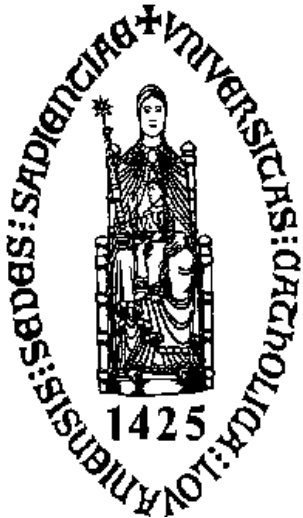


---

---

# CMOS ADC & DAC Principles

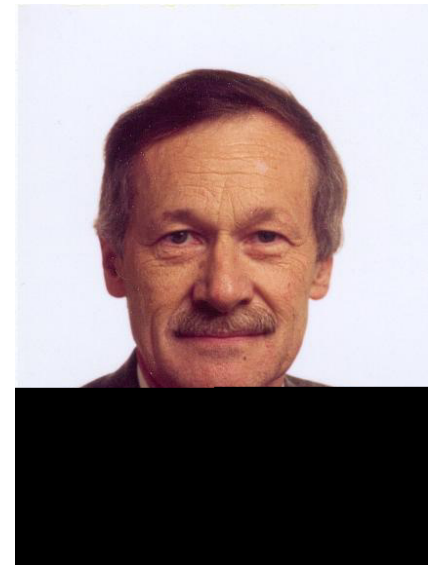


**Willy Sansen**

**KULeuven, ESAT-MICAS**

**Leuven, Belgium**

[willy.sansen@esat.kuleuven.be](mailto:willy.sansen@esat.kuleuven.be)



---

# Table of contents

---

- **Definitions**

- **Digital-to-analog converters**

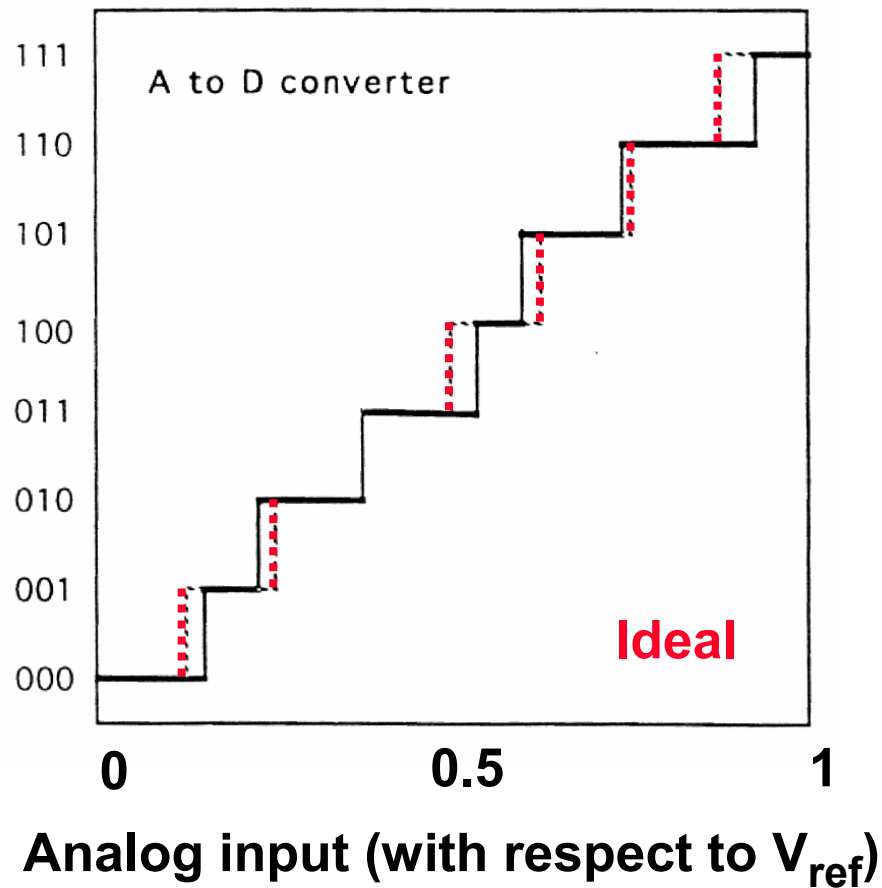
- Resistive
- Capacitive
- Current steering

- **Analog-to-digital converters**

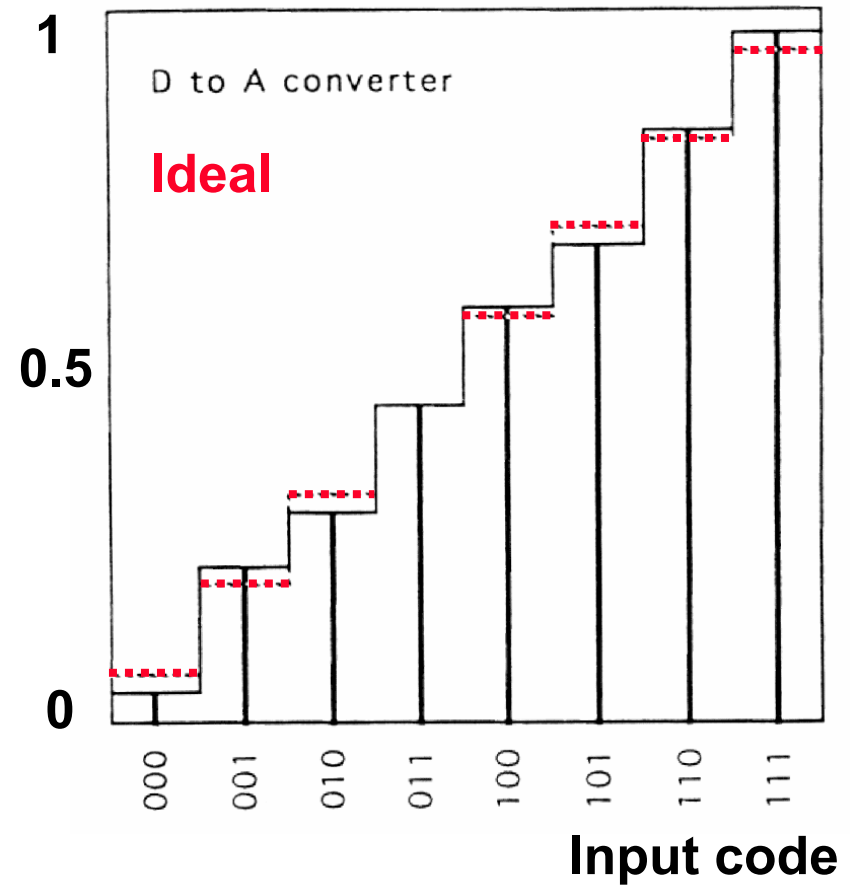
- Integrating
- Successive approximation
- Algorithmic
- Flash / Two-step
- Interpolating / Folding
- Pipeline

# ADC & DAC

## Output code



### Analog output (wrt $V_{ref}$ )

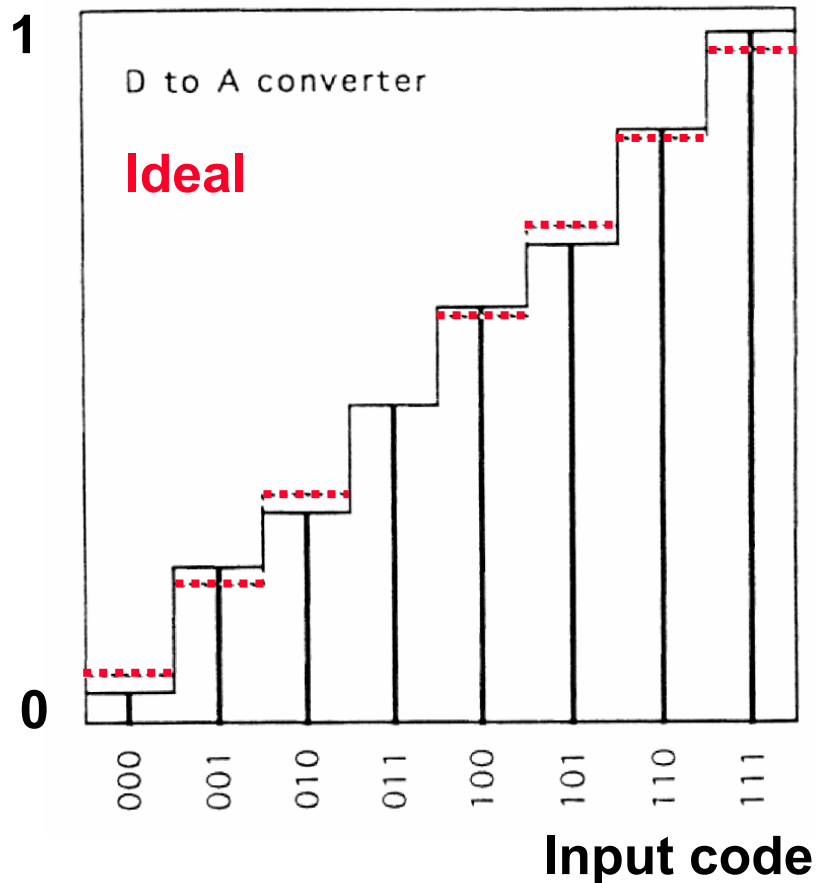


---

# DACs Resolution

---

Analog output (wrt  $V_{ref}$ )



$$V_{OUT} = V_{REF} B_{IN}$$

$$= V_{REF} \left( \frac{b_1}{2^1} + \frac{b_2}{2^2} + \frac{b_3}{2^3} + \dots + \frac{b_N}{2^N} \right)$$

