Some

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Je reviendrai et je serai des millions. «Spartacus»

0.1 chap1

Comparison of MOST and Bipolar transistor models

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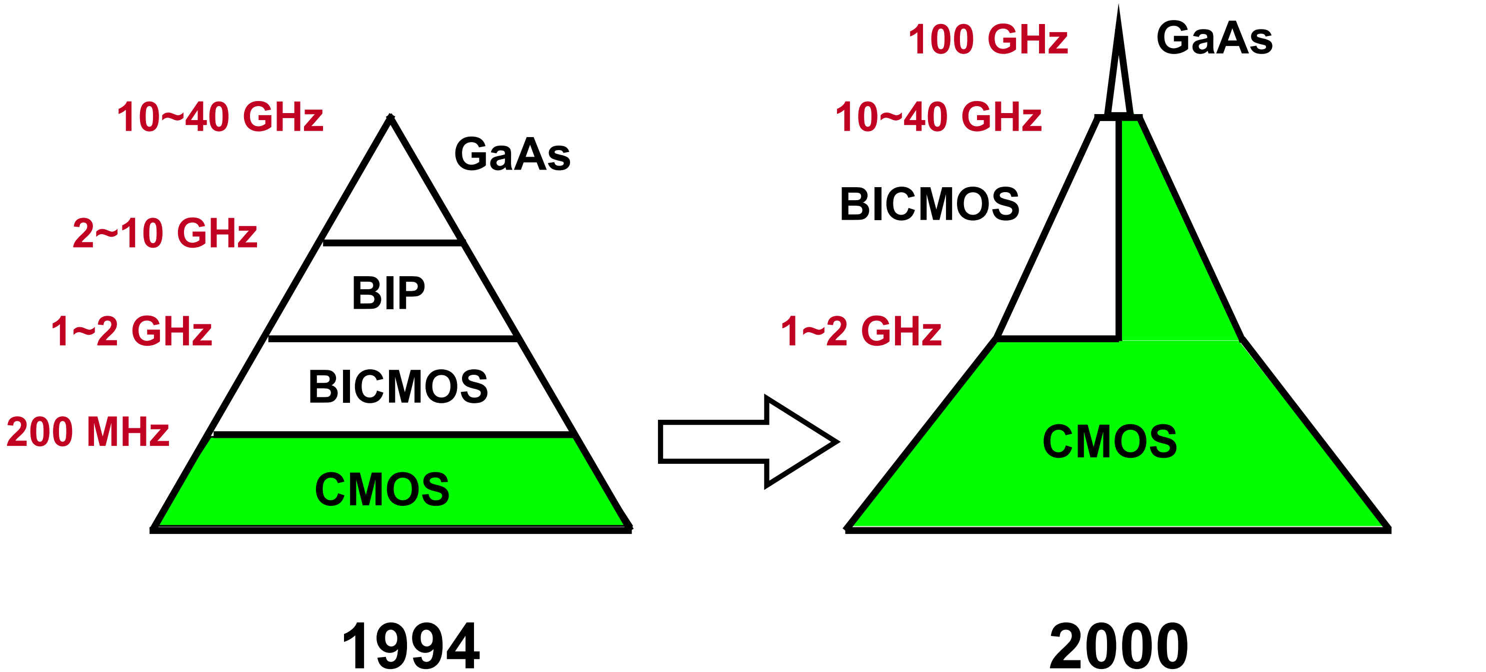
Ref.: W. Sansen : Analog Design Essentials, Springer 2006

From Bipolar to MOST transistors



Ref.Toshiba

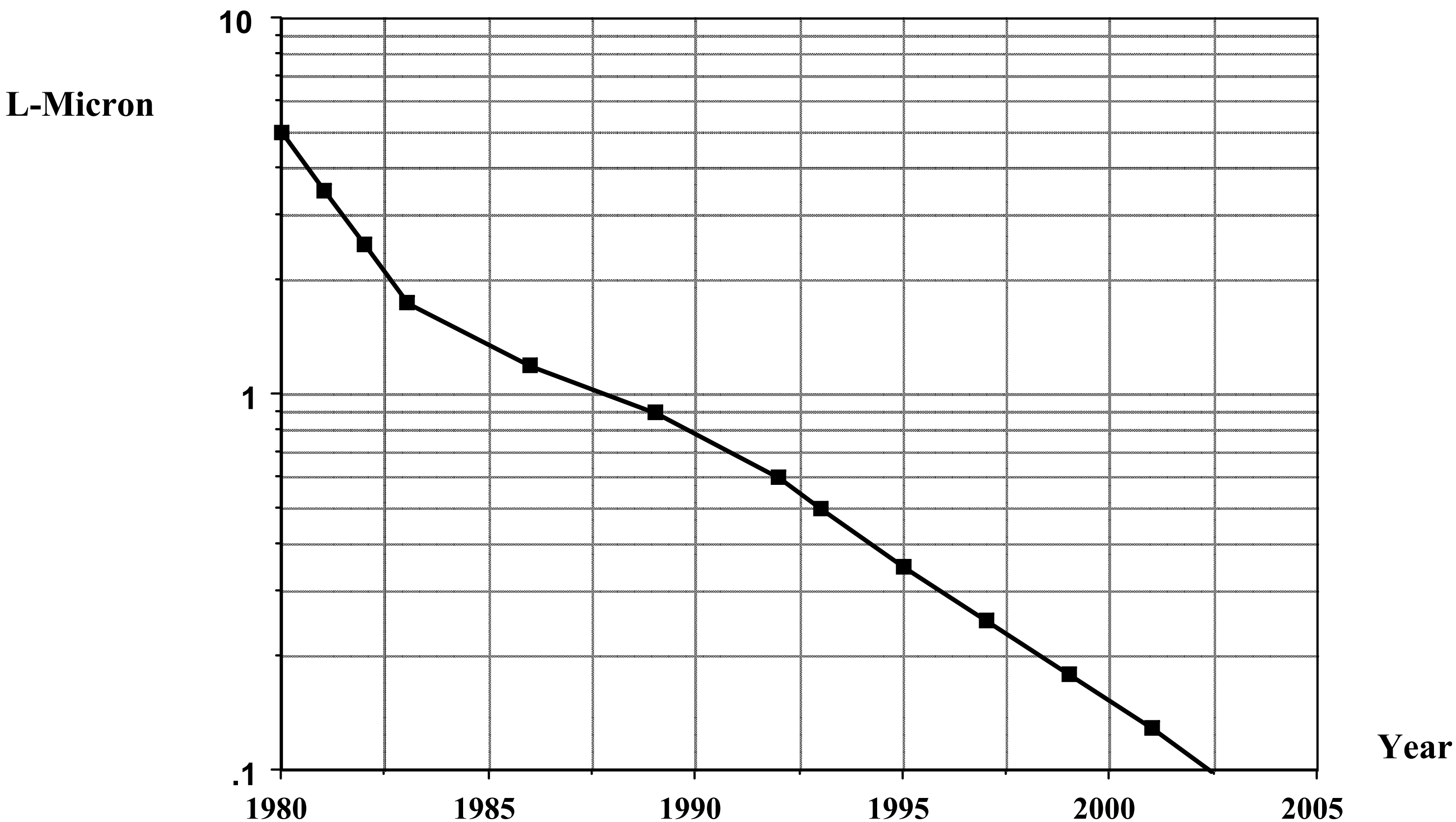
The SIA roadmap

Year	Lmin μm	Bits/chip Gb/chip	Trans/chip millions/chip	Clock MHz	Wiring
1995	0.35	0.064	4	300	4 - 5
1998	0.25	0.256	7	450	5
2001	0.18	1	13	600	5 - 6
2004	0.13	4	25	800	6
2007	0.09	16	50	1000	6 - 7
2010	0.065	64	90	1100	7 - 8

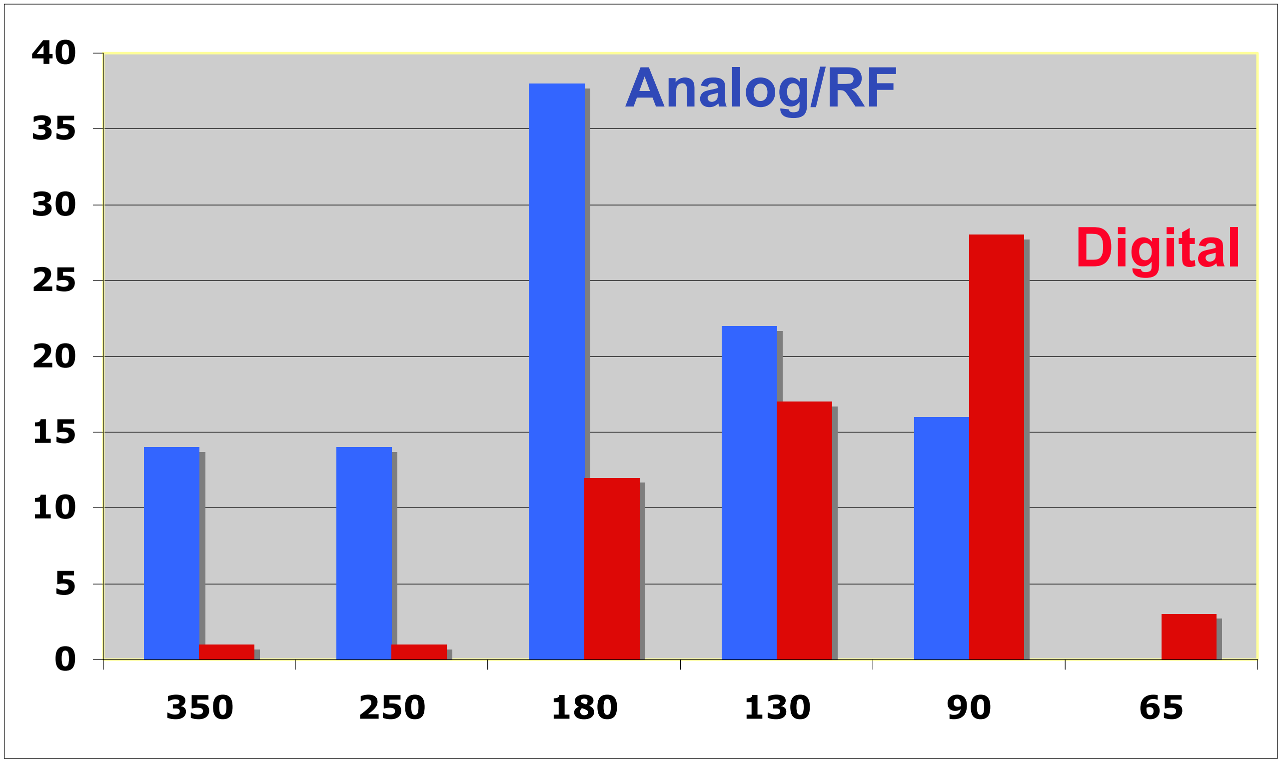
2003

Semiconductor Industry Association

The law of Moore

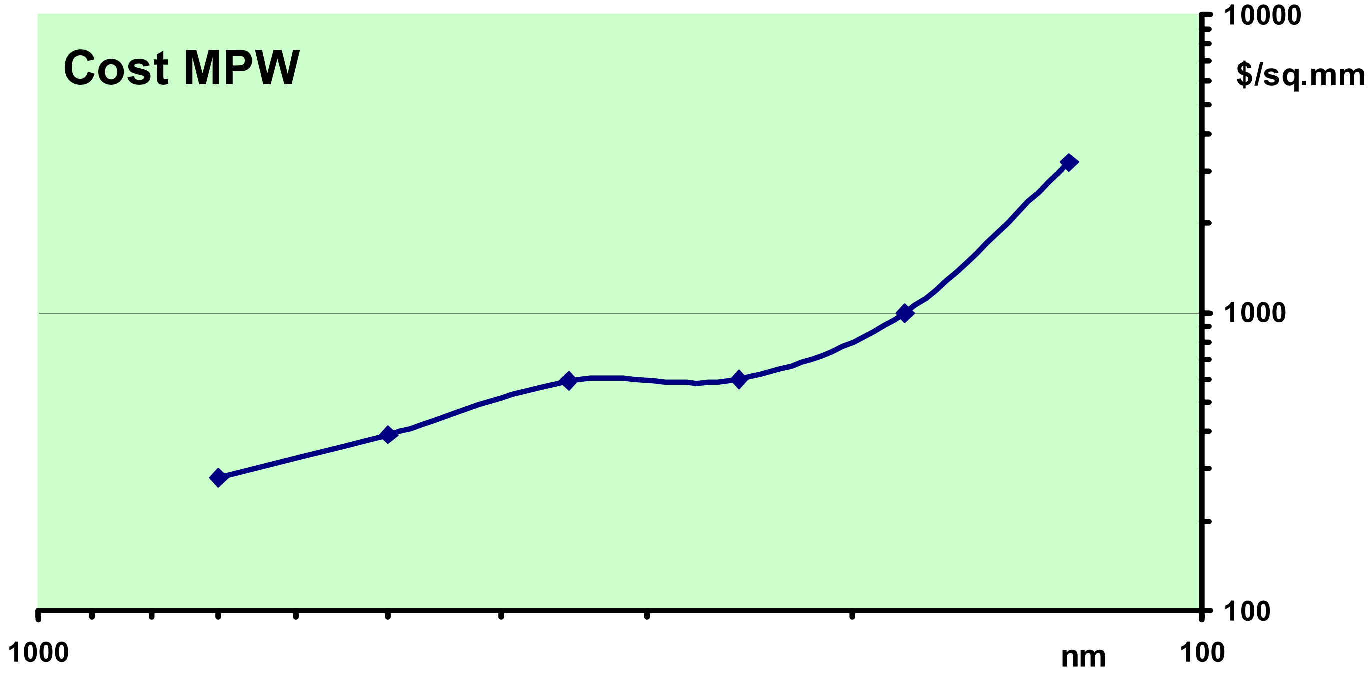


ISSCC 2005 paper distribution

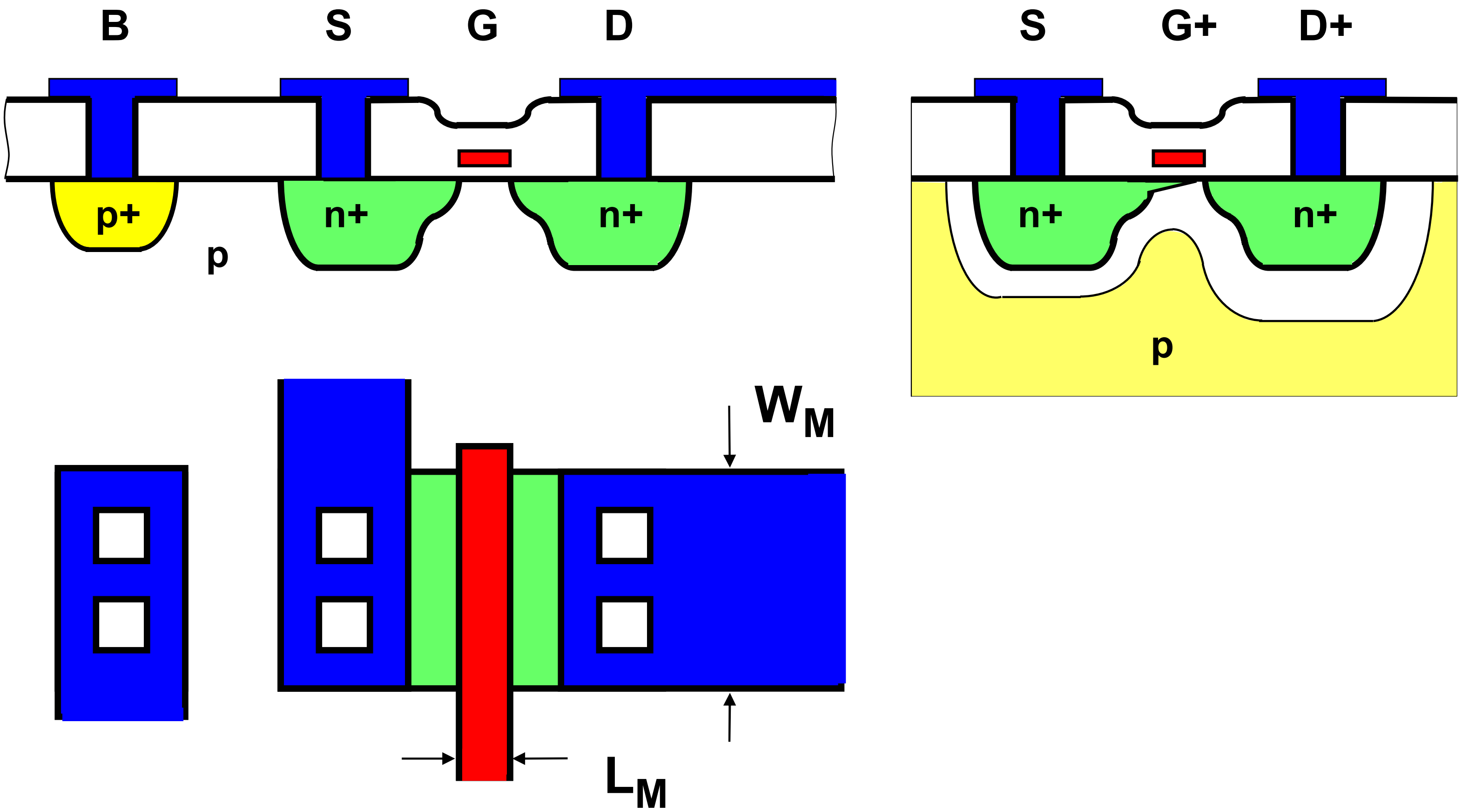


nm L_{min}

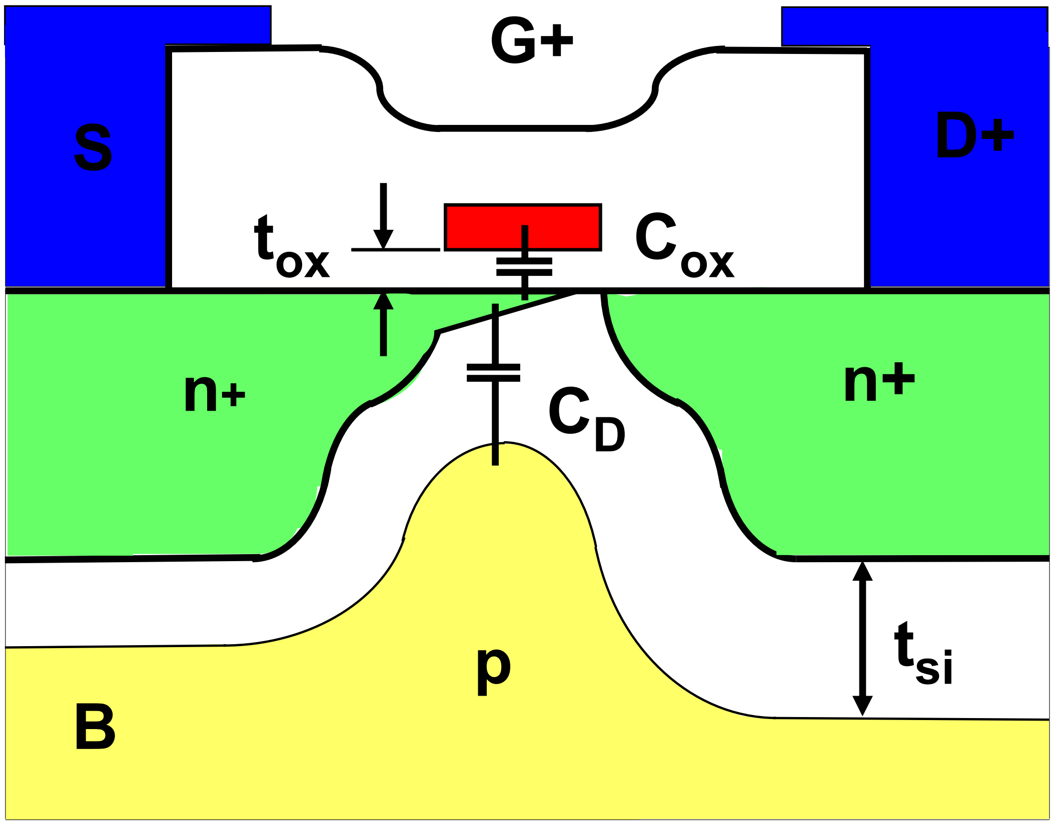
Price MPW silicon for different L (nm)



MOST layout



MOST layout : C_{ox} and C_D



$$C_D = \frac{\varepsilon_{si}}{t_{si}}$$

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$$

$$\frac{C_D}{C_{ox}} = n - 1$$

MOST layout : C_{ox} and C_D values

$$C_D = \frac{\varepsilon_{si}}{t_{si}} \qquad t_{si} = \sqrt{\frac{2\varepsilon_{si}(\phi - V_{BD})}{qN_B}}$$

ε_{si} = 1 pF/cm

ε_{ox} = 0.34 pF/cm

$\phi \approx 0.6$ V

$q = 1.6\ 10^{-19}$ C

$N_B \approx 4\ 10^{17}\ cm^{-3}$

Example : L = 0.35 μm; W/L = 8

V_{BD} = -3.3 V : t_{si} = 0.1 μm

$C_D \approx 10^{-7}\ F/cm^2$

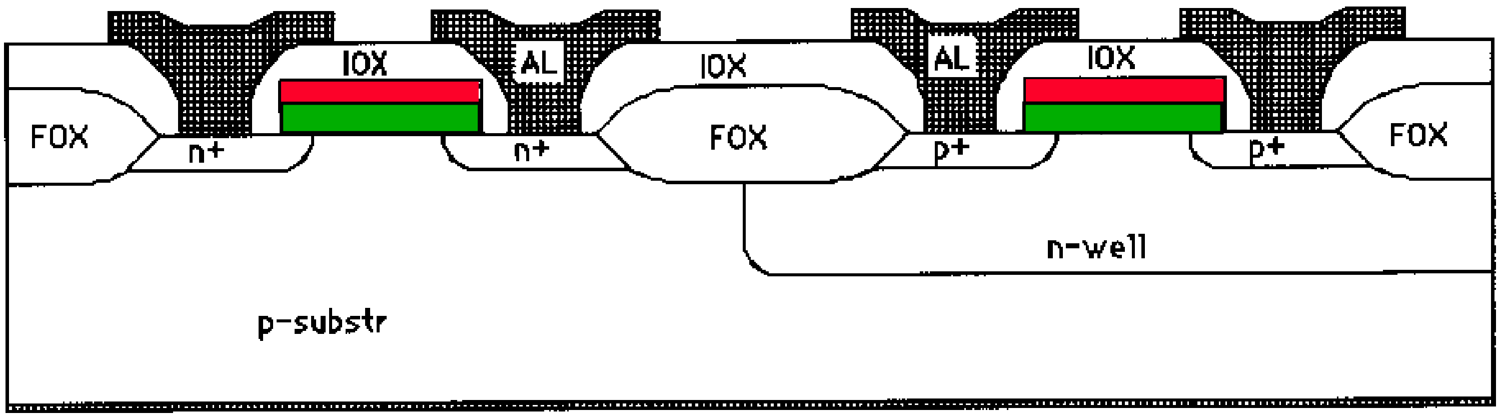
$$t_{ox} = \frac{L_{min}}{50}$$

t_{ox} = 7 nm

$C_{ox} \approx 5\ 10^{-7}\ F/cm^2$

$$\frac{C_D}{C_{ox}} = n - 1 \approx 0.2$$

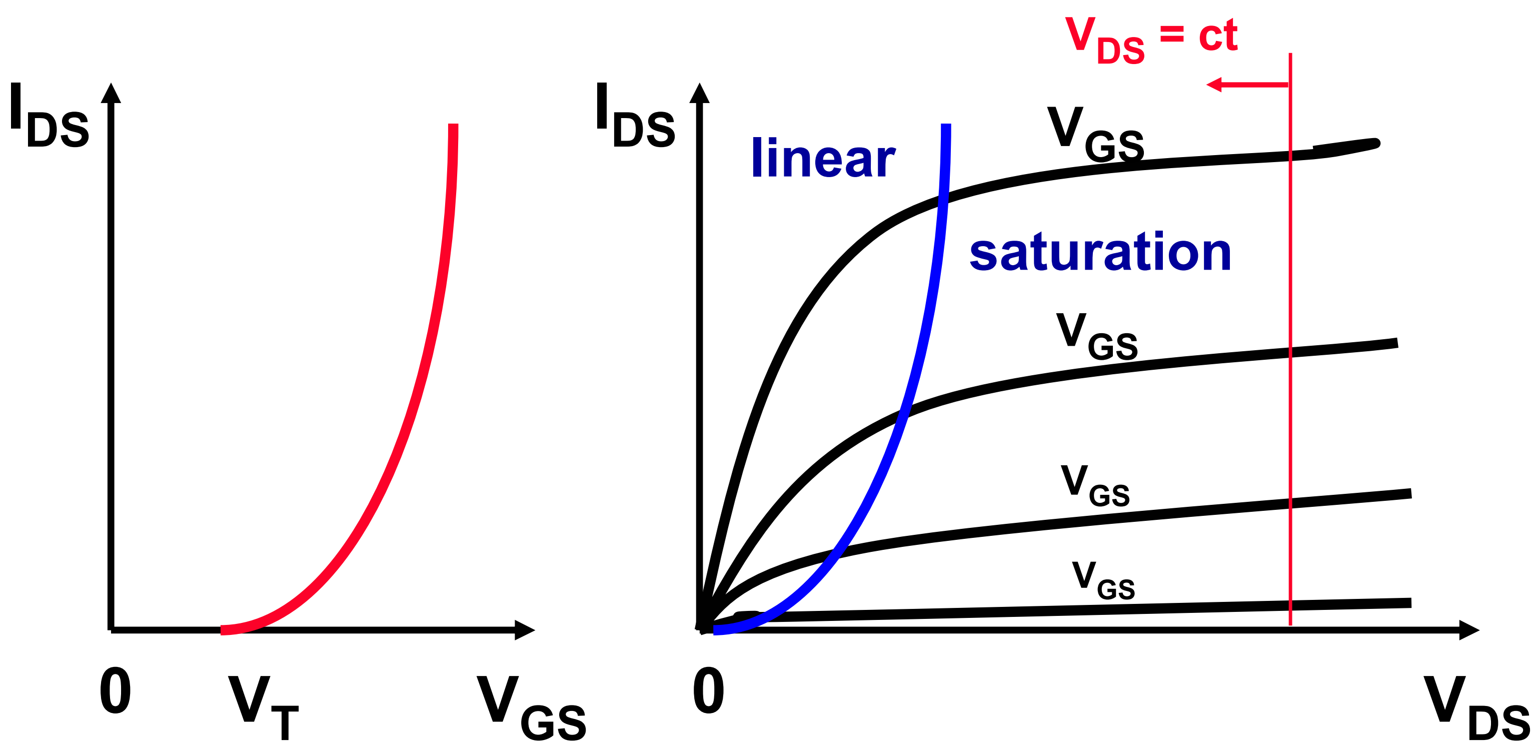
N-well CMOS technology



Gate oxyde

Polysilicon gate

MOST I_{DS} versus V_{GS} and V_{DS}



Saturation because high V_{DS}

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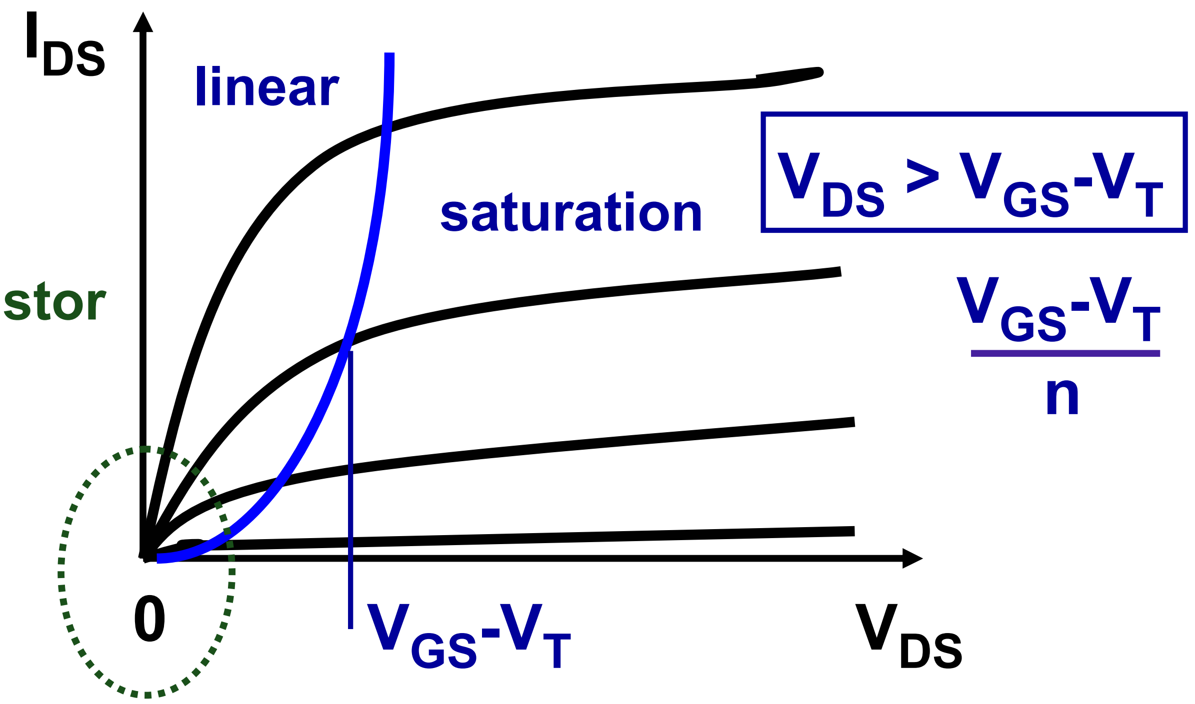
- **Models of Bipolar transistors**
- **Comparison of MOSTs & Bipolar transistors**

MOST I_{DS} versus V_{DS}

Linear resistor

$$R_{on} = \frac{1}{\beta (V_{GS}-V_T)}$$

$$\beta = KP \frac{W}{L}$$



MOST parameters β , KP , C_{ox} , ...

$$\beta = KP \frac{W}{L}$$

$$KP = \mu C_{ox}$$

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$$

$$t_{ox} = \frac{L_{min}}{50}$$

$KP_n \approx 300\ \mu A/V^2$

$C_{ox} \approx 5\ 10^{-7}\ F/cm^2$

$\varepsilon_{ox} = 0.34\ pF/cm$

$\varepsilon_{si} = 1\ pF/cm$

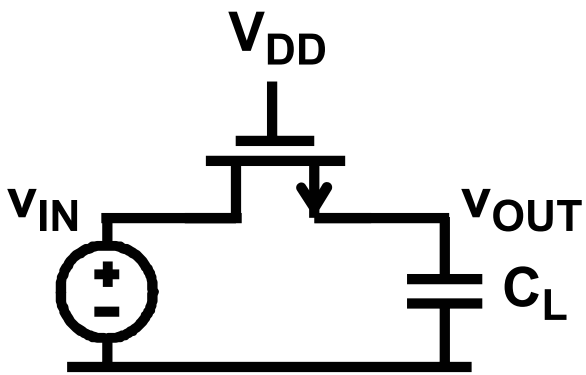
$t_{ox} = 7\ nm$

$L_{min} = 0.35\ \mu m$

$\mu_p \approx 250\ cm^2/Vs$

$\mu_n \approx 600\ cm^2/Vs$

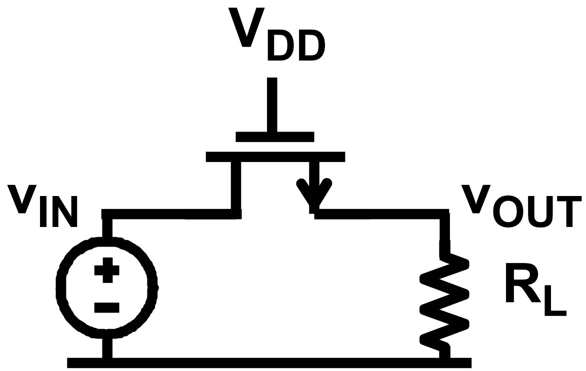
Example : Analog switch on CL

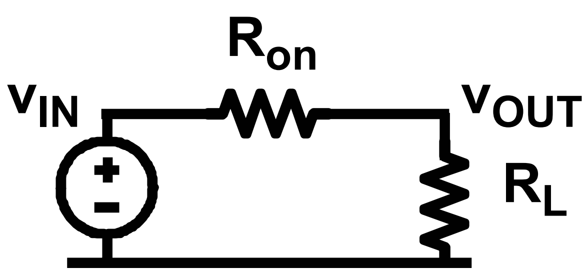


We want to switch 0.6 V to a load capacitance C_L of 4 pF.
We want to do this fast, with time constant 0.5 ns.
Supply voltage V_{DD} = 2.5 V
V_T = 0.5 V
Use standard 0.35 μm CMOS.

Choose minimum channel length and find an average V_{GS} !

Example : Analog switch on RL





We want to switch 0.6 V to a load resistor R_L of 5 kΩ.
W/L = 8
Supply voltage V_{DD} = 2.5 V
0.35 μm CMOS: V_T = 0.5 V
v_{OUT} ? R_{on} ?

Choose minimum channel length !

Body effect - Parasitic JFET

$$V_T = V_{T0} + \gamma \left[\sqrt{|2\Phi_F| + V_{BS}} - \sqrt{|2\Phi_F|} \right]$$

$$n = \frac{\gamma}{\sqrt{|2\Phi_F| + V_{BS}}} = 1 + \frac{C_D}{C_{ox}}$$

$|2\Phi_F| \approx 0.6$ V

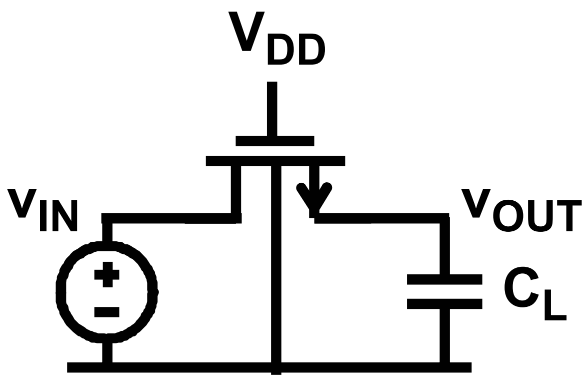
$n \approx 1.2 \ldots 1.5$

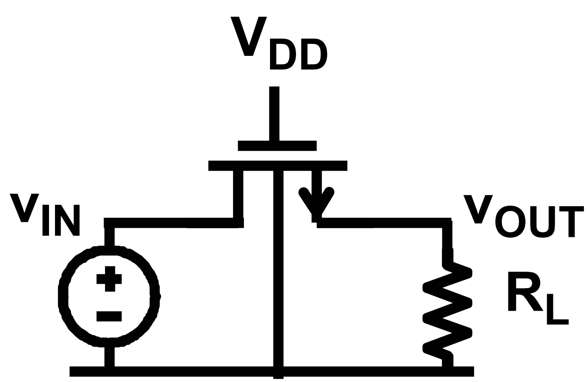
$\gamma \approx 0.5 \ldots 0.8\ V^{1/2}$

Reverse v_{BS} increases $|V_T|$ and decreases $|i_{DS}|$!!!

$n = 1/\kappa$ subthreshold gate coupling coeff. Tsividis

Ex. : Analog switch with nonzero VBS





Switch 0.6 V to a
load capacitance C_L of 4 pF
or a load resistor R_L of 5 kΩ.
W/L = 8 (R_{on} = 125 Ω @ V_{BS} = 0)
Supply voltage V_{DD} = 2.5 V
0.35 μm CMOS: V_T = 0.5 V
v_{OUT} ? for γ = 0.5 V^{-1}

Start with V_{BS} = 0.

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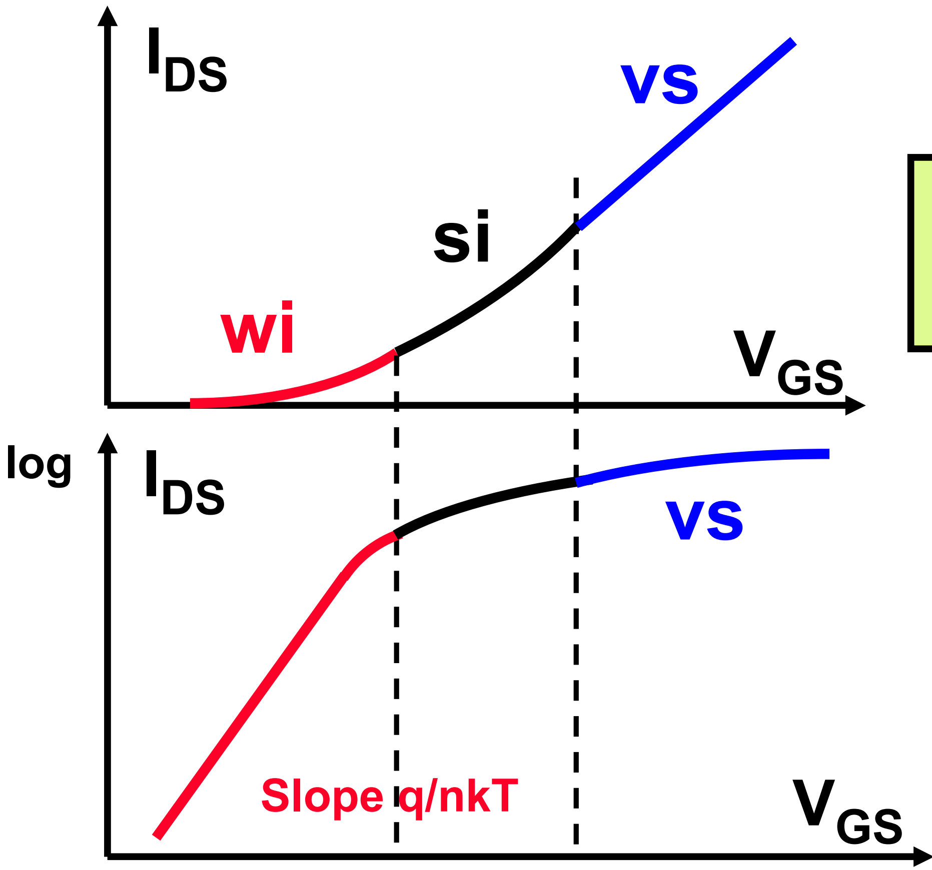
MOST I_{DS} versus V_{GS}



$I_{DS} \sim (V_{GS}-V_T)$

$$I_{DS} = K'_n \frac{W}{L}(V_{GS}-V_T)^2$$

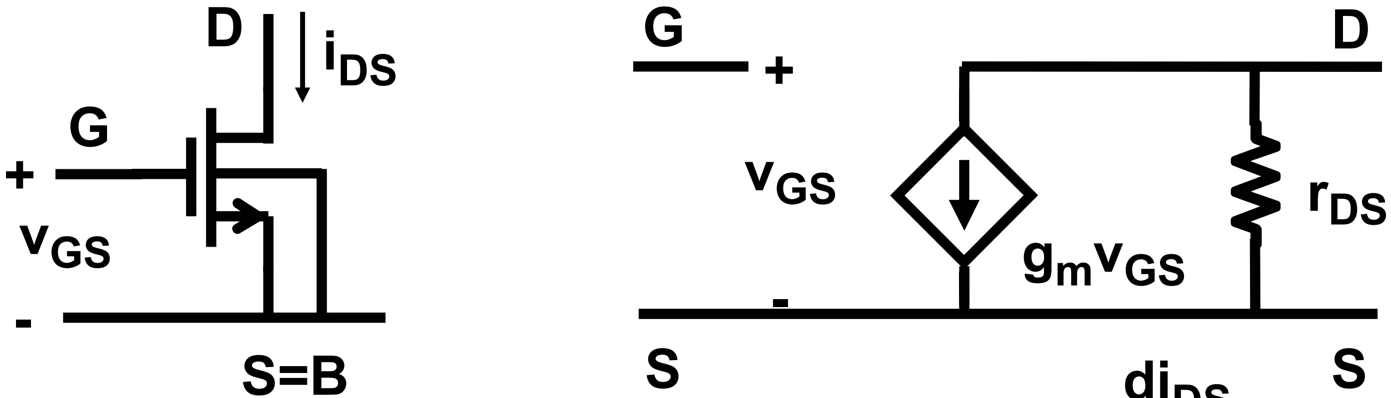
$K' = \frac{KP}{2\,n}$ n = ??

$K'_n \approx 100\ \mu A/V^2$

$K'_p \approx 40\ \mu A/V^2$

$$I_{DS} \sim \exp \frac{V_{GS}}{nkT/q}$$

MOST small-signal model : g_m & r_{DS}



$$g_m = \frac{di_{DS}}{dv_{GS}}$$

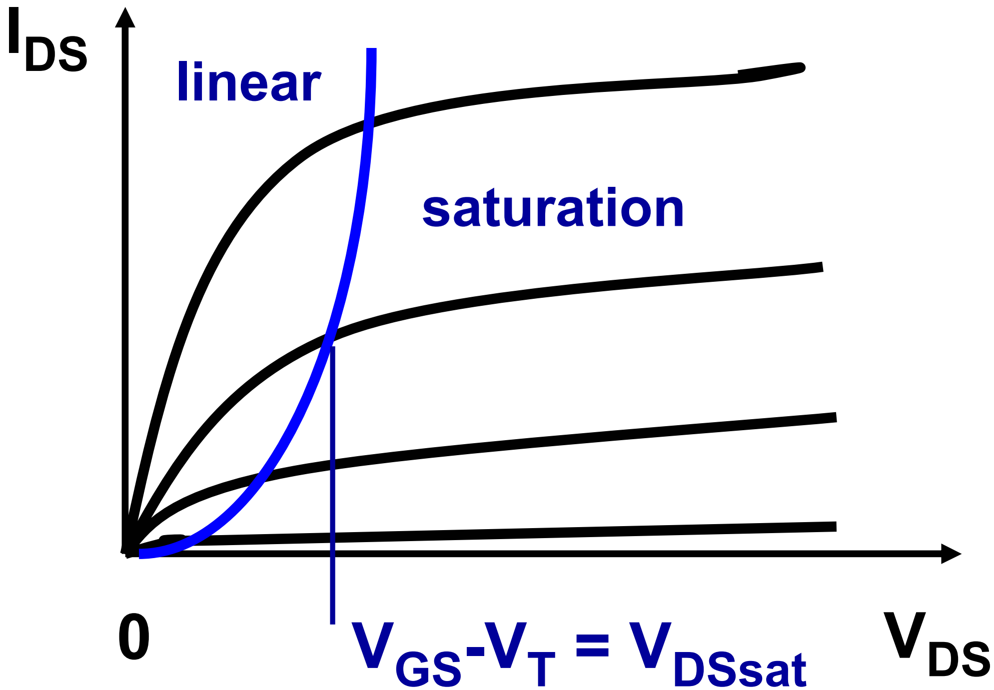
$$g_m = 2K'_n \frac{W}{L} (V_{GS}-V_T) = 2\sqrt{K'_n \frac{W}{L} I_{DS}} = \frac{2\, I_{DS}}{V_{GS}-V_T}$$

The transconductance g_m

Is $g_m \sim \sqrt{I_{DS}}$

or $\sim I_{DS}$?

MOST small-signal model : r_{DS}



$$r_{DS} = r_o = \frac{V_E L}{I_{DS}}$$

$$\lambda = \frac{1}{V_E L}$$

V_{En} = 4 V/µmL

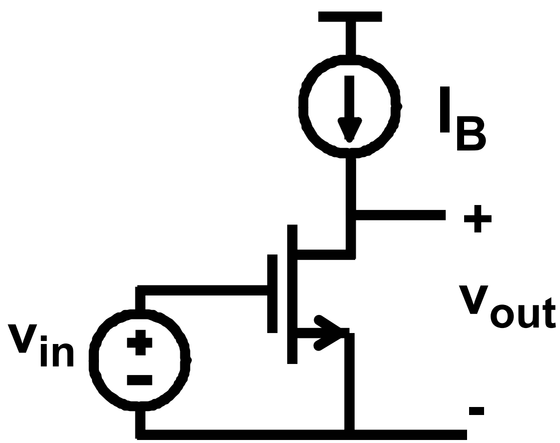
L = 1 µm

I_{DS} = 100 µA

r_o = 40 kΩ

$$I_{DS} = K'_n \frac{W}{L} (V_{GS}-V_T)^2 (1 + \lambda V_{DS})$$

MOST single-transistor gain A_v



$$A_v = g_m r_{DS} = \frac{2\ V_E\ L}{V_{GS}-V_T}$$

$$A_v \approx 100$$

If $\quad V_E L \approx 10$ V

and $\quad V_{GS}-V_T \approx 0.2$ V

Design for high gain :

	High gain	High speed
$V_{GS}-V_T$	Low (0.2 V)	
L	High	

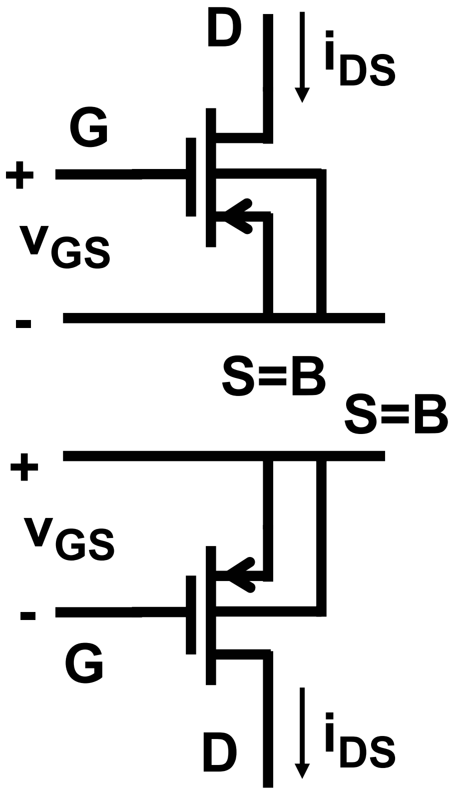
$V_{GS}-V_T$ sets the ratio g_m/I_{DS} !

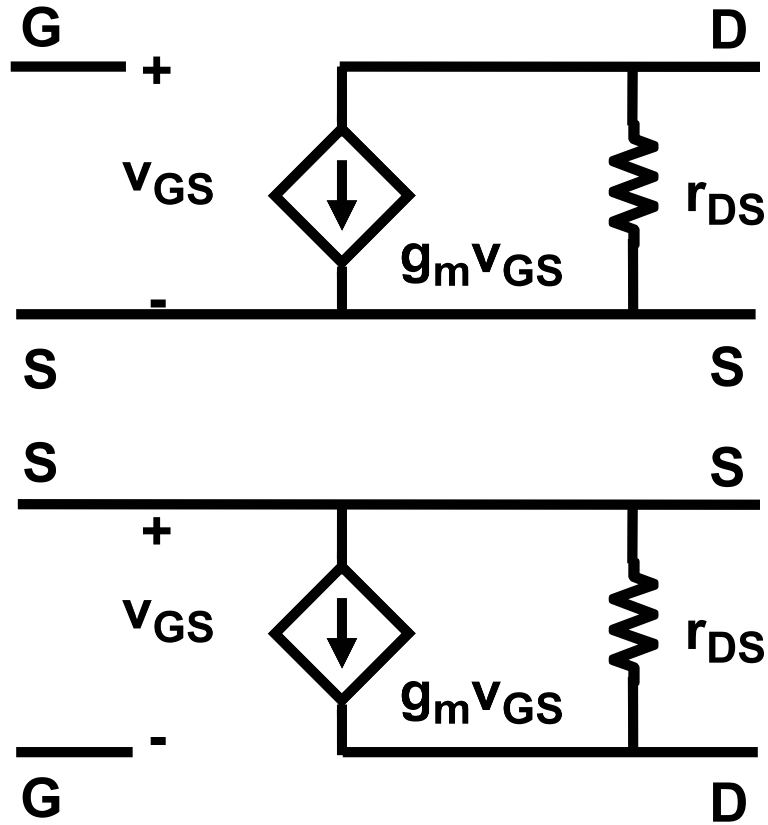
Example: single-transistor amplifier

We want to realize a three-stage amplifier with a total gain of 10.000.
We use three single-transistor stages in series.
What minimum lengths do we have to use in an advanced 65 nm CMOS technology with V_E = 4 V/μm ?

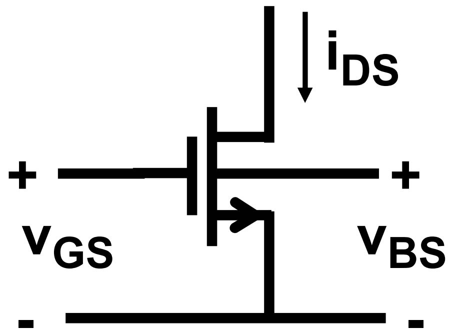
Choose
V_{GS}-V_T = 0.2 V !

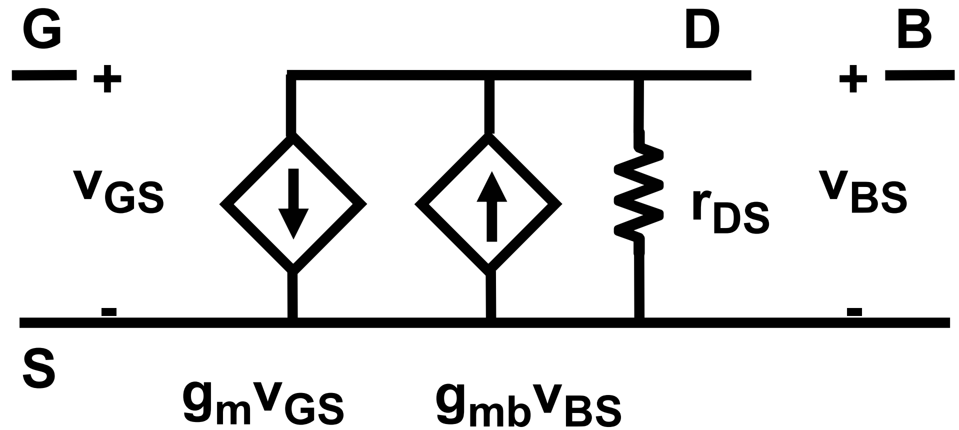
pMOST small-signal model





MOST small-signal model: g_m & g_{mb}





$$g_m = \frac{di_{DS}}{dv_{GS}} \qquad g_{mb} = \frac{di_{DS}}{dv_{BS}}$$

$$\boxed{\frac{g_{mb}}{g_m} = \frac{C_D}{C_{ox}} = n - 1}$$

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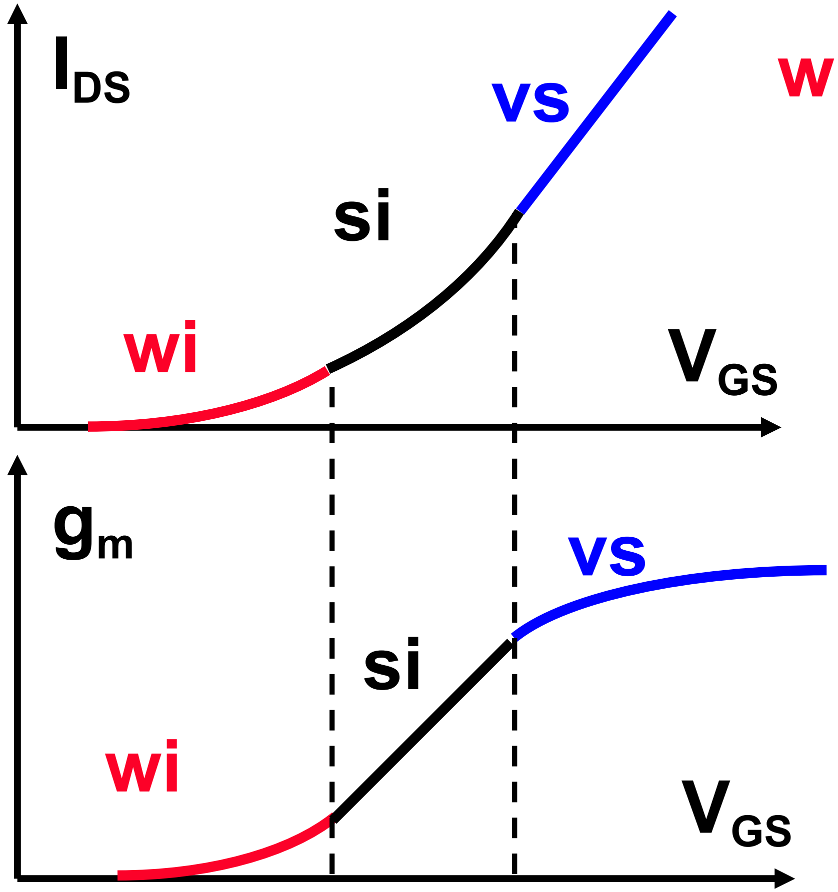
I_{DS} & g_m versus V_{GS} : weak inversion



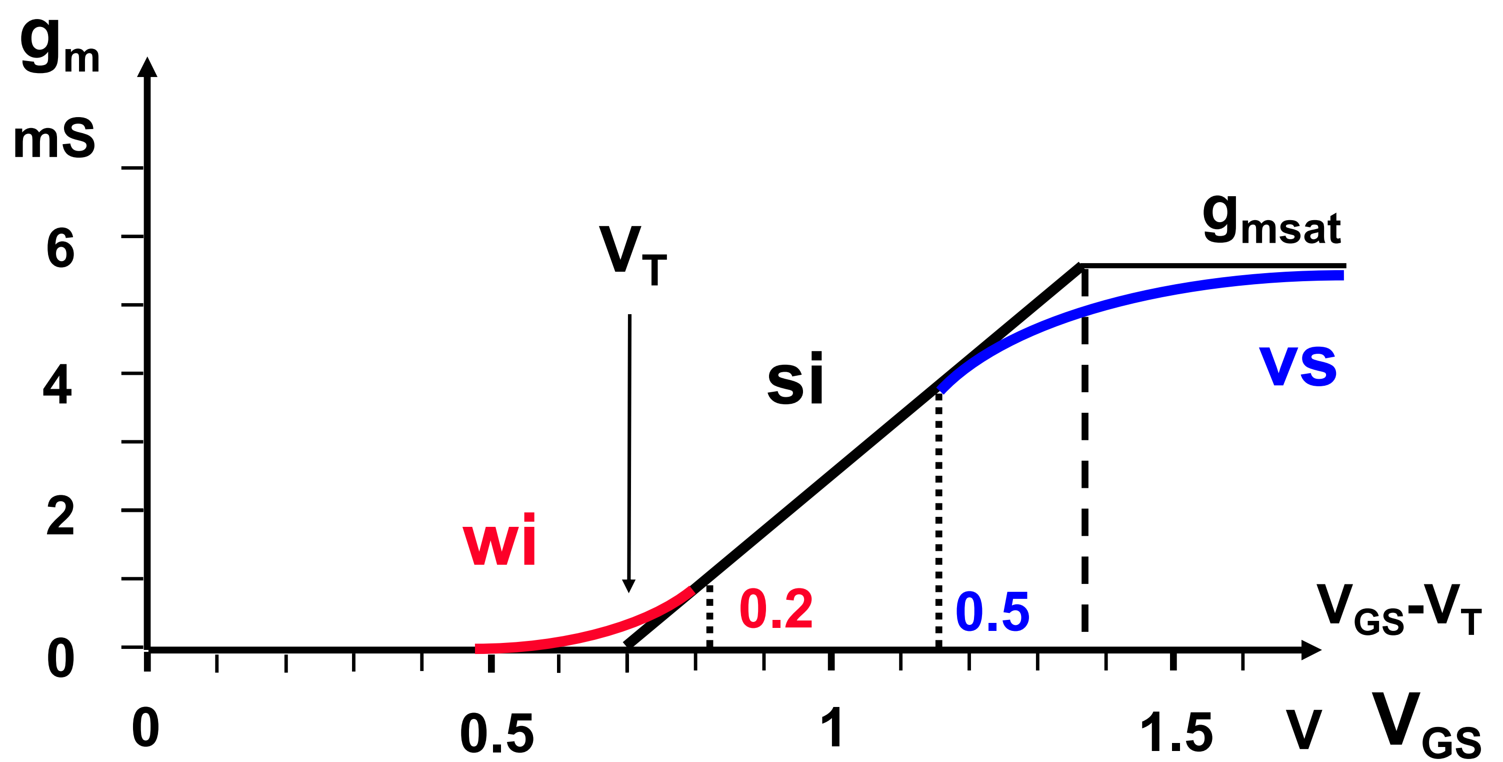
wi : weak inversion

$$I_{DSwi} = I_{D0}\,\frac{W}{L}\,\exp\frac{V_{GS}}{nkT/q}$$

Subthreshold slope :
nkT/q ln(10)

$$g_{mwi} = \frac{I_{DSwi}}{nkT/q}$$

Transconductance g_m versus V_{GS}



Transition voltage V_{GSt} between wi & si

$$I_{DSwi} = I_{D0} \frac{W}{L} \exp^{\frac{V_{GS}}{nkT/q}}$$

$$I_{DS} = K'_n \frac{W}{L} (V_{GS}-V_T)^2$$

$$g_{mwi} = \frac{I_{DSwi}}{nkT/q}$$

$$g_m = \frac{2\, I_{DS}}{V_{GS}-V_T}$$

$$\frac{g_{mwi}}{I_{DSwi}} = \frac{1}{nkT/q}$$

$$\frac{g_m}{I_{DS}} = \frac{2}{V_{GS}-V_T}$$

$$\boxed{(V_{GSt}-V_T)_t = 2n \frac{kT}{q}}$$

Transition Voltage V_{GSt} for different L

$$\boxed{(V_{GSt}-V_T)_t = 2n\,\frac{kT}{q}}$$

$$I_{DSt} \approx K'_n\,\frac{W}{L}\left(2n\,\frac{kT}{q}\right)^2$$

Is independent of channel length L

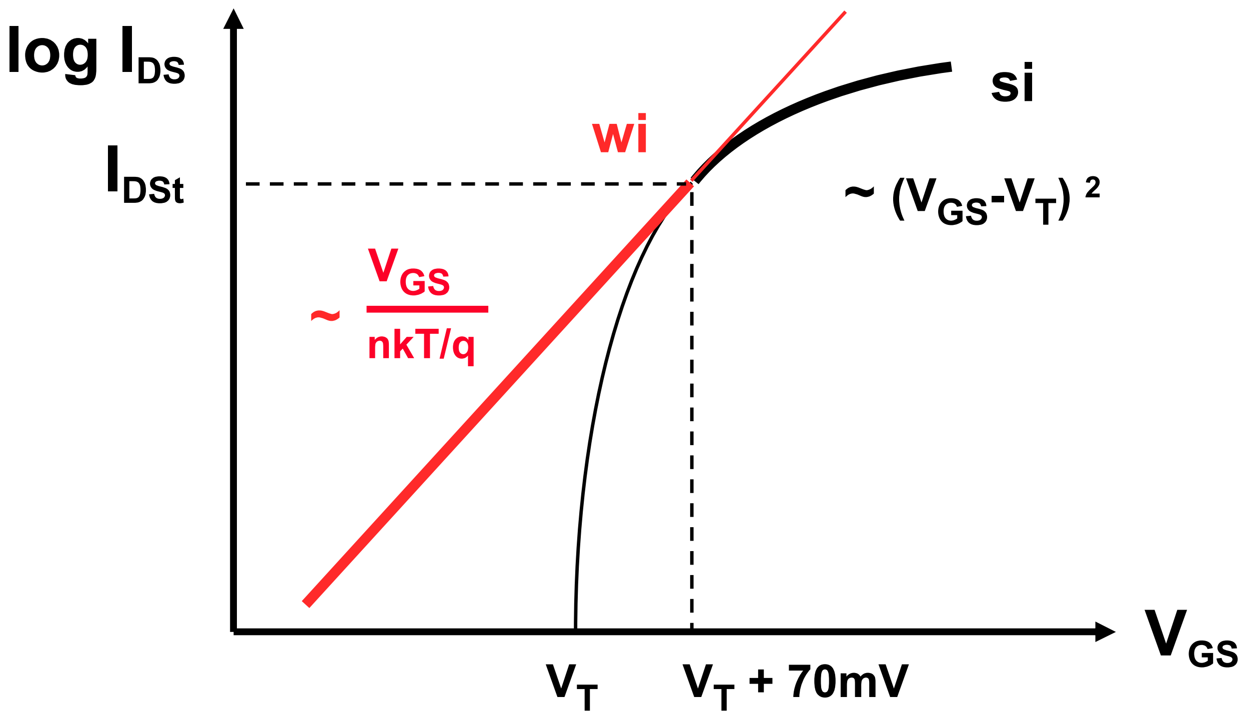
Is still true in … years !

$$(V_{GSt}-V_T)_t = 2n\,\frac{kT}{q} \approx 70\ \text{mV}$$

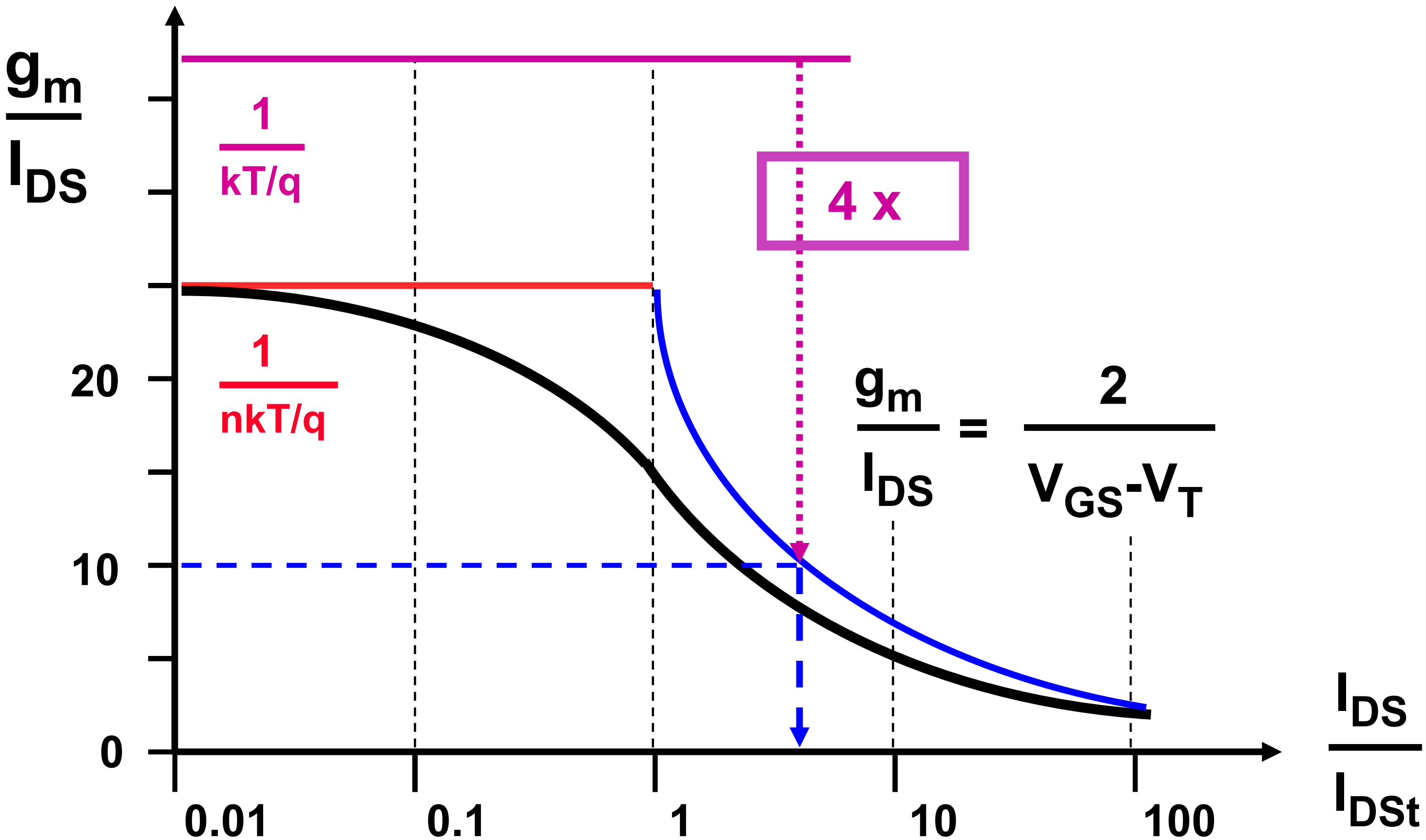
$$I_{DSt} \approx 2\ \mu\text{A for}\ \frac{W}{L} = 10$$

for nMOST

Transition wi - si



Ratio g_m/I_{DS} at the transition wi - si



EKV model for smooth wi-si transition

$$I_{DS} = K' \frac{W}{L} V_{GSTt}^2 \left[\ln (1 + e^v) \right]^2 \qquad V_{GST} = V_{GS} - V_T \qquad K' = \frac{KP}{2n}$$

$$v = \frac{V_{GST}}{V_{GSTt}} \qquad V_{GSTt} = (V_{GS} - V_T)_t = 2n \frac{kT}{q}$$

$$\approx 70 \text{ mV}$$

Small v : $\ln (1 + e^v) \approx e^v$

$$I_{DS} = K' \frac{W}{L} V_{GSTt}^2 e^{2v} = \underbrace{K' \frac{W}{L} V_{GSTt}^2}_{I_{DSt}} \exp \left(\frac{V_{GS} - V_T}{n\, kT/q} \right)$$

Large v : $\ln (1 + e^v) \approx v$

$$I_{DS} = K' \frac{W}{L} V_{GSTt}^2 v^2 = K' \frac{W}{L} (V_{GS} - V_T)^2$$

Enz, AICSP '95,
83-114
Cunha, JSSC Oct.98
1510-1519

Transition current I_{DSt} between wi & si

$$I_{DSt} = I_{DS}\Big|_{\substack{V=1\\ i=1}} = K' \frac{W}{L} V_{GSTt}^{2}$$

$I_{DSt} = 2\ \mu A$ for $W/L = 10$

$$i = \frac{I_{DS}}{I_{DSt}} = [\ \ln(1 + e^{v})\ ]^{2}$$

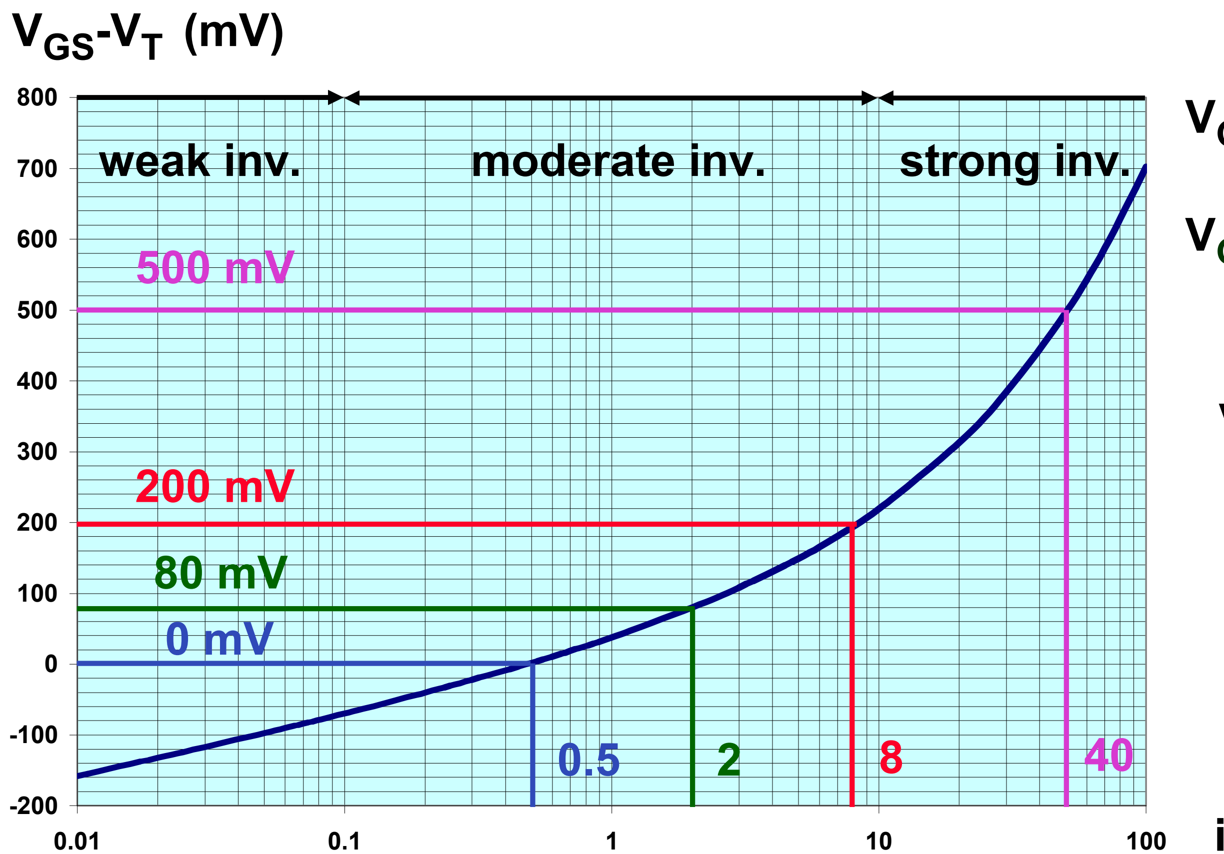
inversion coefficient

$$v = \ln(e^{\sqrt{i}} - 1)$$

$$V_{GS} - V_T = V_{GSTt} \ln(e^{\sqrt{i}} - 1)$$

$$V_{GSTt} = 2n \frac{kT}{q} \approx 70\ mV$$

Relation V_{GS}-V_T and inversion coefficient i



V_{GS}-V_T =

V_{GSTt} ln (e $^{\sqrt{i}}$ - 1)

$$V_{GSTt} = 2n \frac{kT}{q}$$

$$i = \frac{I_{DS}}{I_{DSt}}$$

Transconductance g_m between wi & si

$$i = \frac{I_{DS}}{I_{DSt}} = [\ \ln(1 + e^{v})\ ]^2 \qquad g_m \approx \ldots.$$

$$GM = \frac{g_m}{I_{DS}} \frac{nkT}{q} = \frac{1 - e^{-\sqrt{i}}}{\sqrt{i}}$$

Large i : $GM = \dfrac{1}{\sqrt{i}}$

Small i : $GM = 1 - \dfrac{\sqrt{i}}{2}$

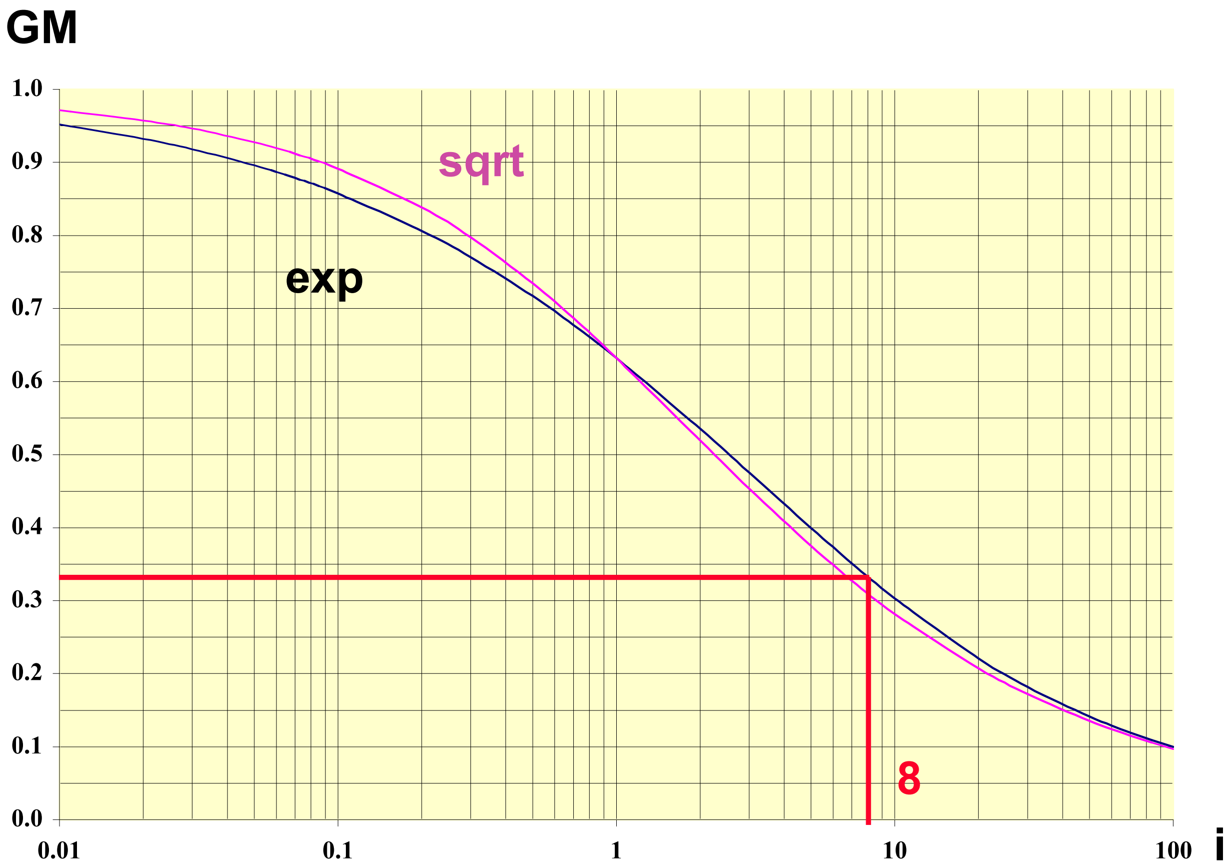
Alternative approximation :

$$GM = \frac{1}{\sqrt{1 + 0.5\sqrt{i} + i}}$$

Large i : $GM = \dfrac{1}{\sqrt{i}}$

Small i : $GM = 1 - \dfrac{\sqrt{i}}{4}$

GM versus inversion coefficient i



$$GM = \frac{g_m}{I_{DS}} \frac{nkT}{q}$$

$$GM =$$

$$\frac{1 - e^{-\sqrt{i}}}{\sqrt{i}}$$

$$\frac{1}{\sqrt{1 + 0.5\sqrt{i} + i}}$$

$$i = \frac{I_{DS}}{I_{DSt}}$$

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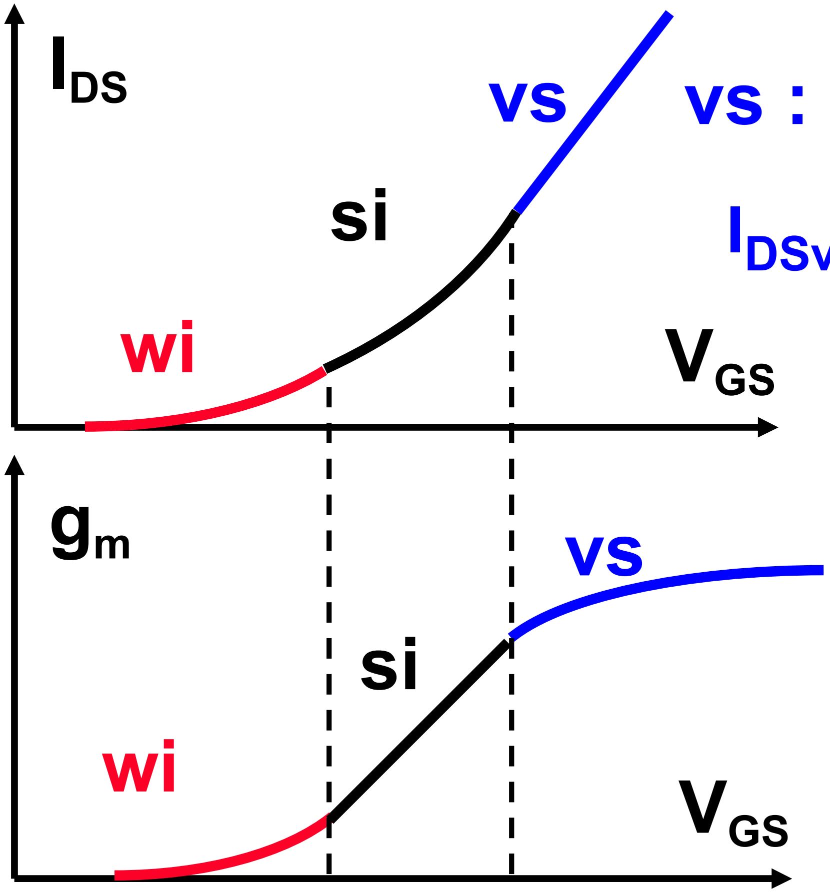
- **Comparison of MOSTs & Bipolar transistors**

I_{DS} & g_m vs V_{GS} : velocity saturation



vs : velocity saturation

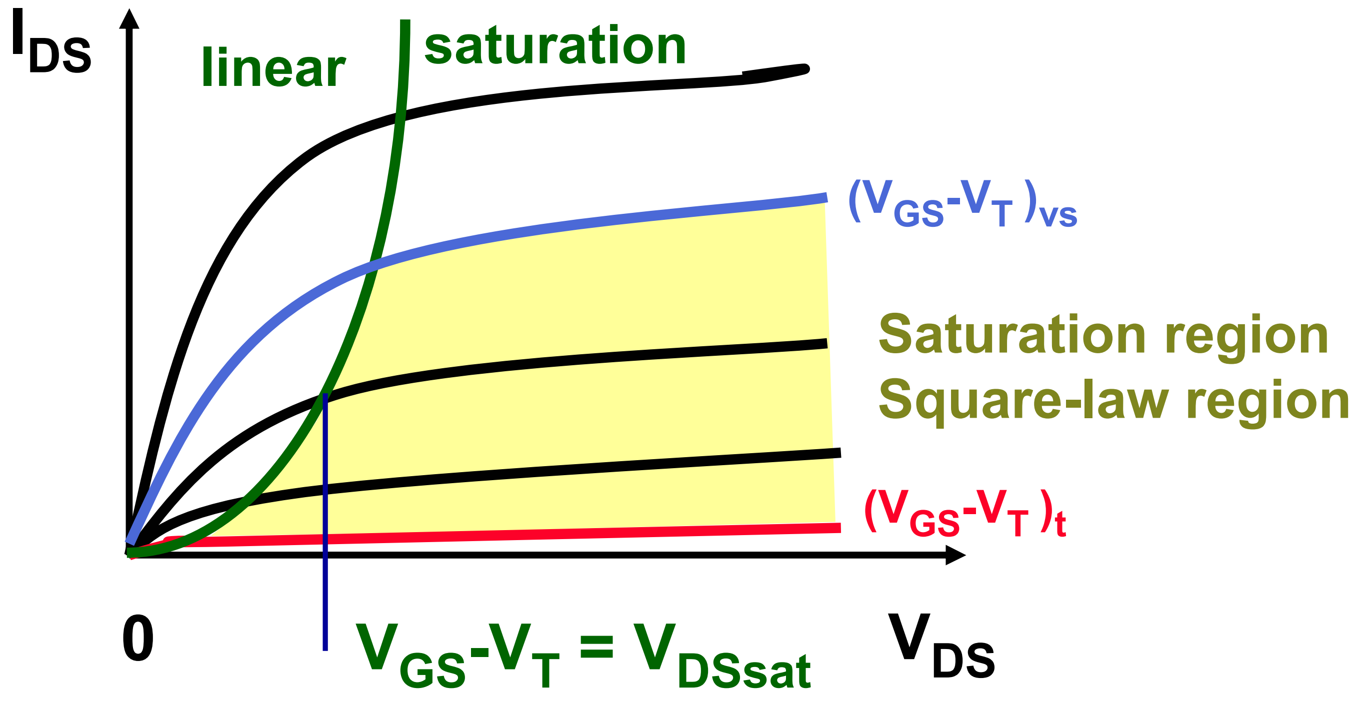
$$I_{DSvs} = WC_{ox}v_{sat}\,(V_{GS}-V_T)$$

$$v_{sat} \approx 10^7 \text{ cm/s}$$

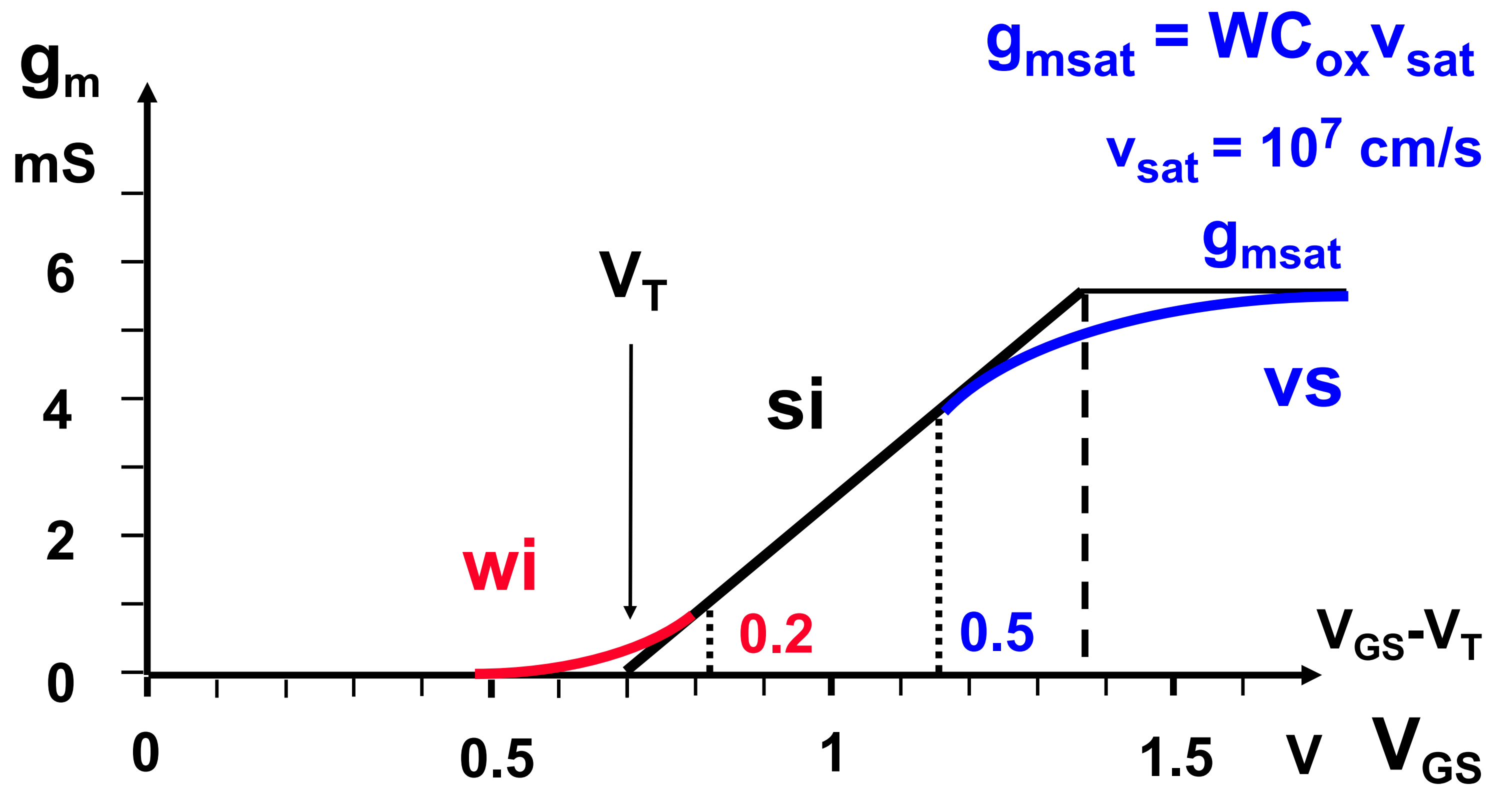
$$g_{msat} = WC_{ox}v_{sat}$$

is absolute max. !

The saturation region and velocity saturation



Transconductance g_m versus V_{GS}



Velocity saturation : v_{sat} & θ

[large V_{GS}]

$$I_{DS} = \frac{K'_n \frac{W}{L} (V_{GS}-V_T)^2}{1 + \theta (V_{GS}-V_T)} \approx \frac{K'_n}{\theta} \frac{W}{L} (V_{GS}-V_T)$$

$$g_{msat} \approx 2K'_n \frac{W}{L} (V_{GS}-V_T)^2 \frac{1 + \frac{\theta}{2}(V_{GS}-V_T)}{[1 + \theta (V_{GS}-V_T)]^2} \approx \frac{K'_n}{\theta} \frac{W}{L}$$

$= WC_{ox}v_{sat}$

$$\boxed{\theta L = \frac{\mu}{2n} \frac{1}{v_{sat}}} = \frac{1}{E_c}$$

E_c is the vertical critical field !

$\theta L \approx 0.2\ \mu m/V$: For $L = 0.13\ \mu m$ $\theta \approx 1.6\ V^{-1}$

Velocity saturation : θ & R_S & v_{sat}

$$I_{DS} = \frac{K'_n \frac{W}{L} (V_{GS}-V_T)^2}{1 + \theta (V_{GS}-V_T)}$$

[large V_{GS}]

$$g_{msat} \approx \frac{K'_n}{\theta} \frac{W}{L}$$

$$g_{mRs} = \frac{g_m}{1 + g_m R_S} \approx \frac{1}{R_S}$$

$$\boxed{R_S = \frac{\theta}{K'_n W/L}}$$

$$R_S \approx \frac{\mu}{2n} \frac{1}{W K'_n v_{sat}} \approx \frac{1}{W C_{ox} v_{sat}}$$

Transition Voltage V_{GSTvs} between si and vs

$$I_{DS} = \frac{K'_n \frac{W}{L} (V_{GS}-V_T)^2}{1 + \theta (V_{GS}-V_T)}$$

$$I_{DSsat} = WC_{ox}v_{sat} (V_{GS}-V_T)$$

$$g_{msat} = WC_{ox}v_{sat} \approx \frac{K'_n}{\theta} \frac{W}{L}$$

$$\boxed{(V_{GS}-V_T)_{vs} = \frac{1}{\theta} \approx 2nL \frac{v_{sat}}{\mu}}$$

Is proportional to channel length L !!!

$\approx 5\,L \approx 0.62\ \mathrm{V}$ if $L = 0.13\ \mu\mathrm{m}$

Transition Current I_{DSvs} between si and vs

$$I_{DSvs} \approx K' WL \left(\frac{2n\, v_{sat}}{\mu}\right)^2 \approx 100\, n\, \varepsilon_{ox} W \frac{v_{sat}^{\,2}}{\mu}$$

$$\frac{I_{DSvs}}{W} \approx 10 \;\; A/cm$$

$$K' = \frac{\mu C_{ox}}{2n}$$

$$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}} \qquad t_{ox} = \frac{L}{50}$$

W = 2.6 μm & L = 0.13 μm :

$I_{DSvs} \approx 2.6$ mA

$v_{sat} = 10^7$ cm/s

n = 1.4

μ= 500 cm^2/Vs

Transconductance g_m between si and vs

$g_{msat} = W\, C_{ox}\, v_{sat}$

$g_{msat} \approx 17\ 10^{-5}\ W/L\ S/cm$

$$g_{mK'} = 2K' \frac{W}{L} \underbrace{(V_{GS}-V_T)}_{V_{GST}}$$

$g_{mK'} \approx 1.2\ 10^{-9}\ V_{GST}\ W/L^2\ S/cm$

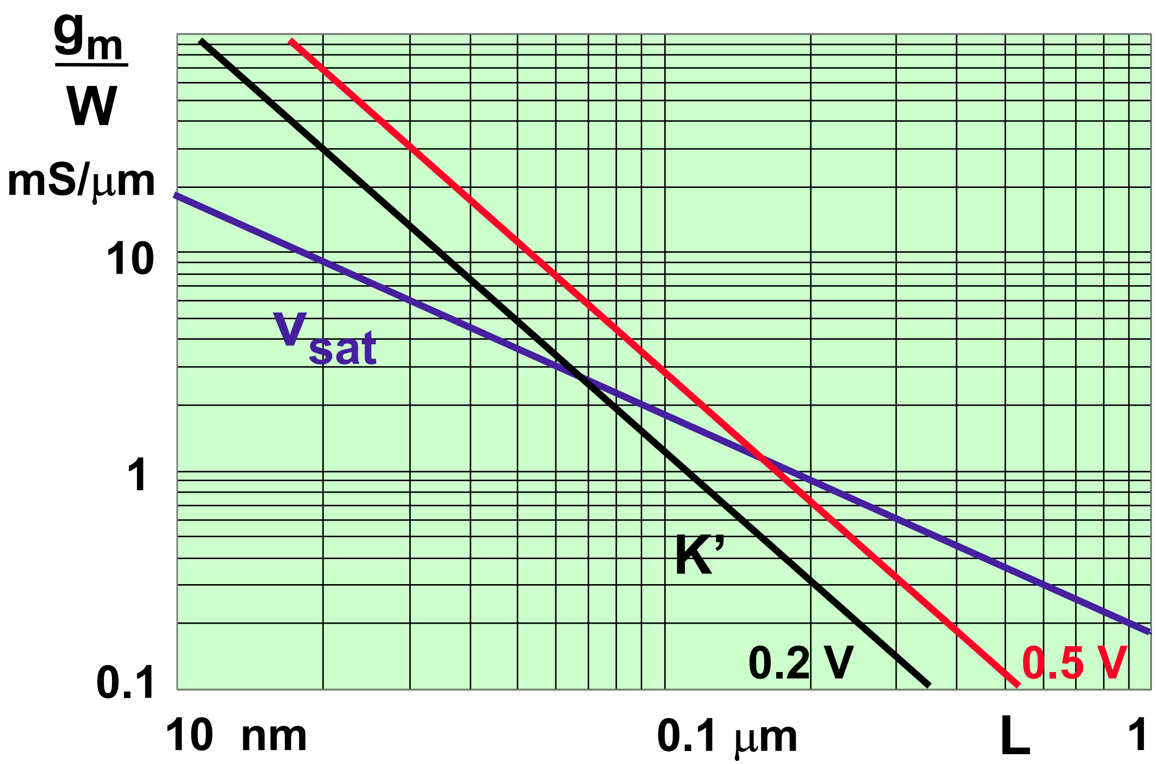
$$\frac{1}{g_m} = \frac{1}{g_{mK'}} + \frac{1}{g_{msat}}$$

$$\boxed{g_m \approx \frac{W}{L} \frac{17\ 10^{-5}}{1 + 2.8\ 10^4\ L / V_{GST}}}$$

L in cm

If V_{GST} = 0.2 V, v_{sat} takes over for L < 65 nm (If 0.5 V for L < 0.15 μm)

Now in velocity saturation ?



$V_{GS}-V_T \approx 0.2\ V$

(0.5 V)

$$\frac{g_m}{W} = \frac{2K'}{L}(V_{GS}-V_T)$$

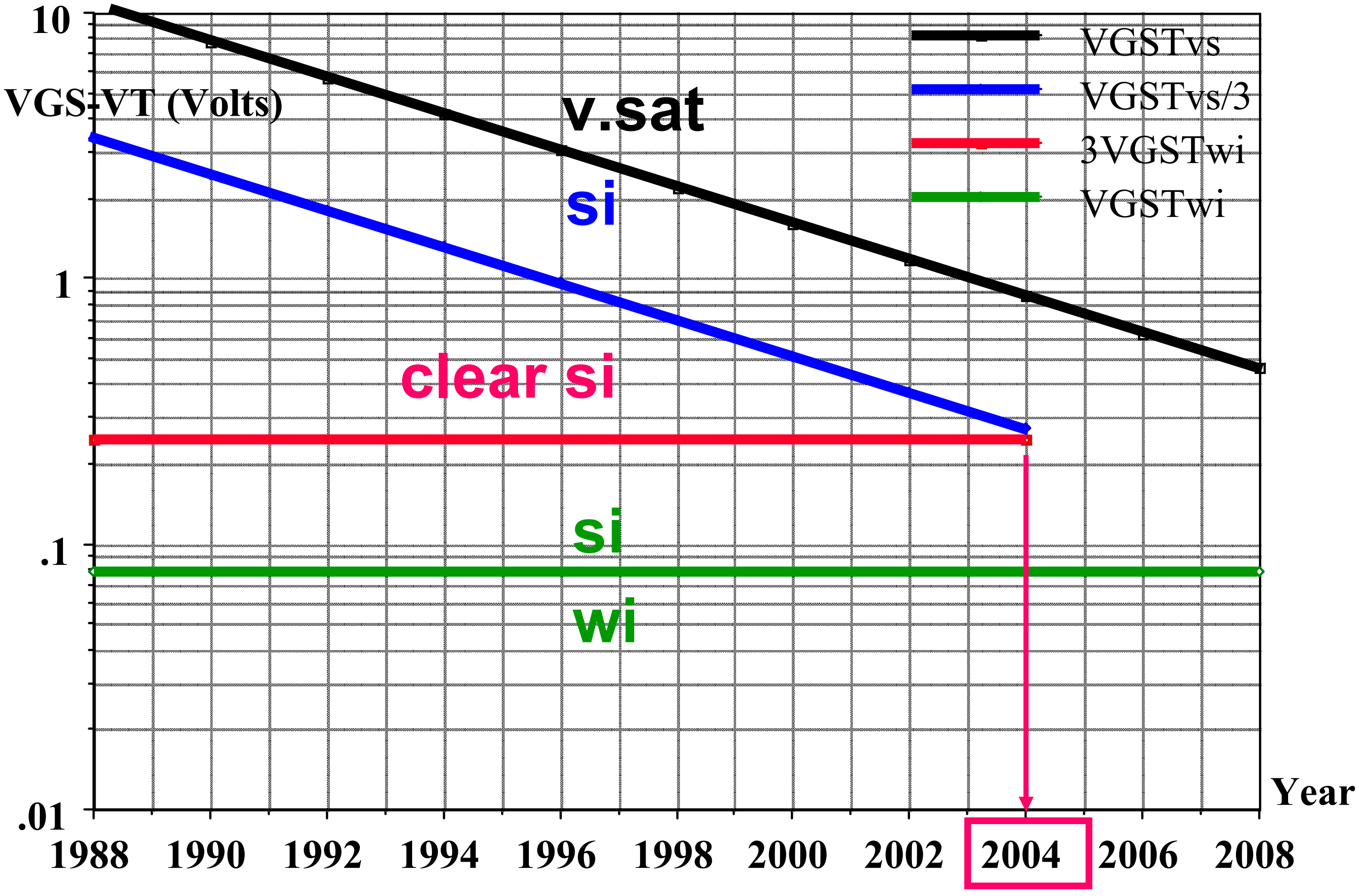
$$\frac{g_m}{W} \approx \frac{0.06\ V_{GST}}{L^2}\ \text{in } \mu m$$

$$\frac{g_m}{W} = C_{ox}\, v_{sat}$$

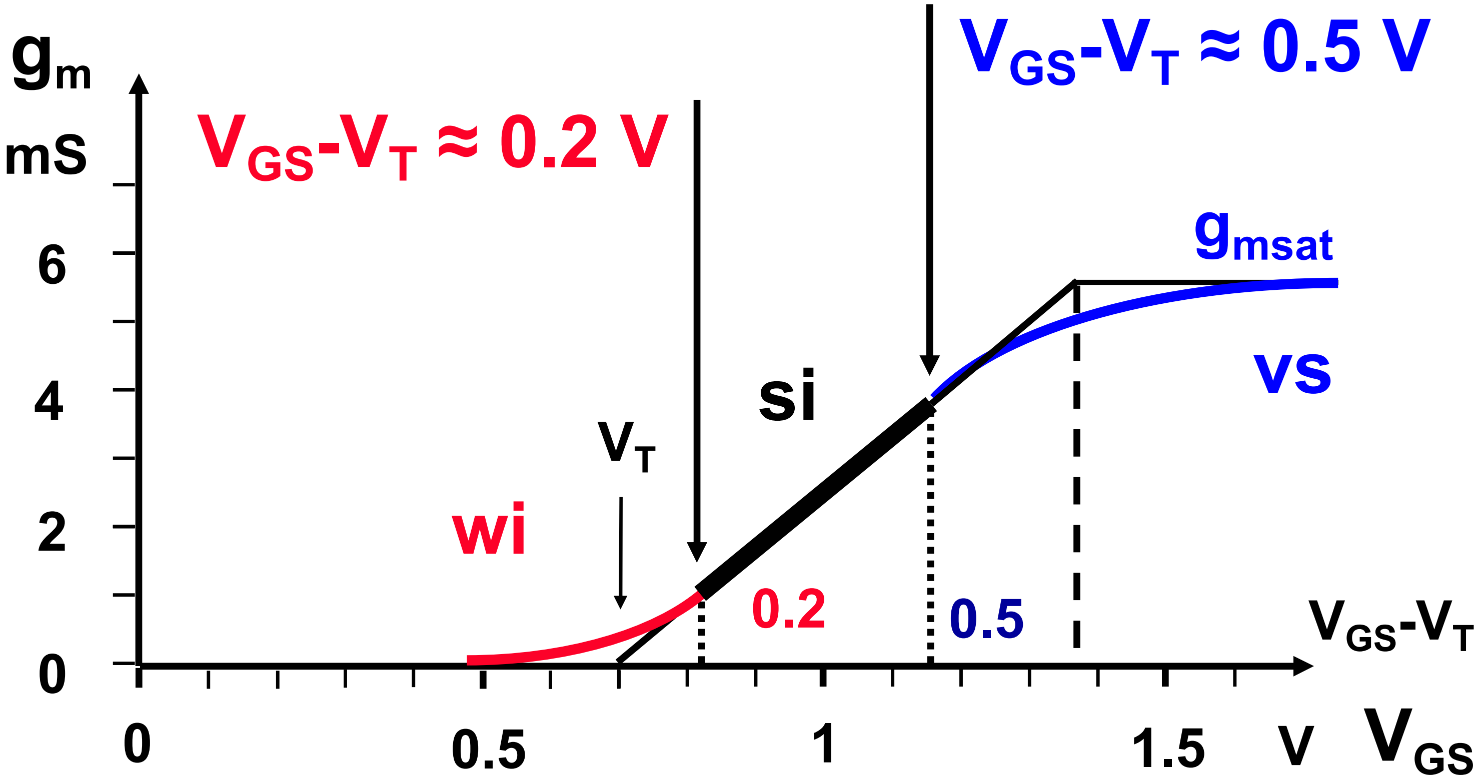
$$\frac{g_m}{W} \approx \frac{0.17}{L}$$

in μm

Range of V_{GS}-V_T values for si vs time

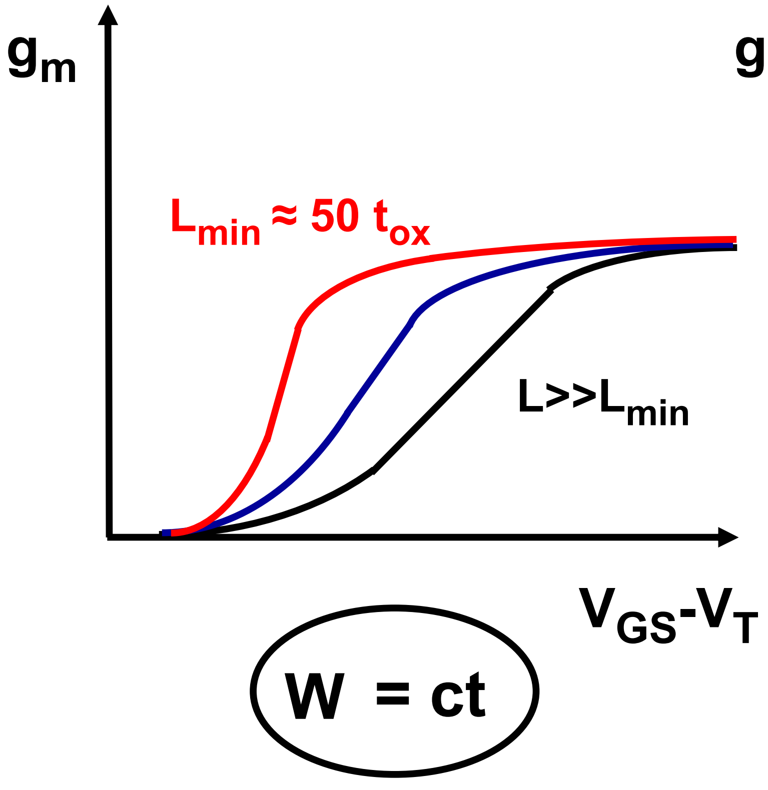


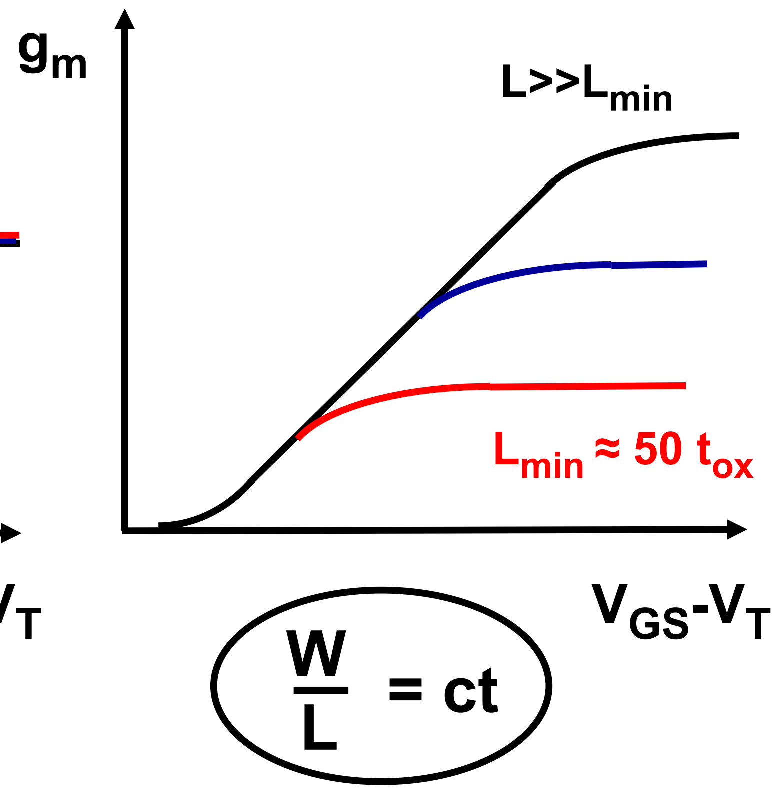
MOST operating region in si



g_m vs V_{GS} for different L (same t_{ox})

Exercise :





g_m vs V_{GS} for different t_{ox} ($\approx L_{min}/50$)

Exercise :

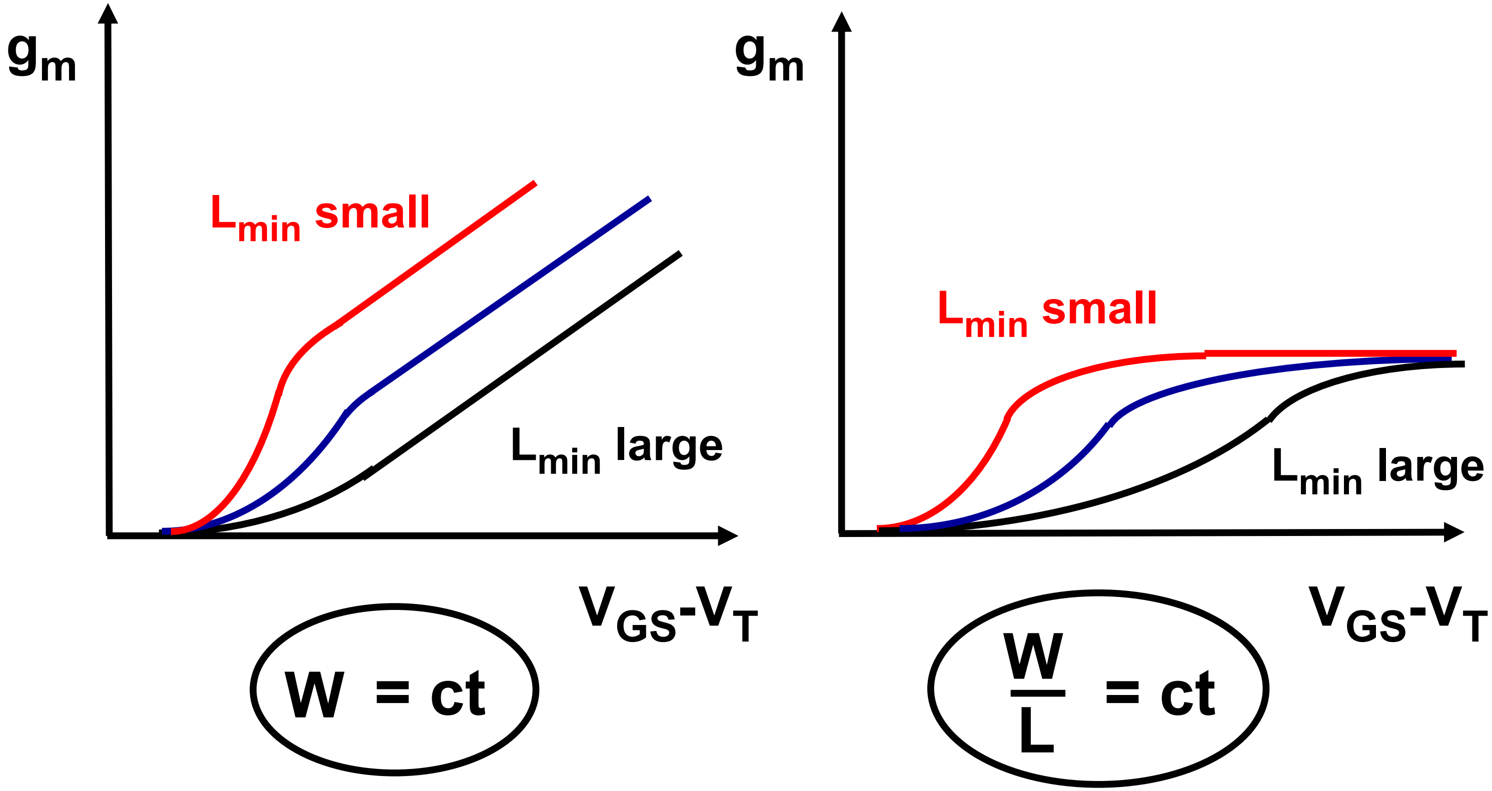


Table : MOST I_{DS} , g_m & g_m/I_{DS}

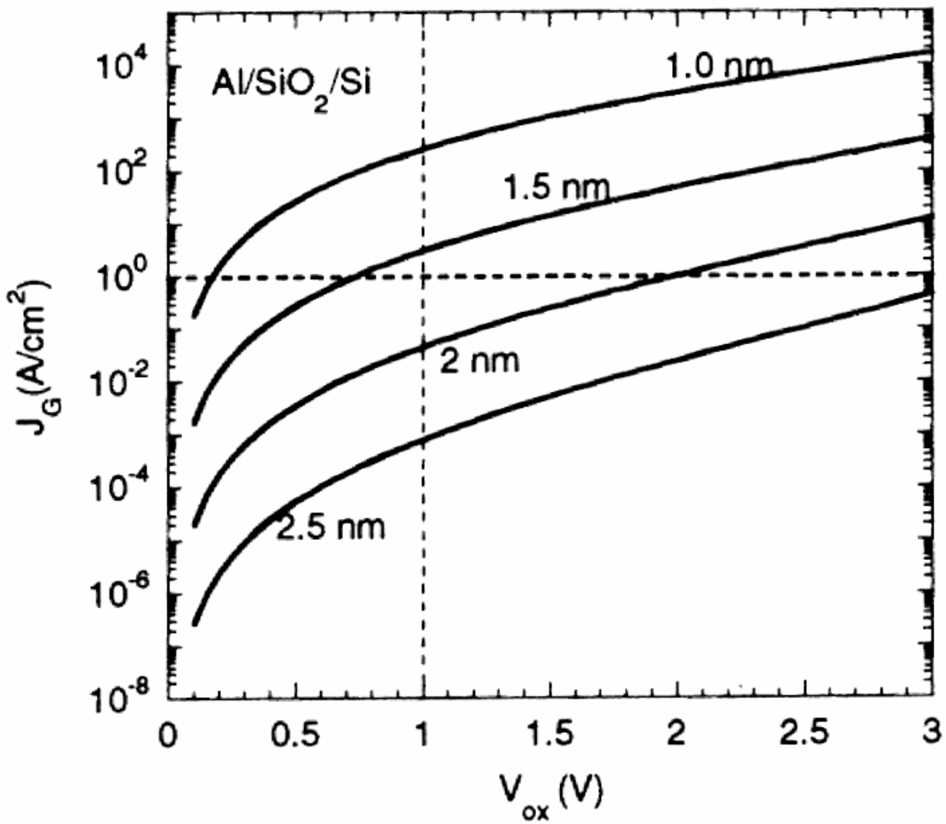
Summary :

TABLE 1-4 EXPRESSIONS OF I_{DS}, g_m AND g_m/I_{DS} FOR MOST

	I_{DS}	g_m	$\frac{g_m}{I_{DS}} = f(v_{GS}-V_T)$	$\frac{g_m}{I_{DS}} = f(I_{DS})$
wi	$I_{D0}\frac{W}{L}\exp\left(\frac{v_{GS}}{nkT/q}\right)$ (1-25*a*)	$\frac{I_{D0}}{nkT/q}\frac{W}{L}\exp\left(\frac{v_{GS}}{nkT/q}\right)$ (1-25*b*)	$\frac{1}{nkT/q}$ (1-26*b*)	$\frac{1}{nkT/q}$ (1-26*b*)
ws			$(v_{GS}-V_T)_{ws} = 2n\frac{kT}{q}$	$I_{DSws} = \frac{KP}{2n}\frac{W}{L}\left(2n\frac{kT}{q}\right)^2$
si	$\frac{KP}{2n}\frac{W}{L}(v_{GS}-V_T)^2$ (1-18*c*)	$2\frac{KP}{2n}\frac{W}{L}(v_{GS}-V_T)$ (1-22*a*)	$\frac{2}{v_{GS}-V_T}$ (1-26*a*)	$2\sqrt{\frac{KP}{2n}\frac{W}{L}\frac{1}{I_{DS}}}$ (1-26*a*)
sv			$(v_{GS}-V_T)_{sv} = \frac{2nLC_{ox}v_{sat}}{KP}$	$I_{DSsv} = \frac{2WLC_{ox}^2v_{sat}^2}{KP/2n}$
vs	$WC_{ox}v_{sat}(v_{GS}-V_T)$ (1-38*b*)	$WC_{ox}v_{sat}$ (1-39)	$\frac{1}{v_{GS}-V_T}$	$\frac{WC_{ox}v_{sat}}{I_{DS}}$

Ref.: Laker, Sansen : Design of analog …, MacGrawHill 1994; Table 1-4

Gate current



For 0.1 μm CMOS :

$t_{ox} \approx 2$ nm

$J_G \approx 4\ 10^{-2}$ A/cm^2

For 10 x 0.5 μm

$I_G \approx 2$ nA

J_G (A/cm^2)

$\approx 4.5\ 10^5 \exp(-\dfrac{L}{6.5})$

L in nm

Ref. Koh, Tr ED 2001, 259-
Annema, JSSC Jan.05, 135.

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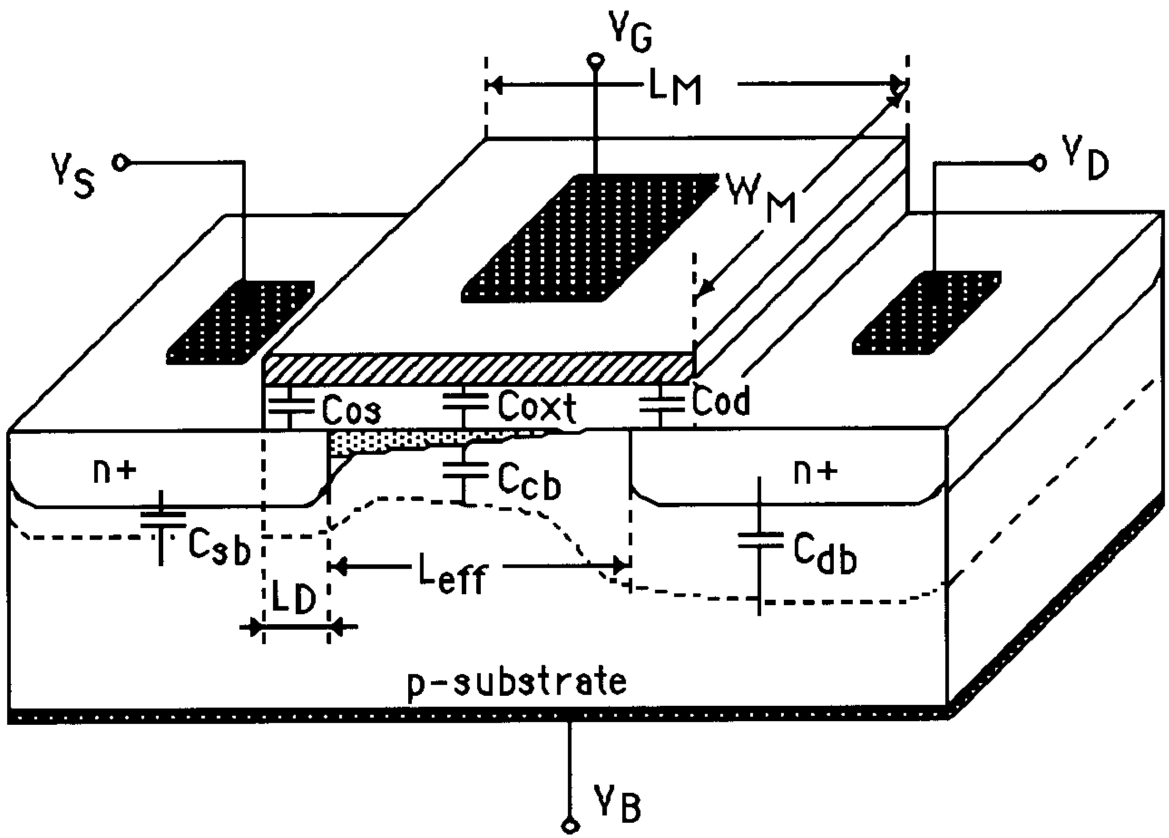
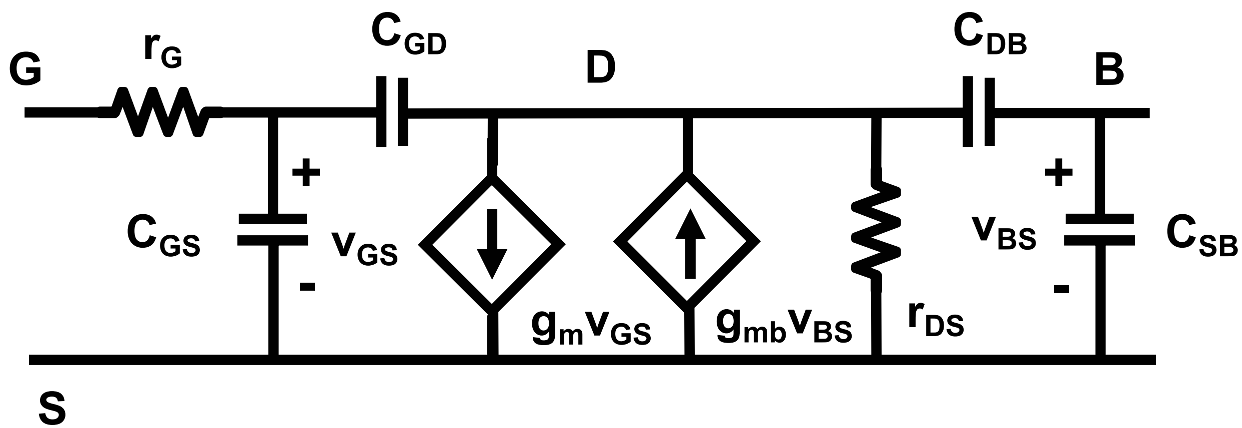
MOST capacitances



MOST capacitances C_{GS} & C_{GD}

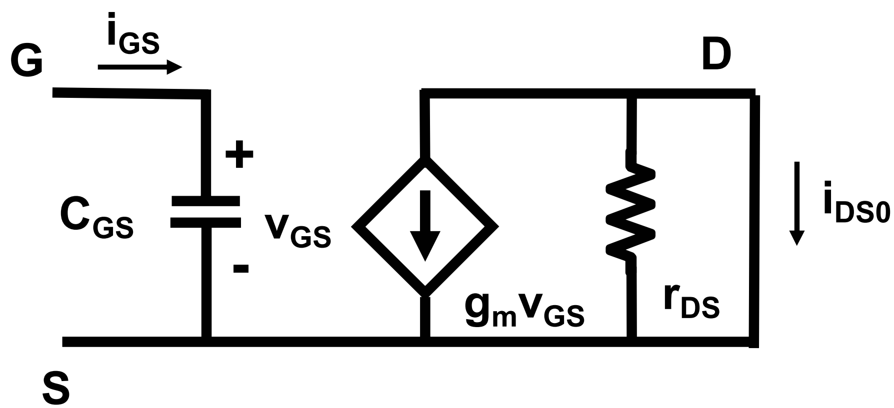


$$C_{GS} \approx \frac{2}{3} WLC_{ox} \quad \approx 2W \text{ fF}/\mu\text{m for } L_{min}$$

$$L_{min}C_{ox} \approx L_{min} \frac{\varepsilon_{ox}}{t_{ox}} \approx 50\ \varepsilon_{ox} \approx 2 \text{ fF}/\mu\text{m}$$

$$C_{GD} = WC_{gdo}$$

MOST f_T where $i_{DS} = i_{GS}$



$$i_{GS} = v_{GS} C_{GS} s$$

$$i_{DS} = g_m v_{GS}$$

$$C_{GS} = \frac{2}{3} WLC_{ox} \quad g_m = 2K' \frac{W}{L} (V_{GS}-V_T) \quad K' = \frac{\mu C_{ox}}{2n}$$

$$\boxed{f_T = \frac{g_m}{2\pi C_{GS}} = \frac{1}{2\pi} \frac{3}{2n} \frac{\mu}{L^2} (V_{GS}-V_T)}$$

or $\boxed{\approx \dfrac{v_{sat}}{2\pi L}}$

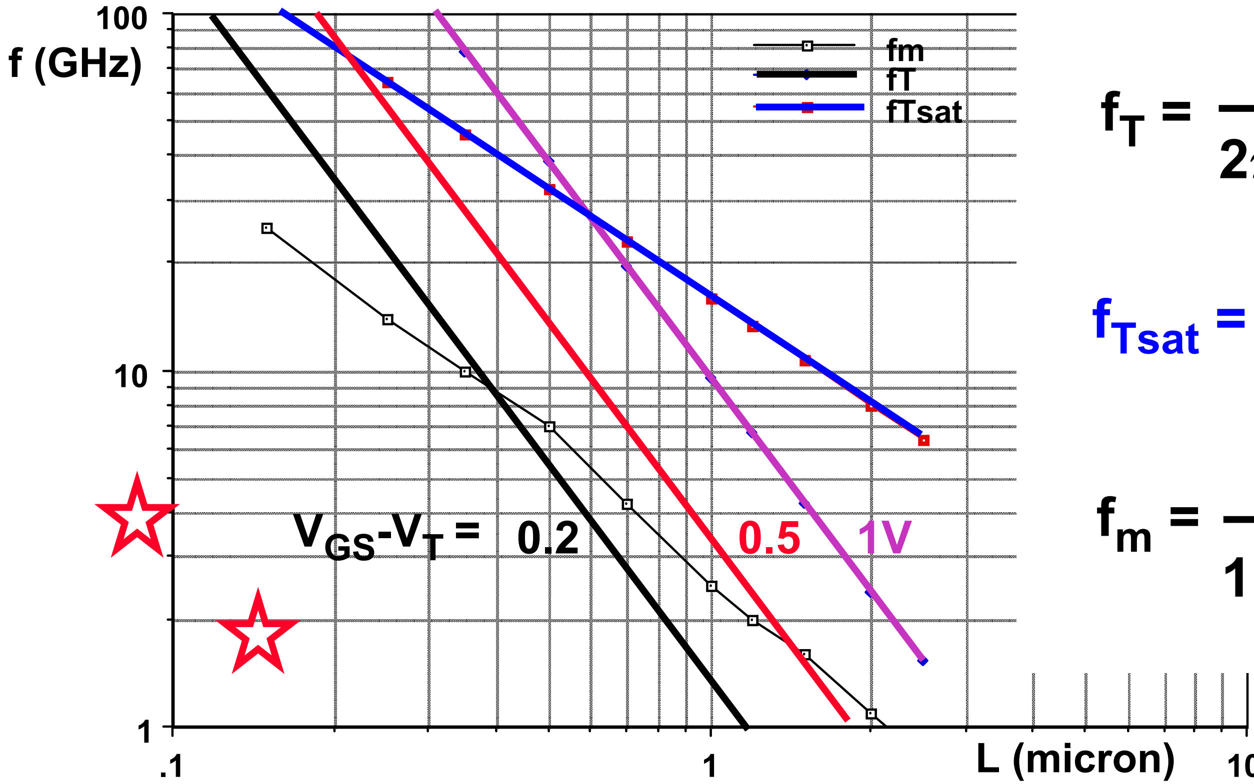
$$f_{max} \approx \sqrt{f_T / 8\pi r_G C_{GD}}$$

Design for high speed :

	High gain	High speed
V_{GS}-V_T	Low (0.2 V)	High (0.5 V)
L	High	Low

V_{GS}-V_T sets the ratio g_m/I_{DS} !

Maximum f_T values versus channel length L



Processors

$$f_T = \frac{\mu}{2\pi L^2} \underbrace{(V_{GS}-V_T)}_{0.2 \ldots 1\ V}$$

$$f_{Tsat} = \frac{v_{sat}}{2\pi L}$$

$$f_m = \frac{f_T}{1 + \alpha_{BD}}$$

$$\alpha_{BD} \approx \frac{C_{BD}}{C_{ox}}$$

f_T model in si and velocity saturation

$$f_T = \frac{g_m}{2\pi\, C_{GS}}$$

$C_{GS} = kW$ $\quad$ $k = 2\ fF/\mu m = 2\ 10^{-11}\ F/cm$

$$g_m = \frac{W}{L}\ \frac{17\ 10^{-5}}{1 + 2.8\ 10^4\ L / V_{GST}}$$

L in cm

$$\boxed{f_T = \frac{1}{L}\ \frac{13.5}{1 + 2.8\ L / V_{GST}}\ \text{GHz}}$$

L in μm

If V_{GST} = 0.2 V, v_{sat} takes over for L < 65 nm

If V_{GST} = 0.5 V for L < 0.15 μm

f_T model in si and weak inversion

$$f_T = \frac{g_m}{2\pi\, C_{GS}}$$

$$GM = \frac{g_m}{I_{DS}} \frac{nkT}{q} = \frac{1 - e^{-\sqrt{i}}}{\sqrt{i}}$$

$$g_m = \frac{I_{DS}}{nkT/q} \frac{1 - e^{-\sqrt{i}}}{\sqrt{i}} \quad \text{but } I_{DS} = i\, I_{DSt}$$

$$g_m = \frac{I_{DSt}}{nkT/q} \sqrt{i}\, (1 - e^{-\sqrt{i}})$$

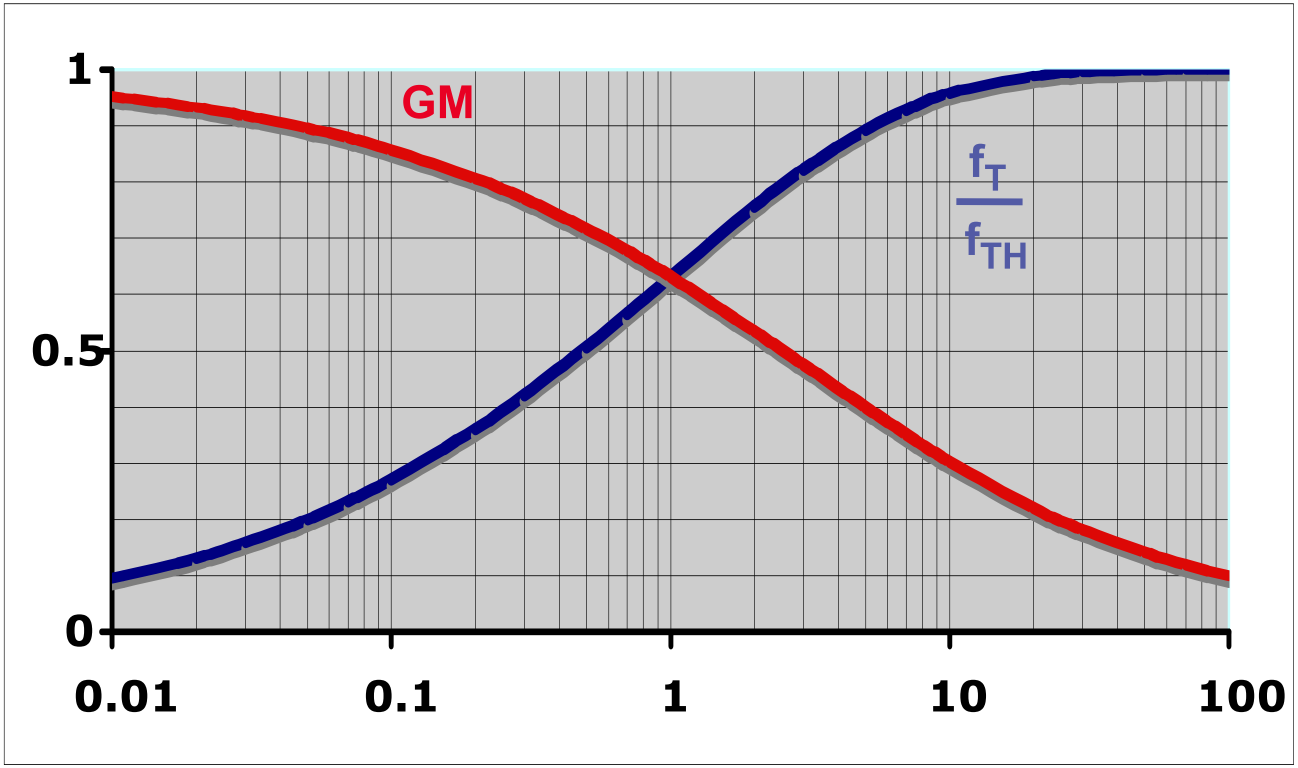
$$\frac{f_T}{f_{TH}} = \sqrt{i}\, (1 - e^{-\sqrt{i}})$$

$\approx$ i for small i !

$$f_{TH} = \frac{I_{DSt}}{2\pi\, C_{GS}\, nkT/q} = \frac{K'\, V_{GSTt}^2\, W/L}{2\pi\, WL\, C_{ox}\, nkT/q}$$

$$= \frac{4\, K'\, nkT/q}{2\pi\, C_{ox}\, L^2} = \frac{2\, \mu\, kT/q}{2\pi\, L^2}$$

f_T versus inversion coefficient i



$$\frac{f_T}{f_{TH}} = \sqrt{i}\,(1 - e^{-\sqrt{i}})$$

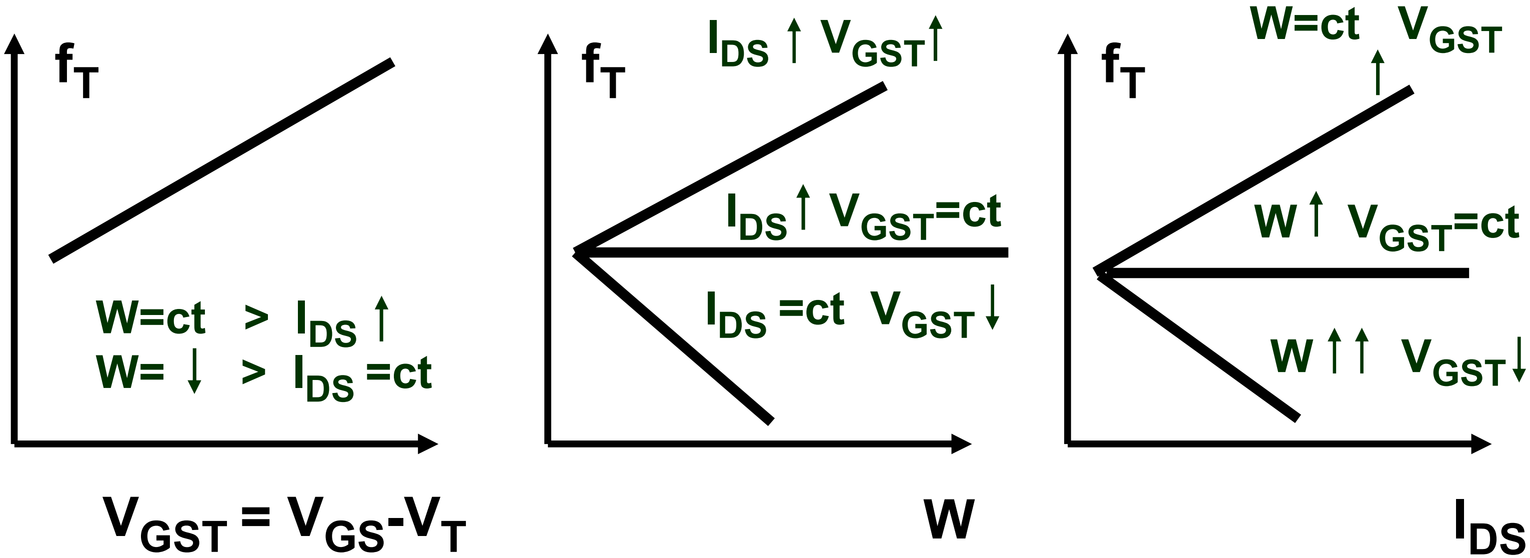
$$GM = \frac{1 - e^{-\sqrt{i}}}{\sqrt{i}}$$

$$i = \frac{I_{DS}}{I_{DSt}}$$

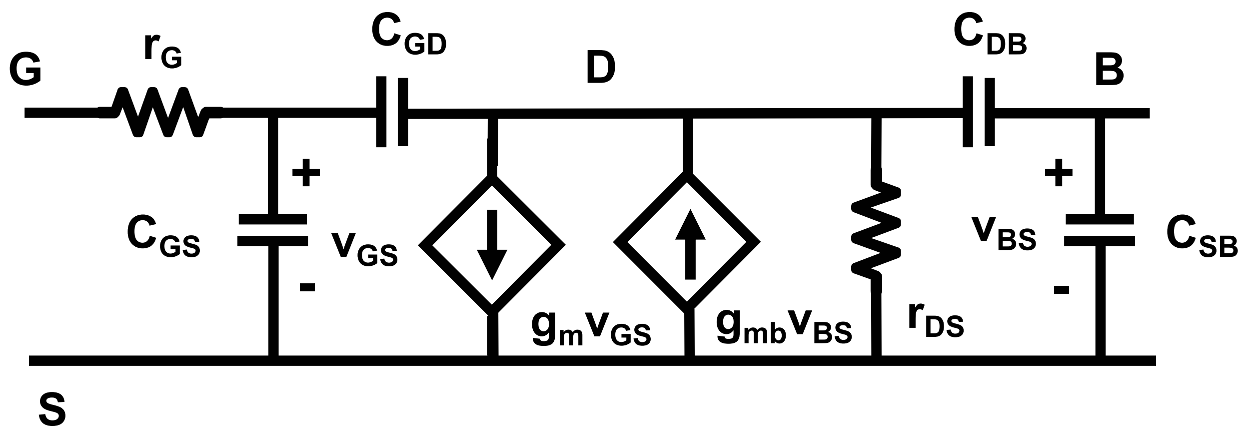
Exercise: MOST f_T or not f_T ?

all $L = L_{min}$

$$f_T = \frac{1}{2\pi}\frac{\mu}{L^2}(V_{GS}-V_T) = \frac{\sqrt{K'I_{DS}}}{\pi\, C_{ox}\sqrt{WL^3}} = \frac{I_{DS}}{\pi W L C_{ox}(V_{GS}-V_T)}$$



MOST capacitances C_{SB} & C_{DB}

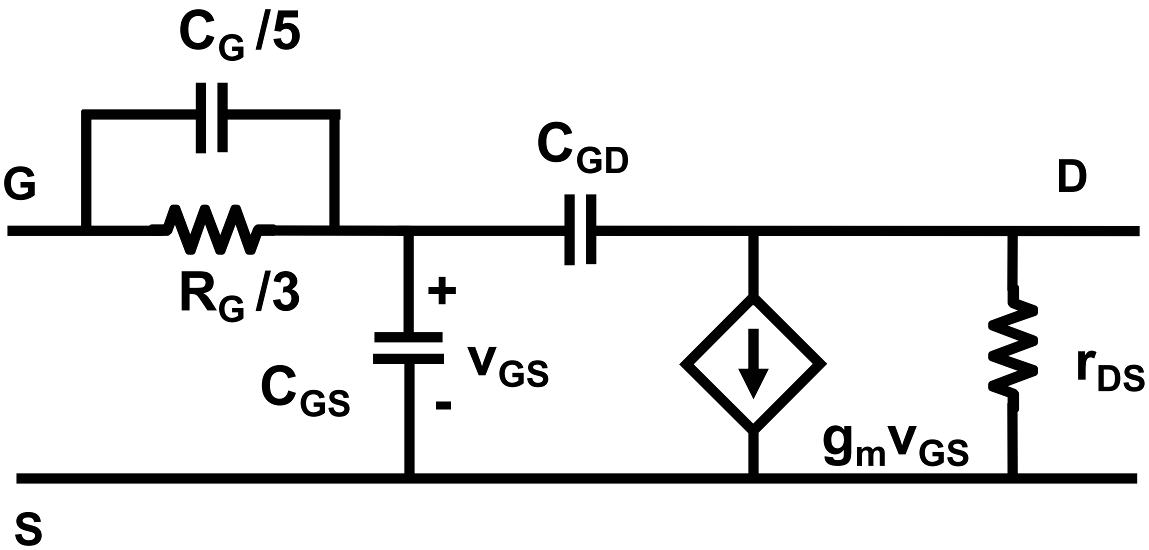


$$C_{SB} = \frac{C_{jSB0}}{\sqrt{1 + V_{SB}/\phi_{jS}}}$$

$$\phi_{jS} \approx \phi_{jD} \approx 0.5 \ldots 0.7 \text{ V}$$

$$C_{DB} = \frac{C_{jDB0}}{\sqrt{1 + V_{DB}/\phi_{jD}}}$$

RF MOST model



$$C_G = C_{GS} + C_{GD}$$

Ref. Sansen, etal, ACD, XDSL, RFMOS models, Kluwer 1999

Ref. Tin, Tr. CAD, April 1998, 372

Single-page MOST model

$$I_{DS} = K'_n \frac{W}{L} (V_{GS}-V_T)^2$$

$V_{GS}-V_T \approx 0.2\ V$

$K'_n \approx 100\ \mu A/V^2$
$K'_p \approx 40\ \mu A/V^2$

$$g_m = 2K'_n \frac{W}{L} (V_{GS}-V_T) = 2\sqrt{K'_n \frac{W}{L} I_{DS}} = \frac{2\ I_{DS}}{V_{GS}-V_T}$$

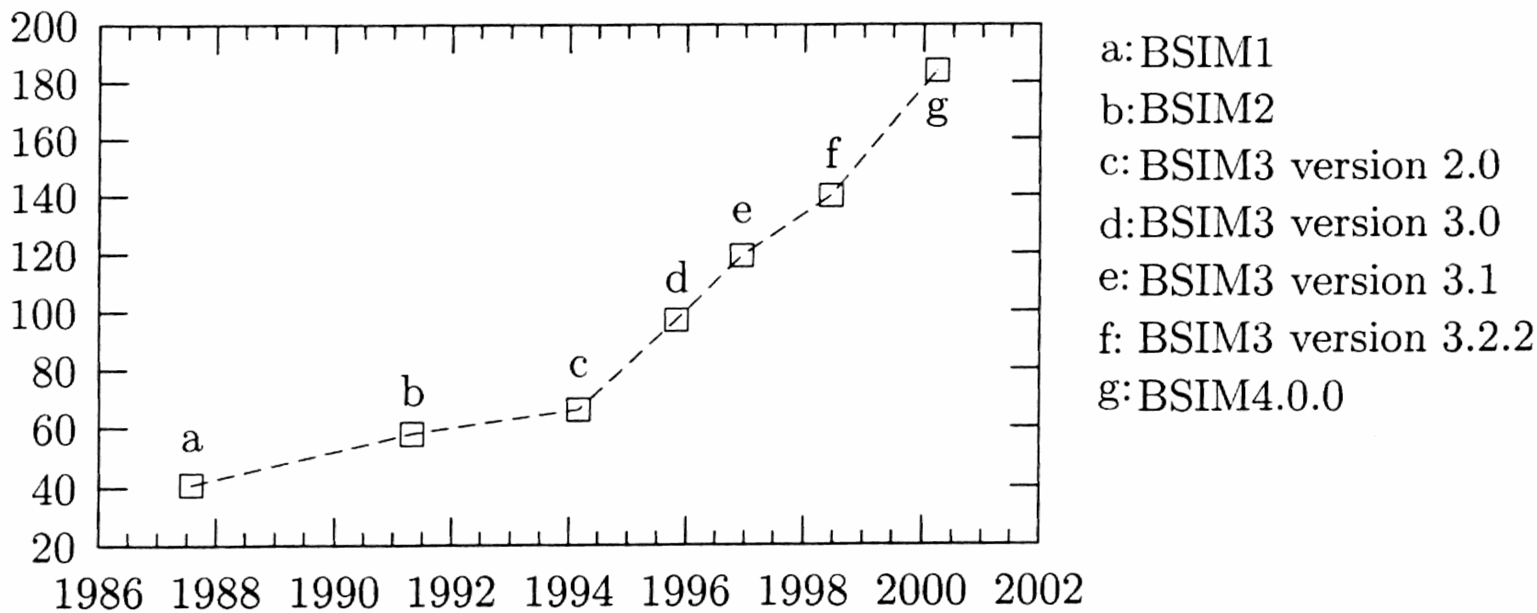
$$r_{DS} = r_o = \frac{V_E L}{I_{DS}}$$

$V_{En} \approx 5\ V/\mu mL$ $V_{Ep} \approx 8\ V/\mu mL$

$v_{sat} = 10^7\ cm/s$

$$f_T = \frac{1}{2\pi} \frac{3}{2n} \frac{\mu}{L^2} (V_{GS}-V_T) \quad \text{or now} \approx \frac{v_{sat}}{2\pi\ L}$$

Growing number of parameters !



BSIM4 : http://www-device.eecs.berkeley.edu/bsim/bsim_ent.html

Model 11 : http://www.semiconductors.philips.com/Philips_Models/mos_models/model11/index.html

EKV : http://legwww.epfl.ch/ekv/model.html

Benchmark tests

1. Weak inversion transition for I_{DS} and g_m/I_{DS} ratio
2. Velocity saturation transition for I_{DS} and g_m/I_{DS} ratio
3. Output conductance around V_{DSsat}
4. Continuity of currents and caps around zero V_{DS}
5. Thermal and 1/f noise
6. High frequency input impedance (s_{11}) and transimpedance (s_{21})

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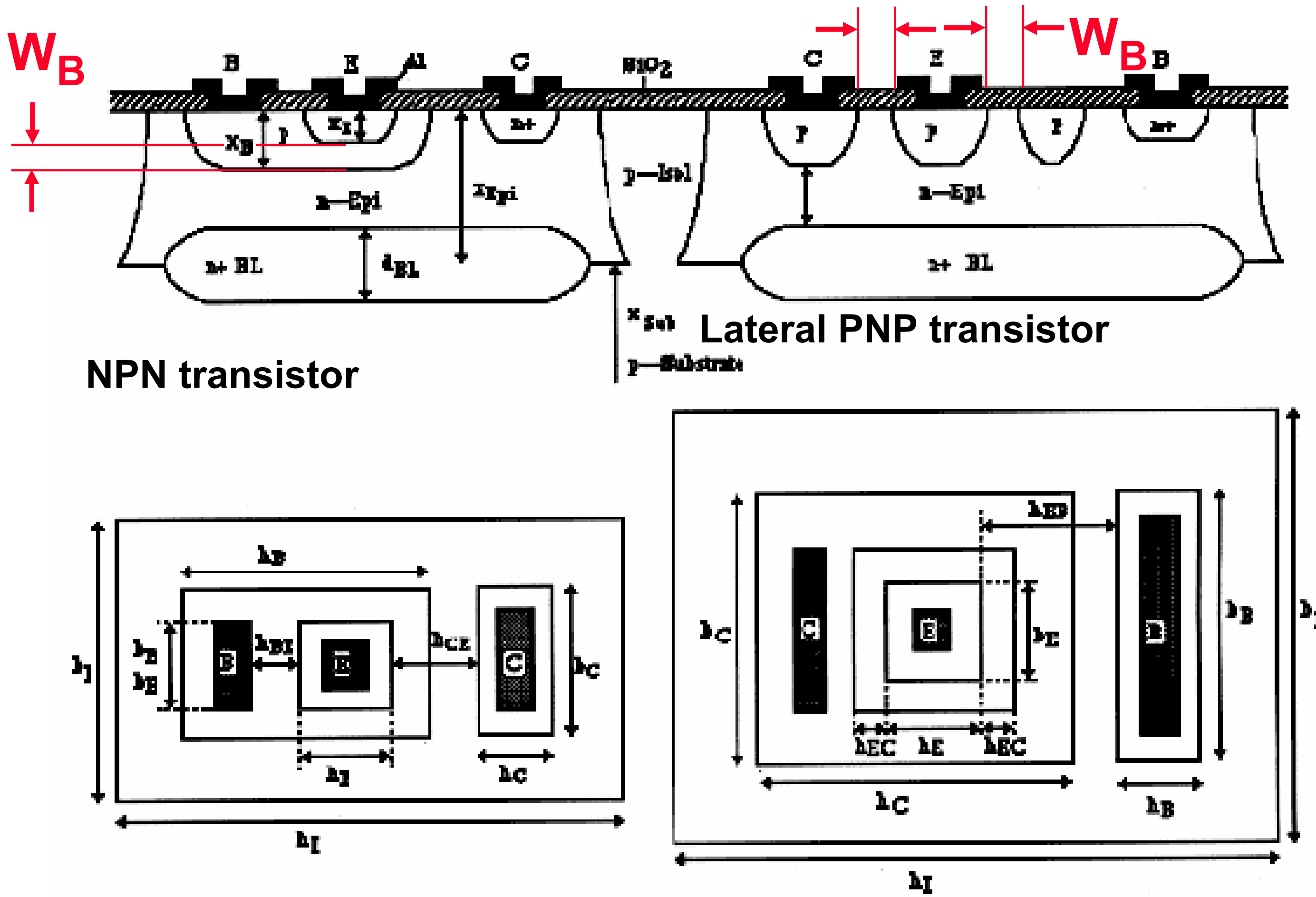
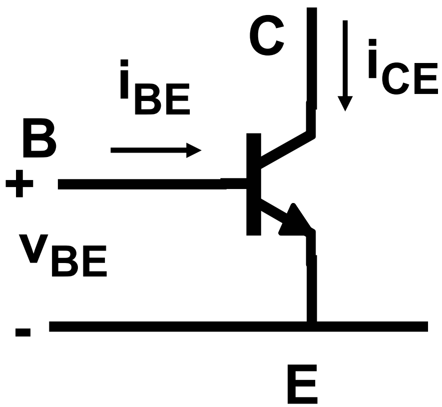
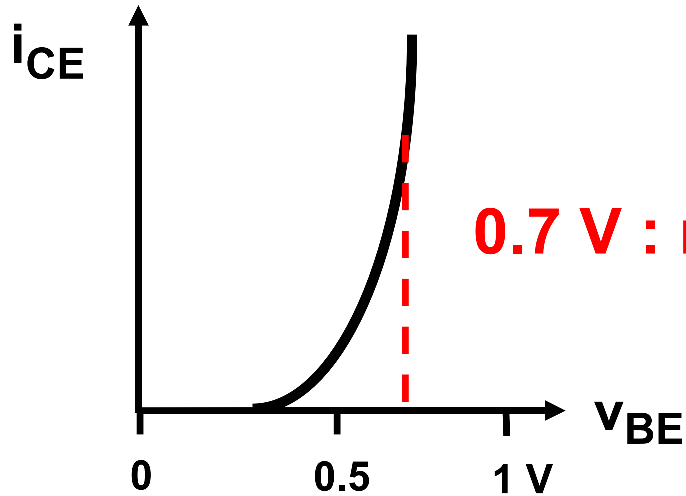
- Comparison of MOSTs & Bipolar transistors

□Bipolar transistors



Bipolar transistor I_{CE} versus V_{BE}





0.7 V : not more
not less

$$I_{CE} = I_S \exp \frac{V_{BE}}{kT/q}$$

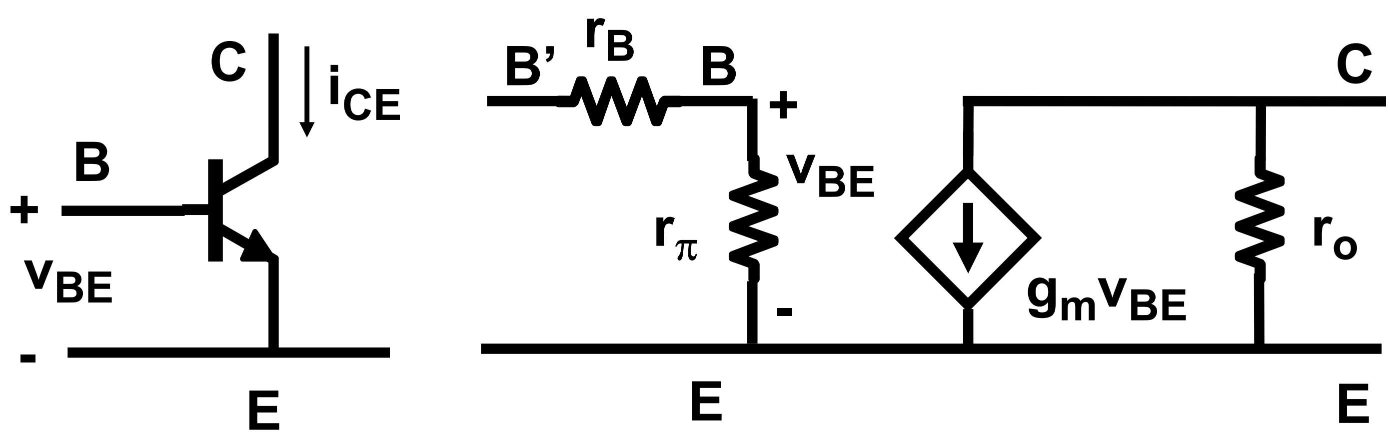
$I_S \approx 10^{-15}$ A   kT/q = 26 mV at 300 K

$$I_{BE} = \frac{I_{CE}}{\beta}$$

is leakage current

$\beta \approx$ 10 … 1000

Bipolar transistor small-signal model : g_m & r_o

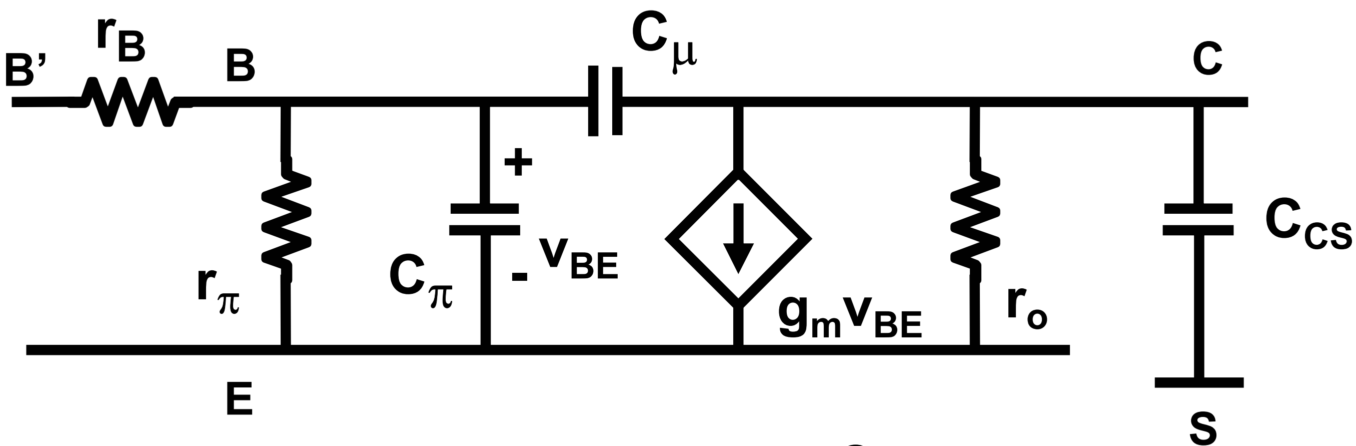


$$g_m = \frac{di_{CE}}{dv_{BE}} = \frac{I_{CE}}{kT/q} \qquad \frac{g_m}{I_{CE}} = \frac{1}{kT/q} \approx 40\ V^{-1}$$

$$r_\pi = \frac{dv_{BE}}{di_{BE}} = \beta\ \frac{dv_{BE}}{di_{CE}} = \frac{\beta}{g_m} \qquad r_o = \frac{V_E}{I_{CE}}$$

$V_{En} \approx 20$ V
$V_{Ep} \approx 10$ V

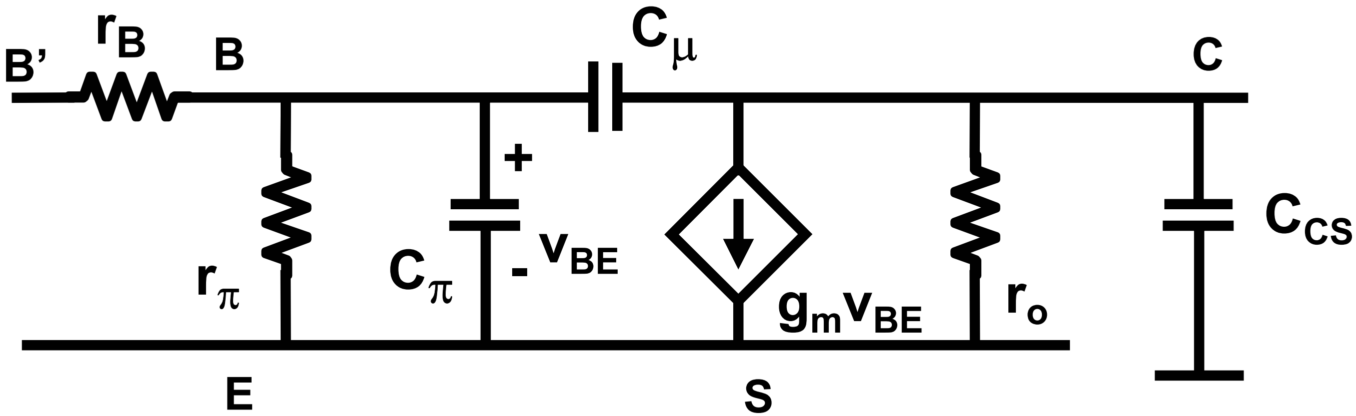
Bipolar transistor capacitance C_π



$$C_\pi = C_{jBE} + C_D \qquad C_{jBE} = \frac{C_{jBE0}}{\sqrt{1 + V_{BE}/\phi_{jE}}} \qquad \phi_{jE} \approx 0.7\ \mathrm{V}$$

C_D is the diffusion capacitance

Diffusion capacitance C_D

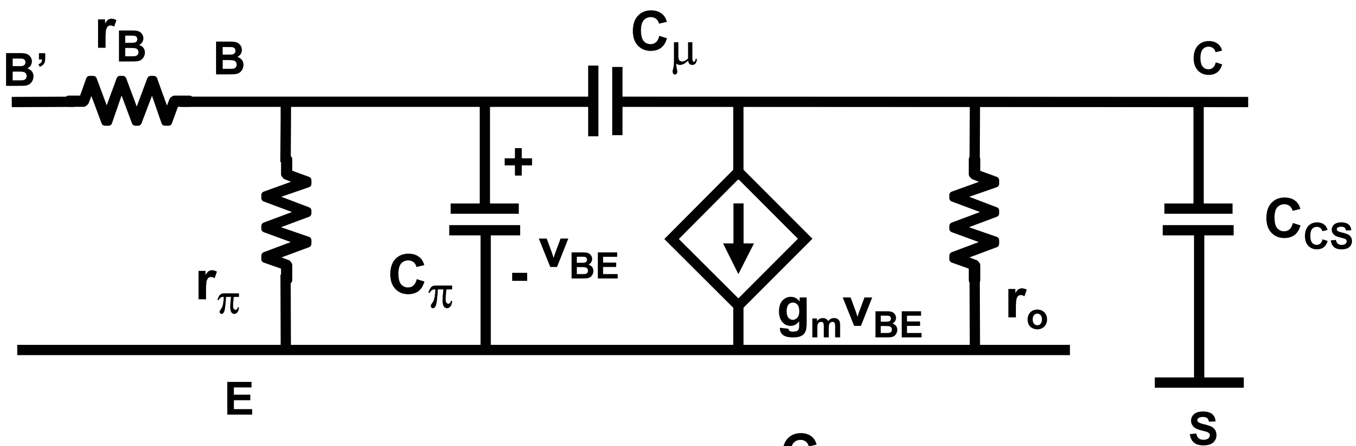


$$C_D = \frac{Q_B}{v_{BE}} = \tau_F \frac{di_{CE}}{dv_{BE}} = \tau_F g_m = \tau_F \frac{I_{CE}}{kT/q}$$

Base transit time $\tau_F = \dfrac{W_B^2}{2D_n}$ or now $\approx \dfrac{W_B}{v_{sat}}$

$\approx$ 10 … 200 ps

Bipolar transistor capacitances C_μ & C_{CS}

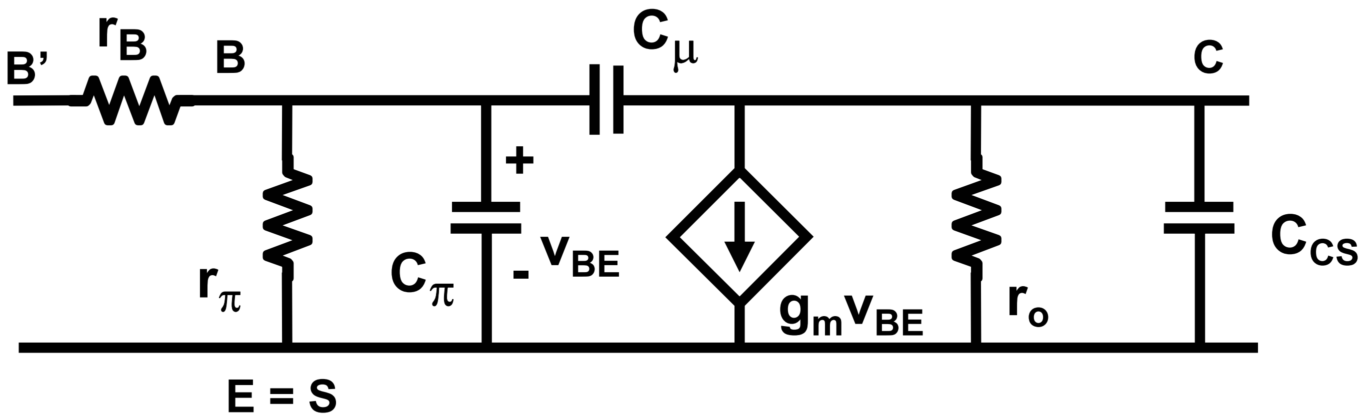


$$C_\mu = C_{jBC} \qquad C_{jBC} = \frac{C_{jBC0}}{\sqrt{1 + V_{BC}/\phi_{jC}}}$$

$$C_{CS} = C_{jCS} \qquad C_{jCS} = \frac{C_{jCS0}}{\sqrt{1 + V_{CS}/\phi_{jS}}}$$

$\phi_{jC} \approx \phi_{jS} \approx 0.5$ V

Bipolar transistor f_T

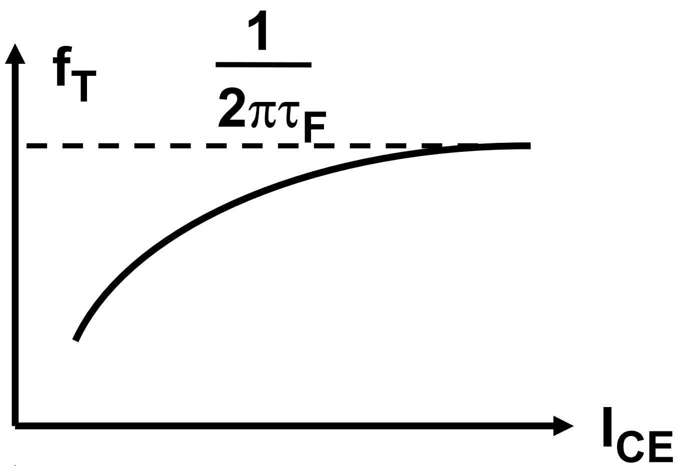


$$f_T = \frac{g_m}{2\pi C_\pi} = \frac{1}{2\pi} \frac{1}{\tau_F + \frac{C_{jBE}+C_\mu}{g_m}} \quad \text{or} \approx \frac{v_{sat}}{2\pi W_B}$$

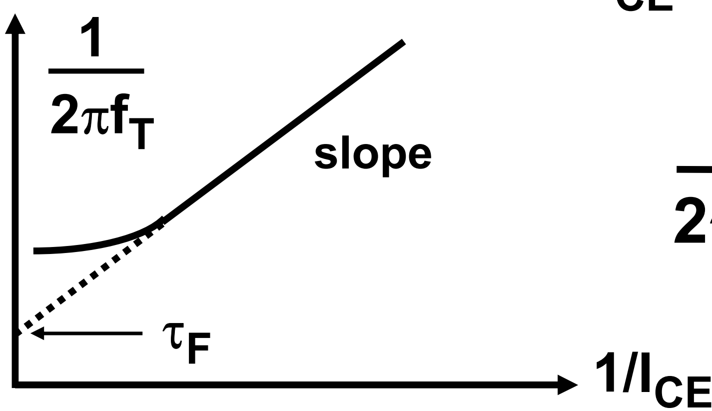
For a current drive !

$$f_{max} \approx \sqrt{f_T / 8\pi\, r_B C_\mu}$$

Bipolar transistor f_T versus I_{CE}



$$f_T = \frac{1}{2\pi} \frac{1}{\tau_F + \frac{C_{jBE}+C_\mu}{g_m}}$$



$$\frac{1}{2\pi f_T} = \tau_F + \underbrace{(C_{jBE} + C_\mu)\ \frac{kT}{q}}_{\text{slope}}\ \frac{1}{I_{CE}}$$

Single-page Bipolar transistor model

$$I_{CE} = I_S \exp \frac{V_{BE}}{kT/q}$$

$I_S \approx 10^{-15}$ A $kT/q = 26$ mV at 300 K

$$g_m = \frac{I_{CE}}{kT/q} \qquad r_o = \frac{V_E}{I_{CE}}$$

$V_{En} \approx 20$ V $V_{Ep} \approx 10$ V

$$f_T = \frac{1}{2\pi} \frac{1}{\tau_F + \frac{C_{je}+C_{jc}}{g_m}}$$

$$\text{or} \approx \frac{v_{sat}}{2\pi W_B}$$

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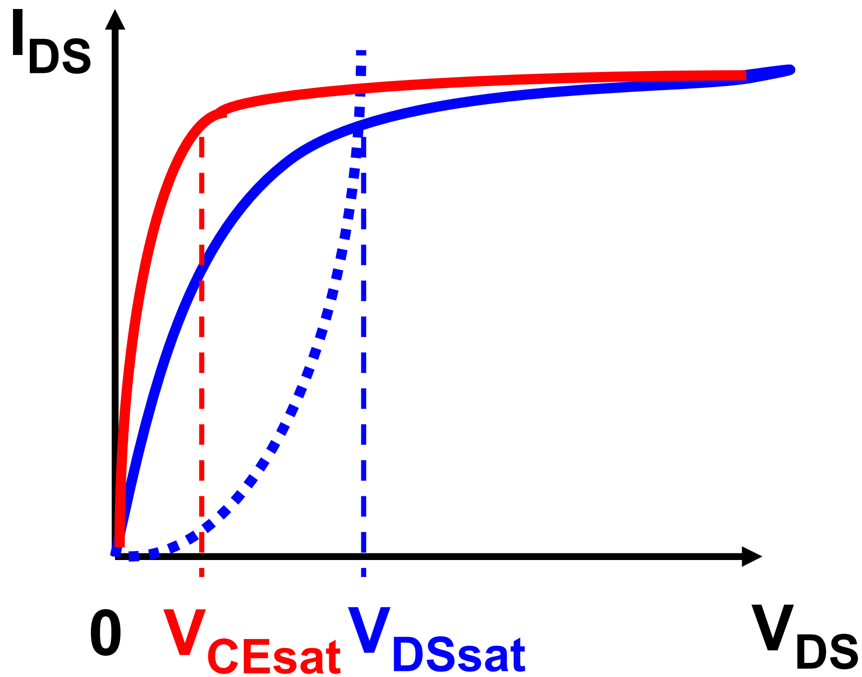
Comparison MOST - Bipolar

TABLE 2-8 COMPARISON OF MOSTS AND BIPOLAR TRANSISTORS

	Specification		MOST	Bipolar transistor	
1.	I_{IN}		0	I_C/β	β **?**
	R_{IN}		∞	$r_\pi + r_B$	
2.	$V_{DS\,sat}$		$V_{GS} - V_T = \sqrt{\dfrac{I_{DS}}{K'W/L}}$	few kT/q	
3.	$\dfrac{g_m}{I}$	wi	$\dfrac{1}{nkT/q}$	$\dfrac{1}{kT/q}$	$n = 1 + \dfrac{C_D}{C_{ox}}$
		si	$\dfrac{2}{V_{GS} - V_T}$	$\dfrac{1}{kT/q}$	**4… 6 x**
		vs	$\dfrac{1}{V_{GS} - V_T}$	$\dfrac{1}{kT/q}$	

Ref. Laker Sansen Table 2-8

Comparison MOST - Bipolar : minimum V_{DS}



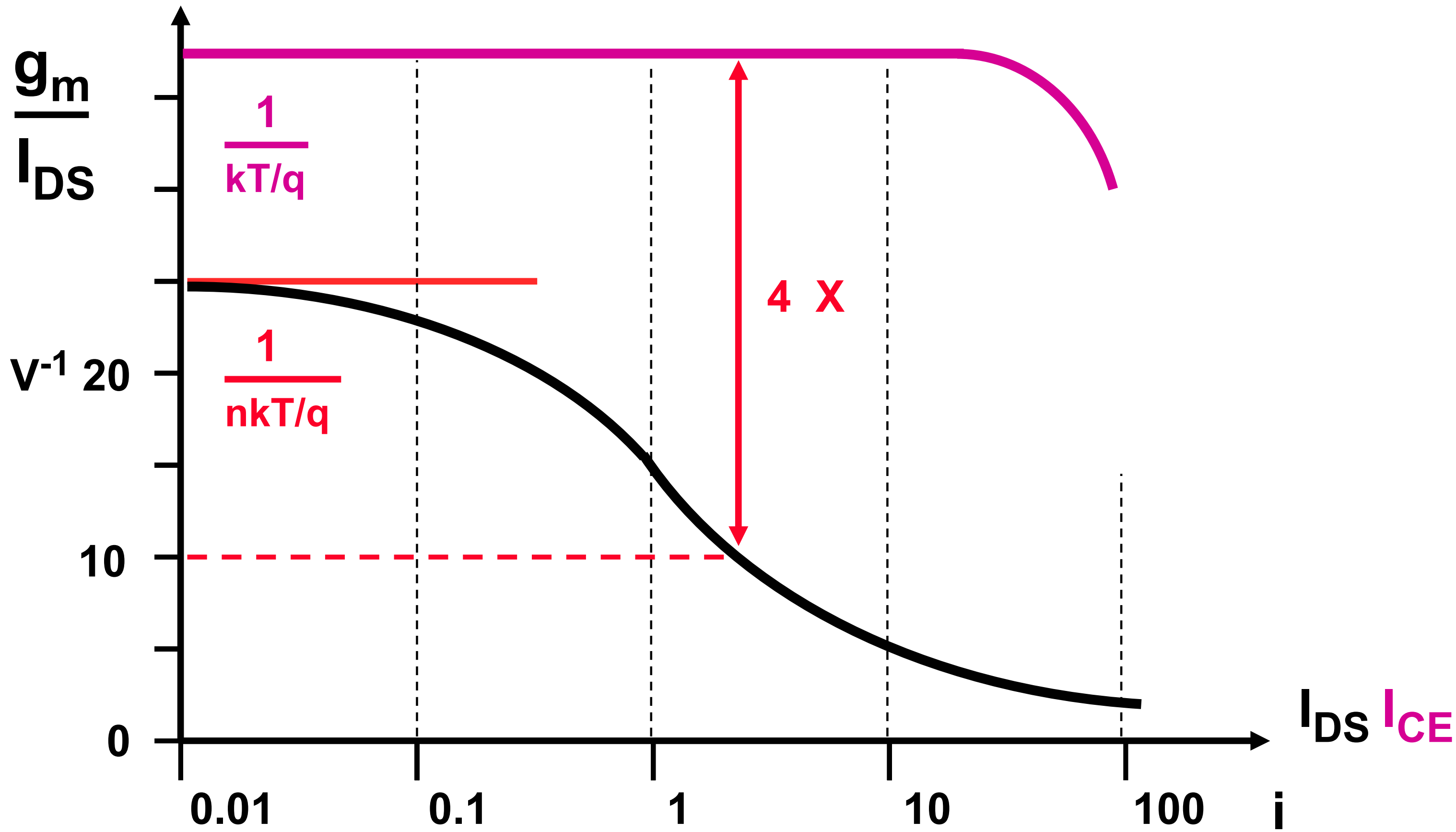
$V_{DSsat} \approx V_{GS} - V_T$

$$V_{GS} - V_T \approx \sqrt{\frac{I_{DS}}{K'_n \frac{W}{L}}}$$

$V_{CEsat} \approx kT/q$'s

Ref. Laker - Sansen Table 2-8

Comparison MOST - Bipolar : g_m/I_{DS} ratio



Design plan for g_m :

$$I_{DS} = K'_n \frac{W}{L} (V_{GS}-V_T)^2$$

$$g_m = 2K'_n \frac{W}{L} (V_{GS}-V_T) = 2\sqrt{K'_n \frac{W}{L} I_{DS}} = \frac{2\, I_{DS}}{V_{GS}-V_T}$$

4 variables with 2 equations >> 2 free variables

Choose V_{GS}-V_T and L !

Comparison MOST - Bipolar

				MOST	Bipolar
4.	Design planning			$L, V_{GS} - V_T$	kT/q
5.	I-range			1 decade	7 decades
6.	Max f_T		low I	C_{GS}, C_{GD}	C_{jEt}, C_{μ}
			high I	v_{sat}/L_{eff}	v_{sat}/W_B
7.	Noise	$\overline{dv_i^2}$	Therm.	$4kT\left(\frac{2/3}{g_m} + R_G\right)$	$4kT\left(\frac{1/2}{g_m} + R_B\right)$
			$1/f$	$10\times$	
	Offset			$10\times$	

$v_{sat} \approx 10^7$ cm/s

Ref. Laker Sansen Table 2-8

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Ref.: W. Sansen : Analog Design Essentials, Springer 2006

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0.2 chap2

Amplifiers,
Source followers & Cascodes

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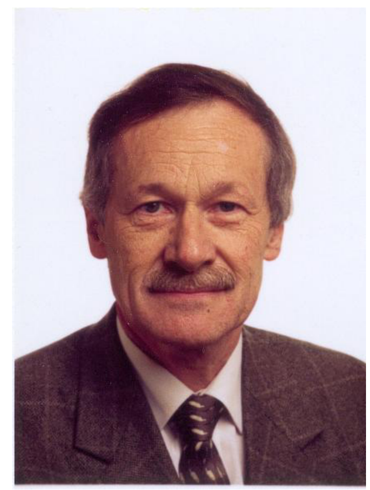
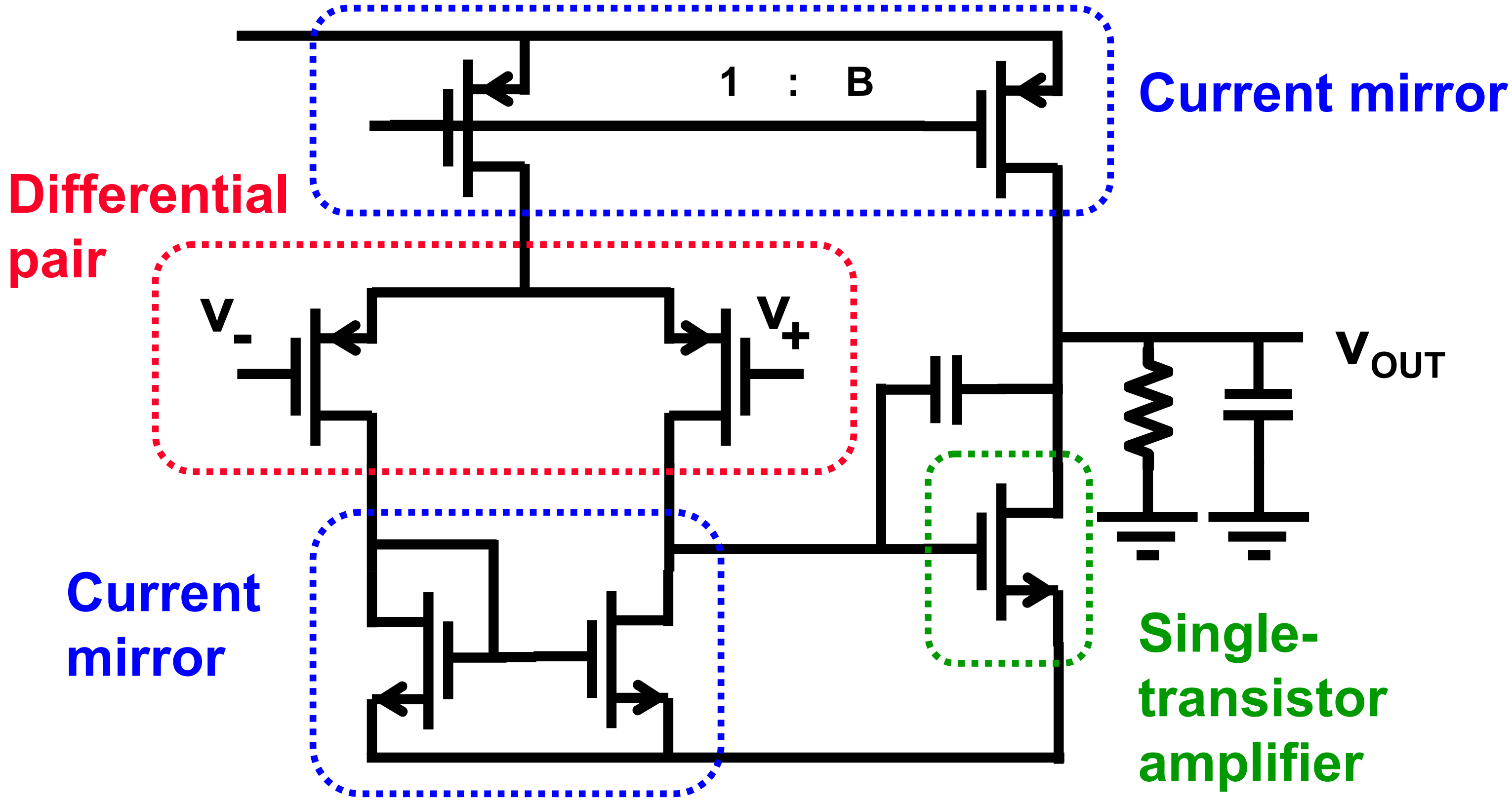


Operational amplifier



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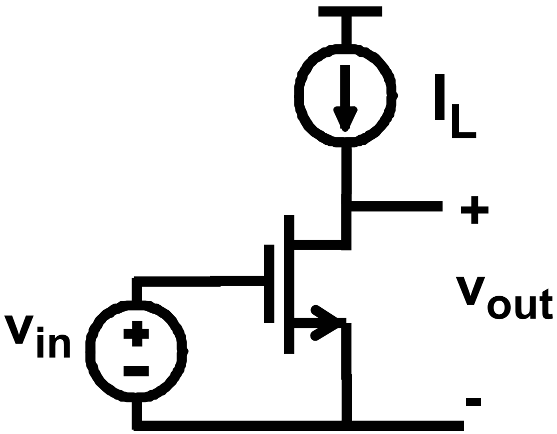
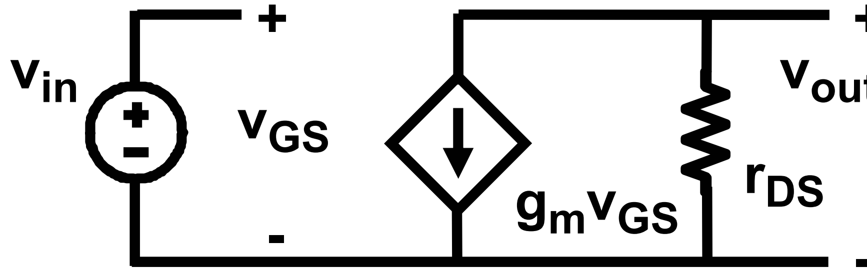
- ❑ **Source followers**
- ❑ **Cascodes**

Single-transistor amplifier - 1





$$A_v = g_m r_{DS} = \frac{2\,I_{DS}}{V_{GS}-V_T}\,\frac{V_E\,L}{I_{DS}} = \frac{2\,V_E\,L}{V_{GS}-V_T}$$

$A_v \approx 100$ if $V_E L \approx 10$ V and $V_{GS}-V_T \approx 0.2$ V

Single-transistor amplifier - 2

High gain ?

Low V_{GS}-V_T and large L !!!

MOST or bipolar amplifier ?

MOST $$A_v = \frac{V_E L}{(V_{GS}-V_T)/2}$$

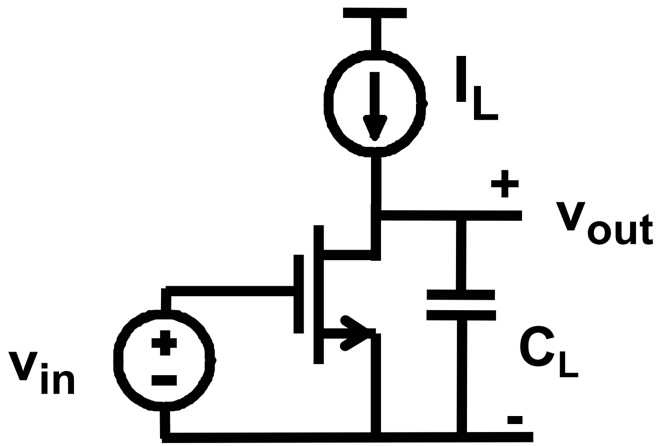
$A_v \approx 100$ if $V_E L \approx 10$ V and $V_{GS}-V_T \approx 0.2$ V

Bipolar $$A_v = \frac{V_E}{kT/q}$$

3 vs 2 stages for 10^6

$A_v \approx 1000$ if $V_E \approx 26$ V since $kT/q = 26$ mV

Gain, Bandwidth and Gain-bandwidth



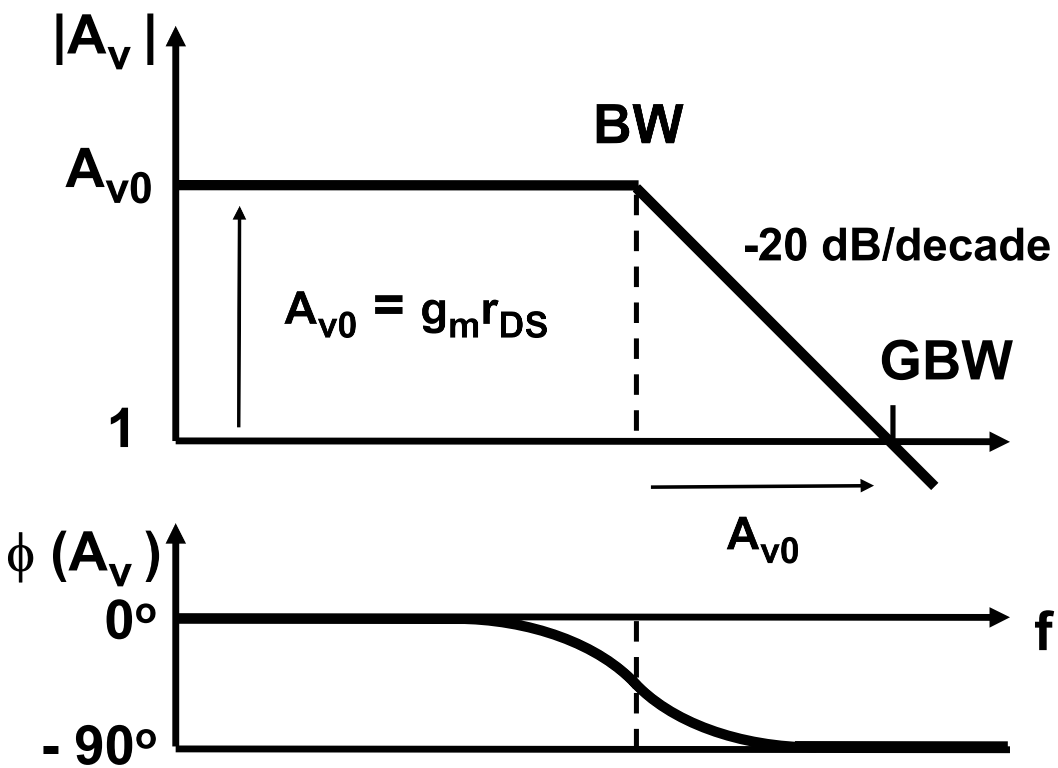
$$A_{v0} = g_m r_{DS}$$

$$BW = \frac{1}{2\pi\, r_{DS} C_L}$$

$$\boxed{GBW = \frac{g_m}{2\pi\, C_L}}$$

For all single-stage Operational amplifiers

Gain A_v , BW and GBW



$$GBW = \frac{g_m}{2\pi\ C_L}$$

$\phi\ (A_v) = -45^o$ at BW

Single-transistor amplifier : Exercise

GBW = 100 MHz for C_L = 3 pF

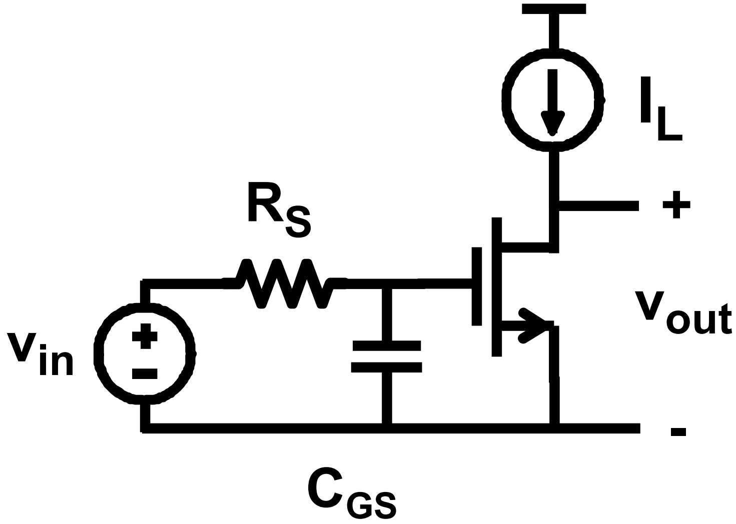
Techno.: $K'_n \approx 50\ \mu A/V^2$

$L_{min} = 0.5\ \mu m$

I_{DS} ? L ? W ?

$$\frac{GBW.C_L}{I_{DS}} ?$$

Gain, Bandwidth and Gain-bandwidth



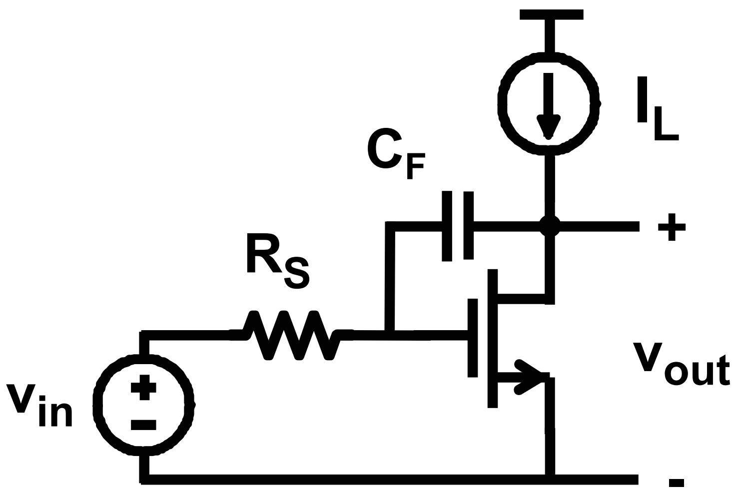
$$A_{v0} = g_m r_{DS}$$

$$BW = \frac{1}{2\pi R_S C_{GS}}$$

$$GBW = \frac{g_m}{2\pi C_{GS}} \frac{r_{DS}}{R_S} = f_T \frac{r_{DS}}{R_S} \sim \frac{1}{WC_{ox}} \frac{1}{V_{GS}-V_T}$$

W ? L ? V_{GS}-V_T?

Gain, Bandwidth and Gain-bandwidth

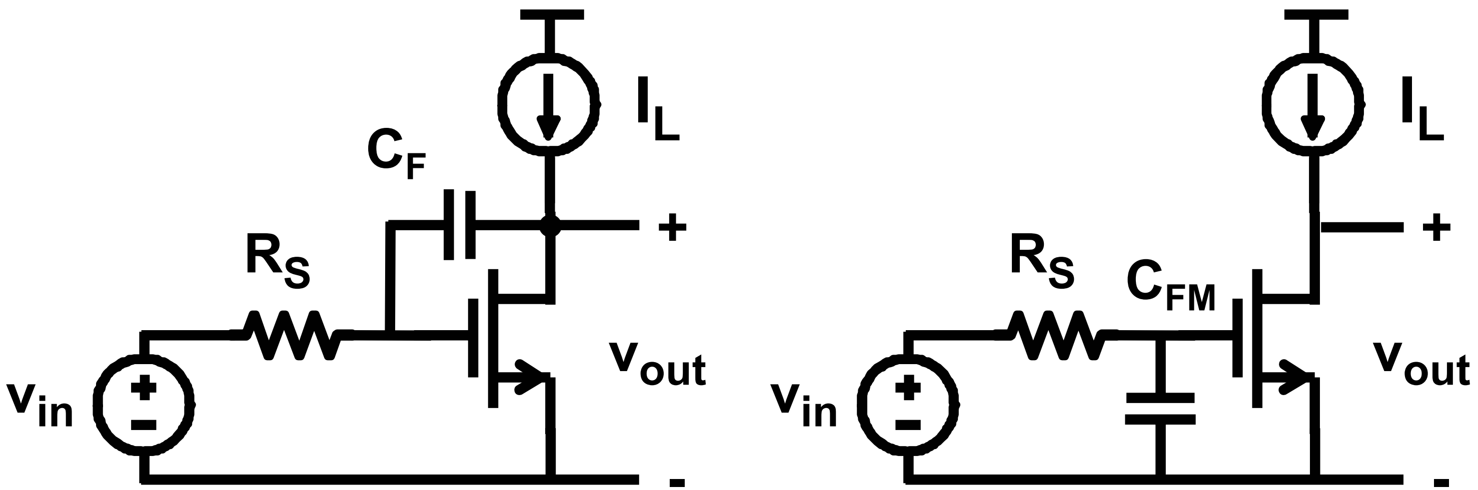


$$A_{v0} = g_m r_{DS}$$

$$BW = \frac{1}{2\pi\, R_S\, A_{v0}\, C_F}$$

$$GBW = \frac{1}{2\pi\, R_S\, C_F}$$

Miller effect

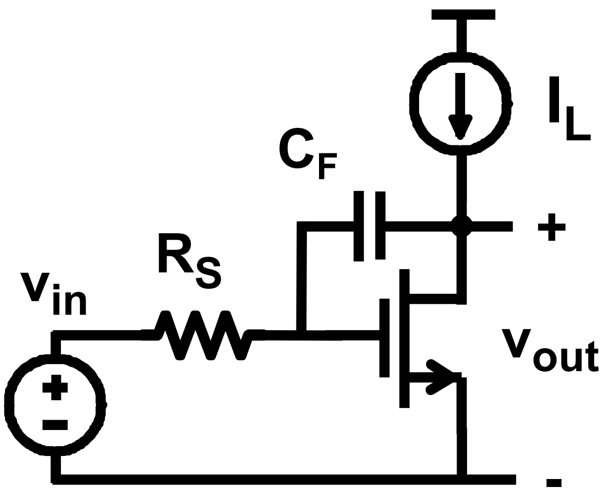


$$A_{v0} = g_m r_{DS}$$

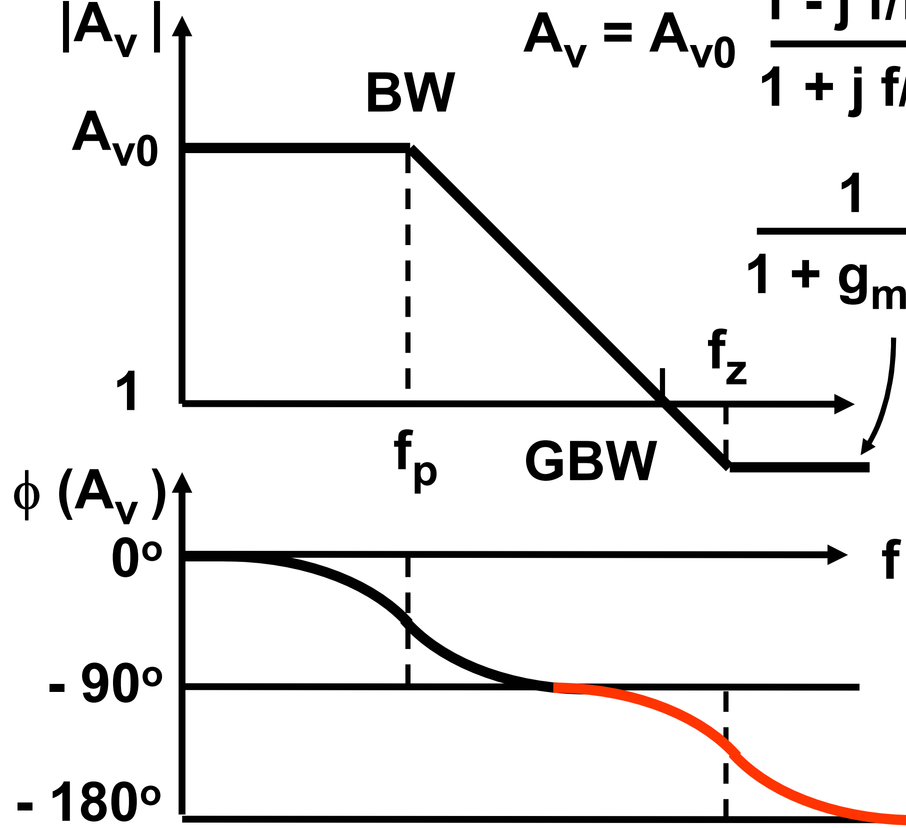
$$C_{FM} = (1+A_{v0})\ C_F$$

Miller, Dependence of the input impedance of a three-electrode vacuum tube upon the load in the plate circuit, Scient. Papers Bur. Standards, 1920, 367-385.

Miller capacitance feedback effects



$$f_z = \frac{g_m}{2\pi C_F}$$

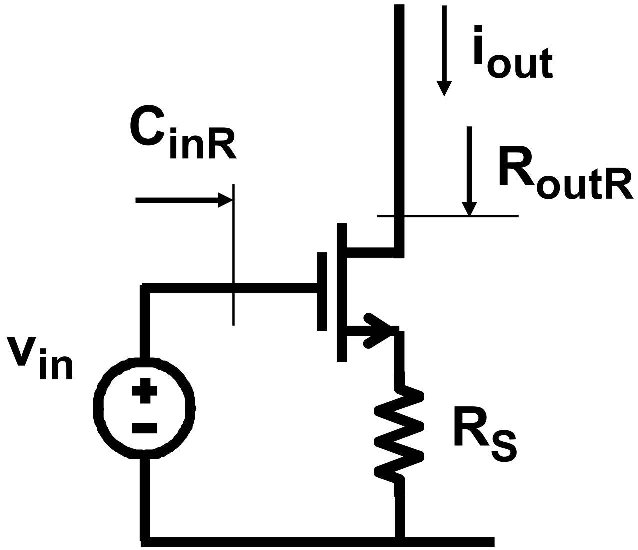


$$A_v = A_{v0}\,\frac{1 - j\,f/f_z}{1 + j\,f/f_p}$$

$$\frac{1}{1 + g_m R_S}$$

For phase, a positive zero is like a negative pole !!!

Amplifier with local R- (series) feedback



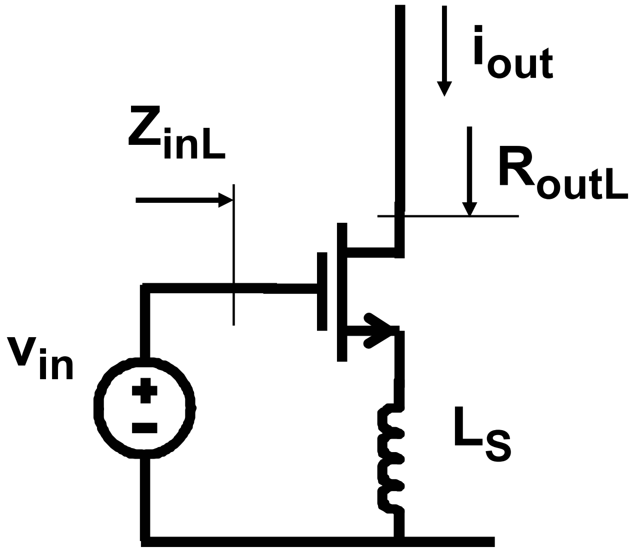
$$g_{mR} = \frac{g_m}{1 + g_m R_S} \sim \frac{1}{R_S}$$

$$R_{outR} = r_{DS}\,(1 + g_m R_S)$$
$$\approx (g_m r_{DS})\, R_S$$

$$C_{inR} = \frac{C_{GS}}{1 + g_m R_S}$$

But R_S gives extra noise !

Amplifier with local L- feedback



$$g_{mL} = \frac{g_m}{1 + g_m L_S s}$$

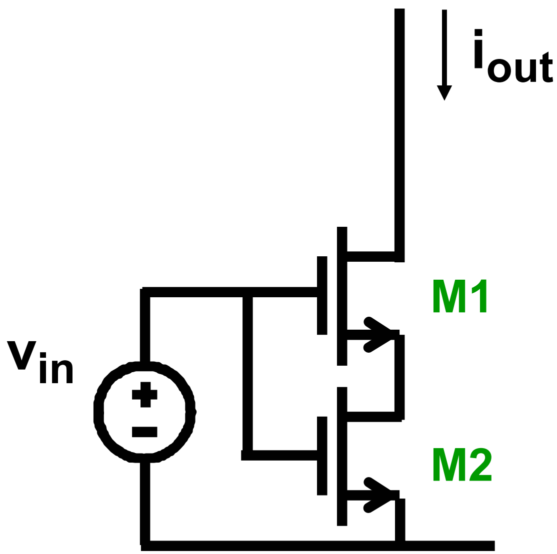
$$R_{outL} = r_{DS}\,(1 + g_m L_S s)$$

$$Z_{inL} = g_m \frac{L_S}{C_{GS}} + \frac{1 + L_S C_{GS} s^2}{s\, C_{GS}}$$

$$Z_{inL} = L_S\, \omega_T + L_S s + \frac{1}{s\, C_{GS}}$$

No extra noise !

