艺指标计算相应的直流偏置电流和典型晶体管尺寸。

$$L_{min} = 0.35 \mu m \quad K_P_n \approx 300 \mu A/V^2$$

## 》》单晶体管放大器的频率特性

- 如果只有大的输入电容

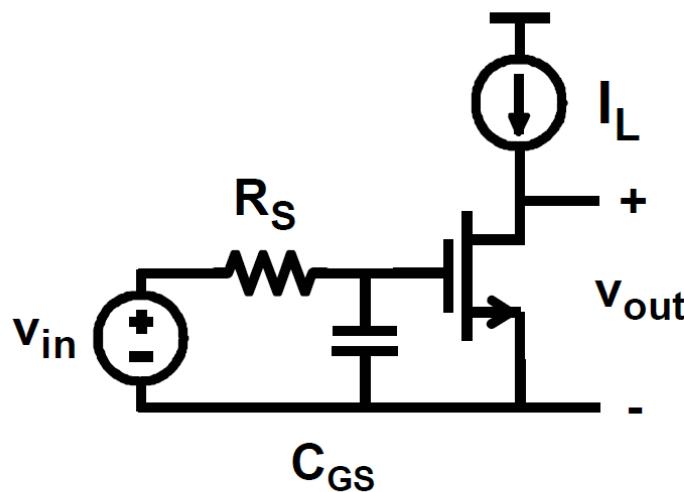


$$A_{v0} = g_m r_{DS}$$

$$BW = \frac{1}{2\pi R_s C_{GS}}$$

## 》》单晶体管放大器的频率特性

- 如果只有大的输入电容



$$A_{v0} = g_m r_{DS}$$

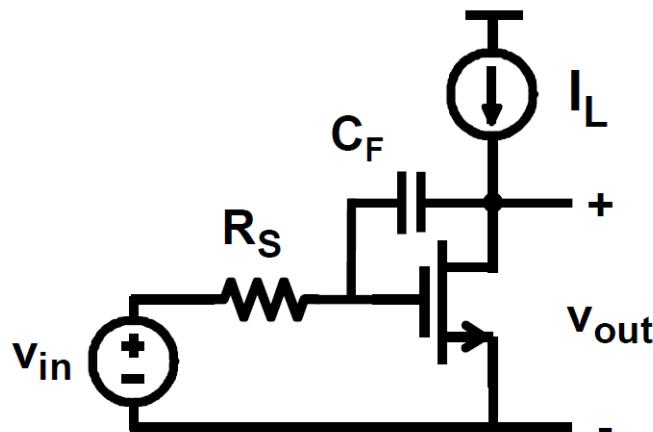
$$BW = \frac{1}{2\pi R_s C_{GS}}$$

GBW和长度L无关!

$$GBW = \frac{g_m}{2\pi C_{GS}} \frac{r_{DS}}{R_s} = f_T \frac{r_{DS}}{R_s} \sim \frac{1}{WC_{ox}} \frac{1}{V_{GS}-V_T}$$

## 》》单晶体管放大器的频率特性

- 如果只有大的反馈电容



$$A_{v0} = g_m r_{DS}$$

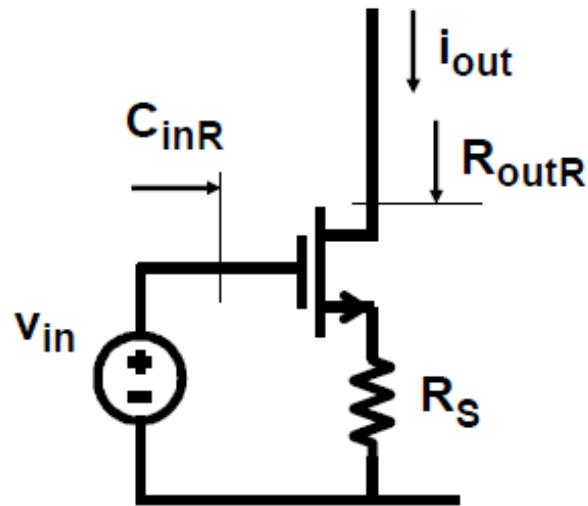
$$BW = \frac{1}{2\pi R_S A_{v0} C_F}$$

$$GBW = \frac{1}{2\pi R_S C_F}$$

GBW和晶体管的参数无关!

# »» Source Degeneration

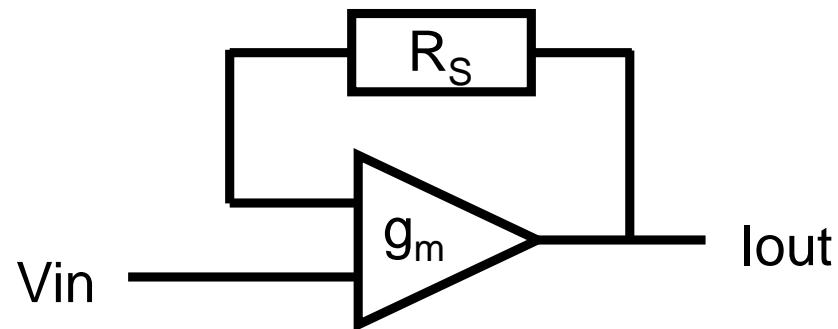
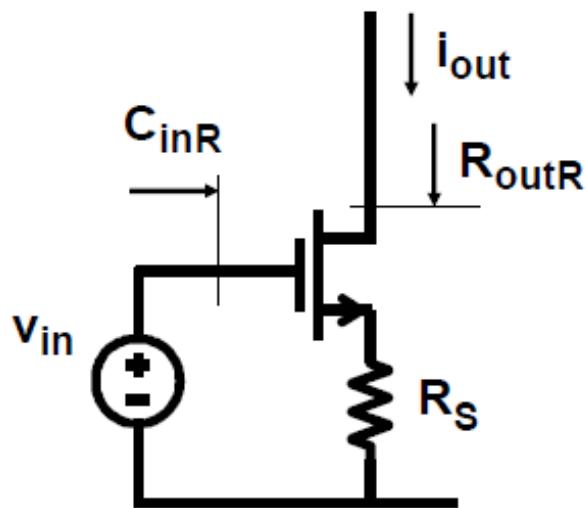
---



Source Degeneration

是一种**负反馈**形式

# »» Source Degeneration



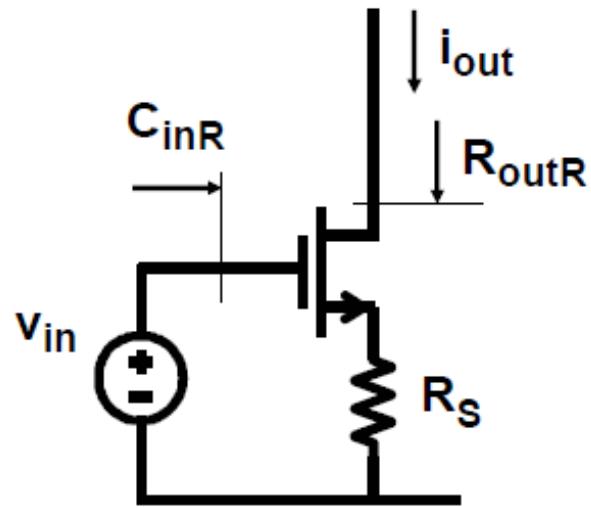
$$G_m = \frac{g_m}{1 + g_m R_s}$$

Source Degeneration  
是一种**负反馈**形式

$$C_{inR} = \frac{C_{GS}}{1 + g_m R_s}$$

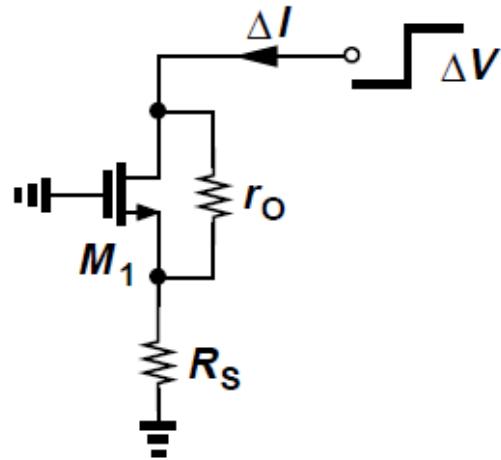
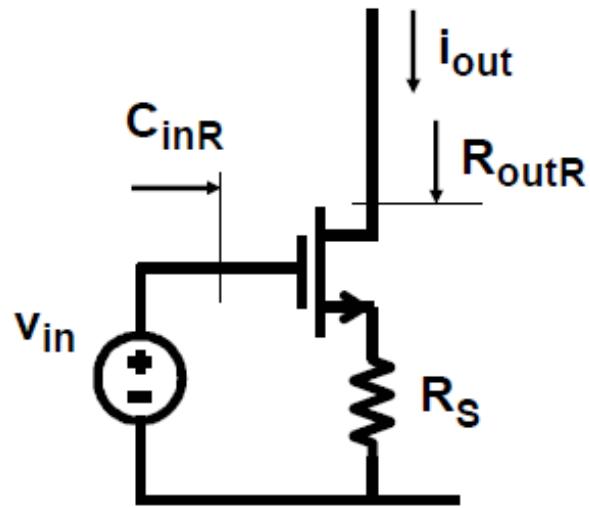
## »» Source Degeneration

---



- 在输出端施加电压变化 $\Delta V$ 并测量输出电流的最终变化 $\Delta I$

# »» Source Degeneration



$$\bullet R_{outR} = r_{DS} (1 + g_m R_s) \approx (g_m r_{DS}) R_s$$

$R_s$ 造成额外的噪声



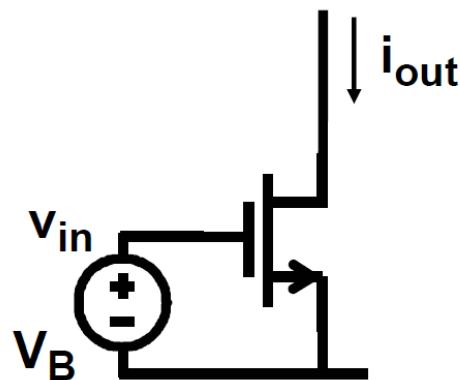
# 模拟电路的基本结构

---

1. 单晶体管放大器
2. 源极跟随器
3. Cascode (共源共栅极)
4. 电流镜
5. 差分对

# 》》 源极跟随器 (Source Follower)

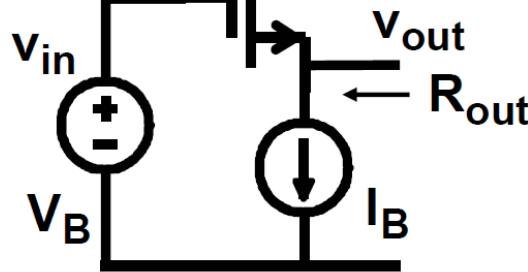
Common source



$$i_{out} = g_m v_{in}$$

Amplifier

Common drain

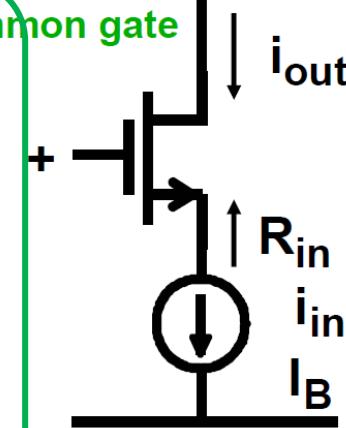


$$v_{out} = v_{in}$$

$$R_{out} \approx 1/g_m$$

Source follower  
Voltage buffer

Common gate

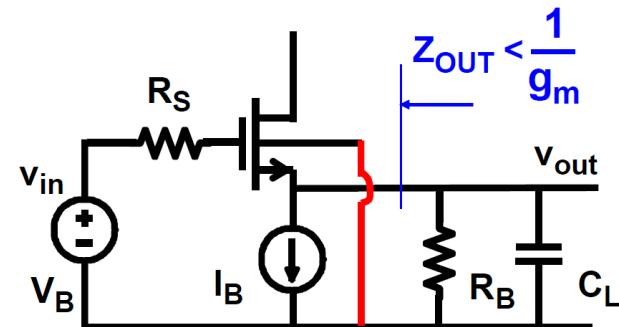
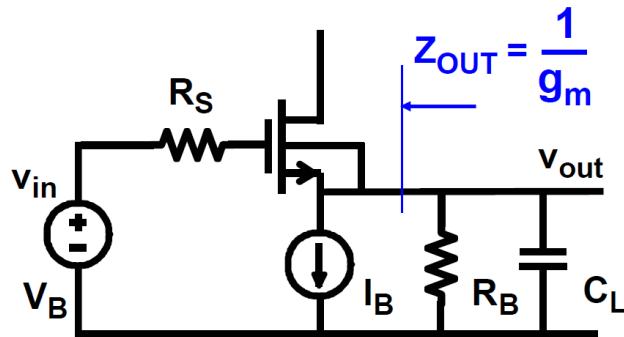


$$i_{out} = i_{in}$$

$$R_{in} \approx 1/g_m$$

Cascode  
Current buffer

# 》》 源极跟随器 (Source Follower)



$I_B$  为常数  
 ↓  
 $V_{GS}$  为常数  
 ↓  
 $\Delta V_{OUT} = \Delta V_{IN}$   
 ↓  
 $A_V = 1$

$V_{GS}$  不为常数  
 ↓  
 $A_V = \frac{1}{n} < 1$



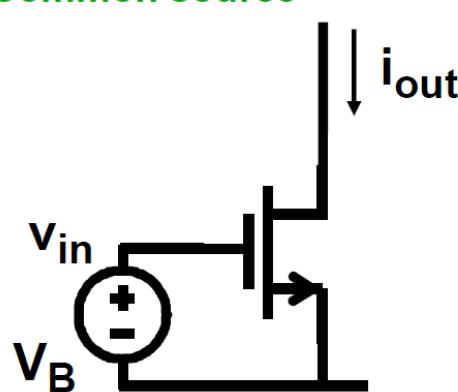
# 模拟电路的基本结构

---

1. 单晶体管放大器
2. 源极跟随器
3. **Cascode (共源共栅极)**
4. 电流镜
5. 差分对

# »» Cascode

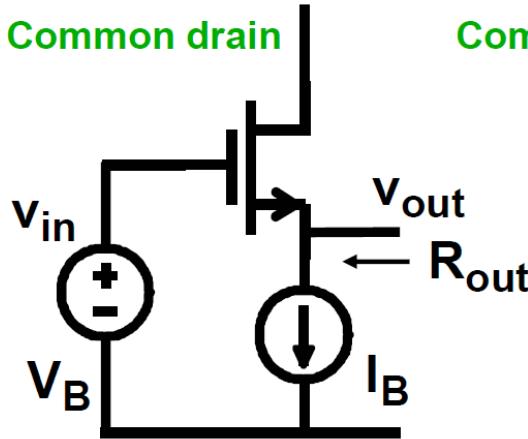
Common source



$$i_{out} = g_m v_{in}$$



Common drain

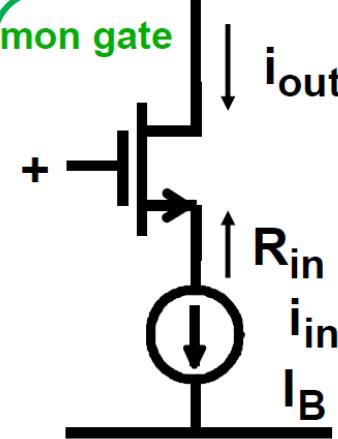


$$v_{out} = v_{in}$$

$$R_{out} \approx 1/g_m$$

Amplifier

Common gate



$$i_{out} = i_{in}$$

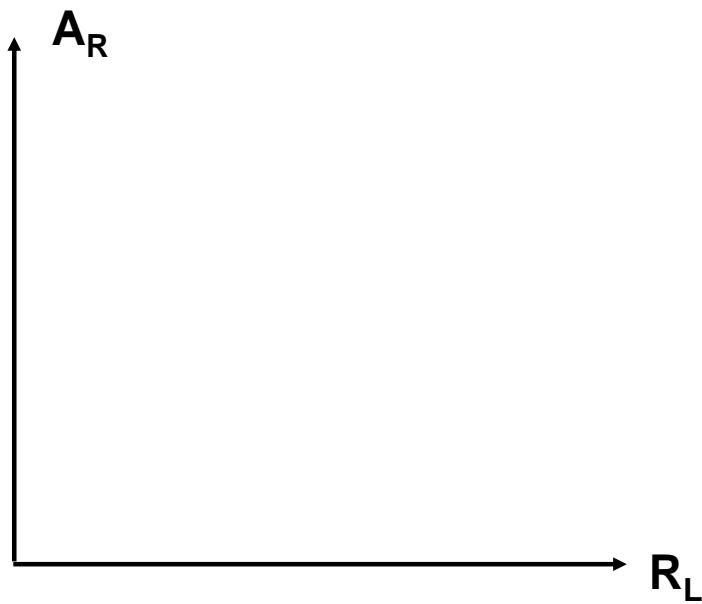
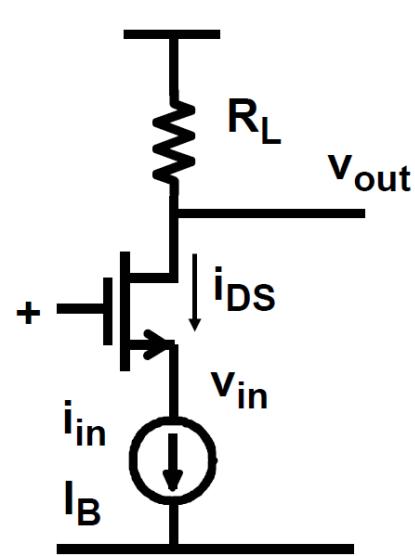
$$R_{in} \approx 1/g_m$$

Source follower  
Voltage buffer

Cascode  
Current buffer

# »» Cascode

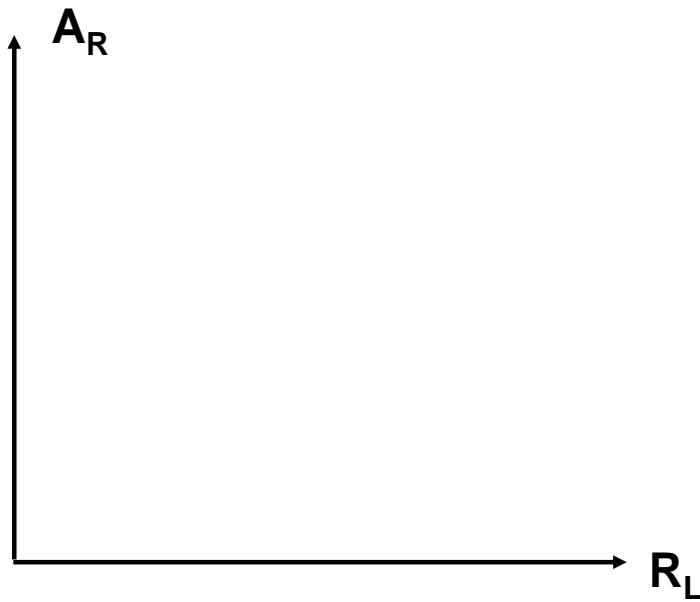
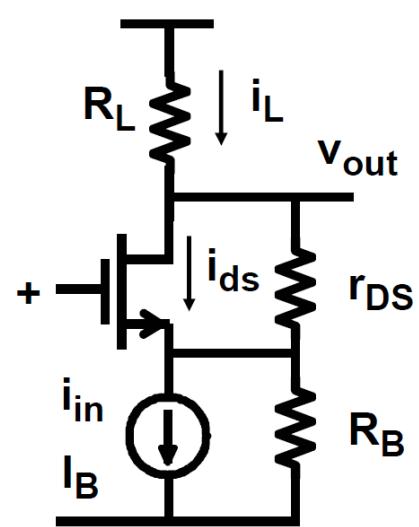
---



- 定义跨阻增益  $A_R = v_{out}/i_{in}$

# »» Cascode

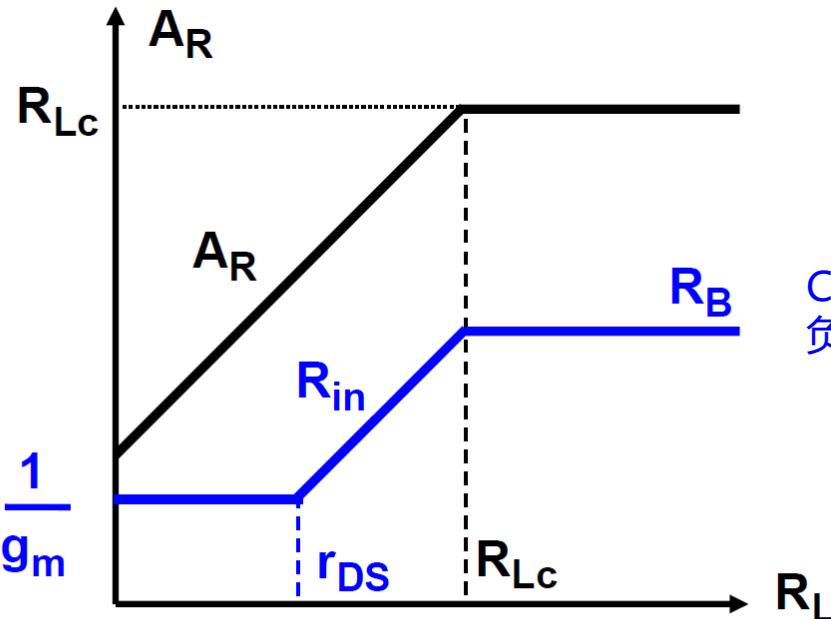
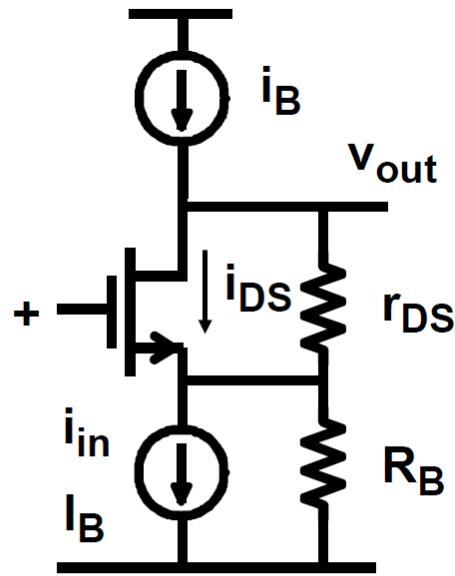
---



- 分析输入电阻  $R_{in}$

# »» Cascode

---



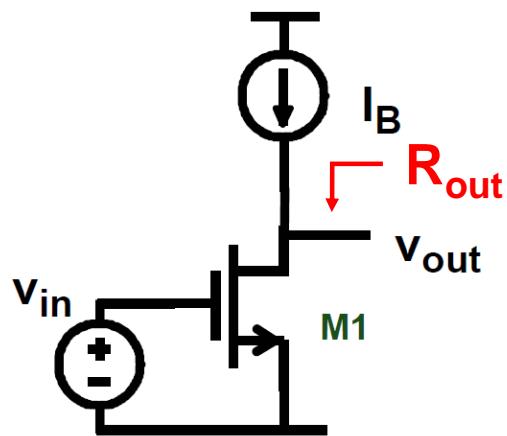
$$A_R = \frac{v_{out}}{i_{in}}$$

$$R_{in} = \frac{v_{in}}{i_{in}}$$

$$R_{in} = \frac{r_{DS} + R_L}{g_m r_{DS}} || R_B$$

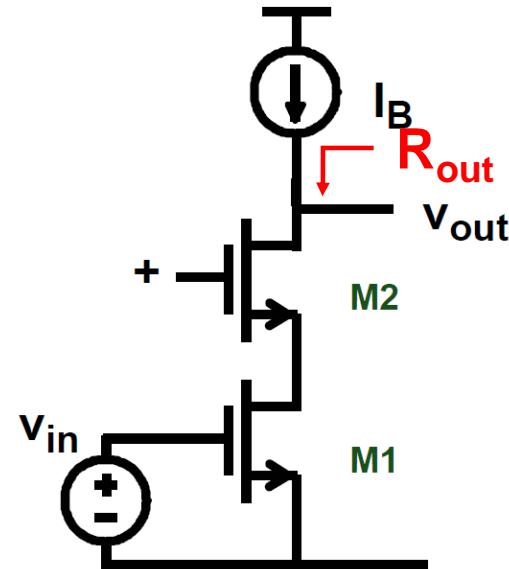
# »» Cascode vs 单晶体管

---



$$A_v = (g_m r_{DS})_1$$

$$R_{out} = r_{DS1}$$

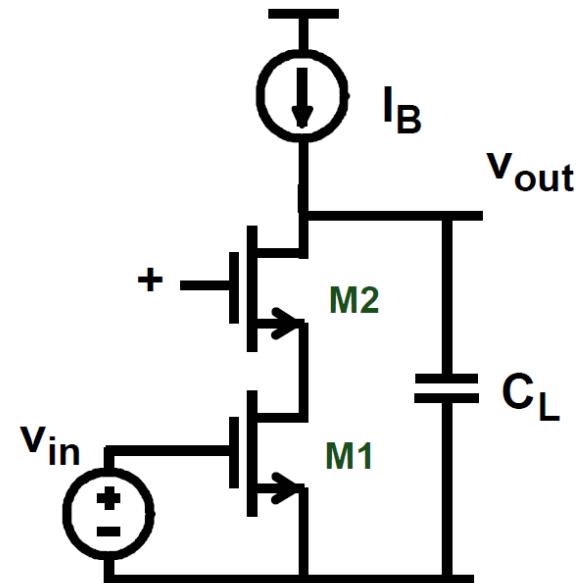
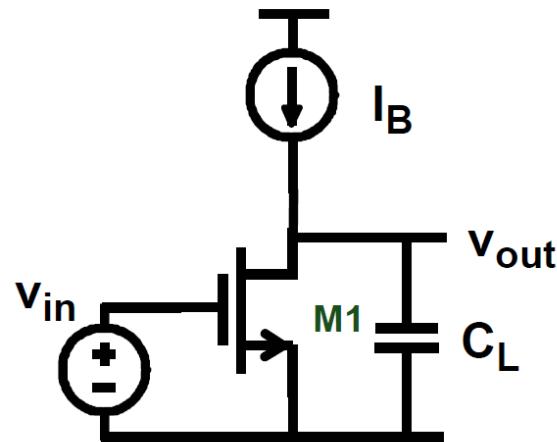


$$A_v = (g_m r_{DS})_1 (g_m r_{DS})_2$$

$$R_{out} = r_{DS1} (g_m r_{DS})_2$$

# »» Cascode vs 单晶体管

---

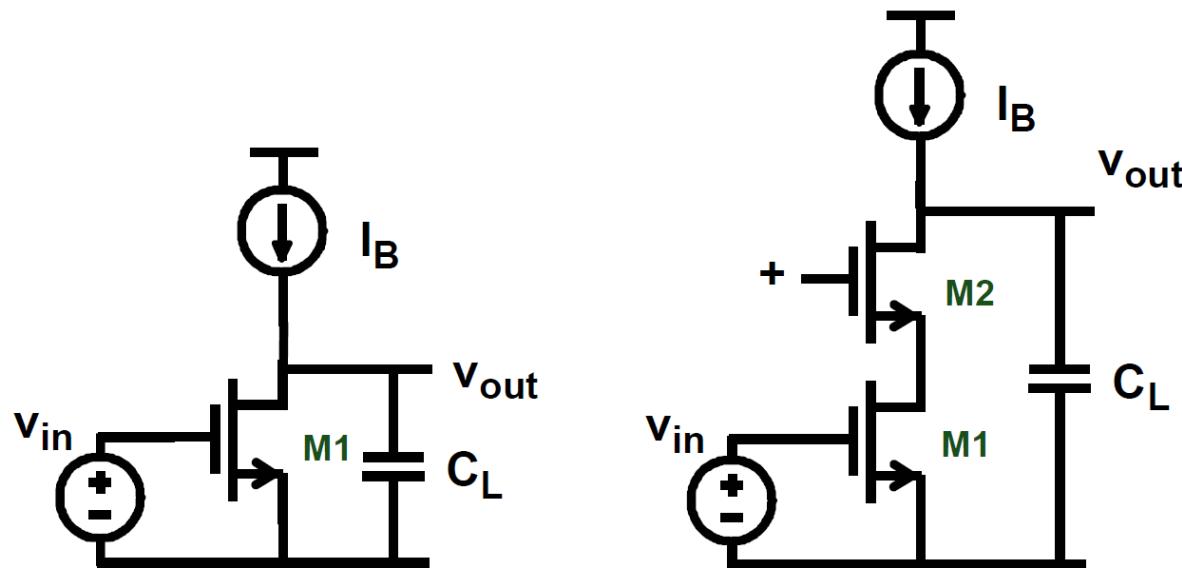


**BW =**

**GBW =**

# »» Cascode vs 单晶体管

---

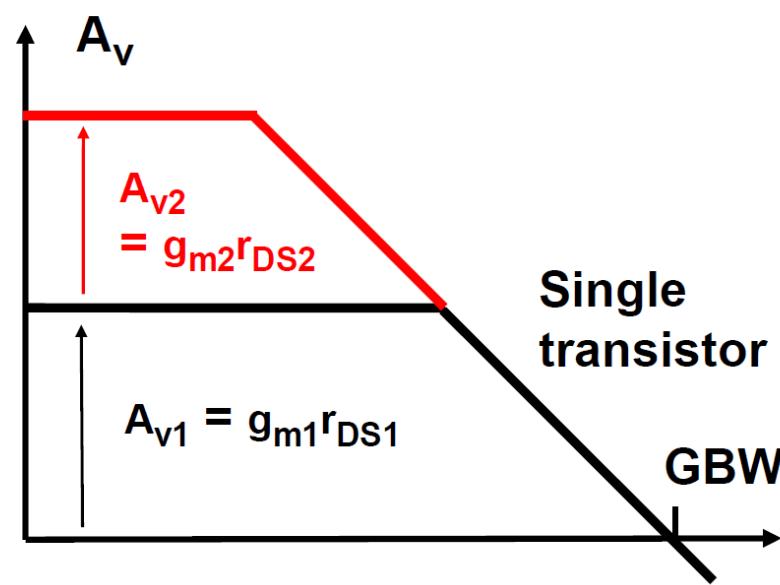
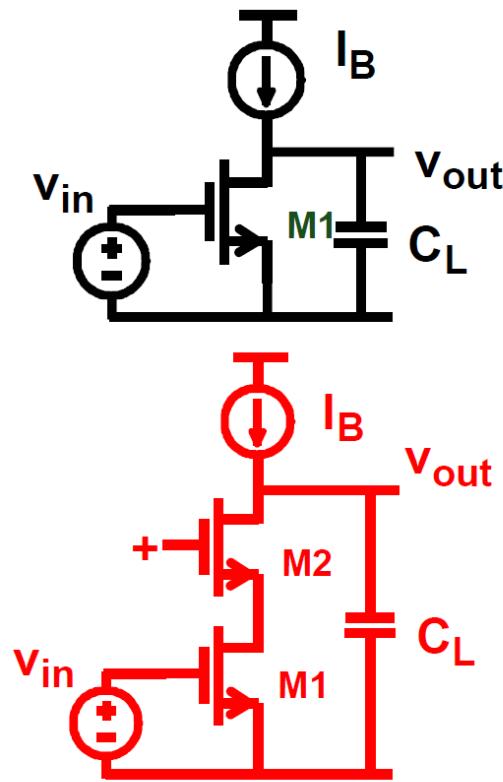