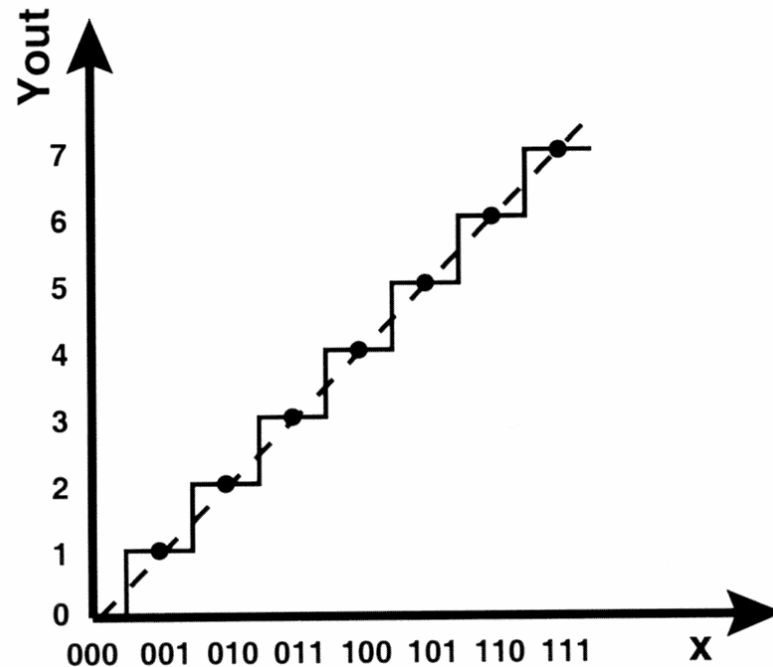
$$V_{LSB} = \frac{V_{REF}}{2^N}$$

Resolution N

$b_1$  is Most Significant bit (MSB)

$b_N$  is Least Significant bit (LSB)

# The quantisation error of a DAC



$$P_{\text{Noise}} = \frac{1}{\Delta} \int_{-\Delta/2}^{\Delta/2} \epsilon^2 d\epsilon = \frac{\Delta^2}{12}$$

$$V_{\text{ptp}} = 2^N \Delta$$

$$P_{\text{Signal}} = \frac{V_{\text{ptp}}^2}{8}$$

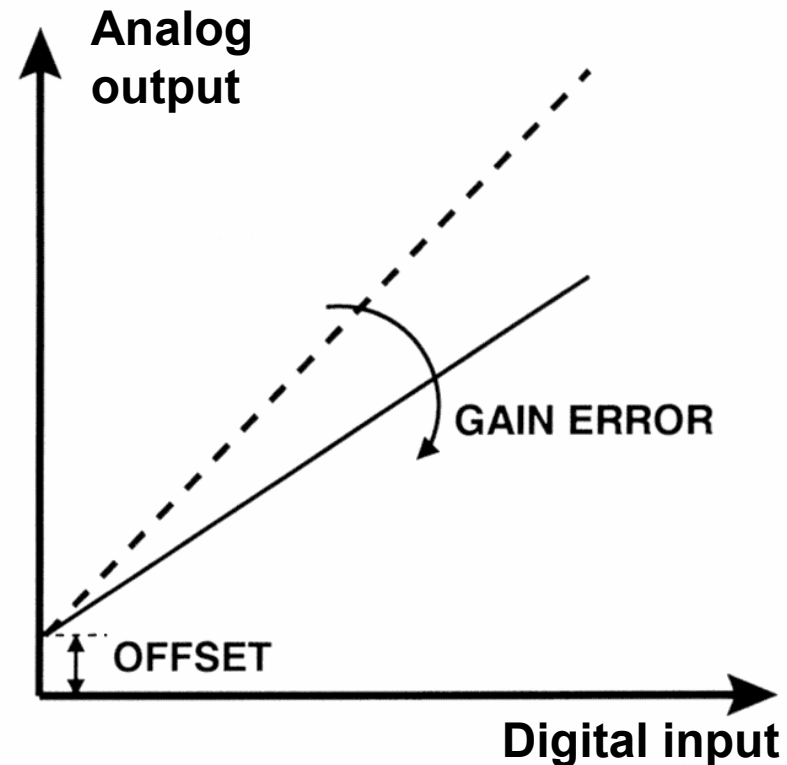
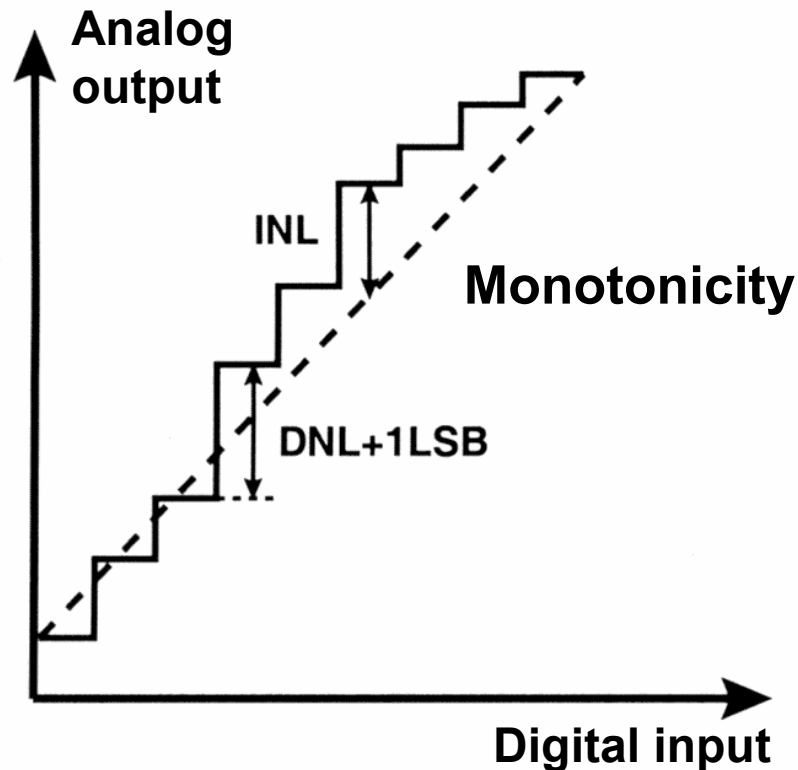
$$\text{SNR} = \frac{3}{2} 2^{2N}$$