Amplifier with local MOST-R- Feedback



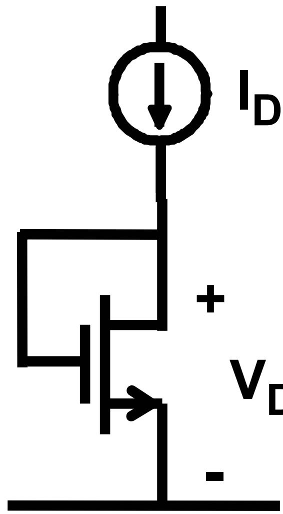
$$V_{DS2} = V_{GS2} - V_{GS1} \approx 0.2\ V$$

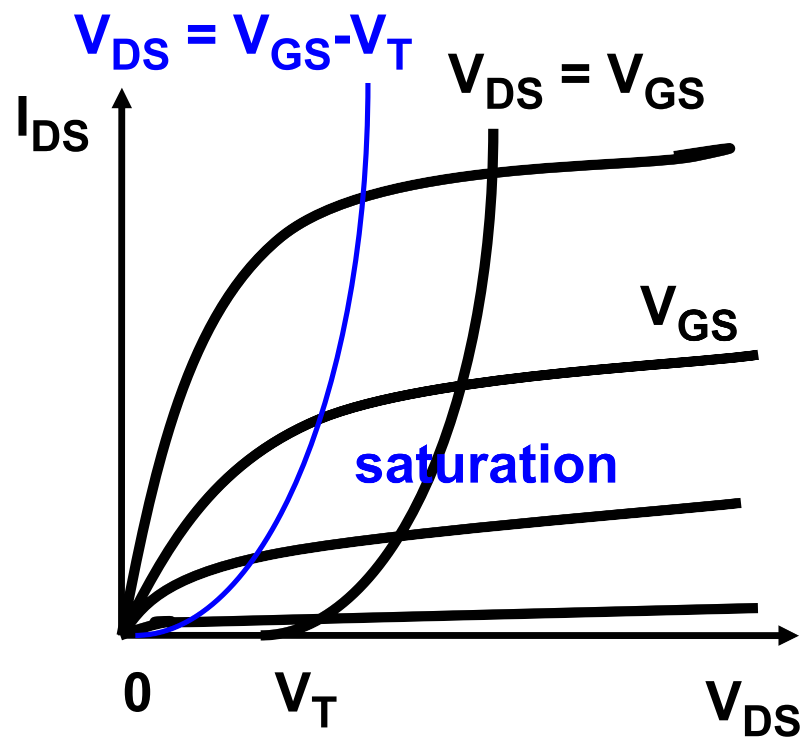
$$r_{DS2} = \frac{1}{KP\ W_2/L_2\ (V_{GS2}-V_T)}$$

$$R_{outR} = r_{DS1}\ (1 + g_{m1}\ r_{DS2})$$

$$C_{inR} = \frac{C_{GS1}+C_{GS2}}{1 + g_{m1} r_{DS2}}$$

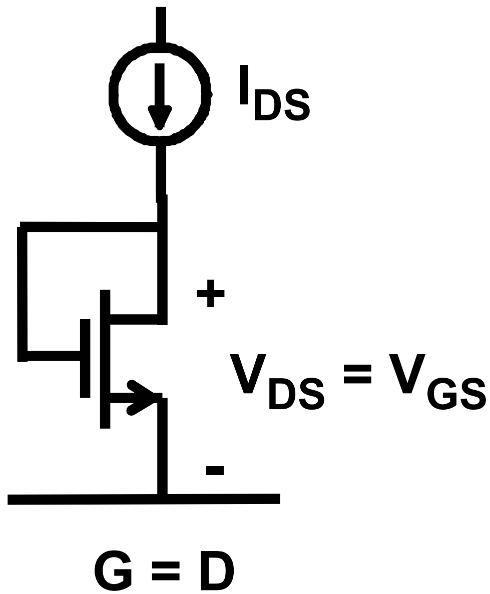
Diode-connected MOST : parallel Feedback

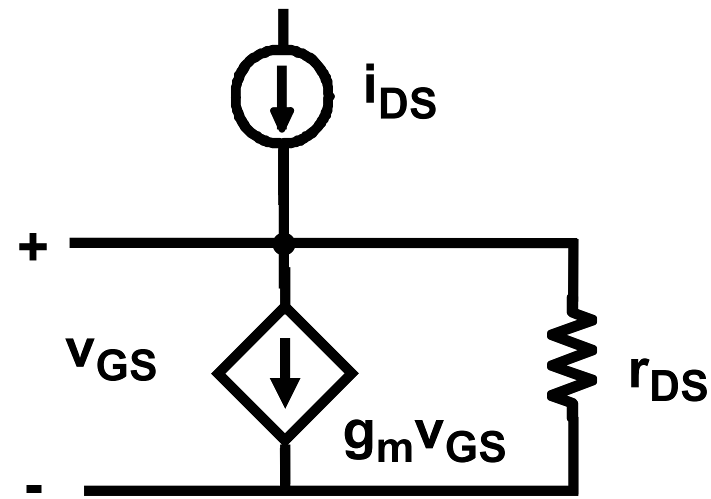




$$I_{DS} = K'_n \frac{W}{L} (V_{DS}-V_T)^2$$

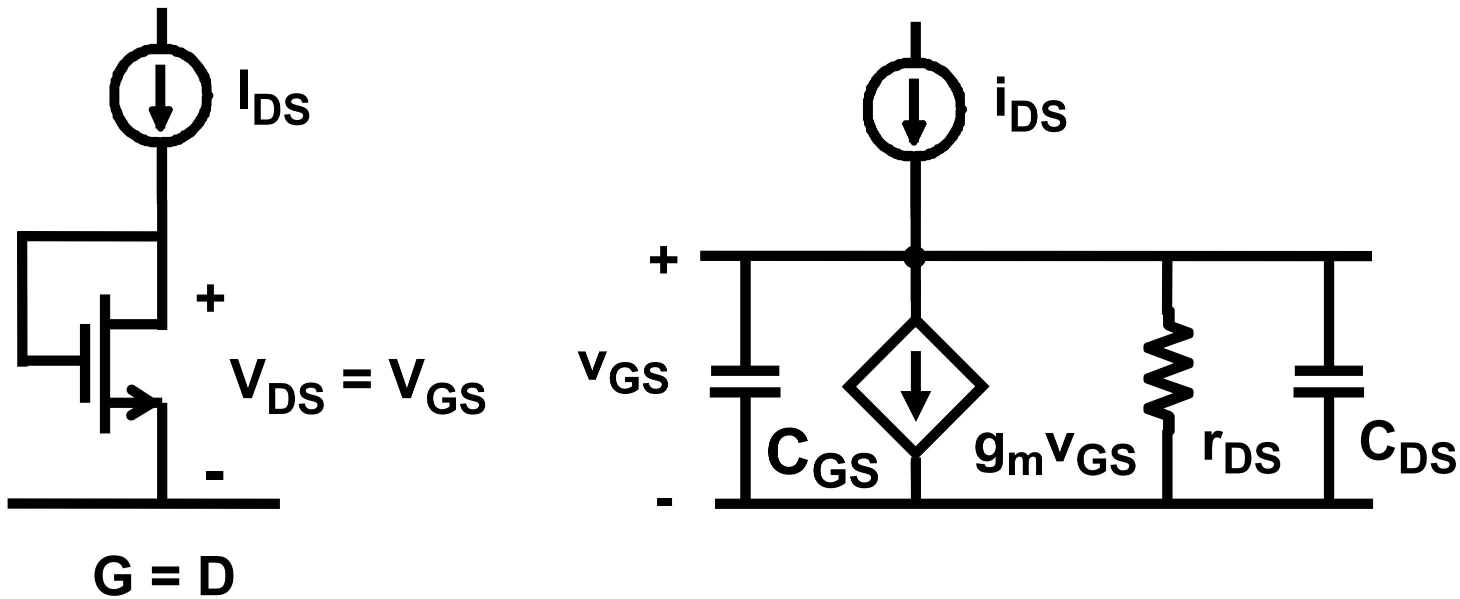
Diode-connected MOST: small-signal





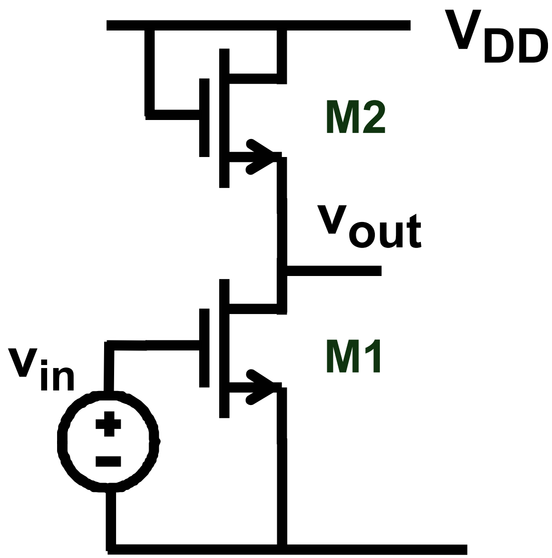
$$r_{ds} = 1/g_m \ // \ r_{DS} \approx 1/g_m$$

Diode-connected MOST at high frequencies



$$BW = \frac{g_m}{2\pi\,(C_{GS}+C_{DS})} \approx \frac{f_T}{2}$$

Wideband amplifier

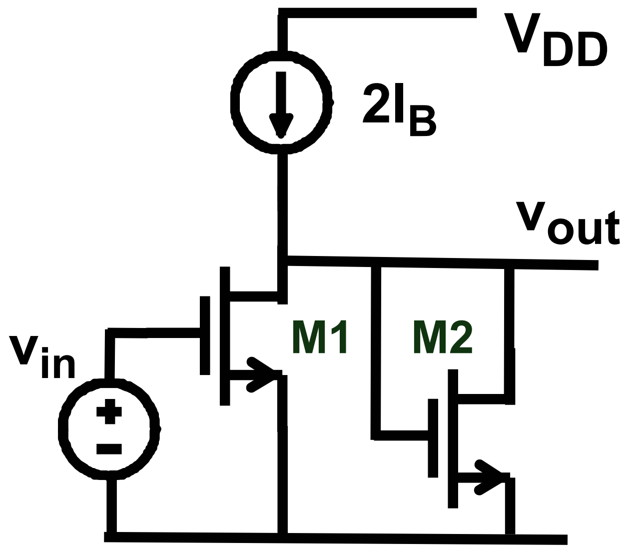


$$V_{OUT} = V_{DD} - V_{GS2}(V_{OUT})$$

$$A_{v0} = \frac{g_{m1}}{g_{m2}} = \sqrt{\frac{(W/L)_1}{(W/L)_2}} = \frac{V_{GS2} - V_T}{V_{GS1} - V_T}$$

$$R_{OUT} = 1/g_{m2}$$

Linear wideband amplifier



$$V_{OUT} = V_{GS2}$$

$$A_{v0} = \frac{g_{m1}}{g_{m2}} = \sqrt{\frac{(W/L)_1}{(W/L)_2}} = \frac{V_{GS2} - V_T}{V_{GS1} - V_T}$$

$$R_{OUT} = 1/g_{m2}$$

Current mirror with only nMOSTs

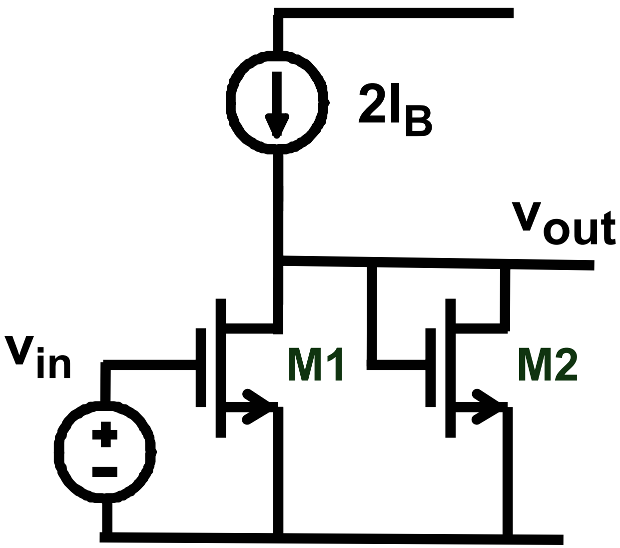
Same V_{OUTDC} as V_{INDC}

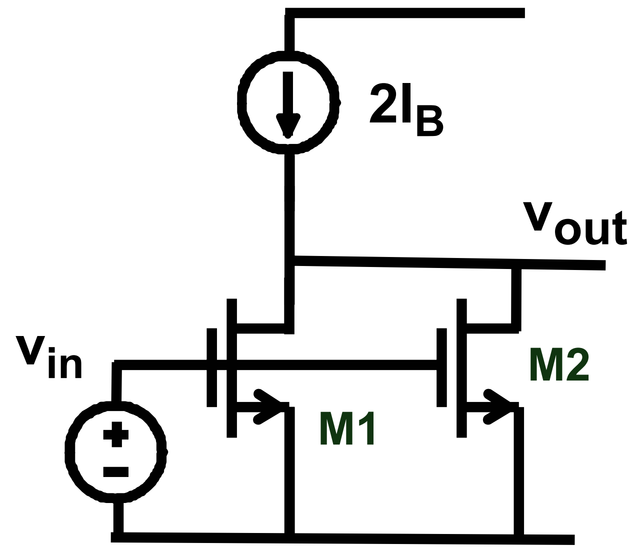
No body bias effect

Good PSRR

Double power consumption

Wideband amplifiers





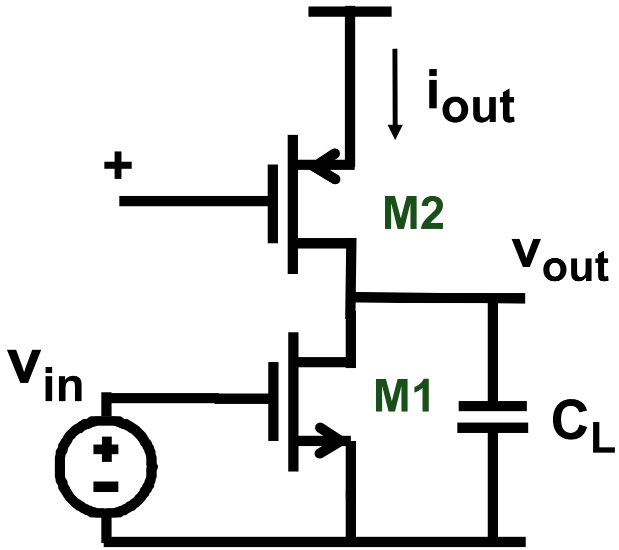
$$A_{v0} = \frac{g_{m1}}{g_{m2}} = \sqrt{\frac{(W/L)_1}{(W/L)_2}} = \frac{V_{GS2} - V_T}{V_{GS1} - V_T}$$

$$R_{out} = 1/g_{m2}$$

$$A_{v0} = g_m R_{out}$$

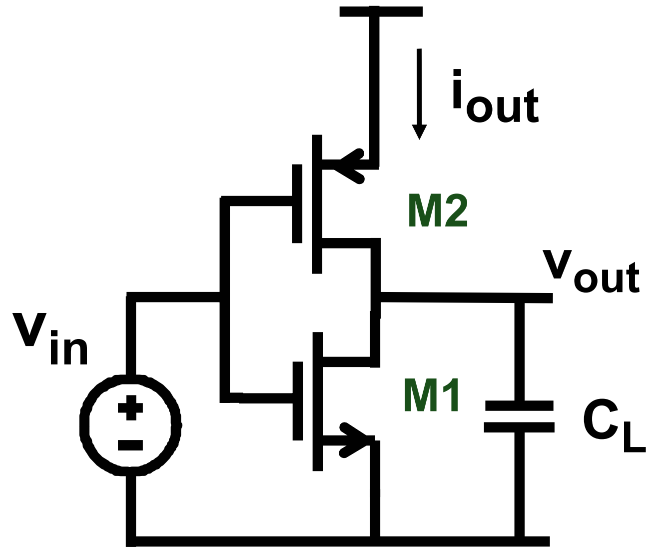
$$R_{out} = r_{DS1}//r_{DS2}$$

Class A versus class AB amplifier



$$v_{out} = A_v\, v_{in}$$

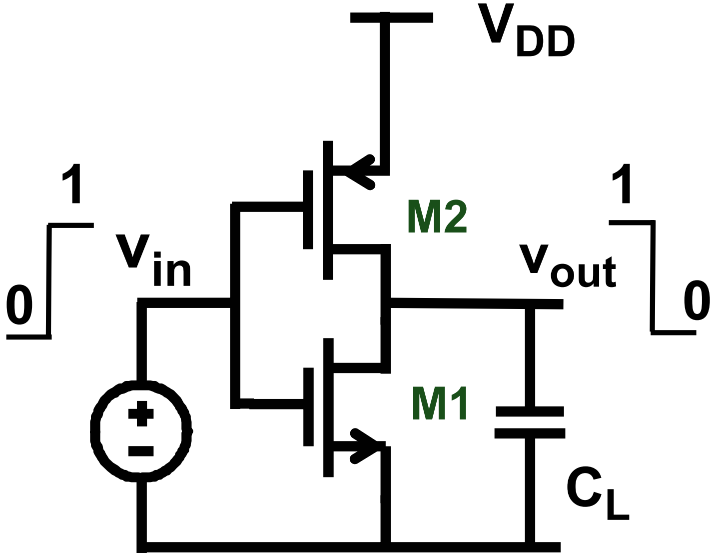
Class A stage



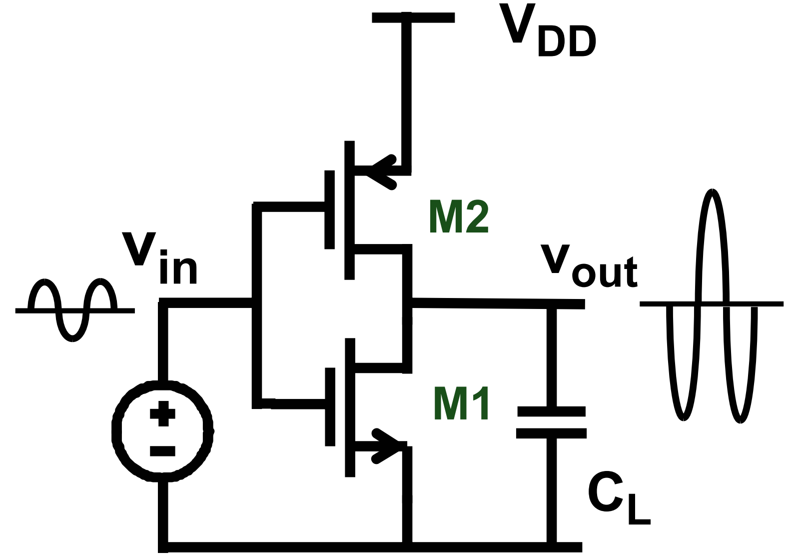
$$v_{out} = A_v\, v_{in}$$

Class AB stage

CMOS inverter-amplifier

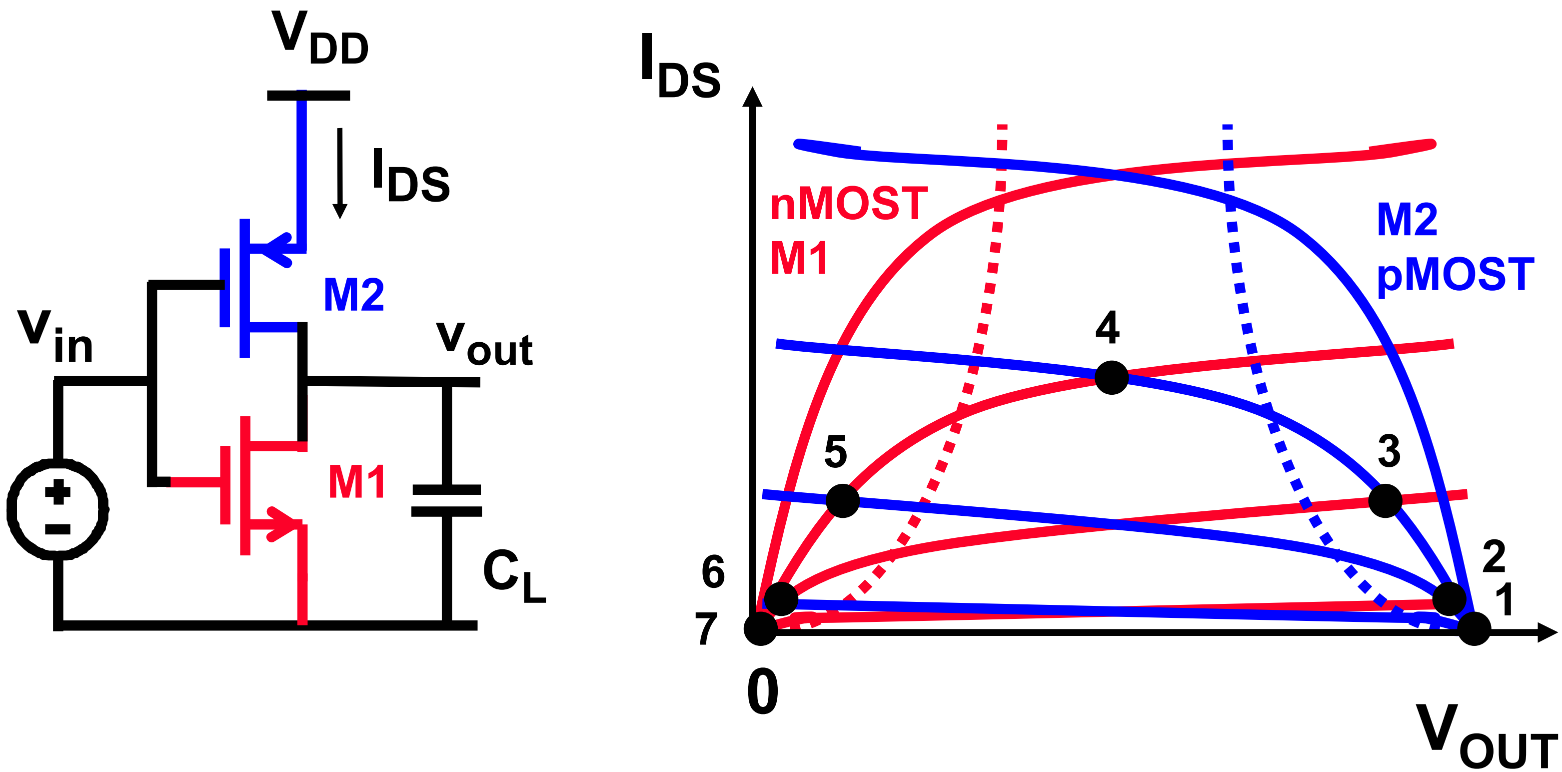


Digital invertor



Analog amplifier

Operating points nMOST & pMOST



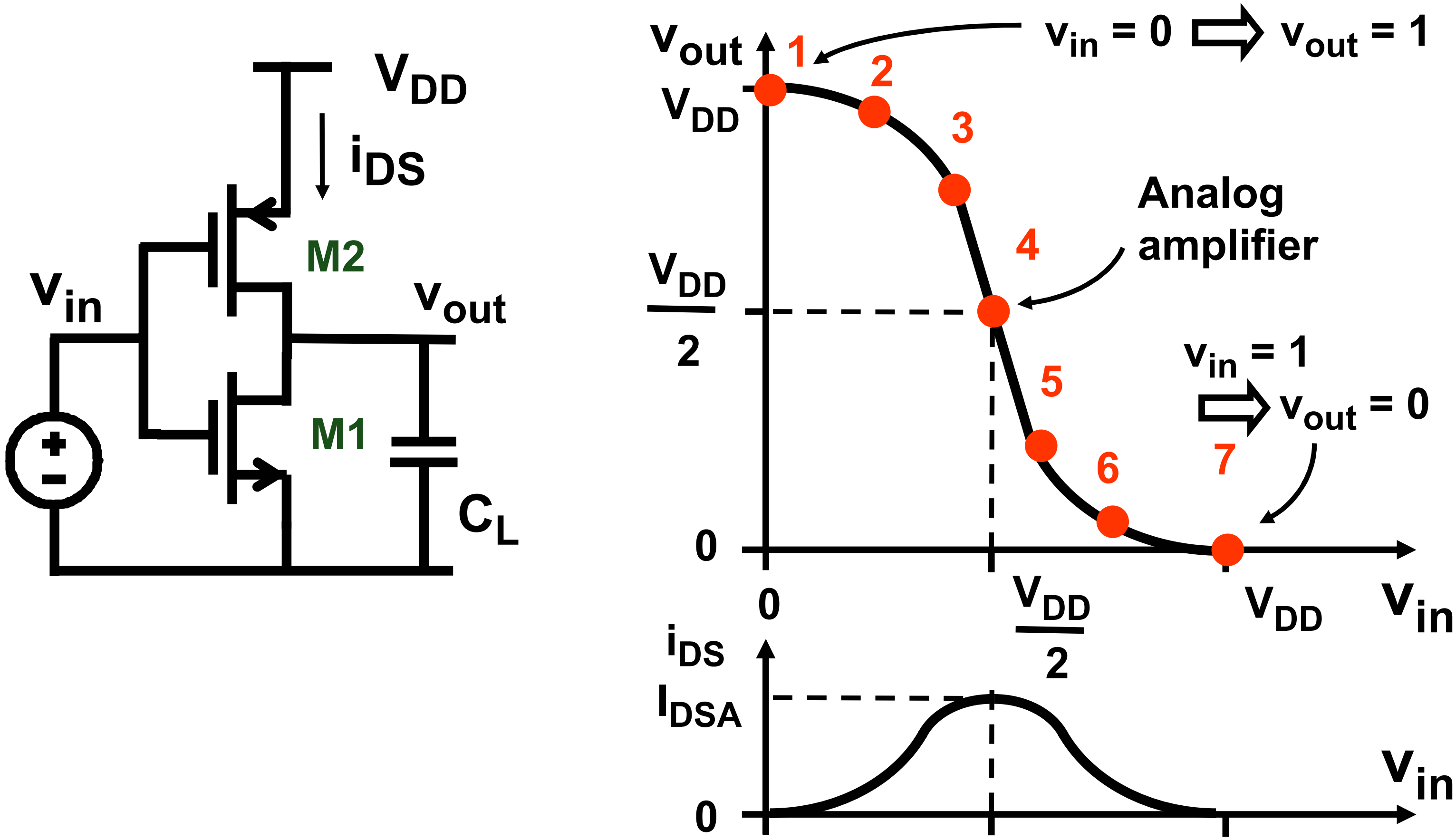
$V_{DD} = V_{DSn} + V_{DSp}$

$= V_{GSn} + V_{GSp}$

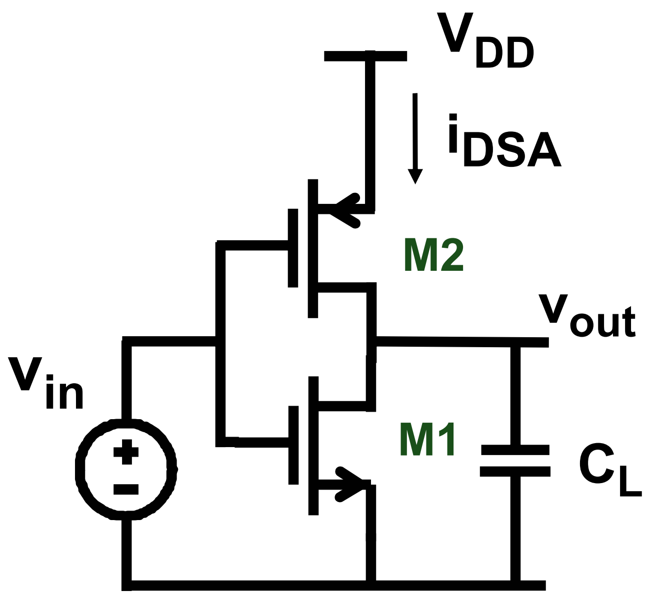
$V_{DSn} = V_{OUT}$ $\quad V_{DSp} = V_{DD} - V_{OUT}$

$V_{GSn} = V_{IN}$ $\quad V_{GSp} = V_{DD} - V_{IN}$

Transfer characteristic



Analog amplifier : DC



$$V_{in} = \frac{V_{DD}}{2} \Rightarrow V_{out} = \frac{V_{DD}}{2}$$

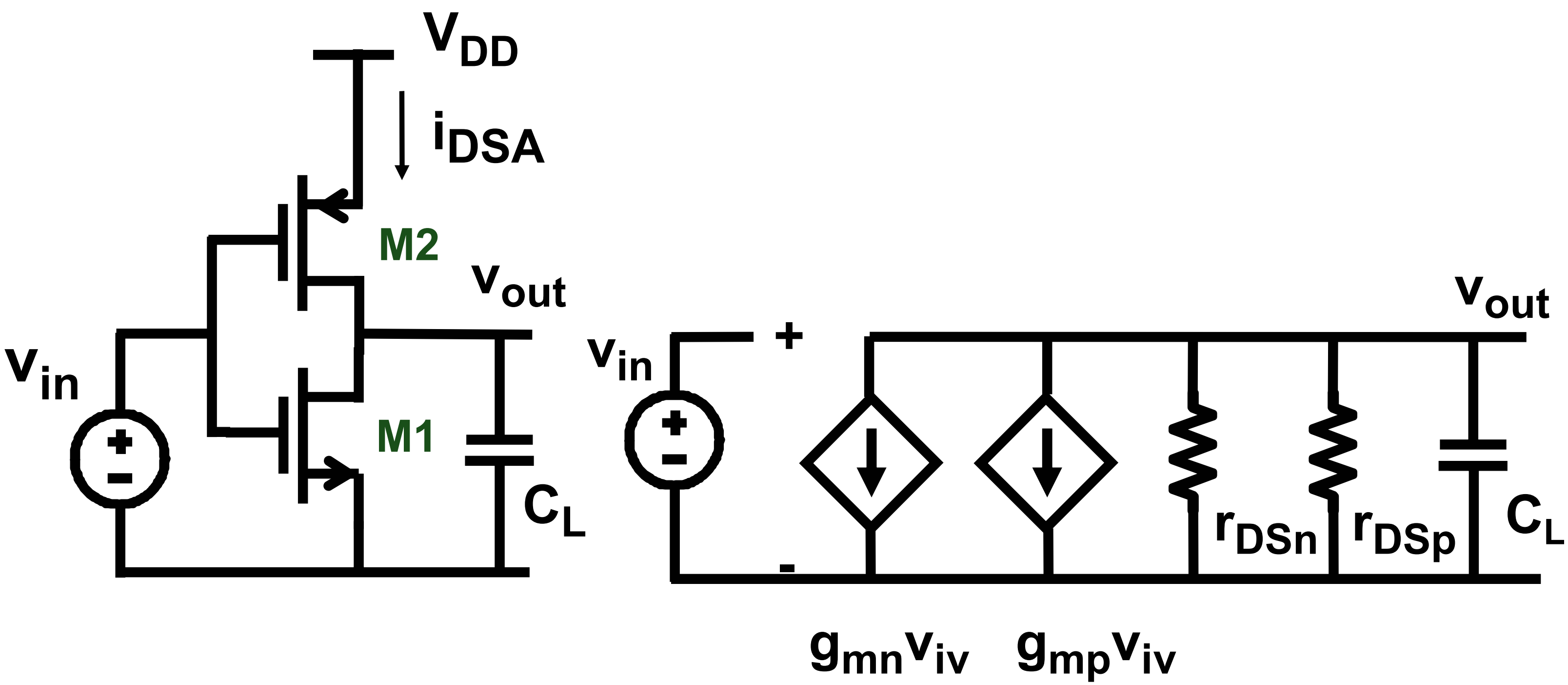
$$I_{DSn} = K'_n \frac{W_n}{L_n} (V_{in} - V_T)^2$$

$$I_{DSp} = K'_p \frac{W_p}{L_p} (V_{DD} - V_{in} - V_T)^2$$

$$\Rightarrow K'_n \frac{W_n}{L_n} = K'_p \frac{W_p}{L_p}$$

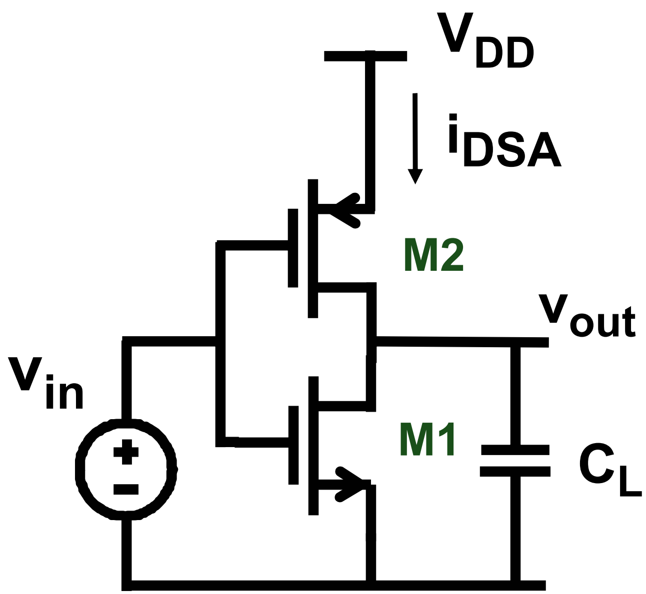
$$I_{DS} = K'_n \frac{W_n}{L_n} \left(\frac{V_{DD}}{2} - V_T\right)^2$$

Analog amplifier : AC model



For the same I_{DS} en V_{GS}-V_T : $g_{mn} = g_{mp} = g_m$

Analog amplifier: AC gain Av



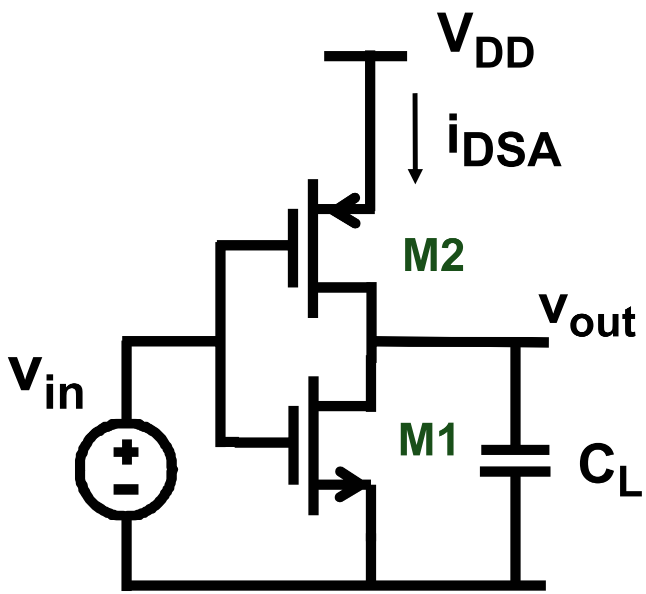
If $V_{En}L_n = V_{Ep}L_p = V_E$

$$g_{DSn} = g_{DSp} = g_{DS}$$

$$(g_{DS} = 1/r_{DS})$$

$$A_{v0} = -\frac{2g_m}{2g_{DS}} = -\frac{2V_E}{\frac{V_{DD}}{2} - V_T}$$

Analog amplifier : BW & GBW



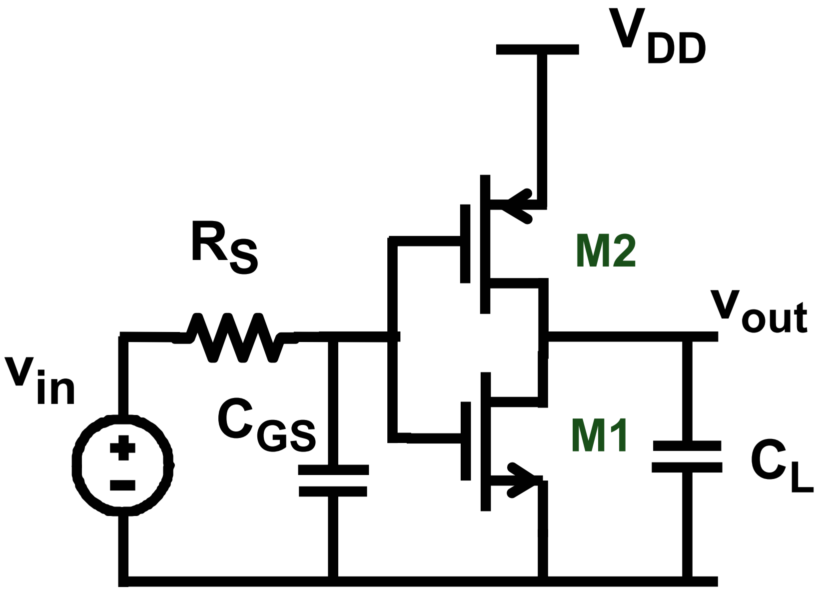
$$A_{v0} = 2g_m R_{out}$$

$$R_{out} = \frac{r_{DS}}{2}$$

$$BW = \frac{1}{2\pi\, R_{out} C_L}$$

$$\boxed{GBW = \frac{2g_m}{2\pi\, C_L}}$$

Analog amplifier: poles due to C_{GS}



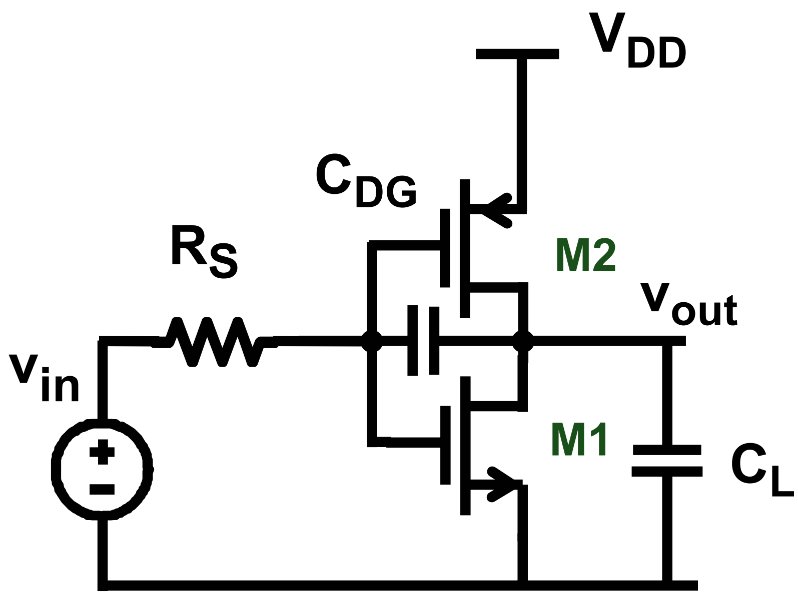
$$A_{v0} = 2g_m R_{out}$$

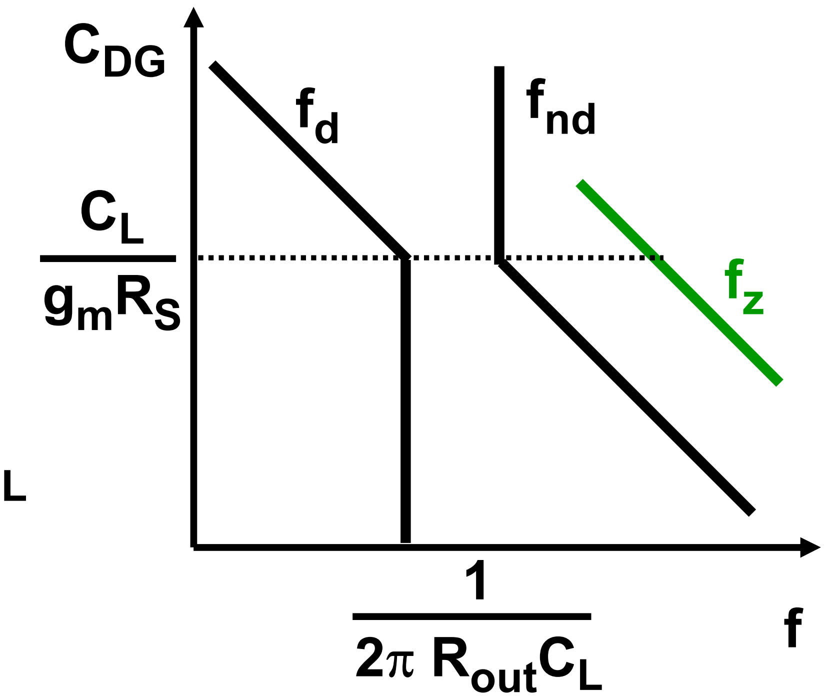
$$GBW = \frac{2g_m}{2\pi\, C_L}$$

$$C_{GSt} = C_{GS1} + C_{GS2}$$

But if $R_S C_{GSt} > r_{DS} C_L$: $\quad GBW = f_T \dfrac{r_{DS}}{R_S}$

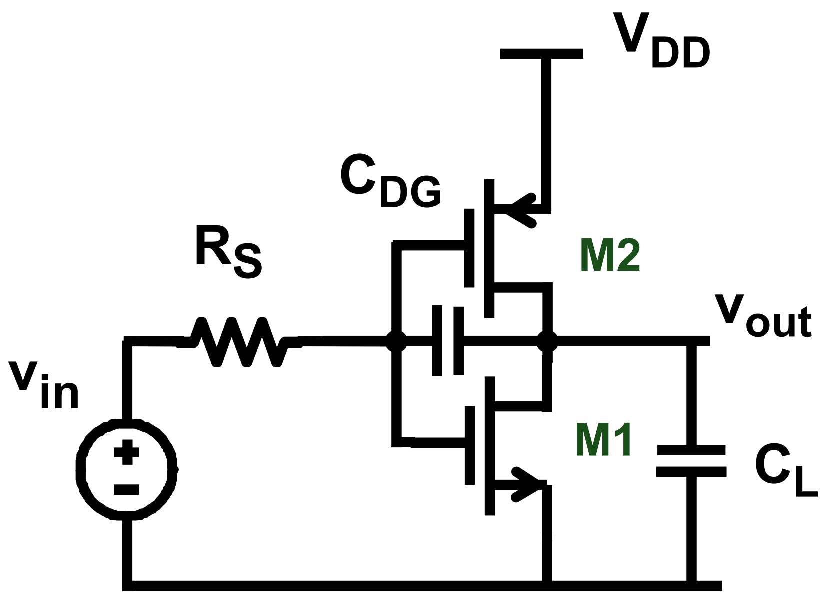
Analog amplifier: poles due to C_{DG}





$$\frac{v_{out}}{v_{in}} = \frac{A_{v0}\,(1 - sC_{DGt}/g_m)}{1 + s\,(R_{out}C_L + A_{v0}R_SC_{DGt}) + s^2 R_SR_{out}C_{DGt}C_L}$$

Analog amplifier: other poles



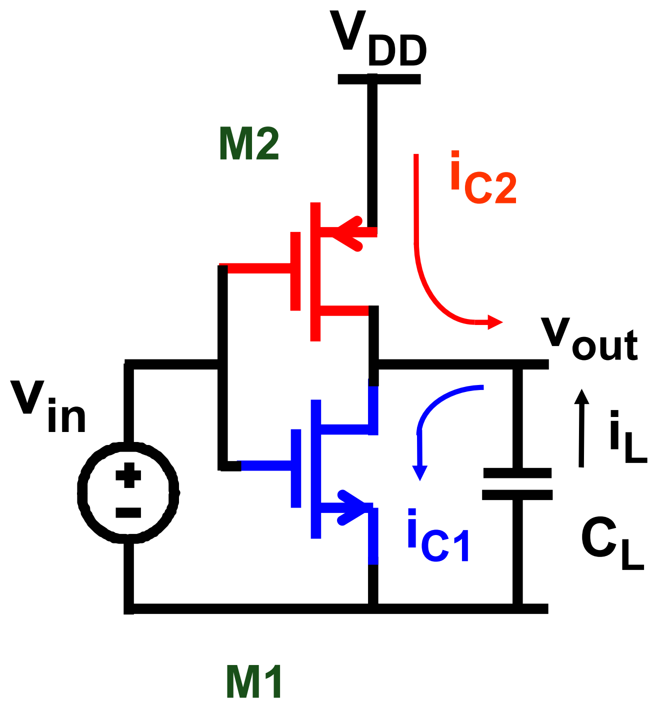
$$A_{v0} = 2g_m R_{out}$$

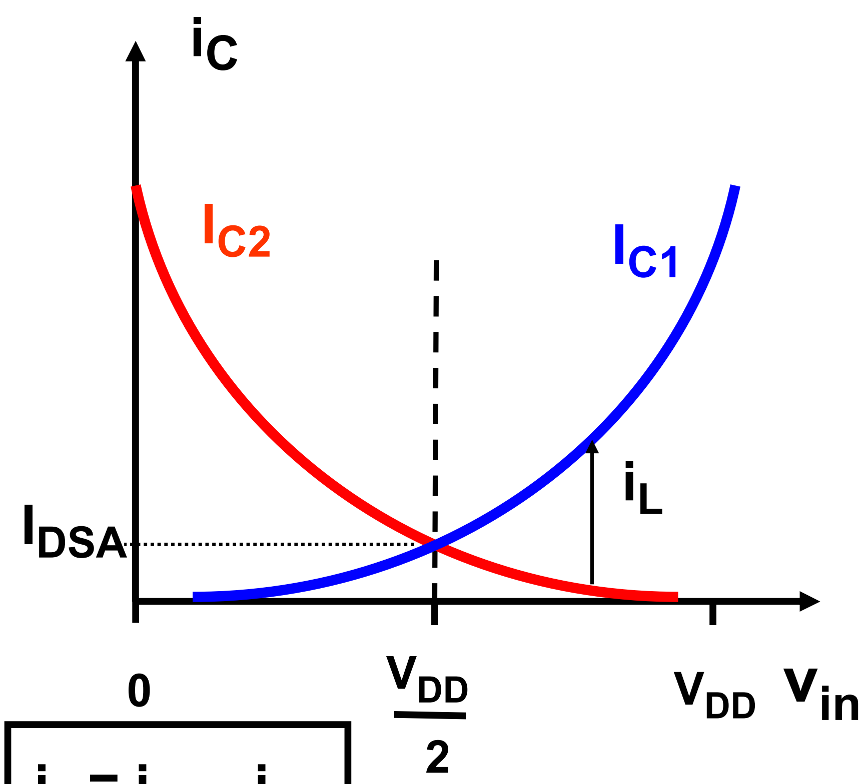
$$GBW = \frac{2g_m}{2\pi\, C_L}$$

$$C_{DGt} = C_{DG1} + C_{DG2}$$

But if $R_S C_{DGt} > \dfrac{1}{2\pi\, GBW}$: $\quad GBW = \dfrac{1}{2\pi\, R_S\, C_{DGt}}$

Class AB operation





$$\boxed{i_L = i_{C2} - i_{C1}}$$

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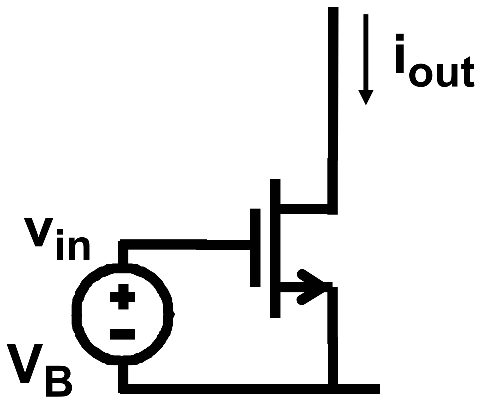
Single-transistor stages

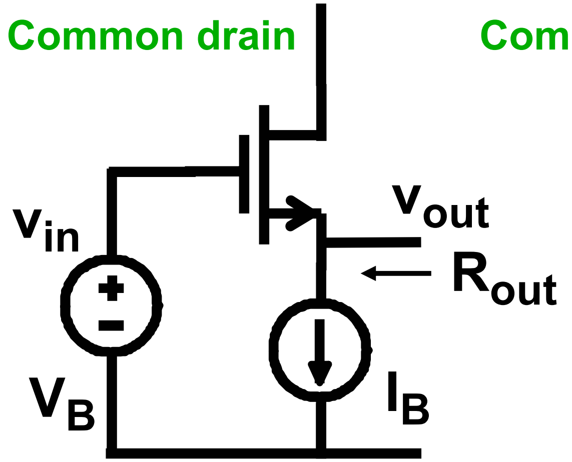
Common source



$$i_{out} = g_m v_{in}$$

Amplifier

Common drain



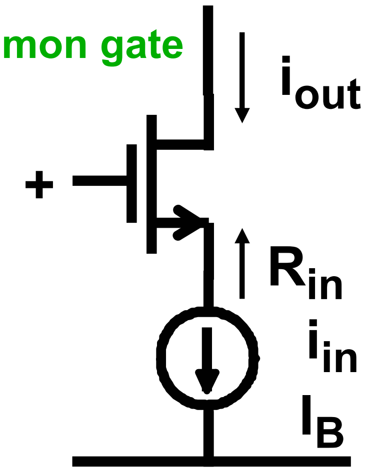
$$v_{out} = v_{in}$$

$$R_{out} \approx 1/g_m$$

Source follower

Voltage buffer

Common gate



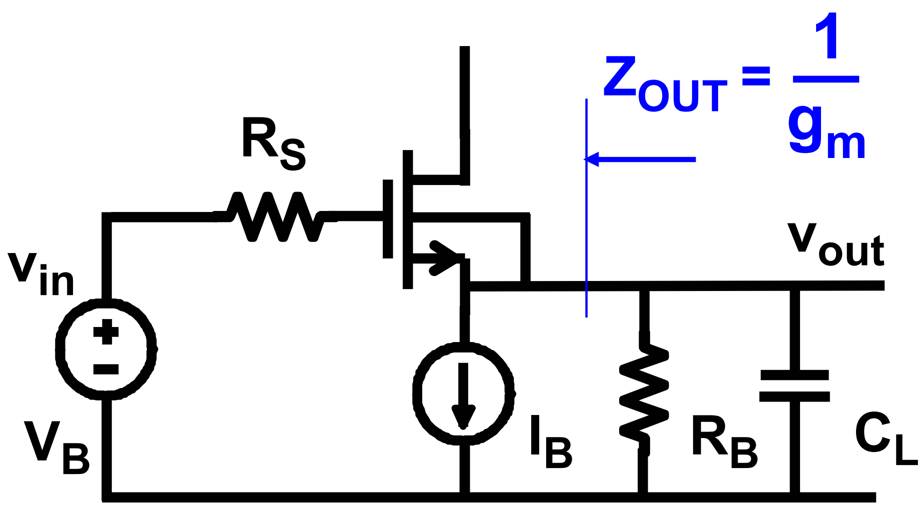
$$i_{out} = i_{in}$$

$$R_{in} \approx 1/g_m$$

Cascode

Current buffer

Source follower with $V_{BS} = 0$ (p-well)



$$V_{GS} = V_{T0} + \sqrt{\frac{I_B}{K'W/L}}$$

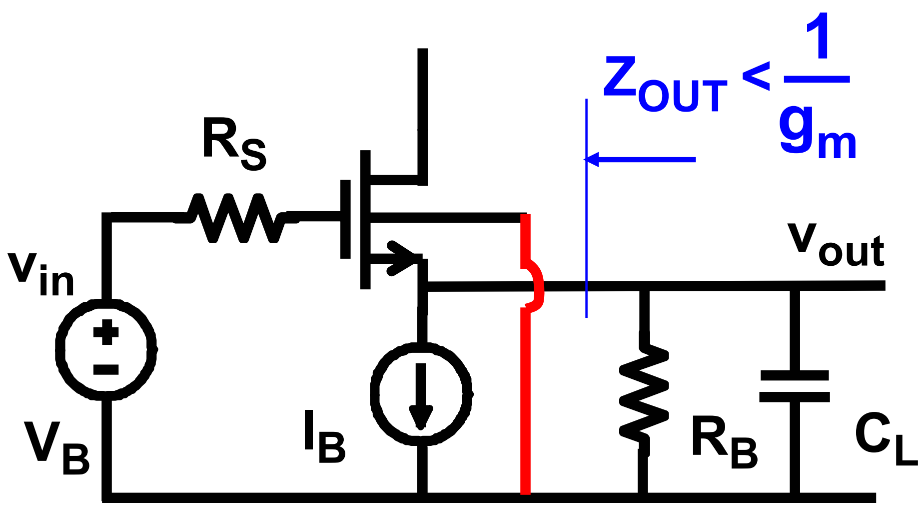
$V_{GS} = ct$ if $I_B = ct$

$$V_{OUT} = V_{IN} - V_{GS}$$

$$\Delta V_{OUT} = \Delta V_{IN}$$

$$\boxed{A_v = 1}$$

Source follower with $V_{BS} \neq 0$ (n-well)



$$V_{GS} = V_T + \sqrt{\frac{I_B}{K'W/L}}$$

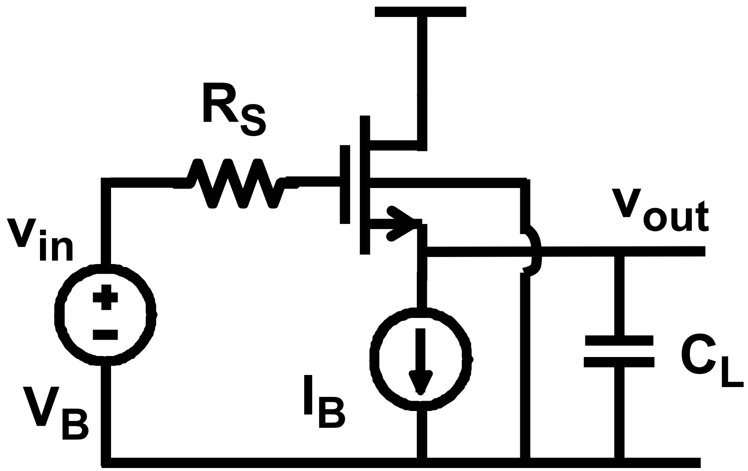
$V_{GS} \neq ct$

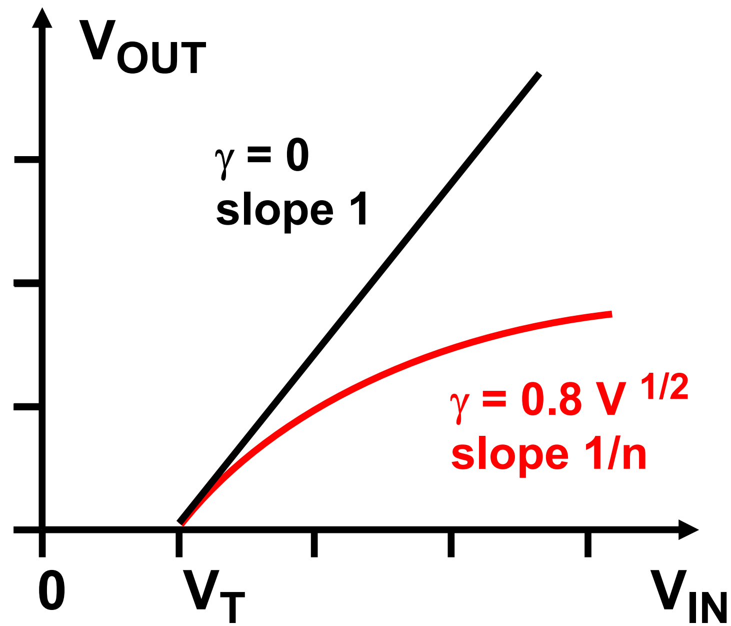
$V_{OUT} = V_{IN} - V_{GS}$

$$V_T = V_{T0} + \gamma \left[\sqrt{|2\Phi_F| + V_{OUT}} - \sqrt{|2\Phi_F|} \right]$$

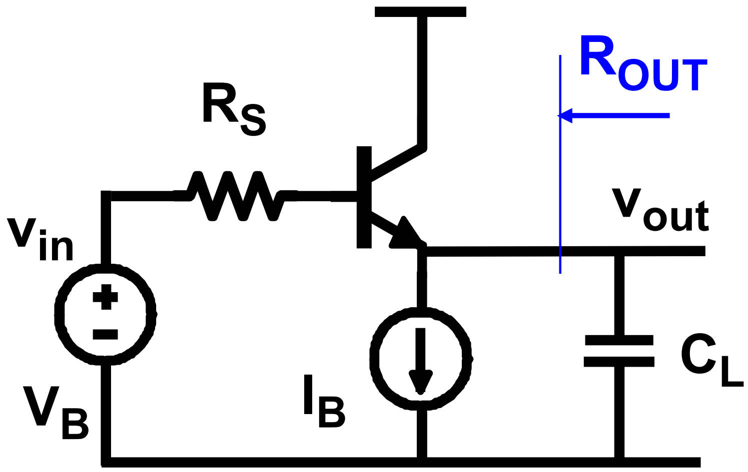
$$\boxed{A_v = \frac{1}{n}}$$

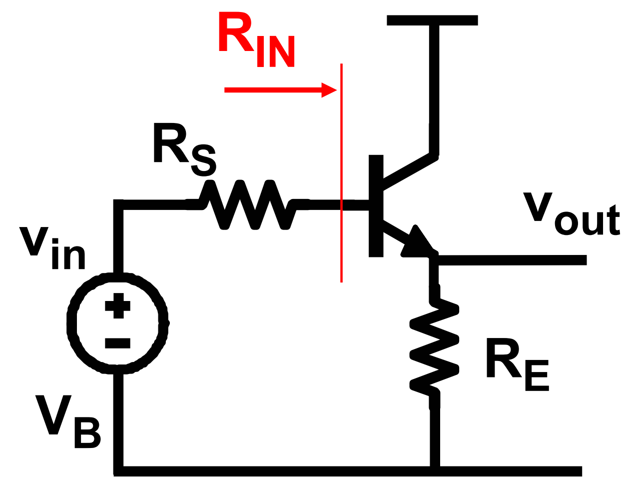
Source follower non-linearity





Emitter follower





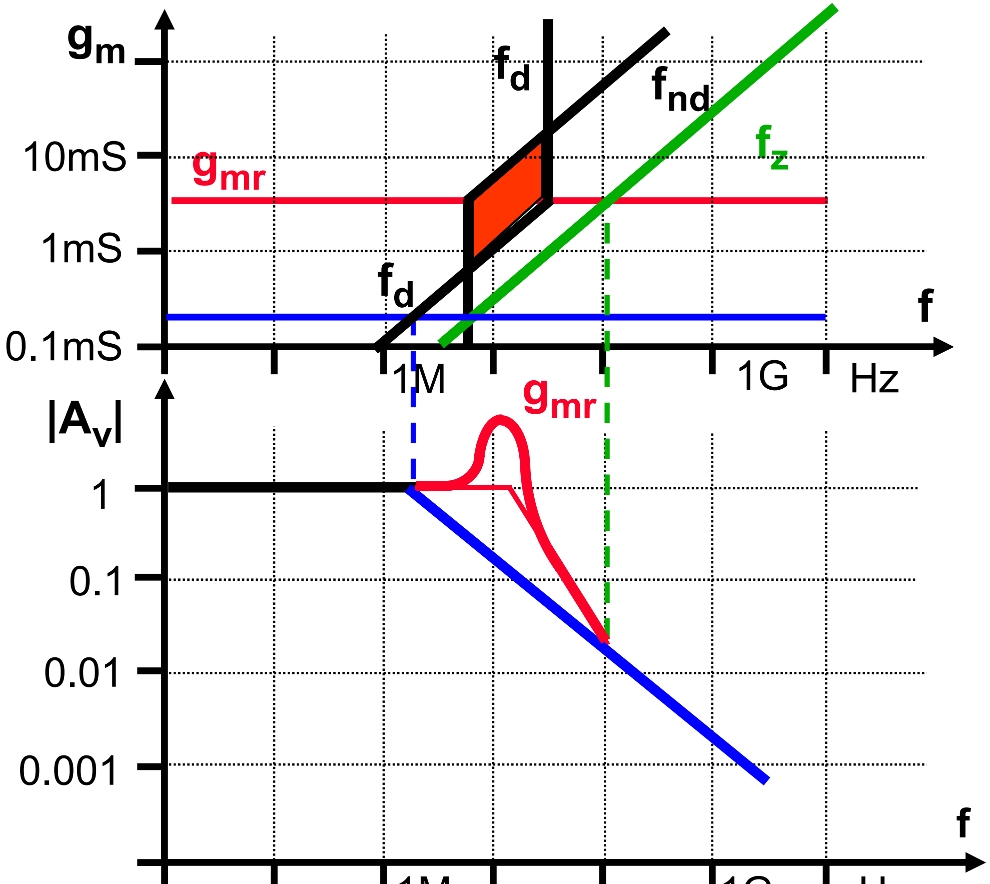
$\boxed{A_v = 1}$

$$R_{OUT} = \frac{1}{g_m} + \frac{R_S + r_B}{\beta+1}$$

$$R_{IN} = r_\pi + r_B + (\beta+1)R_E$$

Limited isolation !

Source follower with C_L load



$$A_v = \frac{(1 + s\,C_{GS}/g_m)}{1 + s\,B + s^2\,C^2 R_S/g_m}$$

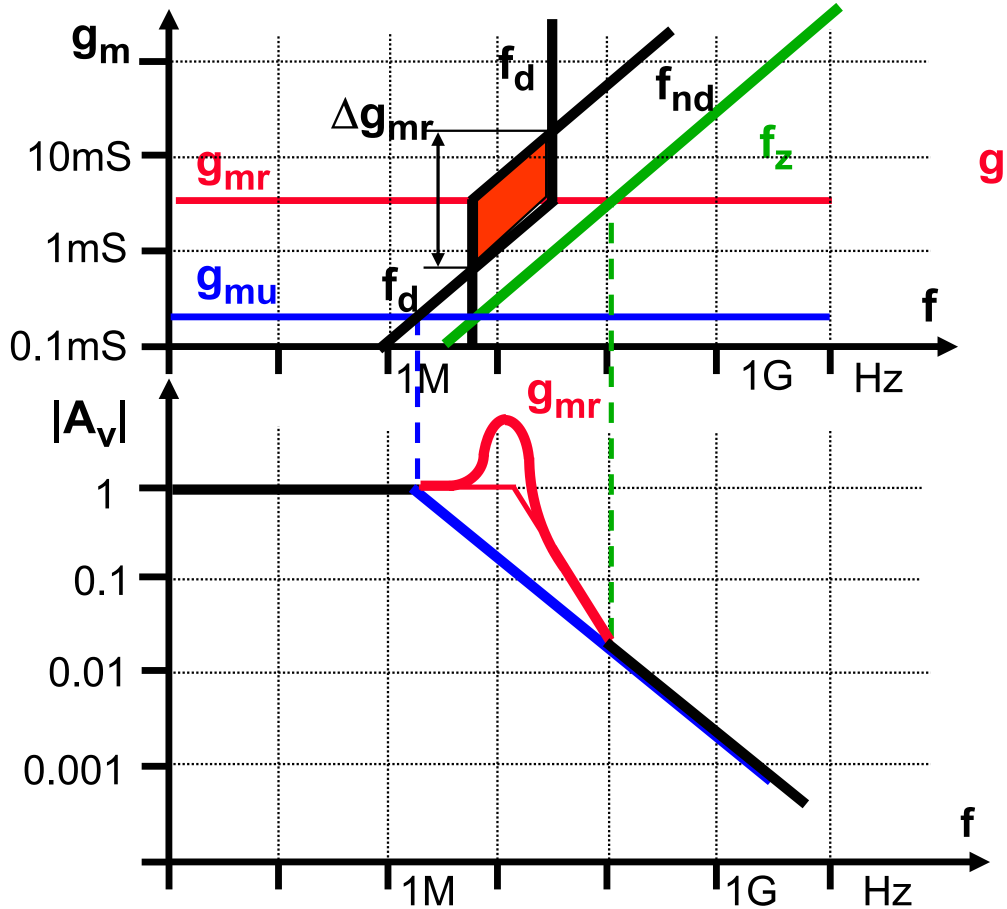
$$B = R_S C_{DG} + \frac{C'_{DS}}{g_m}$$

$$+ \frac{C_{GS}}{g_m}\left(1 + \frac{R_S}{r_{DS}}\right)$$

$$C^2 = C'_{DS}C_{DG} + C'_{DS}C_{GS} + C_{DG}C_{GS}$$

$$C'_{DS} = C_L + C_{DS}$$

Source follower with C_L load



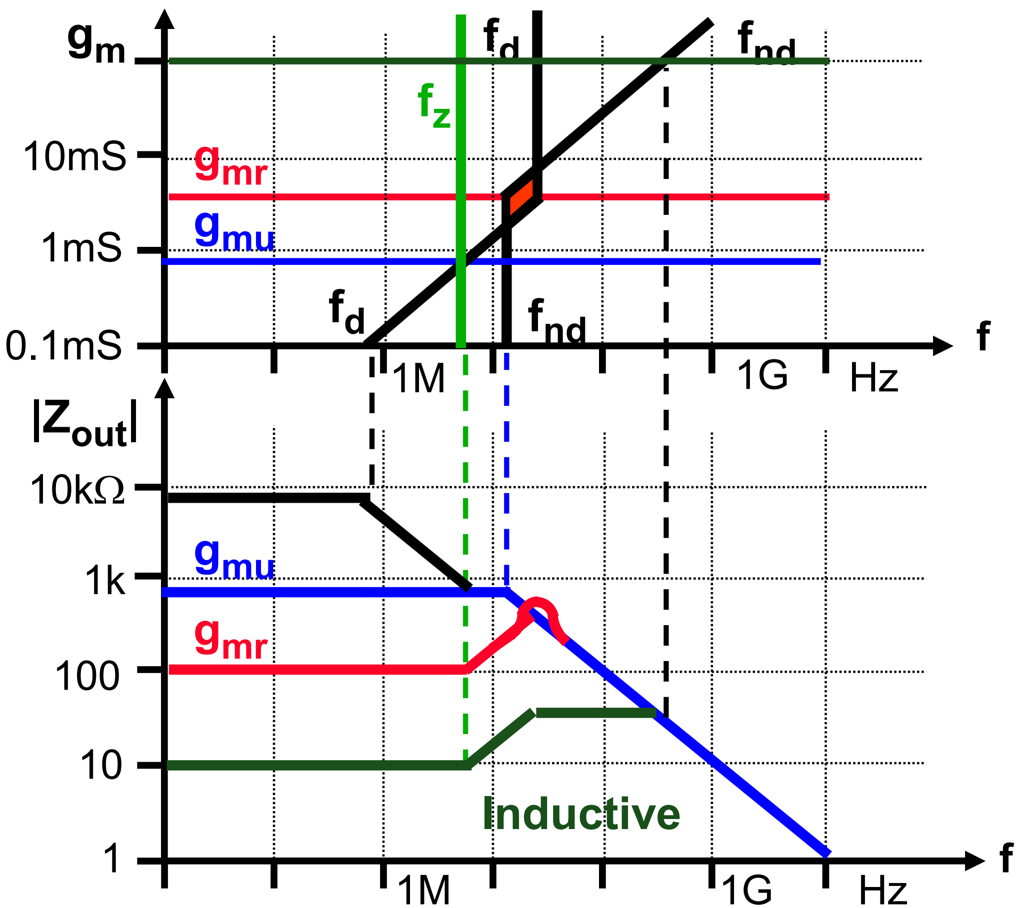
$$g_{mr} = \frac{1}{R_S}\,\frac{C_L + C_{DS} + C_{GS}}{C_{DG}}$$

$$\Delta g_{mr} = \frac{C_{DGt}}{C_{DG}}$$

$$C_{DGt} = \frac{C'_{DS}\,C_{GS}}{C'_{DS} + C_{GS}}$$

$$g_{mu} = \frac{1}{R_S}$$

Source follower : Output impedance



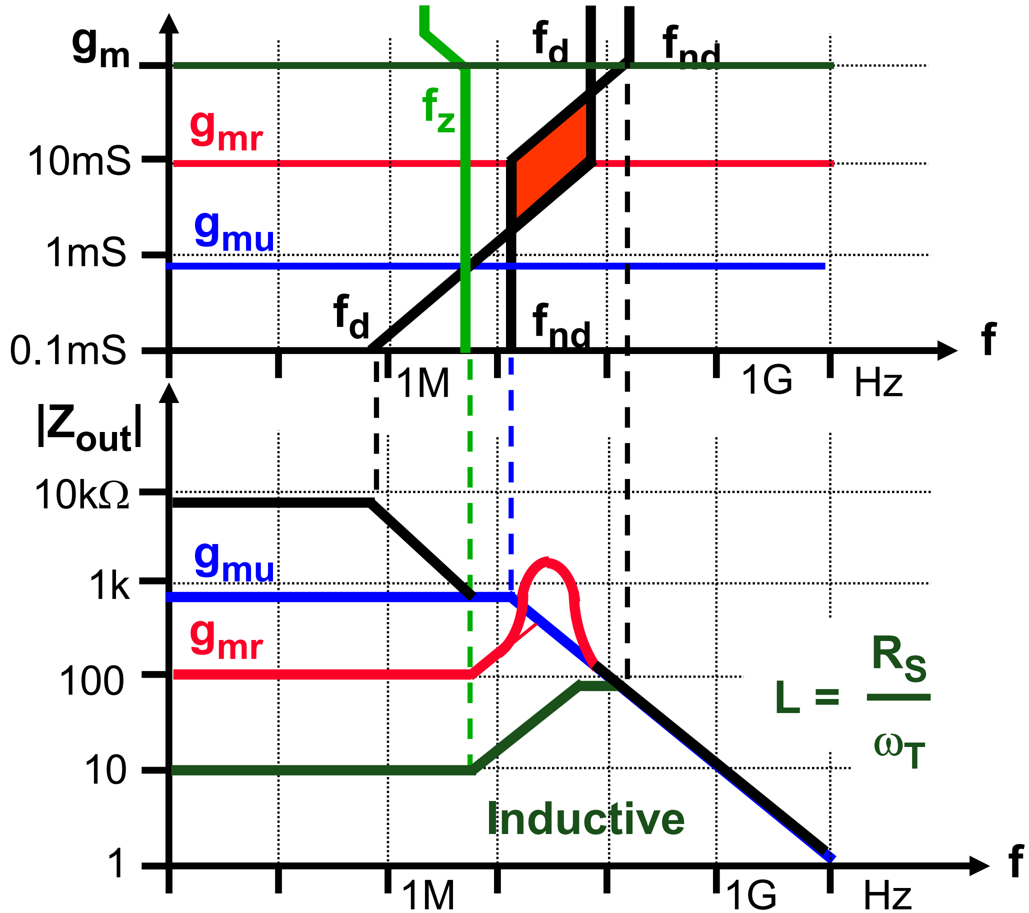
$$g_{mr} = \frac{1}{R_S} \frac{C_{GS} + C_{DS}}{C_{DG}}$$

$$g_{mu} \approx \frac{1}{R_S} \frac{C_{GS} + C_{DS}}{C_{GS} + C_{DG}}$$

$$f_z = \frac{1}{2\pi R_S C_{GS}}$$

$$f_{d,higm} = \frac{1}{2\pi R_S C_{DG}}$$

Emitter follower : Output impedance

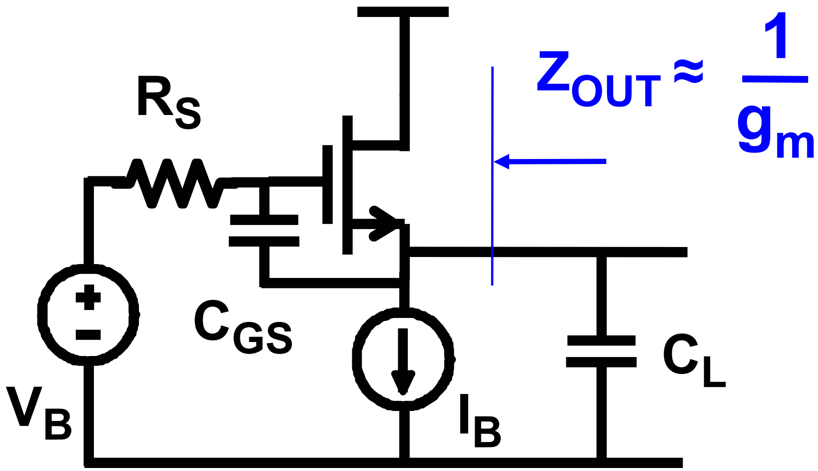


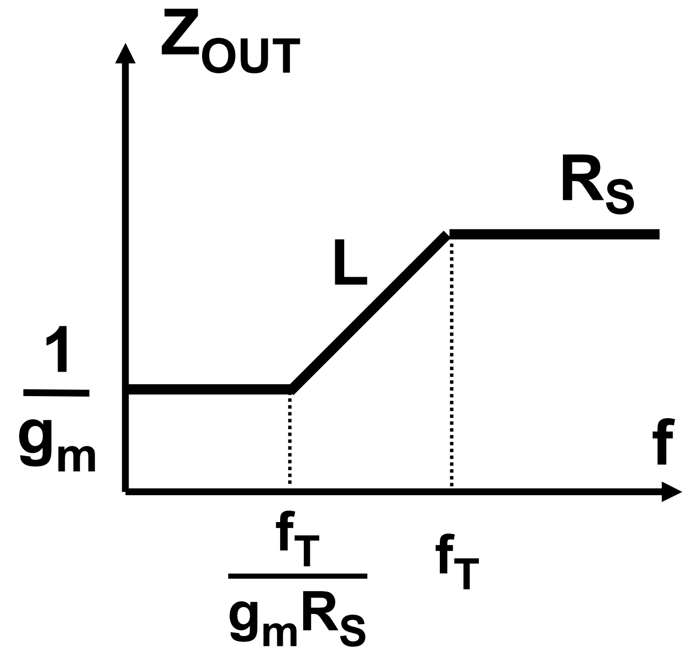
$$g_{mr} = \frac{1}{R_S} \frac{C_\pi + C_{CE}}{C_\pi + C_\mu}$$

$$g_{mu} \approx \frac{1}{R_S} \frac{C_{jE} + C_{CE}}{C_{jE} + C_\mu}$$

$$f_z = \frac{1}{2\pi R_S // r_\pi (C_\pi + C_\mu)}$$

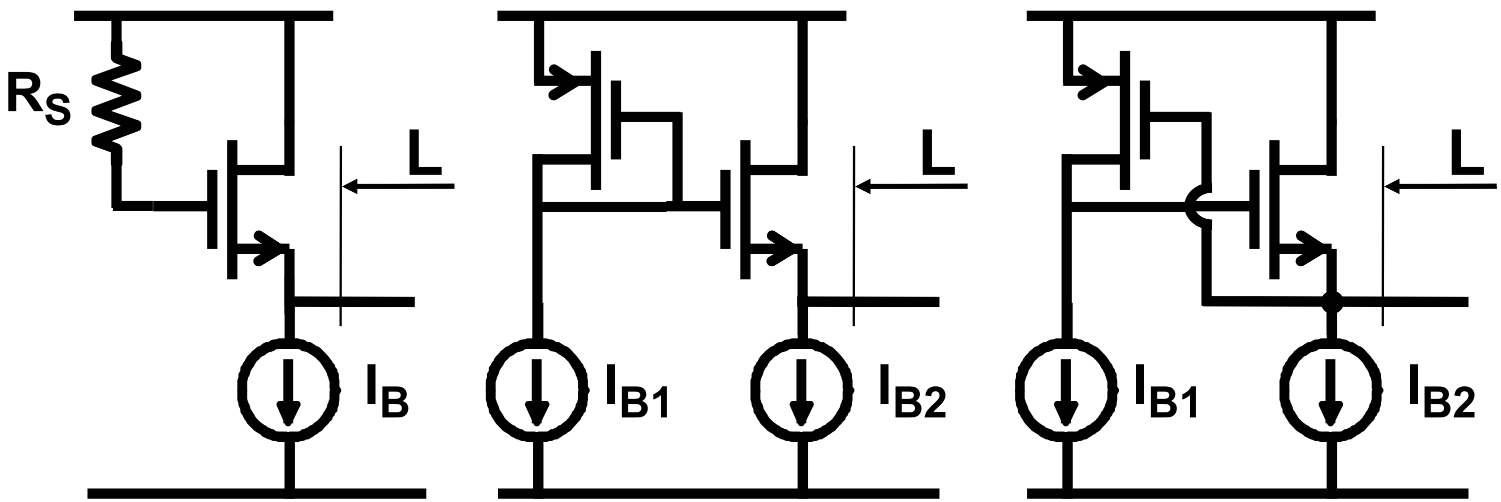
Source follower as active L





$$Z_{OUT} \approx \frac{1}{g_m}(1 + R_S C_{GS} s) \qquad L \approx \frac{R_S}{2\pi f_T} \quad \text{up to} \quad f_T = \frac{g_m}{2\pi C_{GS}}$$

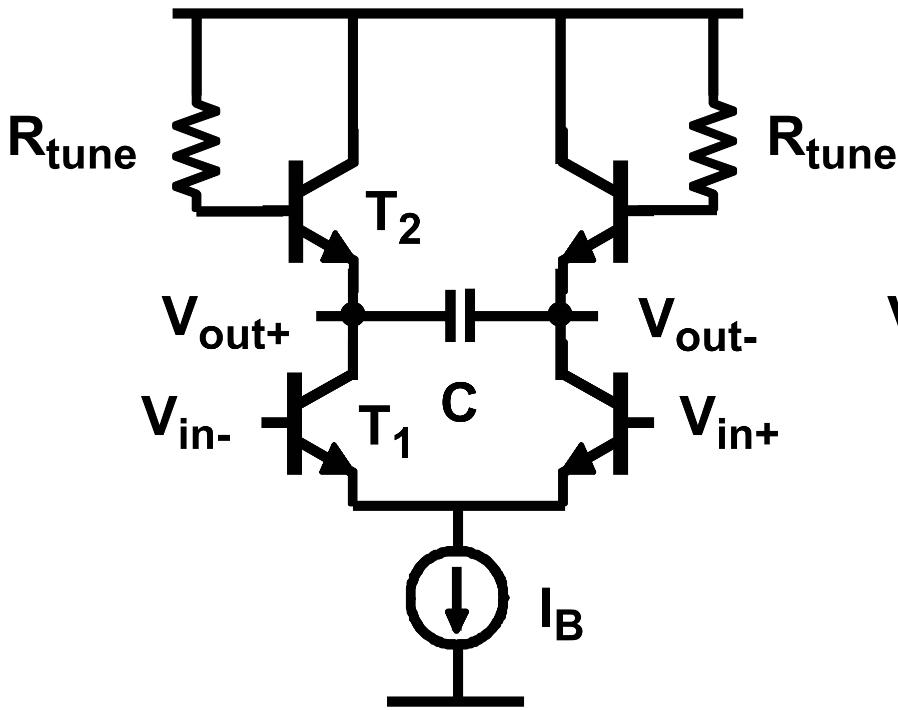
Source follower as active L



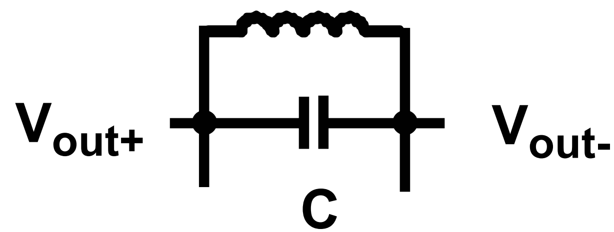
$$L \approx \frac{R_S}{2\pi f_T} \qquad L \approx \frac{1/g_{mp}}{2\pi f_{Tn}} \qquad L \approx \frac{1/g_{mp}}{2\pi f_{Tn}}$$

$$V_{DSn} = V_{GSn} \qquad V_{DSn} = V_{GSn} + V_{GSp} \qquad V_{DSn} = V_{GSp}$$

Floating inductor with parallel C



$$L = \frac{R_{tune}}{\omega_{T2}}$$



$$A_v = \frac{g_{m1}}{g_{m2}}$$

with HF peaking !

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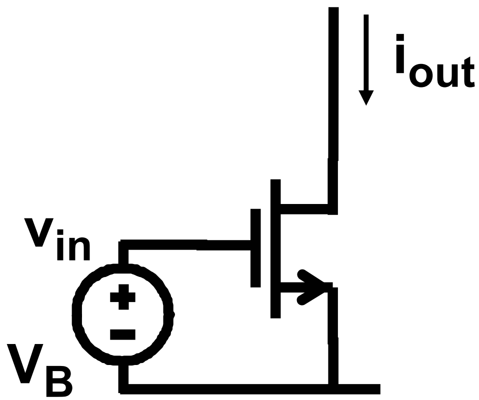
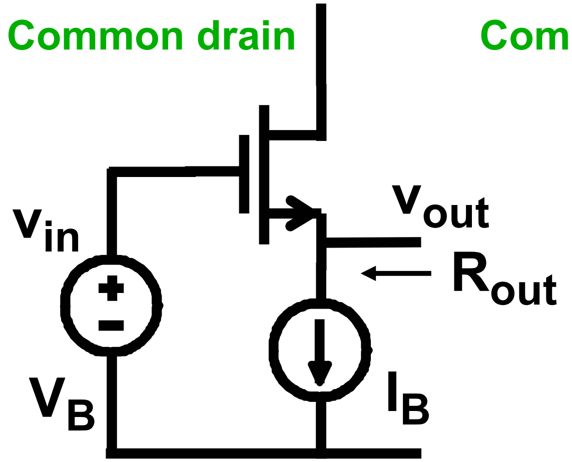
- Single-transistor amplifiers
- Source followers
- Cascodes

Single-transistor stages

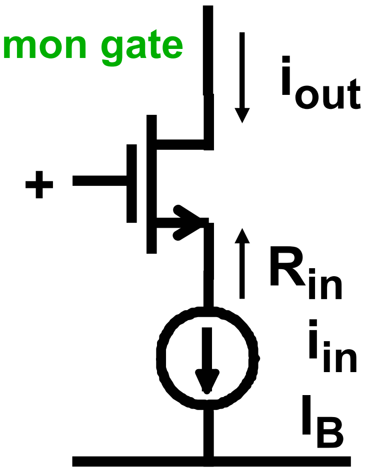
Common source



Common drain



Common gate



$i_{out} = g_m v_{in}$

$v_{out} = v_{in}$

$R_{out} \approx 1/g_m$

$i_{out} = i_{in}$

$R_{in} \approx 1/g_m$

Amplifier

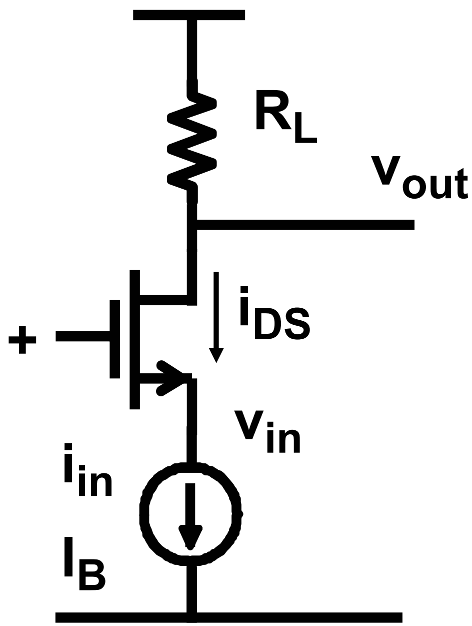
Source follower

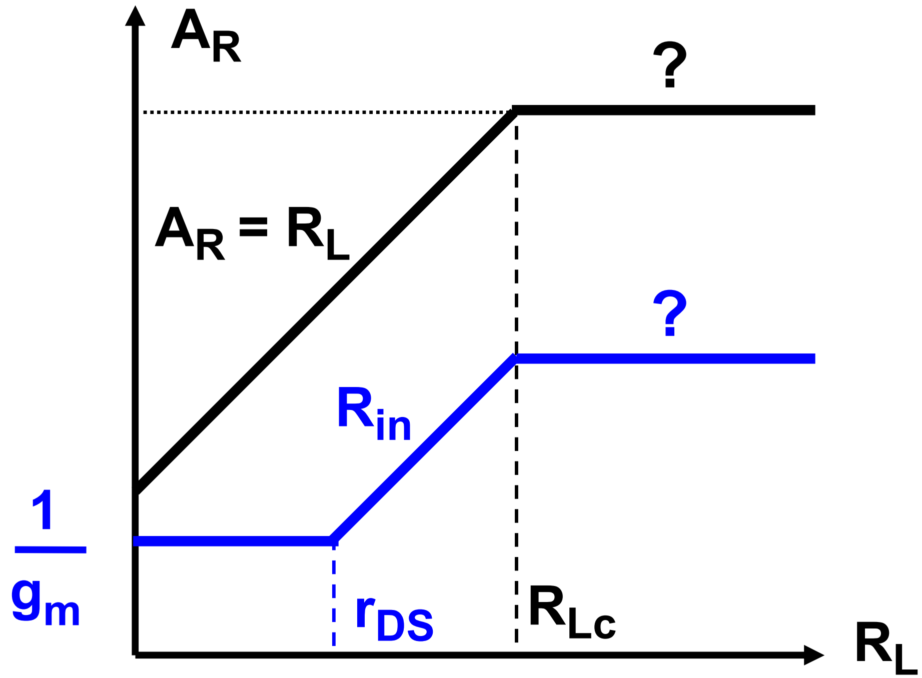
Voltage buffer

Cascode

Current buffer

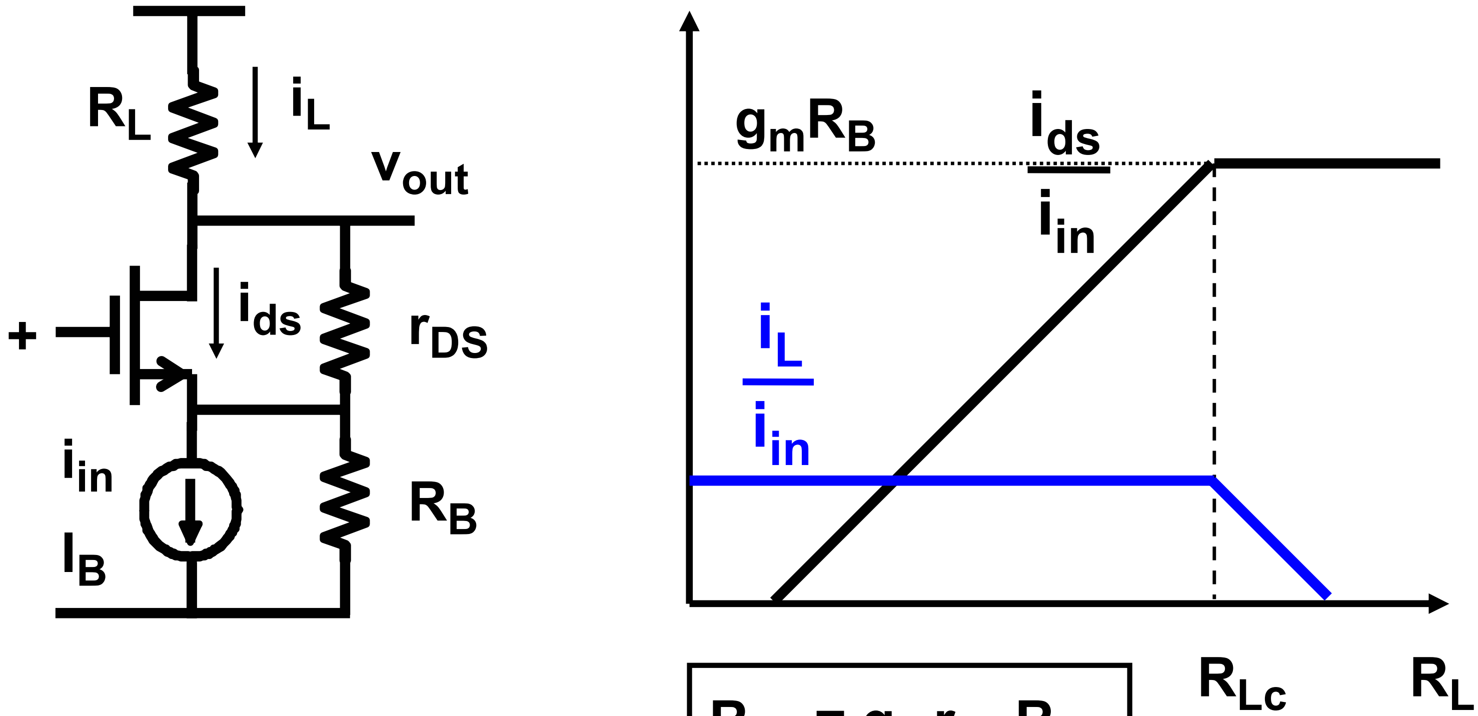
Cascode with resistive load





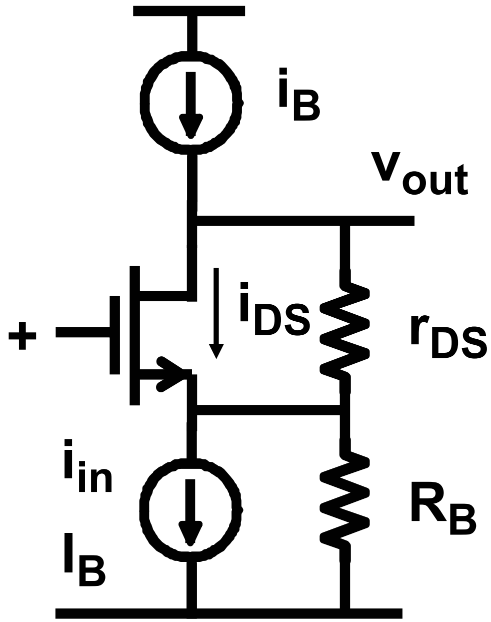
$$A_R = \frac{v_{out}}{i_{in}} \qquad R_{in} = \frac{v_{in}}{i_{in}}$$

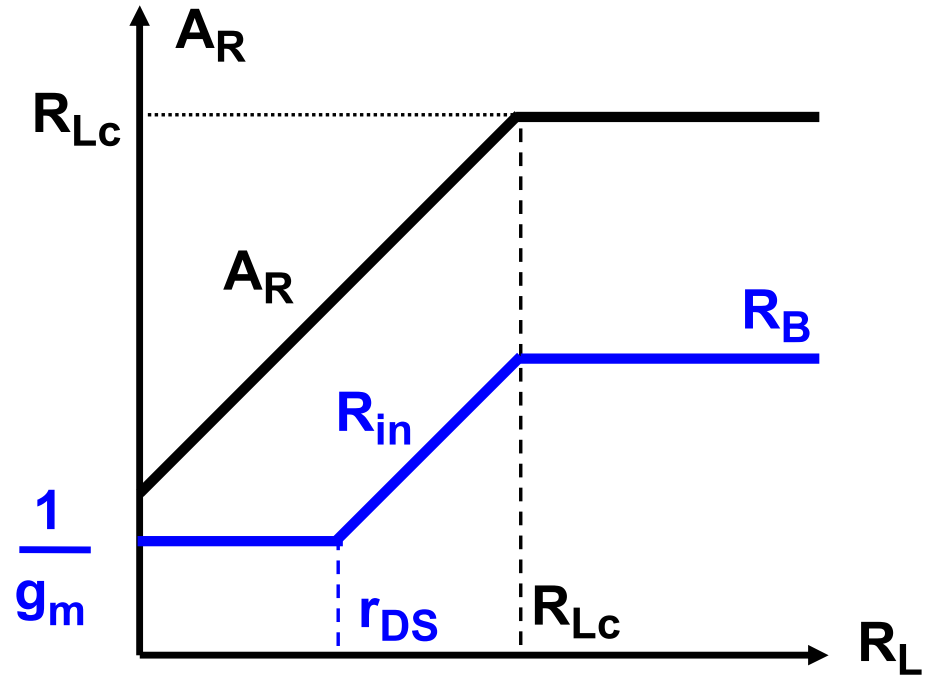
Cascode with resistive load



$$R_{Lc} = g_m r_{DS} R_B$$

Cascode with active load

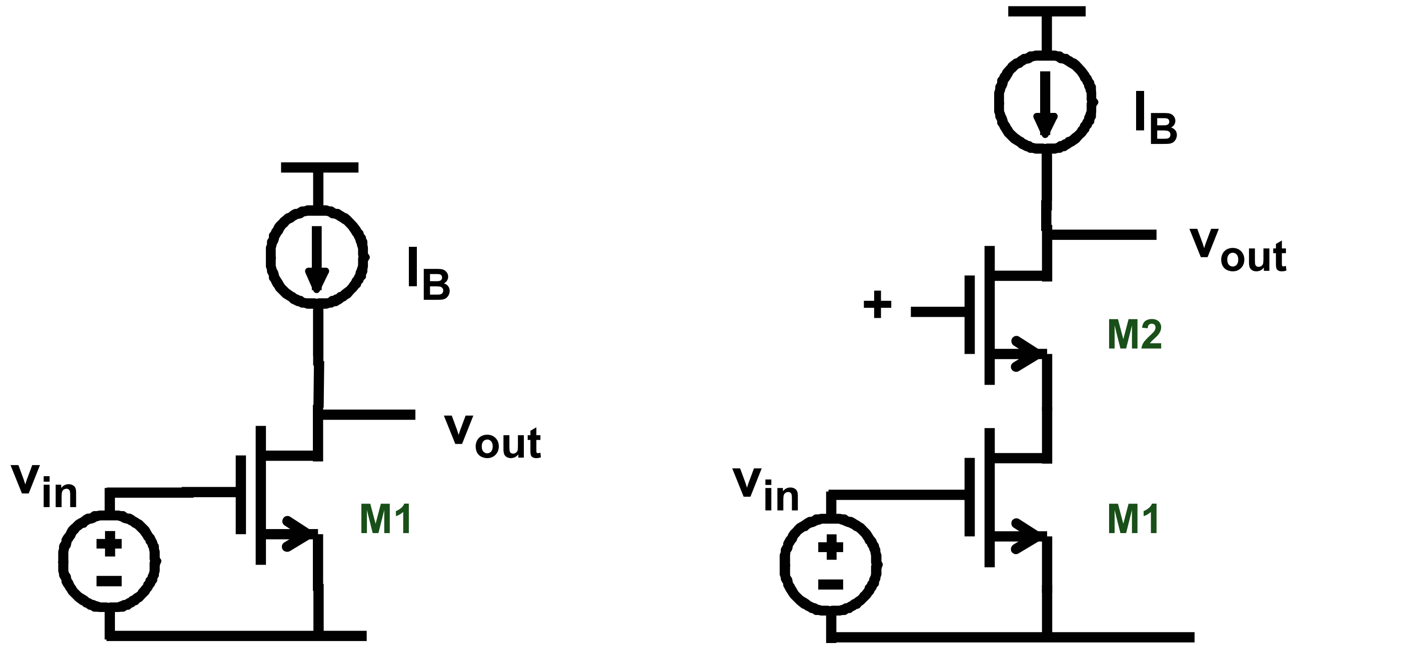




$$A_R = \frac{v_{out}}{i_{in}} \qquad R_{in} = \frac{v_{in}}{i_{in}}$$

$$\boxed{R_{Lc} = g_m r_{DS} R_B} \approx 100\ R_B$$

Cascode versus single-transistor



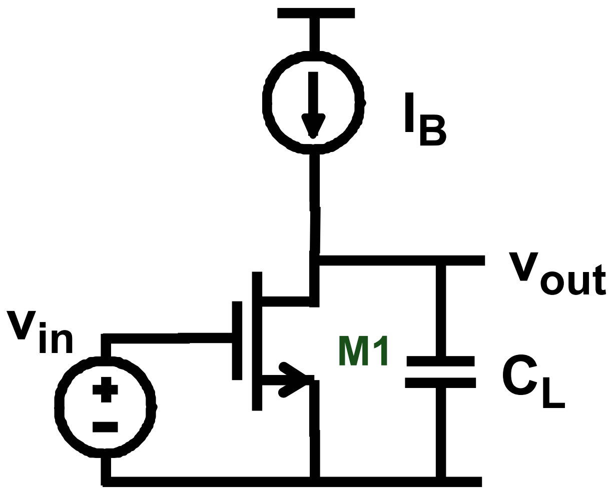
$A_v = (g_m r_{DS})_1$

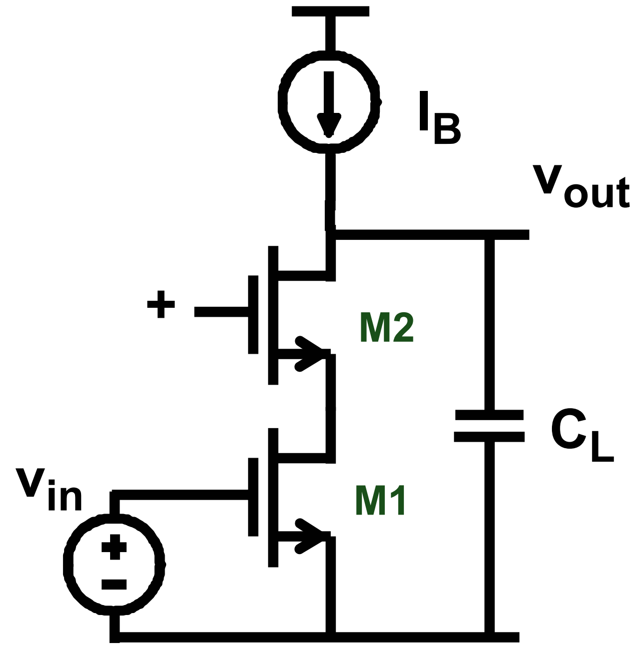
$R_{out} = r_{DS1}$

$A_v = (g_m r_{DS})_1 (g_m r_{DS})_2$

$R_{out} = r_{DS1} (g_m r_{DS})_2$

Cascode versus single-transistor



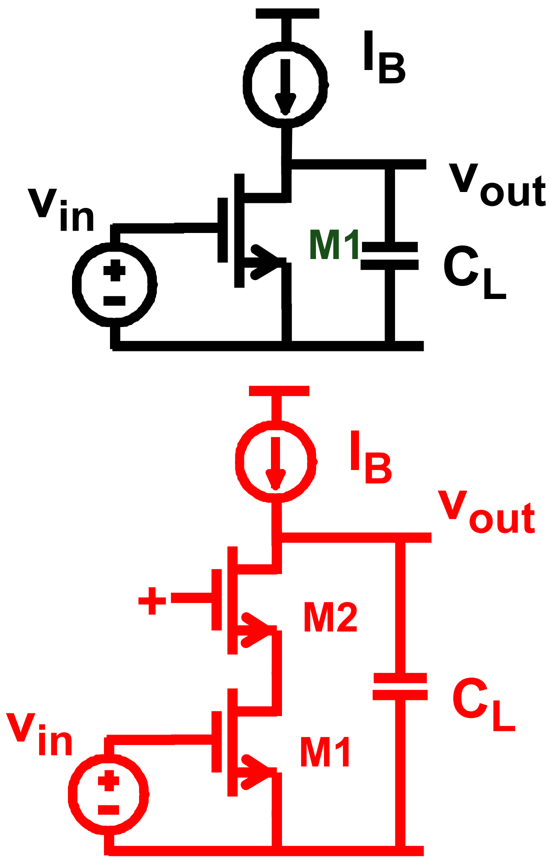


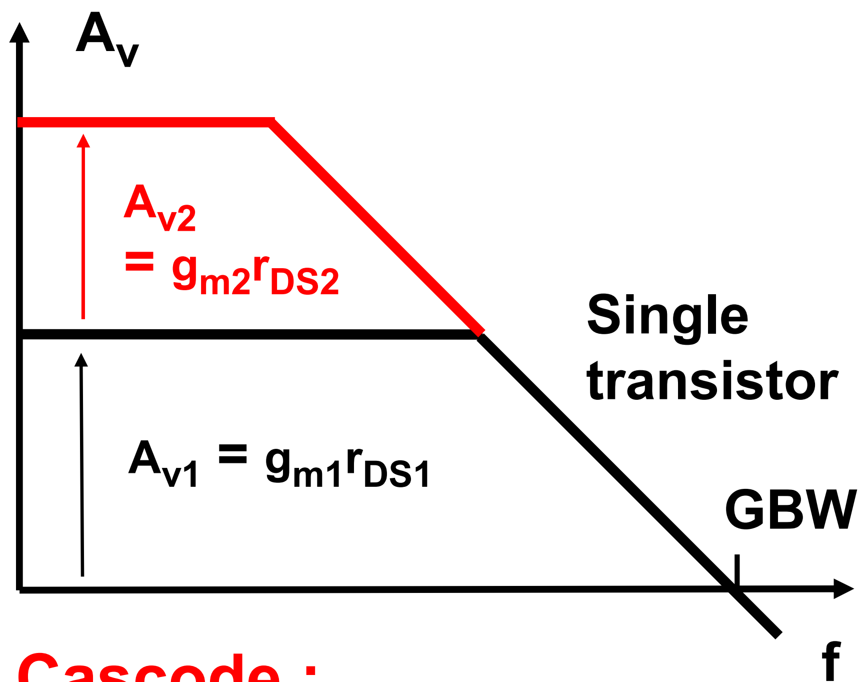
$$BW = \frac{1}{2\pi\, R_{out} C_L}$$

$$GBW = \frac{g_{m1}}{2\pi\, C_L}$$

for both !

Cascode versus single-transistor



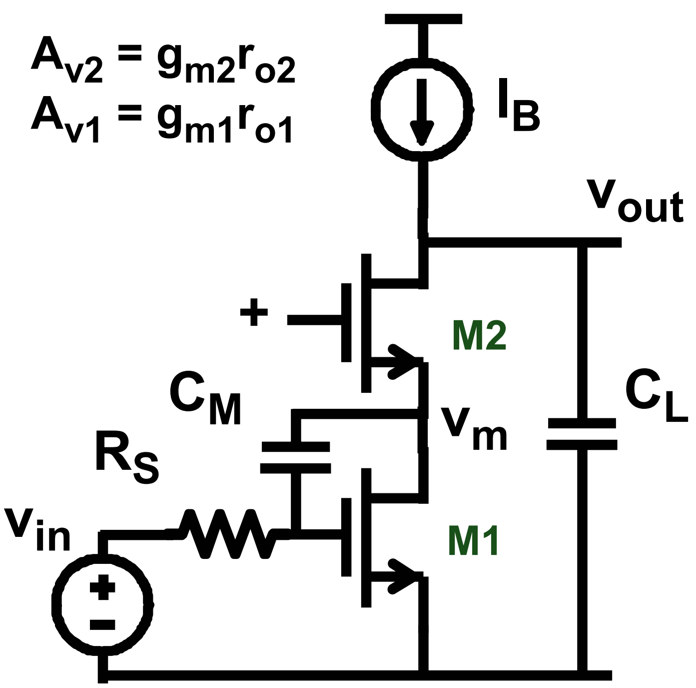


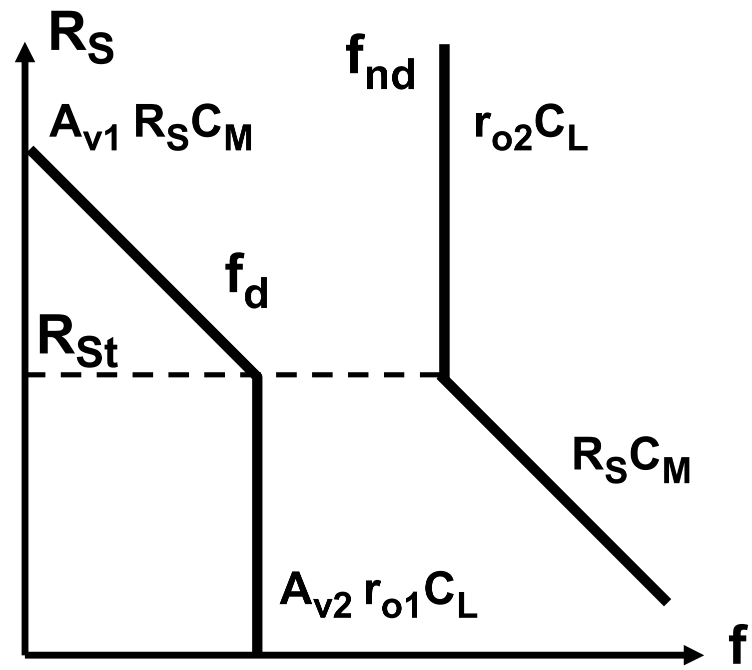
Cascode :
High gain
At low freq.

$$GBW = \frac{g_{m1}}{2\pi\, C_L}$$

Miller effect in cascode ?

$A_{v2} = g_{m2} r_{o2}$
$A_{v1} = g_{m1} r_{o1}$

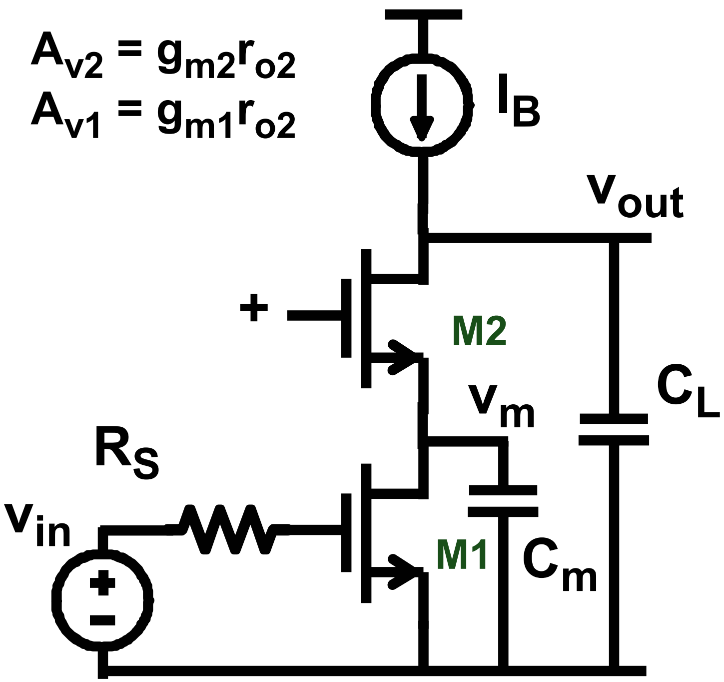




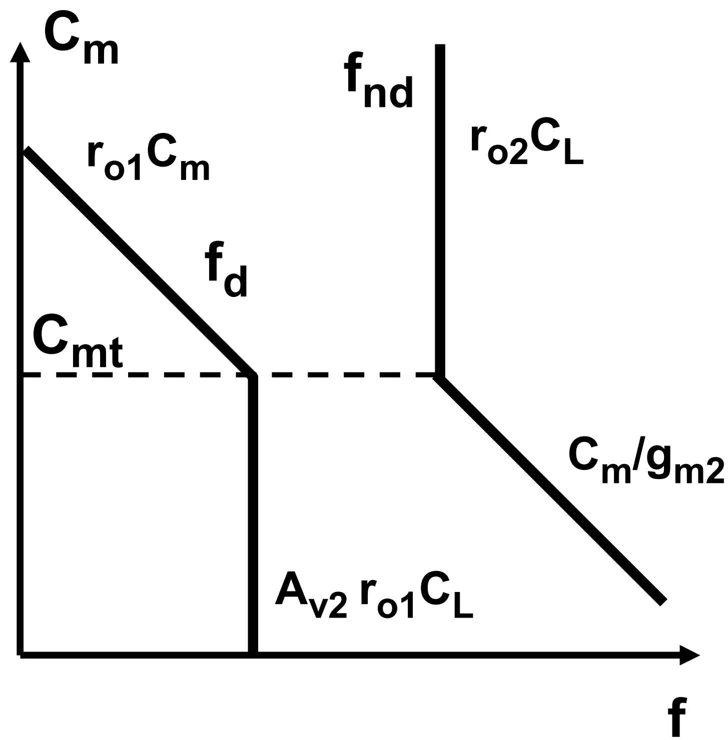
$$GBW = \frac{g_{m1}}{2\pi\, C_L}$$

No Miller if $R_S < R_{St} = r_{o2} \frac{C_L}{C_M} \frac{g_{m2}}{g_{m1}}$

Cascode with capacitance Cm at middle point

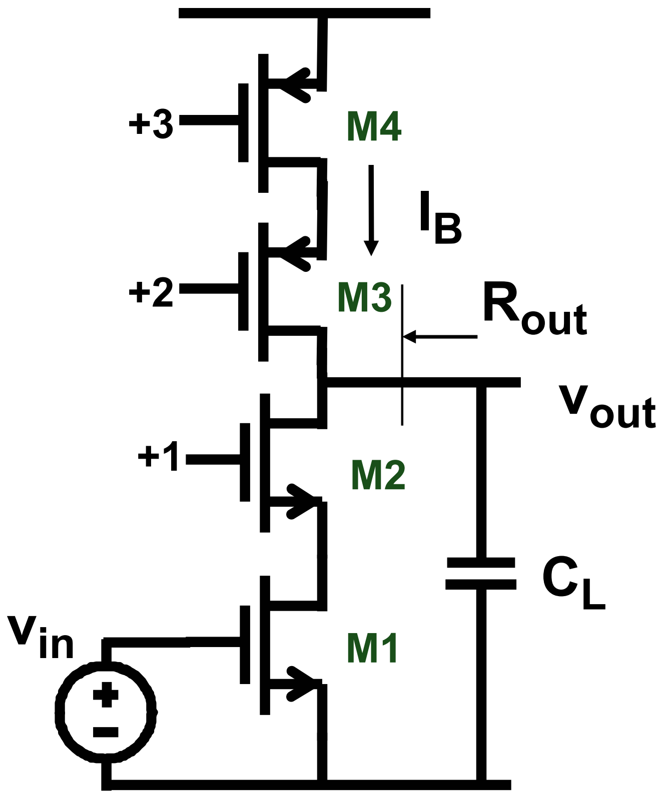


$$GBW = \frac{g_{m1}}{2\pi\, C_L}$$



$$C_{mt} = g_{m2} r_{o2}\, C_L = A_{v2}\, C_L$$

Telescopic Cascode



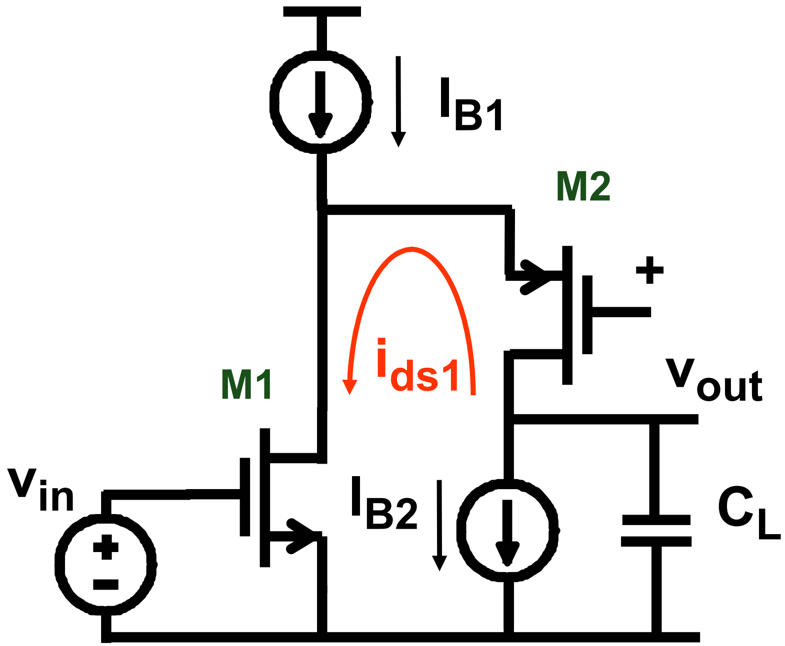
$$A_v = g_{m1} R_{out}$$

$$R_{out} = \frac{1}{2} r_{DS1}\, g_{m2}\, r_{DS2}$$

$$BW = \frac{1}{2\pi R_{out} C_L}$$

$$GBW = \frac{g_{m1}}{2\pi C_L}$$

Folded Cascode



$I_{DS1} = I_{B1} - I_{B2} \approx I_{B1} / 2$

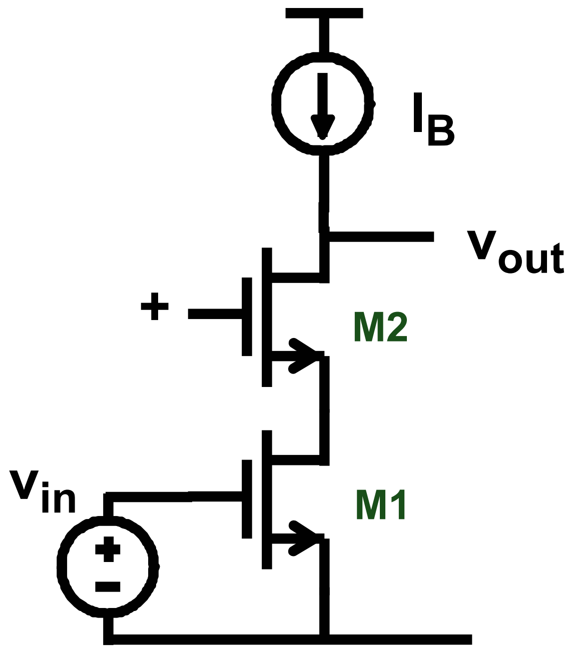
$$A_v = g_{m1} R_{out}$$

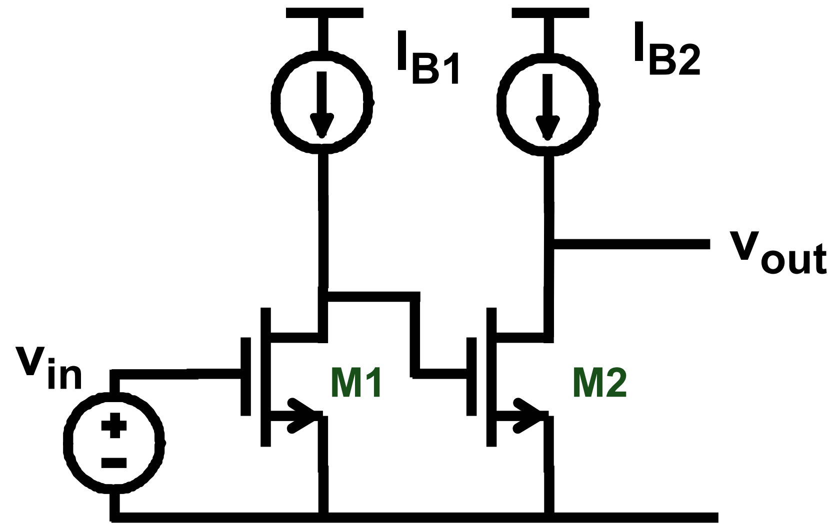
$$R_{out} = r_{DS1}\, g_{m2}\, r_{DS2}$$

$$BW = \frac{1}{2\pi\, R_{out} C_L}$$

$$GBW = \frac{g_{m1}}{2\pi\, C_L}$$

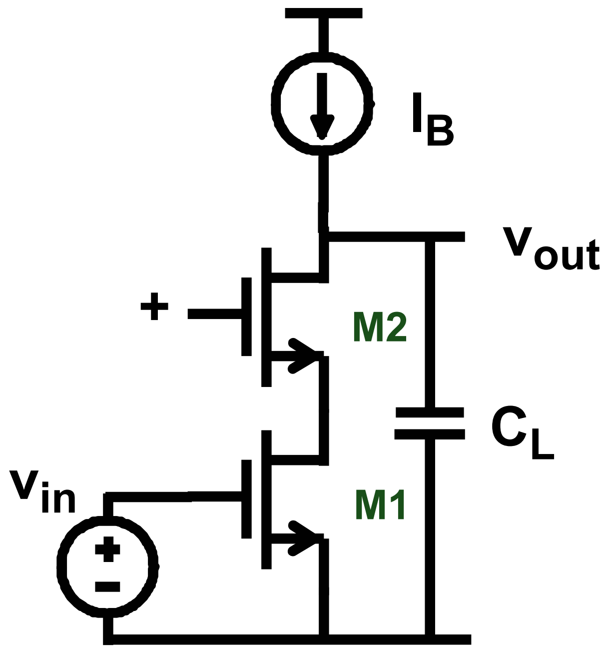
Cascode versus cascade





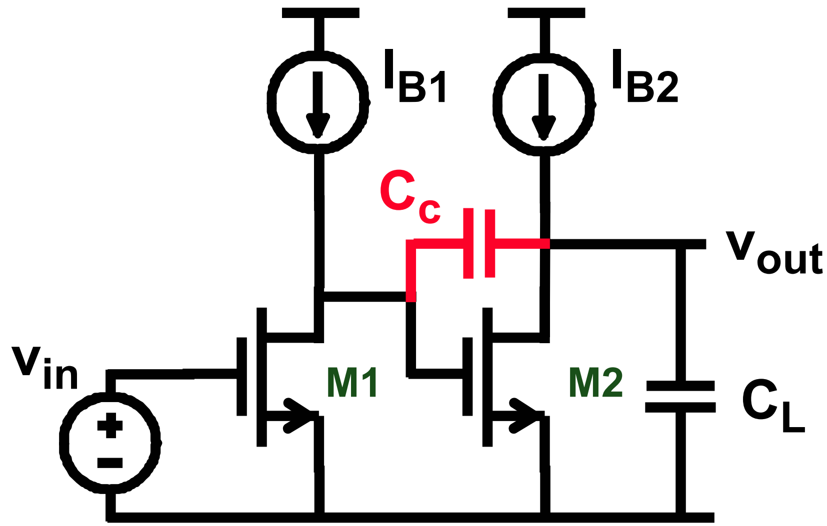
$$A_v = (g_m r_{DS})_1 (g_m r_{DS})_2 \qquad A_v = (g_m r_{DS})_1 (g_m r_{DS})_2$$

Cascode versus cascade



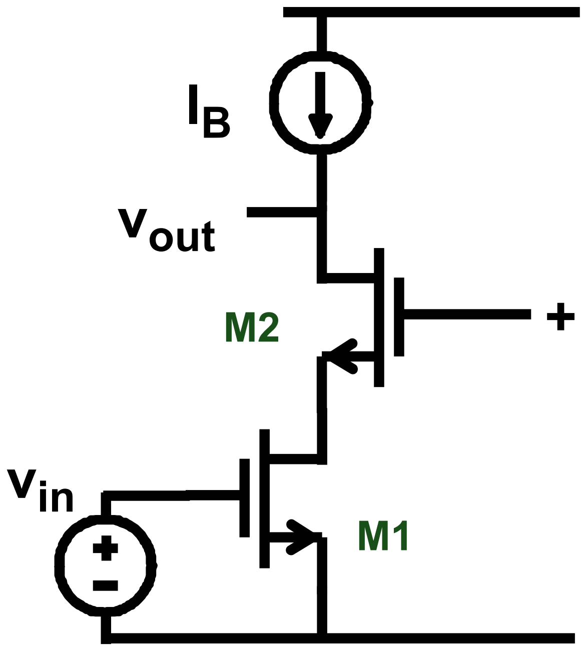
$$GBW = \frac{g_{m1}}{2\pi\, C_L}$$

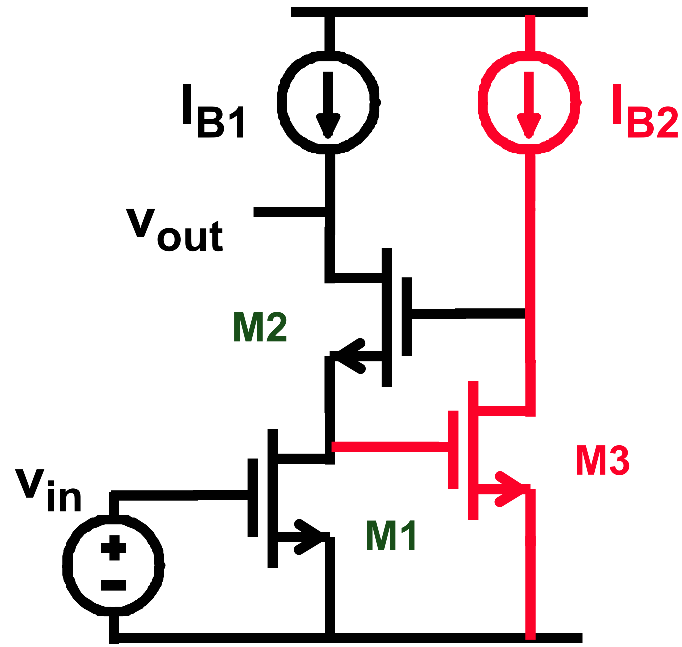
Two-stage Miller amplifier



$$GBW = \frac{g_{m1}}{2\pi\, C_c}$$

Regulated cascode or gain boosting

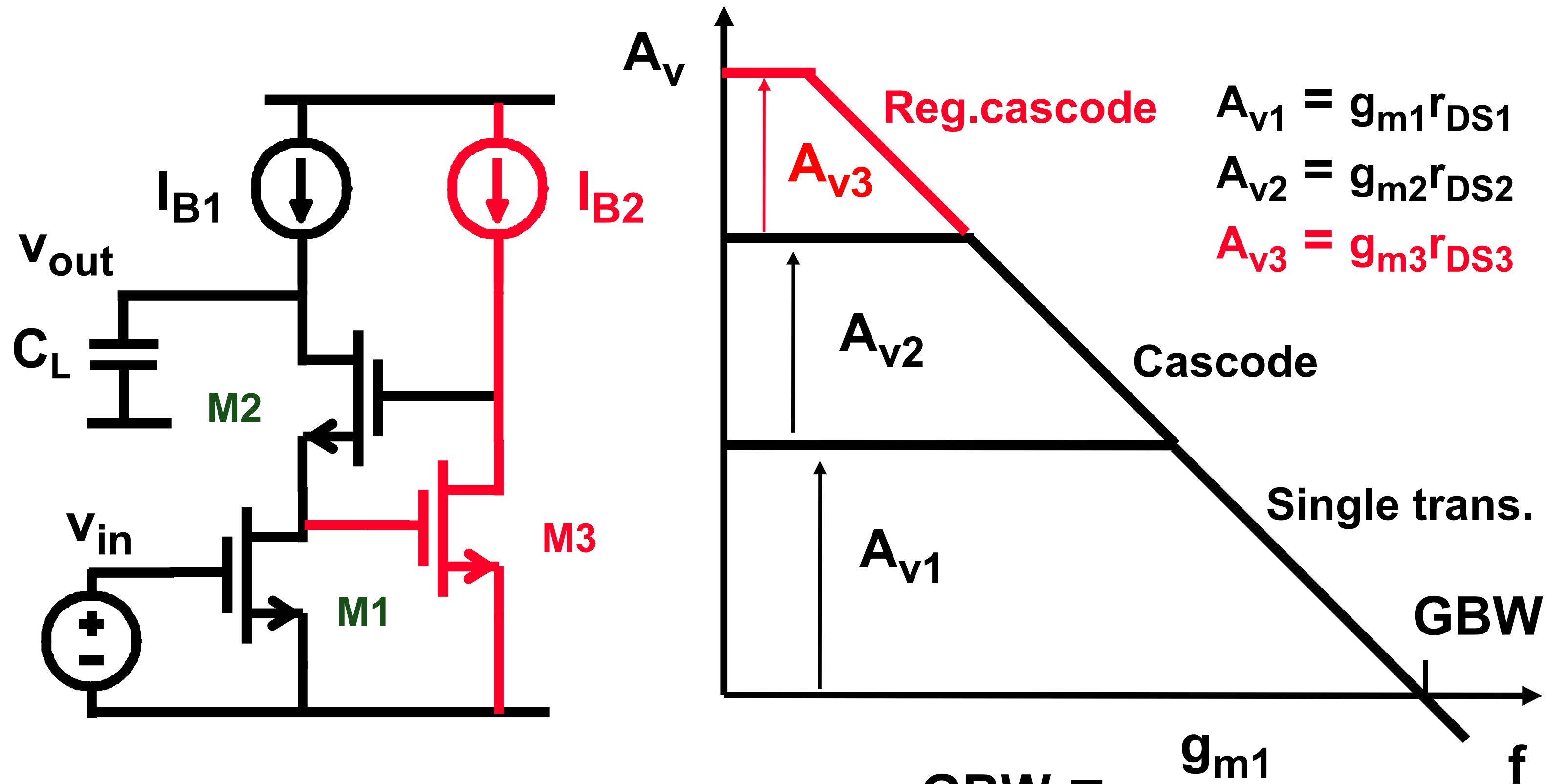




$A_v = (g_m r_{DS})_1 (g_m r_{DS})_2$ $\quad$ $A_v = (g_m r_{DS})_1 (g_m r_{DS})_2 (g_m r_{DS})_3$

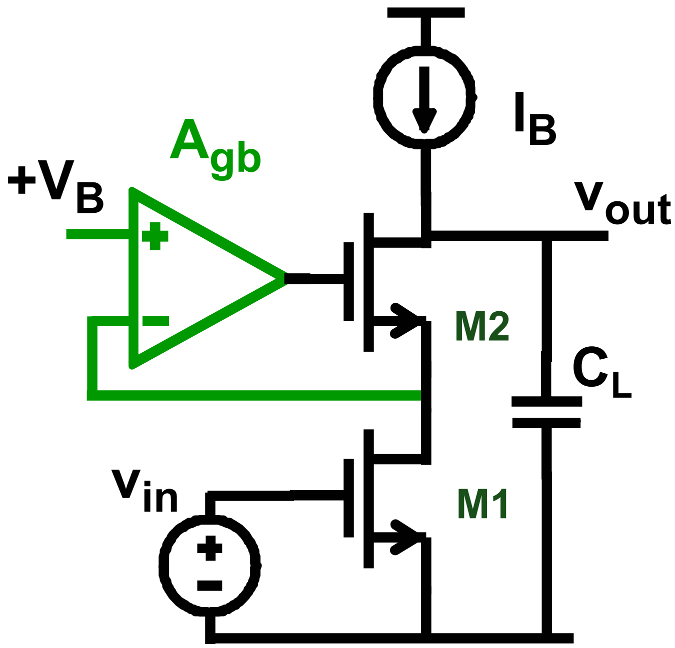
Hosticka, JSSC Dec.79, pp. 1111-1114; Sackinger, JSSC Febr.90, pp. 289-298;
Bult JSSC Dec.90, pp. 1379-1384

Regulated cascode, Cascode & single-transistor

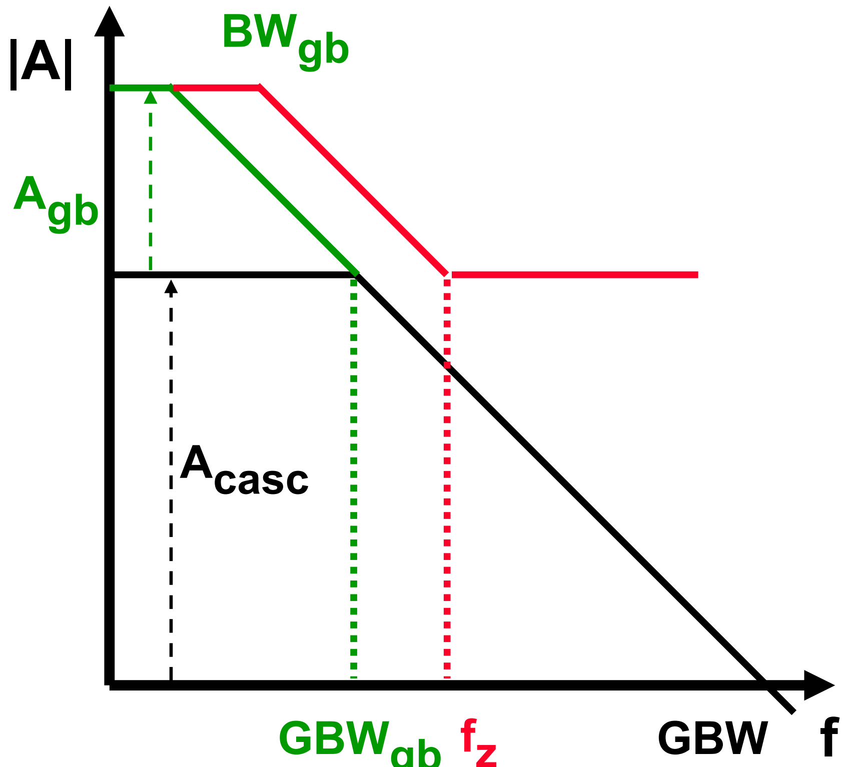


$$GBW = \frac{g_{m1}}{2\pi C_L}$$

Gain boosting

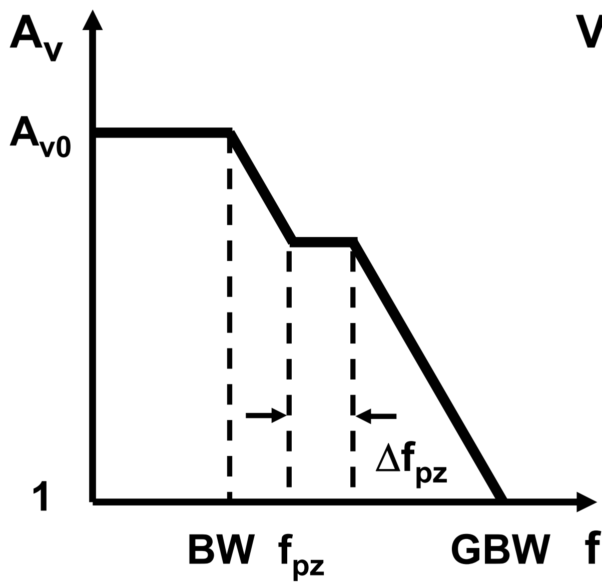


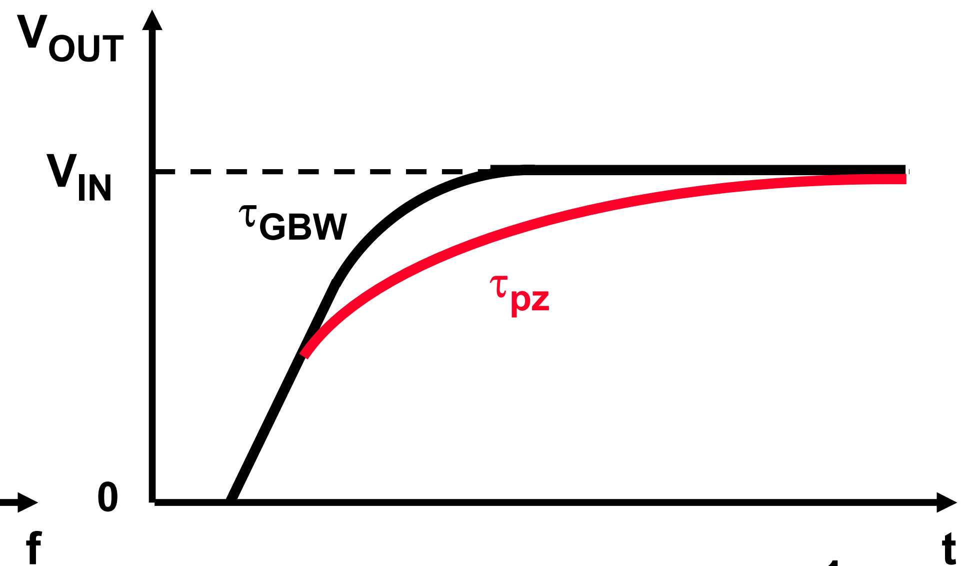
$$A_v = A_{gb}(g_m r_{DS})_1 (g_m r_{DS})_2$$



$$= \frac{g_{m1}}{2\pi\, C_L}$$

Pole-zero doublet and settling time





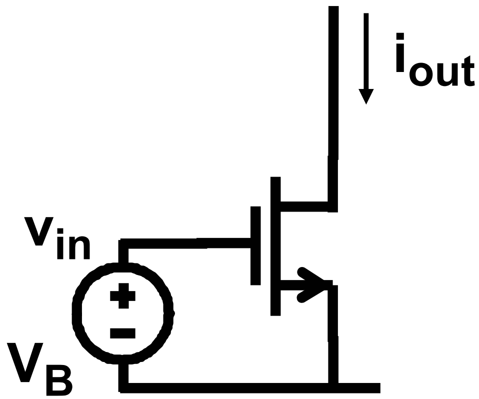
$$f_{pz} = \frac{1}{2\pi\,\tau_{pz}}$$

$$V_{OUT} = V_{IN}\left[1 - \exp\left(-\frac{t}{\tau_{GBW}}\right) - \frac{\Delta f_{pz}}{GBW}\exp\left(-\frac{t}{\tau_{pz}}\right)\right]$$

$$GBW = \frac{1}{2\pi\,\tau_{GBW}}$$

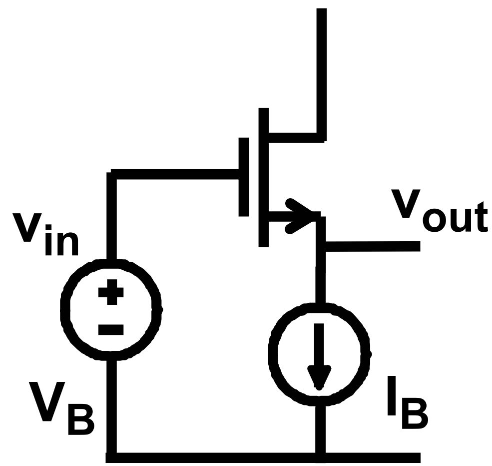
Kamath, etal, JSSC Dec.74, pp. 347-352

Single-transistor stages



$i_{out} = g_m v_{in}$

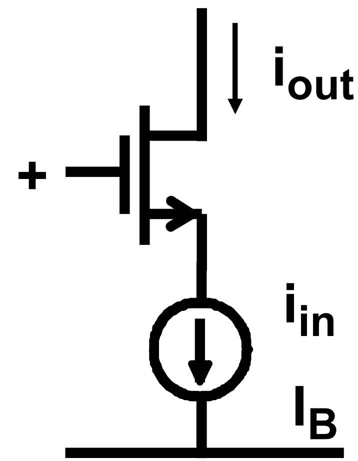
Amplifier



$v_{out} = v_{in}$

$Z_{out} \approx 1/g_m$

Source follower

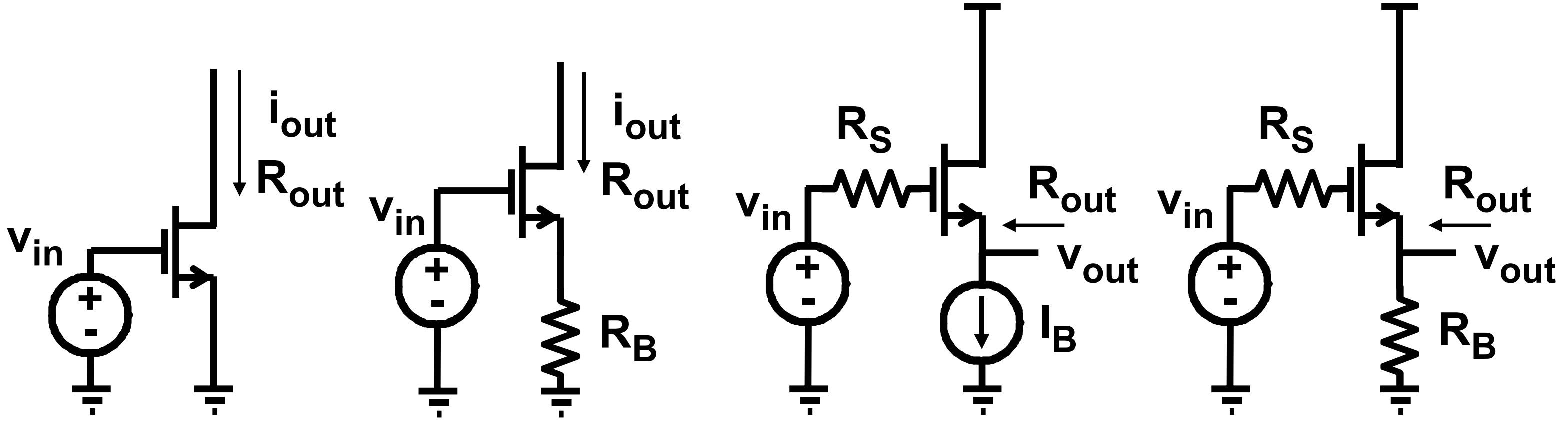


$i_{out} = i_{in}$

$Z_{in} \approx 1/g_m$

Cascode

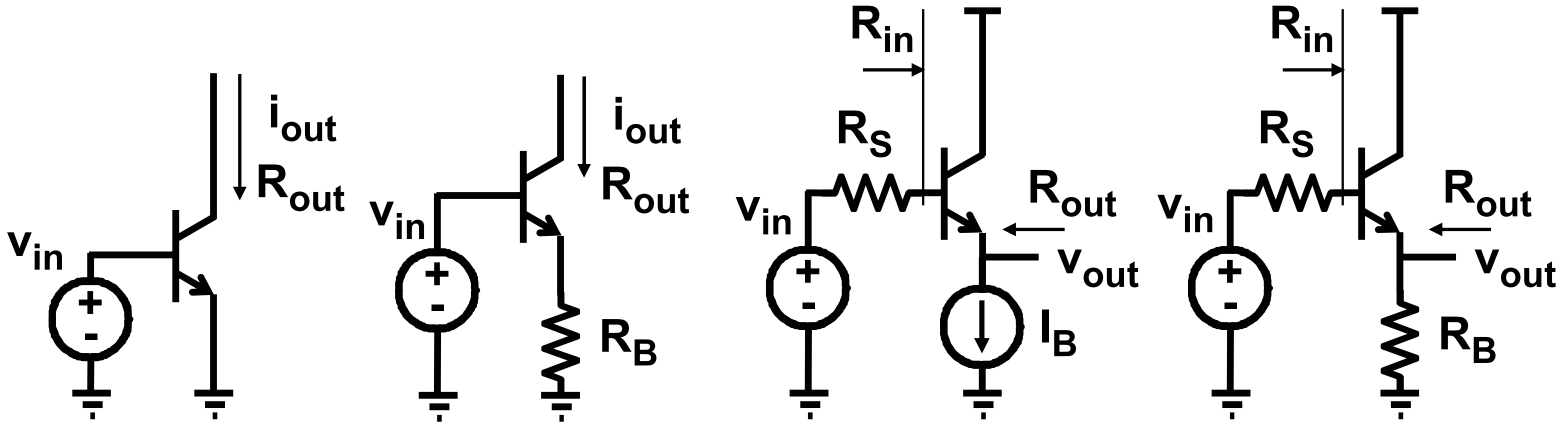
MOST amplifier & follower



$R_B > 1/g_m$ $R_B > 1/g_m$

A_G	g_m	$1/R_B$	A_V	1	1
R_{in}	∞	∞		∞	∞
R_{out}	r_o	$g_m R_B r_o$		$1/g_m$	$1/g_m$

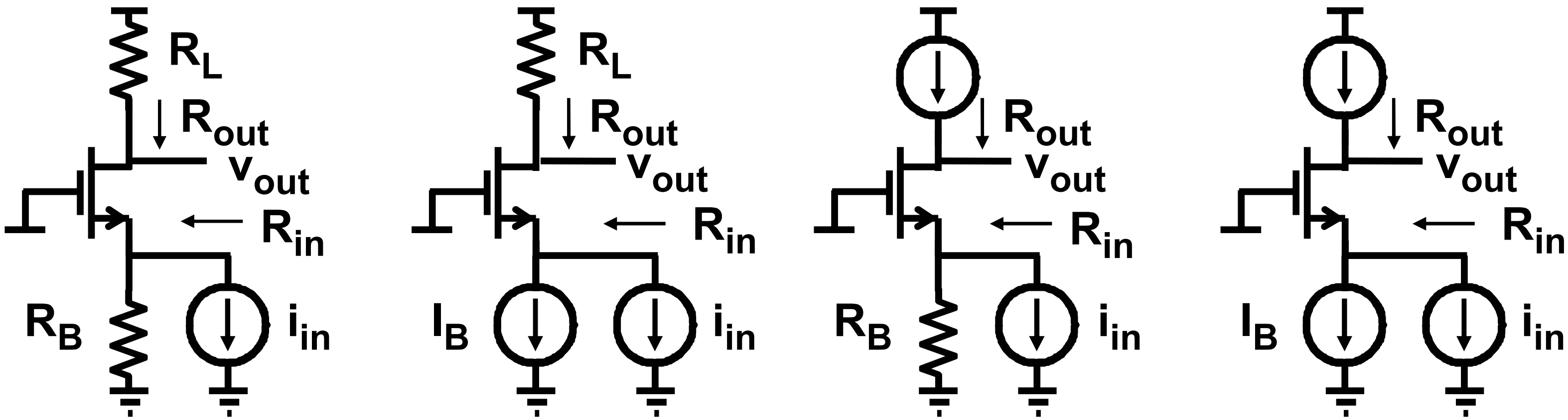
Bipolar transistor ($\beta >> 1$)



$R_B > 1/g_m$ $\qquad$ $R_B > 1/g_m$

A_G	g_m	$1/R_B$	A_V	1	1
R_{in}	r_B+r_π	$r_B+r_\pi+\beta R_B$		$r_B+r_\pi+\beta r_o$	$r_B+r_\pi+\beta R_B$
R_{out}	r_o	$g_m R_B r_o$		$1/g_m + R_S/\beta$	$1/g_m + R_S/\beta$

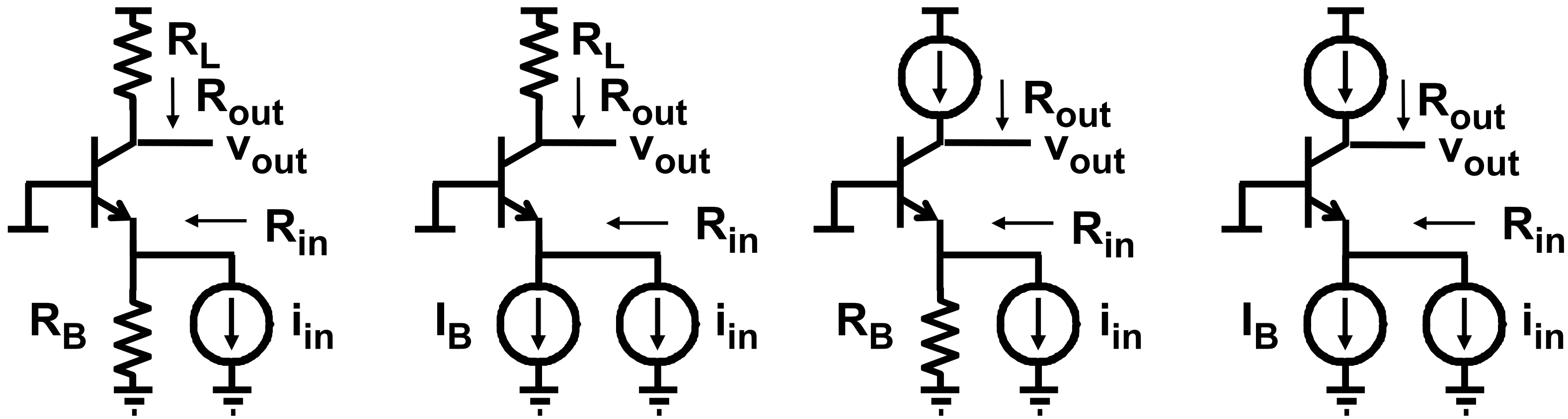
In- & output resistances MOST cascode



$R_B > 1/g_m$ $\qquad$ $R_B > 1/g_m$

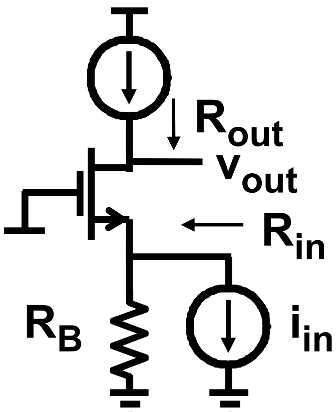
A_R	R_L	R_L	$g_m r_o R_B$	-
R_{in}	$1/g_m$	$1/g_m$	R_B	∞
R_{out}	$g_m r_o R_B$	∞	$g_m r_o R_B$	∞

In- & output resistances Bipolar trans. cascode



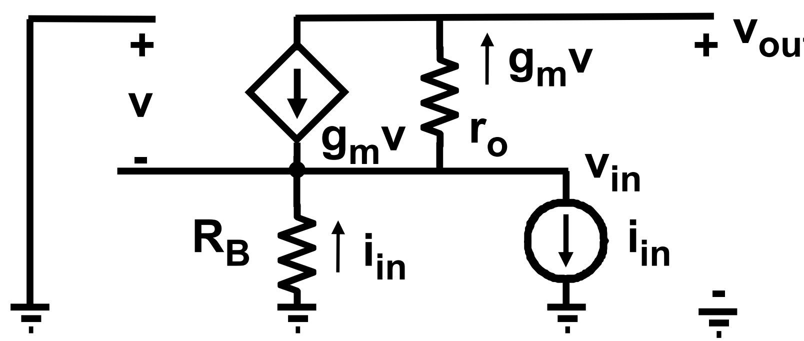
	$R_B > 1/g_m$		$R_B > 1/g_m$	
A_R	R_L	R_L	$g_m r_o R_B$	-
R_{in}	$1/g_m$	$1/g_m$	$R_B//(r_B+r_\pi)$	r_B+r_π
R_{out}	$g_m r_o R_B$	$\approx \beta r_o$	$g_m r_o(R_B//(r_B+r_\pi))$	$\approx \beta r_o$

Calculation of A_R for a MOST cascode



$R_B > 1/g_m$

$$A_R = g_m r_o R_B$$



$$\begin{cases} v = - v_{in} \\ v_{out} = v_{in} - g_m v \; r_o \\ v_{in} = - R_B \; v_{in} \end{cases}$$

yields $\quad v_{out} = - R_B \; i_{in} \, (1 + g_m r_o)$

and $\quad g_m r_o >> 1$

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- Single-transistor amplifiers
- Source followers
- Cascodes

0.3 chap3

Differential
Voltage & Current amplifiers

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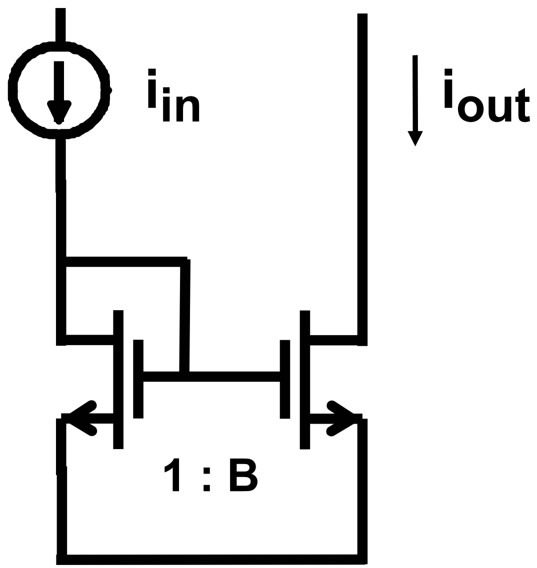
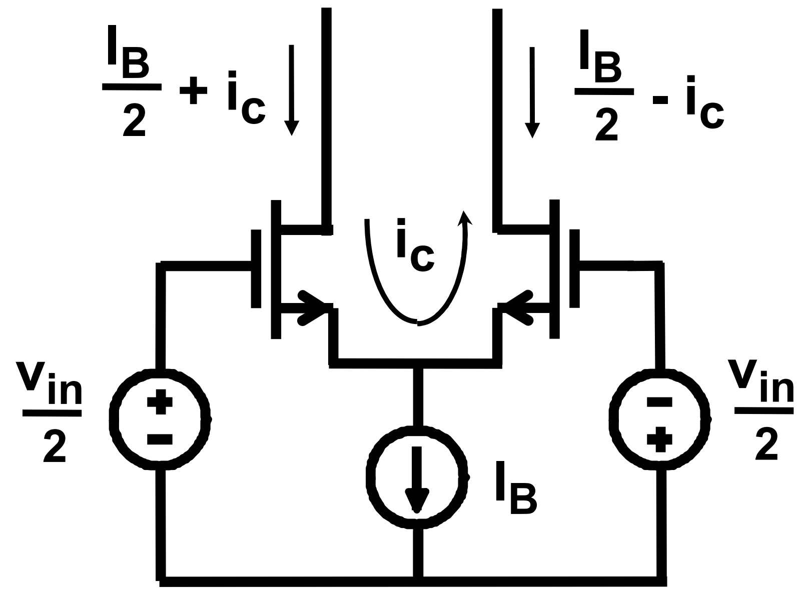
Two-transistor circuits





$$i_{out} = B\, i_{in}$$

$$i_c = g_m \frac{v_{in}}{2}$$

Current mirror/amp.

Differential Voltage amp.

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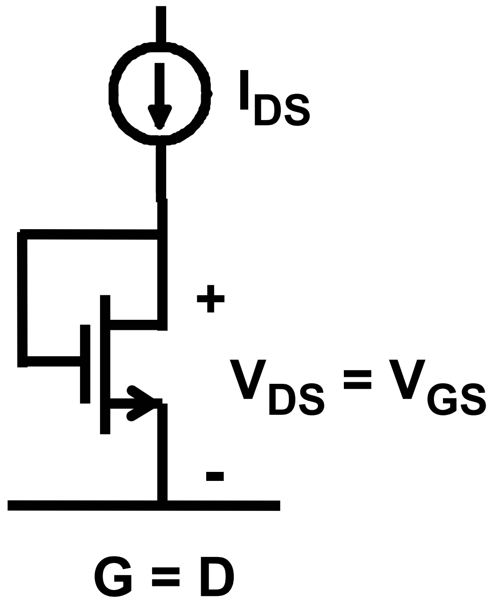
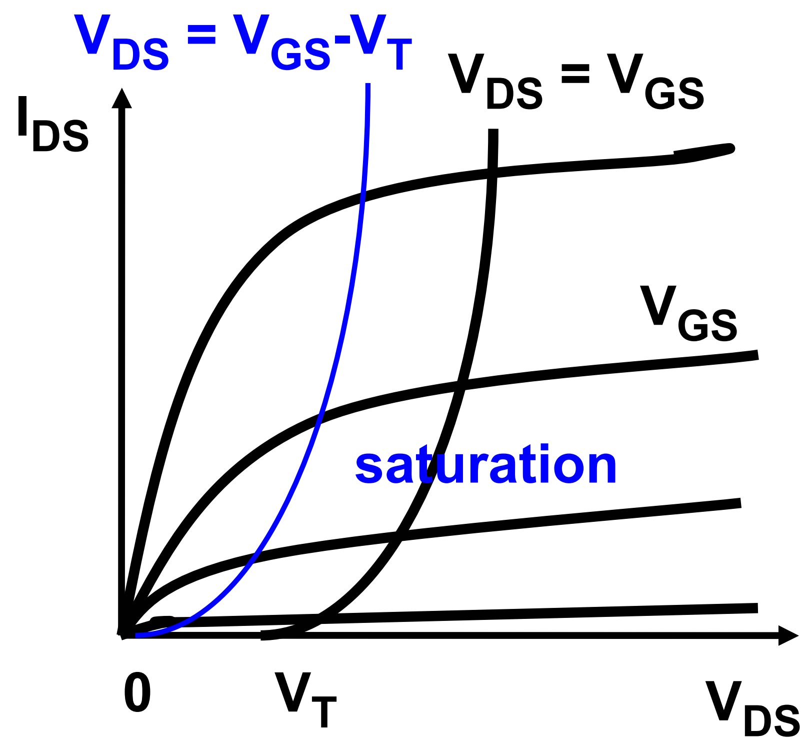
- ❑ **Current mirrors**
- ❑ **Differential pairs**
- ❑ **Differential voltage and current amps**

Diode-connected MOST

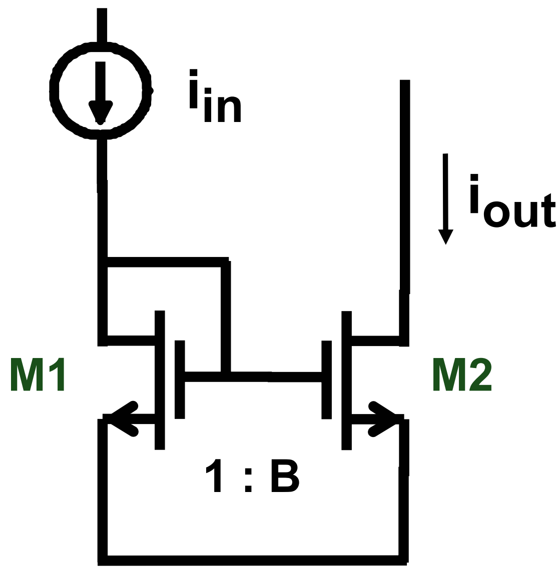


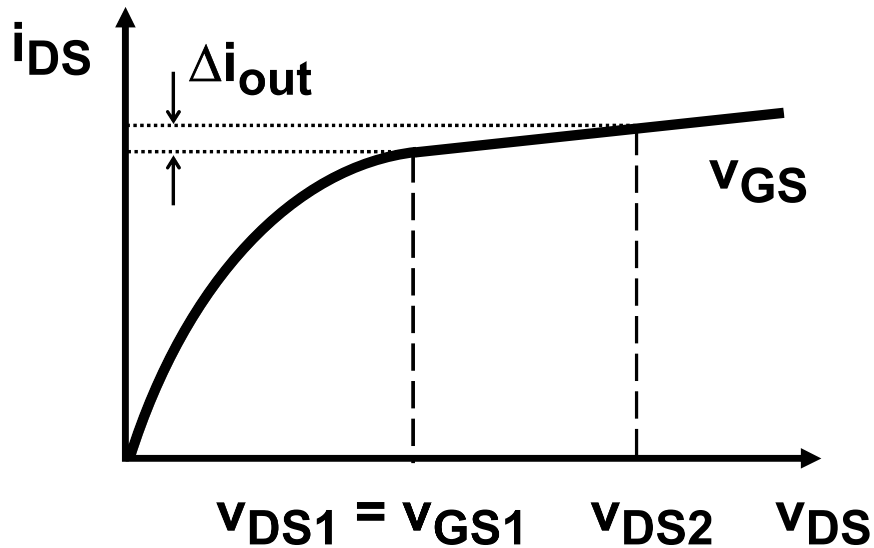


$$I_{DS} = K'_n \frac{W}{L} (V_{DS}-V_T)^2$$

$$g_m = di_{DS} / dv_{DS}$$

Current mirror

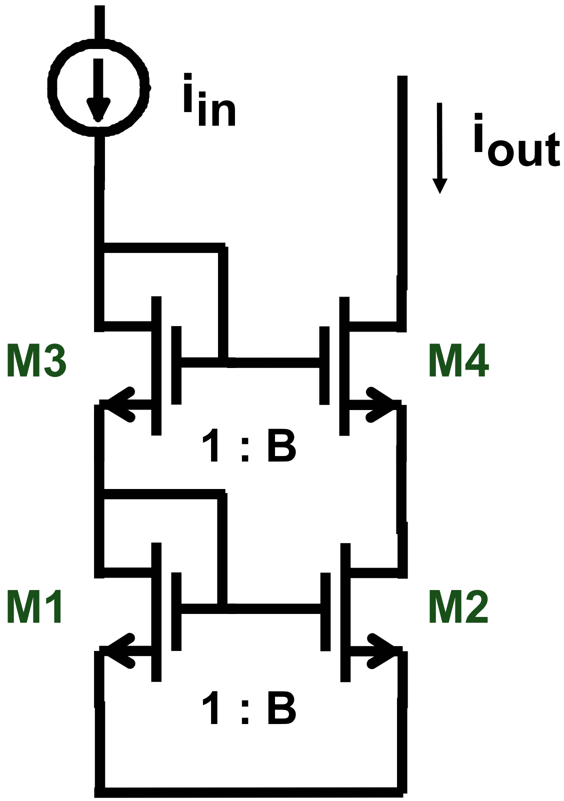


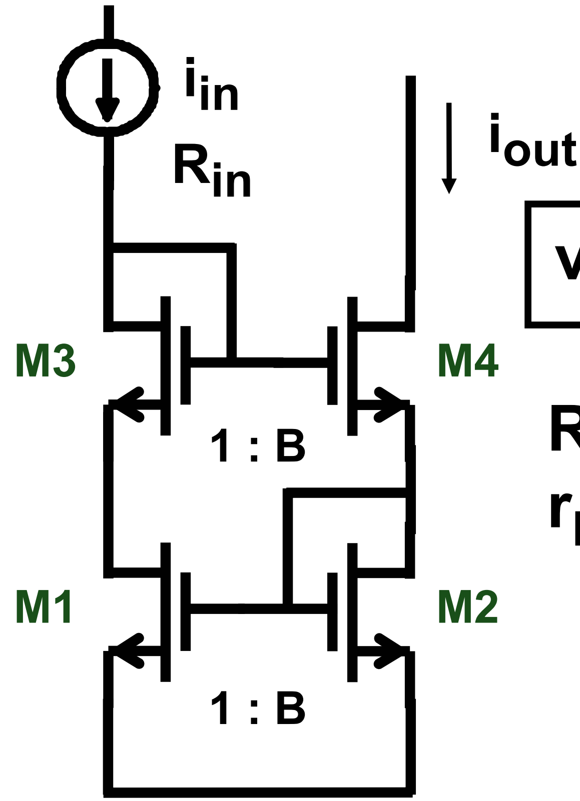


$$i_{out} = B\ i_{in}$$

$$\frac{\Delta i_{out}}{i_{out}} = \frac{v_{DS2} - v_{DS1}}{V_E L_2}$$

Improved current mirrors





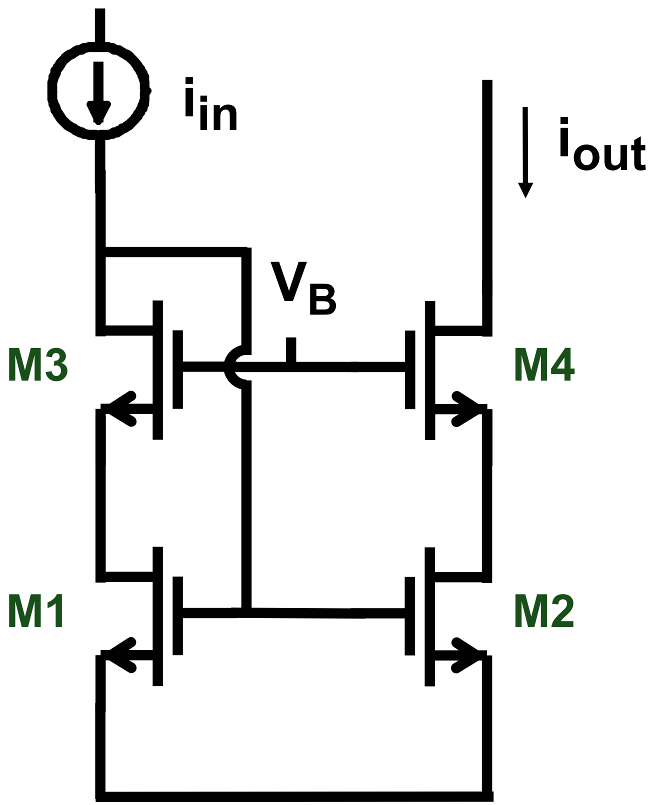
Feedback $T \approx g_{m1}R_{in}$

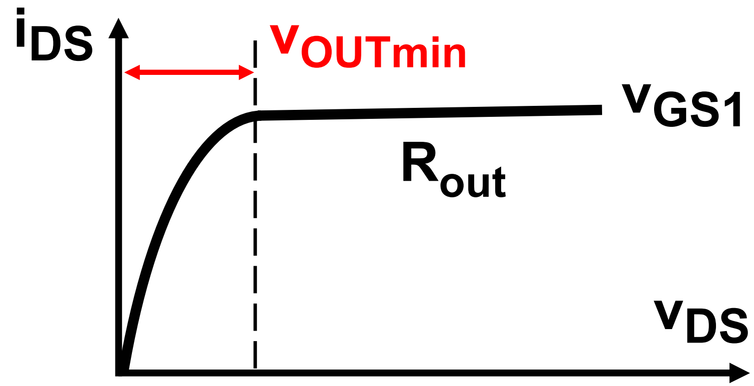
$\boxed{v_{DS2} = v_{DS1}}$

$R_{out} =$
$r_{DS2}\, g_{m4} r_{DS4}$

v_{OUTmin}
$= v_{GS} + v_{DS}$
$\approx 0.9 + 0.2 = 1.1$ V
Is too large !!!

Low-voltage current mirror





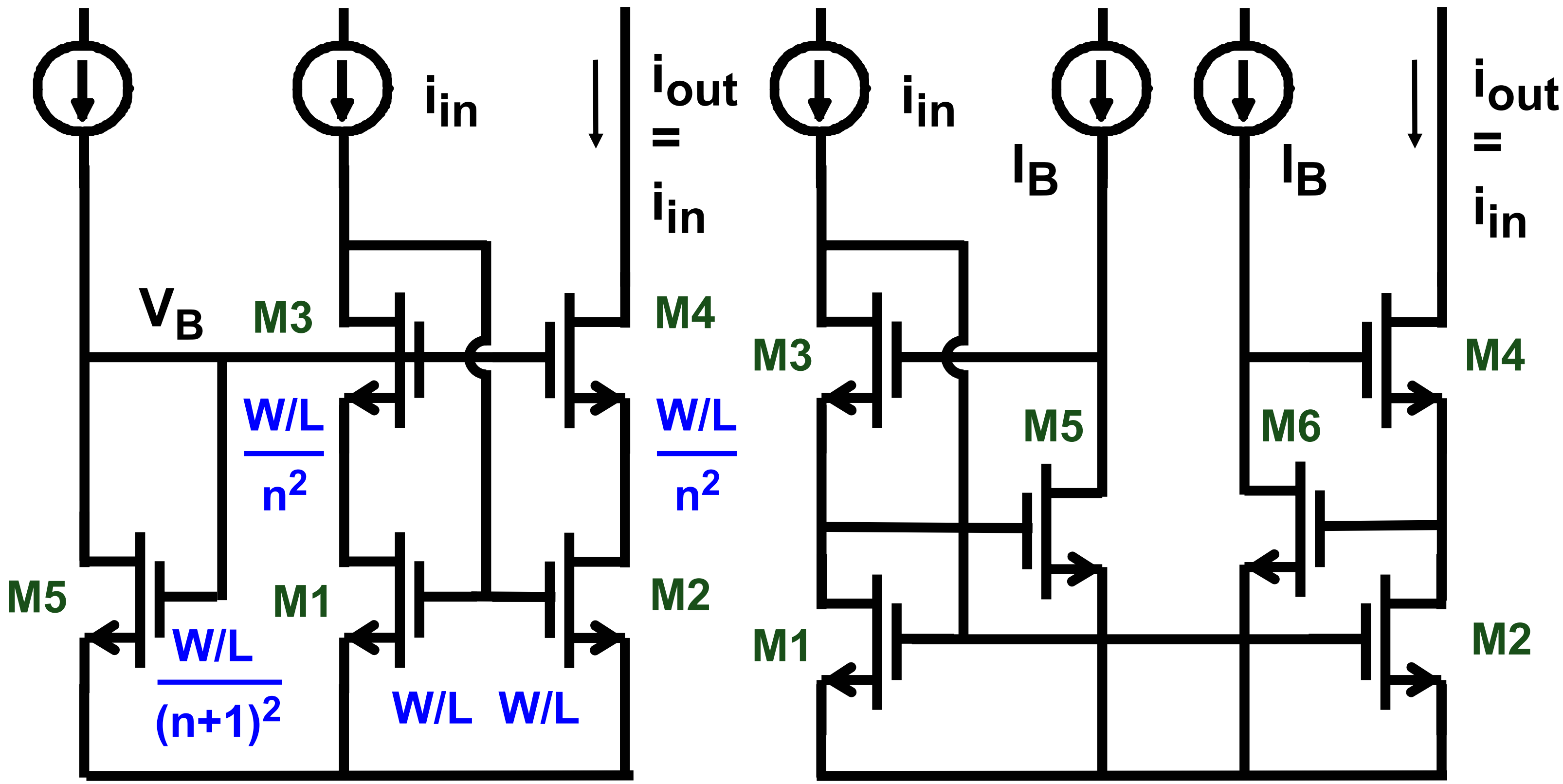
$$\boxed{v_{DS2} = v_{DS1}}$$

$$R_{out} = r_{DS2}\, g_{m4} r_{DS4}$$

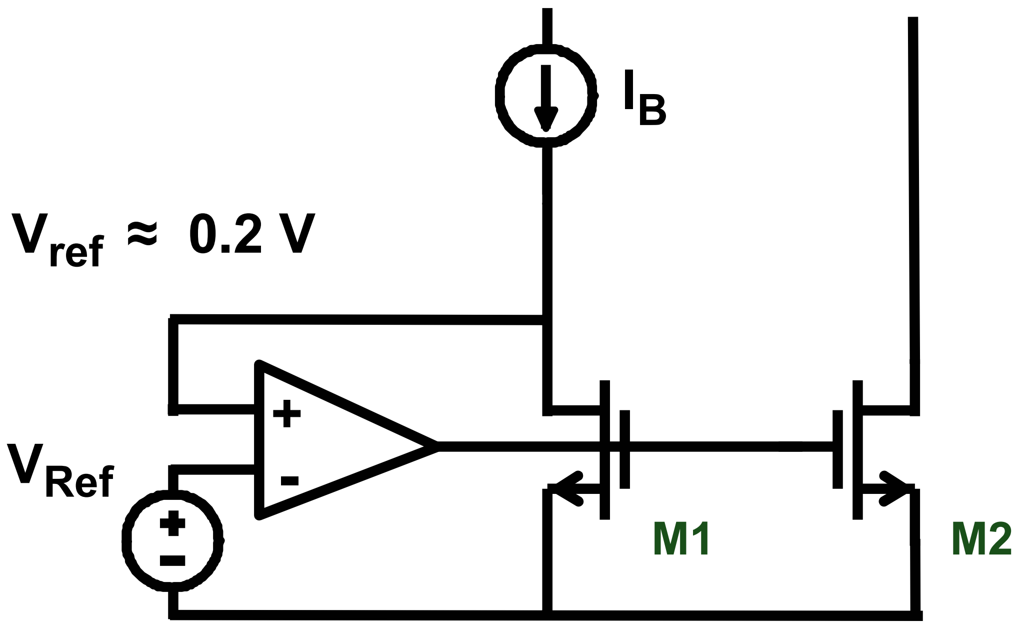
$$v_{OUTmin} = v_{DS2} + v_{DS4}$$

$\approx 0.2 + 0.2 = 0.4$ V is low !

Examples of low-voltage current mirrors



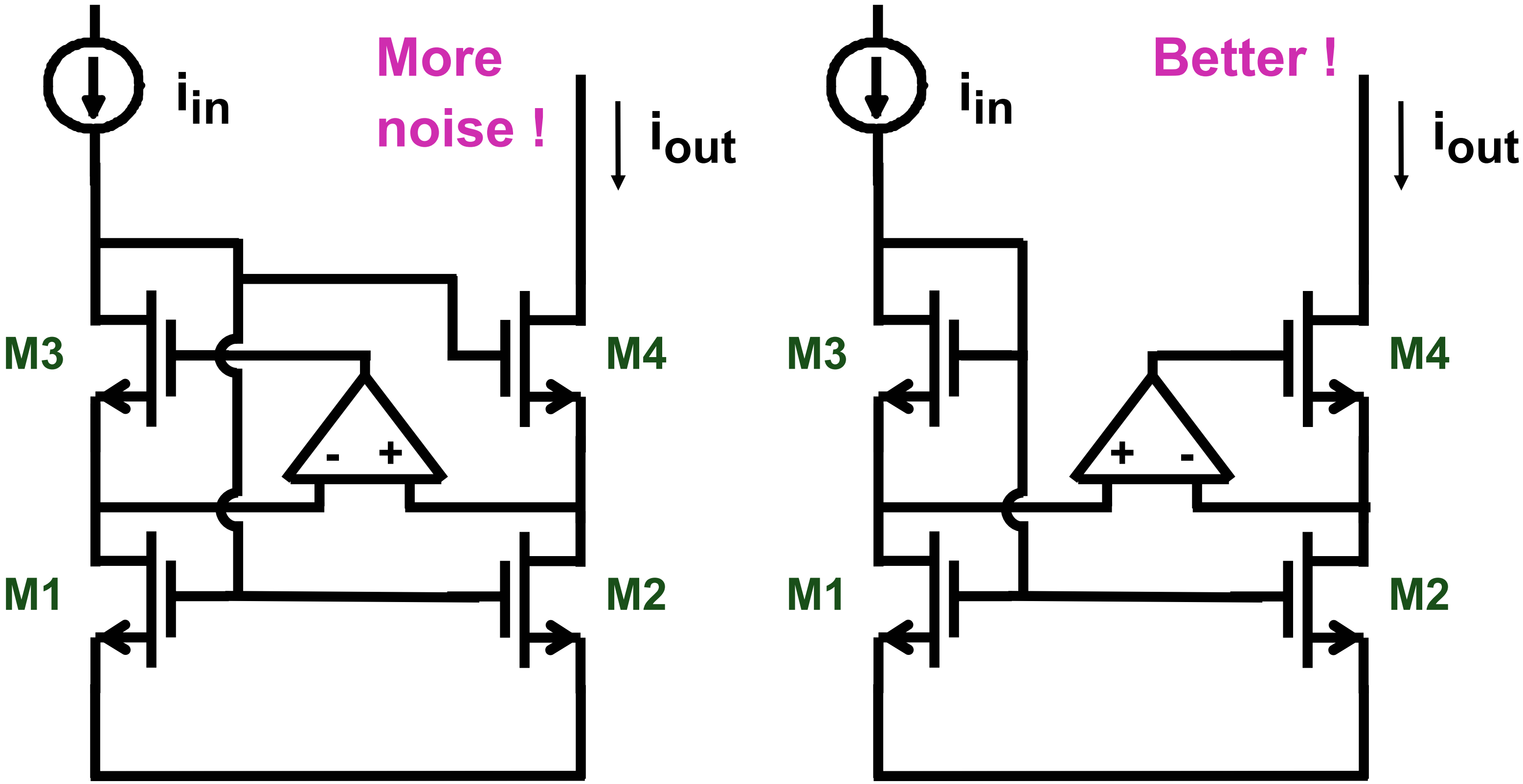
Low-voltage diode-connected MOST



$V_{DSmin} \approx 0.2$ V

$R_{out} = r_{DS2}$

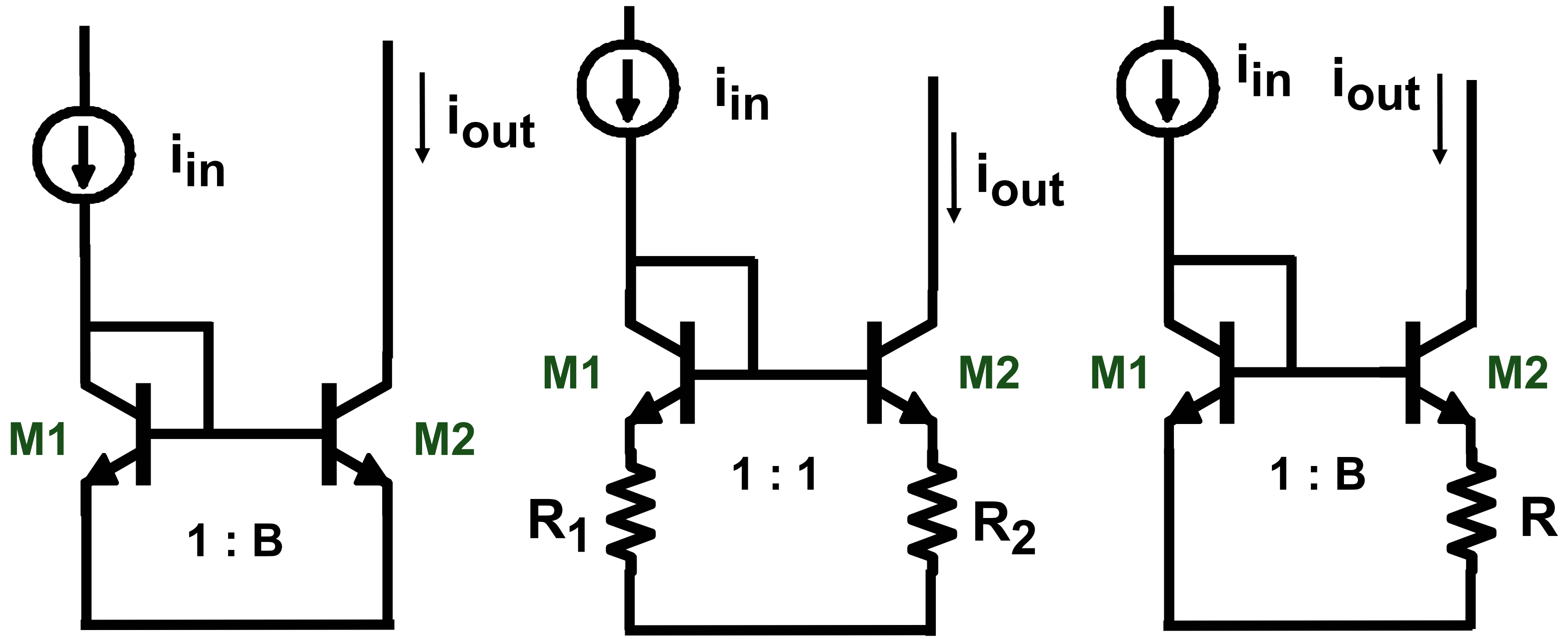
Lowest-voltage current mirrors



$v_{OUTmin} \approx 50$ mV

Ref.: Charlon, .., ESSCIRC 2004

Current mirror



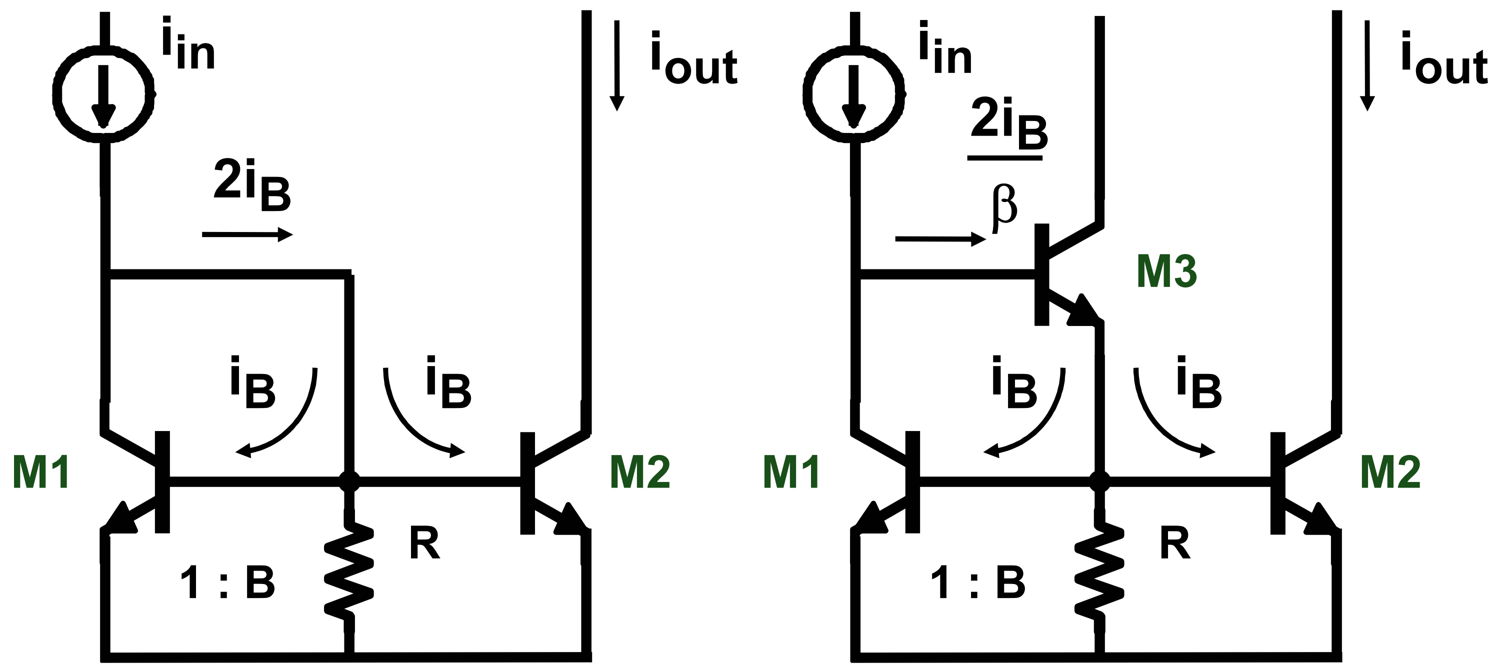
$$i_{out} = B\, i_{in}$$

$$\frac{i_{out}}{i_{in}} = \frac{R_1}{R_2}$$

$$i_{out} = \frac{kT/q}{R} \ln B \frac{i_{in}}{i_{out}}$$

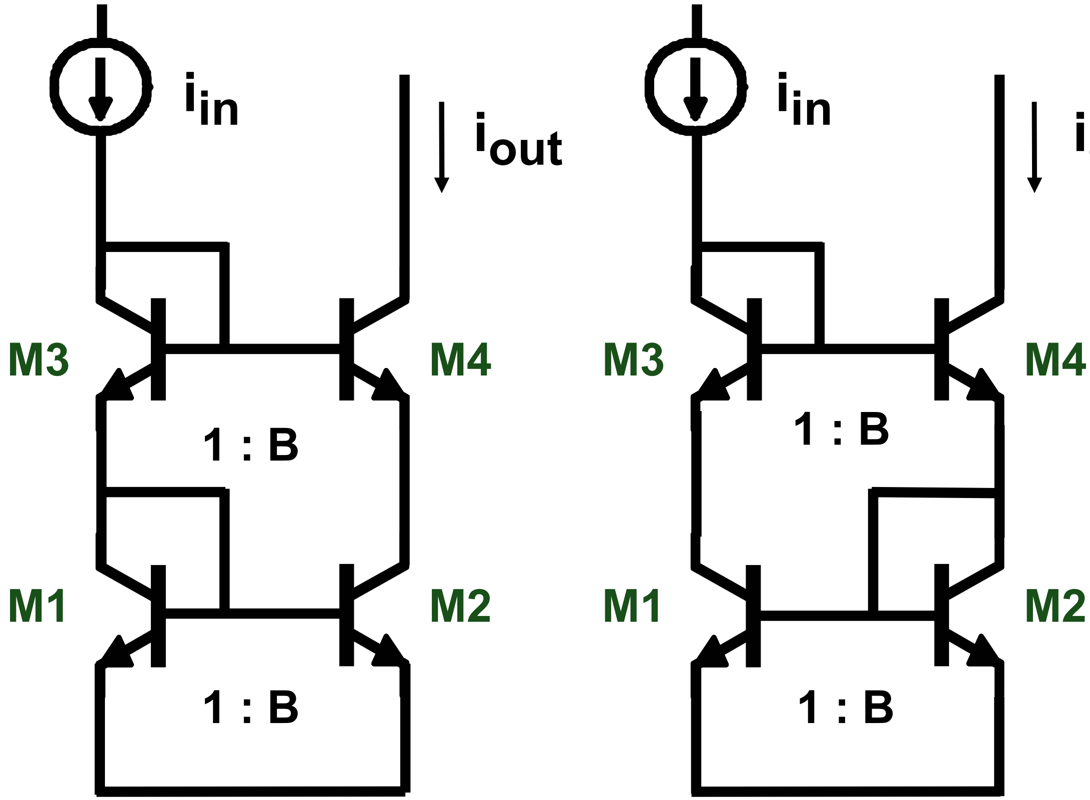
Ref.: Widlar, JSSC Aug 69, 184-191

Improved current mirrors



$$\text{Error} \sim \frac{2}{\beta} \qquad\qquad \text{Error} \sim \frac{2}{\beta^2}$$

Improved current mirrors



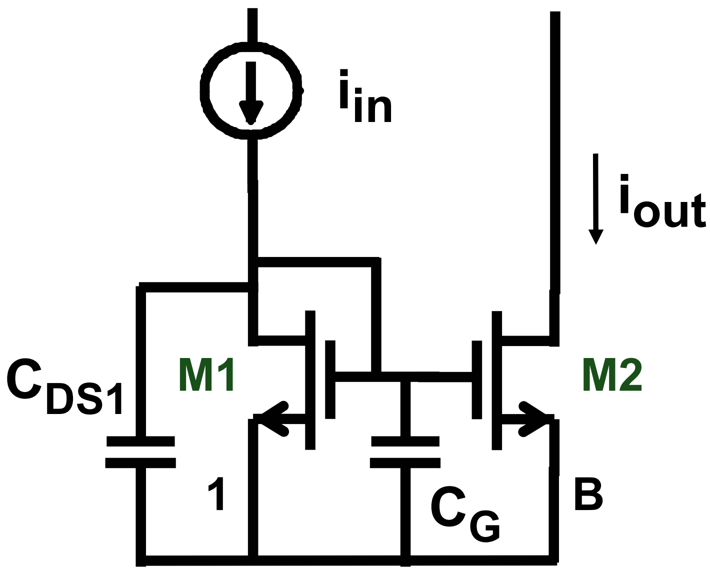
$\boxed{v_{CE2} = v_{CE1}}$

$R_{out} =$
$r_{o2}\, g_{m4} r_{o4}$

v_{OUTmin}
$= v_{BE} + v_{CE}$
$\approx 0.7+0.1= 0.8$ V
Is too large !

Ref.: Wilson, JSSC Dec.68, 341-348

Current mirror at high frequencies



$$i_{out} = B\, i_{in}$$

$$R_{out} = r_{DS}$$

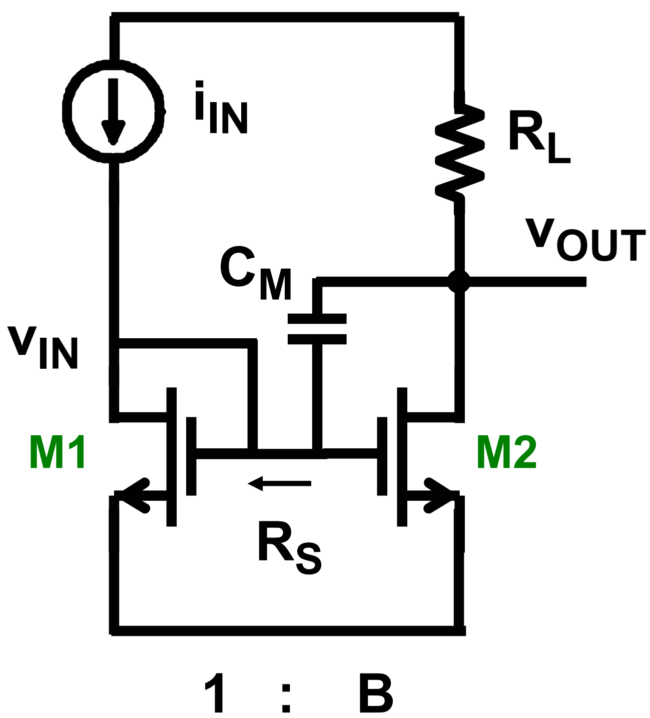
$$C_G = (1 + B)\, C_{GS} + C_{DS1}$$

$$BW = \frac{g_m}{2\pi\,(C_G + C_{DS1})}$$

$$\approx f_T \frac{1}{(2 + B)}$$

Ref.: Gilbert, JSSC Dec.68, 353-365

Current Miller effect



$$A_i = B$$

$$R_{IN} = \frac{1}{g_{m1}}$$

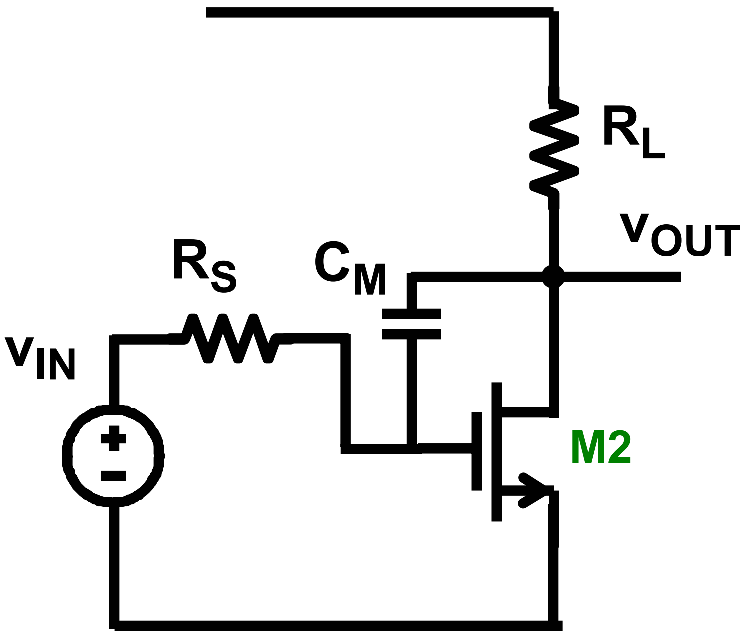
$$R_S = 1/g_{m1}$$

$$v_{IN} \approx i_{IN} R_S$$

$$B = \frac{g_{m2}}{g_{m1}}$$

Ref.: Rincon-Mora, JSSC Jan. 2000, 26-32

Current Miller equivalent circuit



Miller effect :

$$f_{-3dB} = \frac{1}{2\pi\, R_S A_{v2} C_M}$$

$R_S = 1/g_{m1}$ $\quad A_{v2} = g_{m2} R_L$

$$f_{-3dB} = \frac{1}{2\pi\, (1+B) C_M R_L}$$

$$f_z = -\frac{g_{m2}}{2\pi\, C_M}$$

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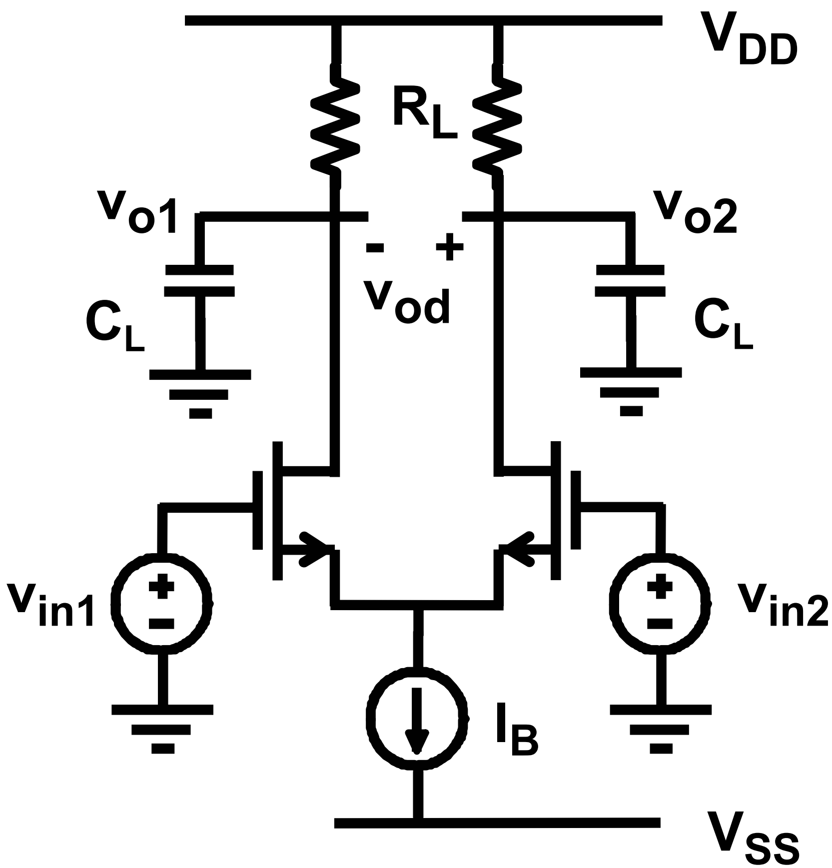
Voltage differential amplifier



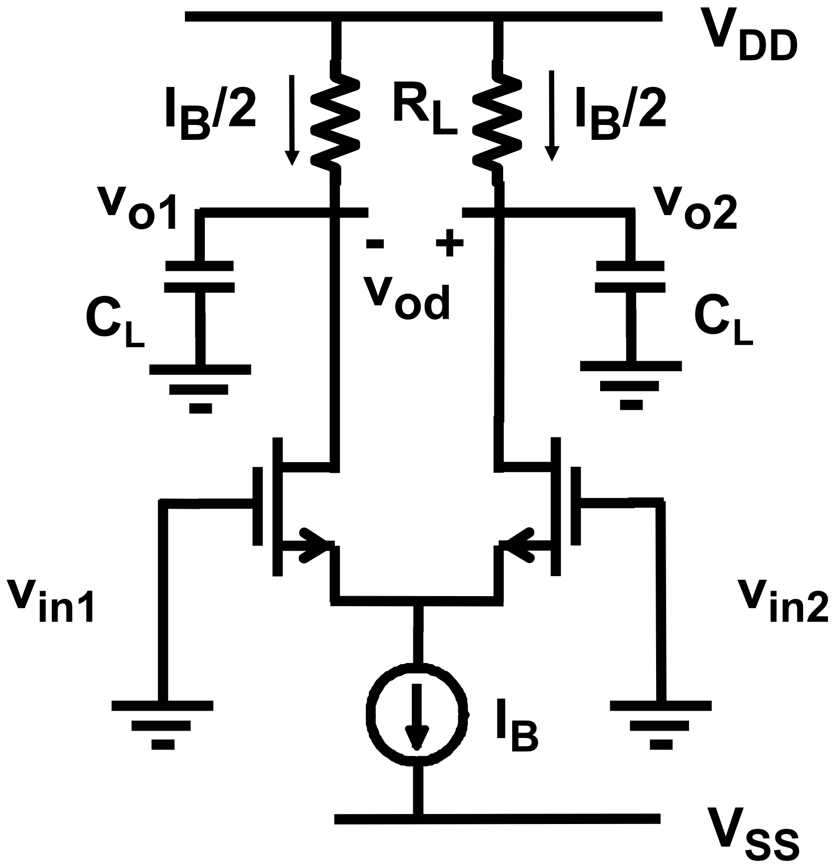
Two equal transistors

Redefine v_{in} & v_o :

$$\left[\begin{array}{l} v_{ind} = v_{in1} - v_{in2} \\ v_{inc} = \dfrac{v_{in1} + v_{in2}}{2} \end{array}\right.$$

$$\left[\begin{array}{l} v_{od} = v_{o1} - v_{o2} \\ v_{oc} = \ldots \end{array}\right.$$

Voltage differential amplifier : DC

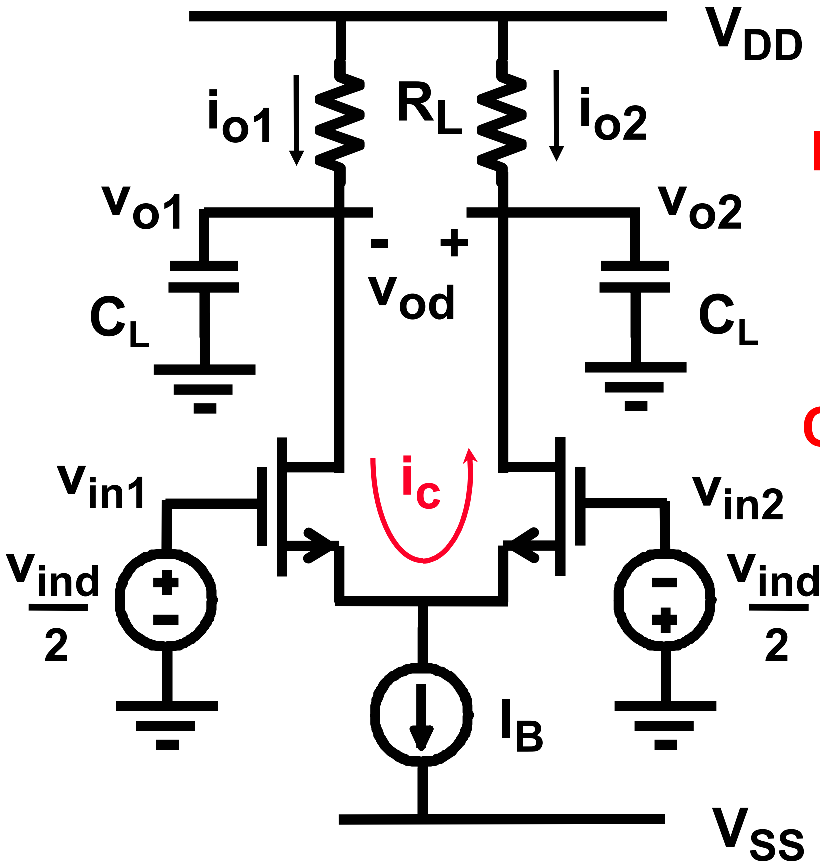


$v_{in1} = v_{in2} = 0$

$v_{o1} = v_{o2}$
$= V_{DD} - R_L\, I_B/2$

$v_{od} = v_{o1} - v_{o2} = 0$

Voltage differential amplifier : AC Gain



Differential input voltage

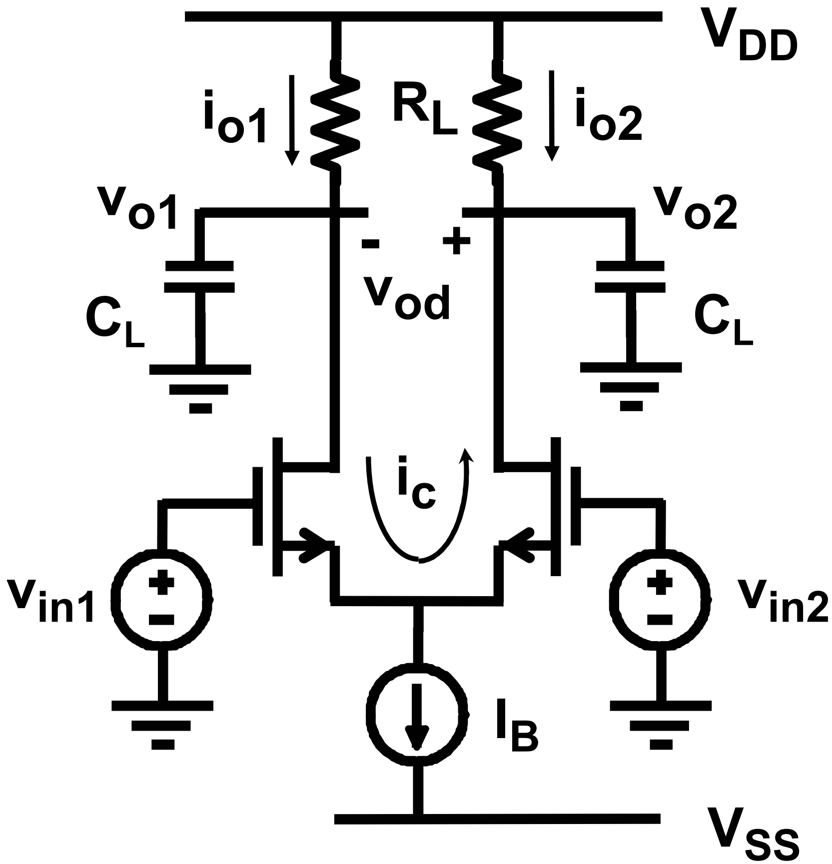
$$v_{ind} = v_{in1} - v_{in2}$$

Circular current $i_c = g_m \frac{v_{ind}}{2}$

$$v_{od} = 2\, R_L\, i_c$$

$$\boxed{A_v = \frac{v_{od}}{v_{ind}} = g_m R_L}$$

Voltage differential amplifier



$$A_v = g_m R_L$$

Same as single-tr. !!