$$BW = \frac{1}{2\pi R_{out} C_L}$$

$$GBW = \frac{g_{m1}}{2\pi C_L} \quad \text{for both !}$$

- 增益带宽积不变！

# »» Cascode vs 单晶体管

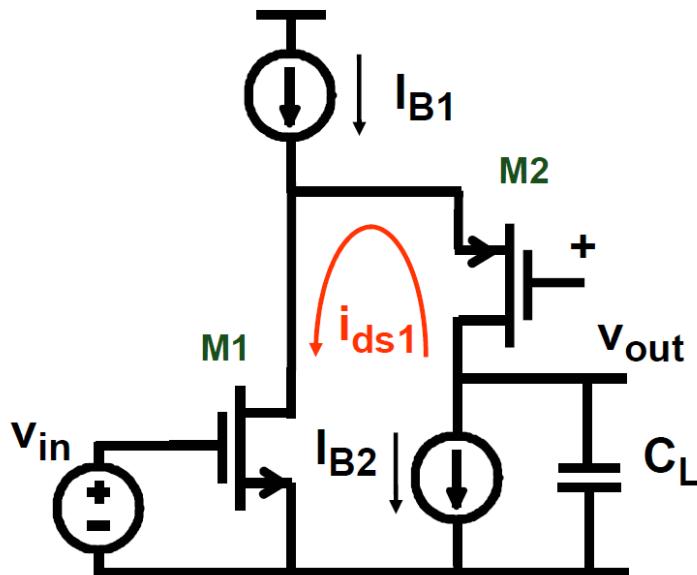


在低频区域提供更高的增益，没有额外的电流消耗

$$GBW = \frac{g_m}{2\pi C_L}$$

## 》》 折叠式Cascode

---



$$I_{DS1} = I_{B1} - I_{B2} \approx I_{B1} / 2$$

$$A_v = g_{m1} R_{out}$$

$$R_{out} = r_{DS1} g_{m2} r_{DS2}$$

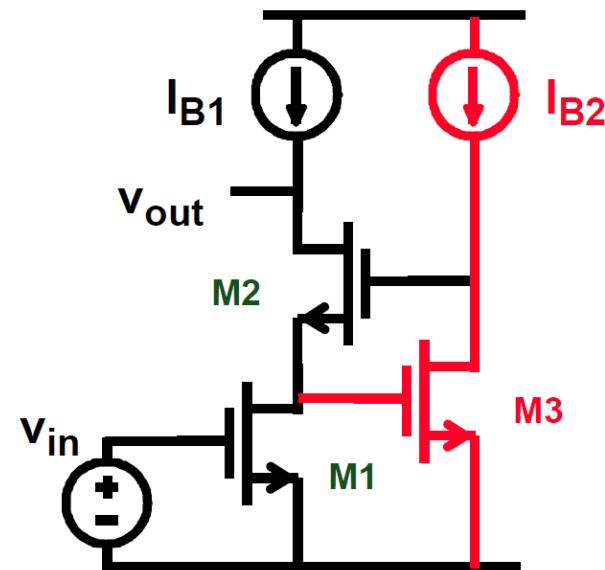
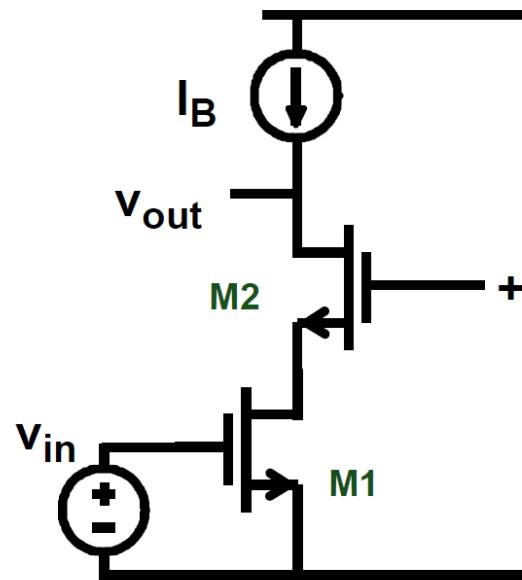
$$BW = \frac{1}{2\pi R_{out} C_L}$$

$$GBW = \frac{g_{m1}}{2\pi C_L}$$

- 主要参数指标与套筒式一致，功耗是其两倍！

# »» 调节式 (Regulated) Cascode

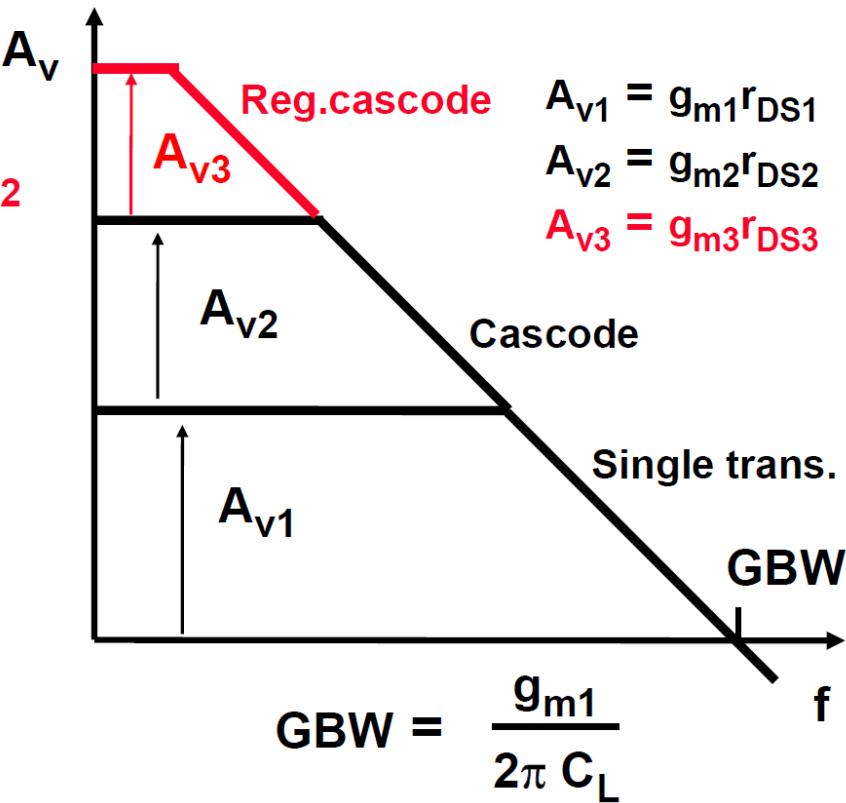
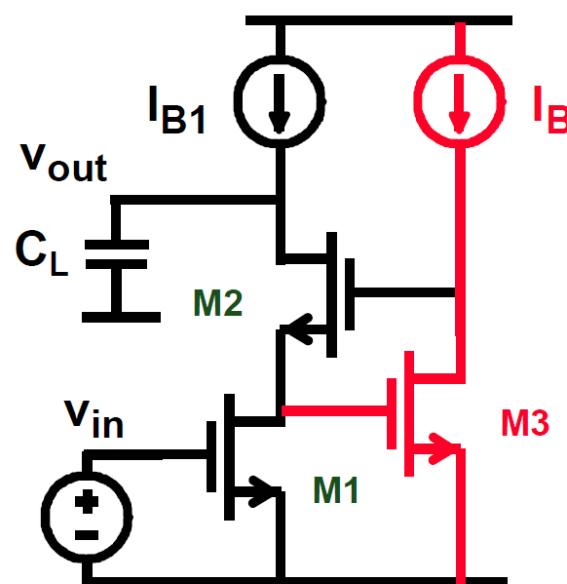
---



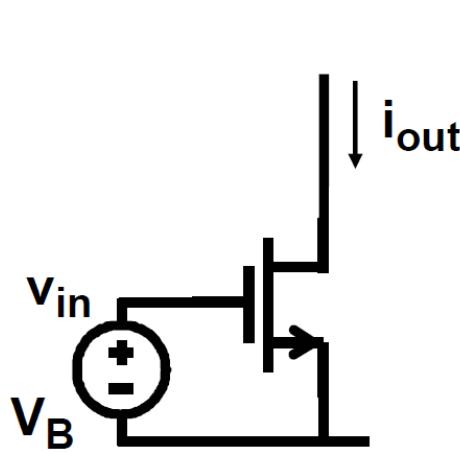
$$A_v = (g_m r_{DS})_1 (g_m r_{DS})_2$$

$$A_v = (g_m r_{DS})_1 (g_m r_{DS})_2 (g_m r_{DS})_3$$

# 》》 调节式 (Regulated) Cascode

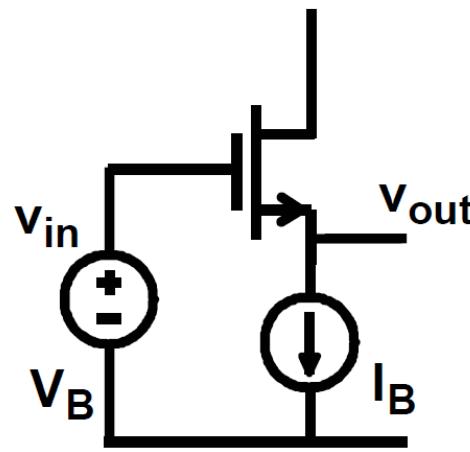


## 》》 单晶体管结构对比



$$i_{out} = g_m v_{in}$$

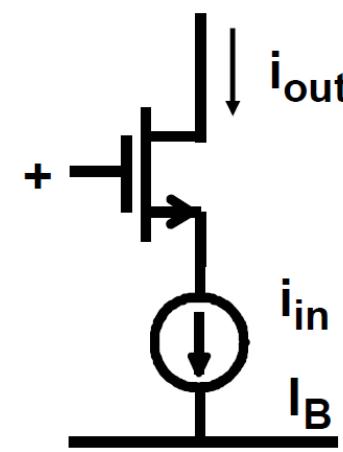
Amplifier



$$v_{out} = v_{in}$$

$$Z_{out} \approx 1/g_m$$

Source follower



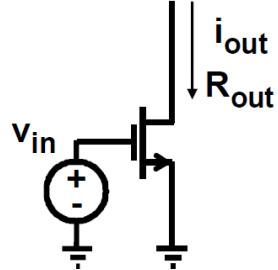
$$i_{out} = i_{in}$$

$$Z_{in} \approx 1/g_m$$

Cascode

# 》》低频特性对比

- 放大器

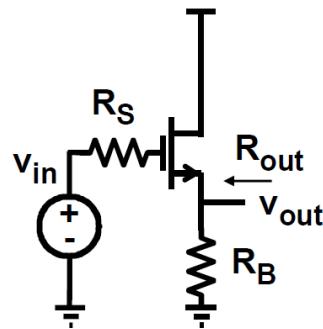


$$A_G \quad g_m$$

$$R_{in} \quad \infty$$

$$R_{out} \quad r_o$$

- 源极跟随器



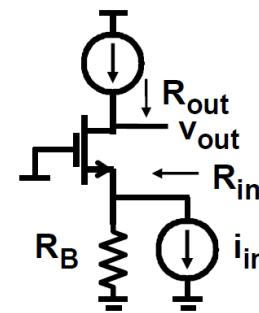
$$R_B > 1/g_m$$

$$A_V \quad 1$$

$$\infty$$

$$1/g_m$$

- Cascode



$$R_B > 1/g_m$$

$$A_R \quad g_m r_o R_B$$

$$R_B$$

$$g_m r_o R_B$$

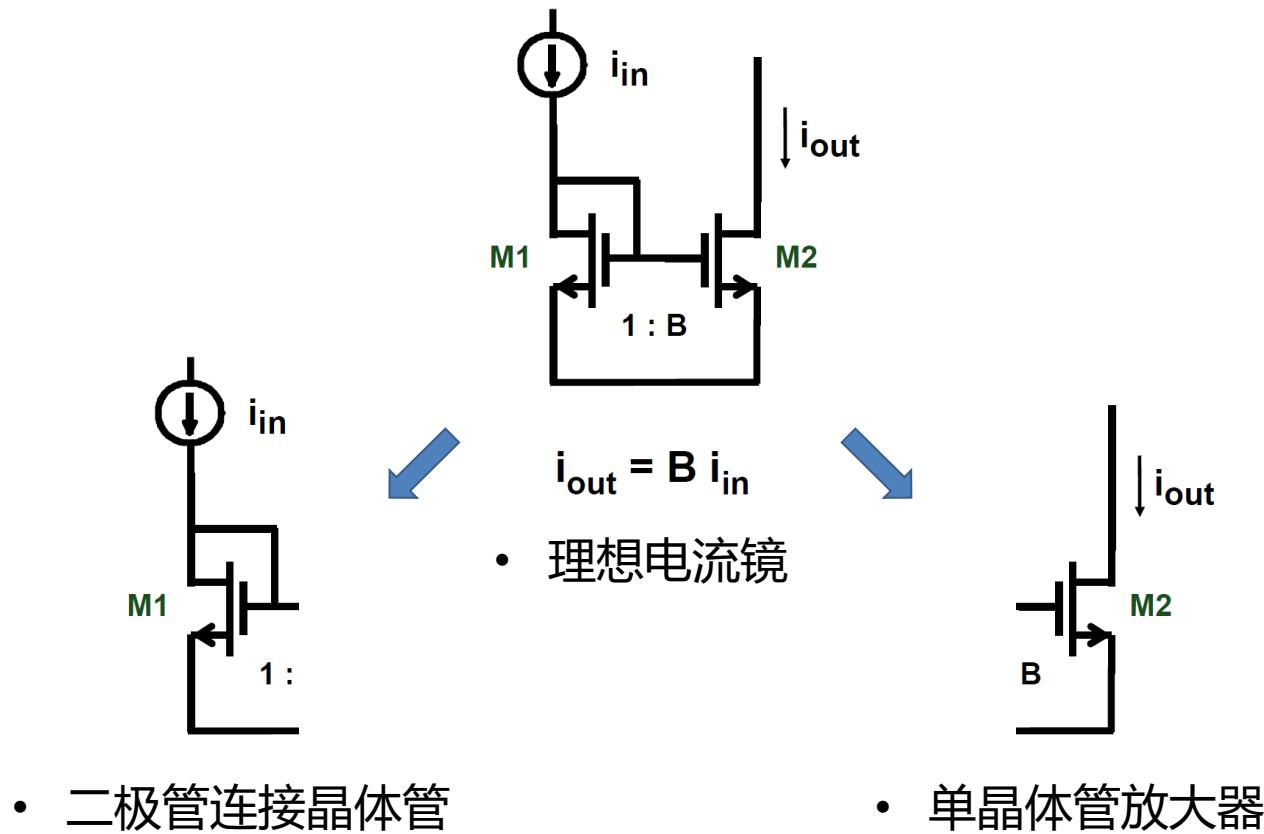


# 模拟电路的基本结构

---

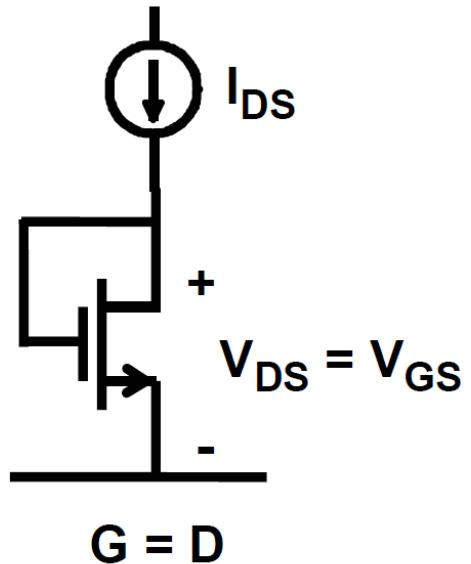
1. 单晶体管放大器
2. 源极跟随器
3. Cascode (共源共栅极)
- 4. 电流镜**
5. 差分对

# 》》 电流镜 (Current Mirror)

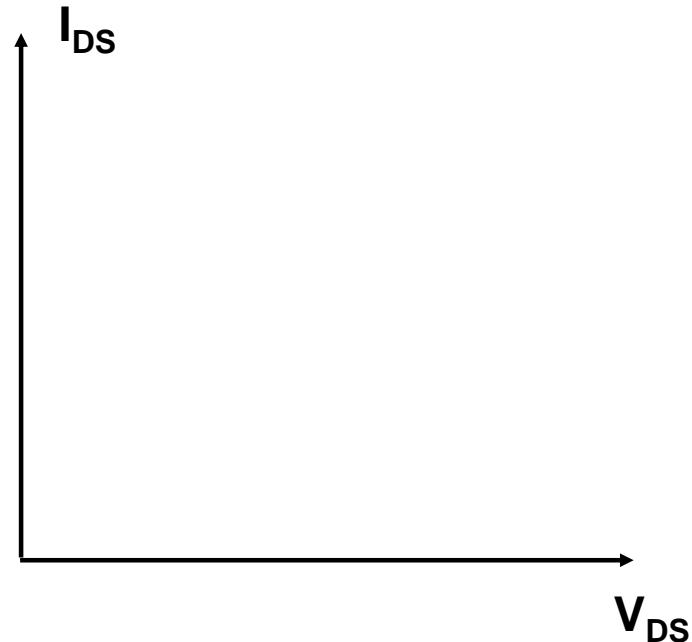


## 》》 二极管连接晶体管 (Diode-connected)

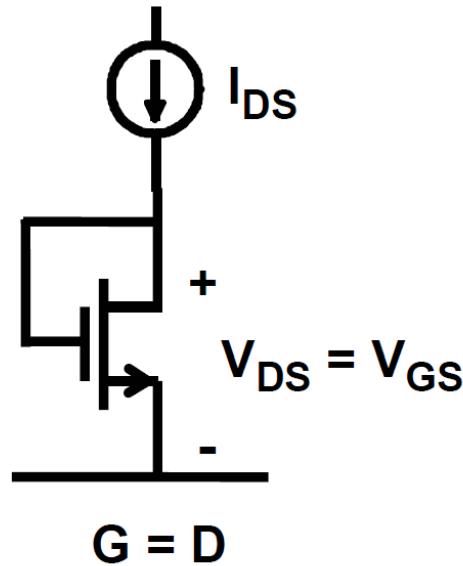
---



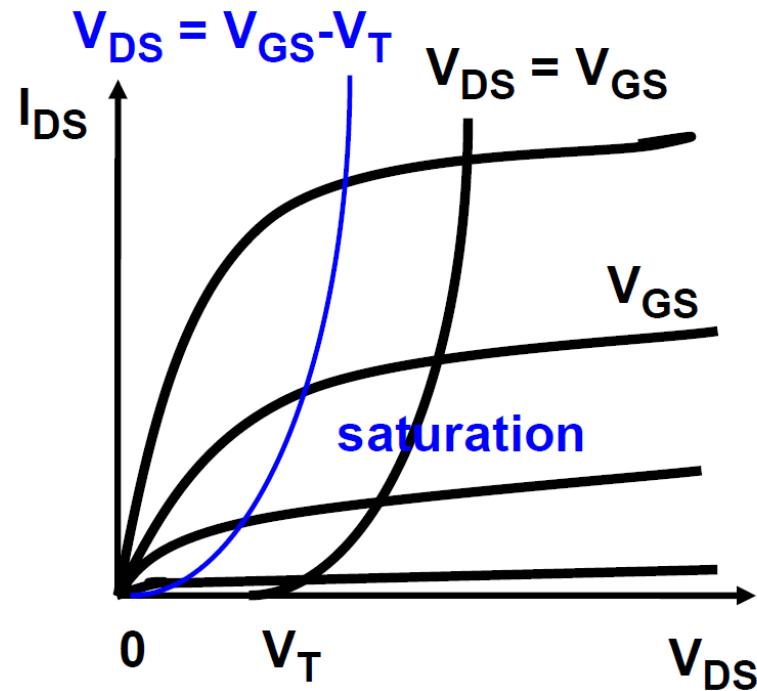
$$I_{DS} = K_n \frac{W}{L} (V_{DS} - V_T)^2$$



## 二极管连接晶体管 (Diode-connected)



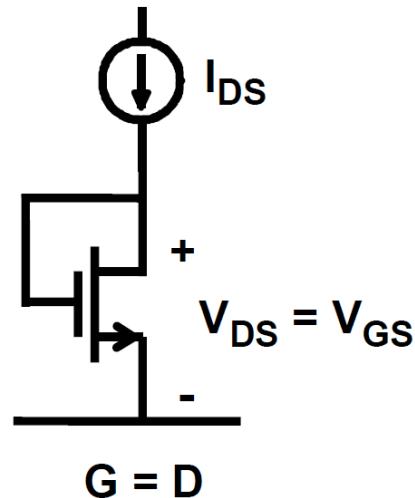
$$I_{DS} = K_n \frac{W}{L} (V_{DS} - V_T)^2$$



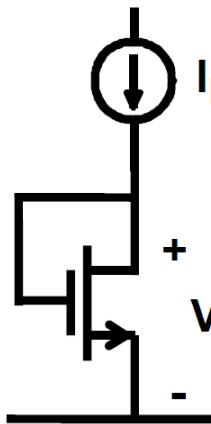
## 》》二极管连接的小信号模型

---

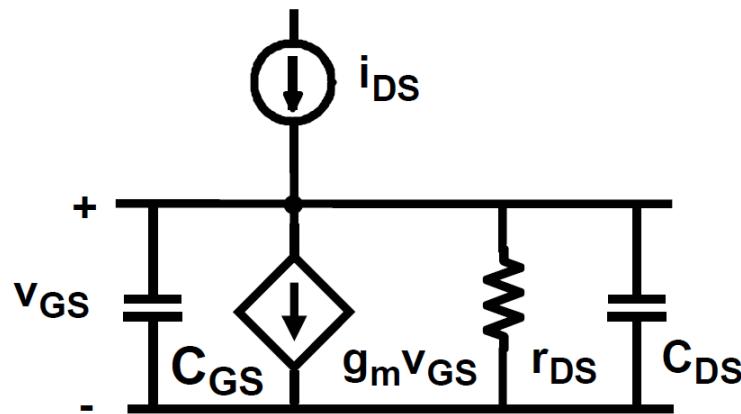
口求二极管连接方式的输入电阻和带宽?



## 二极管连接的小信号模型



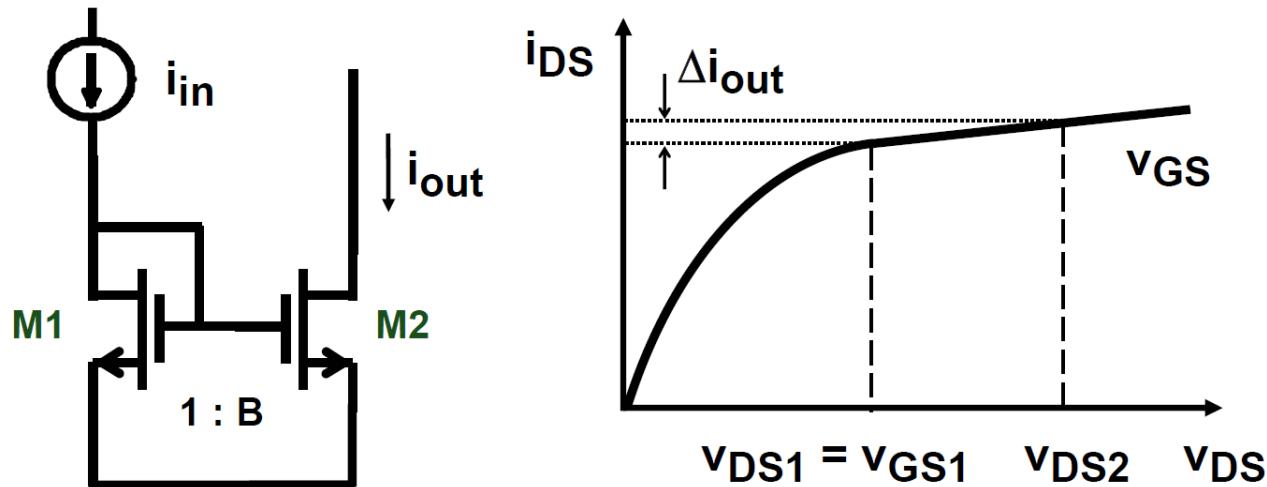
$$G = D$$



输入电阻  $r_{ds} = 1/g_m \parallel r_{DS} \approx \boxed{1/g_m}$

带宽  $BW = \frac{g_m}{2\pi(C_{GS}+C_{DS})} \approx \frac{f_T}{2}$

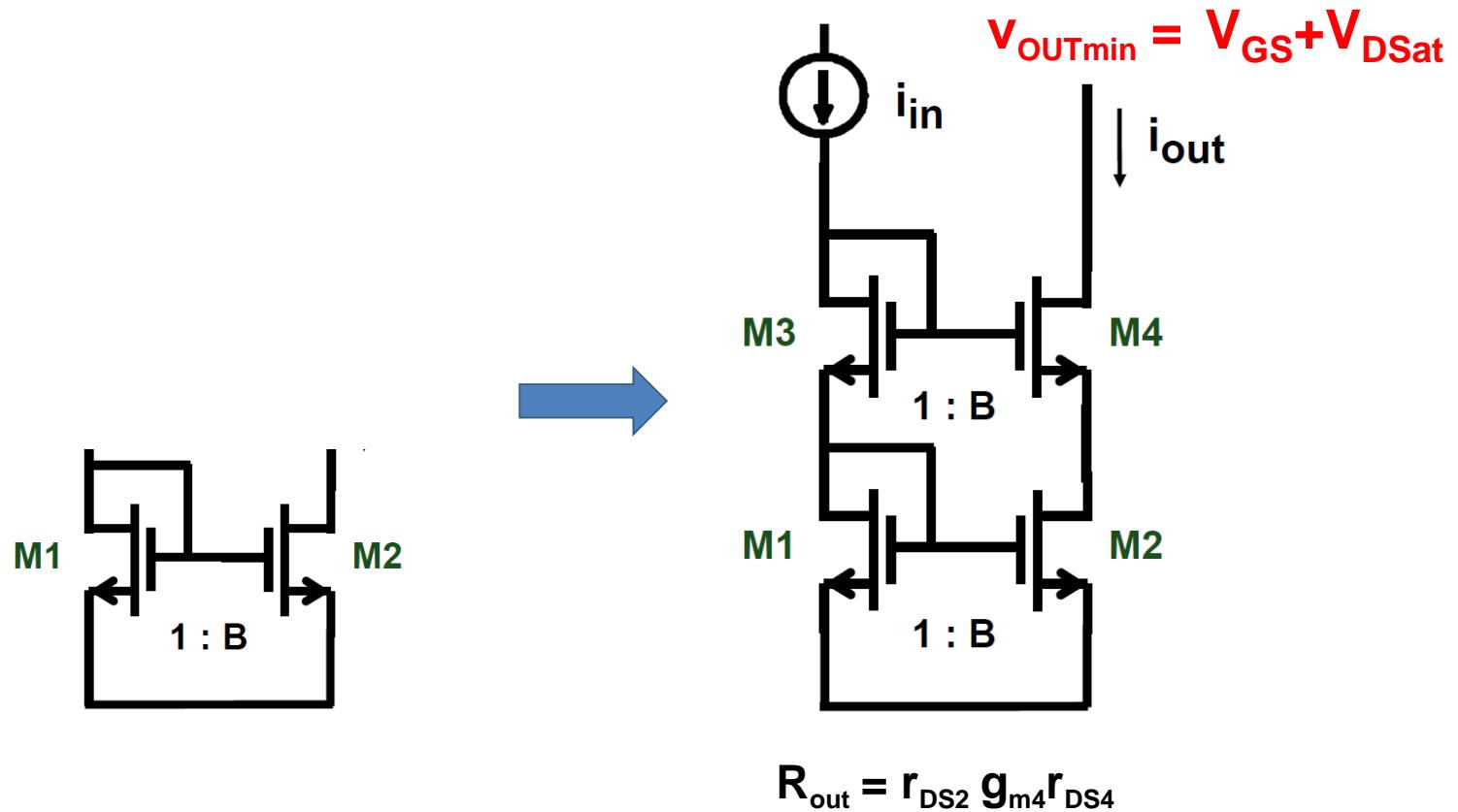
## 》》 电流镜的输出特性



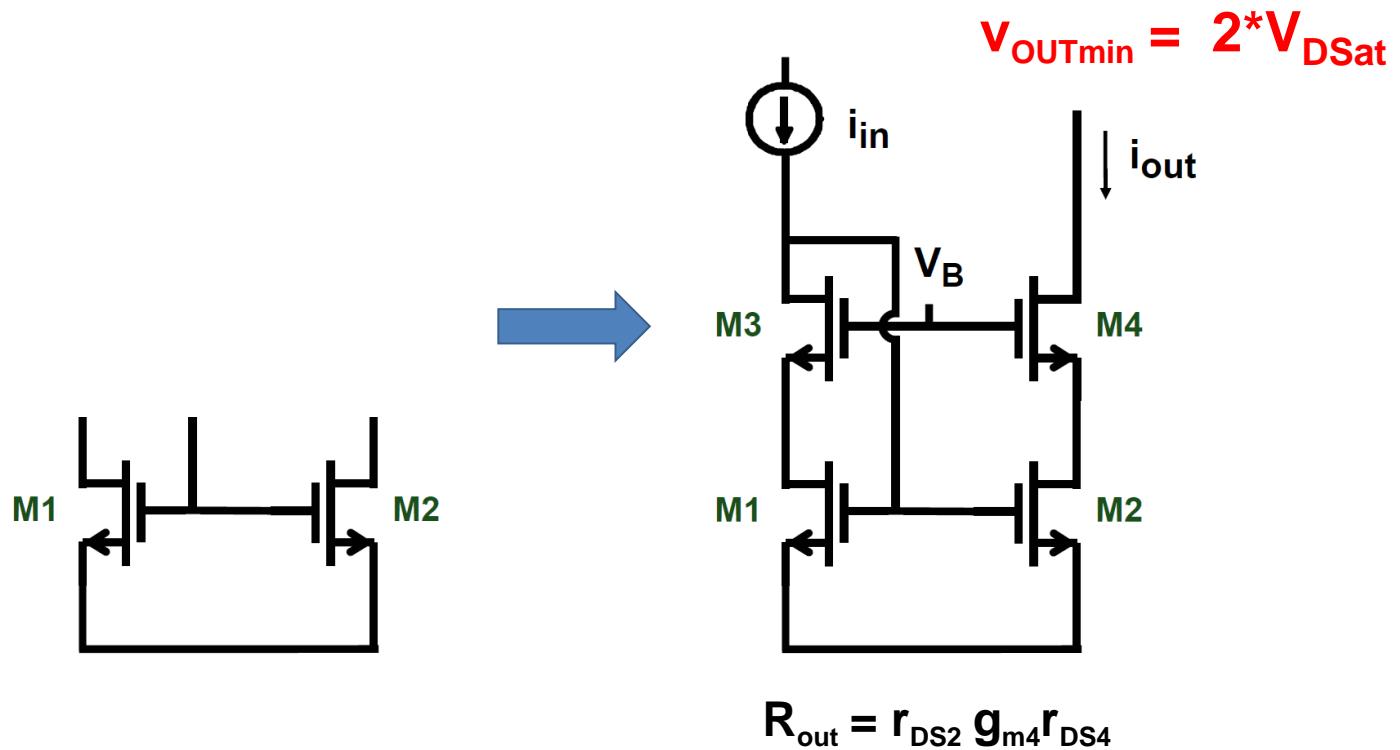
- 电流镜的输出阻抗即是单晶体管放大器的输出阻抗
- 电流镜的输出精度可以描述为