$$\text{SNR} = 6N + 1.76 \text{ dB}$$

---

## Static specs : INL & DNL

---



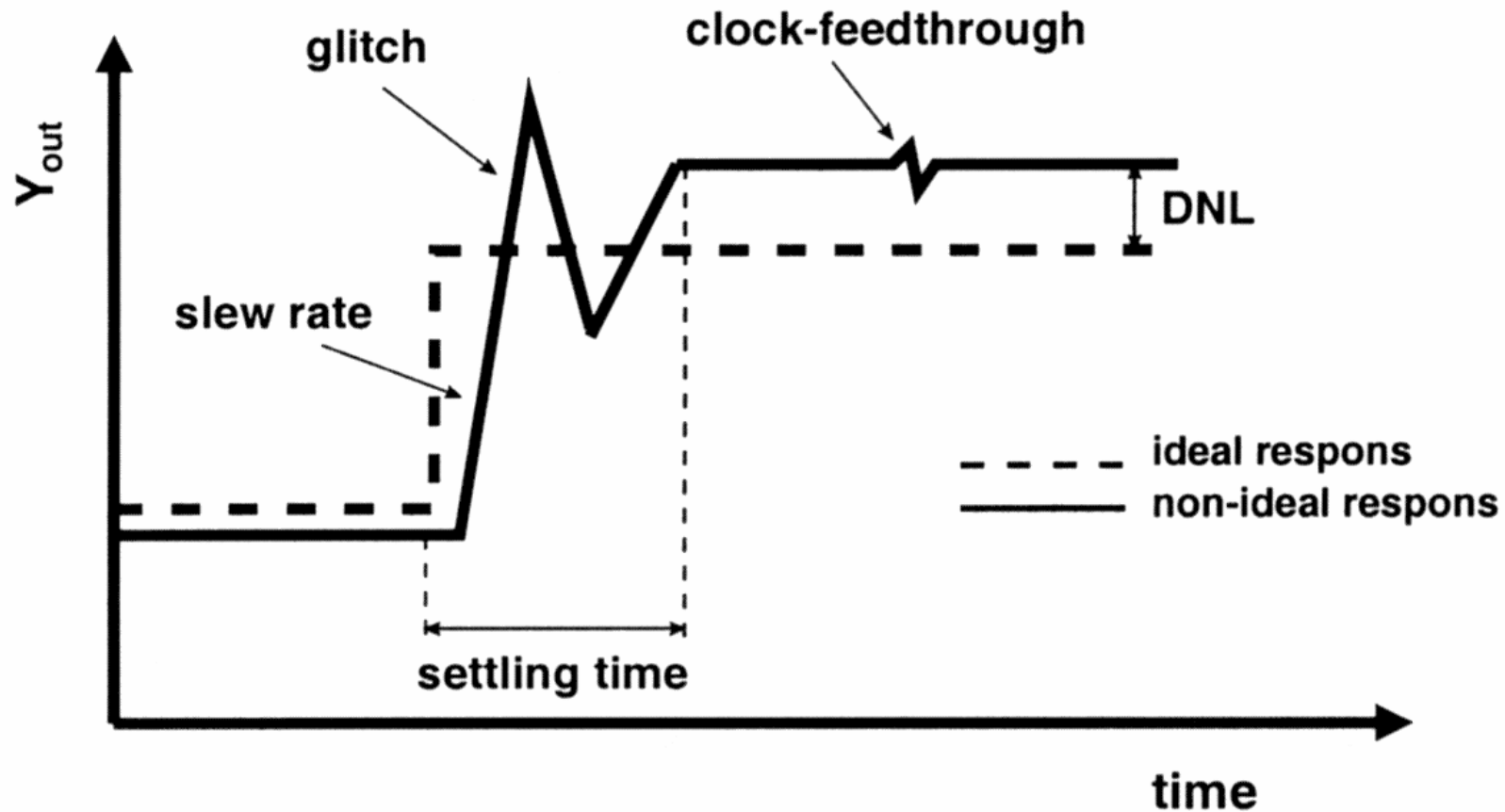
**Differential Nonlinearity :  $DNL = Y_{OUT}(B) - Y_{OUT}(B-1) - 1 \text{ LSB}$**