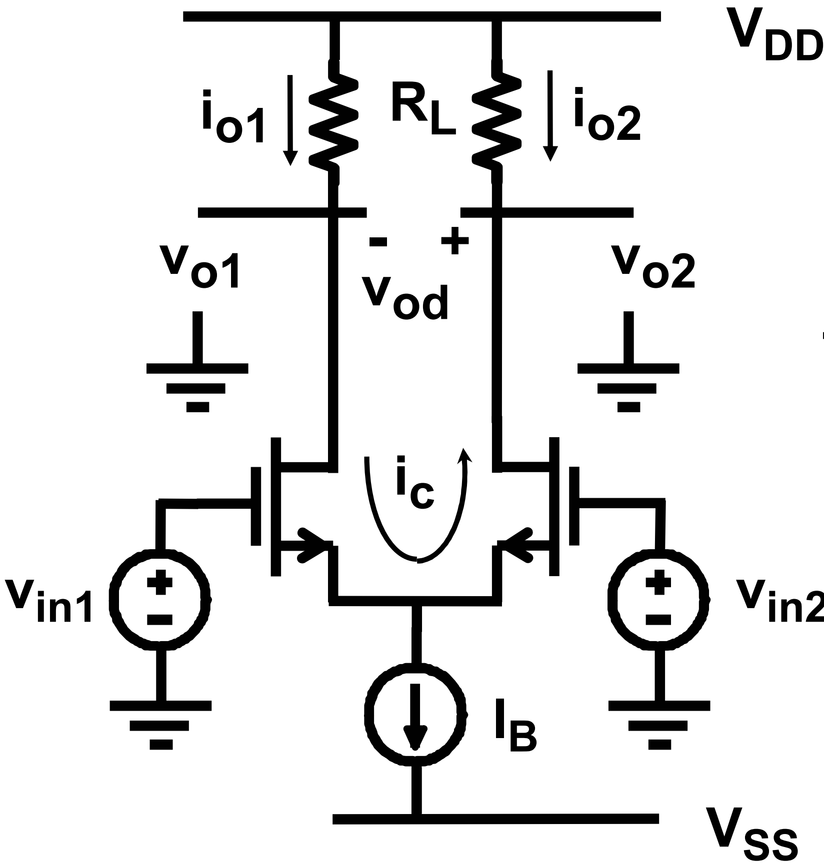
Independent of :

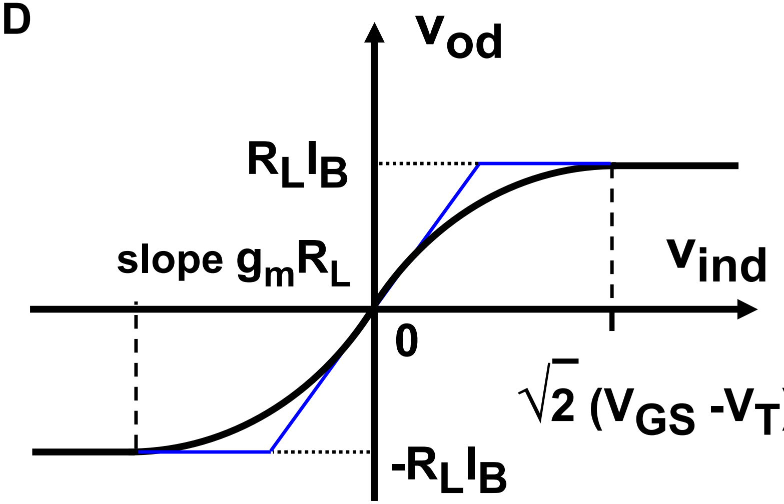
Noise on V_{DD} : $PSRR_{DD}$

Noise on V_{SS} : $PSRR_{SS}$

Noise on Ground : CMRR

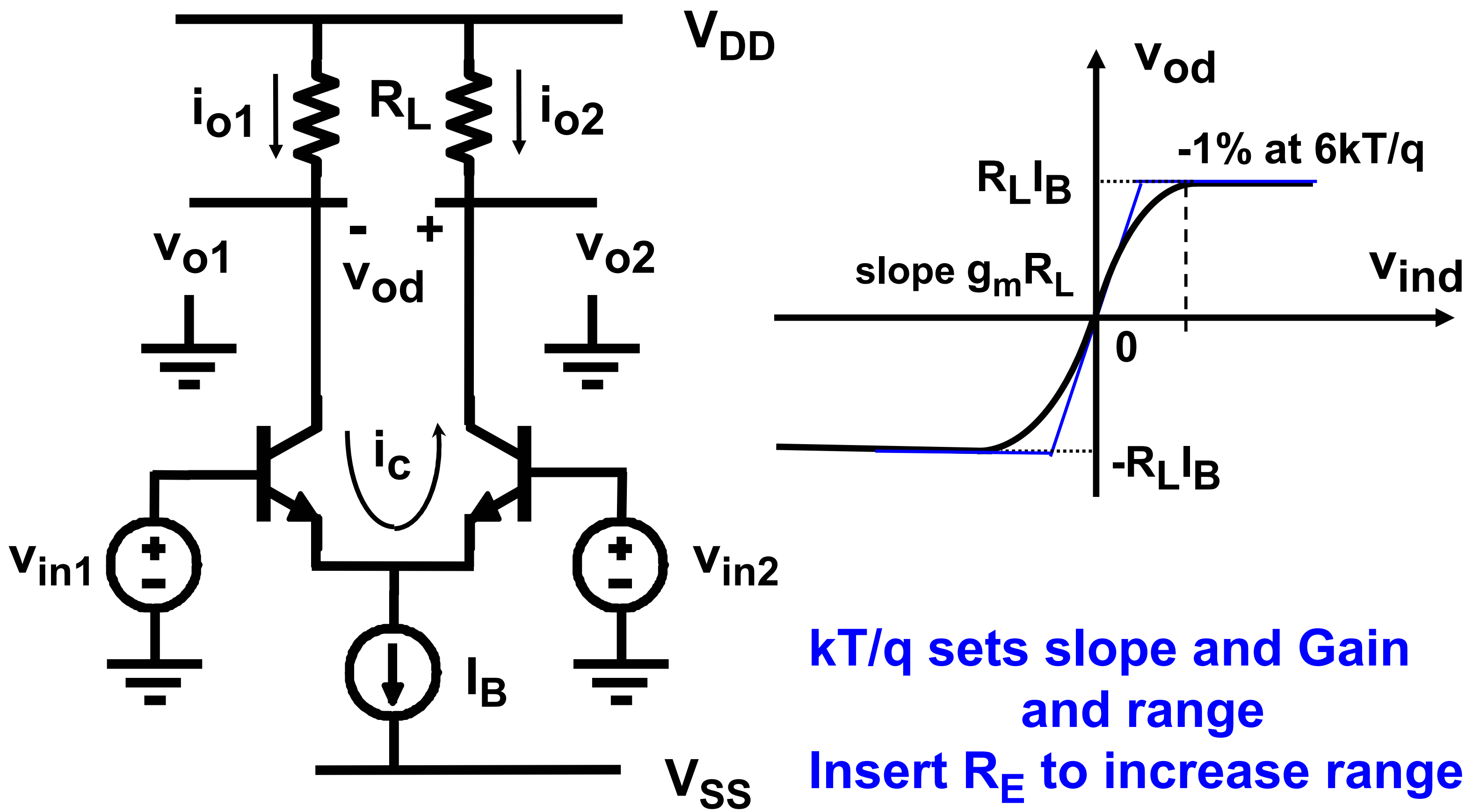
CMOS Voltage differential amplifier : DC range





$V_{GS} - V_T$ sets slope
and range
and …. Gain !

Bipolar Voltage diff. amplifier : DC range



kT/q sets slope and Gain and range

Insert R_E to increase range !

MOST Voltage diff. amplifier : large input signals

$$\frac{i_{Od}}{I_B} = \frac{v_{Id}}{(V_{GS}-V_T)} \sqrt{1 - \frac{1}{4}\left(\frac{v_{Id}}{V_{GS}-V_T}\right)^2}$$

v_{Id} is the differential input voltage

i_{Od} is the differential output current ($g_m v_{Id}$) or

twice the circular current $g_m v_{Id}$ /2

I_B is the total DC current in the pair

Note that $g_m = \dfrac{I_B}{V_{GS} - V_T} = K' \, W/L \, (V_{GS} - V_T)$

Bipolar Voltage diff. amp. : large input signals

$$\frac{i_{Od}}{I_B} = \tanh \frac{V_{Id}}{2\ kT/q} \qquad \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{2e^x - 1}{2e^x + 1}$$

v_{Id} is the differential input voltage

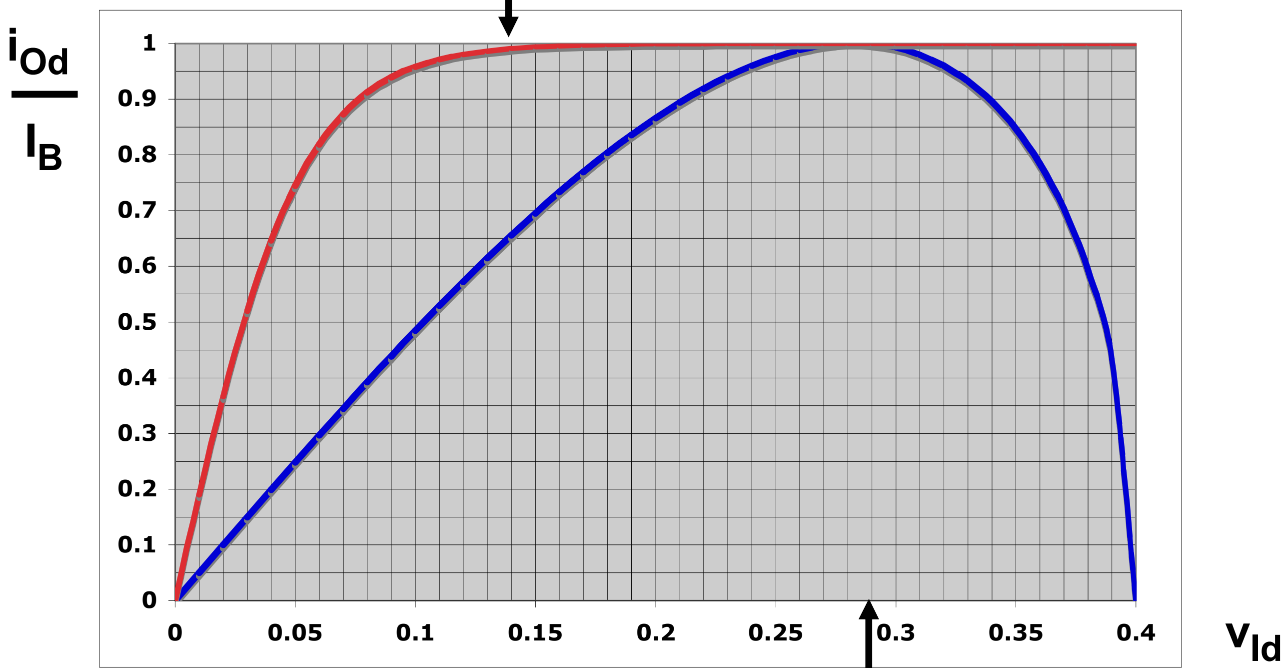
i_{Od} is the differential output current ($g_m v_{Id}$) or twice the circular current $g_m v_{Id}/2$

I_B is the total DC current in the pair

Note that $g_m = \dfrac{I_B}{2\ kT/q}$

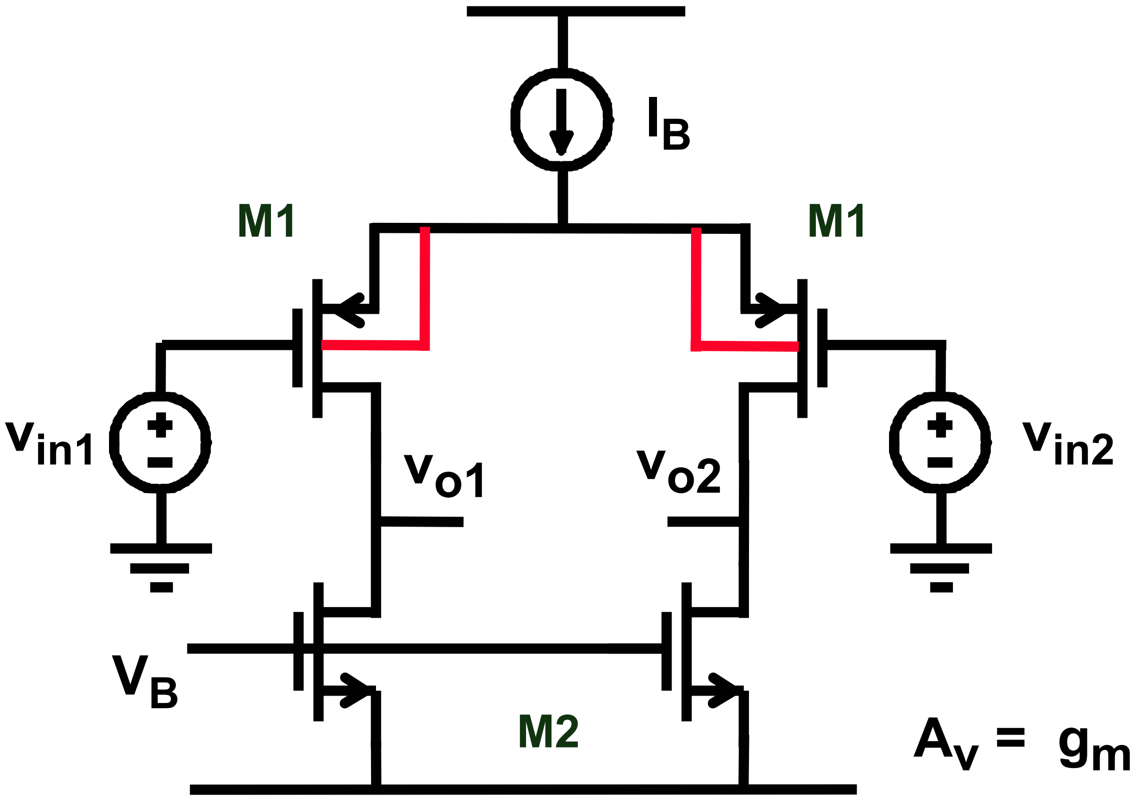
Voltage differential amplifier: transfer function

-1% $V_{Id} \approx 2.67 \times 2kT/q = 0.14$ V



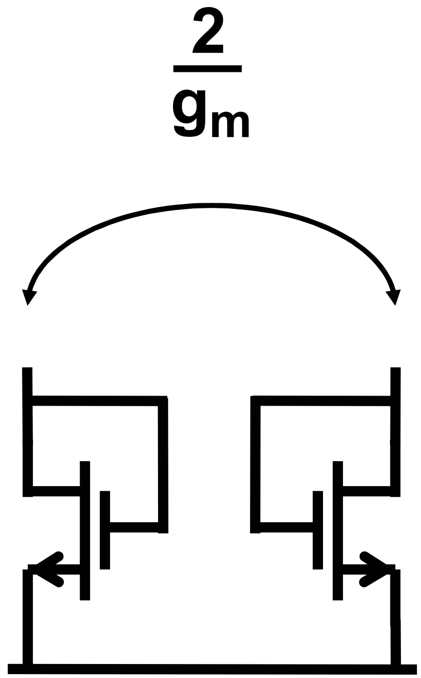
$V_{Id} = \sqrt{2}\,(V_{GS}-V_T) = \sqrt{2} \times 0.2$ V

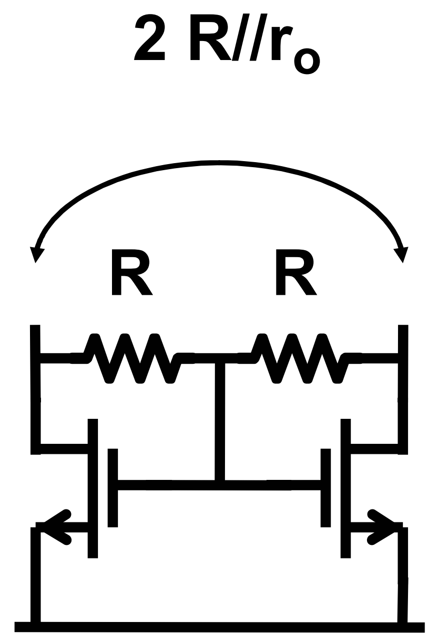
Voltage differential amplifier with $g_m r_{DS}$ gain



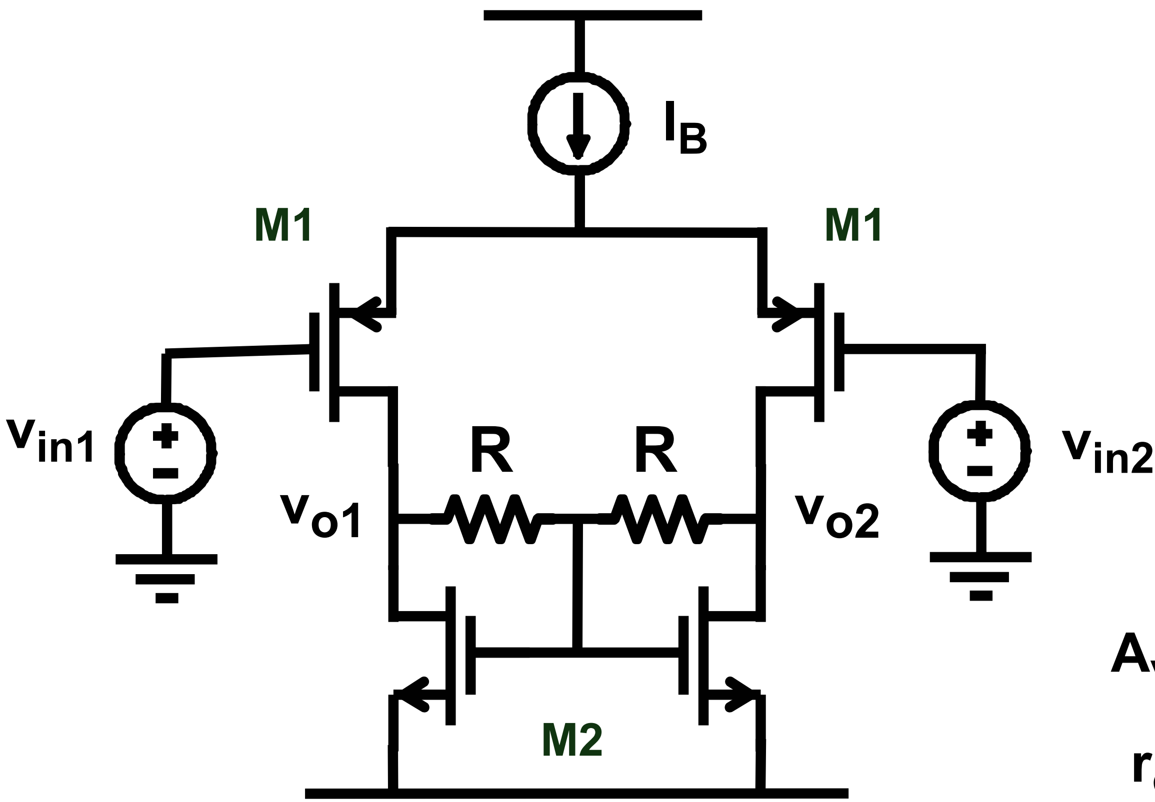
$$A_v = g_{m1}(r_{DS1}//r_{DS2})$$

Diode-connected MOSTs with resistors





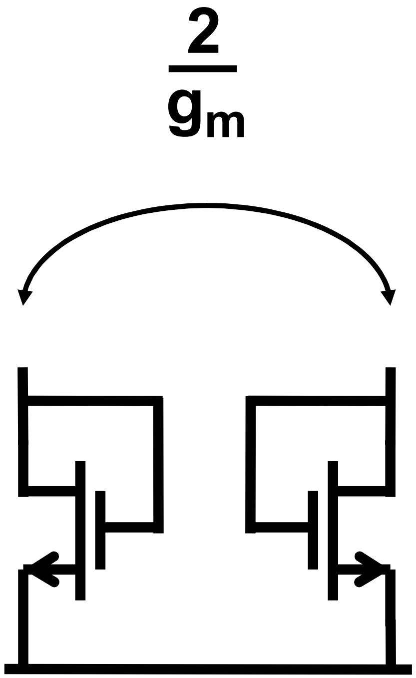
Voltage differential amplifier with high gain



$$A_v = g_{m1}\,(R\,//\,r_o)$$

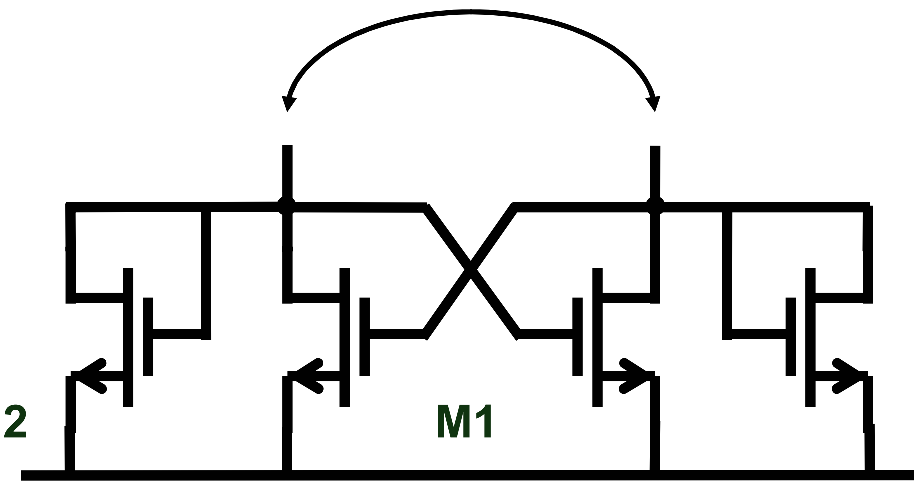
$$r_o = r_{o1}//r_{o2}$$

Differential diode-connected MOSTs



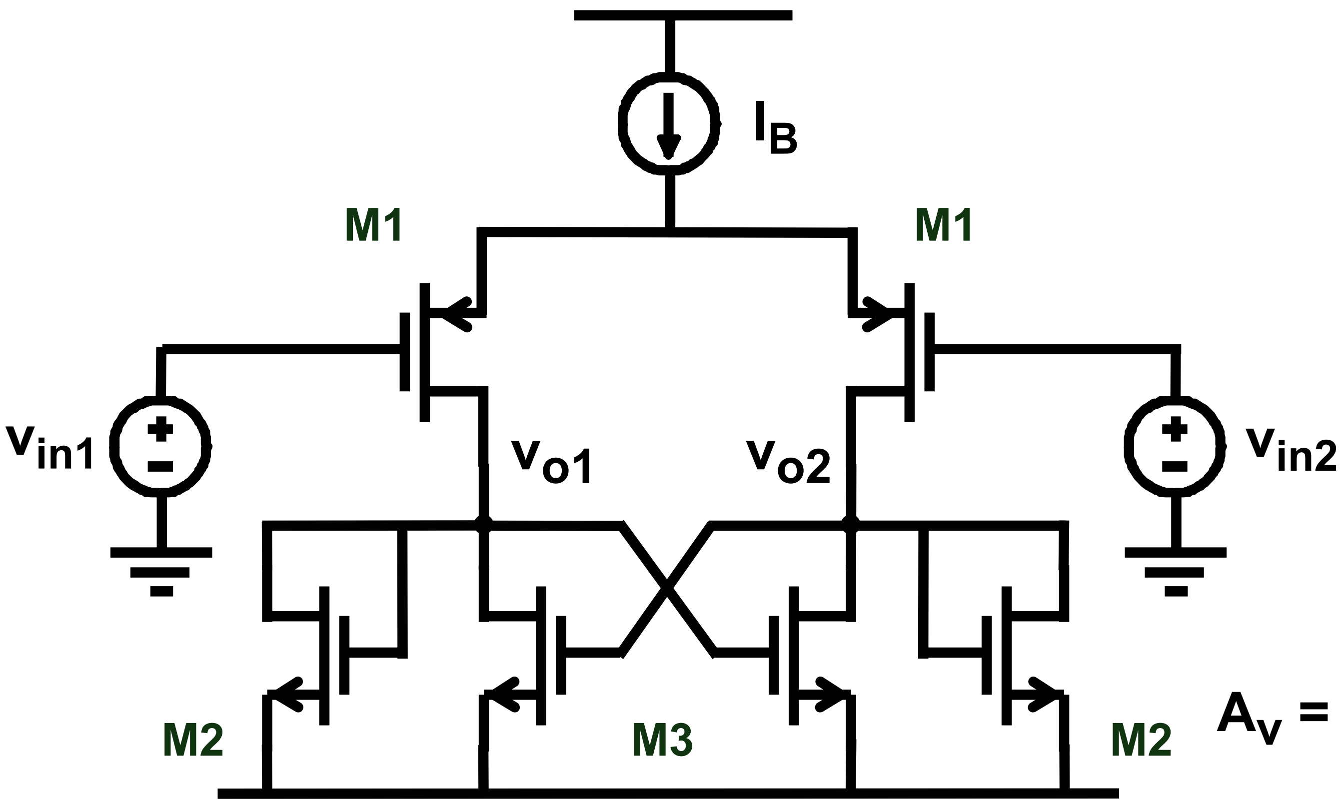
Differential diode-connected MOSTs

$$\frac{2}{g_{m2} - g_{m1}}$$



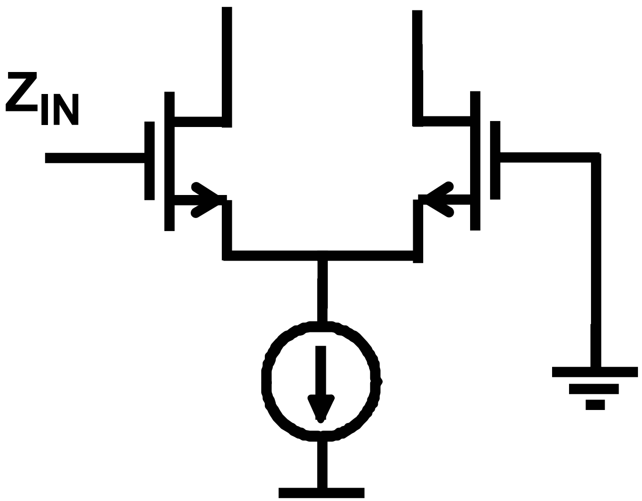
Values close to ∞ !

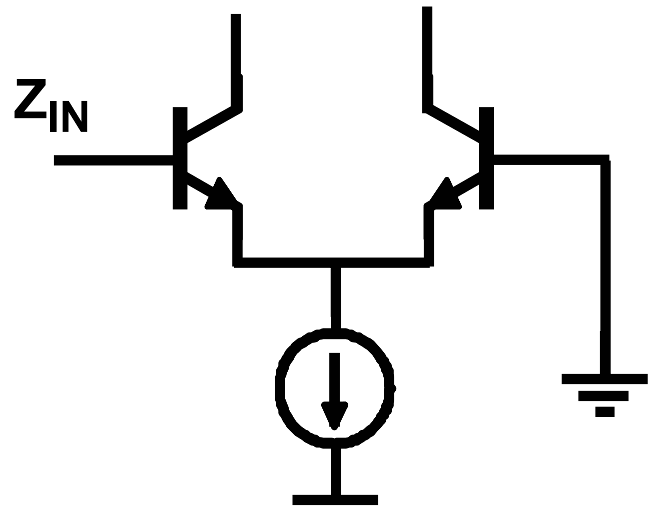
High gain because of current cancellation



$$A_v = \frac{g_{m1}}{g_{m2} - g_{m3}}$$

Input impedance

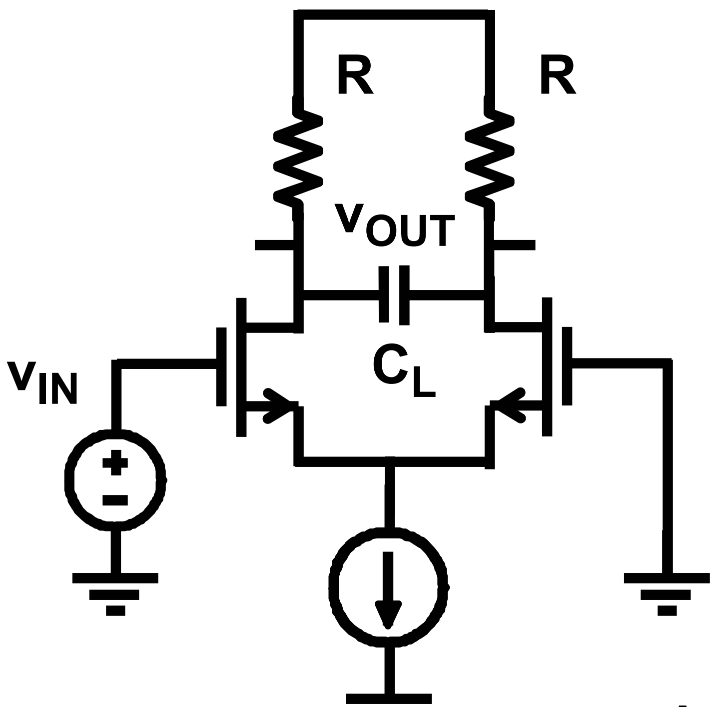


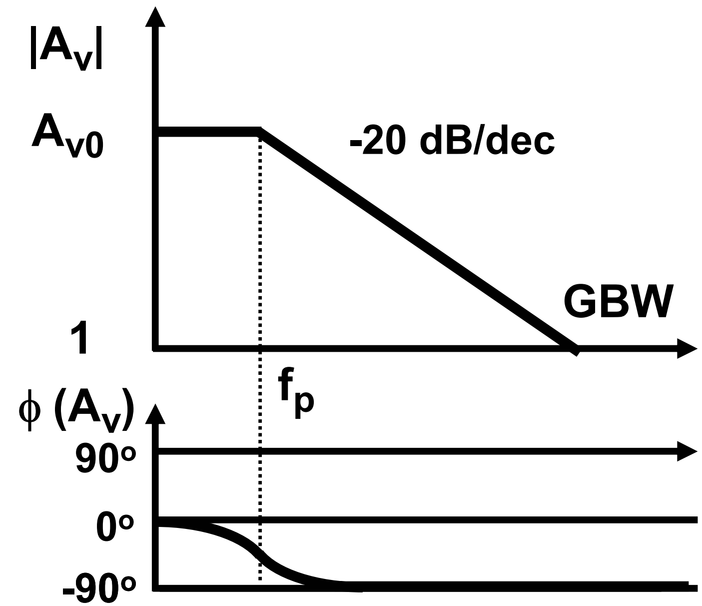


$$C_{IN} = \frac{C_{GS}}{2}$$

$$R_{IN} = 2\ r_{\pi} \qquad C_{IN} = \frac{C_{\pi}}{2}$$

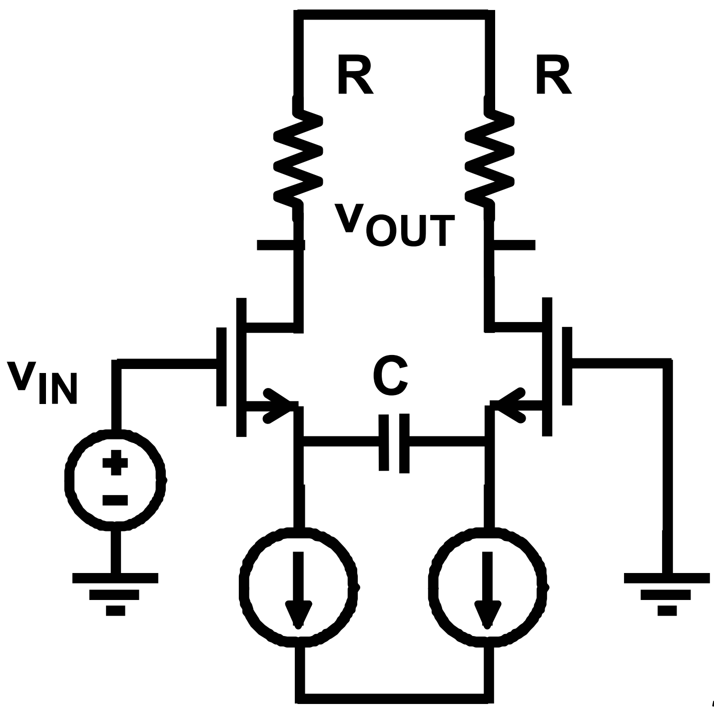
Low-Pass Voltage Differential amplifier

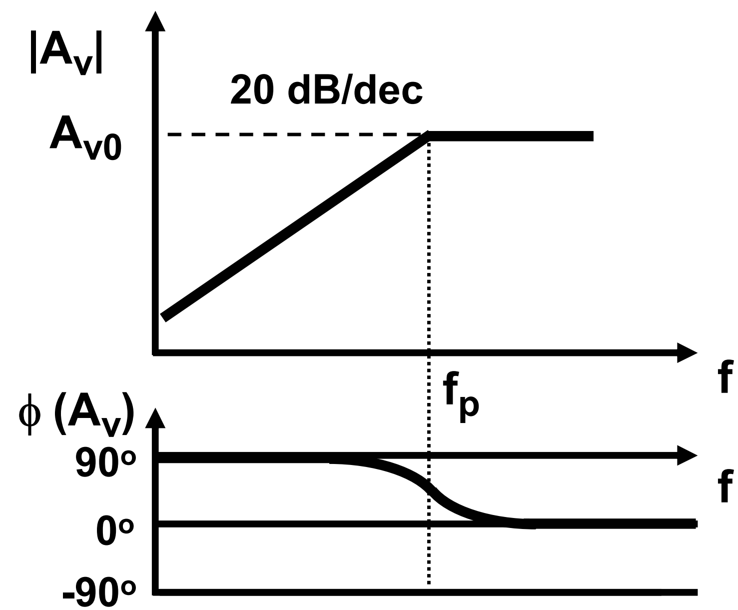




$A_{v0} = g_m R$ $\qquad A_V = \dfrac{A_{v0}}{(1 + j\dfrac{f}{f_p})}$ $\qquad f_p = \dfrac{1}{2\pi\, 2RC_L}$ $\qquad GBW = \dfrac{g_m}{2\pi\, 2C_L}$

High-Pass voltage differential amplifier



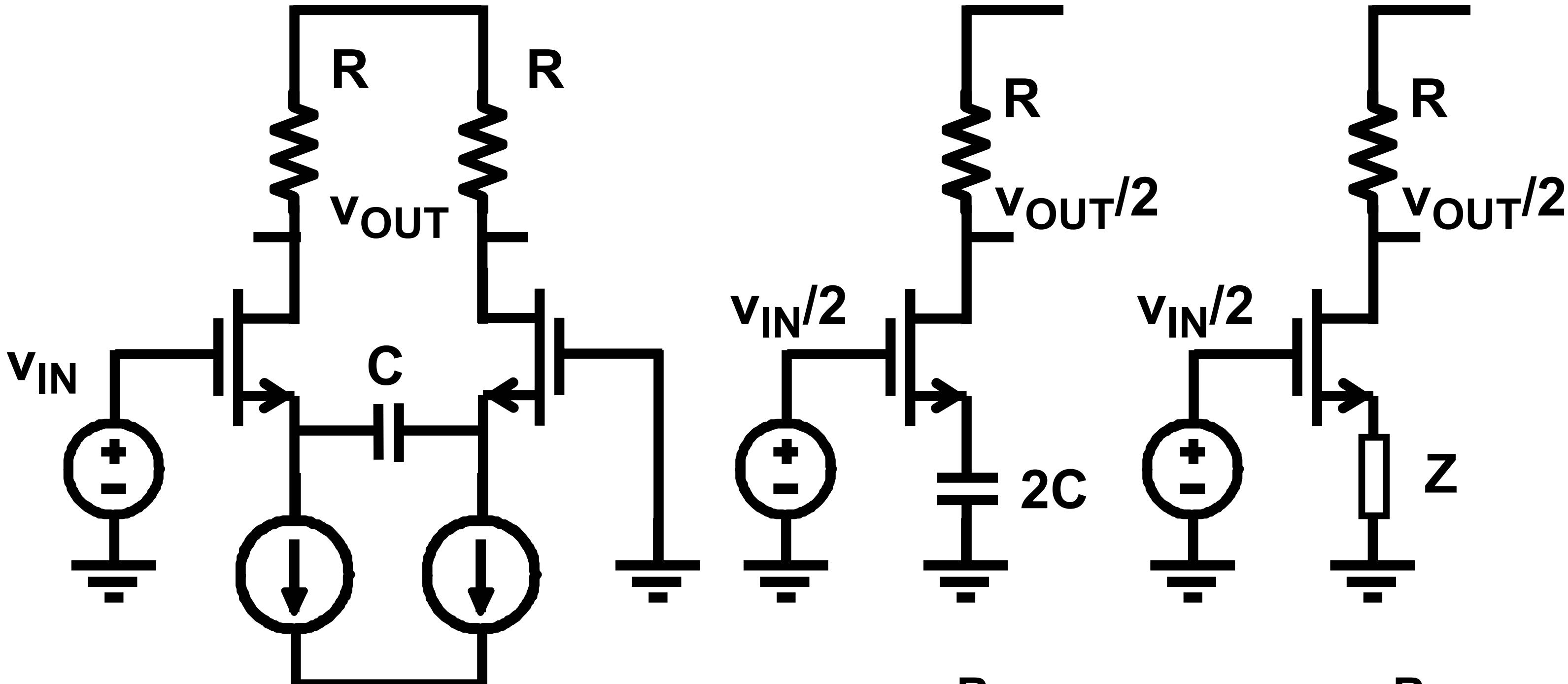


$$A_{v0} = g_m R$$

$$A_v = A_{v0}\,\frac{j\,\dfrac{f}{f_p}}{\left(1 + j\,\dfrac{f}{f_p}\right)}$$

$$f_p = \frac{g_m}{2\pi\,2C}$$

Calculation High-Pass differential amplifier



$$A_v = \frac{-\,g_m R}{\left(1 + \frac{g_m}{2Cs}\right)} \qquad A_v = \frac{-\,g_m R}{1 + g_m Z}$$

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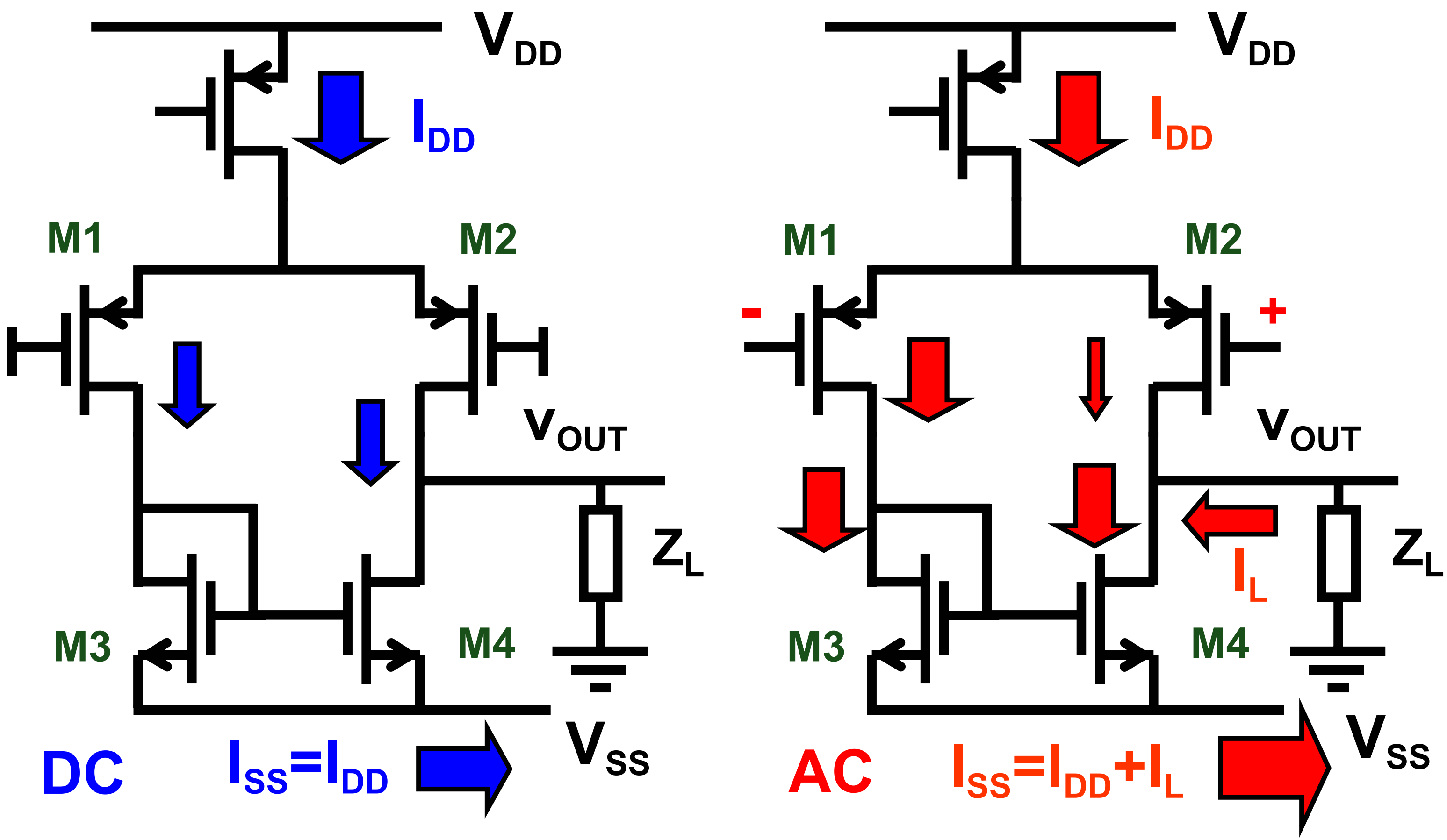
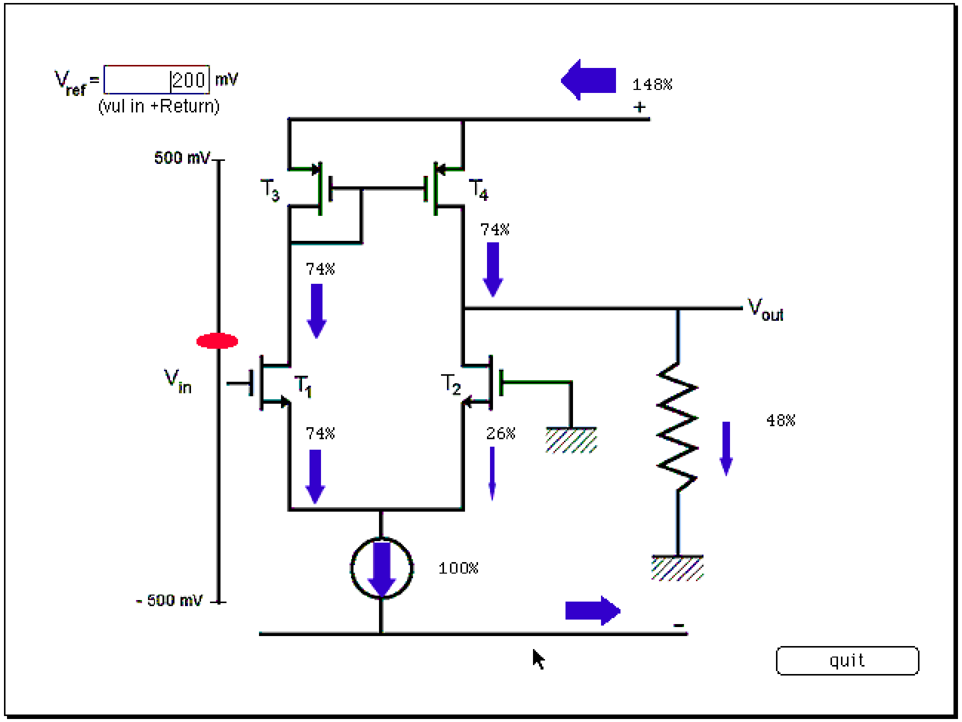
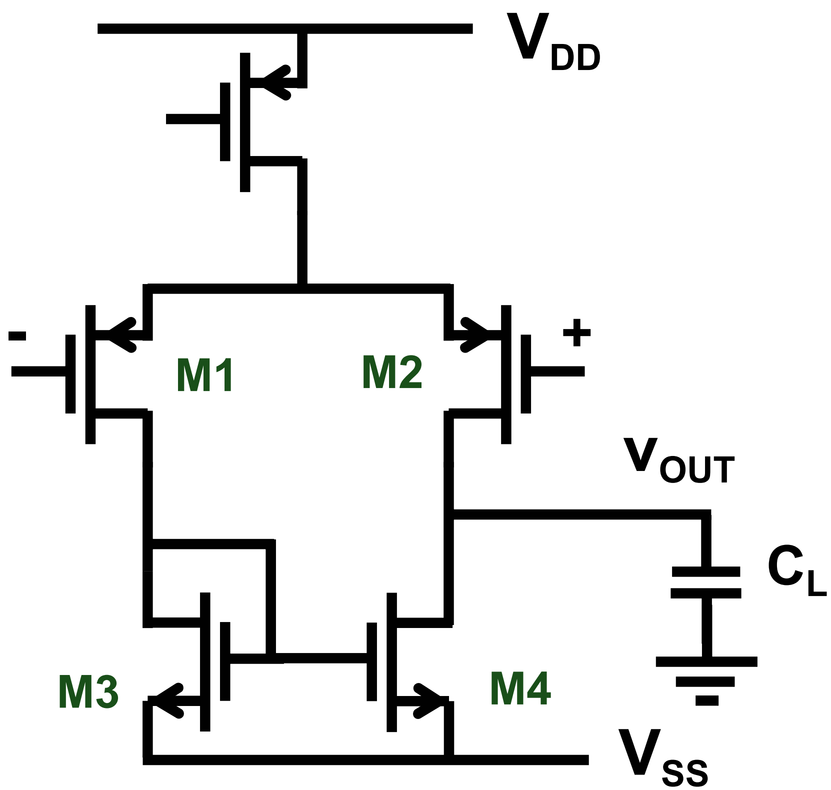
Operational Transconductance Amplifier (OTA)



Single-stage OTA: operation



Single-stage OTA



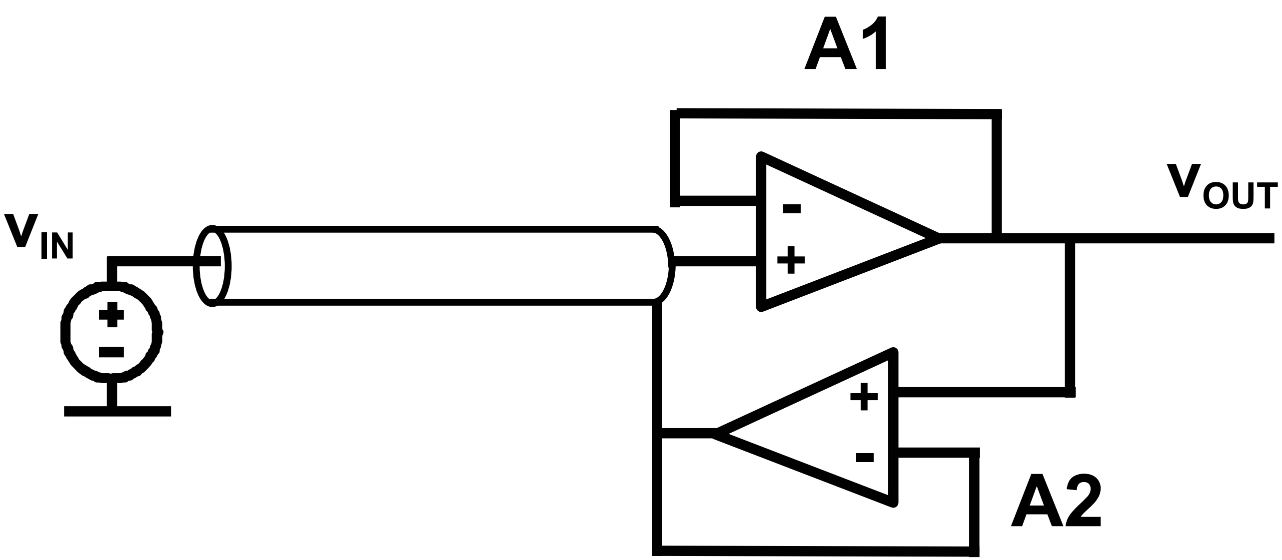
$$A_v = g_{m1} R_{out}$$

$$R_{out} = r_{DS2} // r_{DS4}$$

$$BW = \frac{1}{2\pi R_{out} C_L}$$

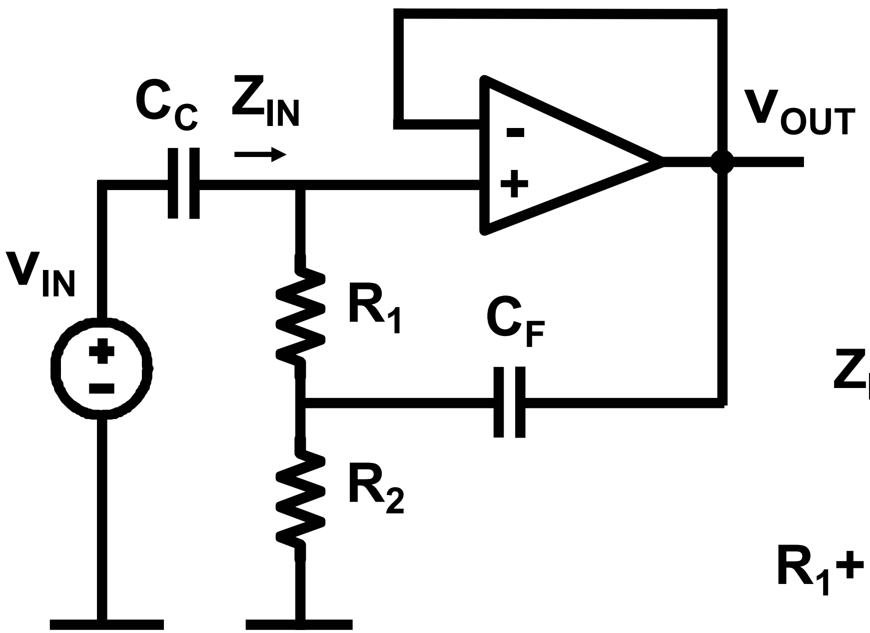
$$GBW = \frac{g_{m1}}{2\pi C_L}$$

Bootstrapping for low input capacitance



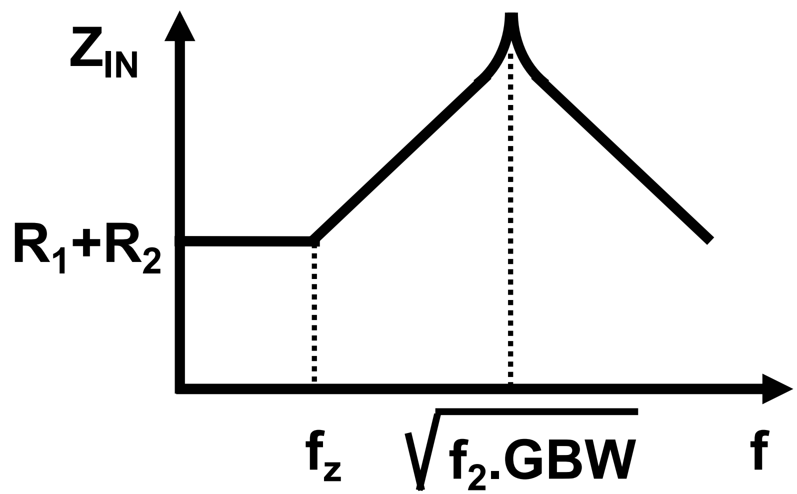
$C_{coax} \approx 0$!!!

Bootstrapping for high input impedance

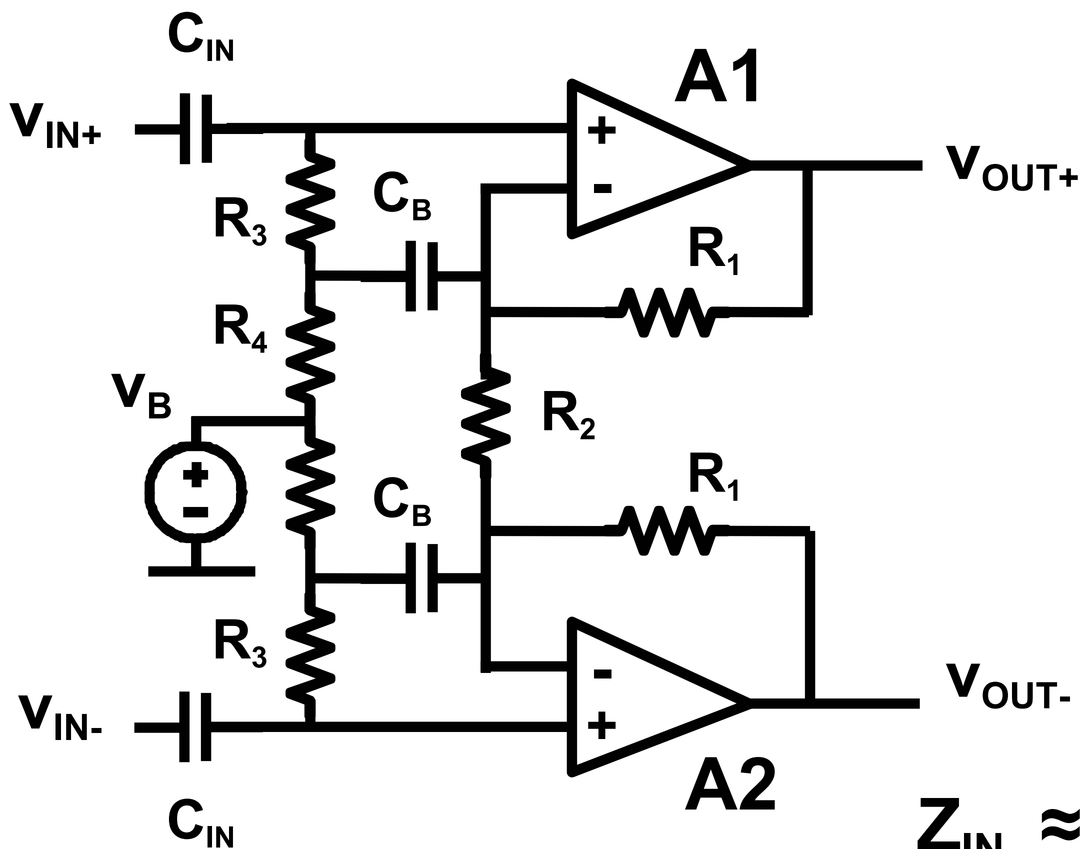


$$f_z = \frac{1}{2\pi\,(R_1+R_2)C_F}$$

$$f_2 = \frac{1}{2\pi\,R_2C_F}$$

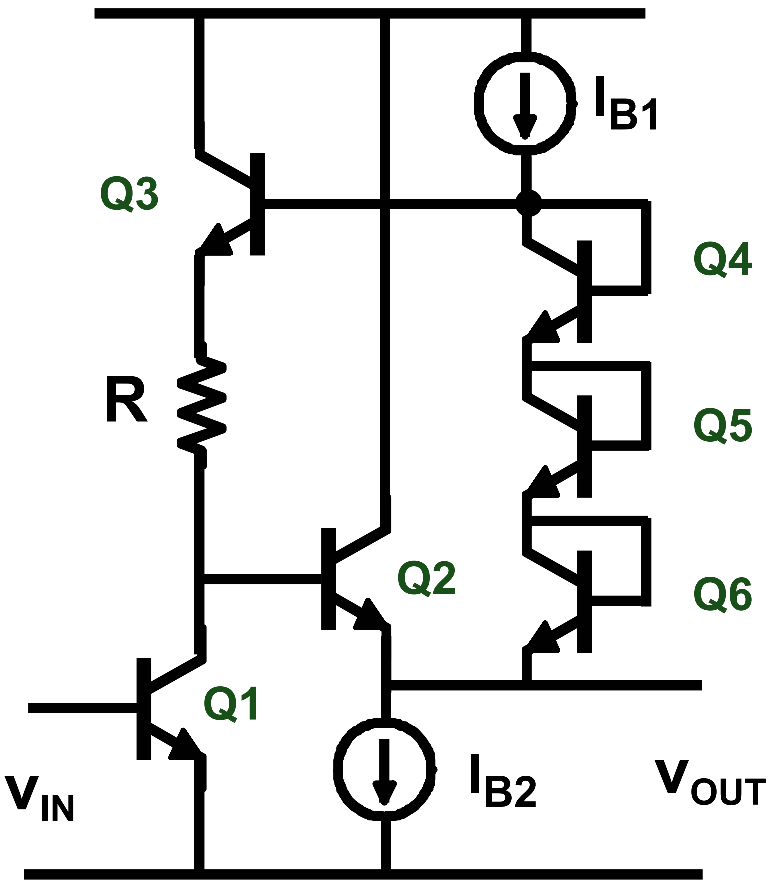


Bootstrapping for high input impedance



$Z_{IN} \approx \infty$!!!

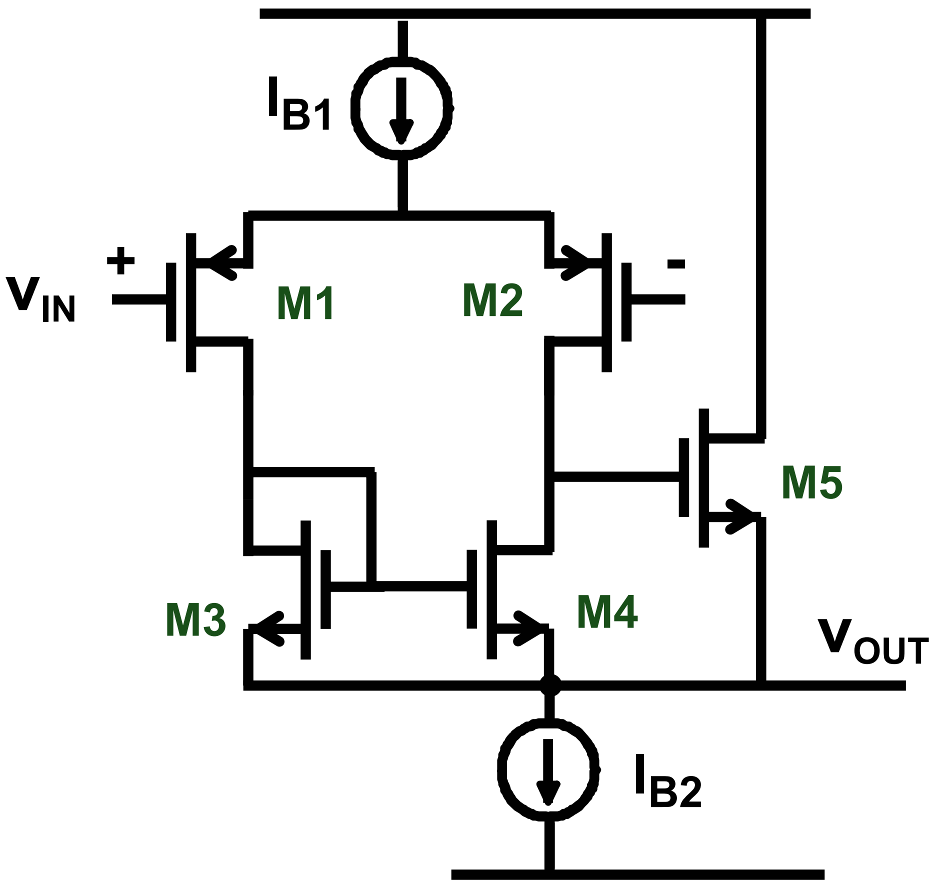
Bootstrapping out a load resistance R



R is bootstrapped out : Very high gain !

Ref.: Nordholt
JSSC June 85, 688-696

Bootstrapping out an output resistance

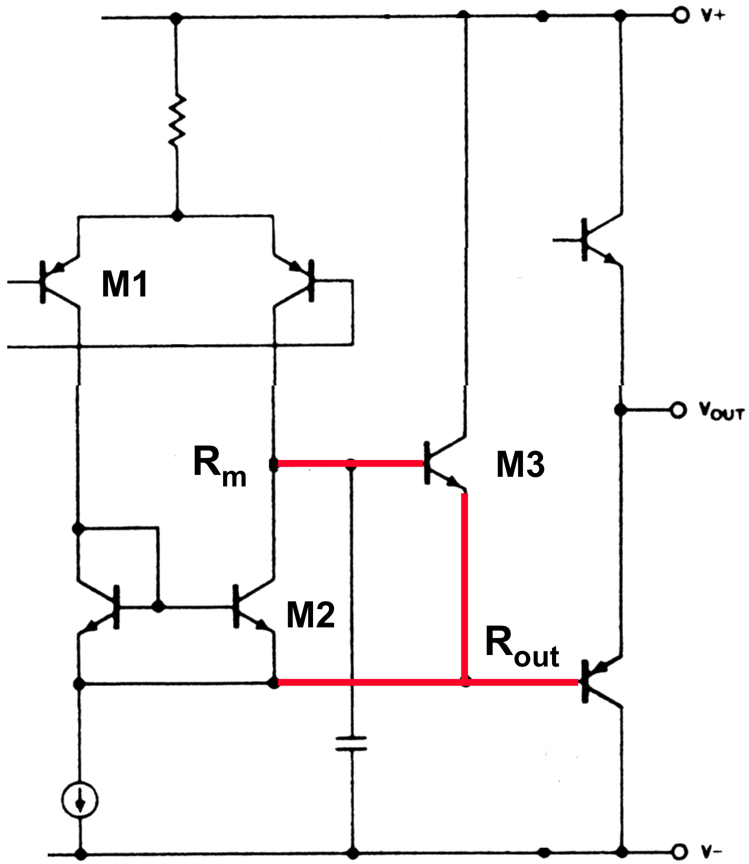


r_{o4} is bootstrapped out !

$A_v \approx g_{m1}\, r_{o2}$

Same GBW !

Bootstrap for high gain A_{v2}



R_m -> $x\,\beta_3$

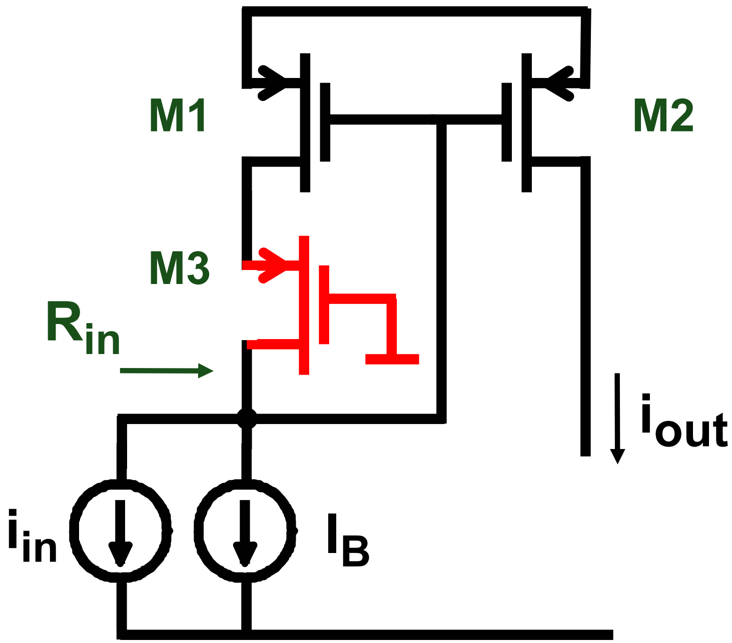
R_{out} -> $x\,\dfrac{1}{\beta_3}$

$A_{v2} \approx g_{m1}\, r_{o2}\, x\, \beta_3$

Same GBW !

Ref.De Man JSSC June 77, pp. 217-222
LT1008, LT1012

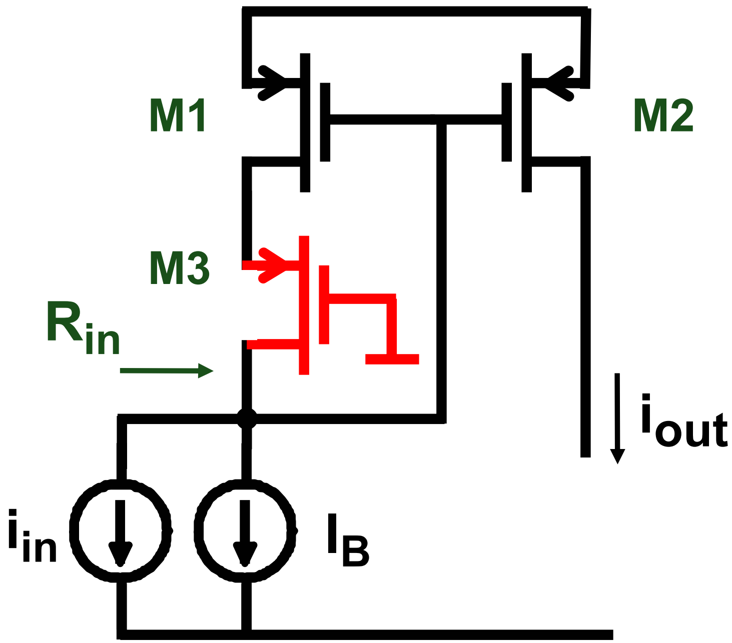
Current differential amplifier

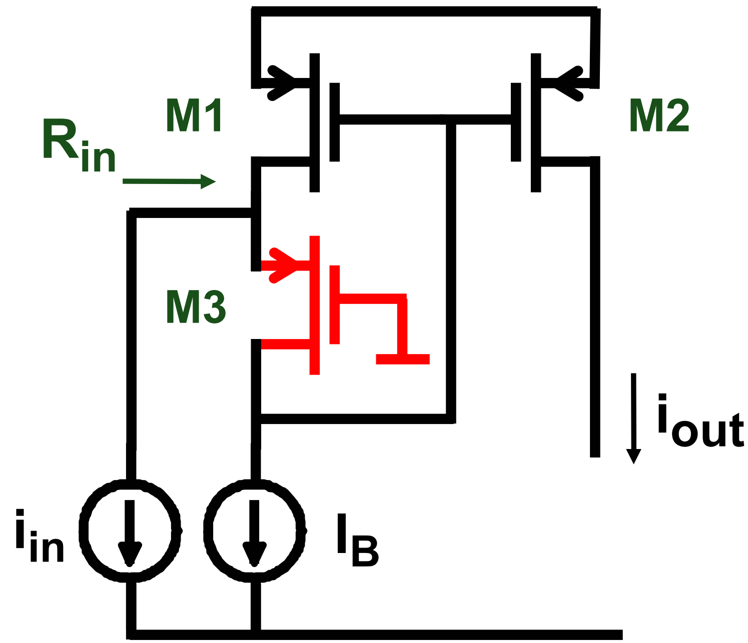


$i_{out} = I_B + i_{in}$ $\quad R_{in} = \frac{1}{g_{m1}}$

Is the same !

Current differential amplifier

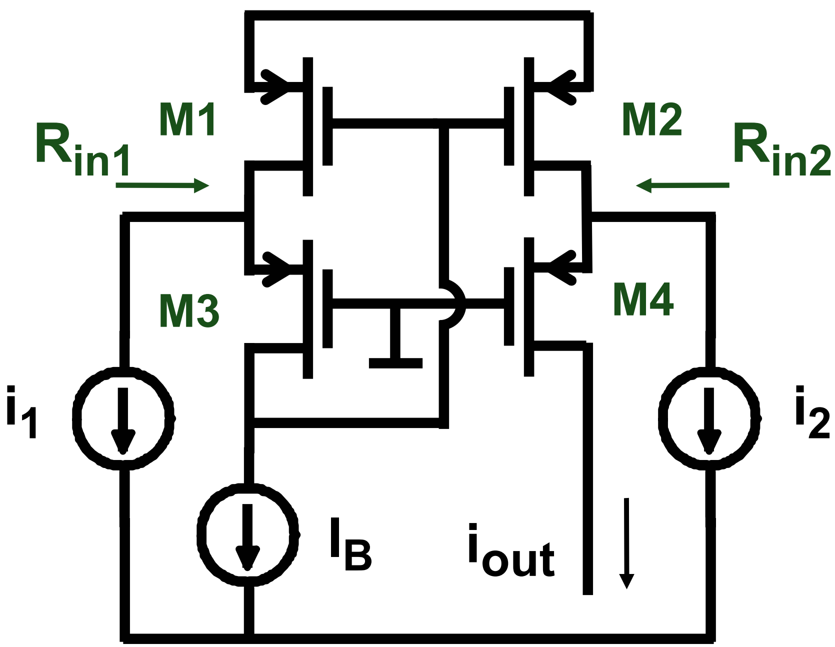




$i_{out} = I_B + i_{in}$ $R_{in} = \frac{1}{g_{m1}}$

$R_{in} = \frac{1}{g_{m1}} \frac{1}{g_{m3} r_{o3}}$

Current differential amplifier



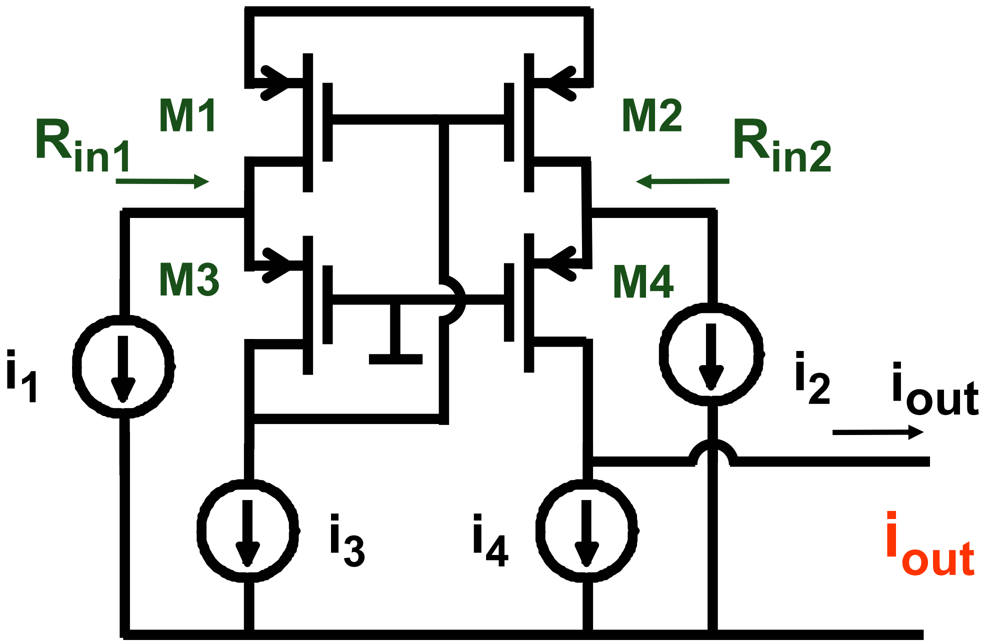
$$i_{out} = I_B + i_1 - i_2$$

$$R_{in1} = \frac{1}{g_{m1}} \frac{1}{g_{m3} r_{o3}}$$

$$R_{in2} = \frac{1}{g_{m4}}$$

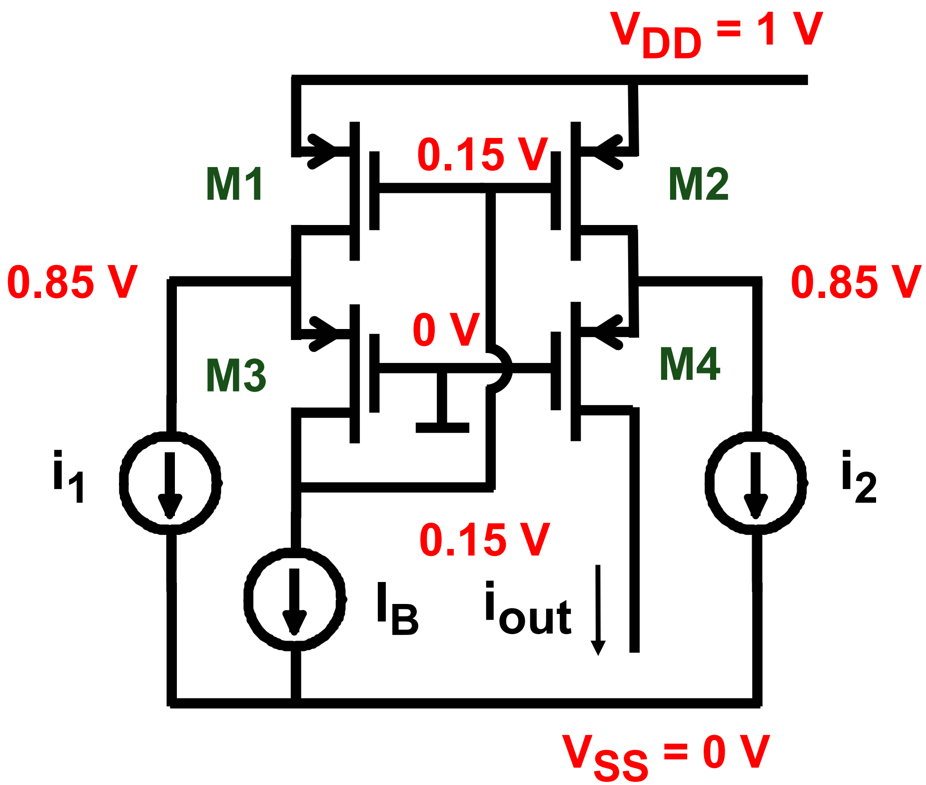
Ref. Fischer, JSSC June 87, 330-340

4-input current amplifier



$i_{out} = i_1 - i_2 + i_3 - i_4$

Low voltage operation



$i_{out} = I_B + i_1 - i_2$

$V_{GS} = 0.85$ V

$V_{DSsat} = 0.15$ V

$V_{outmax} = 0.7$ V

For $V_T = 0.7$ V

$V_{DDmin} \approx 0.6$ V

For $V_T = 0.3$ V

$V_{DDmin} \approx 0.6$ V

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0.4 chap4

Noise performance of elementary transistor stages

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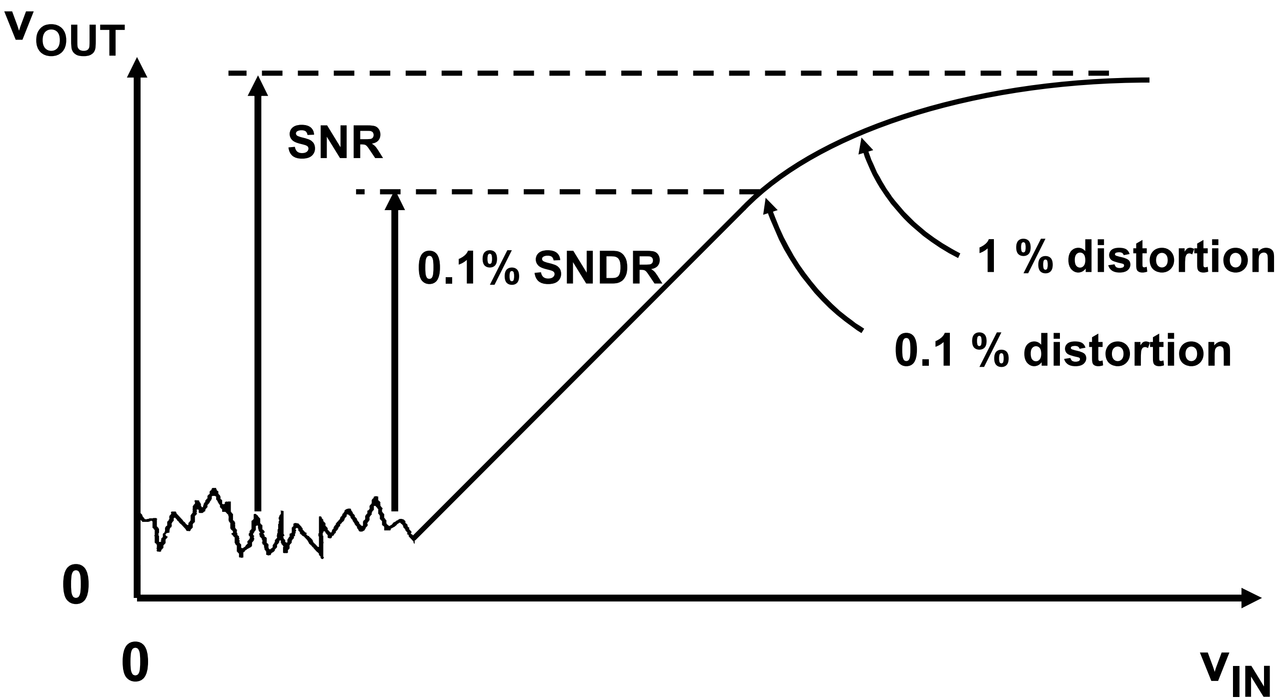
SNR and SNDR



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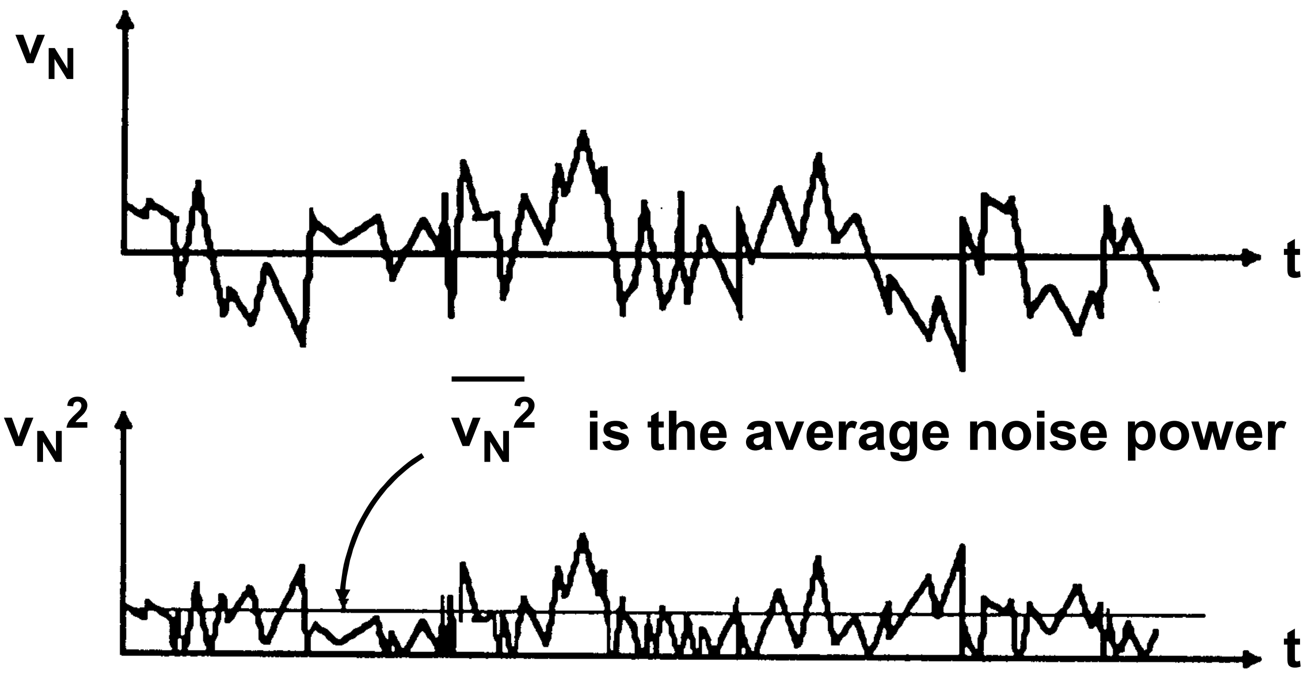
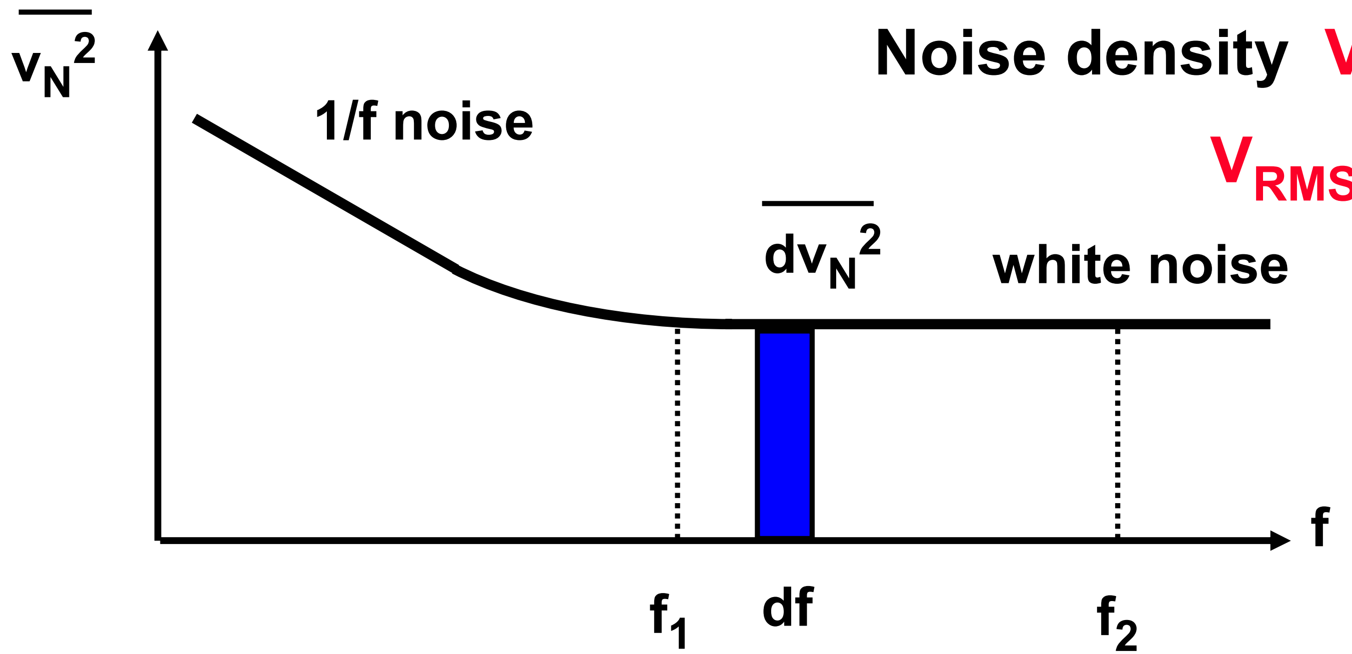
Noise versus time



$\overline{v_N^2}$ is the average noise power

Ref. Van der Ziel (Prentice Hall 1954, Wiley 1986), Ott (Wiley 1988)

Noise versus frequency



Noise density V^2/Hz

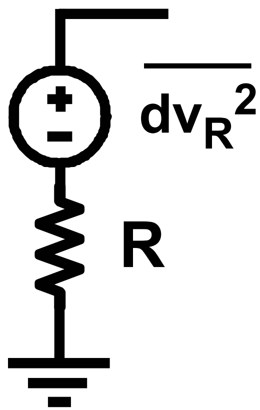
$V_{RMS}/\sqrt{Hz}$

Integrated noise

V_{RMS}

$$\overline{v_{12}} = \sqrt{\overline{\overline{v_N^2}}} = \sqrt{\int_{f_1}^{f_2} \overline{dv_N^2}\, df} = \sqrt{(f_2 - f_1)\, \overline{dv_N^2}}$$

Noise of a resistor is thermal noise

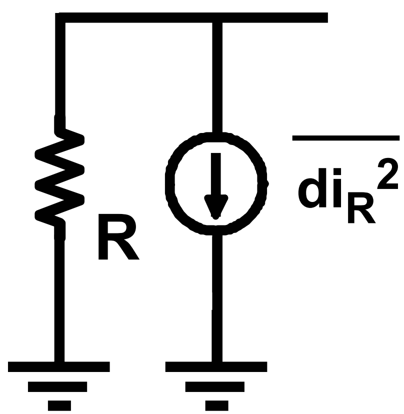


$$\overline{dv_R^2} = 4kT\,R\,df$$ is white

depends on T, not on I_R

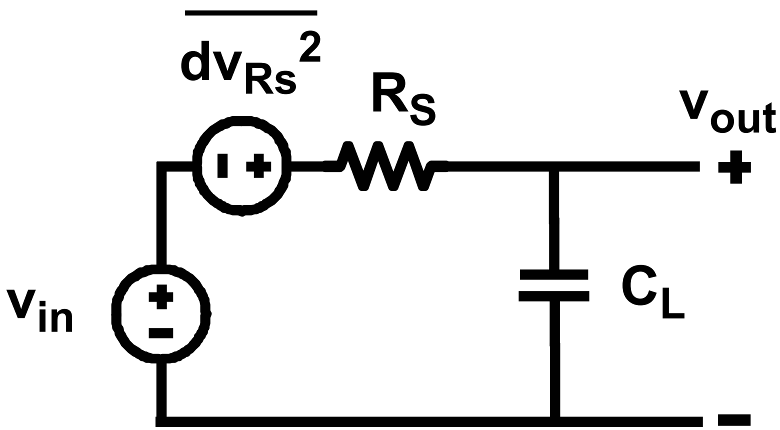
for R = 1 kΩ $\sqrt{\overline{dv_R^2}} = 4\ nV_{RMS} / \sqrt{Hz}$

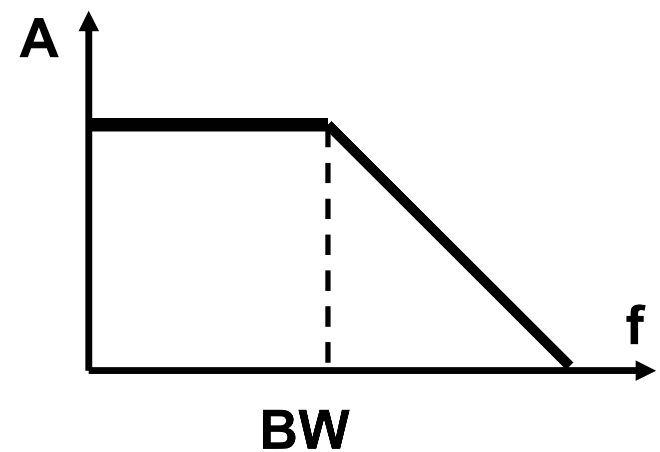
at T = 300 K or 27°C



$$\overline{di_R^2} = \frac{\overline{dv_R^2}}{R^2} = \frac{4kT}{R}\,df$$ is white

Integrated Noise of Resistor - 1



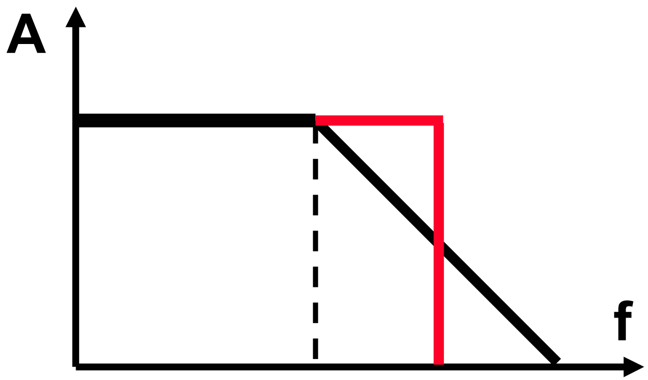


$$\overline{dv_{Rs}^2} = 4kT\,R_S\,df$$

$$BW = \frac{1}{2\pi\,R_S C_L}$$

$$\overline{v_{Rs}^2} = \int_0^\infty \frac{\overline{dv_{Rs}^2}}{1 + (f/\,BW)^2}$$

Integrated Noise of Resistor - 2



BW $\quad BW_n = \frac{\pi}{2} BW$

$$\overline{v_{Rs}^{2}} = \int_0^\infty \frac{\overline{dv_{Rs}^{2}}}{1 + (f/\ BW)^{2}}$$

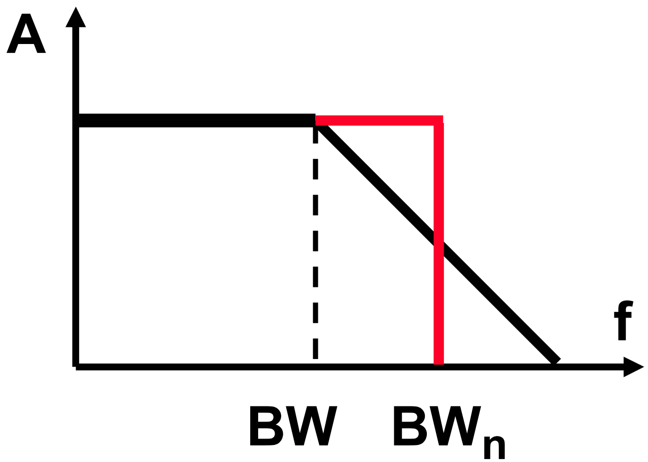
$$\boxed{\int_0^\infty \frac{dx}{1 + x^{2}} = \frac{\pi}{2}}$$

$$\overline{v_{Rs}^{2}} = 4kT\ R_S BW\ \frac{\pi}{2}\ df$$

$$\overline{v_{Rs}^{2}} = \frac{kT}{C_L}$$

C_L = 1pF $\quad v_{Rs}$= 65 μV_{RMS}

Noise density vs integrated noise



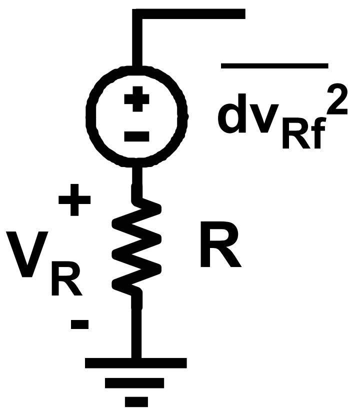
$$\overline{dv_{Rs}^{2}} = 4kT\,R_S\,df$$

$$\overline{v_{Rs}^{2}} = \int_0^\infty \frac{\overline{dv_{Rs}^{2}}}{1 + (f/\,BW)^2} = \frac{kT}{C_L}$$

Noise density (V^2/Hz) ~ R_S (or $1/g_m$)

Integrated noise (V_{RMS}) ~ $1/C_L$

A resistor also has 1/f noise



$$\overline{dv_{Rf}^2} = V_R^2 \frac{KF_R R_\square}{A_R} \frac{df}{f}$$ is 1/f

$KF_{RSi} \approx 2\ 10^{-21}\ Scm^2$

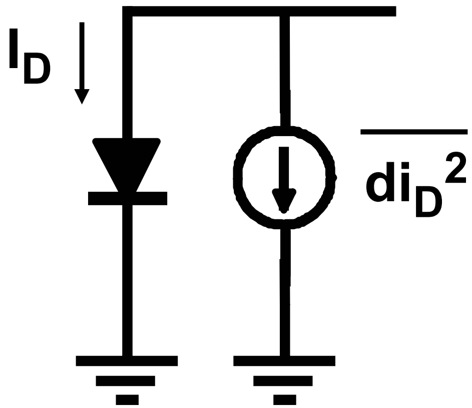
$KF_{Rpoly} \approx 10\ KF_{RSi}$

for R = 1 kΩ with 20 □'s of 50 Ω/ □ and 1 μm wide and V_R= 0.1 V

$\sqrt{\overline{dv_{Rf}^2}}$ = 16 $nV_{RMS}/\sqrt{Hz}$ at 1 Hz

Ref. Vandamme, ESSDERC '04

Noise of a diode is shot noise



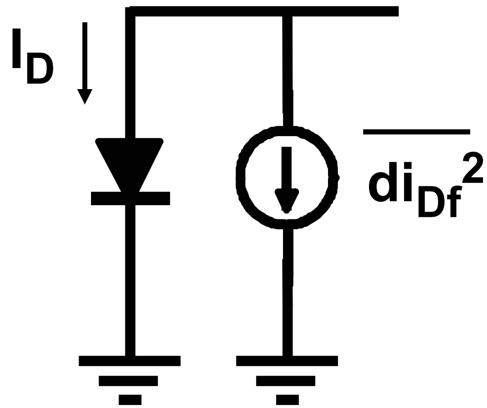
$\overline{di_D^2} = 2q\, I_D\, df$ is white

$q = 1.6\ 10^{-19}$ C

depends on I_D, not on T

for $I_D = 50\ \mu A$ $\sqrt{\overline{di_D^2}} = 4\ pA_{RMS} / \sqrt{Hz}$

A diode also has 1/f noise



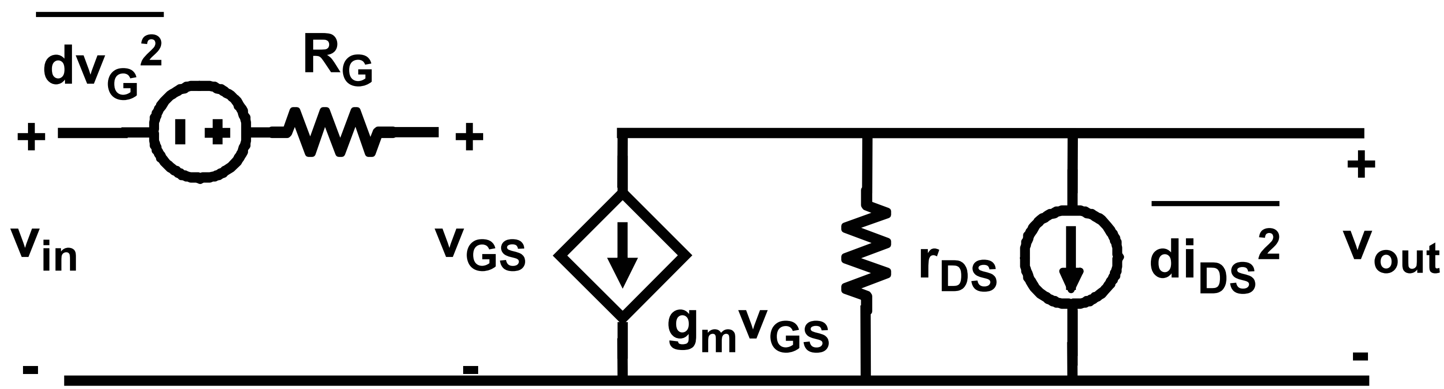
$$\overline{di_{Df}^2} = I_D \frac{KF_D}{A_D} \frac{df}{f}$$ is 1/f

$KF_D \approx 10^{-21}\ Acm^2$

For a diode of $A_D = 5 \times 2\ \mu m = 10\ \mu m^2$ and $I_D = 0.1$ mA

$\sqrt{\overline{di_{Df}^2}} = 1\ nA_{RMS} / \sqrt{Hz}$ at 1 Hz

Noise of a MOST

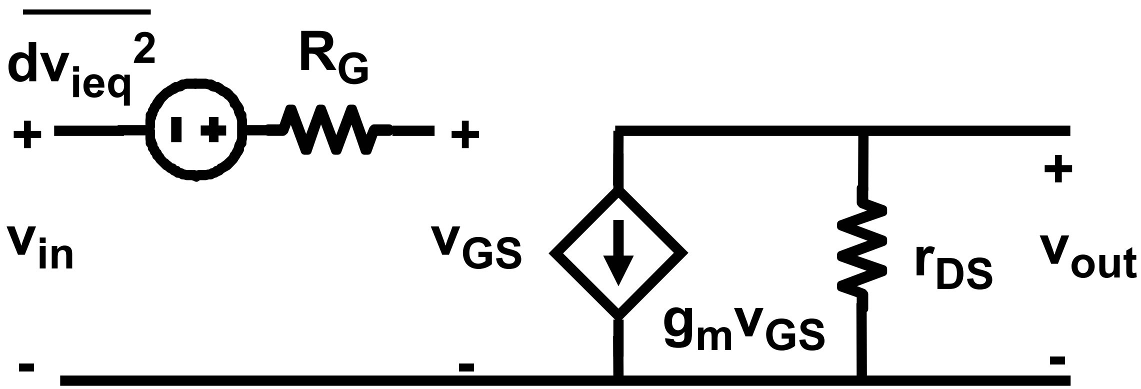


$$\overline{dv_G^2} = 4kT\, R_G\, df$$

$$\overline{di_{DS}^2} = \frac{4kT}{R_{CH}}\, df = 4kT \frac{2}{3} g_m\, df$$

Ref. Van der Ziel, Prentice Hall 1954, Wiley 1986.

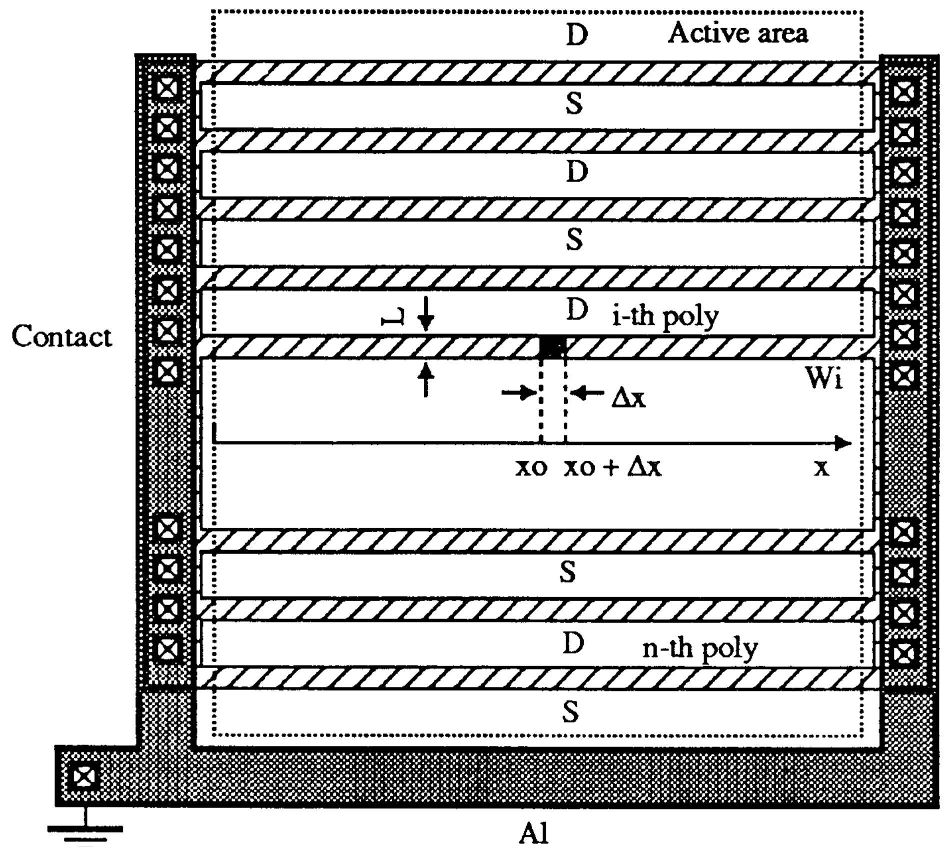
MOST: equivalent input noise : white



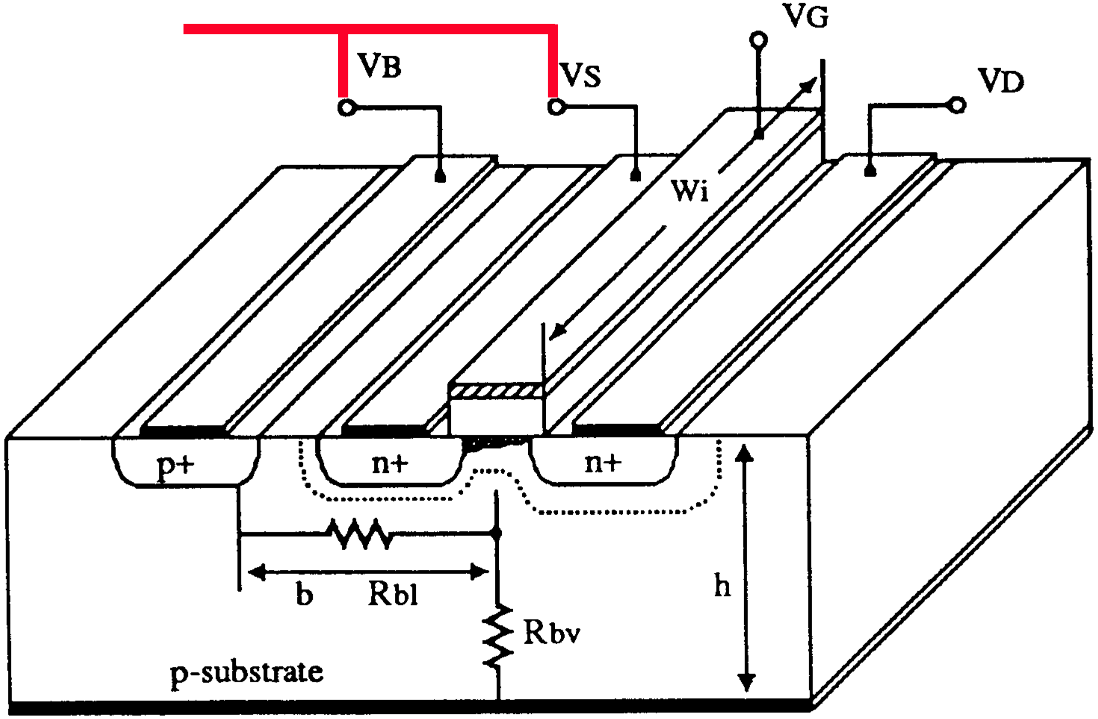
$$\overline{dv_{ieq}^2} = 4kT\,(R_{eff})\,df \qquad R_{eff} = \frac{2/3}{g_m} + R_G$$

Hi Freq.: $\overline{di_{ieq}^2} = (C_{GS}\,\omega)^2\,\overline{dv_{ieq}^2}$ is correlated

Poly Gate resistance r_G in a MOST

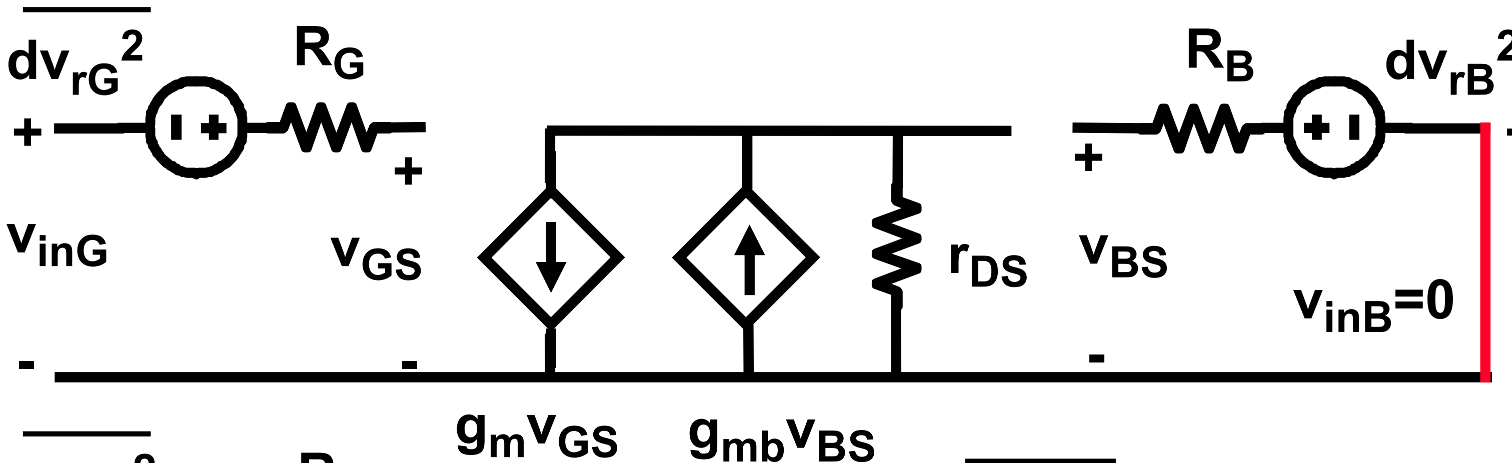


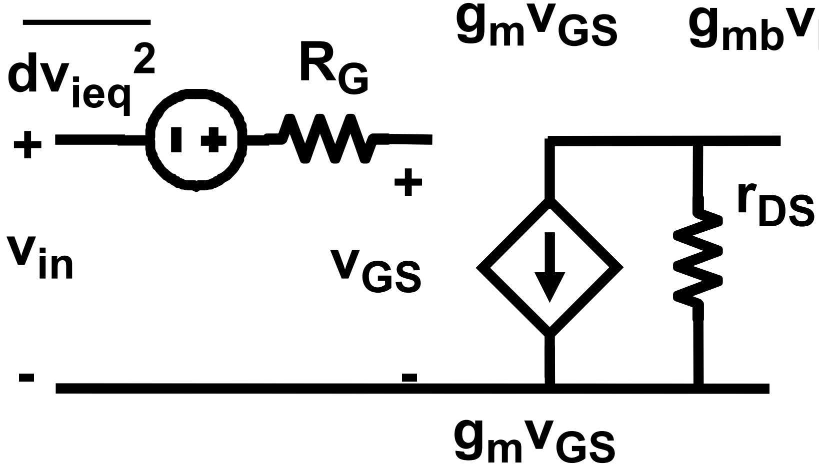
Substrate resistances r_B in a MOST



Ref. Chang, Kluwer 1991

Noise by the Bulk resistance



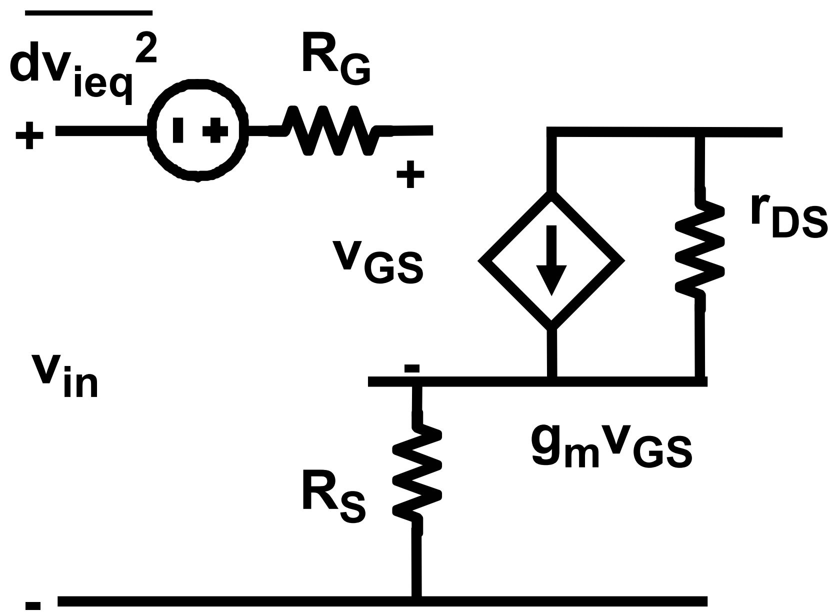


$$\overline{dv_{ieq}^{2}} = 4kT\,(R_{eff})\,df$$

$$R_{eff} = \frac{2/3}{g_m} + R_G + R_B\,(n-1)^2$$

$$(n-1) = C_D/C_{ox} = g_{mb}/g_m$$

Noise by the Source resistance

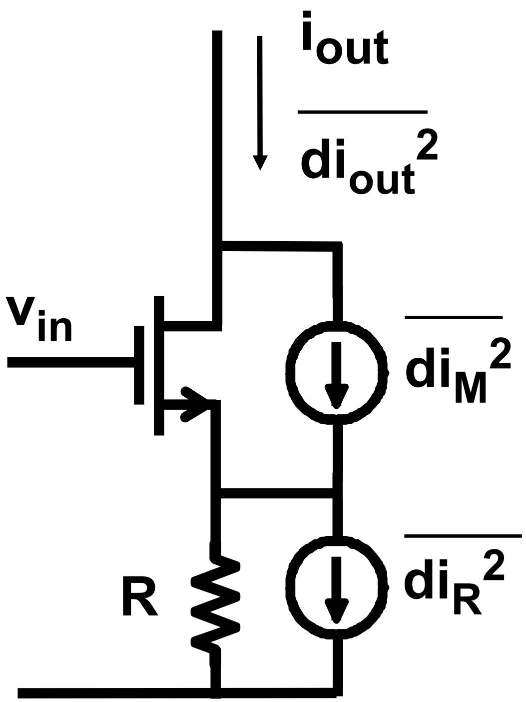


Noise of R_S
= noise R_G

$$\overline{dv_{ieq}^2} = 4kT\,(R_{eff})\,df$$

$$R_{eff} = \frac{2/3}{g_m} + R_G + R_S + R_B\,(n-1)^2$$

Noise by Source resistor R



$g_m R >> 1$

$$i_{out} = \frac{v_{in}}{R}$$

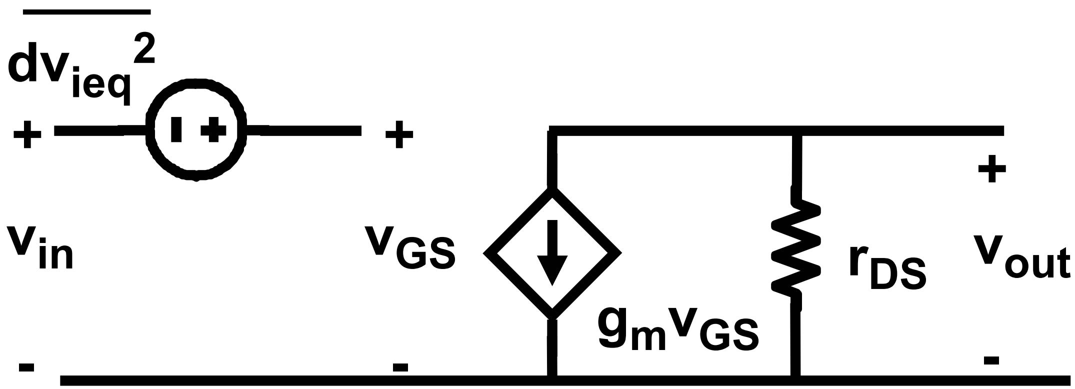
$$\overline{di_M^2} = 4kT\ 2/3\ g_m\ df \qquad \overline{di_{outM}^2} = \frac{\overline{di_M^2}}{(g_m R)^2}$$

$$\overline{di_R^2} = \frac{4kT}{R}\ df \qquad \overline{di_{outR}^2} = \overline{di_R^2}$$

$$\overline{di_{out}^2} = \frac{4kT}{R}\left(\frac{2/3}{g_m R} + 1\right) df \approx \frac{4kT}{R}\ df$$

$$\overline{dv_{in}^2} = 4kT\ R\ df$$

MOST: equivalent input noise : Exercise

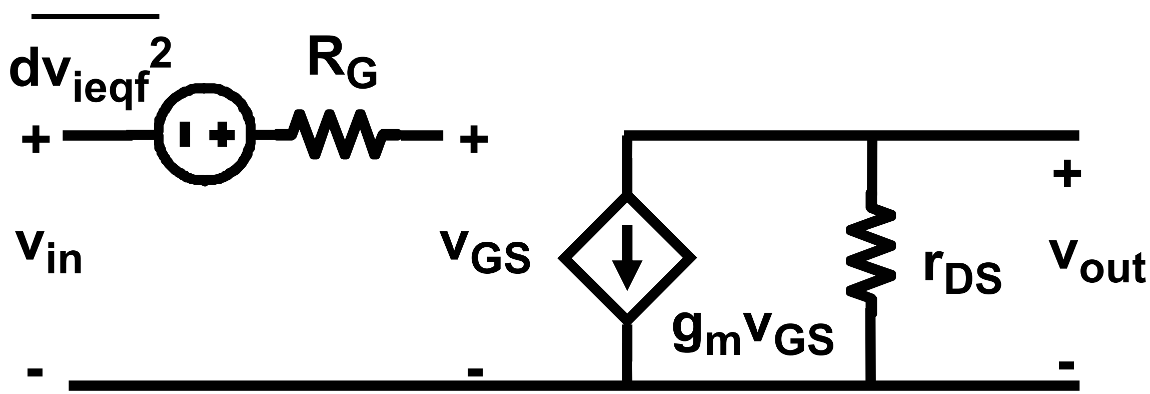


$$\overline{dv_{ieq}^2} \approx 4kT\left(\frac{2/3}{g_m}\right) df$$

$\overline{dv_{ieq}^2} \approx$?

for $I_{DS} = 65\ \mu A$

MOST: equivalent input noise : 1/f noise



$$\overline{dv_{ieqf}^2} = \frac{KF_F}{WL\, C_{ox}^2} \frac{df}{f}$$

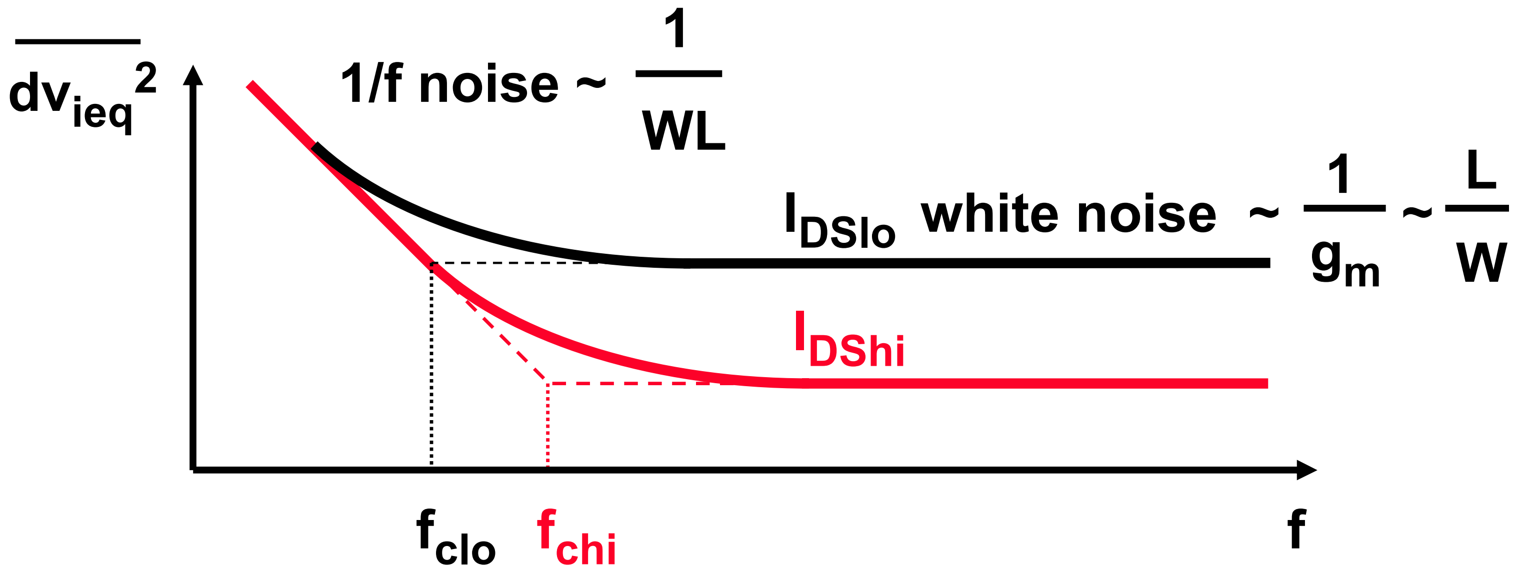
pMOST $KF_F \approx 10^{-32}$ C^2/cm^2

nMOST $KF_F \approx 4\ 10^{-31}$ C^2/cm^2

pJFET $KF_F \approx 10^{-33}$ C^2/cm^2

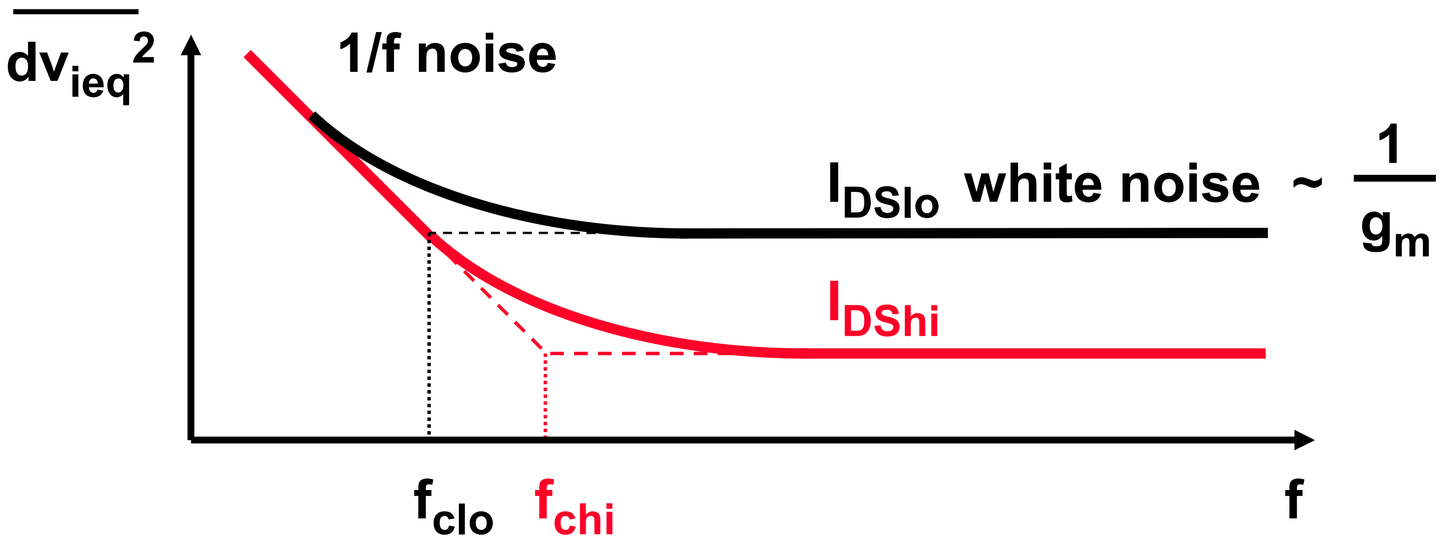
W & L in cm; C_{ox} in F/cm^2

Noise vs current : corner frequency



Corner frequency ~ g_m

Noise vs current : exercise f_c

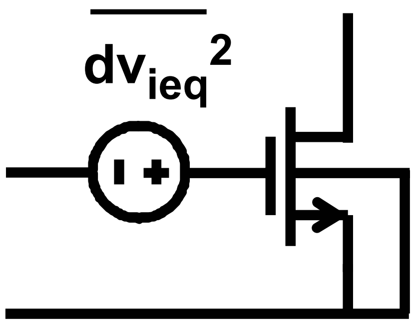


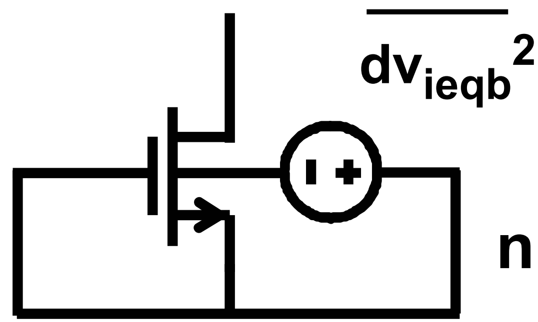
Ex. : f_c ? For $I_{DS} = 65\ \mu A$;

$K'_n = 60\ \mu A/V^2$ and $L = 1\ \mu m$ (0.35 μm process)

$f_c \approx 370$ kHz

Noise seen at the Bulk



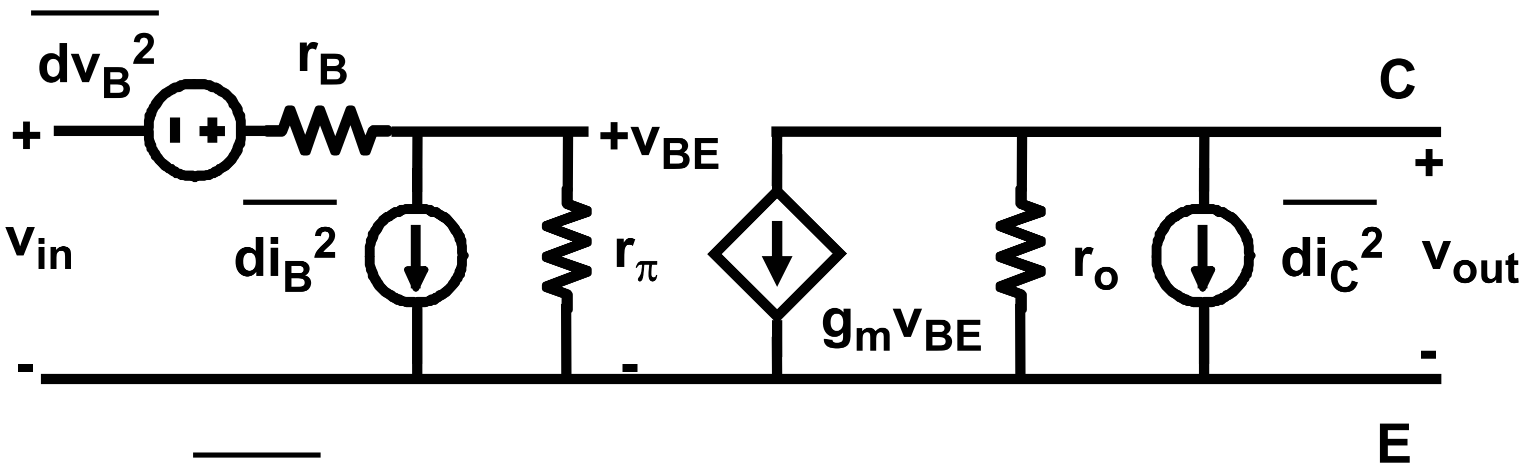


$$n-1 = \frac{g_{mb}}{g_m}$$

$$\overline{dv_{ieq}^2} = 4kT\left(\frac{2/3}{g_m}\right) df \qquad \overline{dv_{ieqb}^2} = 4kT\left(\frac{2/3\ g_m}{g_{mb}^2}\right) df$$

$$\overline{dv_{ieqf}^2} = \frac{KF_F}{WL\ C_{ox}^2}\frac{df}{f} \qquad \overline{dv_{ieqfb}^2} = \frac{KF_F}{WL\ C_{ox}^2}\frac{g_m^2}{g_{mb}^2}\frac{df}{f}$$

Noise of a Bipolar transistor



$$\overline{dv_B^2} = 4kT\, r_B\, df$$

$$\overline{di_B^2} = 2q\, I_B\, df$$

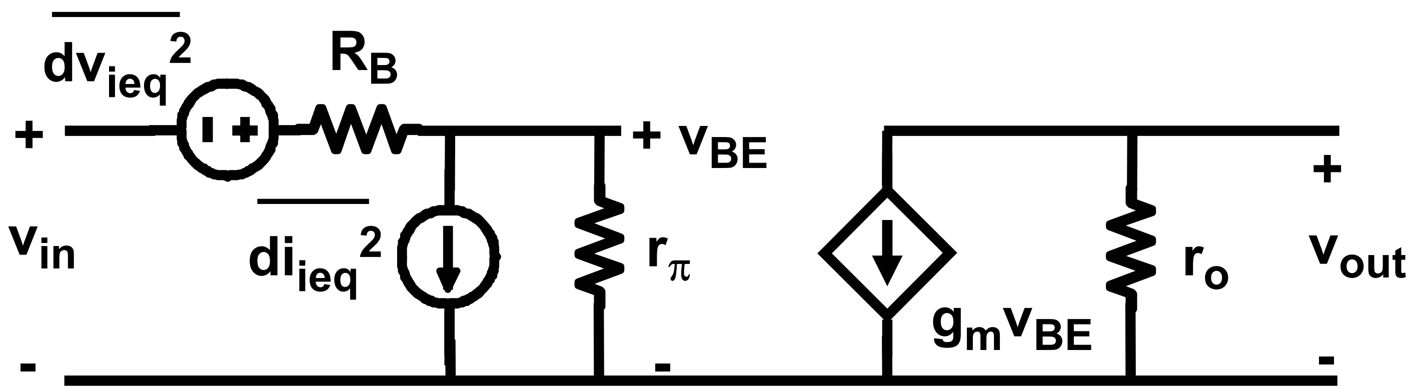
$$\overline{di_C^2} = 2q\, I_C\, df$$

$$\overline{di_{Bf}^2} = \frac{KF_B\, I_B}{A_{EB}} \frac{df}{f}$$

$$KF_B \approx 10^{-21}\ \mathrm{Acm}^2$$

Ref. Van der Ziel (Prentice Hall 1954)

Bipolar trans.: equivalent input noise



$$\overline{dv_{ieq}^2} = 4kT\,(R_{eff})\,df \qquad R_{eff} = \frac{1/2}{g_m} + R_B + R_E$$

$$\overline{di_{ieq}^2} = \overline{di_B^2} = 2q\,I_B\,df$$

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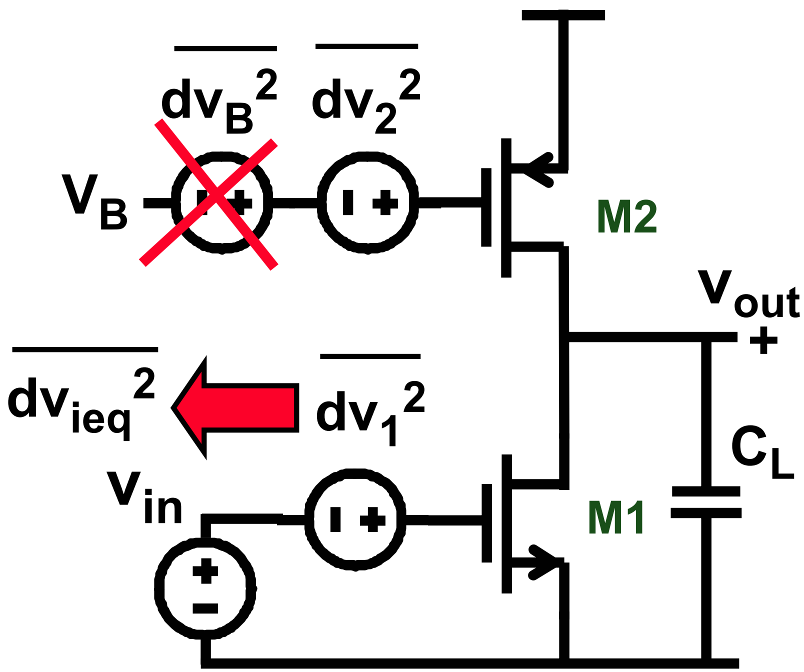
Noise of an amplifier with active load



If $\overline{dv_B^2}$ is negligible :

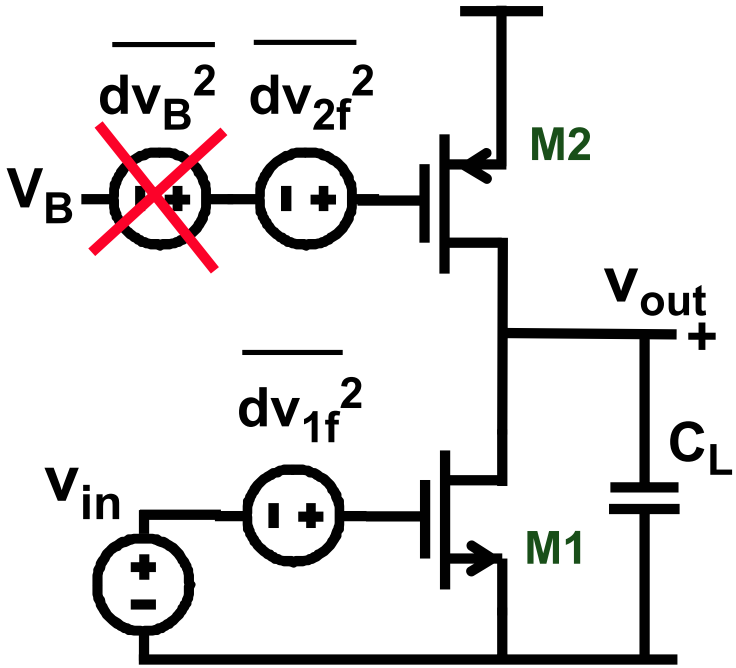
$$\overline{di_{out}^2} = g_{m1}^2\,\overline{dv_1^2} + g_{m2}^2\,\overline{dv_2^2}$$

$$\overline{dv_{ieq}^2} = \overline{dv_1^2} + \overline{dv_2^2}\left(\frac{g_{m2}}{g_{m1}}\right)^2$$

$$\overline{dv_{ieq}^2} = \overline{dv_1^2}\left(1 + \frac{g_{m2}}{g_{m1}}\right)$$

Small g_{m2} : small $(W/L)_2$ or large $(V_{GS} - V_T)_2$

1/f Noise of amplifier with active load



If $\overline{dv_B^2}$ is negligible :

$$\overline{dv_{if}^2} = \overline{dv_{1f}^2} + \overline{dv_{2f}^2}\left(\frac{g_{m2}}{g_{m1}}\right)^2$$

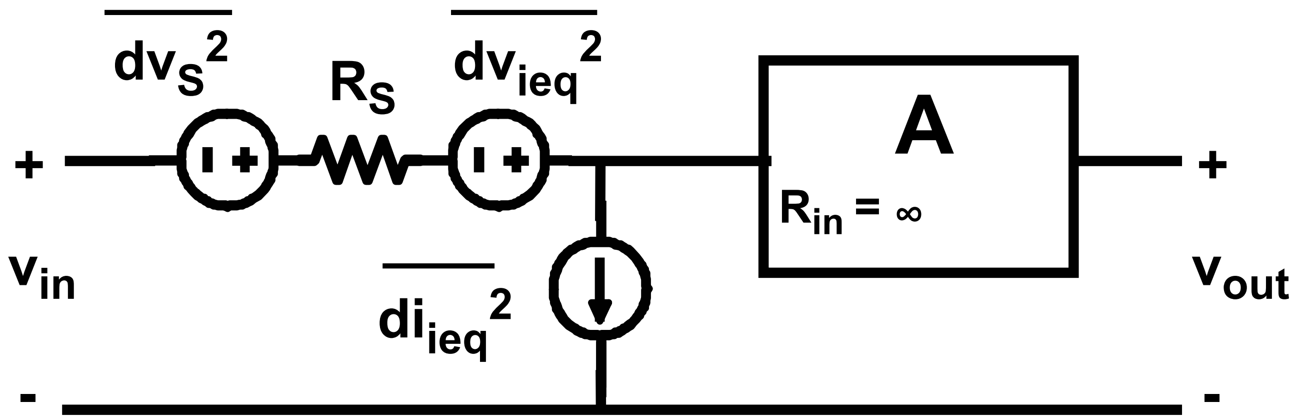
$$\overline{dv_{if}^2} = \overline{dv_{1f}^2}\left[1 + \left(\frac{g_{m2}}{g_{m1}}\right)^2 \left(\frac{\overline{dv_{2f}}}{\overline{dv_{1f}}}\right)^2\right]$$

$$\overline{dv_{if}^2} = \overline{dv_{1f}^2}\left[1 + \frac{KF_2}{KF_1}\,\frac{K'_2}{K'_1}\left(\frac{L_1}{L_2}\right)^2\right]$$

$\overline{dv_{if}^2}$ has minimum at

$$L_{1opt} = L_2\sqrt{\frac{KF_1}{KF_2}\,\frac{K'_1}{K'_2}} \approx 10\,L_2 \quad \text{then} \quad \overline{dv_{if}^2} = 2\,\overline{dv_{1f}^2}$$

Noise figure of an amplifier



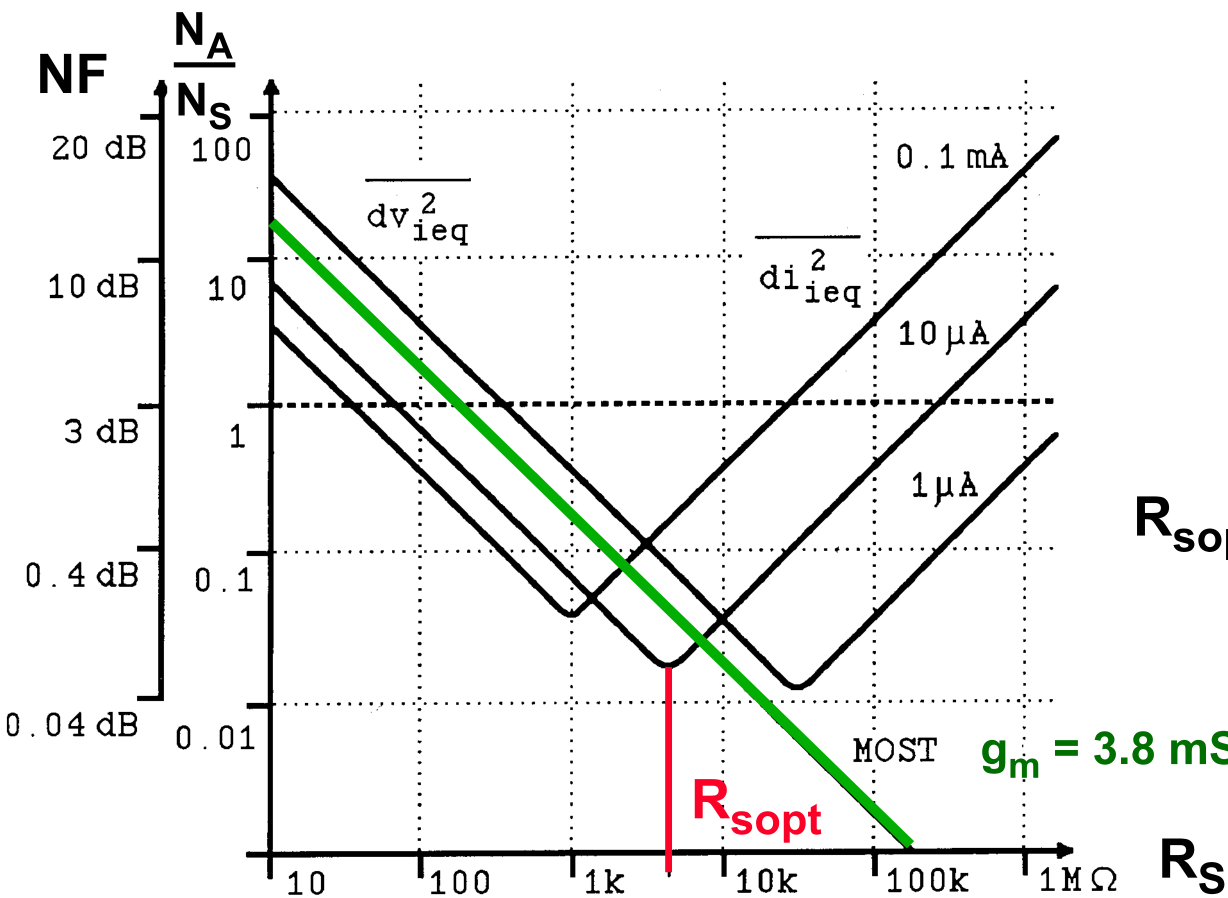
$$NF = \frac{N_S + N_A}{N_S} = 1 + \frac{N_A}{N_S}$$

$$NF = 1 + \frac{\overline{dv_{ieq}^2} + R_S^2\,\overline{di_{ieq}^2}}{4kT\,R_S\,df}$$

Voltage drive $NF \sim \frac{1}{R_S}$

Current drive $NF \sim R_S$

Resistive noise matching



β = 100

r_B = 100 Ω

$$R_{sopt} = \sqrt{\frac{\overline{dv_{ieq}^2}}{\overline{di_{ieq}^2}}}$$

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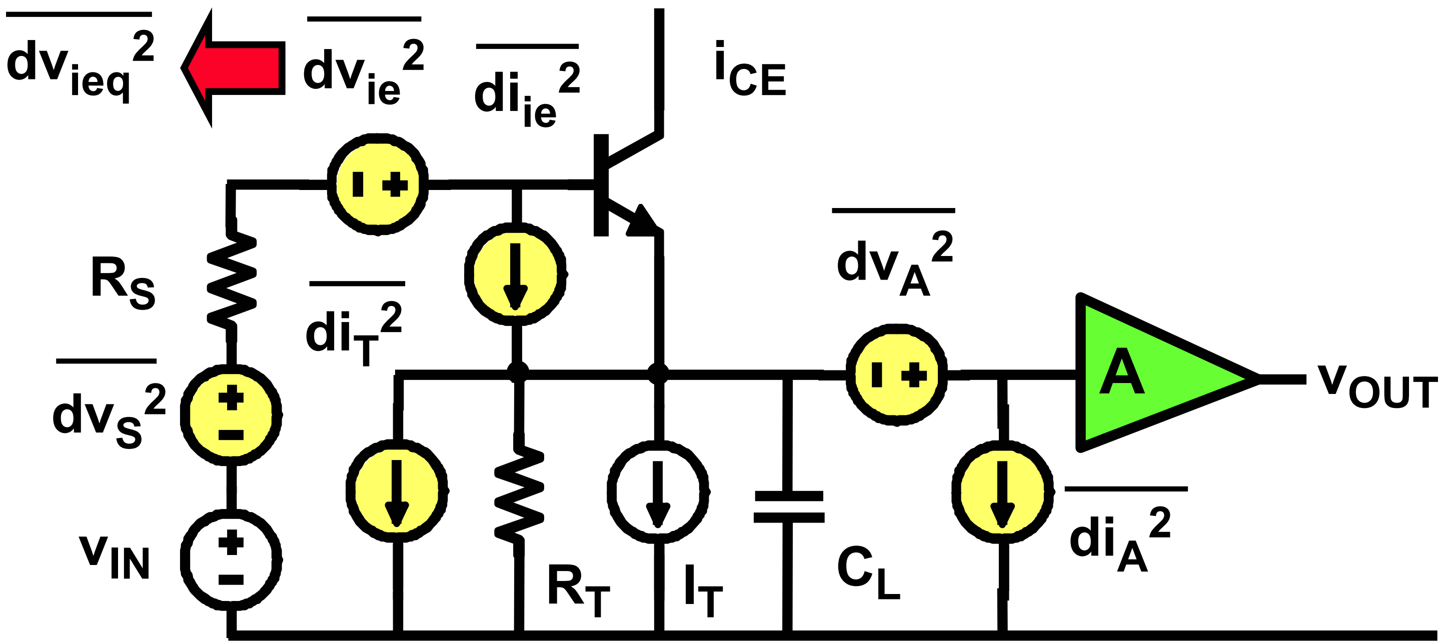
Noise of an emitter follower



$$\overline{dv_{ieq}^2} = \overline{dv_{ie}^2} + \overline{dv_A^2} + \left(R_S - \frac{1}{g_m}\right)^2 \overline{di_{ie}^2} + \frac{\overline{di_T^2} + \overline{di_A^2}}{g_m^2}$$

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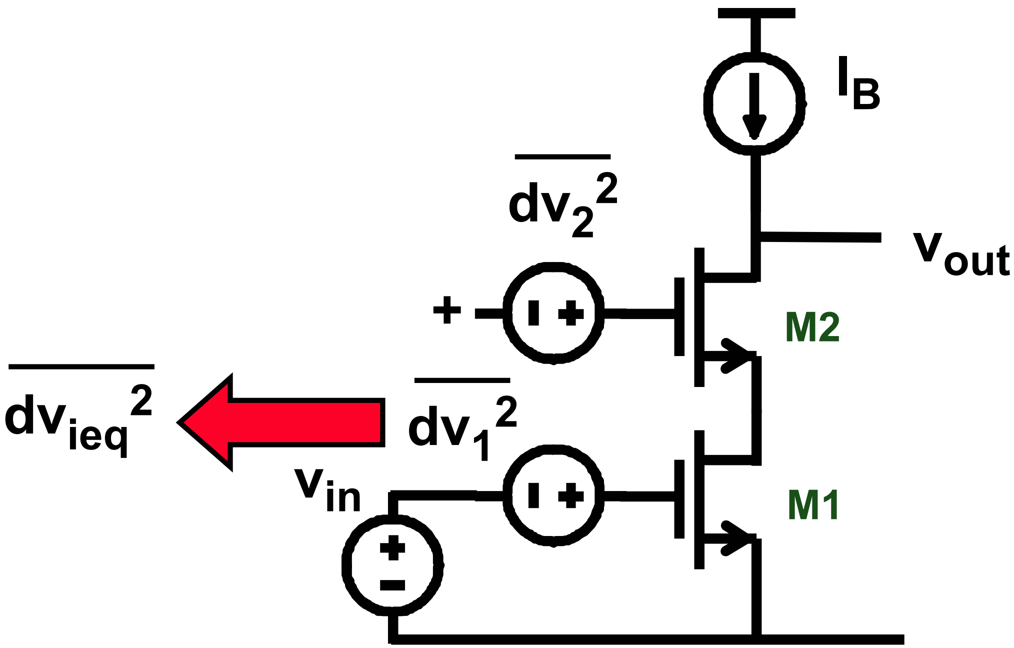
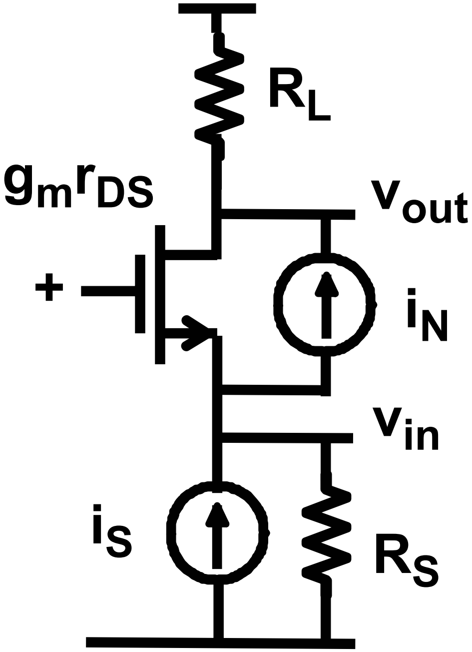
Noise of a cascode amplifier

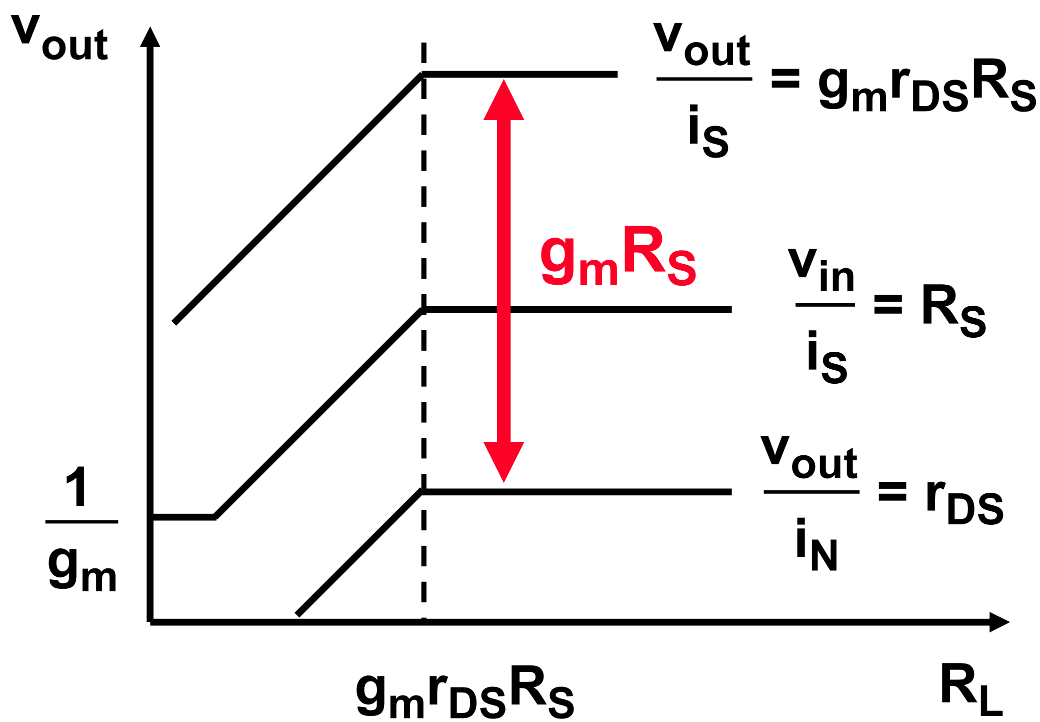


$$\overline{dv_{ieq}^2} = \overline{dv_1^2} + \overline{dv_2^2} \frac{1}{(g_{m1}\, r_{o1})^2} \approx \overline{dv_1^2}$$

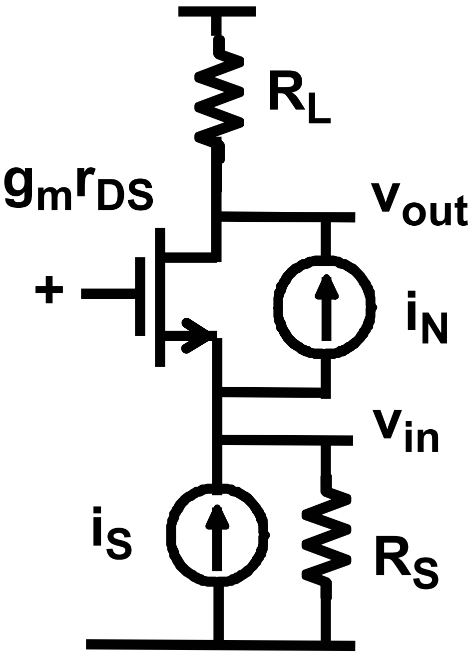
Input referred noise of a cascode

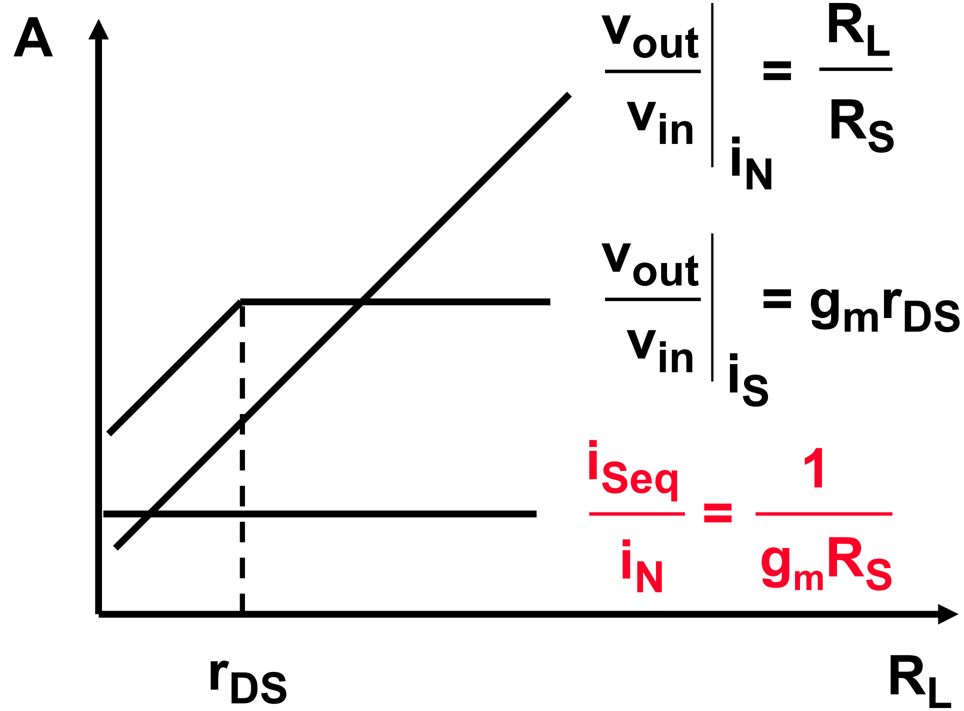


$g_m r_{DS} >> 1$



Noise gains in a cascode





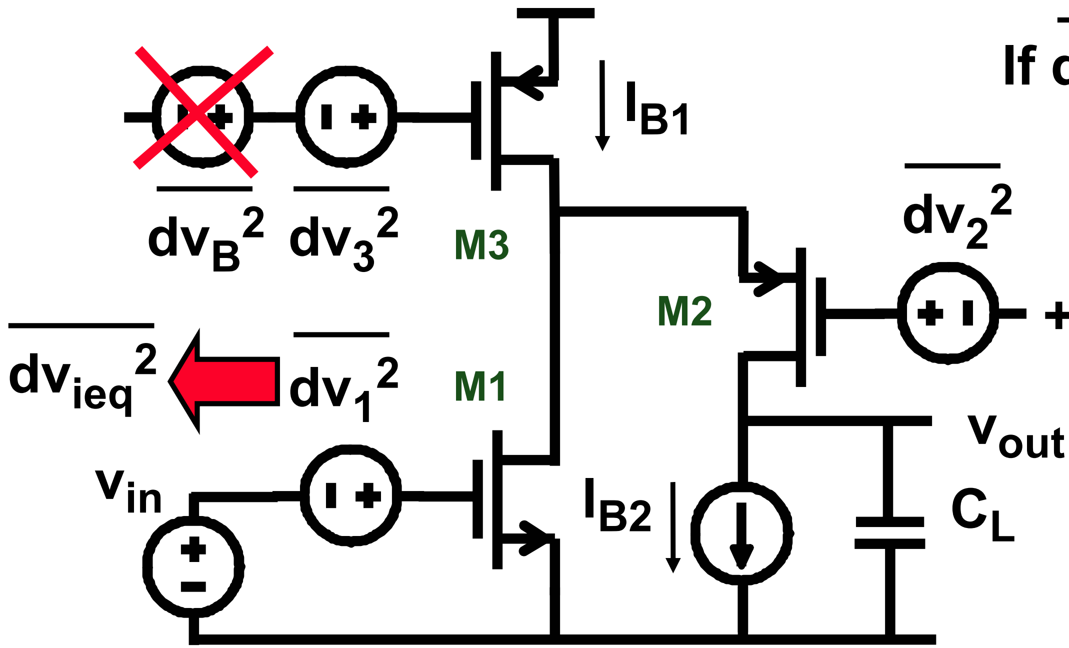
$$\left.\frac{v_{out}}{v_{in}}\right|_{i_N} = \frac{R_L}{R_S}$$

$$\left.\frac{v_{out}}{v_{in}}\right|_{i_S} = g_m r_{DS}$$

$$\frac{i_{Seq}}{i_N} = \frac{1}{g_m R_S}$$

Cascode noise i_N is only negligible if R_S is large !!!

Noise of a folded cascode



If $\overline{dv_B^2}$ is negligible :

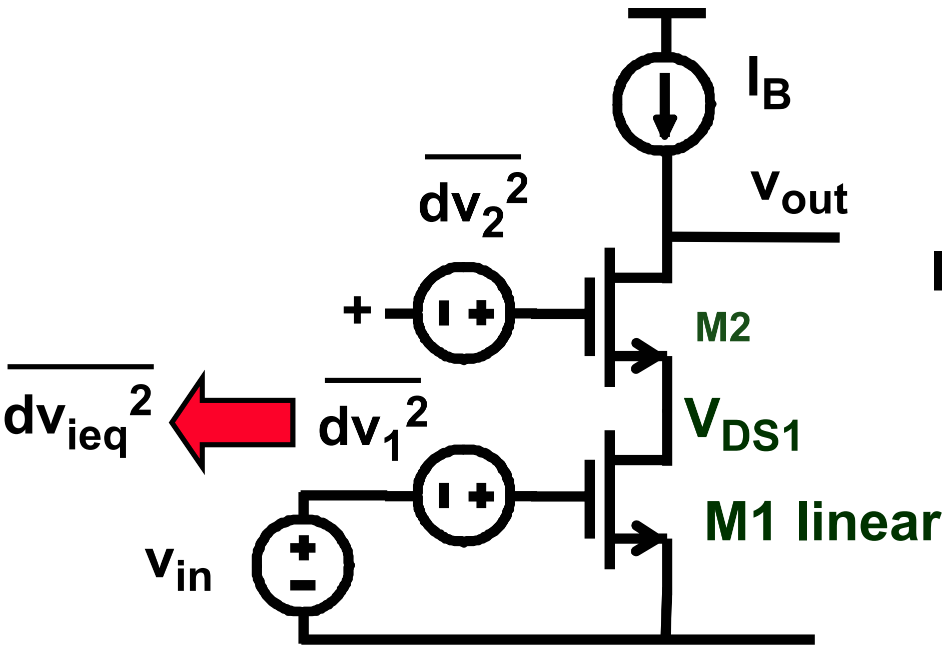
$$\overline{dv_{ieq}^2} = \overline{dv_1^2} + \overline{dv_2^2}\,\frac{1}{(g_{m1}\,r_{o1})^2} + \overline{dv_3^2}\,\frac{(g_{m3})^2}{(g_{m1})^2}$$

Small g_{m3} :

$(W/L)_3 \downarrow$

$(V_{GS}-V_T)_3 \uparrow$

Noise of a cascode with linear M1



$$\alpha_1 = \frac{V_{DS1}}{V_{GS1} - V_T} \qquad \alpha_1 < 0.5$$

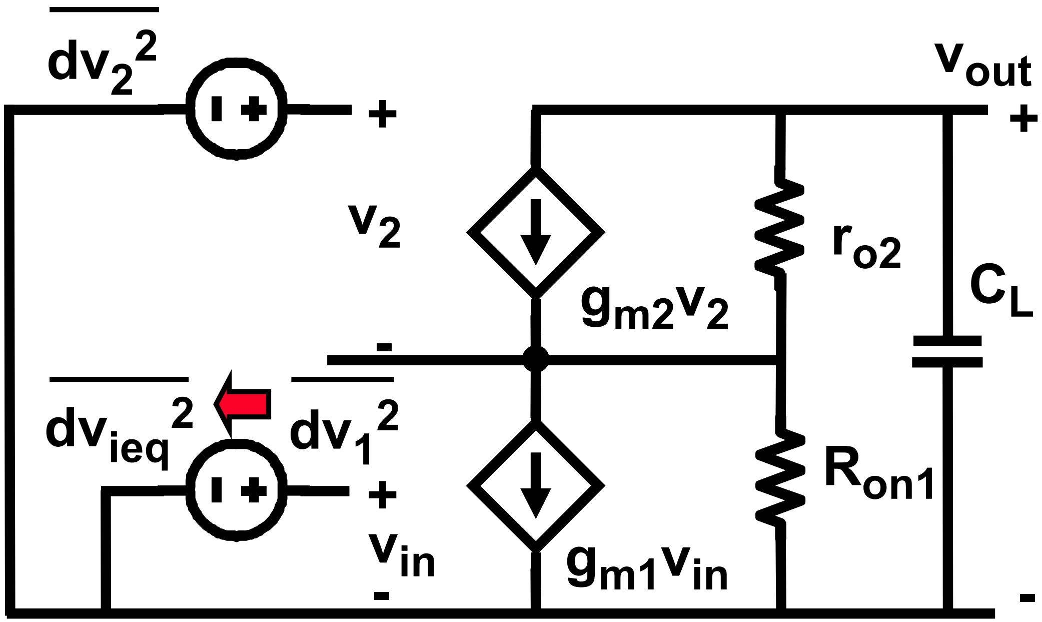
$$I_{DS1} = \beta_1 V_{DS1}(V_{GS1} - V_T)$$

$$R_{on1} = \frac{1}{\beta_1 (V_{GS1} - V_T)}$$

$$A_v = \alpha_1 g_{m2} r_{o2}$$

$$\overline{dv_{ieq}^2} = \frac{4kT}{\alpha_1^2} (R_{on1} + \frac{2/3}{g_{m2}}) \, df$$

Small-signal model of a cascode with linear M1



$$I_{DS1} = \beta_1 V_{DS1}(V_{GS1} - V_T)$$

$$R_{on1} = \frac{1}{\beta_1 (V_{GS1} - V_T)}$$

$$\overline{dv_1^2} = \frac{4kT\,R_{on1}}{\alpha_1^2}$$

$$\overline{dv_2^2} = \frac{4kT\,2/3}{g_{m2}}$$

$$v_{out} / dv_1 = \alpha_1\, g_{m2}\, r_{o2}$$

$$v_{out} / dv_2 = g_{m2}\, r_{o2}$$

$$\overline{dv_{ieq}^2} = \frac{4kT}{\alpha_1^2}\left(R_{on1} + \frac{2/3}{g_{m2}}\right) df$$

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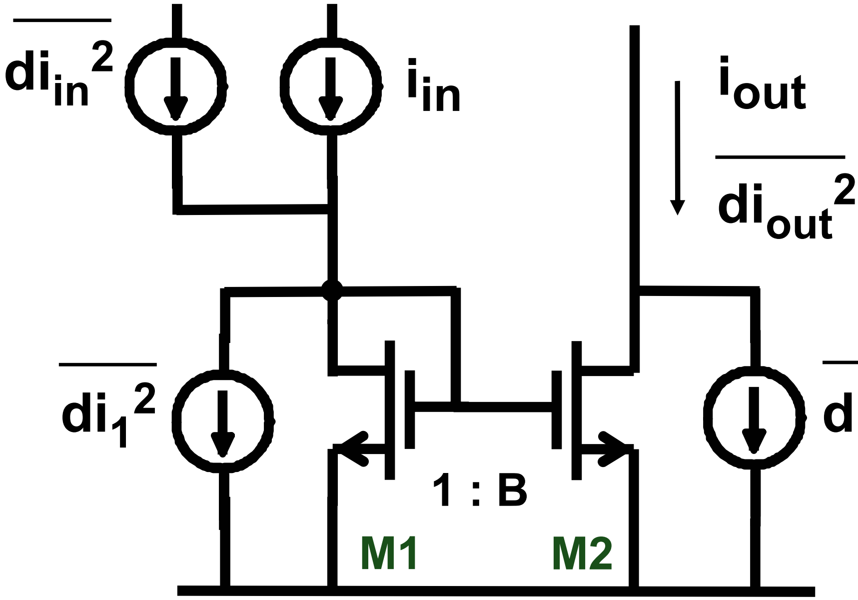
Noise of a current mirror



$$\overline{di_2^2} = \frac{2}{3} 4kT\, g_{m2}\, df$$

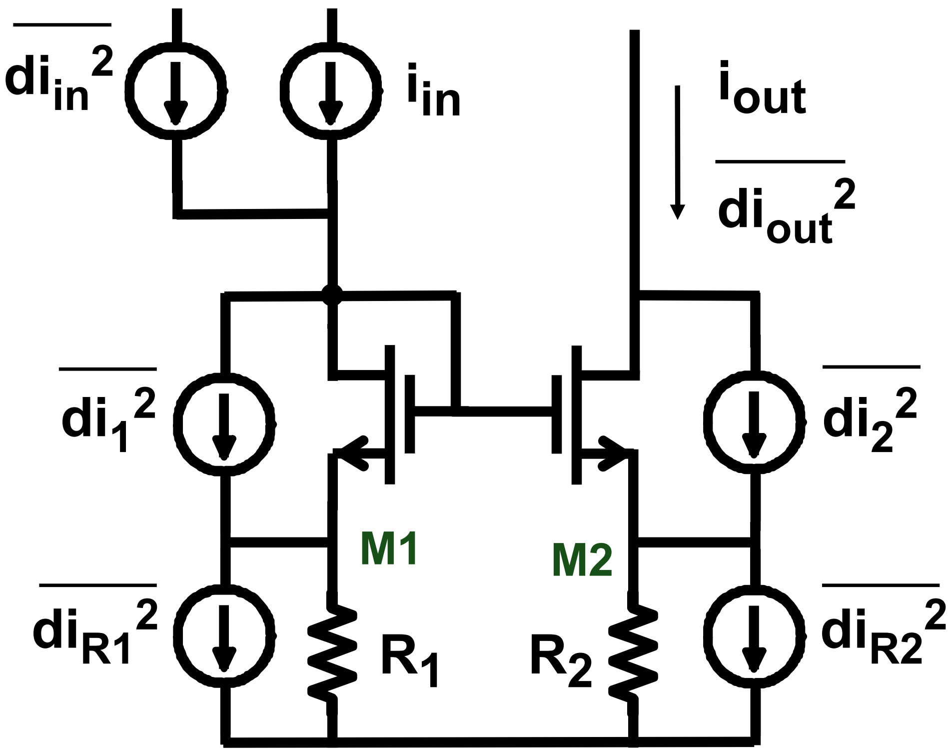
$$\overline{di_{out}^2} = \overline{di_2^2} + B^2 \left(\overline{di_{in}^2} + \overline{di_1^2}\right)$$

Small g_m :

$(W/L) \downarrow$

$(V_{GS}-V_T) \uparrow$

Noise of a current mirror with series R



$$\overline{di_{out}^2} = \overline{di_2^2} + \overline{di_{R2}^2} + \frac{(R_1)^2}{(R_2)^2}\left(\overline{di_1^2} + \overline{di_{R1}^2}\right)$$

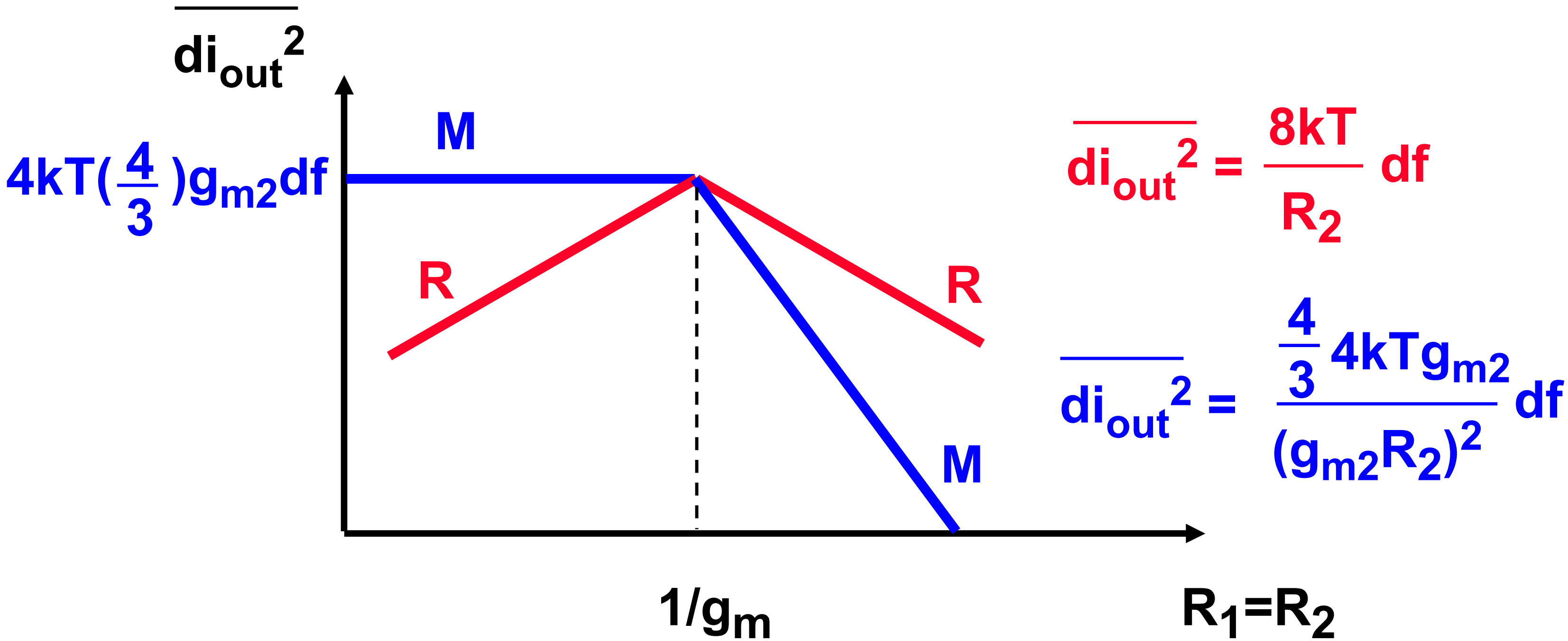
Small g_m :

(W/L) ↓

$(V_{GS}-V_T)$ ↑

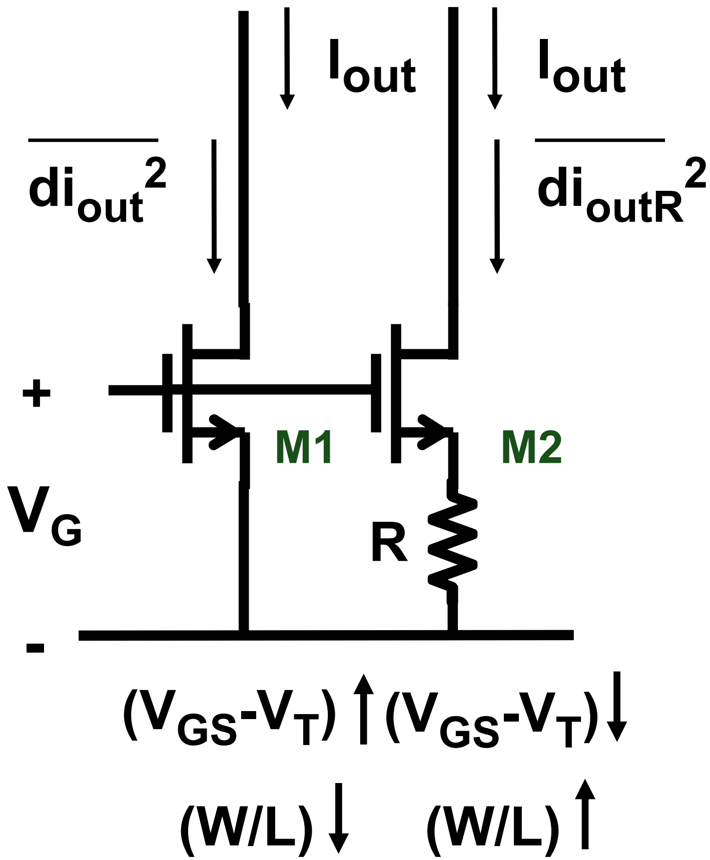
R ↑

Noise of a current mirror with series R



Bilotti, JSSC Dec 75, 516-524

Current mirror with series R



Same I_{out} & same V_G :

$$\boxed{\overline{di_{outR}^2} = \overline{di_{out}^2}}$$

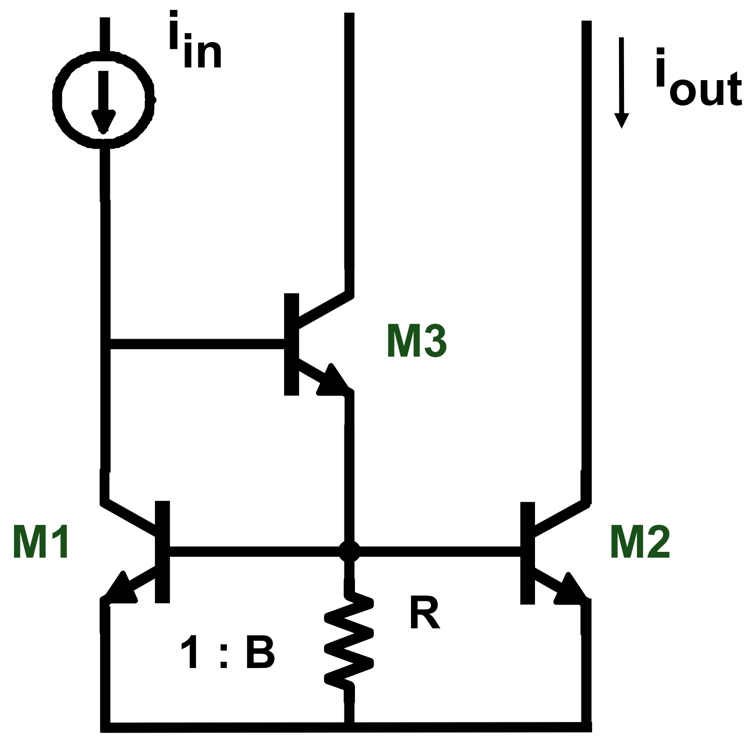
Small g_m :

$(W/L)\downarrow$

$(V_{GS}-V_T)\uparrow$

$V_G\uparrow$

Noise in bipolar current mirror



Noise added by M3 :

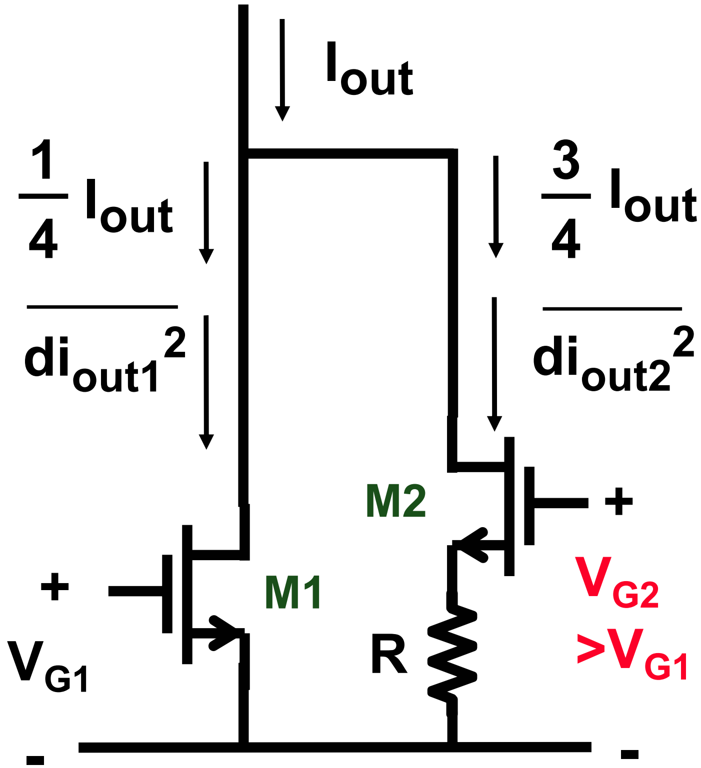
$$\overline{di_{outM3}^{2}} = 2qI_{C3}\, df$$

Noise added by R :

$$\overline{di_{outR}^{2}} = 4kT/R\ \ df$$

Both are divided by $\beta_3^{\,2}$ to be added to the output and are thus negligible !

Low-noise current mirror with series R



Large R or V_{G2}

Same I_{out} & different V_G :

1 MOST: $\overline{di_{out}^2} = \frac{8kT}{3} \frac{2I_{out}\,df}{V_{GS}-V_T}$

2 MOSTs: $V_{G2} > V_{G1}$

$\overline{di_{out}^2} = \overline{di_{out1}^2} + \overline{di_{out2}^2} =$

$$\frac{8kT}{3} \frac{2I_{out}\,df}{V_{G1}-V_T} \left(\frac{1}{4} + \frac{9}{16} \frac{V_{G1}-V_T}{V_{G2}-V_{G1}} \right)$$

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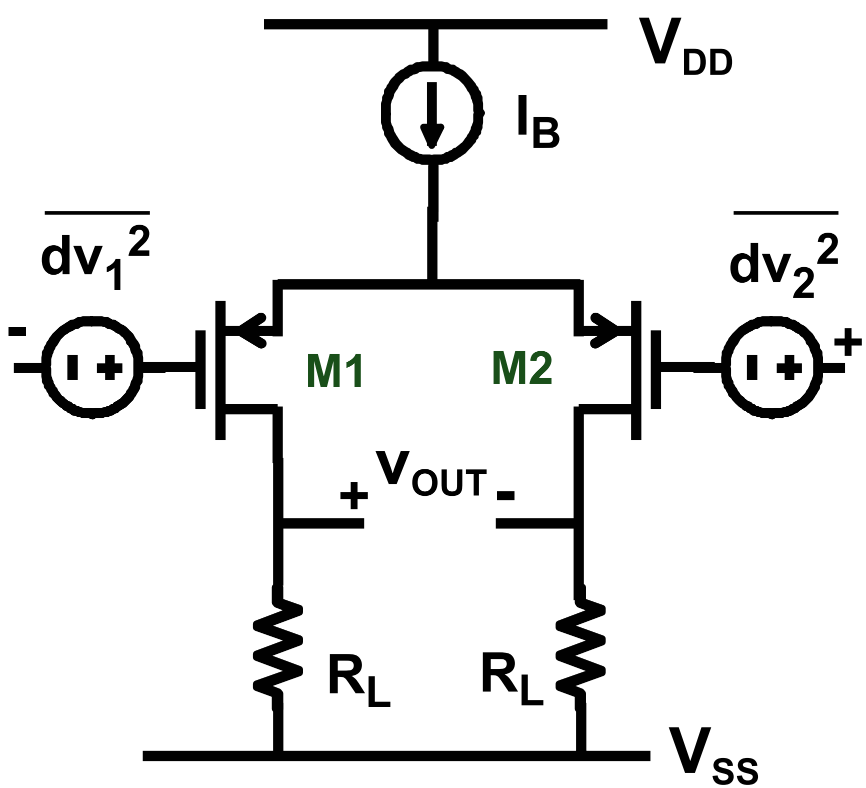
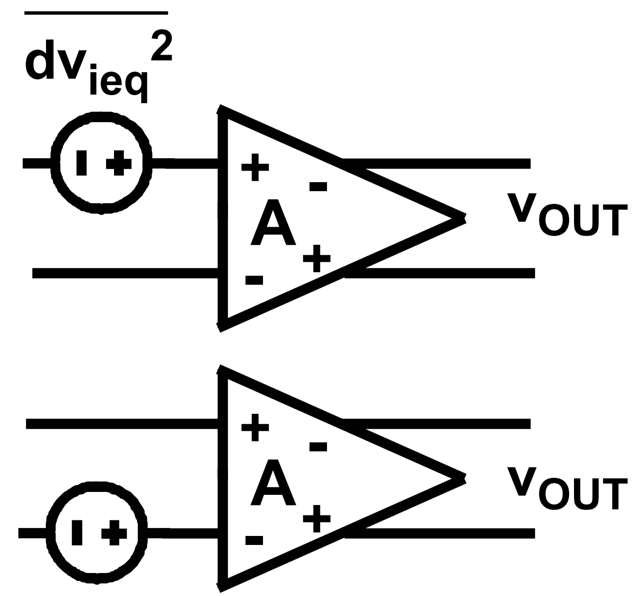
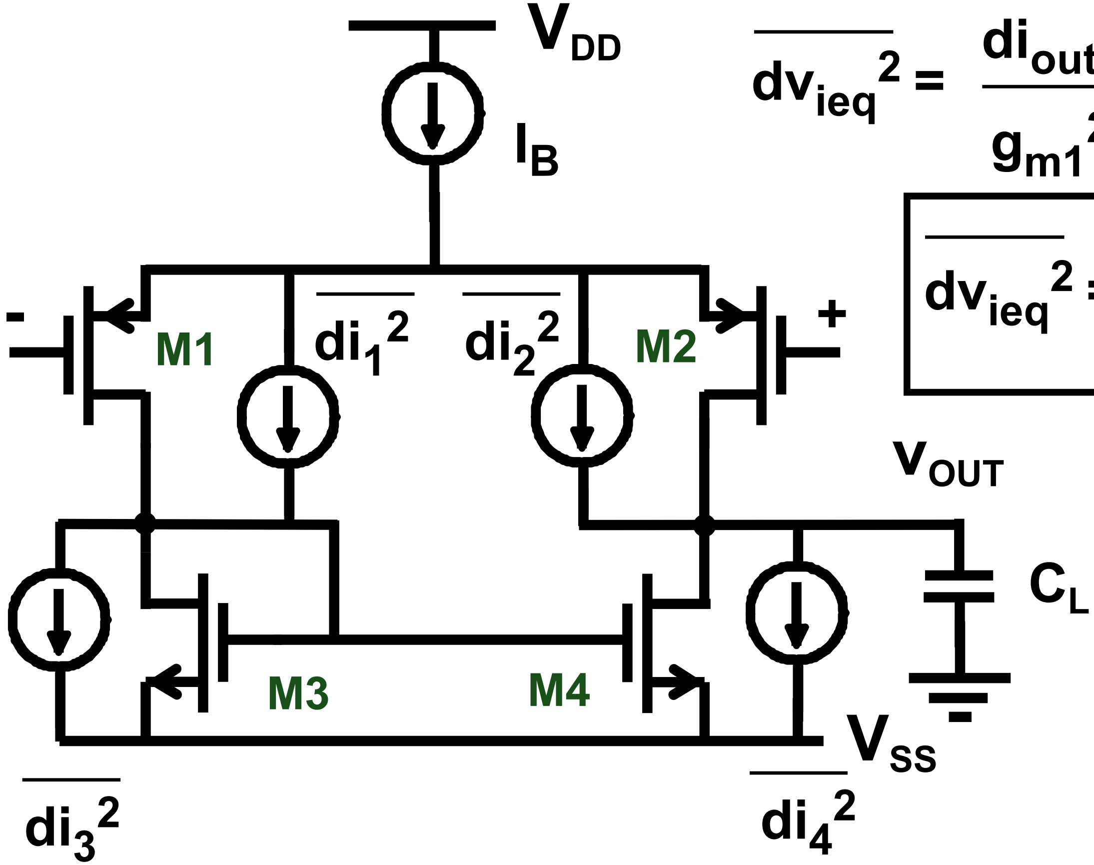
Noise of differential pair





$$\overline{dv_{ieq}^2} = 2\,\overline{dv_1^2}$$

Noise of differential pair with active load



$$\overline{dv_{ieq}^2} = \frac{\overline{di_{out}^2}}{g_{m1}^2} = \frac{2\overline{di_1^2}+2\overline{di_3^2}}{g_{m1}^2}$$

$$\boxed{\overline{dv_{ieq}^2} = 2\overline{dv_1^2}\left(1 + \frac{g_{m3}}{g_{m1}}\right)}$$

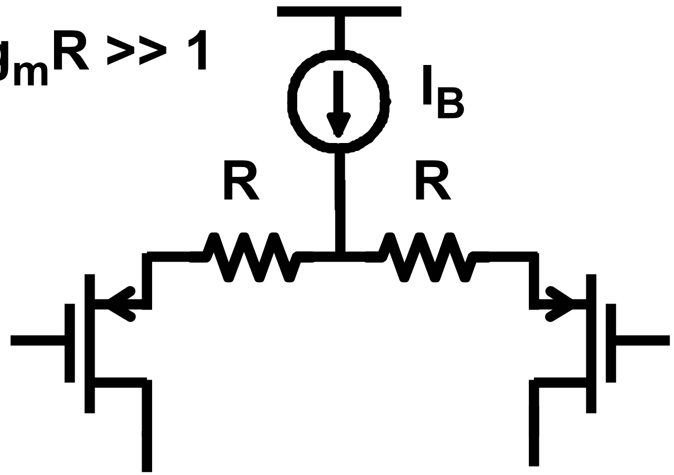
Small g_{m3} :

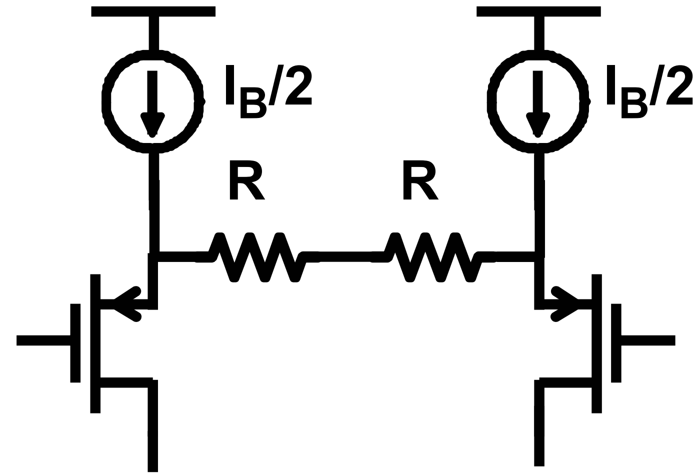
$(W/L)_3 \downarrow$

$(V_{GS}-V_T)_3 \uparrow$

Differential pair with source resistors

$g_m R >> 1$





$$\overline{di_{out}^2} = 2 \, \frac{4kT}{R} \, df$$

$$\overline{dv_{in}^2} = 2\,(4kT\,R\,df)$$

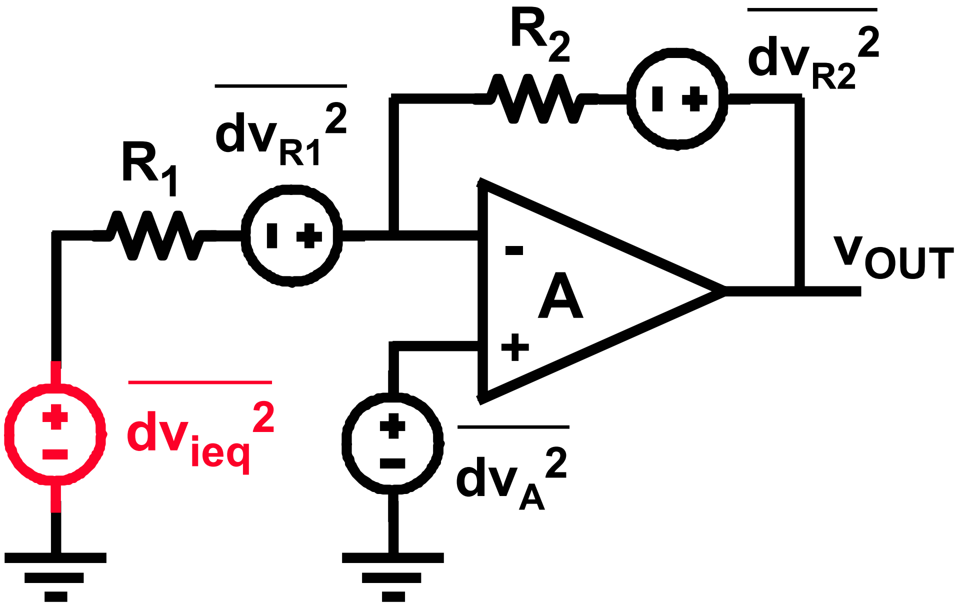
$\overline{di_B^2}$ is negligible

$$\overline{di_{out}^2} = 2\,\left(\frac{4kT}{R}\,df + \overline{di_B^2}\right)$$

$$\overline{di_B^2} = 4kT\,2/3\,g_{mB}\,df$$

$$\overline{dv_{in}^2} = 2\,(4kT\,R\,df)\,(1 + 2/3\,g_{mB}R)$$

Noise of an opamp



$$\overline{dv_{ieq}^2} = \Sigma\, \overline{dv_{out}^2} \left(\frac{R_1}{R_2}\right)^2$$

$$\overline{dv_{out}^2} = \overline{dv_{R1}^2} \left(\frac{R_2}{R_1}\right)^2$$

$$\overline{dv_{out}^2} = \overline{dv_{R2}^2}$$

$$\overline{dv_{out}^2} = \overline{dv_A^2} \left(1 + \frac{R_2}{R_1}\right)^2$$

$$\overline{dv_{ieq}^2} = \overline{dv_{R1}^2} + \overline{dv_{R2}^2} \left(\frac{R_1}{R_2}\right)^2 + \overline{dv_A^2} \left(1 + \frac{R_1}{R_2}\right)^2 \approx \overline{dv_{R1}^2} + \overline{dv_A^2}$$

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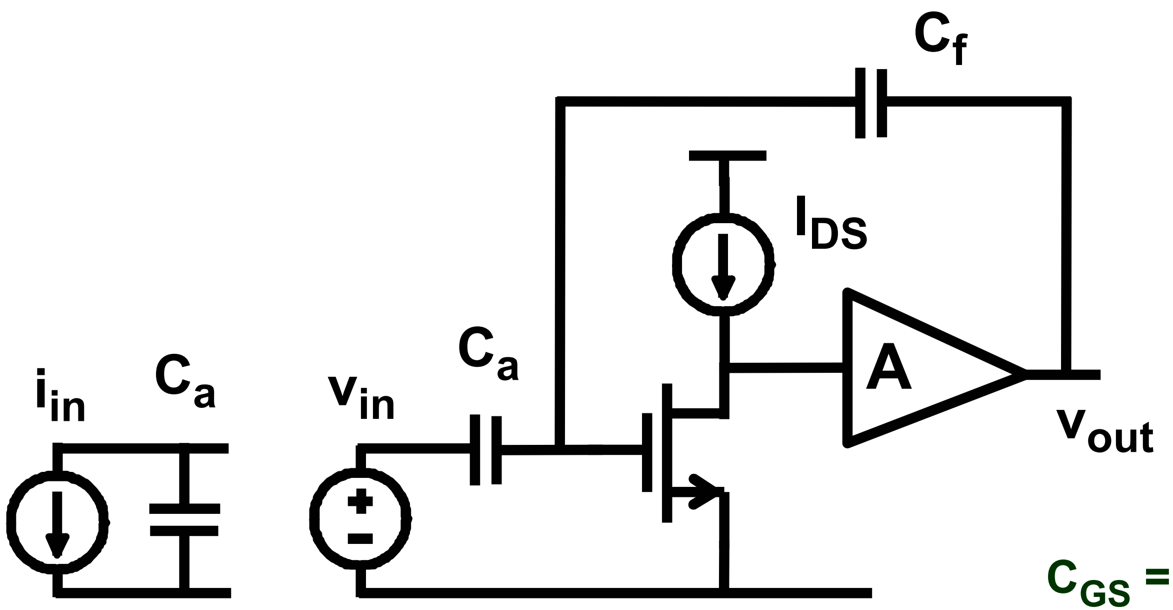
- Definitions of noise
- Noise of an amplifier
- Noise of a follower
- Noise of a cascode
- Noise of a differential pair
- Noise of a current mirror
- Capacitive noise matching

Ref.: Z.Y.Chang, W.Sansen, Low-noise wide-band amplifiers, Kluwer AP, 1991

Capacitive-source amplifier



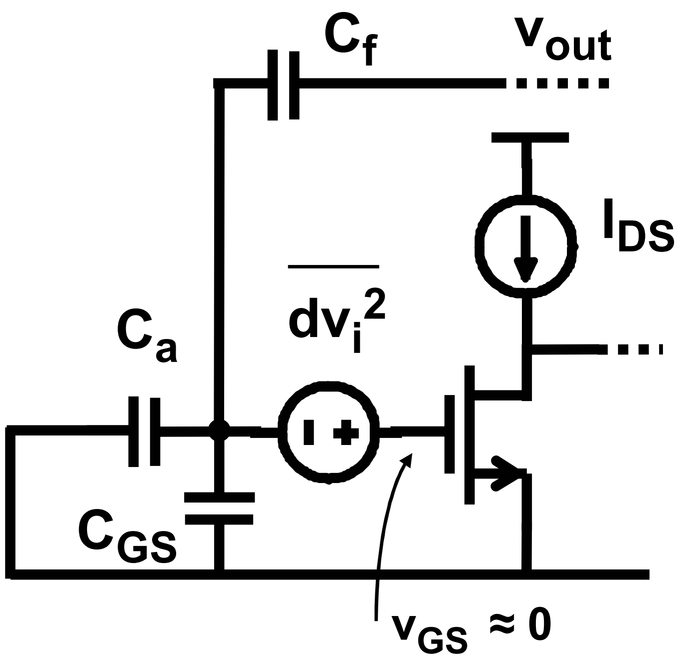
$$A_v = \frac{C_a}{C_f}$$

$C_a = 5$ pF
$C_f = 1$ pF

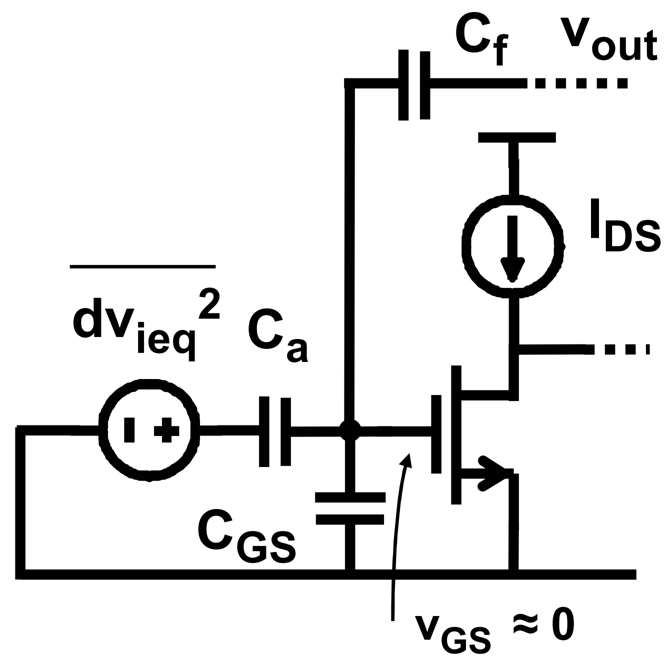
$C_{GS} = kW \quad k \approx 2$ fF/μm

W_{opt} ? I_{DSopt} ? S/N_{opt} for $V_{in} = 10\ mV_{RMS}$?