$$\frac{\Delta i_{out}}{i_{out}} =$$

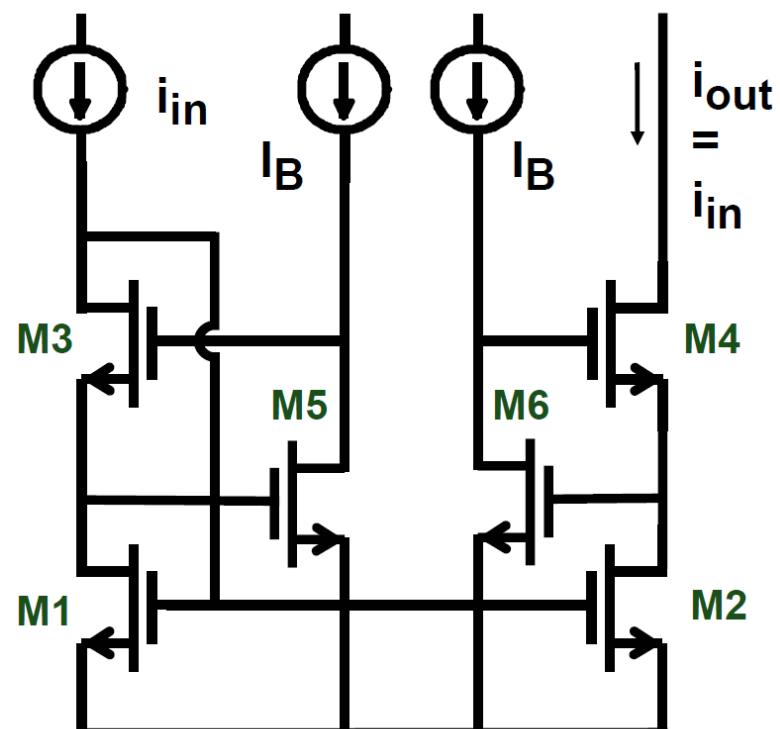
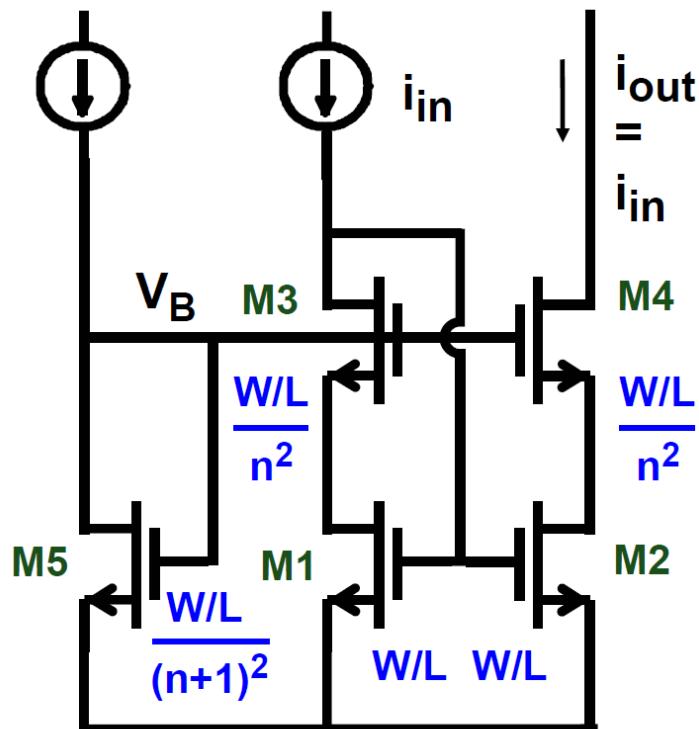
## 》》改进型的电流镜



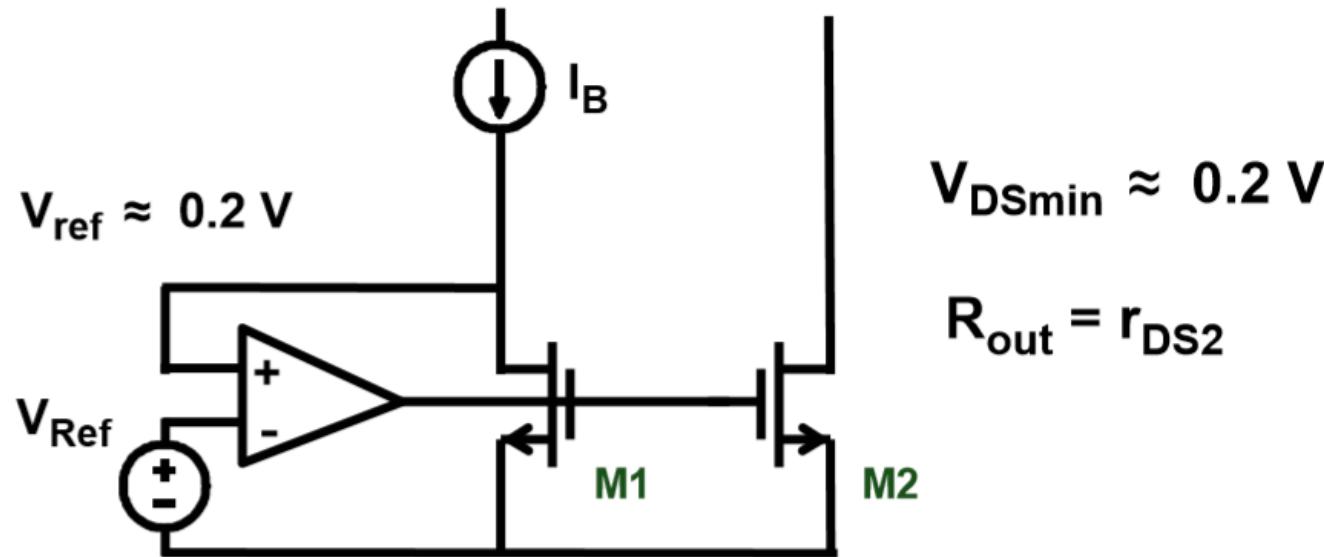
## »» 改进型的电流镜



## 》》 电流镜案例



## » 低电压电流镜

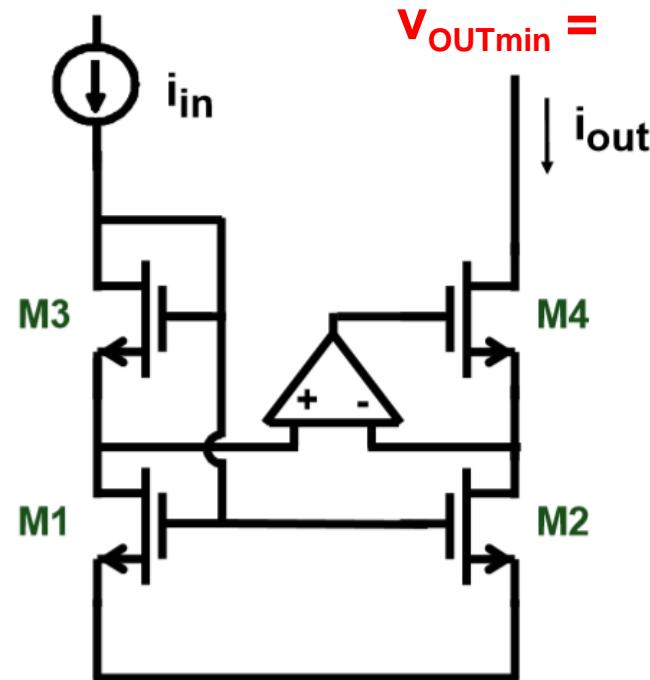
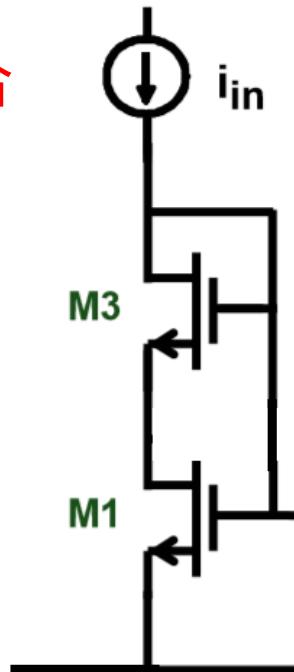


- 该电路**不能**改善输出电阻
- 该电路**可以**改善系统性失配问题

# »» 超低电压电流镜

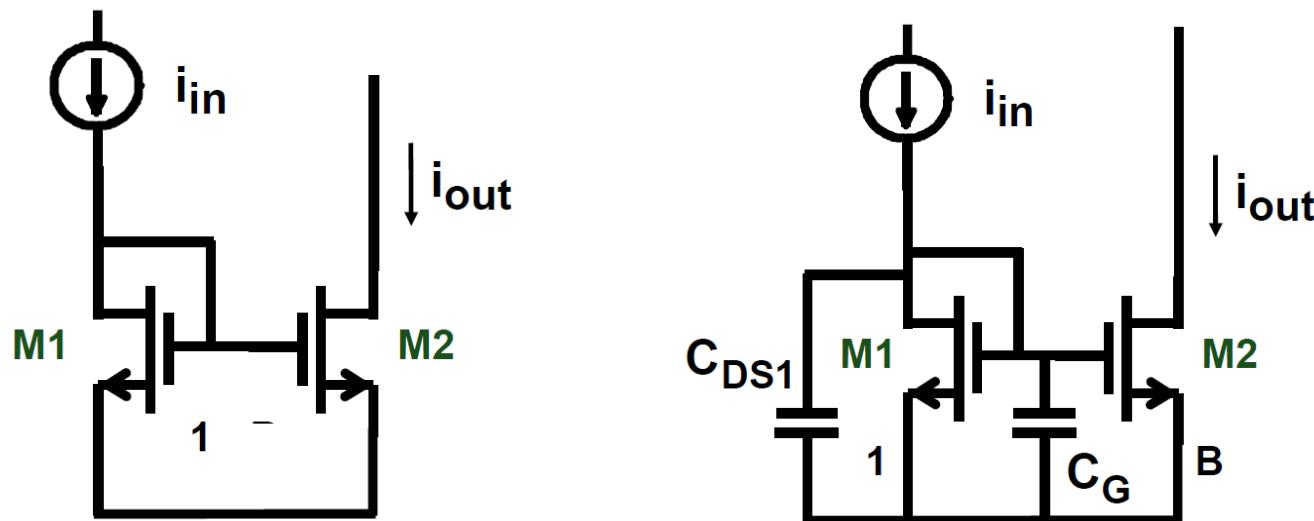
□此结构是否是合适的Cascode?

● 不是!



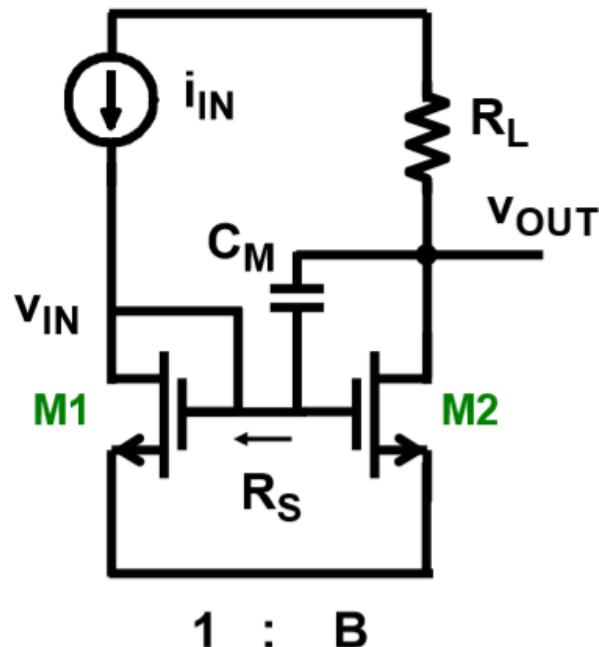
- 电流镜晶体管工作在线性区，通过运放锁定 $V_{ds}$ ，使得两个晶体管工作电压完全一致！

## 》》 电流镜的高频特性



$$BW = \frac{g_m}{2\pi (C_G + C_{DS1})} \approx f_T \frac{1}{(2 + B)}$$

## 》》 电流镜中的密勒效应



Miller effect :

$$f_{-3dB} = \frac{1}{2\pi R_S A_{v2} C_M}$$

$$R_S = 1/g_{m1} \quad A_{v2} = g_{m2} R_L$$

$$f_{-3dB} = \frac{1}{2\pi (1+B) C_M R_L}$$



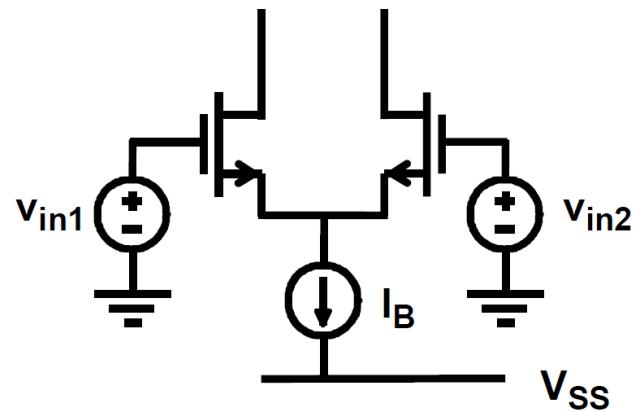
# 模拟电路的基本结构

---

1. 单晶体管放大器
2. 源极跟随器
3. Cascode (共源共栅极)
4. 电流镜
5. 差分对

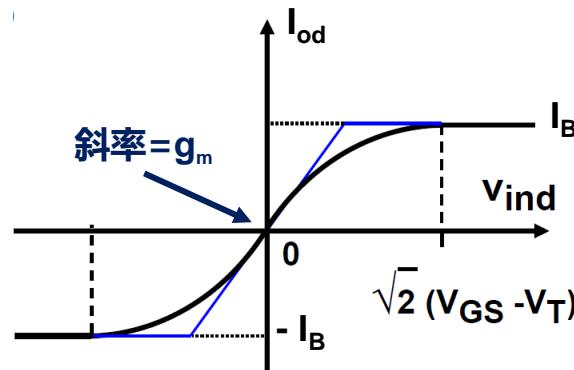
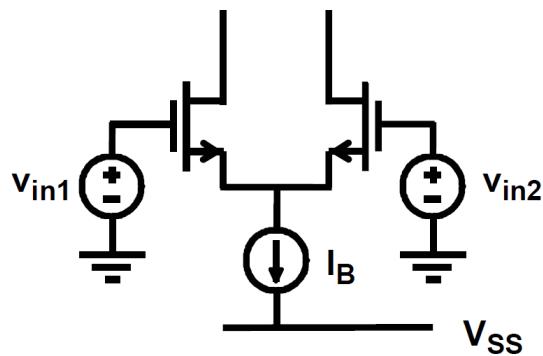
## 》》 差分对 (Differential Pair)

---



- 差分对的本质依旧是**电压输入电流输出**
- 差分对中的信号均以**差分**形式体现
- 差分对的能效是单晶体放大器的**一半**

# 差分对 (Differential Pair)



$$\frac{i_{Od}}{I_B} = \frac{v_{Id}}{(V_{GS}-V_T)} \sqrt{1 + \frac{1}{4} \left( \frac{v_{Id}}{V_{GS}-V_T} \right)^2}$$

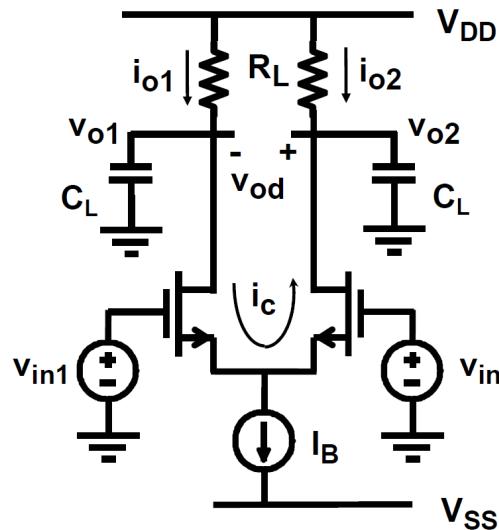
- $V_{Id}$  为差分输入电压
- $I_{Od}$  为差分输出电流
- $I_B$  为差分对总偏置电流

•  $V_{GS}-V_T$  决定  $g_m$  和 直流 范围

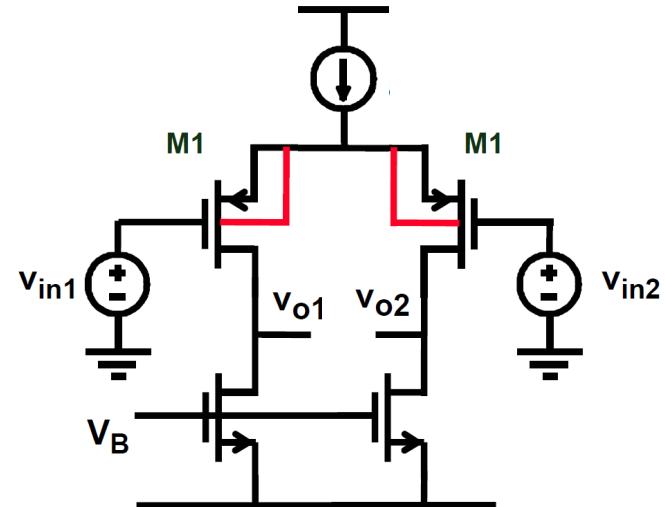
# 》》》 差分放大器

---

- 差分放大器 = 差分对 + 负载



$$A_v = g_m R_L$$

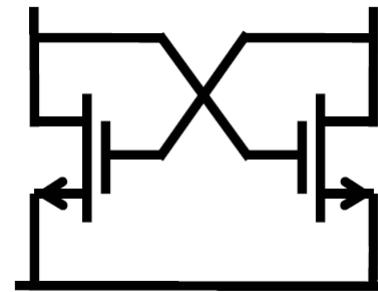
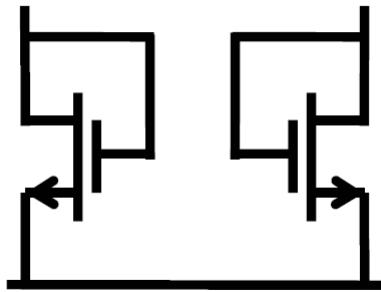


$$A_v = g_{m1} (r_{DS1} // r_{DS2})$$

## »» 特殊形态的负载-交叉耦合对

---

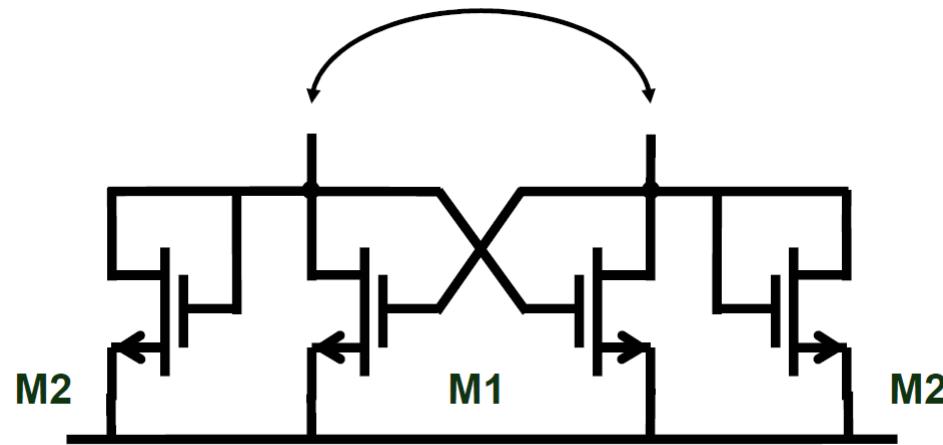
- 交叉耦合对 (Cross Coupled Pair)



## » 特殊形态的负载-交叉耦合对

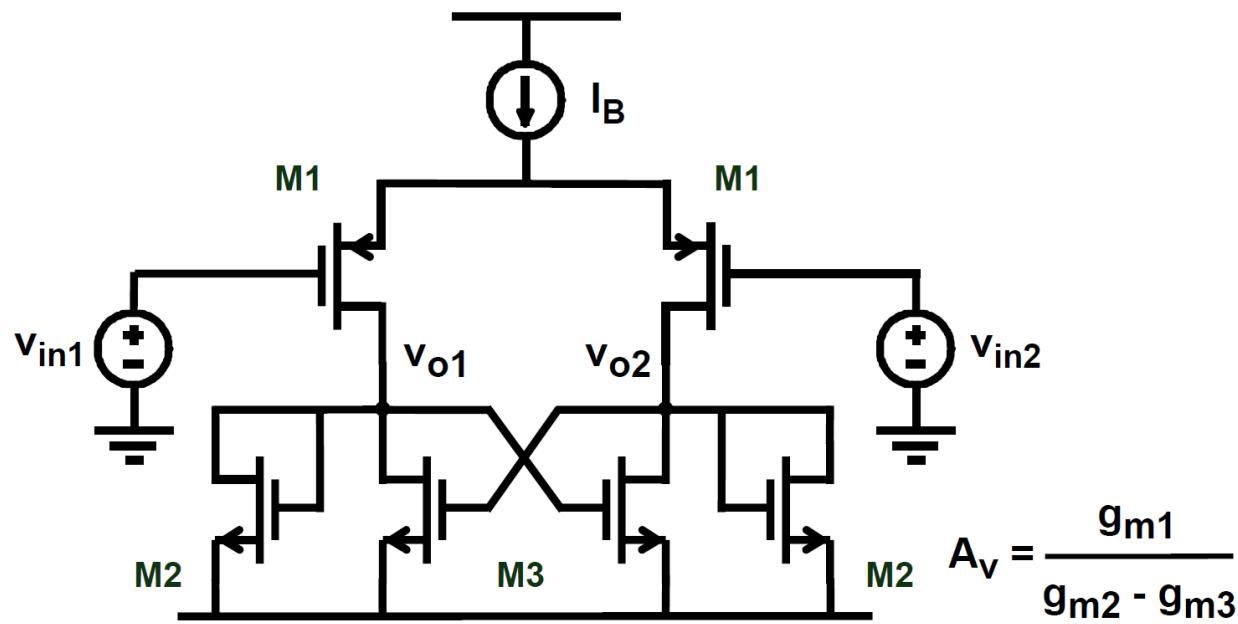
---

差分输入电导:  $g_{m2} - g_{m1}$



实际使用中受到匹配(matching)的限制

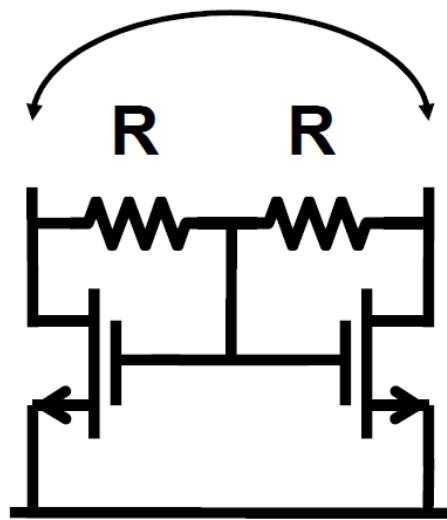
## » 特殊形态的负载-交叉耦合对



实际使用中受到**匹配(matching)**的限制

## »» 特殊形态的负载-共模反馈

---

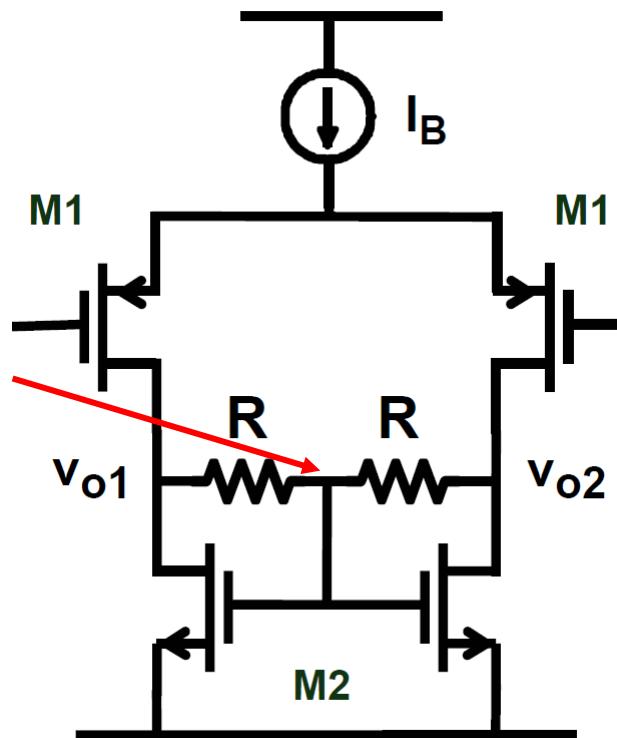


## »» 特殊形态的负载

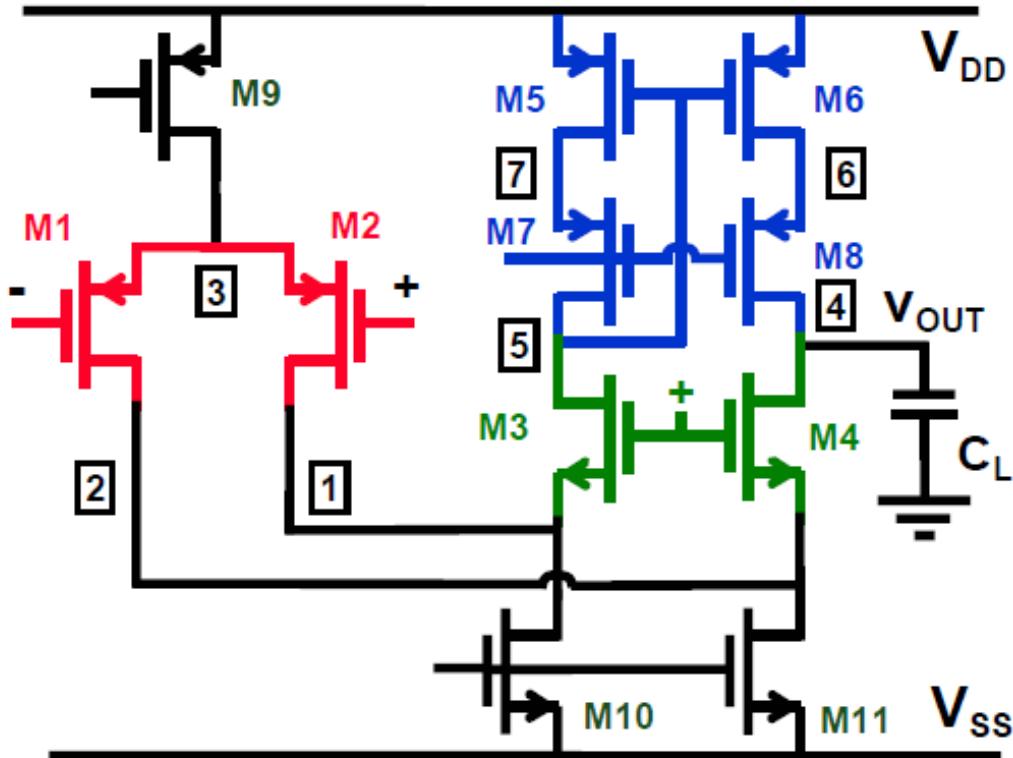
- 本质上是一种**共模反馈**形式

$$A_v = g_{m1} (R // r_o)$$

$$r_o = r_{o1} // r_{o2}$$



## »» Cascode



求该折叠套筒型运放的第一和第二高阻点的阻值，用公式表示。

# 》》》 总结

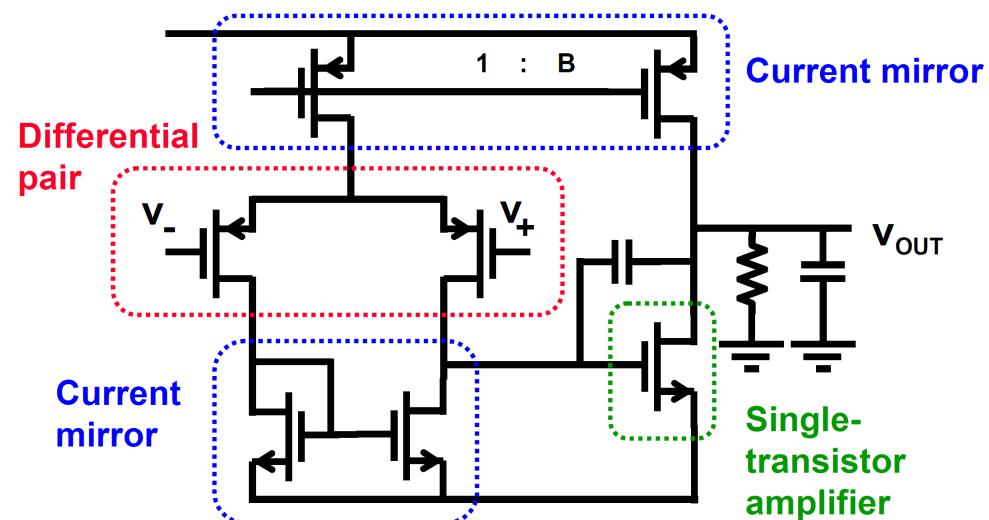
□ 模拟电路的基本构成单元：

## 1. 单晶体管

- 放大器
- 源极跟随器
- Cascode

## 2. 多晶体管

- 差分对
- 电流镜





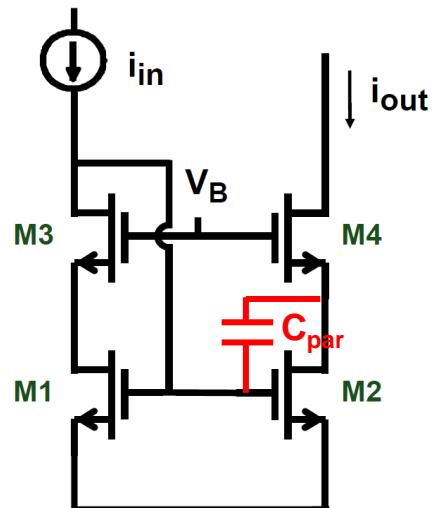
## 作业1

---

- 利用仿真结果，找到我们所使用工艺的 $u_{0p,n}$ ,  $C_{ox}$ ,  $V_{THn}$ ,  $V_{THp}$

## 》》作业2

- 通过公式描述 $V_B$ 的取值范围 (提示：晶体管M1-M4均需要工作在饱和区)
- 在Cascode电流镜中，假设有寄生电容 $C_{par}$ ，利用公式估算并用仿真验证该电流镜的频率特性。 (可以自由设置偏置、晶体管的尺寸以及寄生电容)





## 作业3

---

- 假设差分对偏置电流为200uA,  $W/L=20\mu m/1\mu m$ , 根据理论分析和仿真验证。
  - 计算 $g_m > 99\% * g_{m,max}$ 的区间。
  - 计算差分输出电流为198uA时的差分输入电压。
  - 如果需要把问题2中求得的电压扩大一倍, 差分对的W需要如何修改?