**Integral Nonlinearity :  $INL = Y_{OUT}(B) - Y_{OUT,id}(B)$**

---

# Dynamic specifications

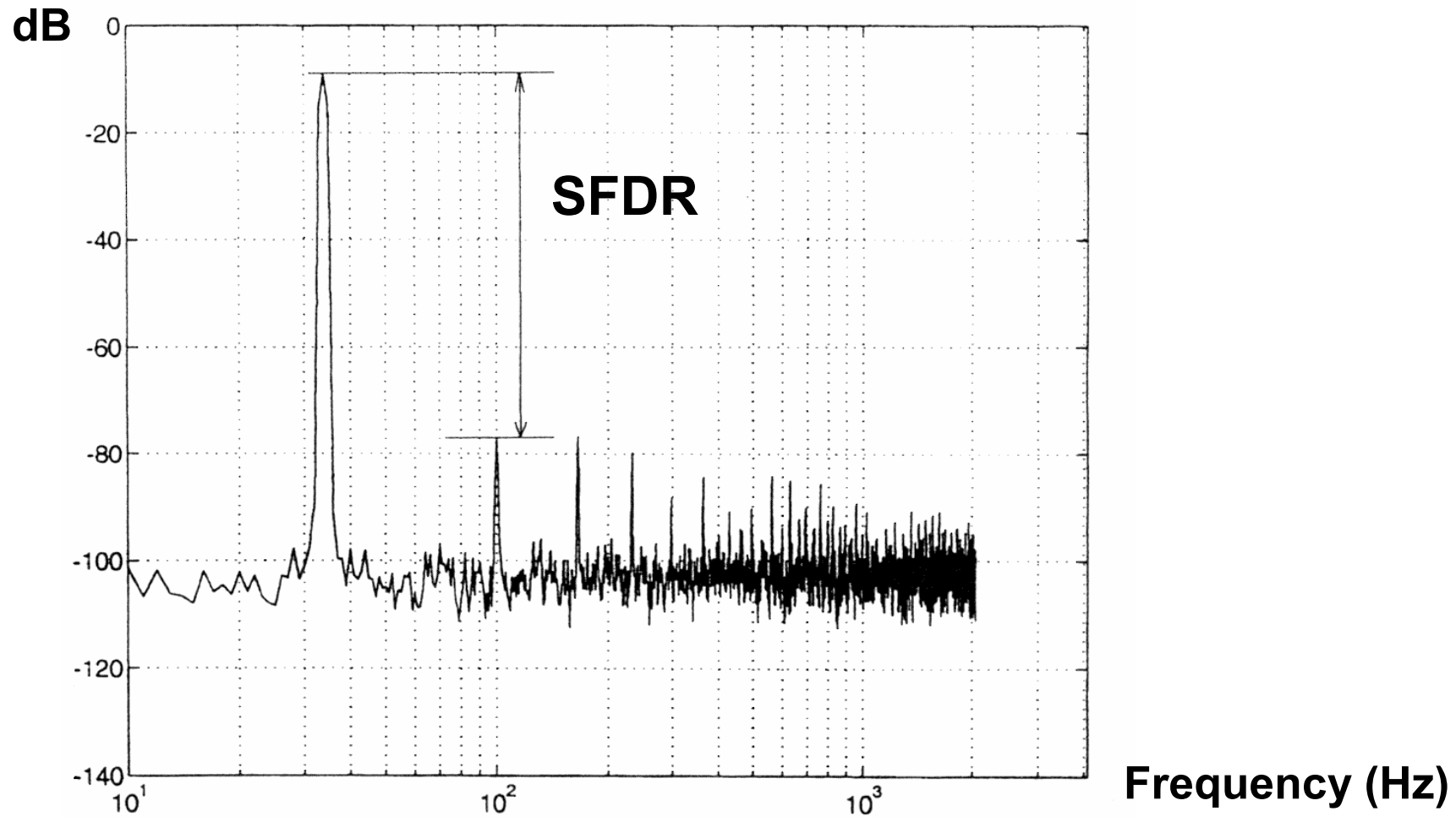
---



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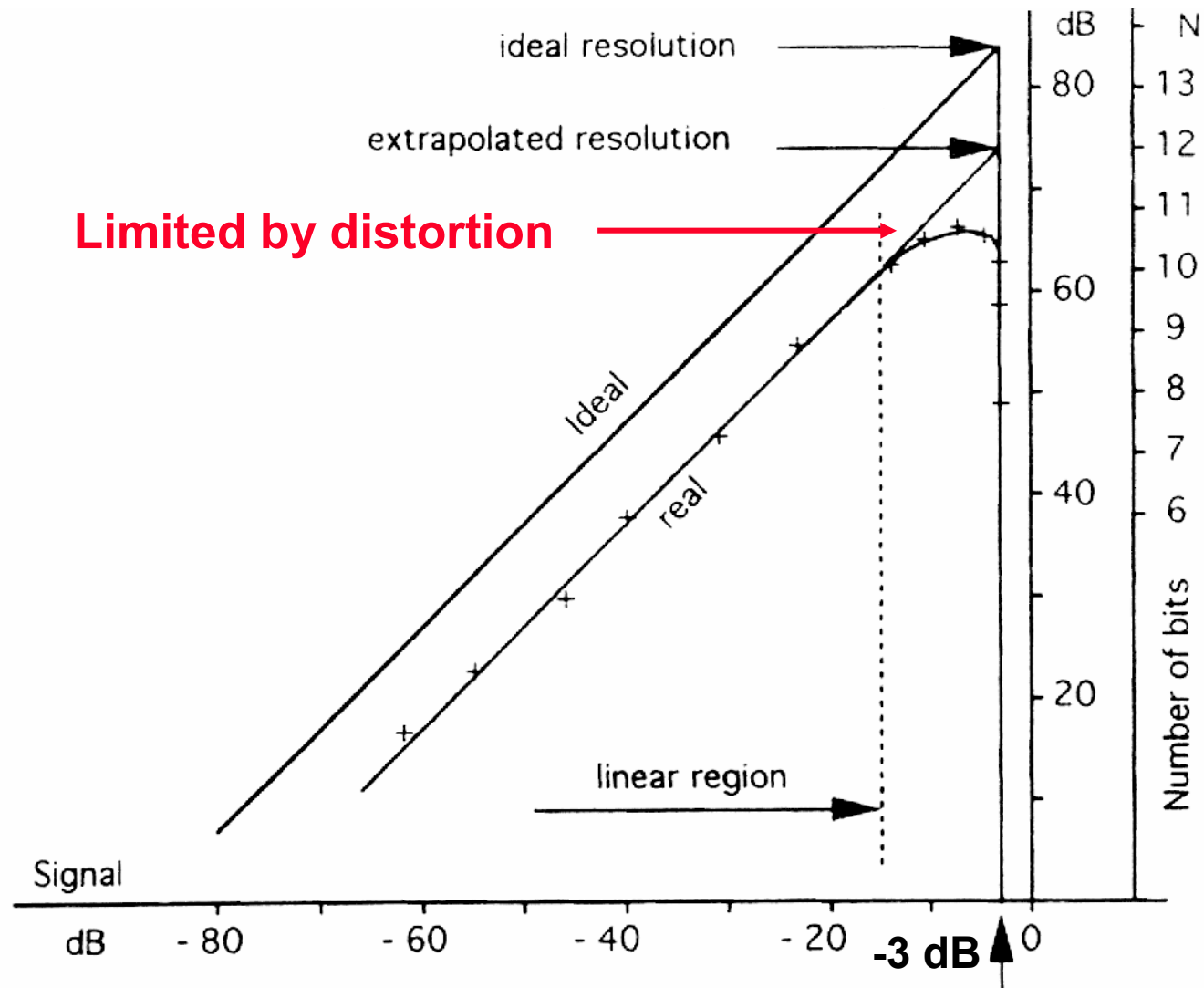
# Spectral content : Spurious free dynamic range

---





# Output SNR versus input Signal



---

# Table of contents

---

- **Definitions**

- **Digital-to-analog converters**

- Resistive
- Capacitive
- Current steering

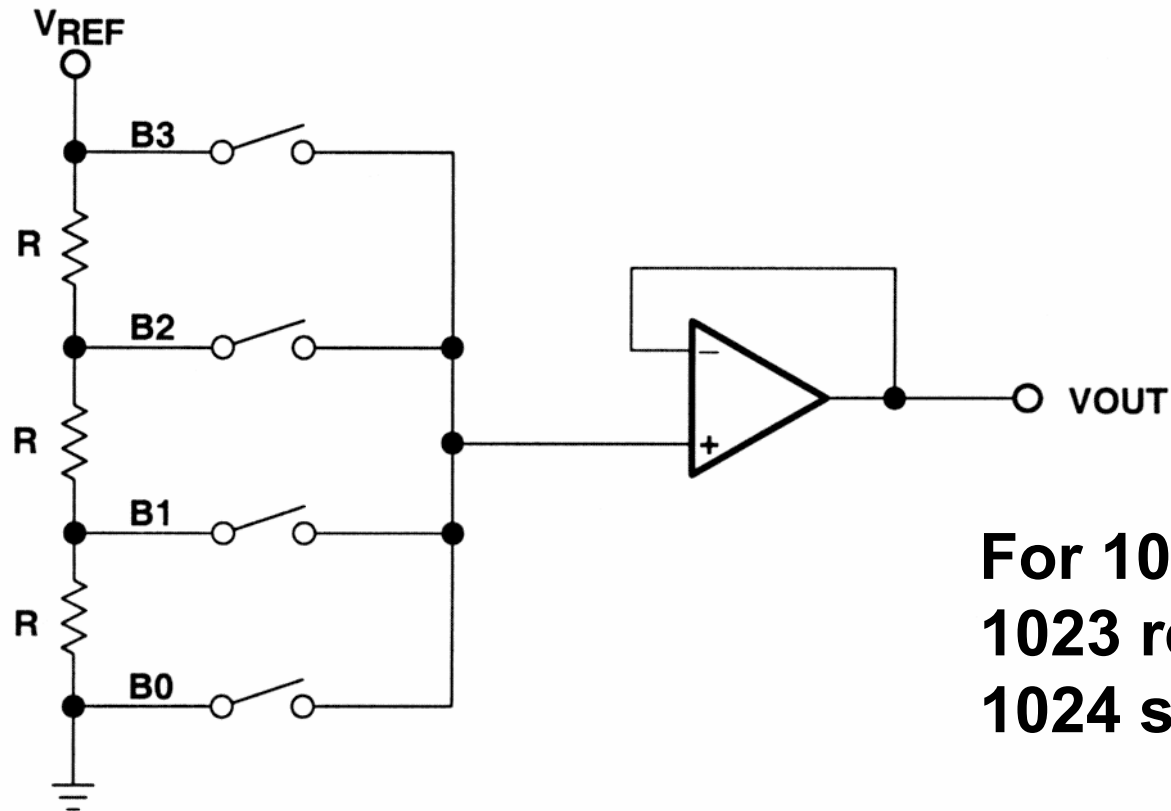
- **Analog-to-digital converters**

- Integrating
- Successive approximation
- Algorithmic
- Flash / Two-step
- Interpolating / Folding
- Pipeline

---

# Resistor string DAC

---



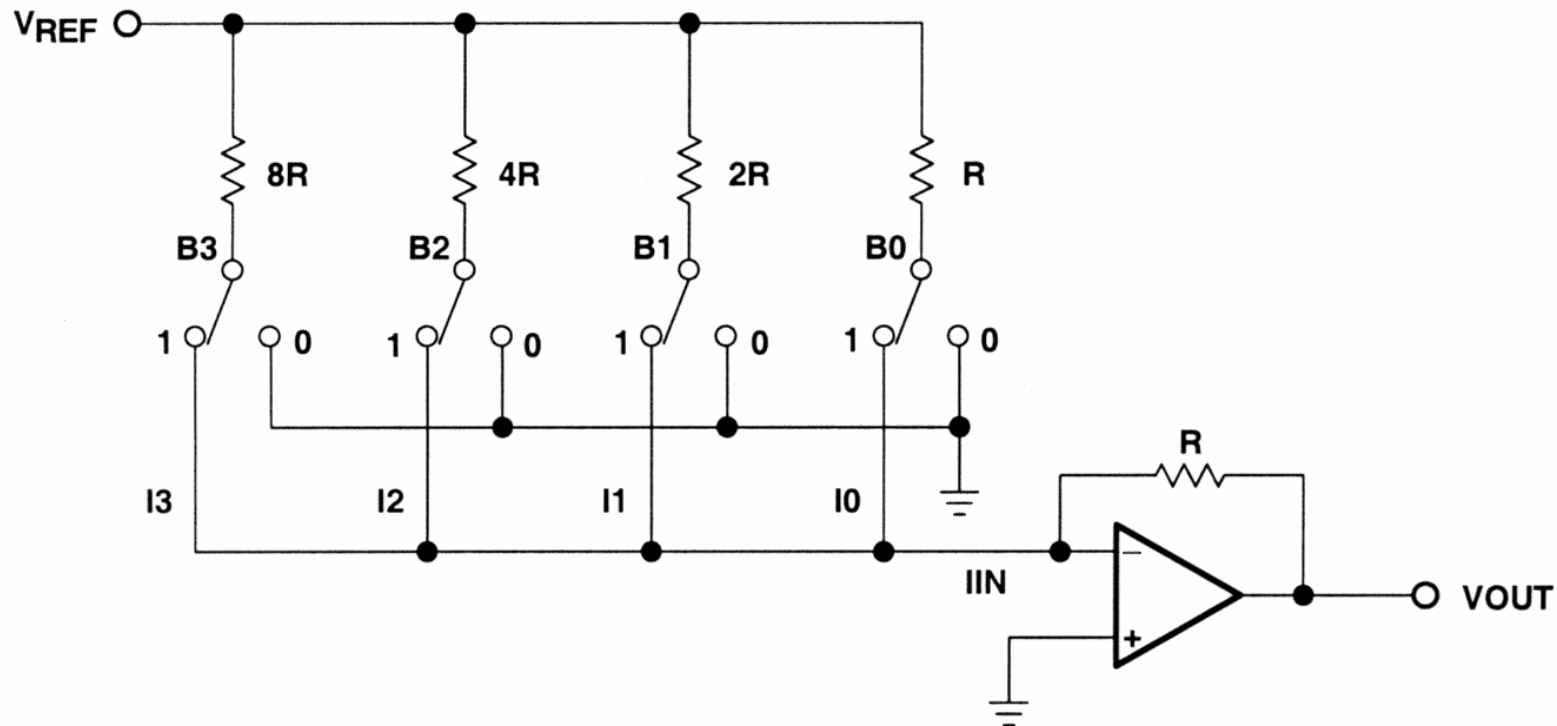
**For 10 bit:  
1023 resistors and  
1024 switches !!**