Capacitive noise matching - 1



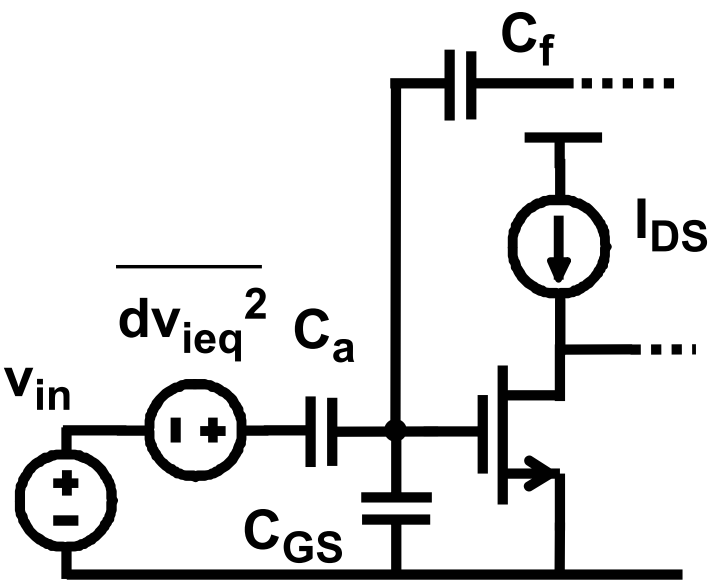
$$\frac{v_{out}}{v_i} = \frac{C_f + C_a + C_{GS}}{C_f}$$



$$\frac{v_{out}}{v_{ieq}} = \frac{C_a}{C_f}$$

No Miller with C_{DG} !!!

Capacitive noise matching - 2



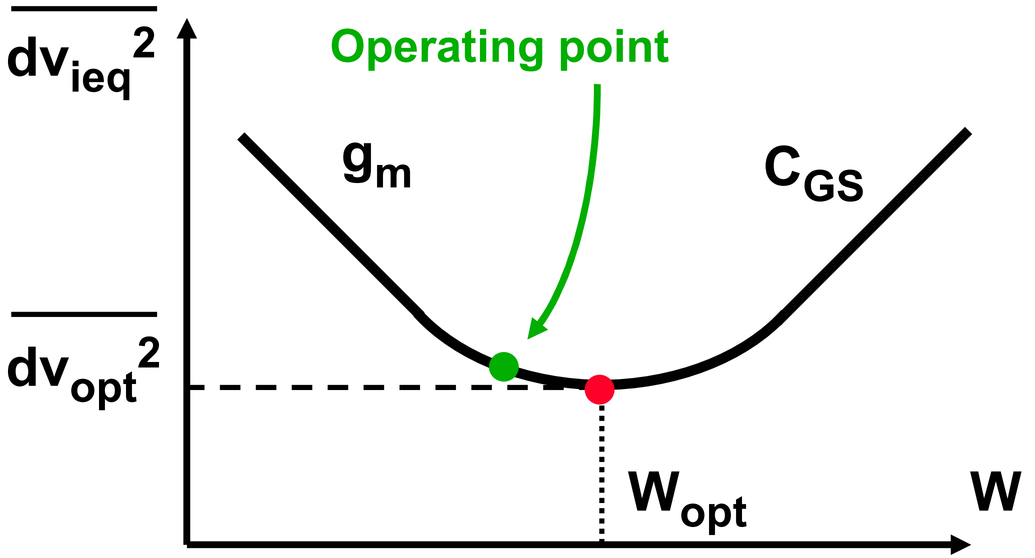
$$\overline{dv_{ieq}^2} = \frac{(C_f+C_a+C_{GS})^2}{C_a^2}\,\overline{dv_i^2}$$

$$\overline{dv_i^2} = \frac{8kT}{3}\,\frac{1}{g_m}\,df$$

$$g_m = 2\,K'_n\,\frac{W}{L}\,(V_{GS}-V_T)$$

$$\overline{dv_{ieq}^2} = \frac{(C_f + C_a + kW)^2}{C_a^2}\,\frac{L}{W}\,\frac{8kT}{3}\,\frac{1}{2\,K'_n\,(V_{GS}-V_T)}$$

Capacitive noise matching - 3

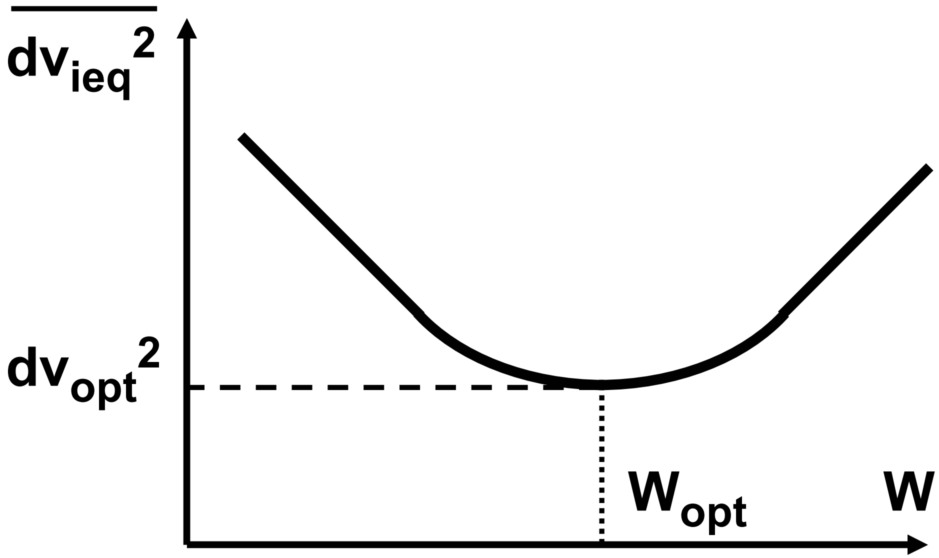


Noise matching where
$C_{GS} = C_f + C_a$

$$W_{opt} = \frac{C_f + C_a}{k}$$

$$\overline{dv_{ieq}^2} = \frac{(C_f + C_a + kW)^2}{C_a^2} \frac{L}{W} \frac{8kT}{3} \frac{1}{2K'_n(V_{GS}-V_T)}$$

Capacitive noise matching - 4



$$W_{opt} = \frac{C_f + C_a}{k}$$

C_{GSopt}

I_{DSopt} , g_{mopt}

$$\overline{dv_{opt}^2} = 4\,\frac{8kT}{3}\,\frac{df}{g_{mopt}}$$

$$BW_n = \frac{\pi}{2}\,BW = \frac{\pi}{2}\,\frac{f_T}{A_v} = \frac{1}{4A_v}\,\frac{g_{mopt}}{C_{GSopt}}$$

$$\frac{S}{N_{opt}} = \frac{10\ mV_{RMS}}{\sqrt{\overline{dv_{opt}^2}\,BW_n}}$$

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0.5 chap5

Stability of Operational amplifiers

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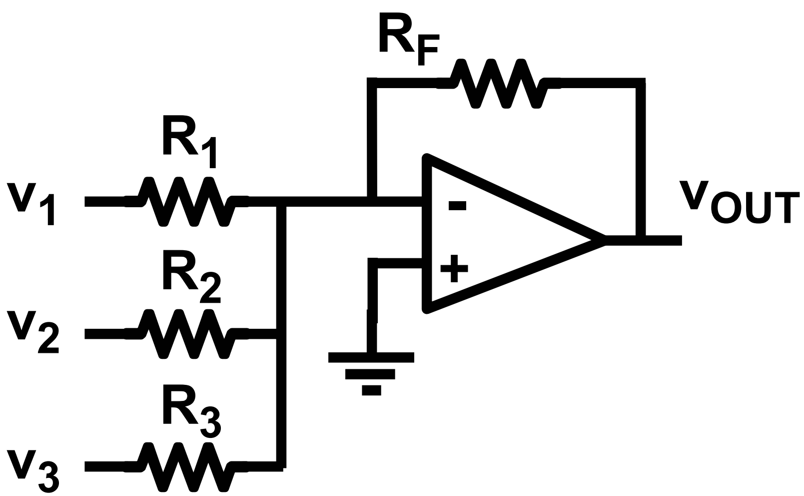
Operational amplifiers do operations

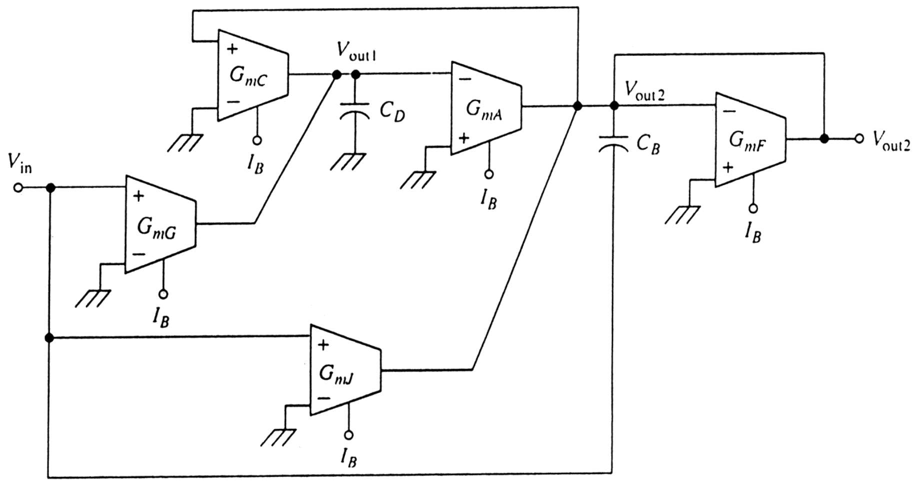


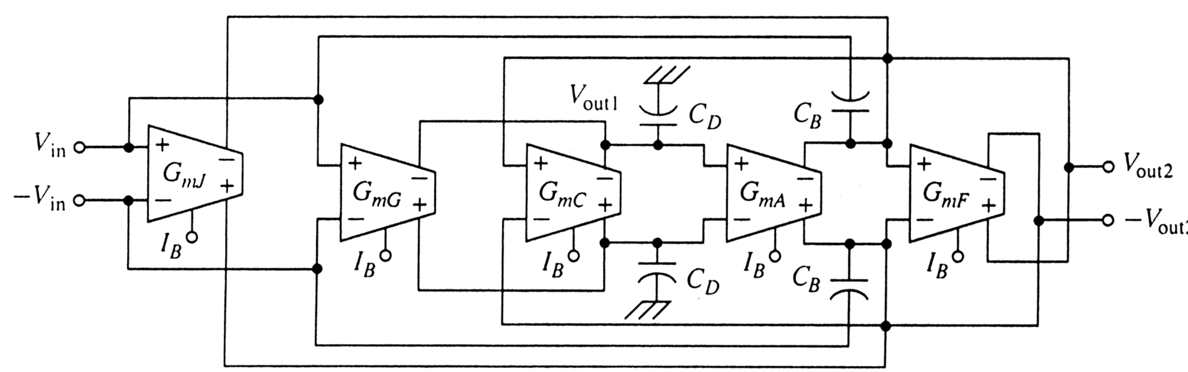
$$-\frac{v_{OUT}}{R_F} = \frac{v_1}{R_1} + \frac{v_2}{R_2} + \frac{v_3}{R_3}$$

Requires High gain
High speed
Low noise
Low power

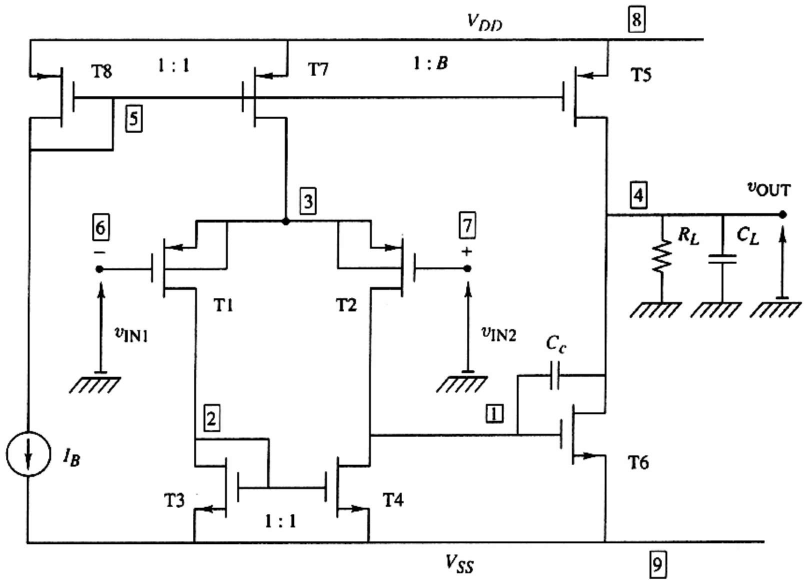
Opamp specs : Voltage gain is large
Differential input voltage ≈ 0
Input current = 0
Bandwidth is high
Gainbandwidth GBW is very, very high

Single-ended or fully differential ?

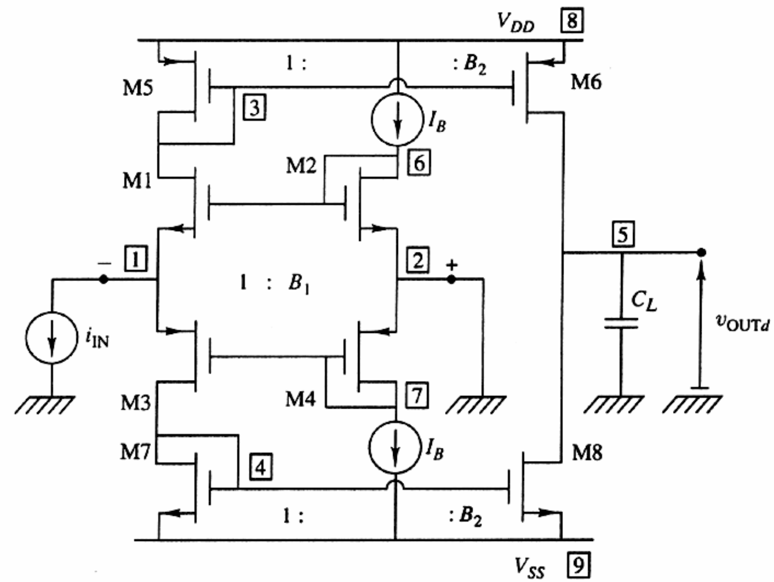




Voltage input or current input ?



Voltage input
Current output

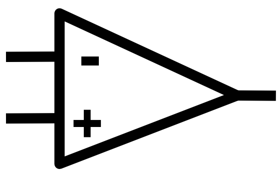


Current input
Current output

Classification

Opamp

Operational amplifier



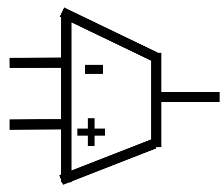
$$A_v = \frac{v_{OUT}}{v_{IN}}$$

$$A_v =$$

GBW

OTA

Operational Transconduct. amplifier

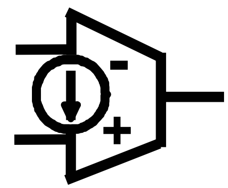


$$A_g = \frac{i_{OUT}}{v_{IN}}$$

$$= A_g R_L$$

OCA

Operational Current amplifier

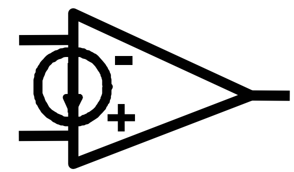


$$A_i = \frac{i_{OUT}}{i_{IN}}$$

$$= A_i \frac{R_L}{R_S}$$

CM amp

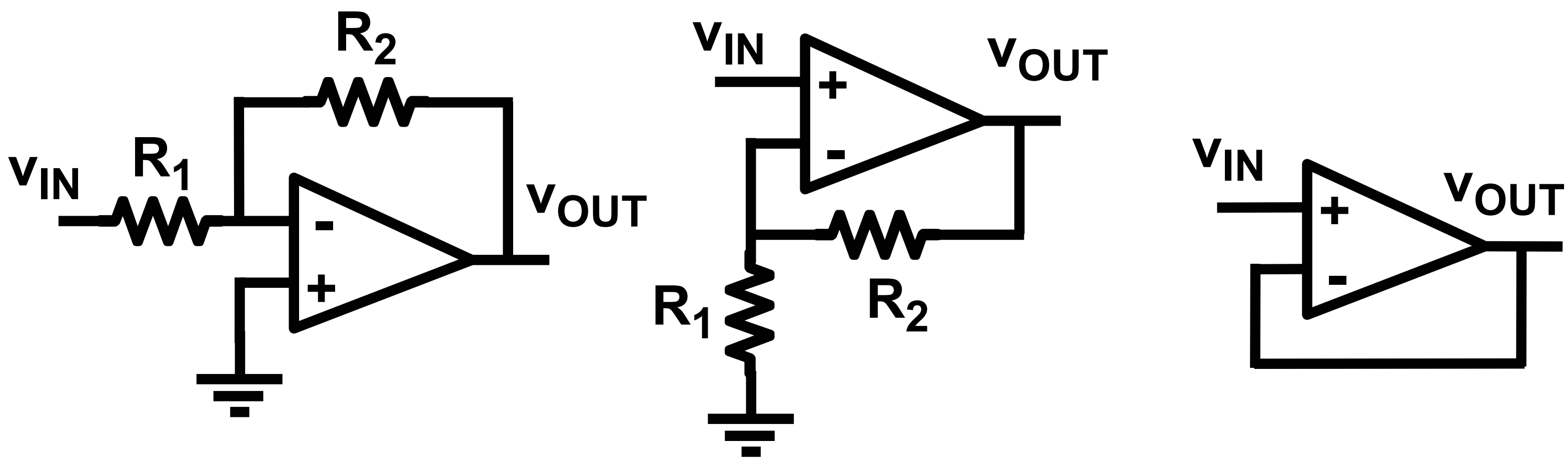
Current Mode amplifier



$$A_r = \frac{v_{OUT}}{i_{IN}}$$

$$= A_r \frac{1}{R_S}$$

Feedback configurations



$$A_v = -\frac{R_2}{R_1}$$

$$R_{IN} = R_1$$

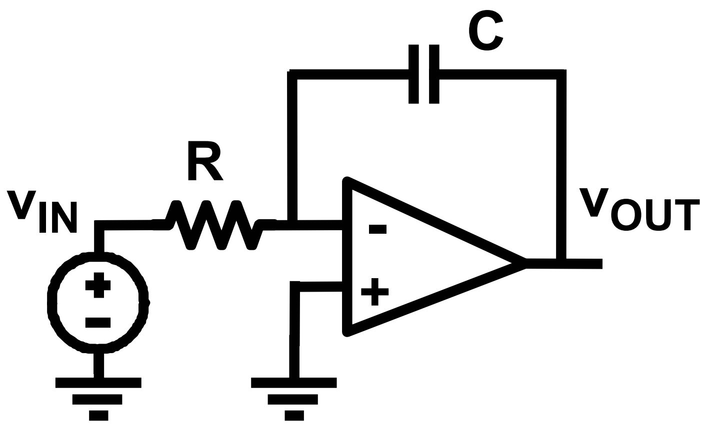
$$A_v = 1 + \frac{R_2}{R_1}$$

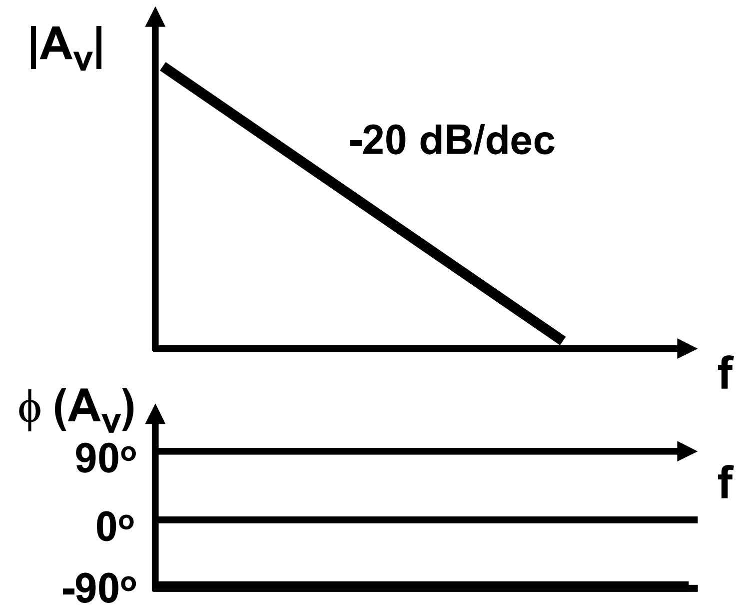
$$R_{IN} = \infty$$

$$A_v = 1$$

$$R_{IN} = \infty$$

Integrator

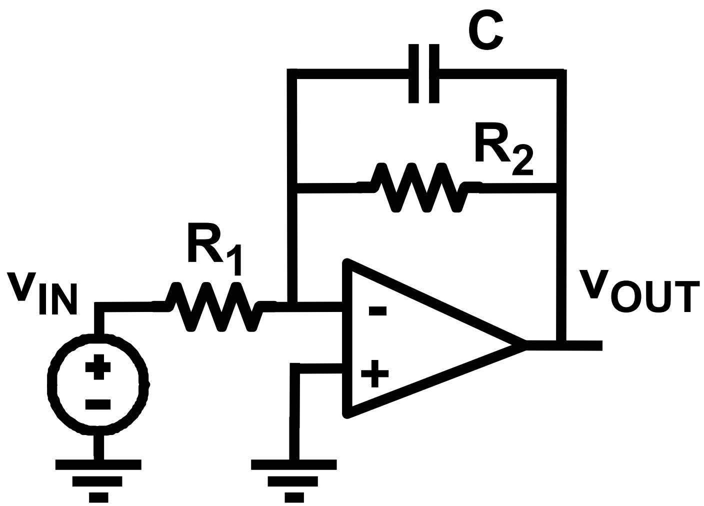


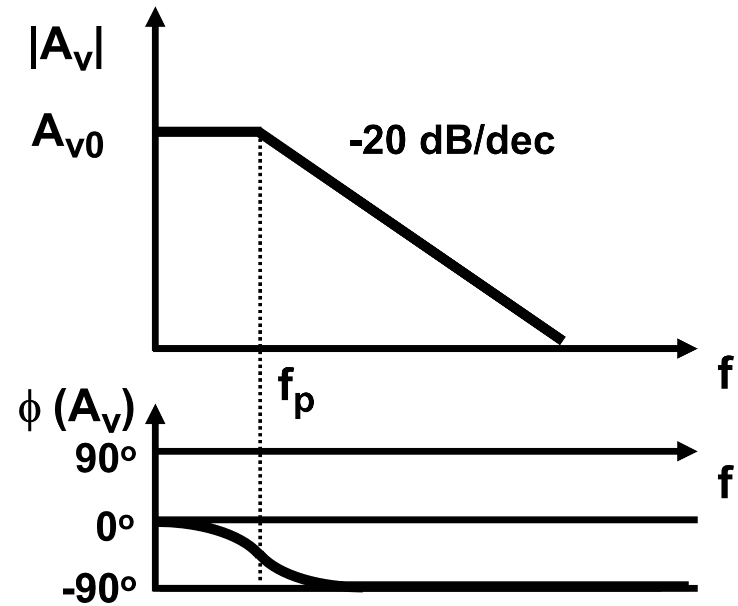


$$A_v = \frac{1}{j\frac{f}{f_p}}$$

$$f_p = \frac{1}{2\pi RC}$$

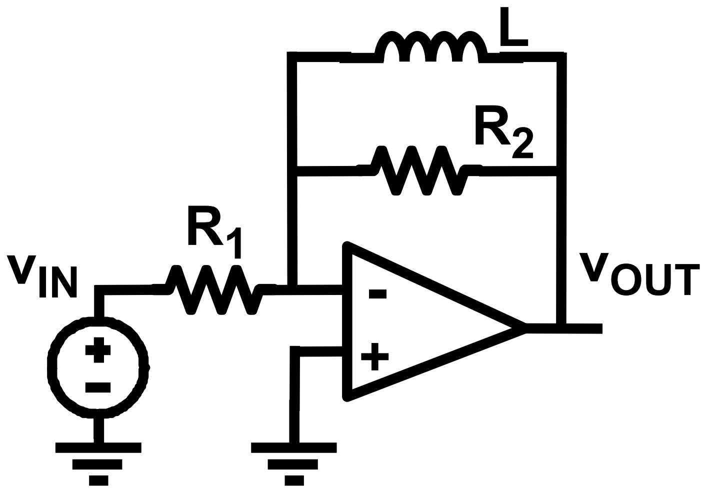
Low-pass filter

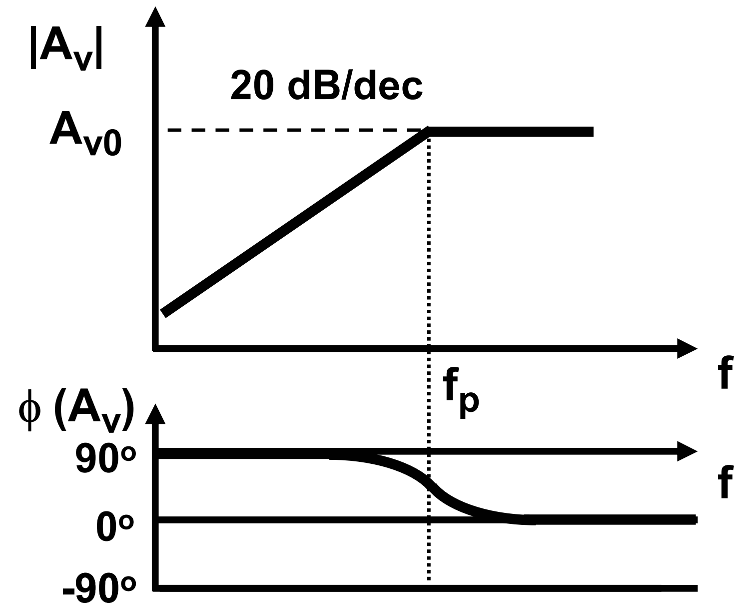




$$A_{v0} = -\frac{R_2}{R_1} \qquad A_v = \frac{A_{v0}}{\left(1 + j\dfrac{f}{f_p}\right)} \qquad f_p = \frac{1}{2\pi R_2 C}$$

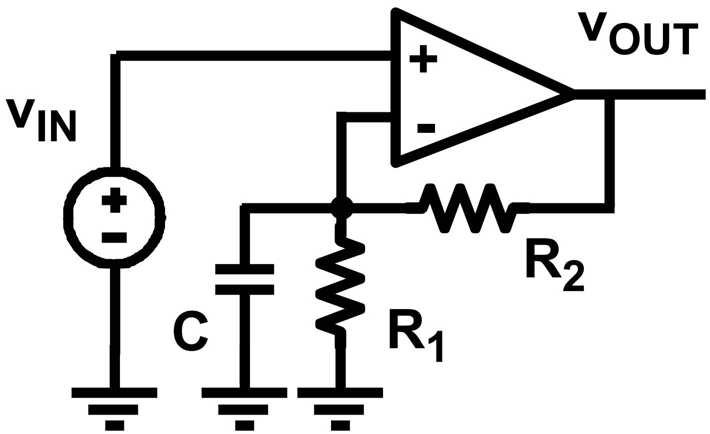
High-pass filter

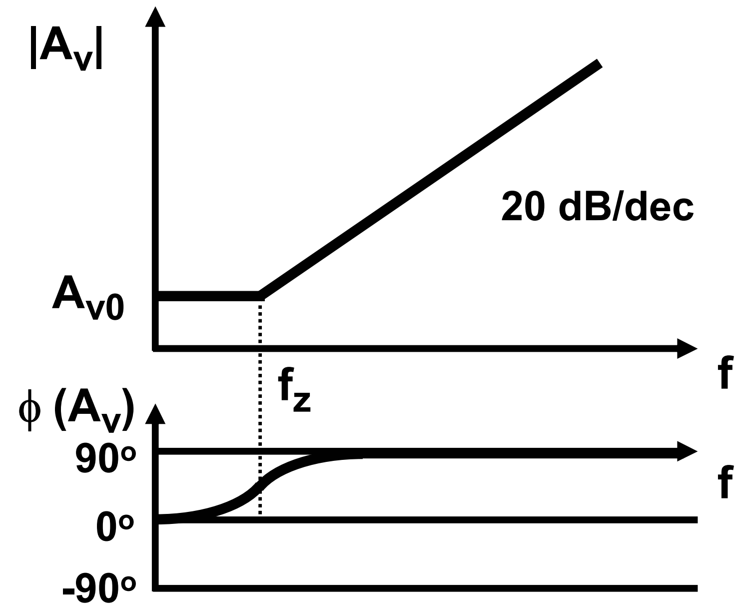




$$A_{v0} = -\frac{R_2}{R_1} \qquad A_v = A_{v0}\,\frac{j\,\dfrac{f}{f_p}}{\left(1 + j\,\dfrac{f}{f_p}\right)} \qquad f_p = \frac{R_2}{2\pi L}$$

High-pass filter





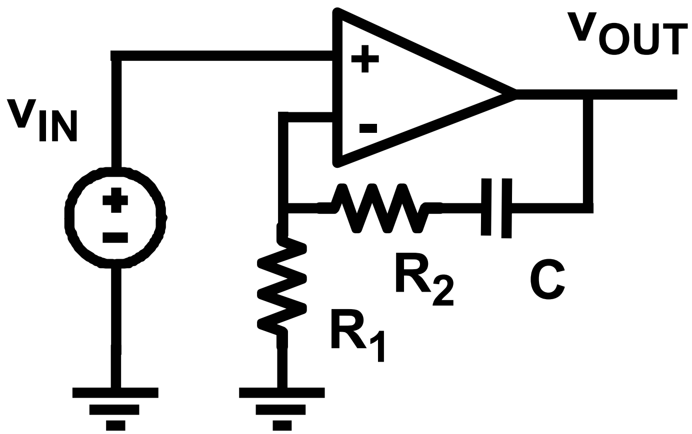
$$A_{v0} = 1 + \frac{R_2}{R_1}$$

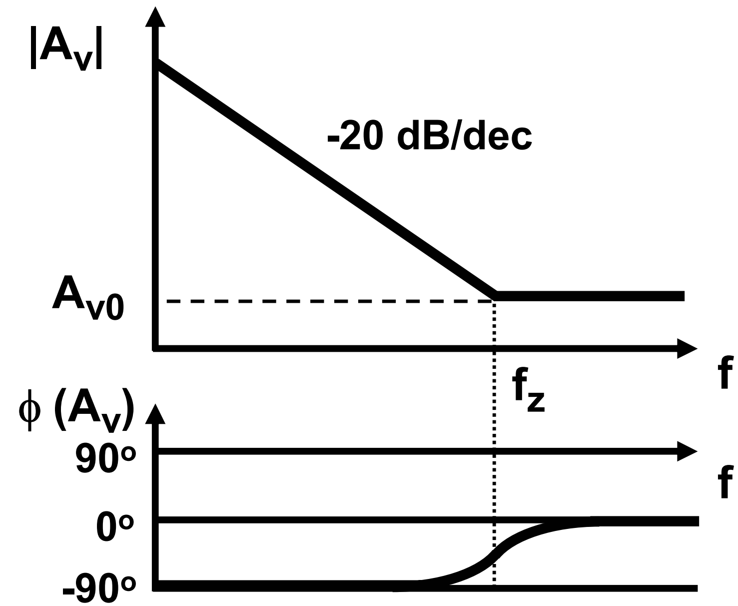
$$A_v = A_{V0}\,(1 + j\,\frac{f}{f_z})$$

$$f_z = \frac{1}{2\pi\,RC}$$

$$R = R_1//R_2 = \frac{R_1R_2}{R_1+R_2}$$

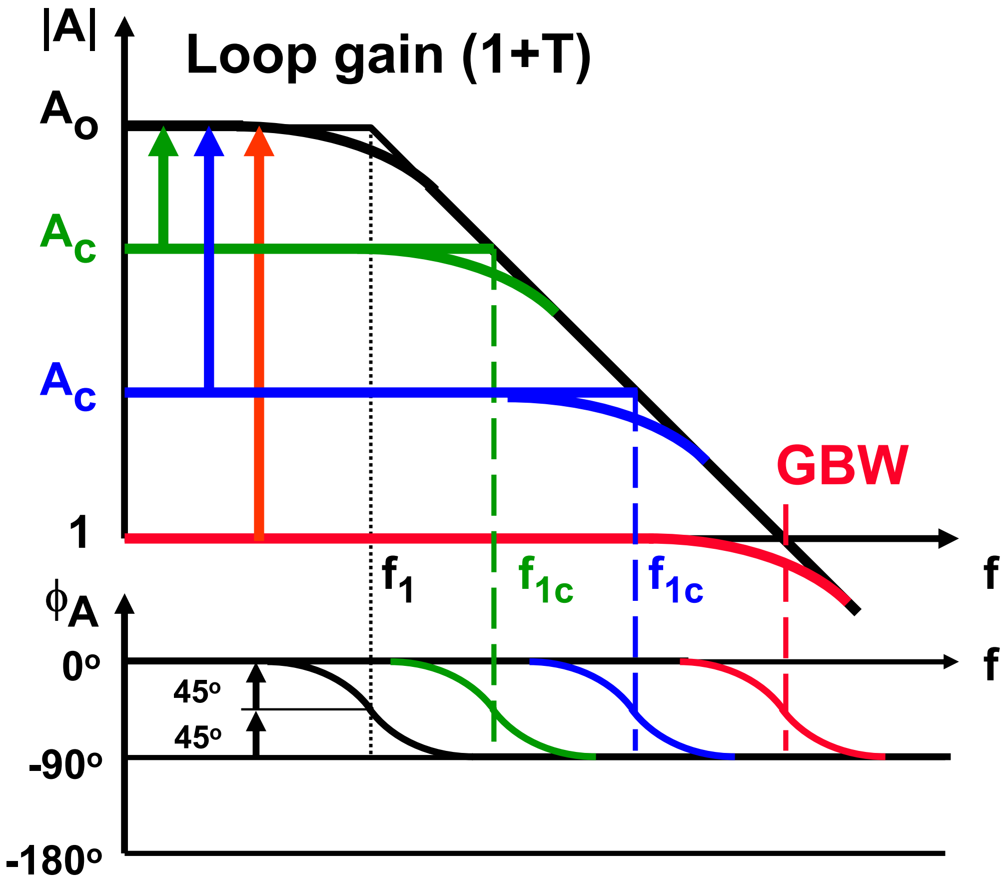
Low-pass filter with finite attenuation





$$A_{v0} = 1 + \frac{R_2}{R_1} \qquad A_v = A_{v0}\,\frac{\left(1 + j\dfrac{f}{f_z}\right)}{j\dfrac{f}{f_z}} \qquad f_z = \frac{1}{2\pi RC} \qquad R = R_1 + R_2$$

Exchange of gain and bandwidth



A_o open loop gain

A_c closed loop gain

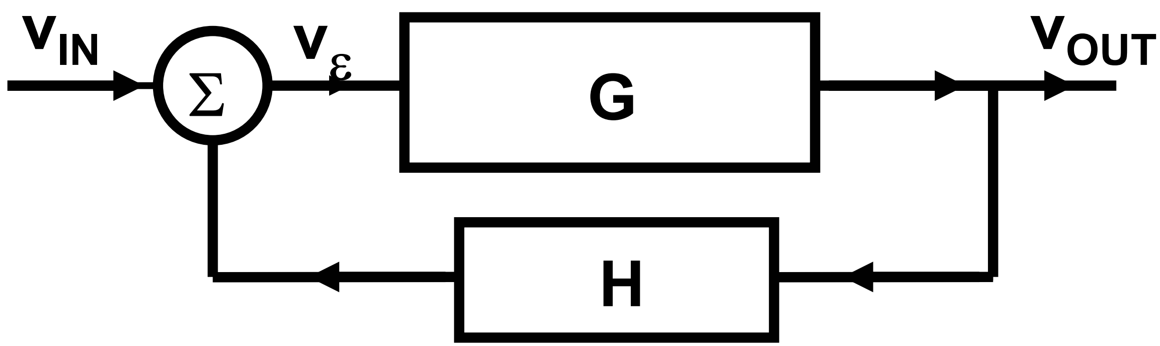
$A_o f_1 =$

$A_c f_{1c} =$

$A_c f_{1c} =$

GBW

Open- and closed-loop gain

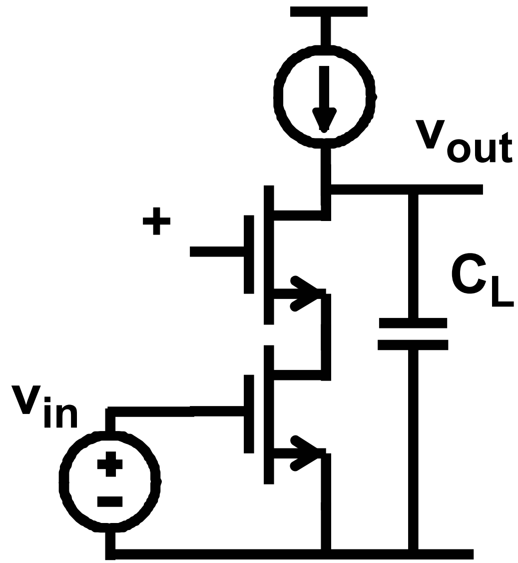


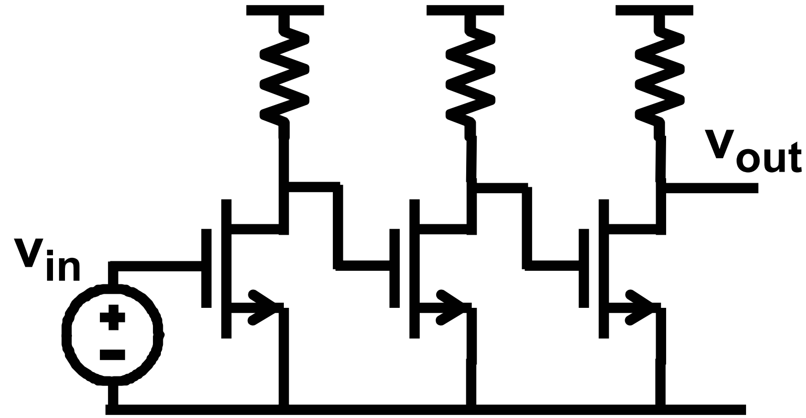
$$\left.\begin{array}{l} v_\varepsilon = v_{IN} - H\, v_{OUT} \\ v_{OUT} = G\, v_\varepsilon \end{array}\right\} \quad A_c = \frac{v_{OUT}}{v_{IN}} = \frac{G}{1 + GH} \approx \frac{1}{H}$$

if the loop gain GH = T >> 1

P. Gray, P.Hurst, S.Lewis, R. Meyer: Design of analog integrated circuits, 4th ed., Wiley 2001

What makes an opamp an opamp ?

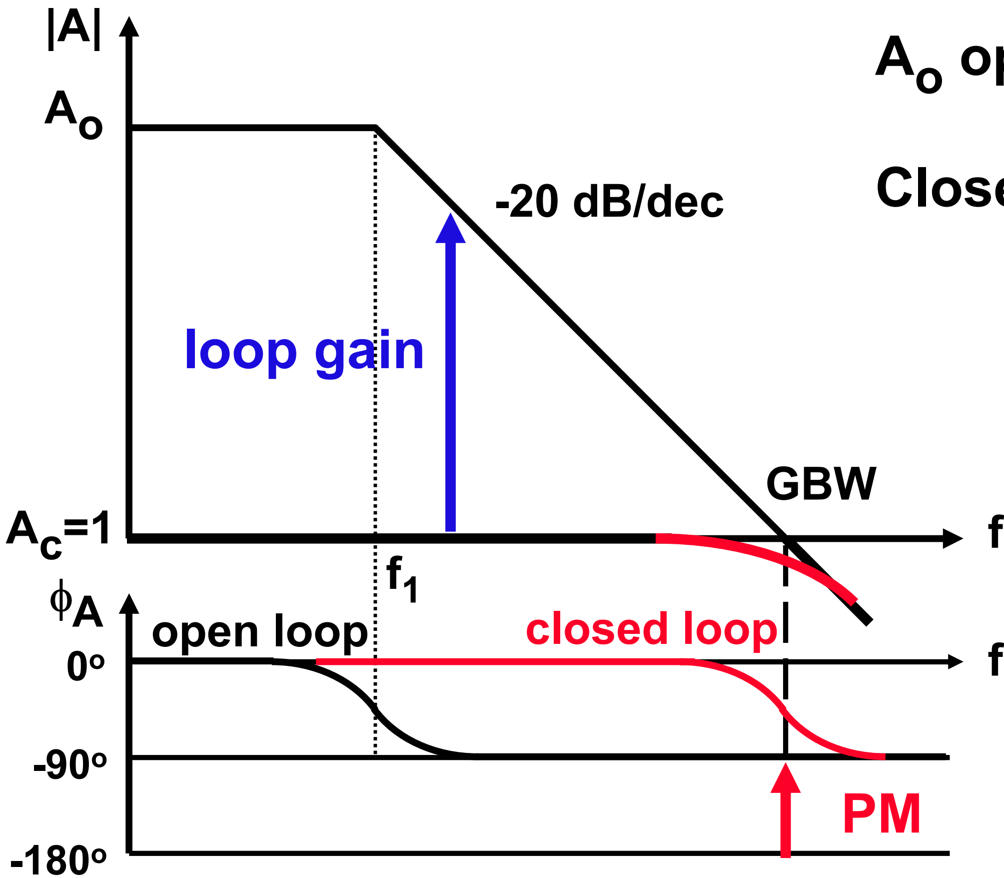




Operational amplifier :
Single-pole amplifier
High impedance = high gain
Exchange Gain-Bandwidth
Stable for all gain values

Wideband amplifier :
Multiple-pole amplifier
Low impedances at nodes
Wide Bandwidth
Stable for one gain only

Single-pole system

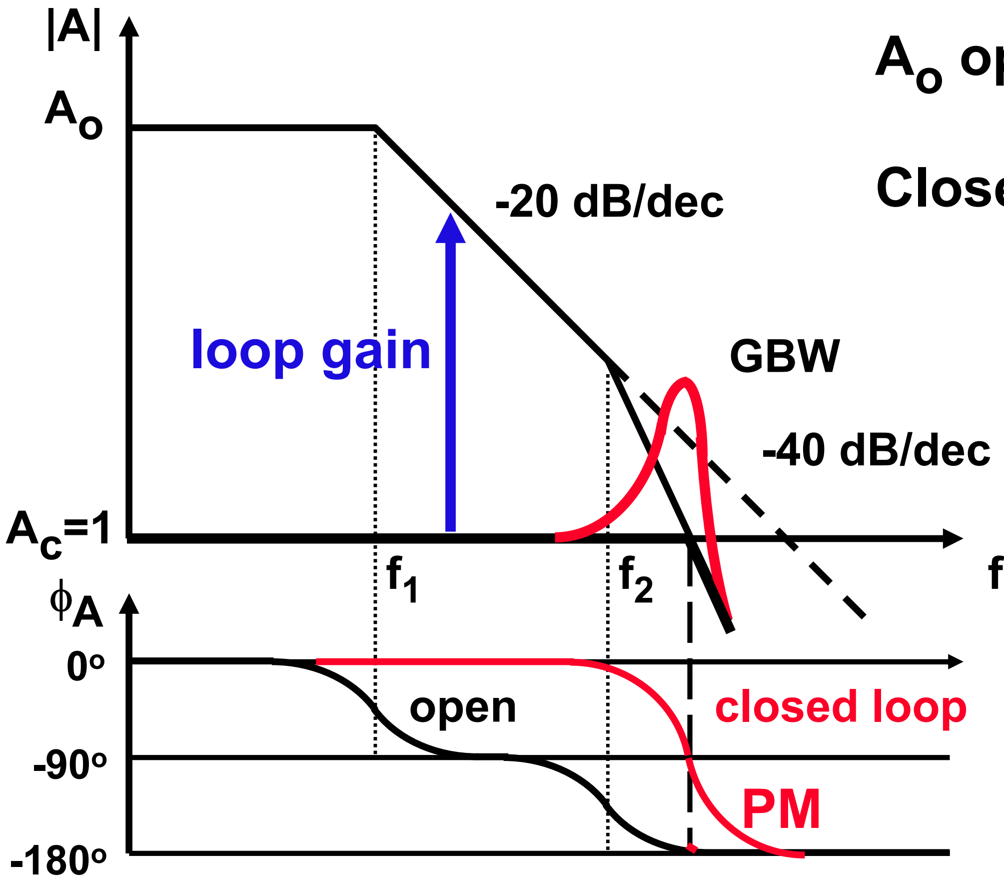


A_o open loop gain

Closed loop gain $A_c = 1$

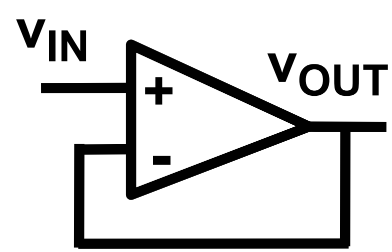
PM phase margin

Two-pole system



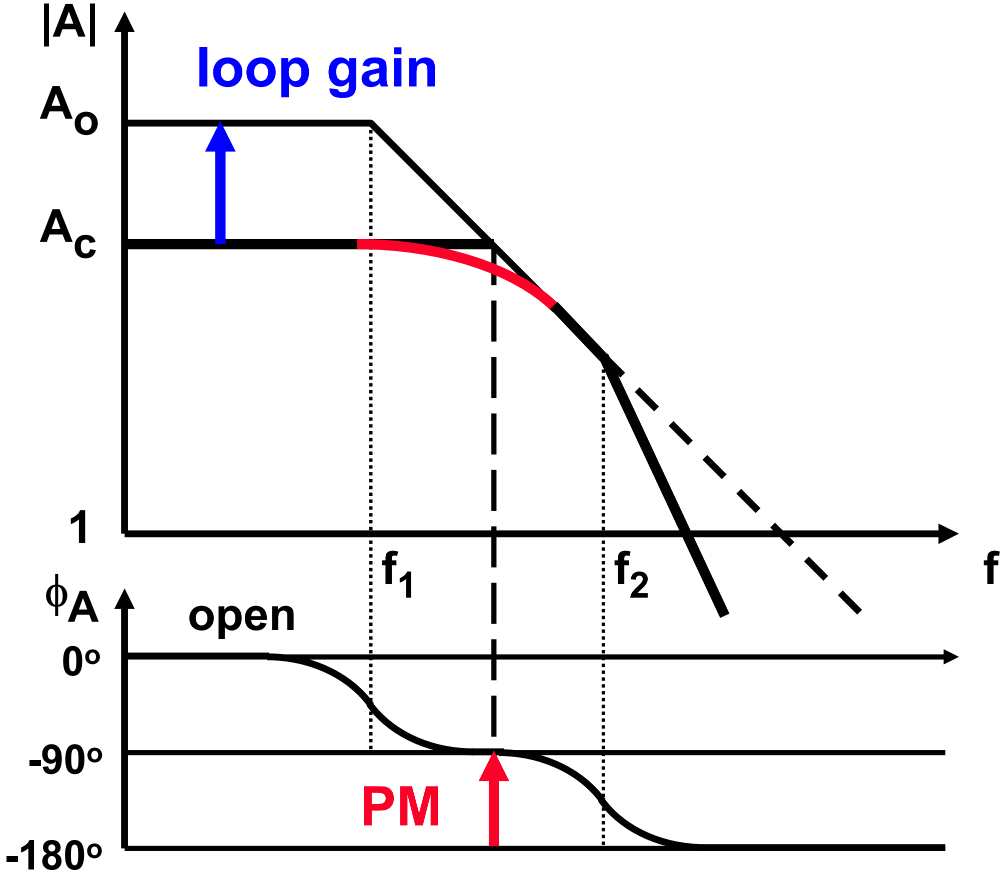
A_o open loop gain

Closed loop gain $A_c = 1$



PM phase margin

Higher loop gain gives less PM

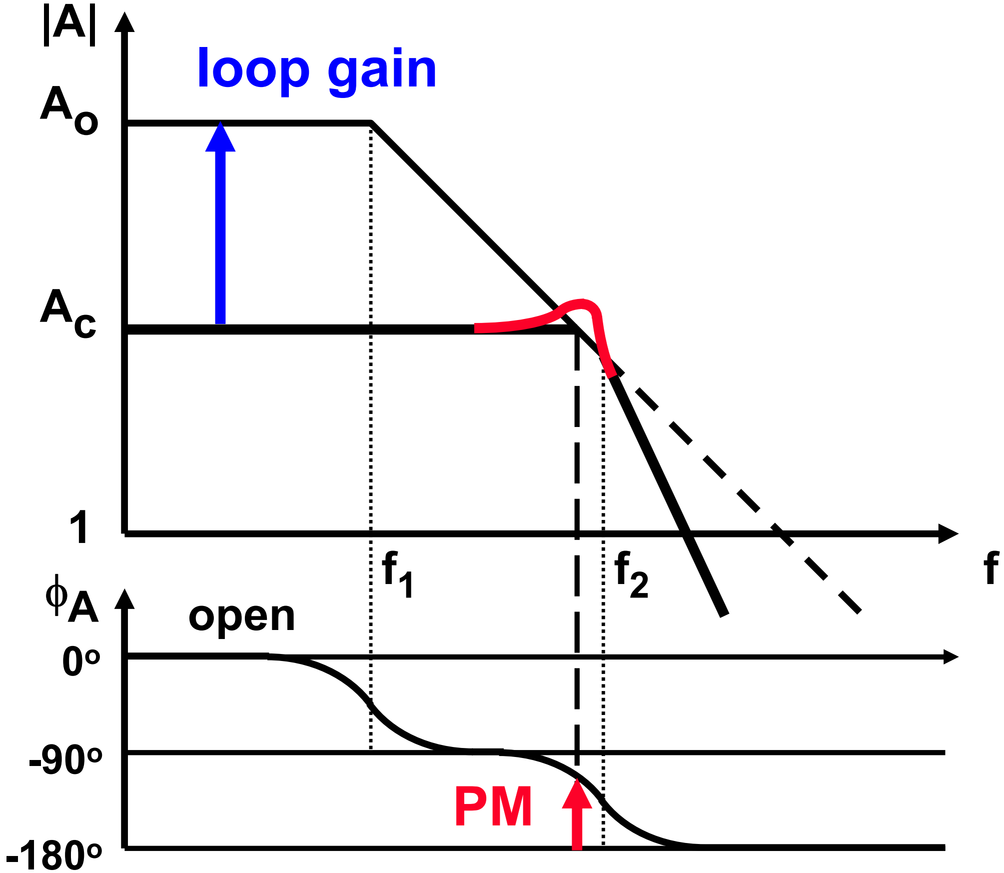


A_o open loop gain

A_c closed loop gain

PM phase margin

Higher loop gain gives less PM

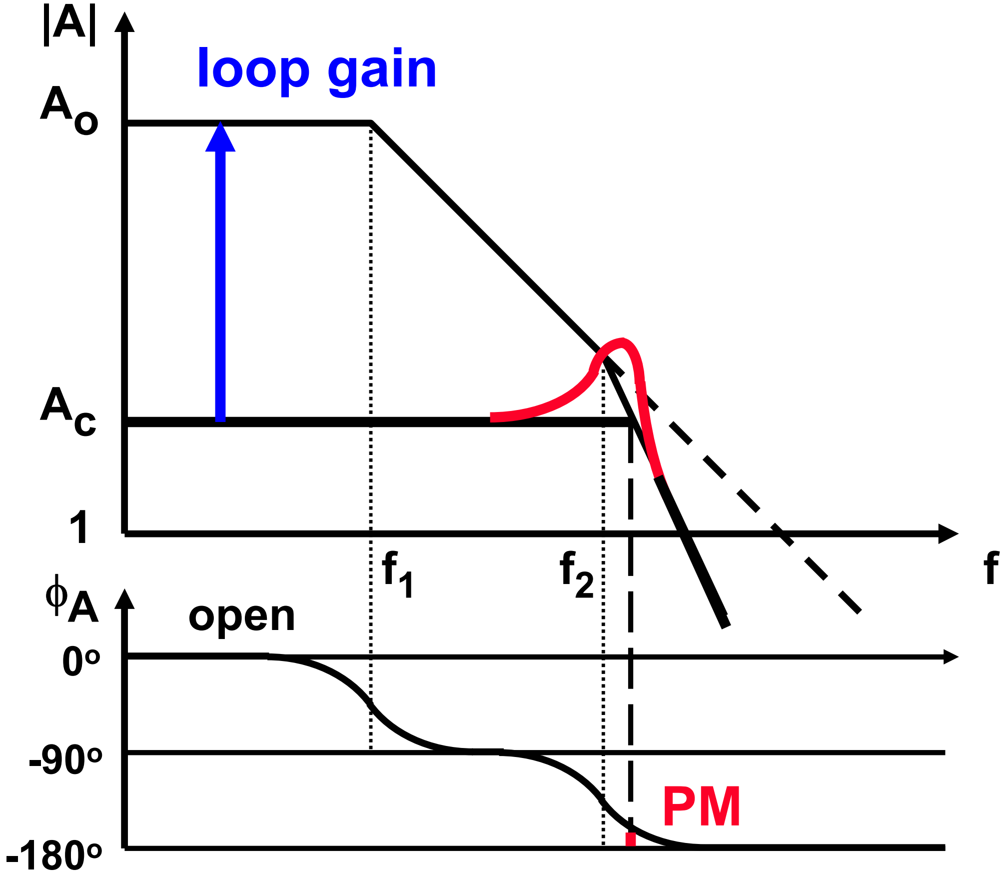


A_o open loop gain

A_c closed loop gain

PM phase margin

Higher loop gain gives less PM

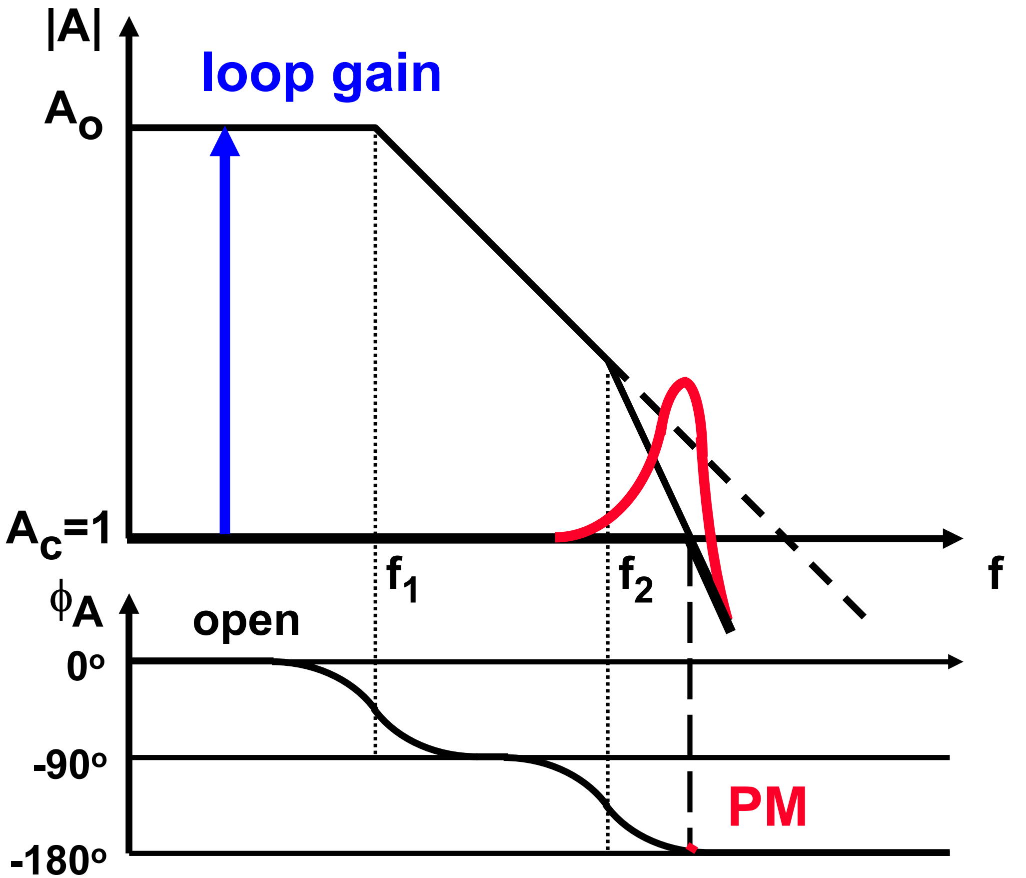


A_o open loop gain

A_c closed loop gain

PM phase margin

Higher loop gain gives less PM



A_o open loop gain

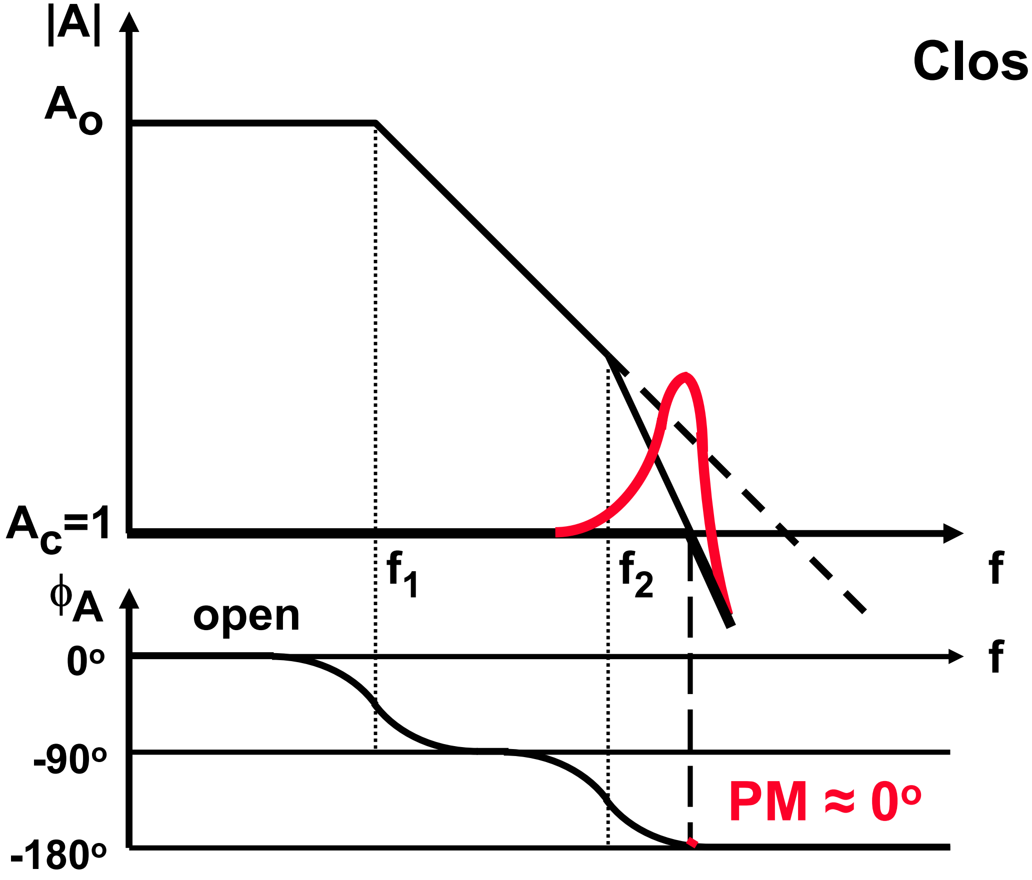
A_c closed loop gain

Worst case for A_c = 1

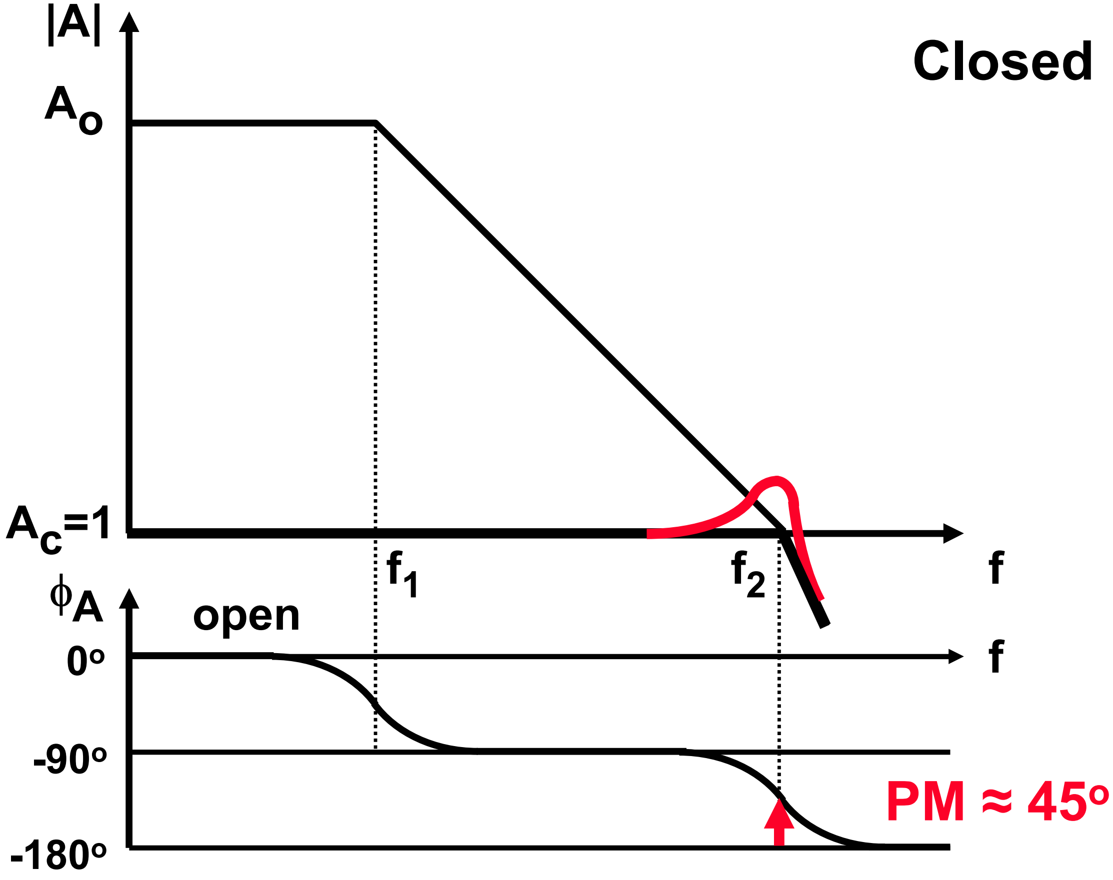
PM phase margin

Increase PM by increasing f_2 : low f_2

Closed loop gain $A_c = 1$



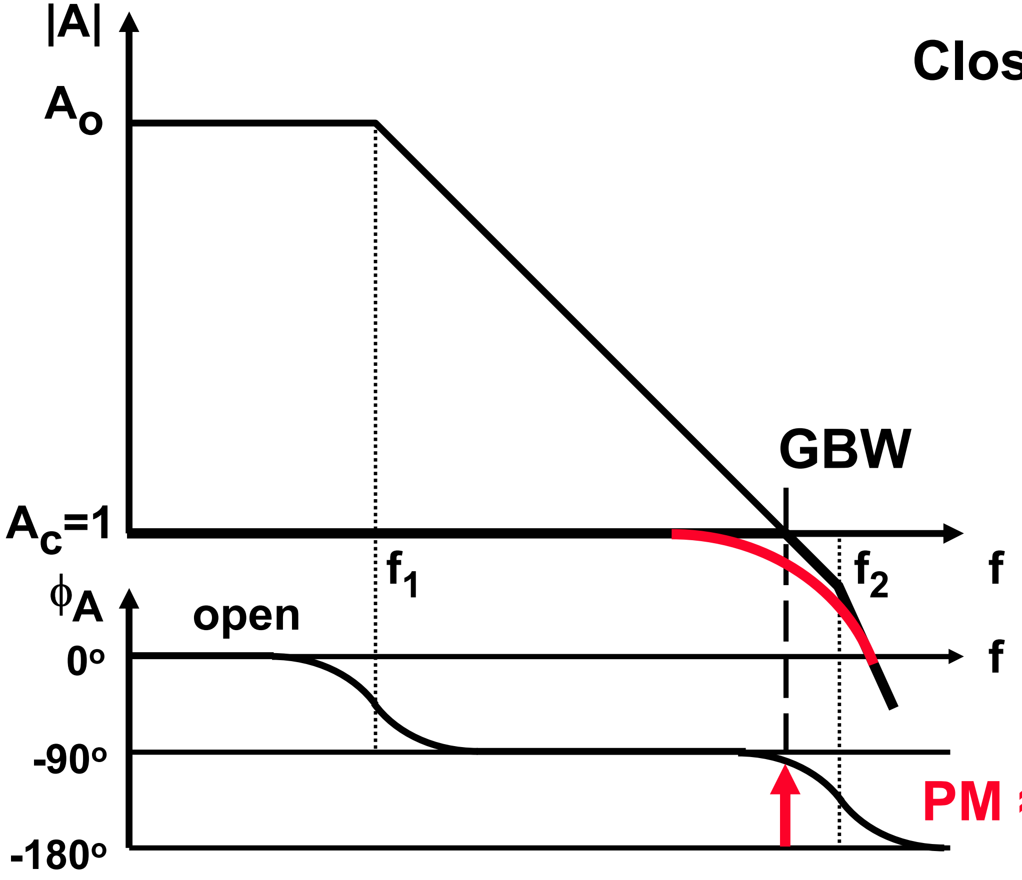
Increase PM by increasing f_2



Closed loop gain $A_c = 1$

Set PM by setting $f_2 \approx 3$ GBW

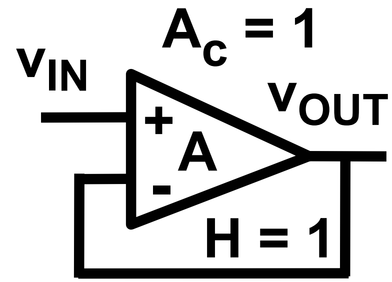
Closed loop gain $A_c = 1$



$f_2 \approx 3$ GBW

Calculate PM for $f_2 \approx 3$ GBW

Open loop gain $A = \dfrac{A_o}{\left(1 + j\dfrac{f}{f_1}\right)\left(1 + j\dfrac{f}{f_2}\right)}$



Closed loop gain $A_c = \dfrac{A}{1+A} \approx \dfrac{1}{1 + j\dfrac{f}{GBW} + j^2\dfrac{f^2}{GBW\, f_2}}$

$$\approx \frac{1}{1 + j\, 2\zeta \dfrac{f}{f_r} + j^2 \dfrac{f^2}{f_r^{\,2}}}$$

ζ is the damping (=1/2Q)

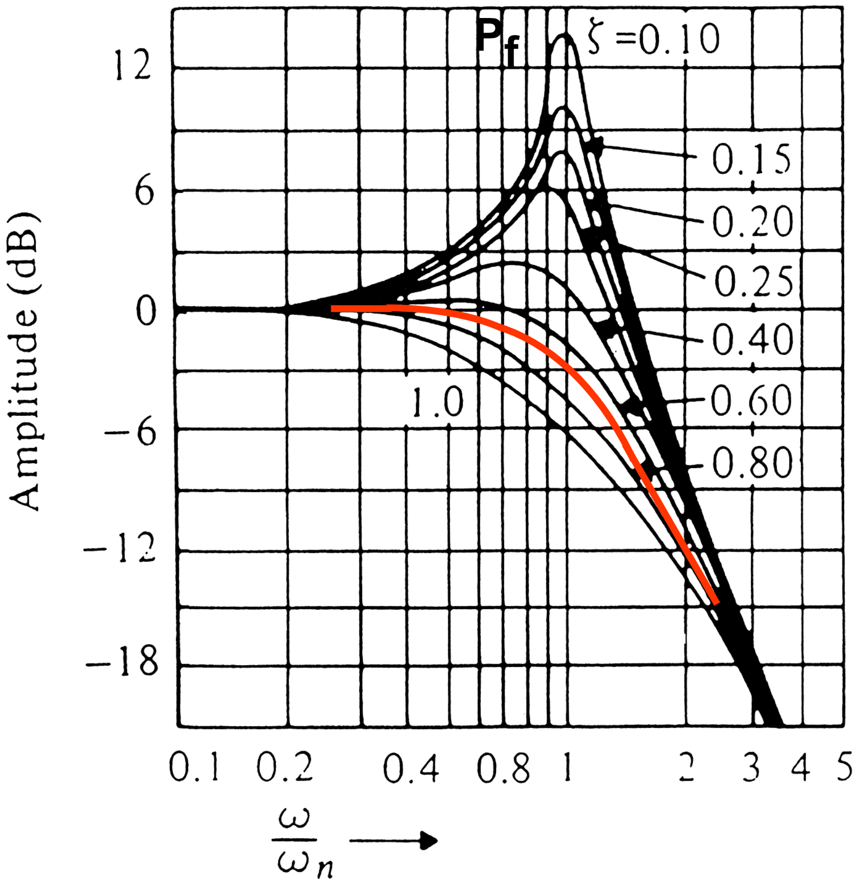
f_r is the resonant frequency

Relation PM, damping and f_2/GBW

$$f_r = \sqrt{\mathrm{GBW}\, f_2} \qquad \mathrm{PM}\,(^\circ) = 90^\circ - \arctan \frac{\mathrm{GBW}}{f_2} = \arctan \frac{f_2}{\mathrm{GBW}}$$

$\frac{f_2}{\mathrm{GBW}}$	PM (°)	$\zeta = \frac{1}{2}\sqrt{\frac{f_2}{\mathrm{GBW}}}$	P_f (dB)	P_t (dB)
0.5	27	0.35	3.6	2.3
1	45	0.5	1.25	1.3
1.5	56	0.61	0.28	0.73
2	63	0.71	0	0.37
3	72	0.87	0	0.04

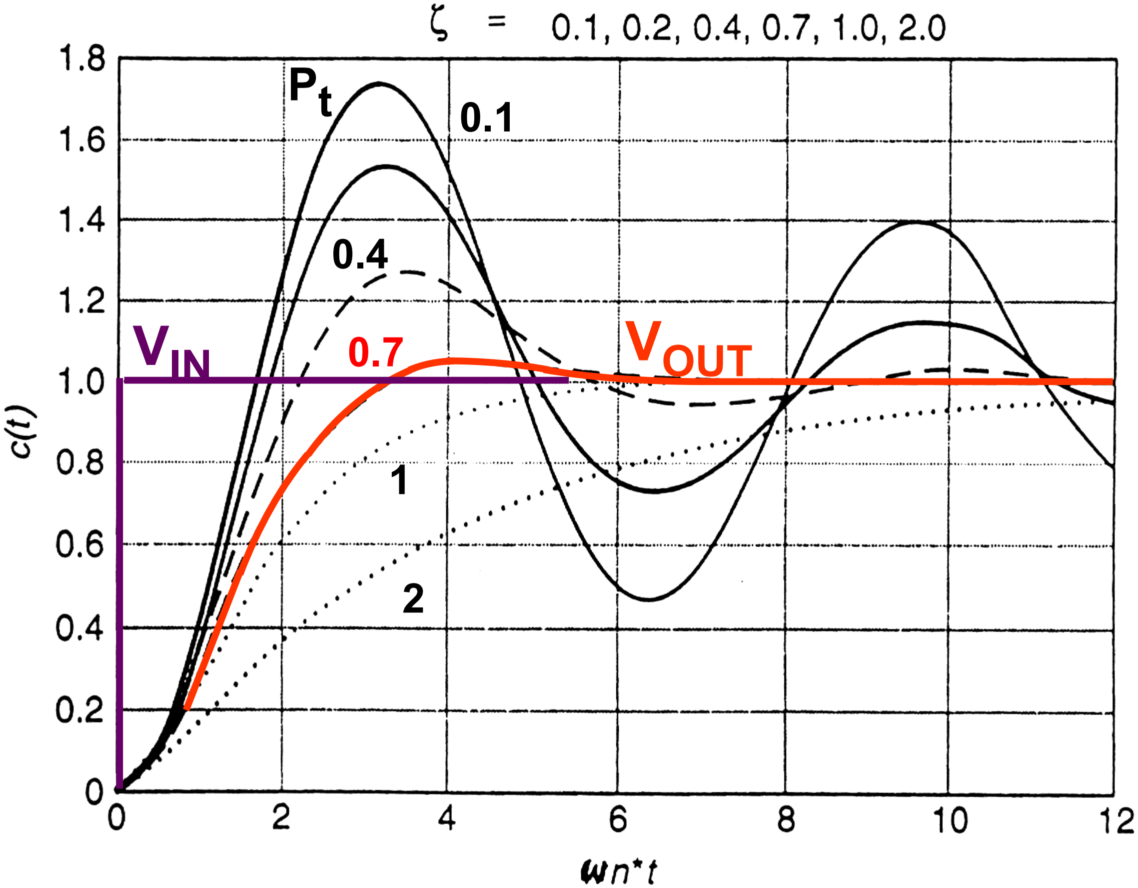
Amplitude response vs frequency



ζ = Q = 0.7

$$P_f = \frac{1}{2\,\zeta\sqrt{1-\zeta^2}}$$

Amplitude response vs time



$\zeta = Q = 0.7$

$$P_t = \frac{1}{1 + e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}}$$

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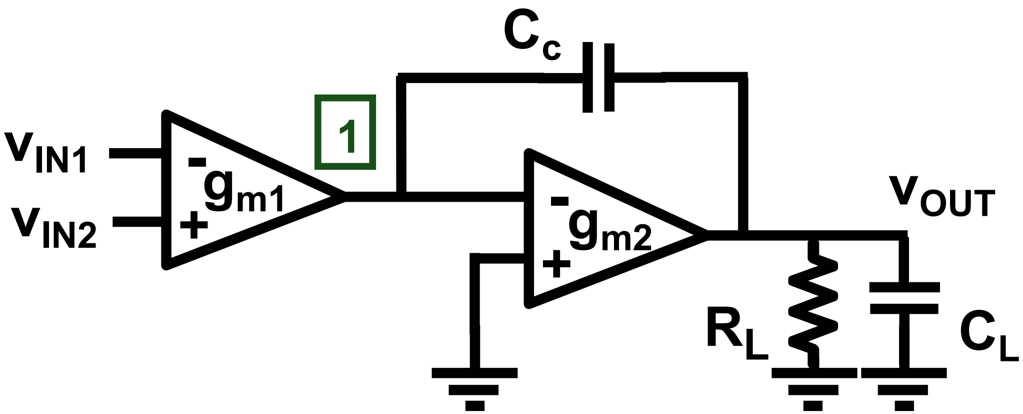
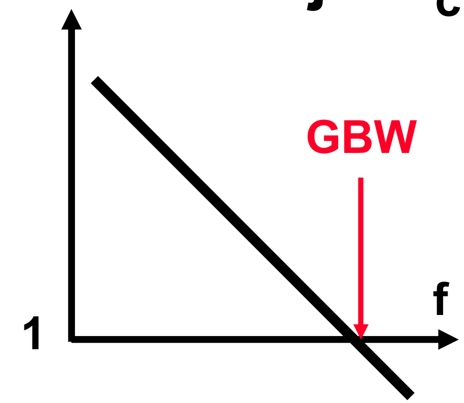
Generic 2-stage opamp

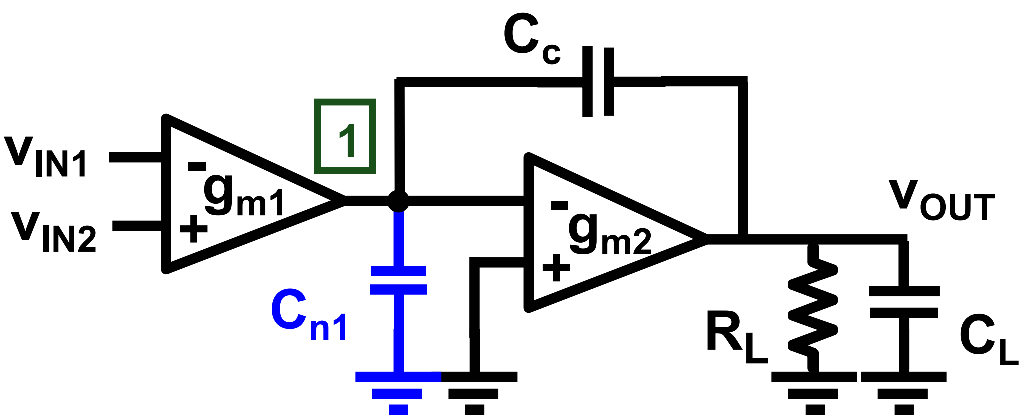


$$A_v = g_{m1} \frac{1}{j\omega\, C_c}$$

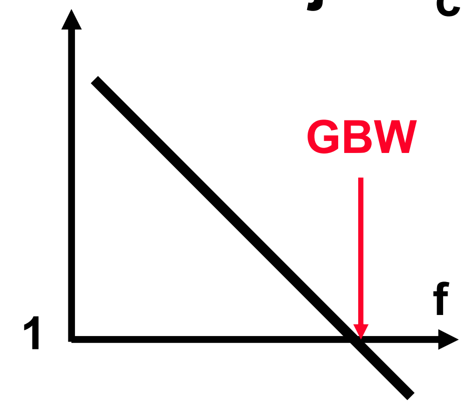


$|A_v| = 1 \Rightarrow$ $\boxed{GBW = \frac{g_{m1}}{2\pi\, C_c}}$ $\boxed{f_{nd} = \frac{g_{m2}}{2\pi\, C_L}}$

Generic 2-stage opamp



$$A_v = g_{m1} \frac{1}{j\omega\, C_c}$$



$|A_v| = 1 \Rightarrow$ $\boxed{GBW = \dfrac{g_{m1}}{2\pi\, C_c}}$

$$\boxed{f_{nd} = \frac{g_{m2}}{2\pi\, C_L} \frac{1}{1 + \dfrac{C_{n1}}{C_c}}}$$

Elementary design of 2-stage opamp

$$GBW = \frac{g_{m1}}{2\pi\, C_c} \qquad f_{nd} = 3\ GBW = \frac{g_{m2}}{2\pi\, C_L}\ \frac{1}{1 + \underbrace{\frac{C_{n1}}{C_c}}_{\approx 0.3}}$$

$$\boxed{\frac{g_{m2}}{g_{m1}} \approx 4\ \frac{C_L}{C_c}}$$

Larger current in 2nd stage !

GBW = 100 MHz for C_L = 2 pF

Solution: choose C_c = 1 pF

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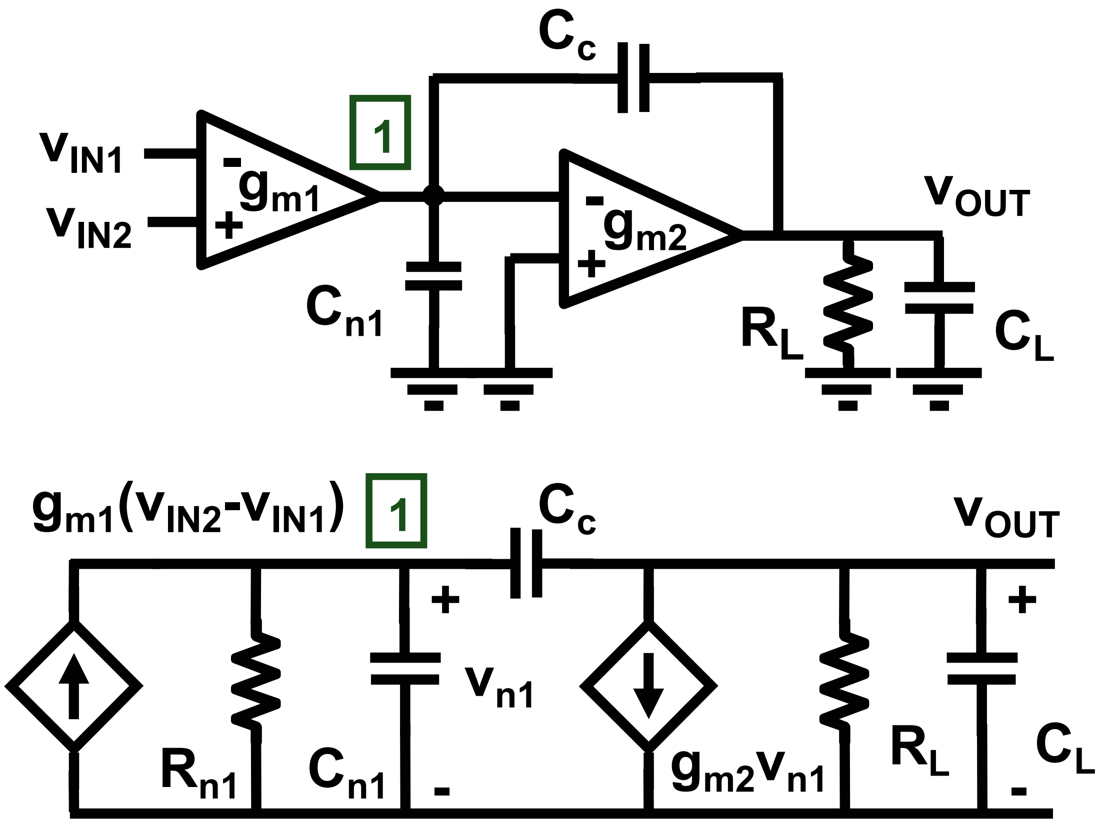
Generic 2-stage opamp : Miller OTA

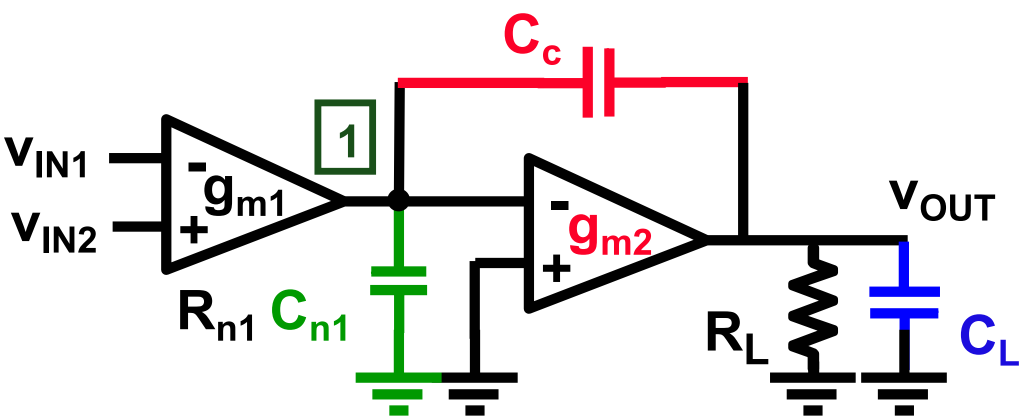


$$A_{v0} = - A_{v1}A_{v2}$$

$$A_{v1} = g_{m1}R_{n1}$$

$$A_{v2} = - g_{m2}R_L$$

Generic two-stage opamp



$$A_{v0} = - A_{v1}A_{v2}$$

$$A_{v1} = g_{m1}R_{n1}$$

$$A_{v2} = g_{m2}R_L$$

$$A_v = A_{v0} \frac{1 - \frac{C_c}{g_{m2}} s}{1 + (R_{n1}C_{n1} + A_{v2}R_{n1}C_c + R_LC_L)s + R_{n1}R_LCCs^2}$$

$$CC = C_{n1}C_c + C_{n1}C_L + C_cC_L$$

Approximate poles and zeros

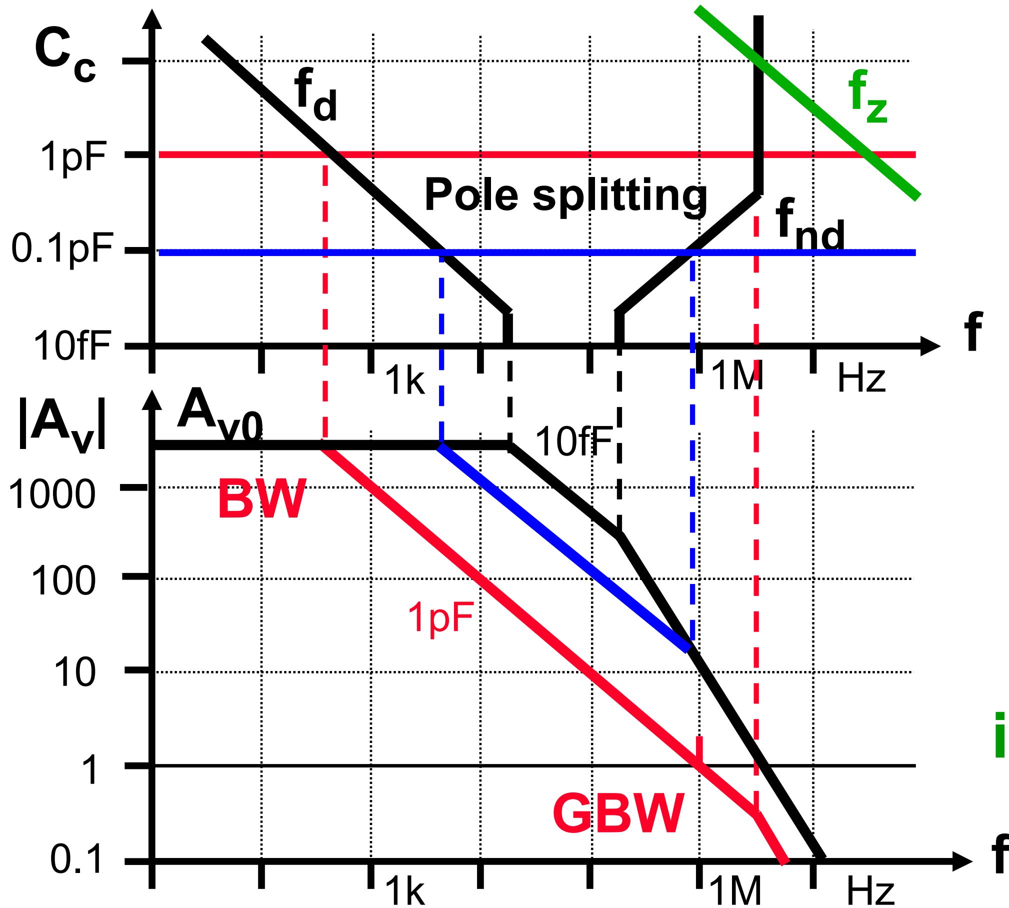
$$A = A_0 \frac{1 - cs}{1 + a\,s + b\,s^2}$$

Zero $s = \frac{1}{c}$

Pole $s_1 = -\frac{1}{a}$

$s_2 = -\frac{a}{b}$ if $s_2 >> s_1$

Miller OTA : pole splitting with C_c



Pole splitting for high C_c :

$$f_d = \frac{1}{2\pi\, A_{v2} R_{n1} C_c}$$

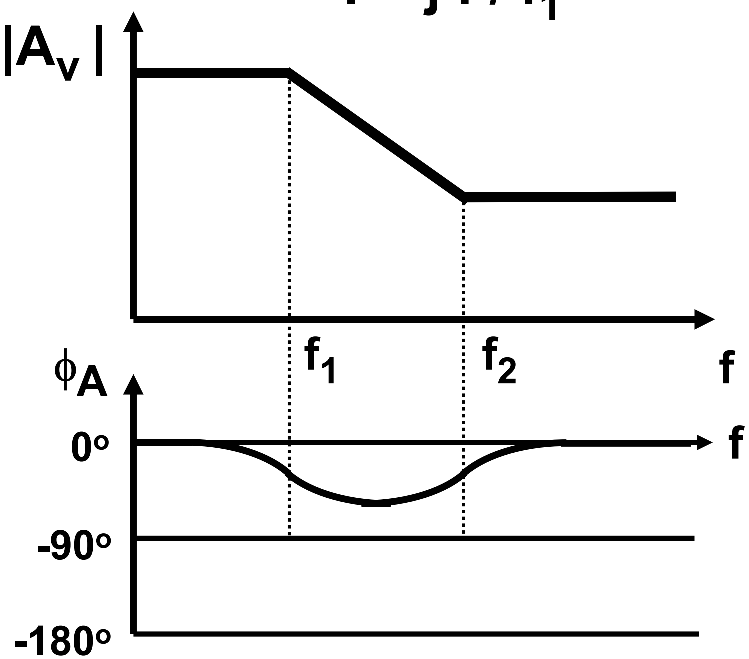
$$f_z = \frac{g_{m2}}{2\pi\, C_c}$$

is a positive zero !

Effect of positive zero

Negative zero

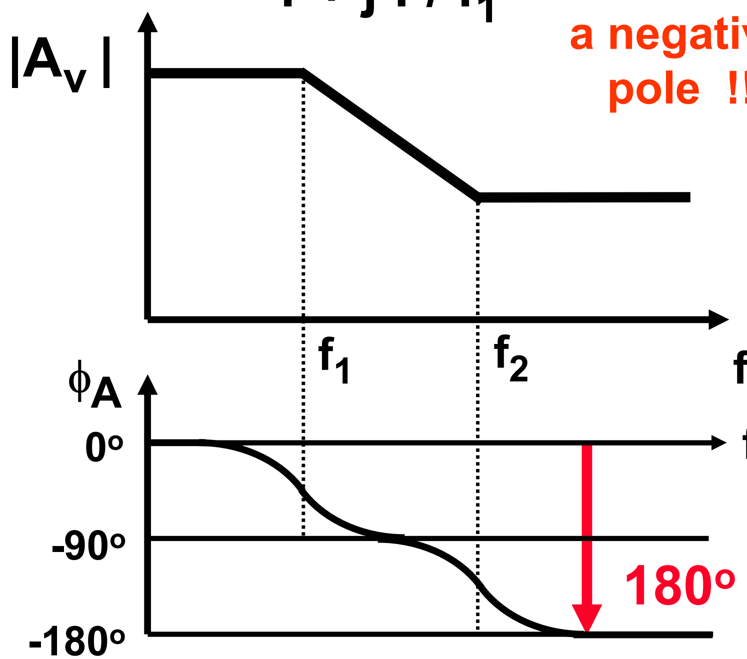
$$A_v = A_{v0} \frac{1 + j f / f_2}{1 + j f / f_1}$$



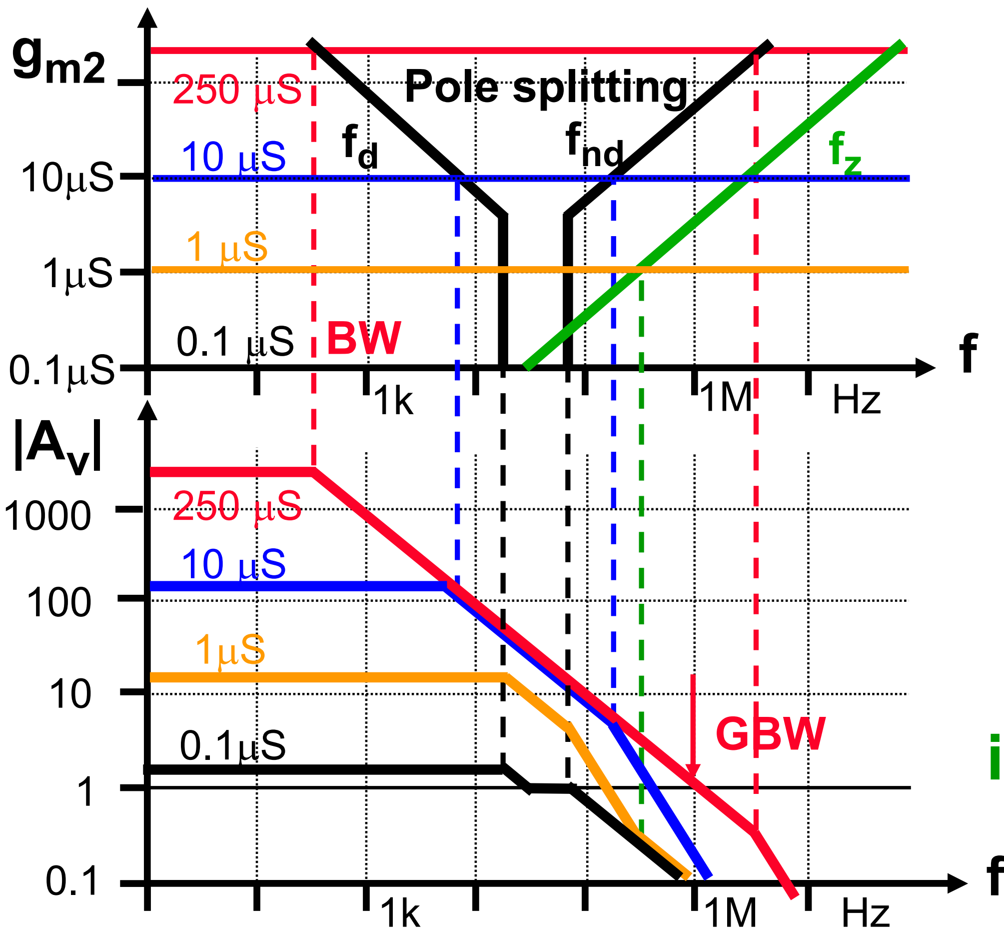
Positive zero

$$A_v = A_{v0} \frac{1 - j f / f_2}{1 + j f / f_1}$$

For phase, a positive zero is like a negative pole !!!



Miller OTA : pole splitting with g_{m2}



Pole splitting for high g_{m2} :

$$f_d = \frac{1}{2\pi\, A_{v2} R_{n1} C_c}$$

$$f_z = \frac{g_{m2}}{2\pi\, C_c}$$

is a positive zero !

Pole splitting by ...

$$\boxed{\frac{g_{m2}}{g_{m1}} \approx 4\ \frac{C_L}{C_c}}$$

or $\quad g_{m2}\, C_c \approx 4\ g_{m1}\, C_L$

$$\boxed{\textbf{both}\quad g_{m2}\, C_c}$$

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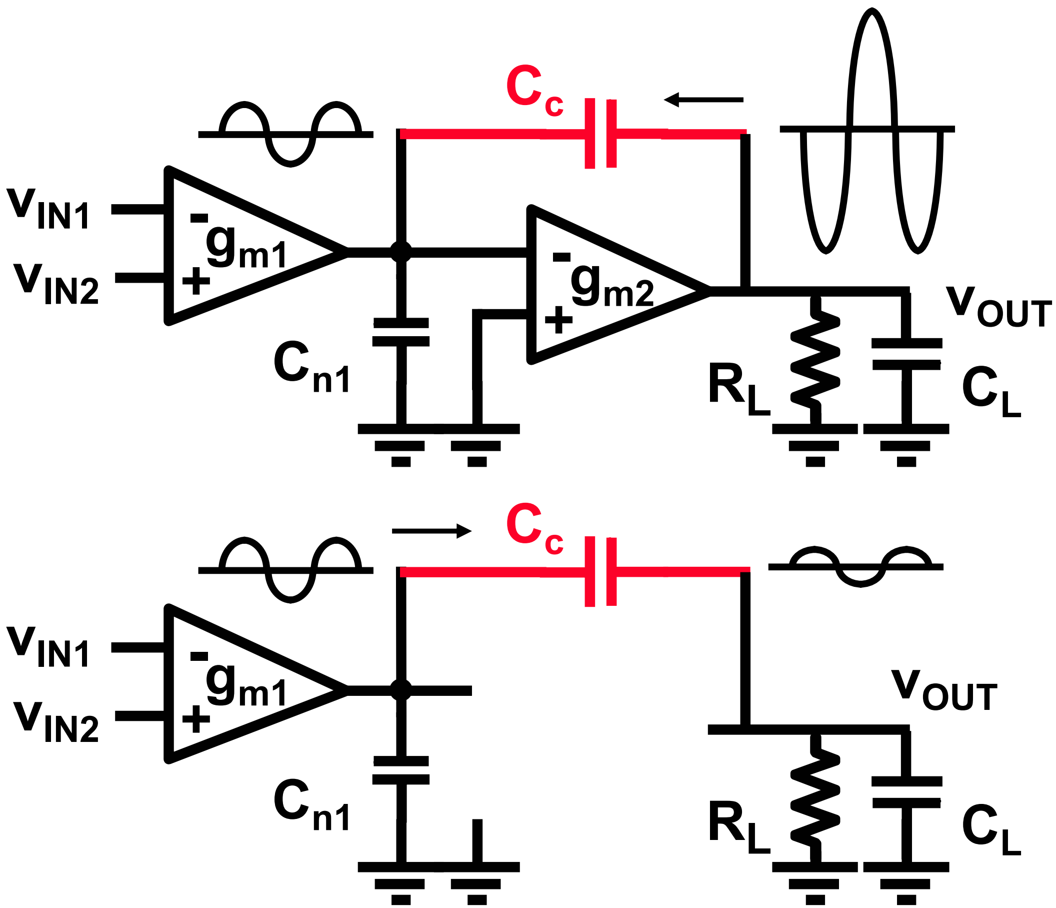
- Use of operational amplifiers
- Stability of 2-stage opamp
- Pole splitting
- Compensation of positive zero
- Stability of 3-stage opamp

Positive zero because feedforward

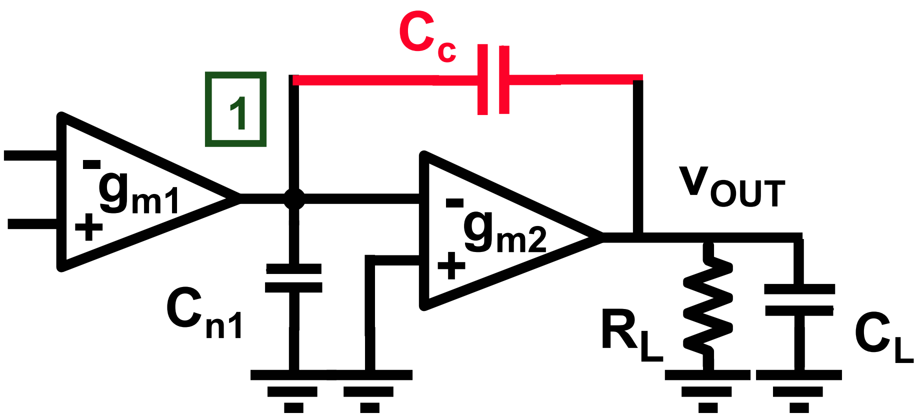


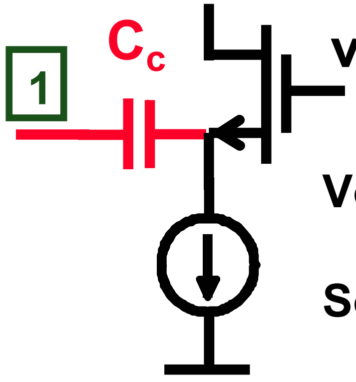
Miller effect Is feedback

Feedforward

Cut !

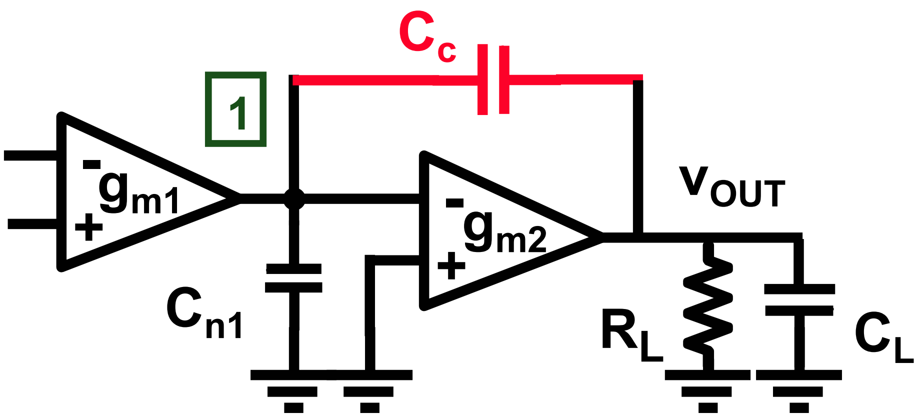
Cut feedforward through C_c - 1

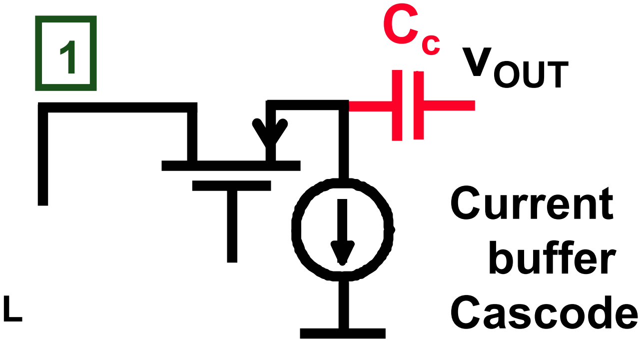


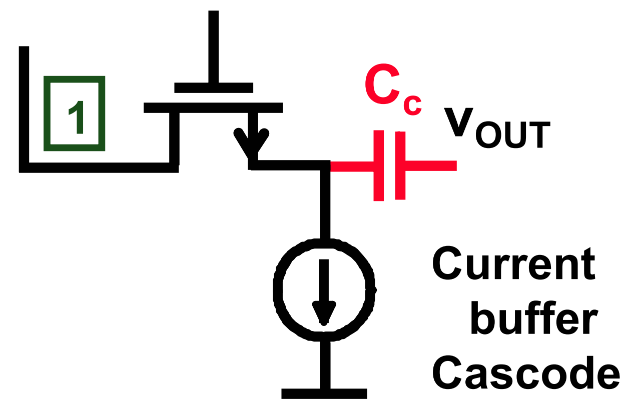


Ref. Tsividis, JSSC Dec.76, 748-753

Cut feedforward through C_c - 2

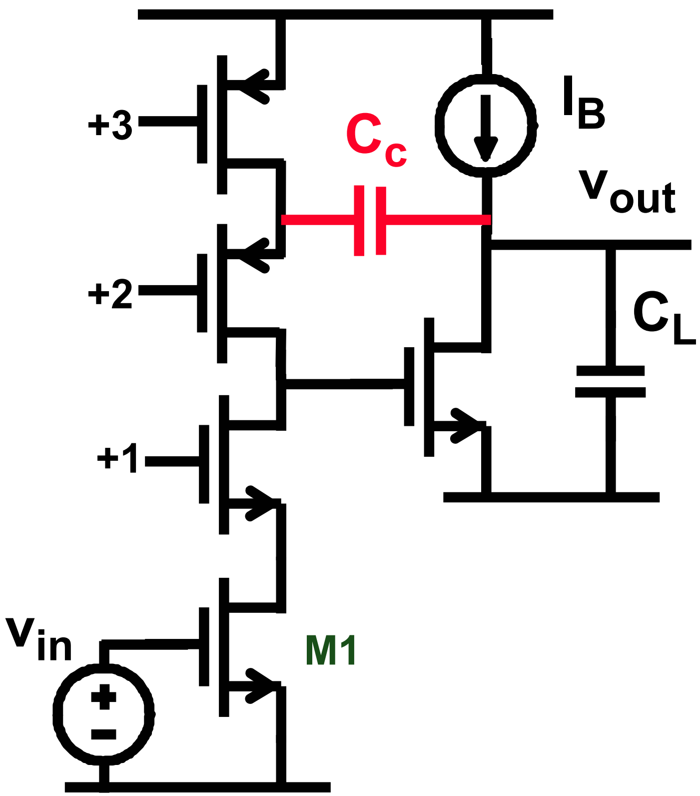


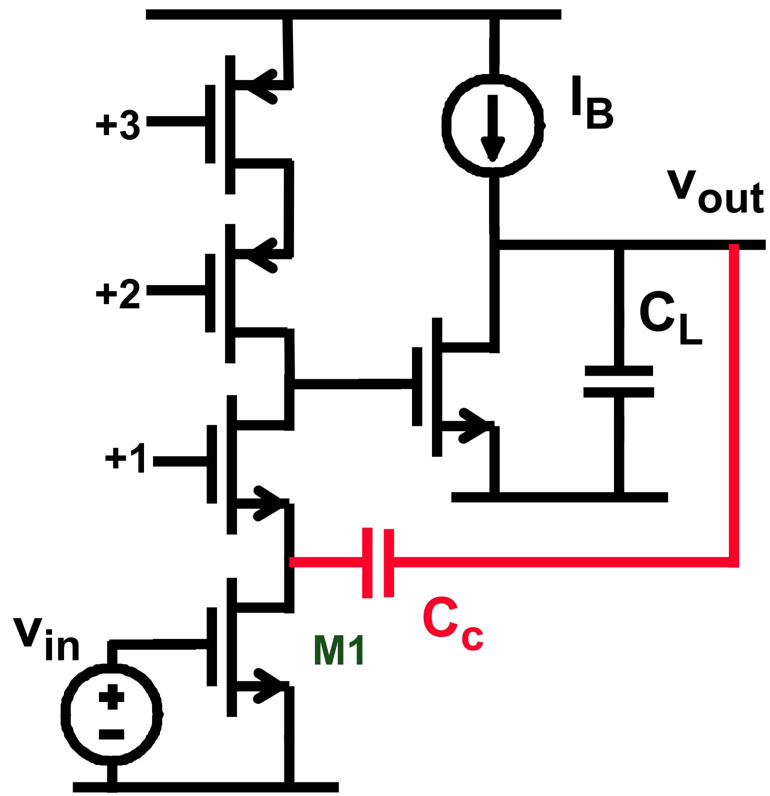




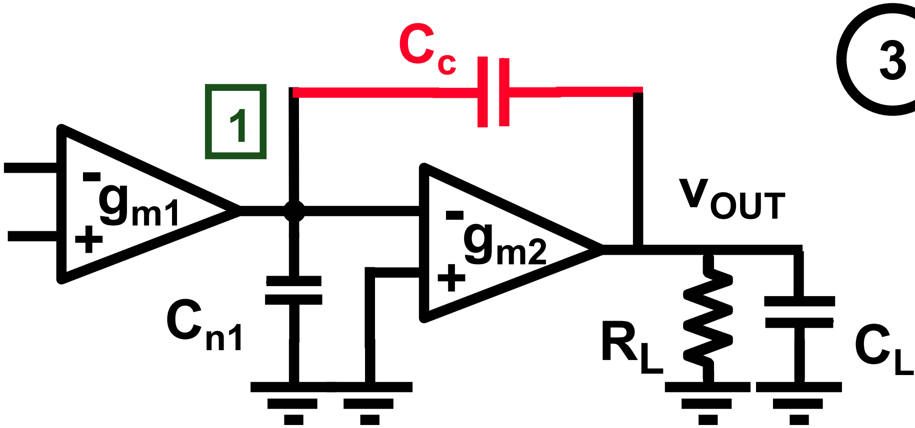
Ref. Ahuja, JSSC Dec 83, 629-633

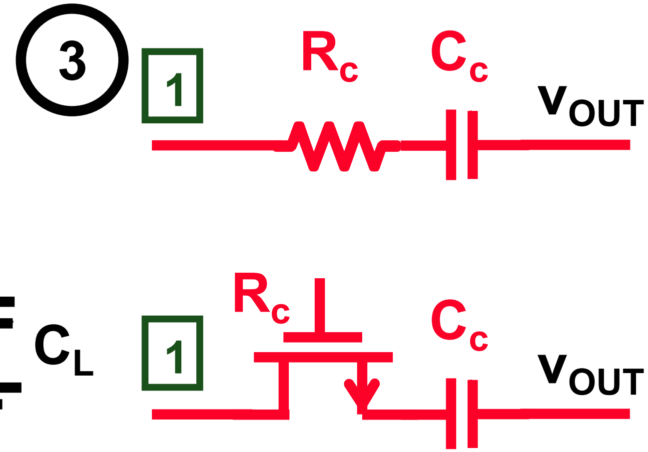
Compensation with cascodes





Cut feedforward through C_c - 3





$$f_z = \frac{1}{2\pi\, C_c\,(1/g_{m2} - R_c)}$$

$R_c = 1/g_{m2}$ **No zero**

$R_c > 1/g_{m2}$ **Negative zero**

Ref. Senderovics, JSSC Dec 78, 760-766

Negative zero compensation

$R_c >> 1/g_{m2}$ $\Rightarrow$ $f_z = - \dfrac{1}{2\pi\, C_c\, R_c}$

$f_z = 3\ GBW$ $\Rightarrow$ $R_c = \dfrac{1}{3\ g_{m1}}$

Final choice :

$$\boxed{\frac{1}{g_{m2}} < R_c < \frac{1}{3g_{m1}}}$$

Exercise of 2-stage opamp

GBW = 50 MHz for C_L = 2 pF

Find I_{DS1}; I_{DS2} ; C_c and R_c !

Choose $C_c = 1$ pF > $g_{m1} = 2\pi\, C_c GBW = 315\ \mu S$

$I_{DS1} = 31.5\ \mu A$ & $1/g_{m1} \approx 3.2\ k\Omega$

f_{nd} = 150 MHz > $g_{m2} = 2\pi\, C_L 4GBW = 8g_{m1} = 2520\ \mu S$

$I_{DS2} = 252\ \mu A$ & $1/g_{m2} \approx 400\ \Omega$

$400\ \Omega < R_c < 1\ k\Omega$: $R_c = 1/\sqrt{2.5} \approx 400\sqrt{2.5} \approx 640\ \Omega \pm 60\%$

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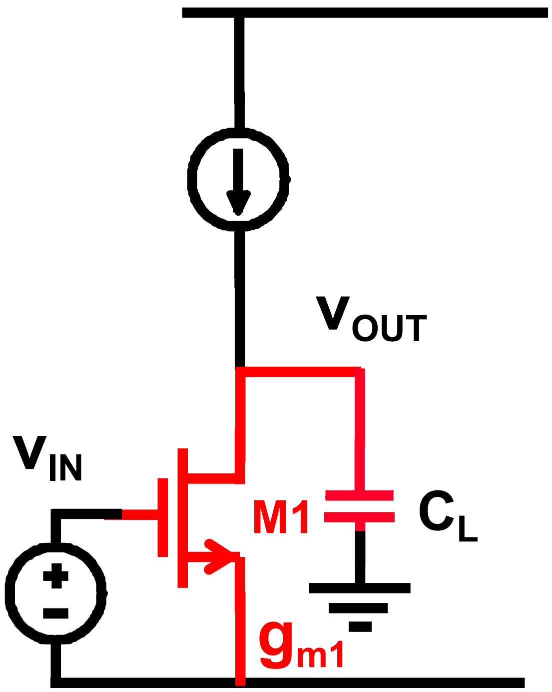
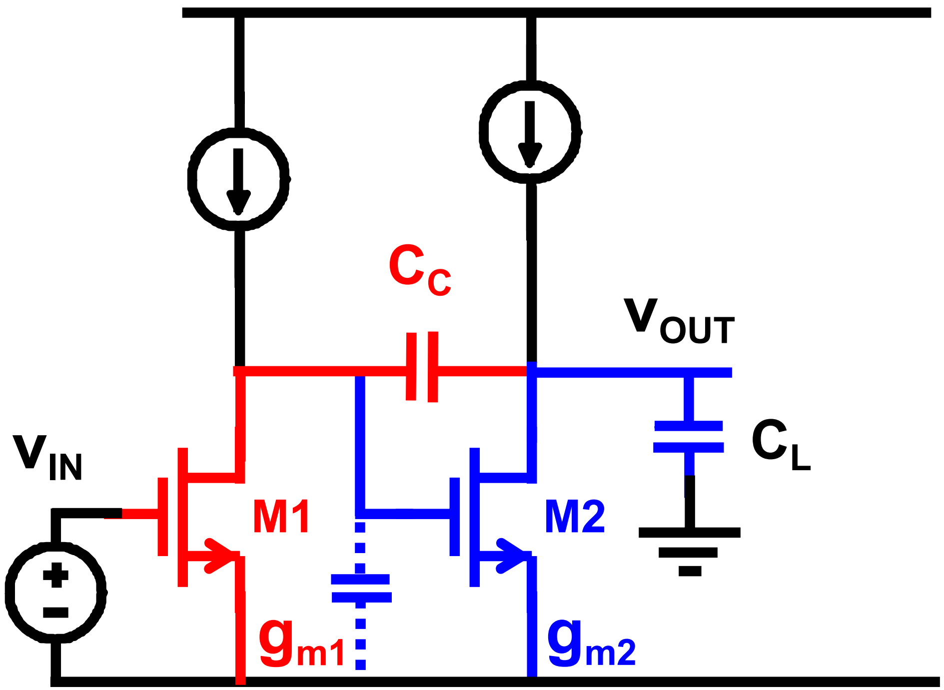
1-stage CMOS OTA



$$GBW = \frac{g_{m1}}{2\pi\, C_L}$$

2-stage Miller CMOS OTA

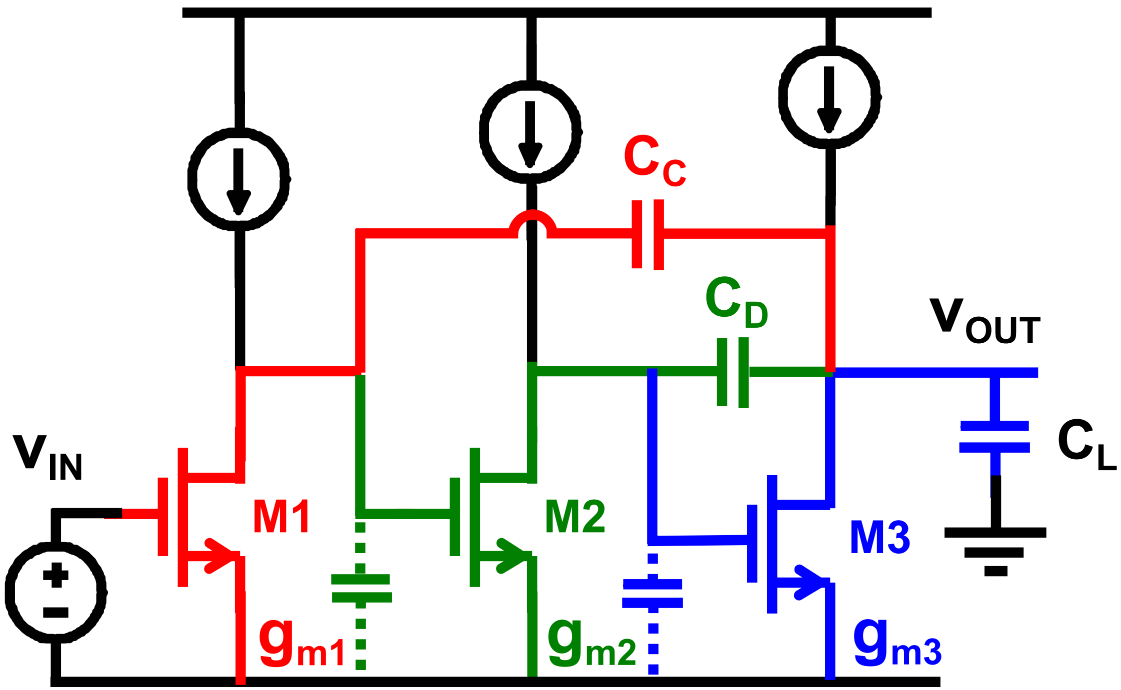


$$GBW = \frac{g_{m1}}{2\pi\, C_C}$$

$$f_{nd1} = \frac{g_{m2}}{2\pi\, C_L}$$

$$f_{nd1} = 3\ GBW$$

3-stage Nested Miller CMOS OTA



$$\mathrm{GBW} = \frac{g_{m1}}{2\pi\, C_C}$$

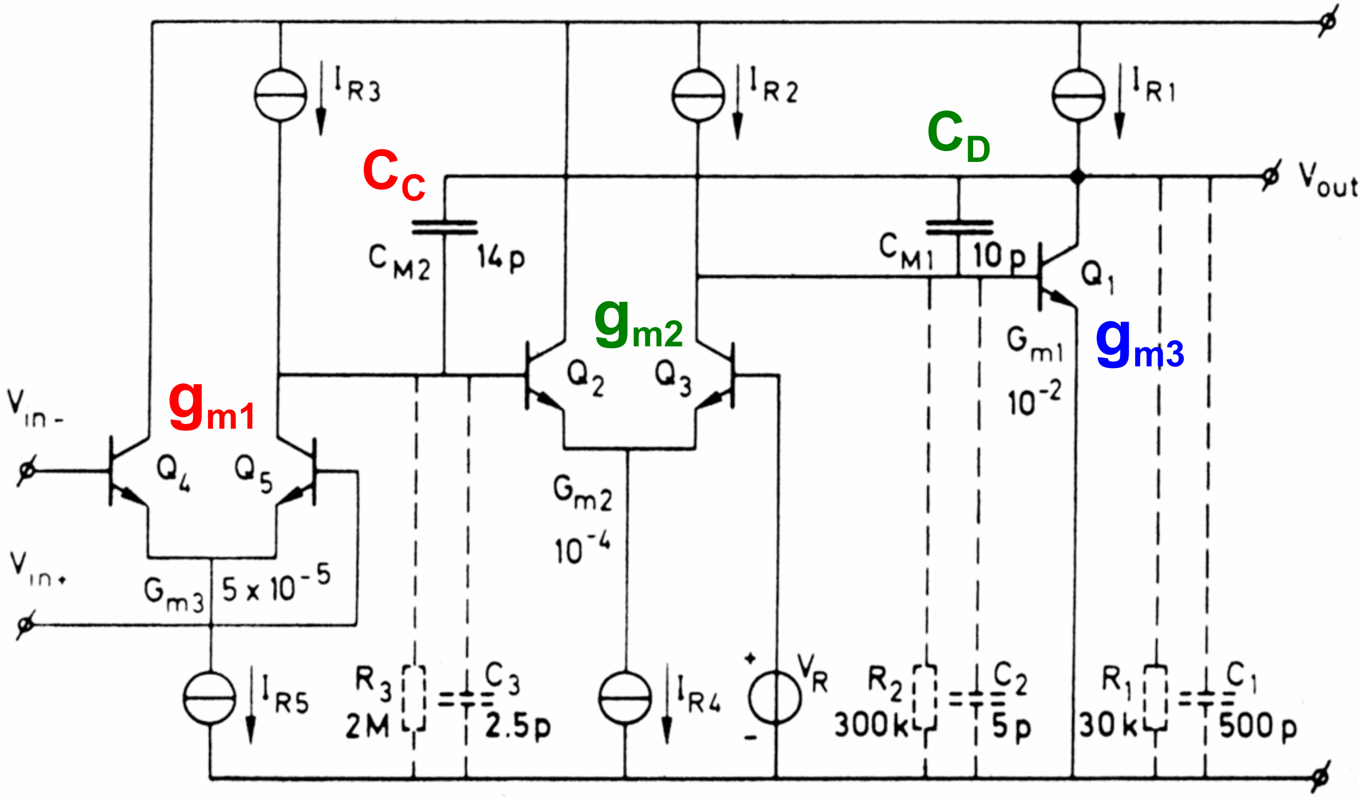
$$f_{nd1} = \frac{g_{m2}}{2\pi\, C_D}$$

$$f_{nd2} = \frac{g_{m3}}{2\pi\, C_L}$$

$f_{nd1} = 3\ \mathrm{GBW}$

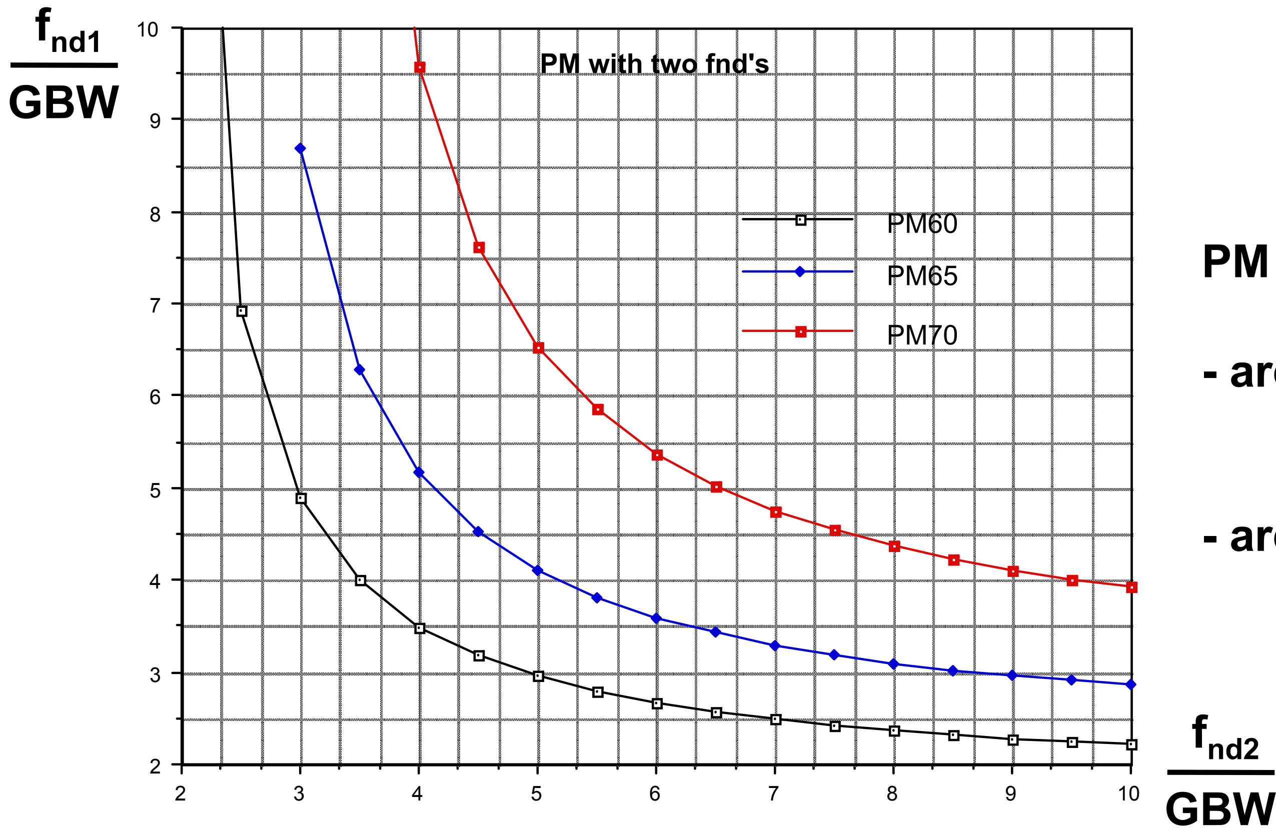
$f_{nd2} = 5\ \mathrm{GBW}$

Nested Miller with differential pair



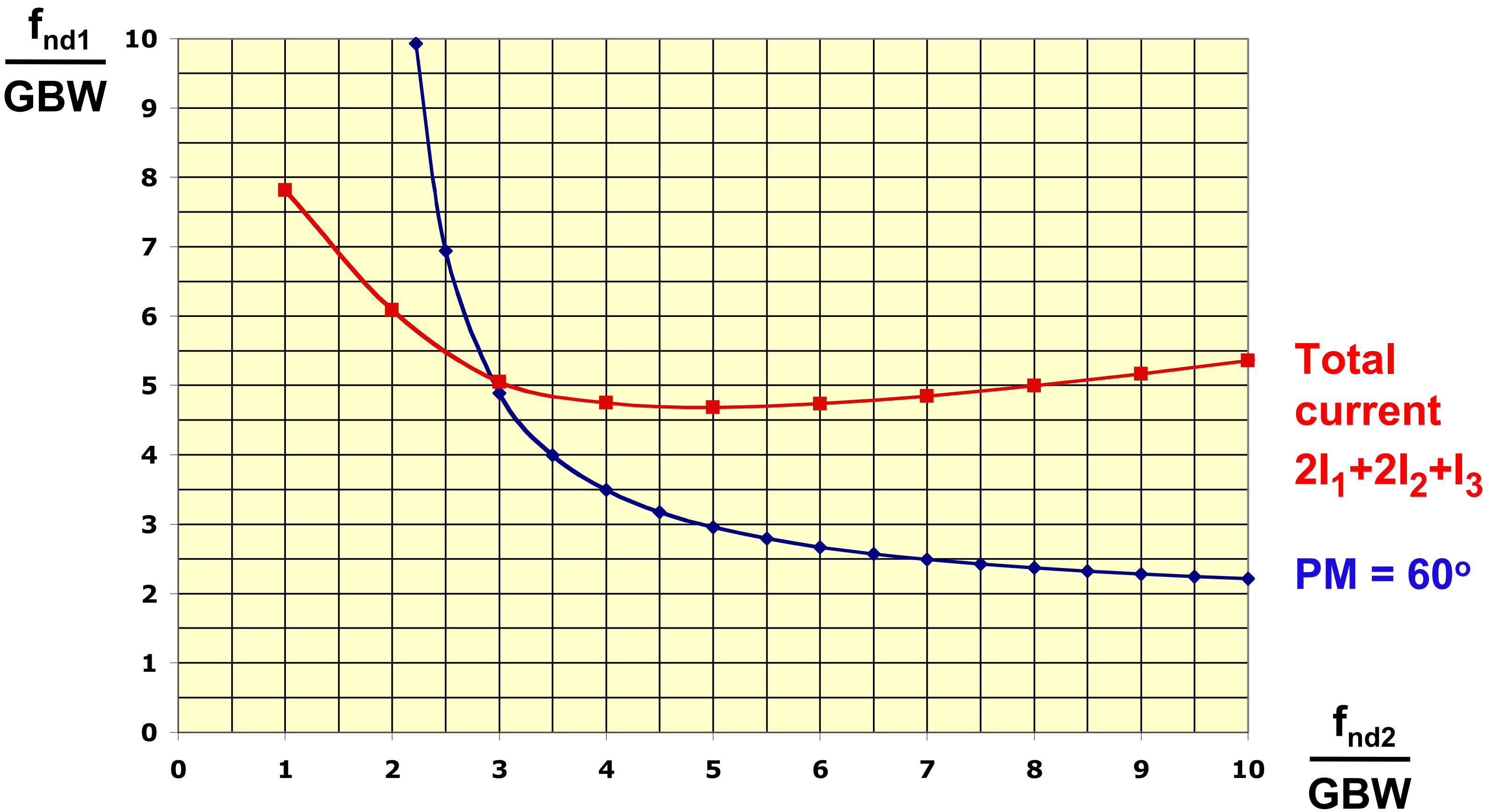
Huijsing, JSSC Dec.85, pp.1144-1150

Relation between the f_{nd}'s



$$PM = 90^o - \arctan\left(\frac{GBW}{f_{nd1}}\right) - \arctan\left(\frac{GBW}{f_{nd2}}\right)$$

Relation f_{nd}'s and power



Elementary design of 3-stage opamp

$$GBW = \frac{g_{m1}}{2\pi\, C_C}$$

$$f_{nd1} = 3\ GBW = \frac{g_{m2}}{2\pi\, C_D}$$

$$f_{nd2} = 5\ GBW = \frac{g_{m3}}{2\pi\, C_L}$$

Choose $C_D \approx C_C$!

$$\frac{g_{m2}}{g_{m1}} \approx 3 \qquad \frac{g_{m3}}{g_{m1}} \approx 5\ \frac{C_L}{C_C}$$

Even larger current in output stage !

Exercise of 3-stage opamp

GBW = 50 MHz for C_L = 2 pF

Find I_{DS1}; I_{DS2} ; I_{DS3} ; C_C and C_D !

Choose $C_C = C_D = 1$ pF > $g_{m1} = 2\pi\, C_C GBW = 315\ \mu S$

$I_{DS1} = 31\ \mu A$

f_{nd1} = 150 MHz > $g_{m2} = 2\pi\, C_D 3GBW = 3g_{m1} = 945\ \mu S$

$I_{DS2} = 95\ \mu A$

f_{nd2} = 250 MHz > $g_{m3} = 2\pi\, C_L 5GBW = 10g_{m1} = 3150\ \mu S$

$I_{DS3} = 315\ \mu A$

Comparison 1, 2 & 3 stage designs

GBW = 50 MHz for C_L = 2 pF

Single stage : I_{DS1} = 31 μA $I_{TOT} = 2I_{DS1}$ = 62 μA

Two stages : Choose C_C = 1 pF

I_{DS1} = 31 μA I_{DS2} = 252 μA $I_{TOT} = 2I_{DS1} + I_{DS2}$ = 314 μA

Three stages : Choose $C_C = C_D$ = 1 pF

I_{DS1} = 31 μA I_{DS2} = 95 μA I_{DS3} = 315 μA

$I_{TOT} = 2I_{DS1} + 2I_{DS2} + I_{DS3}$ = 567 μA

Table of contents

- Use of operational amplifiers
- Stability of 2-stage opamp
- Pole splitting
- Compensation of positive zero
- Stability of 3-stage opamp

0.6 chap6

Systematic Design of Operational Amplifiers

Willy Sansen

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Leuven, Belgium

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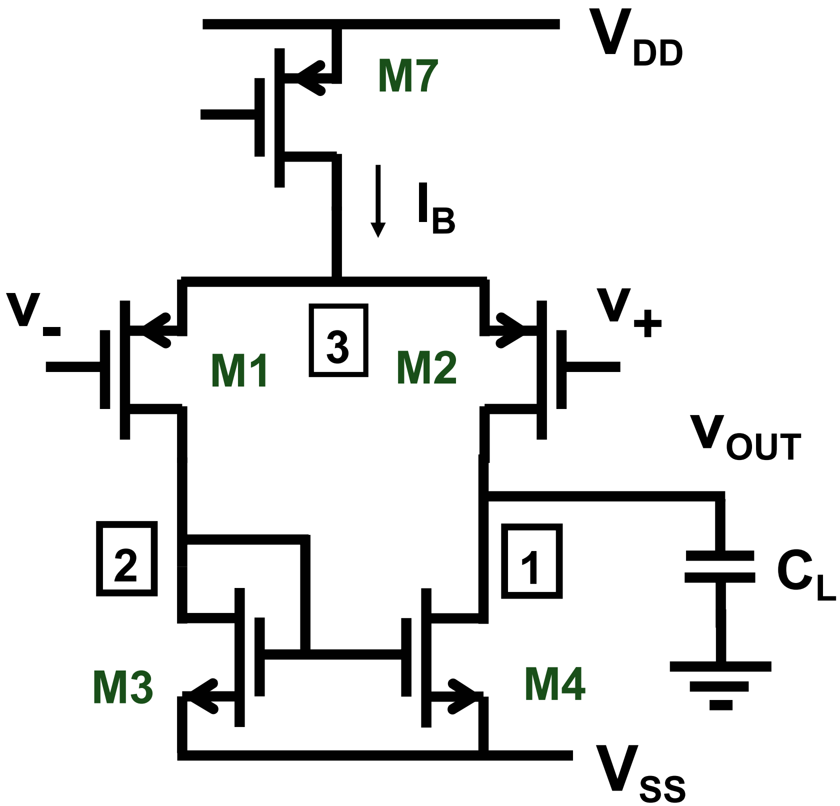
- Design of Single-stage OTA
- Design of Miller CMOS OTA
- Design for GBW and Phase Margin
- Other specs: Input range, output range, SR, ...

Ref.: Sansen : Analog design essentials, Springer 2006

Single-stage CMOS OTA : GBW



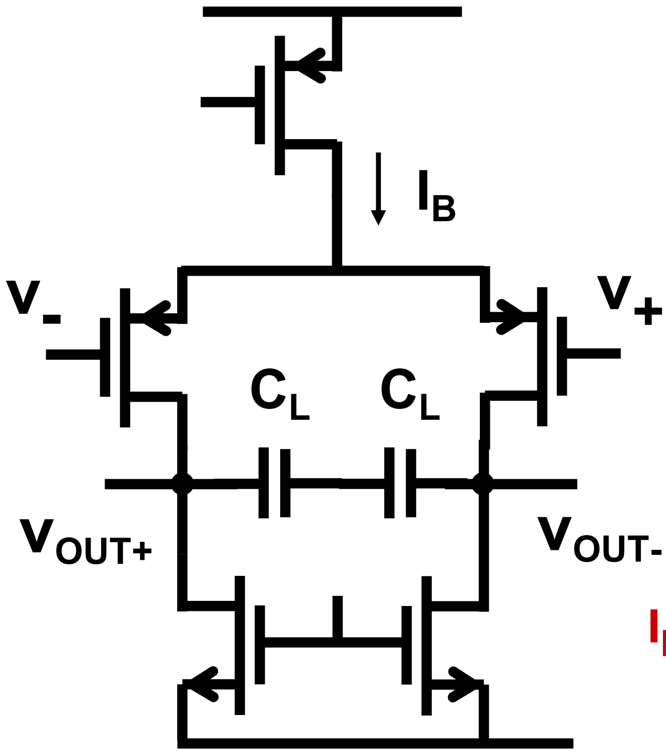
$$A_v = g_{m1} \frac{r_o}{2}$$

if $r_{o2} = r_{o4} = r_o$

$$BW = \frac{1}{2\pi \frac{r_o}{2} (C_L + C_{n1})}$$

$$\boxed{GBW = \frac{g_{m1}}{2\pi (C_L + C_{n1})}}$$

CMOS OTA : Maximum GBW



$$GBW = \frac{g_{m1}}{2\pi C_L} \qquad g_{m1} = \frac{I_B}{V_{GS1}-V_T}$$

$$\boxed{GBW_{max} = \frac{I_B}{V_{GS1}-V_T}\,\frac{1}{2\pi C_L}}$$

0.2 V

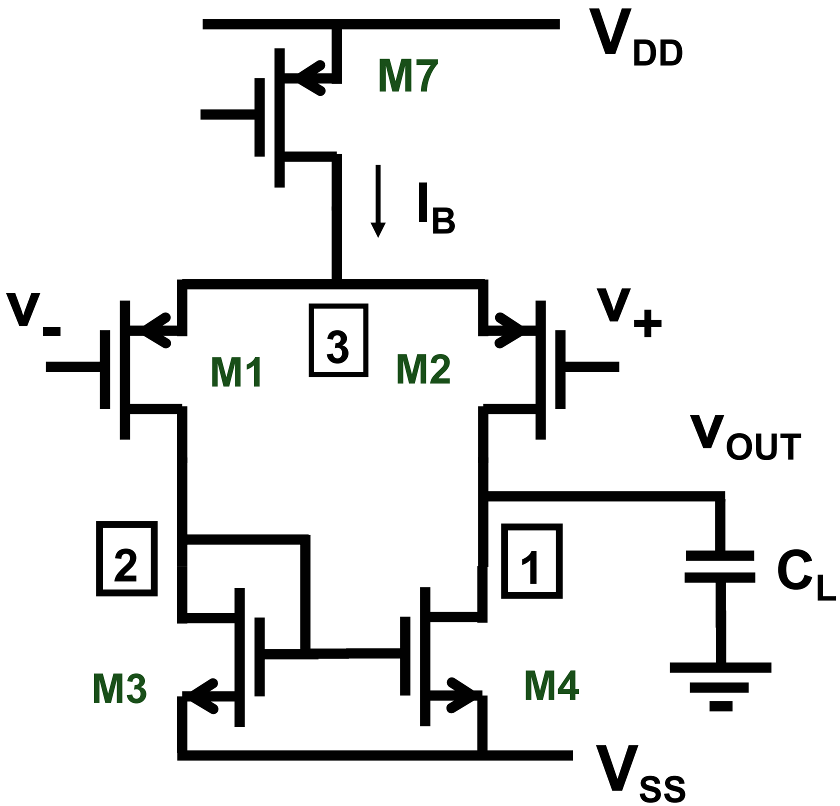
$I_B = 10\ \mu A$ $C_L = 1\ pF$ $GBW_{max} \approx 10\ MHz$

[8]

$$FOM = \frac{GBW.C_L}{I_B} = 1000$$ [800]

MHzpF/mA

Single stage CMOS OTA : f_{nd}



$$\boxed{GBW = \frac{g_{m1}}{2\pi\,(C_L + C_{n1})}}$$

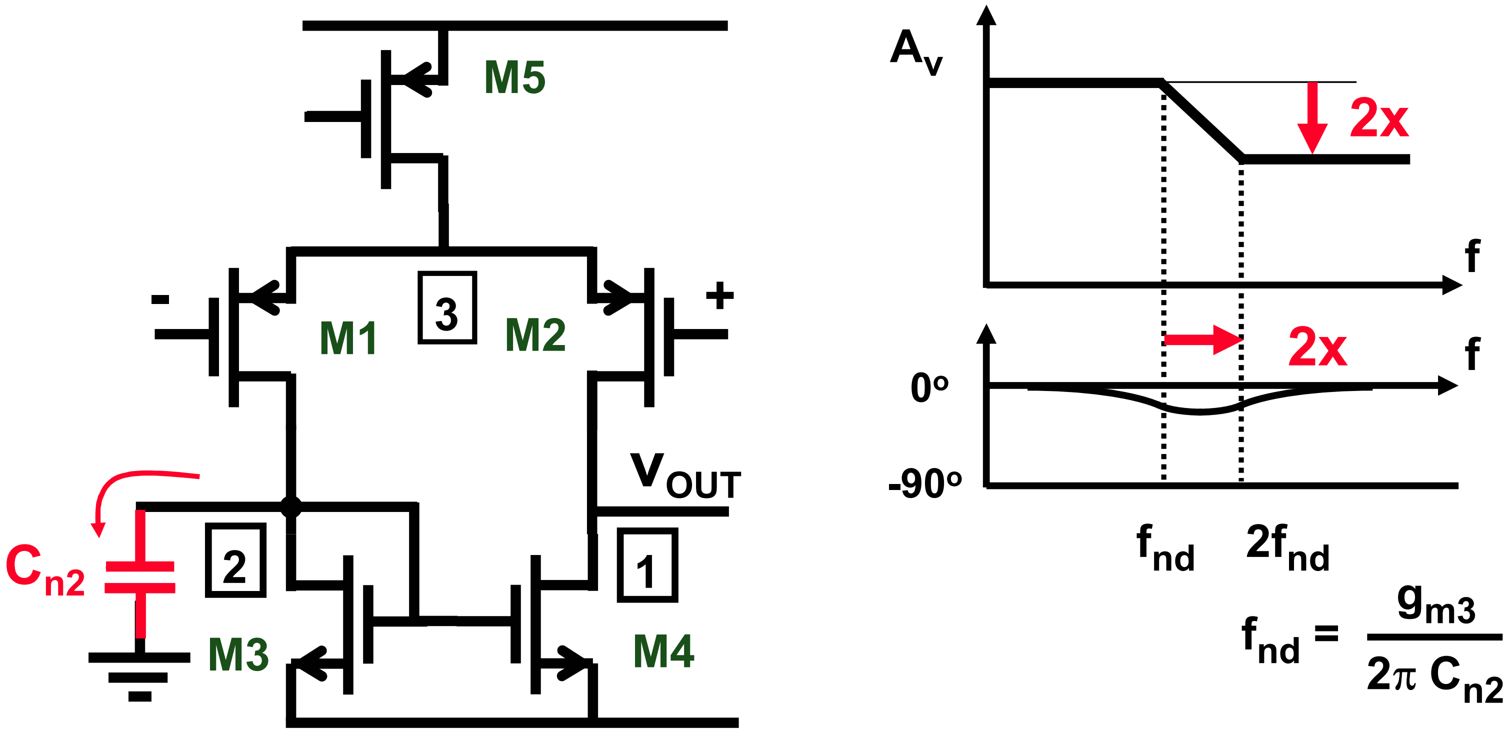
$$f_{nd} = \frac{g_{m3}}{2\pi\,C_{n2}}$$

$$C_{n2} \approx 2C_{GS3} + C_{DB3} + C_{DB1}$$

$$\approx 4\,C_{GS3}$$

$$\boxed{f_{nd} \approx \frac{f_{T3}}{4}}$$

Simple CMOS OTA : f_{nd}



$$f_{nd} = \frac{g_{m3}}{2\pi C_{n2}}$$

$$PM = 90^o - \arctan\frac{GBW}{f_{nd}} + \arctan\frac{GBW}{2\,f_{nd}} \approx 85^o$$

Single stage CMOS OTA : Design 1

GBW = 100 MHz for $C_L = 2$ pF

Techno: $L_{min} = 0.35\ \mu m$; $K'_n = 60\ \mu A/V^2$ & $K'_p = 30\ \mu A/V^2$

I_{DS} ? W ? L ?

$$g_m = GBW\ 2\pi\ C_L = 1.2\ mS$$

$V_{GS}-V_T = 0.2$ V

$$I_{DS} = g_m \frac{V_{GS}-V_T}{2} = \frac{g_m}{10} = 0.12\ mA$$

$$\frac{W}{L} = \frac{I_{DS}}{K'(V_{GS}-V_T)^2} = 100$$

$L_p = L_n = 1\ \mu m$ **GAIN !**

$W_p = 100\ \mu m$; $W_n = 50\ \mu m$

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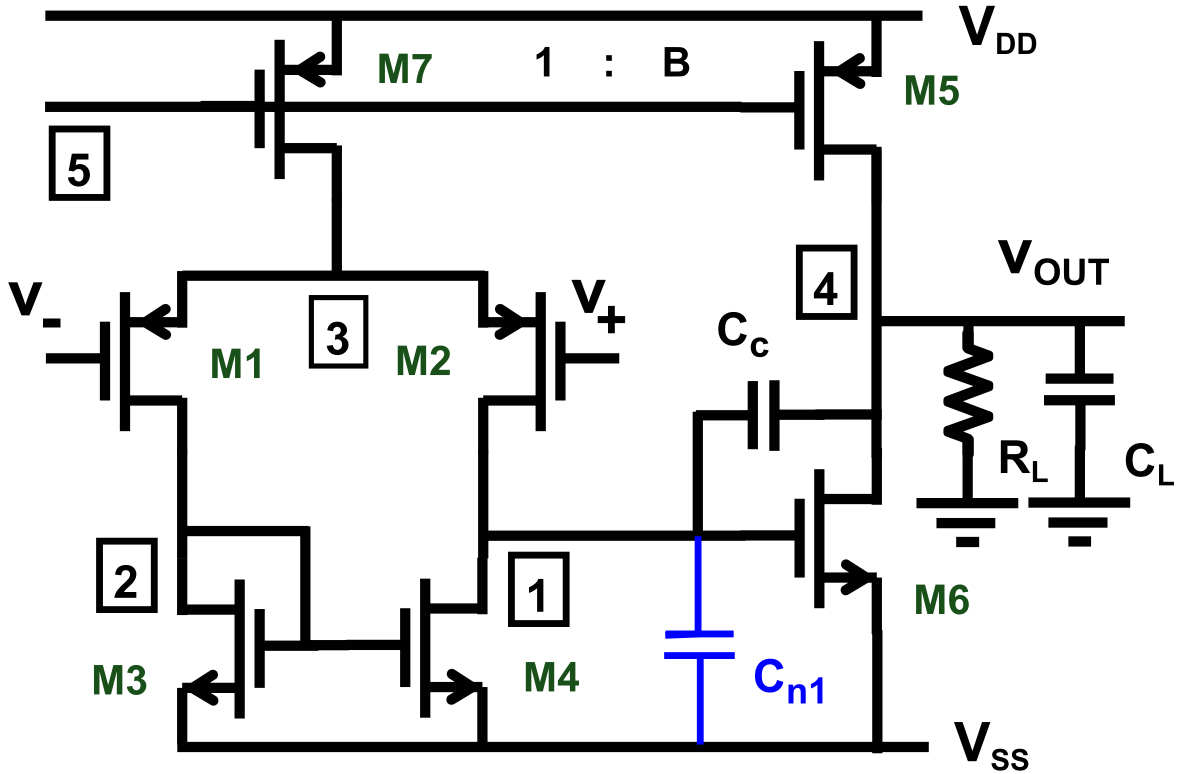
Miller CMOS OTA



Two nodes

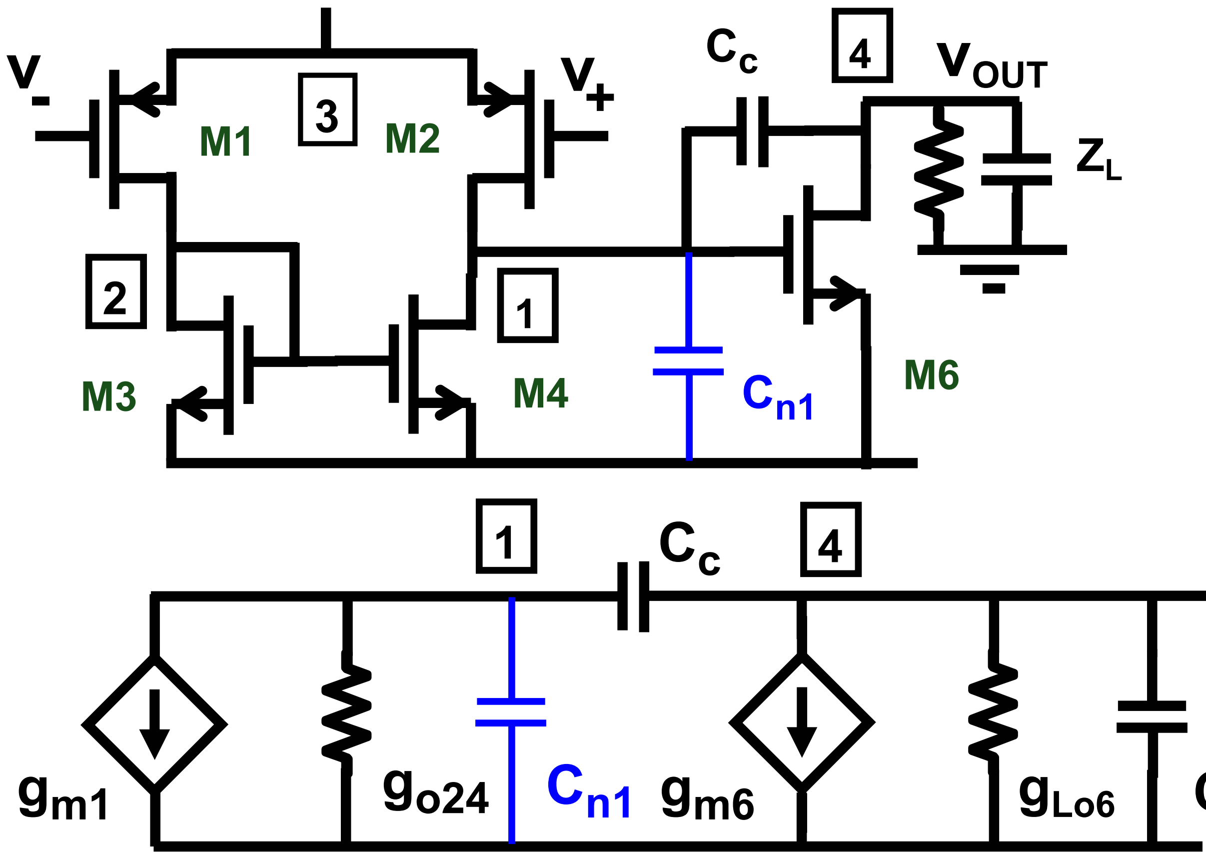
$\boxed{1}$ $\boxed{4}$

with high Impedance

cause two poles

split by C_c

Miller CMOS OTA : small-signal



GBW= 1MHz

C_L = 10 pF

R_L = 10 kΩ

g_{m1} = 7.5 μS

g_{o24} = 0.03 μS

C_{n1} = 0.37 pF

C_c = 1 pF

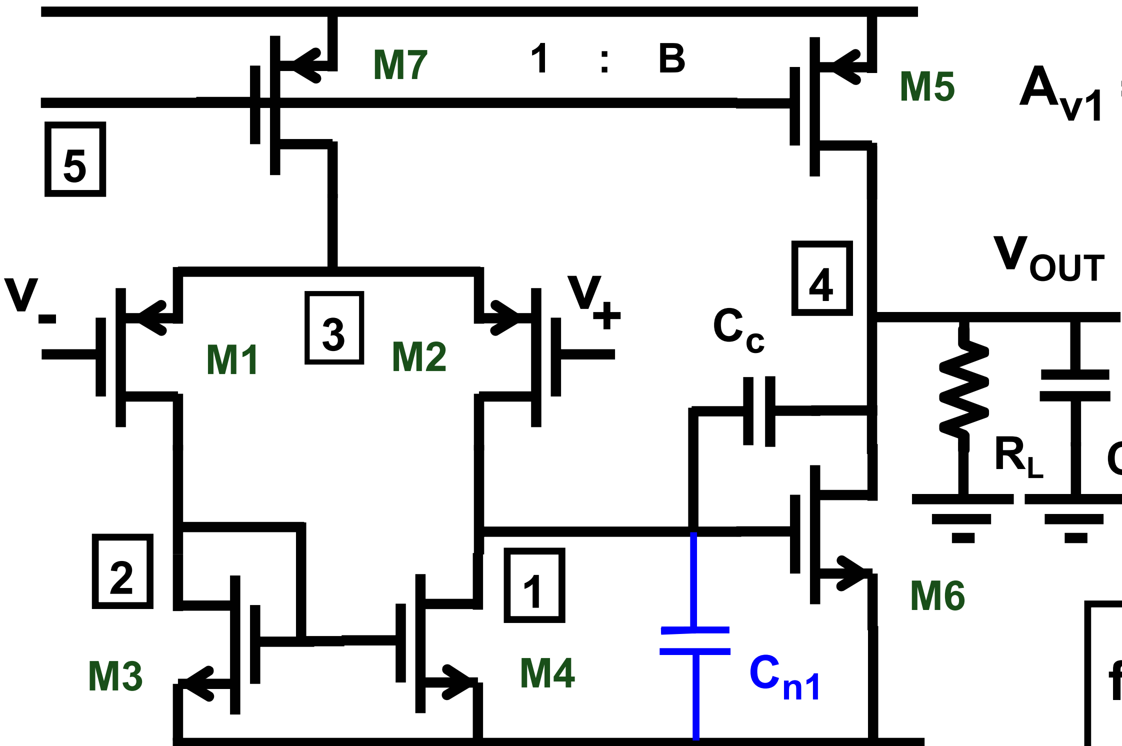
g_{m6} = 246 μS

g_{Lo6} = 120 μS

I_{DS1} = 1.1 μA

I_{DS6} = 25 μA

Miller CMOS OTA : GBW



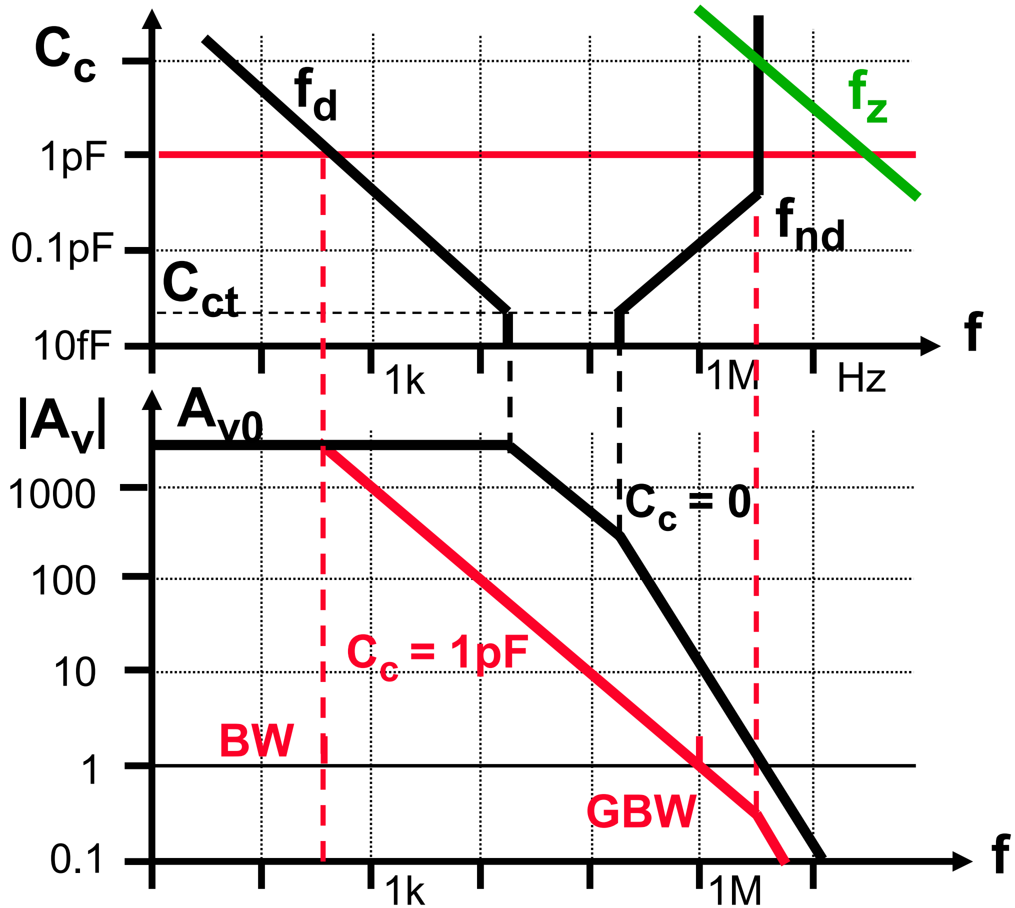
$$A_{v1} = \frac{g_{m1}}{g_{o24}} \qquad A_{v2} = \frac{g_{m6}}{g_{Lo6}}$$

$$BW = \frac{g_{o24}}{2\pi A_{v2} C_c}$$

$$\boxed{GBW = \frac{g_{m1}}{2\pi C_c}}$$

$$\boxed{f_{nd} \approx \frac{g_{m6}}{2\pi C_{Ln4}} \frac{1}{1+\frac{C_{n1}}{C_c}}}$$

Miller CMOS OTA : poles and zero



Pole splitting starts at

$$C_{ct} \approx \frac{C_{n1}}{A_{v2}} \approx 20 \text{ fF}$$

but is sufficient for C_c = 1pF

$$f_z = \frac{g_{m6}}{2\pi\, C_c}$$

Table of contents

- **Design of Single-stage OTA**
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Miller CMOS OTA: Design plan

$$GBW = \frac{g_{m1}}{2\pi\, C_c}$$

GBW = 100 MHz and C_L = 2 pF

$$f_{nd} \approx \frac{g_{m6}}{2\pi\, C_{Ln4}} \, \frac{1}{1+\frac{C_{n1}}{C_c}}$$

Two equations for

Three variables g_{m1}, g_{m6} and C_c ?!?

Solution : choose g_{m1} or g_{m6} or C_c !!!

What is wrong with

choosing $\boxed{C_c = 1\ \text{pF}}$?

Miller CMOS OTA: Design vs C_c

Choose $C_c \approx 3\, C_{n1}$ $\quad$ $GBW = \dfrac{g_{m1}}{2\pi\, C_c}$

$$3GBW \approx \frac{g_{m6}}{2\pi\, C_{Ln4}} \frac{1}{1.3}$$

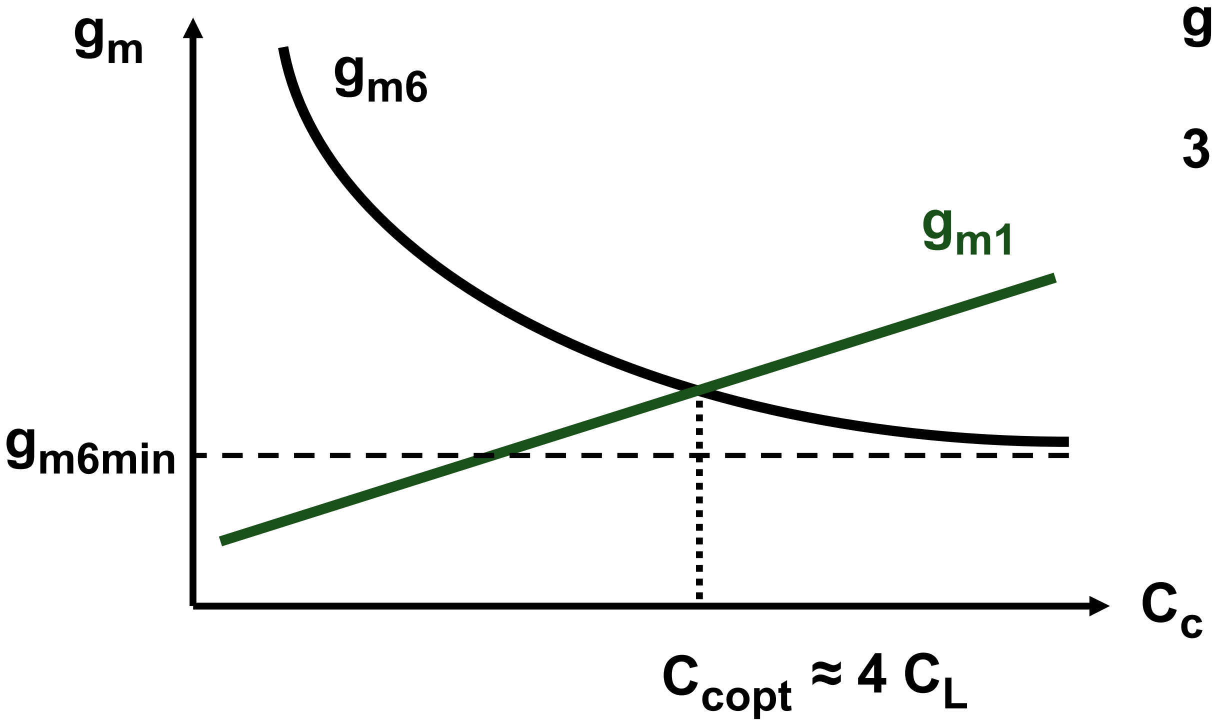
$$\boxed{\frac{g_{m6}}{g_{m1}} \approx 4\, \frac{C_L}{C_c}}$$

$GBW = 100$ MHz and $C_L = 2$ pF

Choose $C_{n1} < C_c < C_L$

Choice $C_c = 1$ pF gives $g_{m1} = 0.6$ mS and $g_{m6} = 4.8$ mS

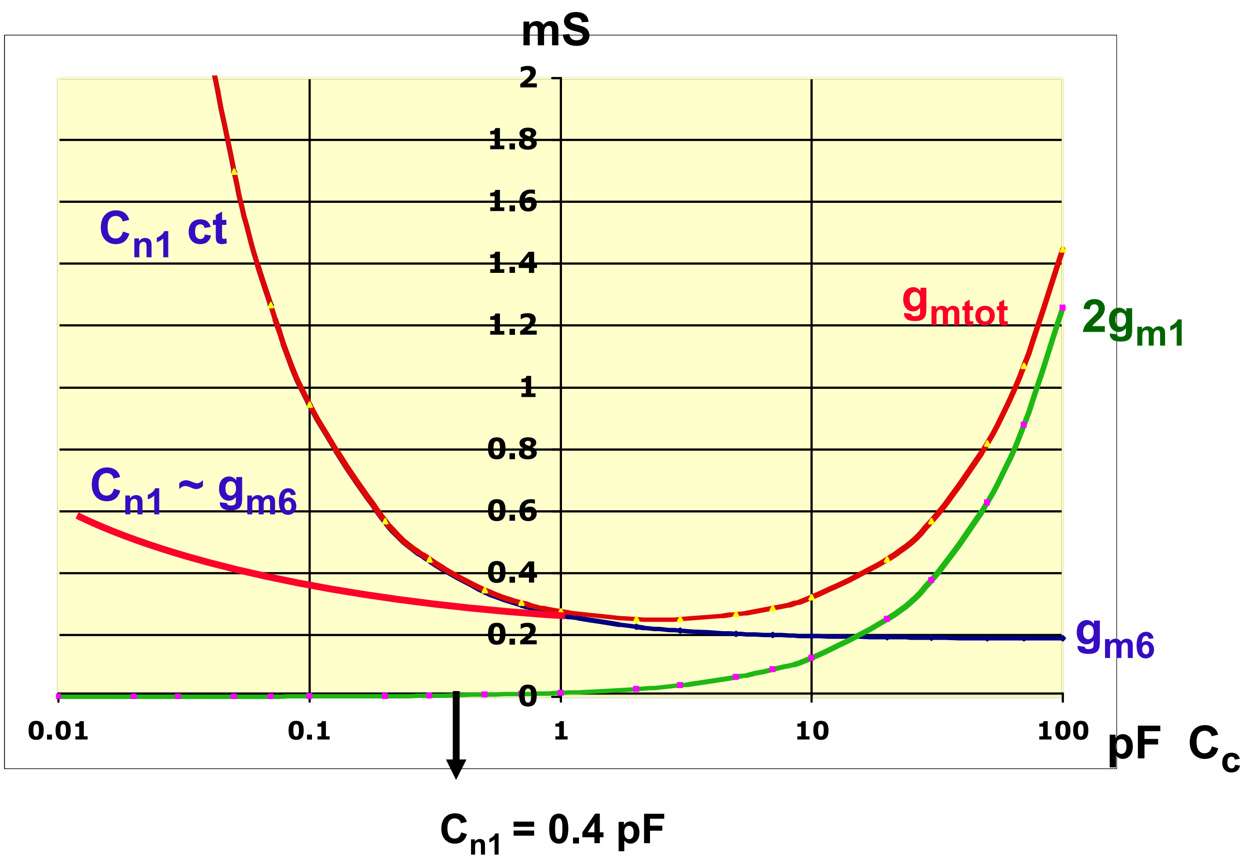
Miller CMOS OTA: Design vs C_c



g_{m6min} =

3 GBW (2π C_L)

1 MHz Miller CMOS OTA: Design vs C_c



GBW = 1 MHz

C_L = 10 pF

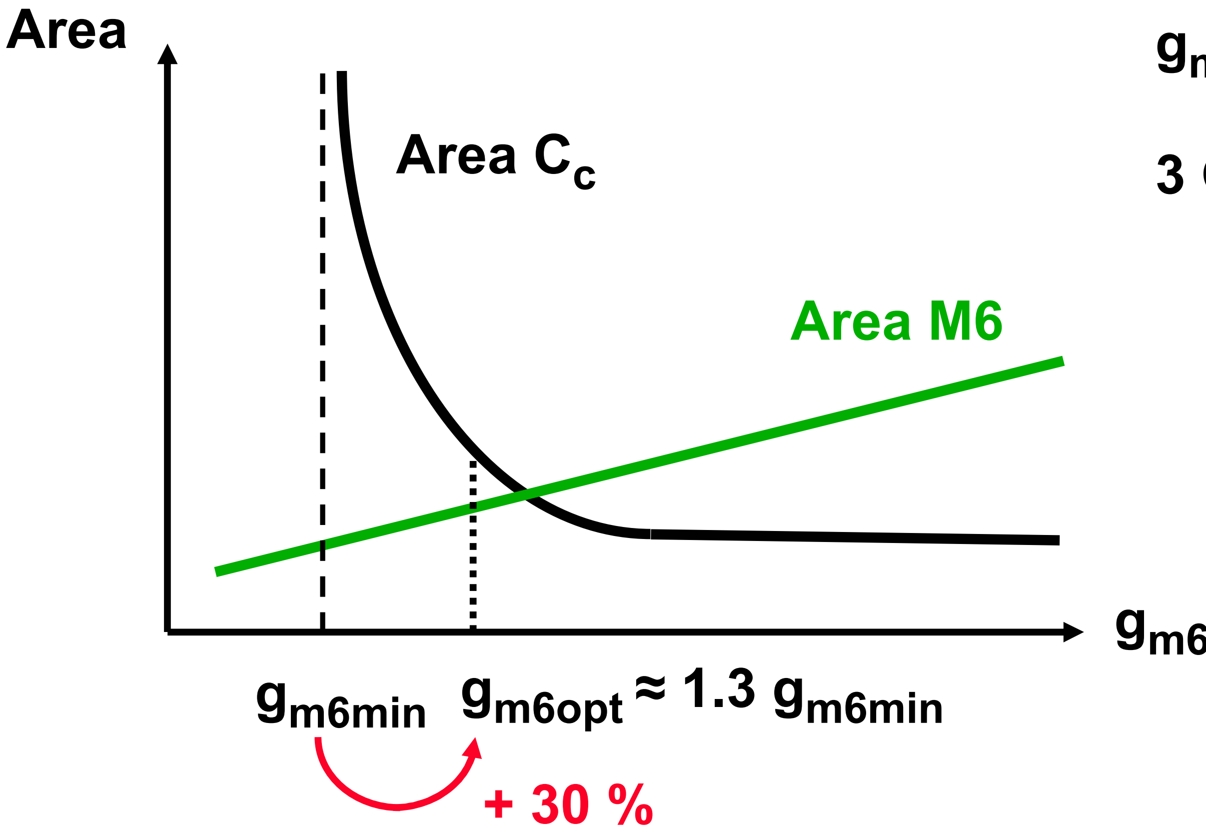
C_{n1} = 0.4 pF

K' = 20 μA/V^2

V_{GS}-V_T = 0.2 V

L = 10 μm

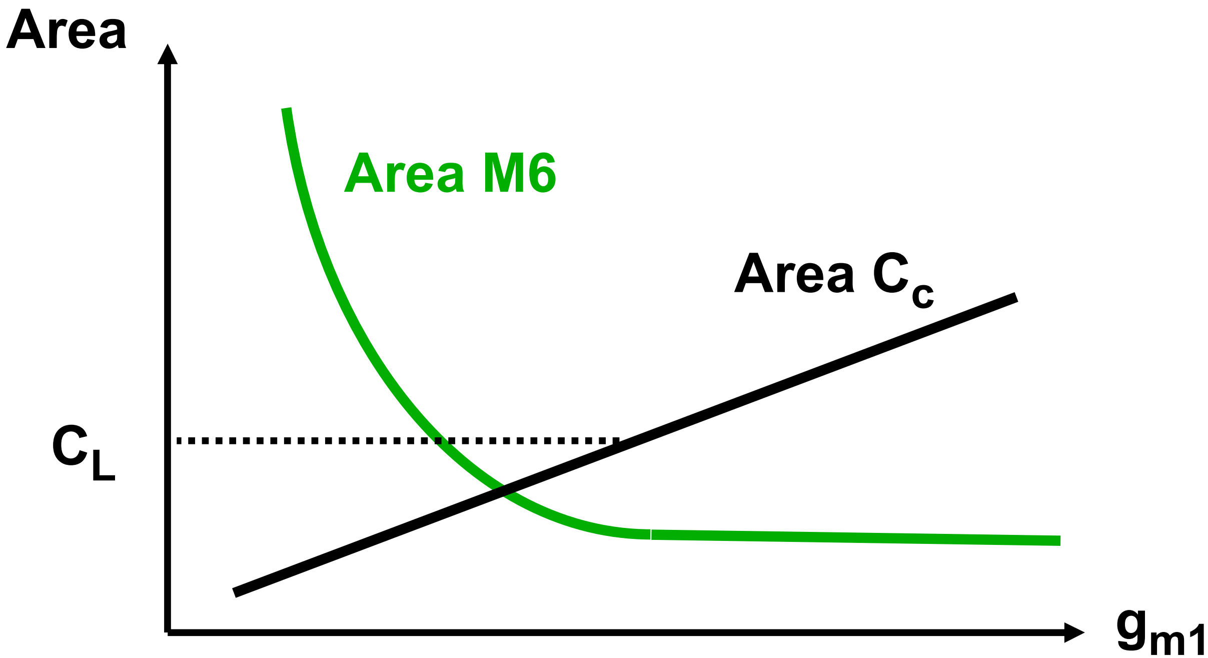
Miller CMOS OTA : Design vs I_{DS6}



g_{m6min} =

3 GBW (2π C_L)

Miller CMOS OTA : Design vs I_{DS1}



Optimum design for high speed Miller OTA - 1

$$GBW = \frac{g_{m1}}{2\pi C_c}$$

$$f_{nd} = \frac{g_{m6}}{2\pi C_L} \frac{1}{1 + C_{n1}/C_c}$$

$$C_L = \alpha C_c \qquad \alpha \approx 2$$

$$C_c = \beta C_{n1} = \beta C_{GS6} \qquad \beta \approx 3$$

$$f_{nd} = \gamma GBW \qquad \gamma \approx 2$$

$$C_{GS} = kW \qquad k = 2\ 10^{-11}\ F/cm$$

$$GBW = \frac{f_{nd}}{\gamma} = \frac{g_{m6}}{2\pi C_L} \frac{1}{\gamma (1 + 1/\beta)}$$

$$C_L = \alpha C_c = \alpha \beta C_{n1} = \alpha \beta C_{GS6} = \alpha \beta kW_6$$

$W_6 \Uparrow$ if $C_L \Uparrow$

Optimum design Miller for high speed OTA - 2

Elimination of C_L yields

$$GBW = \underbrace{\frac{g_{m6}}{2\pi\, kW_6}}_{f_{T6}} \frac{1}{\alpha\, \beta\, \gamma\, (1 + 1/\beta)} \qquad g_m = \frac{W}{L} \frac{17\ 10^{-5}}{1 + 2.8\ 10^4\ L / V_{GST}}$$

W, L in cm

$$GBW = \frac{1}{2\pi\, L_6} \frac{1}{\alpha\, \beta\, \gamma\, (1 + 1/\beta)} \frac{8.5\ 10^6}{1 + 2.8\ 10^4\ L_6 / V_{GST6}}$$

L in cm

GBW is not determined by C_L, only by L (and V_{GST}) !!

f_T is also determined by L !!!

Optimum design Miller for high speed OTA - 3

Substitution for f_T yields

$$f_T = \frac{g_m}{2\pi C_{GS}}$$

$$\boxed{GBW = \frac{f_{T6}}{\alpha\, \beta\, \gamma\, (1 + 1/\beta)}}$$

$$f_T = \frac{1}{L} \frac{1.35}{1 + 2.8\ 10^4\ L / V_{GST}}$$

L in cm

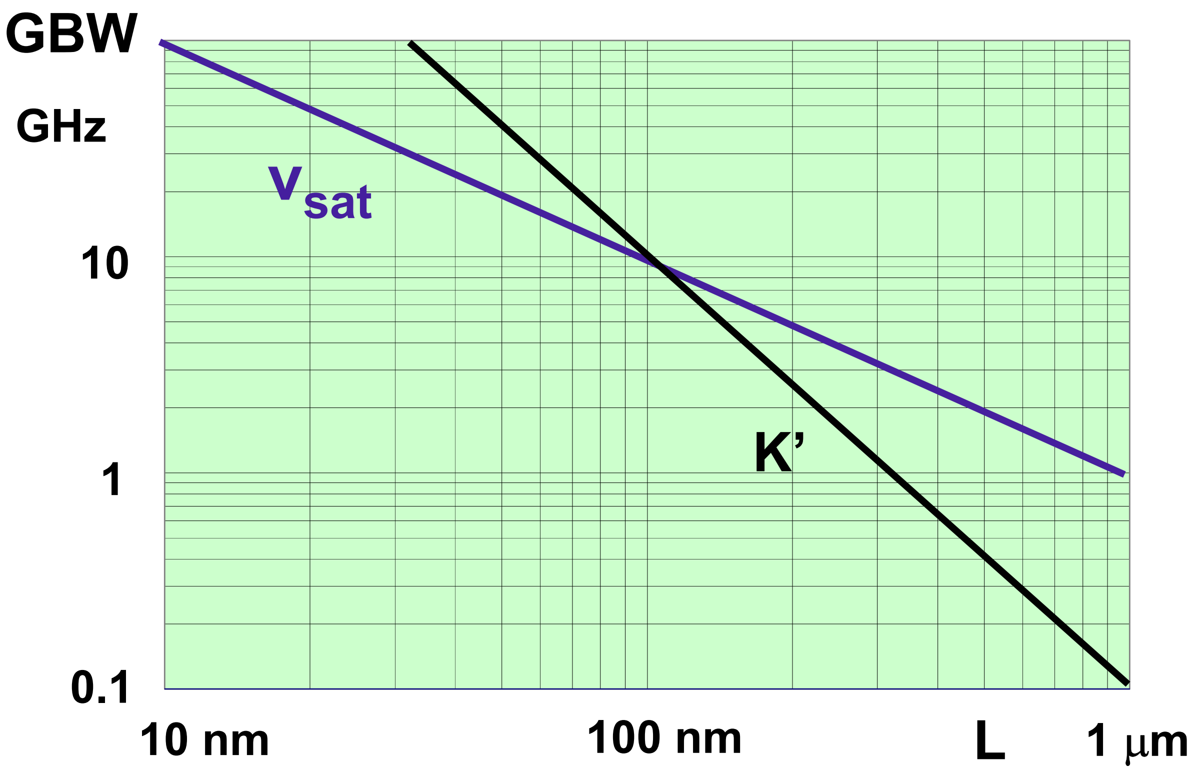
f_T in MHz

GBW is not determined by C_L, only by f_T

f_T is determined by L (and V_{GST}) !!!

If V_{GST} = 0.2 V, v_{sat} takes over for L < 65 nm (If 0.5 V for L < 0.15 μm)

Maximum GBW versus channel length L



V_{GS}-V_T ≈ 0.2 V

$\alpha \approx 2$
$\beta \approx 3$
$\gamma \approx 2$
or 16 x

$$GBW \approx \frac{f_{T6}}{16}$$

Design optimization for high speed Miller OTA

- Choose $\alpha\ \beta\ \gamma$
- Find minimum f_{T6} for specified GBW
- Choose maximum channel length L_6 (max. gain)
  for a chosen $V_{GS6}-V_T$
- W_6 is calculated from C_L ,
  and determines I_{DS6}
- C_c is calculated from C_L through α
- g_{m1} and I_{DS1} are calculated from C_c
- Noise is determined by g_{m1} or C_c

Design Ex. for GBW = 0.4 GHz & C_L = 5 pF

- Choose $\alpha\ \beta\ \gamma$ — 2 3 2
- Minimum f_{T6} for GBW = 0.4 GHz — f_{T6} = 6.4 GHz
- Maximum channel length L_6 — $L_6 = 0.5\ \mu m$
  for a chosen $V_{GS6}-V_T = 0.2$ V
- L_6 is taken to be the minimum L
- W_6 is calculated from C_L , — $W_6 = 417\ \mu m$
  and determines I_{DS6} ($K'_n = 70\ \mu A/V^2$) — $I_{DS6} = 2.3$ mA
  and determines C_{n1} (k = 2 fF/μm) — $C_{n1} = 0.83$ pF
- C_c is calculated from C_L through α — $C_c = 2.5$ pF
- g_{m1} and I_{DS1} are calculated from C_c — $I_{DS1} = 0.63$ mA

Optimum design Miller for low speed OTA

$$\boxed{GBW = \frac{f_{T6}}{\alpha\,\beta\,\gamma\,(1 + 1/\beta)}}$$

$$\frac{f_T}{f_{TH}} = \sqrt{i}\,(1 - e^{-\sqrt{i}}) \approx i \text{ for small } i$$

$$f_{TH} = \frac{2\,\mu\,kT/q}{2\pi\,L^2}$$

GBW is not determined by C_L, only by f_T

f_T is determined by L and i !!!