**0.3**



北京航空航天大學  
BEIHANG UNIVERSITY

| 微电子学院

# CMOS模拟集成电路设计

## 第三章：噪声

胡远奇

©2020



## 噪声 Noise

---

- **噪声的特点**
- 系统中的噪声
- 电阻的热噪声
- 晶体管的噪声源
- 等效噪声的换算



## 噪声

---

- 为什么它很重要?
- 信噪比 (Signal-to-Noise Ratio, SNR)
  - 信号功率  $P_{sig} \sim V_{DD}^2$
  - 噪声功率  $P_{noise} \sim kT/C$
  - $SNR = P_{sig} / P_{noise}$
- 工艺演进
  - $V_{DD}$ 下降 -> SNR下降
  - 保持SNR -> 增加C -> 功耗提升
- 优秀的低功耗设计意味着对噪声的充分了解



## 噪声的类型

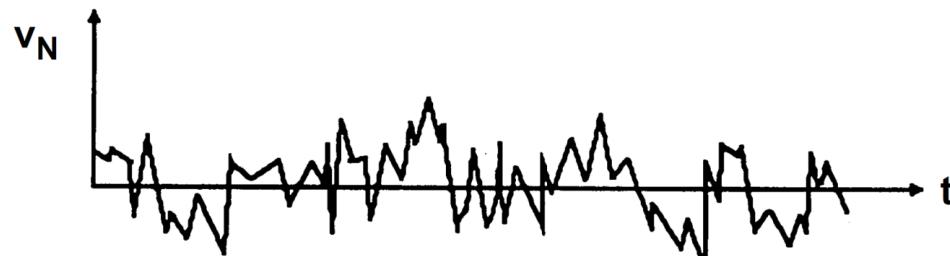
---

- 干扰（人为造成的）
  - 信号耦合
    - 电容
    - 电感
    - 基底
    - 键合线
  - 电源噪声
  - 解决方案：差分电路、版图技巧
- **器件噪声**
  - 由电荷的不连续性造成
  - 根本性的噪声

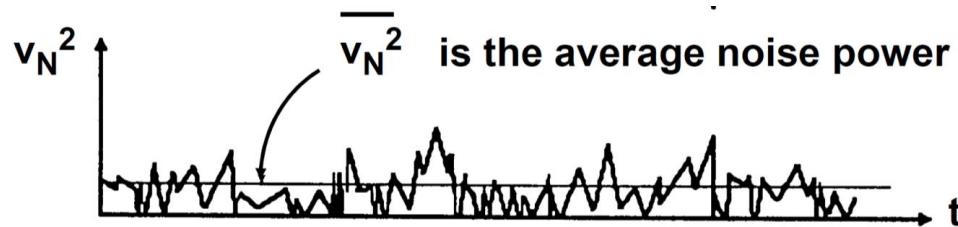
## 》》 噪声

---

- 时域上的表现形式：随机出现，通过**概率**表征



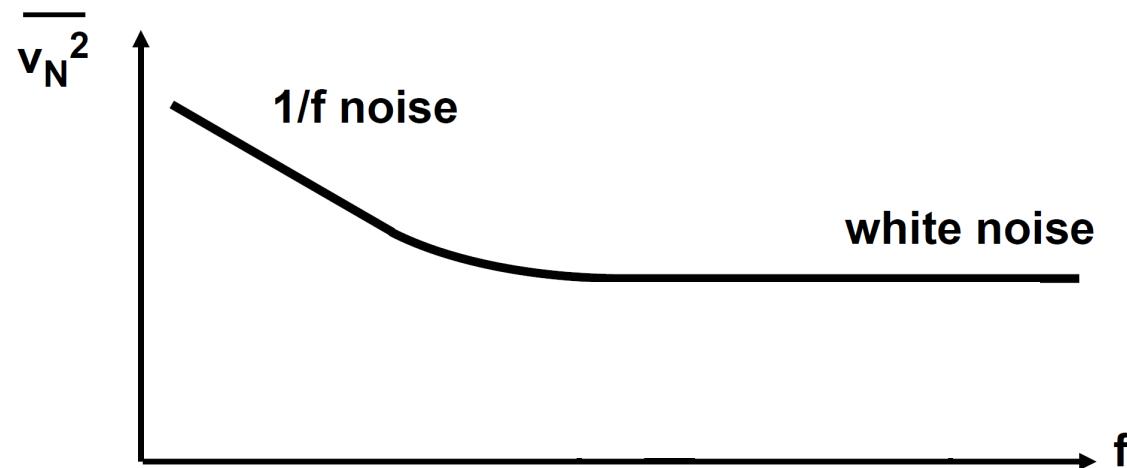
- 噪声的强度通过**平均功率**表达



# 》》 噪声

---

- 频域上的表现形式：

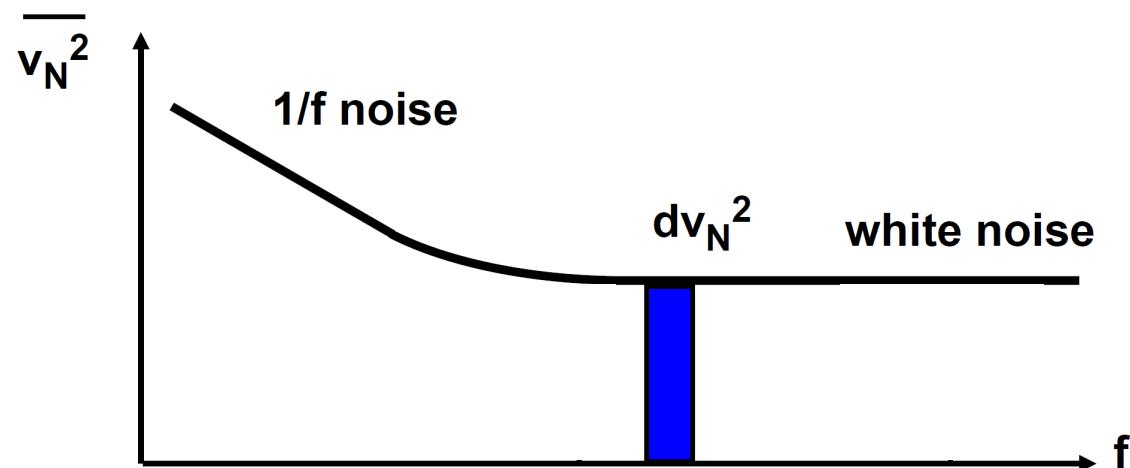


- 低频区域**闪烁噪声**  
**(Flicker Noise)**
- 高频区域**白噪声**

# 》》 噪声

---

- 频域上的表现形式：

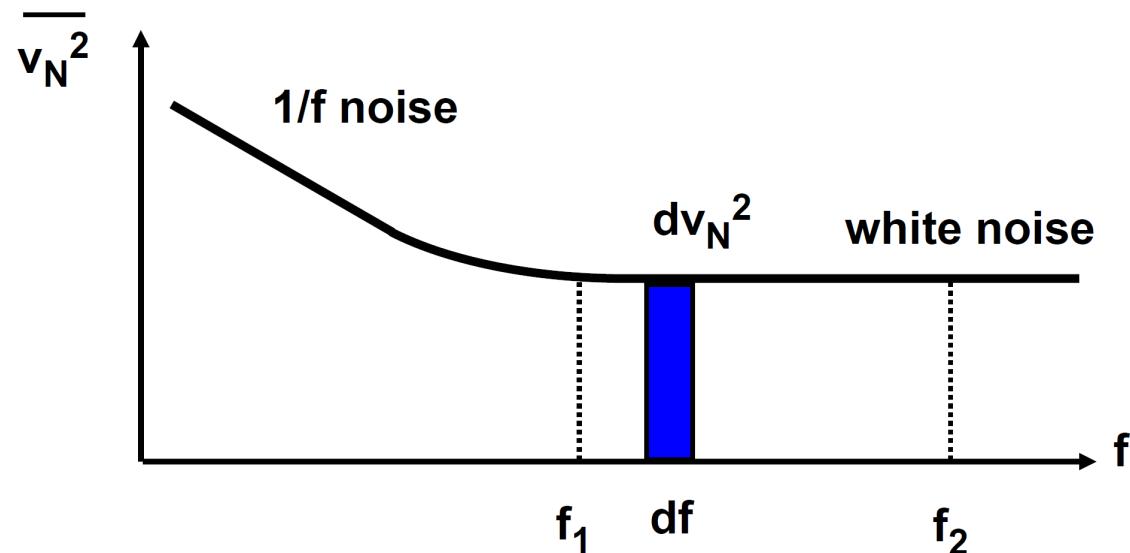


- 噪声的**功率谱密度**:  $V^2/Hz$ 
  - 如何计算总噪声?

# 》》 噪声

---

- 频域上的表现形式：



- 积分噪声： $\overline{v_{N,int}^2} = \int_{f_1}^{f_2} \overline{dv_N^2} df = (f_2 - f_1) \overline{v_N^2}$



## 噪声

---

- 当我们描述一个噪声时，通常的形式是：
  - 特定频率下表述其功率谱密度（均方根）
  - 一定带宽内，表述其积分噪声总量



## 噪声 Noise

---

- 噪声的特点
- **系统中的噪声**
- 电阻的热噪声
- 晶体管的噪声源
- 等效噪声的换算



## 噪声因子(Noise Factor)

- 一个系统噪声性能的衡量指标：

$$\text{噪声因子 } F = \frac{\text{总的噪声输出功率}}{\text{输入噪声源引起的噪声功率}}$$

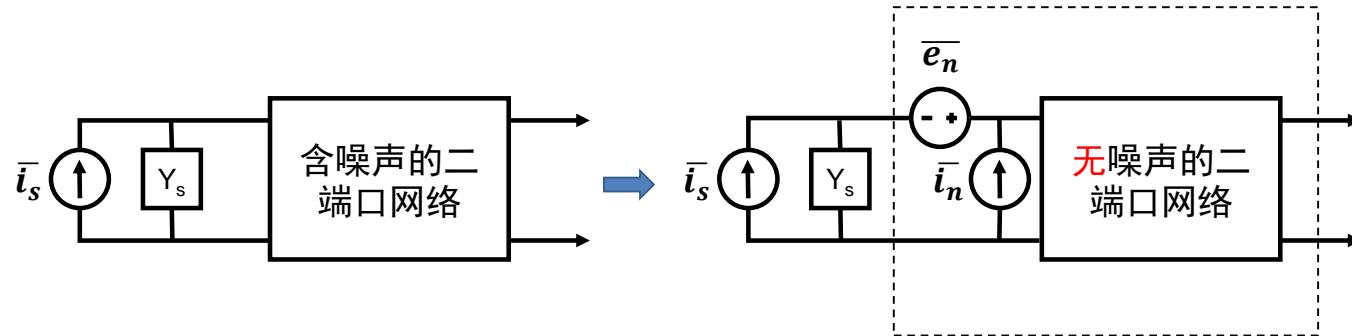


图：噪声源驱动一个含噪声的二端网络

# 》》 噪声因子(Noise Factor)

- 一个系统噪声性能的衡量指标：

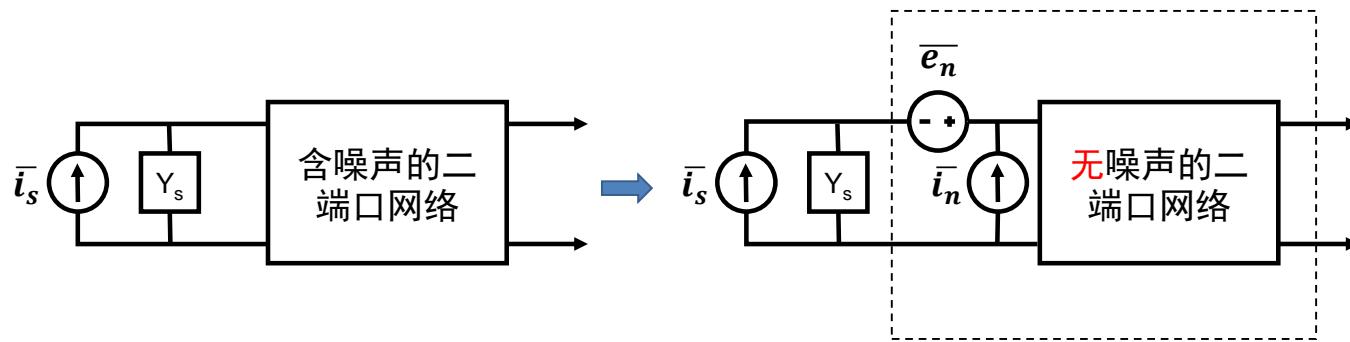
$$\text{噪声因子 } F = \frac{\text{总的噪声输出功率}}{\text{输入噪声源引起的噪声功率}}$$



- 通常我们更关注系统或电路的**输入参考噪声**(Input-Referred Noise)

# 》》 噪声因子(Noise Factor)

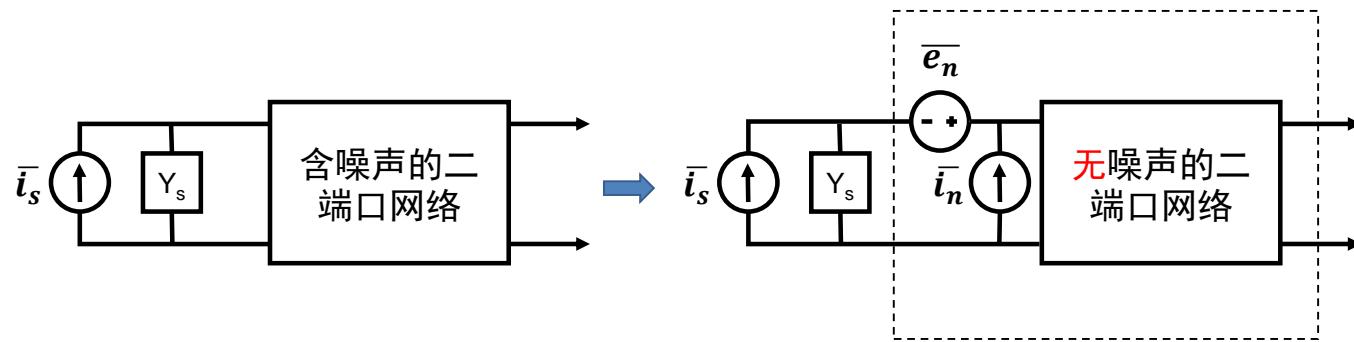
- 一个系统噪声性能的衡量指标：



噪声因子  $F =$

# 》》 噪声因子(Noise Factor)

- 一个系统噪声性能的衡量指标：



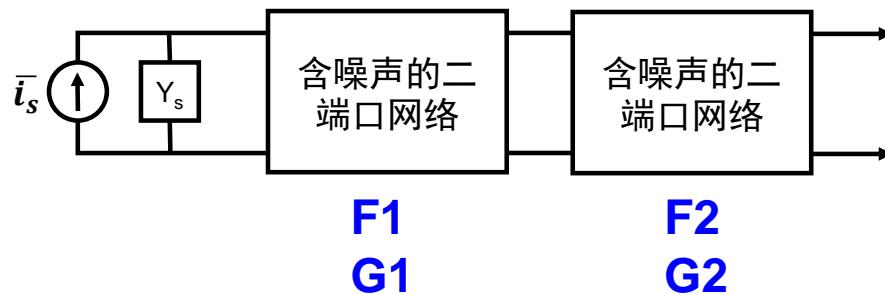
$$\text{噪声因子 } F = \frac{\bar{i}_s^2 + |\bar{i}_n + Y_s \bar{e}_n|^2}{\bar{i}_s^2} = \frac{SNR_{IN}}{SNR_{OUT}}$$

- 注意：输入参考噪声电压/电流  $e_n$  和  $i_n$  可能具有相关性



## 噪声因子(Noise Factor)

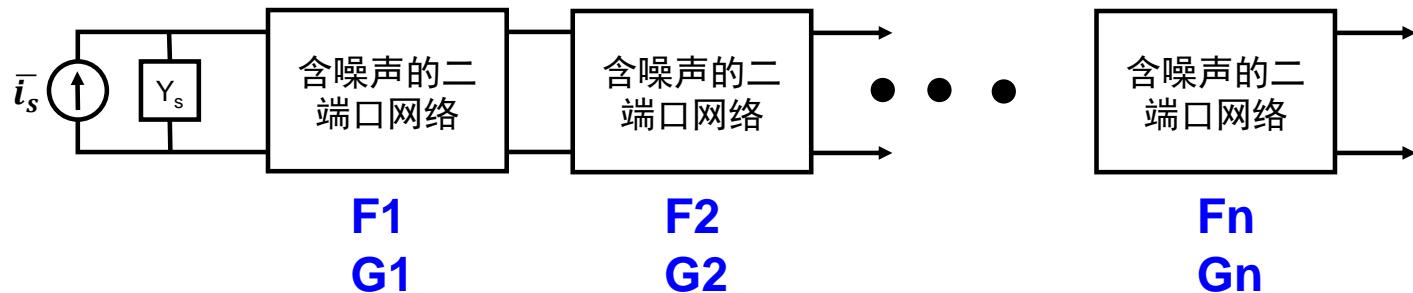
- 一个两级系统的噪声指标：



$$\text{总噪声因子 } F = F_1 + \frac{F_2 - 1}{G_1}$$

# 》》 噪声因子(Noise Factor)

□求一个n级系统的噪声指标?



$$\text{总噪声因子 } F = F_1 + \frac{F_2 - 1}{G_1} + \cdots + \frac{F_n - 1}{G_1 \cdot G_2 \cdot \cdots \cdot G_n}$$

**噪声因子最低的系统在第一级!**



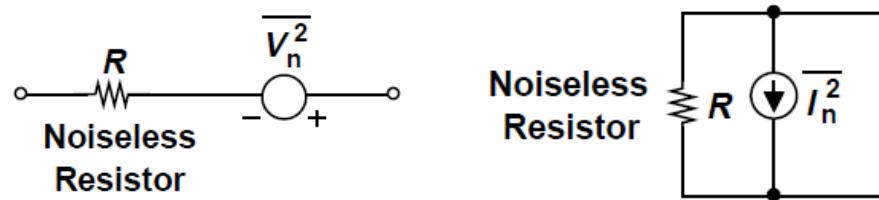
## 噪声 Noise

---

- 噪声的特点
- 系统中的噪声
- **电阻的热噪声**
- 晶体管的噪声源
- 等效噪声的换算

## 》》 电阻的热噪声

- 电阻的热噪声既可以用串联电压源表示，也可以用并联电流源来表示：

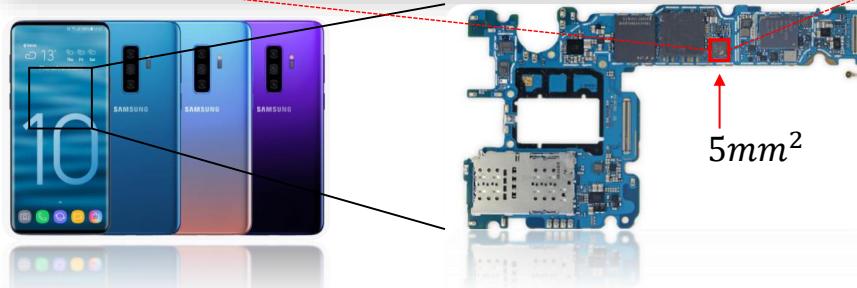
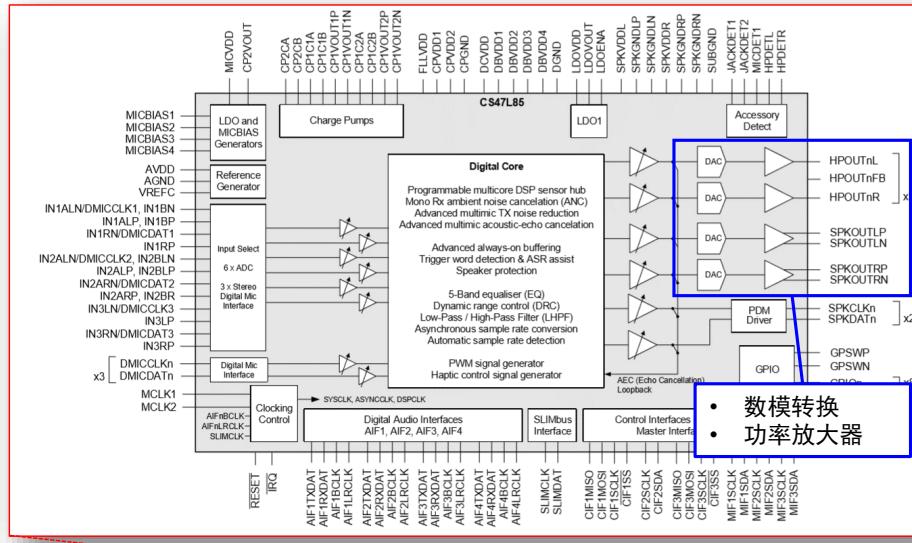


**功率谱:**  $\overline{dV_n^2} = 4kTR$        $\overline{dI_n^2} = 4kT/R$

已知  $R=1\text{k}\Omega$ ,  $T=300\text{K}$ ,  $\sqrt{\overline{dV_n^2}} = 4\text{nV}_{\text{RMS}}/\sqrt{\text{Hz}}$

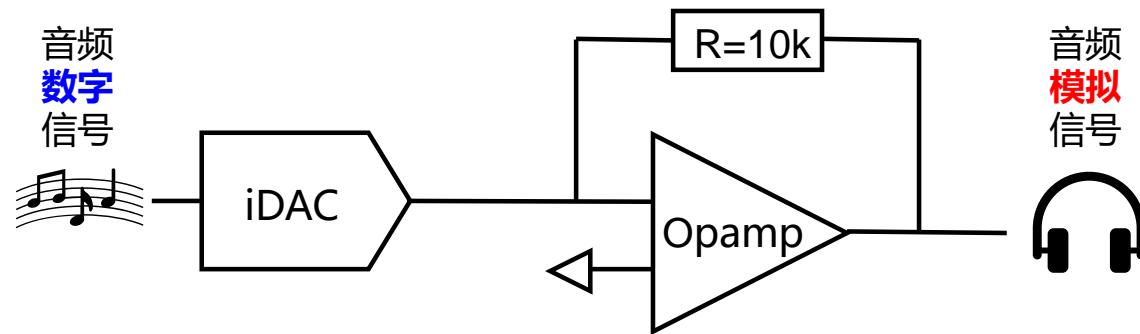


## 电阻的热噪声

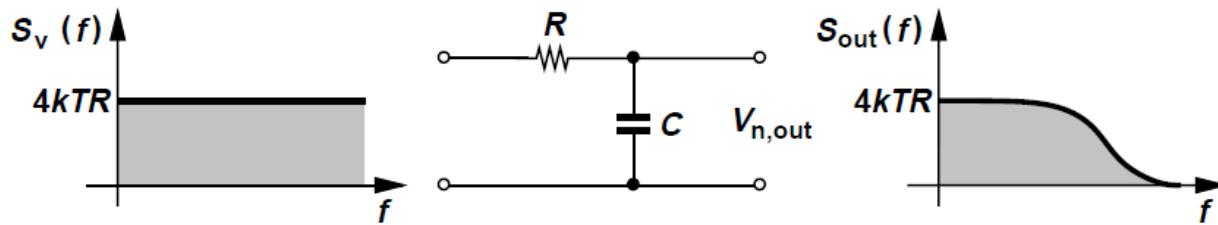


## 》》 电阻的热噪声

有一音频功放系统，它的基本模块如下图所示，求该系统输出的理论最小噪声？ (20-20KHz)



## 》》 电阻的积分噪声



$$\overline{d\nu_n^2} = 4kTR$$

$$H(s) = \frac{1}{1 + s \cdot RC}$$

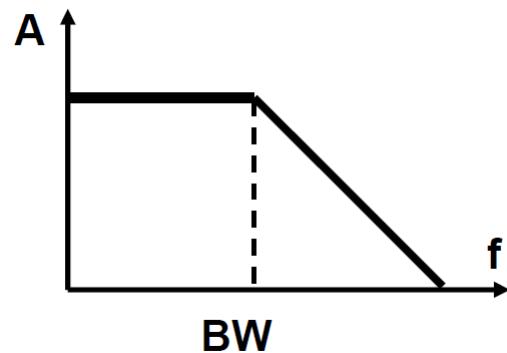


$$\overline{\nu_n^2} = \int_0^{\infty} 4kTR \cdot H^2(s) ds$$



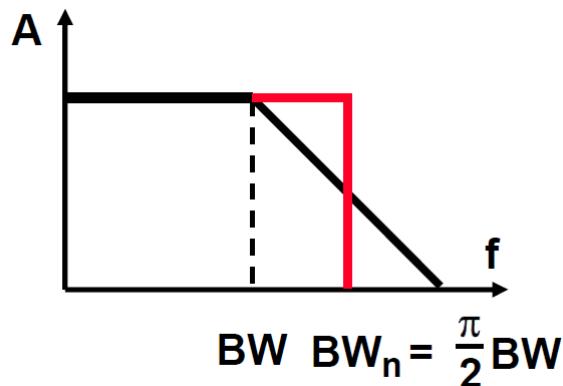
## 电阻的积分噪声

---



$$\int_0^{\infty} \frac{1}{1 + (\omega RC)^2} d\omega$$

## 》》 电阻的积分噪声



$$\int_0^{\infty} \frac{dx}{1+x^2} = \frac{\pi}{2}$$

$$BW_{nL} = \frac{\pi}{2} BW$$

$$\overline{v_n^2} = 4kTR \cdot \frac{1}{2\pi RC} \cdot \frac{\pi}{2} = \frac{kT}{C}$$

**C<sub>L</sub> = 1pF时**  
**v<sub>s</sub> = 65uV<sub>RMS</sub>**

- 所有**开关采样**电路的噪声极限由该公式表达



## 噪声 Noise

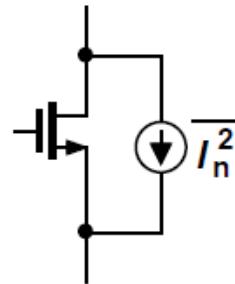
---

- 噪声的特点
- 系统中的噪声
- 电阻的热噪声
- **晶体管的噪声源**
- 等效噪声的换算

## »» 晶体管的噪声

---

- 晶体管的**沟道电阻**和其他电阻一样，也会产生热噪声

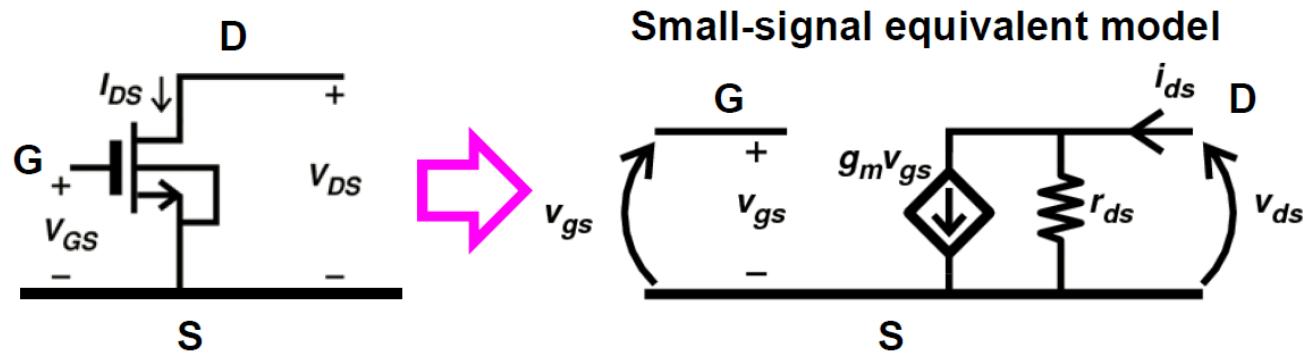


$$\overline{di_n^2} = 4kT/R = 4kT\gamma g_m$$

$$\gamma \approx \frac{2}{3} \text{ for } > 0.35\mu\text{m}; \approx 1 \text{ for } 0.18\mu\text{m}; \approx \frac{4}{3} \text{ for } 0.13\mu\text{m}$$