**Resistive matching !**

---

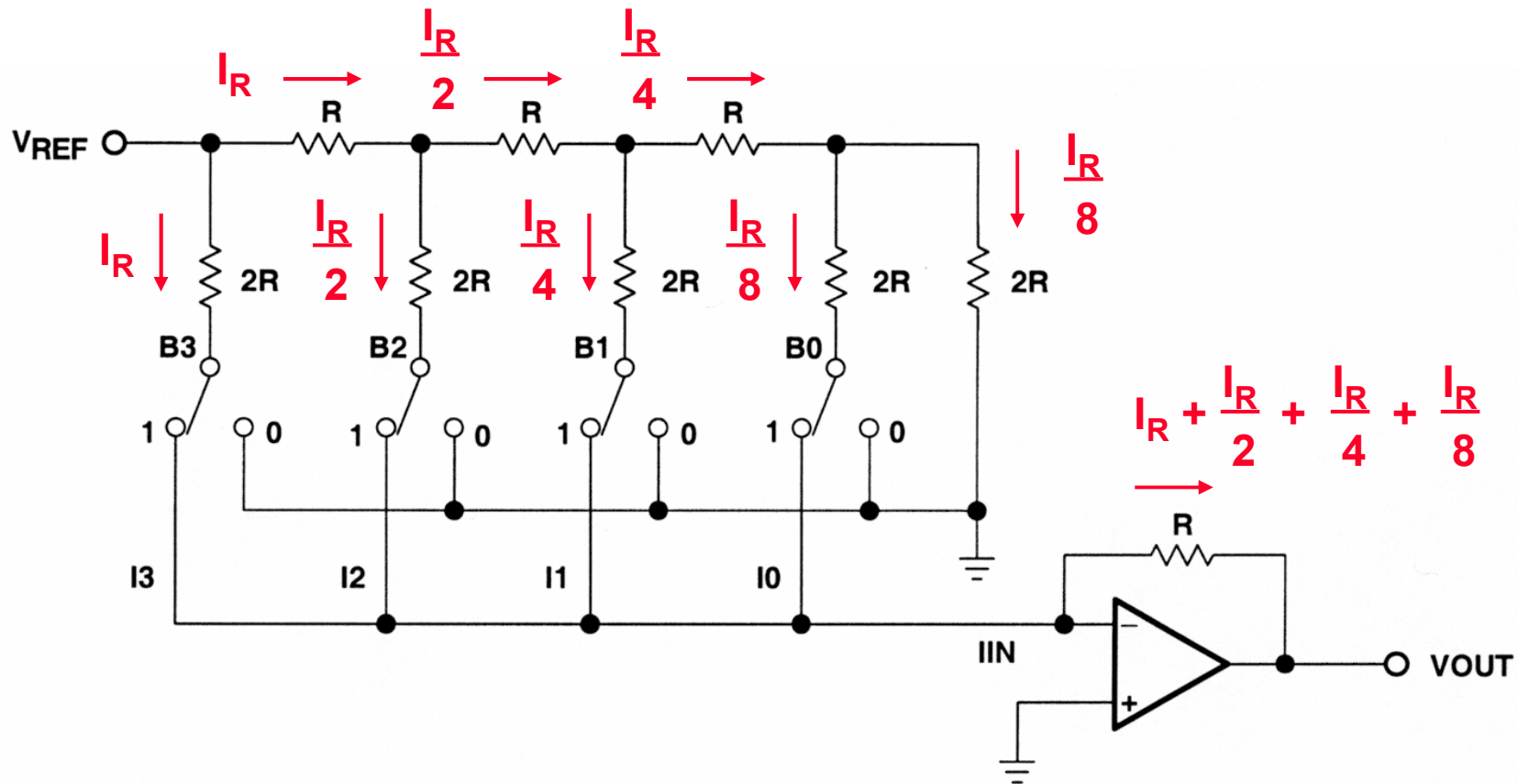
# Binary weighted resistor DAC

---



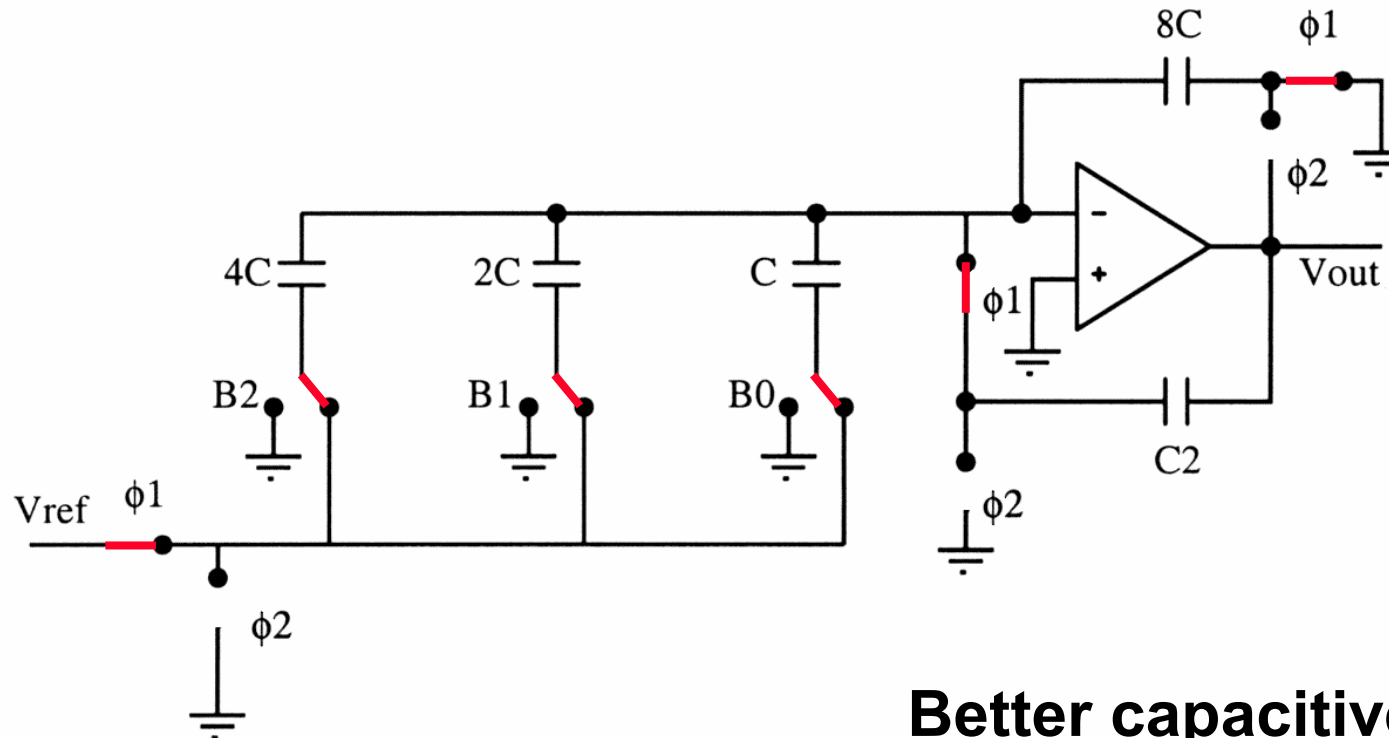
**One Resistor and Switch per bit**  
**No guaranteed monotonicity (glitches !)**