Design optimization for low speed Miller OTA

- Choose α β γ
- Find minimum f_{T6} for specified GBW
- Choose channel length L_6 (max. gain), which gives f_{TH6}
- Calculate i_6
- W_6 is calculated from C_L ,

  and determines I_{DST6} and I_{DS6}
- C_c is calculated from C_L through α
- g_{m1} and I_{DS1} are calculated from C_c
- Noise is determined by g_{m1} or C_c

Design Ex. for GBW = 1 MHz & C_L = 5 pF

• Choose $\alpha\ \beta\ \gamma$	2 3 2
• Minimum f_{T6} for GBW = 1 MHz	f_{T6} = 16 MHz
• Maximum channel length L_6	L_6 = 0.5 μm
gives f_{TH6}	f_{TH6} = 2 GHz
• Inversion coefficient i is	i = 0.008
• W_6 is calculated from C_L ,	W_6 = 417 μm
and determines I_{DST6} (K'_n = 70 $\mu A/V^2$)	I_{DST6} = 0.33 mA
and determines I_{DS6}	I_{DS6} = 2.7 μA
and determines C_{n1} (k = 2 fF/μm)	C_{n1} = 0.83 pF
• C_c is calculated from C_L through α	C_c = 2.5 pF
• g_{m1} and I_{DS1} are calculated from C_c	I_{DS1} = 1.6 μA

Table of contents

- **Design of Single-stage OTA**
- **Design of Miller CMOS OTA**
- **Design for GBW and Phase Margin**
- **Other : SR, Output Impedance, Noise, ...**

Miller CMOS OTA: Specifications 1

1. Introductory analysis

1.1 DC currents and voltages on all nodes

1.2 Small-signal parameters of all transistors

2. DC analysis

2.1 Common-mode input voltage range vs supply Voltage

2.2 Output voltage range vs supply Voltage

2.3 Maximum output current (sink and source)

Miller CMOS OTA: Specifications 2

3. AC and transient analysis

3.1 AC resistance and capacitance on all nodes

3.2 Gain versus frequency : GBW, …

3.3 Gainbandwidth versus biasing current

3.4 Slew rate versus load capacitance

3.5 Output voltage range versus frequency

3.6 Settling time

3.7 Input impedance vs frequency (open & closed loop)

3.8 Output impedance vs frequency (open & closed loop)

Miller CMOS OTA: Specifications 3

4. Specifications related to offset and noise

4.1 Offset voltage versus common-mode input Voltage

4.2 CMRR versus frequency

4.3 Input bias current and offset

4.4 Equivalent input noise voltage versus frequency

4.5 Equivalent input noise current versus frequency

4.6 Noise optimization for capacitive/inductive sources

4.7 PSRR versus frequency

4.8 Distortion

Miller CMOS OTA: Specifications 4

5. Other second-order effects

5.1 Stability for inductive loads

5.2 Switching the biasing transistors

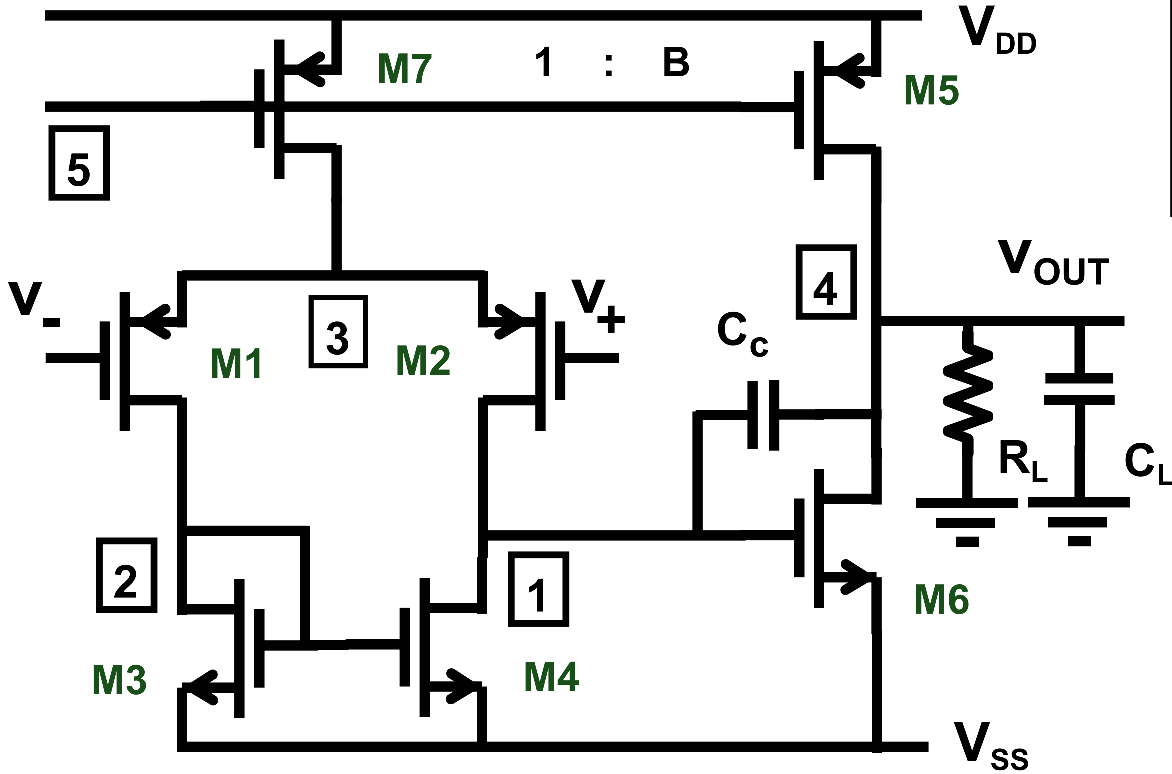
5.3 Switching or ramping the supply voltages

5.4 Different supply voltages, temperatures, ...

M C O : Other specifications

- o Common-mode input voltage range
- o Output voltage range
- o Slew Rate
- o Output impedance
- o Noise

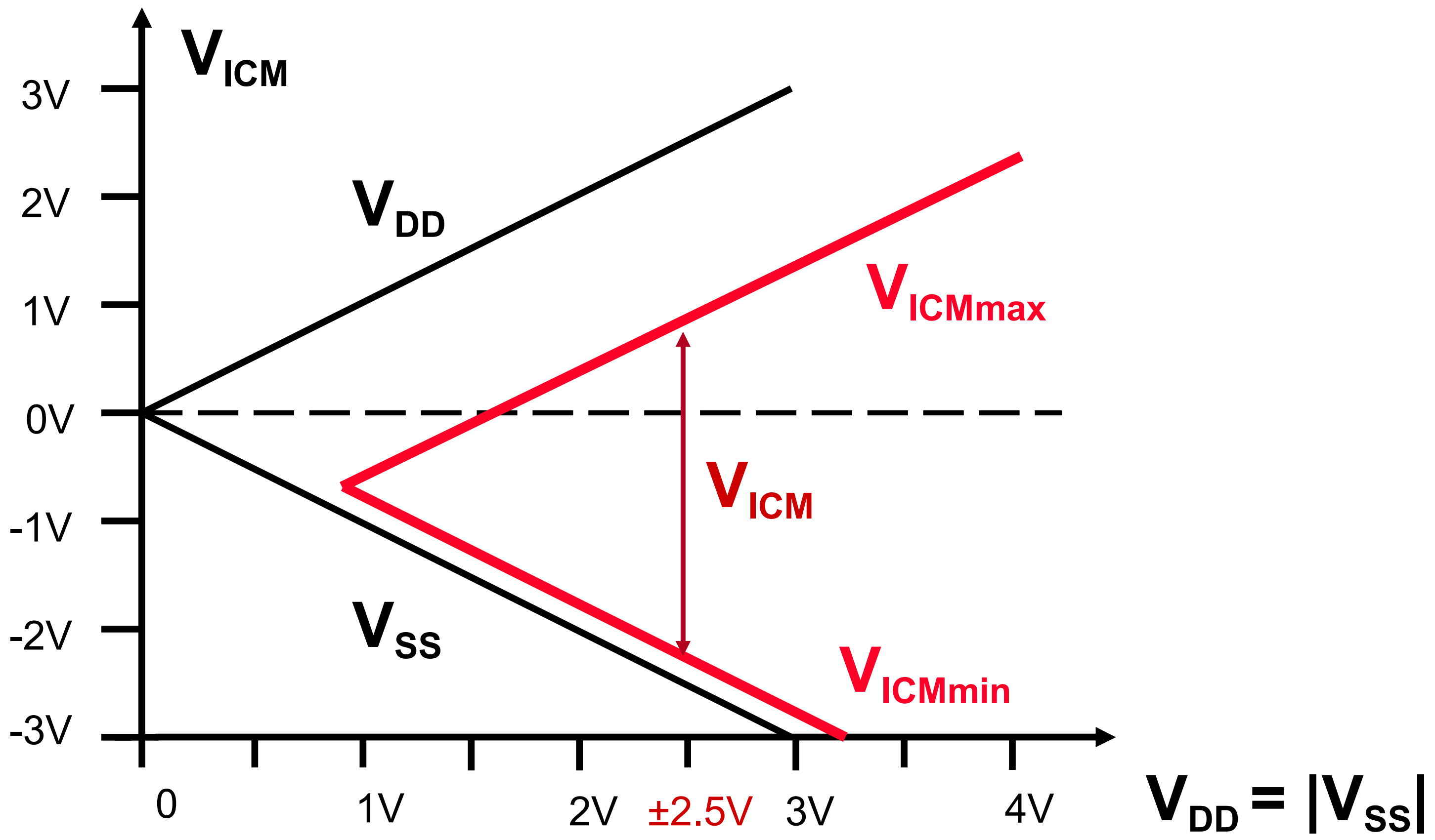
Miller CMOS OTA



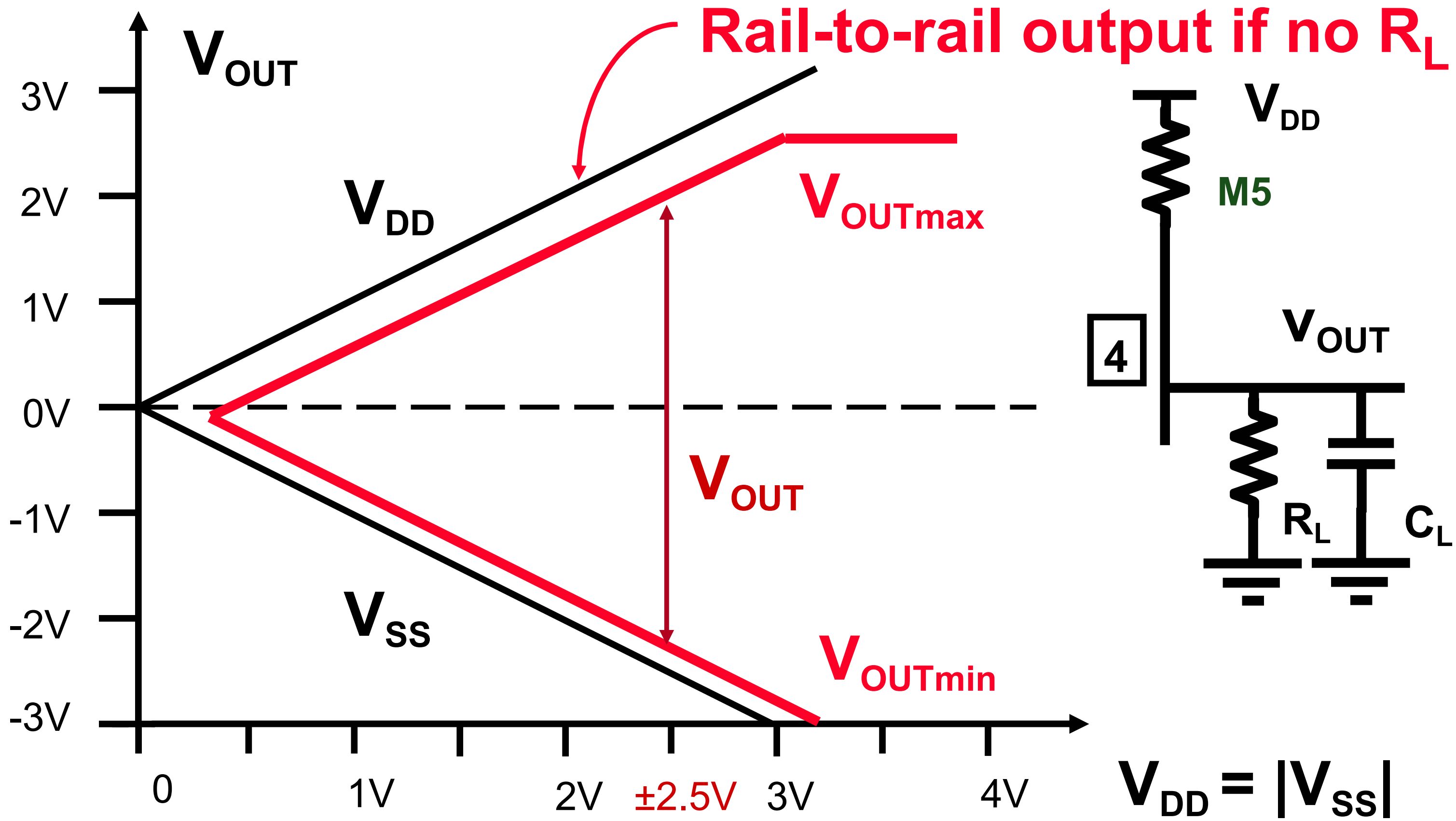
GBW = 1 MHz
C_L = 10 pF
R_L = 10 kΩ

g_{m1} = 7.5 μS
I_{DS1} = 1 μA
g_{o24} = 0.03 μS
g_{m6} = 246 μS
I_{DS6} = 25 μA
C_c = 1 pF

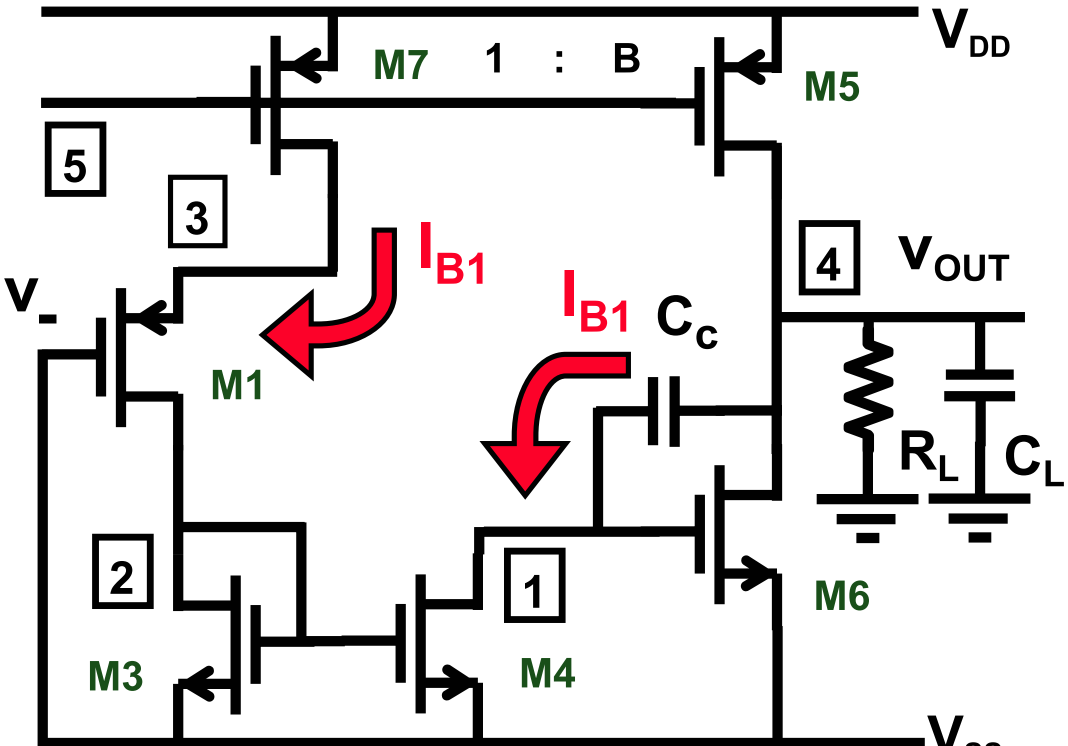
Miller CMOS OTA : CM Input Voltage Range



Miller CMOS OTA : Output Voltage Range



Miller CMOS OTA : Slew Rate - 1



Switch input :

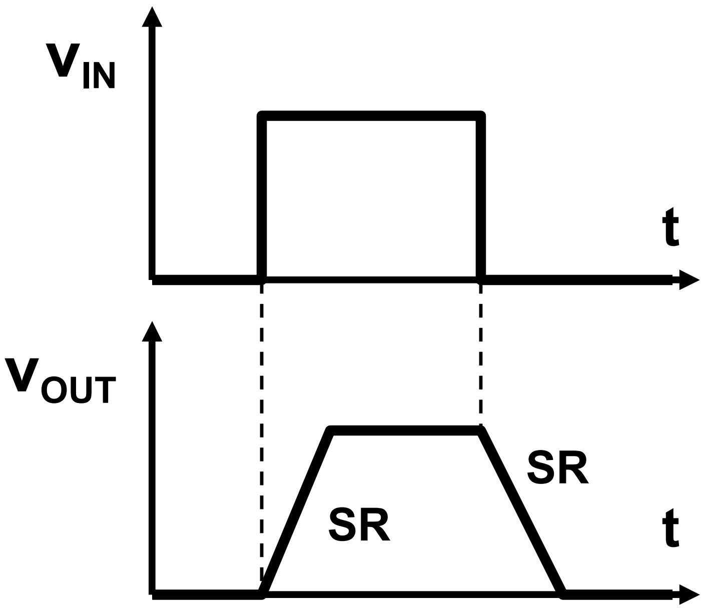
$v_+ > 1$

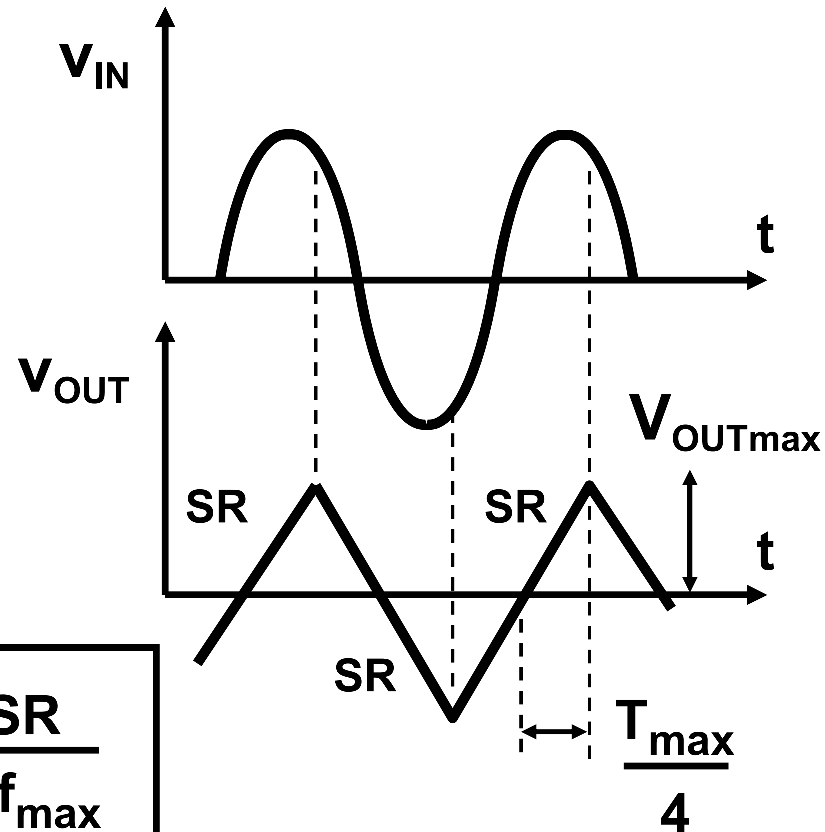
$v_- > 0$

$$SR = \frac{\Delta V_{OUT}}{\Delta t}$$

$$\boxed{SR = \frac{I_{B1}}{C_c}}$$

Miller CMOS OTA : Slew Rate - 2

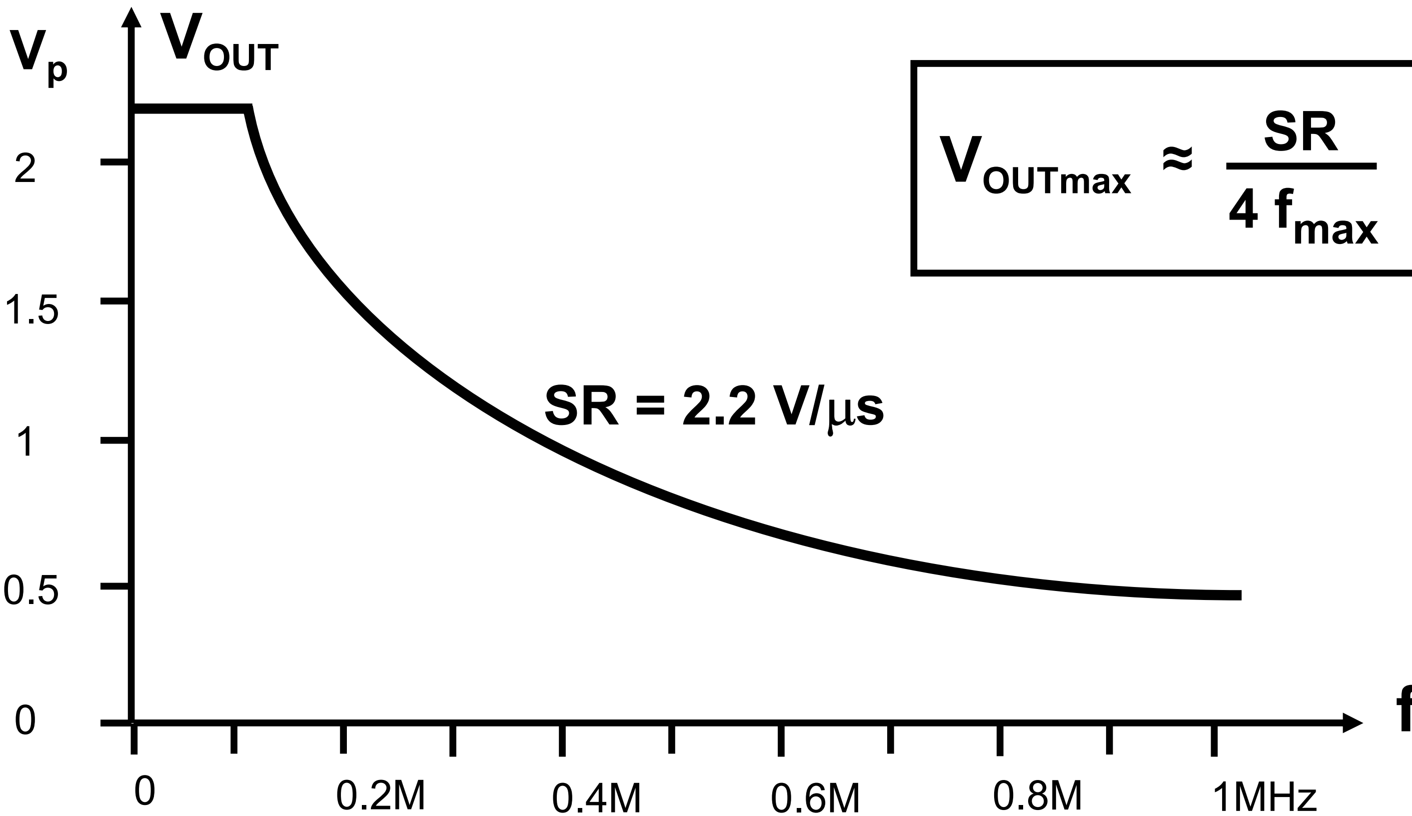




$$\frac{SR}{V_{OUTmax} f_{max}} \approx 4$$

$$\boxed{V_{OUTmax} \approx \frac{SR}{4\, f_{max}}}$$

Miller CMOS OTA : Slew Rate - 3



$$V_{OUTmax} \approx \frac{SR}{4\, f_{max}}$$

Design for GBW or SR ?

$$\boxed{\frac{SR}{GBW} = 4\pi \frac{I_{DS1}}{g_{m1}}}$$

$$\frac{I_{DS1}}{g_{m1}} = \frac{V_{GS1}-V_T}{2} \approx 0.1 \ldots 0.3 \text{ V for MOST (si)}$$

x10

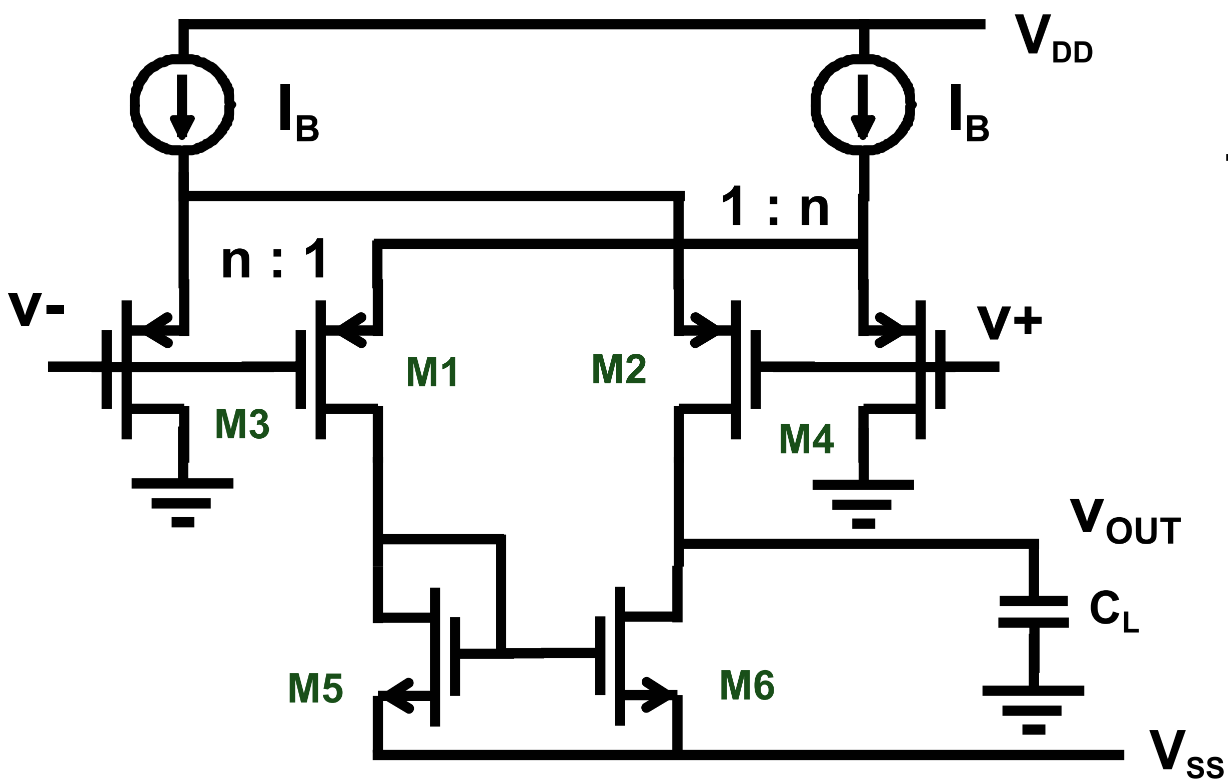
$$\frac{I_{DS1}}{g_{m1}} = \frac{nkT}{q} \approx 30 \ldots 50 \text{ mV for MOST (wi)}$$

$$\frac{I_{CE1}}{g_{m1}} = \frac{kT}{q} \approx 26 \text{ mV for Bipolar trans.}$$

$$\frac{I_{CE1}}{g_{m1}} = (1 + g_{m1}R_E)\frac{kT}{q} \approx \ldots 0.5 \text{ V with } R_E$$

Solomon, JSSC Dec 74, 314-332

High SR by g_m reduction

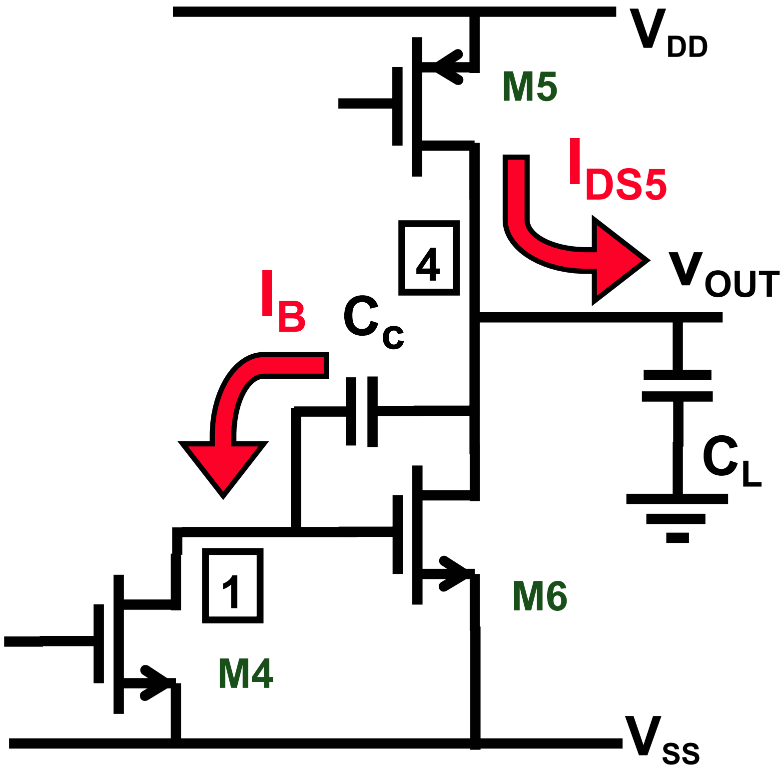


$$\frac{SR}{2\pi\ GBW}$$

$= x\ (n+1)$

Ref. Schmoock, JSSC Dec.75, 407-411

External vs internal Slew Rate



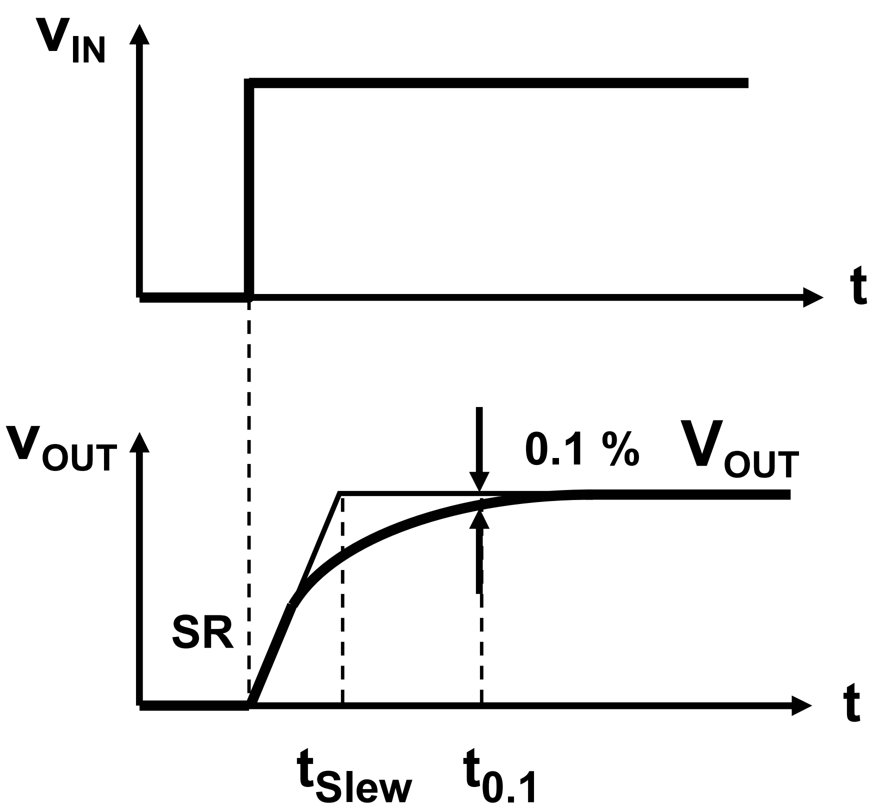
$$SR_{int} = \frac{I_B}{C_c}$$

$$SR_{ext} = \frac{I_{DS5}}{C_L} \quad \text{is larger !}$$

$$\frac{g_{m6}}{g_{m1}} = 4\,\frac{C_L}{C_c} = \frac{I_{DS5}}{I_{DS1}}$$

$$\frac{I_{DS5}}{C_L} \approx 2\,\frac{2I_{DS1}}{C_c}$$

Slew Rate and settling



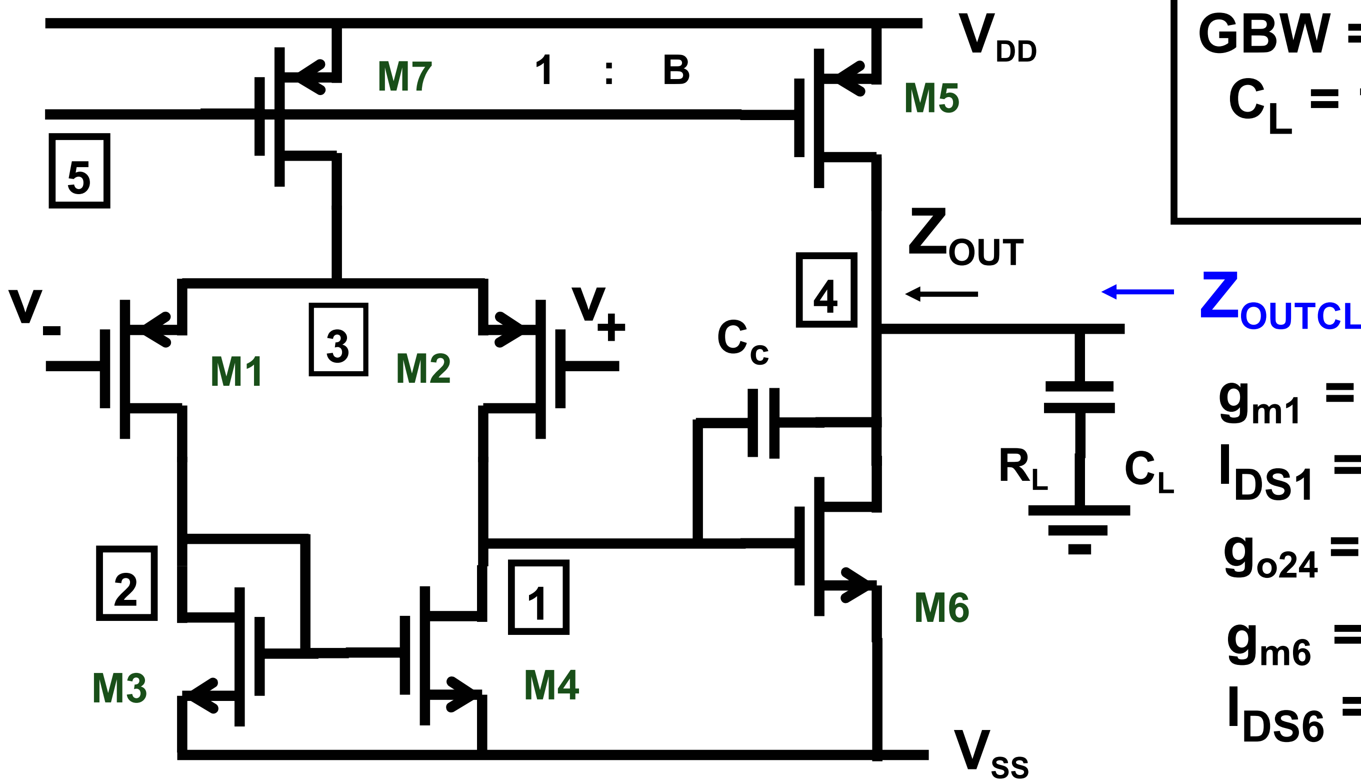
$$t_{TOT} = t_{Slew} + t_{0.1}$$

$$t_{Slew} = \frac{V_{OUT}}{SR}$$

$$t_{0.1} = \frac{7}{2\pi\ BW}$$

$$\ln(1000) \approx 7$$

Miller CMOS OTA Output Impedance



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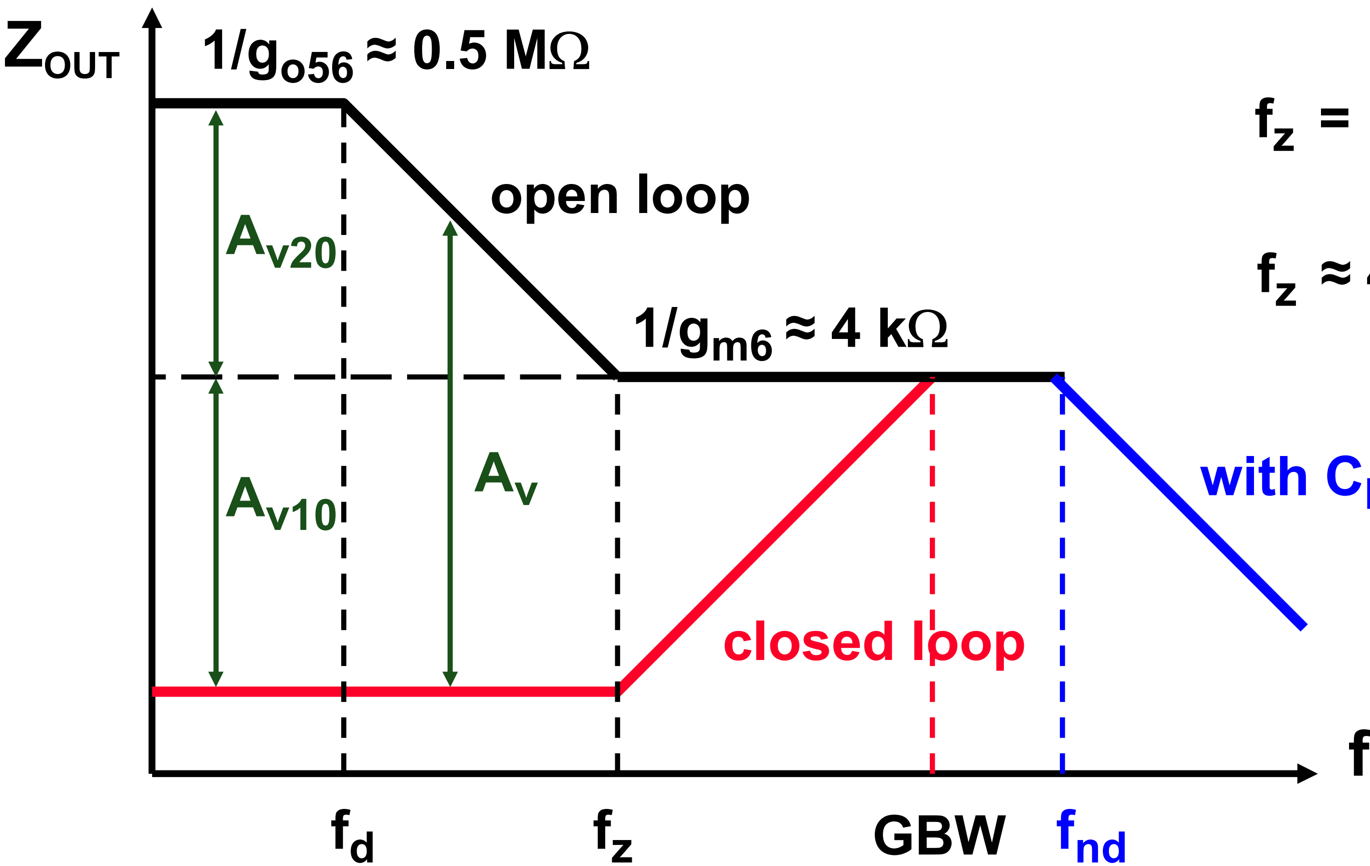
C_L = 10 pF

g_{m1} = 7.5 µS
I_{DS1} = 1 µA
g_{o24} = 0.03 µS
g_{m6} = 246 µS
I_{DS6} = 25 µA
C_c = 1 pF

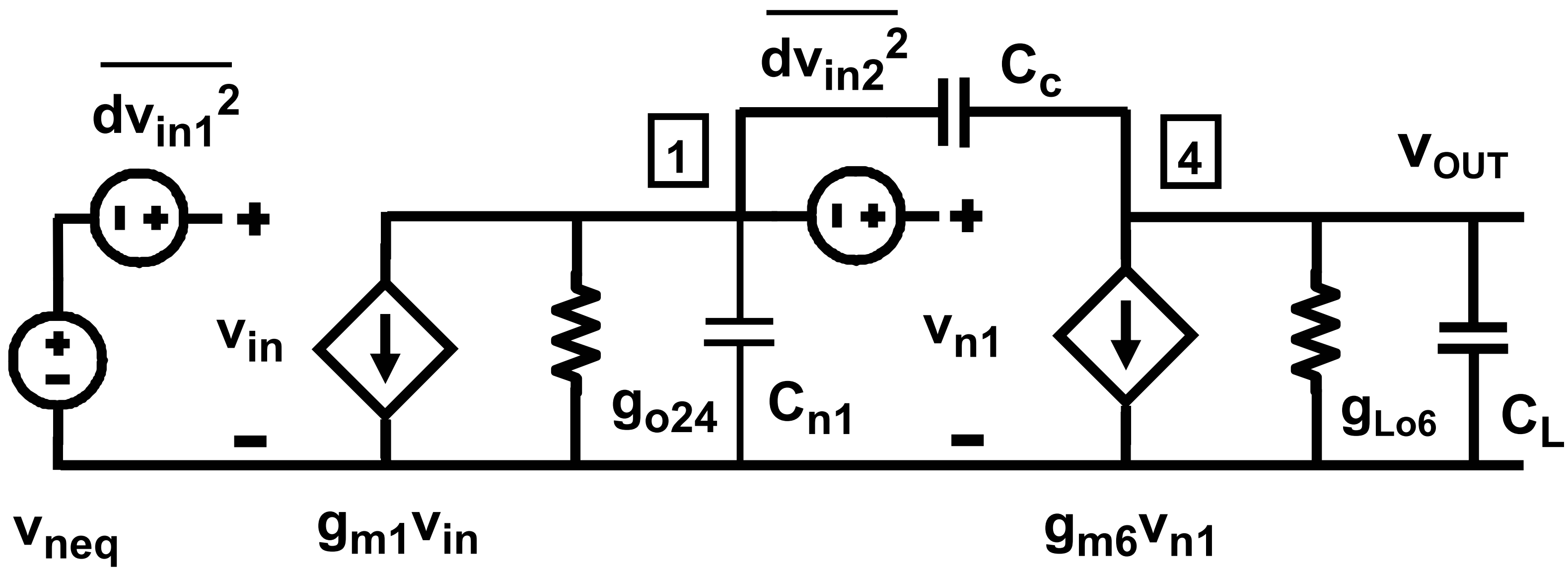
Miller CMOS OTA : Output impedance Z_{OUT}



$$f_z = \frac{g_{o24}}{2\pi C_c}$$

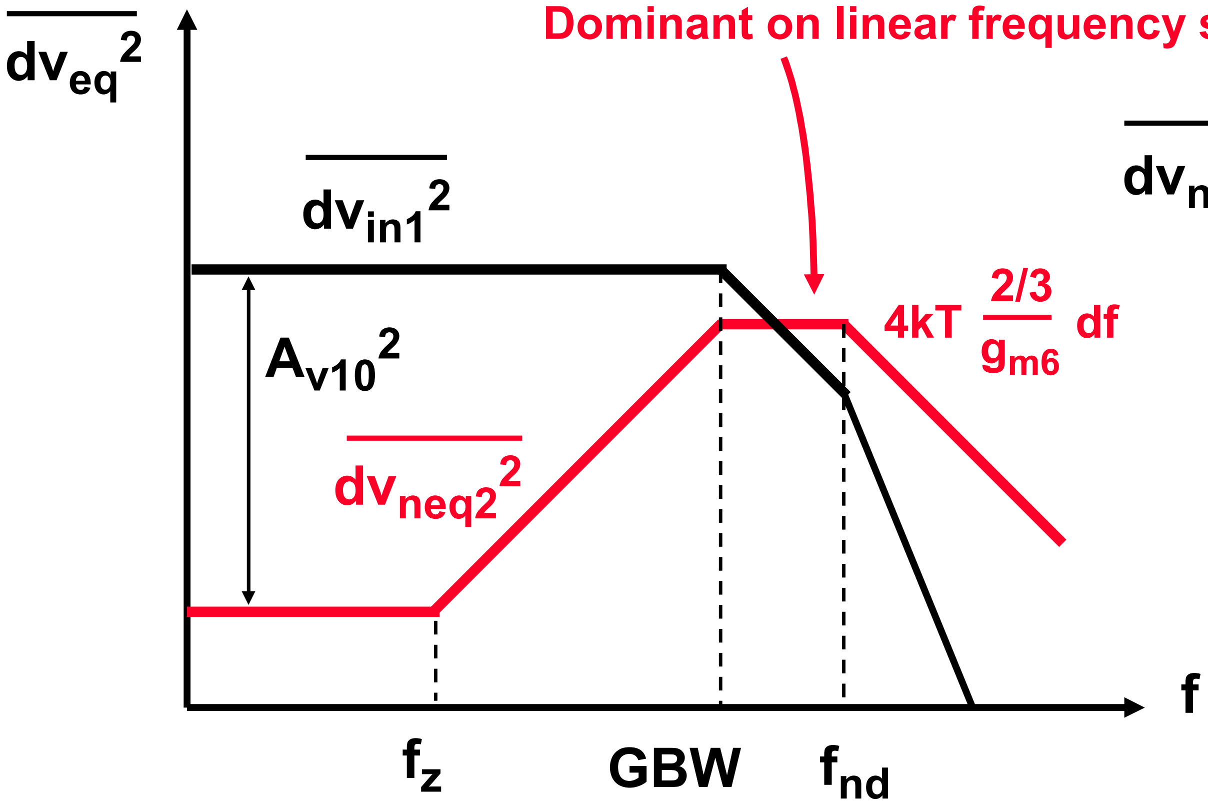
$f_z \approx 4.8$ kHz

Miller CMOS OTA : Noise density 1



$$\overline{dv_{in1}^2} \approx 4kT\,\frac{4/3}{g_{m1}}\,df \qquad \overline{dv_{in2}^2} \approx 4kT\,\frac{2/3}{g_{m6}}\,df$$

Miller CMOS OTA : Noise density 2

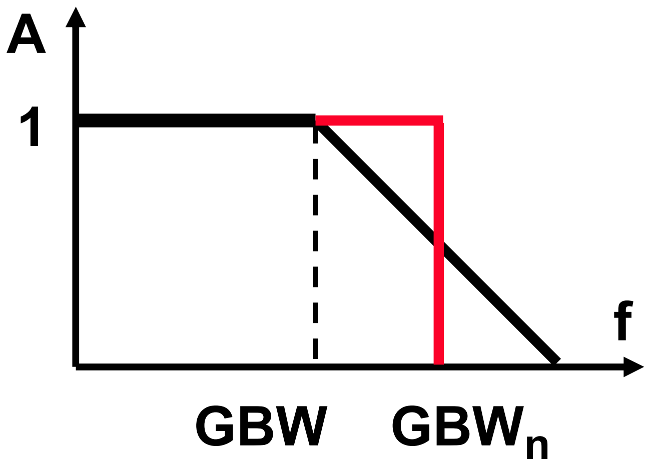


$$\overline{dv_{neq2}^2} = \frac{\overline{dv_{in2}^2}}{|A_{v1}|^2}$$

$$A_{v10} = \frac{g_{m1}}{g_{o24}}$$

$$f_z = \frac{g_{o24}}{2\pi C_c}$$

Miller CMOS OTA : Integrated Noise



$$= \frac{\pi}{2} \text{GBW}$$

$$\overline{v_{nieq}^2} = \int_0^\infty \frac{\overline{dv_{nieq}^2}}{1 + (f/\,GBW)^2}$$

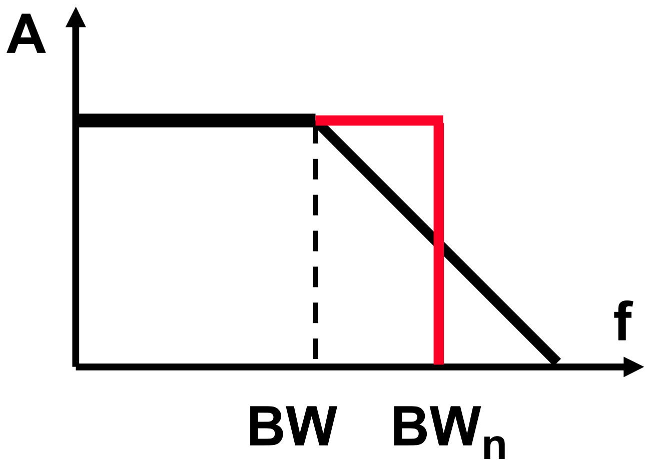
$$\int_0^\infty \frac{dx}{1 + x^2} = \frac{\pi}{2}$$

$$\overline{v_{nieq}^2} = 4kT \frac{4/3}{g_{m1}} \text{GBW} \frac{\pi}{2}$$

$$\boxed{\overline{v_{nieq}^2} = \frac{4}{3} \frac{kT}{C_c}}$$

$C_c = 1pF$ $\quad v_{Rs} = 74\ \mu V_{RMS}$

Noise density vs integrated noise



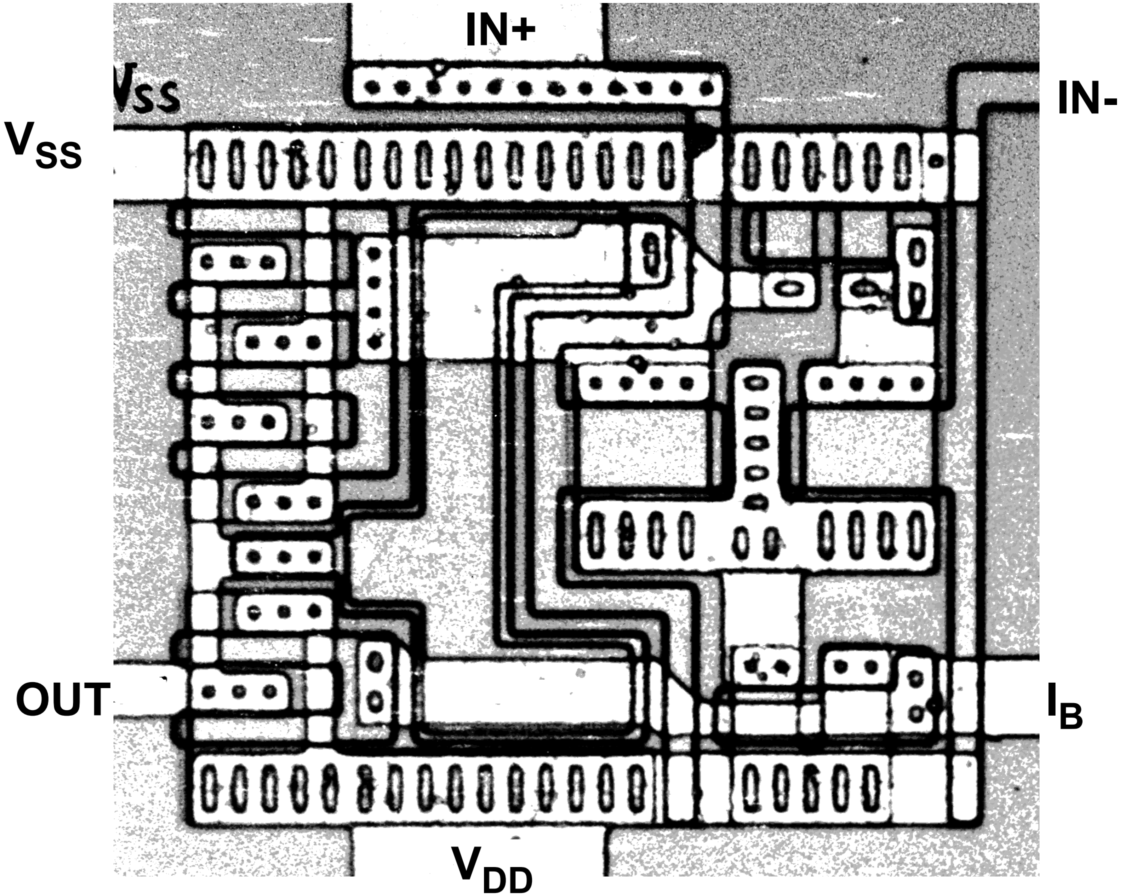
$$\overline{dv_{ni}^2} = 4kT\,\frac{4/3}{g_m}\,df$$

$$\overline{v_{ni}^2} = \int_0^\infty \frac{\overline{dv_{ni}^2}}{1 + (f/\,BW)^2} = \frac{4kT}{3C_c}$$

Noise density (V^2/Hz) ~ $1/g_m$ (or R_S)

Integrated noise (V_{RMS}) ~ $1/C_c$

CMOS Miller OTA layout



GBW = 1 MHz

C_L = 10 pF

SR = 2.2 V/μs

V_{DD} = 5 V

I_{TOT} = 27 μA

370 MHzpF/mA

Miller CMOS OTA : Exercise

GBW = 50 MHz for C_L = 2 pF : use min. I_{DS6} !

Techno: L_{min} = 0.5 μm; K'_n = 50 μA/V² & K'_p = 25 μA/V²

C_{GS} = kW (= C_{ox}WL) and k = 2 fF/ μm

$V_{GS} - V_T = 0.2$ V

Find

g_{m6} I_{DS6} W_6 $C_{n1} = C_{GS6}$ C_c g_{m1} I_{DS1} $\overline{dv_{ineq}^2}$ v_{inRMS}

Conclusion : Table of contents

- Design of Single-stage OTA
- Design of Miller CMOS OTA
- Design for GBW and Phase Margin
- Other specs: Input range, output range, SR, ...

0.7 chap7

Important opamp configurations

Willy Sansen

KULeuven, ESAT-MICAS

Leuven, Belgium

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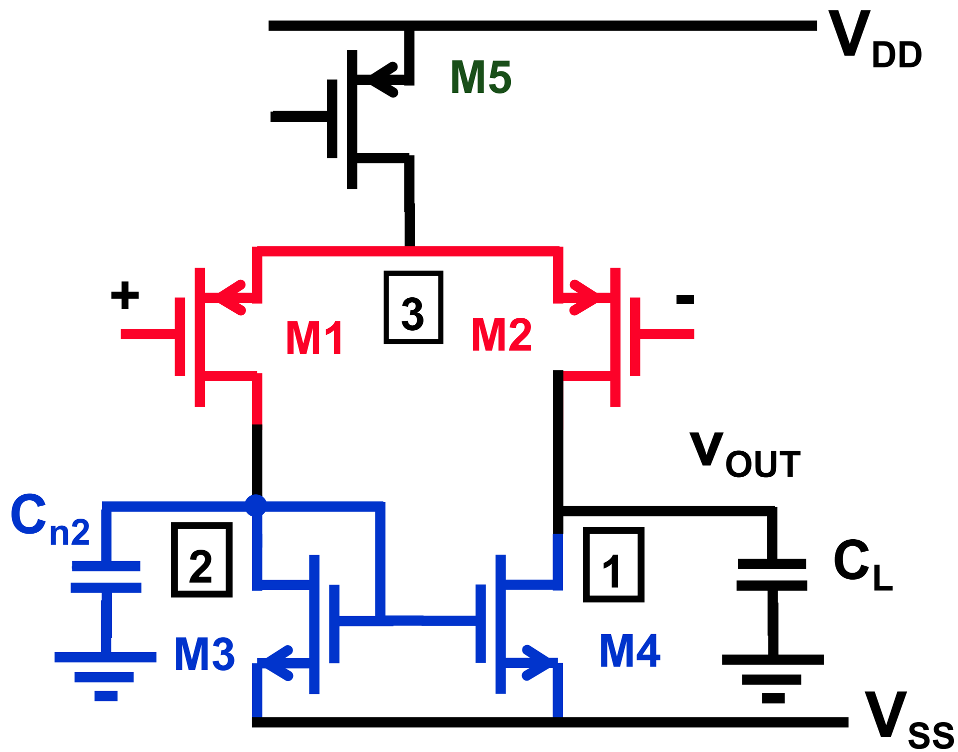
Simple CMOS OTA



$C_{n2} = 2C_{GS3} + C_{DB3} + C_{DB1} \approx 4C_{GS3}$

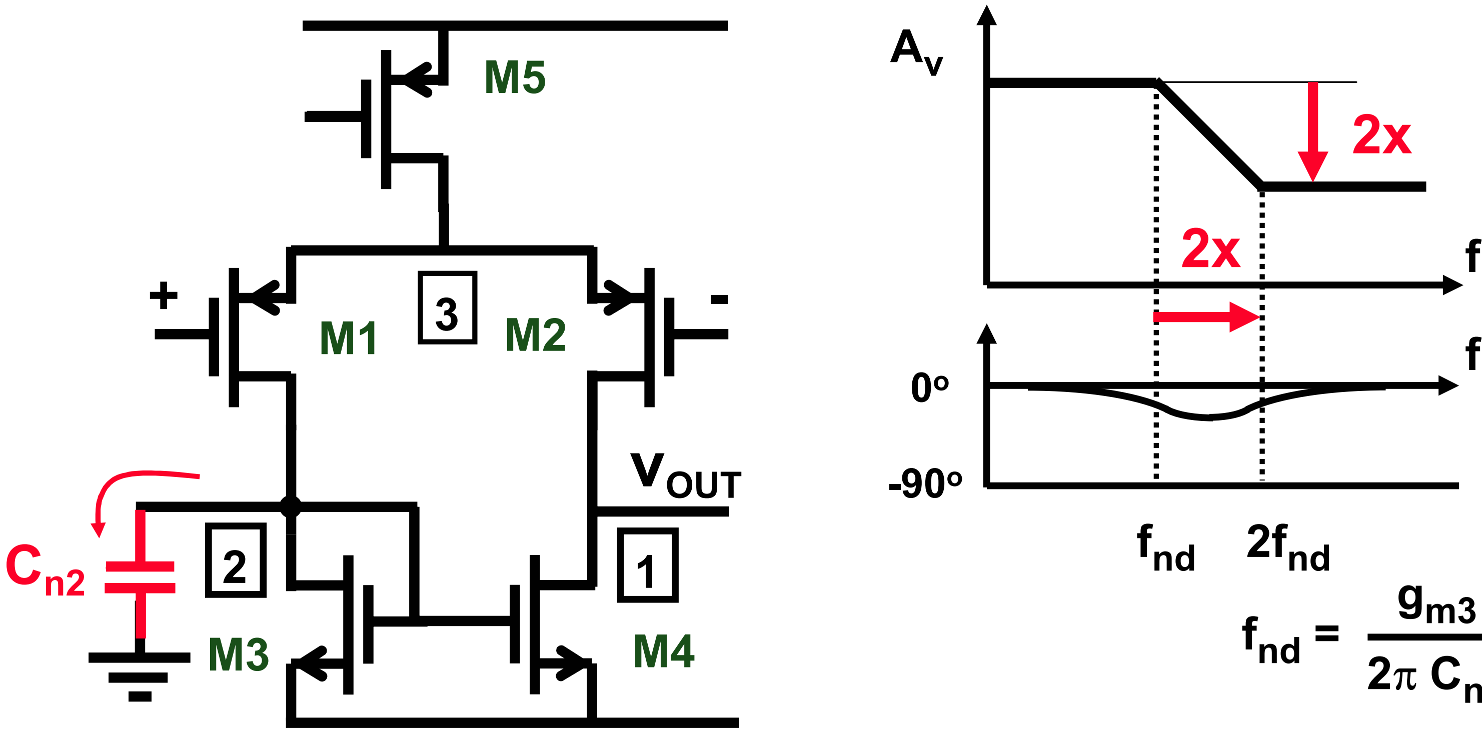
Differential pair
Current mirror

$$GBW = \frac{g_{m1}}{2\pi\, C_L}$$

$$f_{nd} = \frac{g_{m3}}{2\pi\, C_{n2}}$$

$$f_{nd} \approx \frac{f_{T3}}{4}$$?

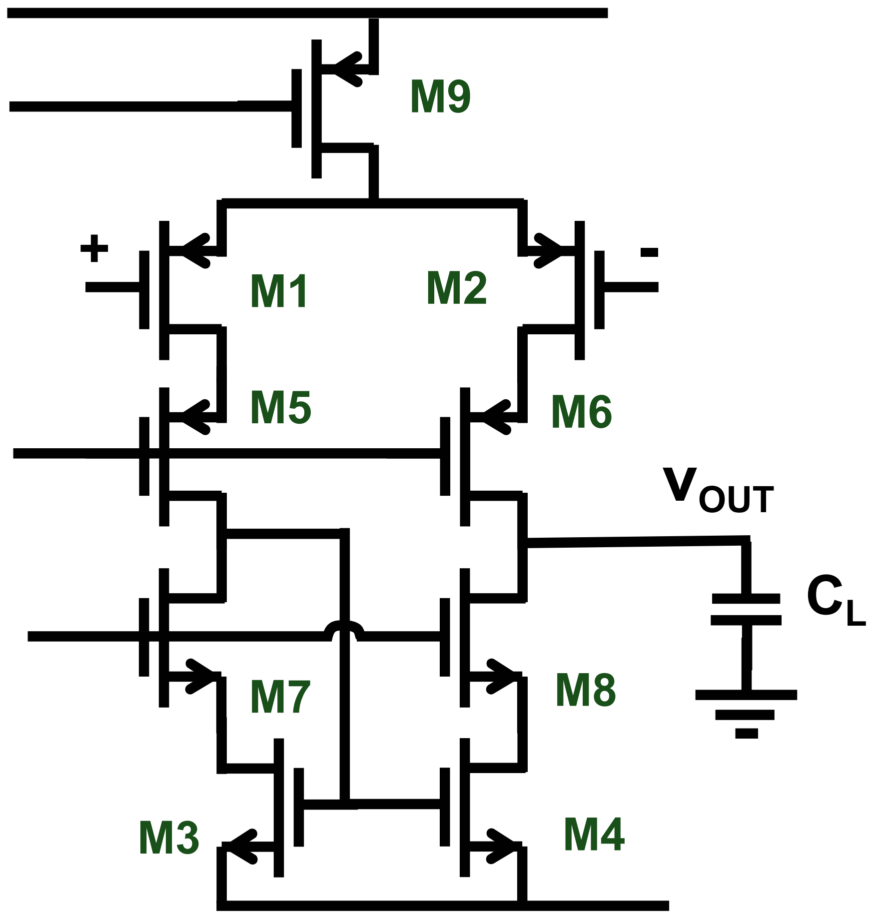
Simple CMOS OTA : f_{nd}



$$f_{nd} = \frac{g_{m3}}{2\pi C_{n2}}$$

$$PM = 90^o - \arctan \frac{GBW}{f_{nd}} + \arctan \frac{GBW}{2 f_{nd}} \approx 85^o$$

Telescopic CMOS OTA

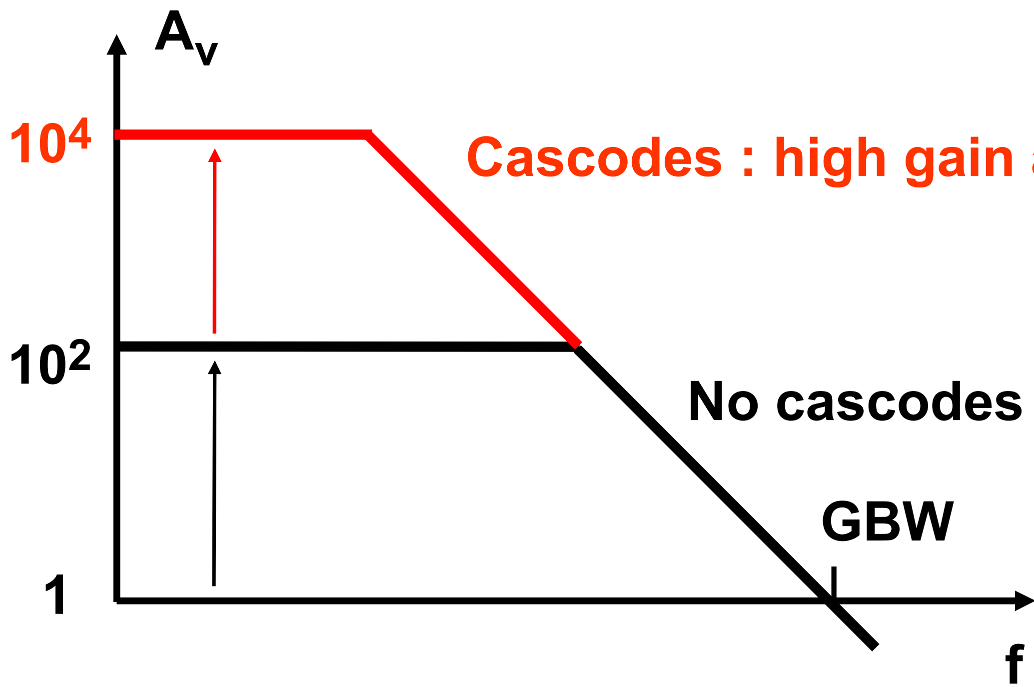


More gain
At low frequencies

$$GBW = \frac{g_{m1}}{2\pi\ C_L}$$

Gulati, JSSC Dec.98, 2010-2019

Cascodes increase gain at low frequencies



$$GBW = \frac{g_{m1}}{2\pi\, C_L}$$

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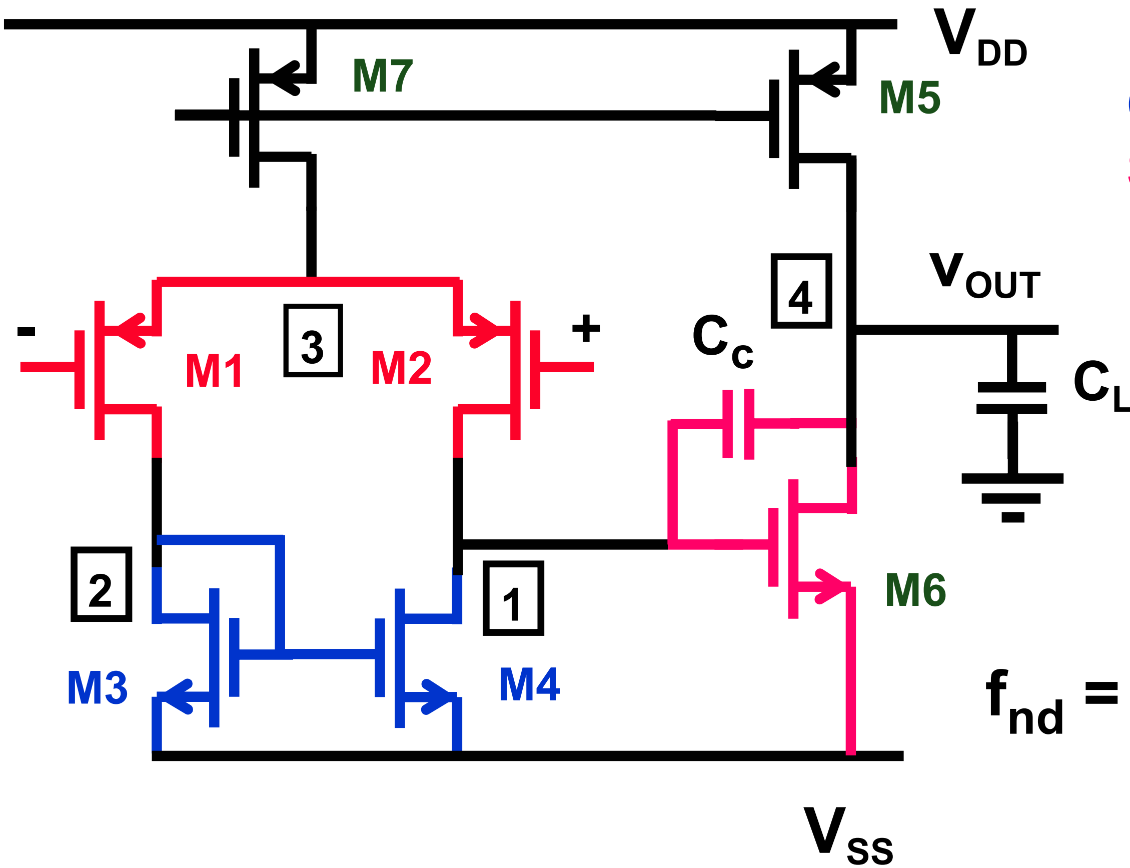
Miller CMOS OTA

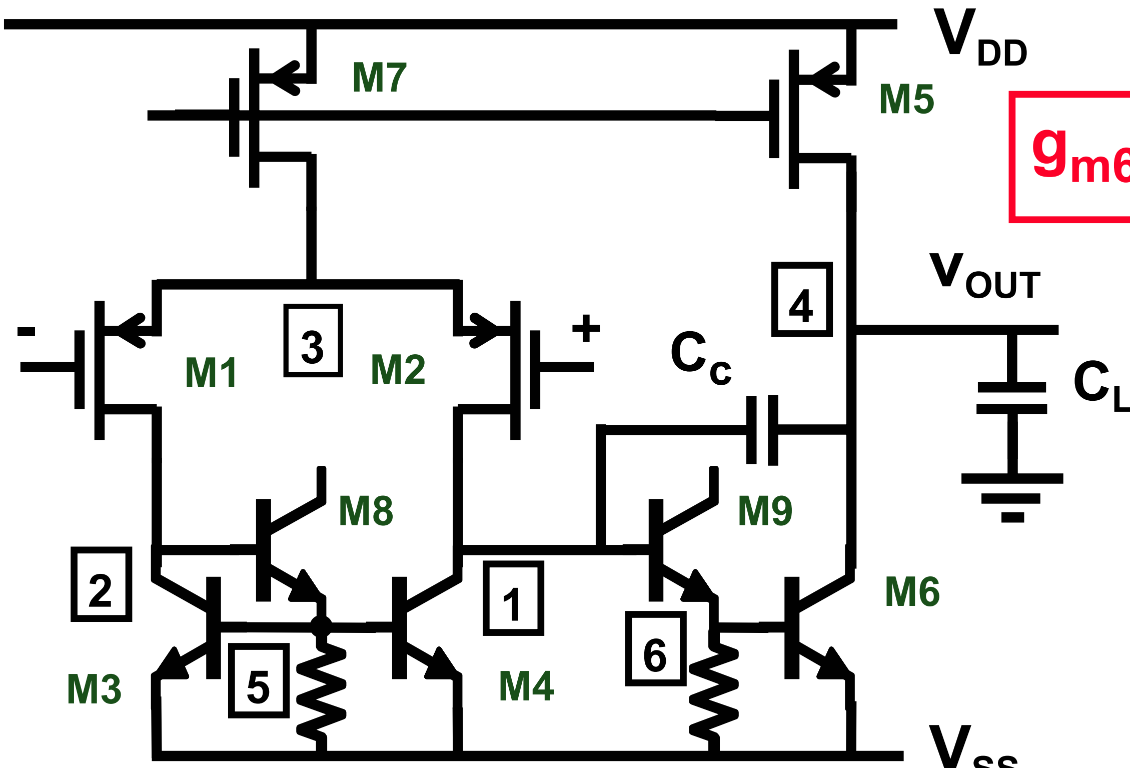


Differential pair
Current mirror
Second stage

$$GBW = \frac{g_{m1}}{2\pi\, C_c}$$

$$f_{nd} = \frac{g_{m6}}{2\pi\, C_L} \; \frac{1}{1 + \frac{C_{n1}}{C_c}}$$

Miller BiCMOS OTA



$g_{m6NPN} > g_{m6NMOS}$

Node 1:
High impedance

Extra nodes 5, 6

f_{nd} ?

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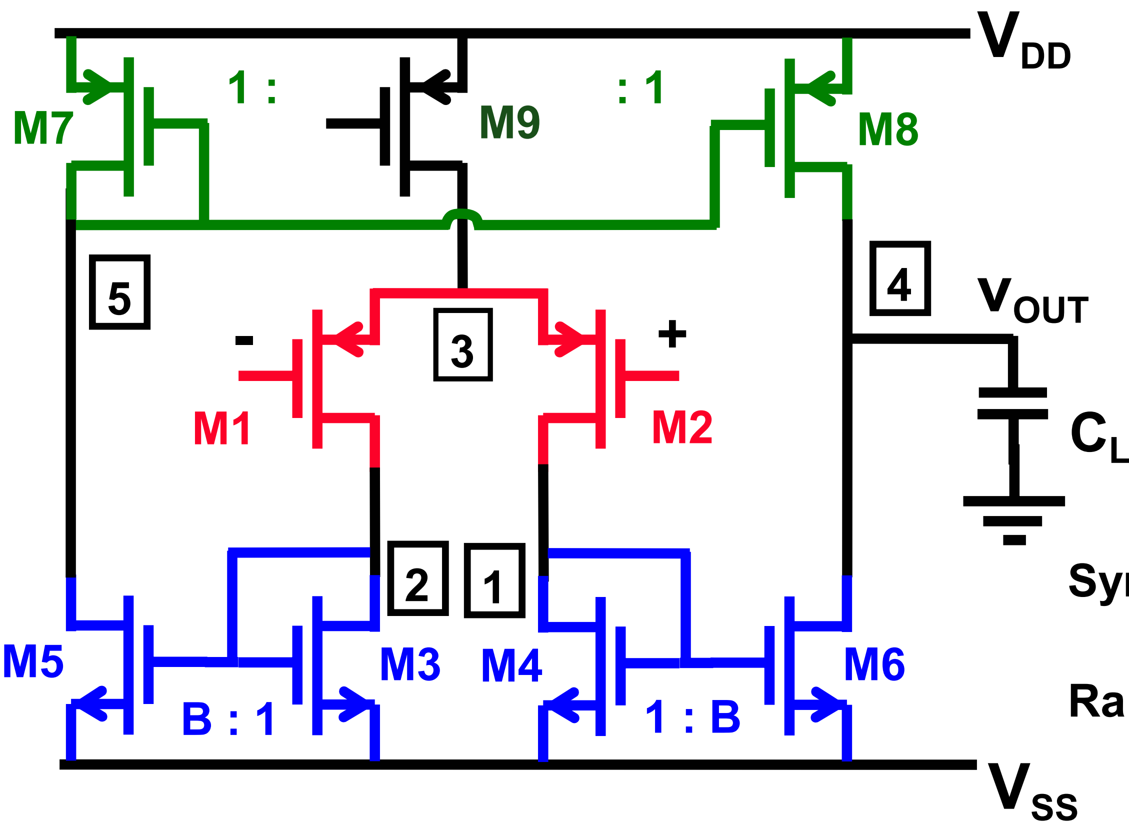
Symmetrical CMOS OTA



Differential pair

3 Current mirrors

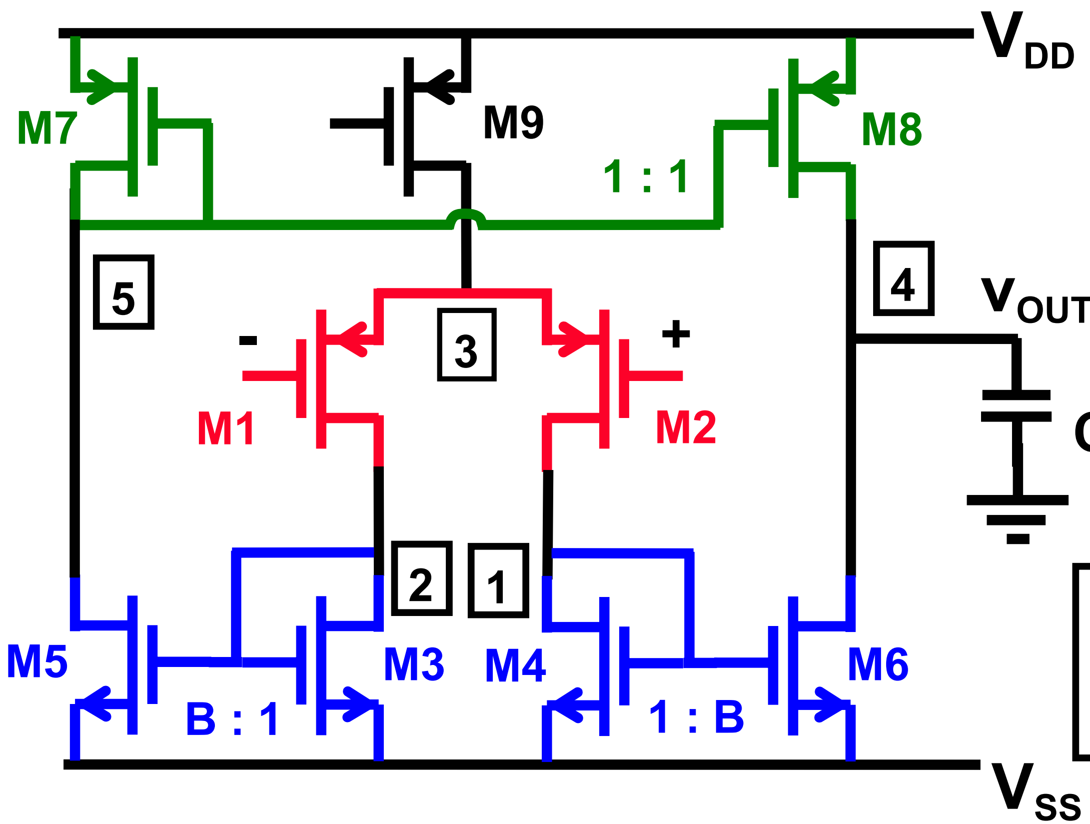
Symmetrical :

Node 1 = Node 2

Rail-to-rail output swing

B ?

Symmetrical CMOS OTA : GBW



$$A_v = g_{m1} B R_{n4}$$

$$= \frac{2V_{En}L_6}{V_{GS1}-V_T}$$

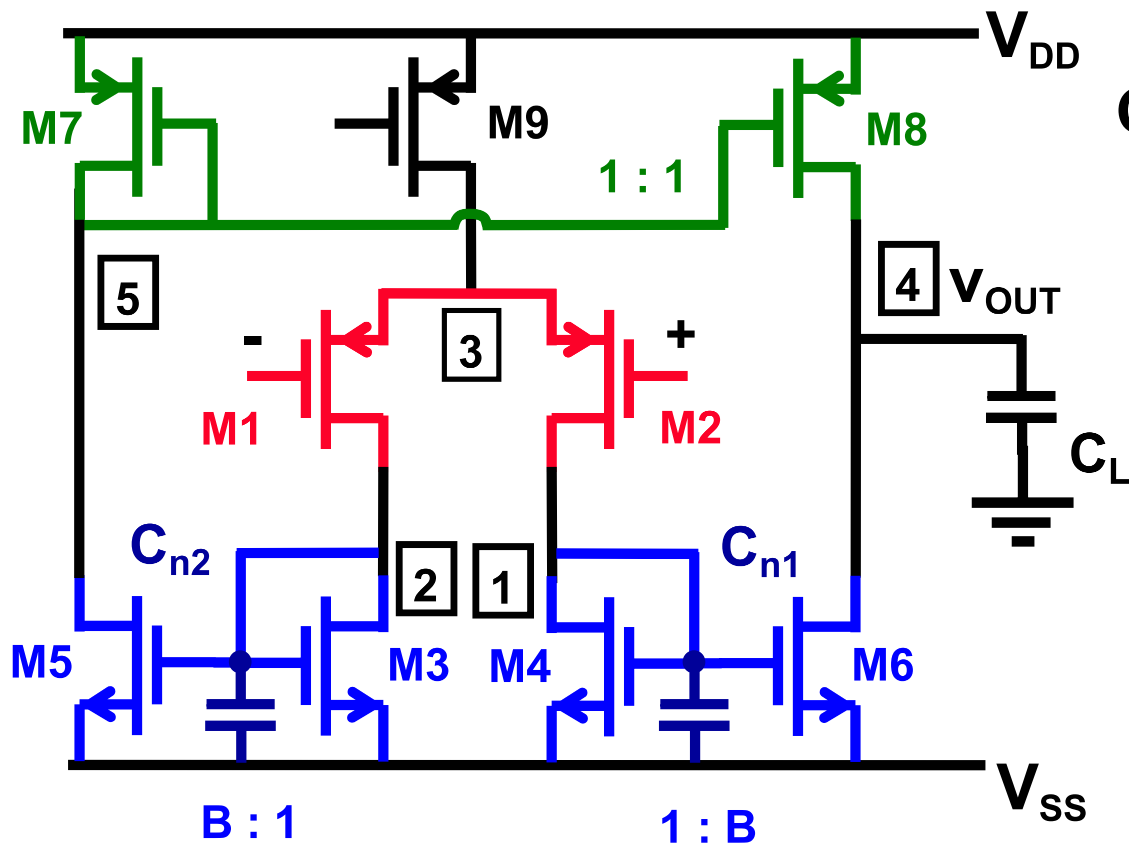
if $L_8 > L_6$

$$BW = \frac{1}{2\pi\, R_{n4} C_L}$$

$$\boxed{GBW = B\,\frac{g_{m1}}{2\pi\, C_L}}$$

B ?

Symmetrical CMOS OTA : $f_{nd1,2}$

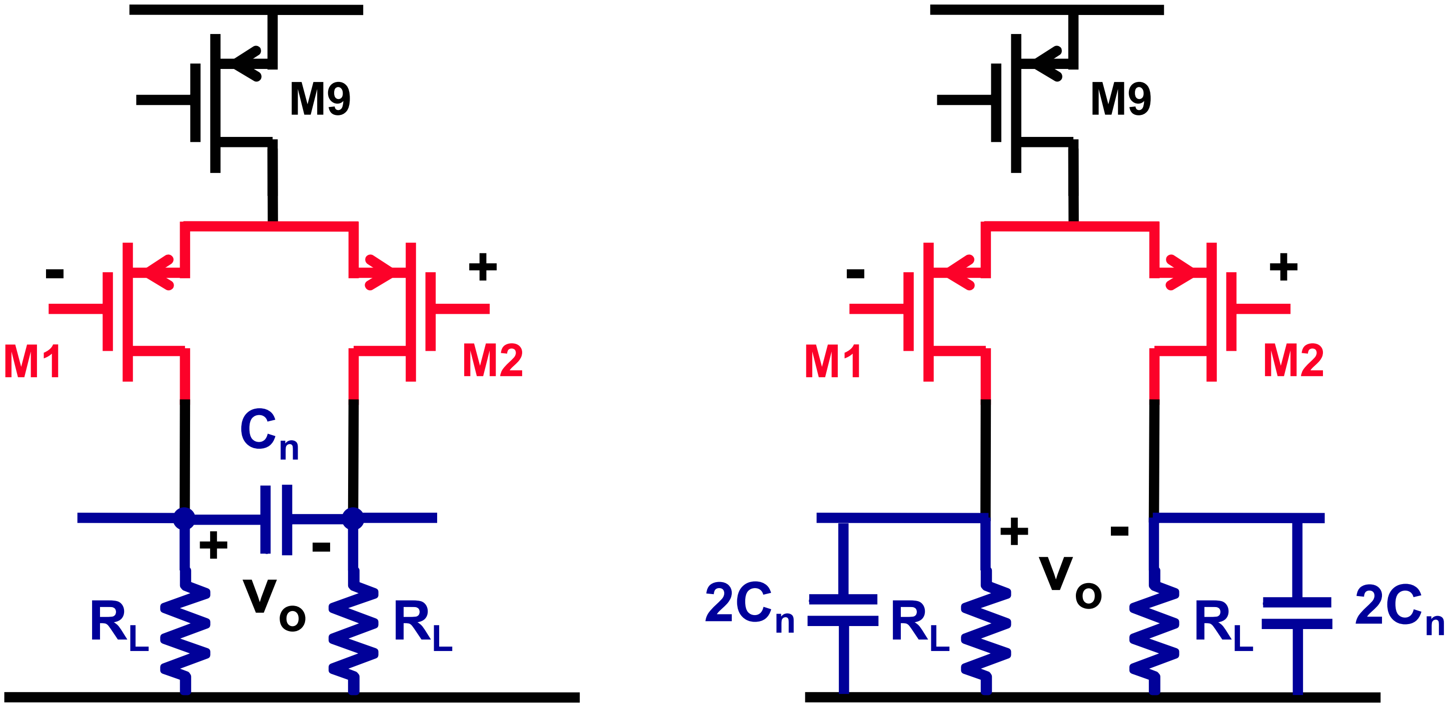


$$GBW = B\,\frac{g_{m1}}{2\pi\, C_L}$$

$$C_{n1} = (1+B)C_{GS4} + C_{DB4} + C_{DB2} \approx (3+B)C_{GS4}$$

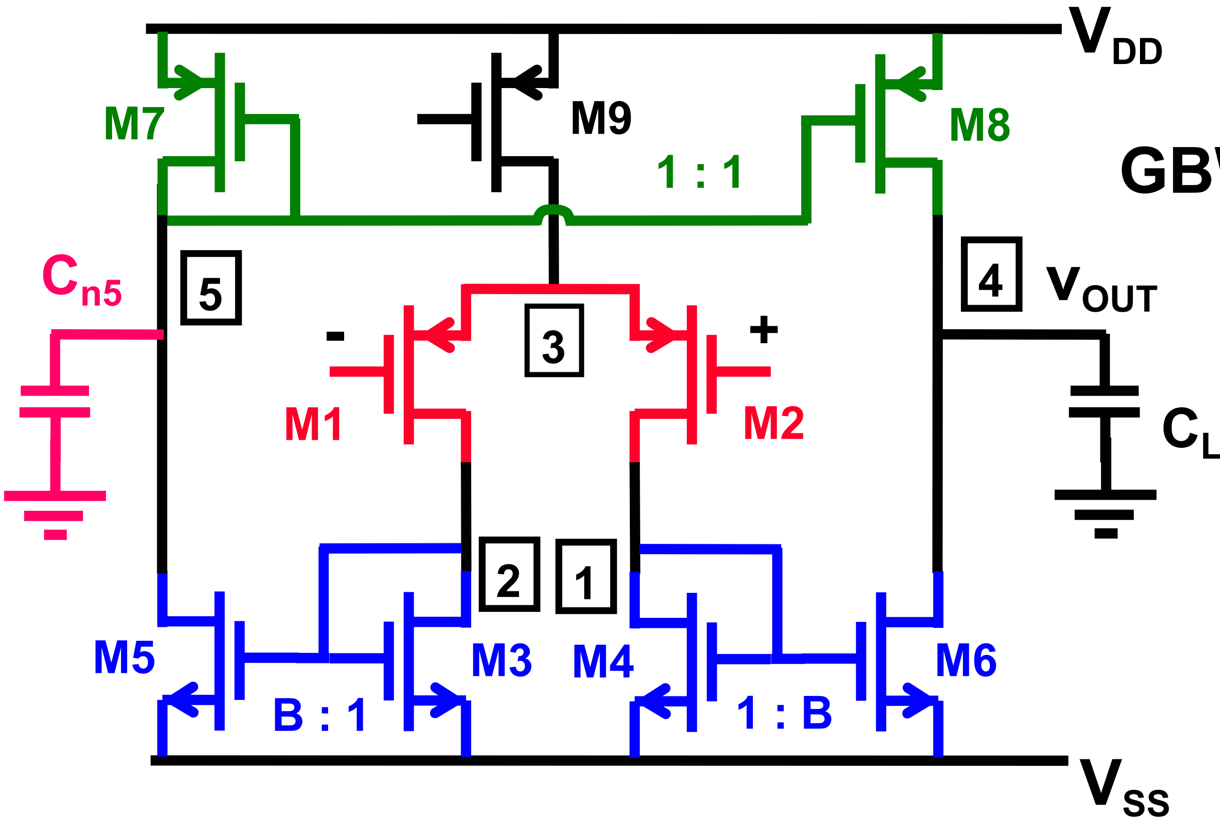
$$f_{nd} = \frac{g_{m4}}{2\pi\, C_{n1}} \approx \frac{f_{T4}}{3+B}$$

Pole at output of a differential pair



One single pole : $f_p = \dfrac{1}{2\pi\, 2R_LC_n}$

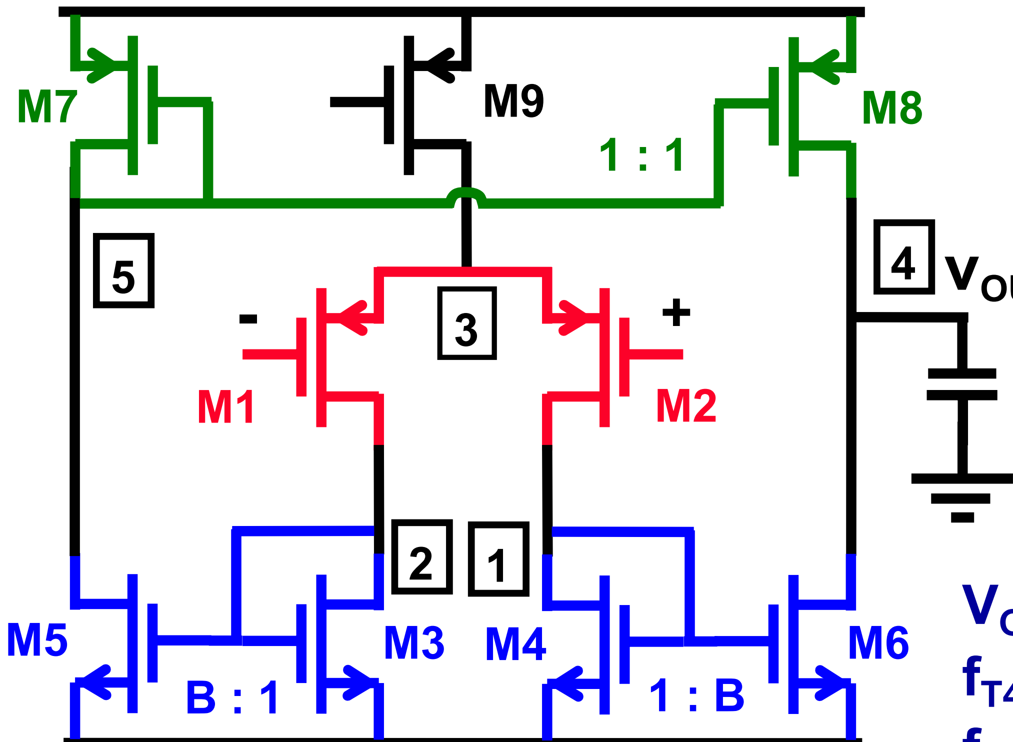
Symmetrical CMOS OTA : f_{nd5}



$$GBW = B \frac{g_{m1}}{2\pi\ C_L}$$

C_{n5} causes pole and zero at 2 x freq.: $\approx$ 3 …6 °

Symmetrical CMOS OTA : Design Example



$$GBW = B \frac{g_{m1}}{2\pi\, C_L}$$

$$f_{nd} \approx \frac{f_{T4}}{3+B}$$

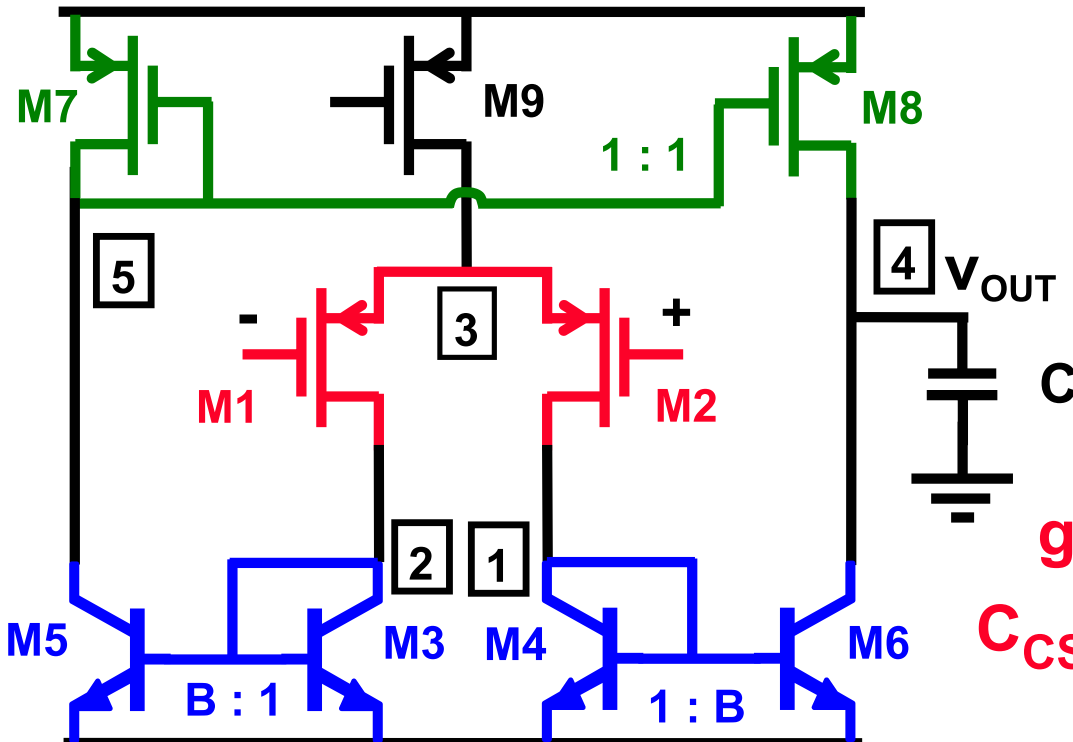
V_{GS}-V_T = 0.5 V; L = 1 μm

f_{T4} = 5 GHz

f_{nd} = 0.6 GHz

B ≈ 5

Symmetrical BiCMOS OTA



$$GBW = B\,\frac{g_{m1}}{2\pi\,C_L}$$

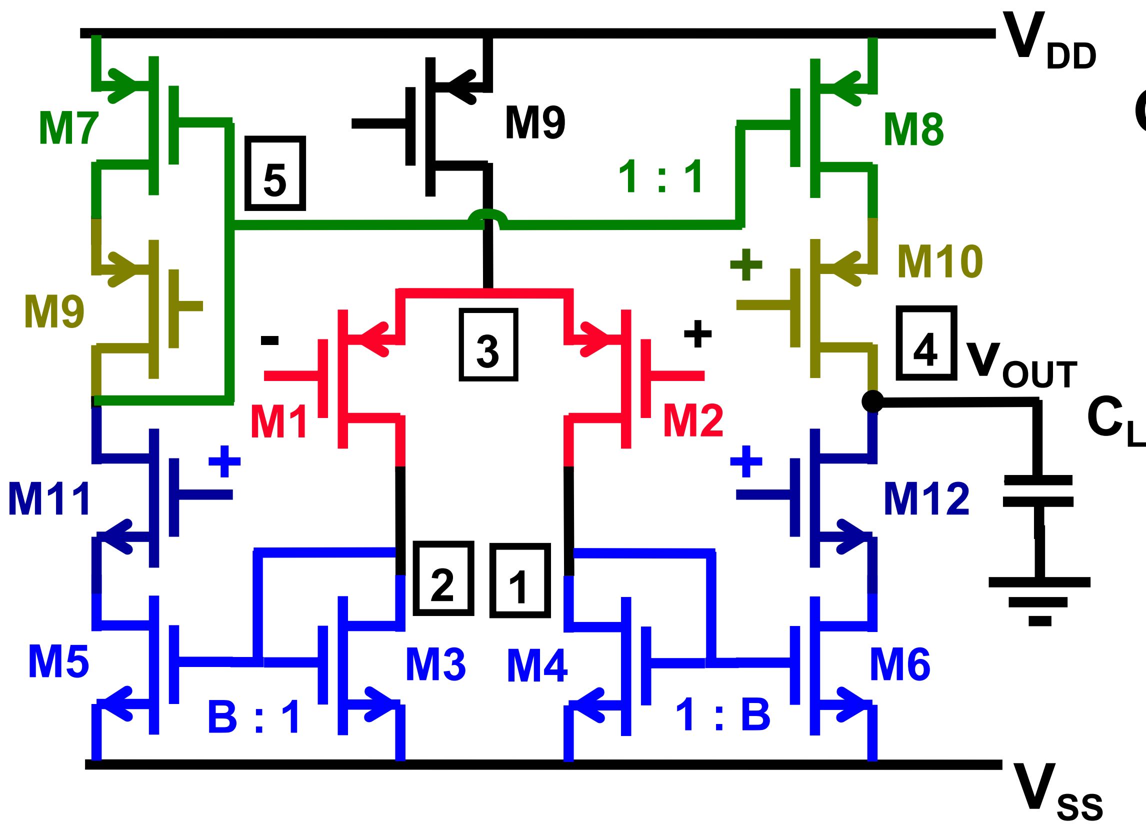
$$f_{nd} \approx \frac{f_{T4}}{3+B}$$

$g_{m4NPN} > g_{m4NMOS}$

$C_{CS4NPN} > C_{DB4NMOS}$

BiCMOS > CMOS ?

Symmetrical CMOS OTA with cascodes

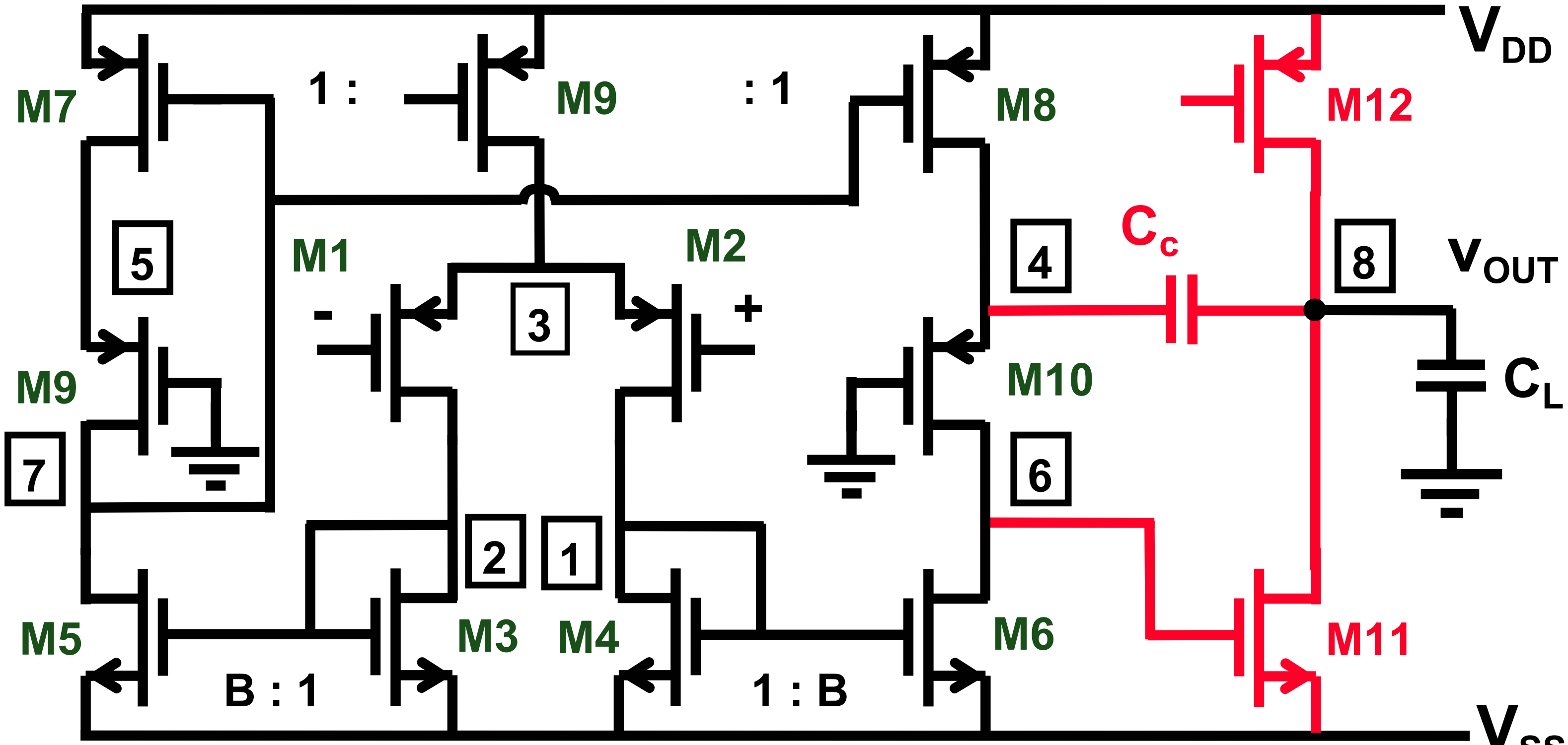


$$GBW = B\,\frac{g_{m1}}{2\pi\,C_L}$$

GBW same but A_v is 100* x higher !!!

* $g_m r_o \approx 100$

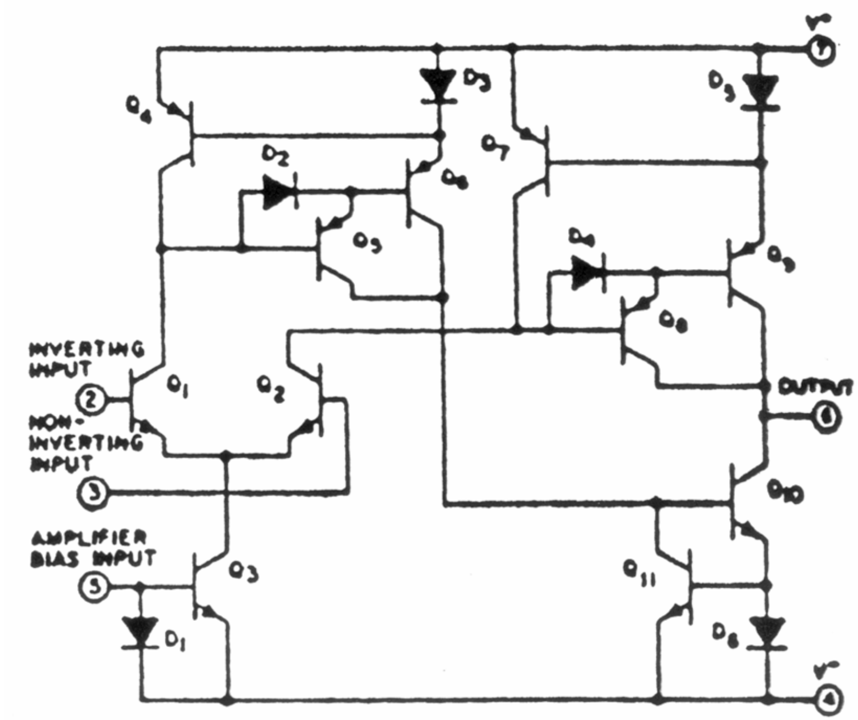
Symmetrical Miller CMOS OTA



$$GBW = B \frac{g_{m1}}{2\pi C_c}$$

No zero !

Bipolar transistor symmetrical amplifier



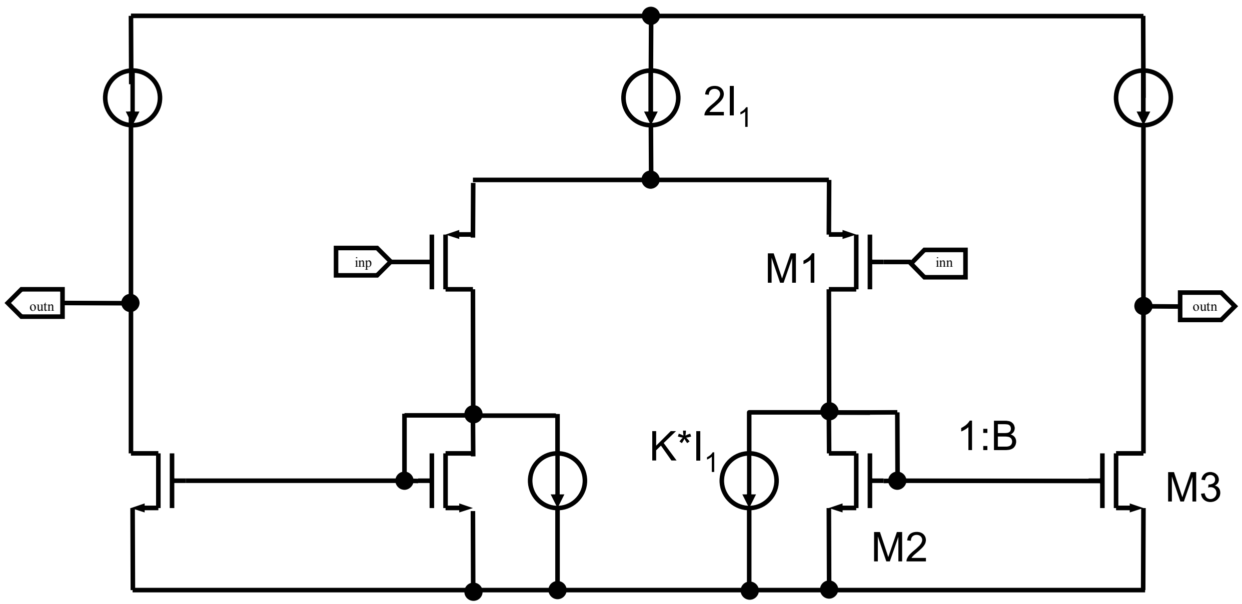
CA3080

0.12 MHz

1.2 V/μs

I_3 = 10 μA

Gain enhancement by current starving



$$A = \frac{2}{(1-k)(V_{GS} - V_T)_1 \cdot \lambda_3} = \frac{A_0}{1-k}$$

Yao, ..., JSSC Nov.04, 1809-1818

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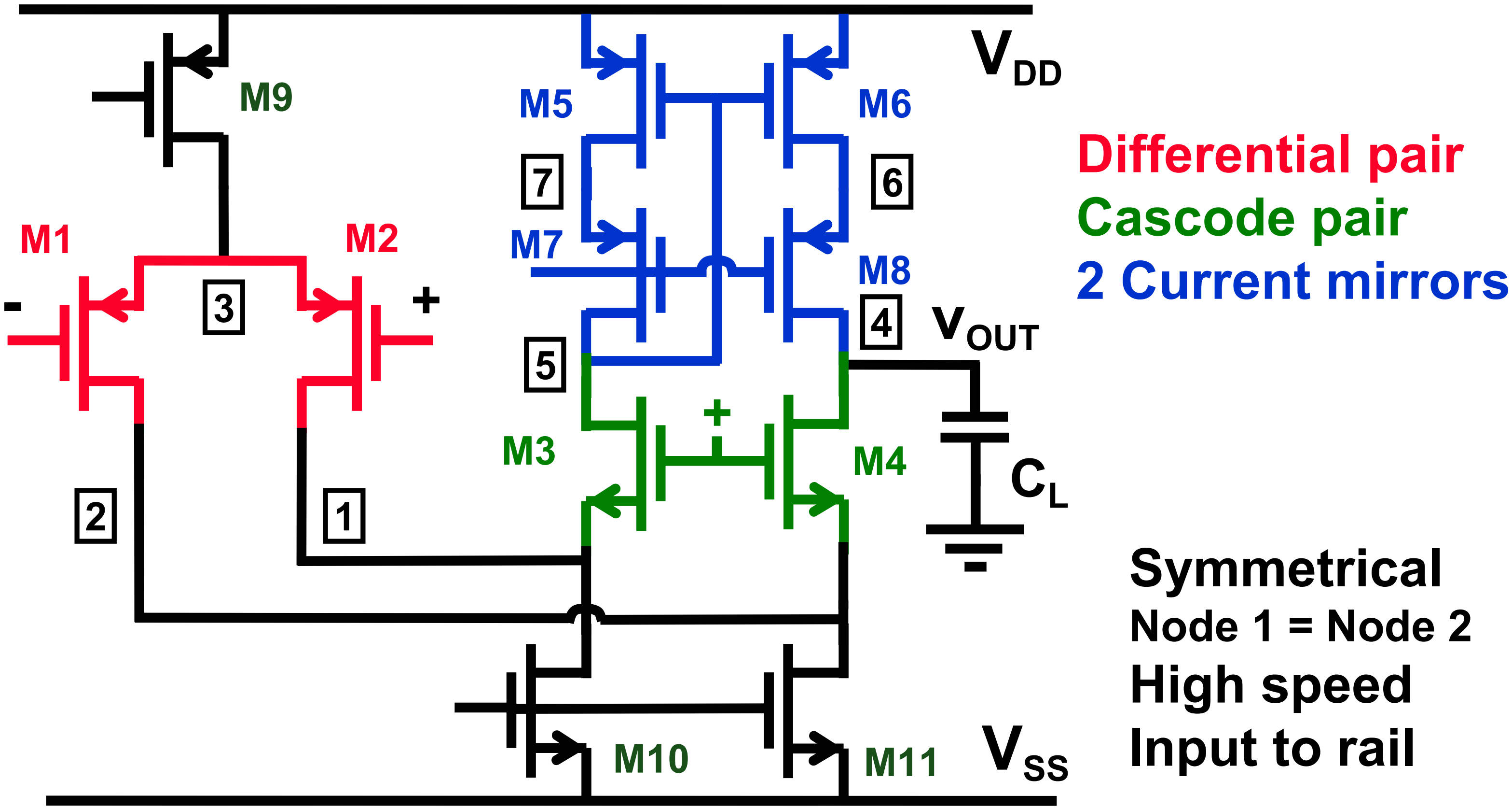
Folded cascode CMOS OTA



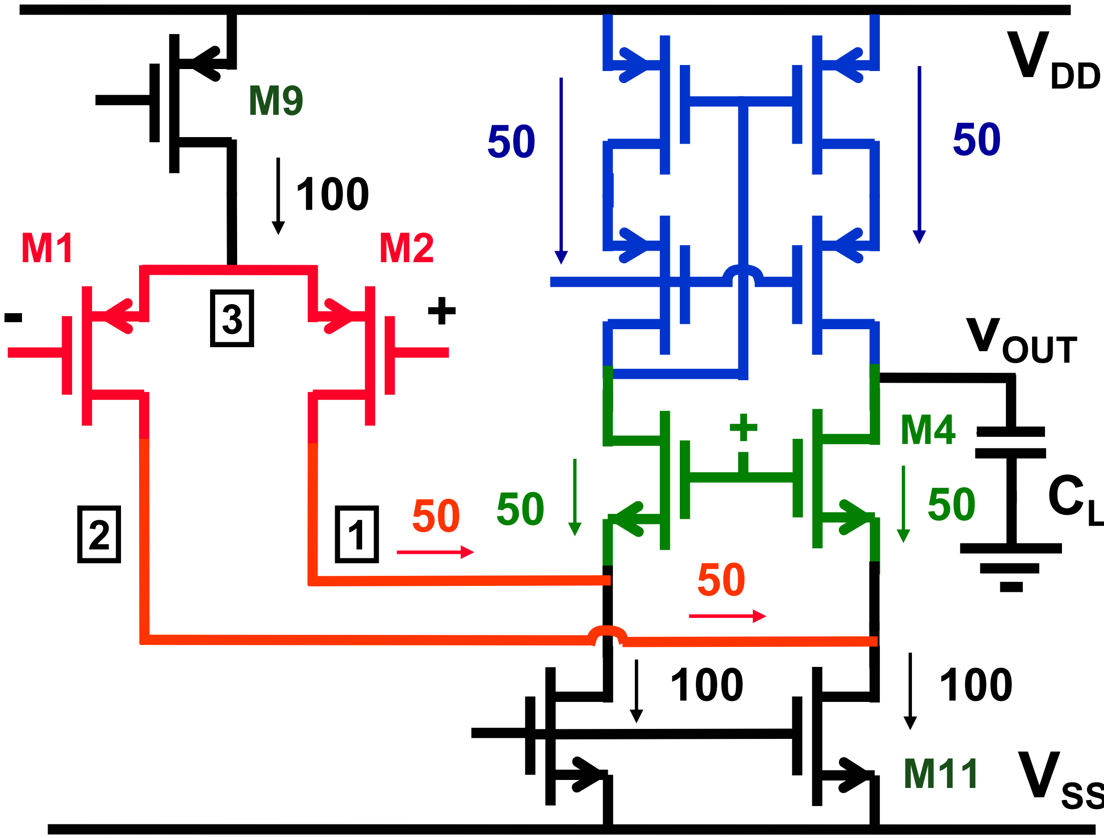
Symmetrical

Node 1 = Node 2

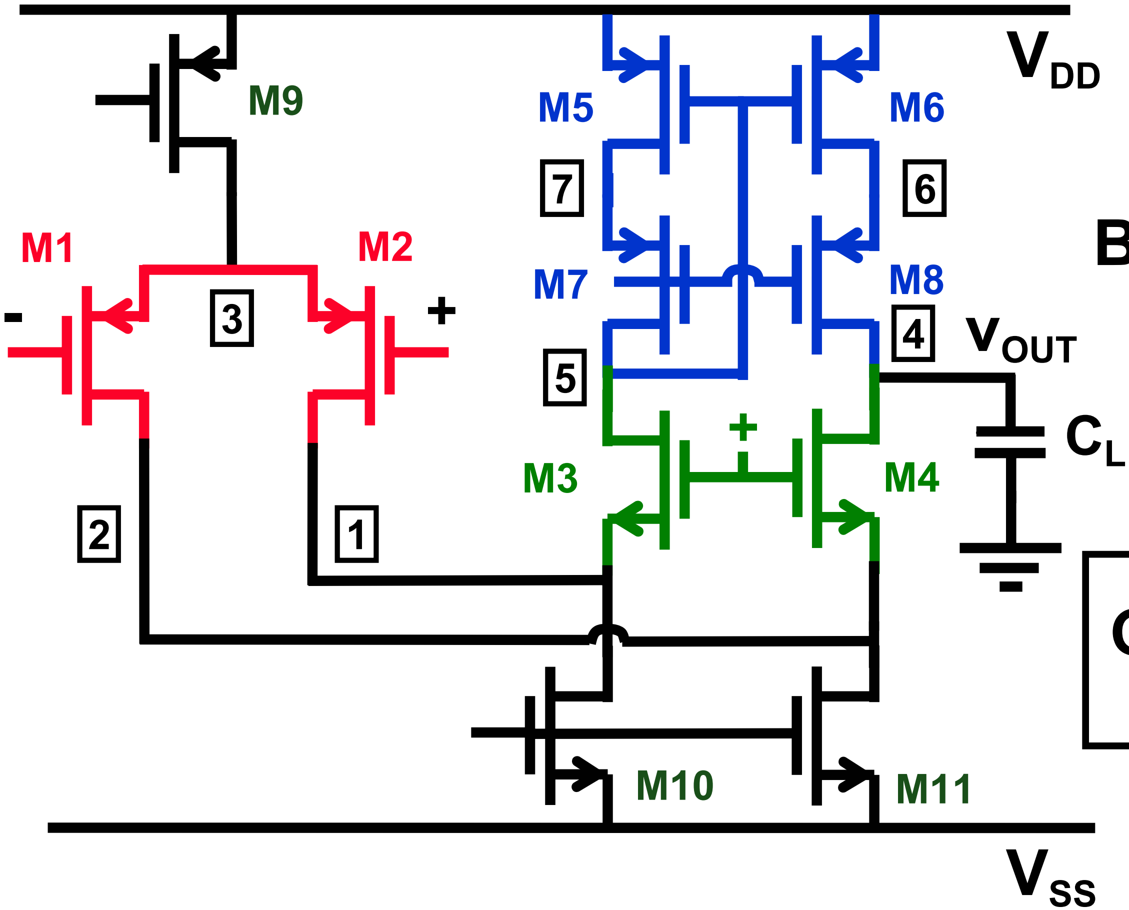
High speed

Input to rail

Folded cascode CMOS OTA : DC



Folded cascode CMOS OTA :



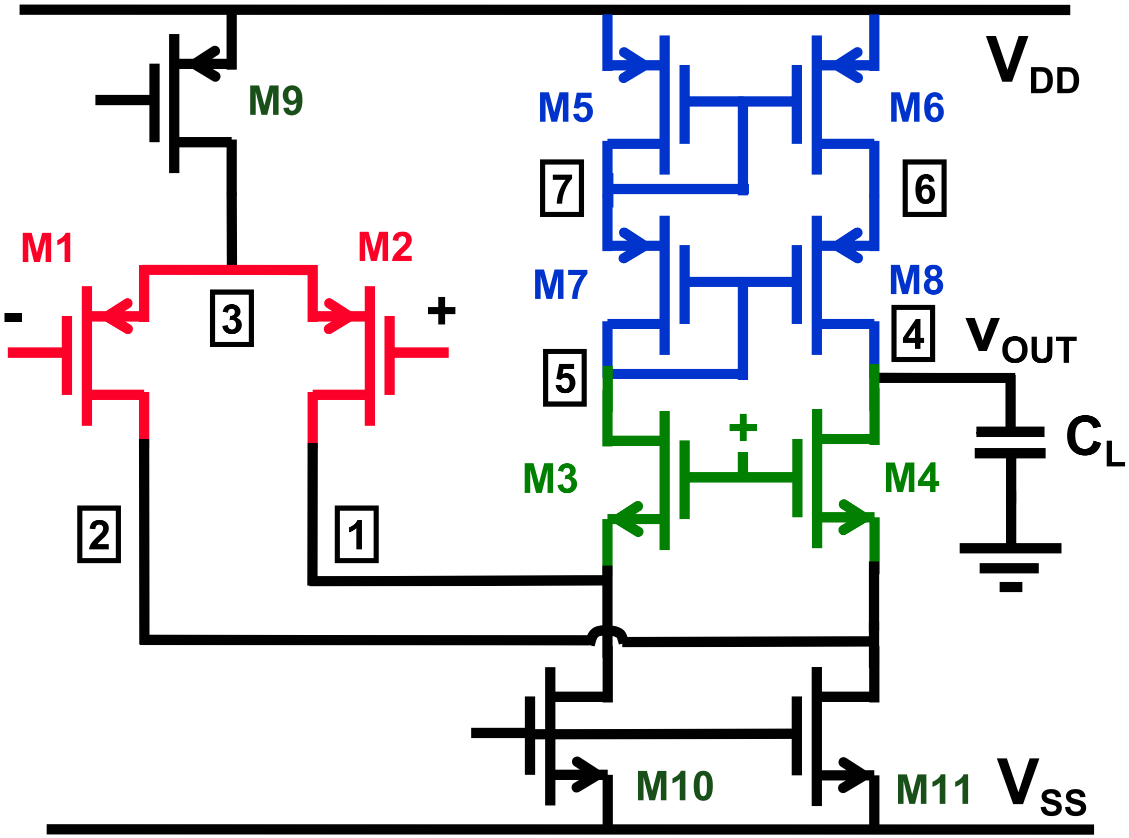
$$A_v = g_{m1} R_{n4}$$

$$BW = \frac{1}{2\pi\, R_{n4} C_L}$$

$$\boxed{GBW = \frac{g_{m1}}{2\pi\, C_L}}$$

Single stage ?

Folded cascode CMOS OTA :



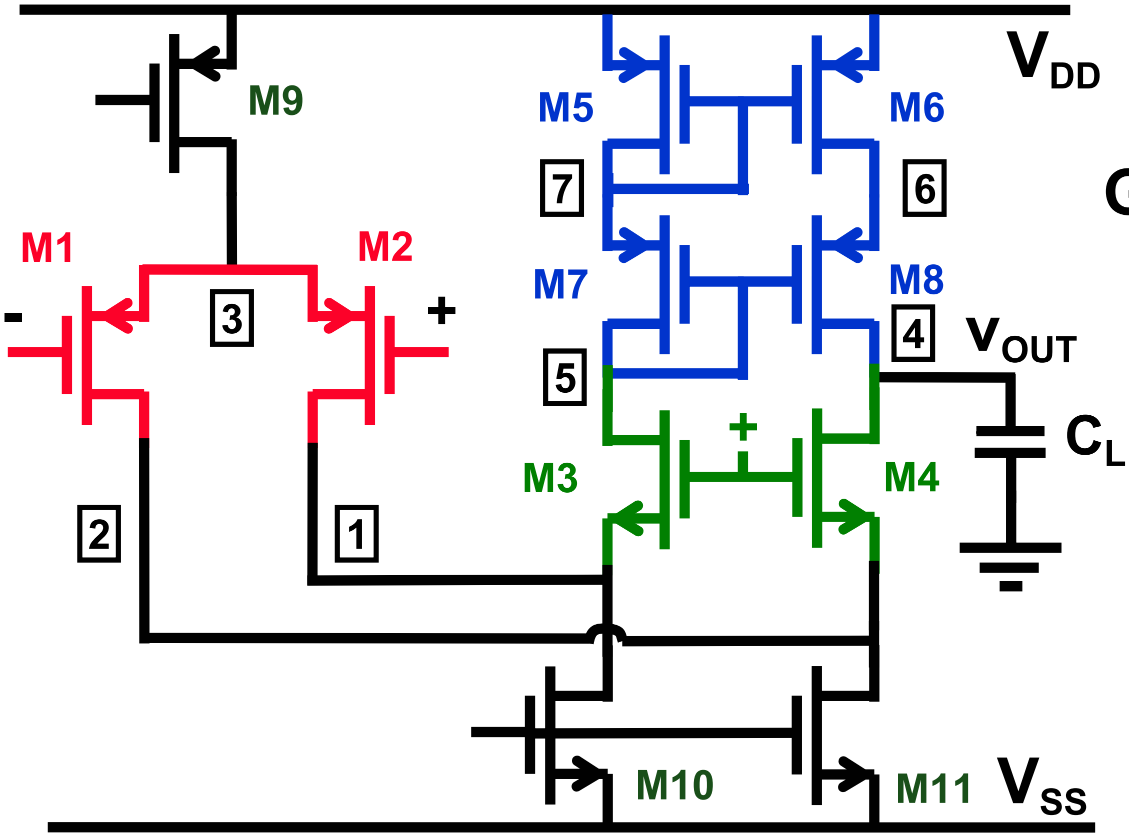
$$GBW = \frac{g_{m1}}{2\pi C_L}$$

$$C_{n1} = C_{GS3} + C_{DB1} + C_{DB10} \approx 3\,C_{GS3}$$

$$f_{nd} = \frac{g_{m3}}{2\pi C_{n1}} \approx \frac{f_{T3}}{3}$$

Hi !

Folded cascode CMOS OTA :

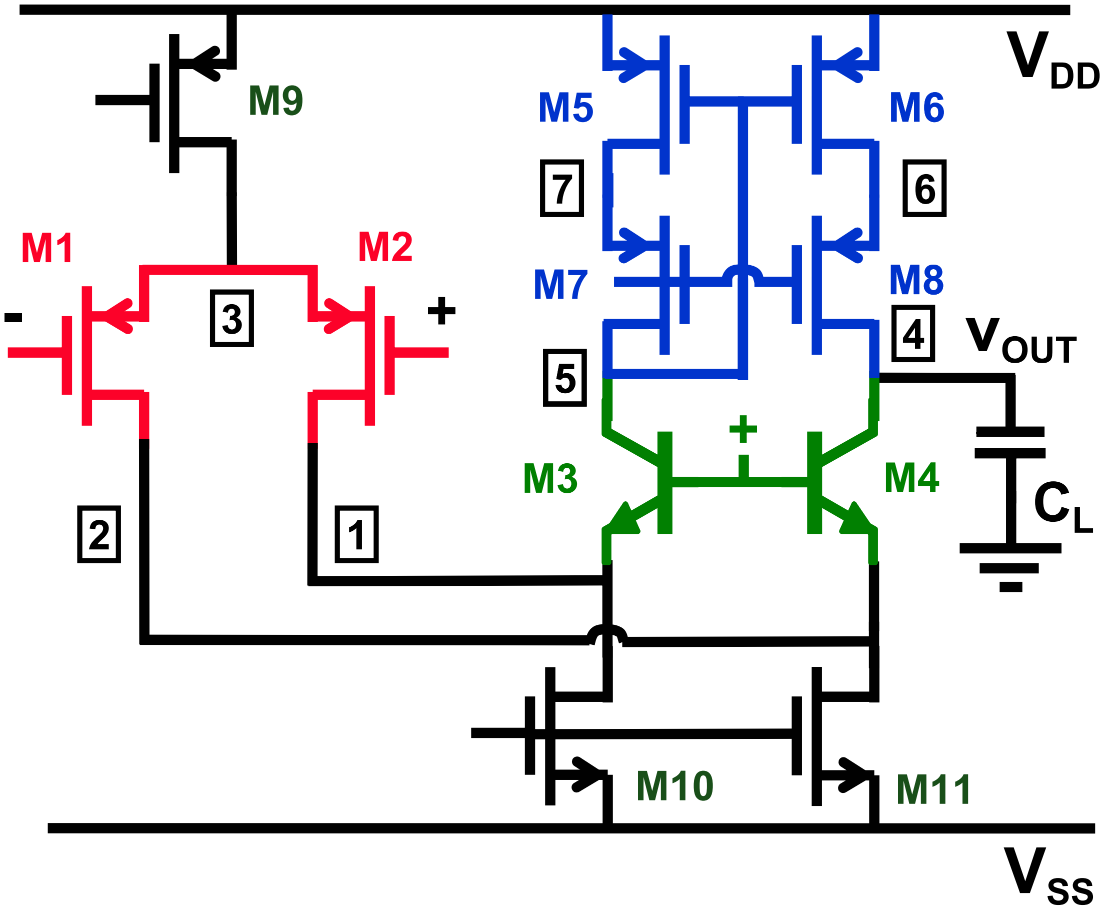


$$GBW = \frac{g_{m1}}{2\pi\ C_L}$$

C_{n5}, C_{n7} ,C_{n6}
Cause pole
and zero
at 2 x freq.:
≈ 5 …10 °

Ref Mallya, JSSC Dec 89, 1737-1740

Folded cascode BiCMOS OTA



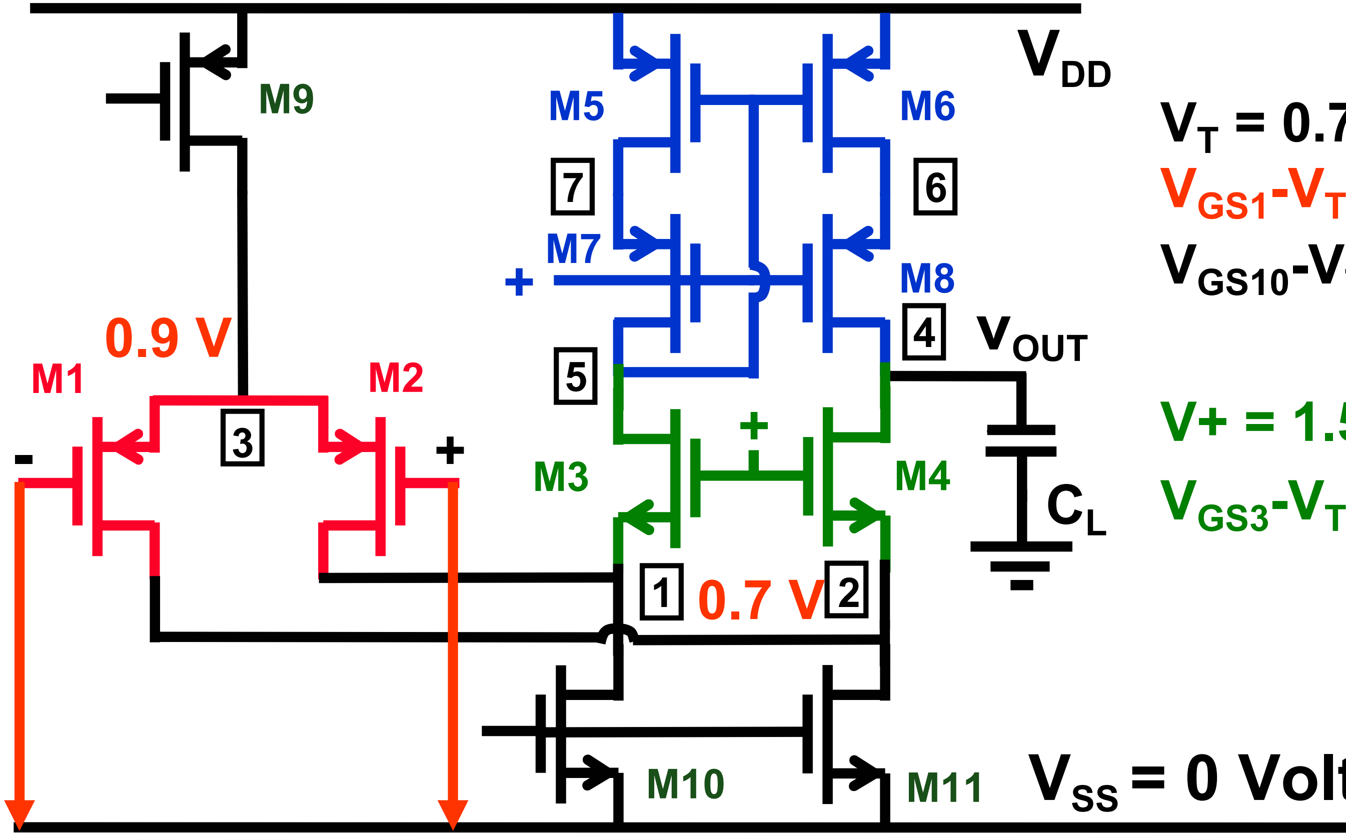
$$GBW = \frac{g_{m1}}{2\pi\, C_L}$$

$$f_{nd} = \frac{g_{m3}}{2\pi\, C_{n1}}$$

$$\approx \frac{f_{T3NPN}}{3}$$

Higher !

Folded cascode OTA: input to V_{ss} rail



$V_T = 0.7$ V

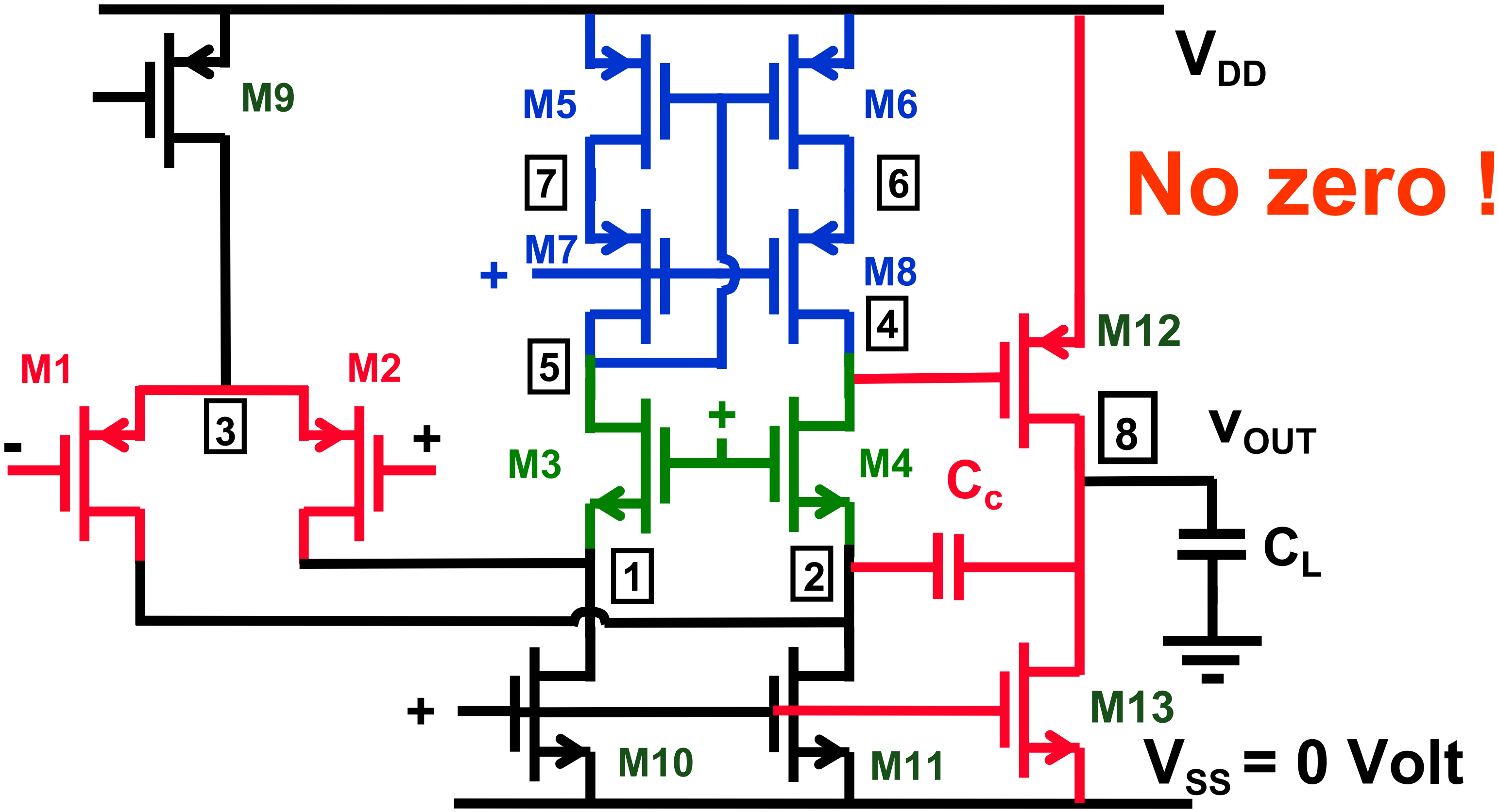
$V_{GS1}-V_T = 0.2$ V

$V_{GS10}-V_T = 0.5$ V

$V+ = 1.5$ V

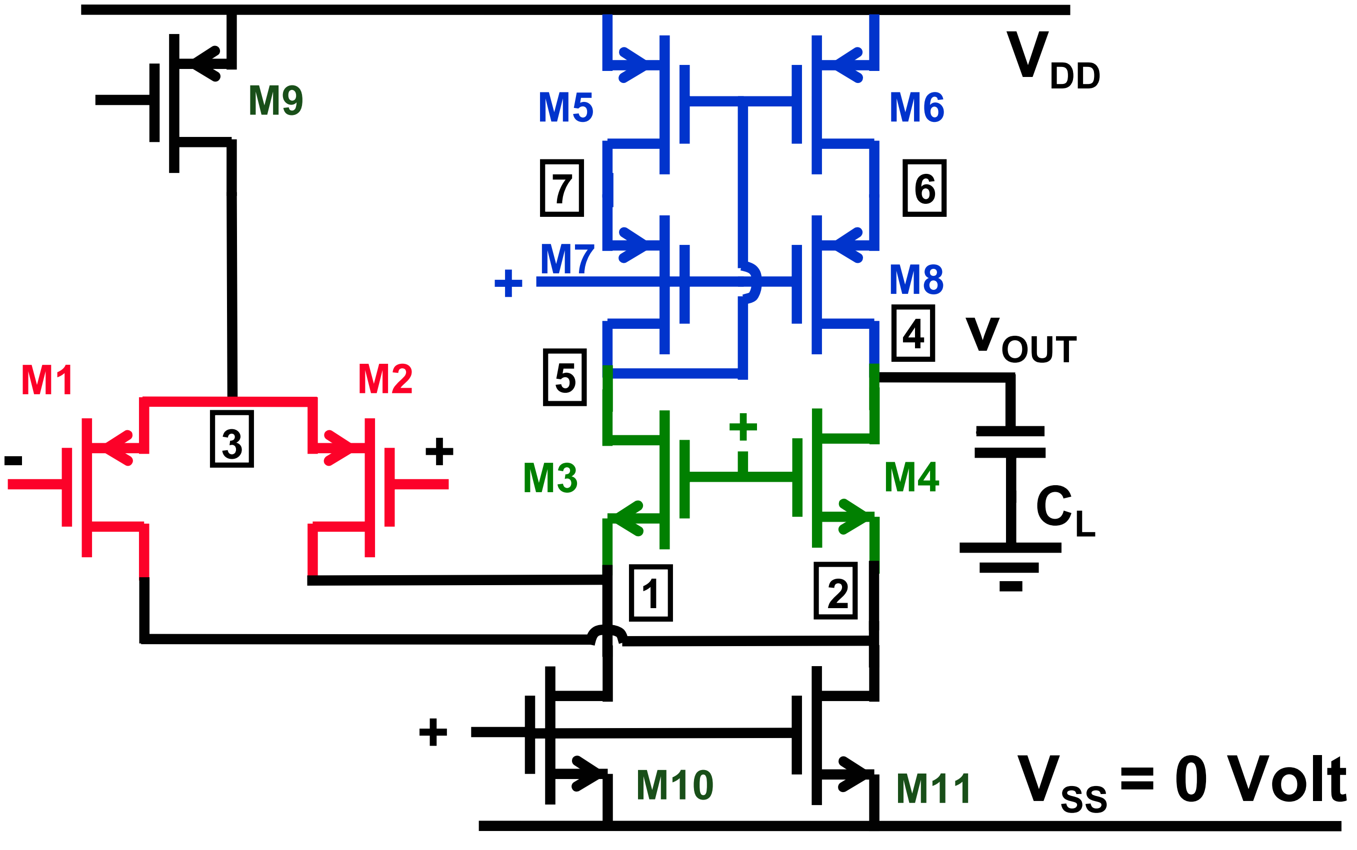
$V_{GS3}-V_T = 0.2$ V

Folded cascode OTA with 2nd stage

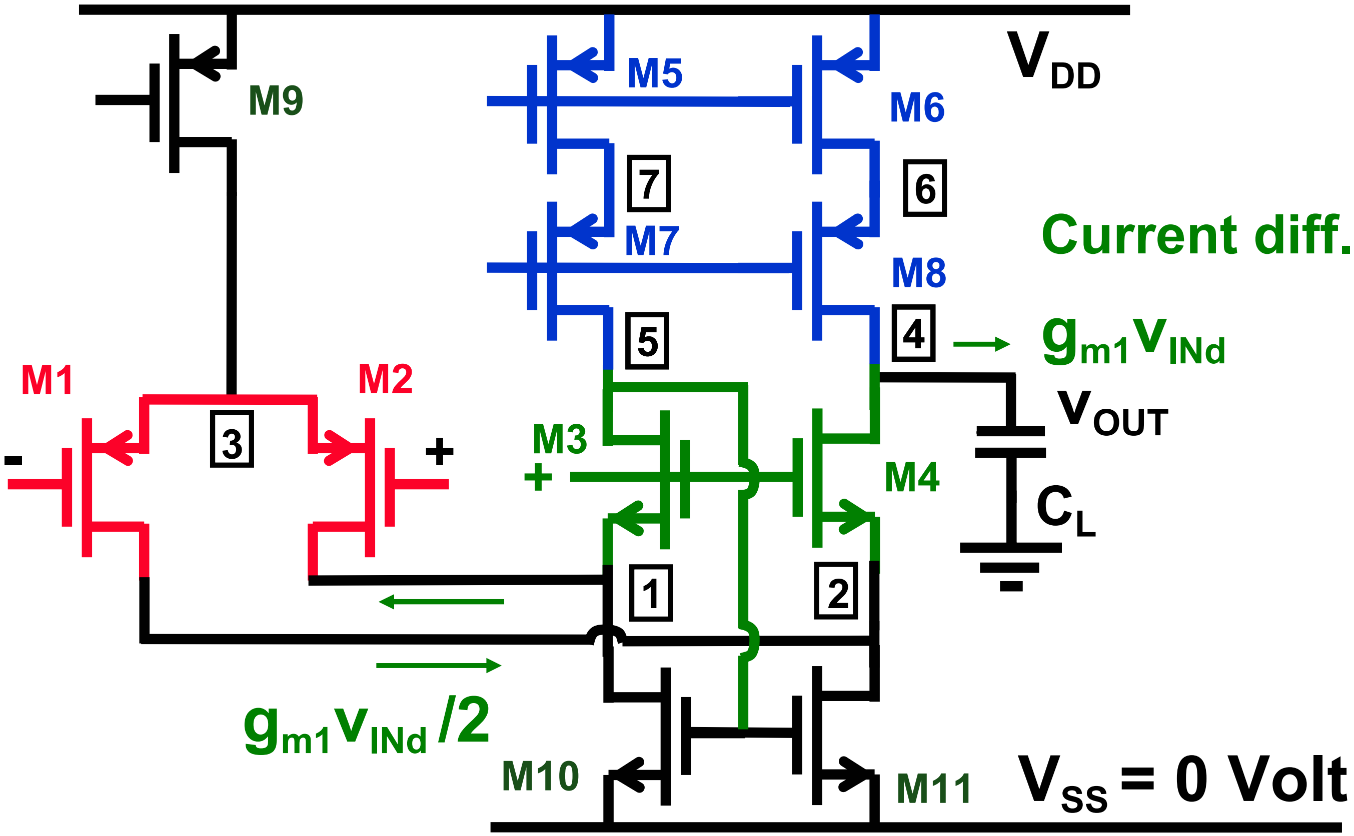


Ribner, JSSC Dec.84, 919-925

Conventional folded cascode OTA



Alternative folded cascode OTA



Comparison amplifiers

	I_{TOT} mA	$\overline{dv_{in,eq}^2}$ $\frac{8/3\ kT\ df}{g_{m1}}$	Swing
Volt. OTA (4 Ts)	0.25	4	avg.
Symmetrical (B= 3)	0.33	16	max.
Telescopic	0.25	4	small
Folded casc.	0.5	4	avg.
Miller 2-stage (C_L/C_c= 2.5)	1.1	4	max.

GBW = 100 MHz C_L = 2 pF V_{GS}-V_T = 0.2 V Fully differential

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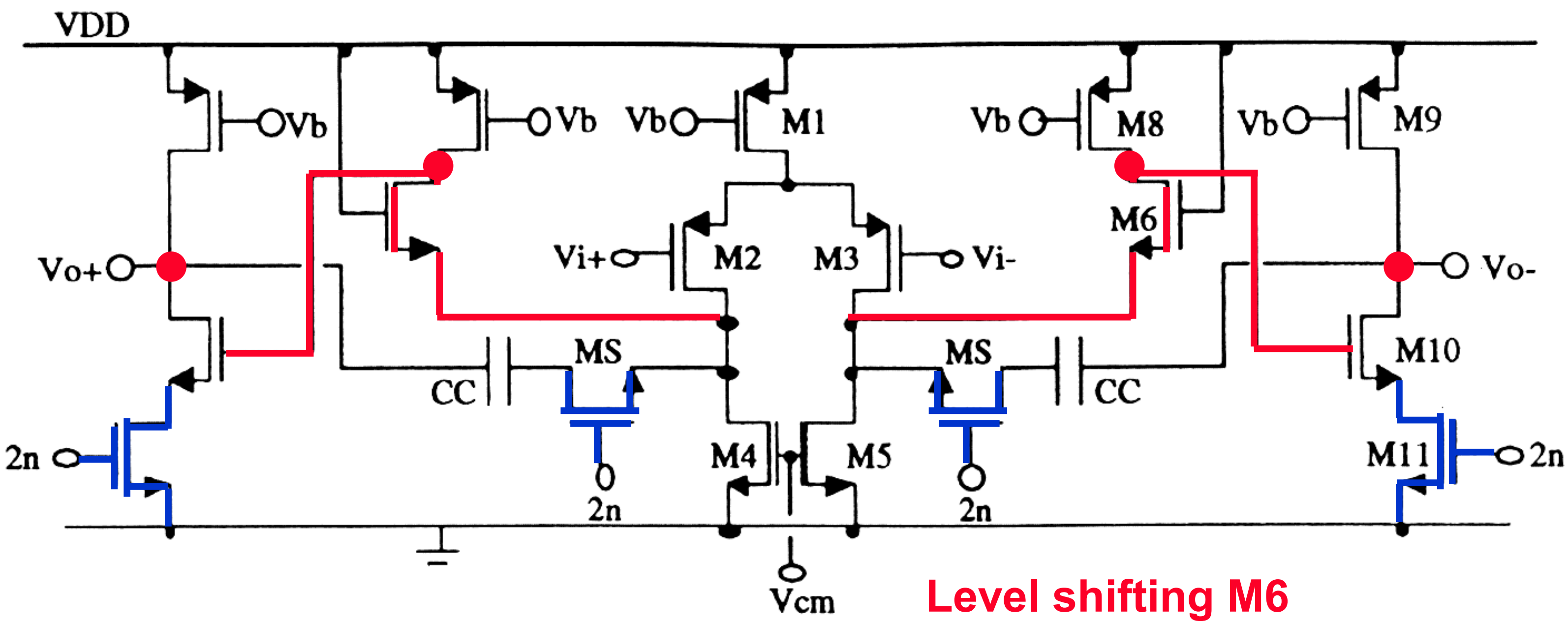
- Simple CMOS OTA
- CMOS Miller OTA
- Symmetrical CMOS OTA
- Folded cascode OTA
- **Other opamps**

Sub-1 Volt OTA

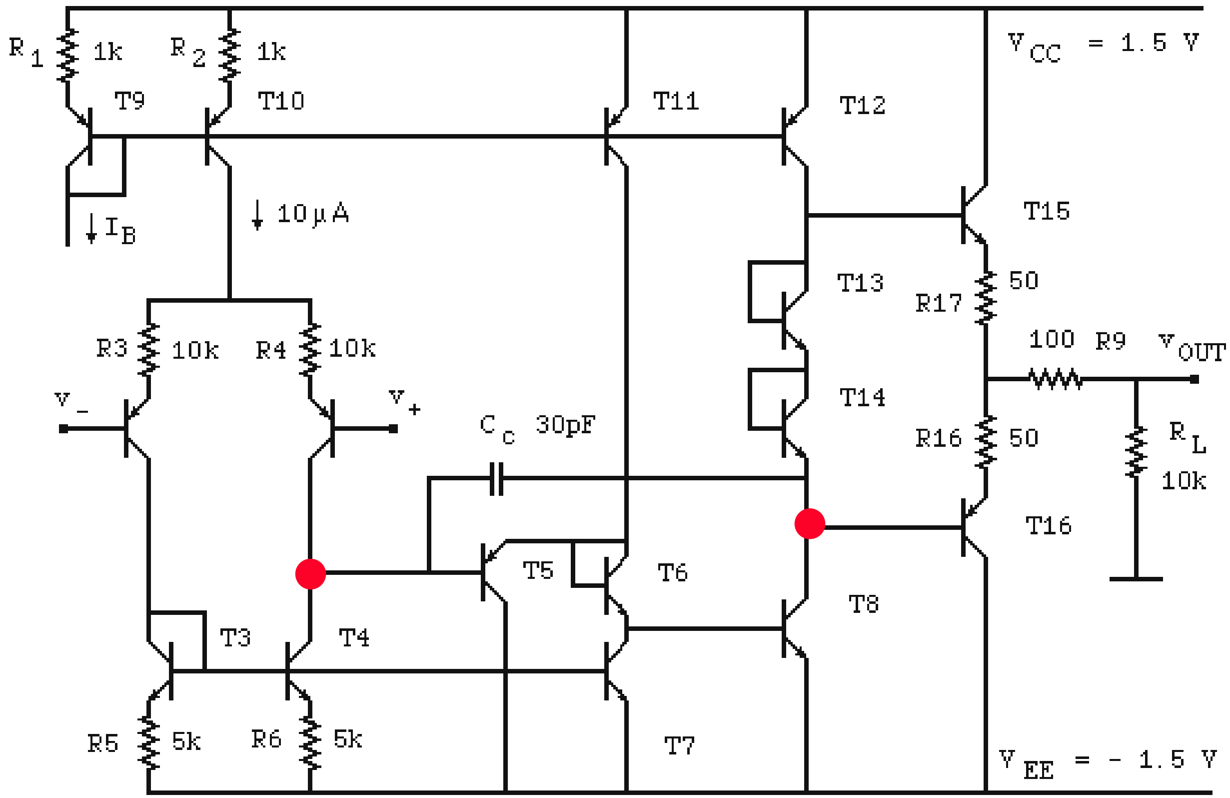


1 V 80 μW (min: V_T+2V_{DSsat})
Fully differential
75 dB 30 MHz (0.1 pF)
< 100 ns

Level shifting M6
4 Switches 2n :
Only 2nd stage switched off !

Baschirotto, .. JSSC Dec.97,pp.1979-1986

LM 4250



GBW = 0.25 MHz

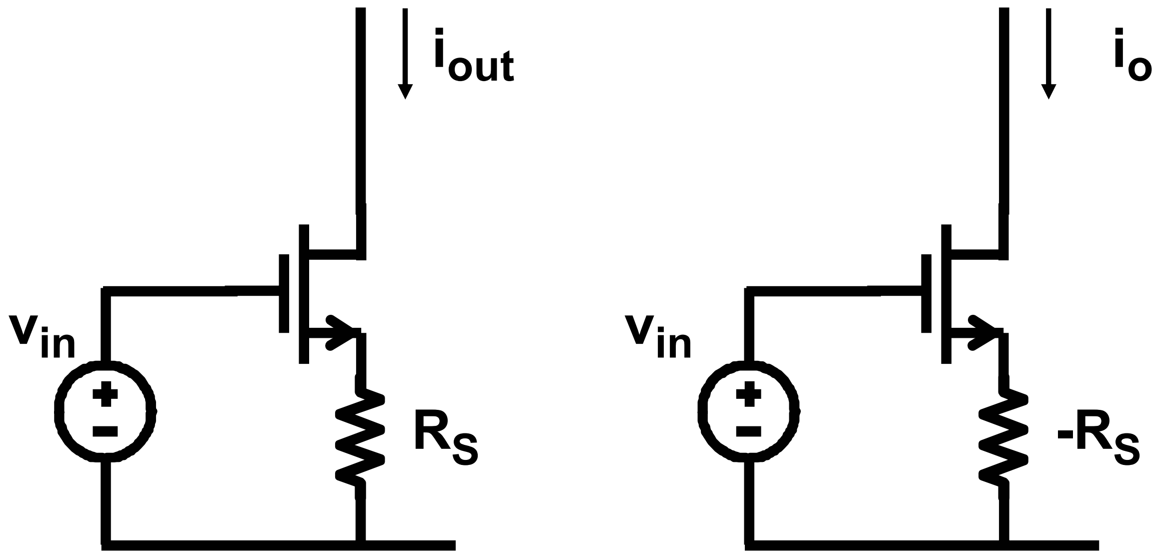
SR = 0.2 V/μs

I_1 = 10 μA

I_{TOT} = 90 μA

38 $nV_{RMS}/\sqrt{Hz}$

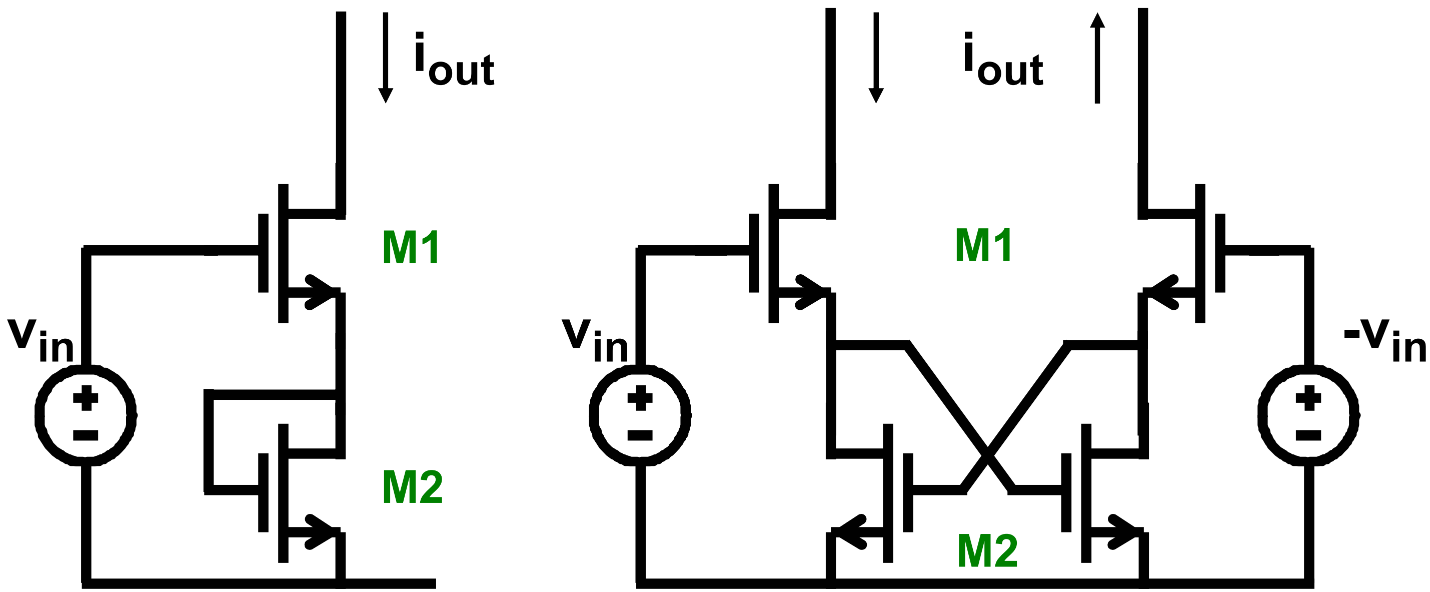
Increased input transconductance - 1



$$g_{mR} = \frac{g_m}{1 + g_m R_S}$$

$$g_{mR} = \frac{g_m}{1 - g_m R_S}$$

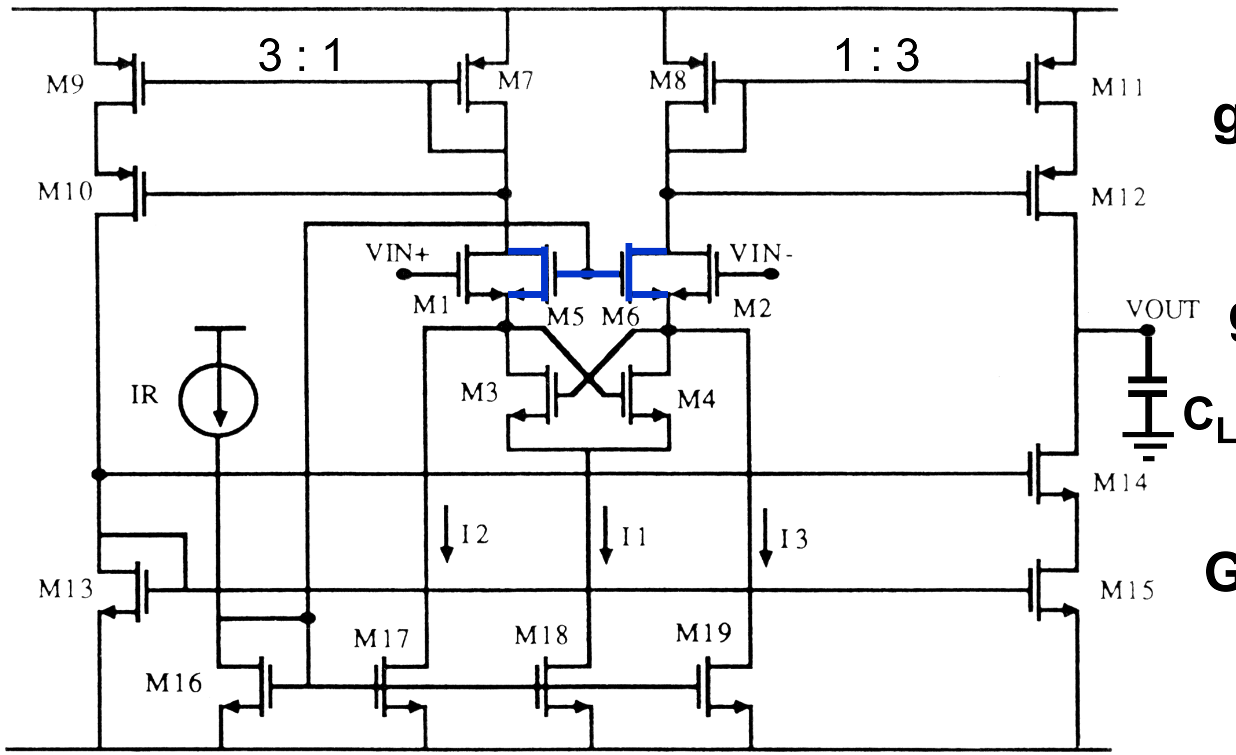
Increased input transconductance - 2



$$g_m = \frac{g_{m1}}{1 + \frac{g_{m1}}{g_{m2}}}$$

$$g_m = \frac{g_{m1}}{1 - \frac{g_{m1}}{g_{m2}}}$$

Increased input transconductance - 3



$$g_m = \frac{g_{m1}}{1 - \frac{g_{m1}}{g_{m3}}}$$

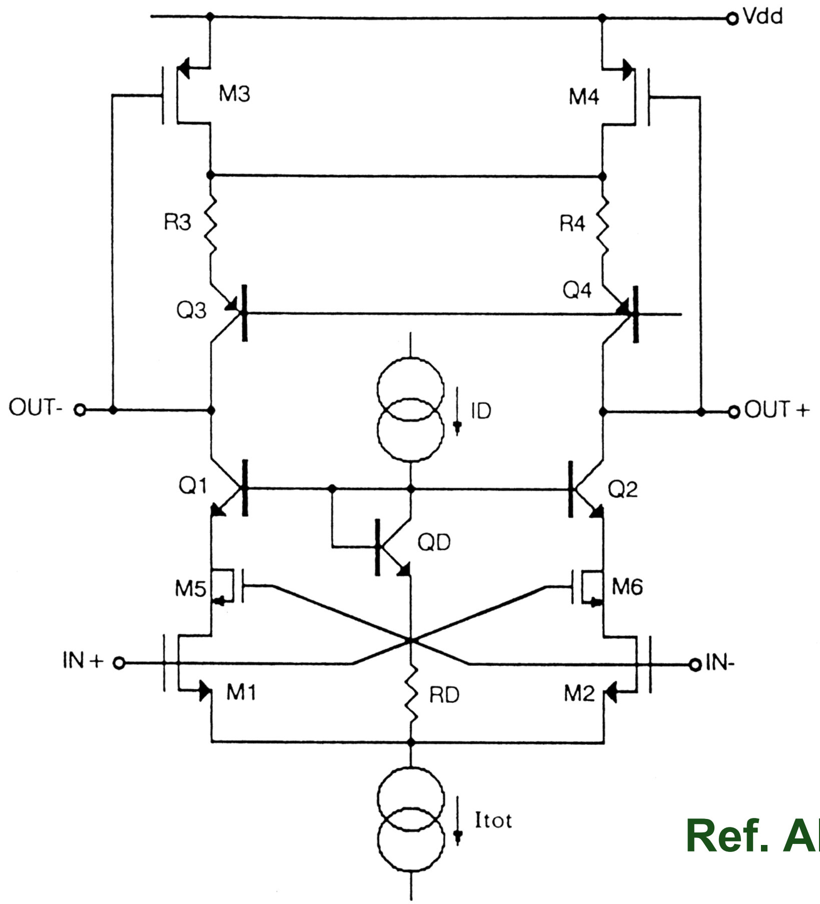
$$g_m \approx 3\ g_{m1}$$

$$GBW = \frac{3\ g_m}{2\pi C_L} \approx \frac{9\ g_{m1}}{2\pi C_L}$$

Ref.: Castello, JSSC June 1990, pp. 669-676

M5,M6 for overdrive !

Transconductor with C□$_{DG}$ compen.



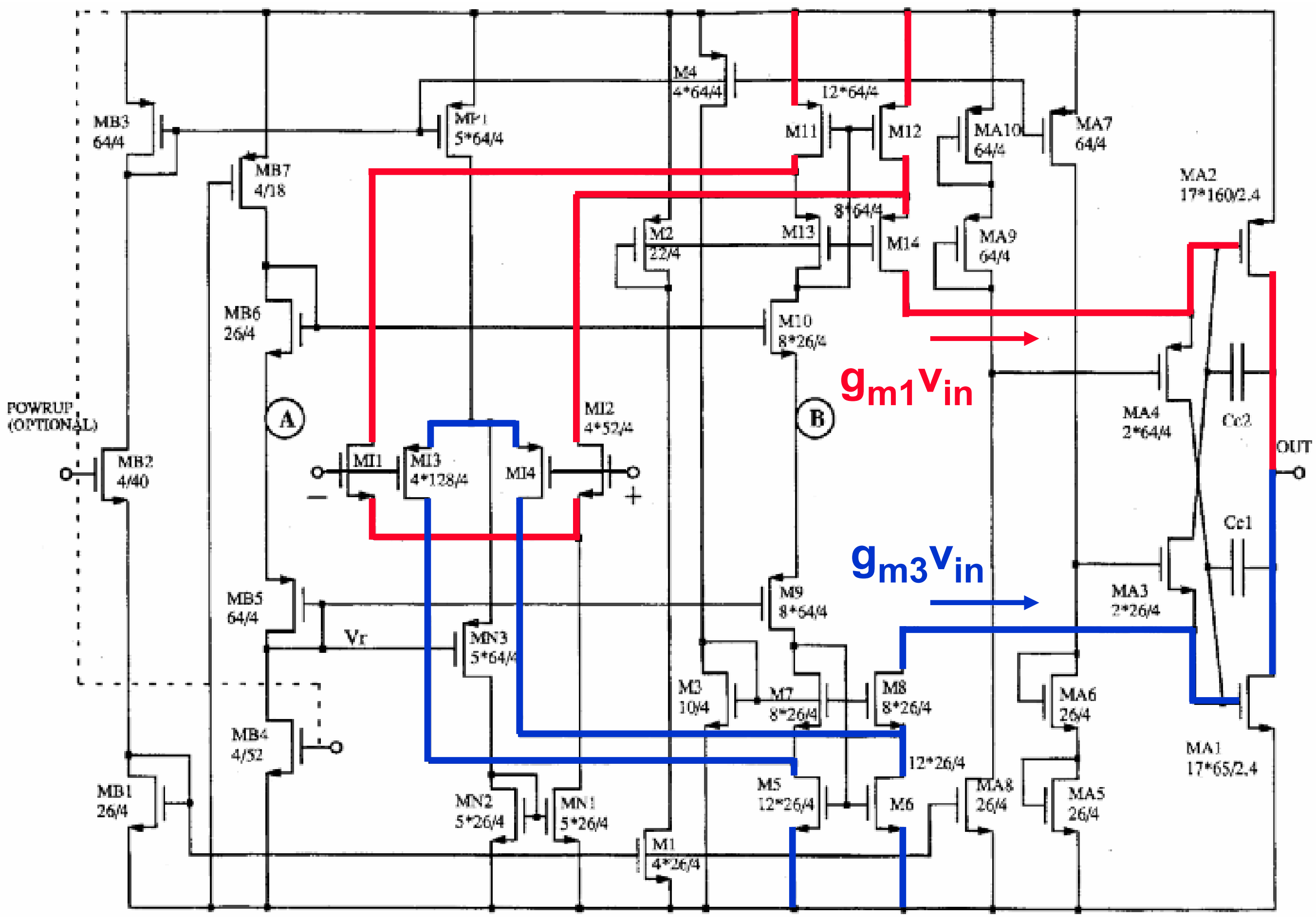
$V_{DS1} = R_D I_D \approx 0.2$ V

$I_{DS1} = \beta_1 V_{DS1}(V_{GS1} - V_T)$

$g_{m1} = \beta_1 V_{DS1}$ is constant

Ref. Alini, JSSC, Dec.92, pp.1905-1915

Ref. : Wu etal, JSSC Jan.1994, pp.63-66



14 MHz / 11pF

5.6 MHz / 100pF

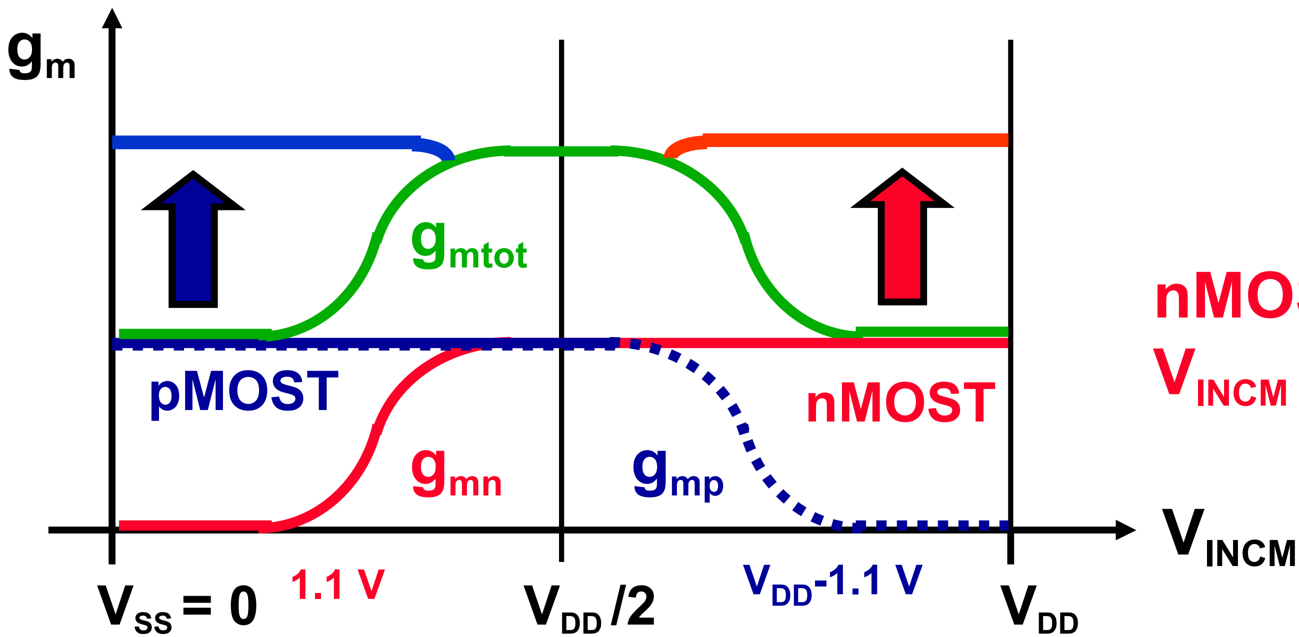
4 V/μs

36 nV/√ Hz

5 V

0.4 mA

Problem: unequal g_{mtot}



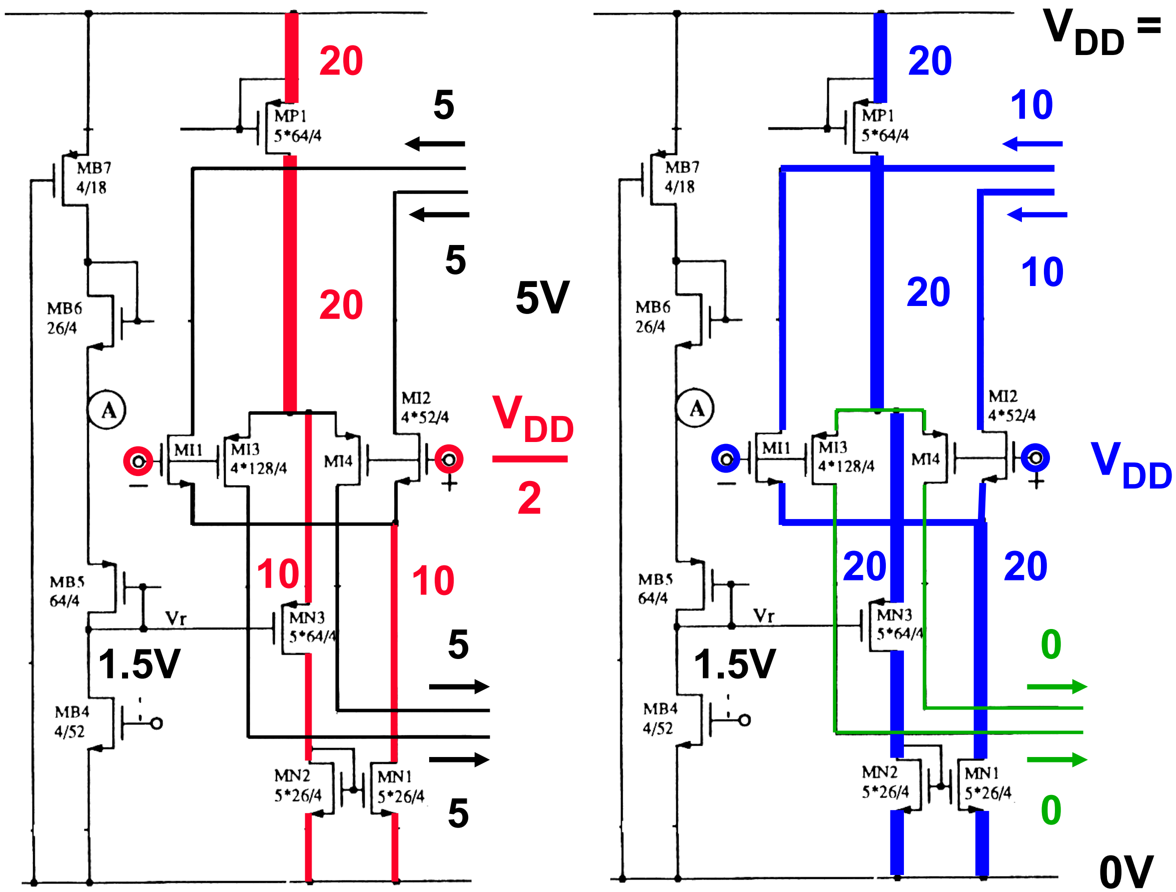
nMOST on : $V_{INCM} > 1.1$ V

pMOST on : $V_{INCM} < V_{DD} - 1.1$ V

Increase g_m:

$g_{mn} + g_{mp} = ct$

Wu : input rail-to-rail stage



V_{+-} high :

pMOSTs off !

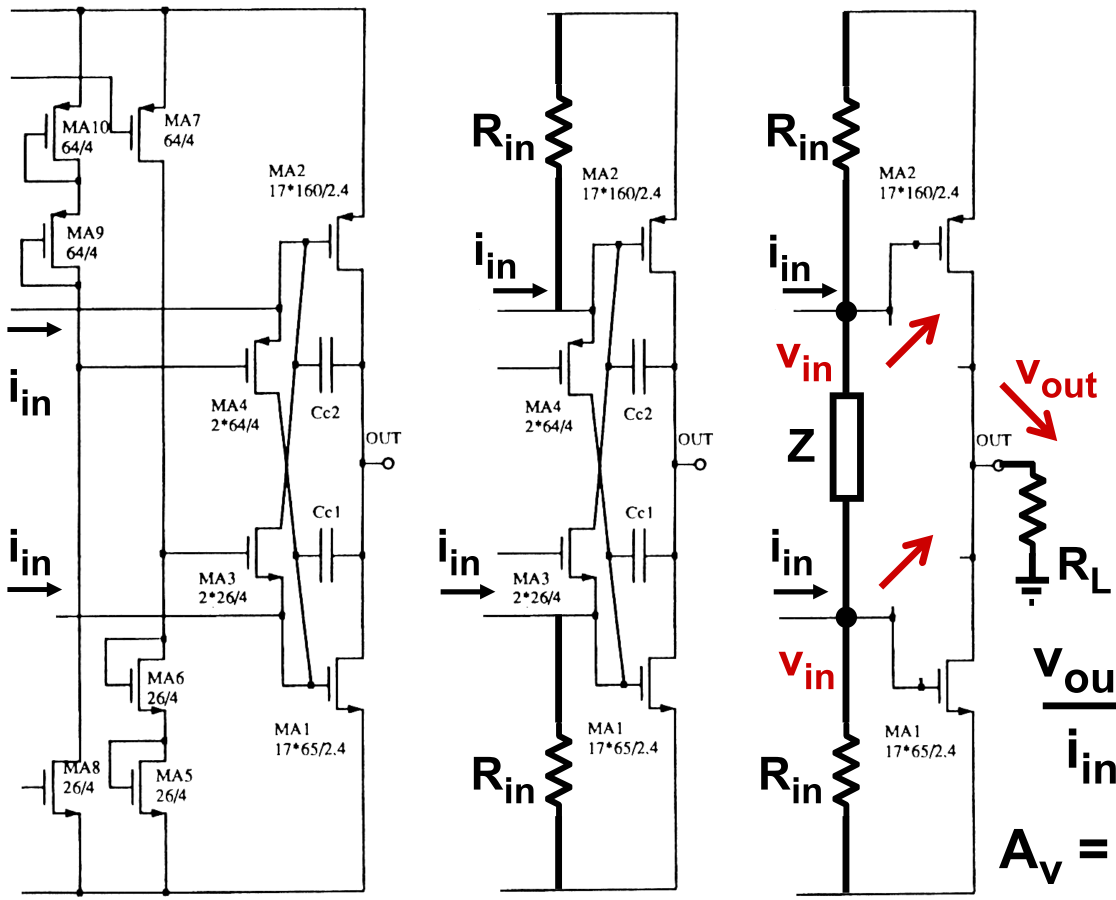
nMOSTs :

$I_{DS1} \times 2$

$g_{m1} \times 2$

Weak inv.

Wu : output stage : gain



$$i_{in} = g_{m1} v_{+-}$$

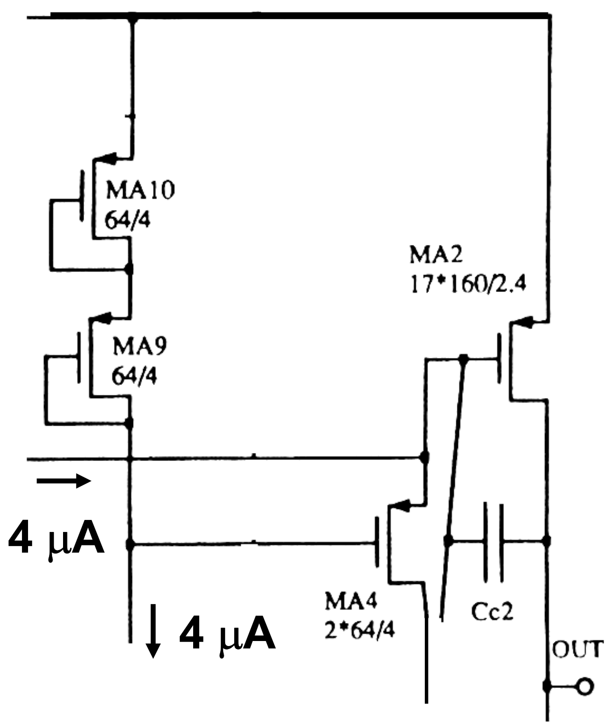
$$\frac{v_{in}}{i_{in}} = R_{in}$$

$$\frac{v_{out}}{v_{in}} = 2 g_{mA1} R_L$$

$$\frac{v_{out}}{i_{in}} = -\,2 R_{in} g_{mA1} R_L$$

$$A_v = 2 g_{m1} R_{in} g_{mA1} R_L$$

Wu : output quiescent current control



$$V_{GS2} + V_{GS4} = V_{GS9} + V_{GS10}$$

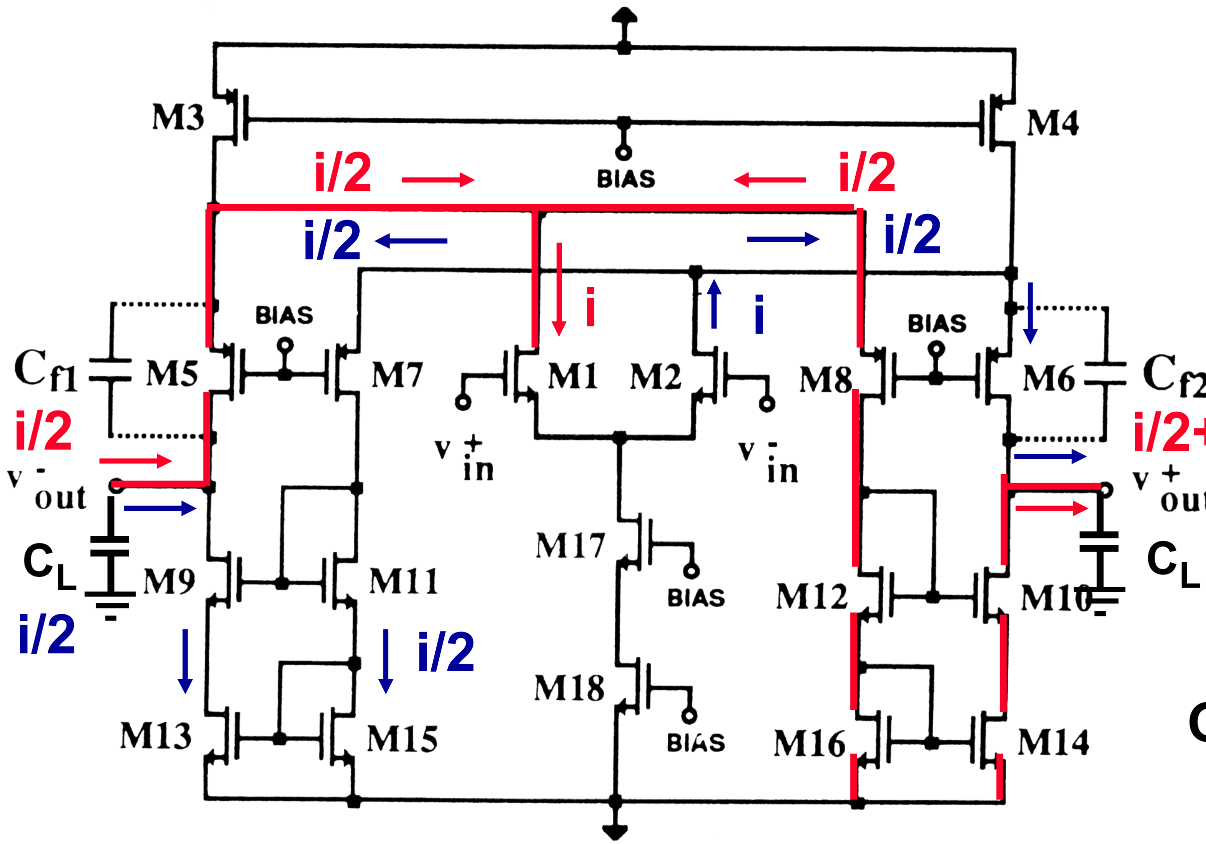
$$M_4 = 2\,M_{10} \quad \& \quad M_9 = M_{10}$$

$$V_{GS2} - V_T = \sqrt{\frac{I_{DS2}}{K'_p\,W/L_2}}$$

$$\frac{I_{DS2}}{I_{DS9}} = \frac{W/L_2}{W/L_9}\left(2 - \frac{1}{\sqrt{2}}\right) \approx 91$$

$I_{DS2} \approx 364\ \mu A$ since $I_{DS9} \approx 4\ \mu A$

Enhanced full-differential folded-cascode

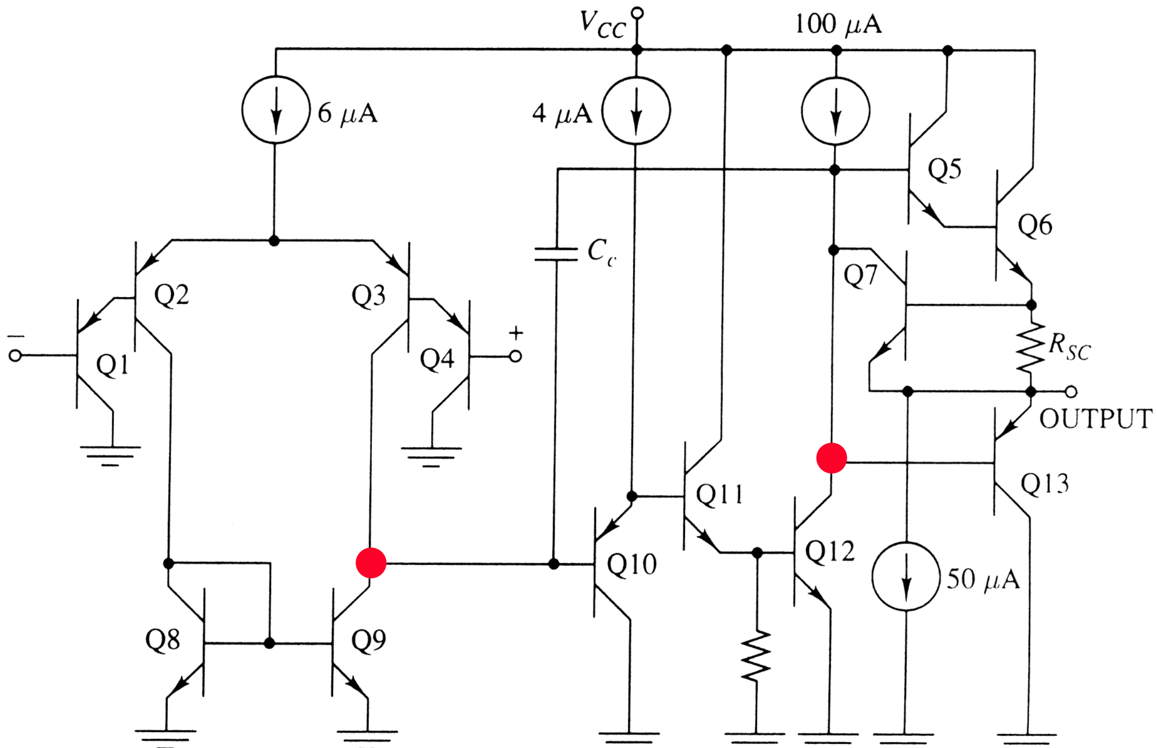


$i = g_{m1} v_{in}$

$$GBW = \frac{g_{m1}}{2\pi C_L}$$

Ref.Nakamura, JSSC April 1992, pp.563-568

Bipolar opamp LM-124



GBW = 0.5 MHz

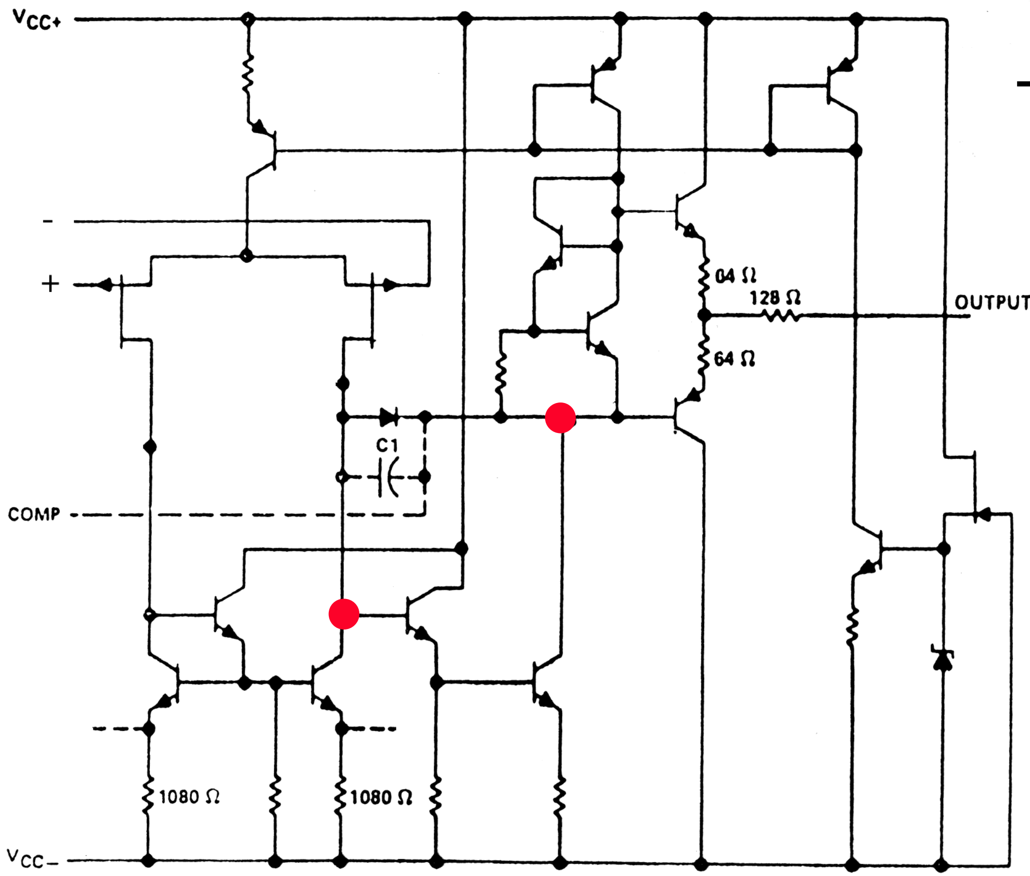
SR = 0.4 V/μs

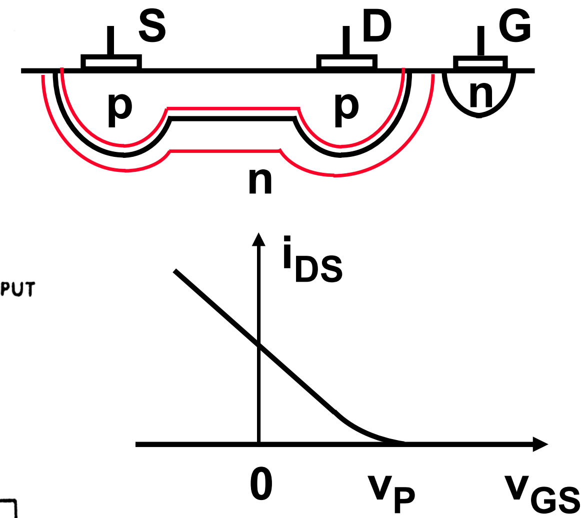
I_1 = 3 μA

I_{TOT} = 650 μA

68 $nV_{RMS}/\sqrt{Hz}$

BiFET opamp TL-070





GBW = 3 MHz

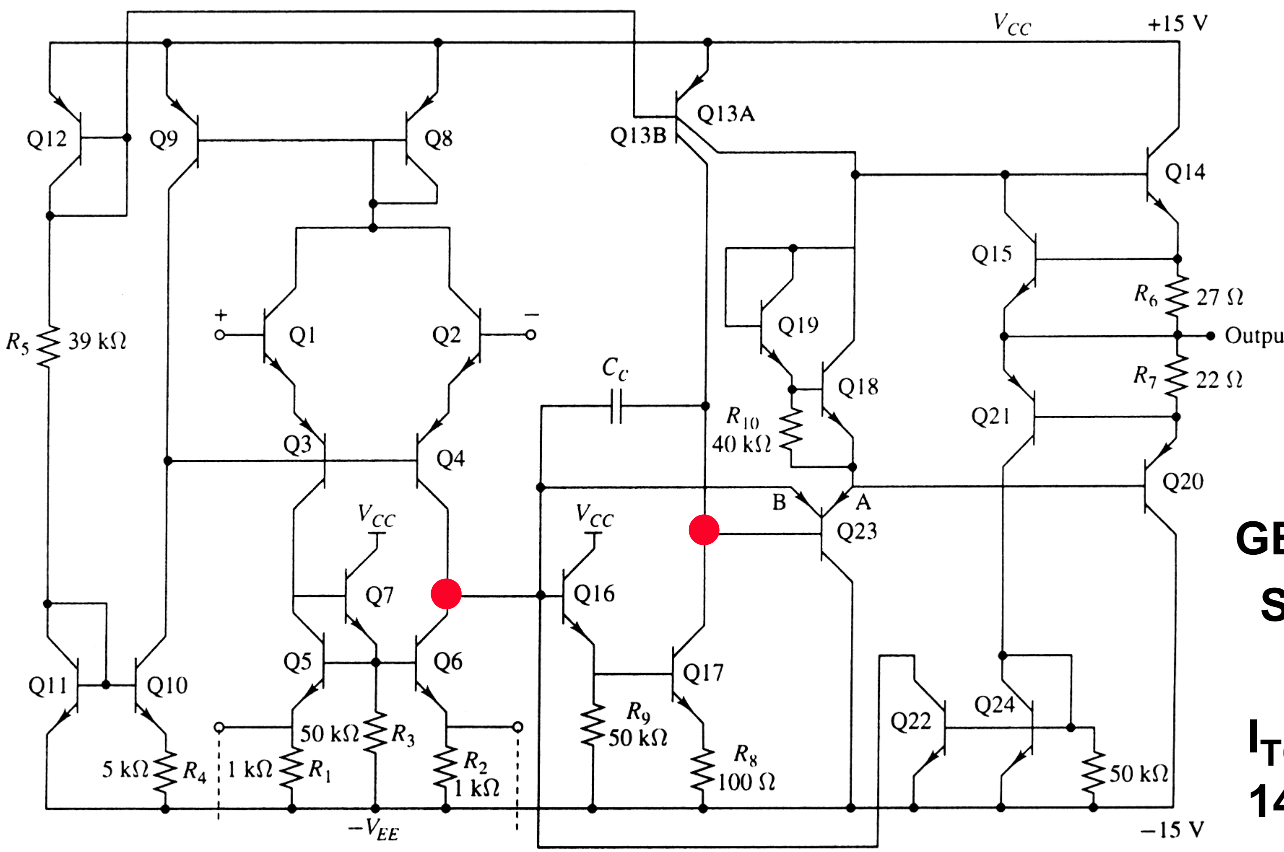
SR = 13 V/μs

I_1 = 100 μA

I_{TOT} = 1400 μA

18 $nV_{RMS}/\sqrt{Hz}$

Bipolar 2-stage opamp 741



GBW = 0.8 MHz

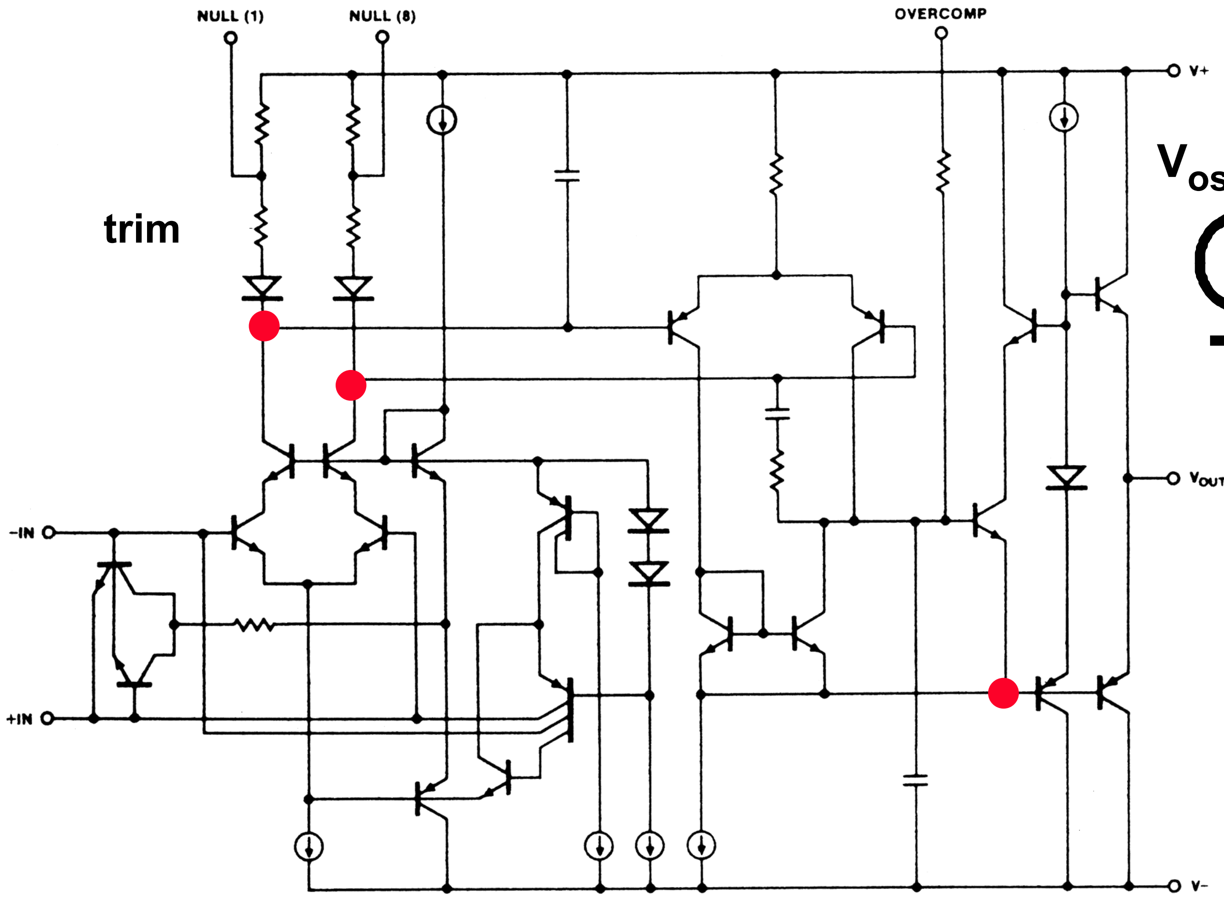
SR = 0.7 V/μs

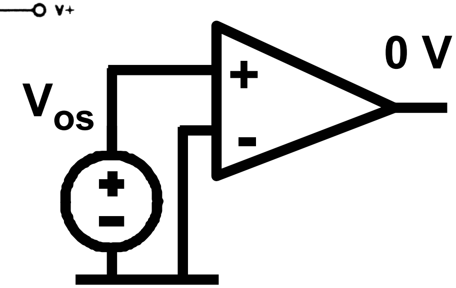
I_1 = 10 μA

I_{TOT} = 1100 μA

14 $nV_{RMS}/\sqrt{Hz}$

Two-stage opamp OP-97





GBW = 0.8 MHz

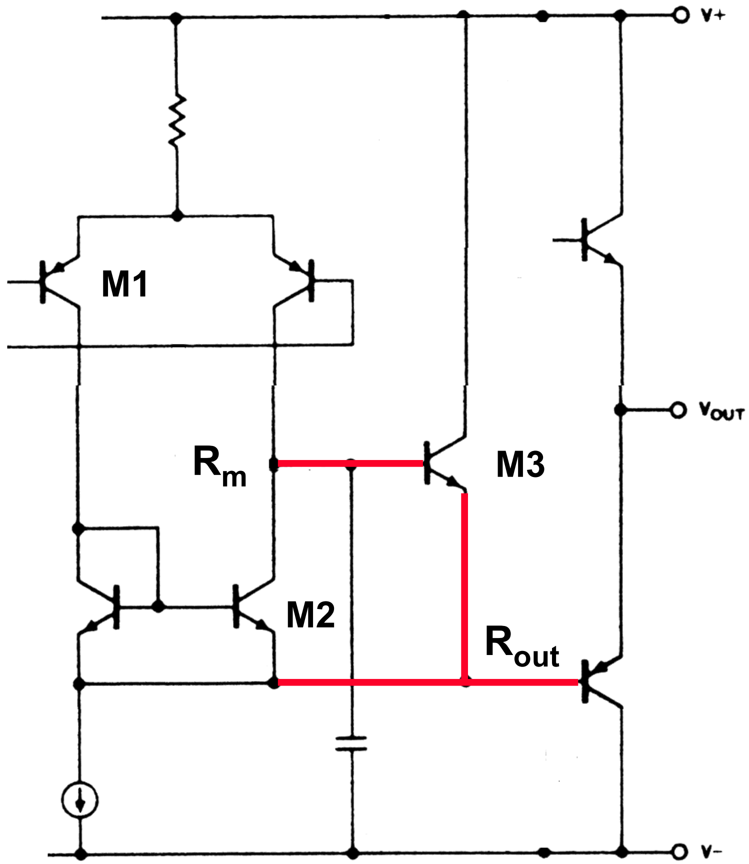
SR = 0.1 V/µs

I_{TOT} = 600 µA

25 µV offset

30 pA input

Bootstrap for high gain A_{v2}



R_m -> x β_3

R_{out} -> x $\frac{1}{\beta_3}$

$A_{v2} \approx g_{m1}\, r_{o2} \times \beta_3$

Same GBW !

Ref.De Man JSSC June 77, pp.217-222

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- Simple CMOS OTA
- CMOS Miller OTA
- Symmetrical CMOS OTA
- Folded cascode OTA
- Other opamps

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Fully-differential amplifiers

Willy Sansen

KULeuven, ESAT-MICAS
Leuven, Belgium

willy.sansen@esat.kuleuven.be

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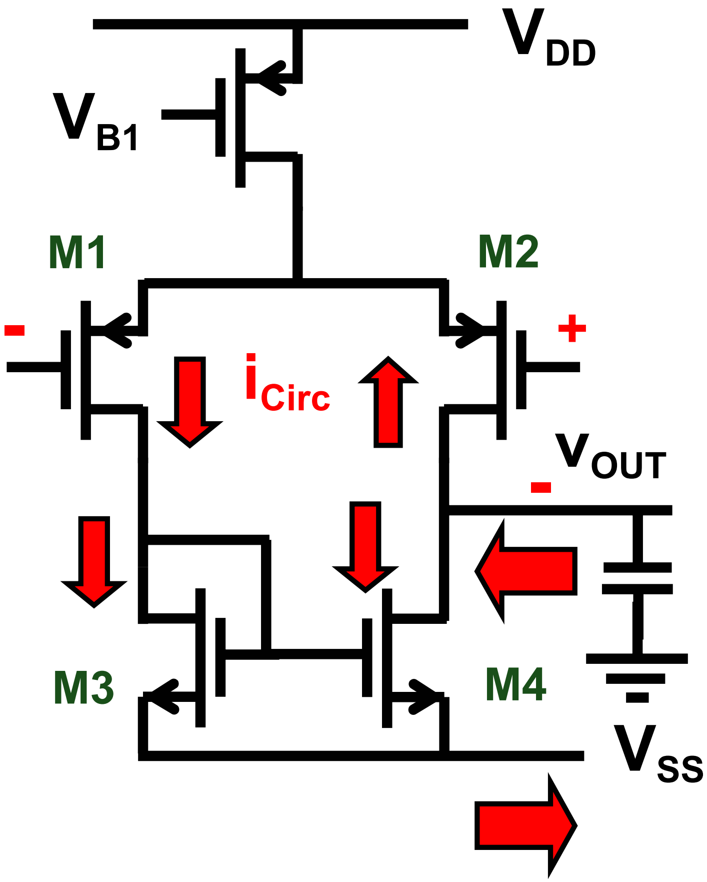
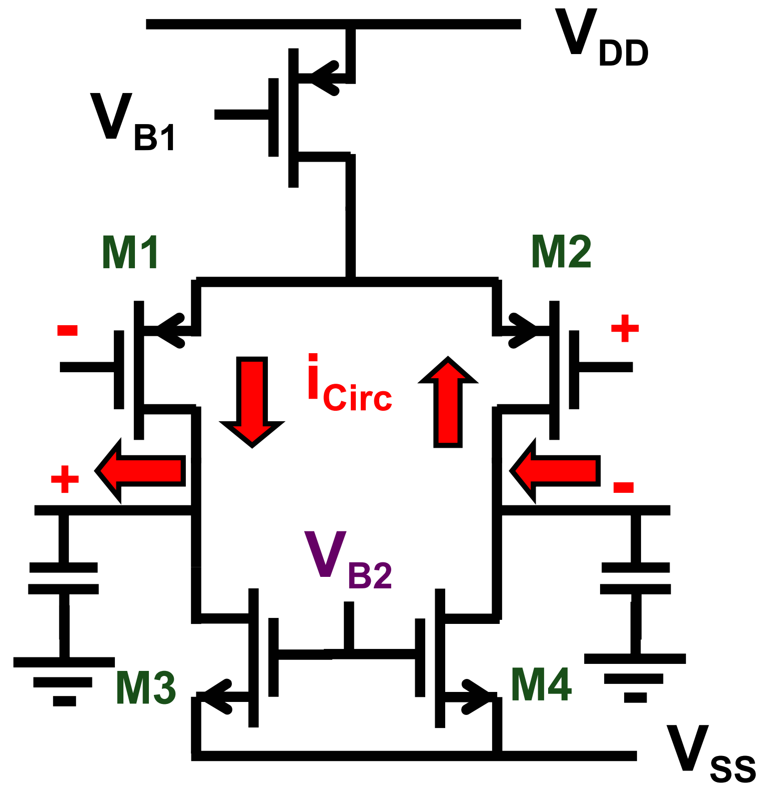
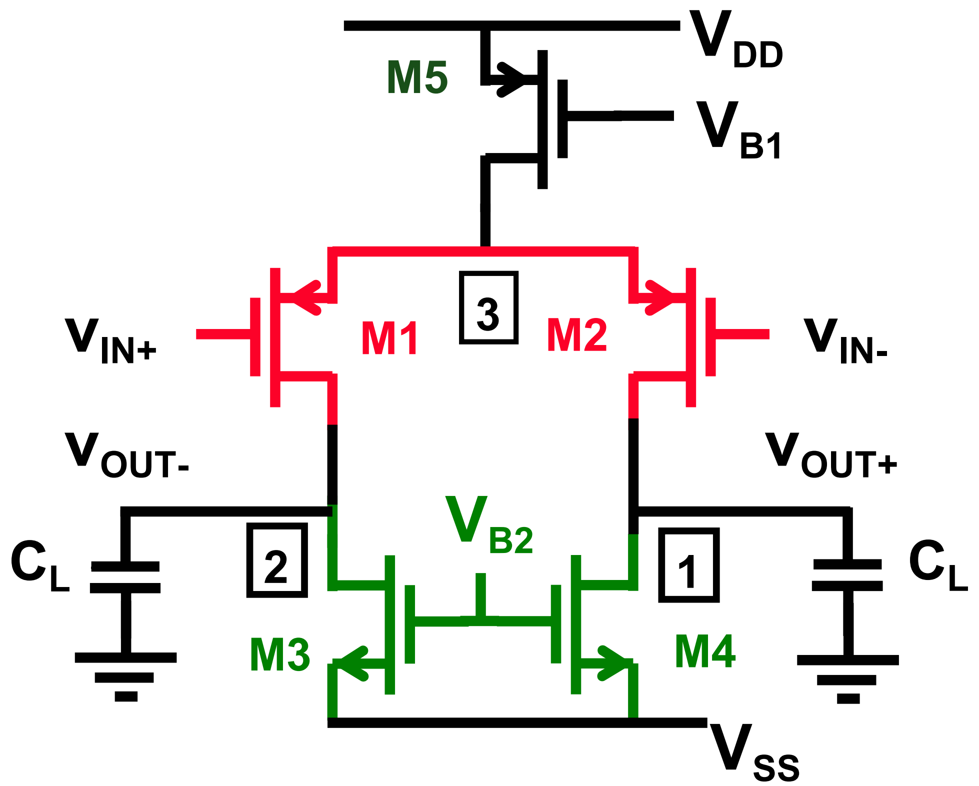
Single-stage OTA





Simple CMOS fully-differential OTA



Differential pair

No current mirror

$$GBW = \frac{g_{m1}}{2\pi\, C_L}$$

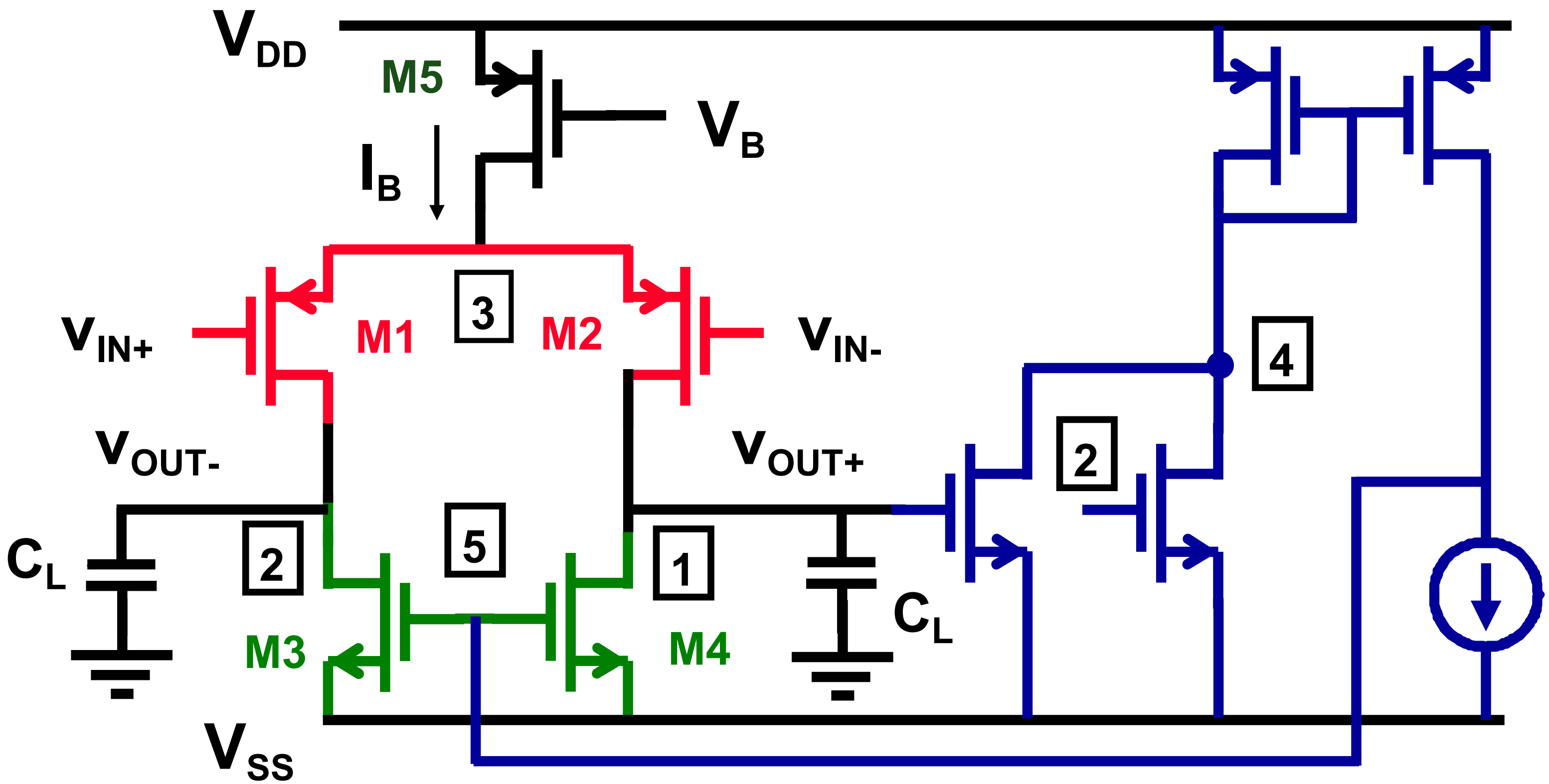
Problem:

keep M1-4 in saturation:

Control V_{OUTCOM}

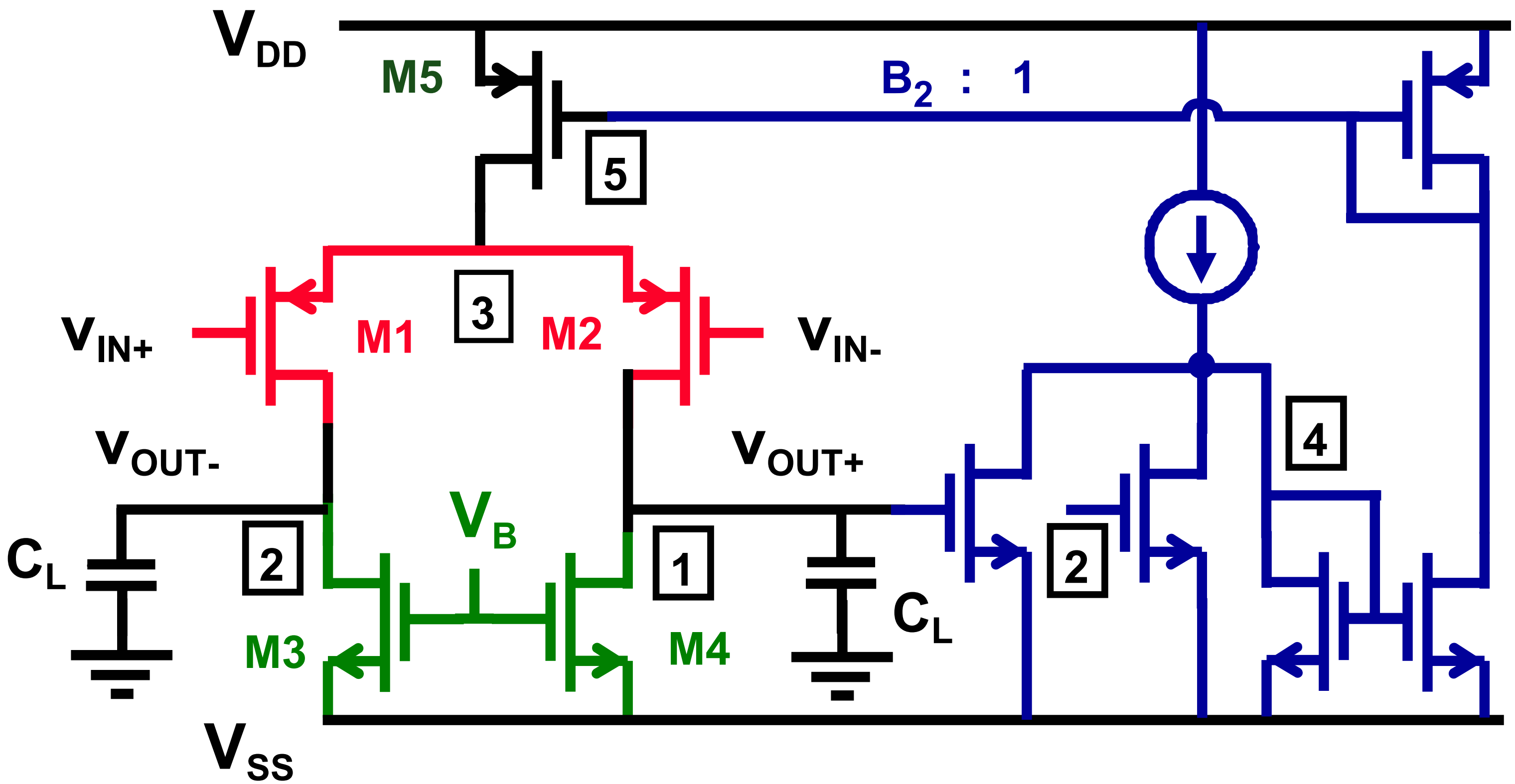
Control I_{DS5}

Simple CMOS fully-diff. OTA with CMFB - 1



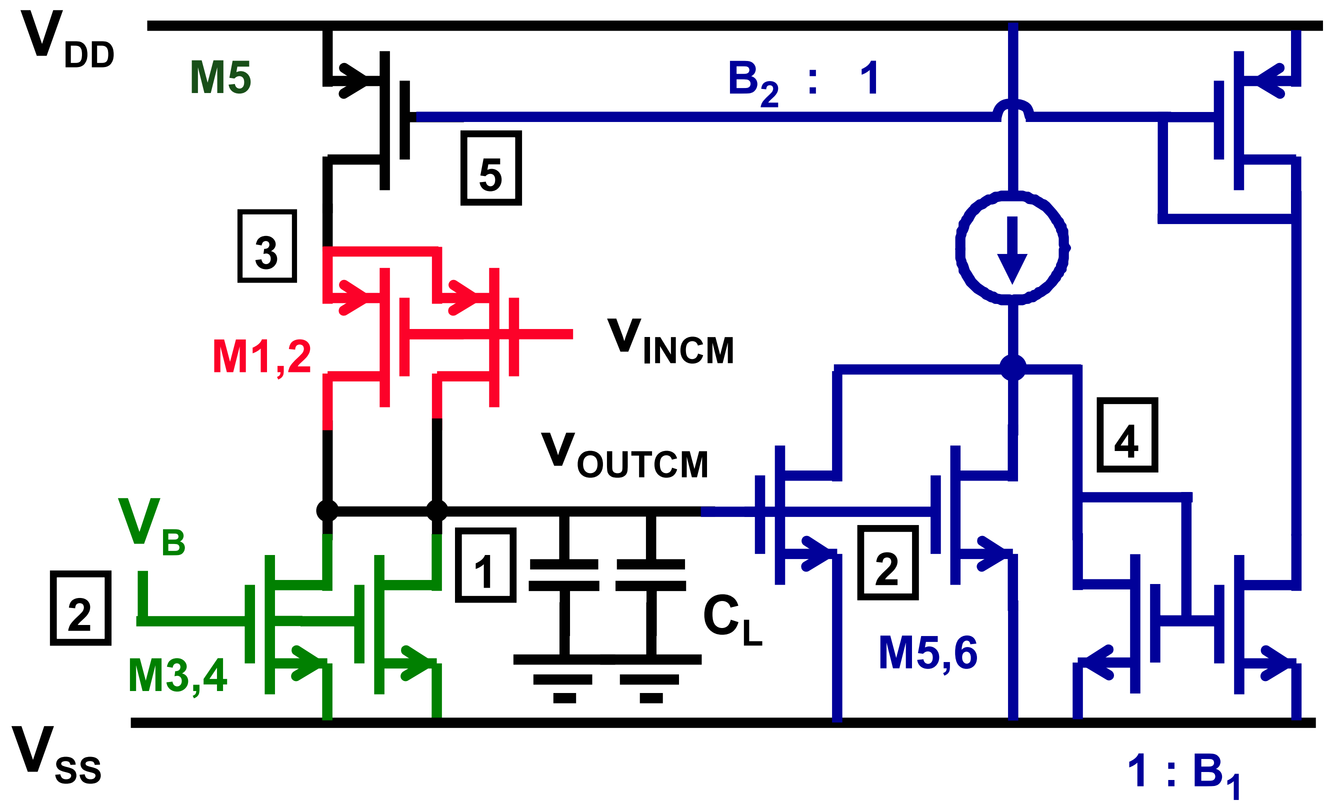
Control V_{OUTCOM}, I_{DS5}

Simple CMOS fully-diff. OTA withy CMFB - 2



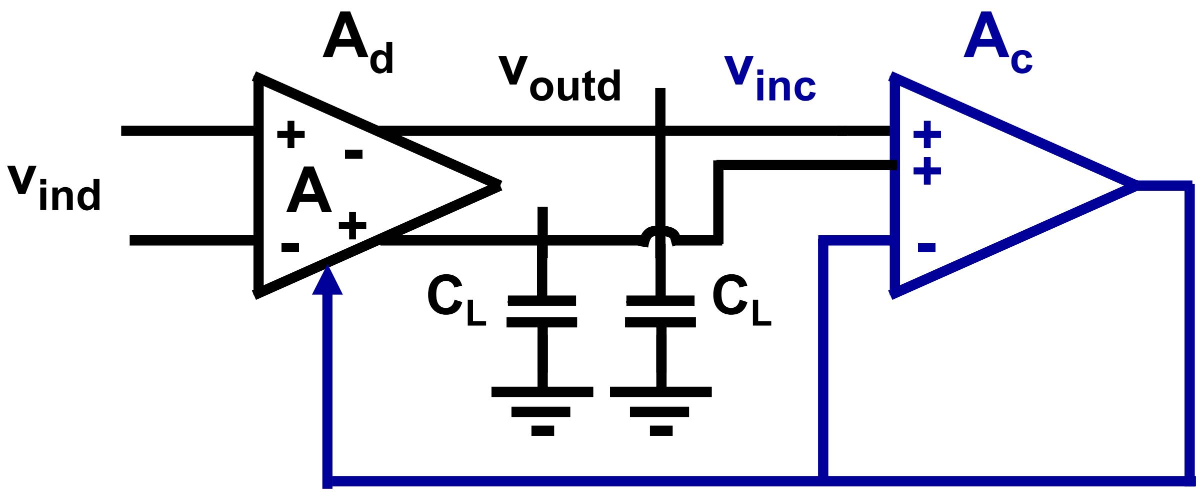
Control V_{OUTCOM}, I_{DS5}

Common-mode feedback equivalent circuit



Control V_{OUTCOM}, I_{DS5}

Common-mode feedback CMFB



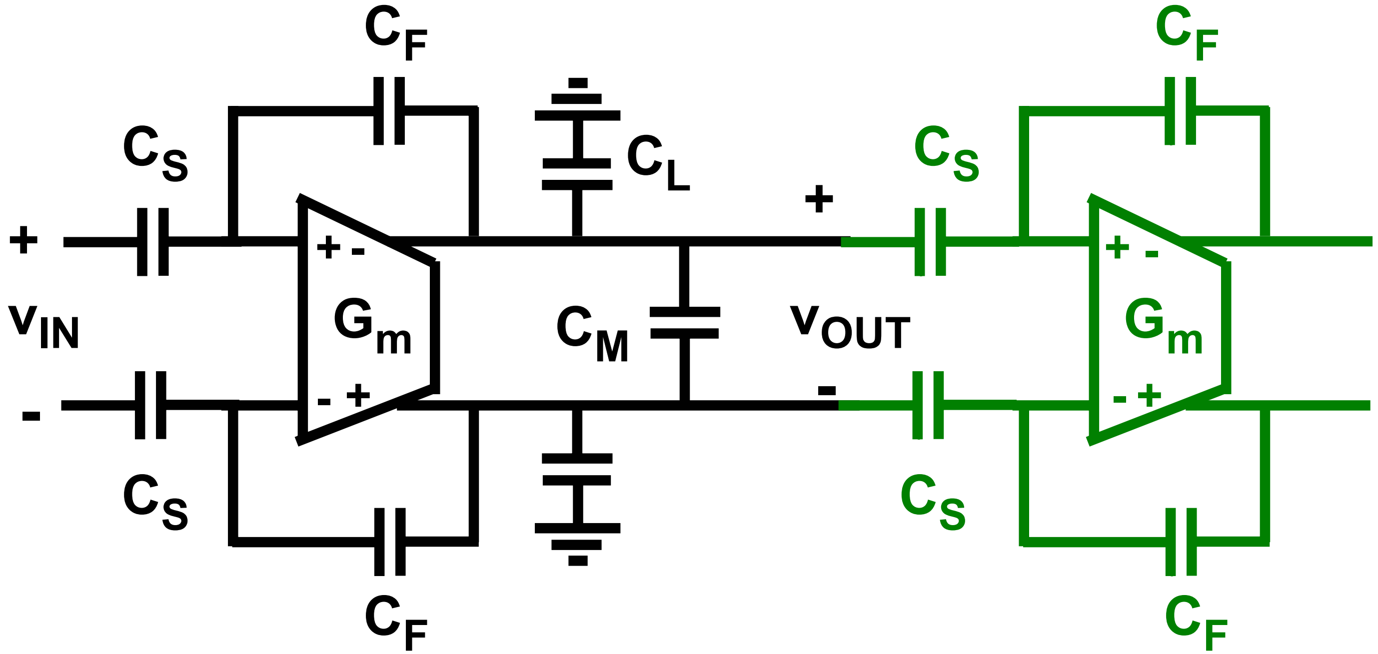
CMFB in unity gain : CMRR = A_{vCM}

Three tasks : 1. Measure the output voltages
2. Cancel out the differential signals
3. Close the CMFB loop

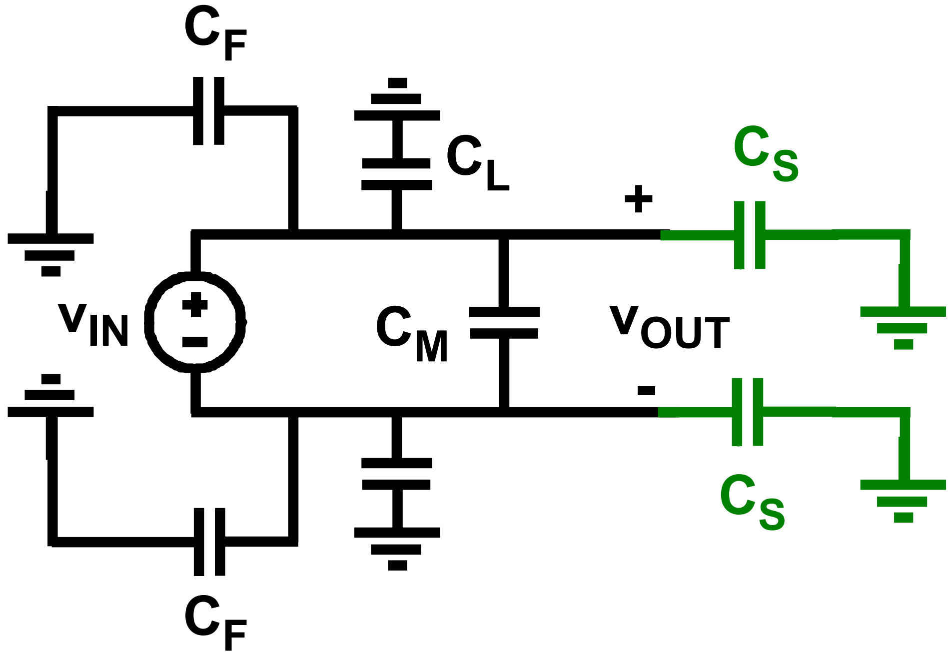
Requirements fully-differential amplifiers

- High speed : $GBW_{CM} > GBW_{DM}$
- Matching
- Output swing limited by :
  - Output swing of differential-mode amp
  - Input range of common-mode amp
- Low power $P_{CM} < P_{DM}$

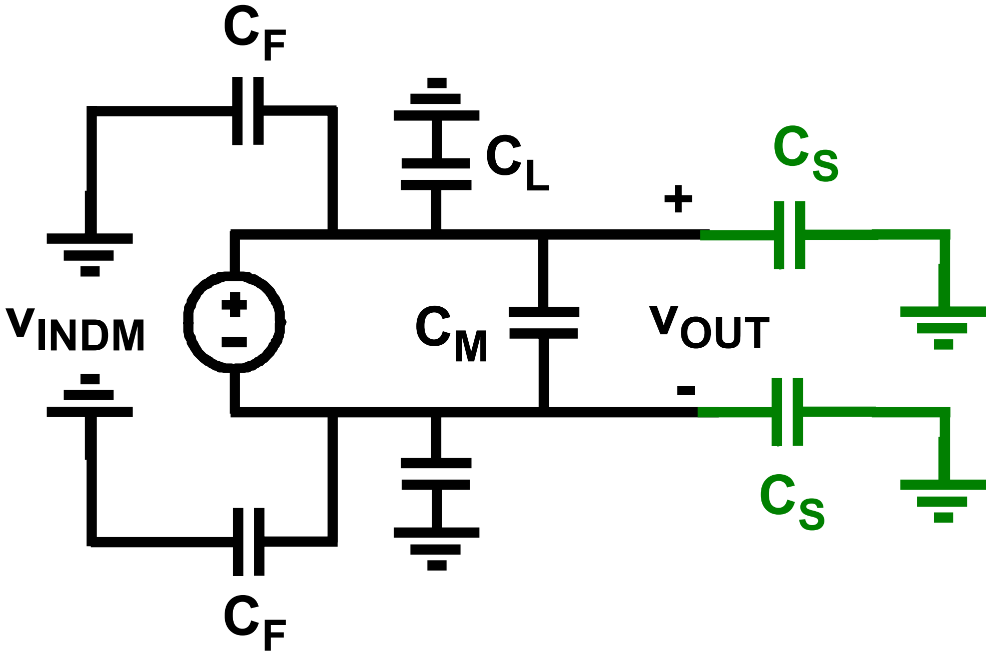
Load capacitance ?