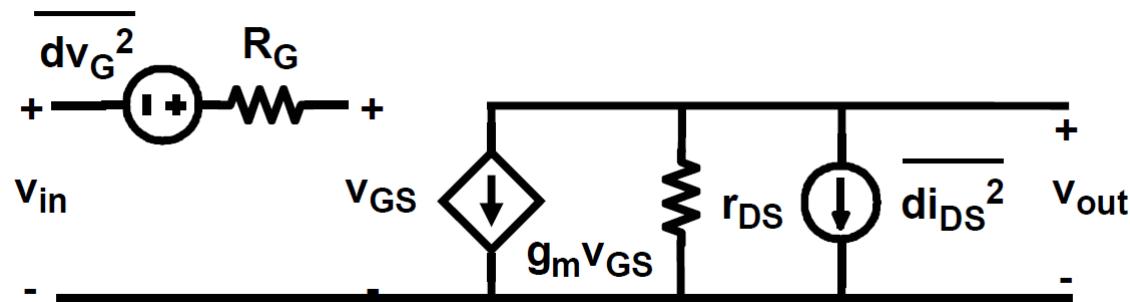
## 》》 晶体管的噪声

- 晶体管的**输入电阻**和其他电阻一样，也会产生热噪声
- 无噪声小信号模型：



## 》》 晶体管的噪声

- 含噪声小信号模型：

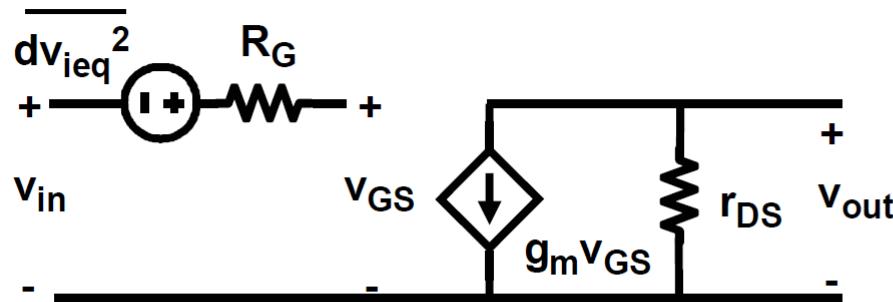


$$\overline{dv_G^2} = 4kT R_G df$$

$$\overline{di_{DS}^2} = \frac{4kT}{R_{CH}} df = 4kT \frac{2}{3} g_m df$$

## 》》 晶体管的噪声

- 输入等效噪声小信号模型：

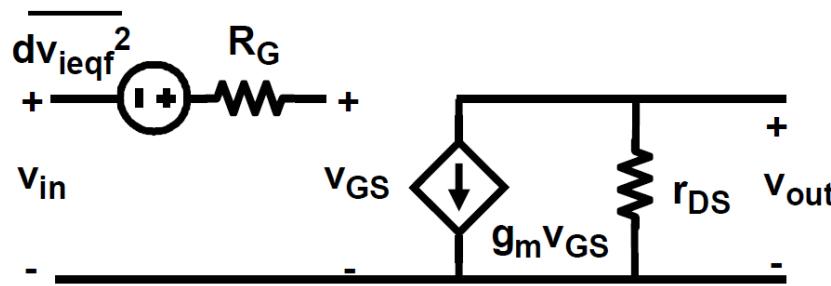


$$\overline{dv_{ieq}^2} = 4kT (R_{eff}) df \quad R_{eff} = \frac{2/3}{g_m} + R_G$$

- 高频时，**电流**可以通过 $C_{GS}$ ，因此也需考虑，同时需注意与输入噪声电压**相关**

## 》》 晶体管的噪声

- 输入等效噪声小信号模型：**1/f 噪声**



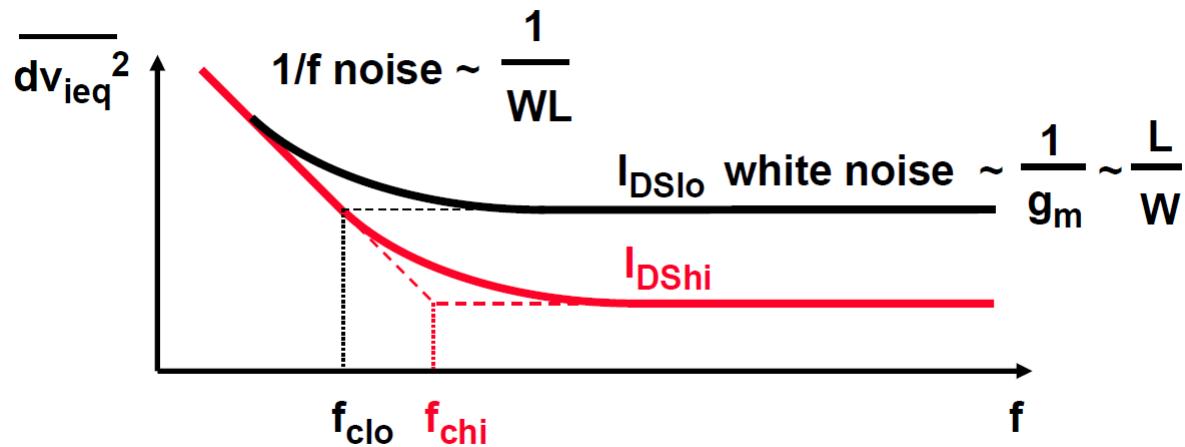
$$\overline{dv_{ieqf}^2} = \frac{KF_F}{WL C_{ox}^2} \frac{df}{f}$$

pMOS  $KF_F \approx 10^{-32} \text{ C}^2/\text{cm}^2$   
nMOS  $KF_F \approx 4 \cdot 10^{-31} \text{ C}^2/\text{cm}^2$

$W \& L \text{ in cm}; C_{ox} \text{ in F/cm}^2$

- NMOS的闪烁噪声是PMOS的**40倍**！

## 》》 晶体管的噪声：转角频率



- 偏置电流越大，白噪声越低，拐角频率越大



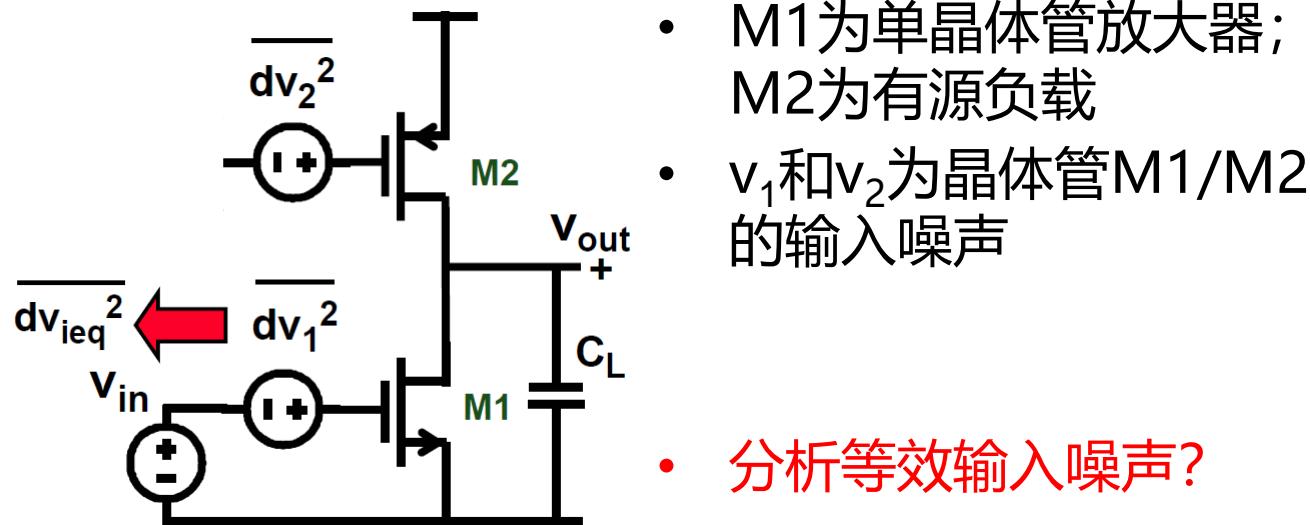
## 噪声 Noise

---

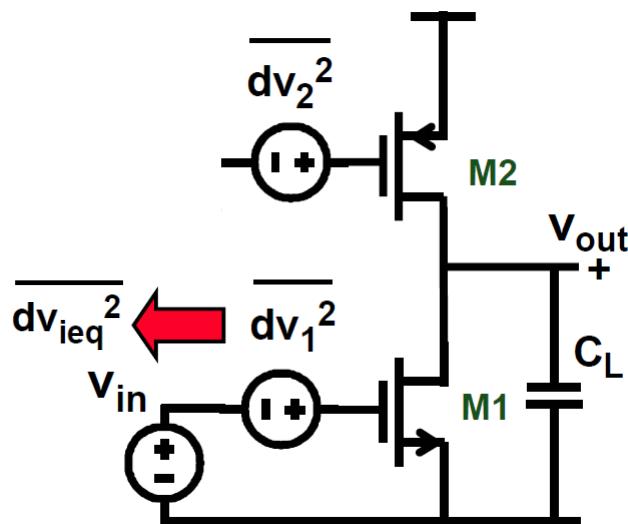
- 噪声的特点
- 系统中的噪声
- 电阻的热噪声
- 晶体管的噪声源
- **等效噪声的换算**

## 》》 放大器的噪声

---



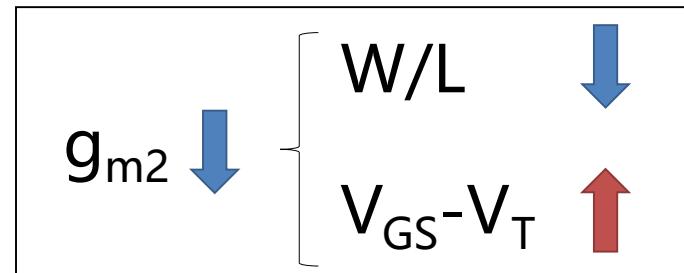
# 》》 放大器的噪声



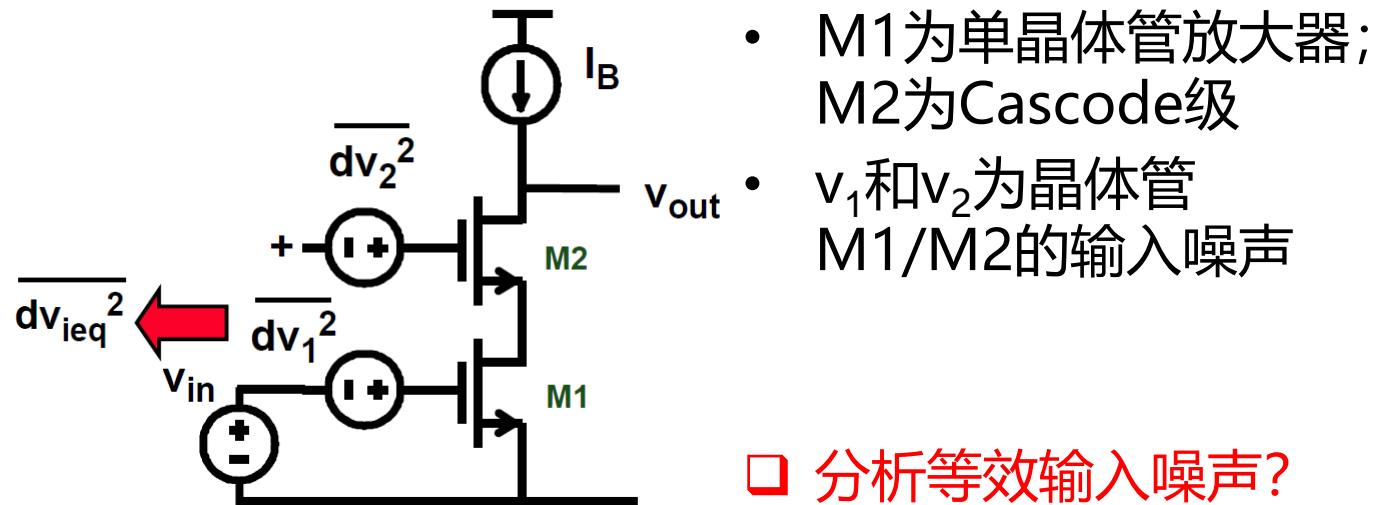
- M1为单晶体管放大器；  
M2为有源负载
- $v_1$ 和 $v_2$ 为晶体管M1/M2  
的输入噪声
- 分析等效输入噪声？

$$\overline{di_{out}^2} = g_{m1}^2 \overline{dv_1^2} + g_{m2}^2 \overline{dv_2^2}$$

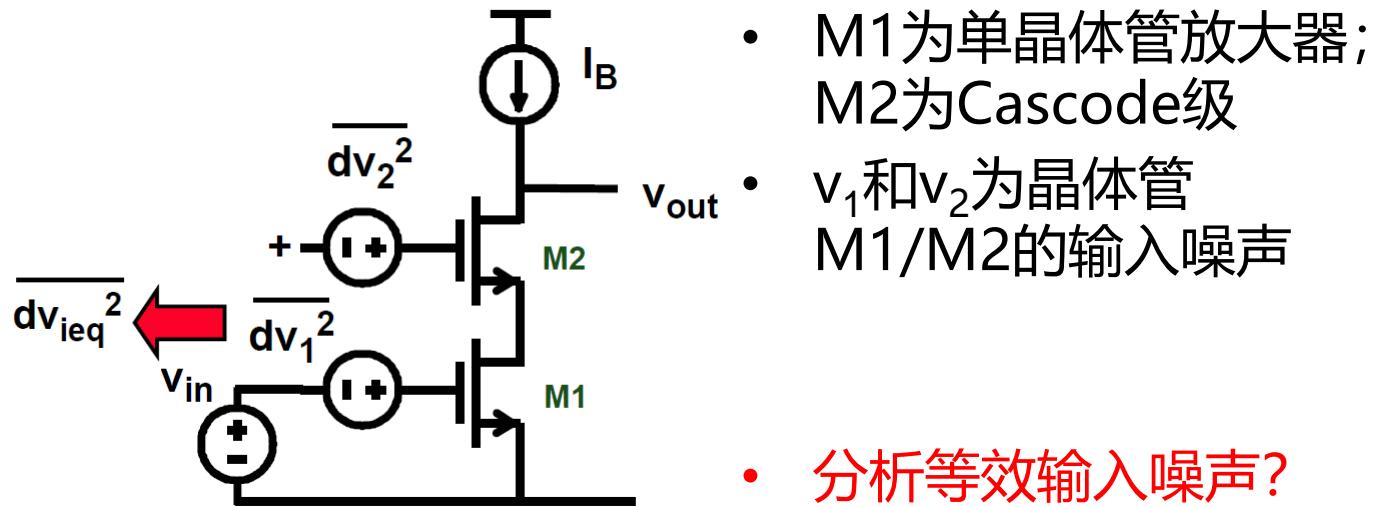
$$\overline{dv_{ieq}^2} = \overline{dv_1^2} \left( 1 + \frac{g_{m2}}{g_{m1}} \right)$$



## »» Cascode的噪声



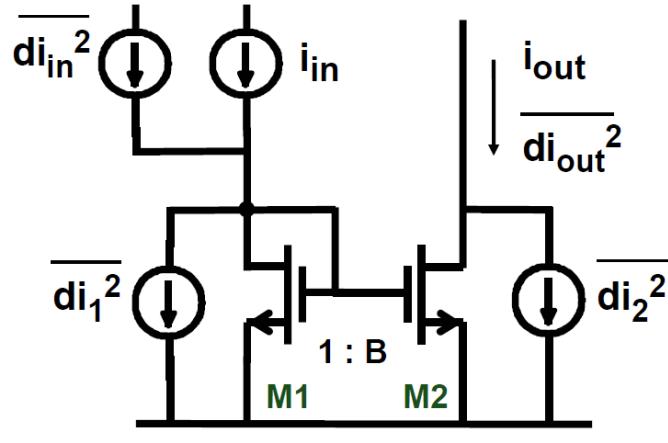
## »» Cascode的噪声



$$\overline{dv_{ieq}^2} = \overline{dv_1^2} + \overline{dv_2^2} \quad \frac{1}{(g_{m1} r_{o1})^2} \approx \overline{dv_1^2}$$

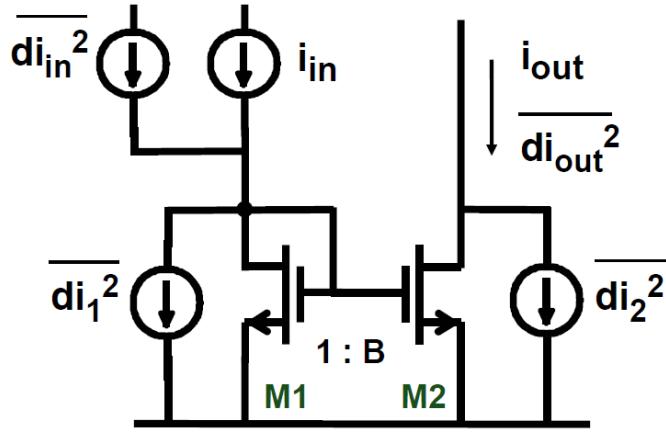
Cascode 晶体  
管M2的噪声忽  
略不计

## 》》 电流镜的噪声



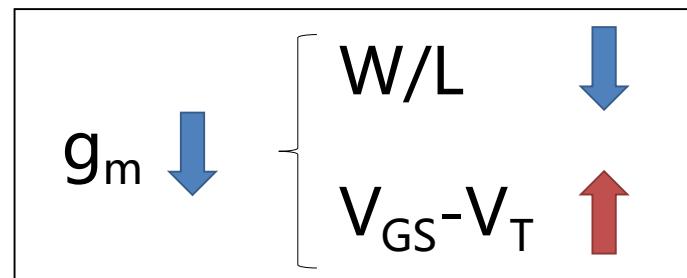
- M1/M2 为放大比例为B的电流镜
- $i_1$ 和 $i_2$ 为晶体管M1/M2的沟道噪声电流
- $i_{in}$ 为输入噪声电流

## 》》 电流镜的噪声

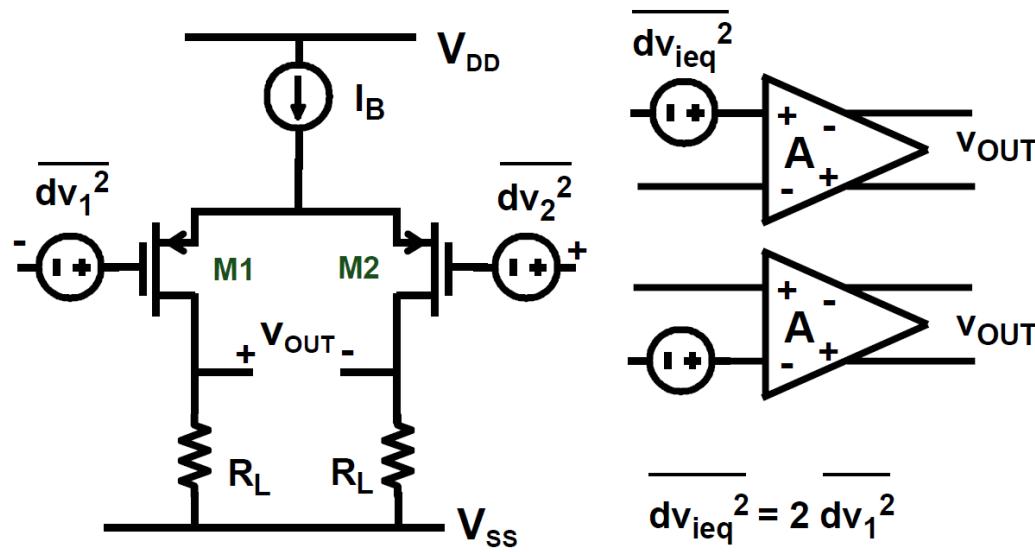


- M1/M2 为放大比例为B的电流镜
- $i_1$ 和 $i_2$ 为晶体管M1/M2的沟道噪声电流
- $i_{in}$ 为输入噪声电流

$$\overline{di_{out}^2} = \overline{di_2^2} + B^2 (\overline{di_{in}^2} + \overline{di_1^2})$$



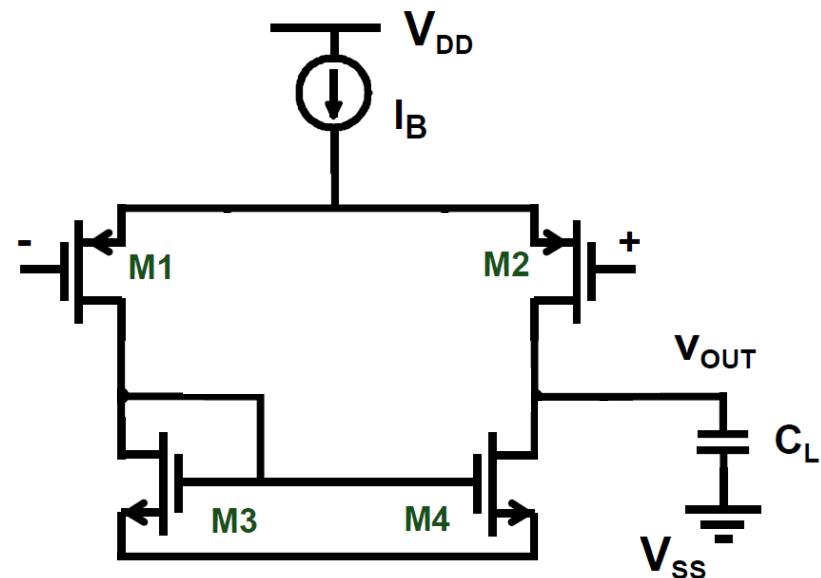
## 》》差分对的噪声



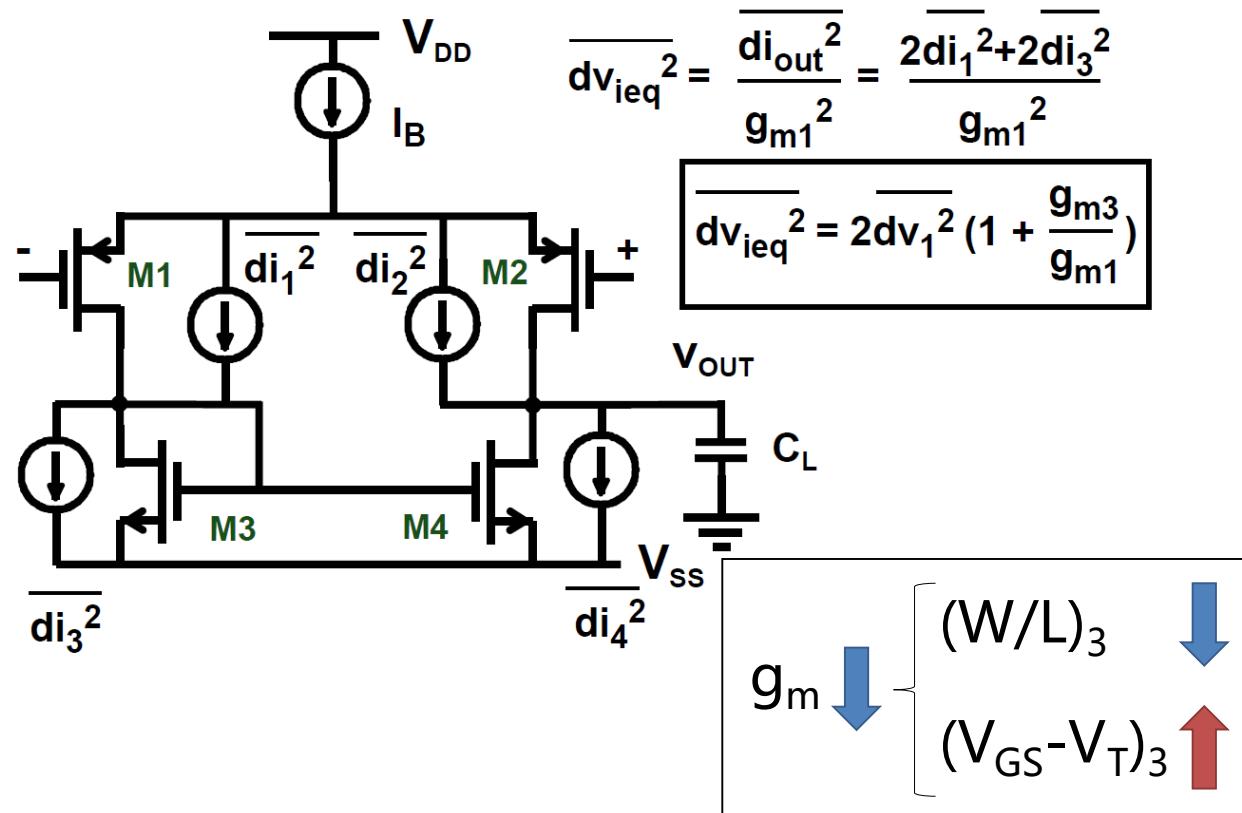
差分对的噪声即为两个输入晶体管噪声的和

## 》》 典型运放的噪声

---

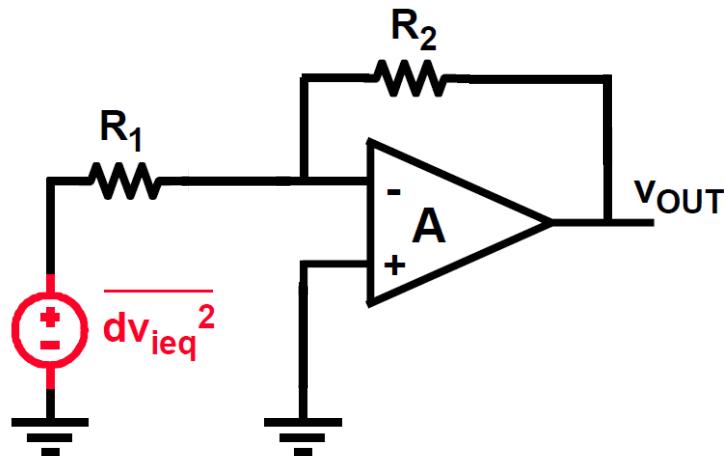


## 典型运放的噪声

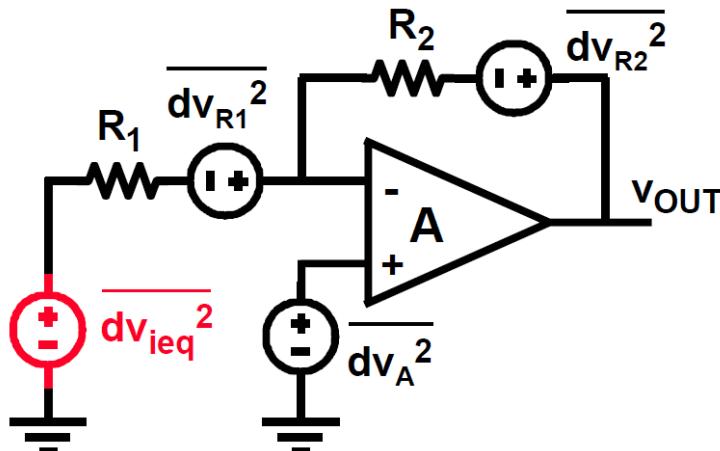


## 》》 典型运放的噪声

---



## 典型运放的噪声



$$\overline{dv_{ieq}^2} = \sum \overline{dv_{out}^2} \left( \frac{R_1}{R_2} \right)^2$$

$$\overline{dv_{out}^2} = \overline{dv_{R1}^2} \left( \frac{R_2}{R_1} \right)^2$$

$$\overline{dv_{out}^2} = \overline{dv_{R2}^2}$$

$$\overline{dv_{out}^2} = \overline{dv_A^2} \left( 1 + \frac{R_2}{R_1} \right)^2$$

$$\overline{dv_{ieq}^2} = \overline{dv_{R1}^2} + \overline{dv_{R2}^2} \left( \frac{R_1}{R_2} \right)^2 + \overline{dv_A^2} \left( 1 + \frac{R_1}{R_2} \right)^2 \approx \overline{dv_{R1}^2} + \overline{dv_A^2}$$



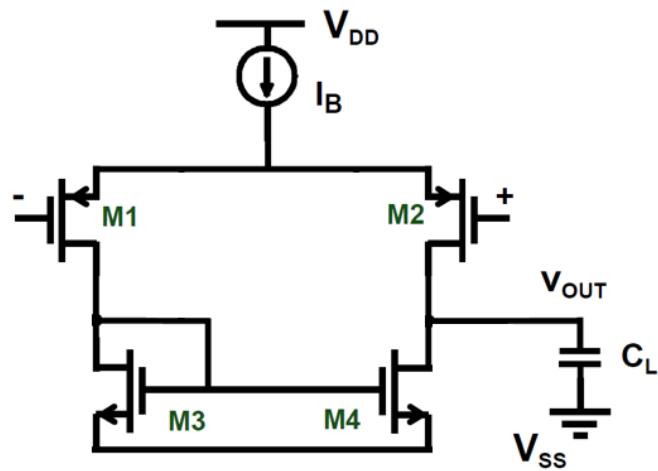
## 设计思路总结

---

1. 噪声通过功率表述
2. 注意噪声源之间的相关性
3. 对于系统噪声性能的分析需要转换成输入参考噪声
4. 输入参考的转换参考电路的跨导计算等

# 》》作业

1. 自由确定晶体管的尺寸，通过仿真寻找NMOS和PMOS的 $1/f$ 噪声系数KF，以及热噪声系数 $\gamma$
2. 对一偏置电流为100uA的五管OTA，通过晶体管的设计，使其等效输入噪声的80%来源于差分对M1和M2。