# R-2R DAC



**Smaller area in Resistors !**

# 3-bit charge redistribution DAC



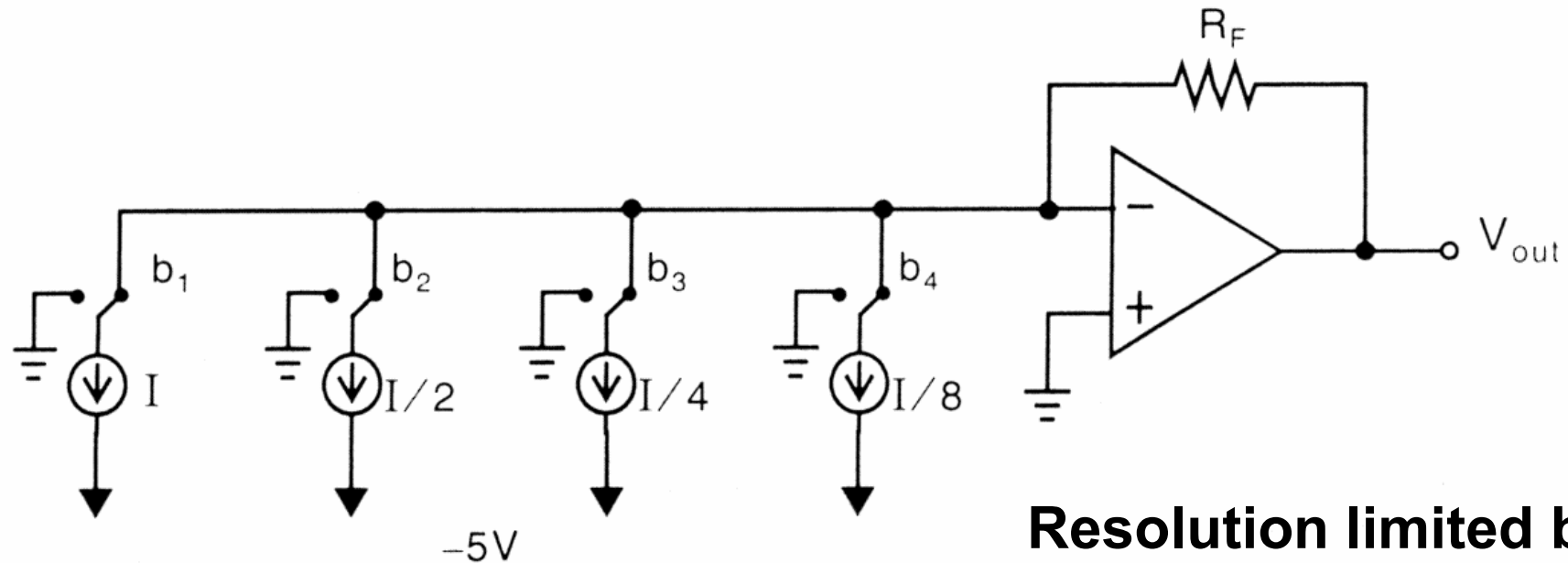
**Better capacitive matching !**

**Phase  $\Phi1$**

---

# 4-bit Current steering DAC

---



**Resolution limited by  
Mismatch in the  
Current sources !**

**Glitches !**

---

# The Binary and thermometer codes

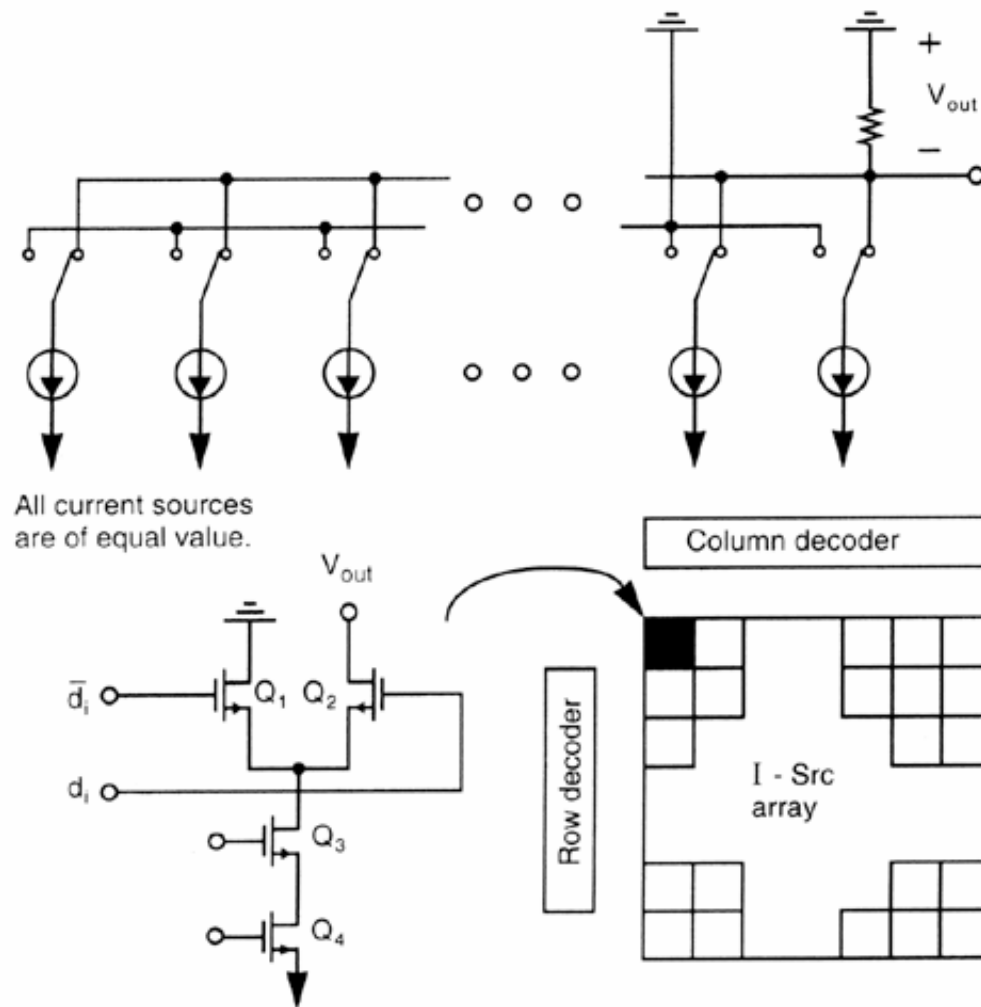
---

Decimal	Binary			Thermometer Code						
	$b_1$	$b_2$	$b_3$	$d_1$	$d_2$	$d_3$	$d_4$	$d_5$	$d_6$	$d_7$
0	0	0	0	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0	1
2	0	1	0	0	0	0	0	0	1	1
3	0	1	1	0	0	0	0	1	1	1
4	1	0	0	0	0	0	1	1	1	1
5	1	0	1	0	0	1	1	1	1	1
6	1	1	0	0	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1