Load capacitance C_{IN}

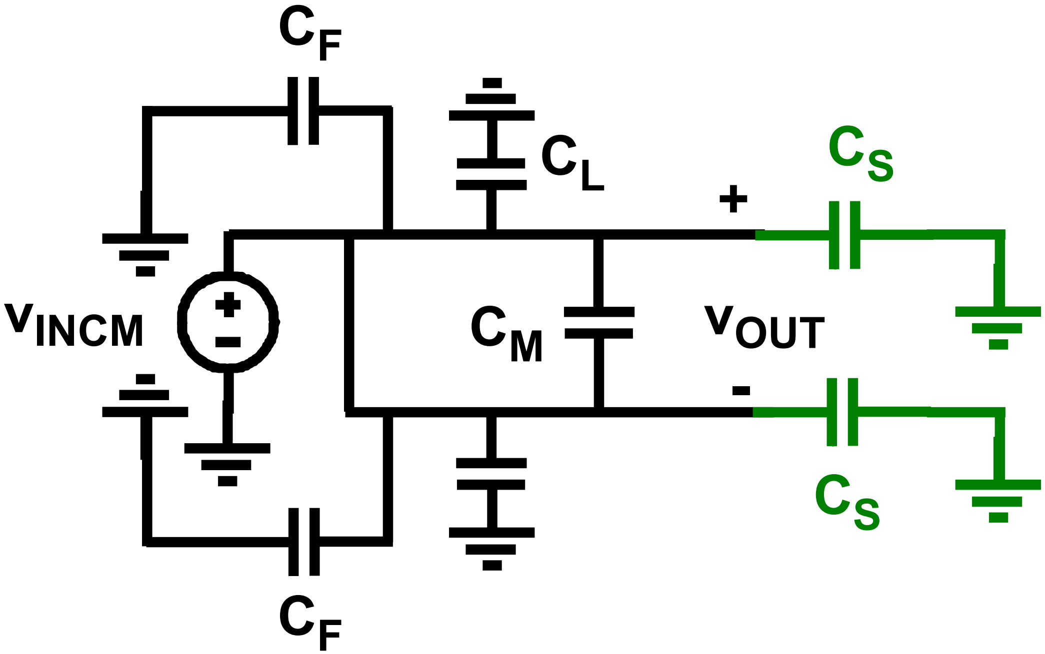


Load capacitance C_{INDM}



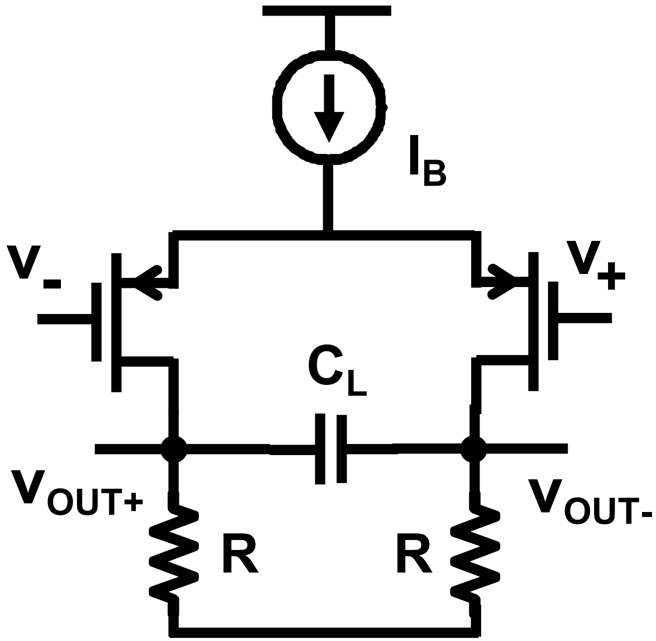
$$C_{INDM} = C_M + \frac{C_F + C_L + C_S}{2}$$

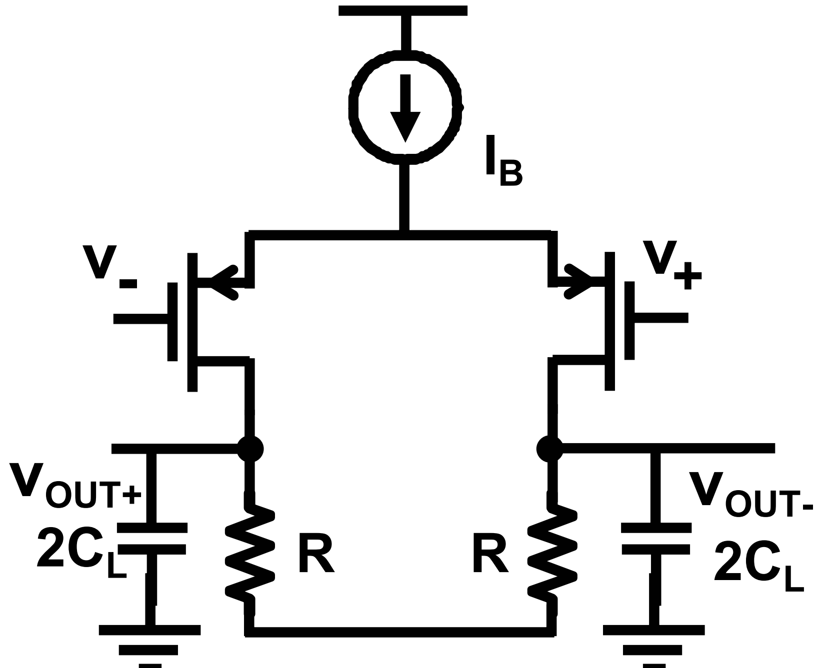
Load capacitance C_{INCM}



$$C_{INCM} = 2\,(C_F + C_L + C_S) \quad > C_{INDM}$$

GBW_{DM} & GBW_{CM}





$$GBW_{DM} = \frac{g_m}{2\pi\, 2C_L}$$

$$C_{LCM} = 0$$

$$GBW_{DM} = \frac{g_m}{2\pi\, 2C_L}$$

$$C_{LCM} = 4\, C_L$$

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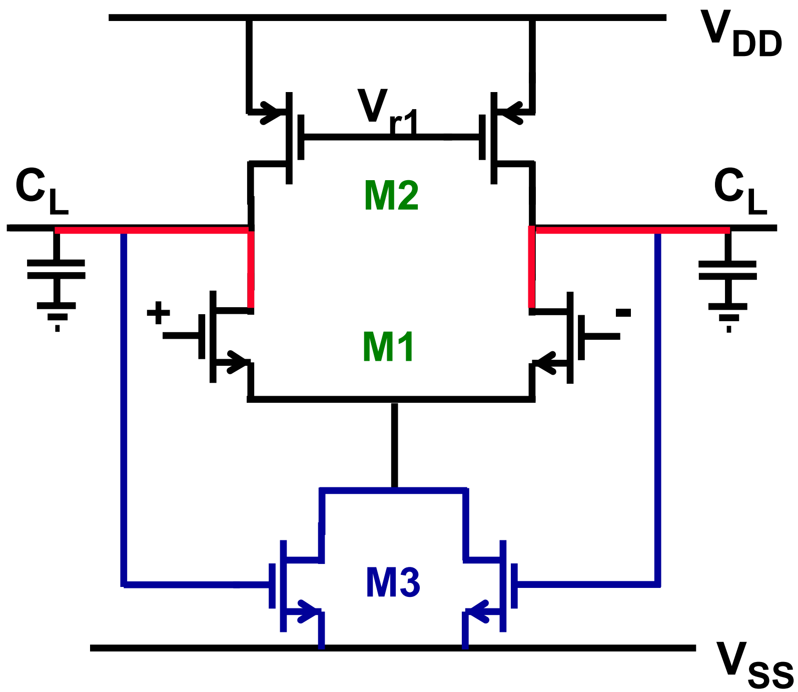
- Requirements
- Fully-diff. amps with linear MOSTs
- FDA's with error amp.& source followers
- Folded cascode OTA without SF's
- Other fully-differential amps
- Exercise

CMFB amplifier with linear MOSTs



Linear MOSTs:

$V_{DS3} \approx 200$ mV

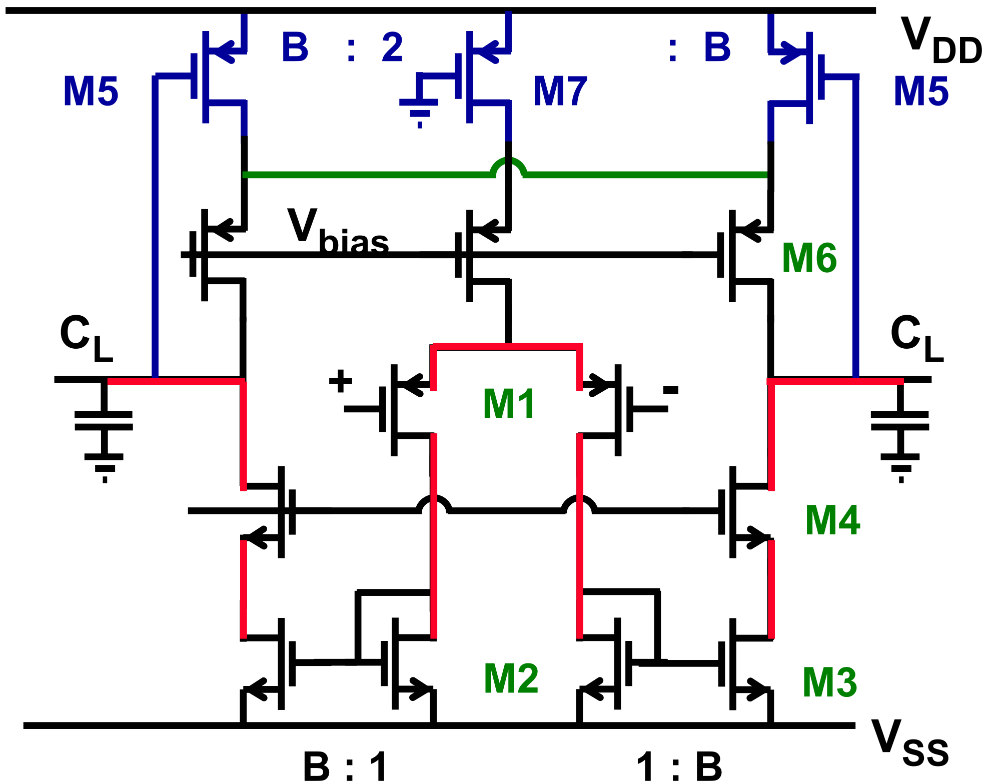
$$I_{DS} = \beta V_{DS}(V_{GS}-V_T)$$
$$g_{m3} = \beta V_{DS3}$$

$$GBW_{DM} = \frac{g_{m1}}{2\pi C_L}$$

$$GBW_{CM} = \frac{g_{m3}}{2\pi C_L}$$

is always smaller !

Fully-differential amp. with linear MOSTs



Linear MOSTs:

$V_{DS5} \approx 200$ mV

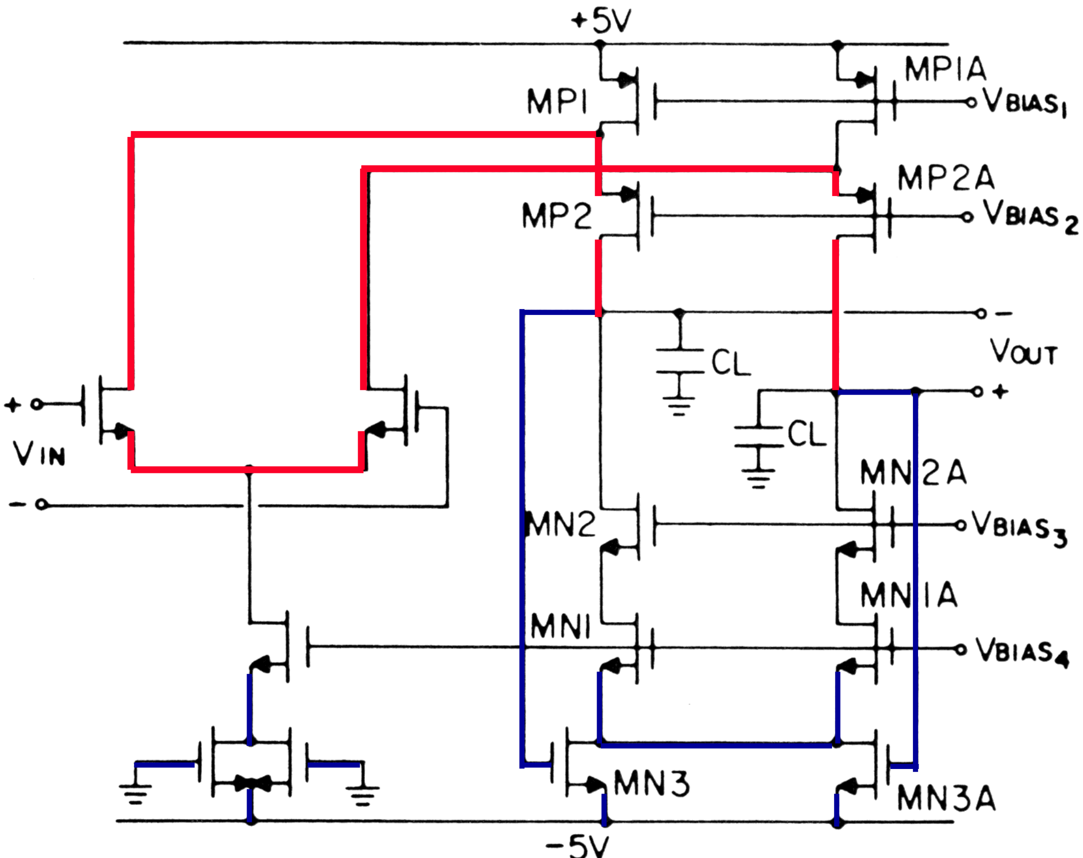
Cancel diff. signals

$$GBW_{DM} = B\frac{g_{m1}}{2\pi\, C_L}$$

$$GBW_{CM} = \frac{g_{m5}}{2\pi\, C_L}$$

is always smaller !
even with M5 in wi !

Fully-differential amp. with linear MOSTs

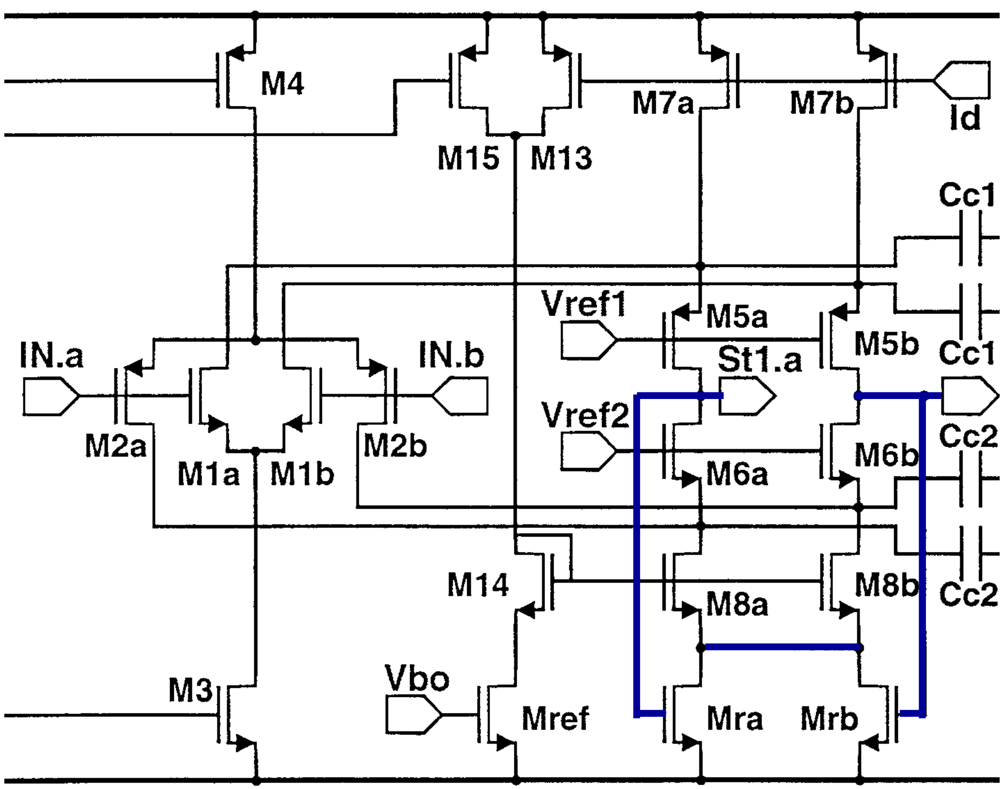


Linear MOSTs:

$V_{DS3} \approx 200$ mV

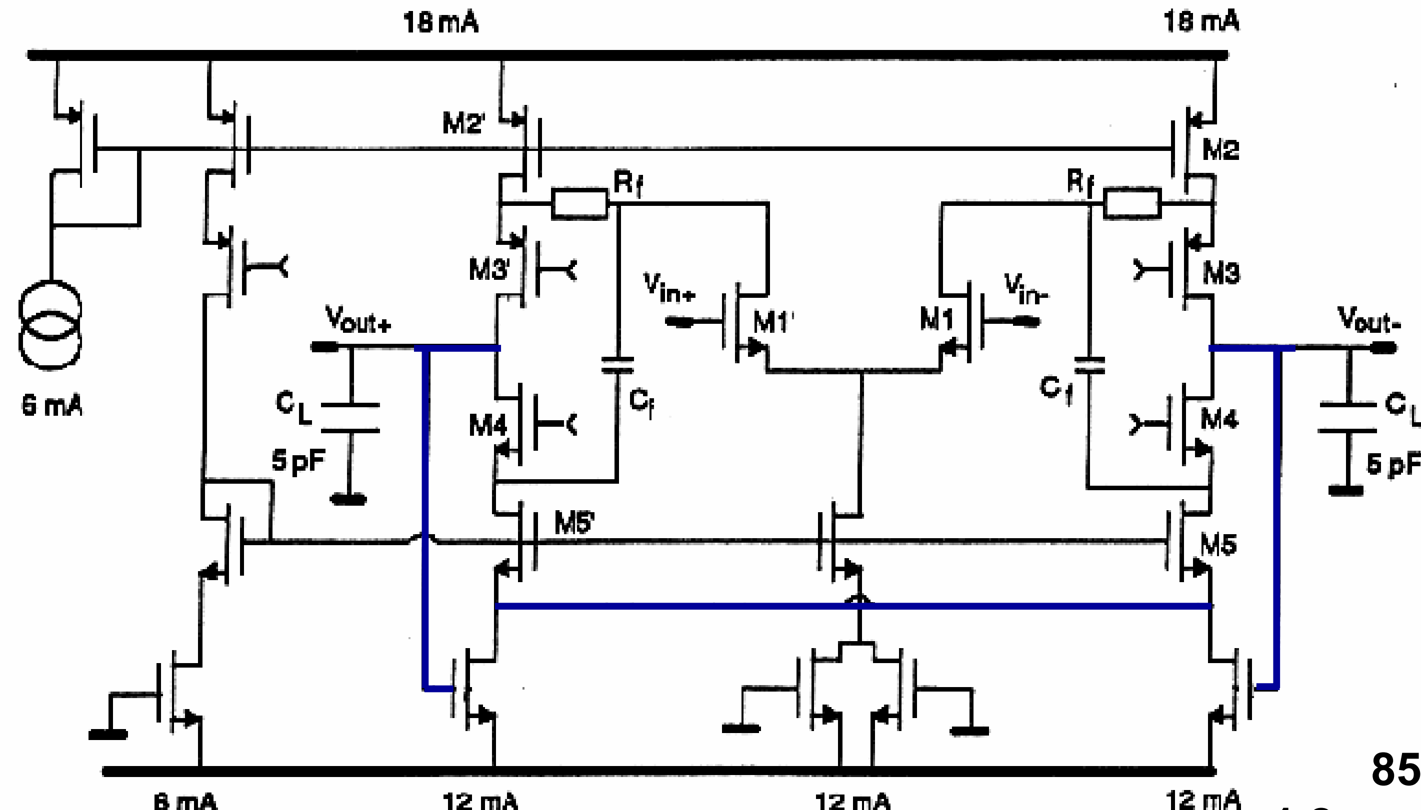
Ref. Choi, JSSC, Dec.83, 652-653

Total amplifier schematic



E.Peeters etal, CICC 1997

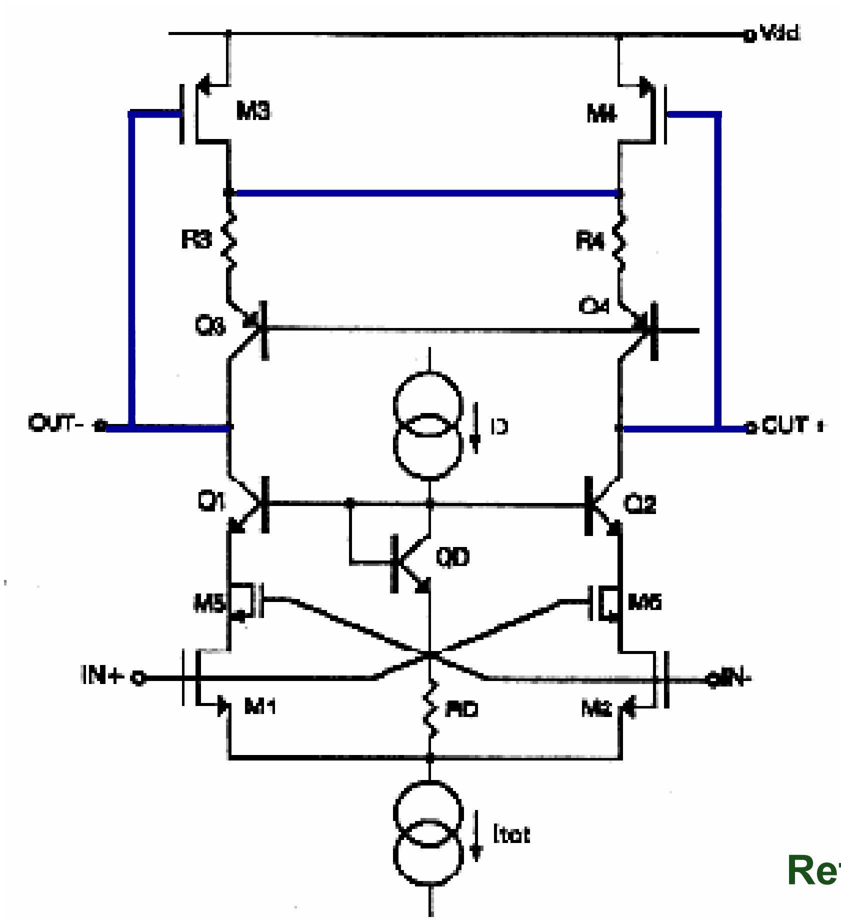
Fully-differential OTA with FF



850 MHz
1.2 μm CMOS

F. Op't Eynde, Kluwer Ac. 1993

Transconductor with C□$_{DG}$ compen.



$V_{DS1} = R_D I_D \approx 0.2$ V

$I_{DS1} = \beta_1 V_{DS1}(V_{GS1} - V_T)$

$g_{m1} = \beta_1 V_{DS1}$ is constant

Ref. Alini, JSSC, Dec.92, pp.1905-1915

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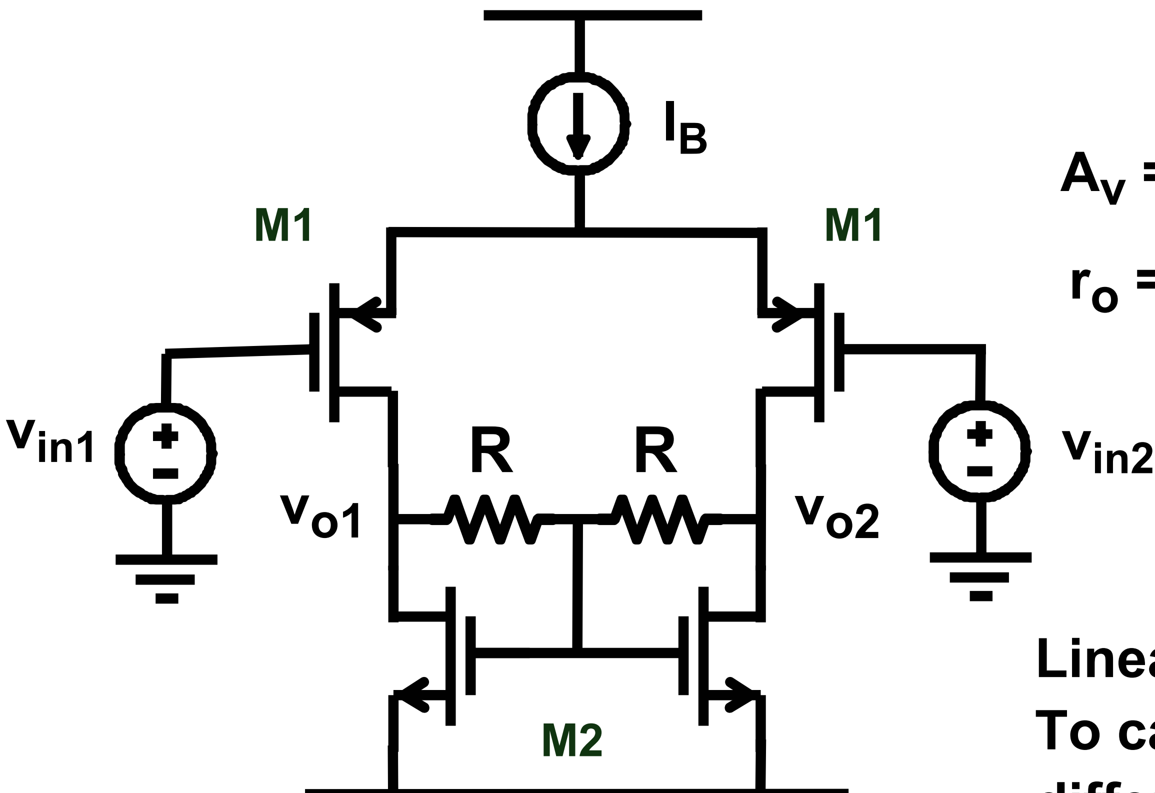
Fully-differential amplifier with resistive CMFB

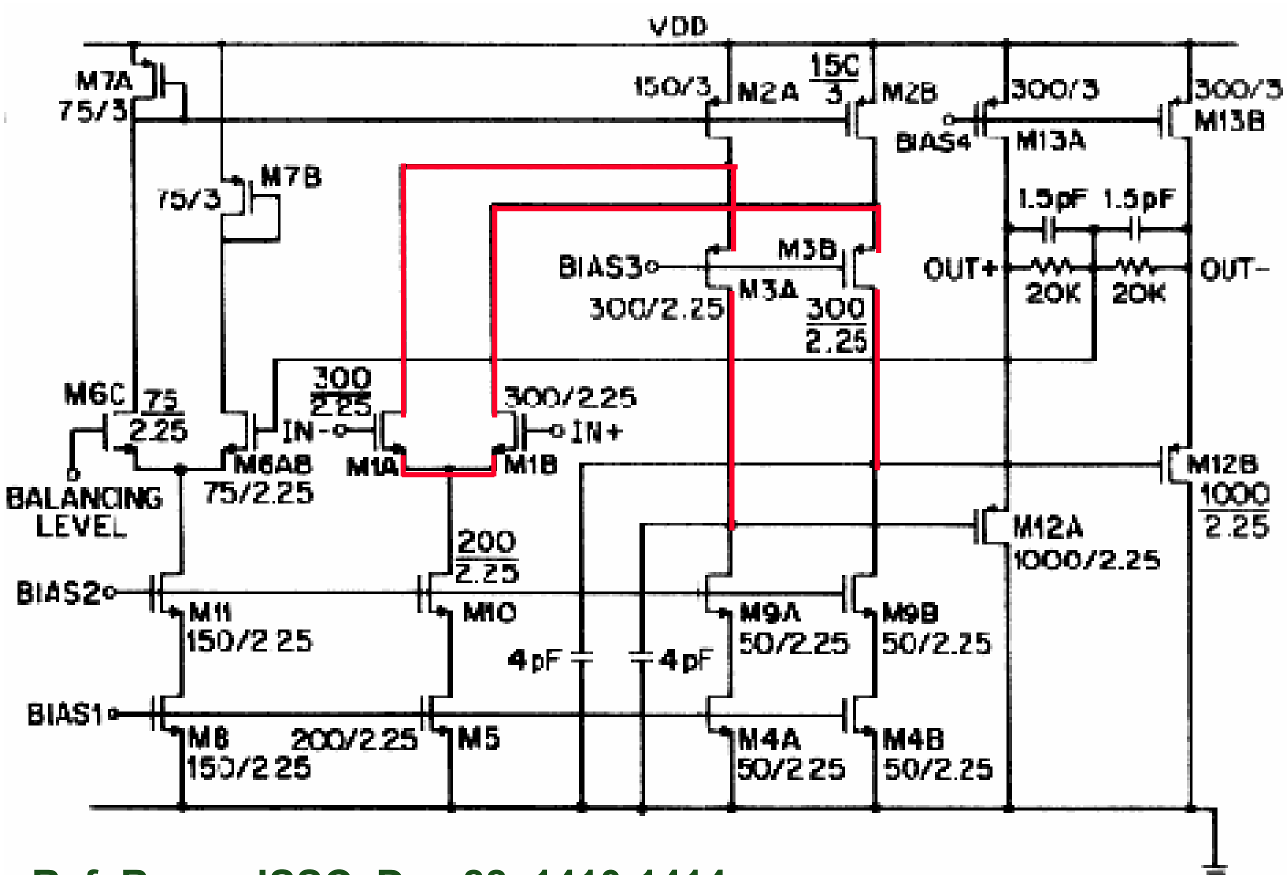


$A_v = g_{m1}\ (R\ //\ r_o\ )$

$r_o = r_{o1}//r_{o2}$

Linear resistors R
To cancel
differential signals

Fully-diff. amp. with source followers: Diff. mode

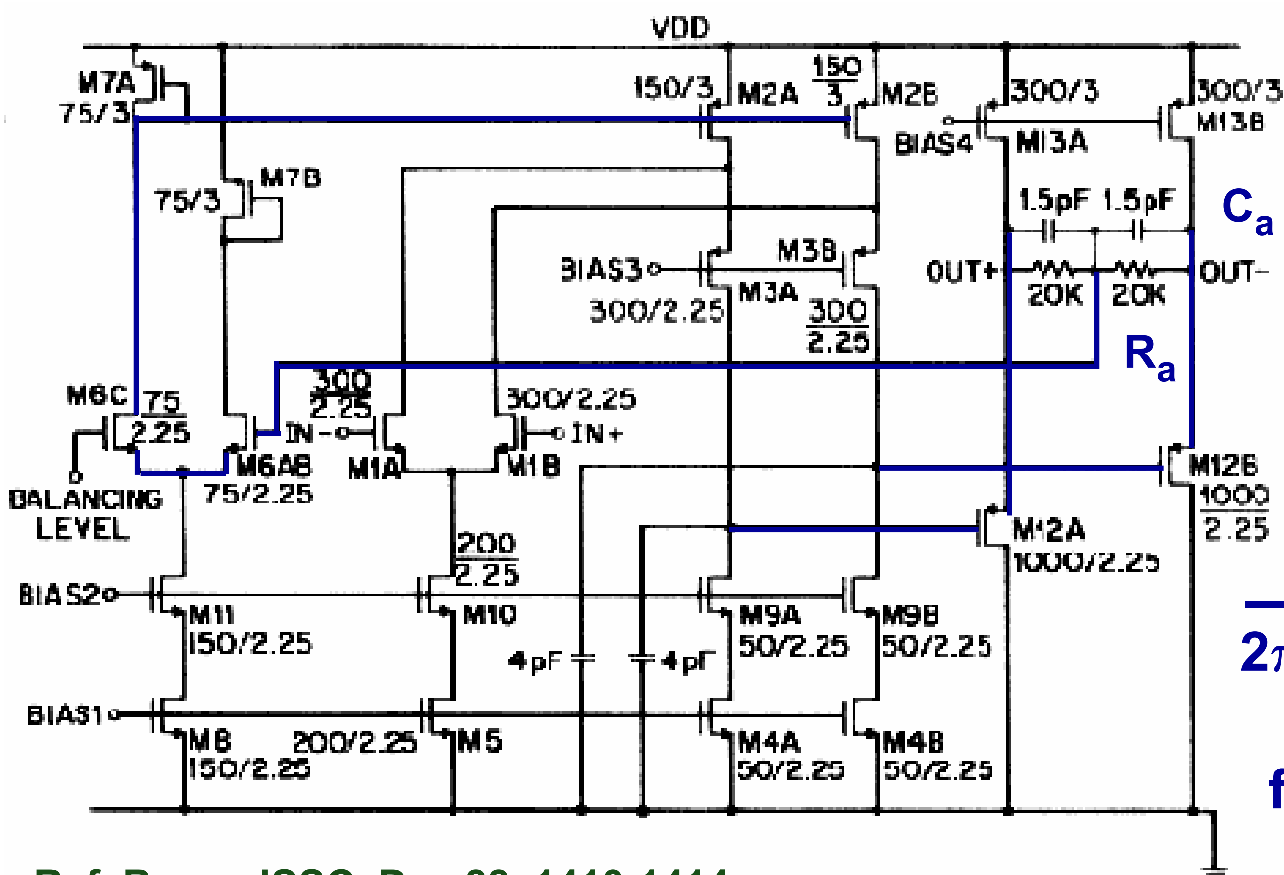


$$GBW_{DM} = \frac{g_{m1}}{2\pi\, C_L}$$

$$C_L = 4\ \text{pF}$$

Ref. Banu, JSSC, Dec.88, 1410-1414

Fully-diff. amp. with source followers: CM



$$GBW_{CM} = \frac{g_{m6}}{4\pi\, C_L}$$

$$f_{ndCM} = \frac{4}{2\pi R_a (C_{GS6} + C_a)}$$

$$f_z = \frac{1}{2\pi R_a C_a}$$

Ref. Banu, JSSC, Dec.88, 1410-1414

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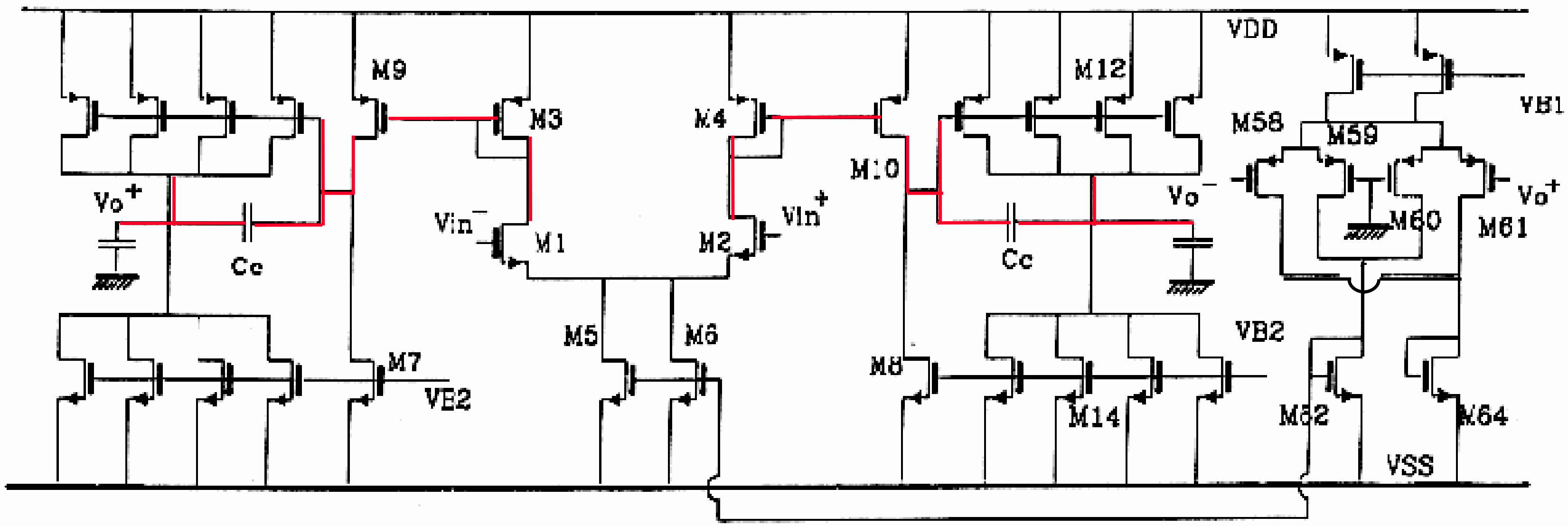
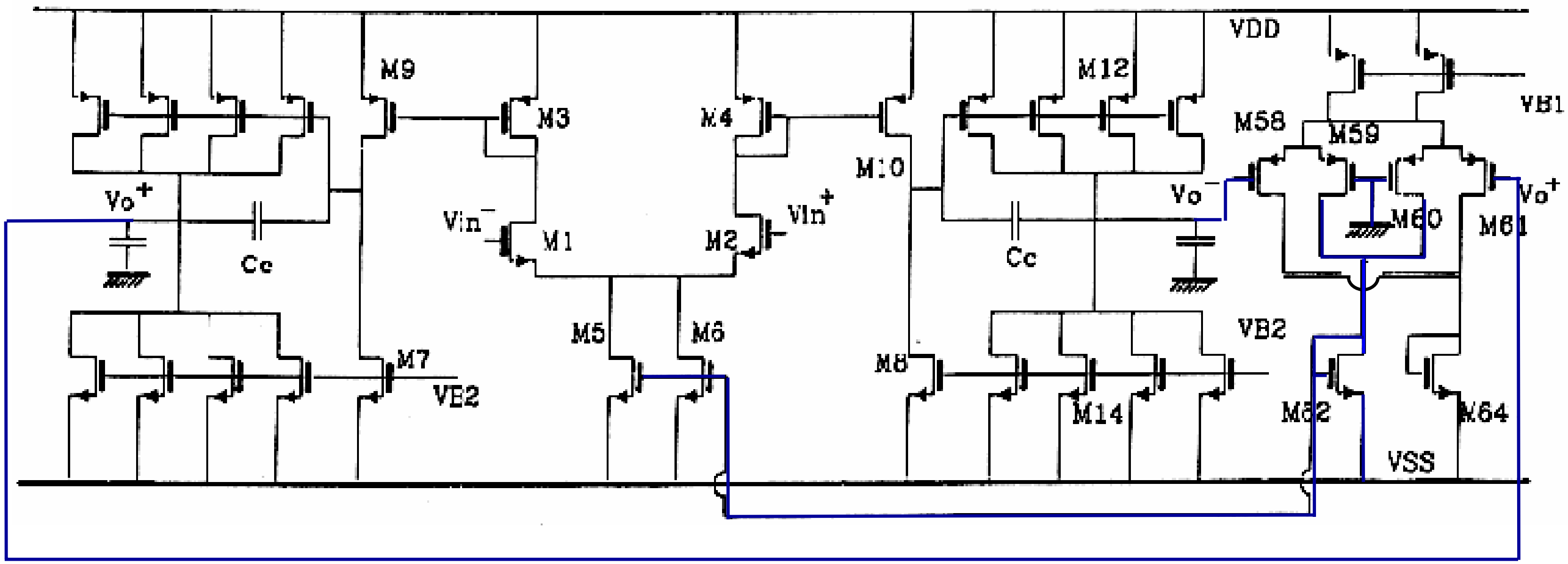
Fully-diff amp. with error amplifier: Diff. mode



$$GBW_{DM} = \frac{g_{m1}}{2\pi\ C_c}$$

Ref. Ribner, CICC 85; Haspeslagh, CICC 88

Fully-diff amp. with error amp. : Common mode

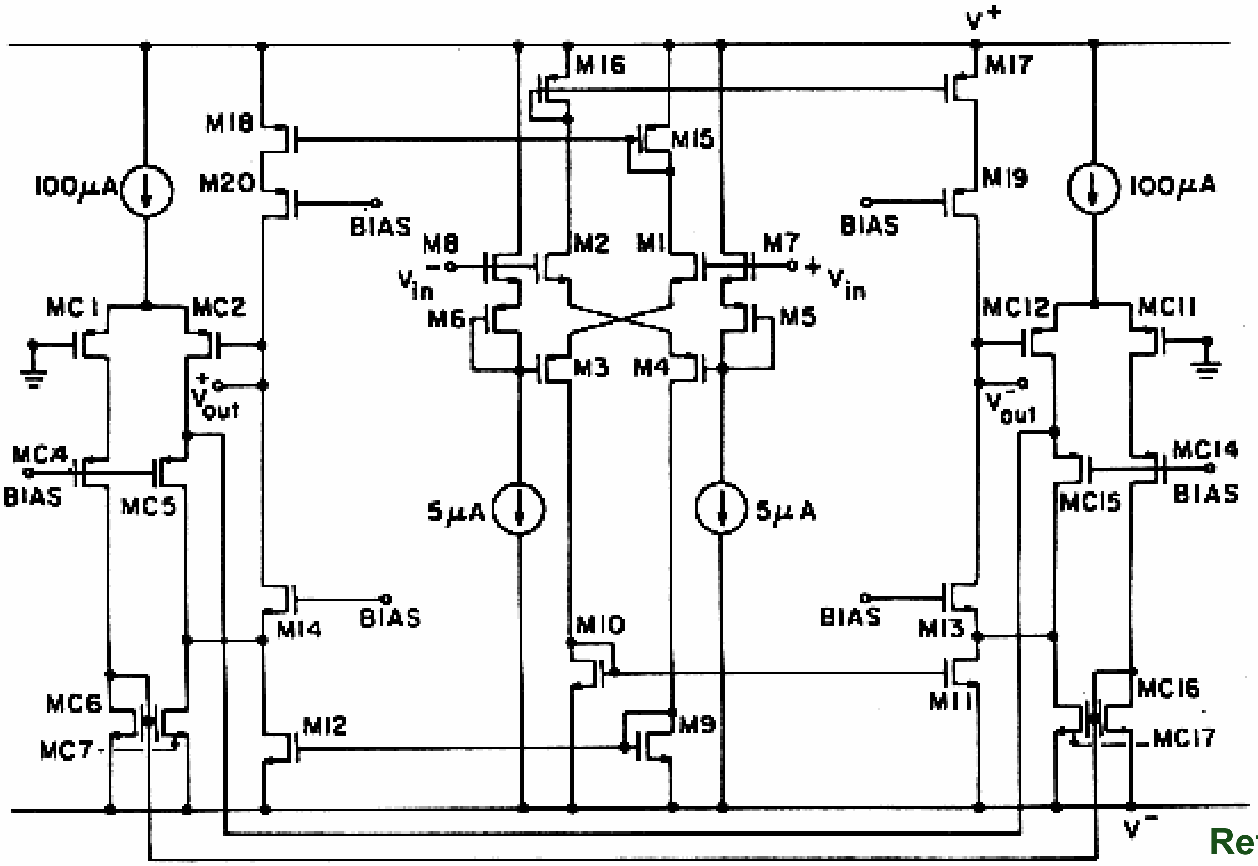


Nonlinear !

$$GBW_{CM} = \frac{g_{m58}}{4\pi\, C_c}$$

Ref. Ribner, CICC 85; Haspeslagh, CICC 88

Class AB fully-differential amplifier



Ref. Lee, JSSC
Dec.85, 1103-1113

Comparison

Criterion	Linear MOST	Error amp Source foll.	Error amp. Quad amp.
GBW_{CM}/GBW_{DM}	< 0.1	> 1	> 1
Required tol.	< 1 %	< 6 %	< 6 %
Diff.output swing Is limited by	$0.8\ V_{DDSS}$ cascodes	$0.4\ V_{DDSS}$ source foll.	$0.4\ V_{DDSS}$ cm input
Power dissipation	1 amp	3 amps	2 amps

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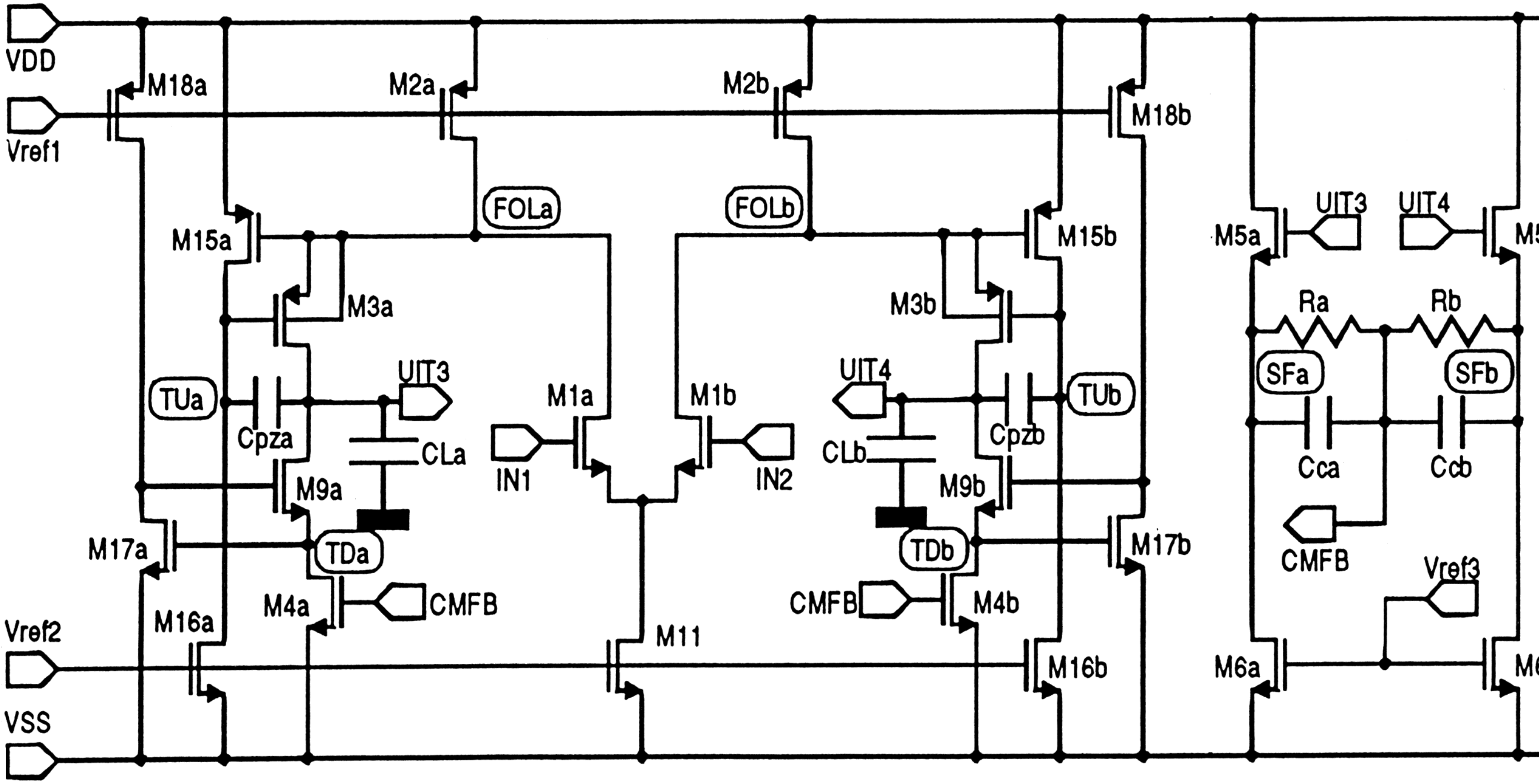
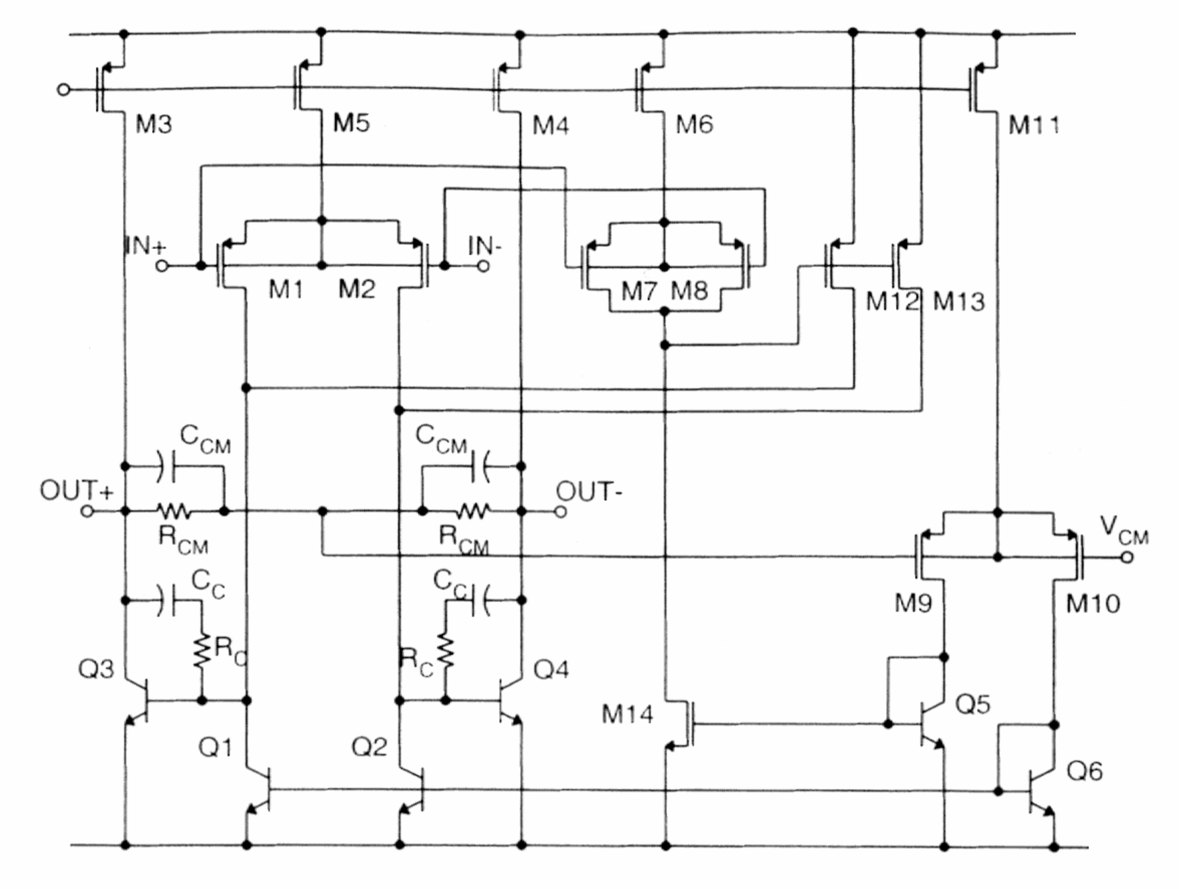
Fully-differential amplifier with gain boosting

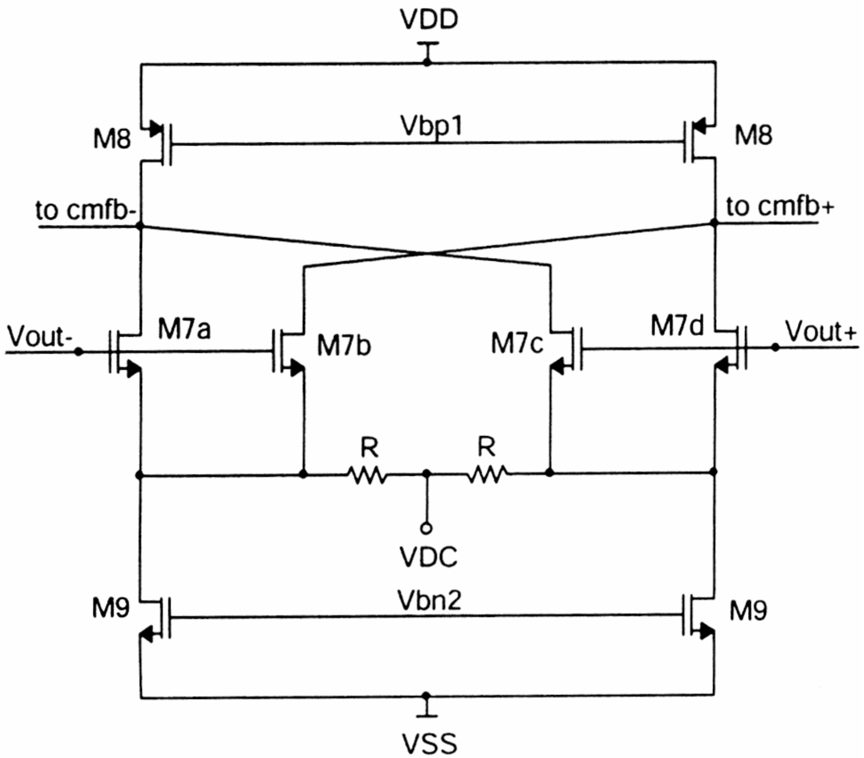


Low-voltage (1.1 V) DIDO



Gata, JSSC Dec.02 1670-1678

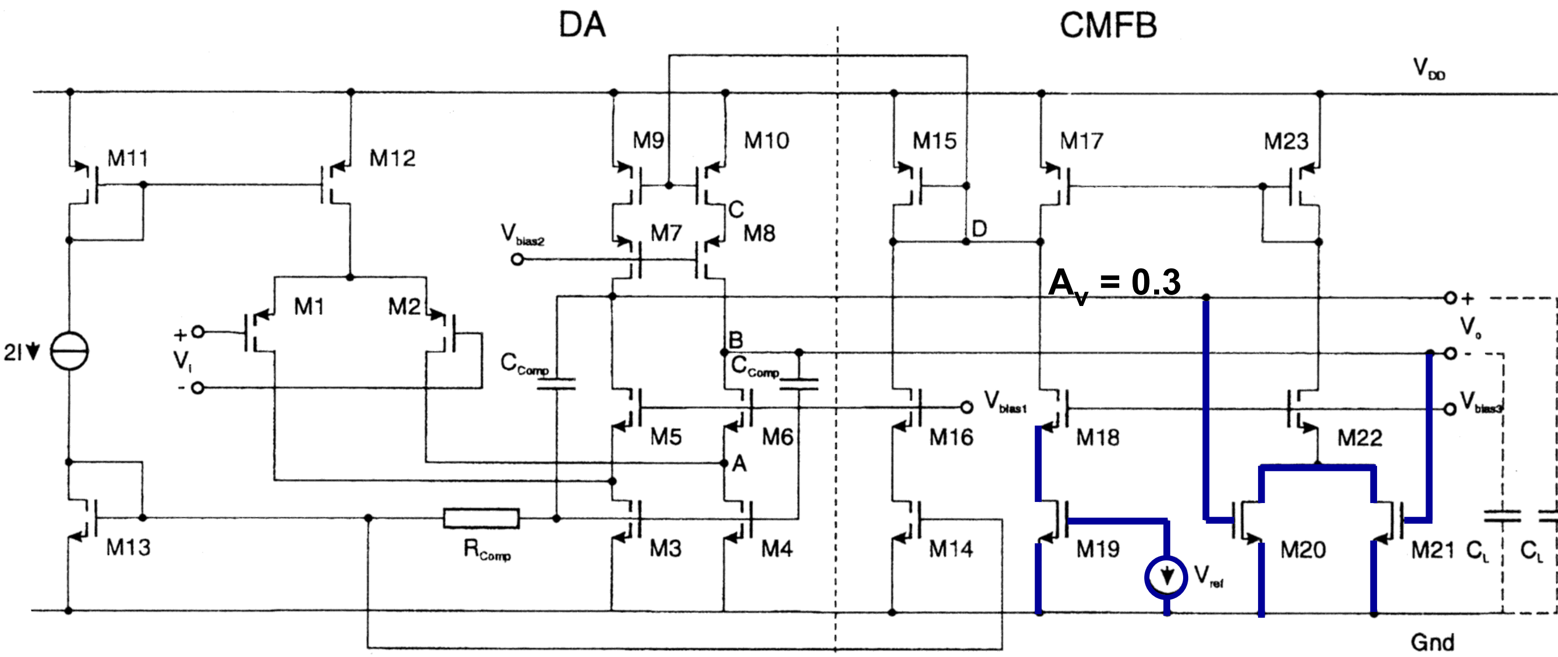
Linear CM amplifier



$$V_{outCM} = VDC + V_{GS7}$$

Ref. Hernandez, JSSC Aug.05, 1610-1617

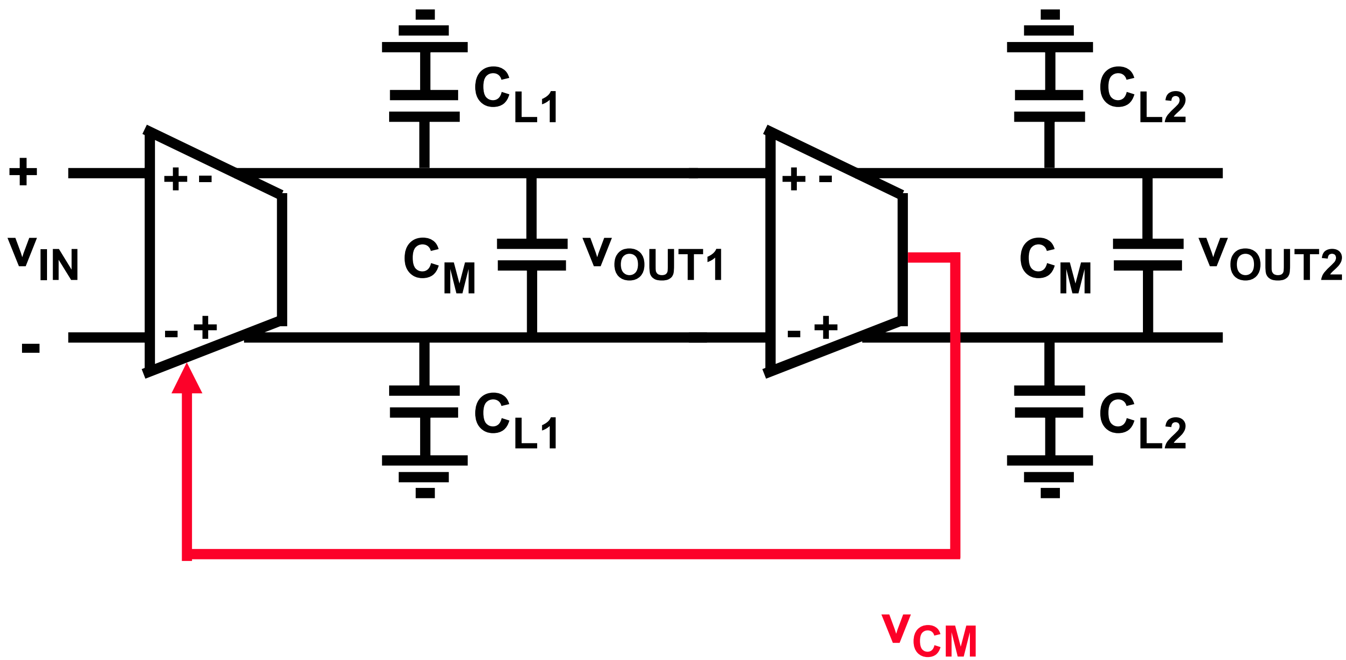
Fully-diff.amp. with separate linear trans.CMFB



24 MHz/ 3 pF   3 V/ 5 mA I_{DS1} = 0.25 mA Comp 4 kΩ/ 2 pF > 20 MHz

Ref. Pasch, AICSP, 2000

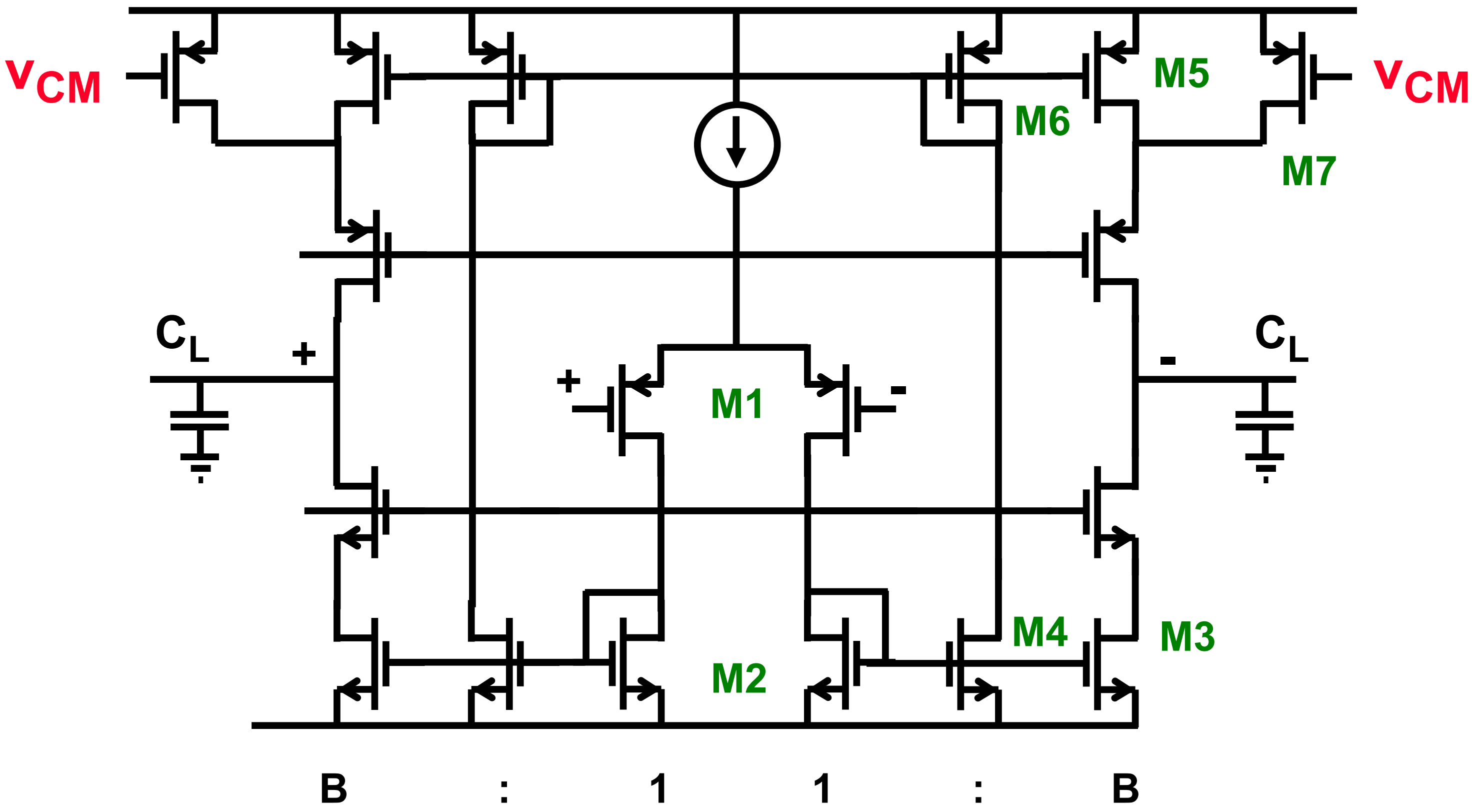
CMFB over 2 or more amplifiers



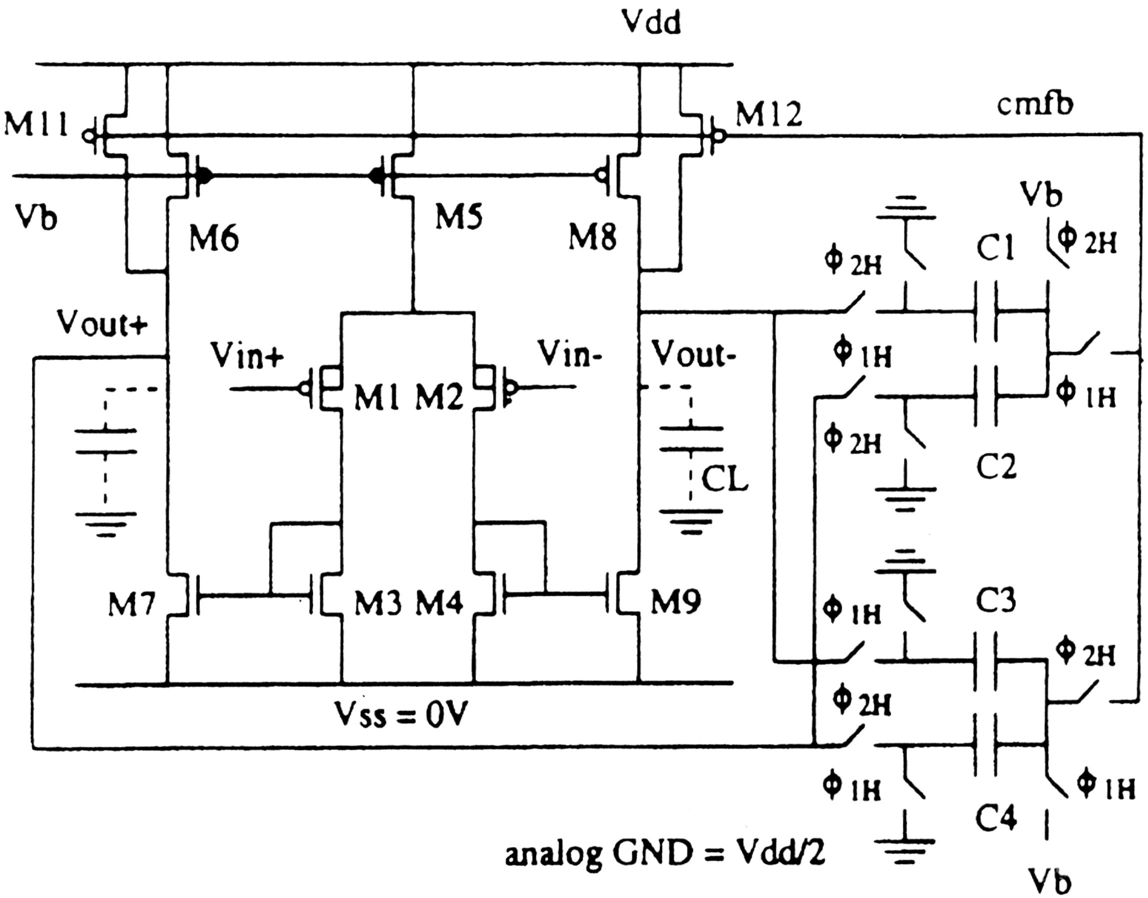
Efficient use of 2nd amplifier !

Ref. Mohieldin, JSSC April 2003, 663-668

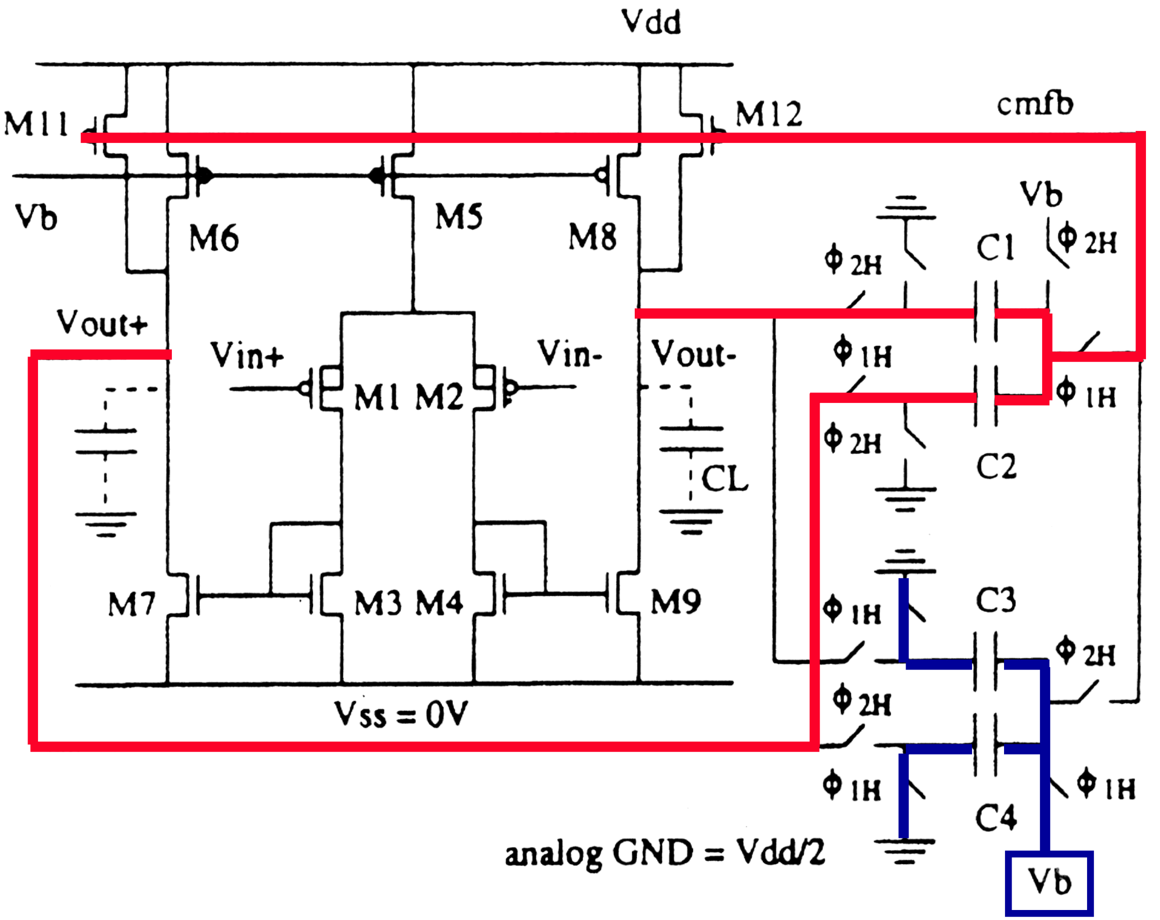
CMFB over 2 pseudo-differential amps



Fully-differential amplifier with SC CMFB

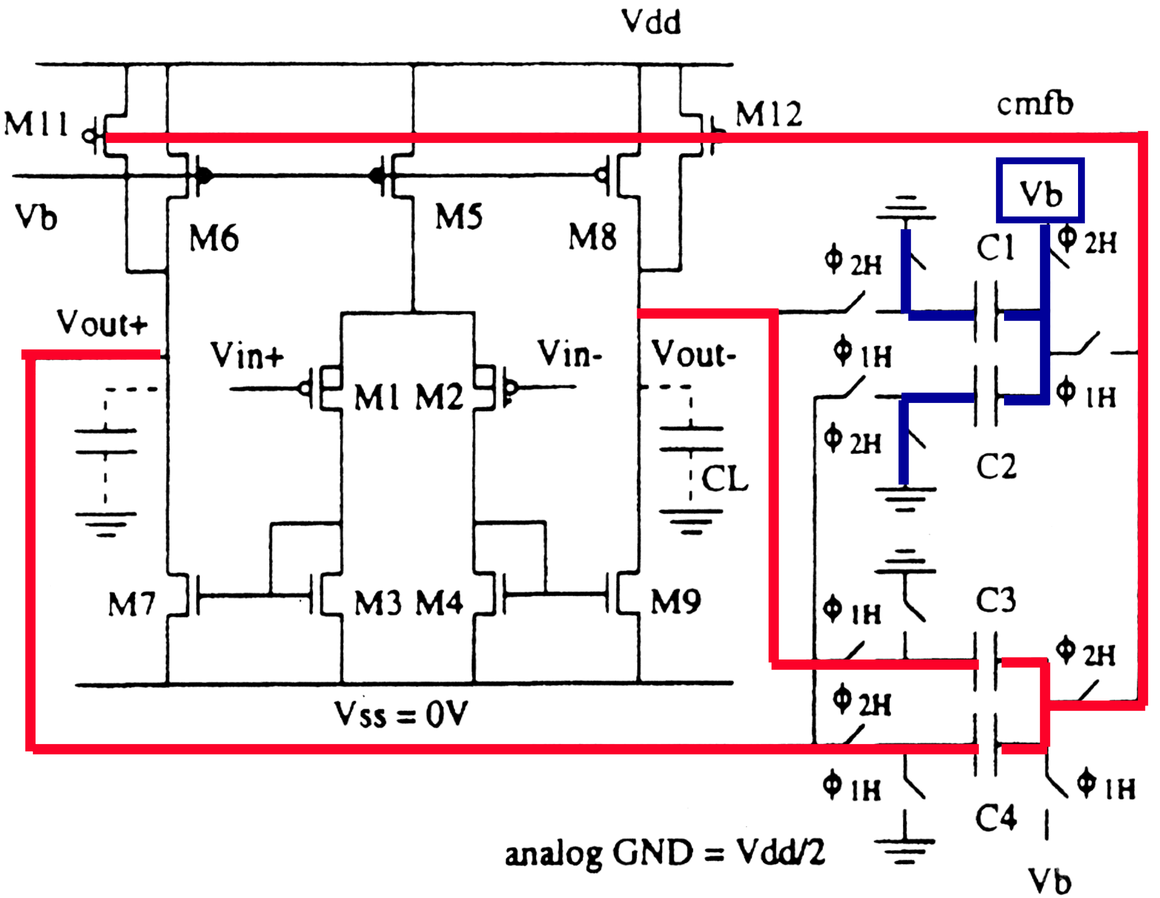


Fully-differential amp. with SC CMFB : Φ1



Switches ϕ_{1H} closed gives CMFB
and precharge C

Fully-differential amp. with SC CMFB : Φ2



Switches ϕ_{2H} closed gives CMFB and precharge C

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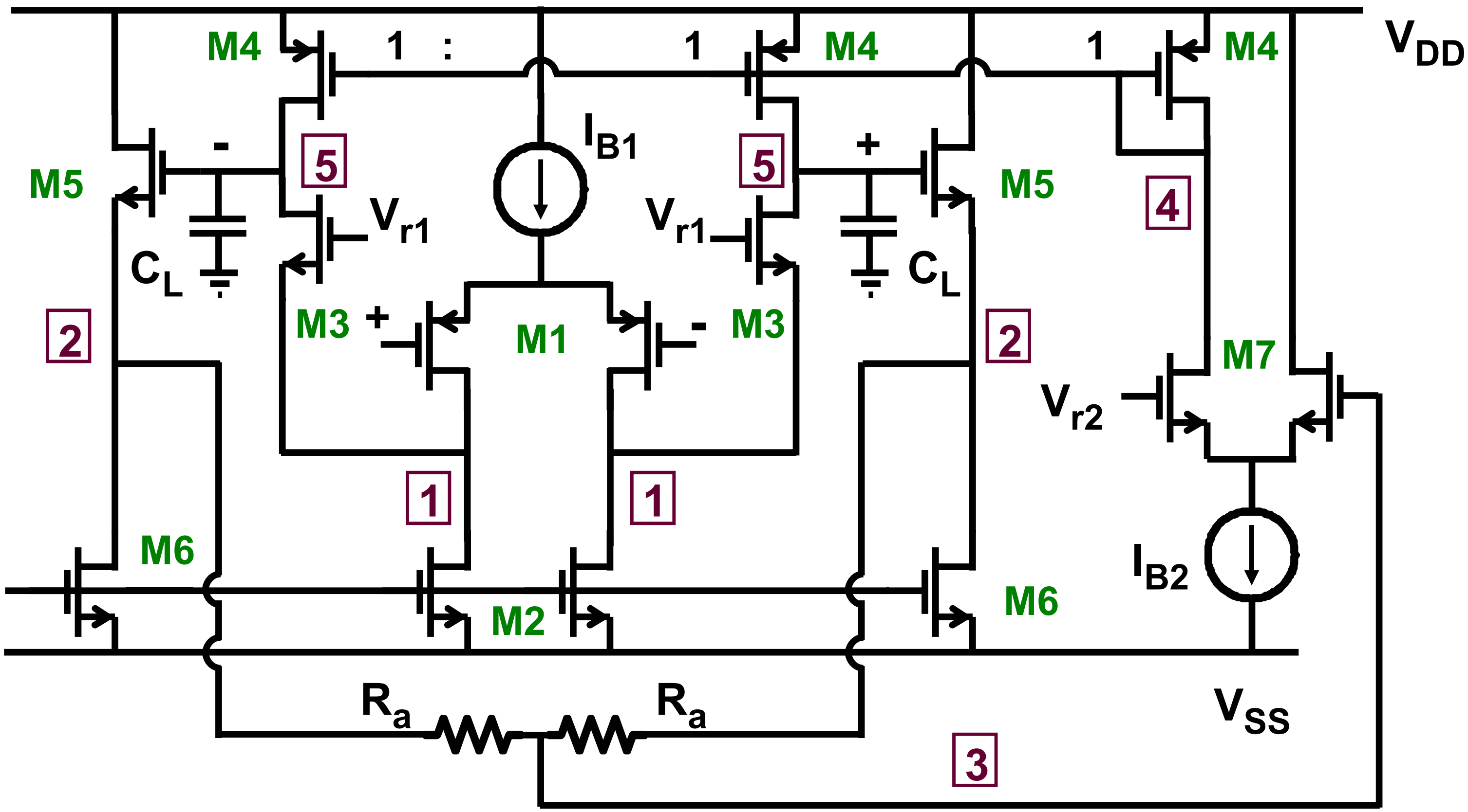
Fully-differental folded cascode with source foll.



Fully-diff. amp. : Specifications

Techn: CMOS

$L_{min} = 0.8\ \mu m$; $V_T = 0.7$ V

$K'_n = 60\ \mu A/V^2$ & $K'_p = 30\ \mu A/V^2$

$V_{En} = 4$ V/µm & $V_{Ep} = 6$ V/µm

Specs:

$GBW_{DM} = 10$ MHz $C_L = 3$ pF

$GBW_{CM} = 20$ MHz

all PM > 70°

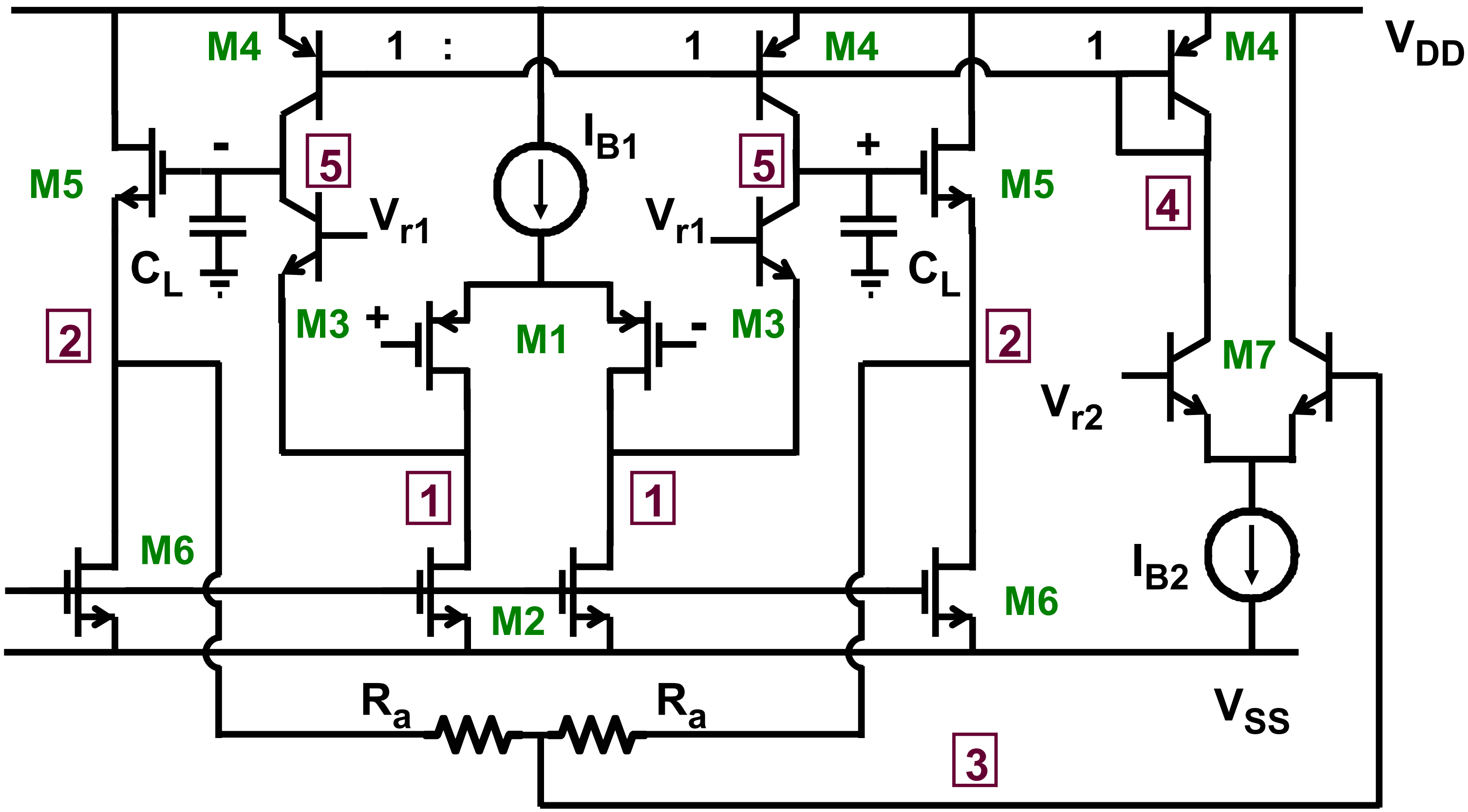
$V_{DD}/V_{SS} = \pm 1.5$ V

Maximum $V_{swingptp} = V_{outmax} - V_{outmin}$

Minimum I_{tot}

Verify: Slew Rate, Noise, ...

Fully-diff. folded cascode in BICMOS



Fully-diff. amp. : Specifications

Techn: **BICMOS** $L_{min} = 0.8\ \mu m$; $V_T = 0.7\ V$

$K'_n = 60\ \mu A/V^2$ & $K'_p = 30\ \mu A/V^2$

$V_{En} = 4\ V/\mu m$ & $V_{Ep} = 6\ V/\mu m$

$f_{Tn} = 12\ GHz$ & $f_{Tp} = 4\ GHz$

Specs: $GBW_{DM} = 10\ MHz$ $C_L = 3\ pF$

$GBW_{CM} = 20\ MHz$

all PM > 70°

$V_{DD}/V_{SS} = \pm 1.5\ V$

Maximum $V_{swingptp} = V_{outmax} - V_{outmin}$

Minimum I_{tot}

Verify: Slew Rate, Noise, ...

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Design of Multistage Operational amplifiers

Willy Sansen

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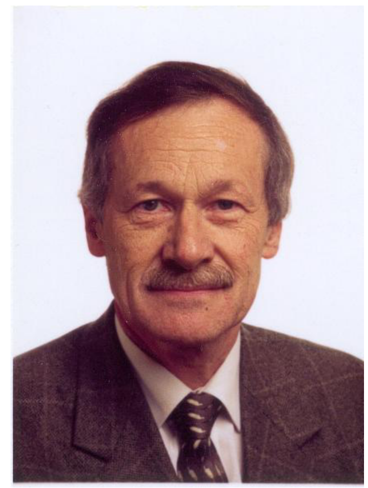

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- Design procedure
- Nested-Miller designs
- Low-power designs
- Comparison

Ref.: W. Sansen : Analog Design Essentials, Springer 2006

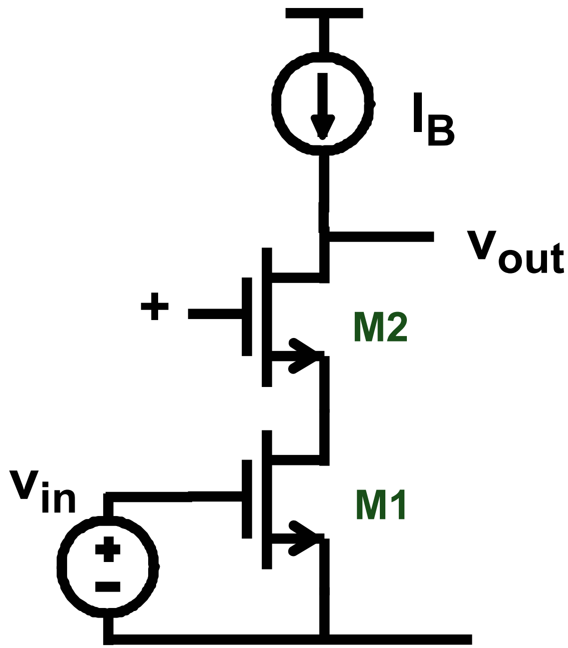
Why three-stage amplifiers ?

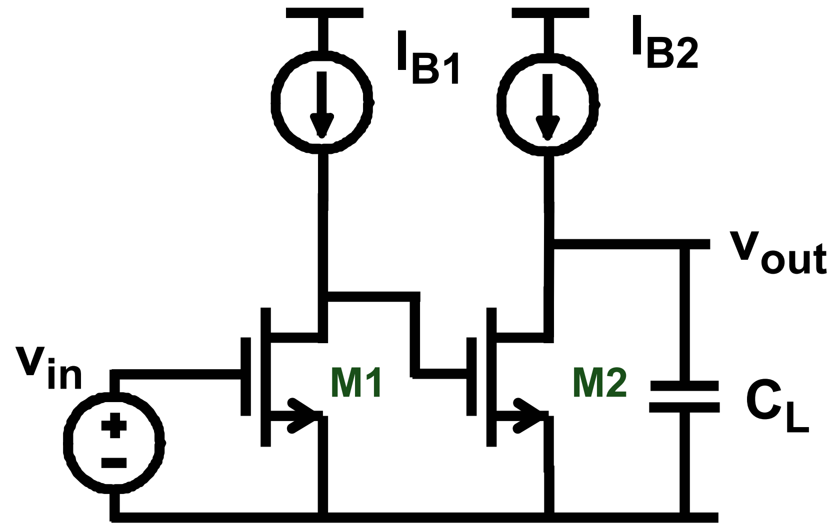
1. Each MOST only gives $g_m r_o \approx$ 15 or 24 dB :
   High gain requires three stages !

2. For drivers (small R_L) : $g_m R_L$ is very low :
   High gain requires three stages !

3. For low V_{DD}, no cascoding but cascading !
   High gain requires three stages !

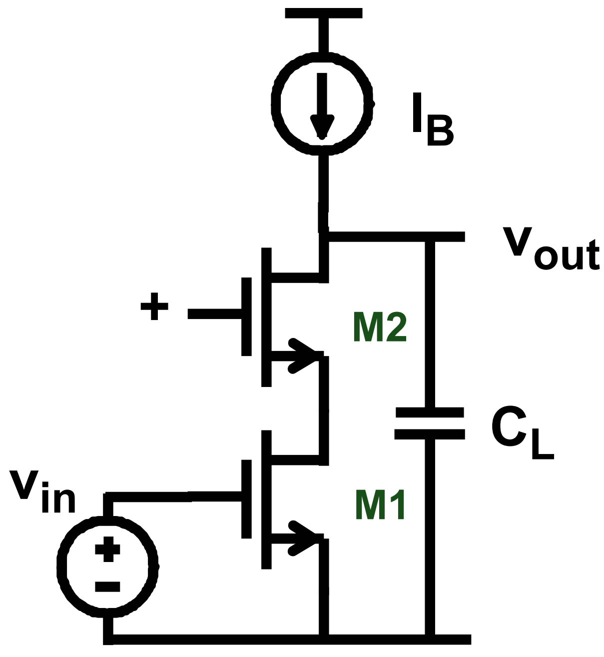
No cascoding but cascading !

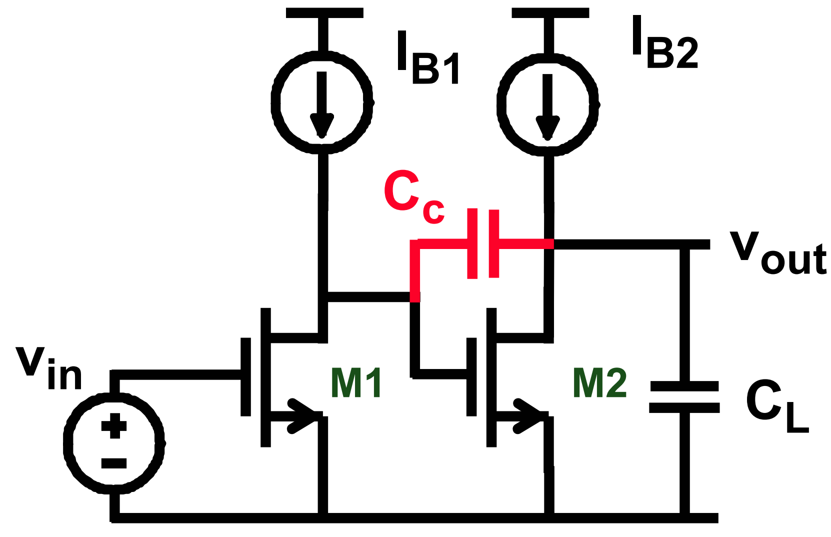




$A_v = (g_m r_{DS})_1 (g_m r_{DS})_2$ $A_v = (g_m r_{DS})_1 (g_m r_{DS})_2$

Cascode versus cascade

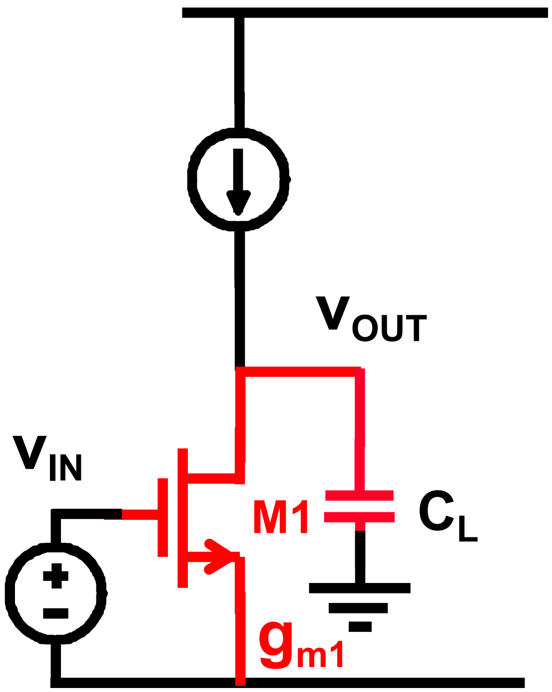




$$GBW = \frac{g_{m1}}{2\pi C_L}$$

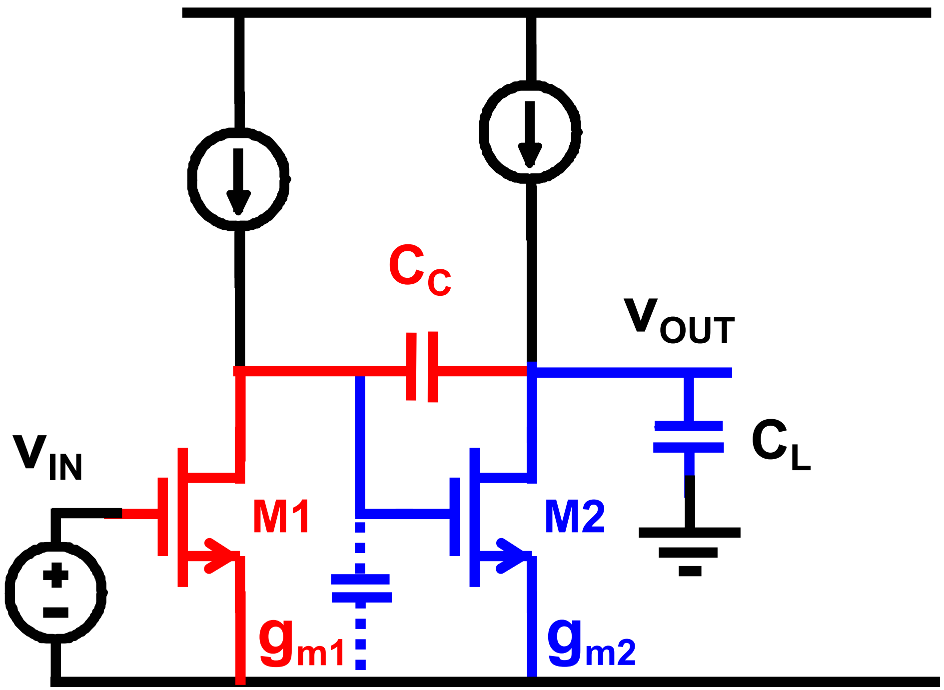
$$GBW = \frac{g_{m1}}{2\pi C_c} < \frac{g_{m2}}{2\pi C_L}$$

1-stage CMOS OTA



$$GBW = \frac{g_{m1}}{2\pi\, C_L}$$

2-stage Miller CMOS OTA

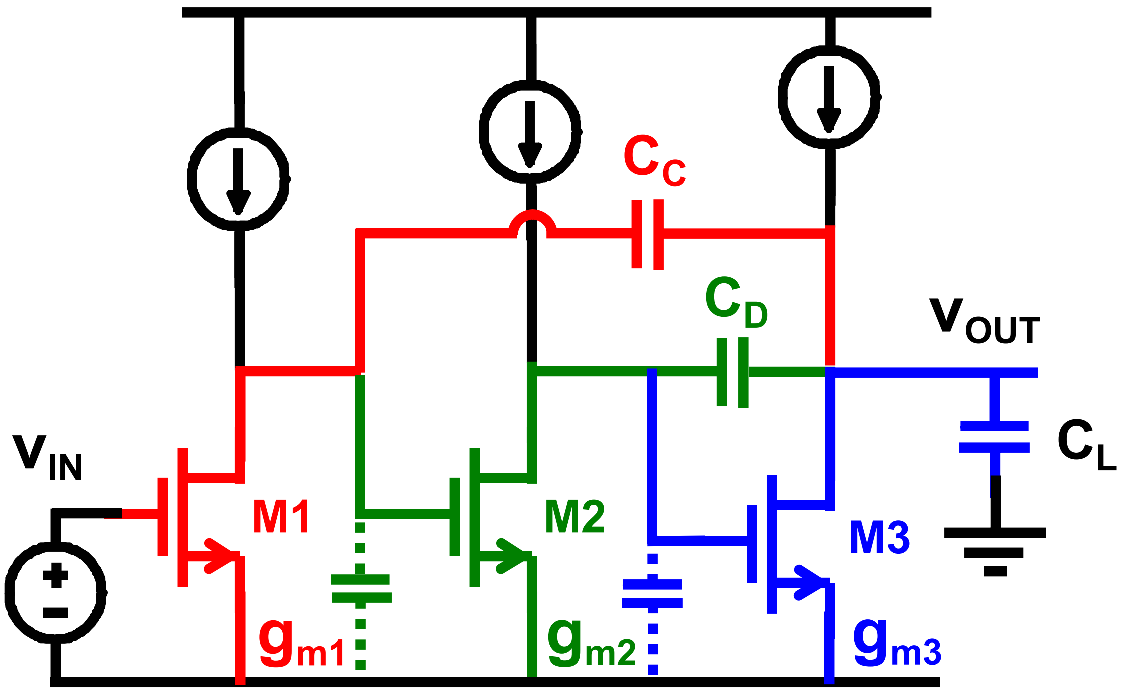


$$GBW = \frac{g_{m1}}{2\pi\ C_C}$$

$$f_{nd1} = \frac{g_{m2}}{2\pi\ C_L}$$

$$f_{nd1} = 3\ GBW$$

3-stage Nested Miller CMOS OTA



$$GBW = \frac{g_{m1}}{2\pi\, C_C}$$

$$f_{nd1} = \frac{g_{m2}}{2\pi\, C_D}$$

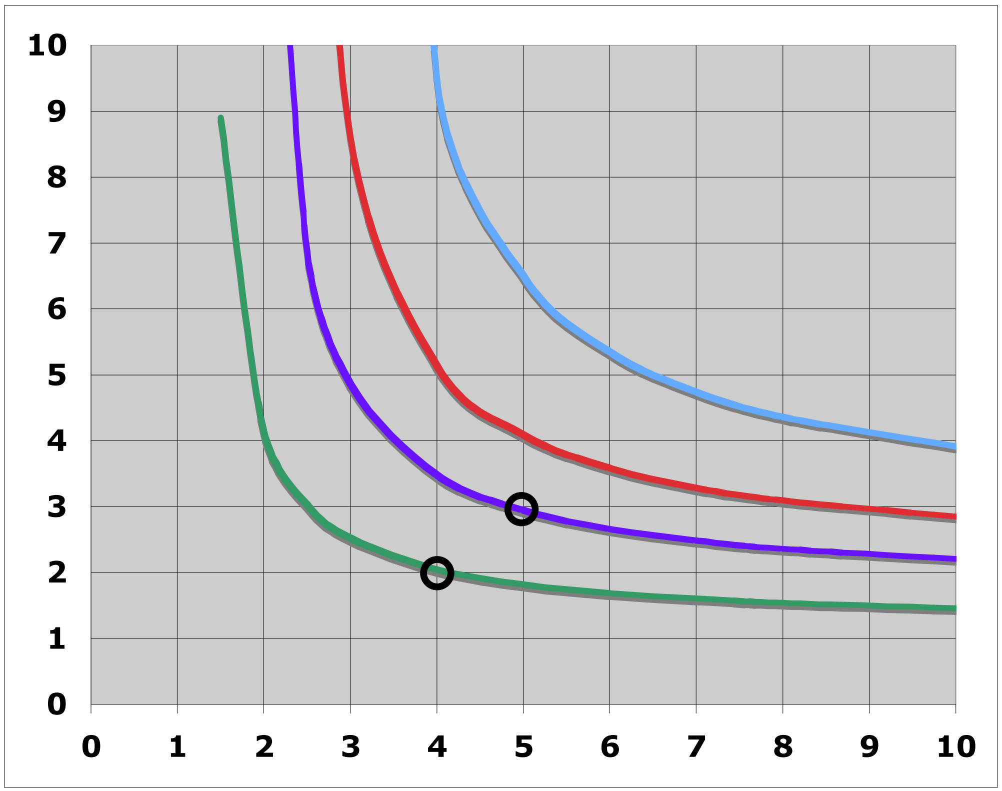
$$f_{nd2} = \frac{g_{m3}}{2\pi\, C_L}$$

$f_{nd1} = 3\ GBW$

$f_{nd2} = 5\ GBW$

3-pole opamp : phase margin PM

$\frac{\omega_1}{\omega_{UG}}$

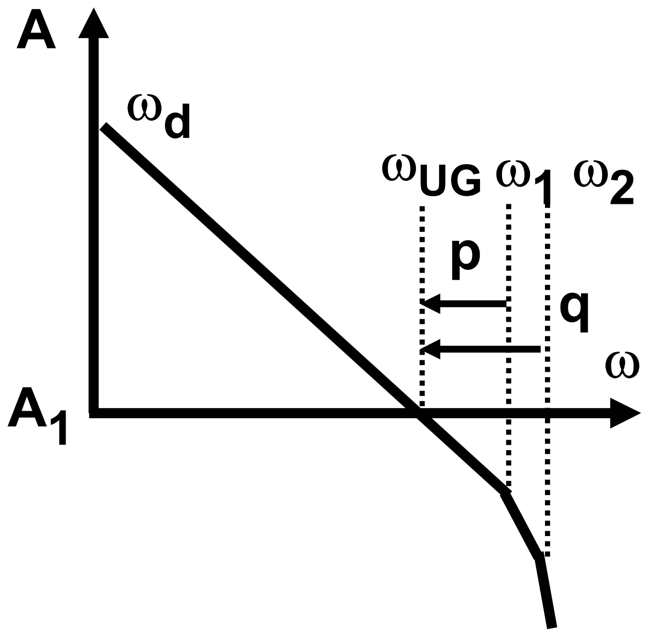


$$PM \approx 90^o - \arctan\left(\frac{\omega_{UG}}{\omega_1}\right) - \arctan\left(\frac{\omega_{UG}}{\omega_2}\right)$$

70°
65°
60°
50° PM

$\frac{\omega_2}{\omega_{UG}}$

Three-pole opamp



Open loop gain

$$A = \frac{\omega_{UG}}{s} \frac{1}{\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)}$$

Closed loop gain (Unity gain)

$$A_1 = \frac{A}{1+A} \approx$$

$$\frac{1}{1 + \frac{s}{\omega_{UG}} + \left(\frac{1}{p} + \frac{1}{q}\right)\left(\frac{s}{\omega_{UG}}\right)^2 + \frac{1}{pq}\left(\frac{s}{\omega_{UG}}\right)^3}$$

$p = \omega_1 / \omega_{UG}$

$q = \omega_2 / \omega_{UG}$

3-pole opamp: $\omega_1 = 3\,\omega_{UG}$ $\omega_2 = 5\,\omega_{UG}$

Open loop

Three poles:

ω_d

ω_1 3x

ω_2 5x

Unity gain

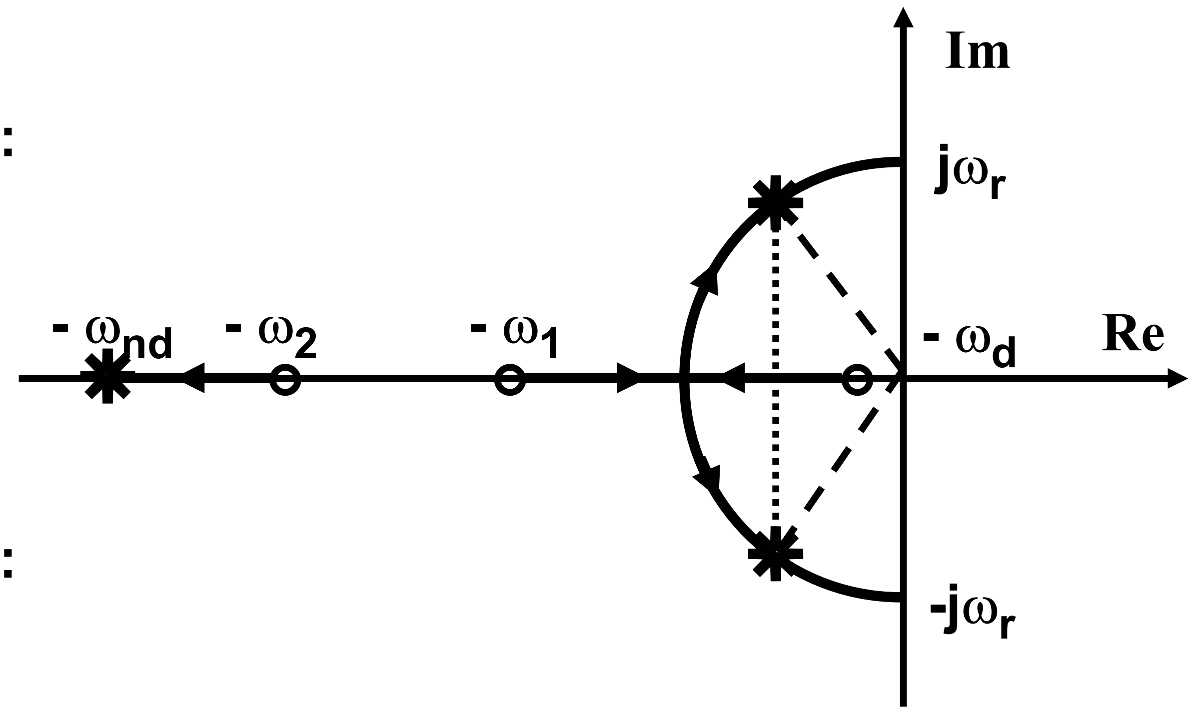
Three poles:

ω_d

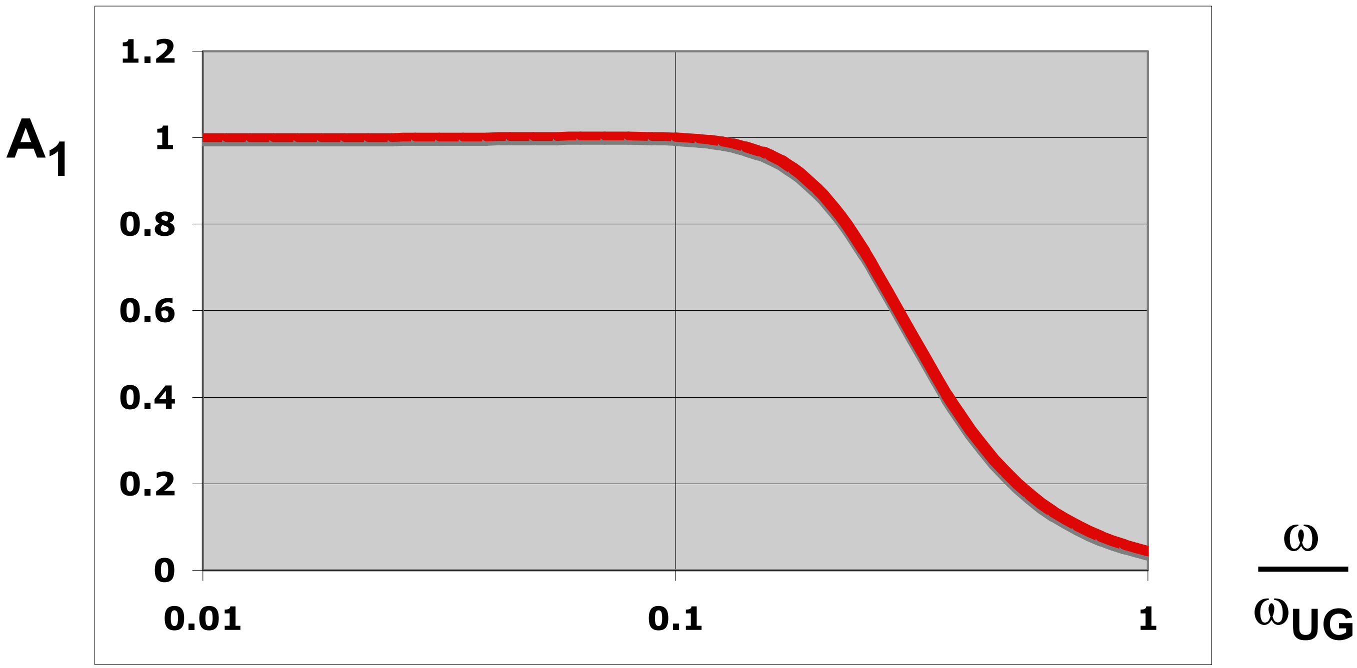
ω_{nd} 6x

ω_r 1± j1.2 x

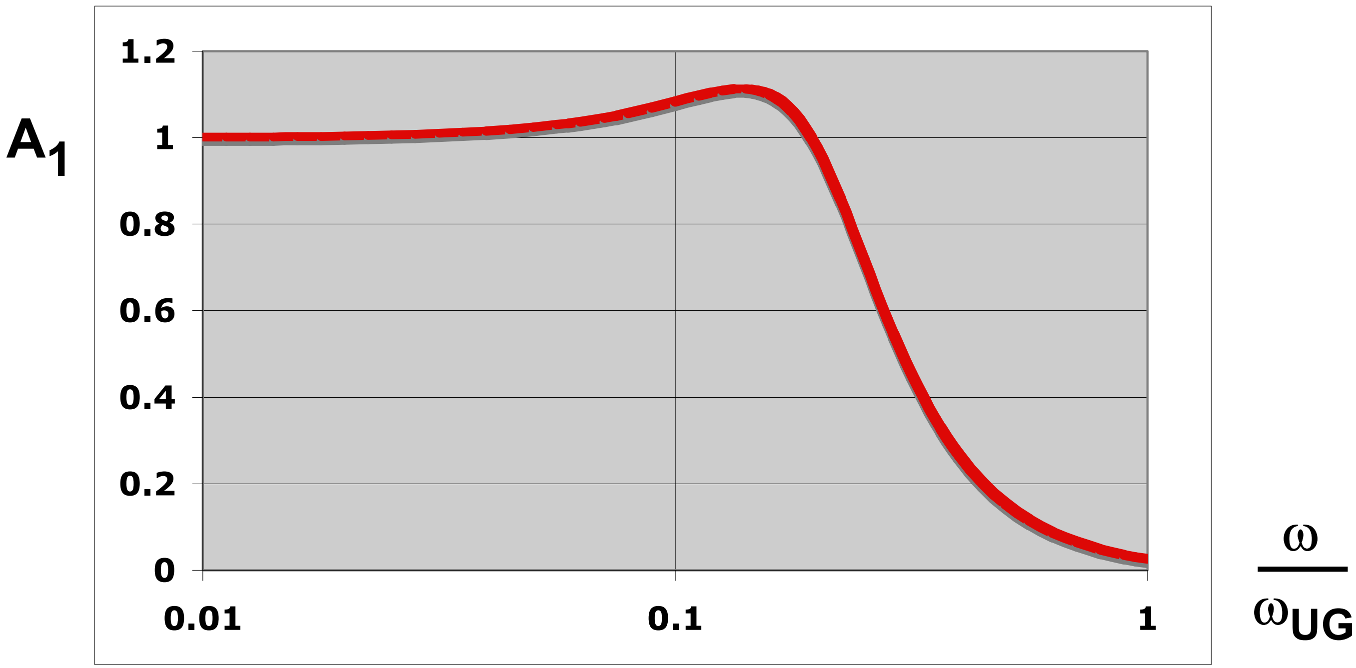
1.6 50°



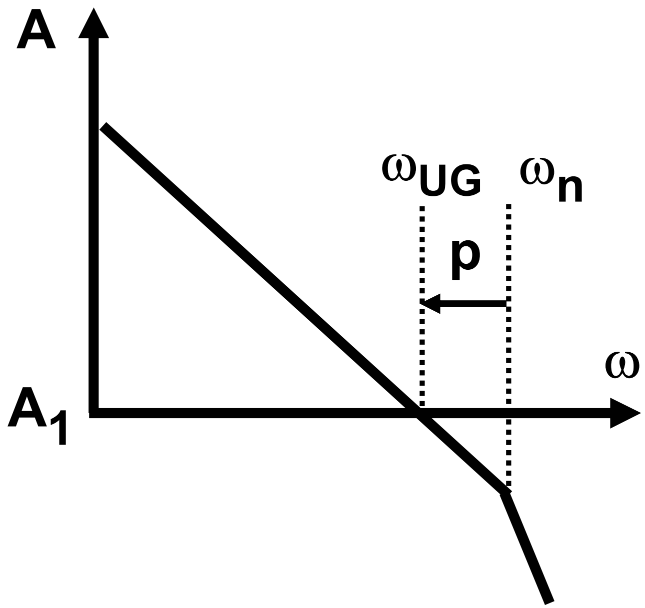
Three-stage with 3/5 on 60^o PM with unity gain



Three-stage with 2/4 on 50º PM with unity gain



Three-pole opamp with complex poles



Open loop gain

$$A = \frac{\omega_{UG}}{s} \frac{1}{1 + 2\zeta \frac{s}{\omega_n} + \left(\frac{s}{\omega_n}\right)^2}$$

Closed loop gain (Unity gain)

$$A_1 = \frac{A}{1+A} \approx$$

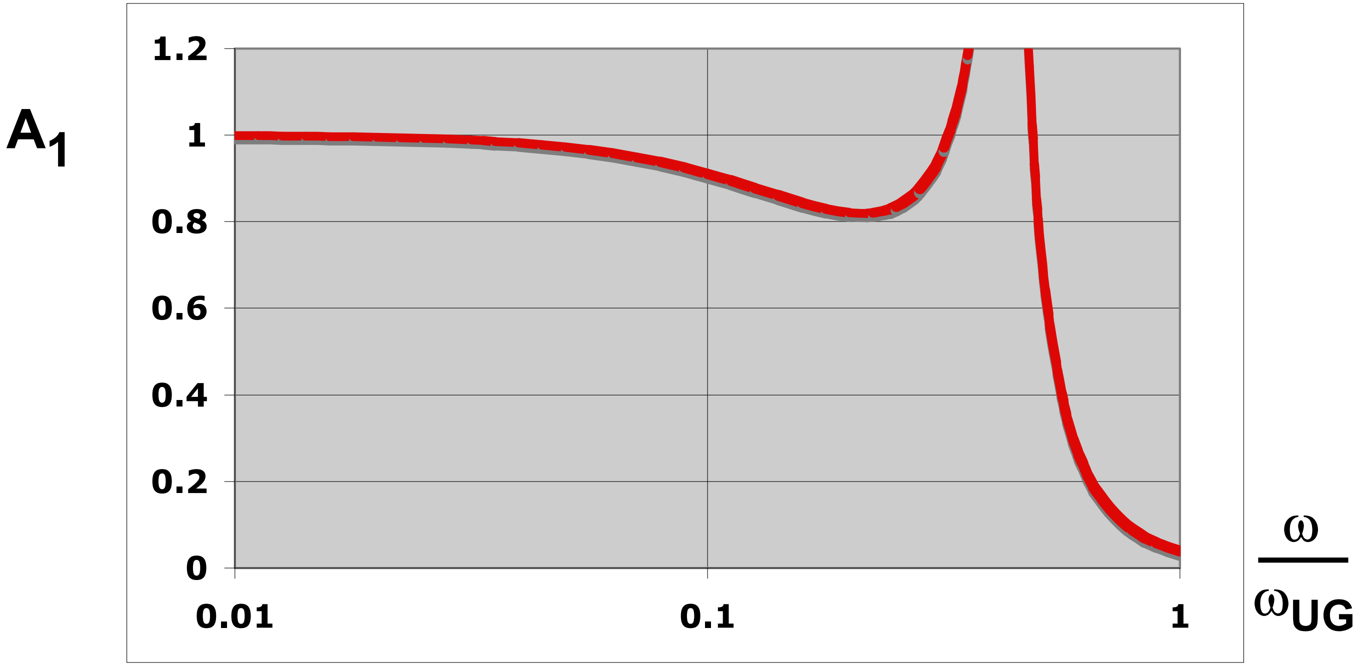
$$\frac{1}{1 + \frac{s}{\omega_{UG}} + \frac{2\zeta}{p}\left(\frac{s}{\omega_{UG}}\right)^2 + \frac{1}{p^2}\left(\frac{s}{\omega_{UG}}\right)^3}$$

Two parameters:

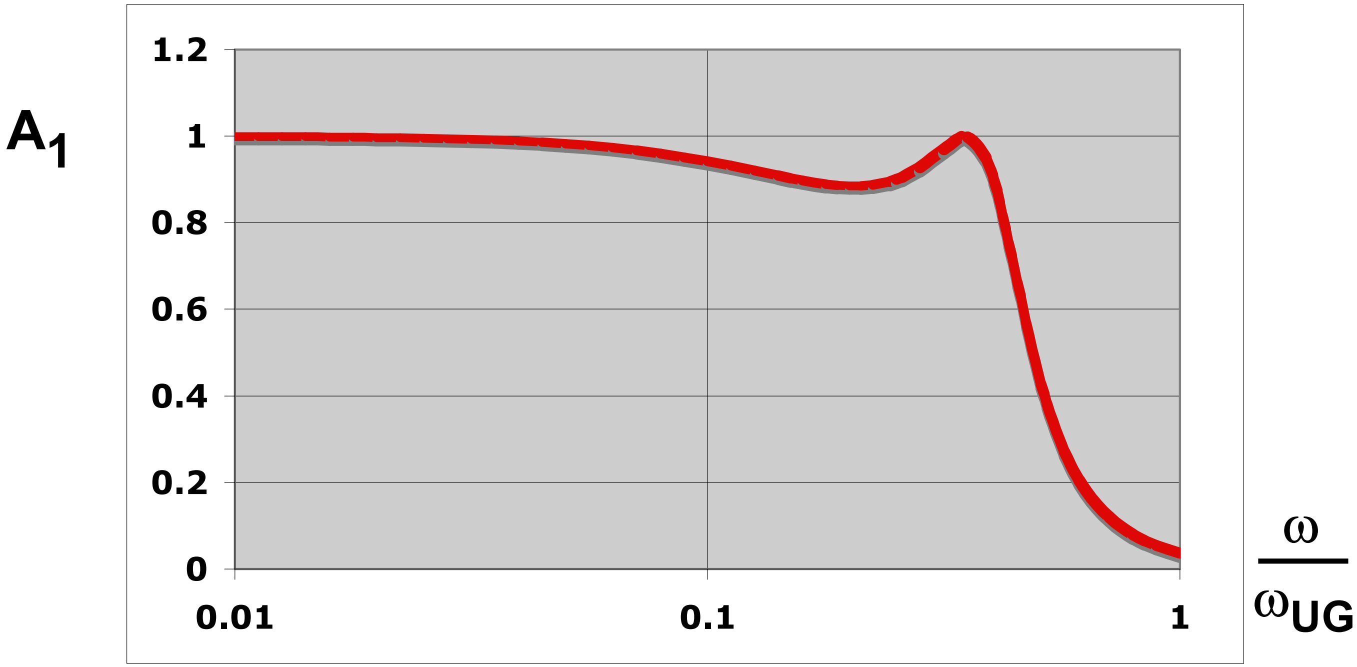
ζ damping (=1/2Q)

$p = \omega_n / \omega_{UG}$

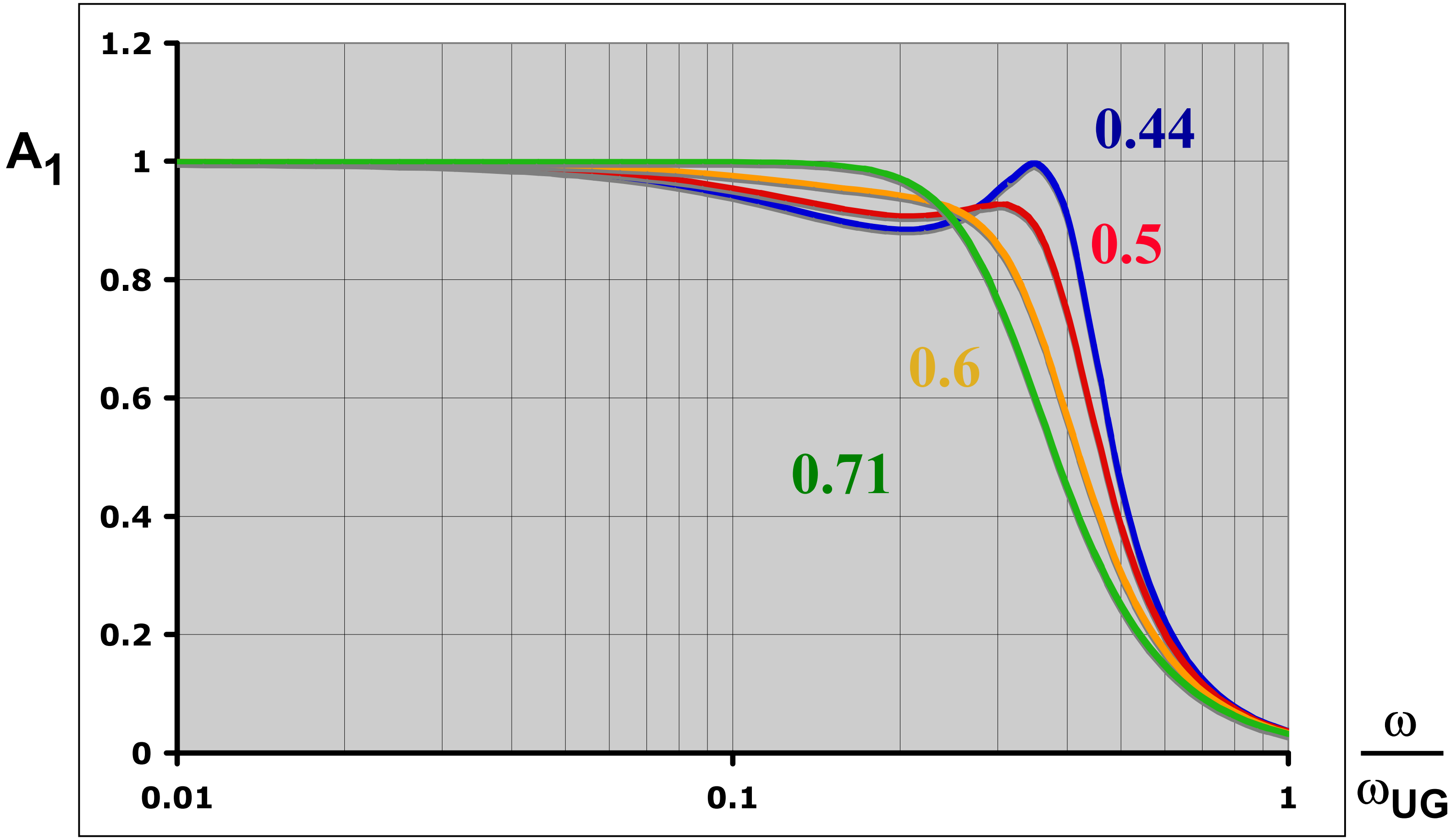
Unity-gain 3-pole opamp: $\zeta = 0.28$ $p = 2.828$



Unity-gain 3-pole opamp: $\zeta = 0.44$ p = 2.828



Unity-gain 3-pole opamp : ζ = …   p = 2.828



Unity-gain 3-pole opamp: $\zeta = 0.71 \quad p = 2.828$

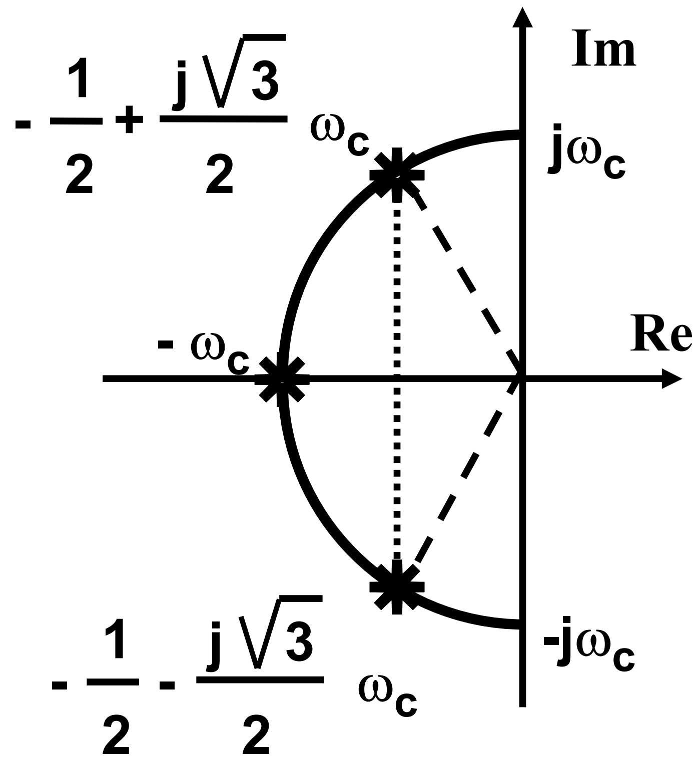
Closed loop

Butterworth response :

maximally flat

Poles :

$$\omega_{-3dB} = \frac{\omega_c}{2}$$



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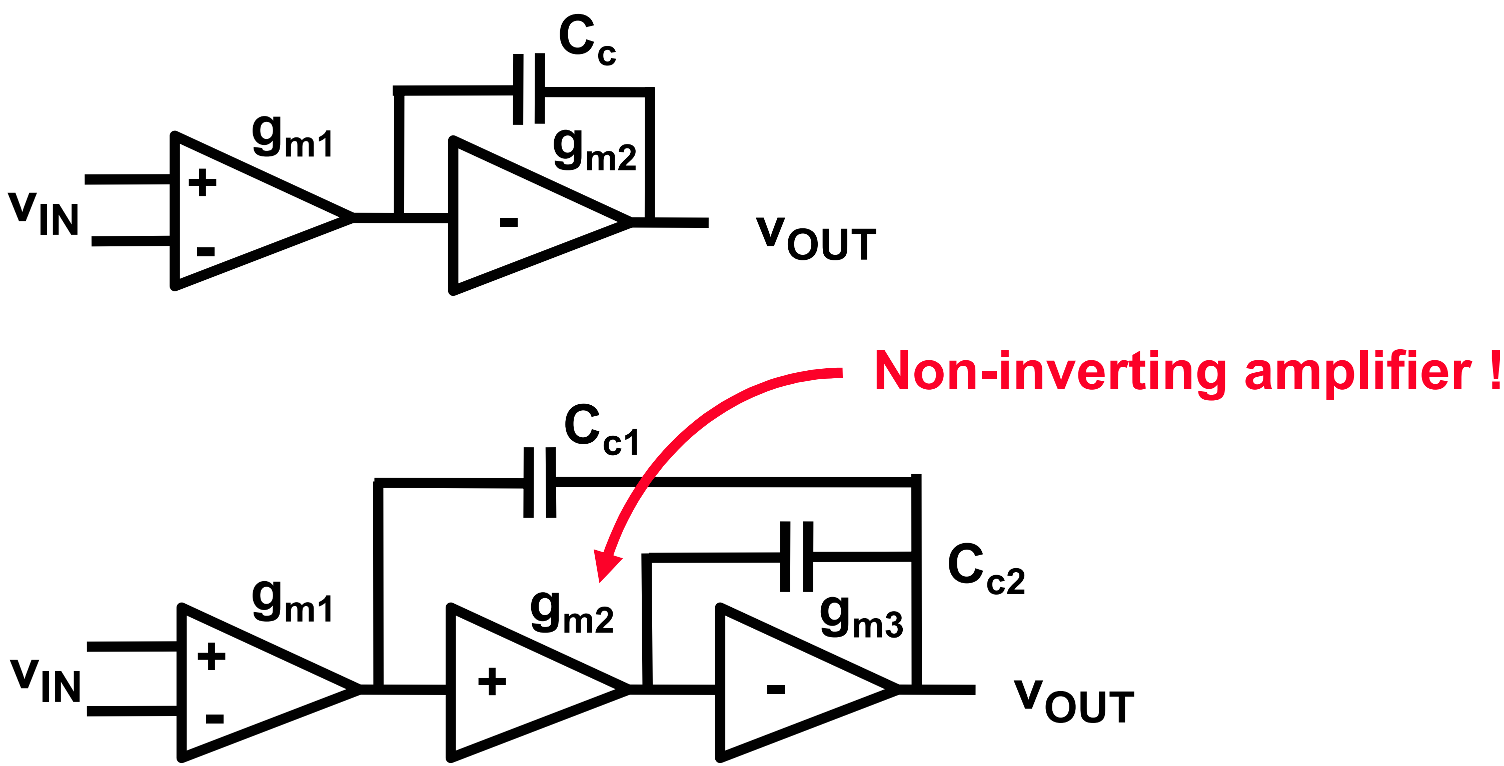
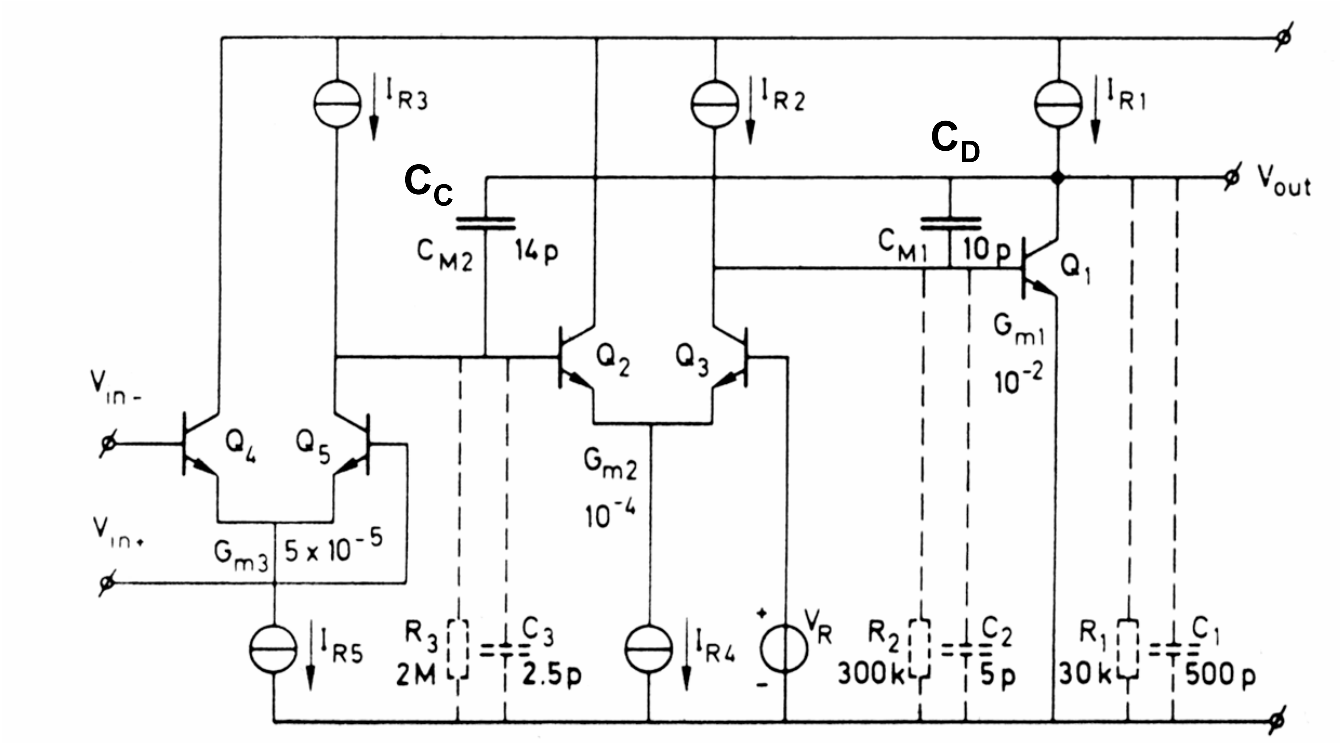
Three-stage configuration



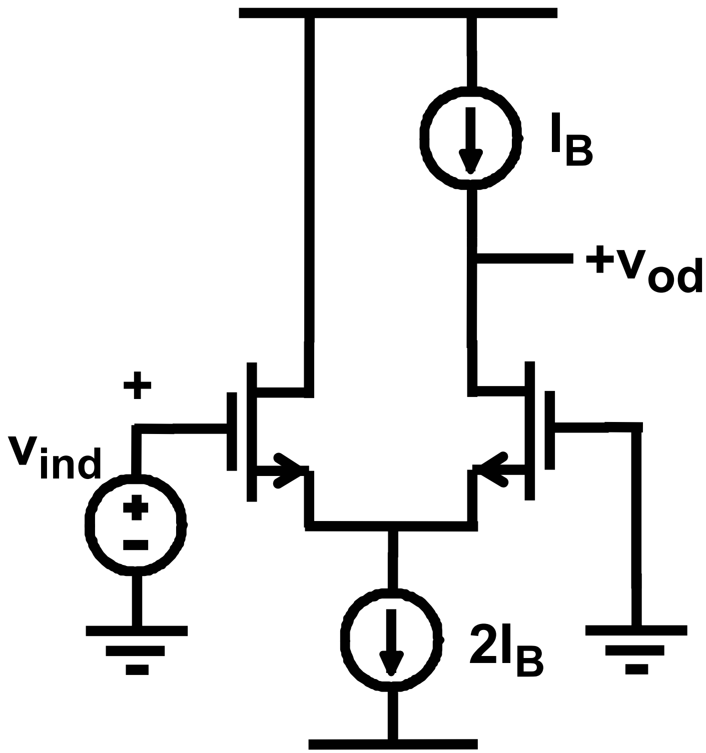
Nested Miller with differential pair as 2nd stage



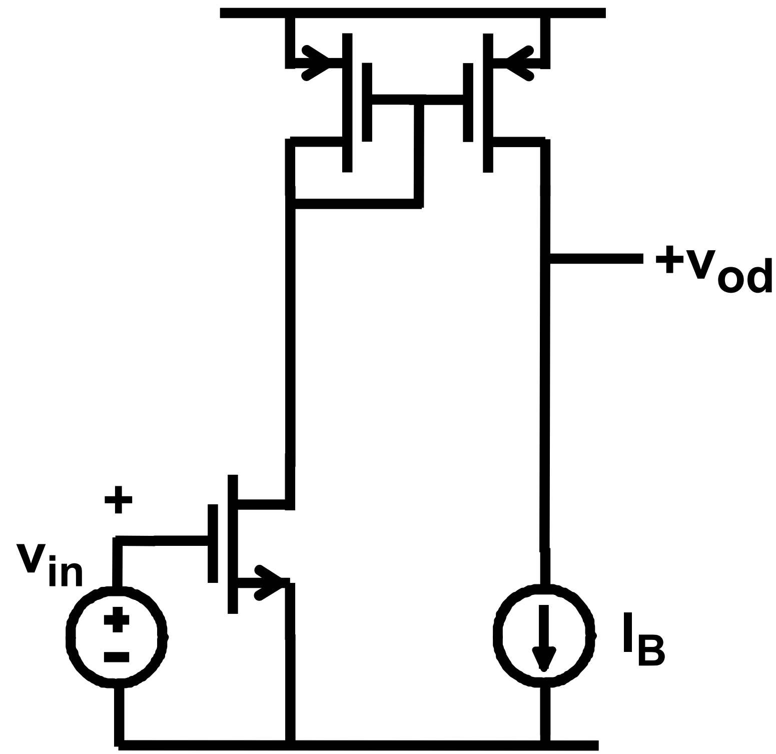
$$\text{GBW} = \frac{g_{m4}}{2\pi\, C_C}$$

Huijsing, JSSC Dec.85, pp.1144-1150

Two ways to non-inverting gain

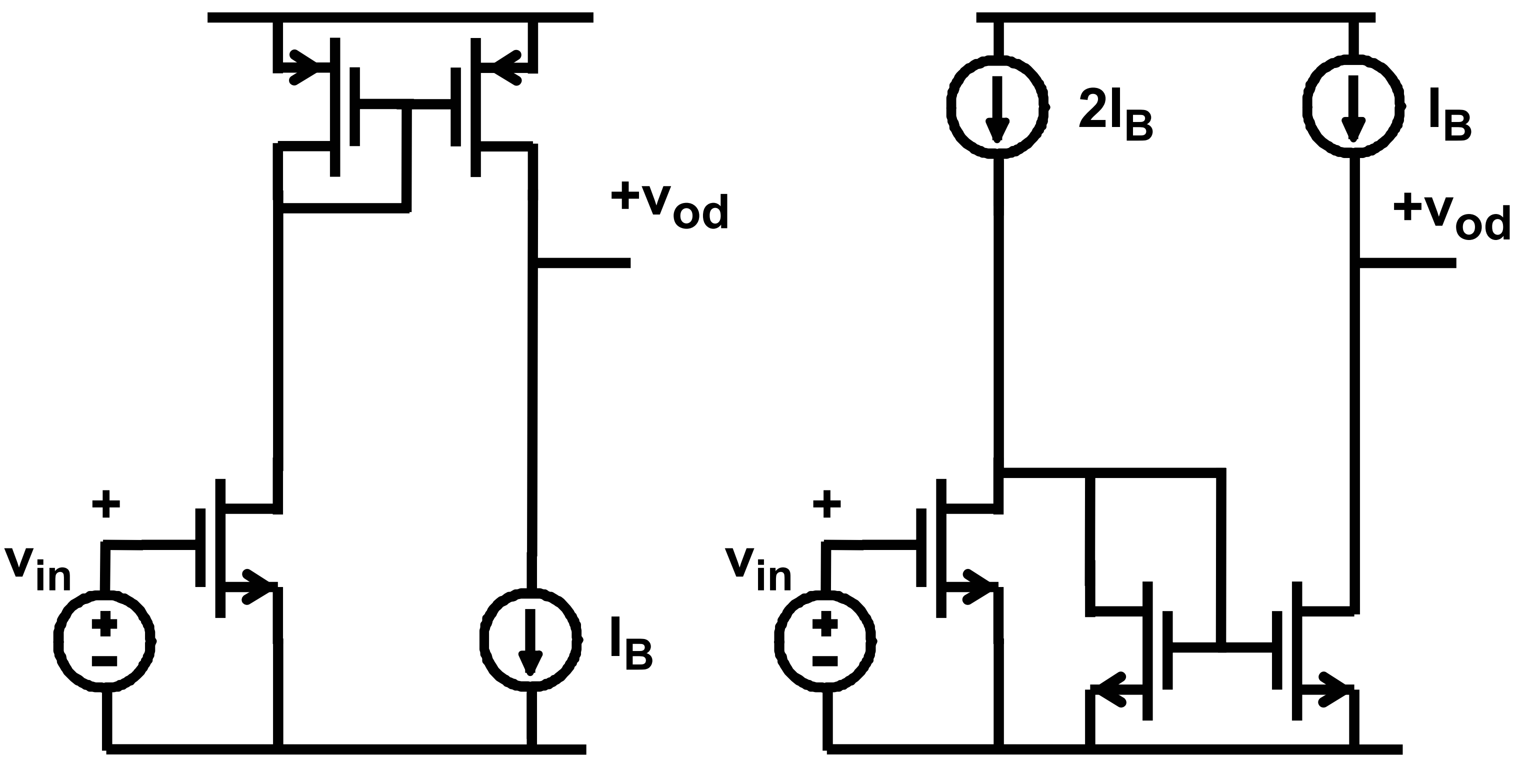


Differential pair



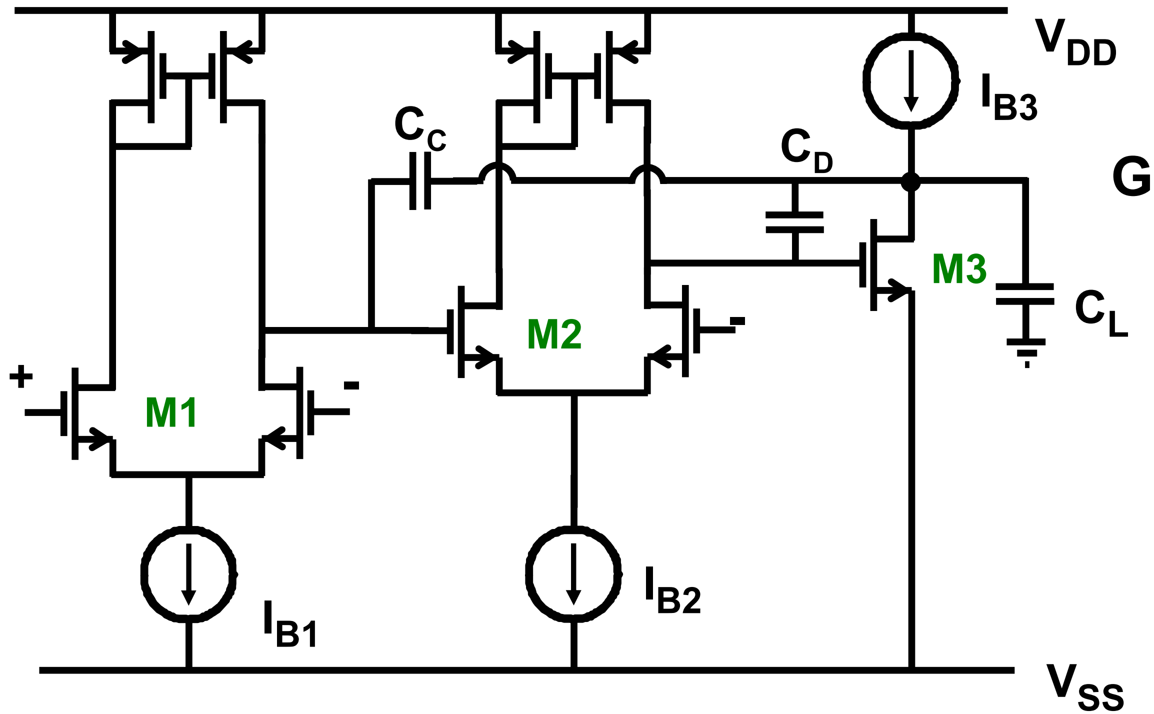
Current mirror

Two types of current mirroring



Current mirror 1   Current mirror 2 (only nMOST)

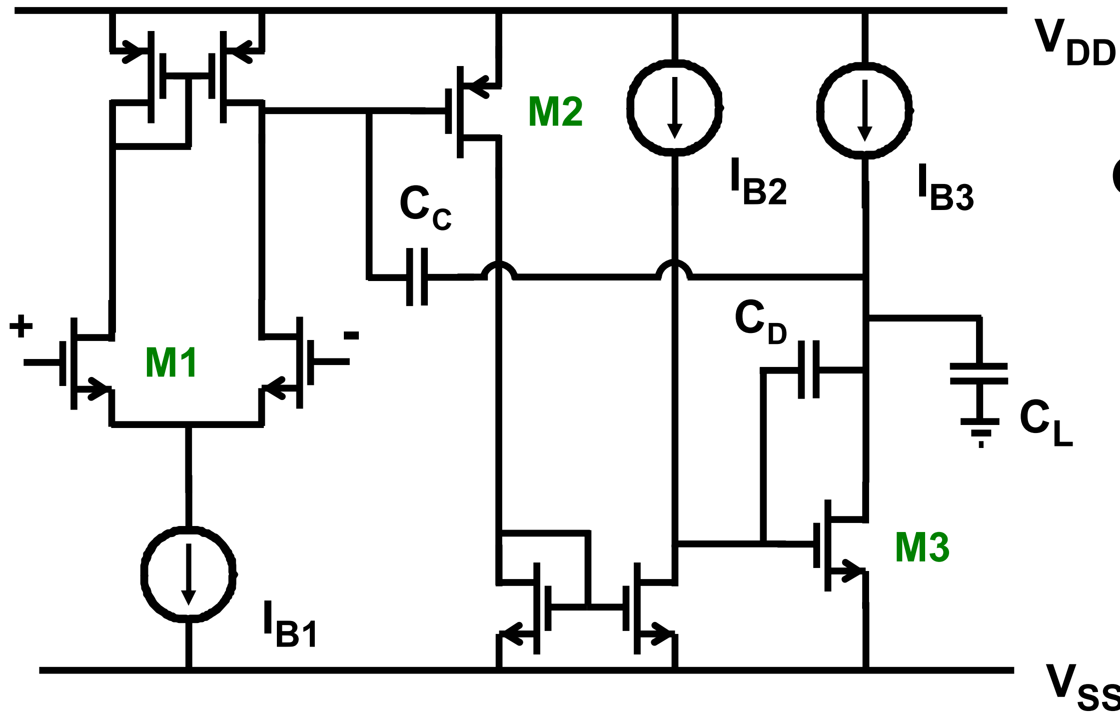
Nested Miller with differential pair as 2nd stage



$$GBW = \frac{g_{m1}}{2\pi\, C_C}$$

"Huijsing"

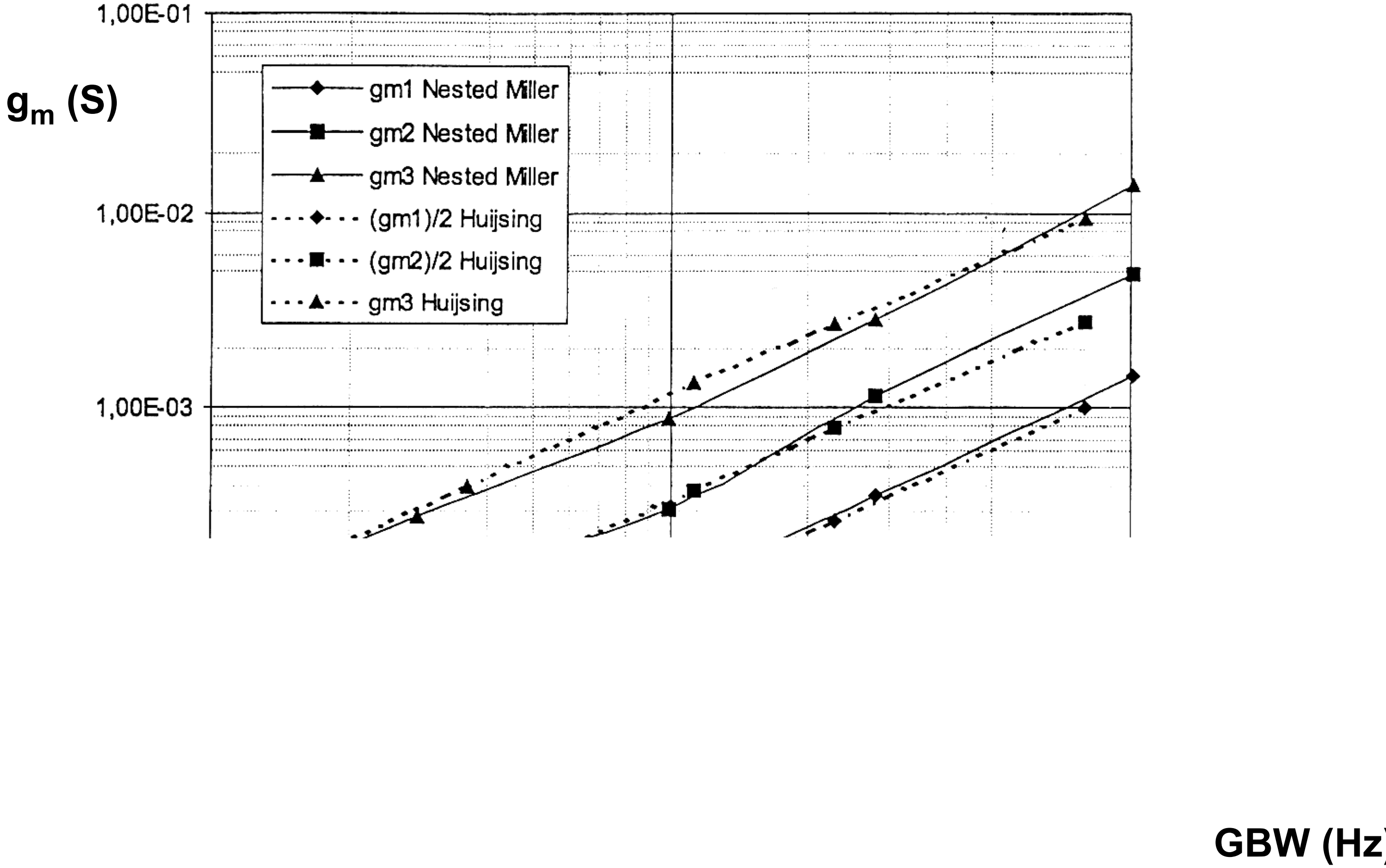
Nested Miller with current mirror as 2nd stage



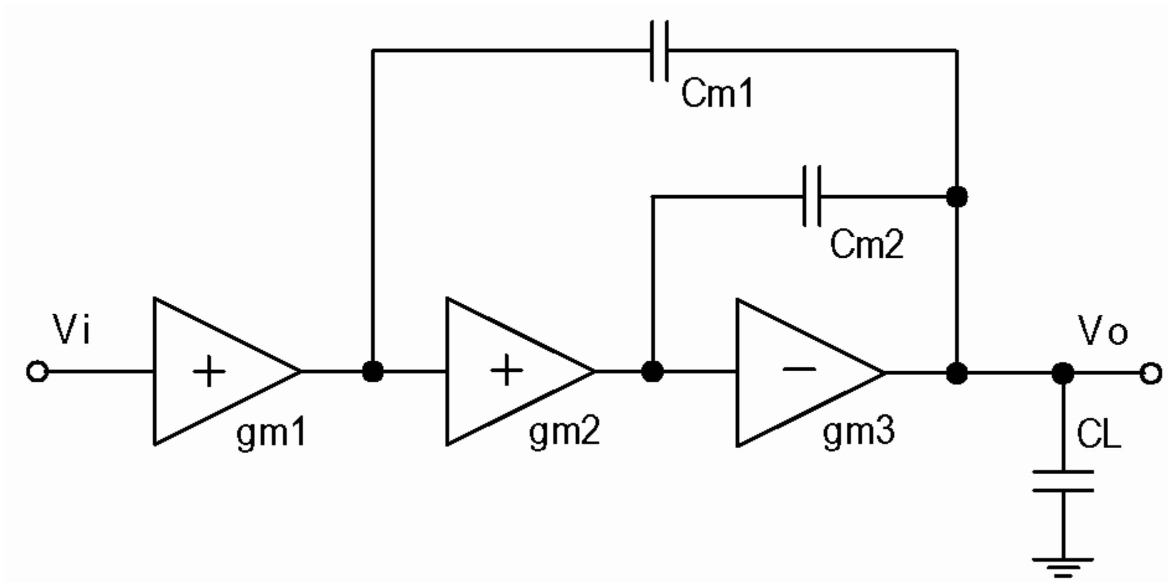
$$GBW = \frac{g_{m1}}{2\pi\ C_C}$$

"Nested Miller"

Comparison of power consumption



Nested-Miller Frequency compensation - NMC



Huijsing, JSSC Dec.85, pp.1144-1150

NMC equations in open loop

$$A_v(s) = \frac{A_{dc}\left(1 + \frac{s}{\omega_3} + \frac{s^2}{\omega_3\omega_4}\right)}{\left(1 + \frac{s}{\omega_d}\right)\left(1 + \frac{s}{\omega_1} + \frac{s^2}{\omega_1\omega_2}\right)}$$

$$\omega_1 = \frac{g_{m2}}{C_{m2}}$$

$$\omega_2 = \frac{g_{m3}}{C_L}$$

$$A_{dc} = g_{m1}g_{m2}g_{m3}R_1R_2R_3$$

$$\omega_3 = -\frac{g_{m3}}{C_{m2}}$$

$$\omega_d = -\frac{1}{C_{m1}g_{m2}g_{m3}R_1R_2R_3}$$

$$\omega_4 = \frac{g_{m2}}{C_{m1}}$$

$$\omega_{UG} = \frac{g_{m1}}{C_{m1}}$$

NMC stability

$$A_v(s) = \frac{A_{dc}\left(1 + \frac{s}{\omega_3} + \frac{s^2}{\omega_3\omega_4}\right)}{\left(1 + \frac{s}{\omega_d}\right)\left(1 + \frac{s}{\omega_1} + \frac{s^2}{\omega_1\omega_2}\right)}$$

$$\omega_1 = \frac{g_{m2}}{C_{m2}}$$

$$\omega_2 = \frac{g_{m3}}{C_L}$$

$$\omega_3 = -\frac{g_{m3}}{C_{m2}}$$

$$\omega_4 = \frac{g_{m2}}{C_{m1}}$$

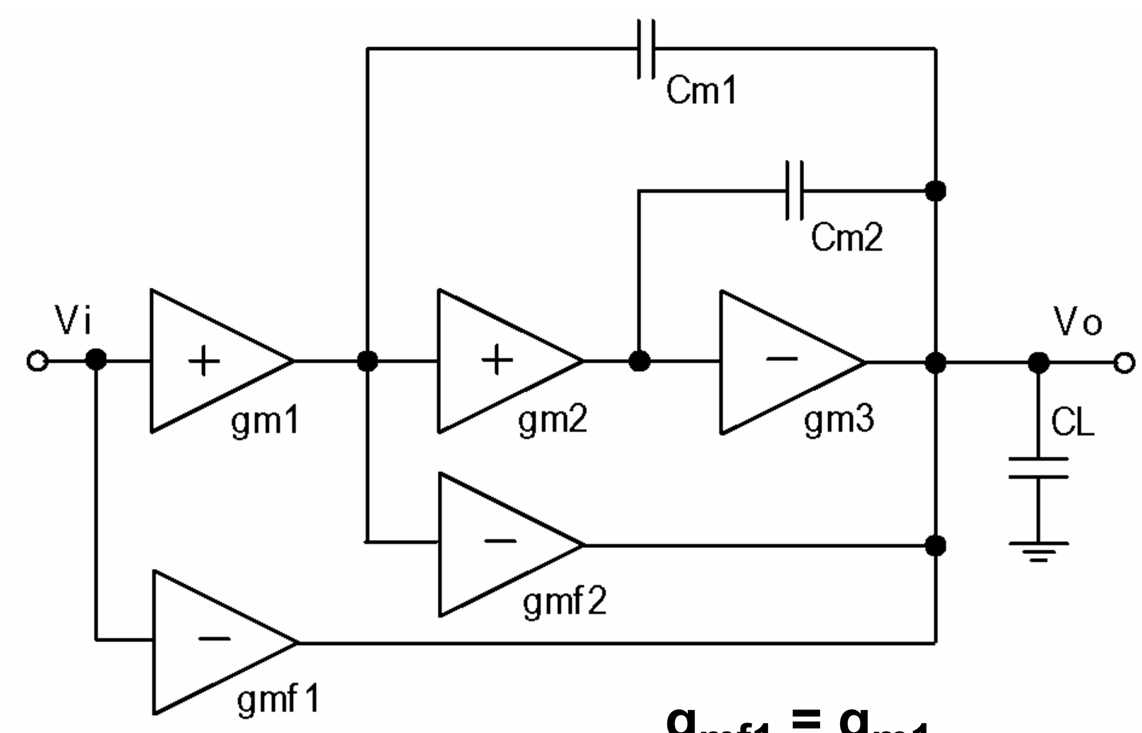
$g_{m1} < g_{m2} < g_{m3}$

Butterworth 3rd order : $\zeta = 0.7$; $p = 2.8$

$$\omega_2 = 2\,\omega_1 = 4\,\omega_{UG}$$

Zero's negligible

Nested Gm-C compensation NGCC



$$\omega_1 \approx \frac{g_{m2}}{C_{m2}}$$

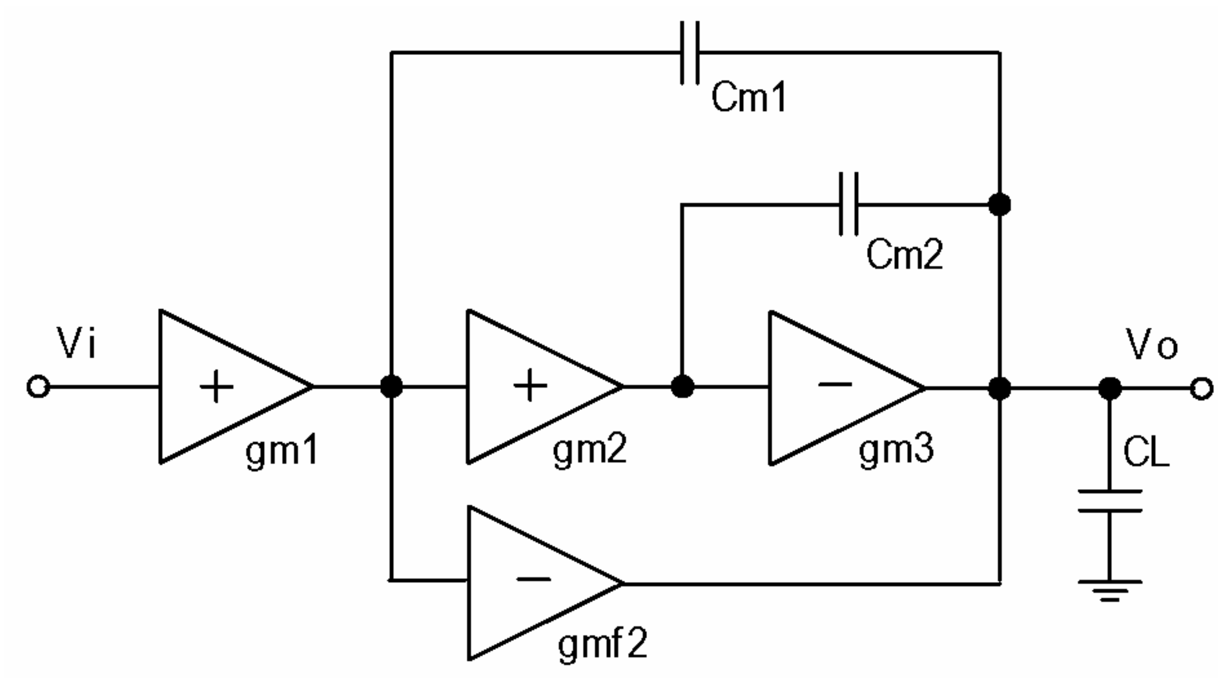
$$\omega_2 \approx \frac{g_{m3}}{C_L}$$

$g_{mf1} = g_{m1}$

$g_{mf2} = g_{m2}$

You, JSSC Dec.97, 2000-2011

Nested-Miller with single Feedforward - NMCF



$$\omega_z \approx \frac{g_{m2}}{C_{m2}}$$

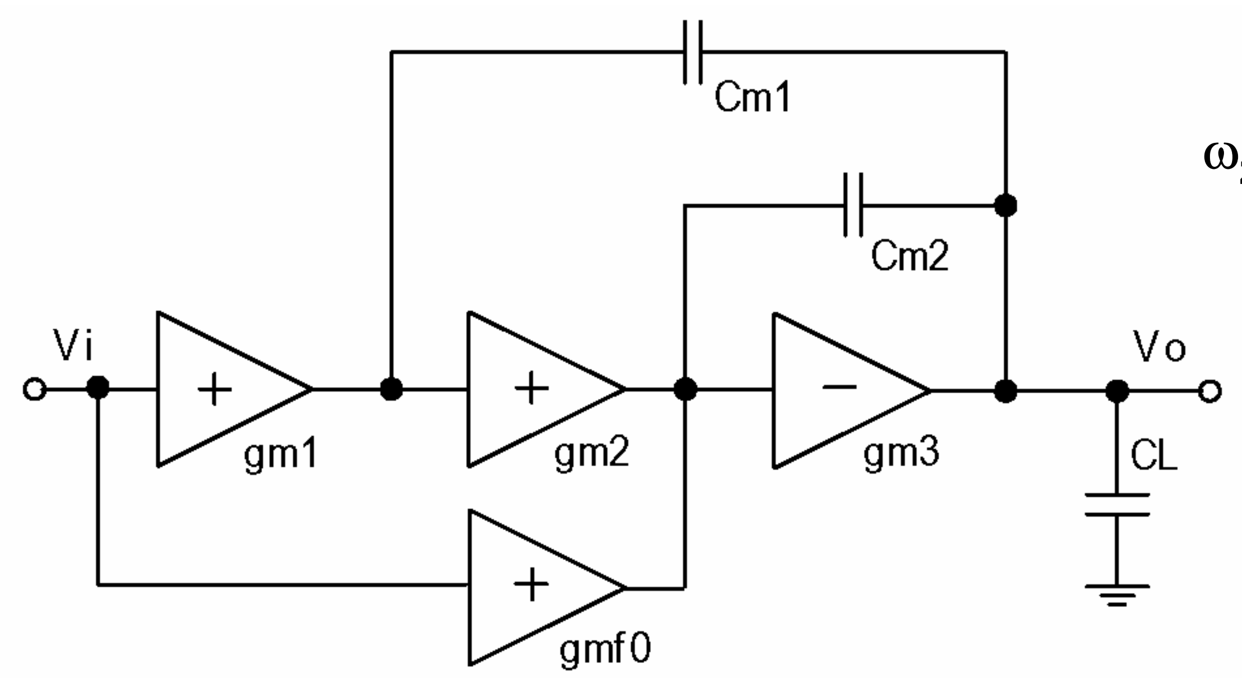
$$\omega_1 \approx \frac{g_{m2}}{2C_{m2}}$$

$$\omega_2 \approx \frac{2g_{m3}}{C_L}$$

$$g_{mf2} = g_{m3}$$

Leung, CAS April 01, 388-394

Multipath Nested-Miller - MNMC



$$\omega_z \approx \frac{g_{m1}\, g_{m2}}{g_{mf0}\, C_{m1}}$$

$$\omega_1 \approx \frac{g_{m2}}{C_{m2}}$$

$$\omega_2 \approx \frac{g_{m3}}{C_L}$$

Eschauzier, JSSC Dec.92, pp.1709-1717

Comparison Nested-Miller solutions

Topology	Stages	PM	GB=2πGBW	T_{eL}	T_{eL} / $T_{eL\ (NMC)}$
Single	One	$<90°$	(g_m/C_L)	1.0	4.0
SMC	Two	$<63°$	$0.5(g_{m2}/C_L)$	0.5	2.0
NMC	Three	$\approx 60°$	$0.25(g_{m3}/C_L)$	0.25	1.0
NGCC	Three	$\approx 60°$	$0.25(g_{m3}/C_L)$	0.25	1.0
NMCF	Three	$>60°$	$<0.5(g_{m3}/C_L)$	<0.5	<2.0
MNMC	Three	$\approx 63°$	$\approx 0.5(g_{m3}/C_L)$	≈ 0.5	≈ 2.0

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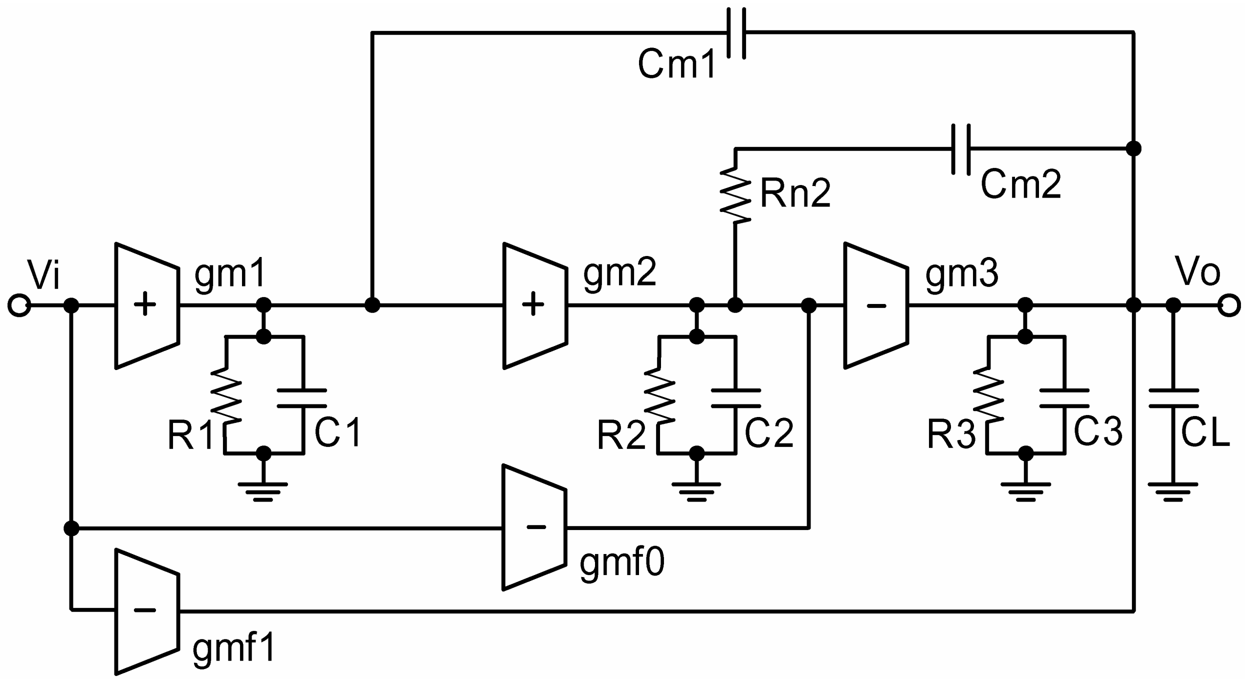
Nested Gm and nulling Res. NMC - NGRNMC



$$\omega_1 \approx \frac{1}{R_{n2}C_{m2}}$$

$$\omega_2 \approx \frac{g_{m2}g_{m3}R_{n2}}{C_L}$$

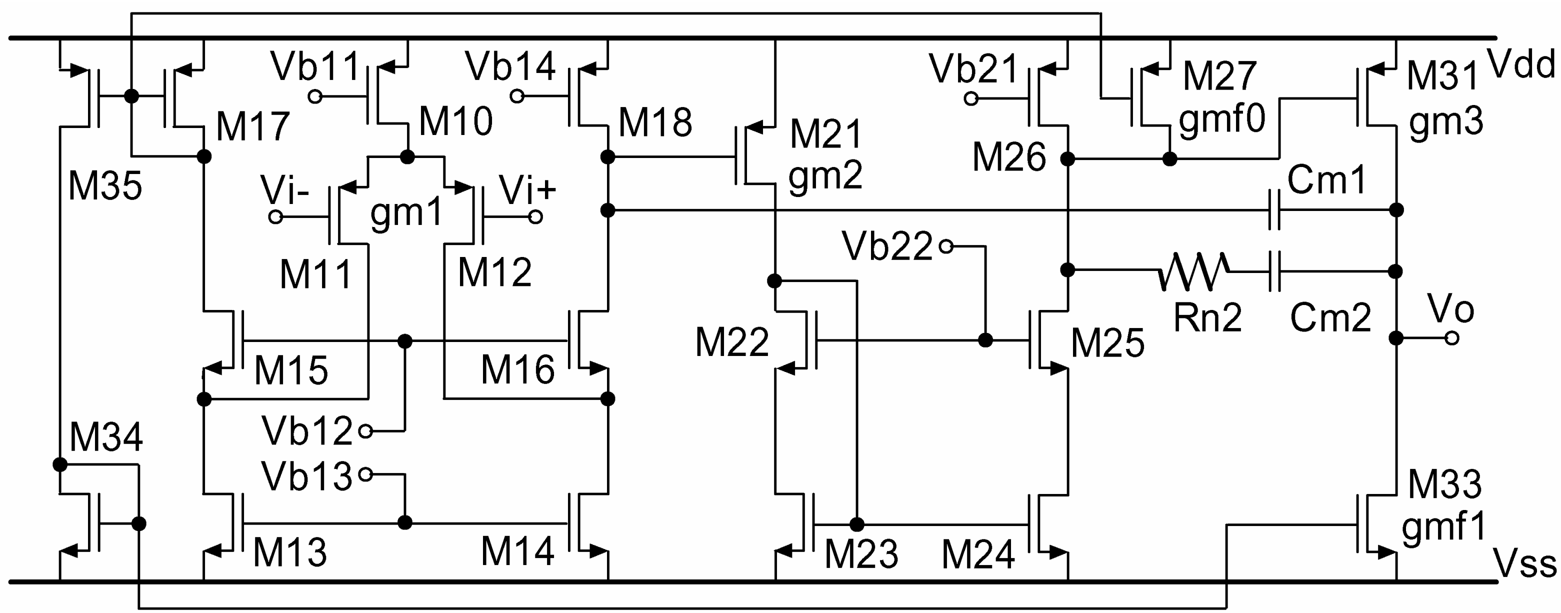
$g_{mf1} = g_{m1}$

$g_{mf0} = \dots$

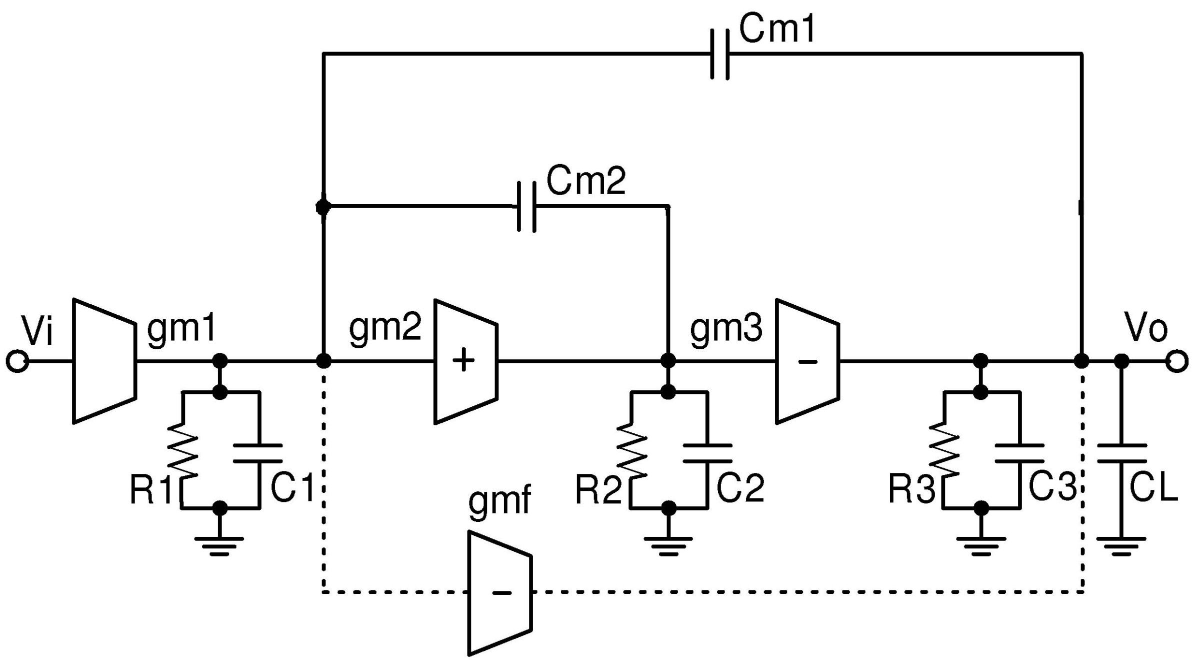
$GBW / GBW_{NMC} \approx 6.8$

Peng, CICC 2002, 329-332

NGRNMC schematic



Positive Feedback Compensation - PFC



$GBW / GBW_{NMC} \approx 6$

Ramos, CICC 2002, 333-336

PFC Equations

$$A_v(s) = \frac{A_{dc}\left(1 + \frac{s}{\omega_1} + \frac{s^2}{\omega_1\omega_3}\right)}{\left(1 + \frac{s}{\omega_d}\right)\left(1 + \frac{s}{\omega_1} + \frac{s^2}{\omega_1\omega_2}\right)}$$

$$\omega_1 = \frac{g_{m2}}{2C_{m2}}$$

$$\omega_2 = \frac{2g_{m3}}{C_L}$$

$$A_{dc} = g_{m1}g_{m2}g_{m3}R_1R_2R_3$$

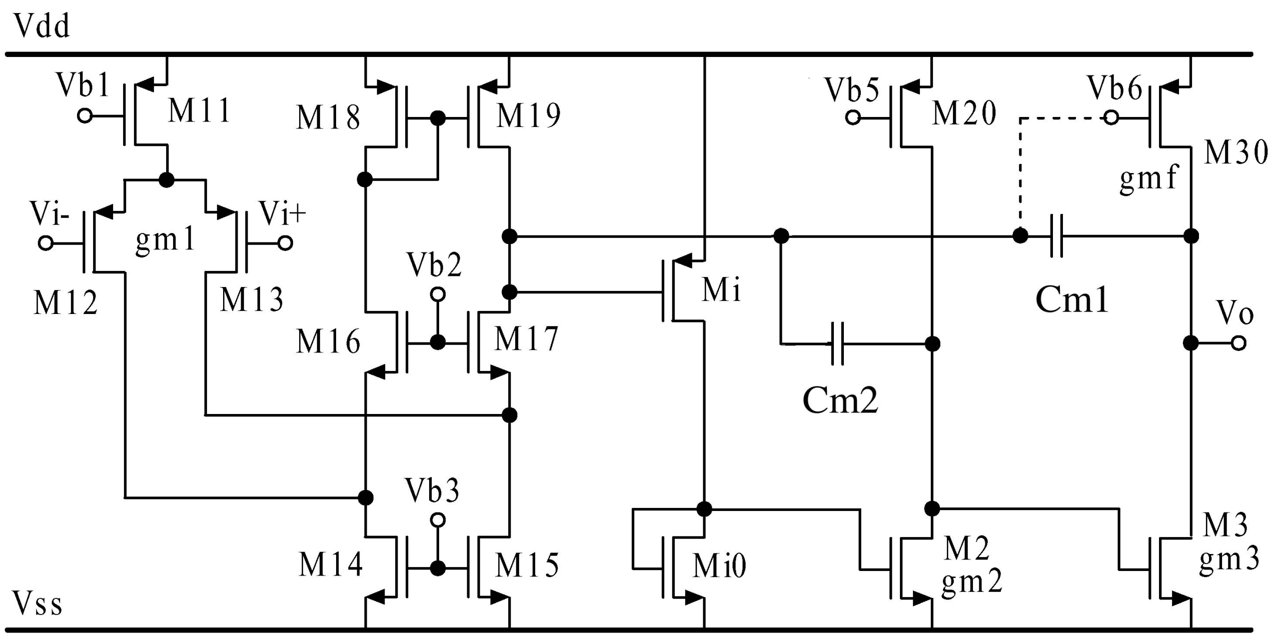
$$\omega_3 = -\frac{2g_{m3}}{C_{m1}}$$

$$\omega_d = -\frac{1}{C_{m1}g_{m2}g_{m3}R_1R_2R_3}$$

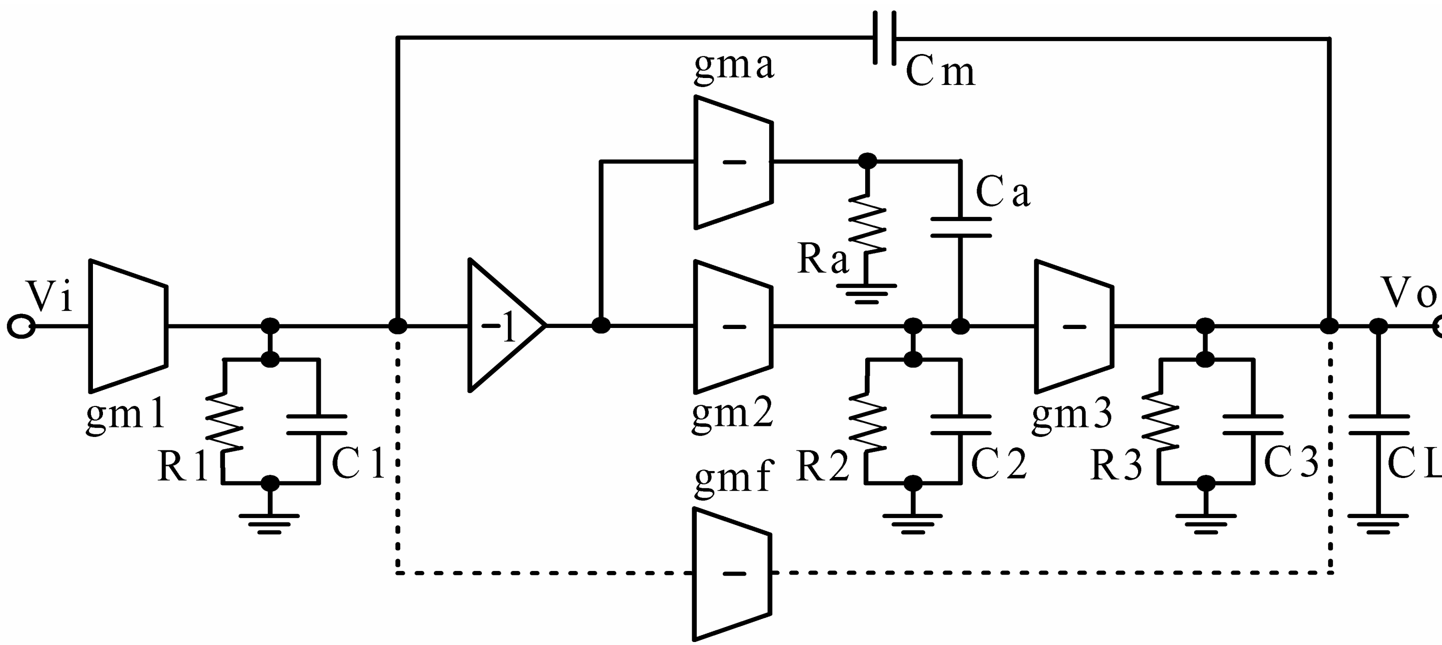
$$\omega_{UG} = \frac{g_{m1}}{C_{m1}}$$

Stability : $\frac{2g_{m3}}{C_L} > \frac{g_{m2}}{C_{m1}}$

PFC schematic



AC Boosting Compensation - ACBC



$$A_{2h} = (g_{m2} + g_{ma})\, R_a$$

Peng, JSSC Nov.04, 2074-2079

ACBC equations

$$A_v(s) = \frac{A_{dc}\left(1 + \frac{s}{\omega_1} + \frac{s^2}{\omega_1\omega_4} + \frac{s^3}{\omega_1\omega_3\omega_4}\right)}{\left(1 + \frac{s}{\omega_d}\right)\left(1 + \frac{s}{\omega_1} + \frac{s^2}{\omega_1\omega_2} + \frac{s^3}{\omega_1\omega_2\omega_3}\right)}$$

$$\omega_1 = \frac{1}{A_{2h}}\frac{g_{m2}}{C_a}$$

$$\omega_2 = A_{2h}\frac{g_{m3}}{C_L}$$

$$A_{dc} = g_{m1}g_{m2}g_{m3}R_1R_2R_3$$

$$\omega_3 = \frac{1}{R_aC_2}$$

$$\omega_d = -\frac{1}{C_{m1}g_{m2}g_{m3}R_1R_2R_3}$$

$$\omega_4 = -A_{2h}\frac{g_{m3}}{C_m}$$

$$\omega_{UG} = \frac{g_{m1}}{C_{m1}}$$

$$A_{2h} = (g_{m2} + g_{ma})R_a$$

ACBC stability

$$A_v(s) = \frac{A_{dc}\left(1 + \frac{s}{\omega_1} + \frac{s^2}{\omega_1\omega_4} + \frac{s^3}{\omega_1\omega_4\omega_3}\right)}{\left(1 + \frac{s}{\omega_d}\right)\left(1 + \frac{s}{\omega_1}\right)\left(1 + \frac{s}{\omega_2}\right)\left(1 + \frac{s}{\omega_3}\right)}$$

$$\omega_1 = \frac{1}{A_{2h}} \frac{g_{m2}}{C_a}$$

$$\omega_2 = A_{2h} \frac{g_{m3}}{C_L}$$

$$\omega_3 = \frac{1}{R_a C_2}$$

$$\omega_4 = - A_{2h} \frac{g_{m3}}{C_m}$$

Stability : $\omega_3 > \omega_2 > \omega_1$

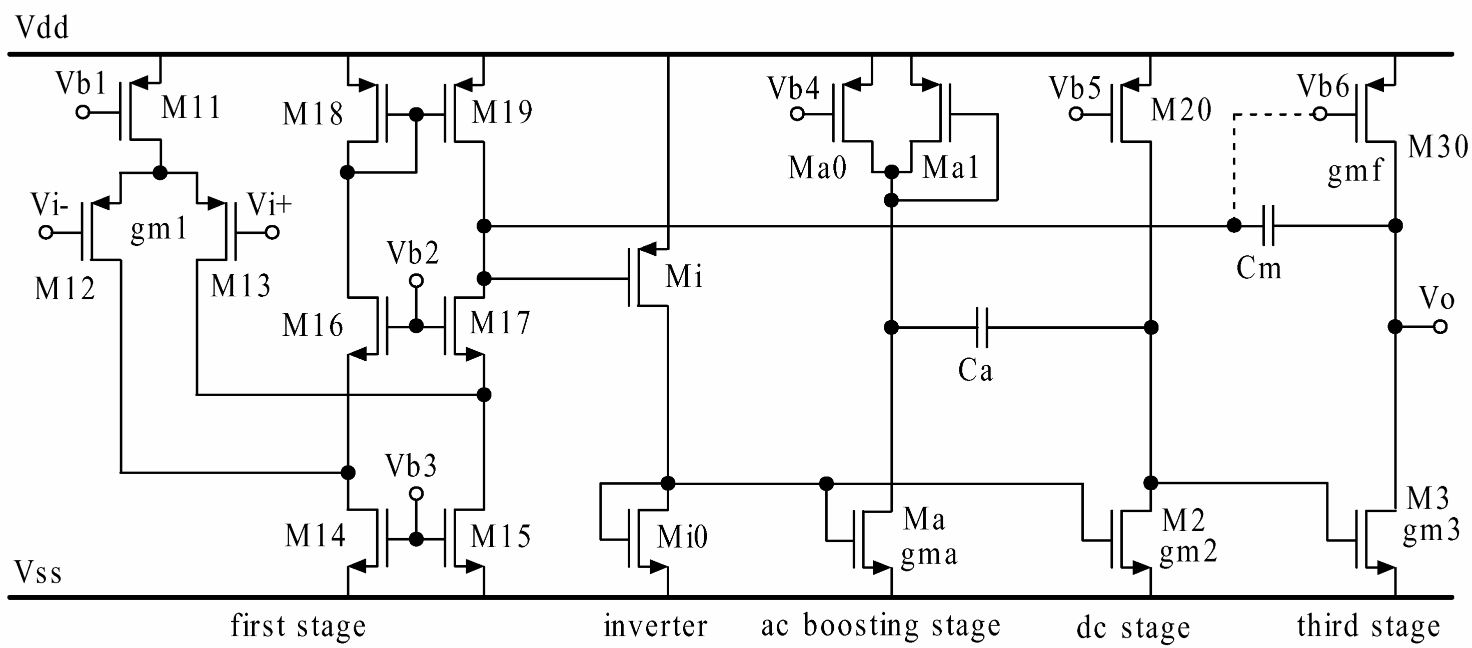
Pole and zero at ω_1 cancel

Design : $\omega_2 \approx 2\,\omega_{UG}$ **for 60° PM**

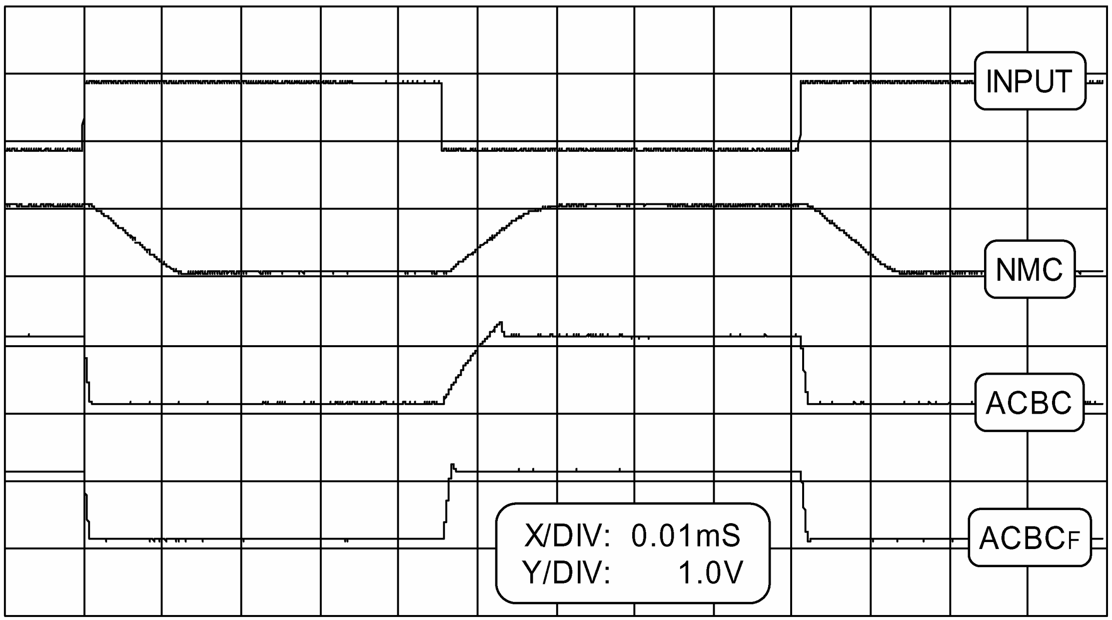
$GBW / GBW_{NMC} \approx 17$

$$A_{2h} = (g_{m2} + g_{ma})\, R_a$$

ACBC schematic



ACBC results



2 MHz

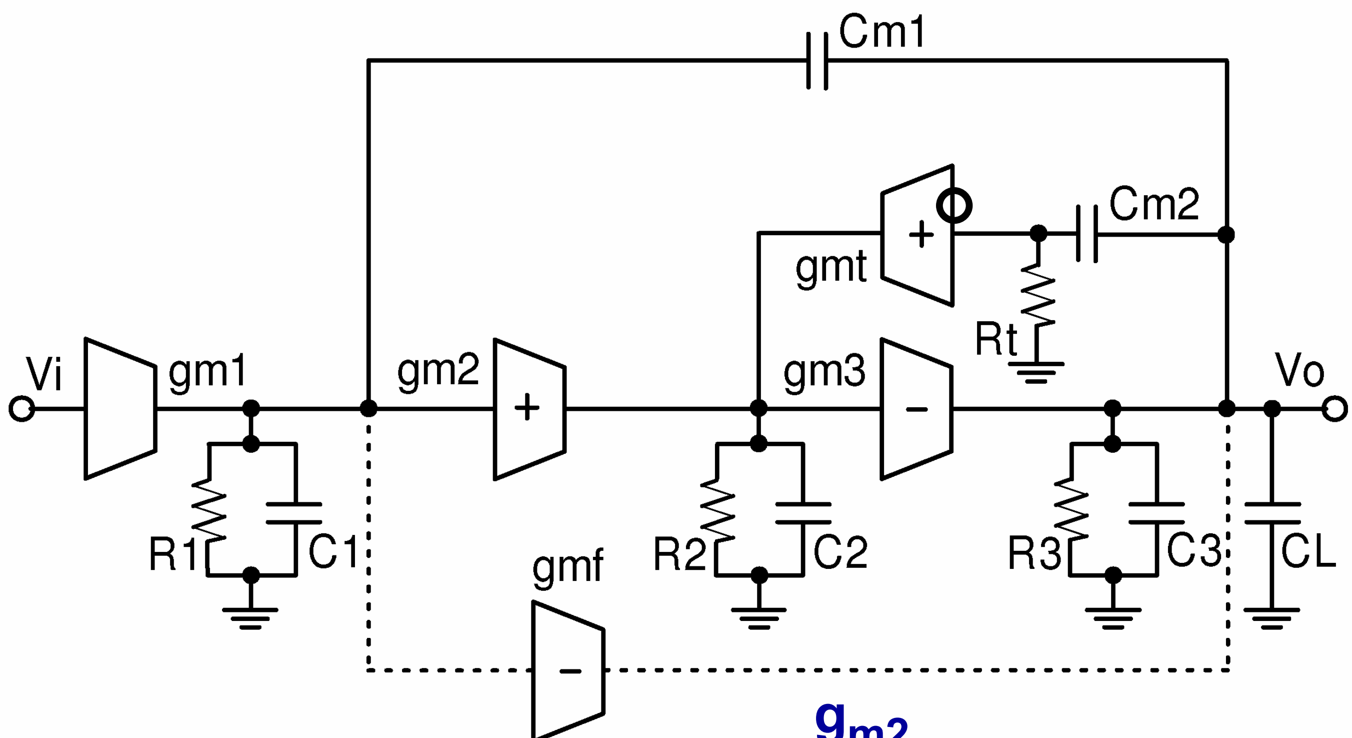
500 pF

0.16 mA

0.2 V/μs

1.2 V/μs$_F$

Transcond. with Cap. Feedback Comp.- TCFC



$$k_t = \frac{g_{m2}}{g_{mt}}$$

Peng, JSSC July 05,1514-1520

TCFC equations

$$A_v(s) = \frac{A_{dc}\left(1 + \frac{s}{\omega_2} + \frac{s^2}{\omega_2^2} + \frac{s^3}{\omega_2^2\omega_4}\right)}{\left(1 + \frac{s}{\omega_d}\right)\left(1 + \frac{s}{\omega_1} + \frac{s^2}{\omega_1\omega_2} + \frac{s^3}{\omega_1\omega_2\omega_3}\right)}$$

$$\omega_1 = \frac{1}{1+k_t}\frac{g_{m2}}{C_{m2}}$$

$$\omega_2 = \frac{1}{k_t}\frac{g_{m2}}{C_{m2}}$$

$$A_{dc} = g_{m1}g_{m2}g_{m3}R_1R_2R_3$$

$$\omega_d = -\frac{1}{C_{m1}g_{m2}g_{m3}R_1R_2R_3}$$

$$\omega_3 = (1+k_t)\frac{C_{m2}}{C_2}\frac{g_{m3}}{C_L}$$

$$\omega_{UG} = \frac{g_{m1}}{C_{m1}}$$

$$k_t = \frac{g_{m2}}{g_{mt}}$$

$$\omega_4 = -k_t\frac{C_{m2}}{C_2}\frac{g_{m3}}{g_{m1}}\omega_{UG}$$

TCFC stability

Stability $(k_t = 2)$:

$$\frac{C_{m2}}{C_2}\frac{g_{m3}}{C_L} > \omega_{UG} \quad \text{since } C_{m2} > C_2$$

$$\omega_1 = \frac{1}{1+k_t}\frac{g_{m2}}{C_{m2}}$$

Design :

$$\omega_2 = \frac{1}{k_t}\frac{g_{m2}}{C_{m2}}$$

$\omega_3 > \omega_1$ since $C_{m2} > C_2$; then $p_{nd} = -\omega_1$

set $\omega_1 \approx 2\,\omega_{UG}$ for 60° PM

$$\omega_3 = (1+k_t)\frac{C_{m2}}{C_2}\frac{g_{m3}}{C_L}$$

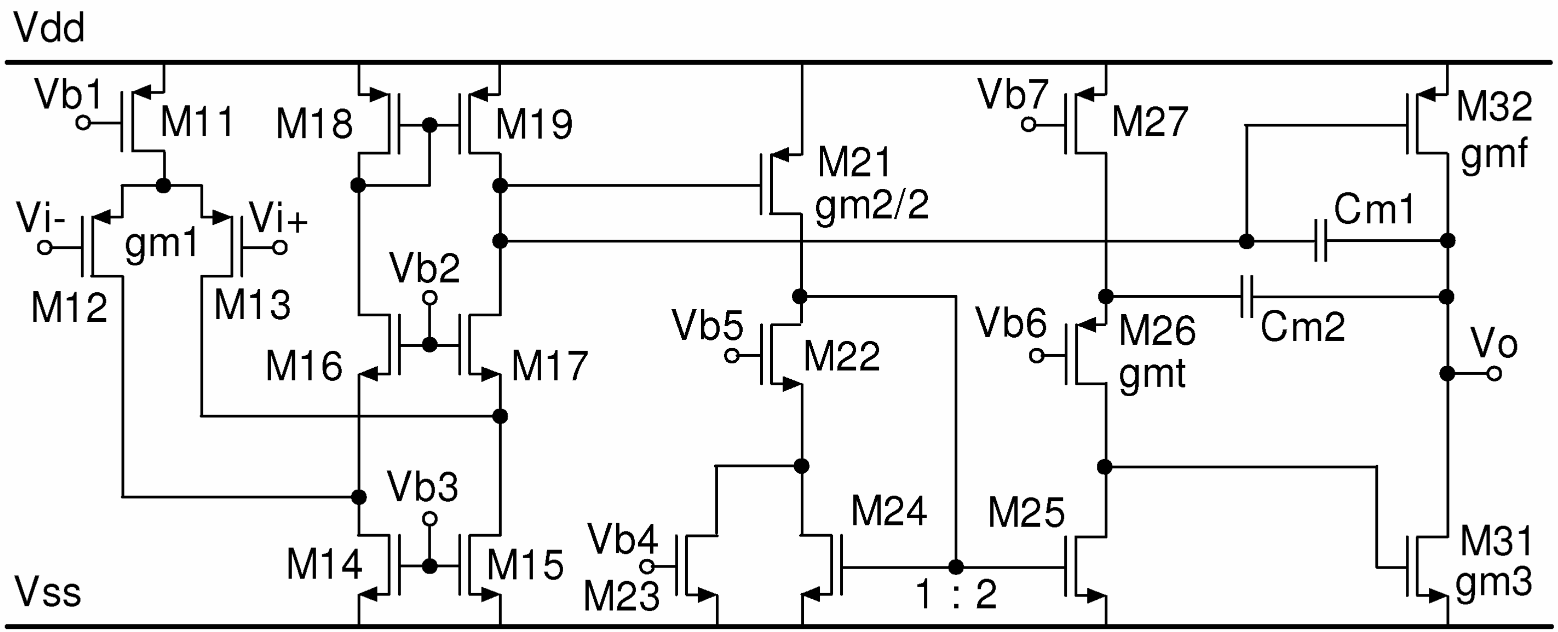
$\omega_4 > \omega_{UG}$; then $z_{nd} = -\omega_2$

is $(1+k_t)/k_t$ larger than ω_1

$$\omega_4 = -k_t\frac{C_{m2}}{C_2}\frac{g_{m3}}{g_{m1}}\omega_{UG}$$

$GBW / GBW_{NMC} \approx 41$

TCFC schematic



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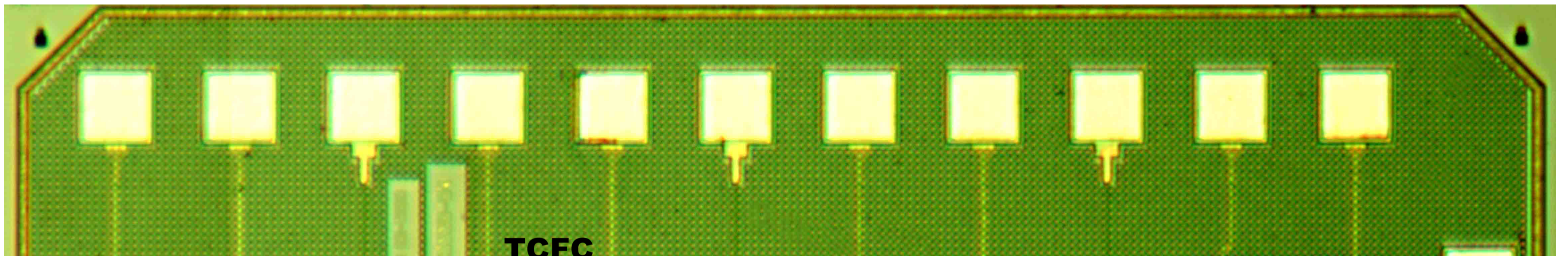
TCFC and other 3-stage opamps



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Comparison

No Capacitance

Nested Miller comp.