**0.4**



北京航空航天大學  
BEIHANG UNIVERSITY

| 微电子学院

# CMOS模拟集成电路设计

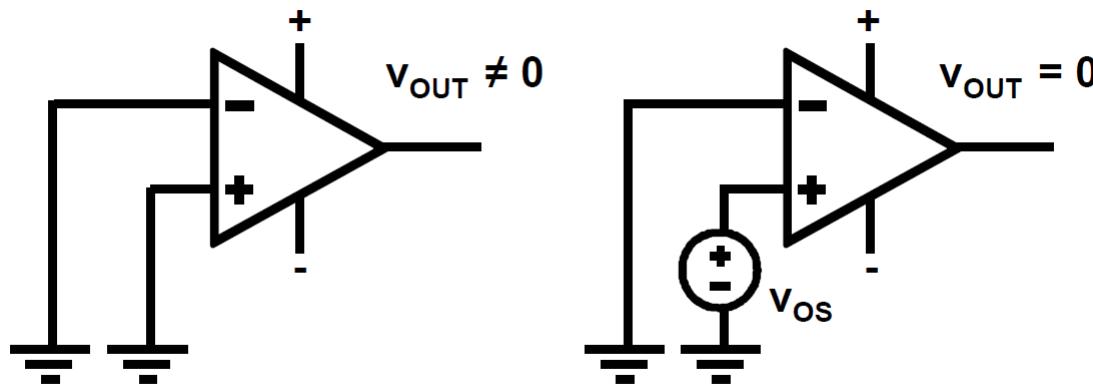
## 第四章：失调与CMRR

胡远奇

©2020

## 》》失调(Offset)的定义

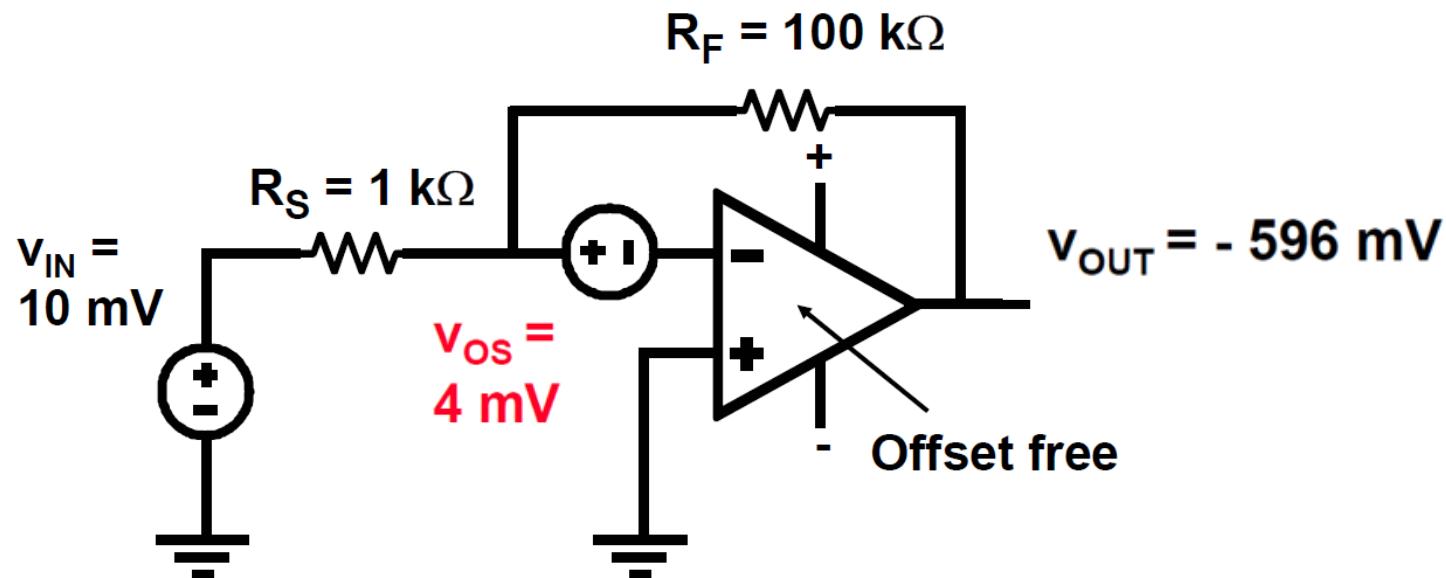
---



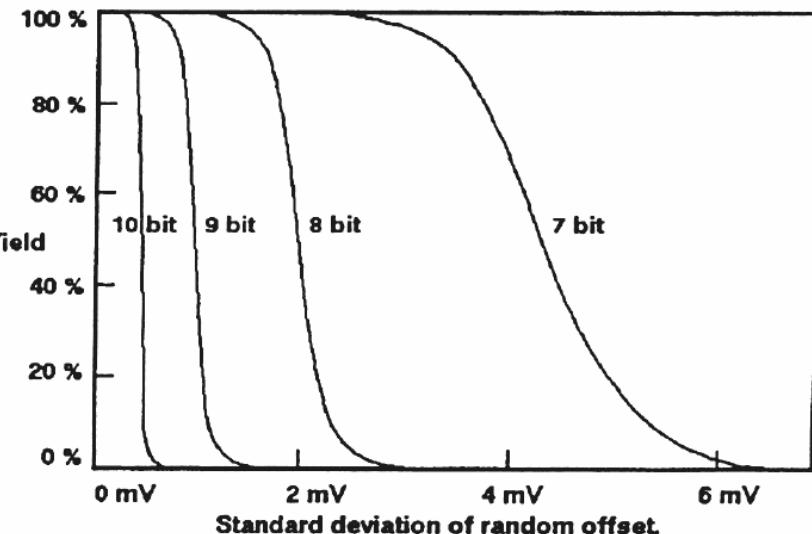
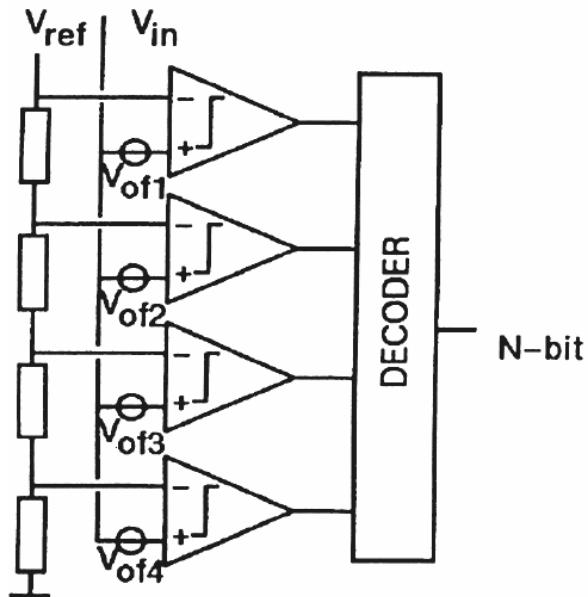
- 失调电压 $V_{OS}$ : 使输出电压为零时的两端输入电压之差
  - 可以加在任意一端

## 》》 失调(Offset)的影响

---



# 》》失调(Offset)的影响



n-bit Flash-ADC with offset

Ref: Pelgrom, IEDM 1998, pp.789.



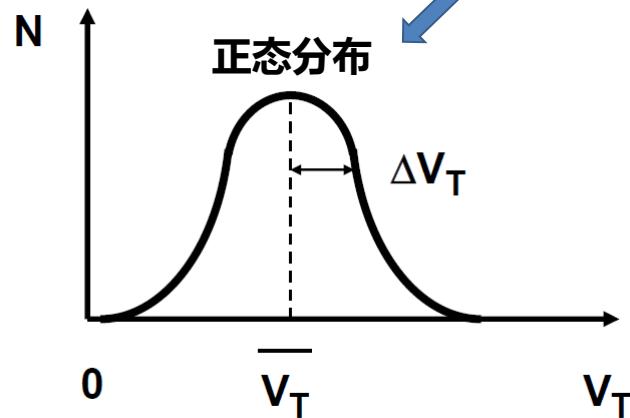
## 失调与CMRR

---

- 随机性失调和 $CMRR_R$
- 系统性失调和 $CMRR_S$
- 设计守则

## » 随机失调：阈值电压的失配

$$I_{DS} = K' W/L (V_{GS} - V_T)^2$$



$$\sigma_{\Delta V_T} = \frac{A_{VT}}{\sqrt{WL}}$$

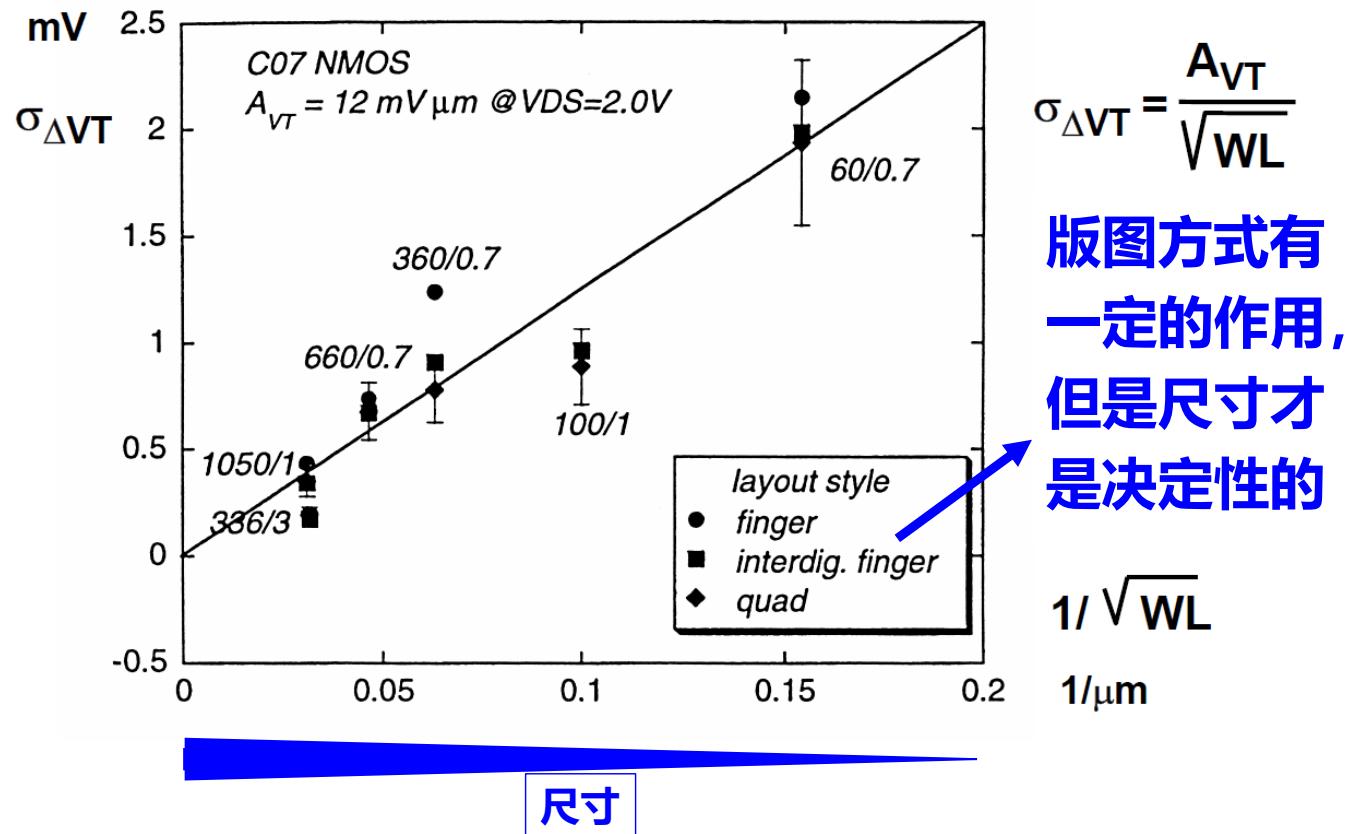
$$A_{VT} \sim t_{ox} \sqrt[4]{N_B}$$

氧化层厚度 & 衬底参杂浓度

$A_{VT} \approx 5 \text{ mV um}$   
for 0.25 um nMOST  
+50 % for pMOST

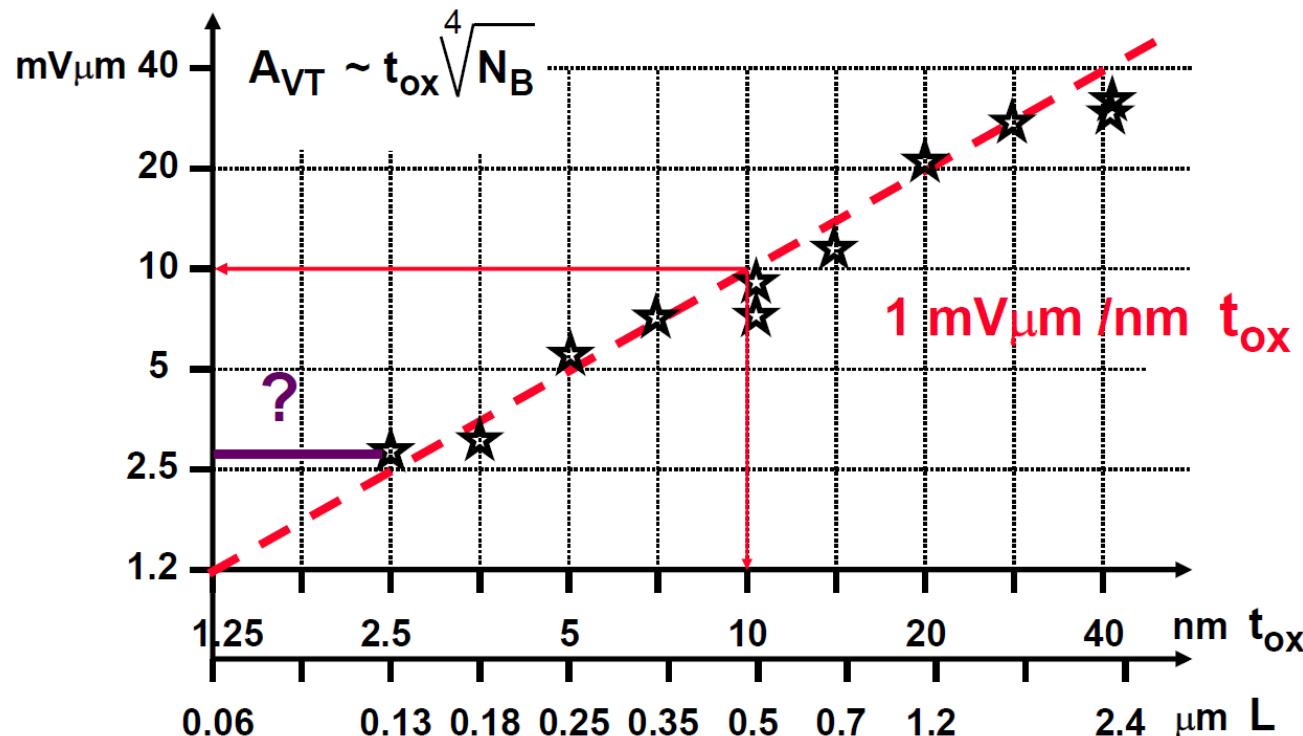
□估算0.25um工艺下、尺寸为16um/1um的PMOS差分对由阈值电压失配造成的大失调电压？（要求99.5%良率）

# 》》 阈值电压失配的标准差





## 阈值电压失配的系数 $A_{VT}$



- 某工厂 55nm 工艺的  $A_{VT} = 2.05e-9$

## » 随机失调：工艺与尺寸的失配

$$I_{DS} = K' W/L (V_{GS} - V_T)^2$$

$$\frac{\Delta K'}{K'} = \frac{A_{K'}}{\sqrt{WL}}$$

$A_{K'} \approx 0.0056 \text{ um}$   
+50 % for pMOS

$$\frac{\Delta W/L}{W/L} = A_{WL} \sqrt{\frac{1}{W^2} + \frac{1}{L^2}}$$

$A_{WL} \approx 0.02 \text{ mV um}$   
+50 % for pMOS

- 参数  $K'$  造成的变化相对较小

- $A_{WL}$  与工艺的相关性不强

## » NMOS的失配参数

---

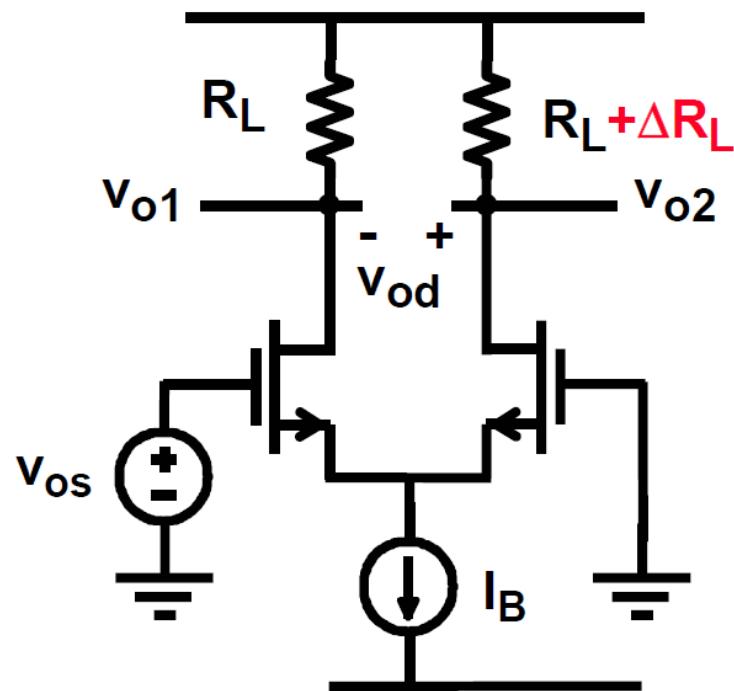
Techno L ( $\mu\text{m}$ )	2.5	1.2	0.7	0.5	0.35	0.25
t <sub>ox</sub> (nm)	50	25	15	11	8	6
A <sub>VT</sub> (mV $\mu\text{m}$ )	30	21	13	7.1	6	⇒ 3
A <sub>WL</sub> (% $\mu\text{m}$ )	2.5	1.8	2.5	1.3	2	⇒ 1.8

A<sub>WL</sub>的单位是um，可以听过设计尺寸或偏置电压减小其影响

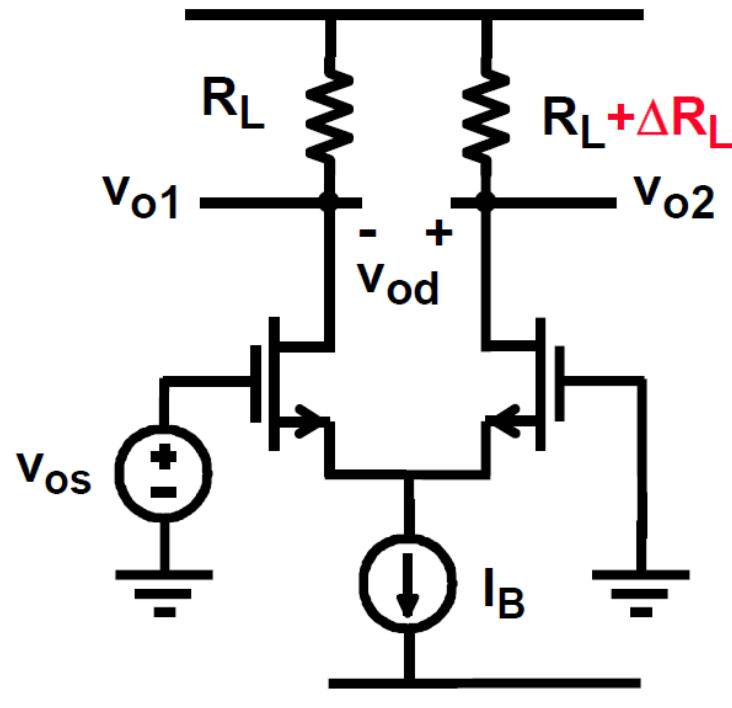
A<sub>VT</sub>的单位是mV\*um，只能通过设计尺寸减小其影响

## 》》差分对中的随机失调

---



## 》》差分对中的随机失调



$$v_{od} = \frac{\Delta R_L}{R_L} \frac{I_B}{2}$$

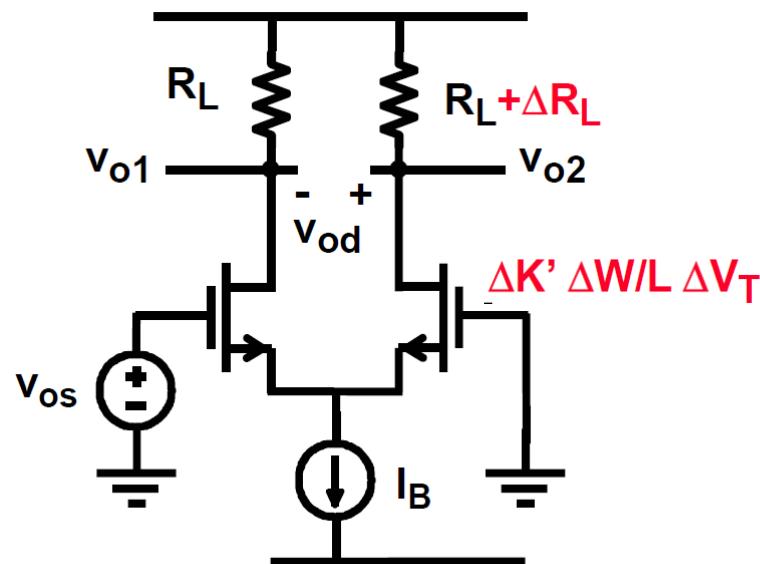
$$v_{os} = \frac{v_{od}}{g_m R_L}$$

$$v_{os} = \frac{\Delta R_L}{R_L} \frac{I_B}{2g_m} \quad \cdot \text{ 需要减小 } V_{GS} - V_T$$

$$v_{os} = \frac{\Delta R_L}{R_L} \frac{V_{GS} - V_T}{2}$$

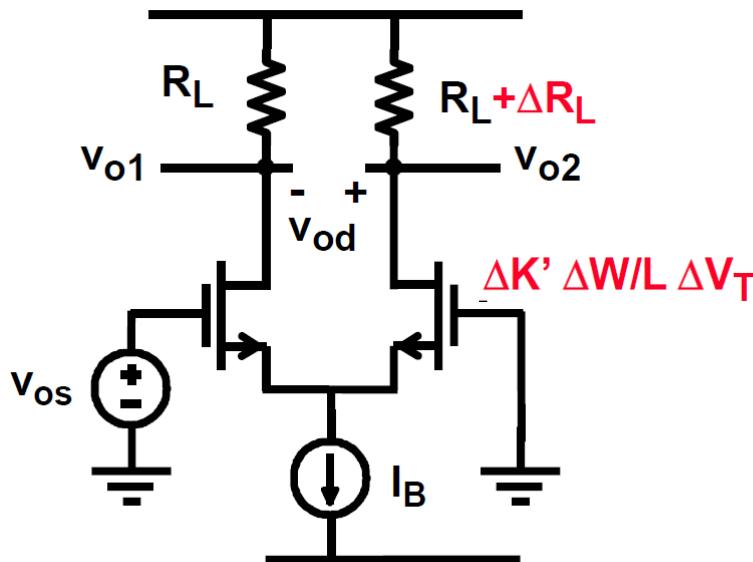
## 》》差分对中的随机失调

---



$$I_{DS} = K' W/L (V_{GS} - V_T)^2$$

# 》》》 差分对中的随机失调



$$I_{DS} = K' W/L (V_{GS} - V_T)^2$$

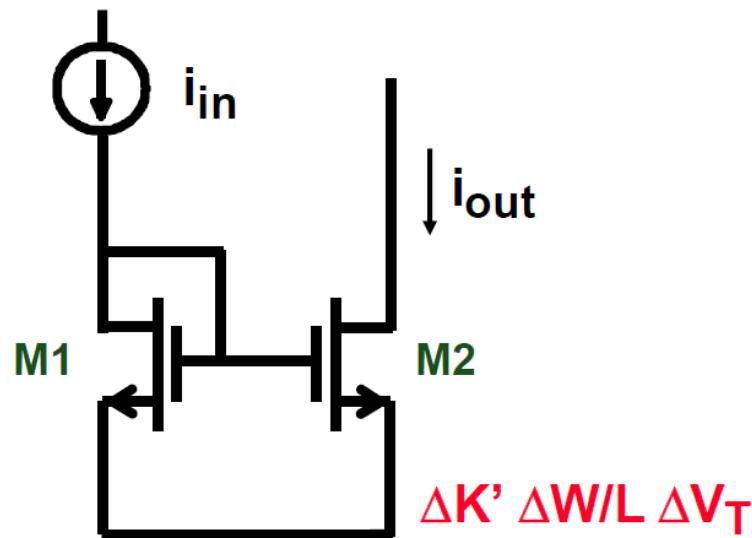
差分对应尽可能的

$$V_{GS} - V_T \downarrow$$

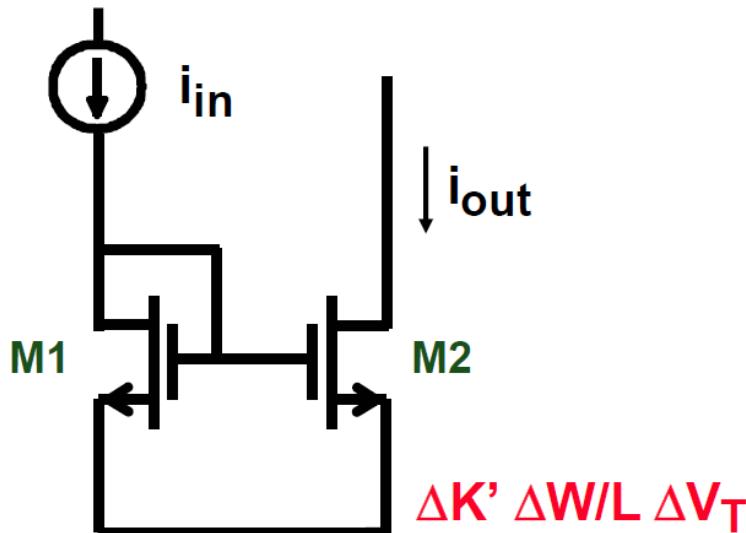
$$v_{OS} = \Delta V_T + \frac{V_{GS} - V_T}{2} \left( \frac{\Delta R_L}{R_L} + \frac{\Delta K'}{K'} + \frac{\Delta W/L}{W/L} \right)$$

## »» 电流镜中的随机失调

---



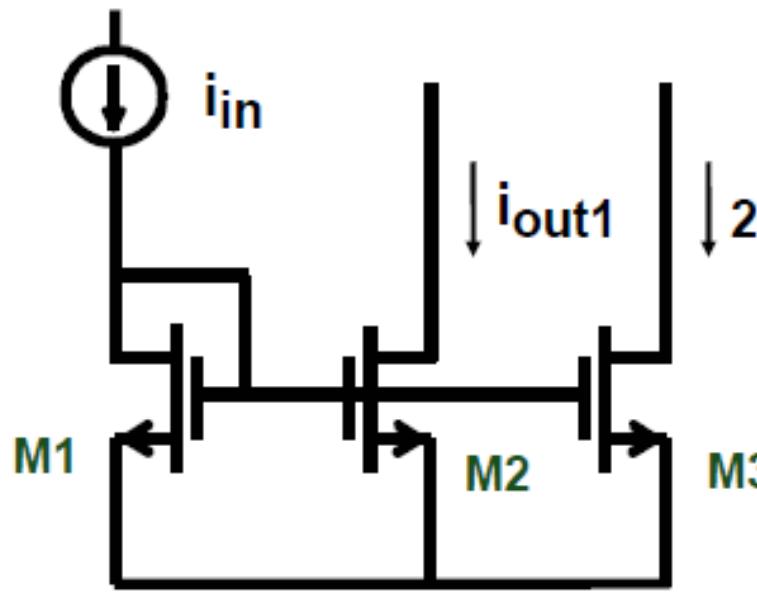
## 》》 电流镜中的随机失调



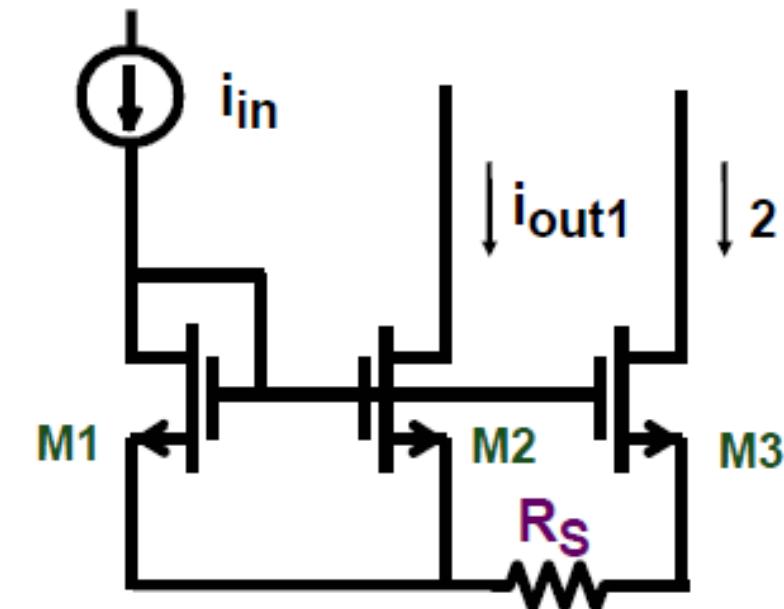
电流镜应尽可能的

$V_{GS} - V_T$  ↑

## » 电流镜中的其他失调因素



$\Delta K' \Delta W/L \Delta V_T$



$\Delta K' \Delta W/L \Delta V_T R_S$

## 》》 输出电流的失配

---

$$I_{DS} = K' W/L (V_{GS} - V_T)^2$$

$$I_{DS} = \frac{\beta}{2} (V_{GS} - V_T)^2$$



定义：总的电流失配由  $\beta$  失配和  $V_T$  失配造成

$$\frac{\Delta I_{DS}}{I_{DS}} =$$

$$\sigma^2 \left( \frac{\Delta I_{DS}}{I_{DS}} \right) =$$

## 》》 输出电流的失配

---

$$I_{DS} = K' W/L (V_{GS} - V_T)^2$$

$$I_{DS} = \frac{\beta}{2} (V_{GS} - V_T)^2$$



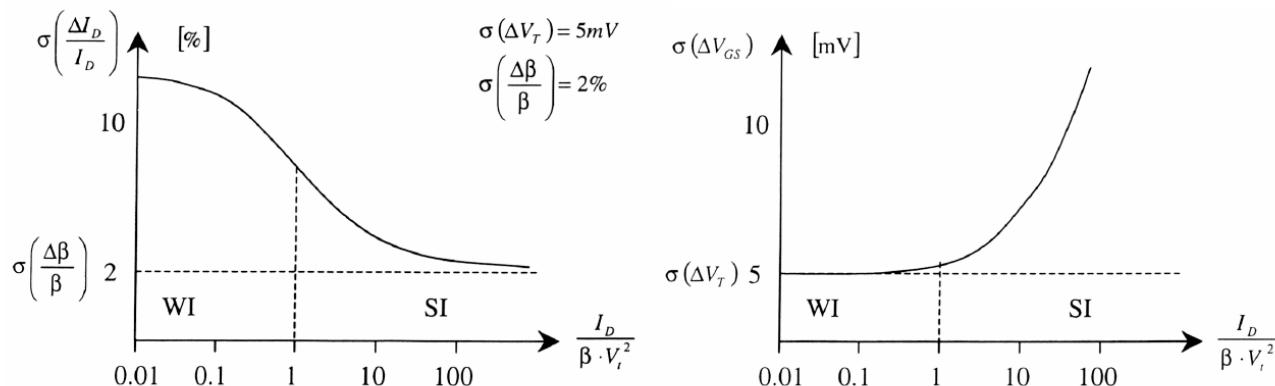
定义：总的电流失配由  $\beta$  失配和  $V_T$  失配造成

$$\frac{\Delta I_{DS}}{I_{DS}} = \frac{\Delta \beta}{\beta} - \Delta V_T \frac{2}{V_{GS} - V_T}$$

$$\sigma^2 \left( \frac{\Delta I_{DS}}{I_{DS}} \right) = \sigma^2 \left( \frac{\Delta \beta}{\beta} \right) + \sigma^2 (\Delta V_T) \frac{4}{(V_{GS} - V_T)^2}$$