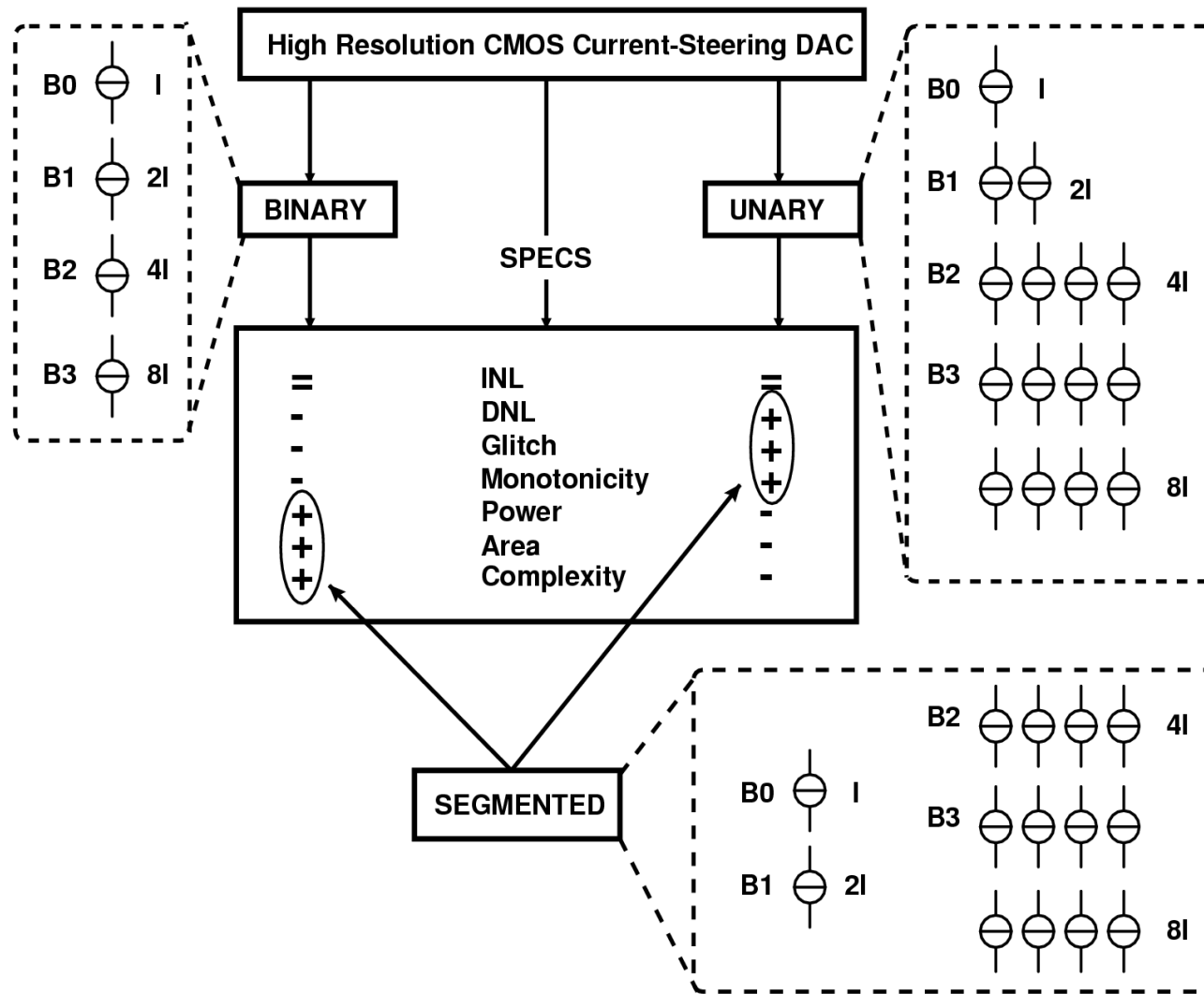
**Monotonicity guaranteed !**



# Thermometer-code Current steering DAC



# Binary, unary, segmented DAC



$$\sigma(\Delta I) =$$

**Binary**

$$\sqrt{2^N - 1} \frac{\sigma(I)}{I} \text{ LSB}$$

**Unary**

$$\frac{\sigma(I)}{I} \text{ LSB}$$

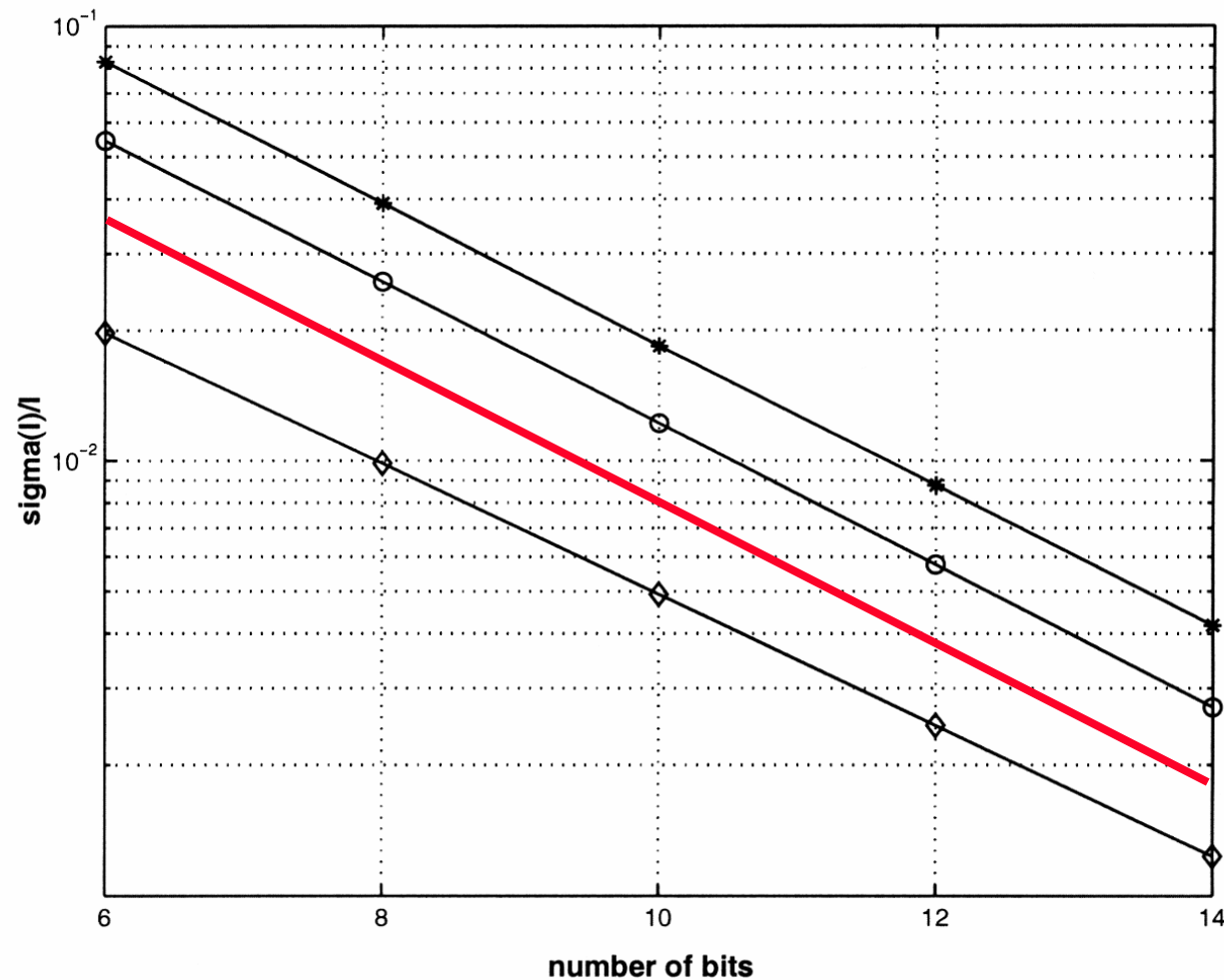
**Segmented**

**B LSBs & N-B MSBs**

$$\sqrt{2^{B+1} - 1} \frac{\sigma(I)}{I} \text{ LSB}$$

Van den Bosch, ...,  
Kluwer 2004

# $\sigma(I)/I$ versus resolution



$$\frac{\sigma(I)}{I} = \frac{1}{2 C \sqrt{2^N}}$$

$C \approx 6.2 \cdot 10^{-4}$   
for INL\_yield = 90%

Yield =

10 %

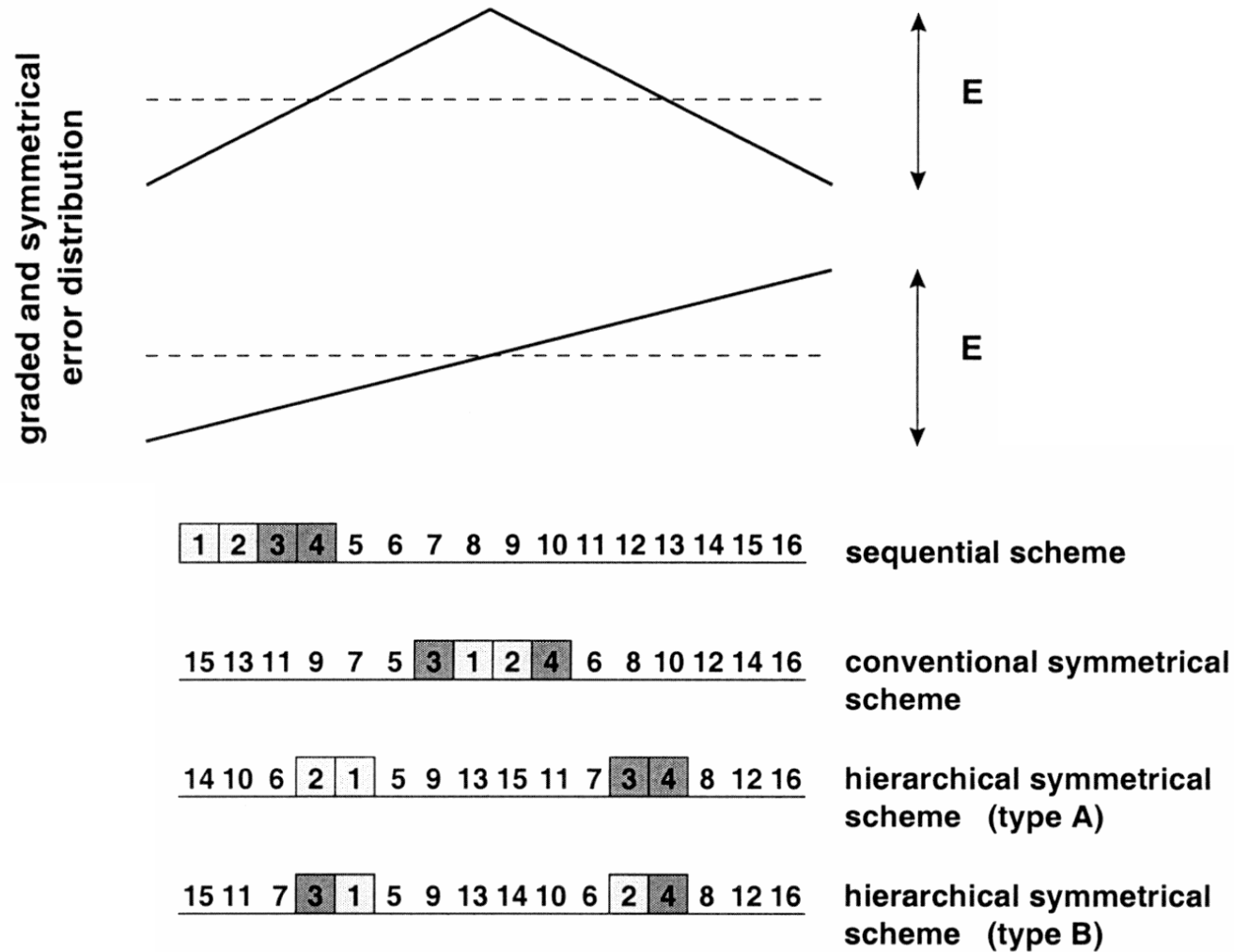
50 %

90 %

99.7 %

Van den Bosch, ...,  
Kluwer 2004

# Switching schemes



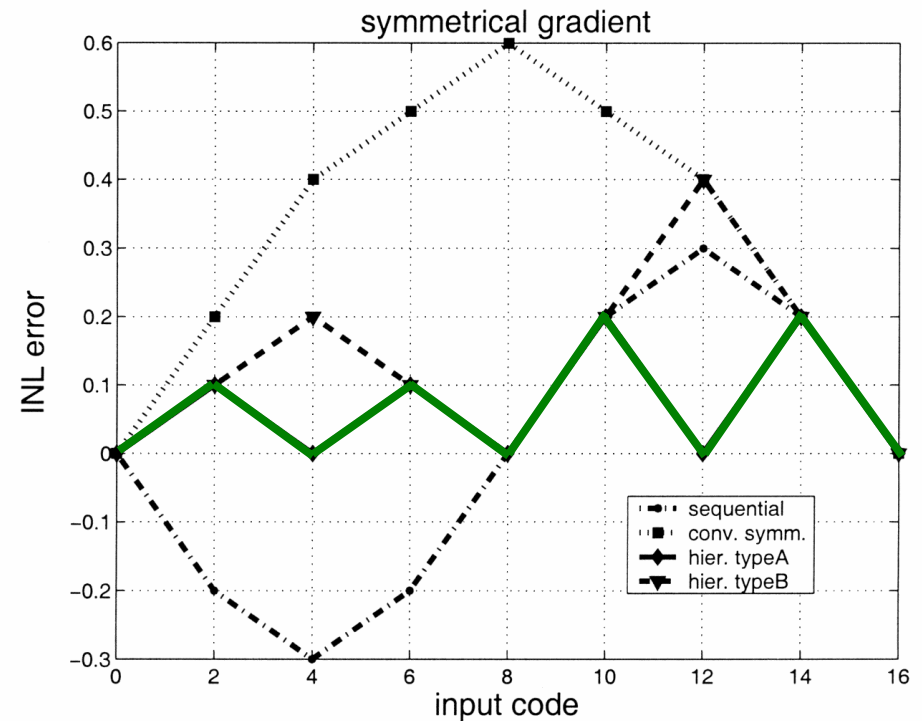
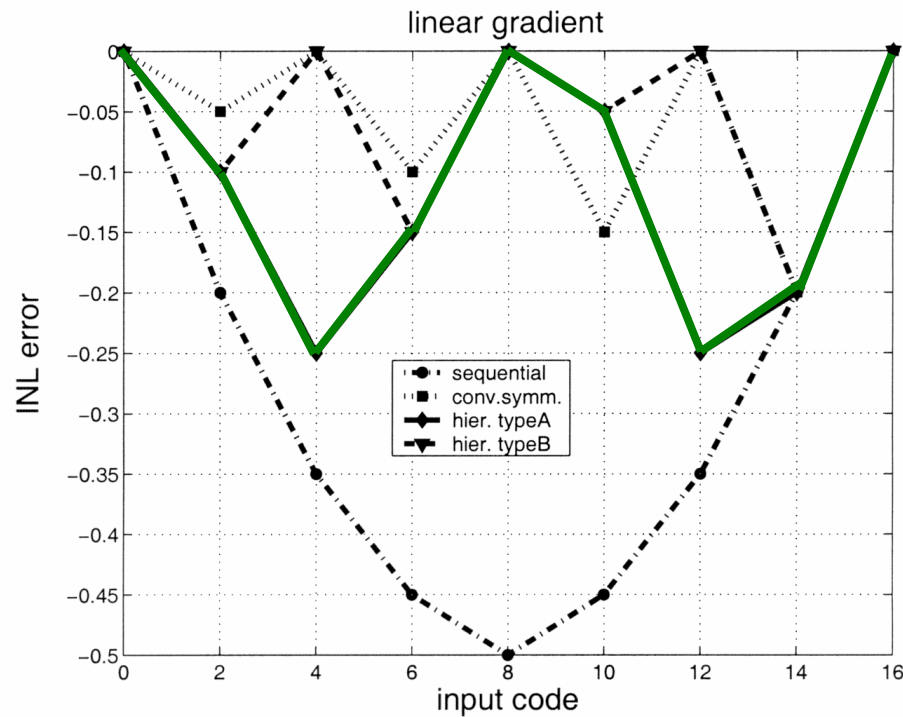
is centroide !

Van den Bosch, ...,  
Kluwer 2004  
Miki, JSSC Dec.86,  
983-988

---