NMC with nulling R

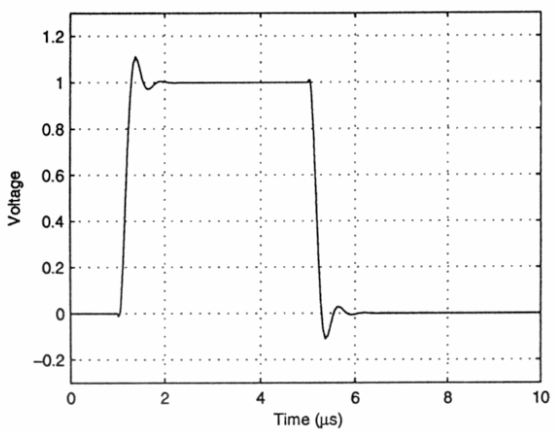
Miller C Substitutions

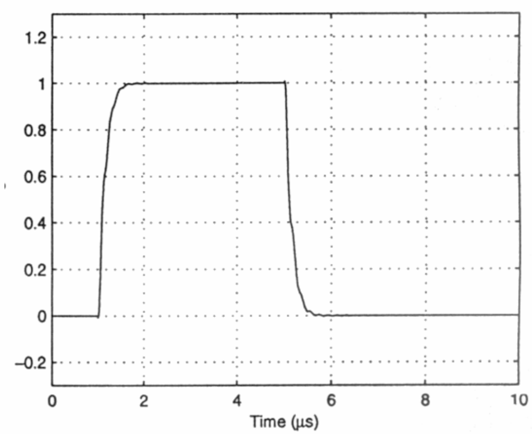
AC Boosting comp.

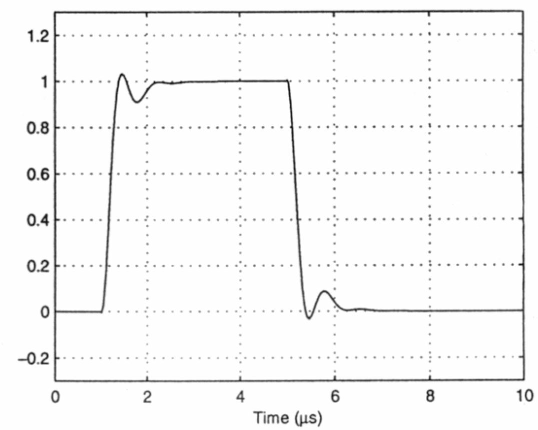
Transconductance Capacitance FC

		IFOMs $\frac{MHz \cdot pF}{mA}$	IFOML $\frac{V/\mu S \cdot pF}{mA}$	FOMs $\frac{MHz \cdot pF}{mW}$	FOML $\frac{V/\mu S \cdot pF}{mW}$	Tech.
NCs						
NCFF	[Tha03]	536	-	214	-	$\frac{0.5\mu m}{CMOS}$
NMs						
NMC	[Esc92]	632	211	79	26	$\frac{3GHz\, f_t}{BJT}$
MNMC	[Esc92]	1053	368	132	46	$\frac{3GHz\, f_t}{BJT}$
NMCF	[Leu01a]	600	246	300	123	$\frac{0.8\mu m}{CMOS}$
NGCC	[You97]	36	148	18	74	$\frac{2\mu m}{CMOS}$
HNMC	[Esc94]	134	134	89	89	$\frac{0.8\mu m}{CMOS}$
MHNMC	[Esc94]	401	467	267	311	$\frac{0.8\mu m}{CMOS}$
DNMC	[Per93]	250	188	50	38	$\frac{1.5\mu m}{CMOS}$
MRs						
NMCNR	[Leu01a]	410	168	205	84	$\frac{0.8\mu m}{CMOS}$
IRNMC	[Ho03]	626	444	209	148	$\frac{0.6\mu m}{CMOS}$
EFC	[Ng99]	817	1200	272	400	$\frac{0.6\mu m}{CMOS}$
NGRNMC	[Peng03]	700	490	280	196	$\frac{0.35\mu m}{CMOS}$
MSs						
PFC	[Ram03b]	1915	709	1276	473	$\frac{0.35\mu m}{CMOS}$
DFCFC	[Leu01a]	1238	628	619	314	$\frac{0.8\mu m}{CMOS}$
ACBs						
ACBC	[Peng04]	5981	2215	2991	1108	$\frac{0.35\mu m}{CMOS}$
$ACBC_F$	[Peng04]	5864	3086	2932	1543	$\frac{0.35\mu m}{CMOS}$
TCFs						
AFC	[Ahu83]	326	-	33	-	$\frac{4\mu m}{CMOS}$
AFFC	[Lee03b]	2700	894	1350	447	$\frac{0.8\mu m}{CMOS}$
DLPC	[Lee03a]	3818	1800	2545	1200	$\frac{0.6\mu m}{CMOS}$
TCFC	[Peng05]	14250	5175	9500	3450	$\frac{0.35\mu m}{CMOS}$

Transient responses in unity gain







NMC **TCFC** **PFC**

All GBW ≈ 1 MHz and C_L = 100 pF

Same C_{m1} = 18 pF ; C_{m2} = 3 pF; minimum currents.

References

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J. Ramos and M. Steyaert, "Three stage amplifier with positive feedback compensation scheme," in *Proc. IEEE Custom Integrated Circuits Conf.*, Orlando, FL, May 2002, pp. 333-336.
X. Peng and W. Sansen, "AC boosting compensation schema for low-power multistage amplifiers", *IEEE J. Solid-State Circuits*, vol. 39, pp. 2074-2079, Nov. 2004.
X. Peng and W. Sansen, "Transconductance with capacitances feedback compensation for multi-stage amplifiers", *IEEE J. Solid-State Circuits*, vol. 40, pp.1514-1520, July 2005.

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- Nested-Miller designs
- Low-power designs
- Comparison

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Current-input Operational Amplifiers

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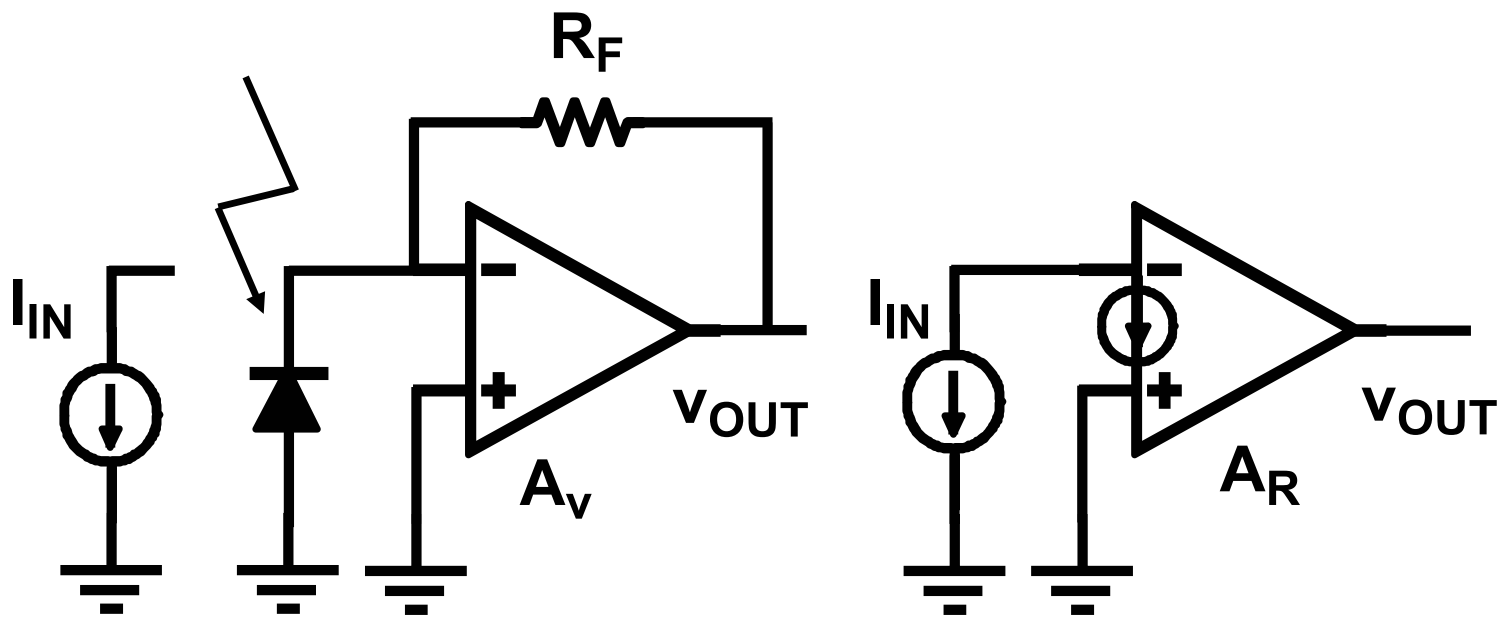
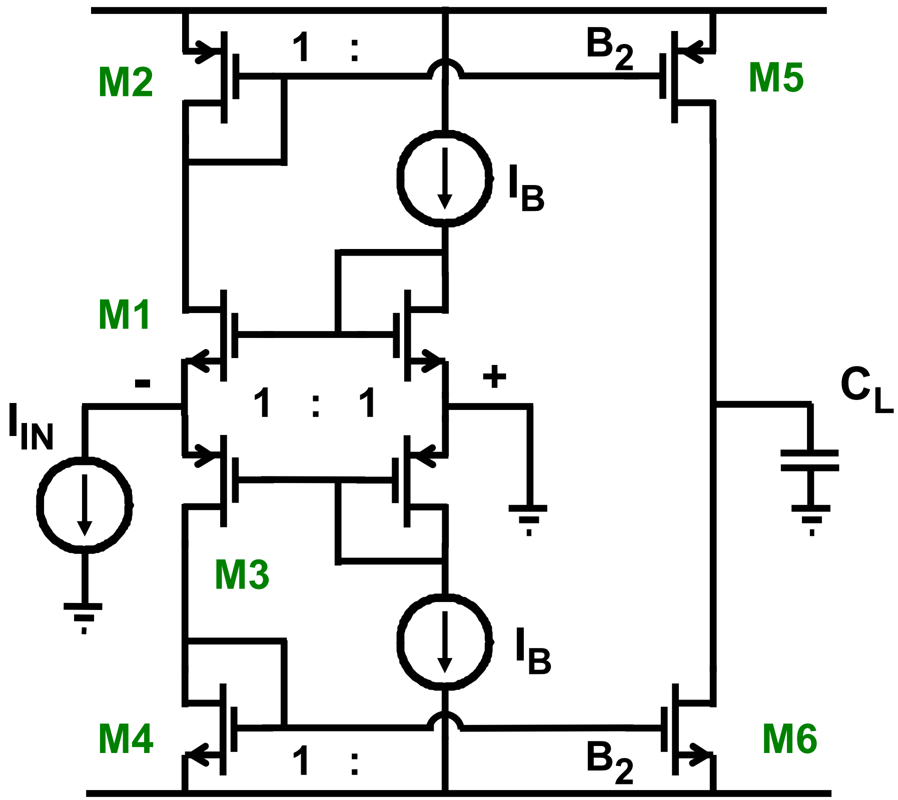
- **Operational current amplifier**
- **Configurations current amplifiers**

Current or voltage amplifier



$v_{OUT} = R_F I_{IN}$ $v_{OUT} = A_R I_{IN}$

Operational current amplifier Gain & Speed



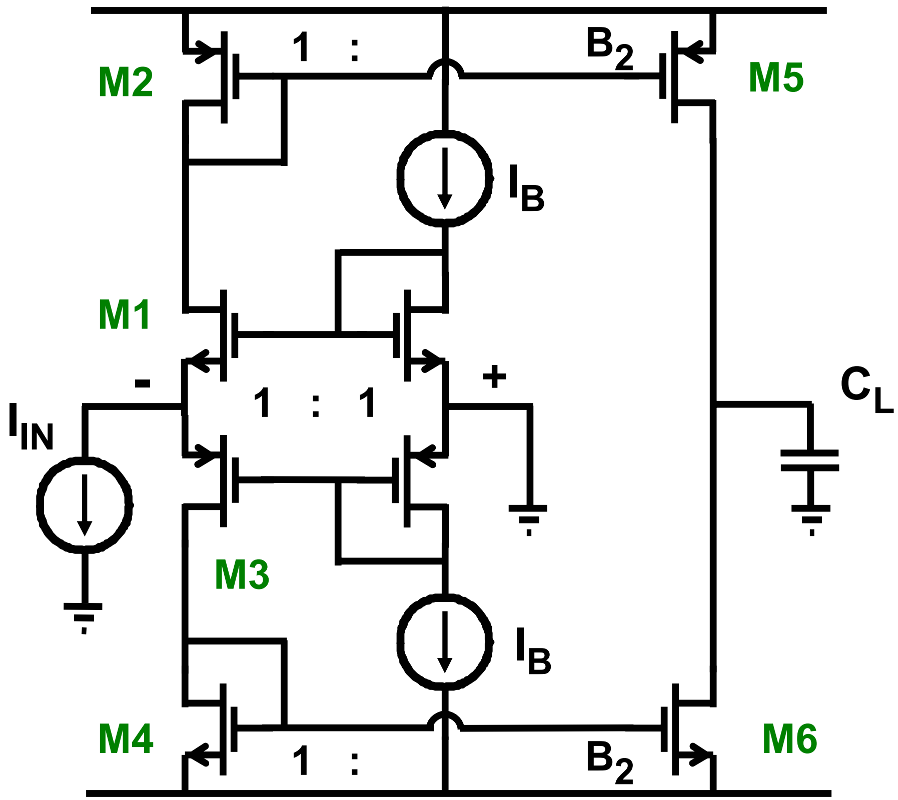
$$A_R = B_2 R_{OUT} = B_2\, r_{o5} // r_{o6}$$

$$BW = \frac{1}{2\pi\, C_L R_{OUT}} \sim I_{DS} \uparrow$$

$$A_R\, BW = \frac{B_2}{2\pi\, C_L}$$

$$SR = B_2 \frac{I_{IN}}{C_L} \qquad I_{DS} \uparrow$$

Operational current amplifier : noise



$$SR = B_2 \frac{I_{IN}}{C_L} \qquad I_{DS} \uparrow$$

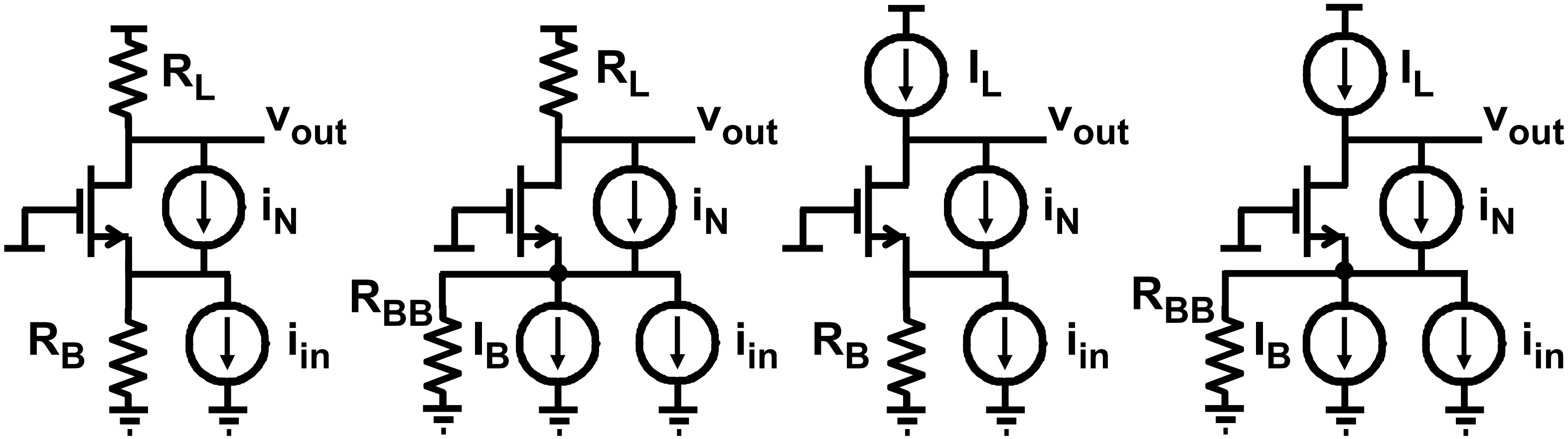
$$\overline{di_{DS}^2} = 4kT \frac{2}{3} g_m \, df$$

$$\overline{di_{IN}^2} \approx 2\, \overline{di_{DS}^2}$$

$$\frac{S}{N} = \frac{I_{IN}}{\sqrt{\overline{di_{IN}^2}\, BW \frac{\pi}{2}}}$$

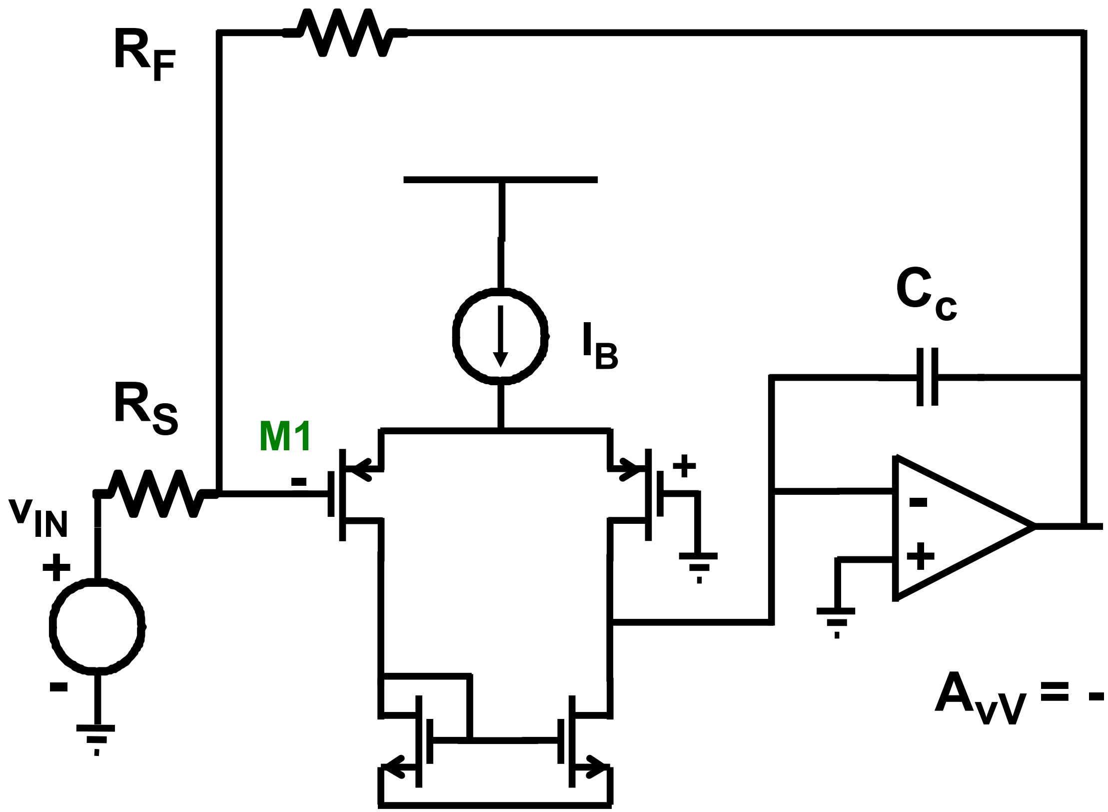
$$\frac{S}{N} \sim \frac{I_{IN}}{\sqrt{I_{DS}}} \qquad I_{DS} \downarrow$$

Gain and noise in MOST cascodes



	$r_{DS} > R_L$	$R_{BB} > R_L$	$g_m r_{DS} > 1$	
$\frac{v_{out}}{i_{in}}$	$g_m R_L \frac{R_B}{1+g_m R_B}$	R_L	$g_m r_{DS} R_B$	$g_m r_{DS} R_{BB}$
$\frac{v_{out}}{i_N}$	$\frac{R_L}{1+g_m R_B}$	$\frac{R_L}{g_m R_{BB}}$	R_B	r_{DS}
$\frac{i_{in}}{i_N}$	$\frac{1}{g_m R_B}$	$\frac{1}{g_m R_{BB}}$	$\frac{1}{g_m r_{DS}}$	$\frac{1}{g_m R_{BB}}$

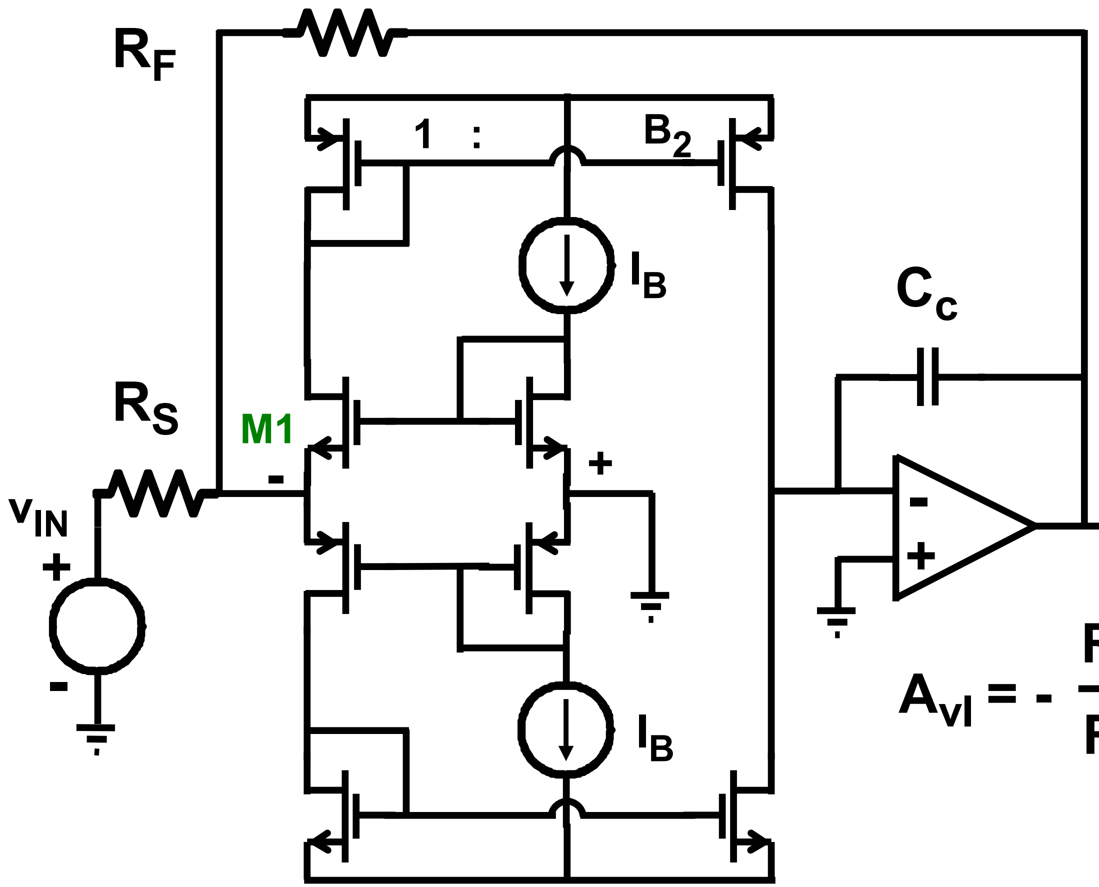
OTA in feedbackloop



$$GBW = \frac{g_{m1}}{2\pi\, C_c}$$

$$A_{vV} = -\frac{R_F}{R_S}\,\frac{1}{1 + \frac{R_F}{R_S}\frac{j\omega C_c}{g_{m1}}}$$

OCA in feedbackloop

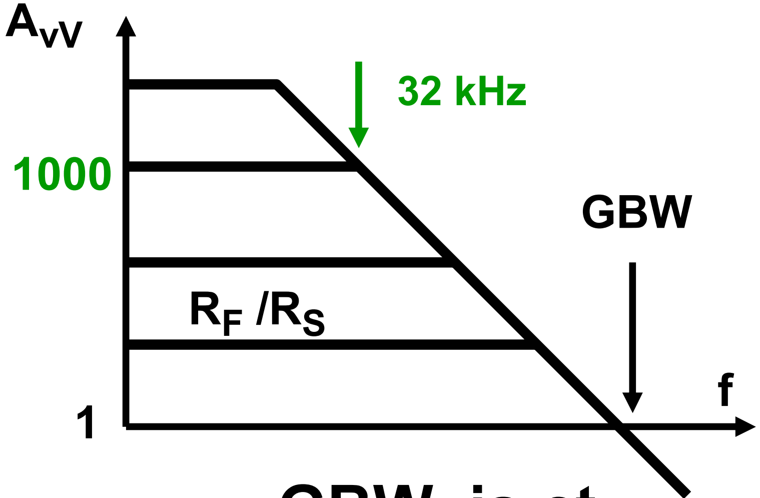


$$A_{vl} = - \frac{R_F}{R_S} \frac{1}{1 + \frac{R_F}{R_S} j\omega R_S C_c}$$

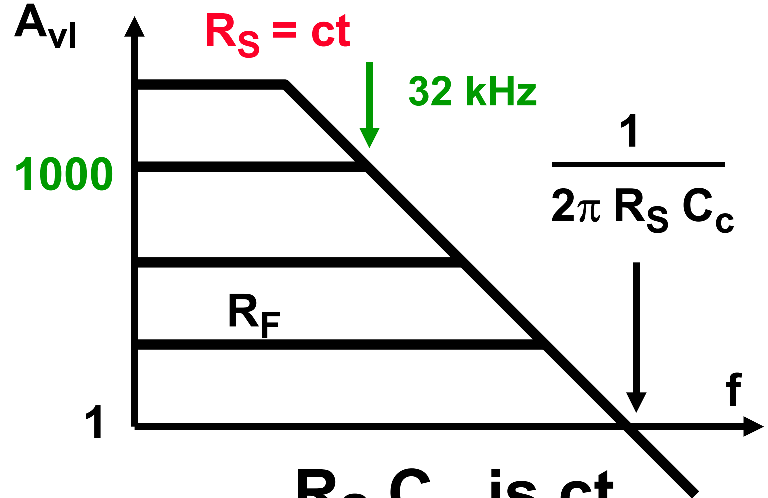
Comparison A_{vV} and A_{vI}

$$A_{vV} = -\frac{R_F}{R_S}\,\frac{1}{1+\dfrac{R_F}{R_S}\,\dfrac{j\omega C_c}{g_{m1}}}$$

$$A_{vI} = -\frac{R_F}{R_S}\,\frac{1}{1+\dfrac{R_F}{R_S}\,j\omega\, R_S\, C_c}$$



GBW is ct



$R_S\,C_c$ is ct

g_{m1} = 0.2 mS; C_c =1 pF; GBW = 32 MHz; A_{vV} = 1000; BW = 32 kHz

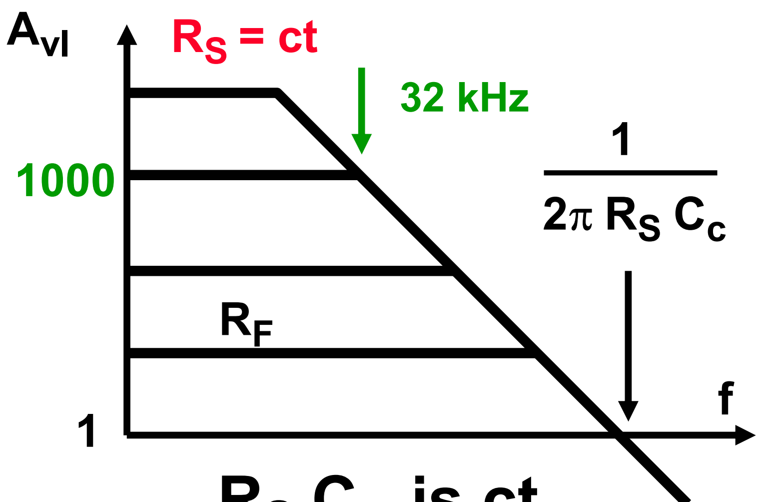
Advantage A_{vI}

$$A_{vI} = -\frac{R_F}{R_S}\,\frac{1}{1+\dfrac{R_F}{R_S}\,j\omega R_S C_c}$$

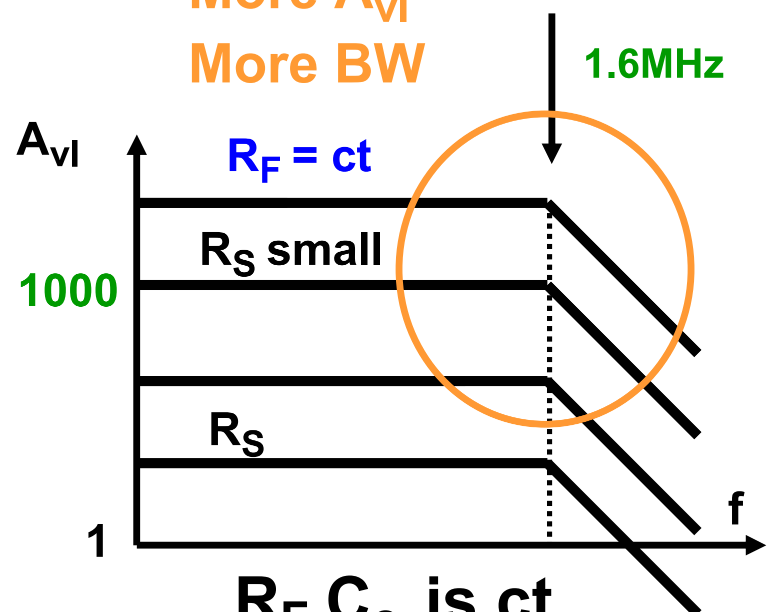
More A_{vI}

More BW

$$\frac{1}{2\pi R_f C_c}$$



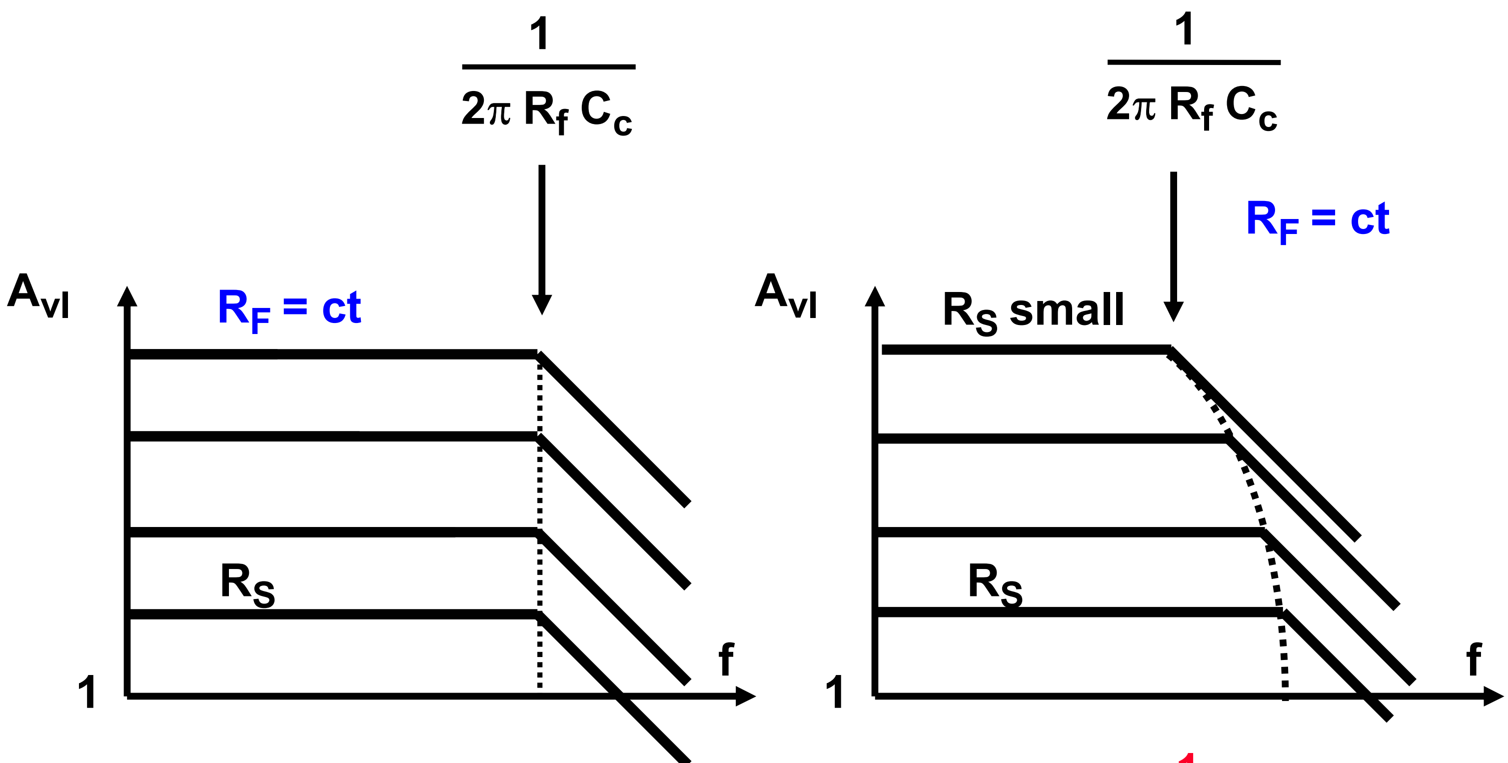
$R_S C_c$ is ct



$R_F C_c$ is ct

C_c =1 pF; R_S = 100 Ω; R_F = 100 k Ω; BW = 1.6MHz

Limitations advantage A_{vI}



R_S limited to $R_S + \dfrac{1}{2g_{m1}}$: **Bipolar**

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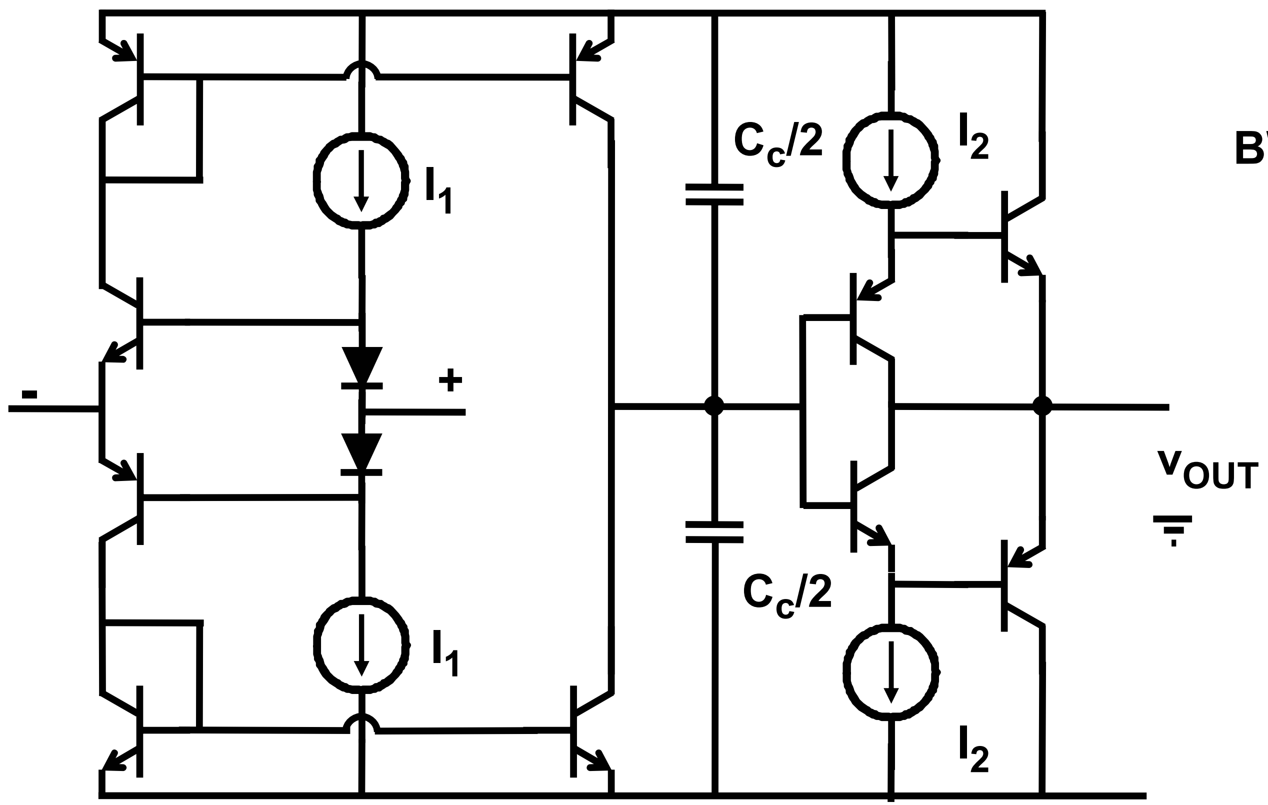
- **Operational current amplifiers**
- **Configurations Current amplifiers**

Single-stage OCA- 1 (AD846)

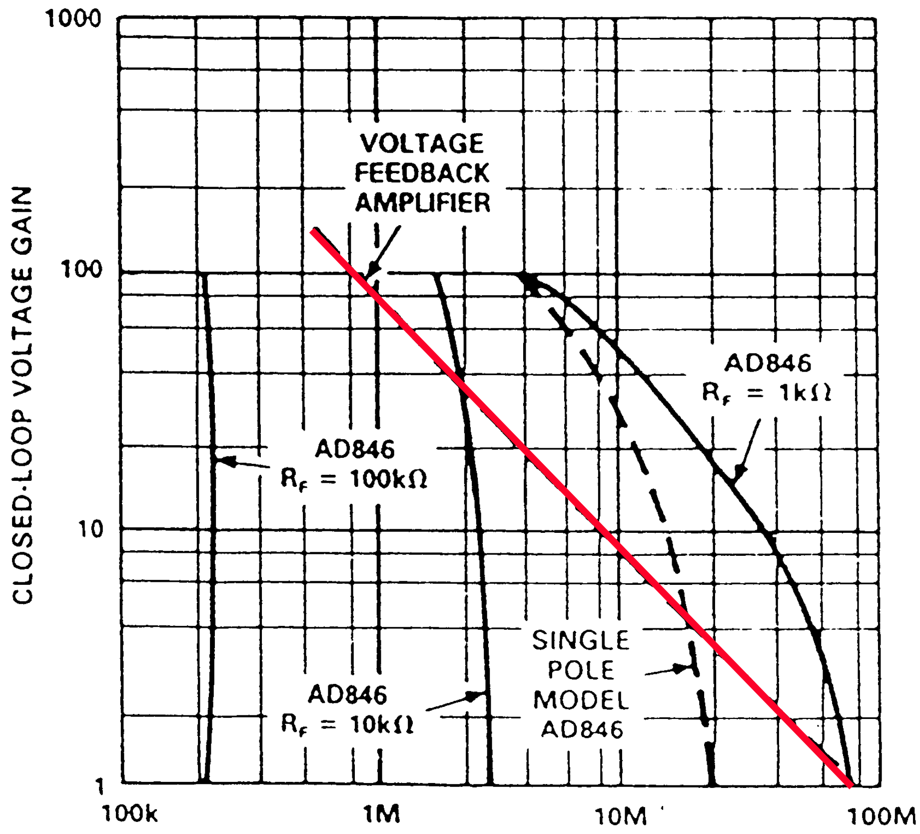


BW = 80 MHz

SR (G=1) = 450 V/μs

6.5 mA (5V)
0.45 μA

Single-stage OCA- 2 (AD846)



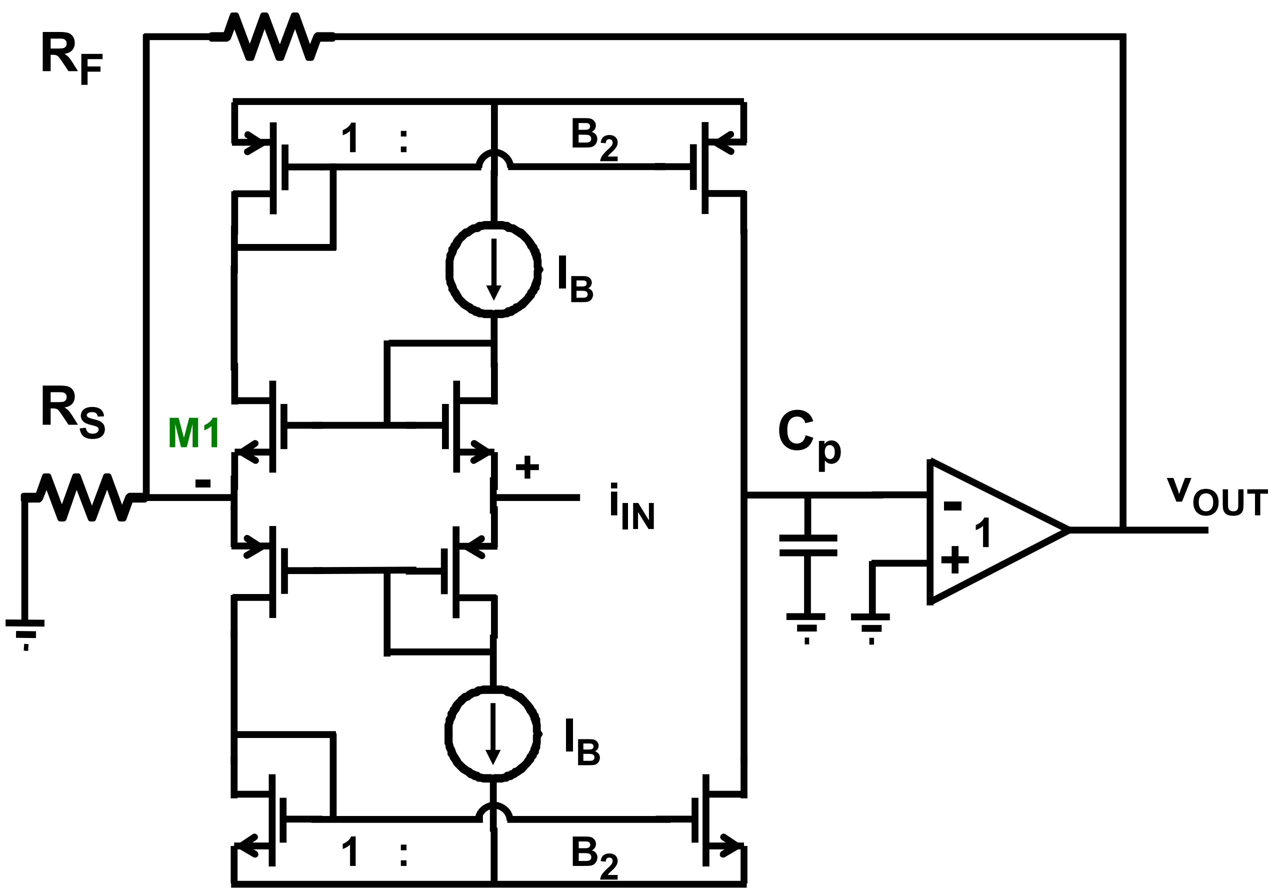
$R_F = 100\ k\Omega$
A = 10 : $R_S = 10\ k\Omega$
A = 100 : $R_S = 1\ k\Omega$

$R_F = 10\ k\Omega$
A = 10 : $R_S = 1\ k\Omega$
A = 100 : $R_S = 100\ \Omega$

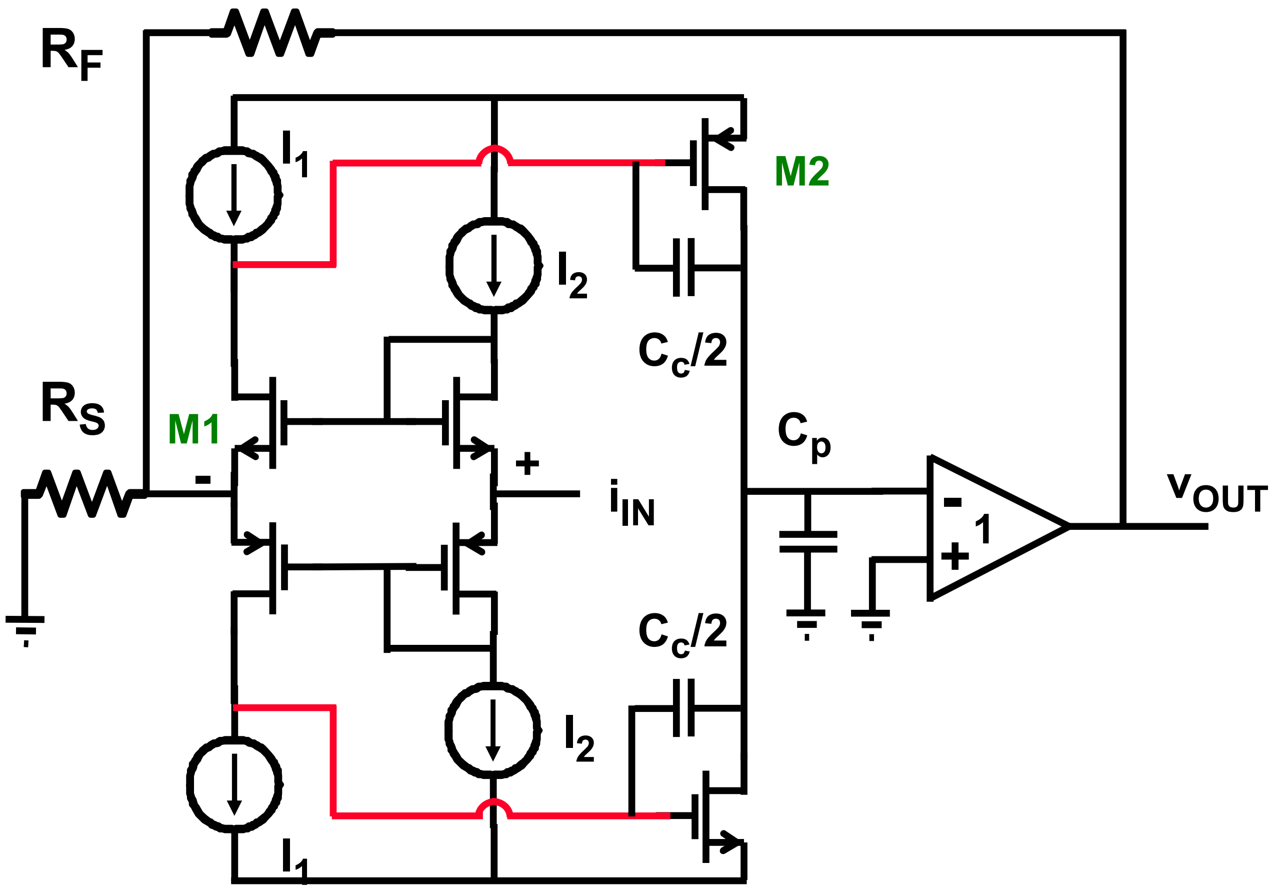
$R_F = 1\ k\Omega$
A = 10 : $R_S = 100\ \Omega$
A = 100 : $R_S = 10\ \Omega$

$$R_S + \frac{1}{g_{m1}} > 10\ \Omega$$

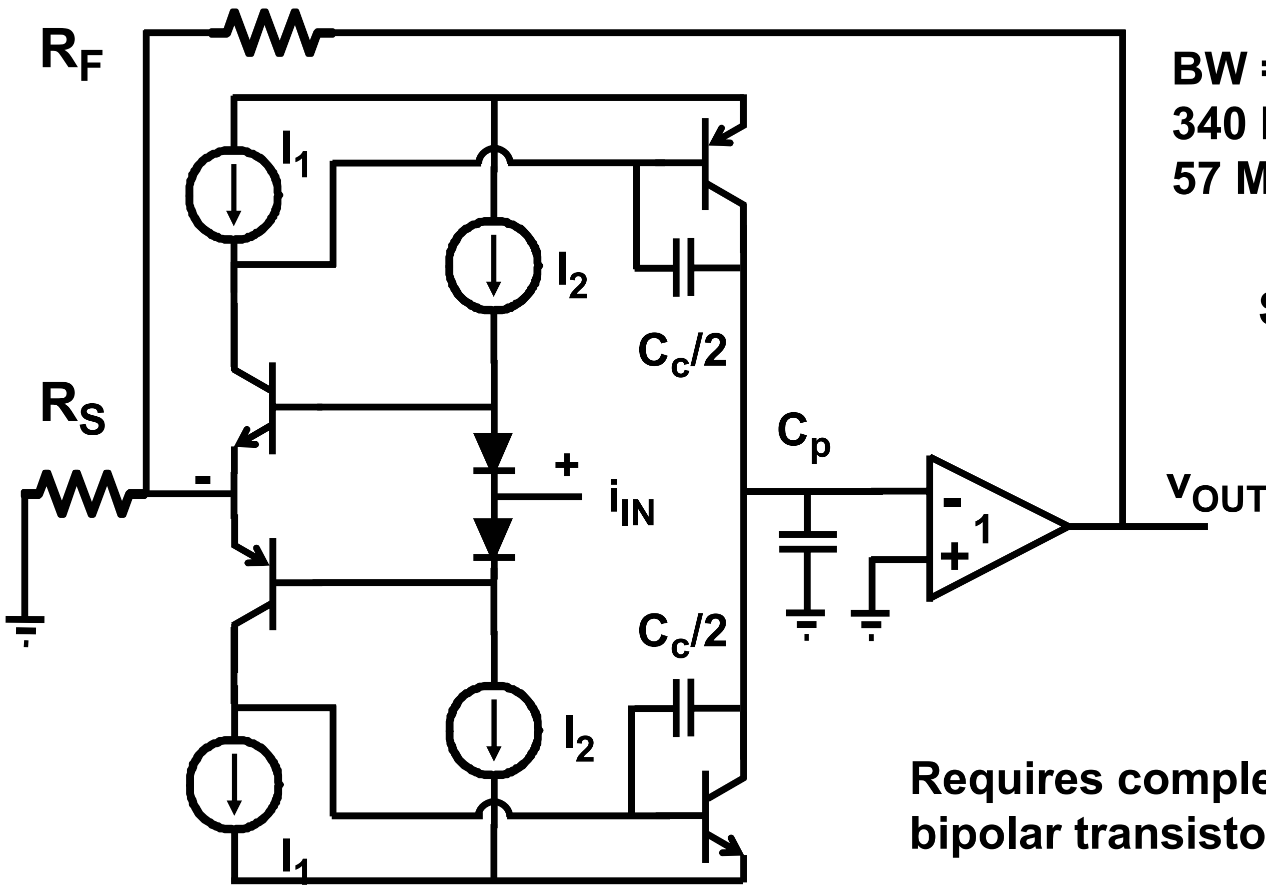
Two-stage OCA- 1 (AD8011 bipolar)



Two-stage OCA- 2 (AD8011 bipolar)



Two-stage OCA :AD8011



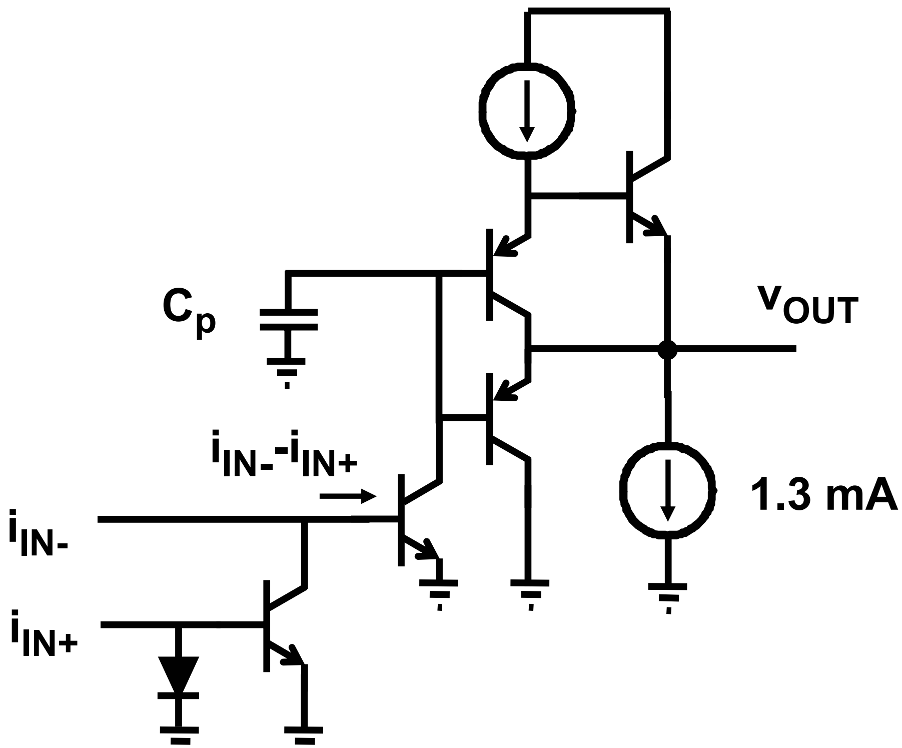
BW =
340 MHz (G=1)
57 MHz (G=10)

SR (G=1)
3500 V/μs

1 mA (5V)
15 μA

Requires complementary
bipolar transistor process

Two-stage OCA (LM3900)

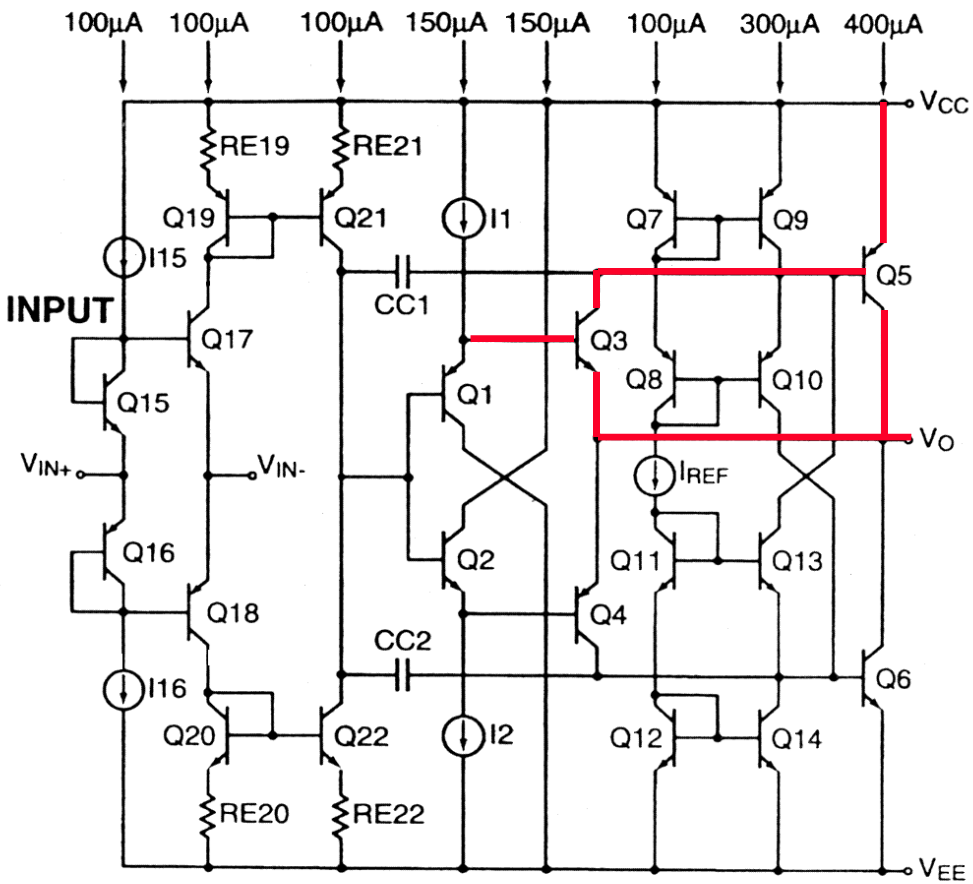


GBW = 2.5 MHz

SR (G=1) =
0.5 V/μs

1.4 mA (4-36V)
0.03 μA

Current feedback opamp



110 MHz/ 1.5 mA

230 V/μs

A_v = 2 in 100 Ω

f_T = 3.8 GHz

Ref.Bales, JSSC Sept. 97, 1470-1474

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- Other current amplifiers

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Rail-to-rail input and output amplifiers

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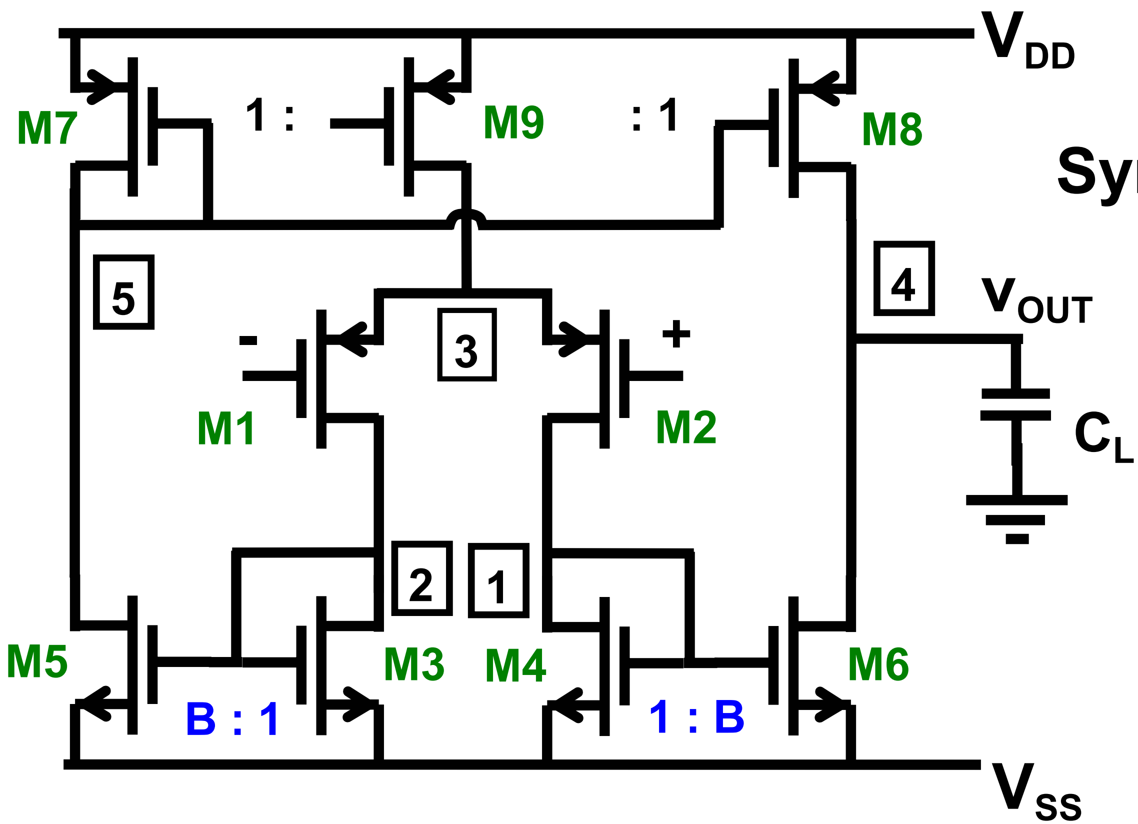
Why rail-to-rail amplifiers ?

- For low supply voltages : use full range for maximum dynamic range
- Fully differential signal processing
- Rail-to-rail output is always required
- But not necessarily rail-to-rail-input !

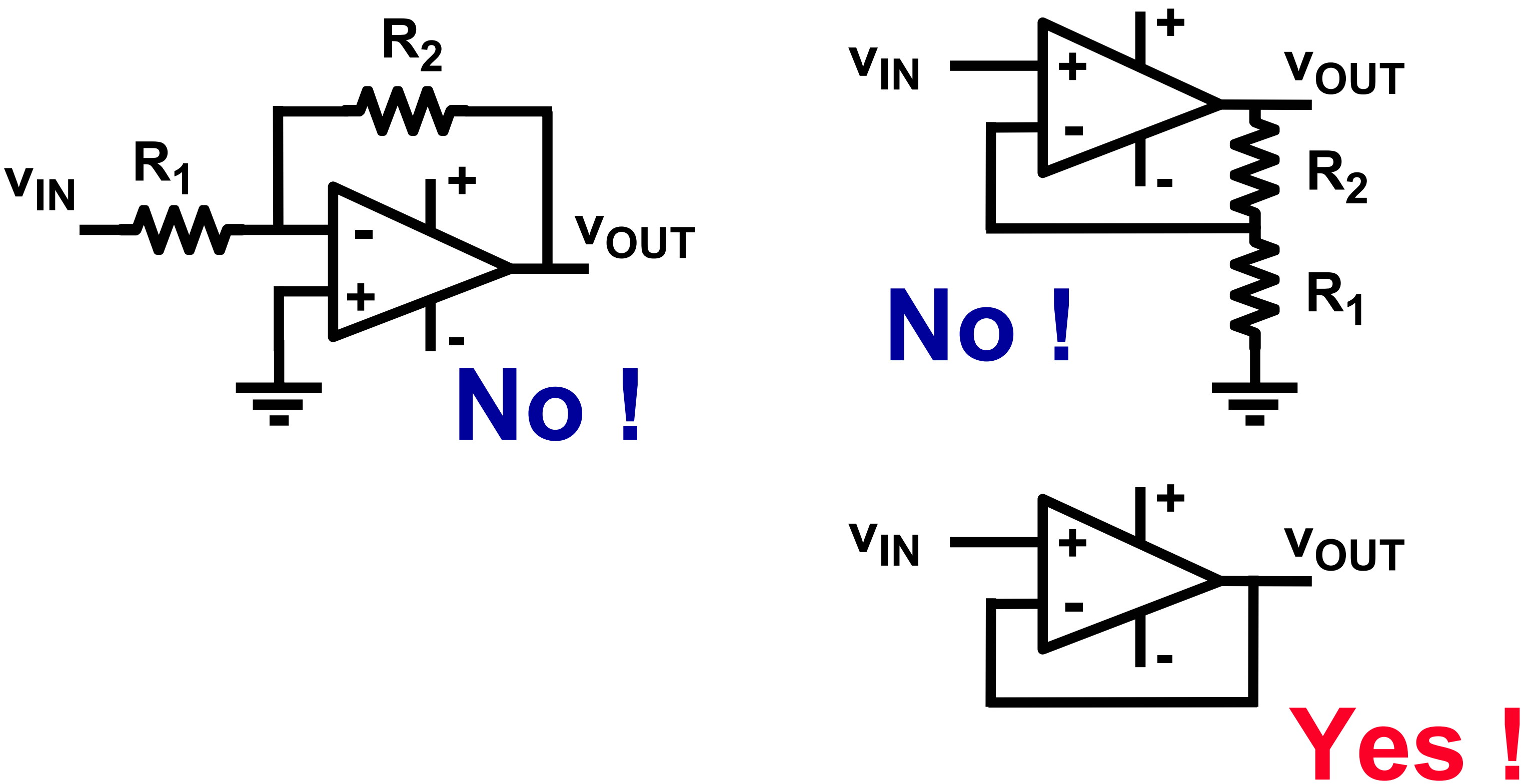
Symmetrical CMOS OTA



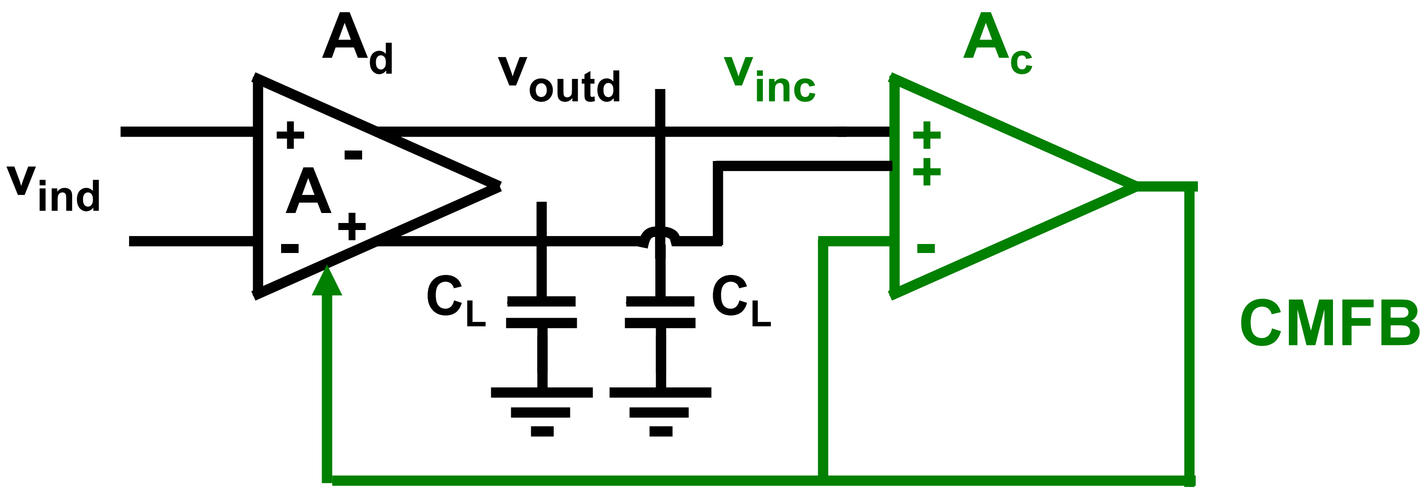
Symmetrical OTA

Rail-to-rail output swing !

When rail-to-rail input ?



Rail-to-rail input for CMFB



For a rail-to-rail output swing in fully-differential amplifiers
A CMFB amplifier is required
With rail-to-rail input capability !

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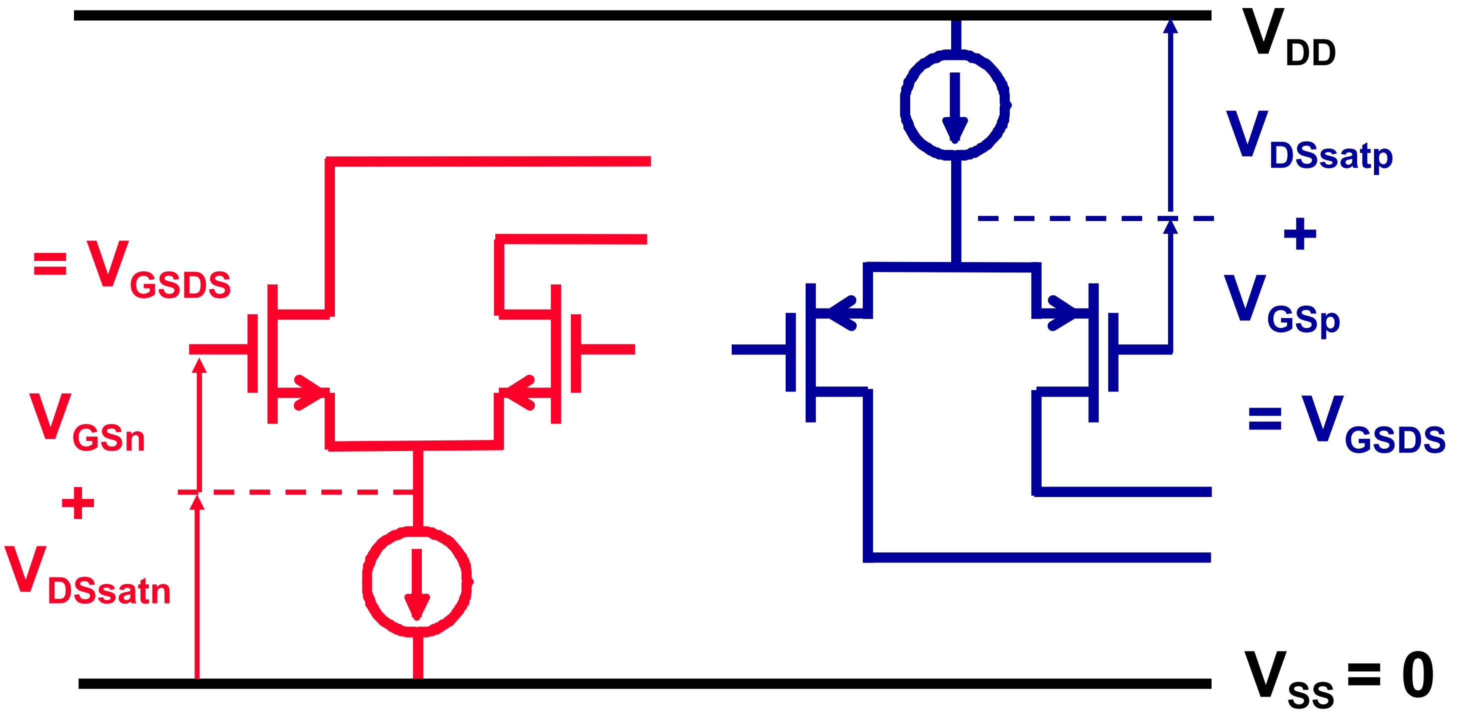
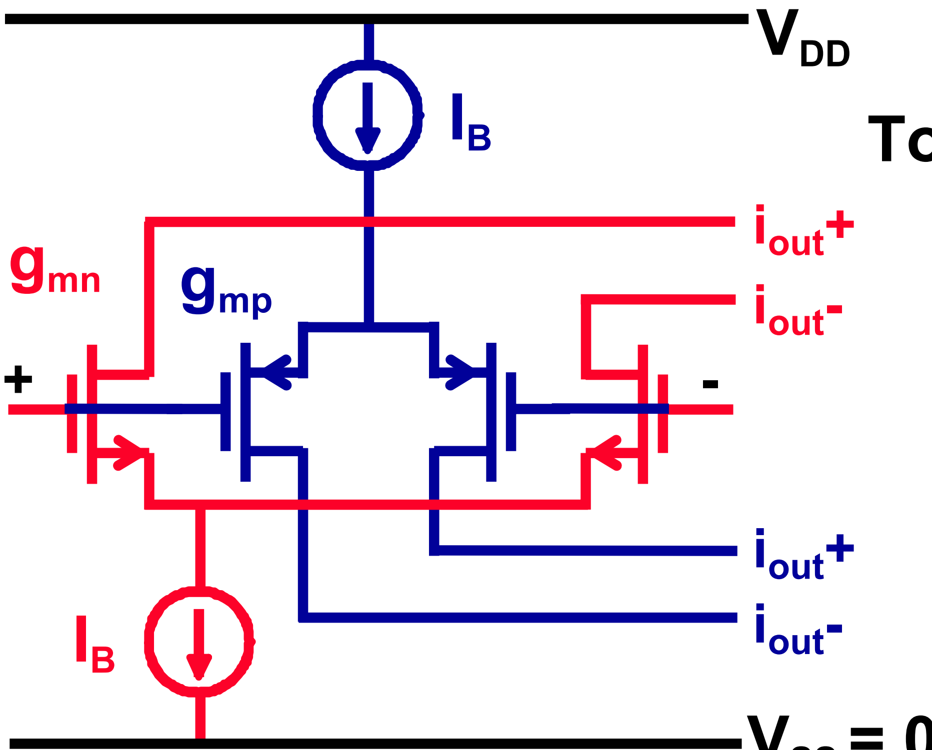
Problem ?

$V_{GS} \approx 0.9\ V$ & $V_{DSsat} \approx 0.2\ V$ >>> $V_{GSDS} = 1.1\ V$



$V_{INCM} > 1.1\ V$ $V_{INCM} < V_{DD} - 1.1\ V$

Problem : limited input CM range



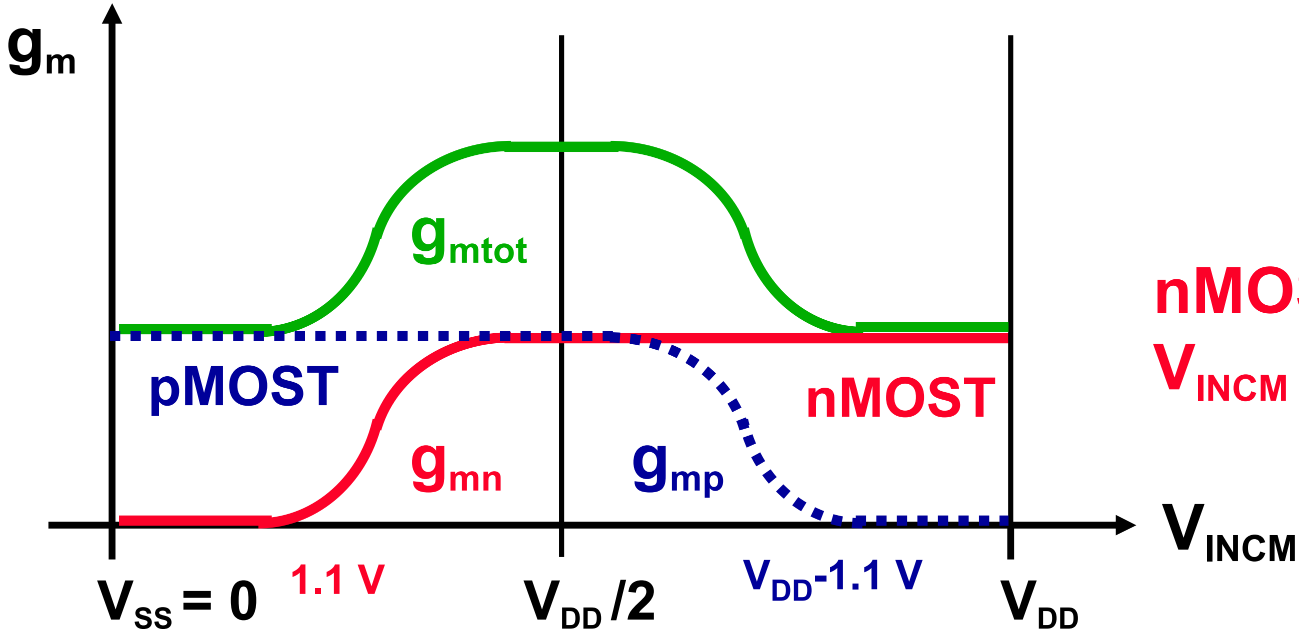
To be operational :

$V_{INCM} > 1.1\ V$

$V_{INCM} < V_{DD} - 1.1\ V$

$V_{DDmin} = 2.2\ V$

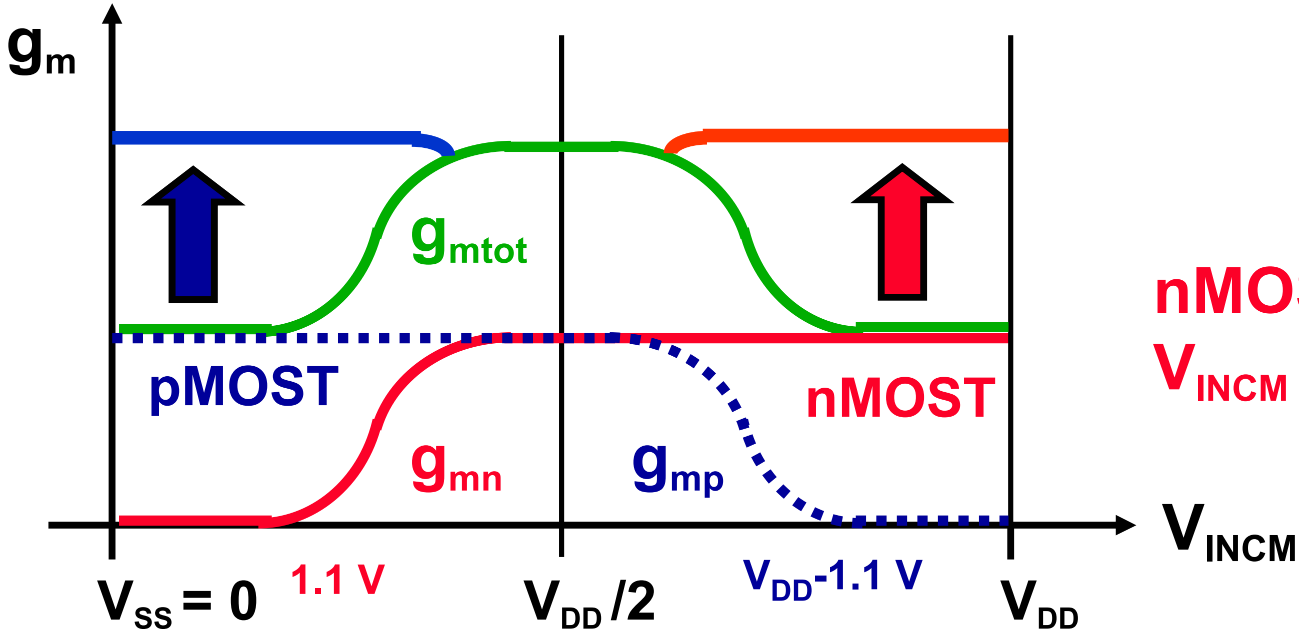
Problem: unequal g_{mtot}



nMOST on : $V_{INCM} > 1.1$ V

pMOST on : $V_{INCM} < V_{DD} - 1.1$ V

Solution : g_m equalization



nMOST on : $V_{INCM} > 1.1$ V

pMOST on : $V_{INCM} < V_{DD} - 1.1$ V

Increase g_m:
$g_{mn} + g_{mp} = ct$

Equalize g_{mtot} in strong inversion

$$g_{mn} + g_{mp} = ct1$$

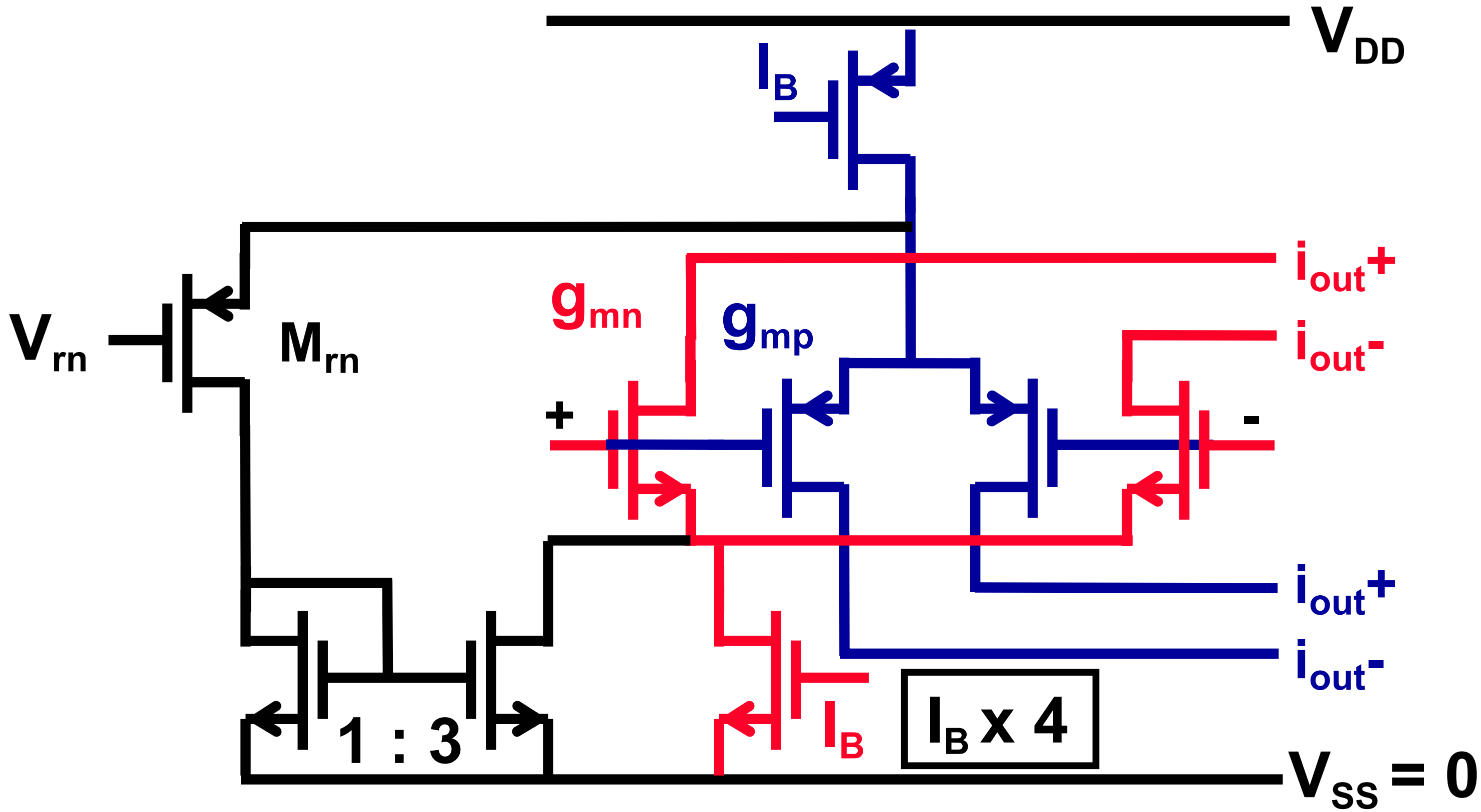
$$\sqrt{2\,K'_n \frac{W_n}{L_n}\,I_{Bn}} + \sqrt{2\,K'_p \frac{W_p}{L_p}\,I_{Bp}} = ct1$$

$$\sqrt{K'_n\,I_{Bn}} + \sqrt{K'_p\,I_{Bp}} = ct2$$

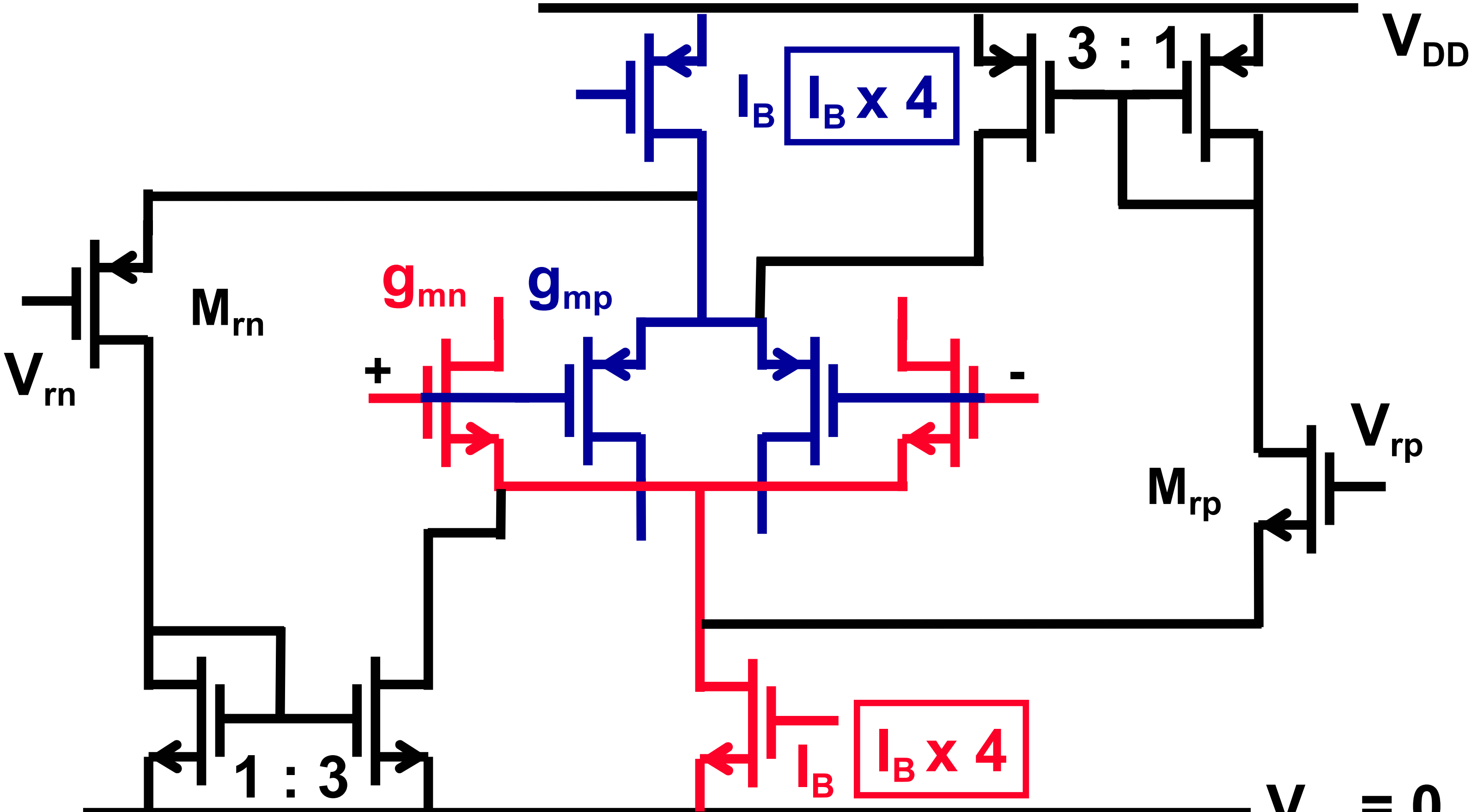
$$\sqrt{I_{Bn}} + \sqrt{I_{Bp}} = ct3$$

3 x Current mirror : $\sqrt{1} + \sqrt{1} = \sqrt{0} + \sqrt{4} \quad > \; 4 - 1 = 3$

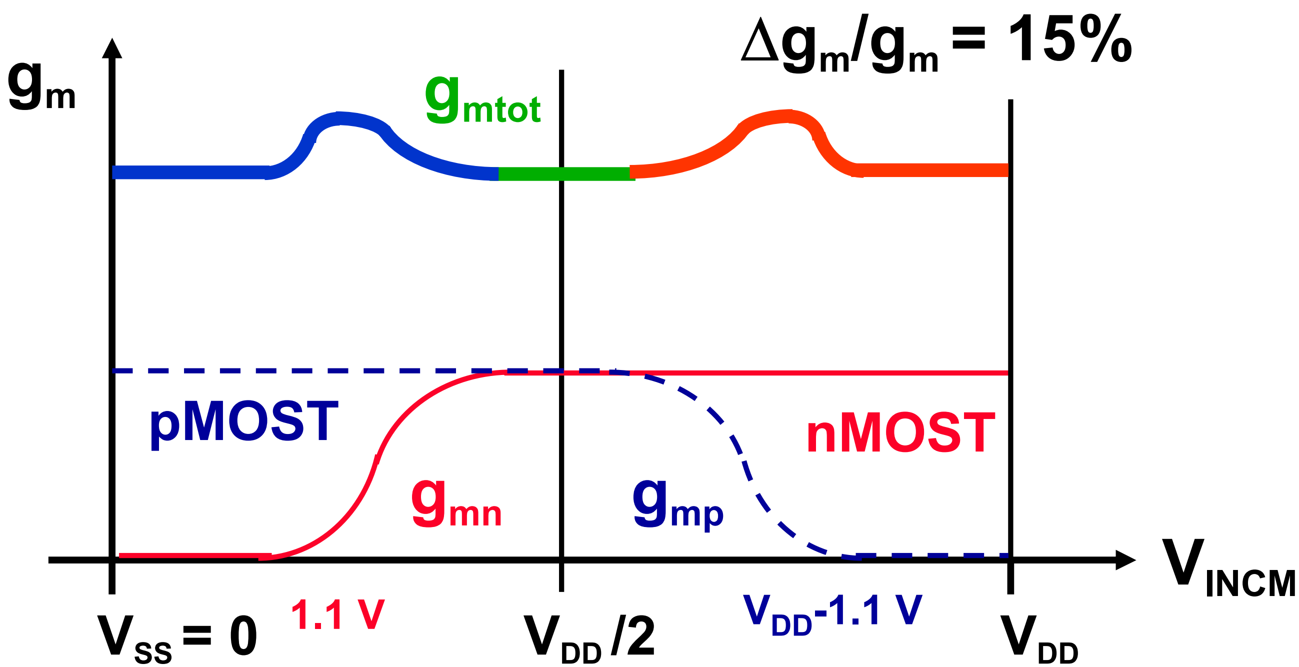
3x Current mirror for nMOSTs



3x Current mirror for all MOSTs

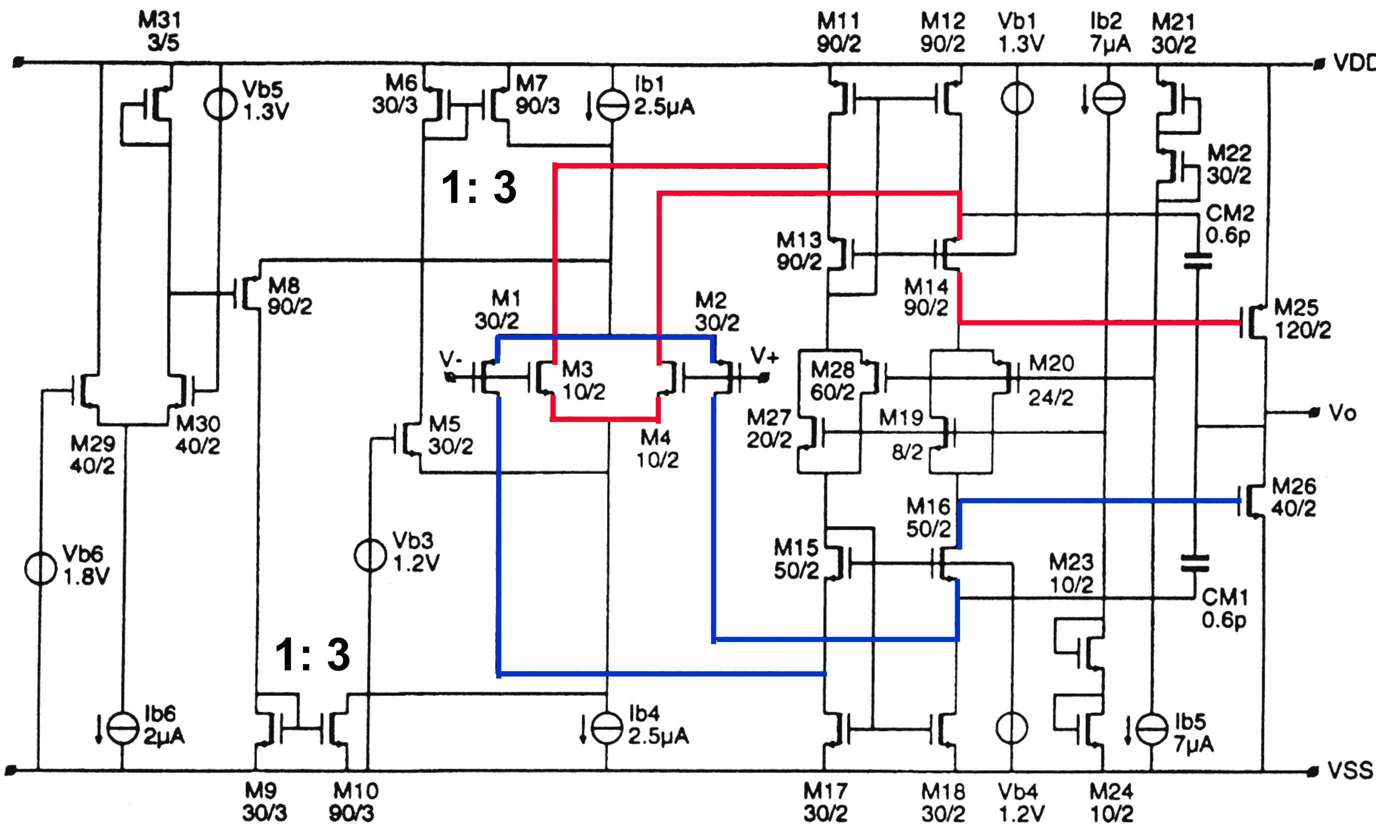


3x Current mirror : performance



$$g_{mtot} \sim \sqrt{(4 - 3x)\, I_B} + \sqrt{x\, I_B} \qquad \Delta g_m/g_m = 15\% \;\; (x=1/3)$$

Rail-to-rail opamp



6.4 MHz / 10 pF

6.4 V/μs

31 nV/√ Hz

3.3 V
0.18 mA
3 mApk

Ref.Hogervorst, JSSC Dec.1994, 1505-1512

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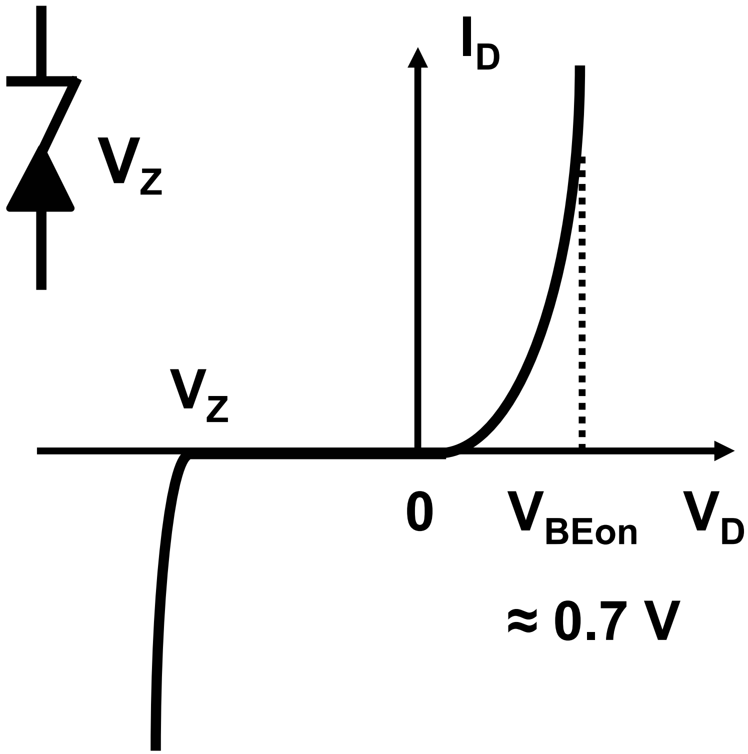
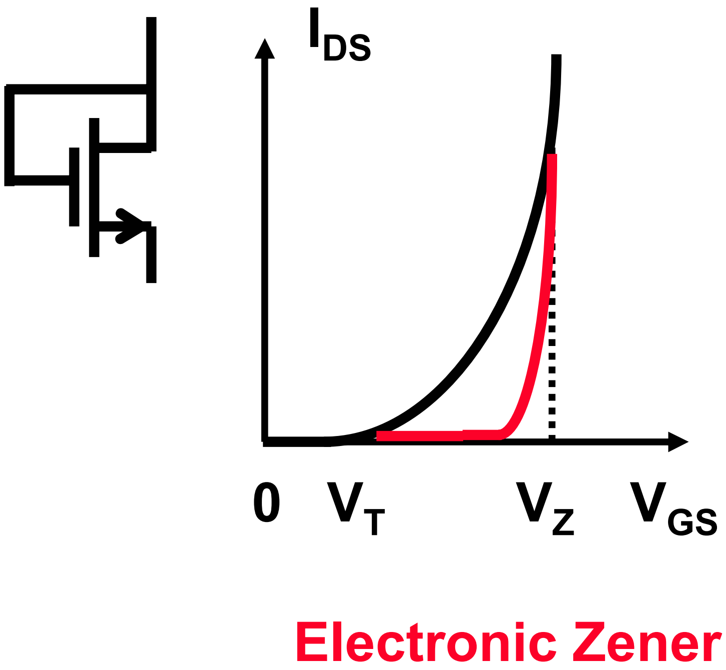
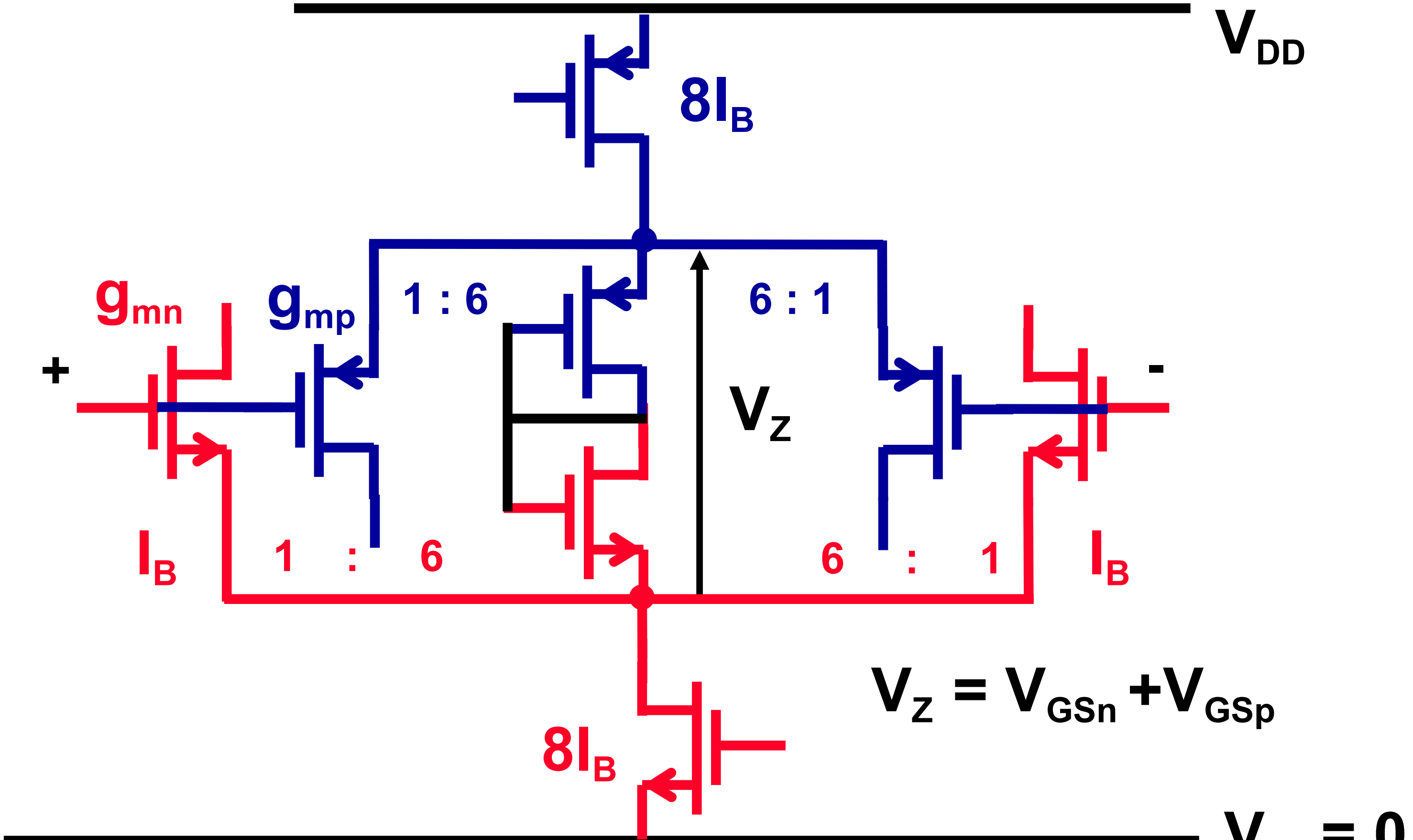
Zener diodes

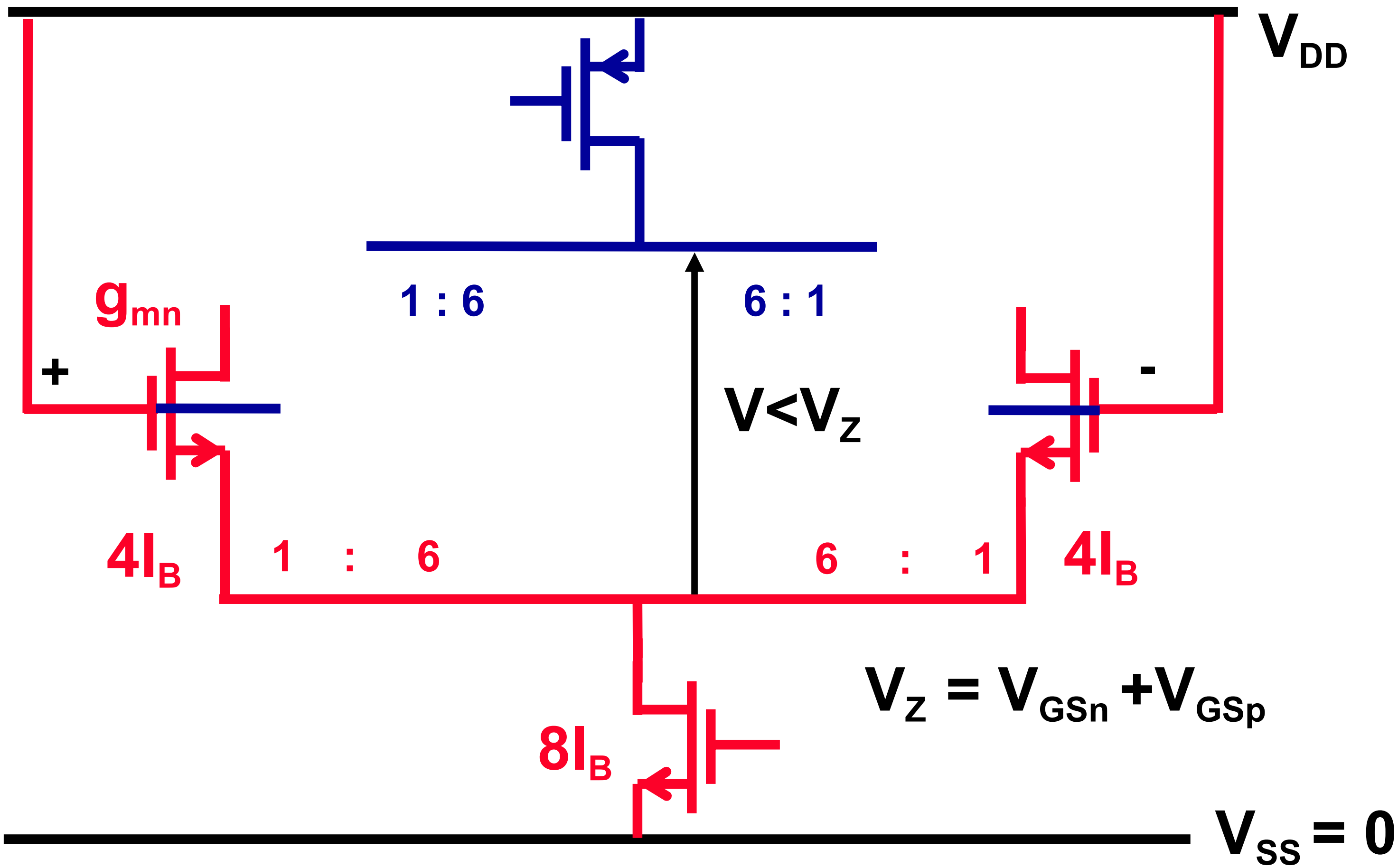




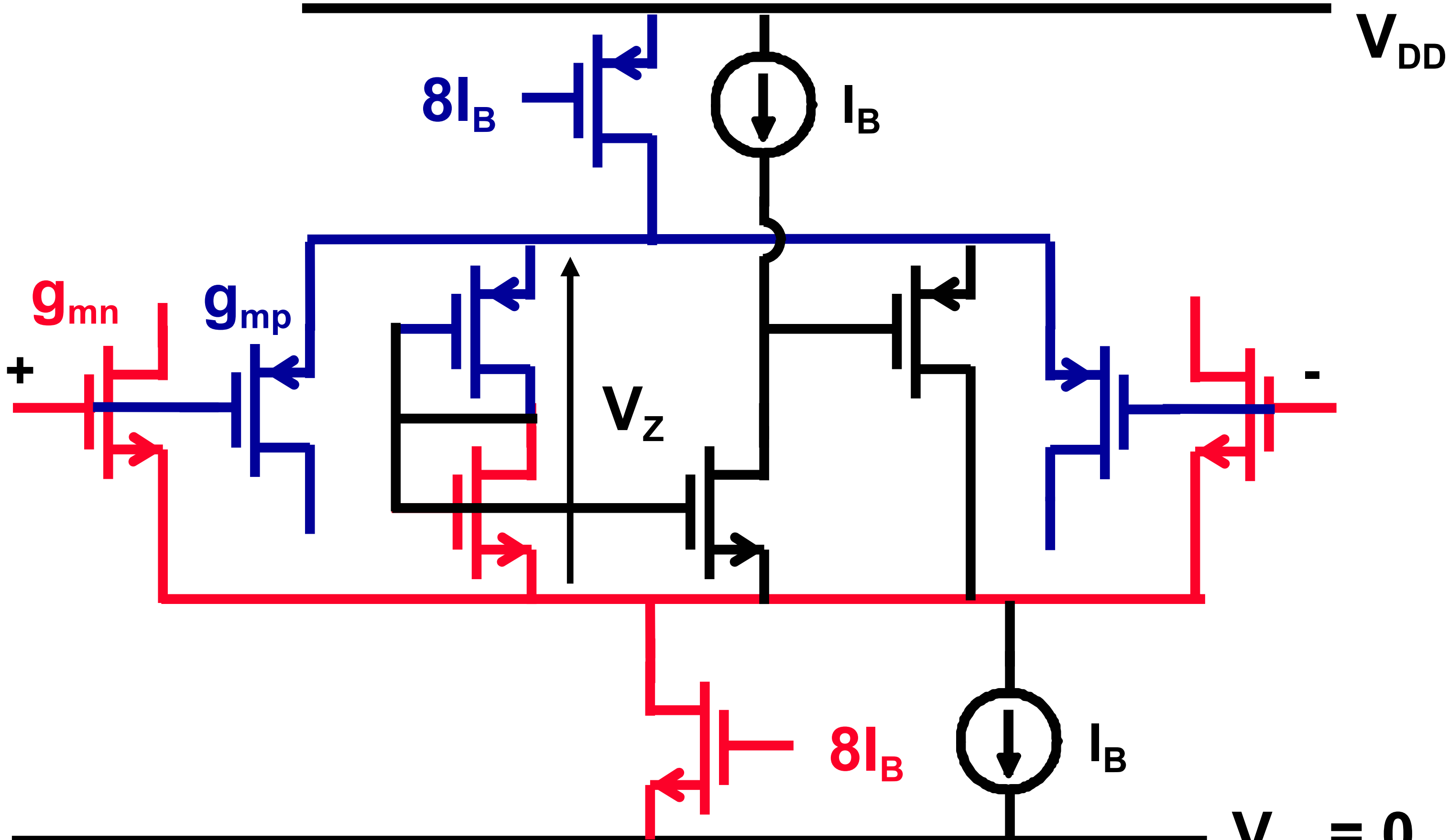
Rail-to-rail amplifier with Zener diode



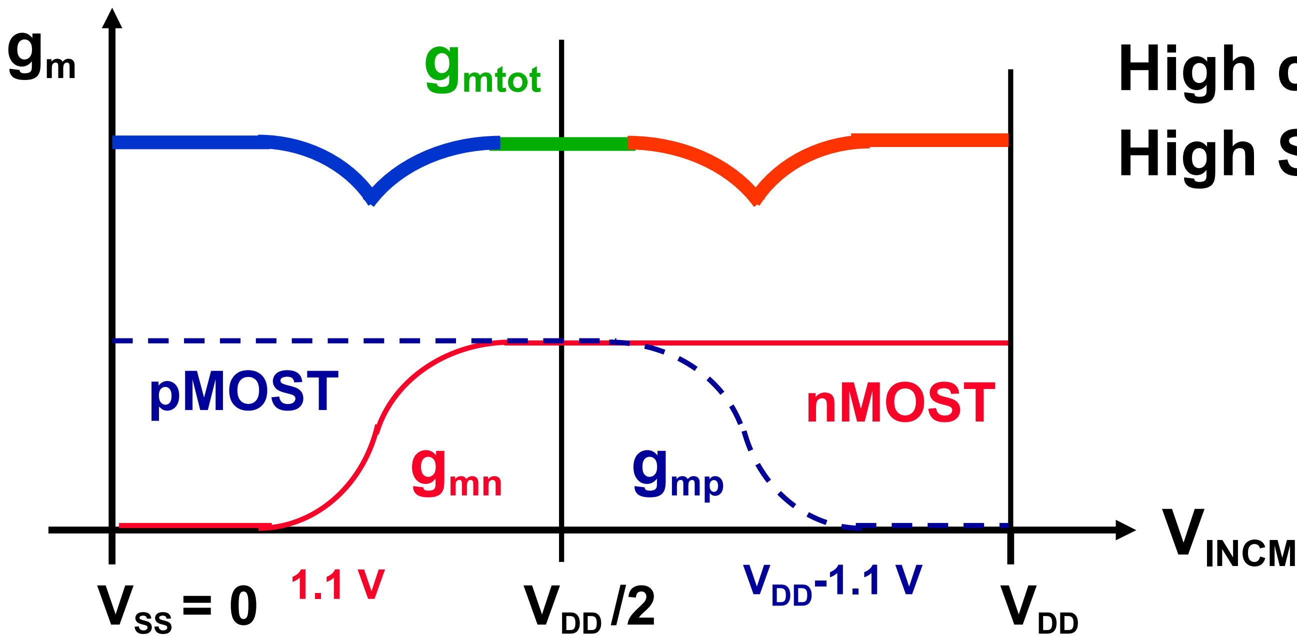
Rail-to-rail amplifier with Zener diode



Rtr amp. with electronic Zener



Rail-to-rail amp. with Zener : performance



High current

High SR !

Zener: $\Delta g_m/g_m = 25\%$ Electronic Zener: $\Delta g_m/g_m = 6\%$

Ref.Hogervorst, JSSC July 1996, 1035-1040

Table of contents

- Why rail-to-rail ?
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Equalize g_{mtot} in weak inversion

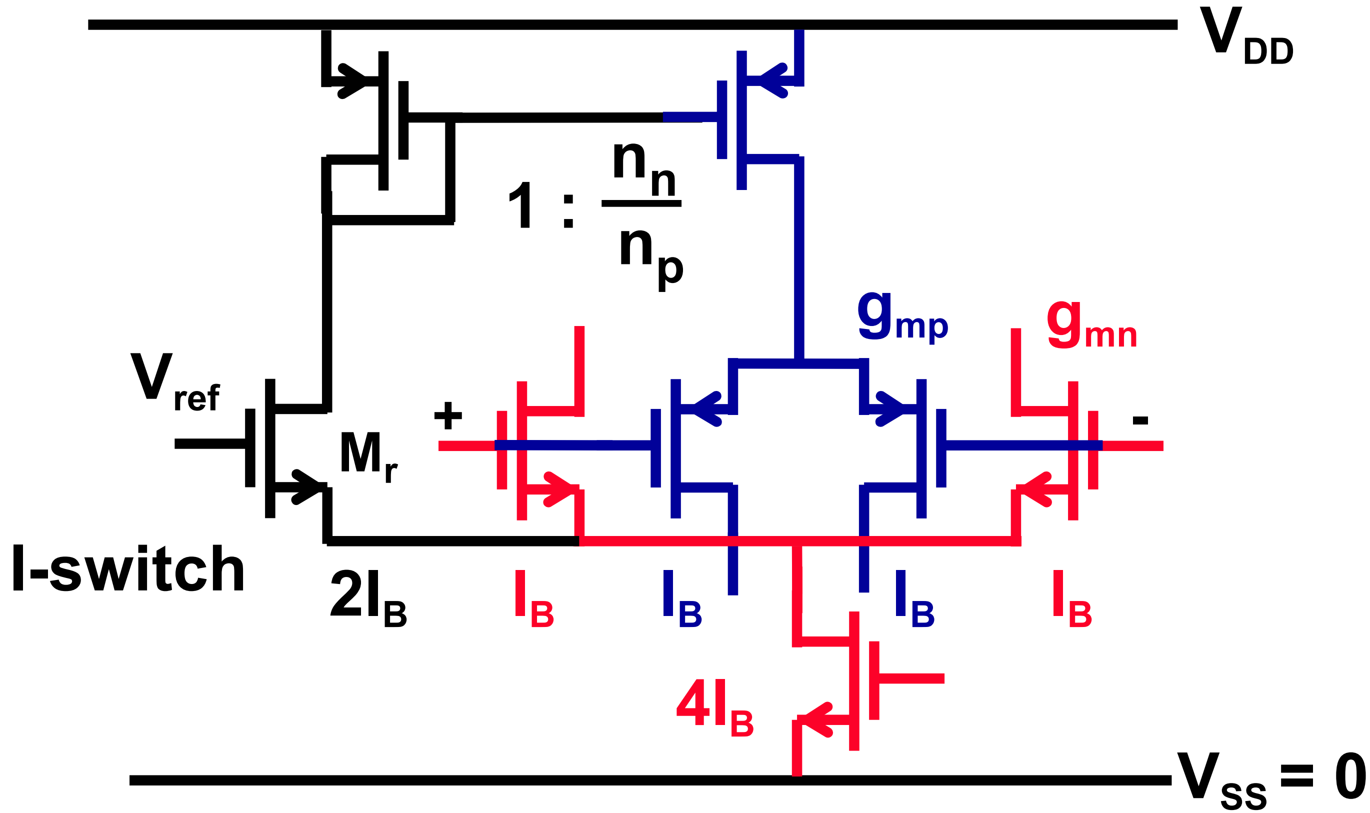
$$g_{mn} + g_{mp} = ct$$

$$\frac{I_{Bn}}{2\, n_n\, kT/q} + \frac{I_{Bp}}{2\, n_p\, kT/q} = ct$$

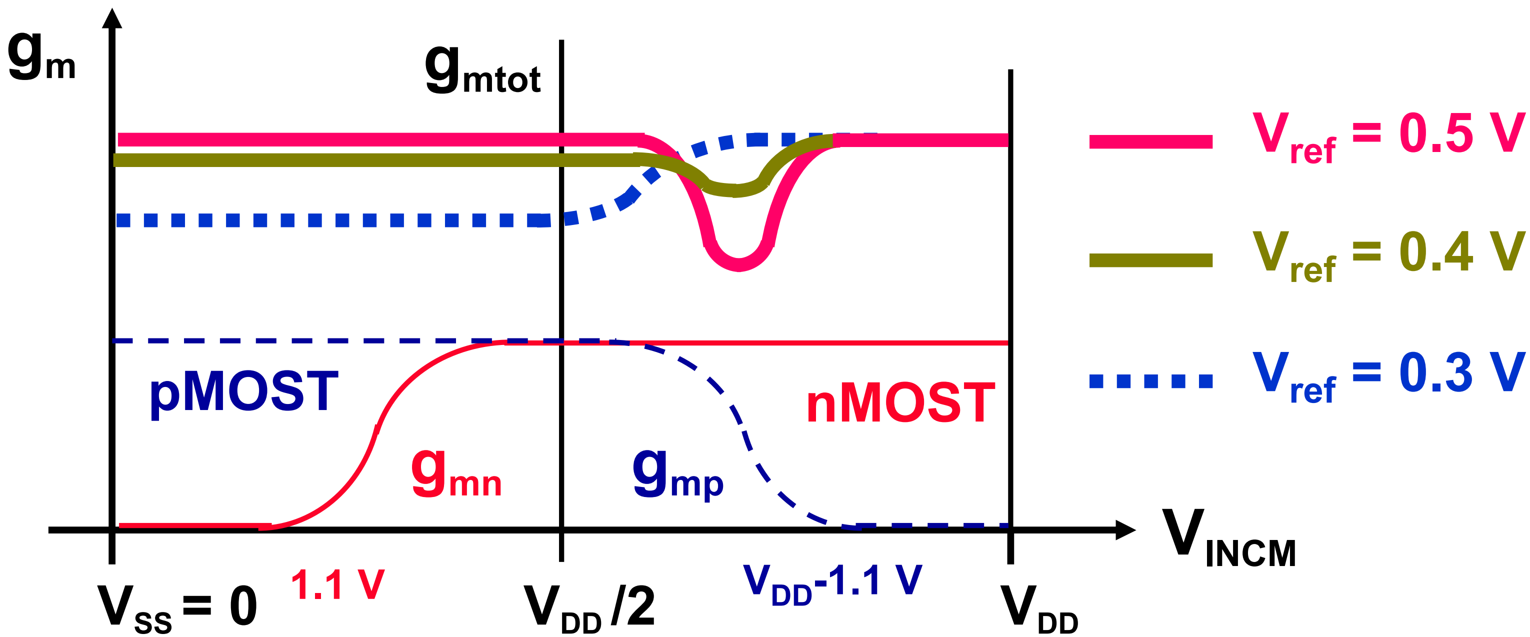
$$I_{Bn} + \frac{n_n}{n_p} I_{Bp} = ct$$

$$n = 1 + \frac{C_D\,(V_{BS})}{C_{ox}}$$

Rail-to-rail amplifier with current switch

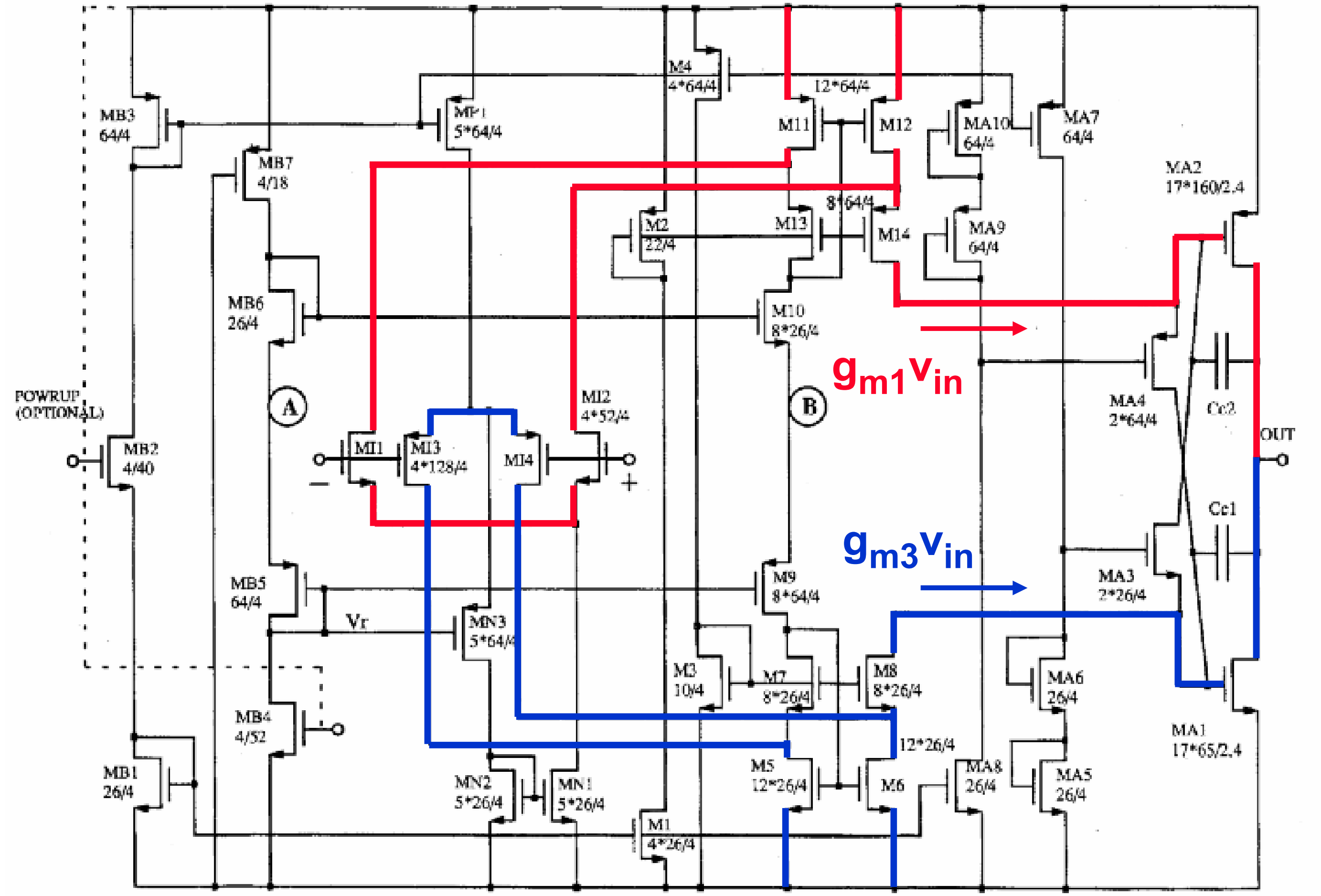


Rtr amp. with I-switch : performance



Current switch : V_{ref} very critical !

Ref. : Wu etal, JSSC Jan.1994, pp.63-66



14 MHz / 11pF

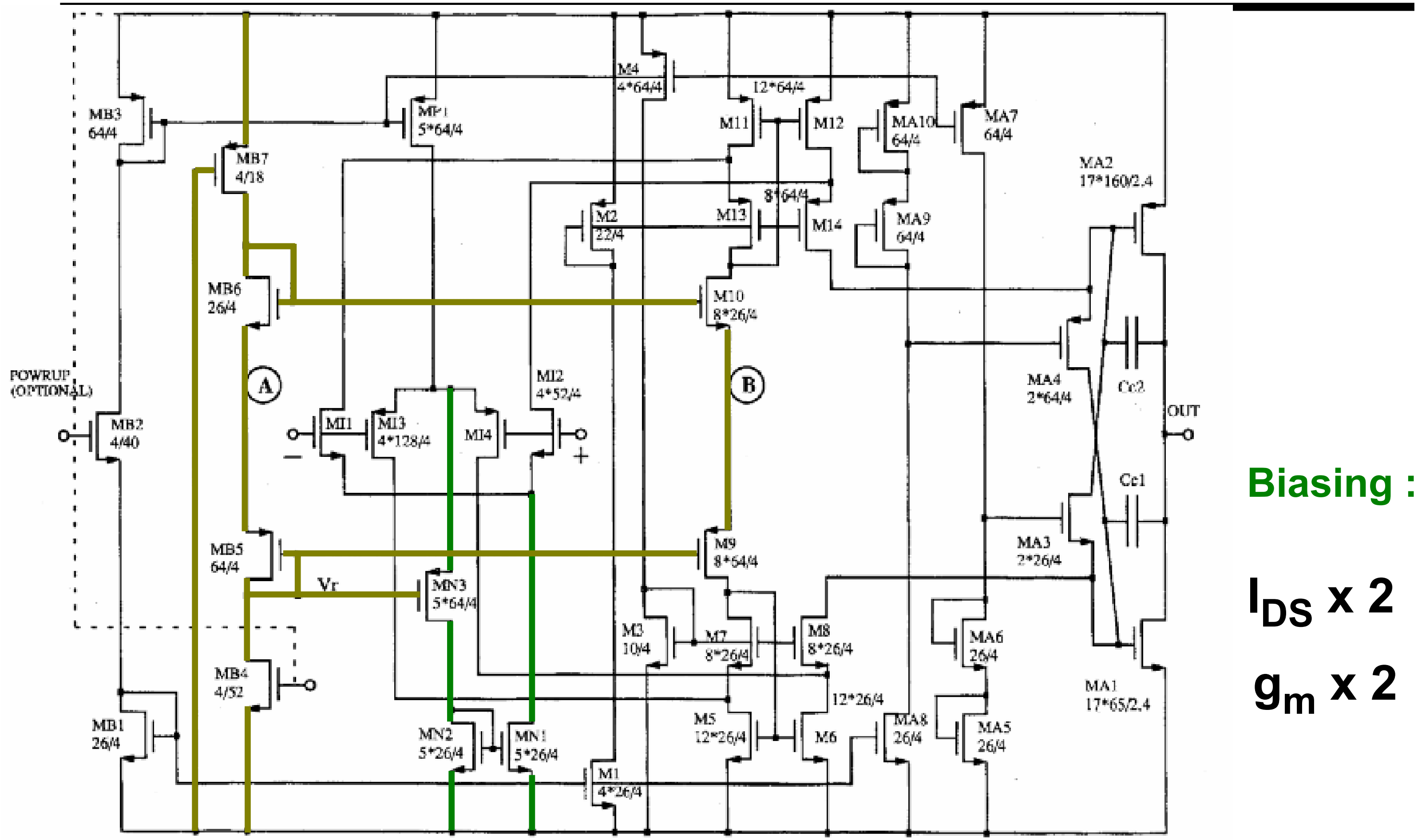
5.6 MHz / 100pF

4 V/μs

36 nV/√ Hz

5 V
0.4 mA

Ref. : Wu etal, JSSC Jan.1994, pp.63-66

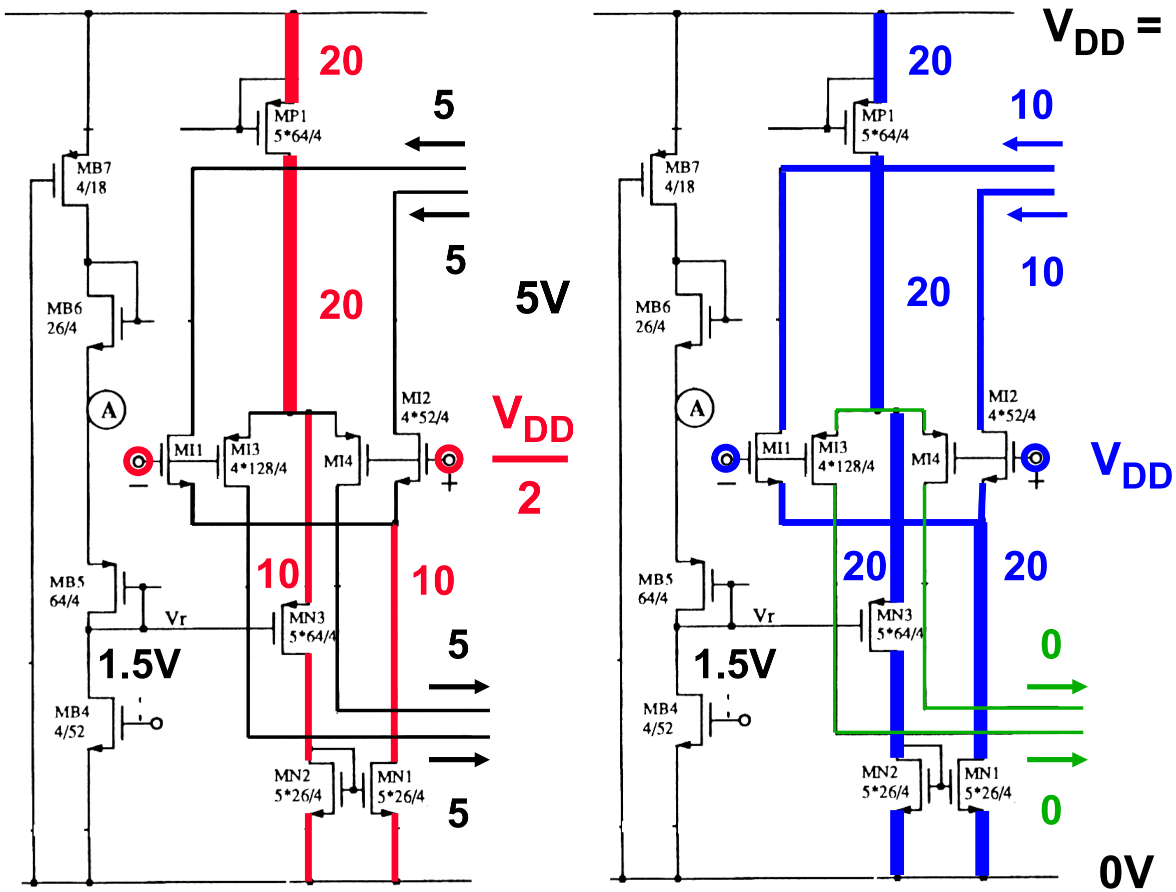


Biasing :

I_{DS} x 2

g_m x 2

Input rail-to-rail stage



V_{+-} high :

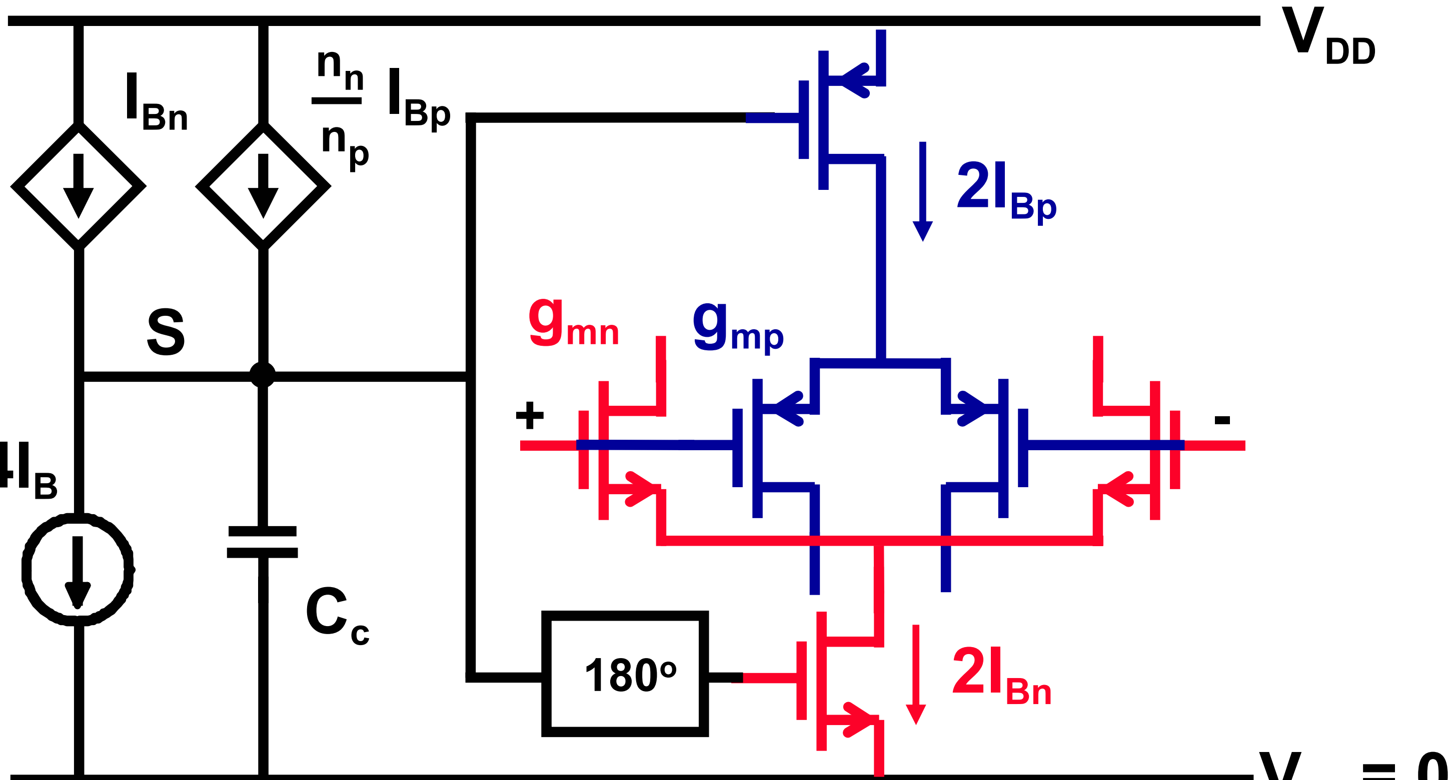
pMOSTs off !

nMOSTs :

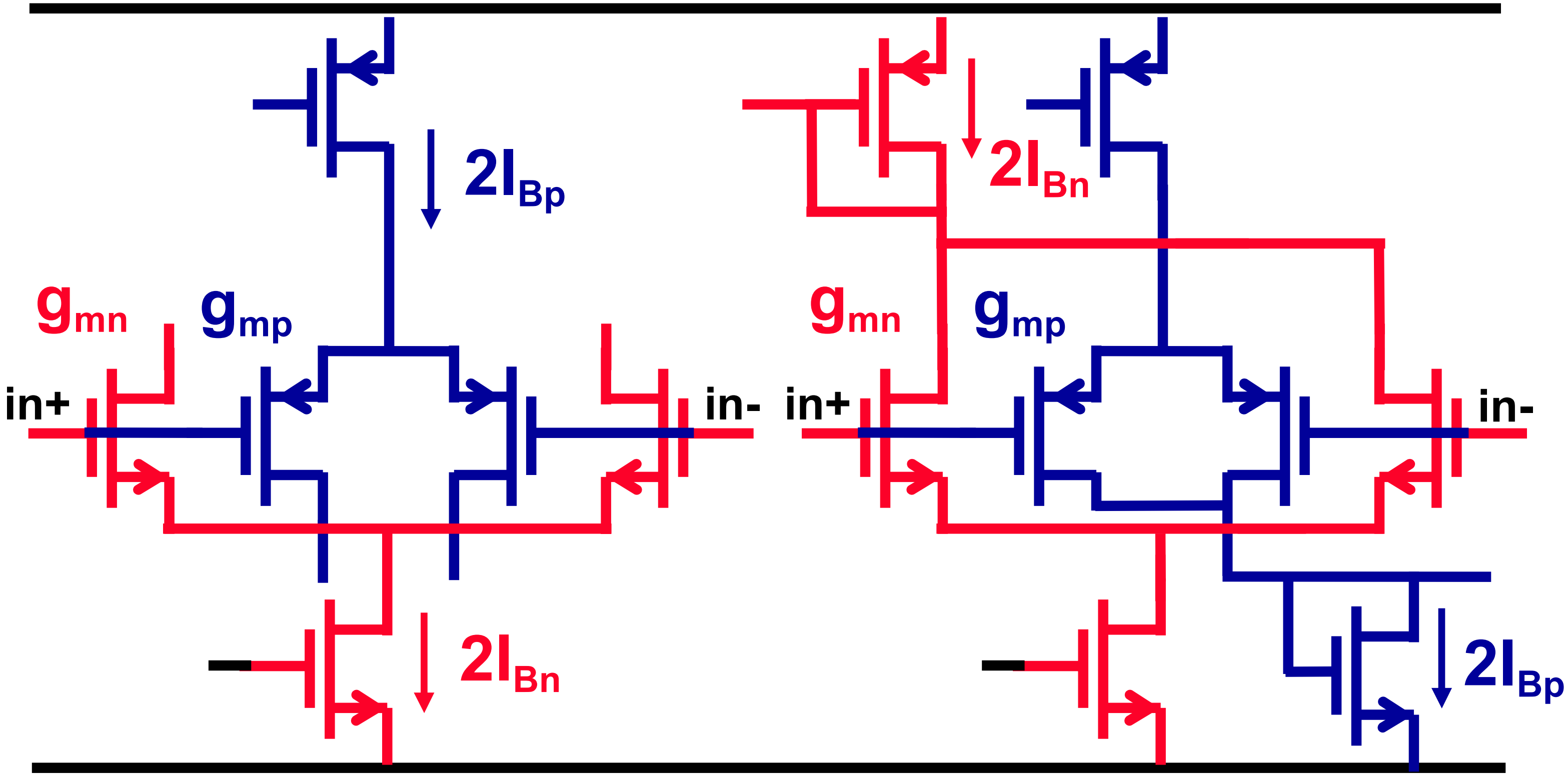
I_{DS1} x 2

g_{m1} x 2

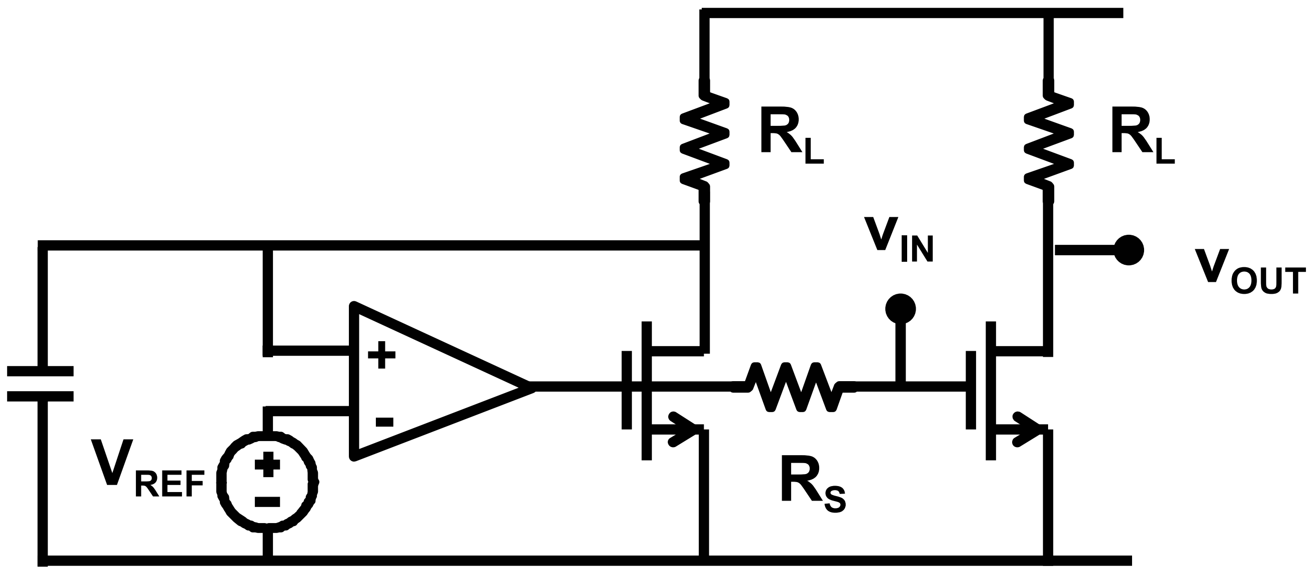
Current regulator FB loop



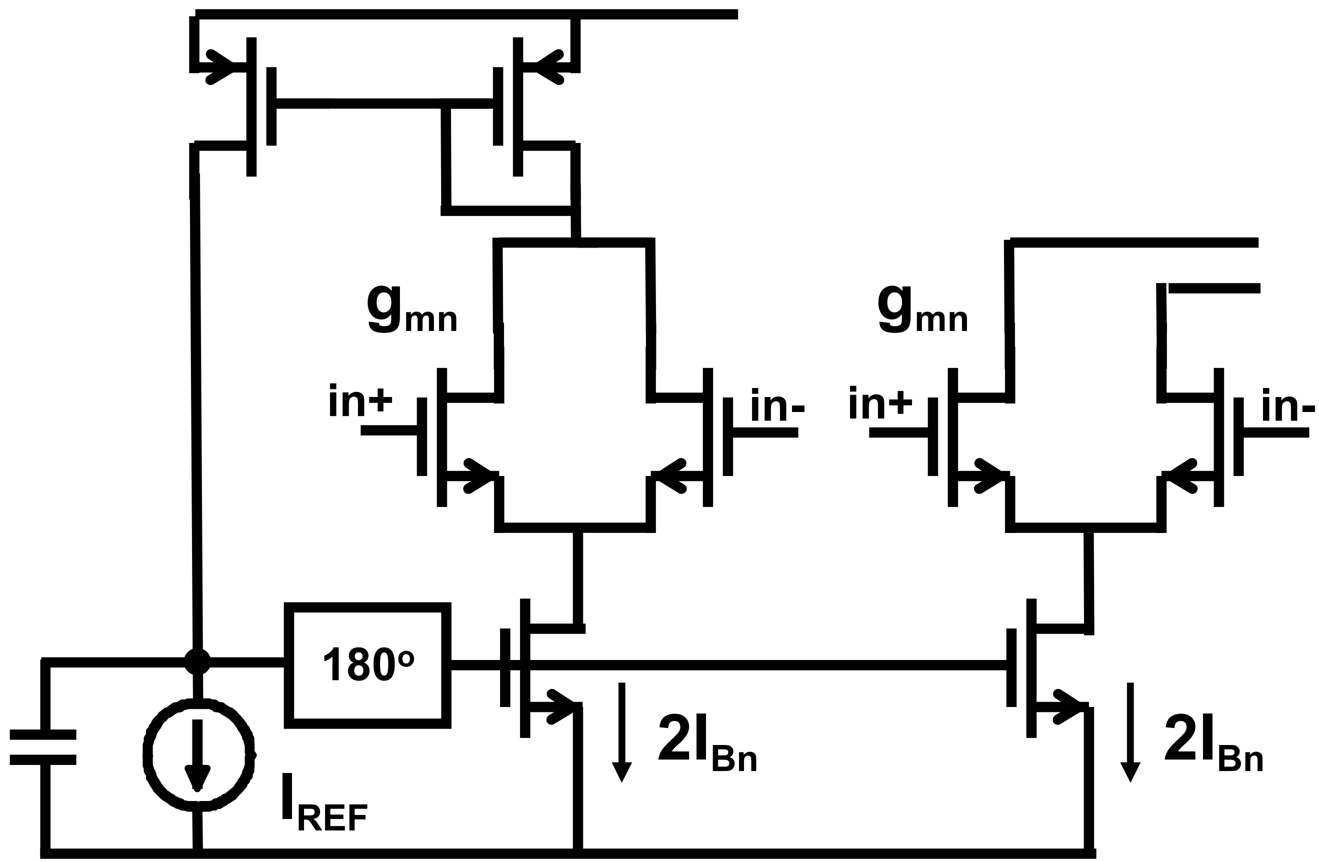
Current regulator FB loop : replica biasing



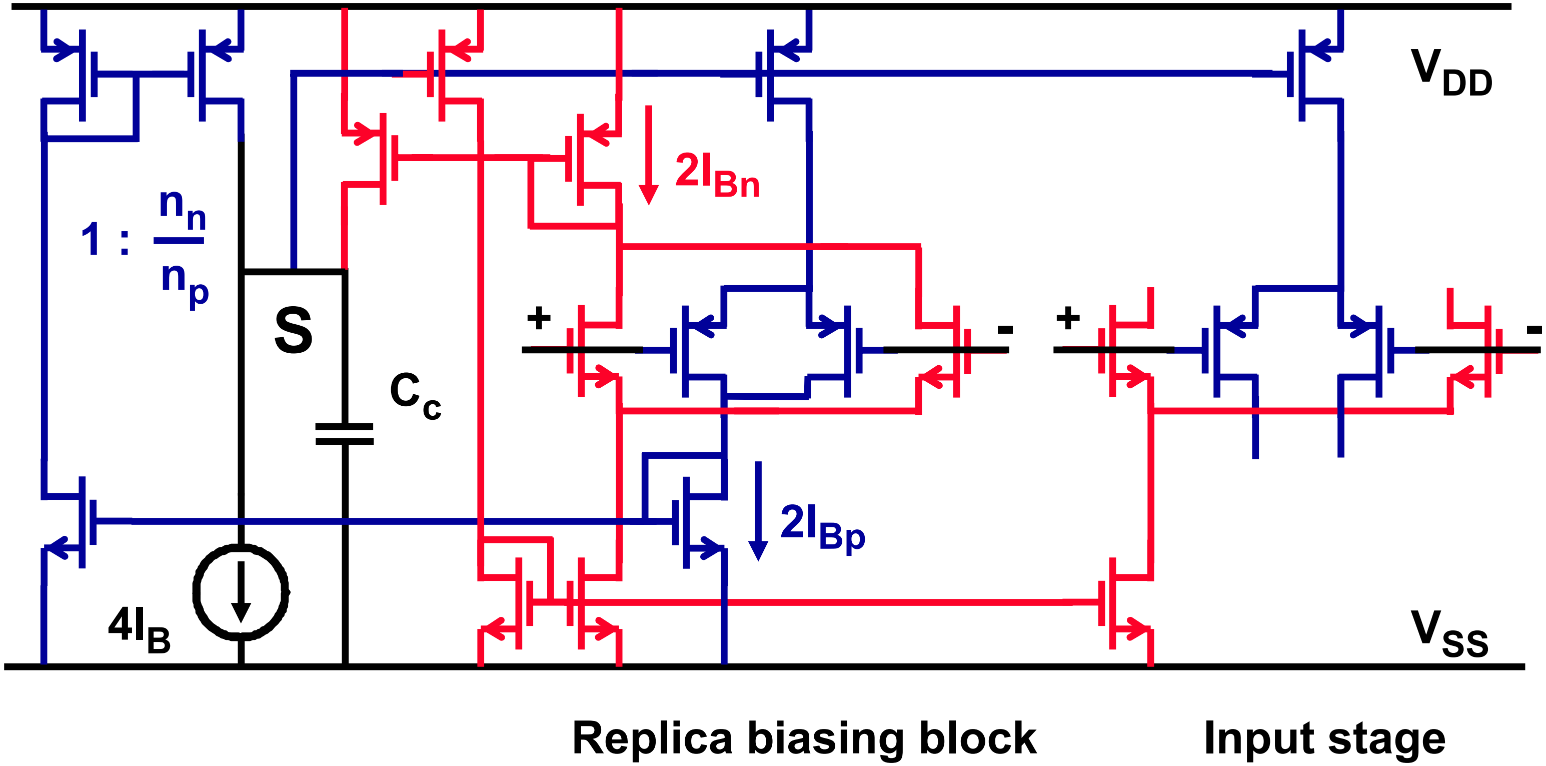
Replica biasing with one transistor



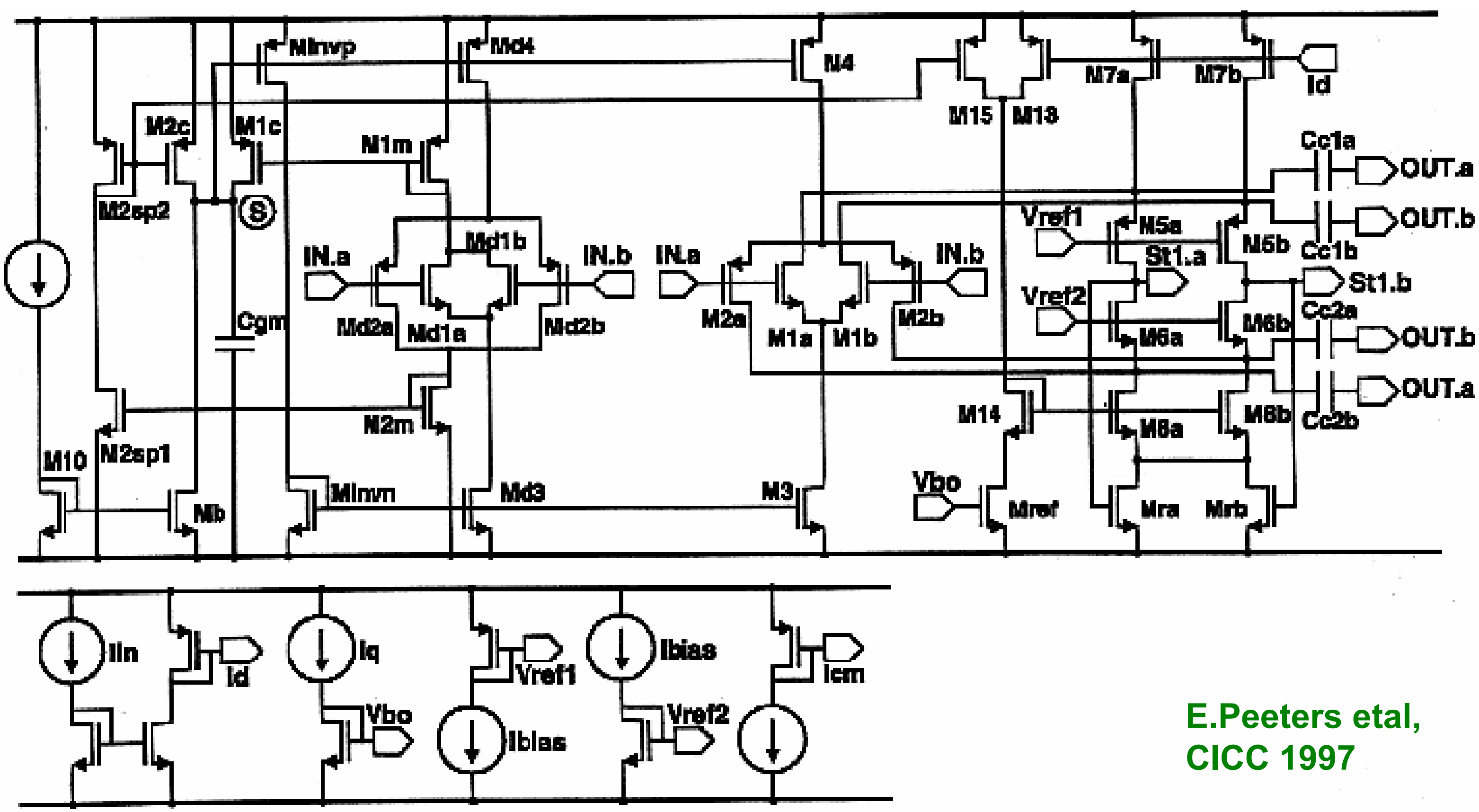
Replica biasing with differential pair



Current regulator rail-to-rail amplifier

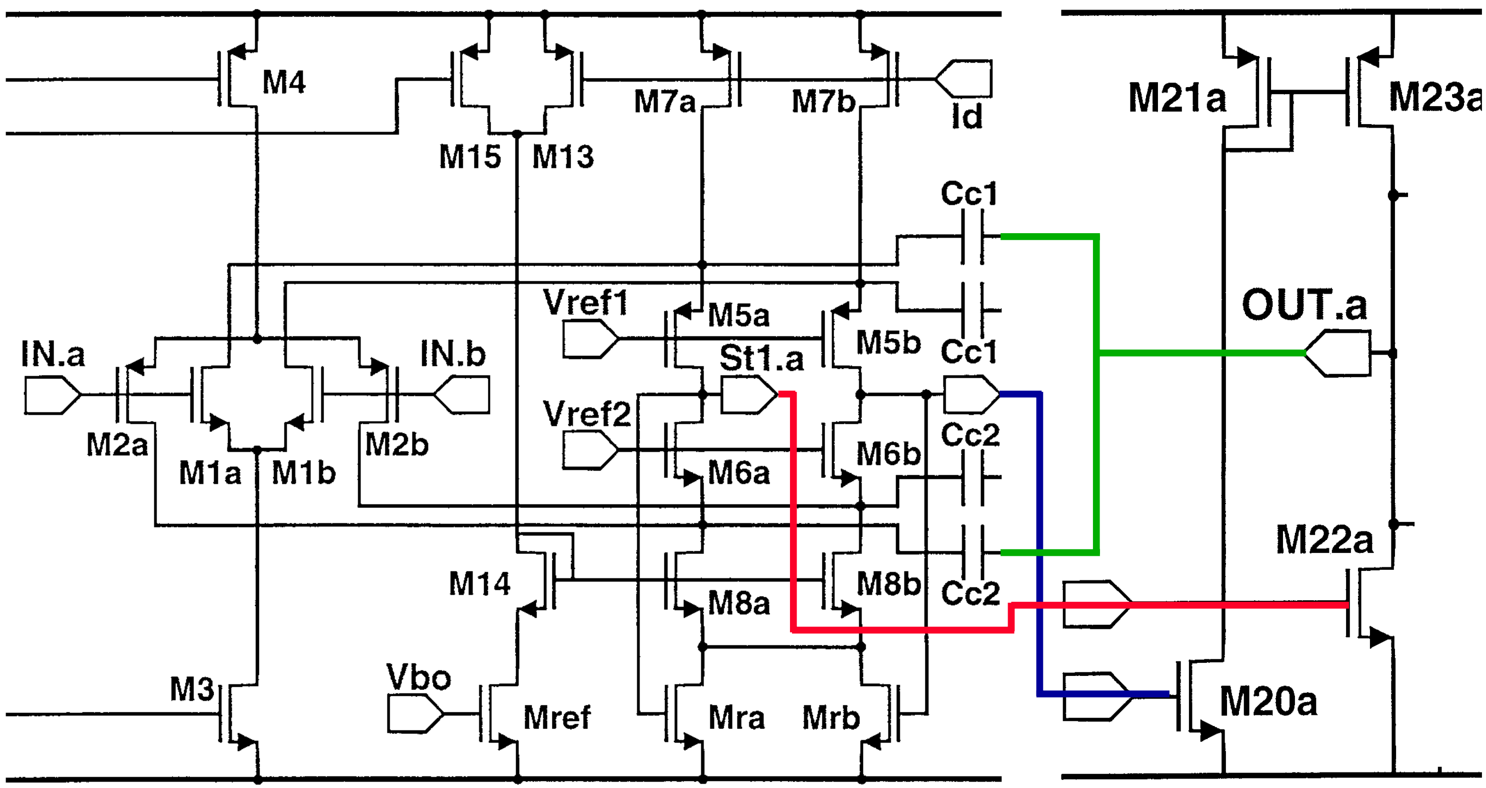


I-regulator rtr amplifier



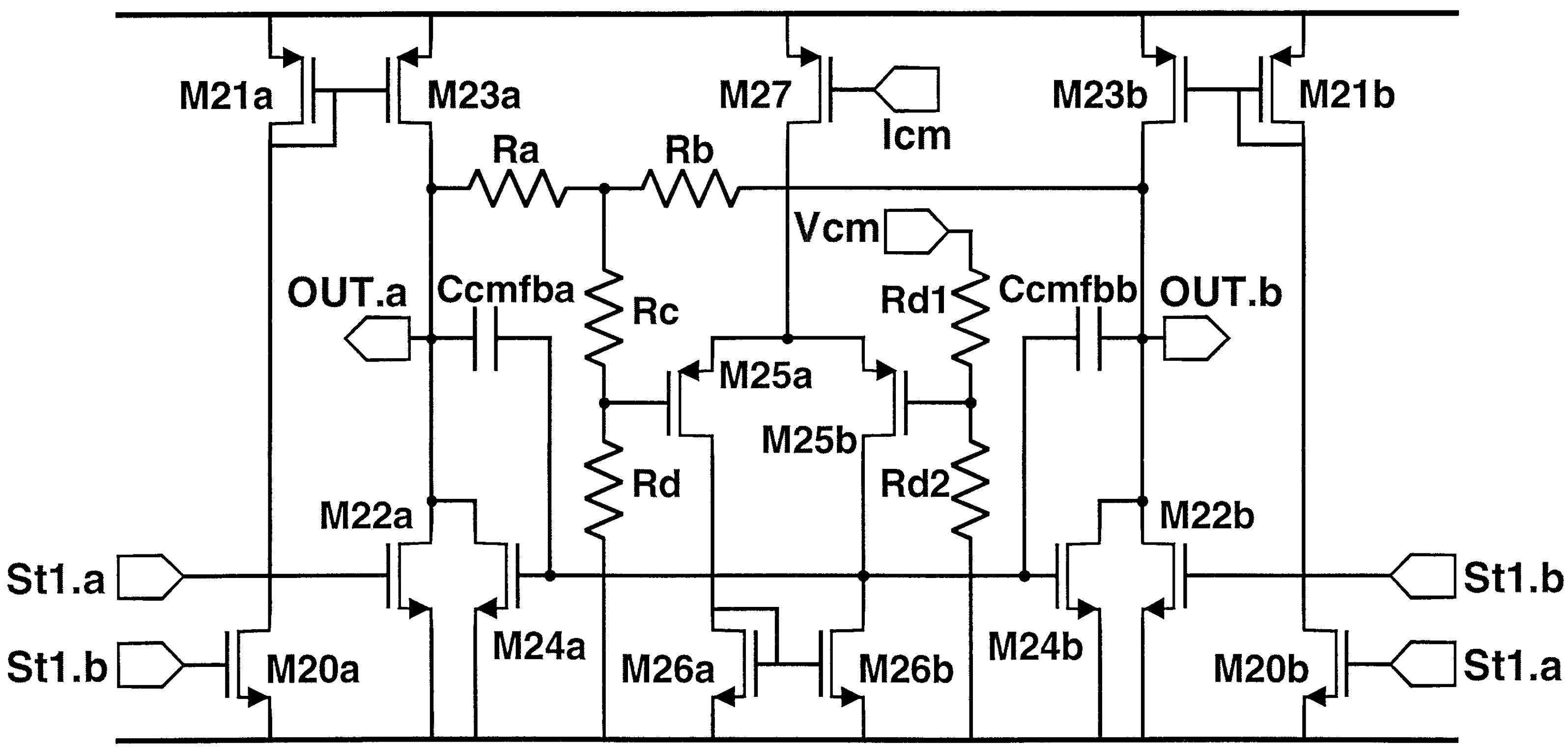
E.Peeters etal,
CICC 1997

Total amplifier schematic

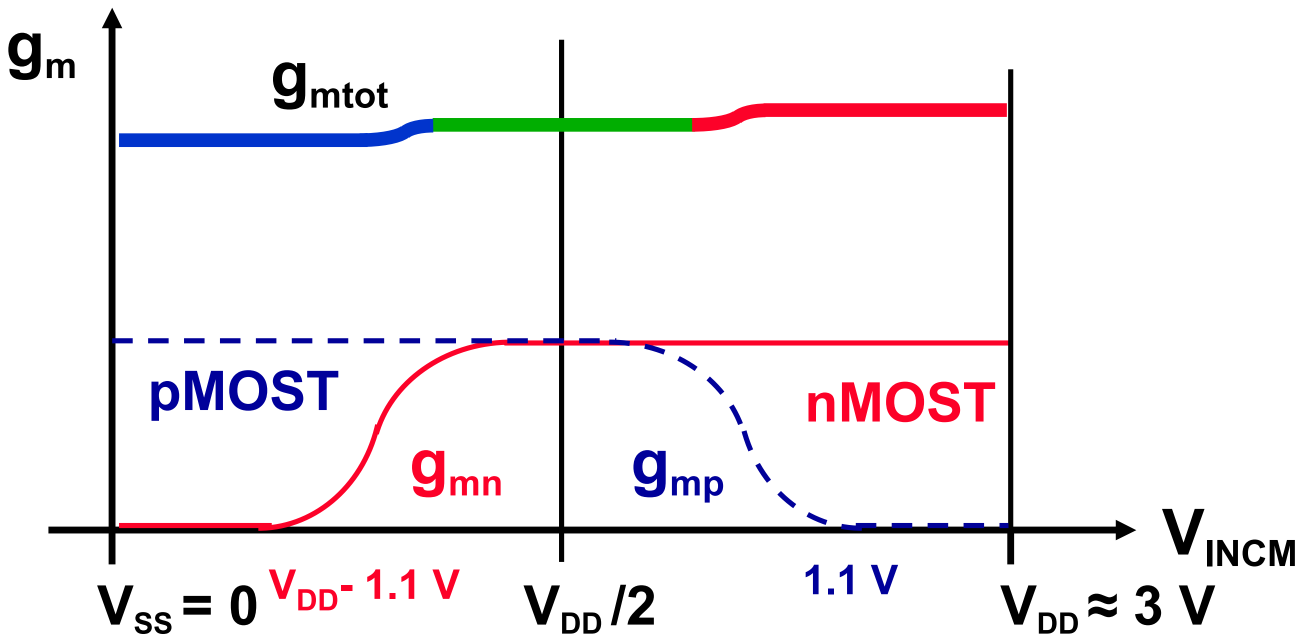


E.Peeters etal, CICC 1997

Output stage

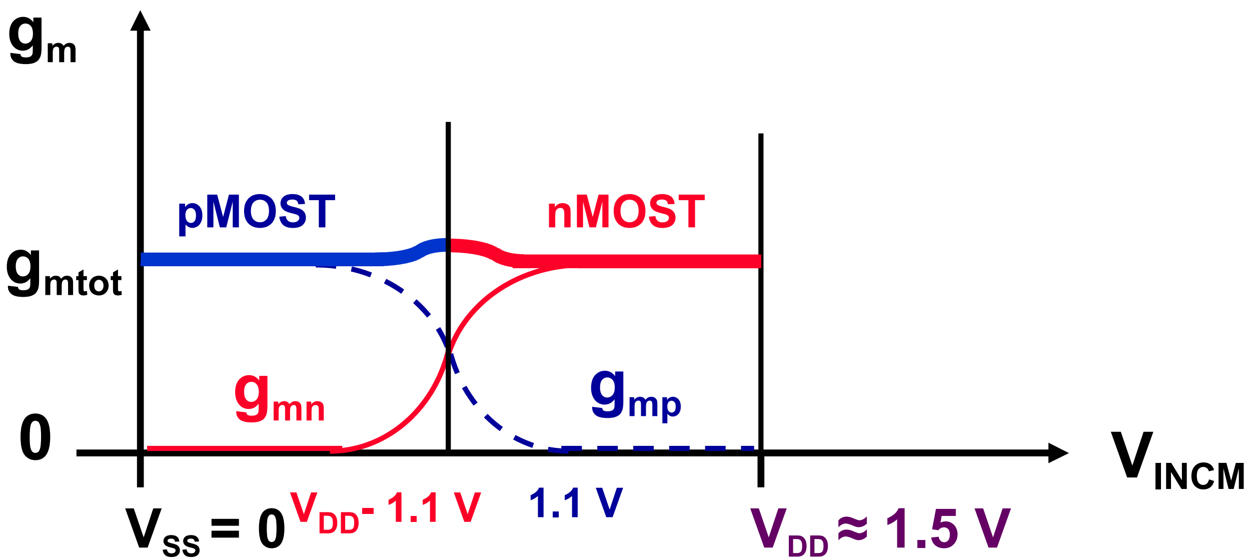


Current-regulator rtr amp. : performance



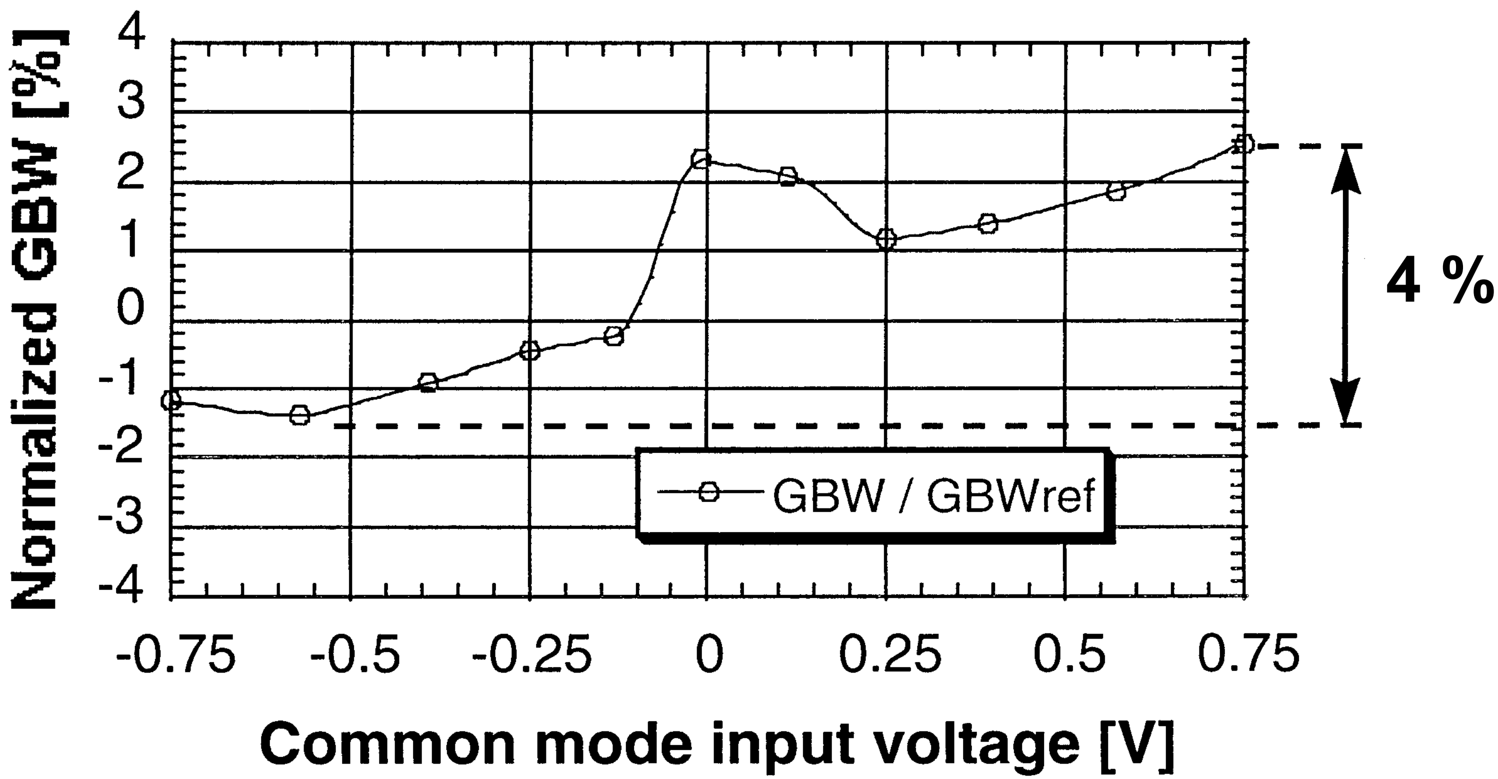
n - mismatch : $\Delta g_m/g_m \approx 4\%$

Current-regulator rtr amp. : towards 1.5 V



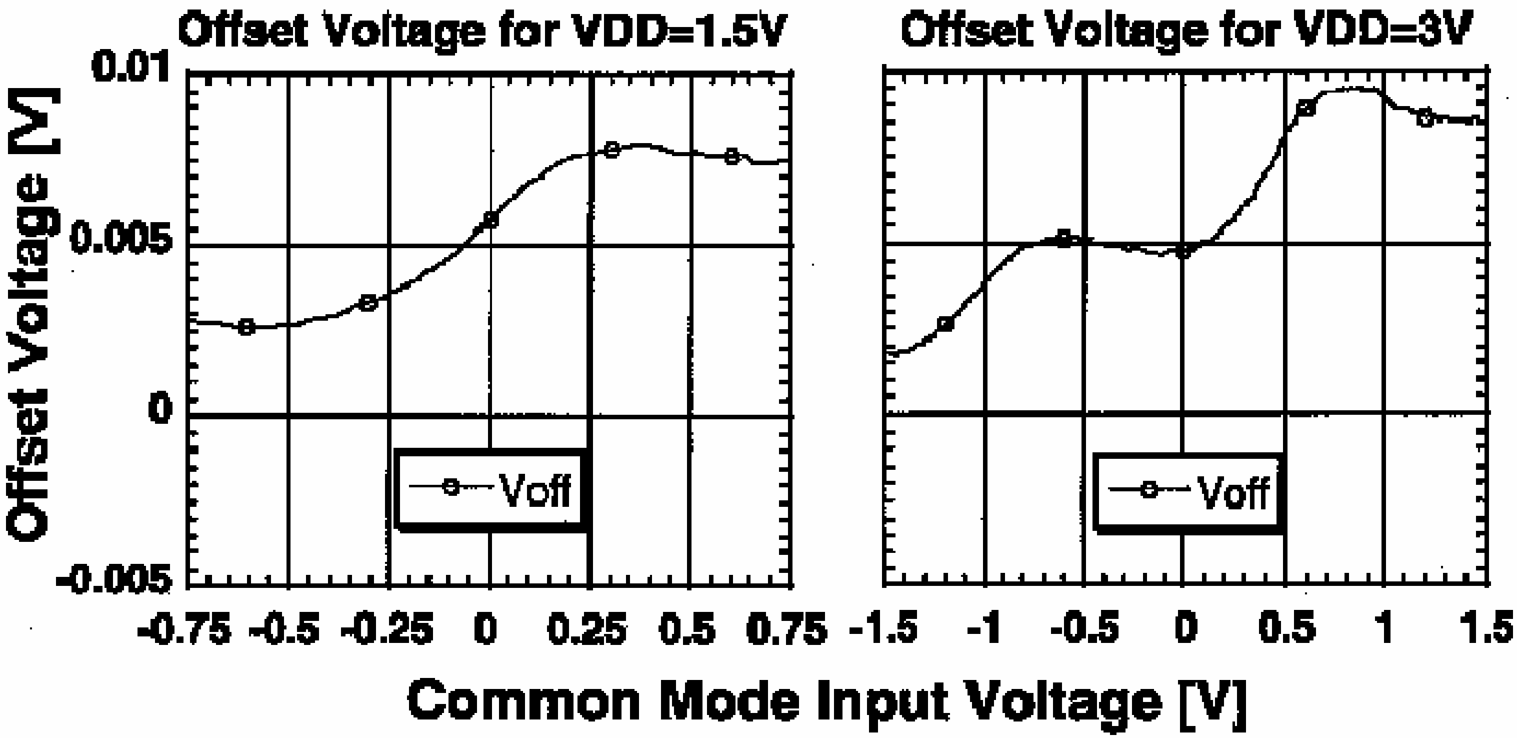
n - mismatch : $\Delta g_m/g_m \approx 4\%$

GBW error

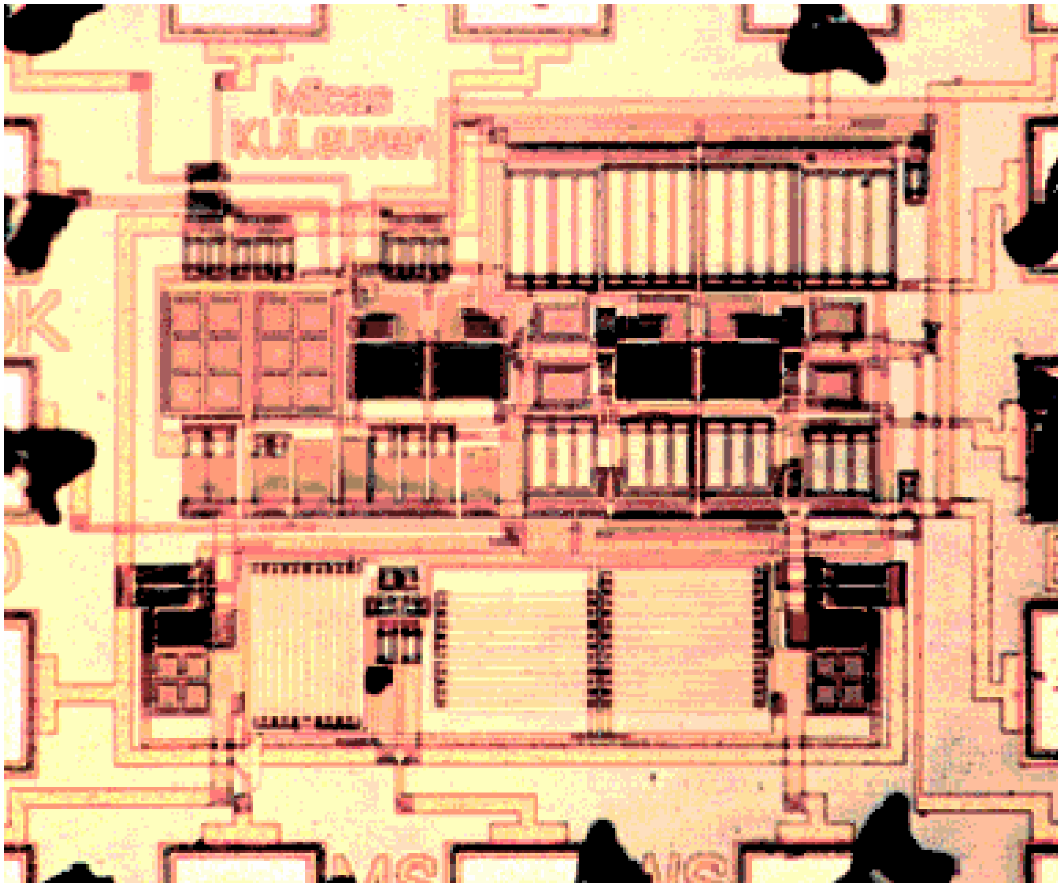


Normalized GBW [%]
4
3
2
1
0
-1
-2
-3
-4
-0.75
-0.5
-0.25
0
0.25
0.5
0.75
Common mode input voltage [V]
GBW / GBWref
4 %

Input offset voltage



Rail-to-rail Opamp with Current regulator



V_{DD} = 1.5 V

I_{TOT} = 0.2 mA

$\Delta g_m/g_m$ = 4 %

GBW = 4.3 MHz

C_L = 15 pF

E.Peeters etal, CICC 1997

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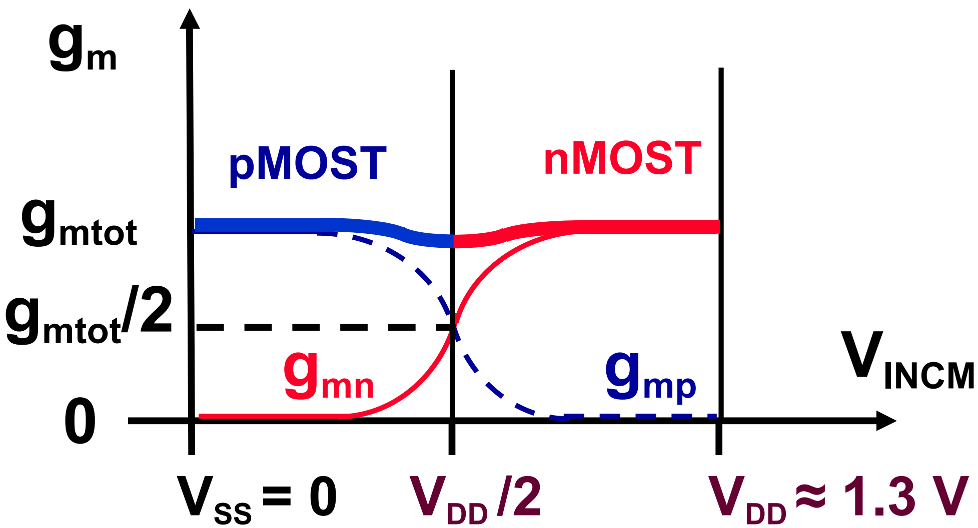
Internal V_{DD} Regulator



Weak inversion :

$$I_{Bn} + \frac{n_n}{n_p} I_{Bp} = \text{ct}$$

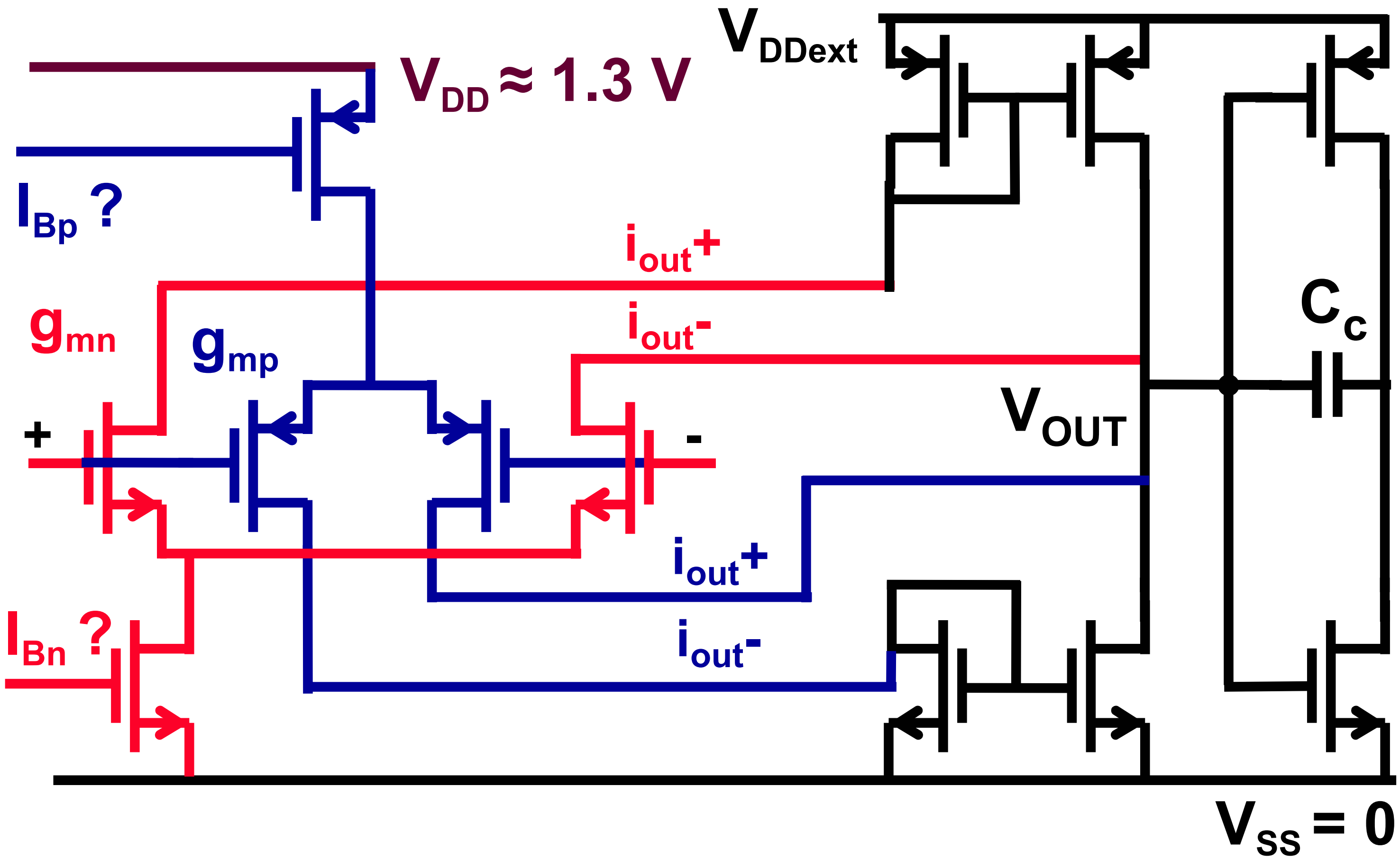
Minimum V_{DD} ?

Minimum $V_{GS}+V_{DSsat}$?