# 输出电流的失配

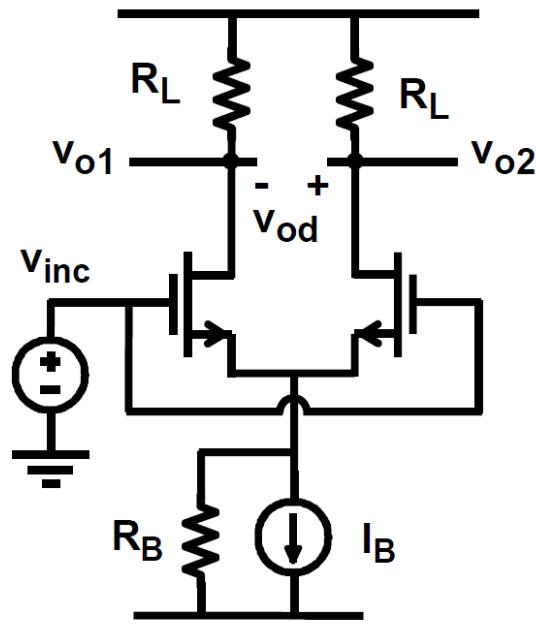
$$\sigma^2 \left( \frac{\Delta I_{DS}}{I_{DS}} \right) = \sigma^2 \left( \frac{\Delta \beta}{\beta} \right) + \sigma^2 (\Delta V_T) \underbrace{\frac{4}{(V_{GS} - V_T)^2}}_{\text{in si}} \quad \text{or} \quad \underbrace{\frac{1}{(nkT/q)^2}}_{\text{in wi}}$$



- 弱反型区中,  $V_T$  失配占主要因素; 强反型区中,  $\beta$  失配占主要因素, 其中主要又是W/L的失配为主

## » 共模抑制比

- 差分对的另一随机指标：共模抑制比（Common Mode Rejection Ration）



**共模增益：**

差模输出  $V_{od}$  / 共模输入  $V_{inc}$

$$A_{dc} = \left. \frac{V_{od}}{V_{ic}} \right|_{V_{id} = 0} \approx 0$$

**共模抑制比：**

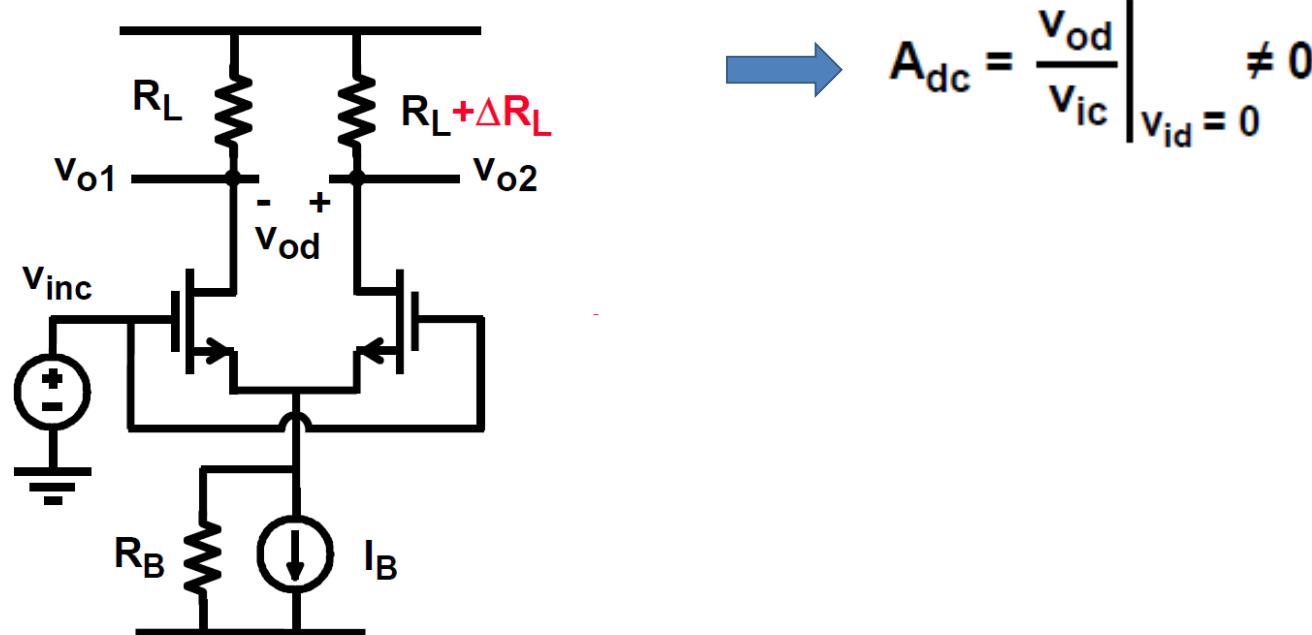
差模增益 / 共模增益

$$CMRR = \frac{A_{dd}}{A_{dc}} \approx \infty$$

## 》》共模抑制比

- 差分对的另一随机指标：共模抑制比（Common Mode Rejection Ration）

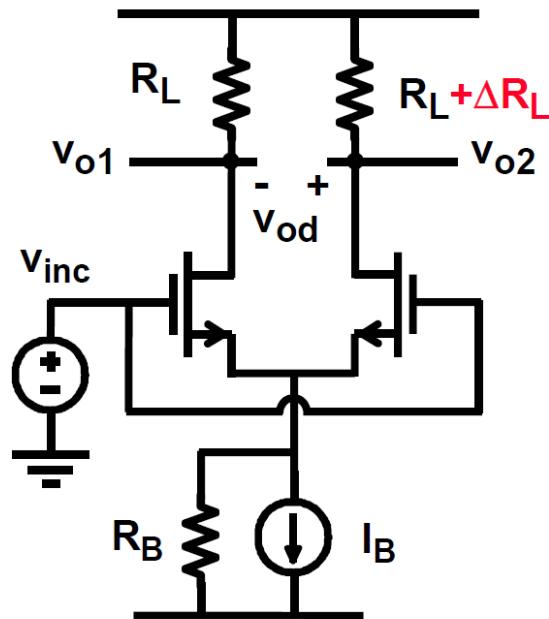
• 负载不对称



## 》》共模抑制比

- 差分对的另一随机指标：共模抑制比（Common Mode Rejection Ration）

• 负载不对称



$$\rightarrow A_{dc} = \left. \frac{v_{od}}{v_{ic}} \right|_{v_{id}=0} \neq 0$$

共模偏置电流： $I_B = V_{INC}/R_B$

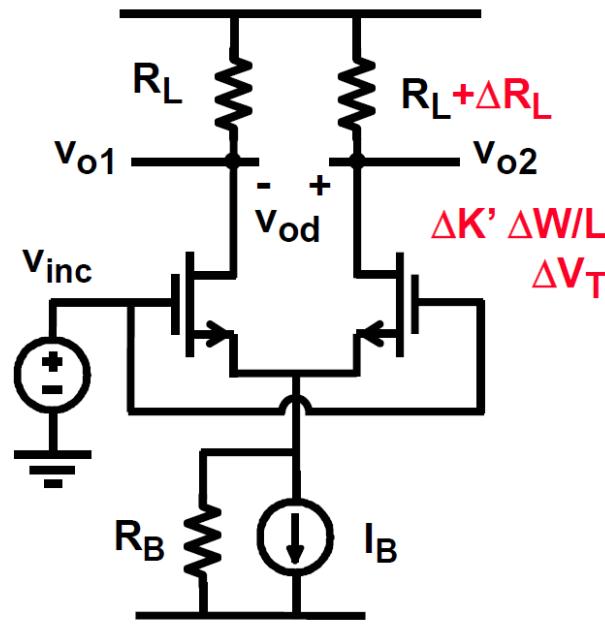
差模输出： $V_{od} = \Delta R_L \cdot I_B / 2$

$$A_{dc} = \frac{\Delta R_L}{2 R_B}$$

$$CMRR = \frac{2 g_m R_B}{\Delta R_L / R_L}$$

# 》》共模抑制比

- 差分对的另一随机指标：共模抑制比（Common Mode Rejection Ration）



- 负载不对称
- 差分对失调

$$CMRR = \frac{2g_m R_B}{\frac{\Delta R_L}{R_L}}$$



$$CMRR = \frac{2g_m R_B}{\frac{\Delta R_L}{R_L} + \frac{2V_{OS}}{V_{GS} - V_T}}$$



## 随机失调与CMRR的关系

---

$$CMRR = \frac{2g_m R_B}{\frac{\Delta R_L}{R_L} + \frac{2V_{OS}}{V_{GS} - V_T}}$$

$$\rightarrow V_{OS} \cdot CMRR = V_{OS} \cdot \frac{2g_m R_B}{\frac{\Delta R_L}{R_L} + \frac{2V_{OS}}{V_{GS} - V_T}}$$

$$\rightarrow V_{OS} \cdot CMRR \approx (V_{GS} - V_T) g_m R_B$$

$$\rightarrow V_{OS} \cdot CMRR \approx V_E L_B$$

减小失调就是提高CMRR



## 随机失调与CMRR的关系

---

$$v_{OSr} \text{ CMRR}_r \approx V_E L_B \approx 10 \text{ V} \quad (\sim L_B)$$

10 mV 60 dB ≈ 10 V as for MOSTs

1 mV 80 dB ≈ 10 V as for Bipolar transistors

10  $\mu$ V 120 dB ≈ 10 V with trimming :  
with laser  
with Zener zap  
with fusible links

**Low offset = High CMRR**

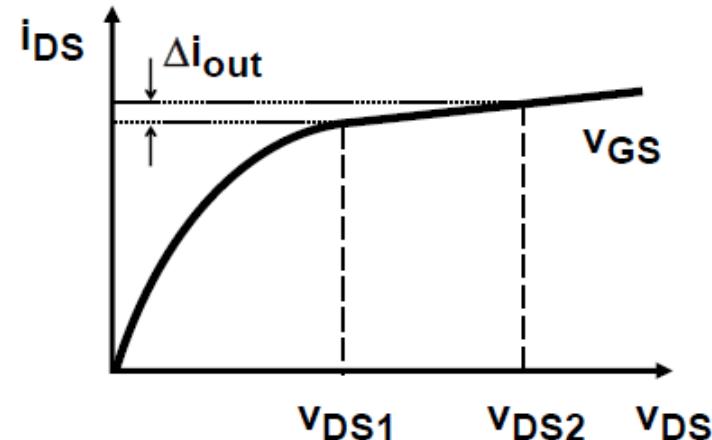
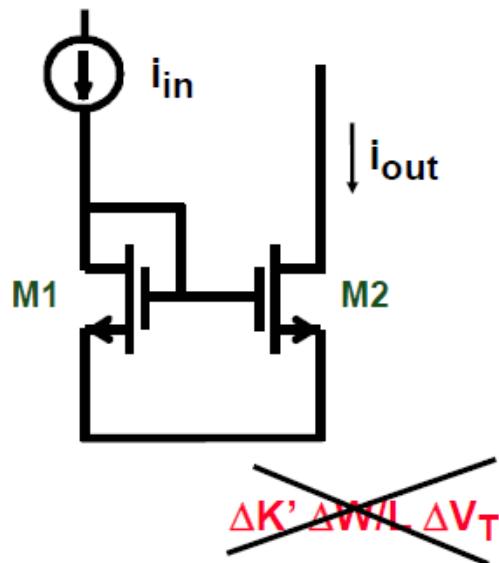


## 失调与CMRR

---

- 随机性失调和 $CMRR_R$
- 系统性失调和 $CMRR_S$
- 设计守则

## 》》 电流镜中的系统失调

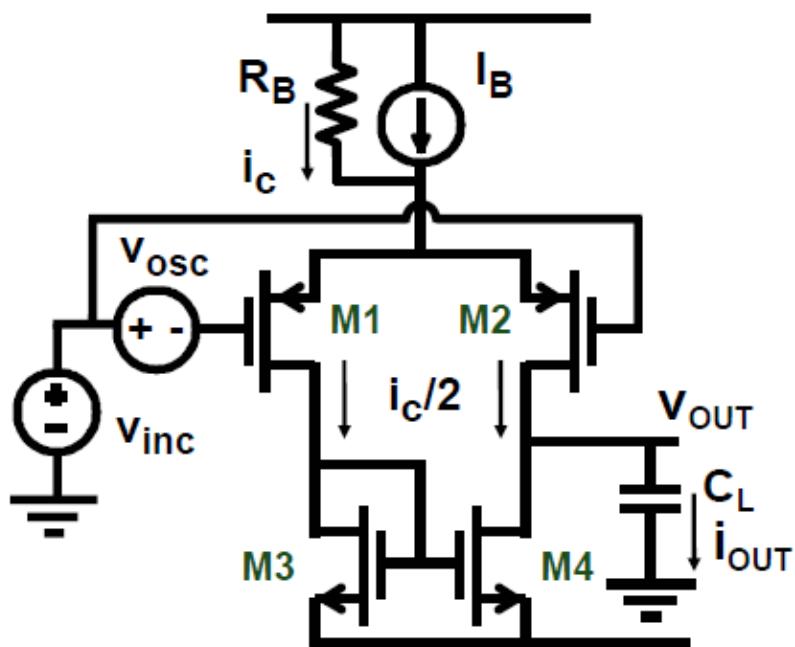


$$I_{out} \neq I_{in}$$

$$\frac{\Delta i_{out}}{i_{out}} = \frac{v_{DS2} - v_{DS1}}{V_E L_2}$$

# 》》》 差分运放中的系统性CMRR

- 不考虑任何随机性失调的情况下：



$$V_{OUT} = V_{GS3}$$

$$\Delta V_{GS3} = \frac{1}{2} \Delta I_B / g_{m3}$$

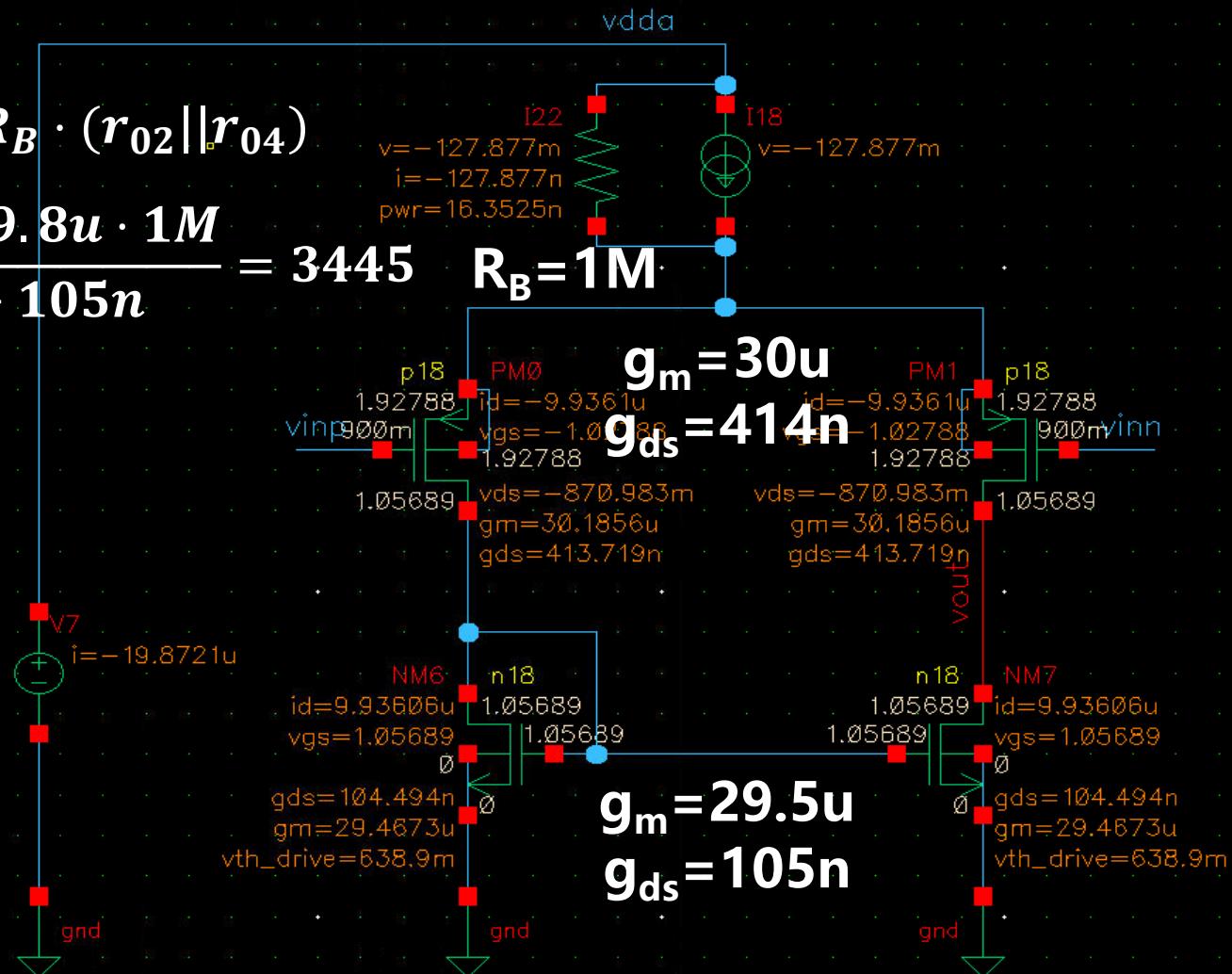
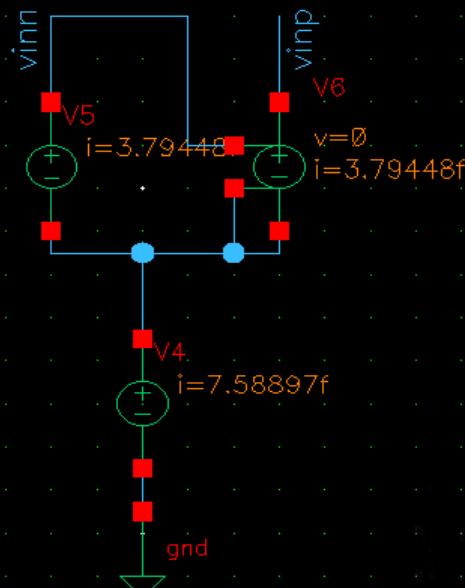
$$A_{dc} = \frac{1}{2g_{m3}R_B}$$

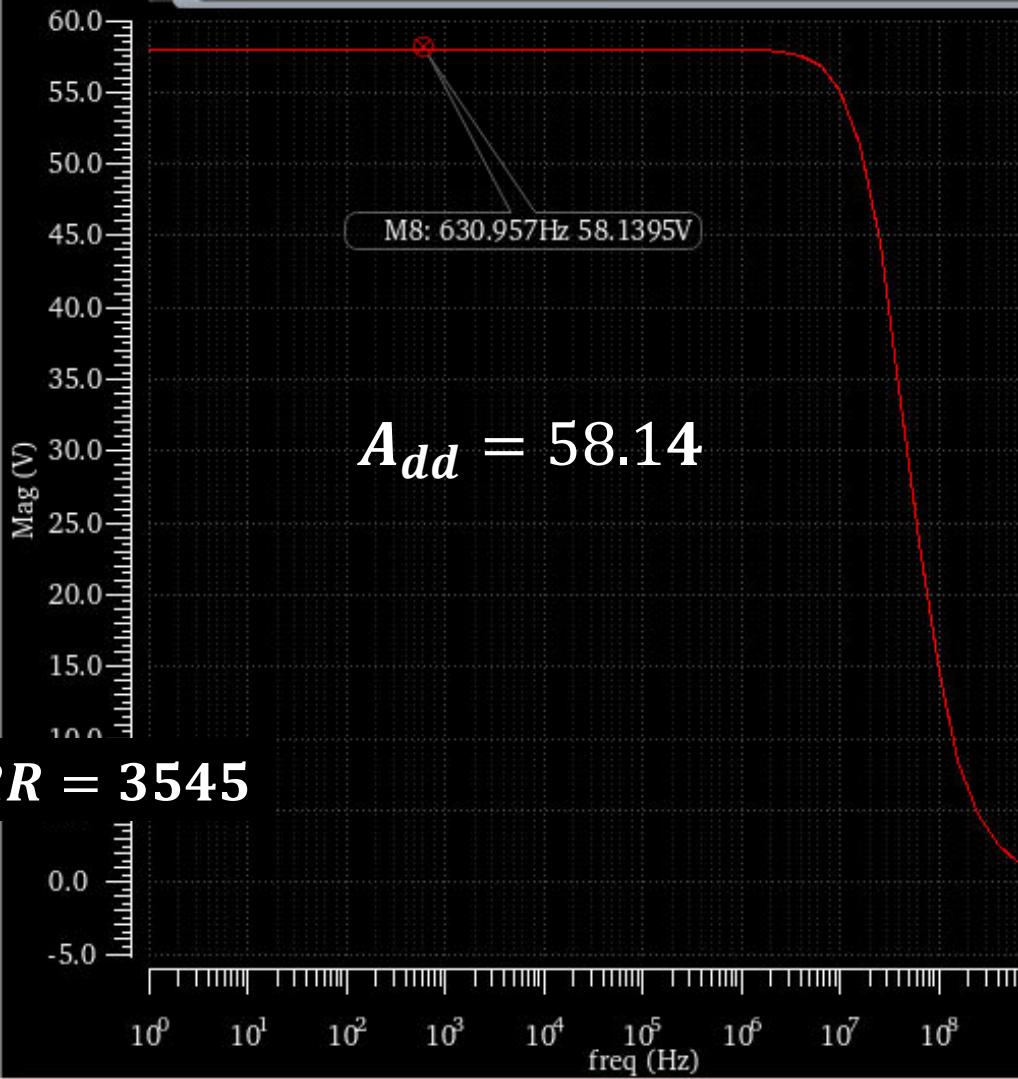
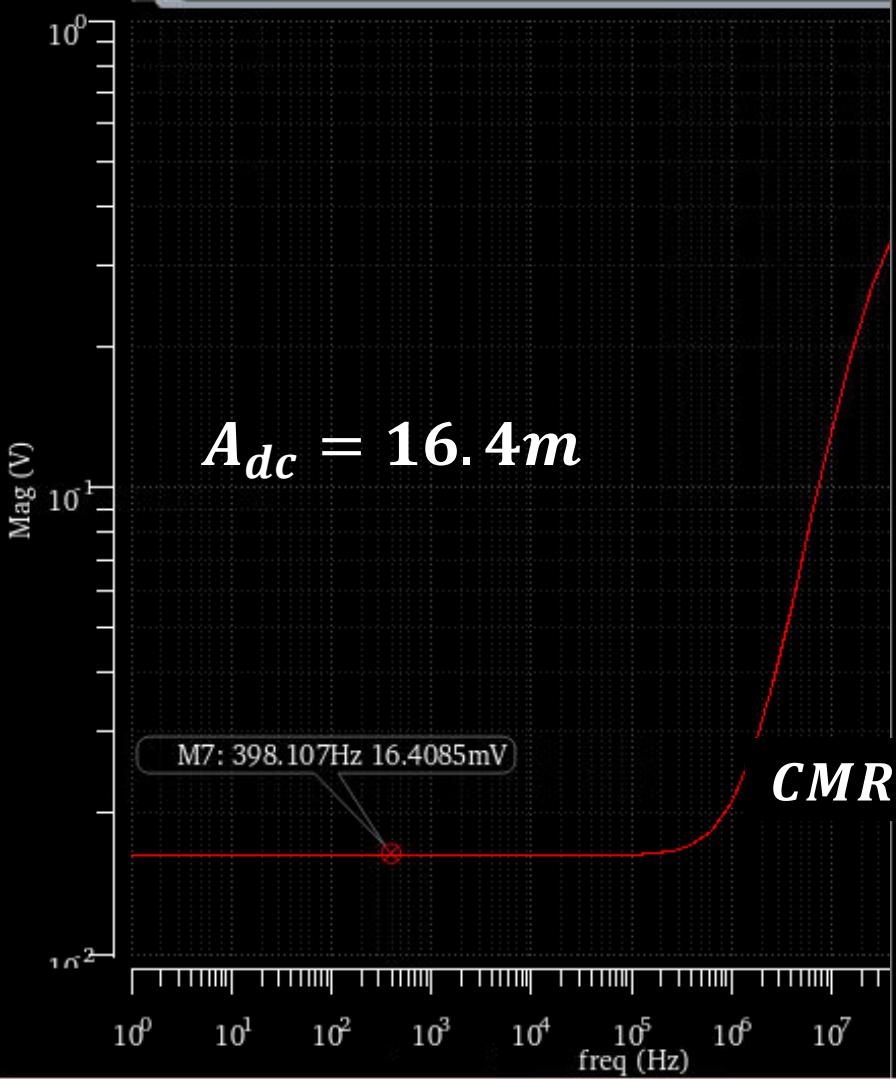
$$A_{dd} = g_{m1} \cdot (r_{02} || r_{04})$$

$$CMRR = 2g_{m1}g_{m3}R_B \cdot (r_{02} || r_{04})$$

$$CMRR = 2g_m1g_m3R_B \cdot (r_{02}||r_{04})$$

$$CMRR = \frac{2 \cdot 30\mu \cdot 29.8\mu \cdot 1M}{414n + 105n} = 3445 \quad R_B=1M$$