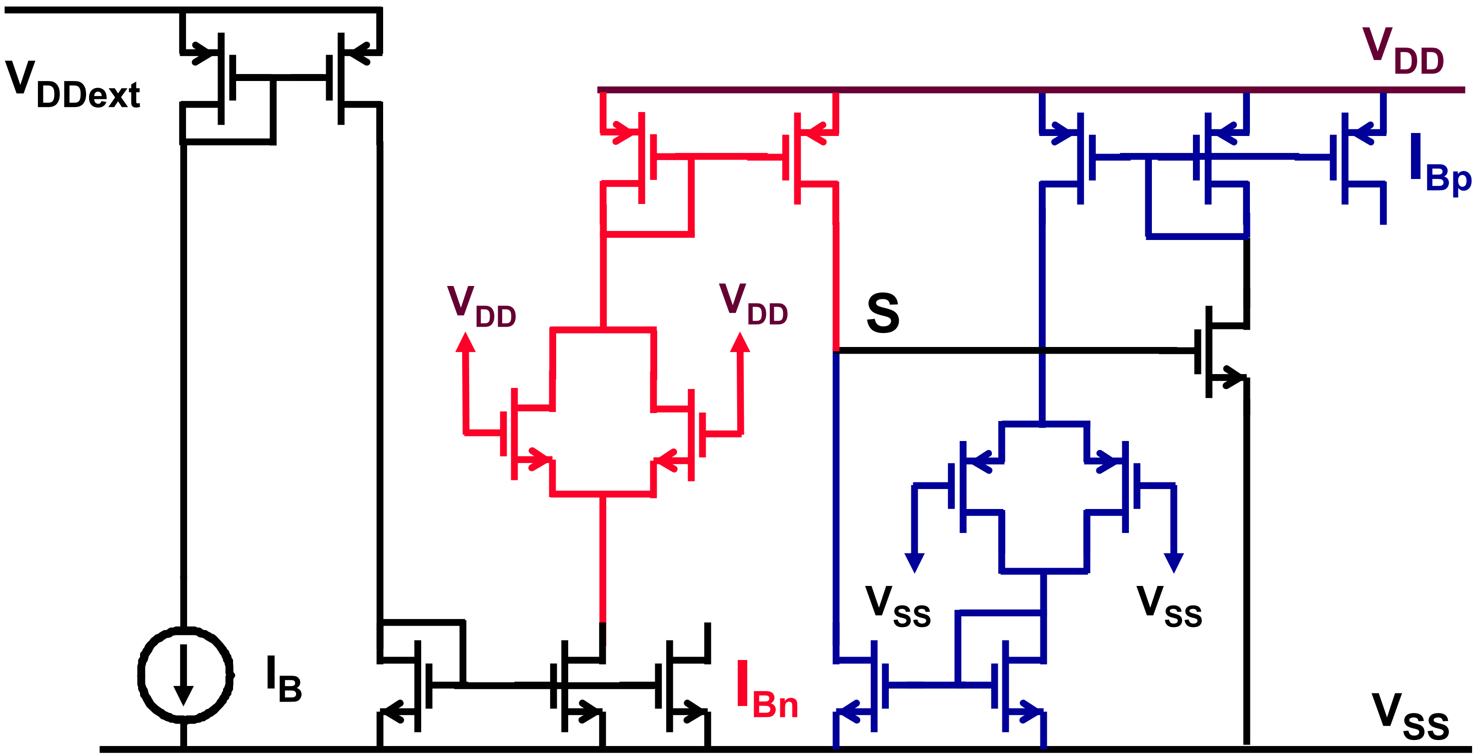
Independent of ΔV_T's !

n - mismatch and g_{mtot} dip : $\Delta g_m/g_m \approx 15$ %

Regulating V_{DD} : total schematic

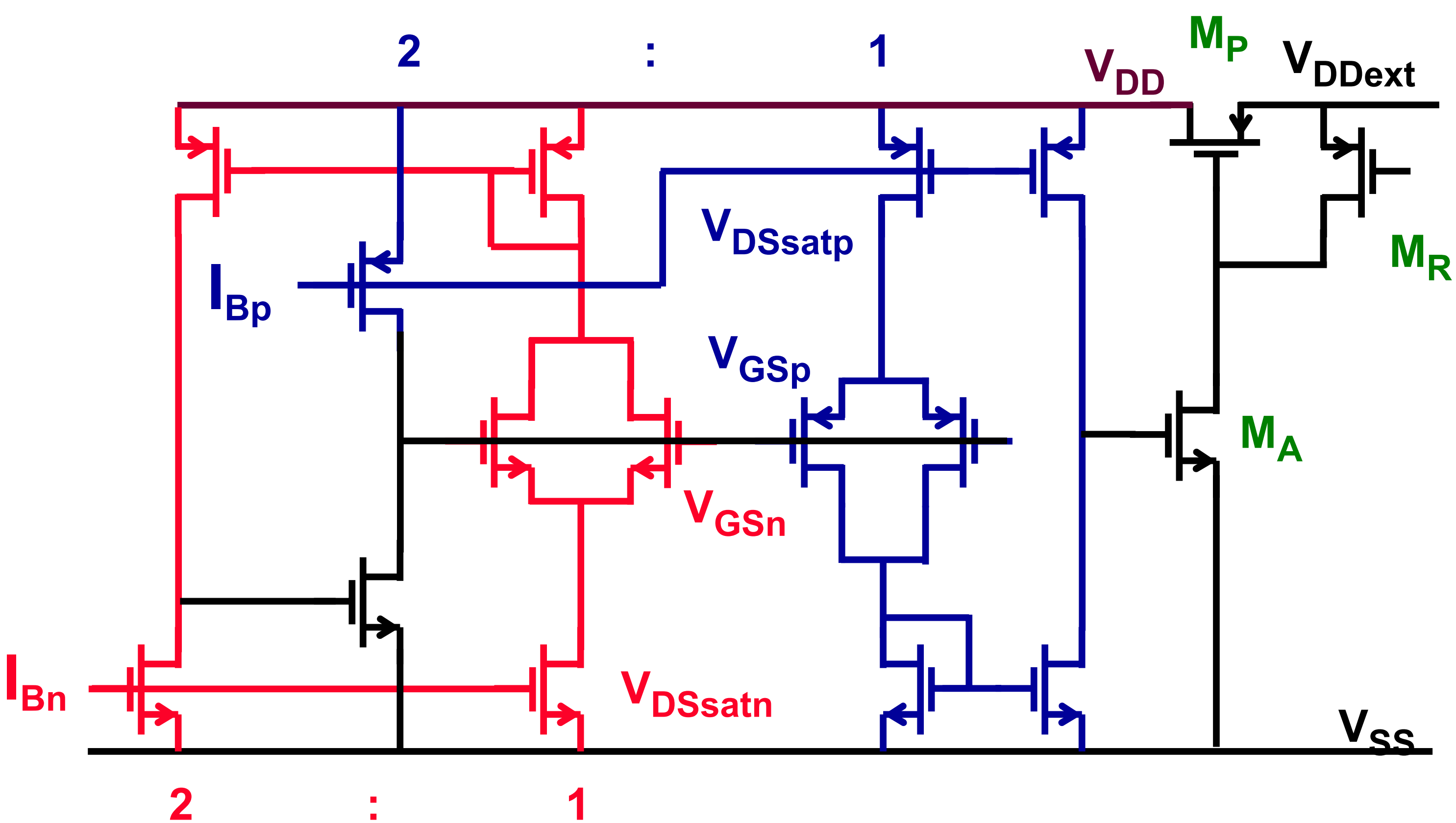


Replica biasing block

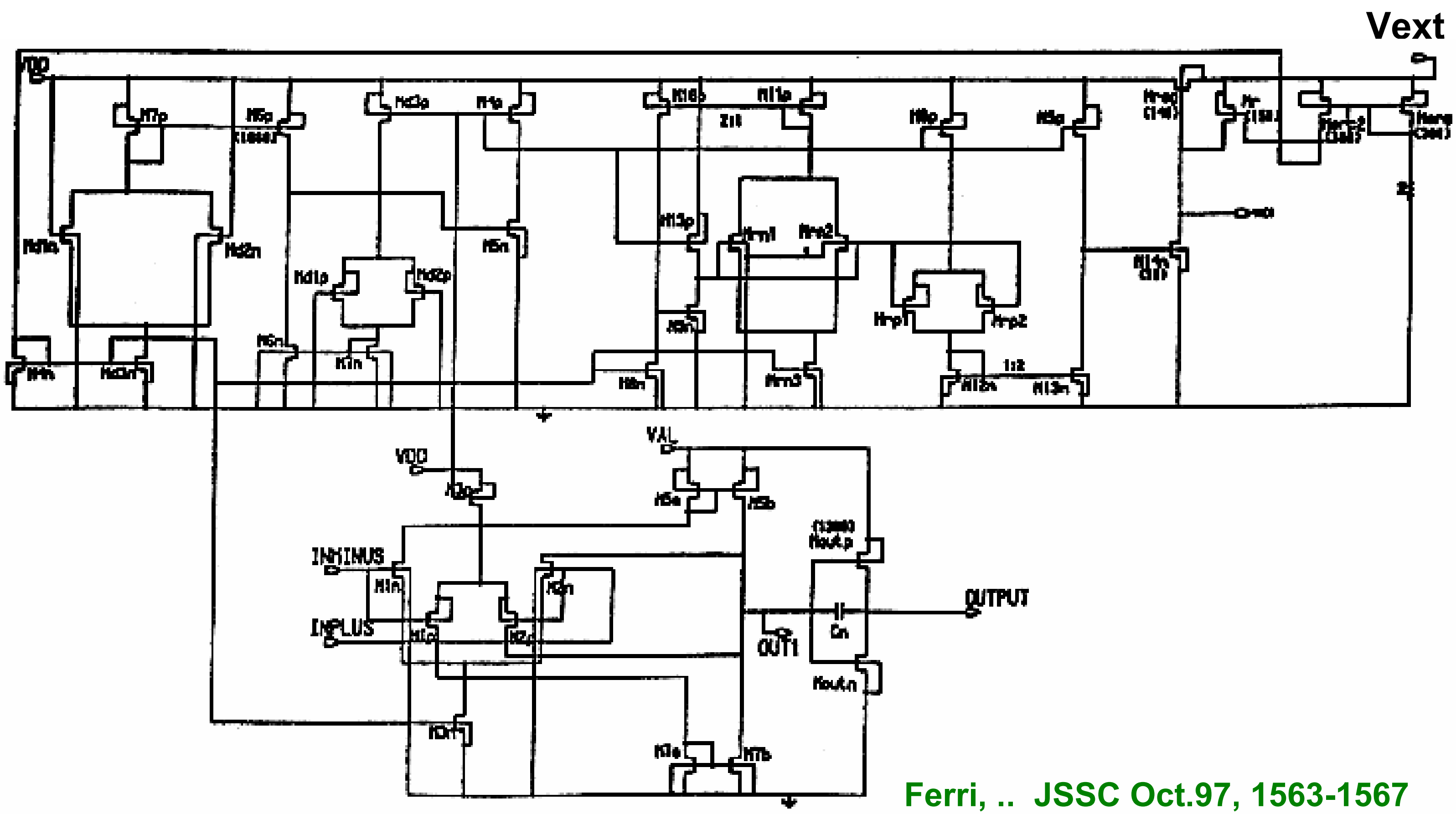


Ferri, .. JSSC Oct.97, 1563-1567

Internal V_{DD} regulator

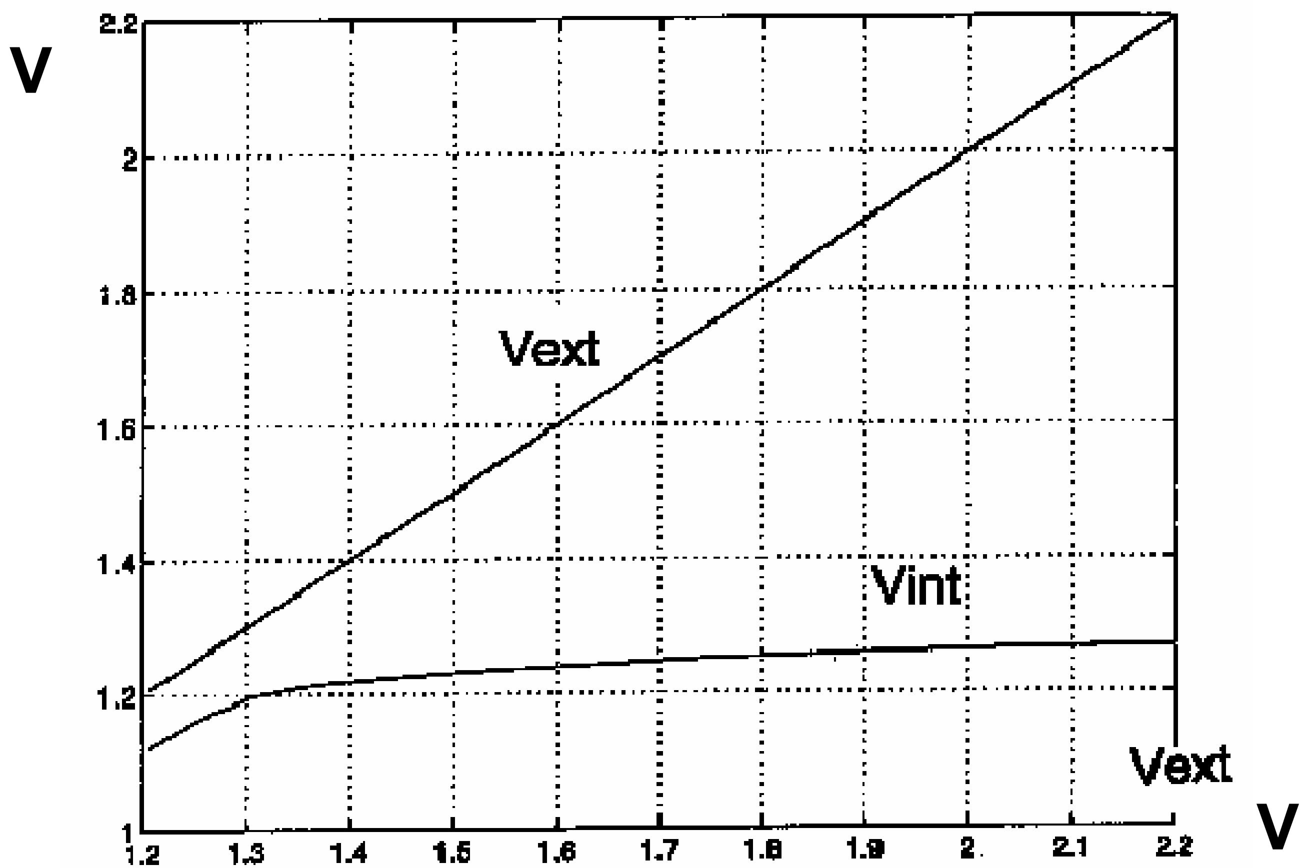


Total amplifier schematic

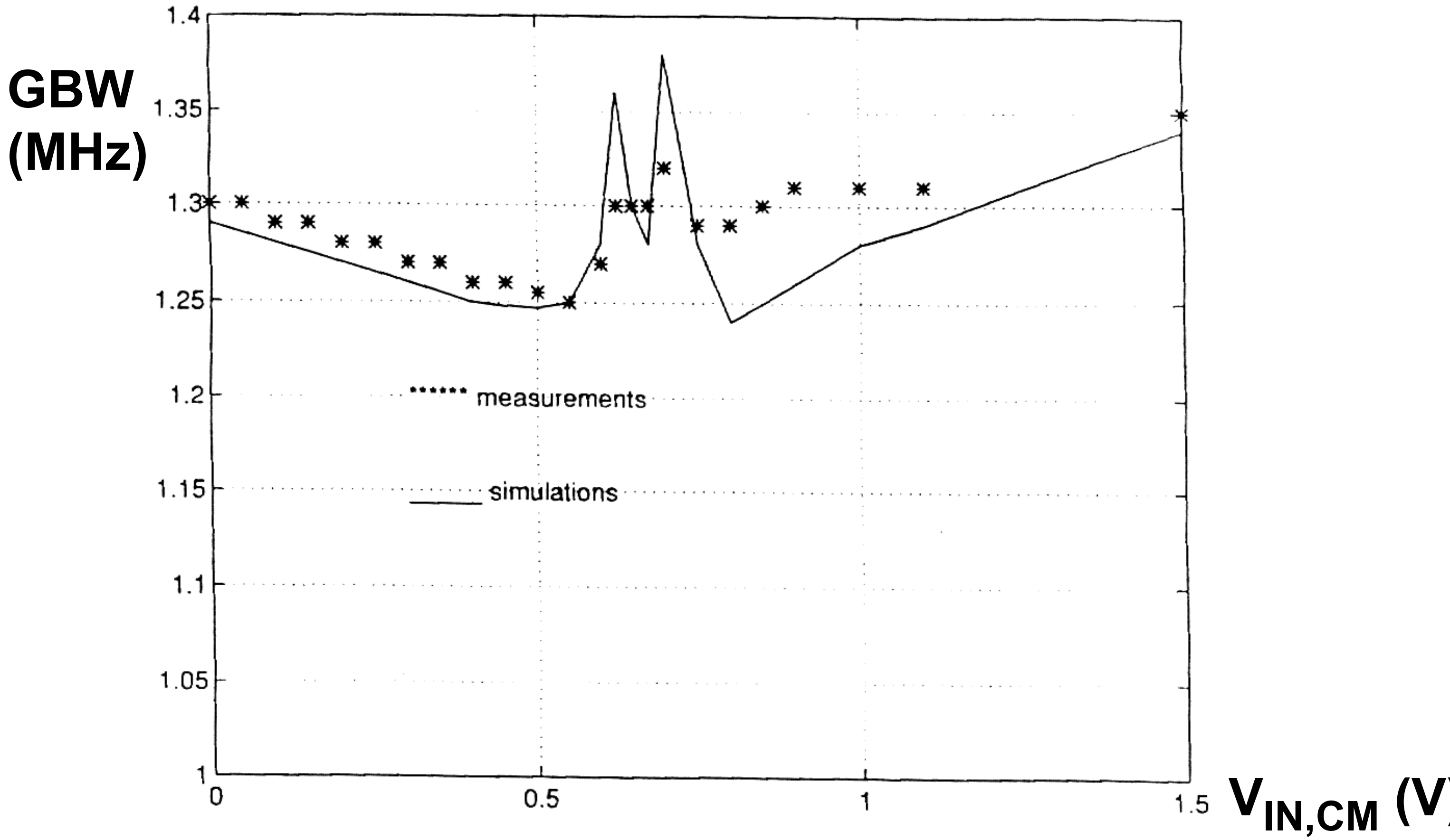


Ferri, .. JSSC Oct.97, 1563-1567

Internal supply voltage



GBW error



GBW (MHz)
1.4
1.35
1.3
1.25
1.2
1.15
1.1
1.05
1
0
0.5
1
1.5
$V_{IN,CM}$ (V)
****** measurements
simulations

Rail-to-rail amp. with V_{DD} regulator : Specs

V_{DDmin} = 1.3 V

GBW = 1.3 MHz in C_L = 15 pF

g_{m1} = 200 μS

I_{DSn1} = 10 μA

W/L_{in} = 830

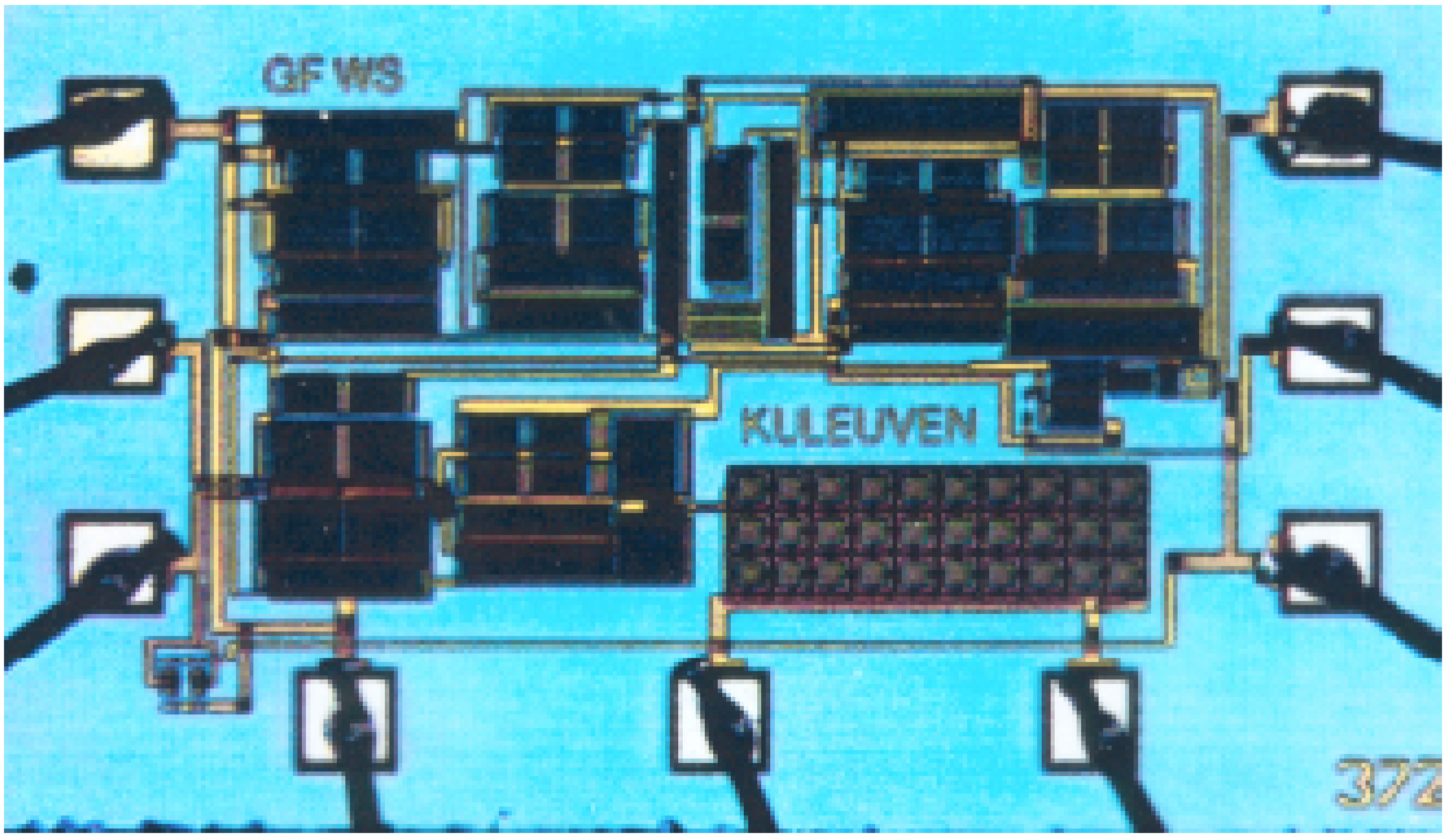
I_{TOT} = 354 μA

$V_{in,eq}$ = 25 $nV_{RMS}/\sqrt{Hz}$

$V_{in,offset}$ = 0.8 mV (3σ = 0.2 mV)

Ferri, .. JSSC Oct.97, 1563-1567

Rtr Opamp with V_{DD}-regulator



Rail-to-rail with V_{DD} regulator : min V_{DD}

$$V_{DDmin} = 2\,(V_{GS} + V_{DSsat})$$

$$= 2\,(V_{GS} - V_T + V_T + V_{GS} - V_T)$$

$$= 2\,[V_T + 2(V_{GS} - V_T)]$$

$$= 2\,[0.6 + 2(0.15)] = 1.8 \text{ V}$$

$$= 2\,[0.3 + 2(0.10)] = 1.0 \text{ V} \quad !!!!$$

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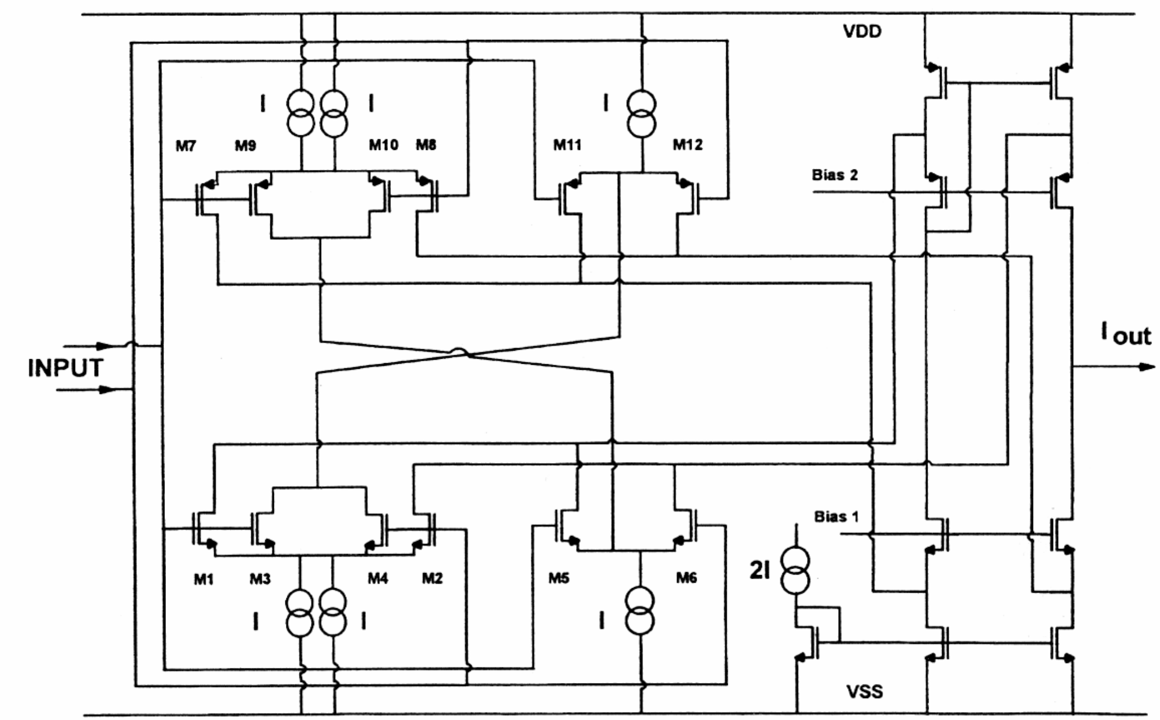
Rail-to-rail opamp with current summation



3.3 V

2.3 mW

(2.2 V min.)

Gm ± 10%

THD :

-55 dB

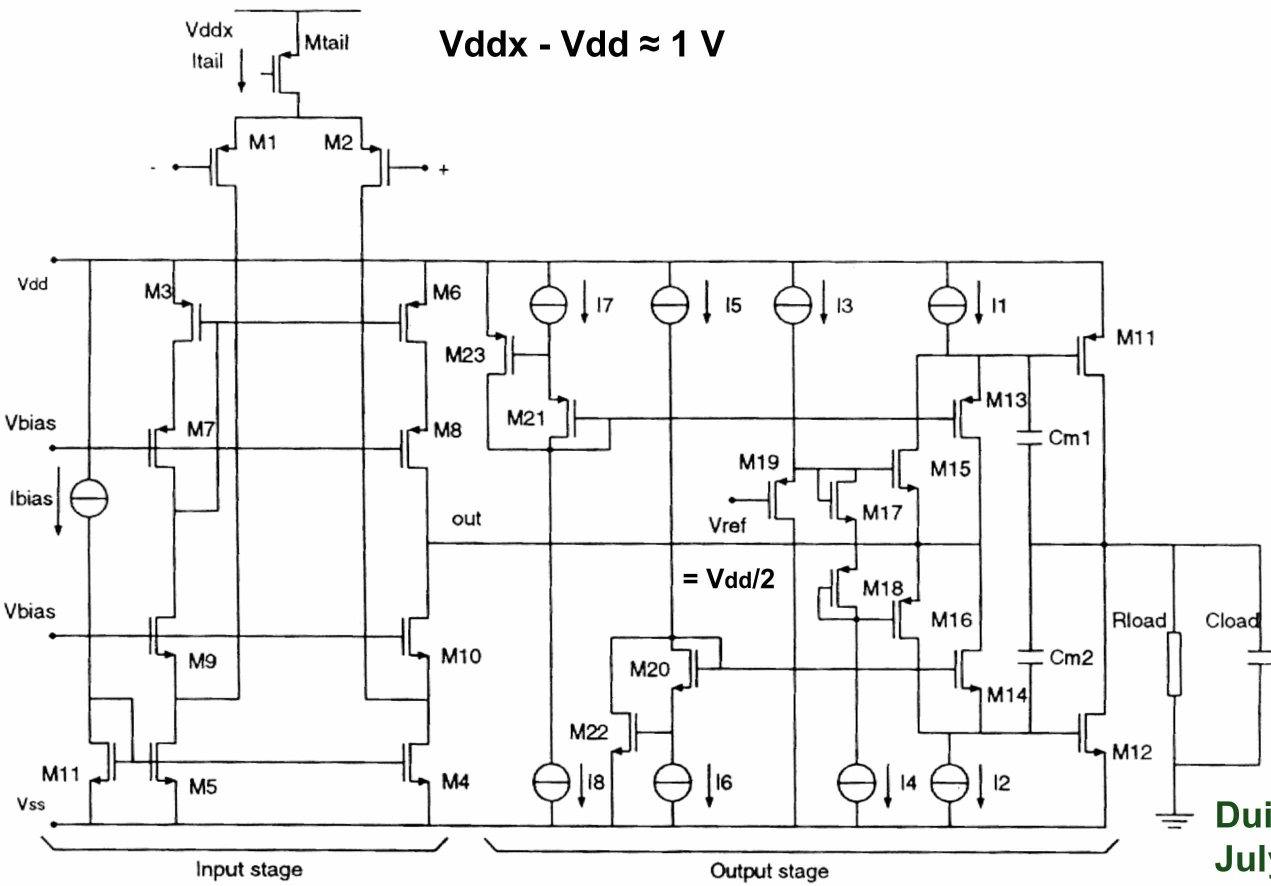
40 MHz

9 mW

0.5 μm CMOS

Redman-White, JSSC May 97, 701-712

Opamp with voltage multiplier



1.8 - 3.3 V

0.75 mA

6.5 MHz

On 3 V :

2.8 V_{ptpt}

THD :

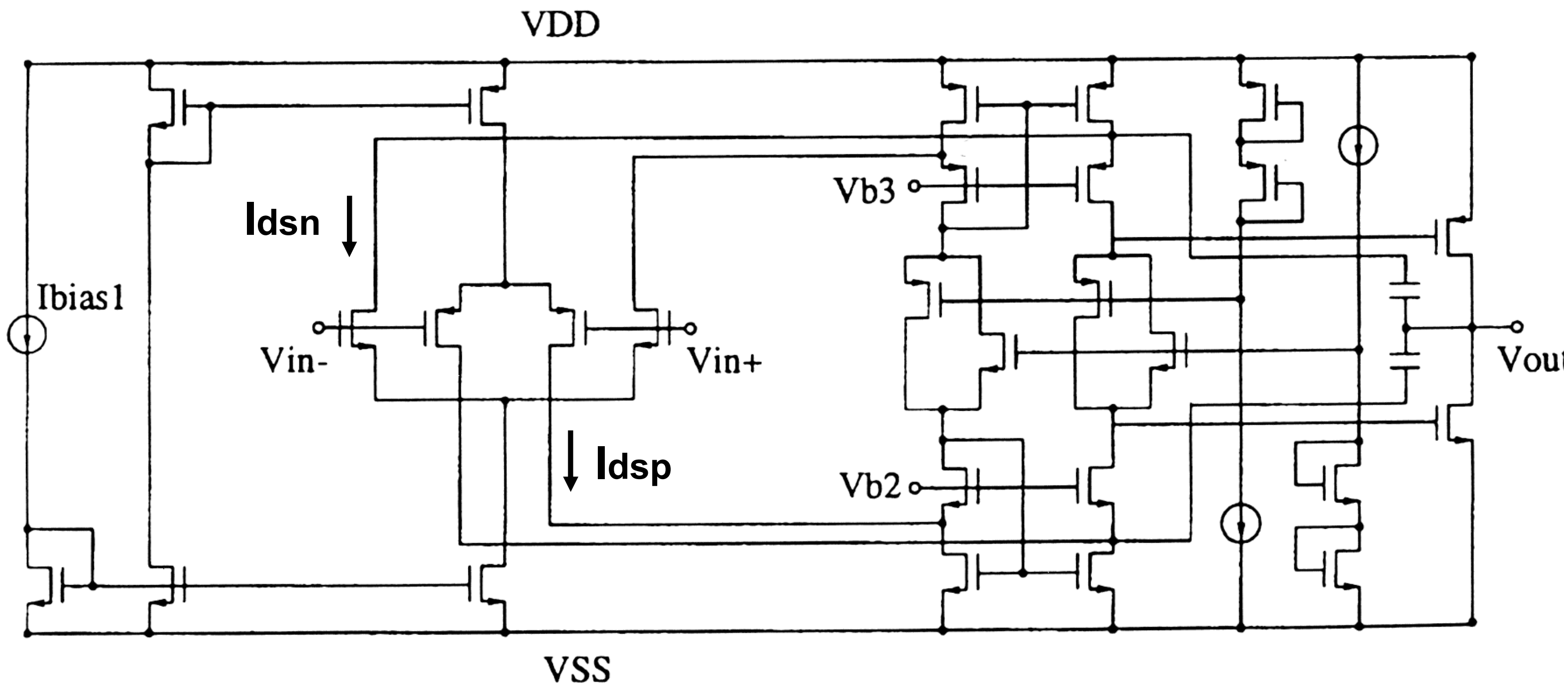
-90 dB /10kΩ

-81 dB/32 Ω

0.5 μm CMOS

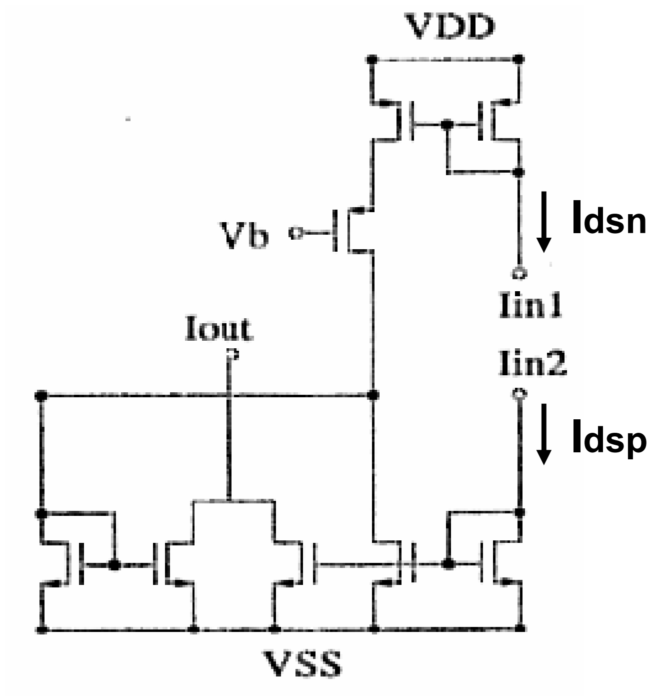
Duisters, .., JSSC July 98,pp.947-955

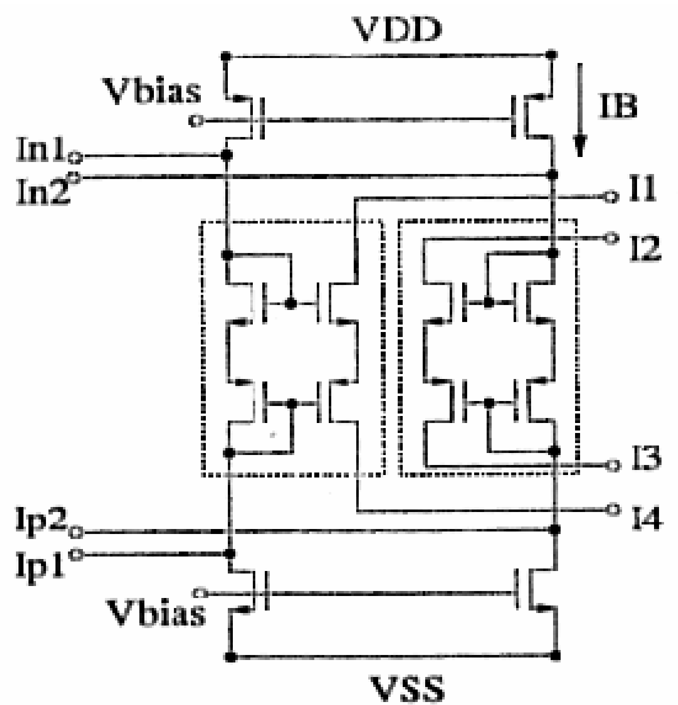
Rail-to-rail opamp with differential signal proc.



Ref.Lin, AICSP 1999, 153-162

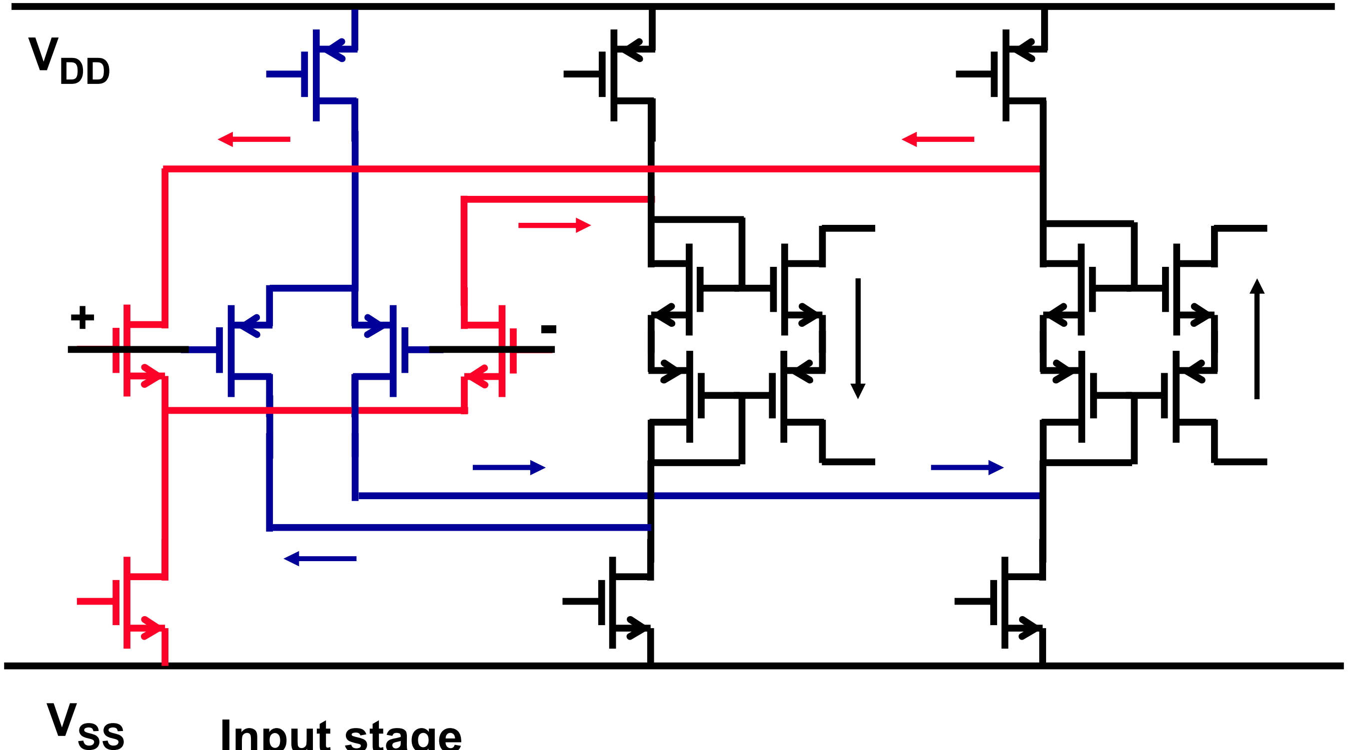
Maximum-current selecting circuits





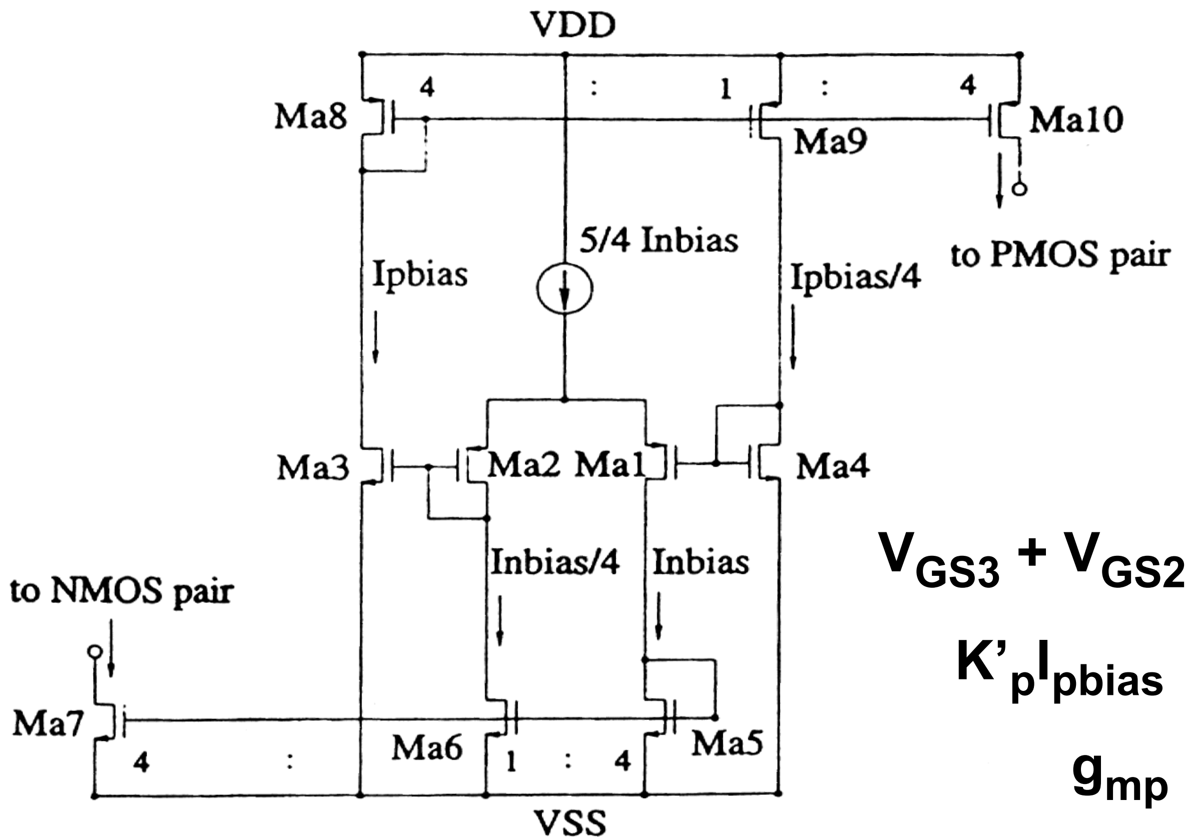
Ref.Lin, AICSP 1999, 153-162

Maximum-current selecting circuit



Input stage

Transconductance equalizer circuit



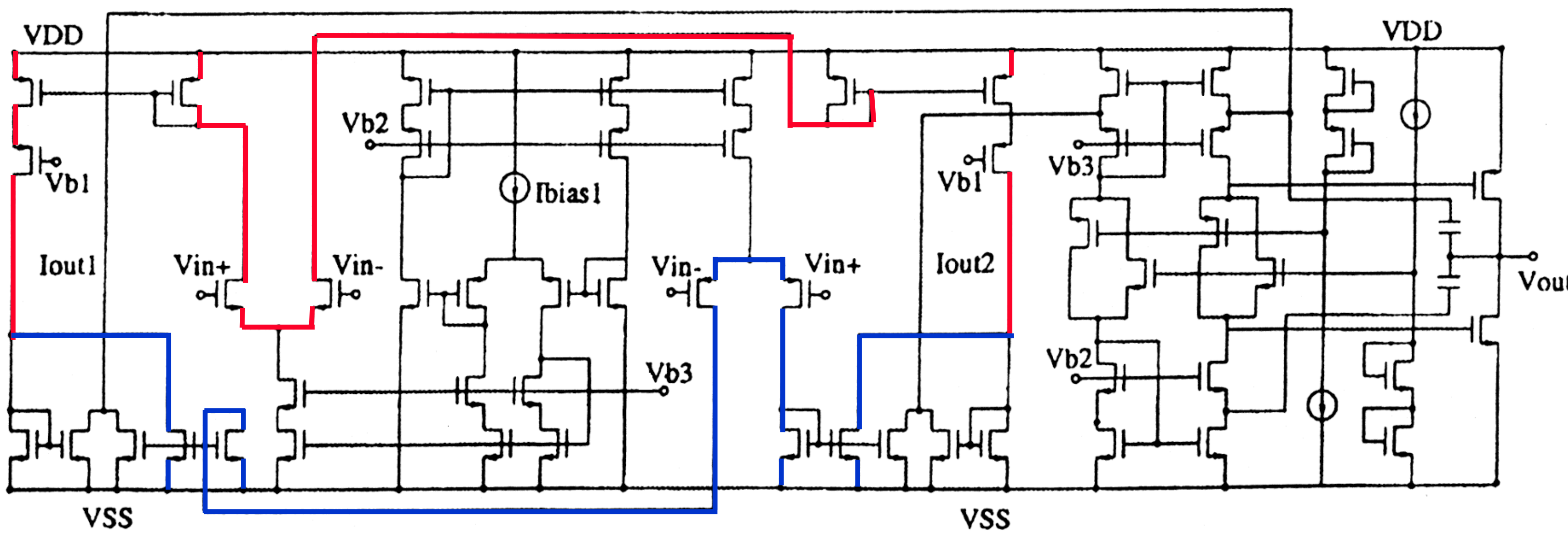
$$V_{GS3} + V_{GS2} = V_{GS1} + V_{GS4}$$

$$K'_p I_{pbias} = K'_n I_{nbias}$$

$$g_{mp} = g_{mn}$$

Ref.Lin, AICSP 1999, 153-162

Rail-to-rail opamp with max.-current selector

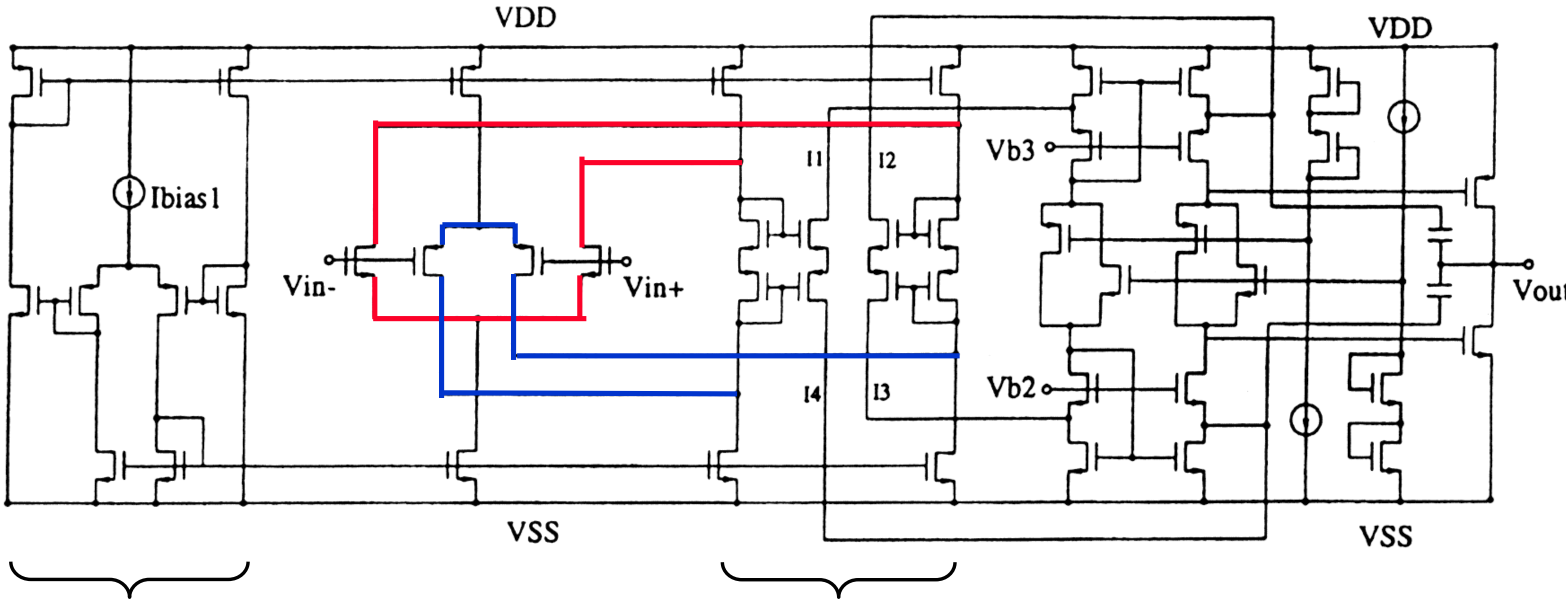


max.curr.selector g_m equalizer max.curr.selector

1.9 MHz 20 pF
3 V/ 0.26 mA

Ref.Lin, AICSP 1999, 153-162

Rail-to-rail opamp with max.-current selector



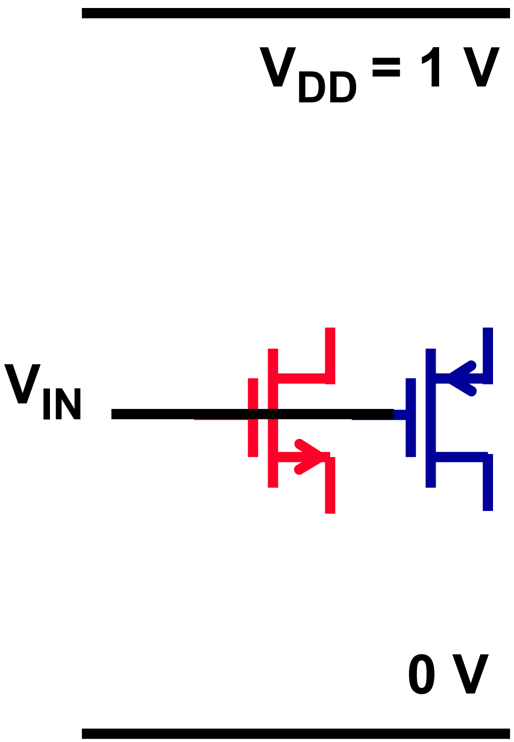
g_m equalizer

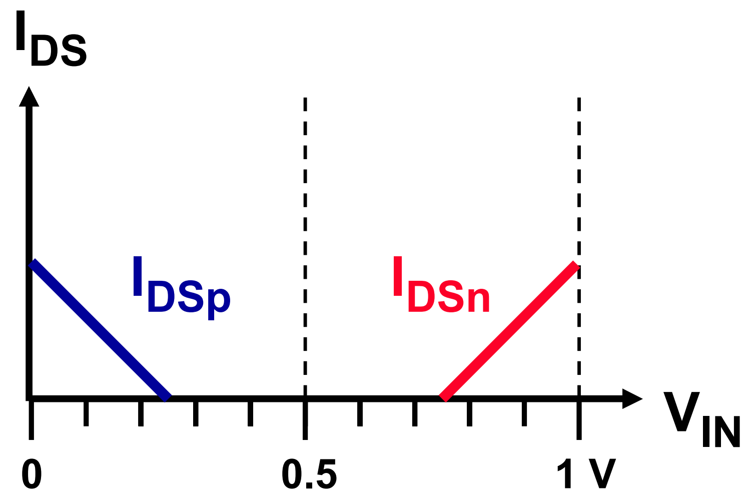
max.curr.selector

1.9 MHz 20 pF
3 V/ 0.26 mA

Ref.Lin, AICSP 1999, 153-162

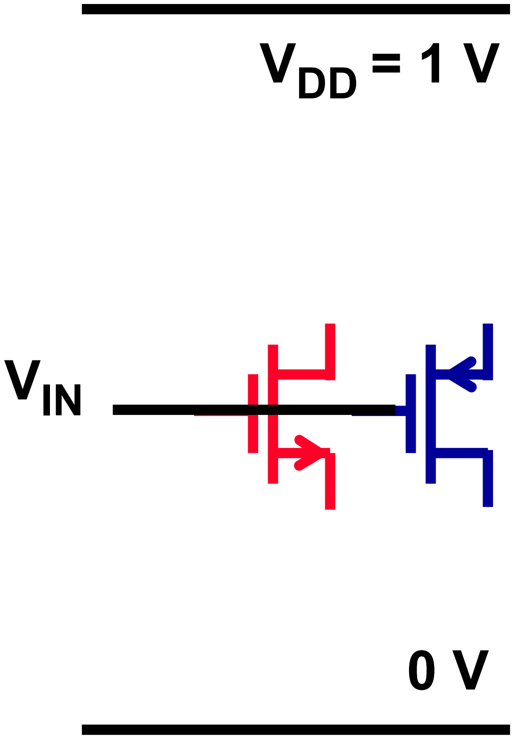
Rail-to-rail opamp on 1 Volt Supply

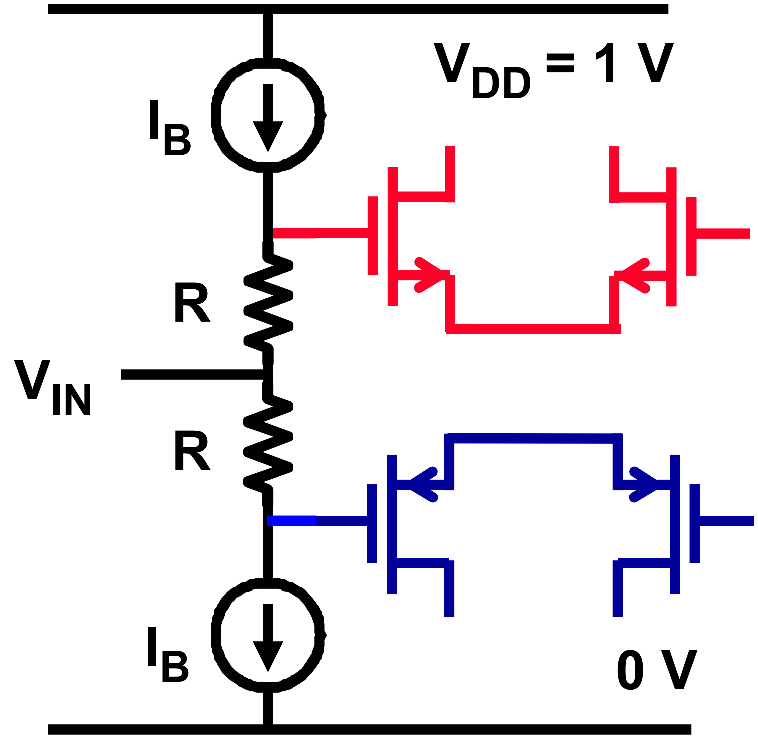




Ref.Duque-Carrillo, JSSC Jan.2000, 33-43

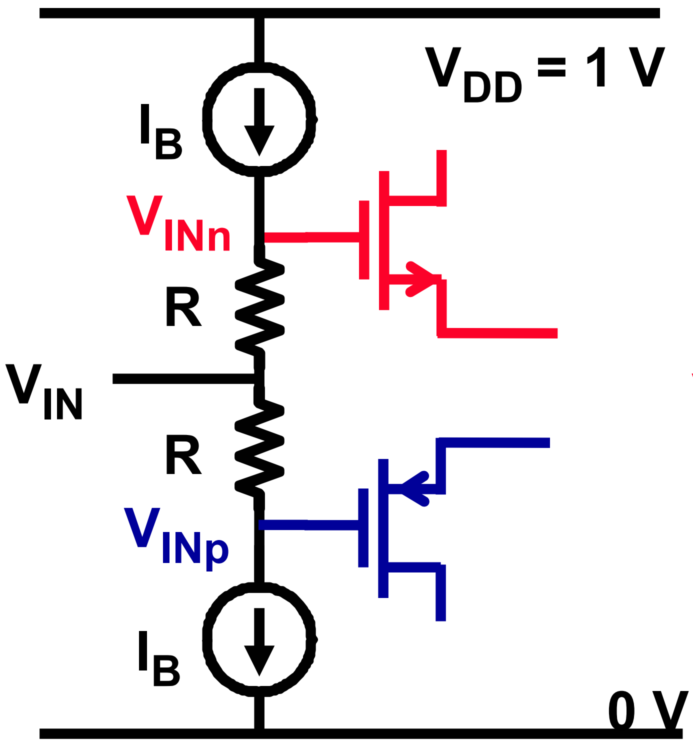
Rail-to-rail opamp on 1 Volt

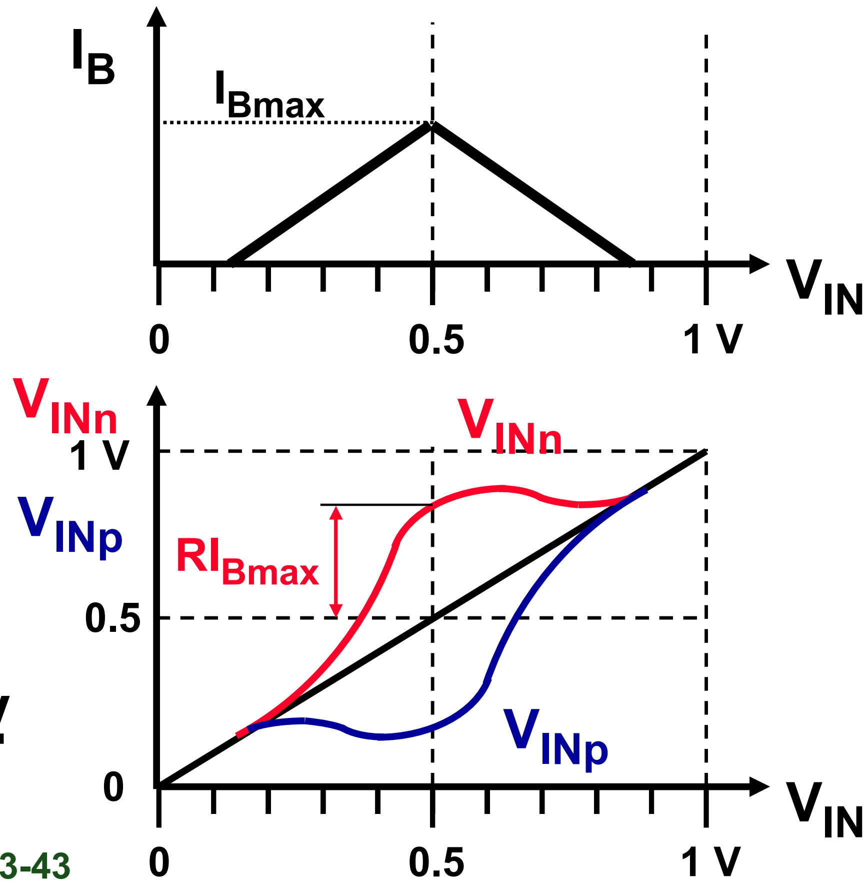




Ref.Duque-Carrillo, JSSC Jan.2000, 33-43

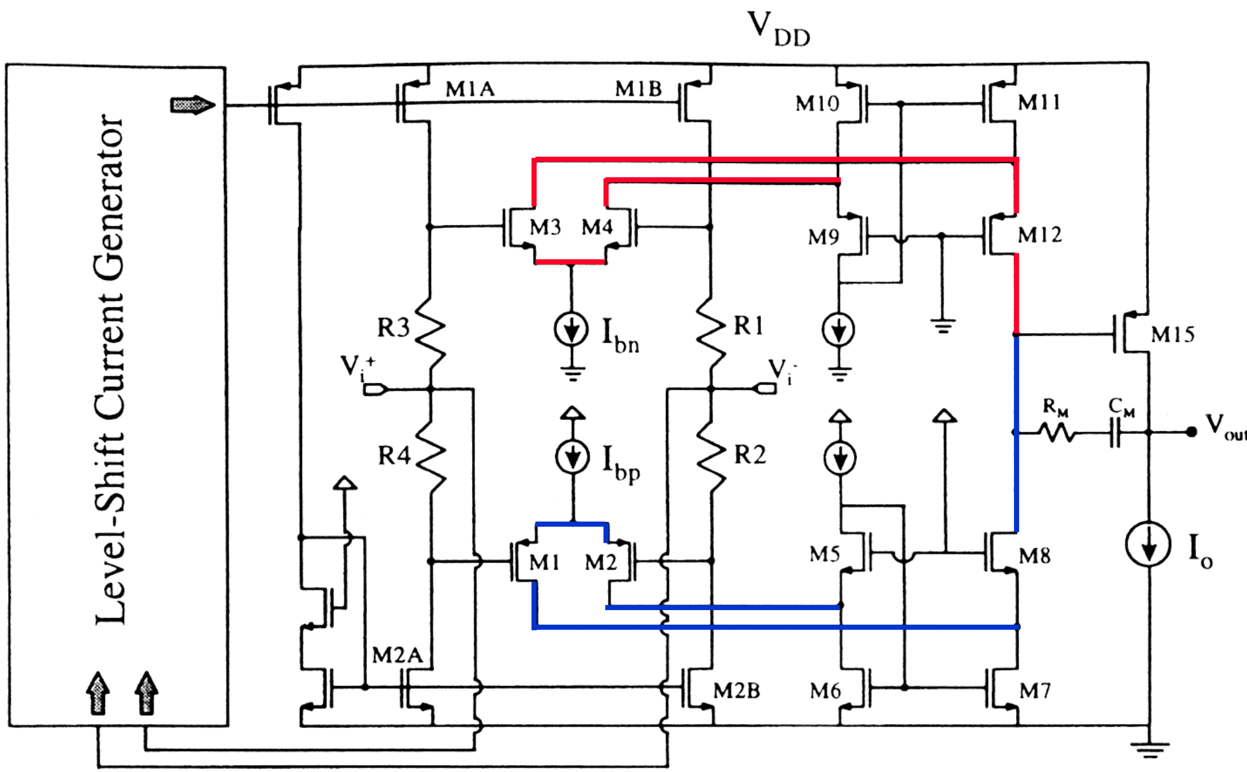
Rail-to-Rail opamp on 1 Volt





Ref.Duque-Carrillo, JSSC Jan.2000, 33-43

RtR opamp : full opamp schematic



2 MHz

15 pF

1 V

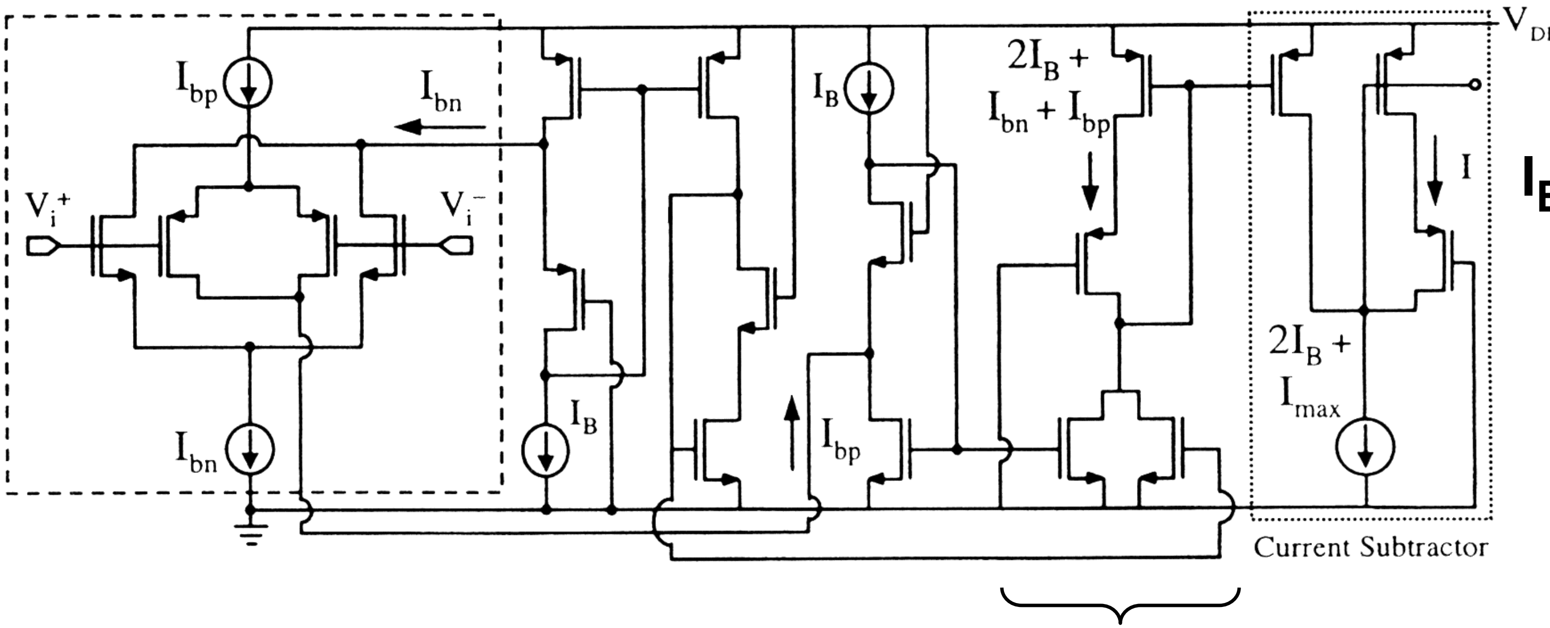
0.4 mA

I_{offset} < 1 μA

Ref.Duque-Carrillo, JSSC Jan.2000, 33-43

R ↑ I_{offset} ↓ noise ↑

RtR opamp : current generator



I_B

current summer

Ref.Duque-Carrillo, JSSC Jan.2000, 33-43

Comparison rail-to-rail input amplifiers

Type	Ref.	$\Delta g_m/g_m$ %	GBW MHzpF/mW	I_{TOT} μA	V_{DDmin} V
3x Curr.mirr.	JSSC-12-94	15	110	150	3
Electr. Zener	JSSC-7-96	6	70	215	2.7
Curr.switch	AICSP-5-94	8	1.1	500	3.3
Curr.regulat.	CICC 97	4	210	200	1.5
Regulat. VDD	JSSC-10-97	6	43	350	1.3
MOST translin.	AICSP-6-94	8	4.2	800	2.5
Improv.CMRR	JSSC-2-95	9	3	1400	5
Max. current	AICSP-1-99	10	77	260	3
Resistive input	JSSC-1-00	x	75	400	1

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References

T. Duisters, etal, ”A -90 dB THD rail-to-rail input opamp using a new local charge pump in CMOS", IEEE Journal Solid-State Circuits, Vol. SC-33, pp. 947-955, July 1998.

R. Duque-Carillo, etal, "A 1 V rail-to-rail operational amplifier in standard CMOS technology", IEEE Journal Solid-State Circuits, Vol. SC-35, pp. 33-43, Jan. 2000.

G.Ferri, W.Sansen, ”A rail-to-rail constant-gm low-voltage CMOS operational transconductance amplifier", IEEE Journal Solid-State Circuits, Vol. SC-32, pp. 1563-1567, Oct.1997.

R. Hogervorst, etal, "A compact power-efficient 3V CMOS rail-to-rail input/output operational amplifier for VLSI cell libraries", IEEE Journal Solid-State Circuits, Vol. SC-29, pp. 1504-1512, Dec.1994.

R. Hogervorst, etal, ”Compact CMOS constant-gm rail-to-rail input stage with gm-control by an electronic Zener diode", IEEE Journal Solid-State Circuits, Vol. SC-31, pp. 1035-1040, July 1996.

R. Lin, etal, "A compact power-efficient 3V CMOS rail-to-rail input/output operational amplifier for VLSI cell libraries", Analog Integrated Circuits and Signal Processing, Kluwer Ac., pp. 153-162, Jan.1999.

References

E. Peeters, etal, "A compact power-efficient 3V CMOS rail-to-rail input/output operational amplifier for VLSI cell libraries", CICC 1997.

W. Wu, etal, ”Digital-compatible high-performance operational amplifier with rail-to-rail input and output stages", IEEE Journal Solid-State Circuits, Vol. SC-29, pp. 63-66, Jan 1994.

0.12 chap12

Class AB and driver amplifiers

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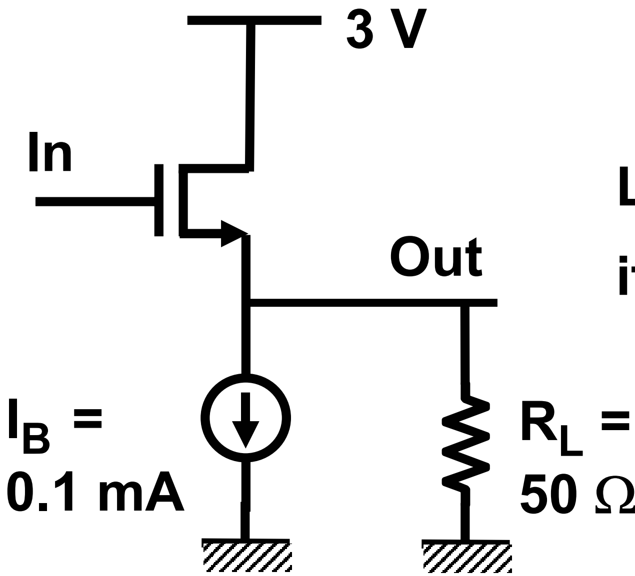
Outline

- Problems of class AB drivers
- Cross-coupled quads
- Adaptive biasing
- I_Q control with translinear circuits, etc.
- Current feedback and other principles
- Low-Voltage realizations

Ref.: W. Sansen : Analog Design Essentials, Springer 2006

CMOS Output stage problem



$I_{max} < I_B$

$V_{Out} = V_{In} - V_{GS}$

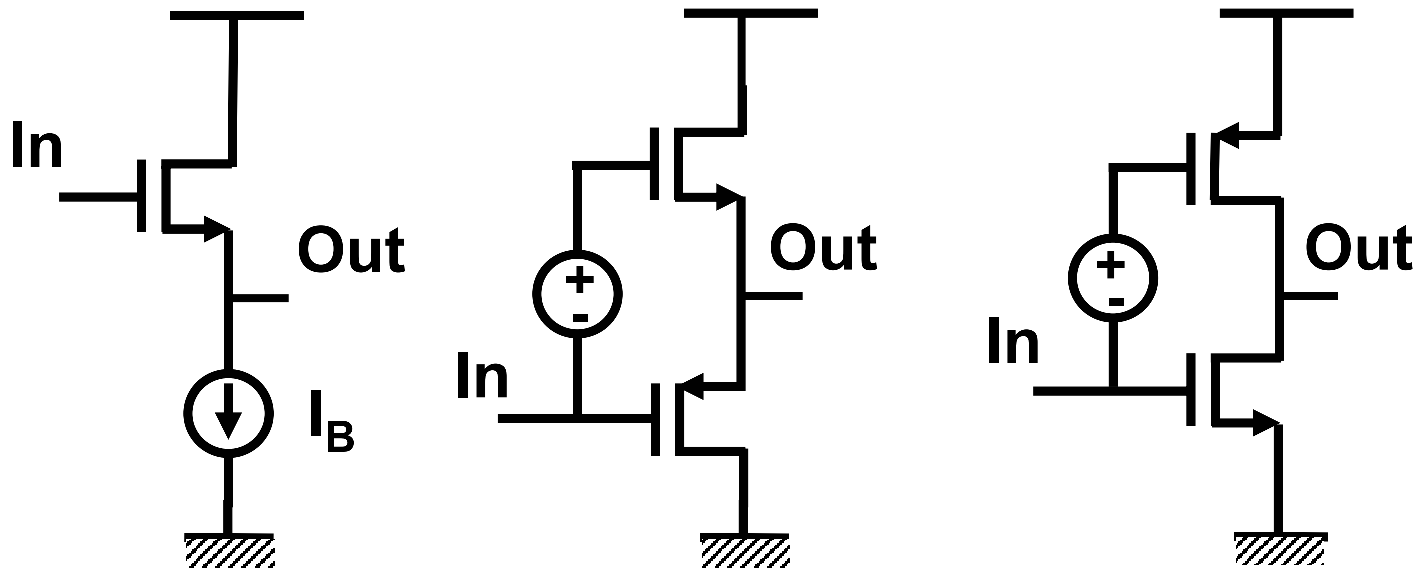
Low power consumption:

if I_B = 0.1 mA : $V_{out,\ peak}$ = 5 mV_{peak}

For $V_{out,\ peak}$ = 1 V_{peak} : I_B = 20 mA

High power consumption !

CMOS Output stages



$I_{max} < I_B$

$V_{Out} = V_{In} - V_{GS}$

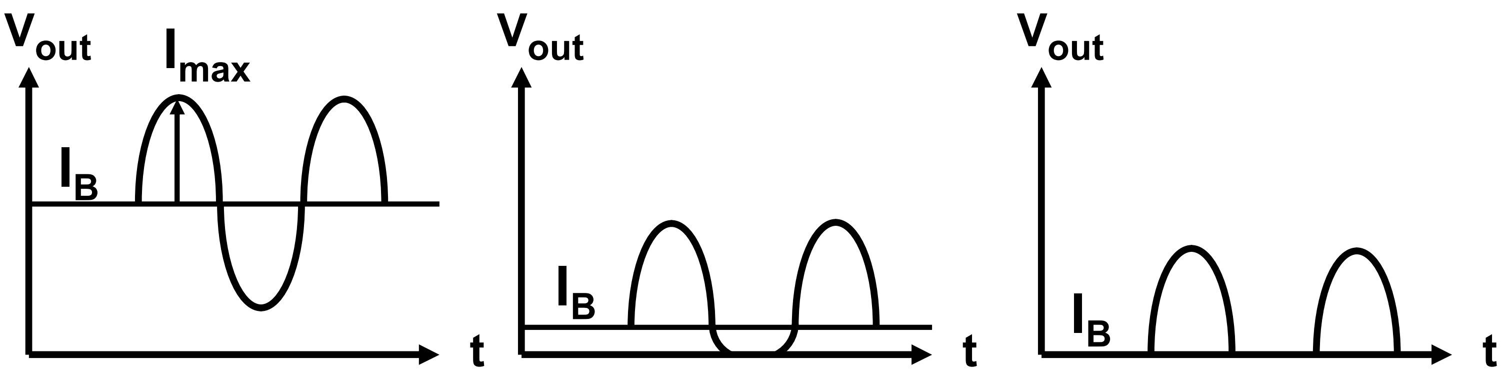
Push-Pull

$V_{out,max} = V_{DD} - 2V_{GS}$

Amplifier

Rail-to-rail

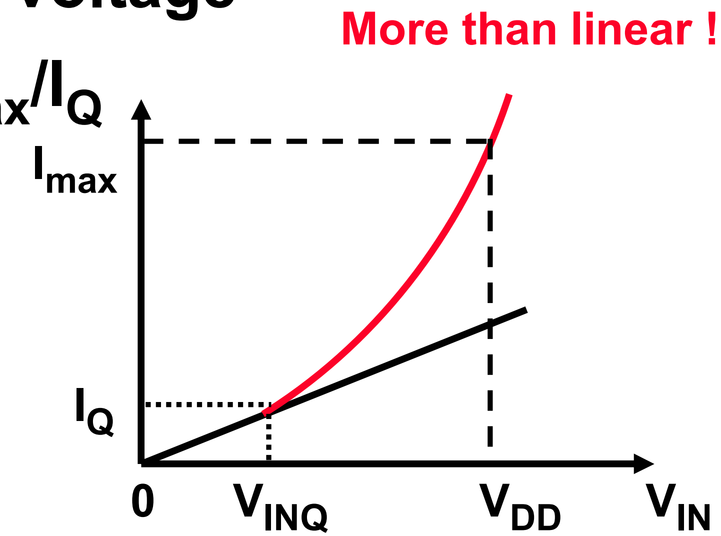
Class A, AB, B, etc



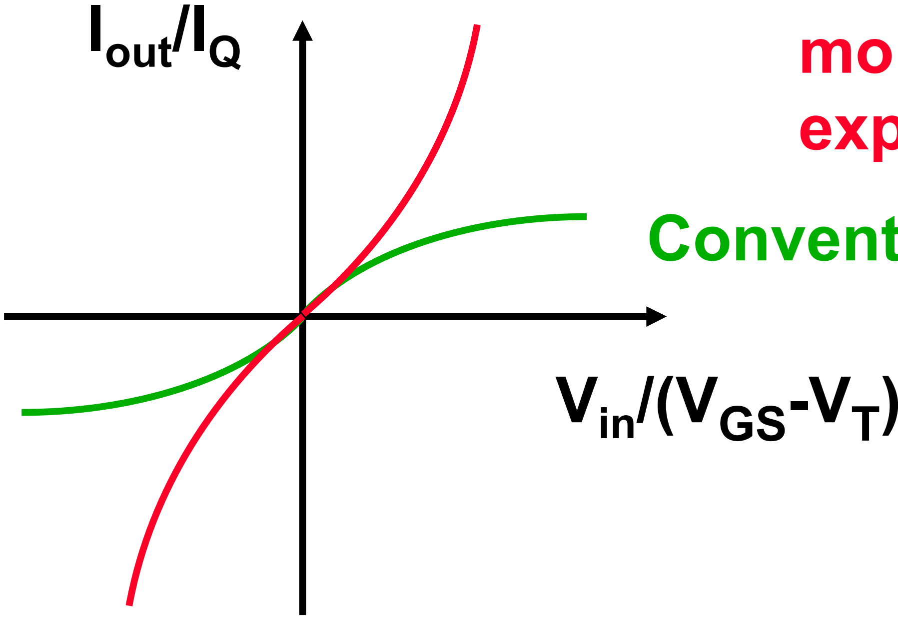
$I_B > I_{max}$	$I_B < I_{max}$	$I_B = 0$
Class A	**Class AB**	**Class B**
High power !		**Distortion !**

Requirements class -AB stages

- Rail-to-rail output swing
- Accurate control of quiescent current I_Q
  - Must be low
  - Independent of supply voltage
- Large drive capability I_{max}/I_Q
- Small area



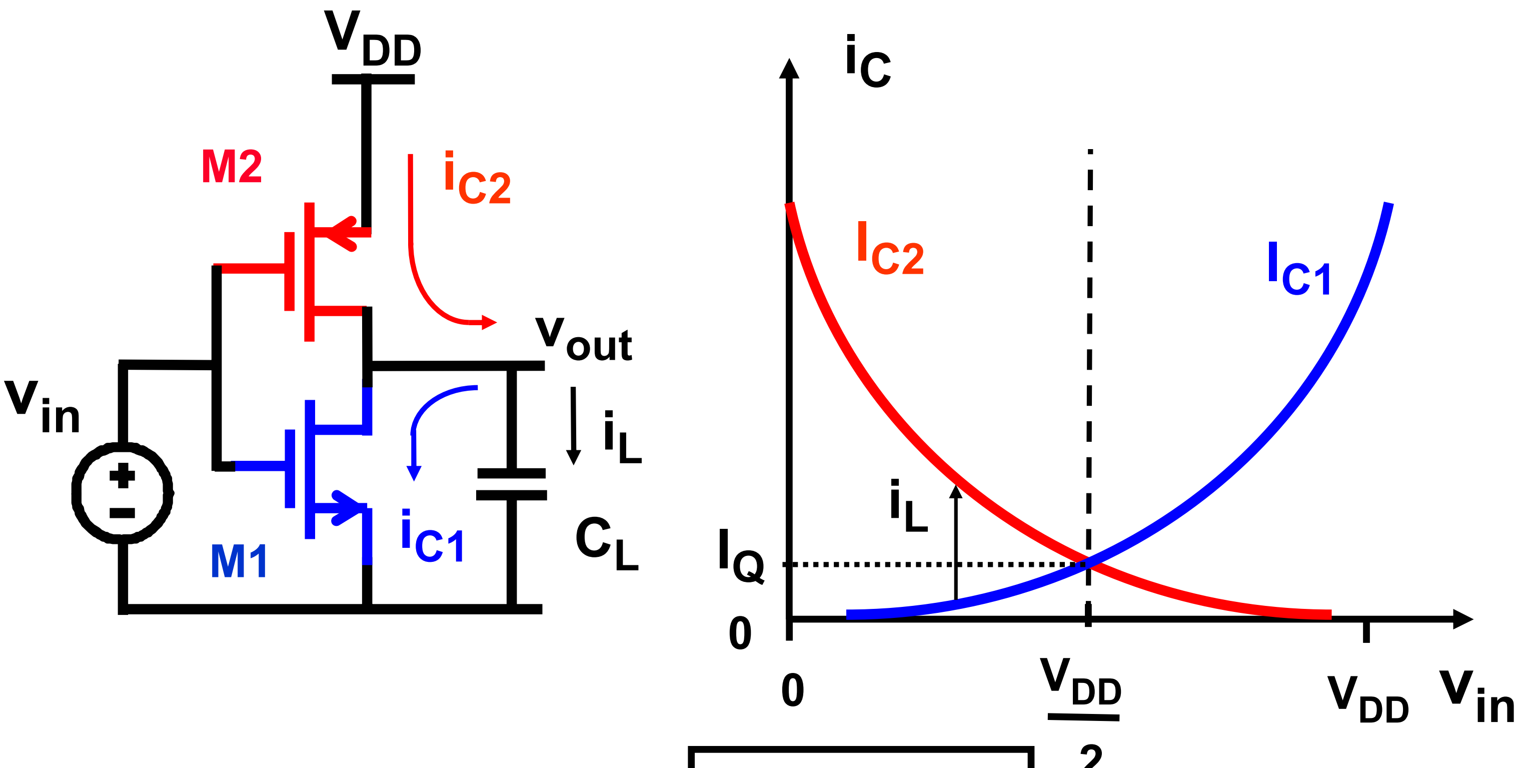
Class -AB stages



Class AB diff stage :
more than linear !
expanding !

Conventional diff. Stage :
limiter !

Simple CMOS class-AB amplifier

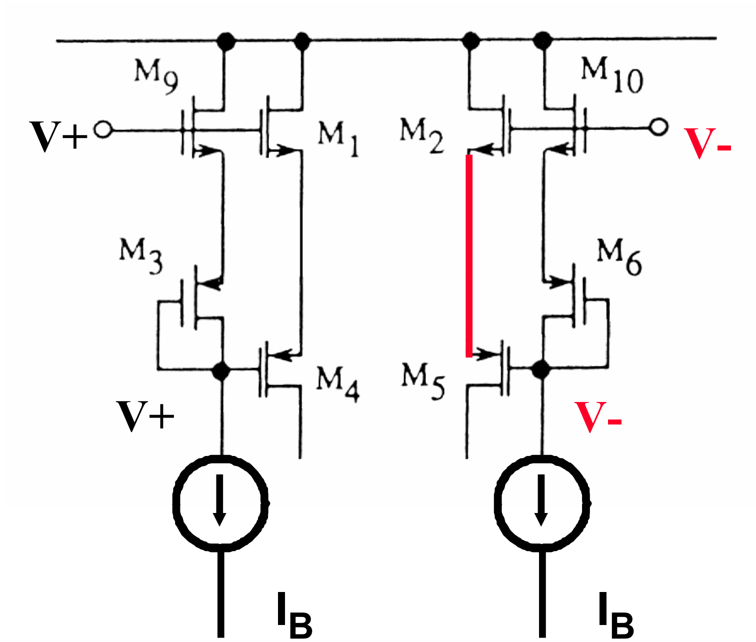


$$\boxed{i_L = i_{C2} - i_{C1}}$$

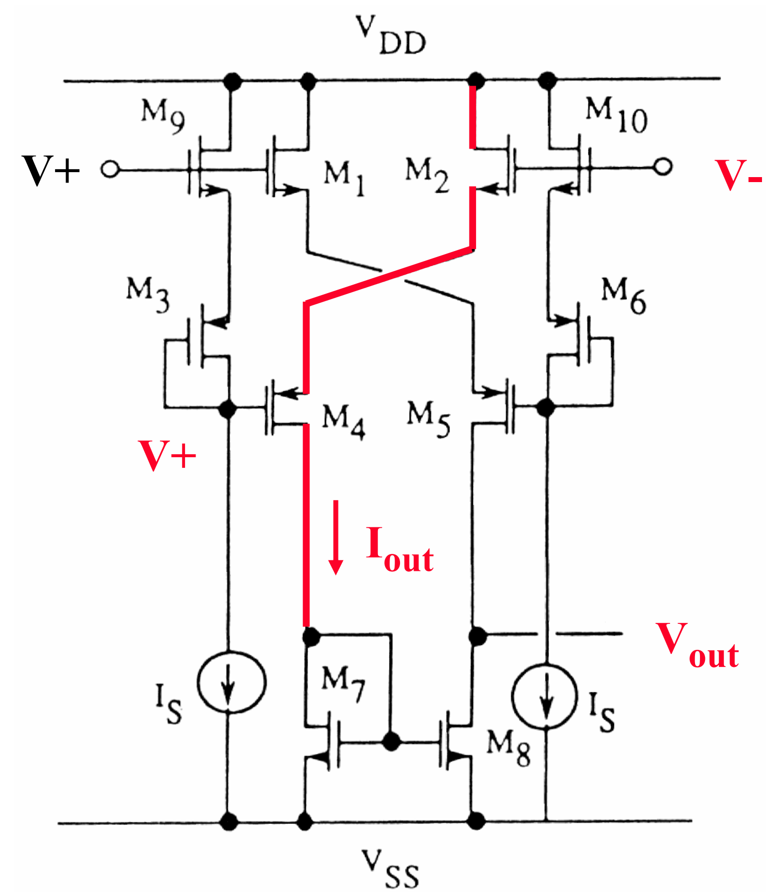
Outline

- Problems of class AB drivers
- Cross-coupled quads
- Adaptive biasing
- I_Q control with translinear circuits, etc.
- Current feedback and other principles
- Low-Voltage realizations

Cross-coupled quad

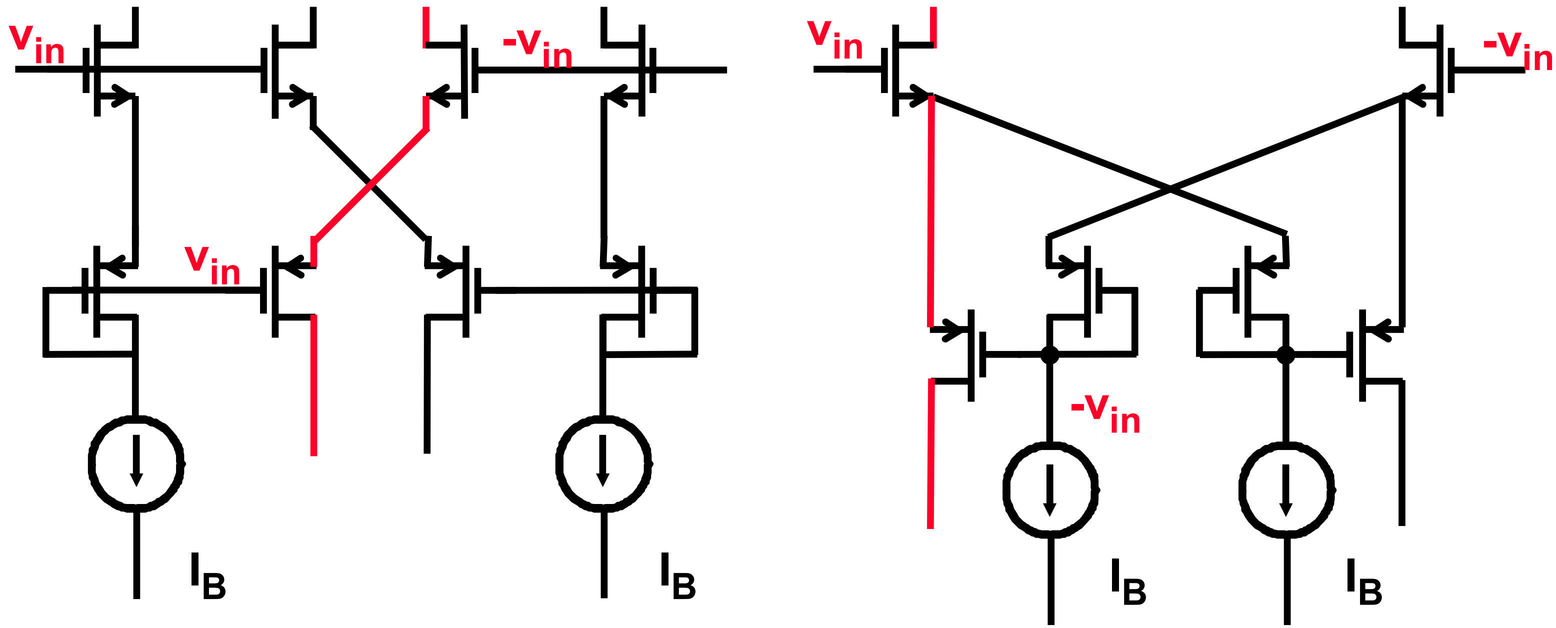


Two super-followers



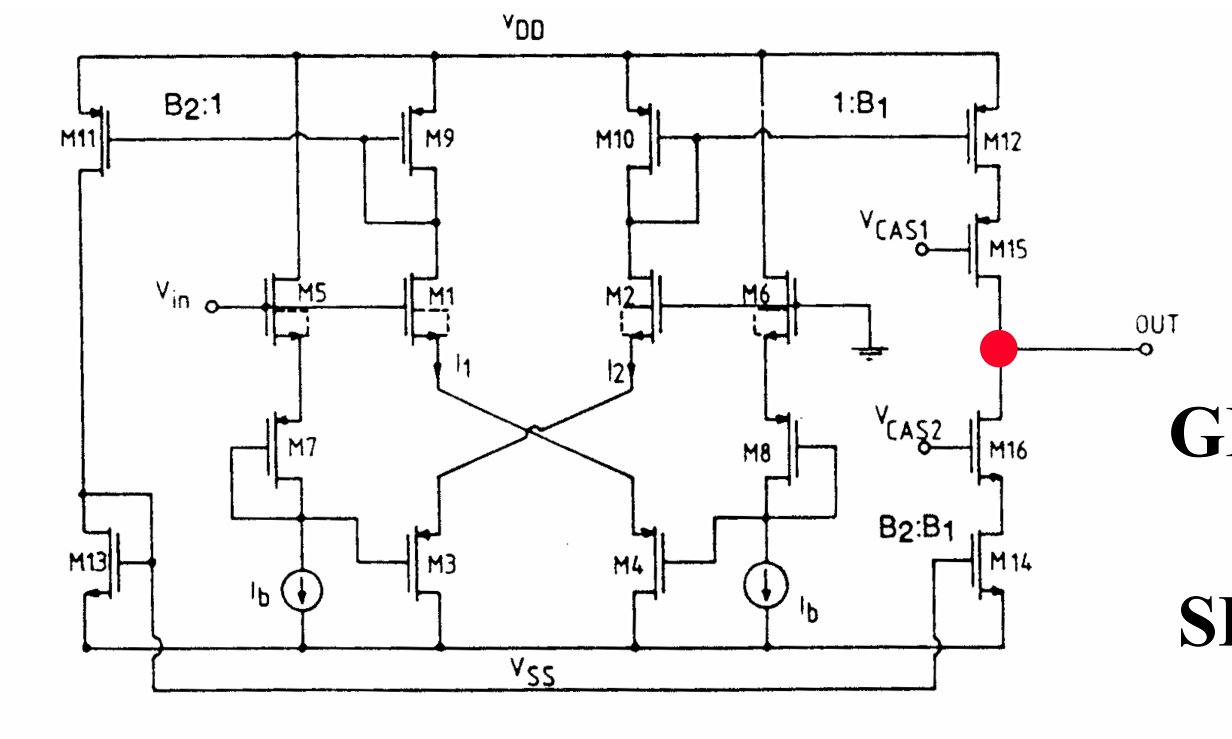
Ref. Castello, JSSC Dec.85, 1122-1132

Other cross-coupled quads



Bipolar Ref. Hearn, JSSC Febr.71

Class AB Input structures

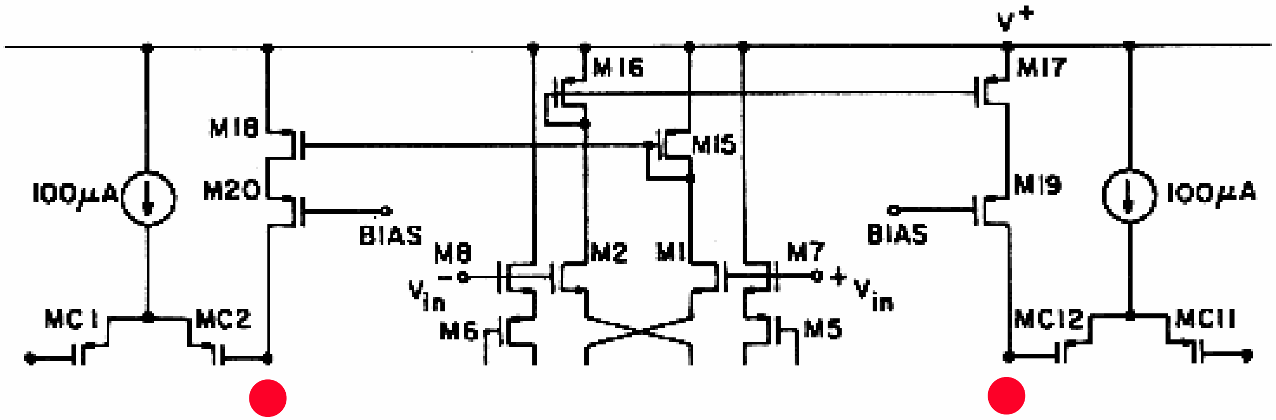


$$GBW = \frac{g_{m1}}{2\pi\ C_L}$$

SR ⇧⇧

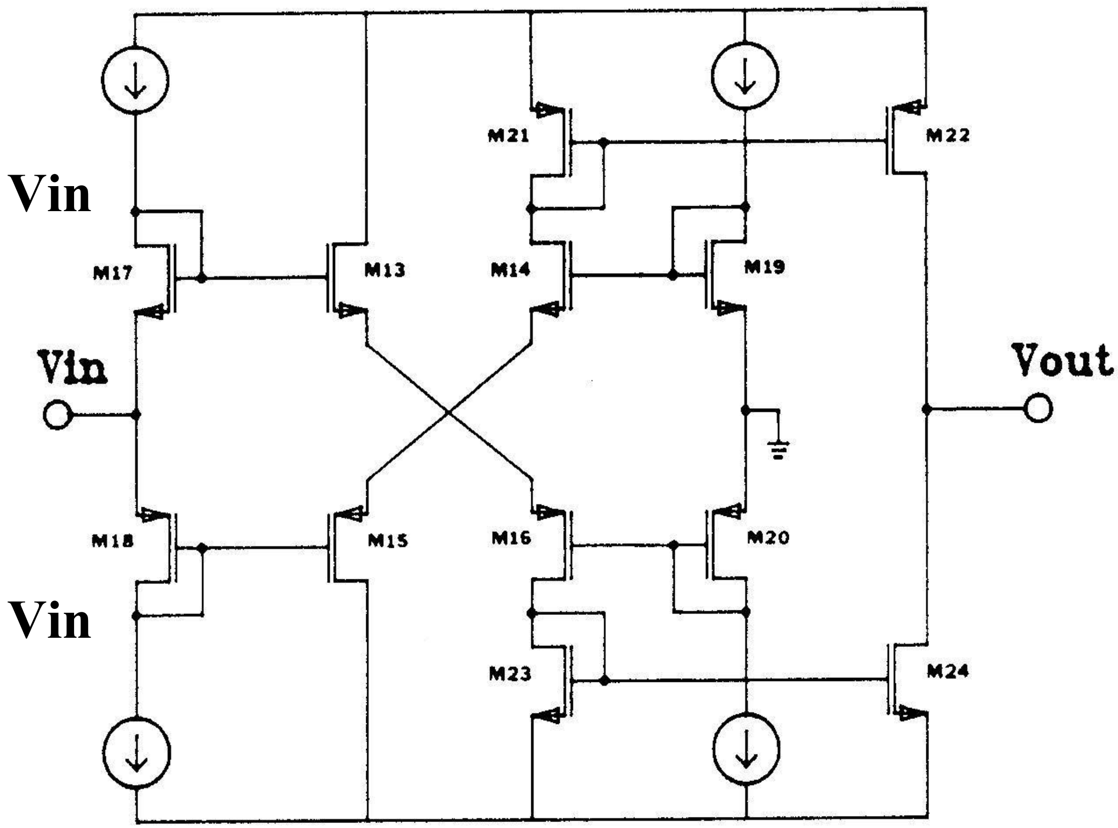
Ref. Halonen, 1987

Class AB fully differential amplifier



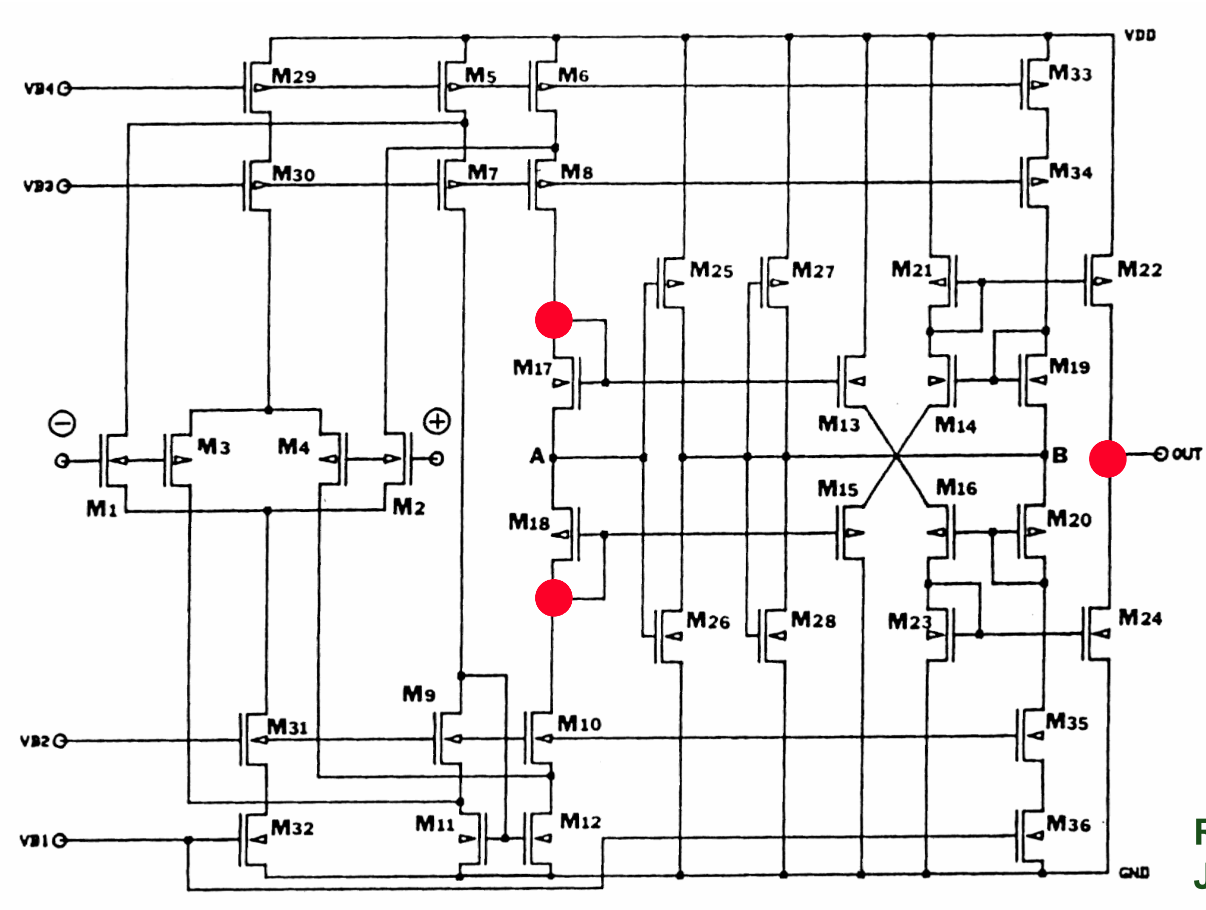
Ref.Lee, JSSC Dec.85, 1103-1113

Double-Push



Ref. Fischer, JSSC June 87, 330-340

Double-Push amp (Fischer)

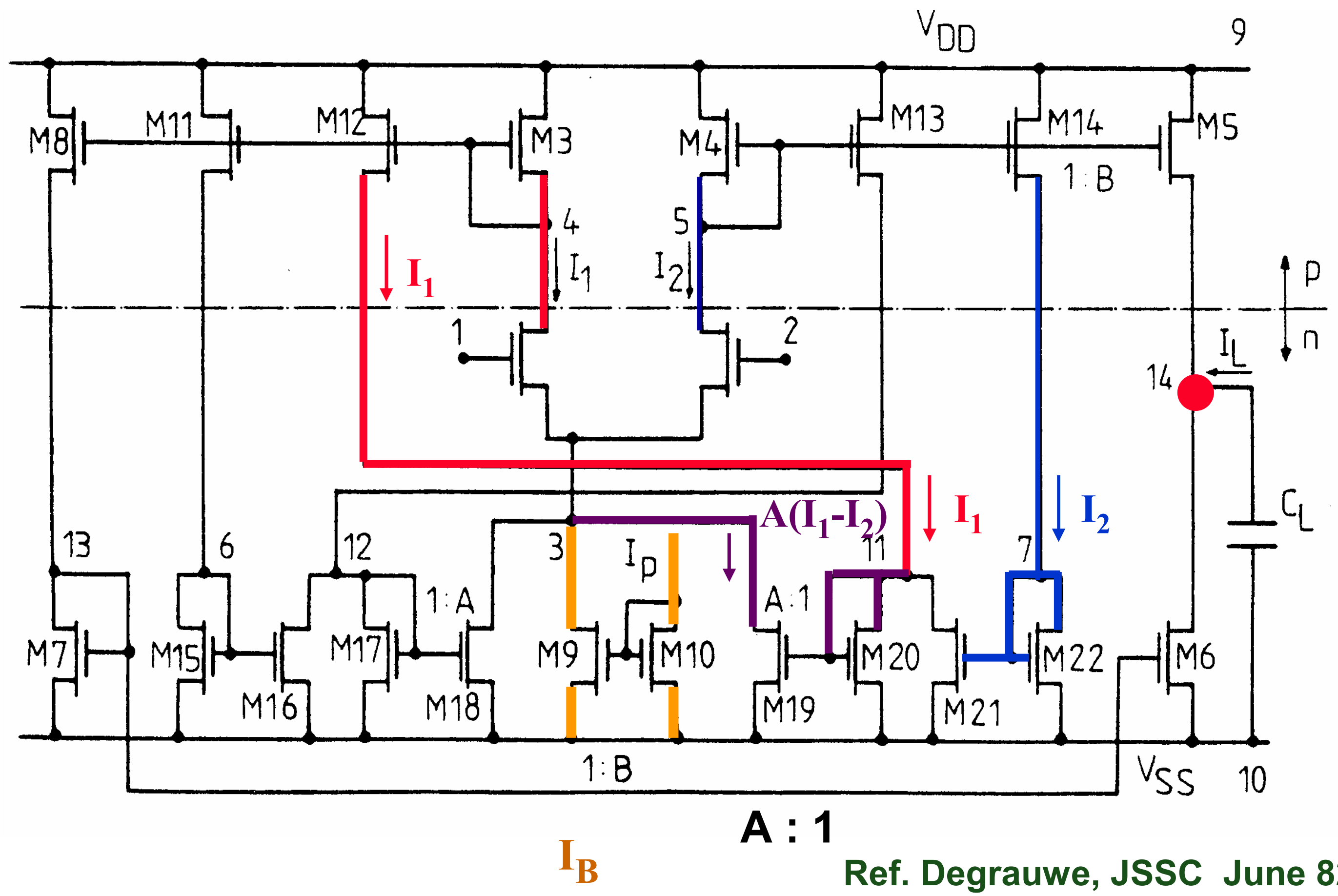


Ref. Fischer, JSSC June 87, 330-340

Outline

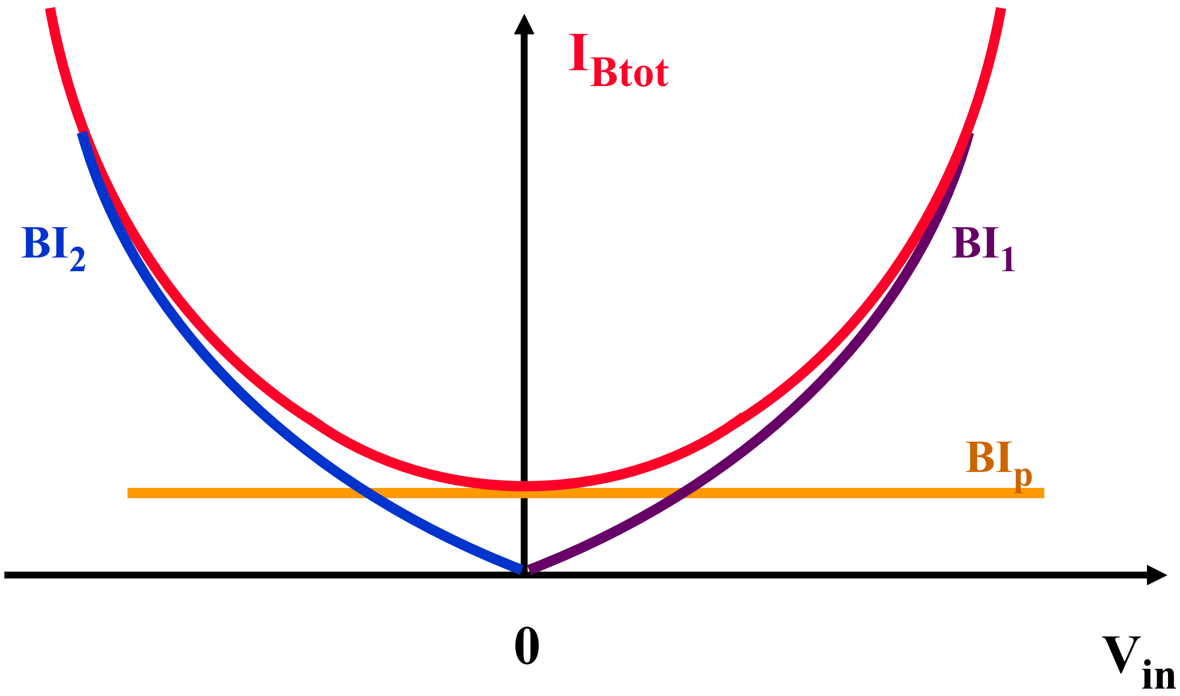
- Problems of class AB drivers
- Cross-coupled quads
- **Adaptive biasing**
- I_Q control with translinear circuits, etc.
- Current feedback and other principles
- Low-Voltage realizations

Adaptive Biasing Amplifier



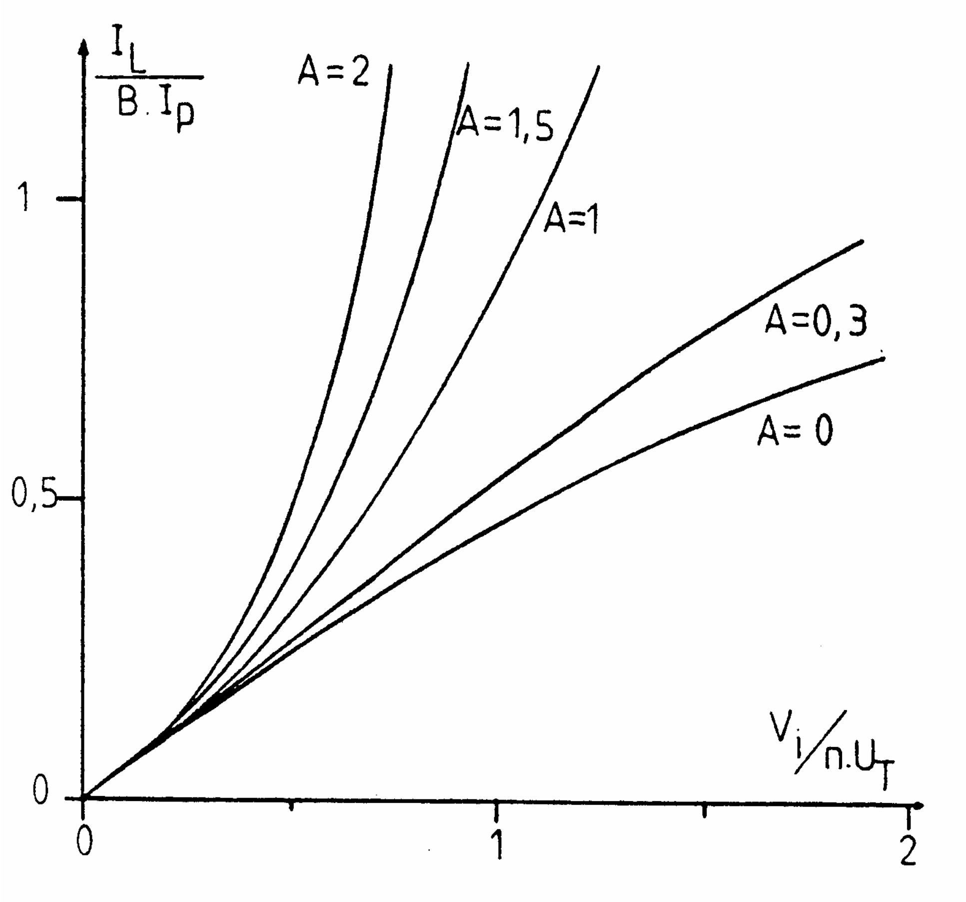
Ref. Degrauwe, JSSC June 82, 522-528

Adaptive Biasing Amplifier : biasing current



Ref. Degrauwe, JSSC June 82, 522-528

Adaptive Bias Amplifier: transfer curve



If A.$\alpha_{mis18\text{-}19} \geq 1$

⇩

UN-stable

e.g. if $\alpha_{mismatch}$ = 10%

⇩

A << 10

Outline

- Problems of class AB drivers
- Cross-coupled quads
- Adaptive biasing
- I_Q control with translinear circuits, etc.
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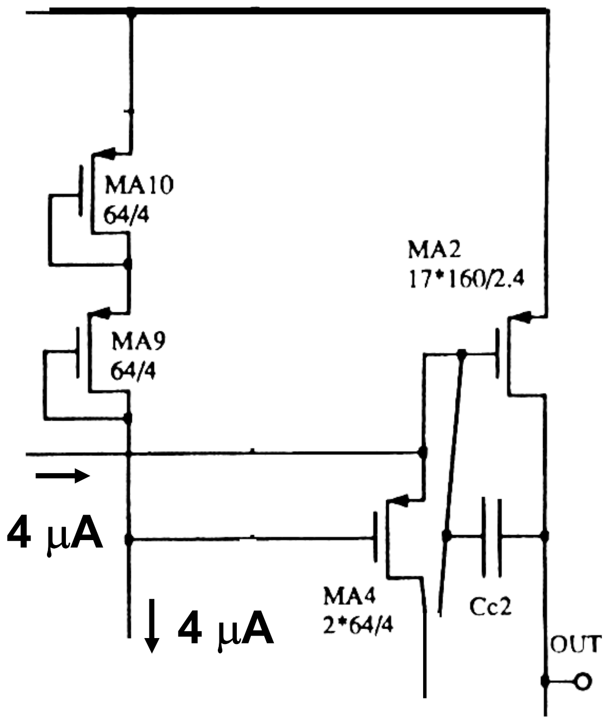
Quiescent current control with translinear loop



$W/L_4 = 2\ W/L_9$ & $W/L_2 = 70.8\ W/L_9$

$I_{DS2} \approx 473\ \mu A$ since $I_{DS9} \approx 4\ \mu A$

Translinear loop :

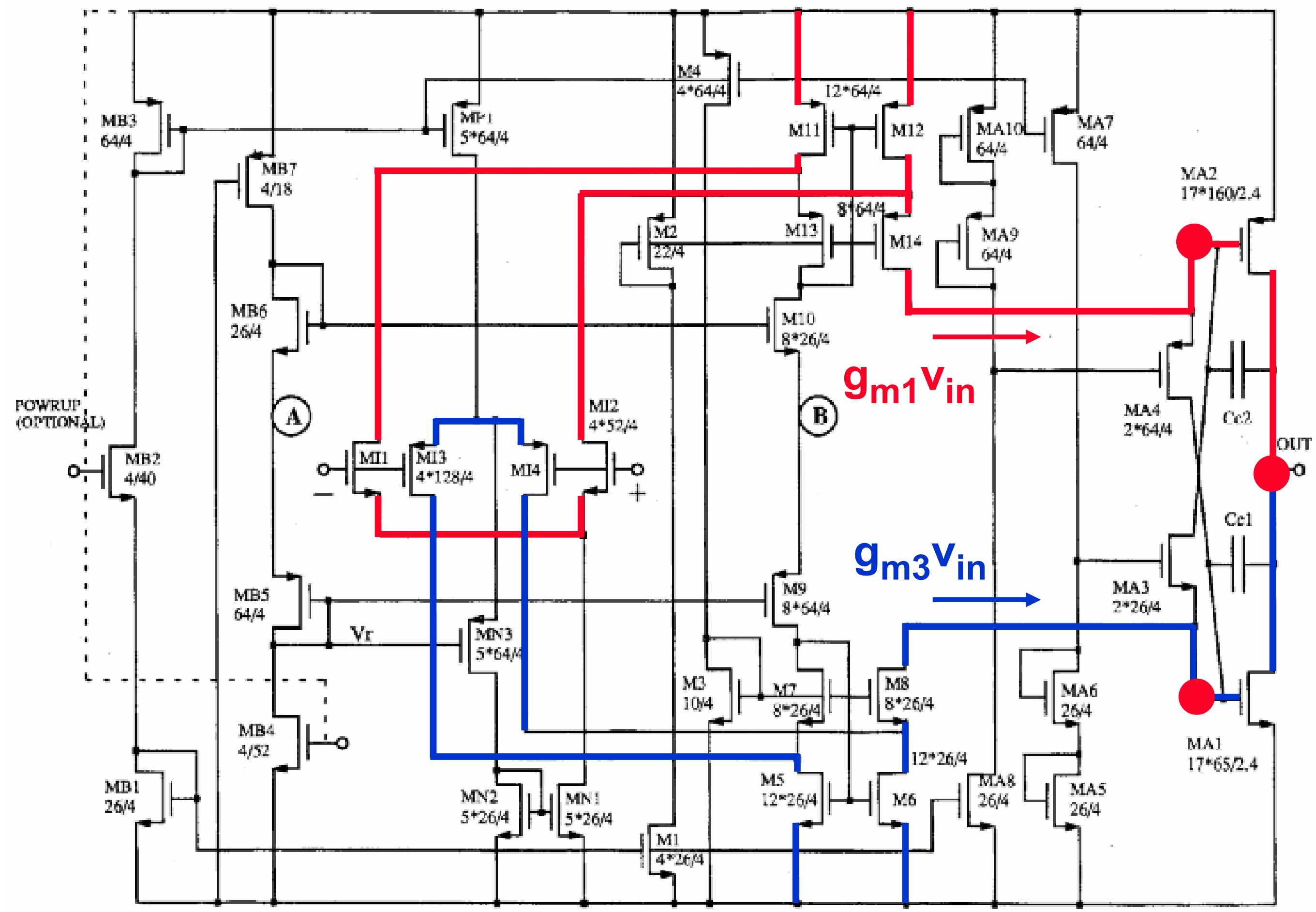
$$V_{GS2} + V_{GS4} = V_{GS9} + V_{GS10}$$

$$V_{GS2} - V_T = \sqrt{\frac{I_{DS2}}{K'_p\ W/L_2}}$$

$$\sqrt{\frac{I_{DS2}}{W/L_2}} + \sqrt{\frac{I_{DS4}}{W/L_4}} = 2\sqrt{\frac{I_{DS9}}{W/L_9}}$$

$$\frac{I_{DS2}}{I_{DS9}} = \frac{W/L_2}{W/L_9}\left(2 - \frac{1}{\sqrt{2}}\right)^2 \approx 118$$

Ref. : Wu etal, JSSC Jan.1994, pp.63-66



14 MHz / 11pF

5.6 MHz / 100pF

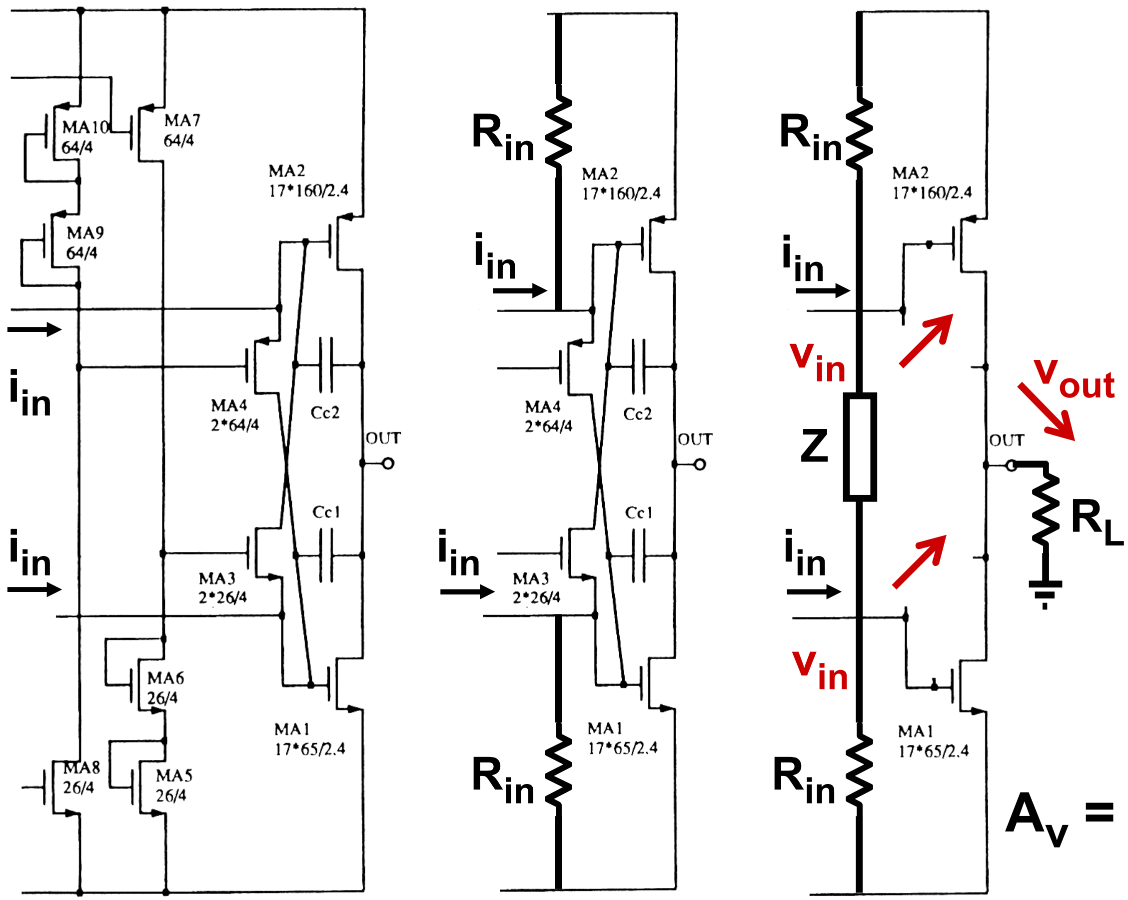
4 V/μs

36 nV/√ Hz

5 V

0.4 mA

Output stage : gain



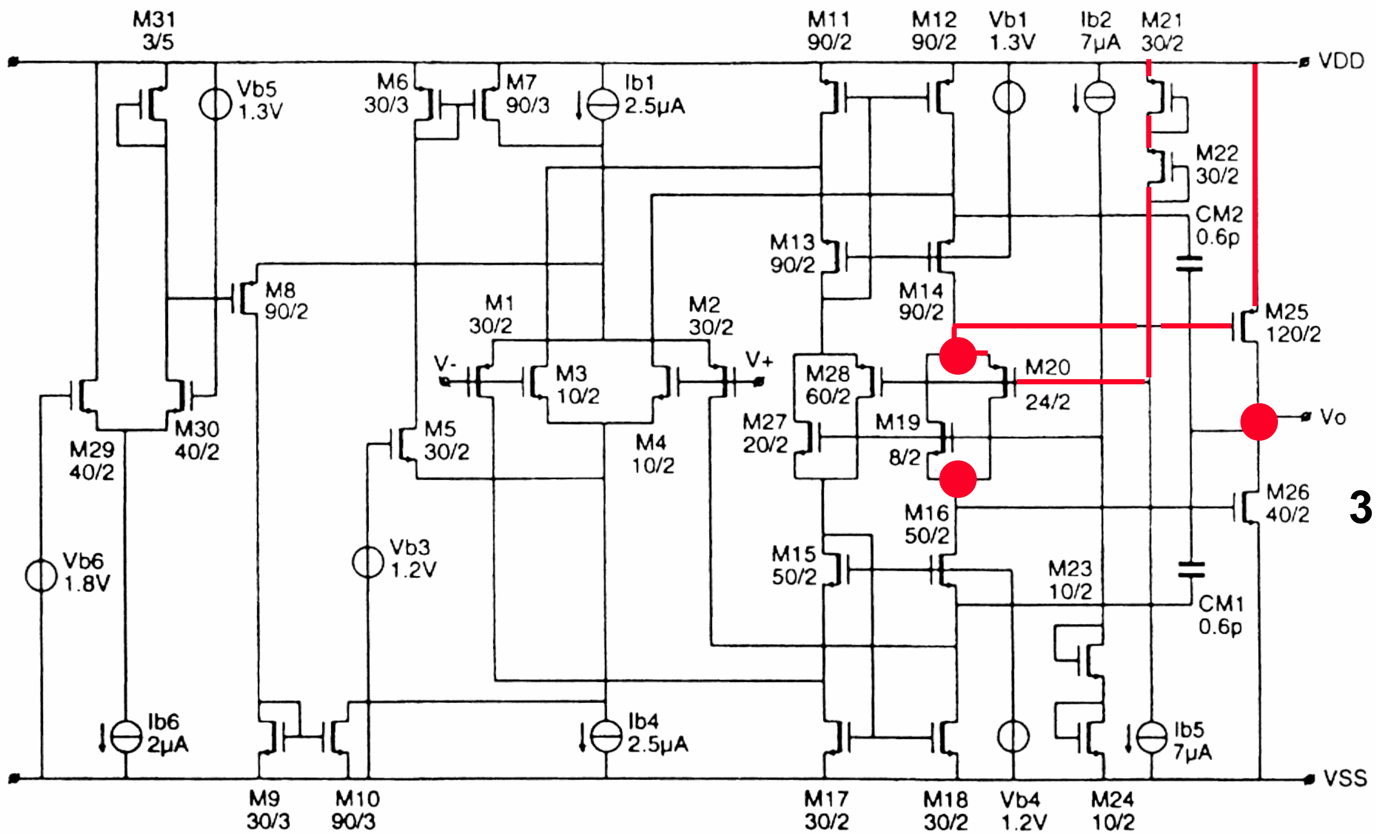
$i_{in} = g_{m1}\, v_{+-}$

$$\frac{v_{in}}{i_{in}} = R_{in}$$

$$\frac{v_{out}}{v_{in}} = 2g_{mA1}R_L$$

$$A_v = 2g_{m1}R_{in}g_{mA1}R_L$$

Class AB amplifier with translinear loop



6.4 MHz / 10 pF

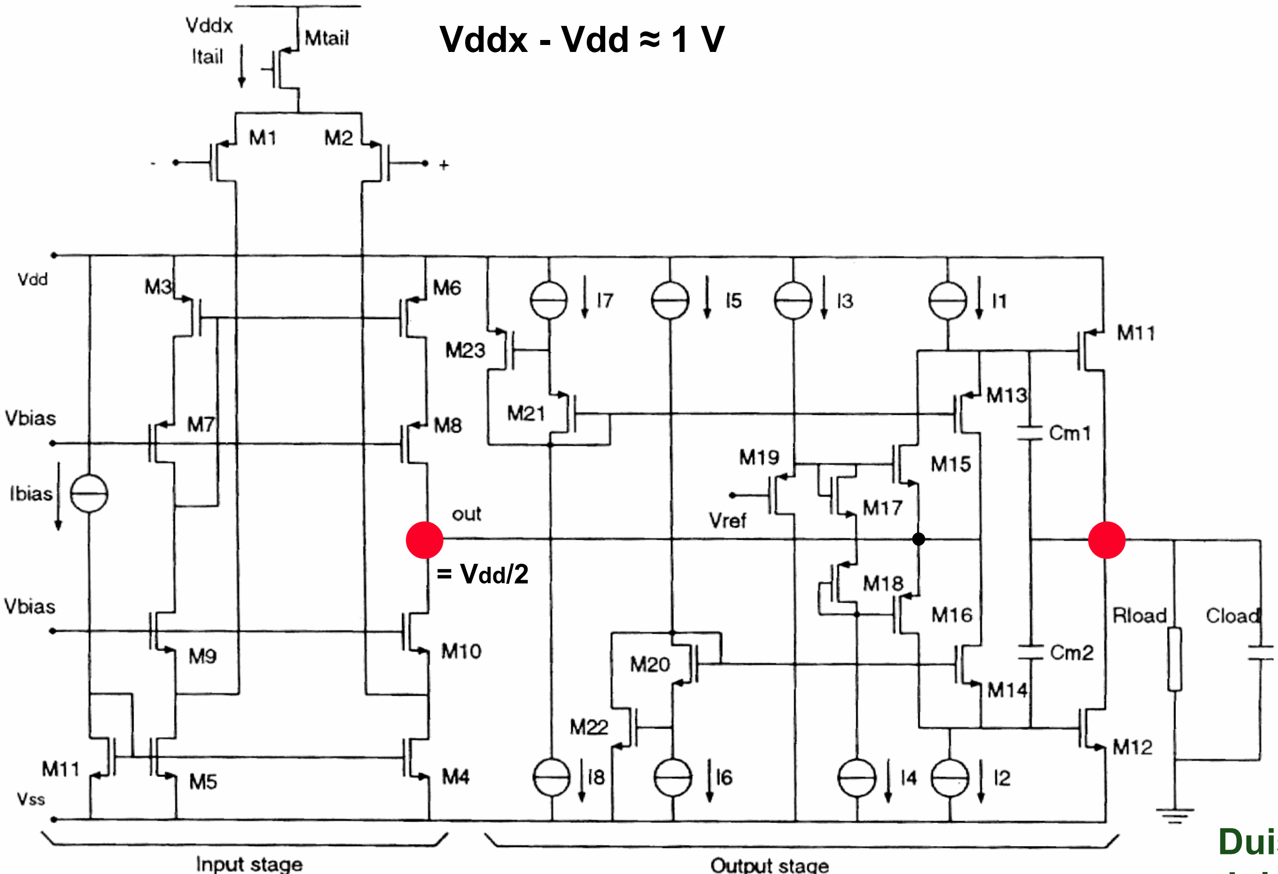
6.4 V/μs

31 nV/√ Hz

3.3 V
0.18 mA
3 mApk

Ref. Hogervorst, JSSC Dec 94, 1504-1512

Class-AB Opamp with voltage multiplier



Vddx - Vdd ≈ 1 V

1.8 - 3.3 V

0.75 mA

6.5 MHz

On 3 V :

2.8 V_{ptpt}

THD :

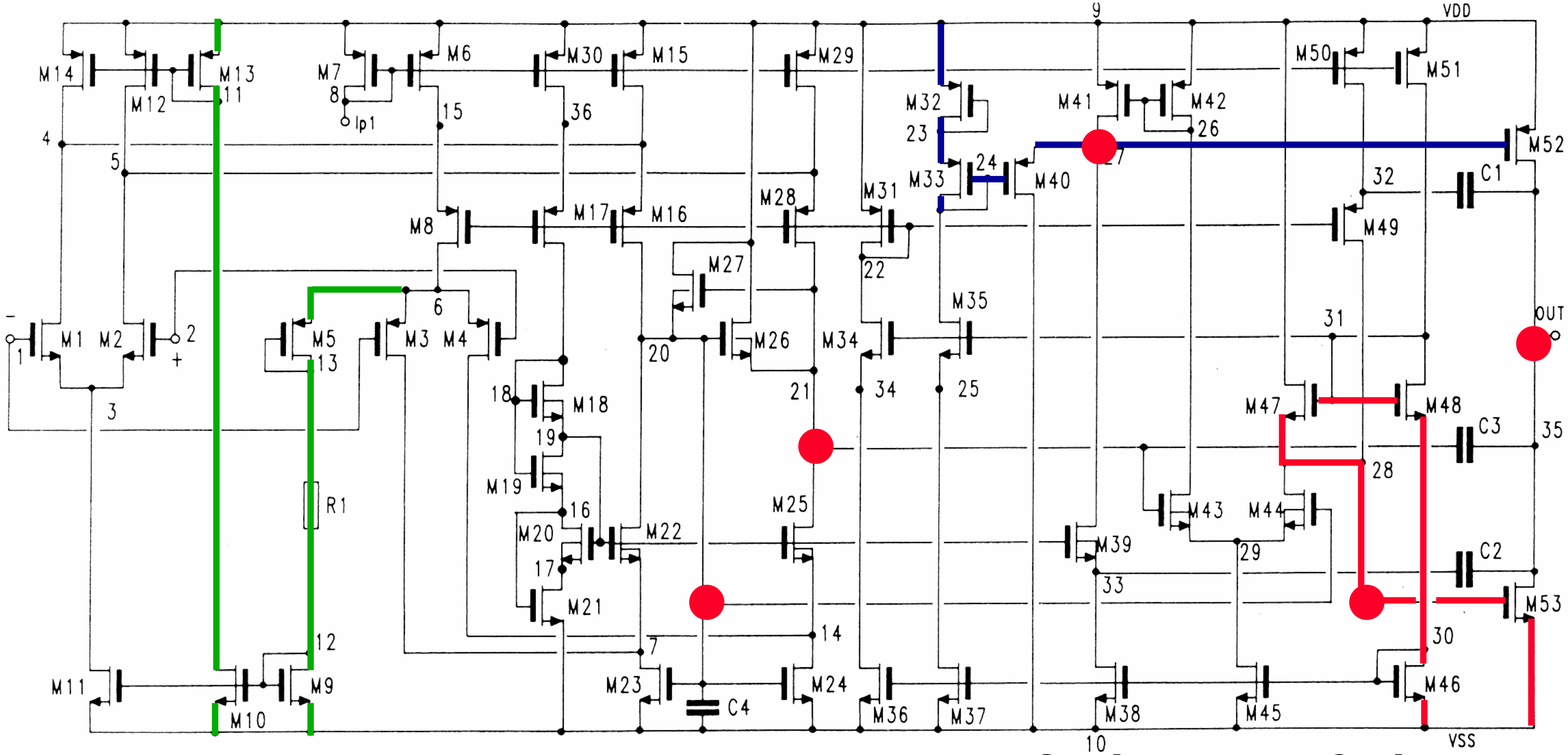
-90 dB /10kΩ

-81 dB/32 Ω

0.5 μm CMOS

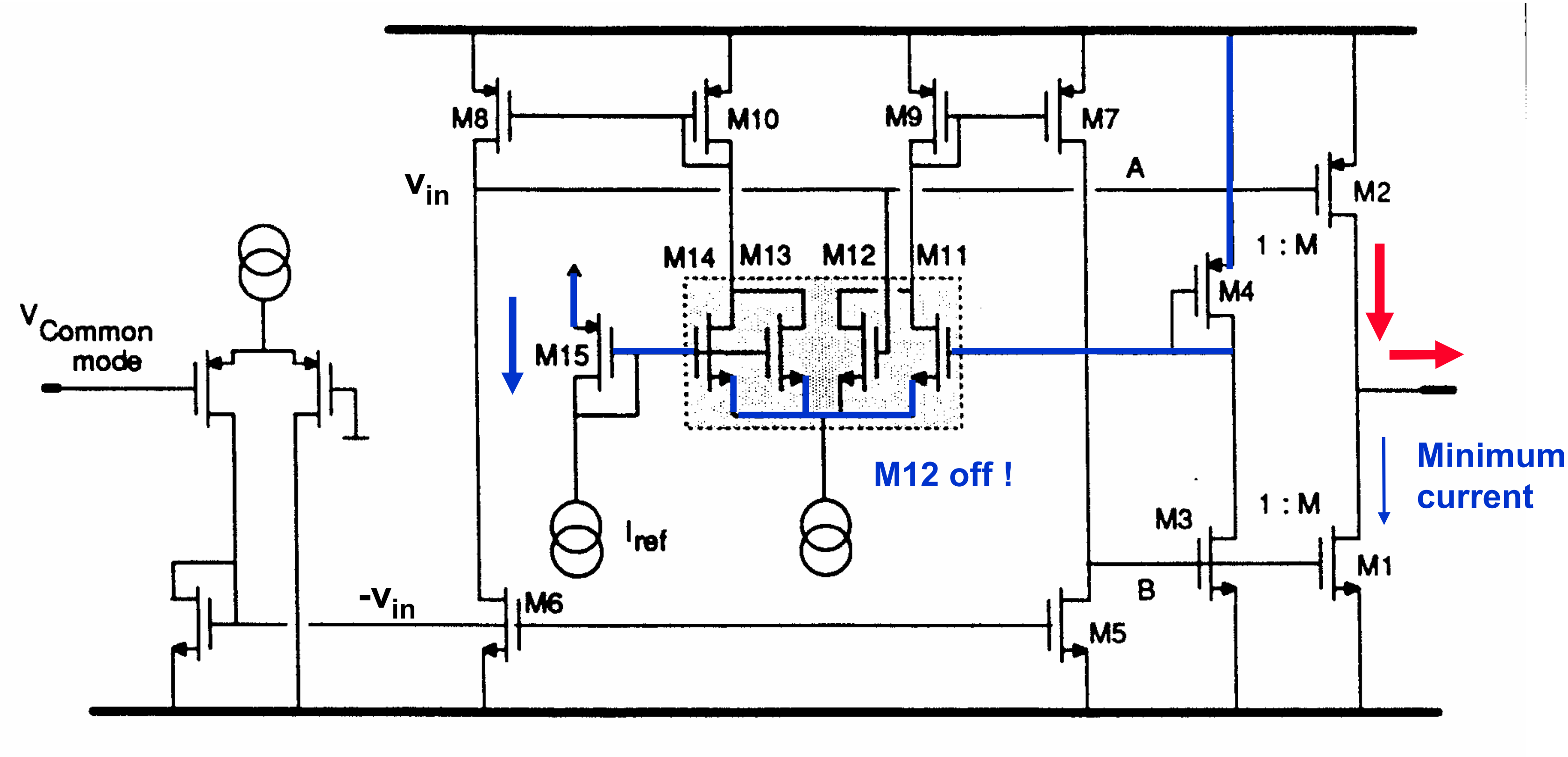
Duisters, .., JSSC
July 98,pp.947-955

Three-stage Modified Current Mirror



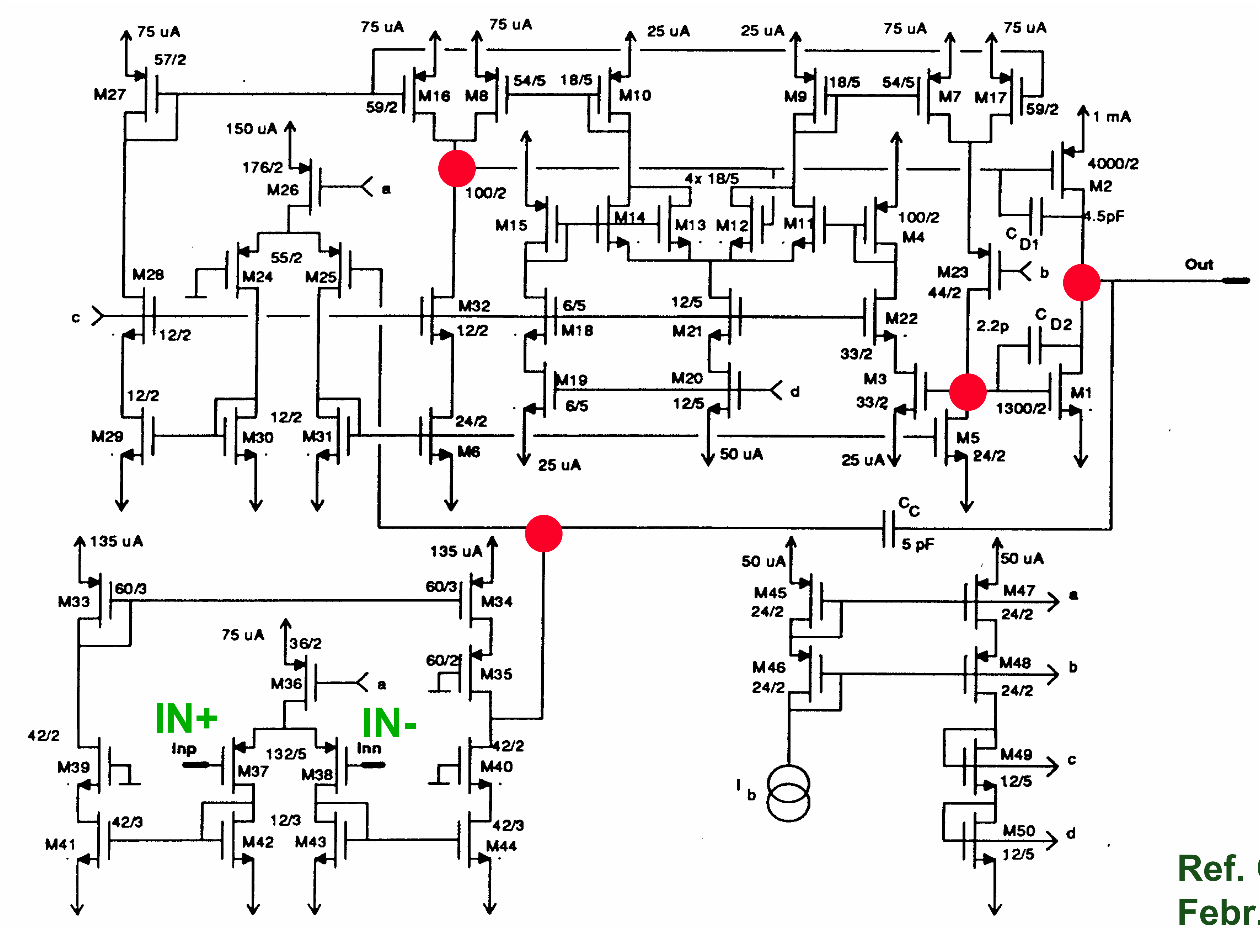
Pardoen, .., JSSC April 90, 501-504

Translinear I_Q Control



Ref. Op 't Eynde, JSSC Febr.90, 265-273

Translinear I_Q Control



3 stage

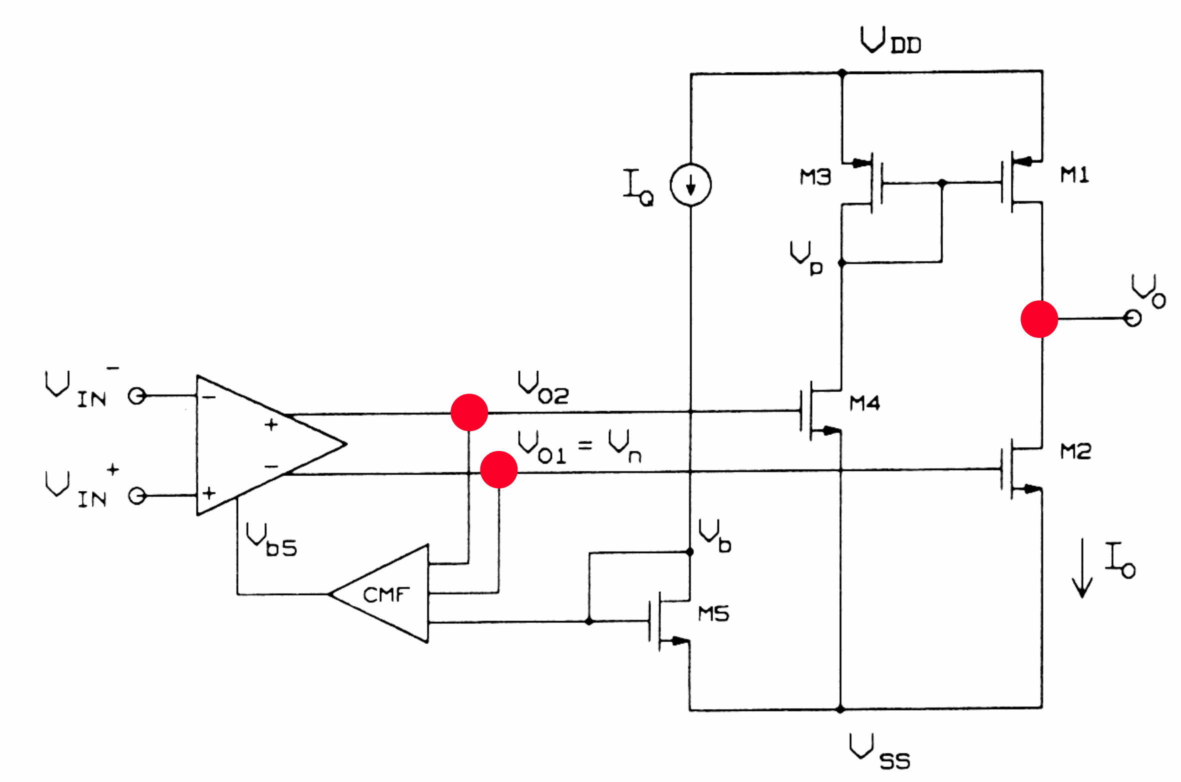
Class-AB

Amplifier

I_Q control

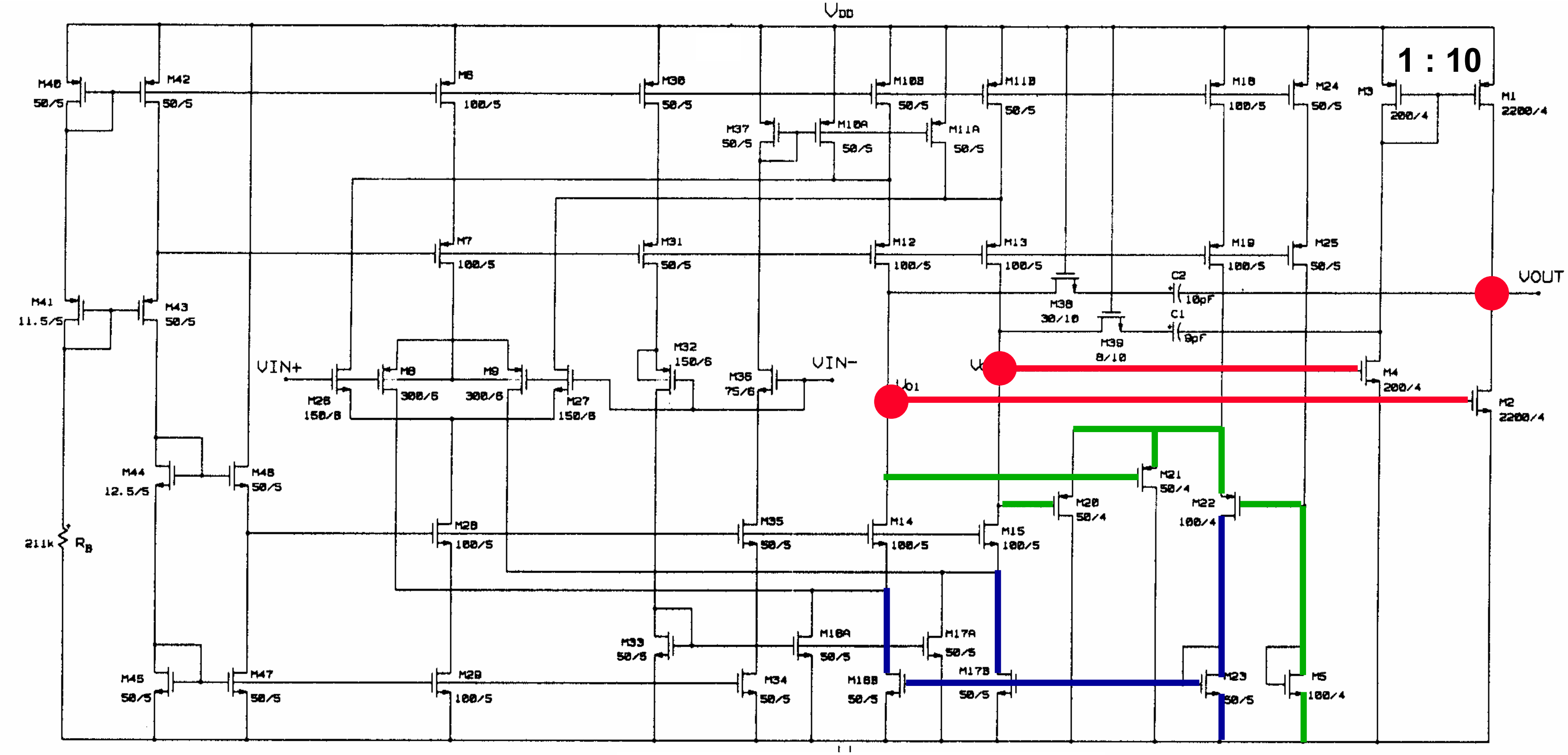
Ref. Op 't Eynde, JSSC Febr.90, 265-273

Class-AB amplifier with differential drive



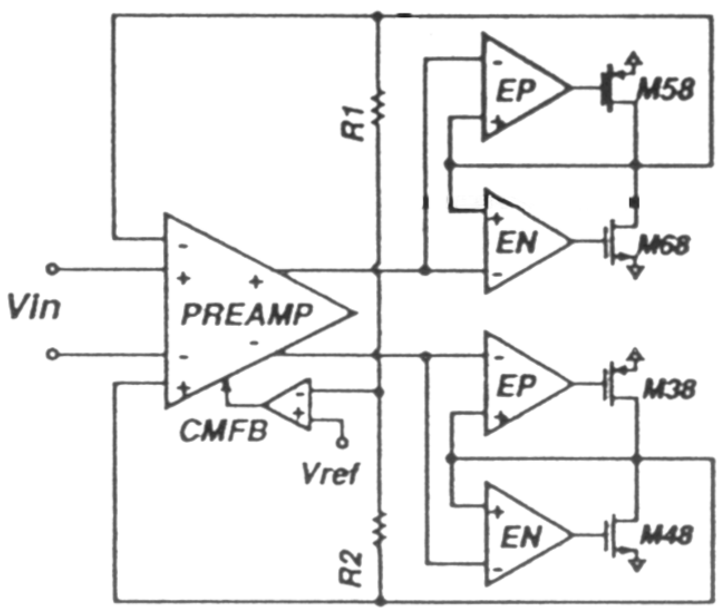
Ref. Babanezhad, JSSC Dec.88, 1414-1417

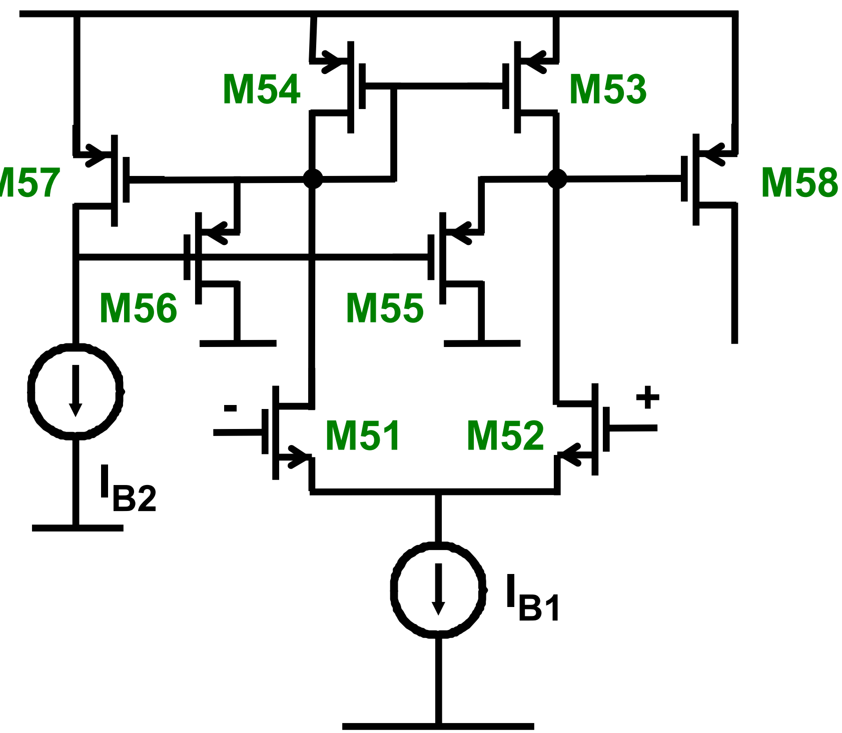
Differentially driven stage



Rail-to-rail input **CMFB + I_Q**

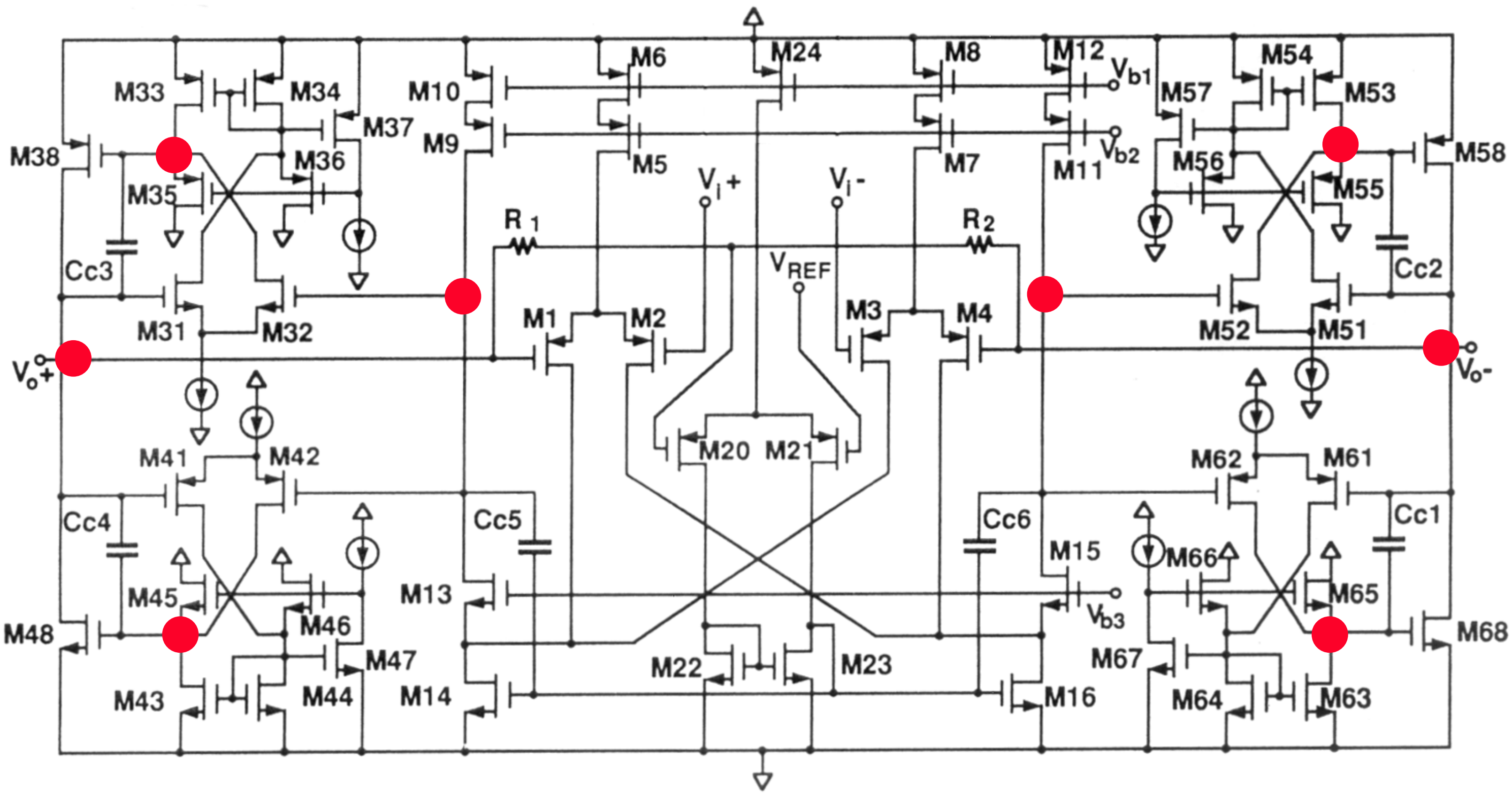
Class-AB amplifier with high linearity





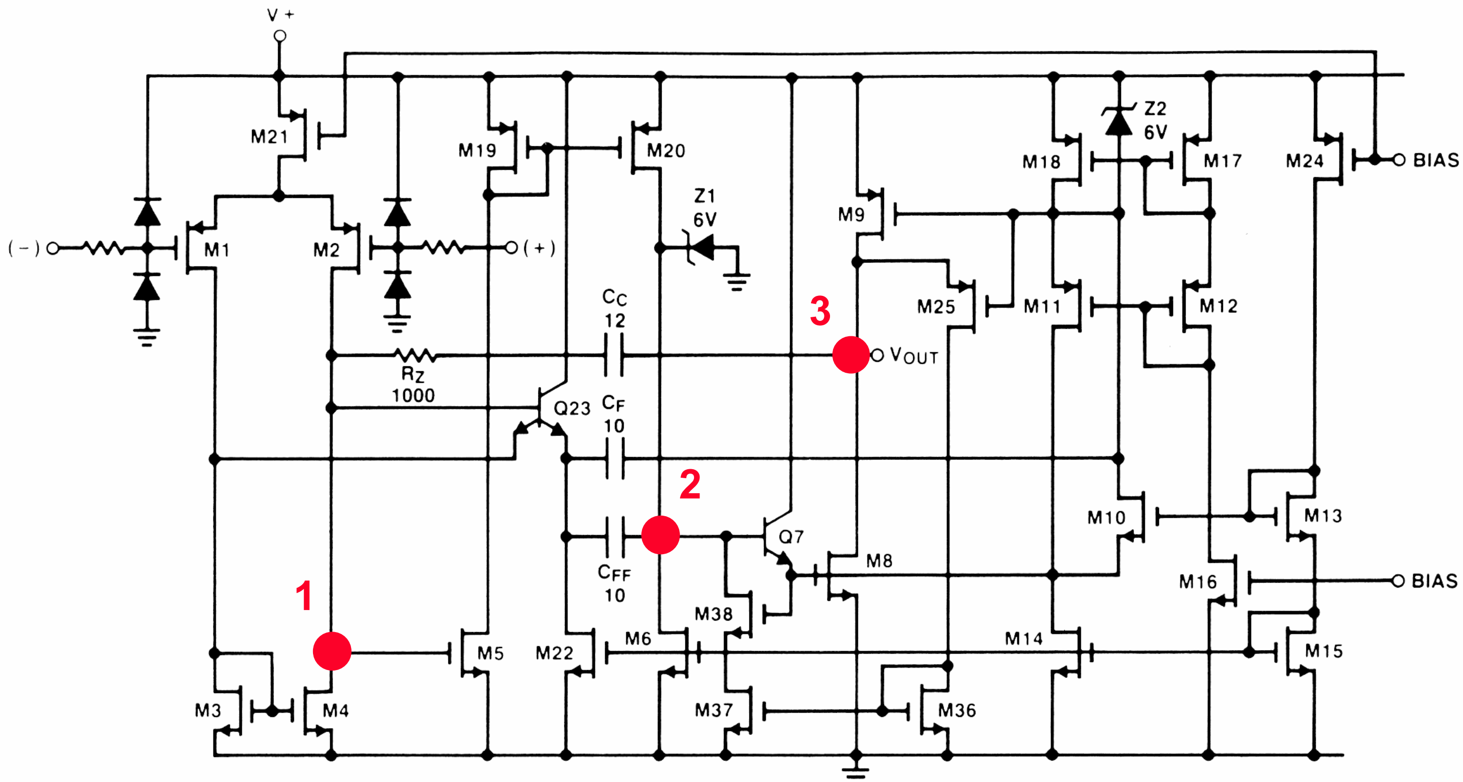
Ref. Khorramabadi, JSSC April 92, 539-544

Class-AB amplifier with high linearity



Ref. Khorramabadi, JSSC April 92, 539-544

Three-stage class AB amplifier with FF



M1, M2	700/13
M3, M4	120/40
M5	40/16
M6	80/9
M8	1500/9
M9	2200/7
M10, M11	100/11
M12, M13	75/11
M14	600/9
M15	75/9
M16	60/30
M17	100/7
M18	400/7
M19, M20	60/14
M21	320/20
M22	155/9
M24	150/20
M25	9/9
M36	9/4
M37	200/4
M38	200/9

Protection : Z1, Z2, Q23, M25, M36, M37, M38

Ref. Monticelli JSSC Dec.86, 1026-1034

Outline

- Problems of class AB drivers
- Cross-coupled quads
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- Current feedback and other principles
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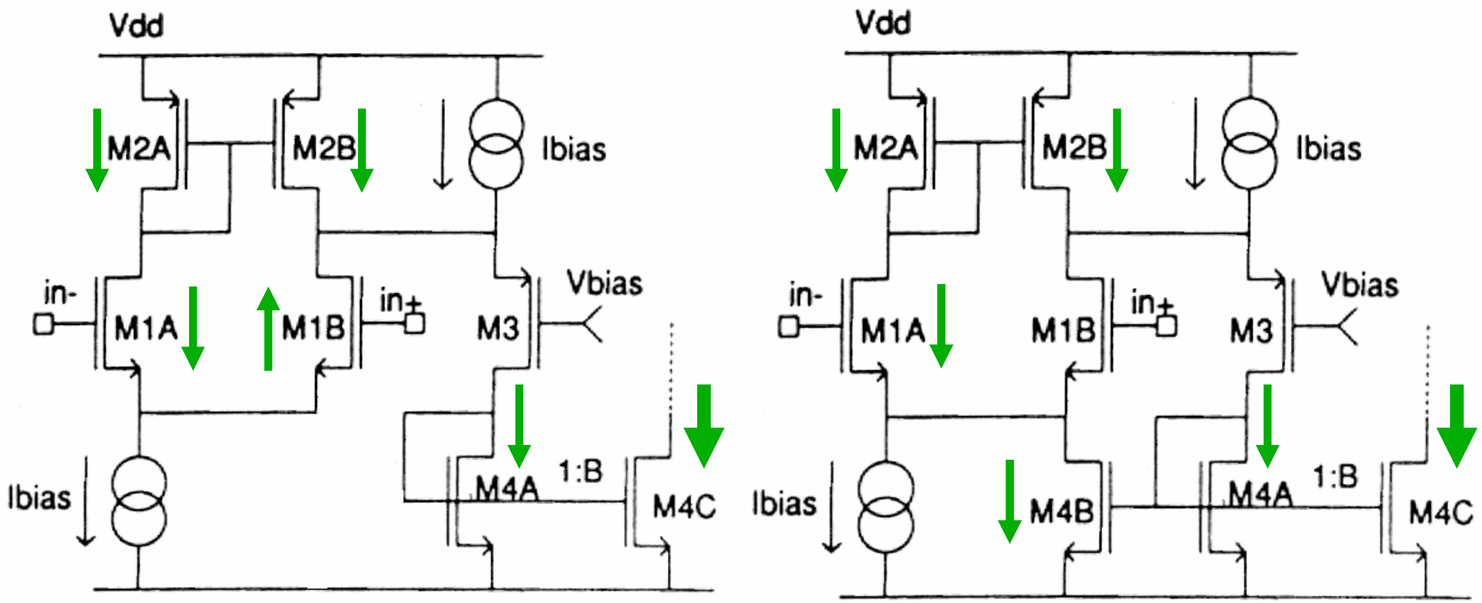
Current feedback



Folded Cascode OTA **Current Feedback**

Ref. Callewaert, JSSC June 90, 684-691

Two-stage Miller Amplifier with current FB

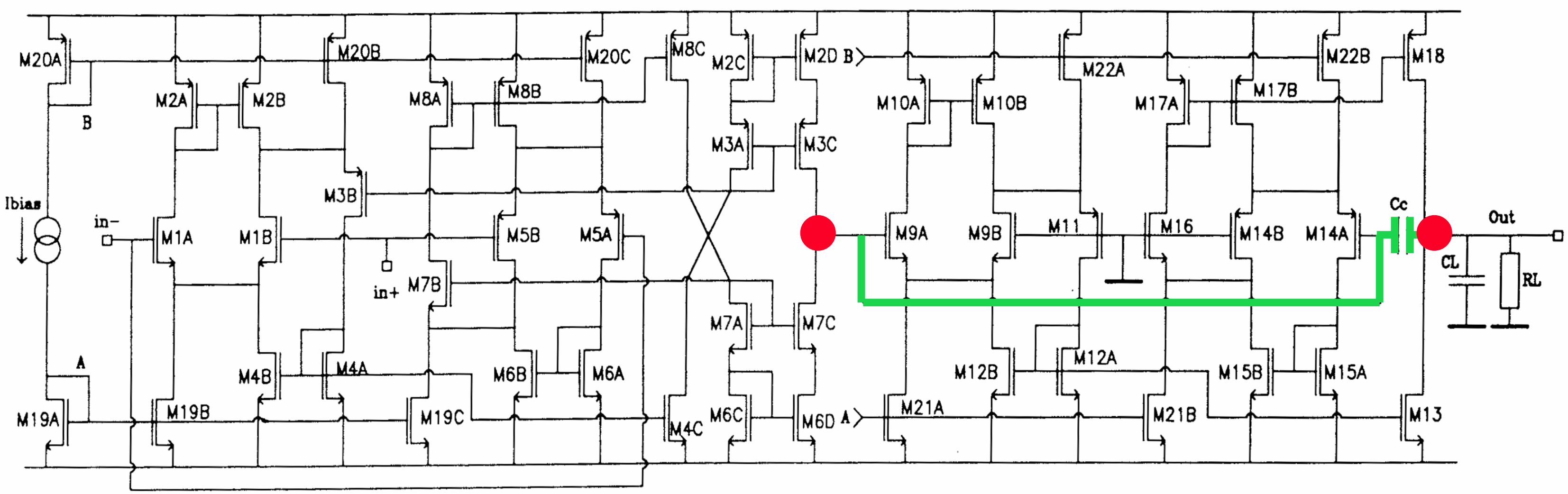
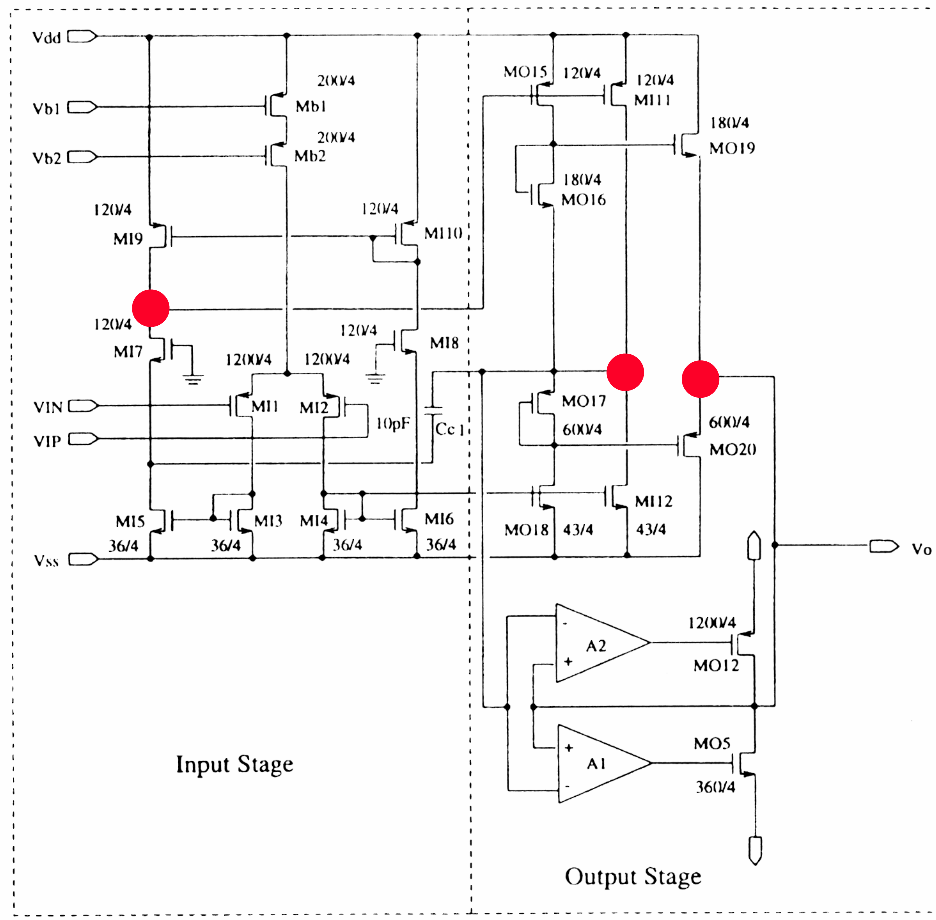


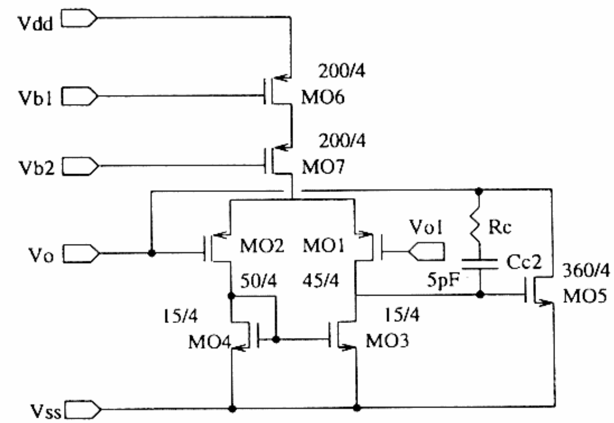
Fig. 11. Circuit diagram of the amplifier with both input and output stages based on the new class AB principle.

4 current feedback stages
2 stage Miller amplifier

Ref. Callewaert, JSSC June 90, 684-691

Low-distortion symmetrical class-AB amplifier





**Class-AB source foll.
In parallel with
Class-AB power amp.**

Ref. Saether, JSSC
Febr.96, 255-258

Outline

- Problems of class AB drivers
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- Adaptive biasing
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- Current feedback and other principles
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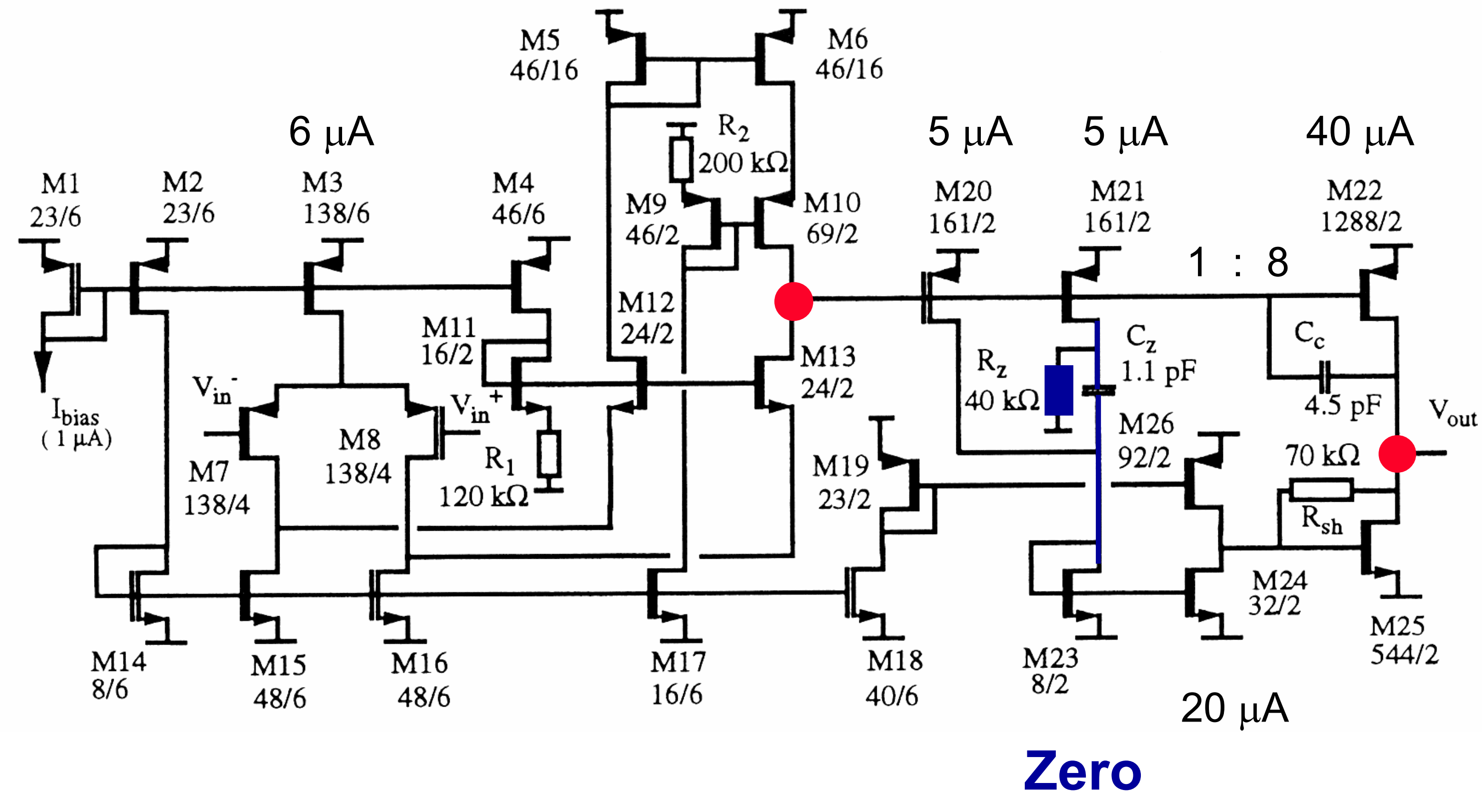
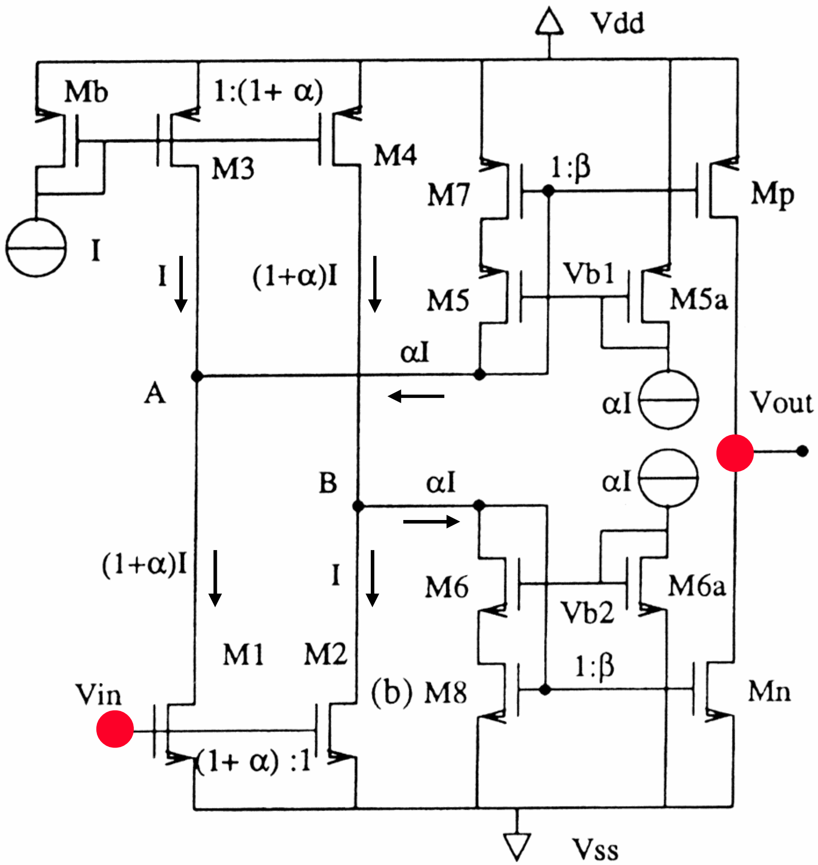
1.5 Vsupply voltage class-AB amp.



1.5 V 90 µA 1 MHz/150 pF

Ref. Van Dongen, JSSC Dec.95, 1333-1337

1.5 V class AB driver principle



Maximum voltage swing on A & B:

$\alpha \approx 0.2$

For larger α :

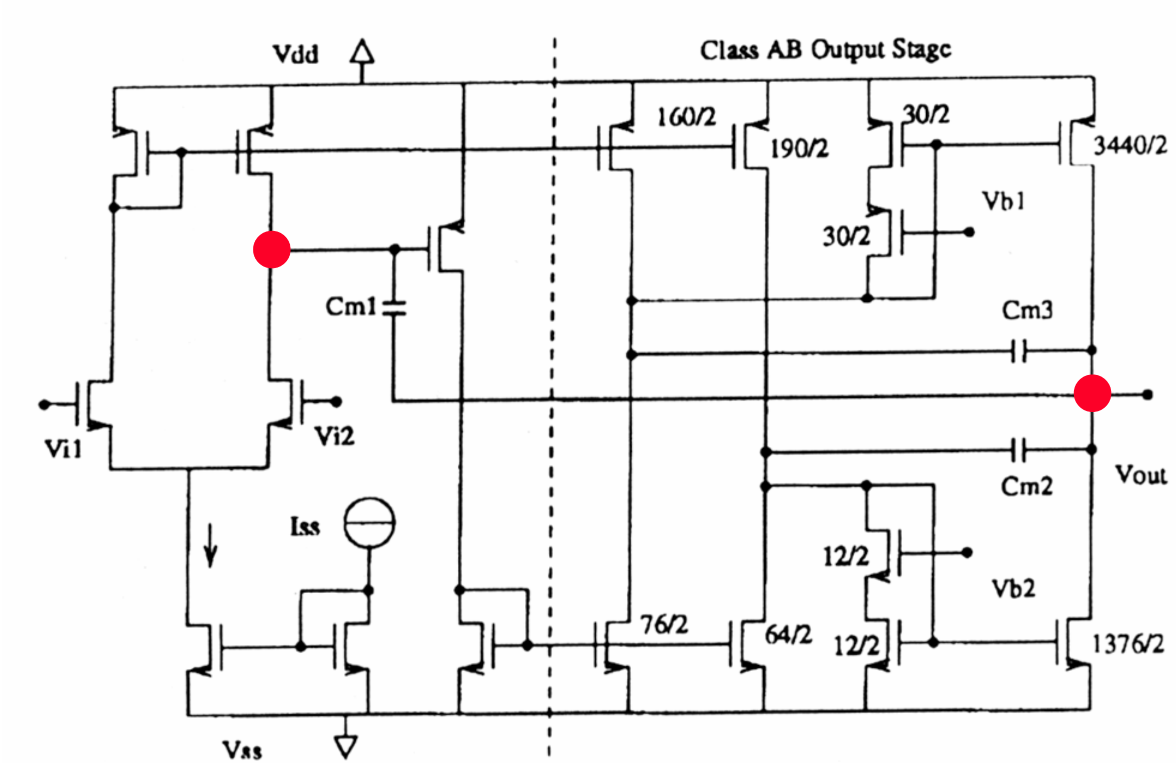
- less gain (more current)

- more mismatch and distortion

$\beta \approx 120$

You, etal, JSSC June 98, pp. 915-920

1.5 V class AB driver opamp



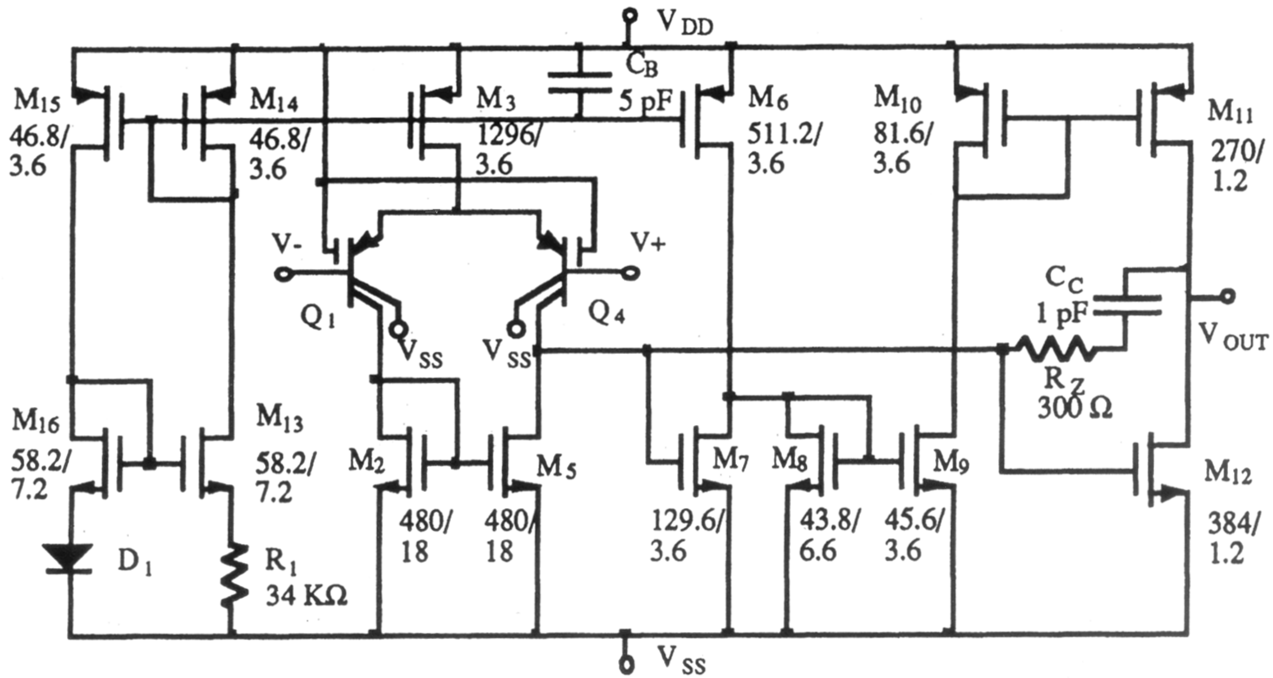
$I_Q = 300\ \mu A$

100 pF // 200 Ω

Two stage Miller compensation

You, etal, JSSC June 98, pp. 915-920

BiCMOS low-voltage opamp

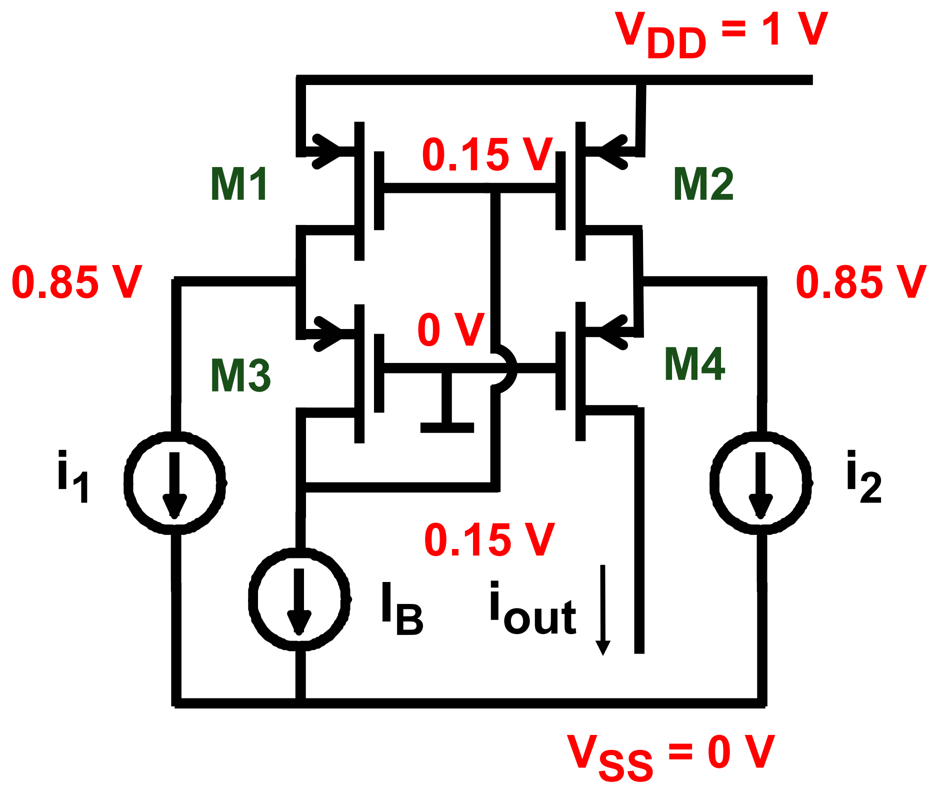


12 MHz   2.1 mA   3.2 nV$_{RMS}$/√Hz

Vittoz, JSSC June 83, pp. 273-279

Holman, JSSC June 95, pp. 710-714

Current differential amplifier for < 1V



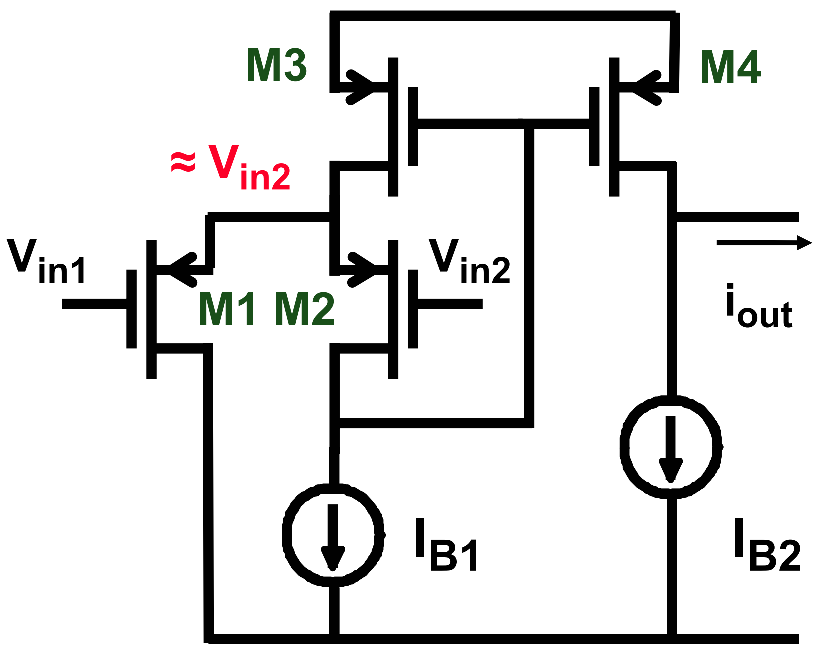
$$i_{out} = I_B + i_1 - i_2$$

$V_{GS} = 0.85$ V

$V_{DSsat} = 0.15$ V

$V_{outmax} = 0.7$ V

Class AB differential Voltage amplifier



M2 is source follower

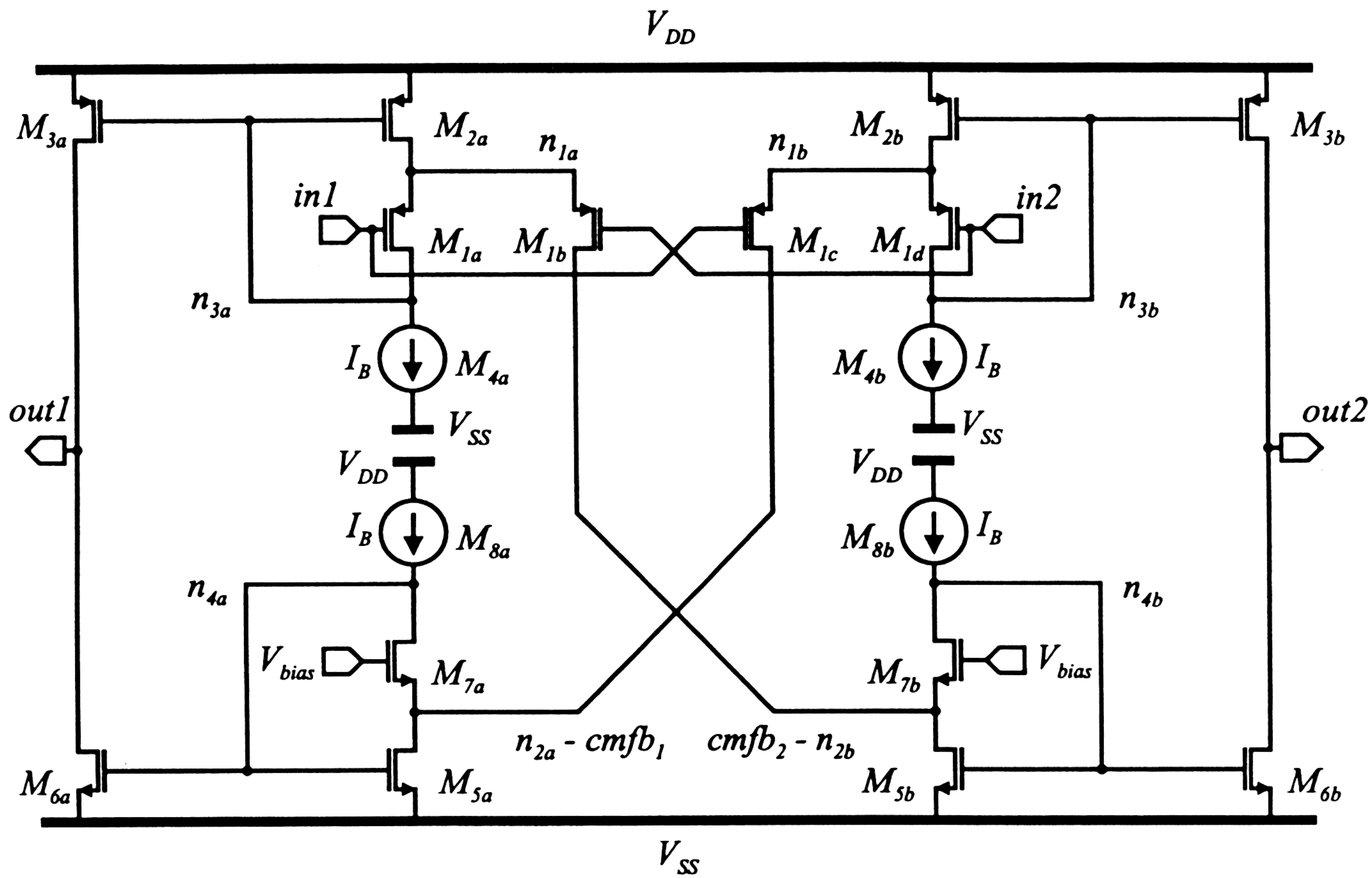
$$V_{GS1} = V_{in1} - V_{in2}$$

$$i_{out} \sim (V_{in1} - V_{in2})^2$$

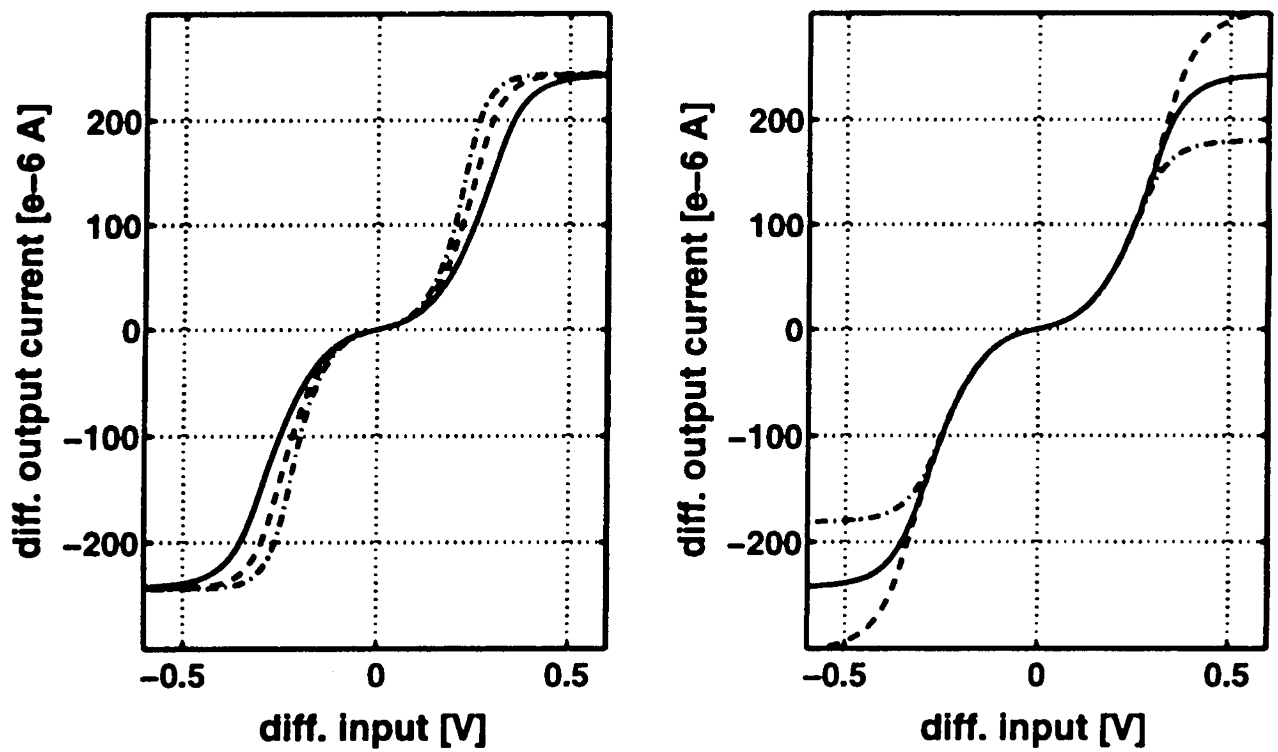
>>> Class AB

Ref. Peluso, JSSC Dec.98, 1887-1897

Differential class-AB OTA on 1 V supply voltage

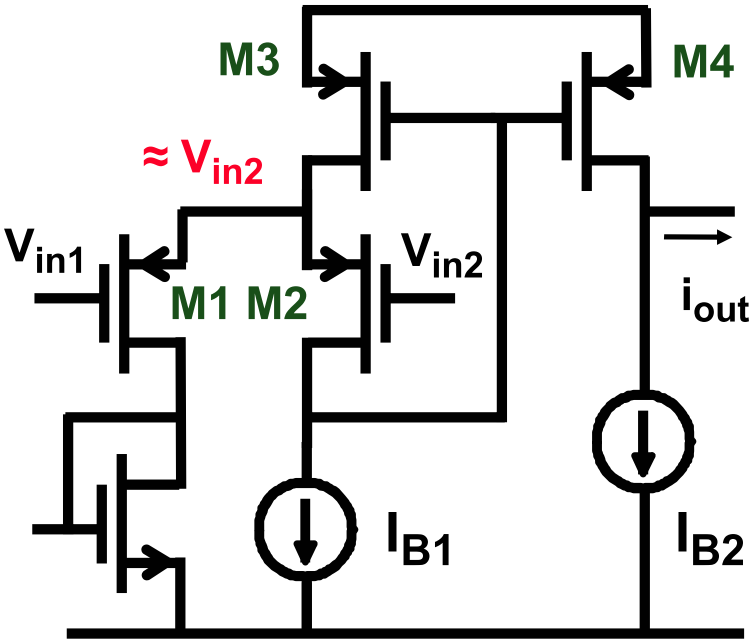


Class-AB characteristic



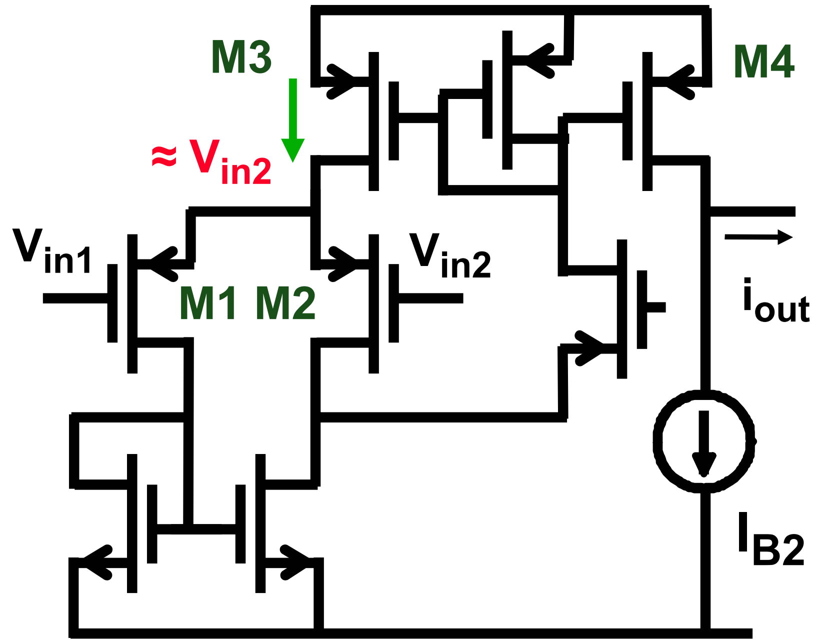
Larger input W/L **Larger current source W/L**

Low-voltage Class AB amplifiers



M2 is source follower
3 trans. carry current
$V_{GS} + V_{DSsat}$

Ref. Peluso, JSSC Dec.98, 1887-1897



M2 is source follower
7 trans. carry current
$V_{GS} + 2V_{DSsat}$

Ref. Callewaert, JSSC June 90, 684-691

Conclusions

- Problems of class AB drivers
- Cross-coupled quads
- Adaptive biasing
- I_Q control with translinear circuits, etc.
- Current feedback and other principles
- Low-Voltage realizations

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