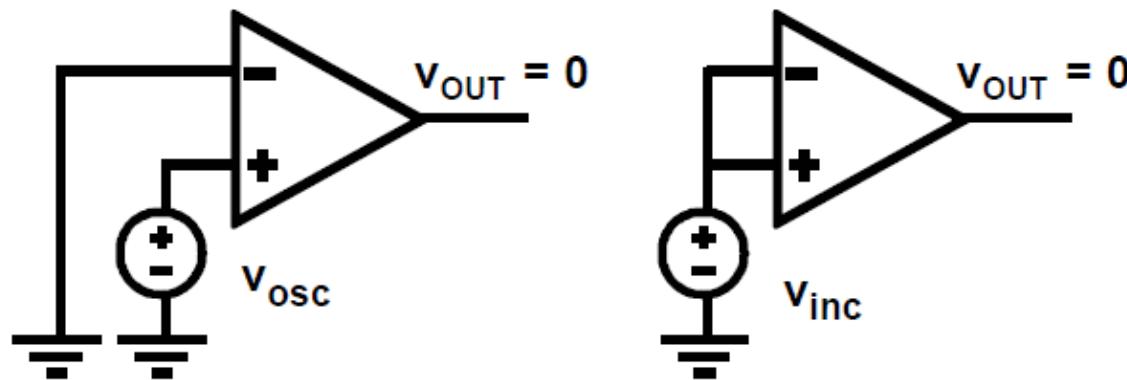




## 》》差分运放中的总CMRR

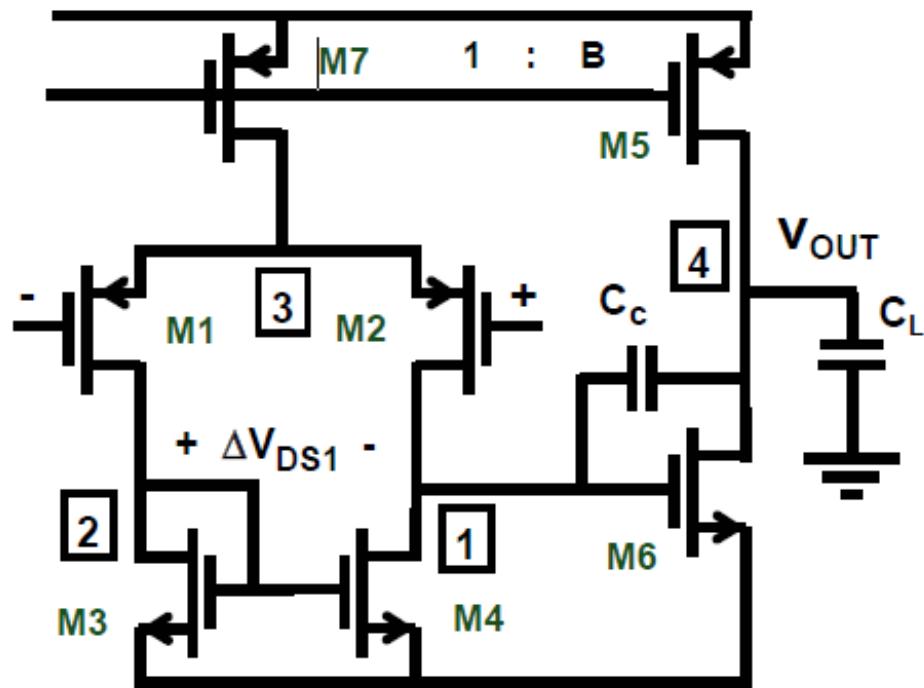
---

$$\frac{1}{\text{CMRR}} = \frac{1}{\text{CMRR}_r} + \frac{1}{\text{CMRR}_s}$$

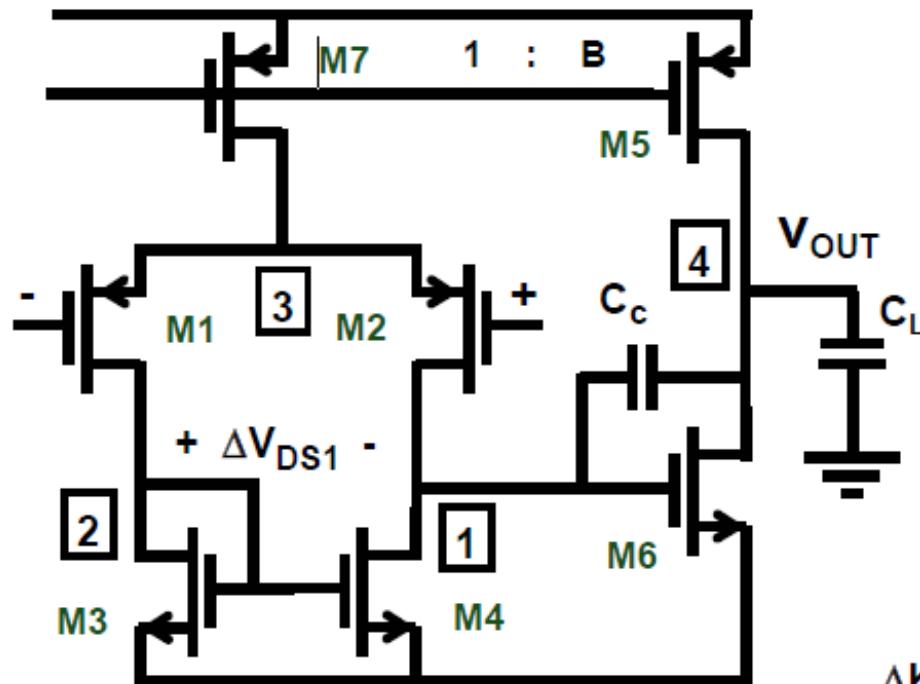


## 》》两级运放中的失调现象

---



## 》》两级运放中的失调现象



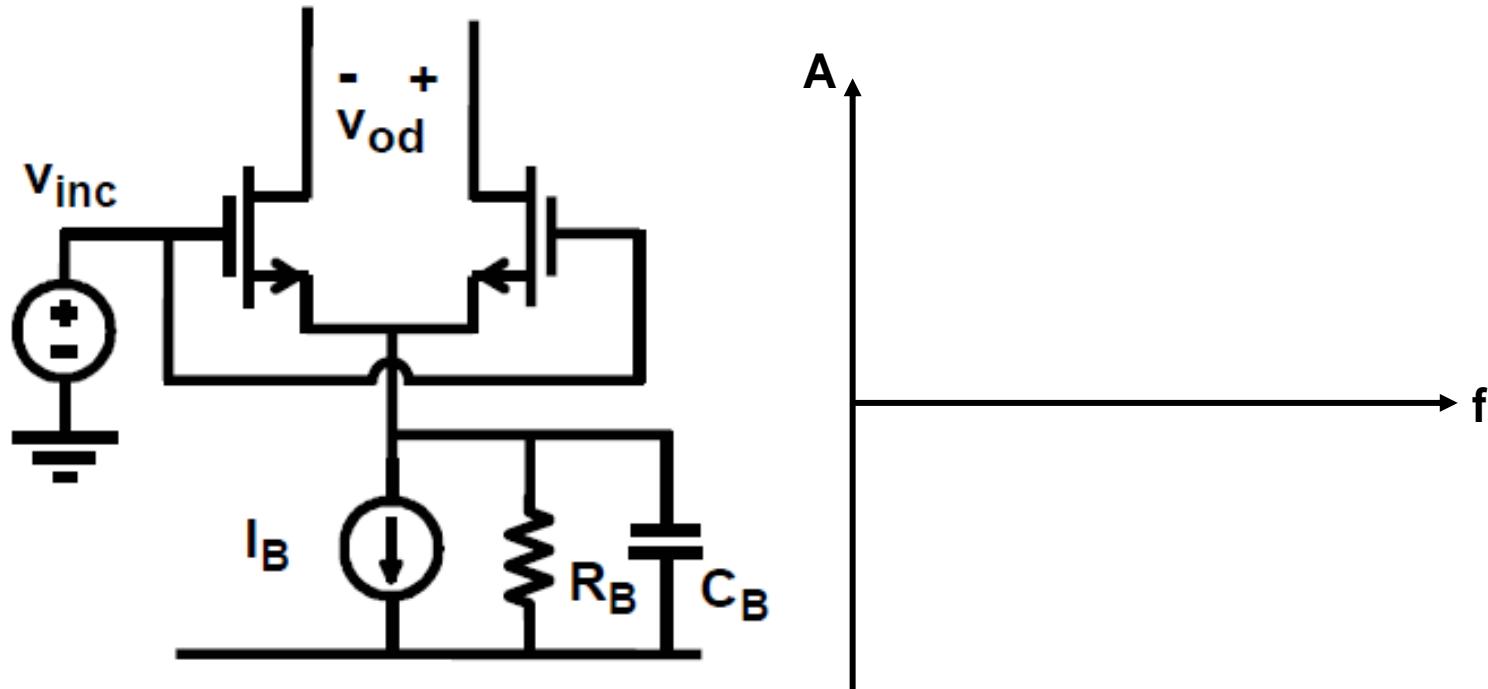
$$A_{v1} = g_{m1} r_{02} // r_{04}$$

$$\begin{aligned} v_{OS} = & \frac{\Delta V_{DS1}}{A_{v1}} + \\ & \Delta V_{T1} + \frac{g_{m3}}{g_{m1}} \Delta V_{T3} * + \\ & + \frac{V_{GS1} - V_T}{2} S \end{aligned}$$

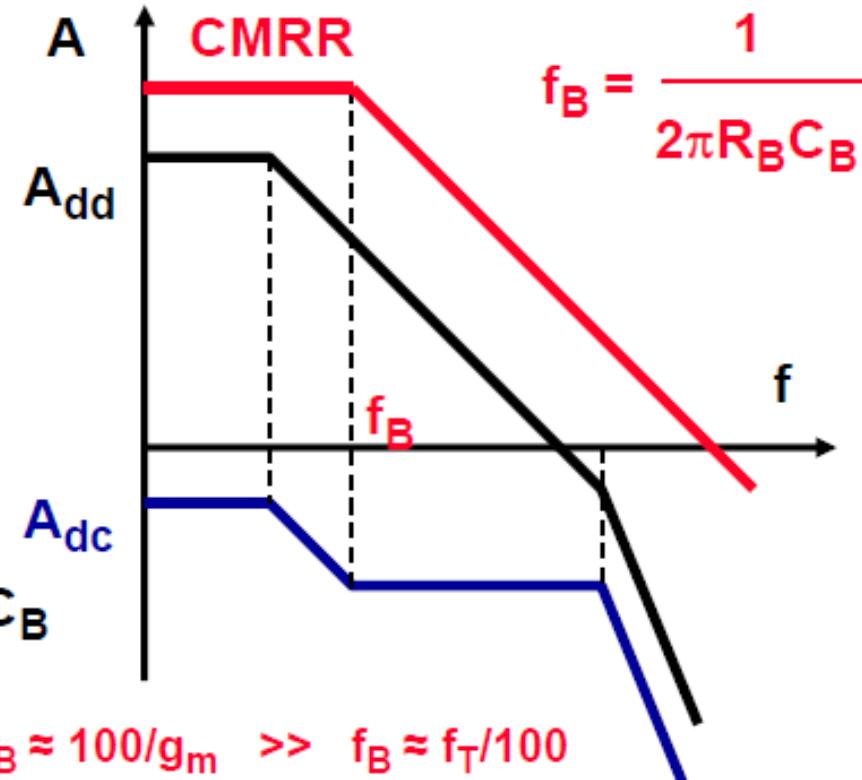
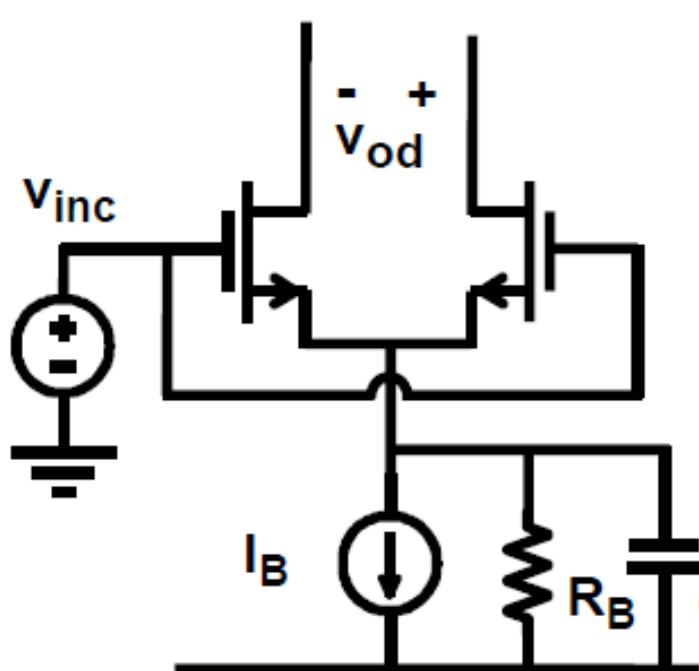
$$S = \frac{\Delta K'_n}{K'_n} + \frac{\Delta K'_p}{K'_p} + \frac{\Delta W/L_1}{W/L_1} + \frac{\Delta W/L_3}{W/L_3}$$

## 》》 CMRR的频率特性

---



## » CMRR的频率特性





## 失调与CMRR

---

- 随机性失调和 $CMRR_R$
- 系统性失调和 $CMRR_S$
- **设计守则**



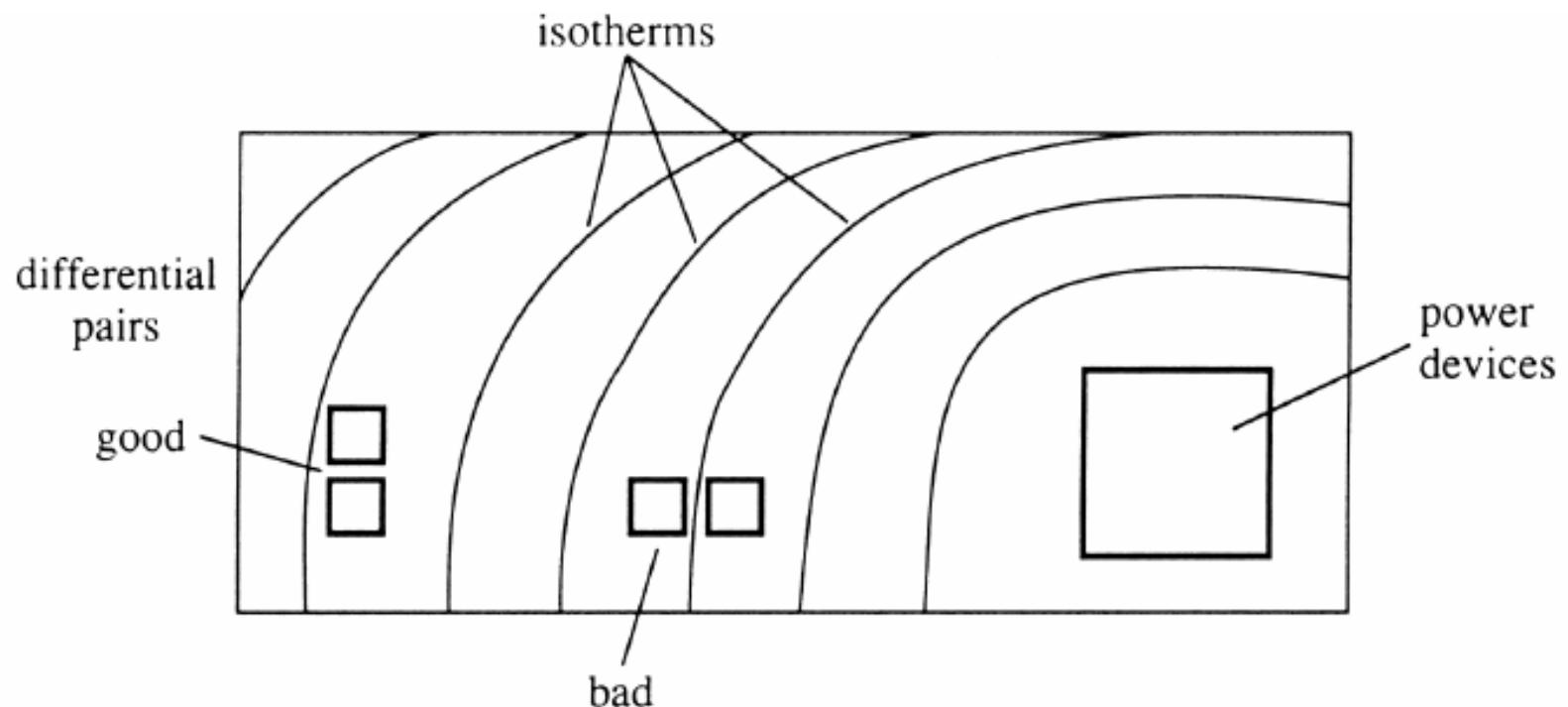
## 设计守则

---

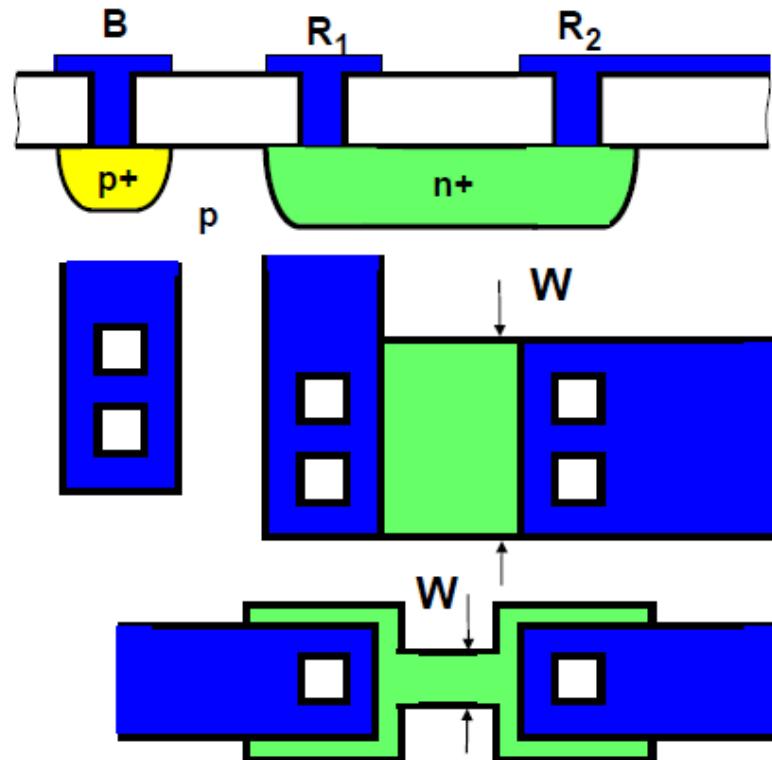
- 相同的特性
- 相同的温度
- 增大尺寸
- 减小间距
- 同样的方向
- 同样的长宽比
- 中心对称
- Dummy

## » 相同的温度

---



## 》》 电阻的版图



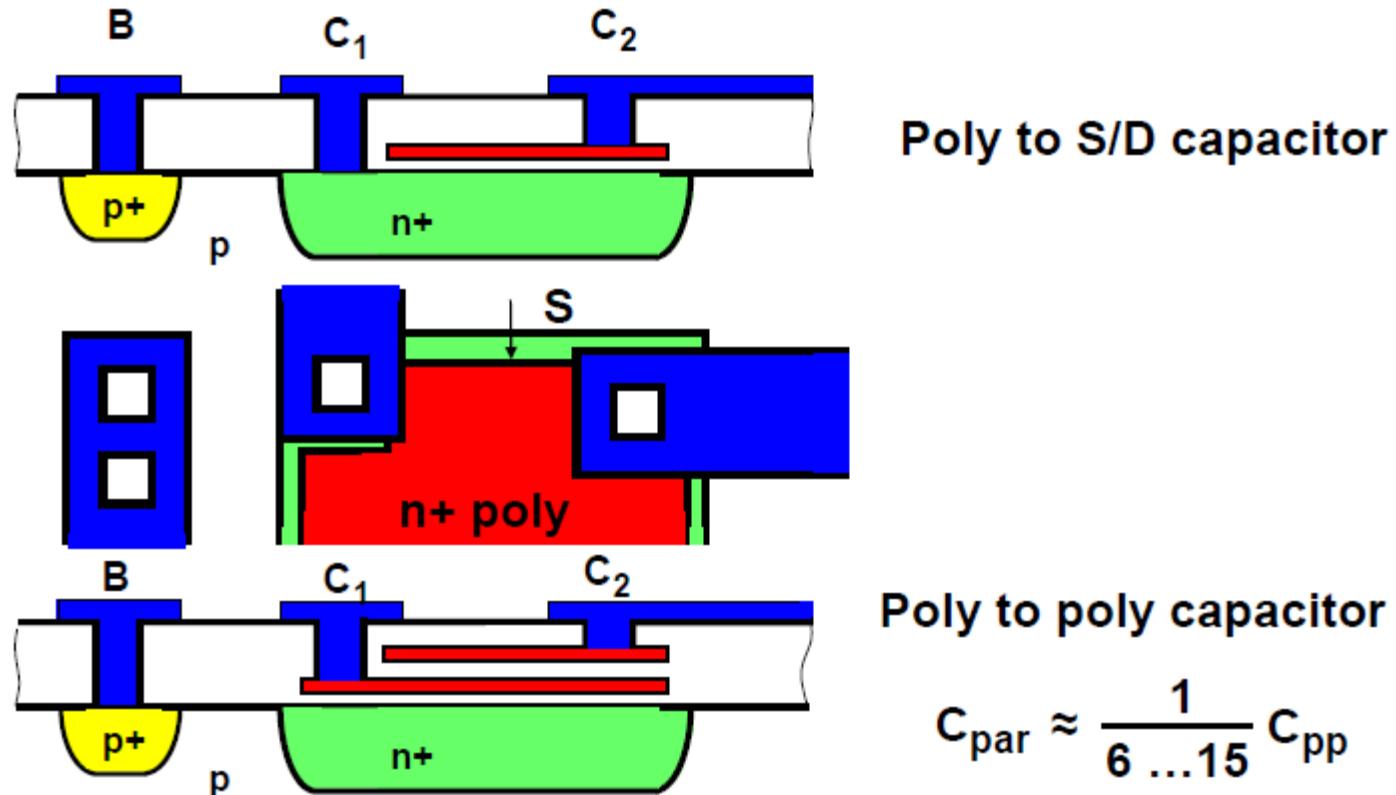
Source/drain  
diffusion  
resistor  
in CMOS

Ref.: Laker, Sansen :  
Design of analog ...,  
MacGrawHill 1994  
Table 2-6

# 》》 电阻的实现方式及对比

Process	Type	$\rho$ □ Ω/□	absolute accuracy percent	temperature coefficient percent/°C	voltage coefficient percent/V	breakdown voltage V
CMOS	S/D diffusion	20-50	20	0.2	0.5	20
	well	2.5k	10	0.3	1	20
	poly gate	50	20	0.2	0.05	40
	poly resistance	1.5k	1	0.05	0.02	20
	aluminum	50m	20	0.01	0.02	90

## 》》电容的版图



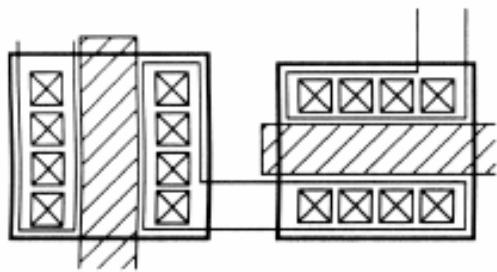
# 电容的实现方式及对比

Process	Type	C Nf/cm <sup>2</sup>	absolute accuracy percent	temperature coefficient percent/°C	voltage coefficient percent/V	breakdown voltage V
CMOS	$C_{ox}(50nm)$	70	5	0.002	0.005	40
	$C_{m,poly}$	12	10	0.002	0.005	40
	$C_{poly,poly}$	56	2	0.002	0.005	40
	$C_{poly,substrate}$	6.5	10	0.01	0.05	20
	$C_{m,substrate}$	5.2	10	0.01	0.05	20
	$C_{poly,substrate}$	6.5	10	0.01	0.05	20

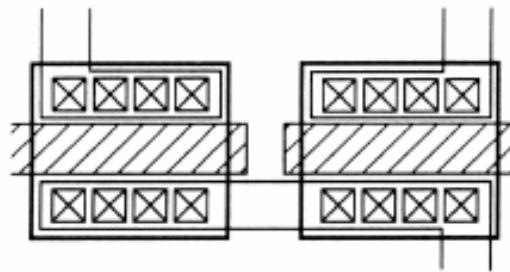
Ref.:Laker,Sanse:  
Design of analog...,  
MacGrawHill 1994  
Table 2-7

## 》》差分对的匹配

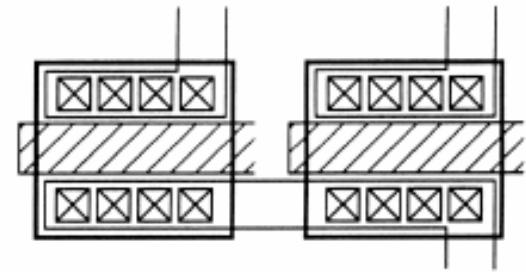
---



**Bad**



**Better**



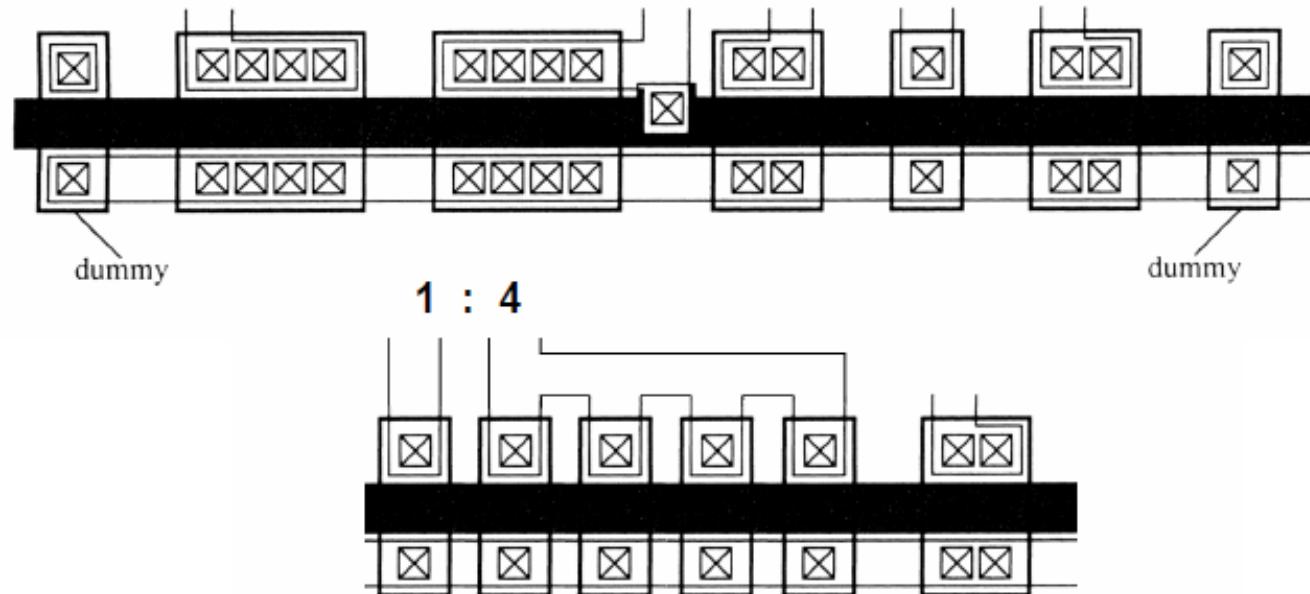
**Better**

# 》》 同样的长宽比

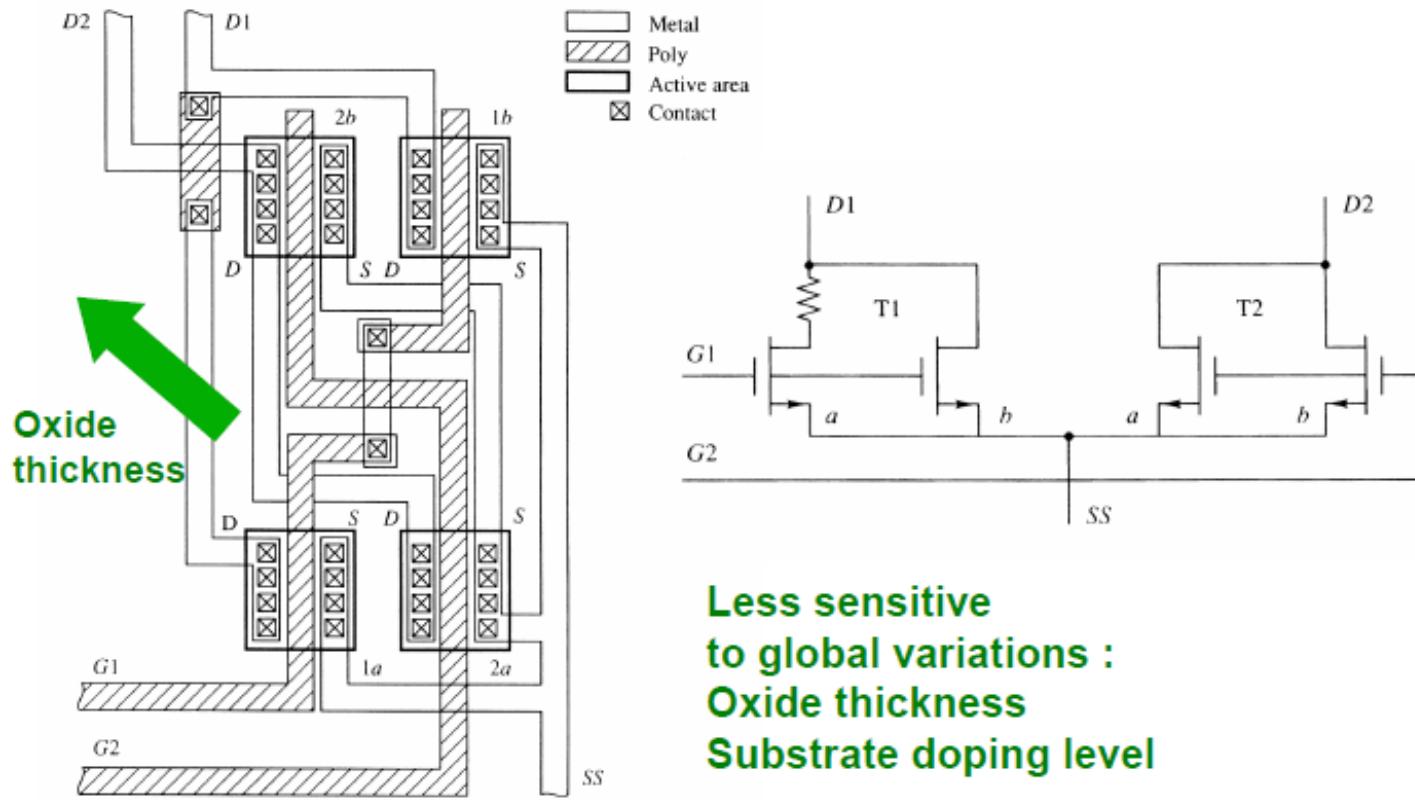
---

Current mirror 4:4:2:1:2 with end dummies.

 Metal       Active area  
 Poly       Contact

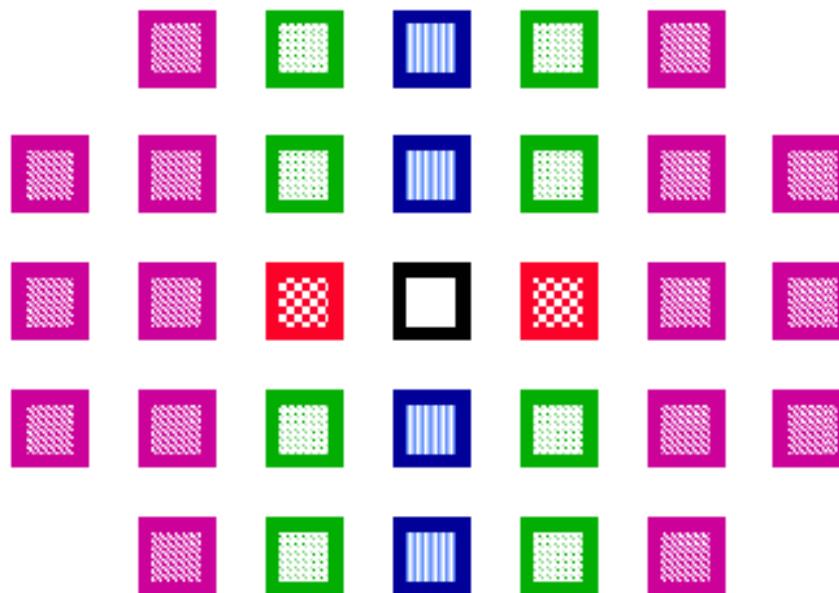


# 》》 中心对称的版图



## 》》 中心对称的版图

---



Ratio:

1  
2  
4  
8  
16



## 设计思路总结

---

1. 与噪声的设计一样，差分对需要 $V_{GST}$ 尽量小，而电流镜则需要 $V_{GST}$ 尽量大
2. 失调的优化与CMRR的优化相同，超高的CMRR往往需要后期的Calibration.
3. 在版图中选择合适的被动器件，以及优秀的版图技巧都能改善失调的现象。

## 》》课后作业

---

- 对一偏置电流为 $100\mu A$ 的五管OTA，共模电压为 $0.9V$ ，设计晶体管的尺寸，
  1. 使其在单位增益负反馈时系统性失调小于 $0.1mV$ ；
  2. 使其随机性失调的标准差 (std) 小于 $1mV$ ；
  3. 使其共模抑制比大于  $50dB$ 。