# INL error for different switching schemes

---



**Hierarchical symmetrical scheme (type A)**

---

# DAC Design: Static Accuracy

---

INL\_yield = percentage of functional D/A converters with an INL specification smaller than half an LSB.

$$\text{INL\_yield} = f(\text{mismatch}) = f\left(\frac{\sigma(I)}{I}\right)$$

$$WL = \frac{1}{2 \left(\frac{\sigma(I)}{I}\right)^2} \left[ A_{\beta}^2 + \frac{4A_{VT}^2}{(V_{GS}-V_T)^2} \right]$$

High yield  
↓  
small  $\frac{\sigma(I)}{I}$   
↓

Large current source area

$$\frac{\sigma(I)}{I} \leq \frac{1}{2 C \sqrt{2^N}}$$



$$\sigma(I_{\text{unit}})/I_{\text{unit}} = 0.25 \%$$

---

## DAC Design: Calculation W and L

---

$$\frac{W}{L} = \frac{I_{\text{LSB}}}{K' (V_{\text{GS}} - V_{\text{T}})^2}$$

$$WL = \frac{1}{2 \left( \frac{\sigma(I)}{I} \right)^2} \left[ A_{\beta}^2 + \frac{4A_{V_{\text{T}}}^2}{(V_{\text{GS}} - V_{\text{T}})^2} \right]$$

$$\sigma(I)/I = 0.0025$$

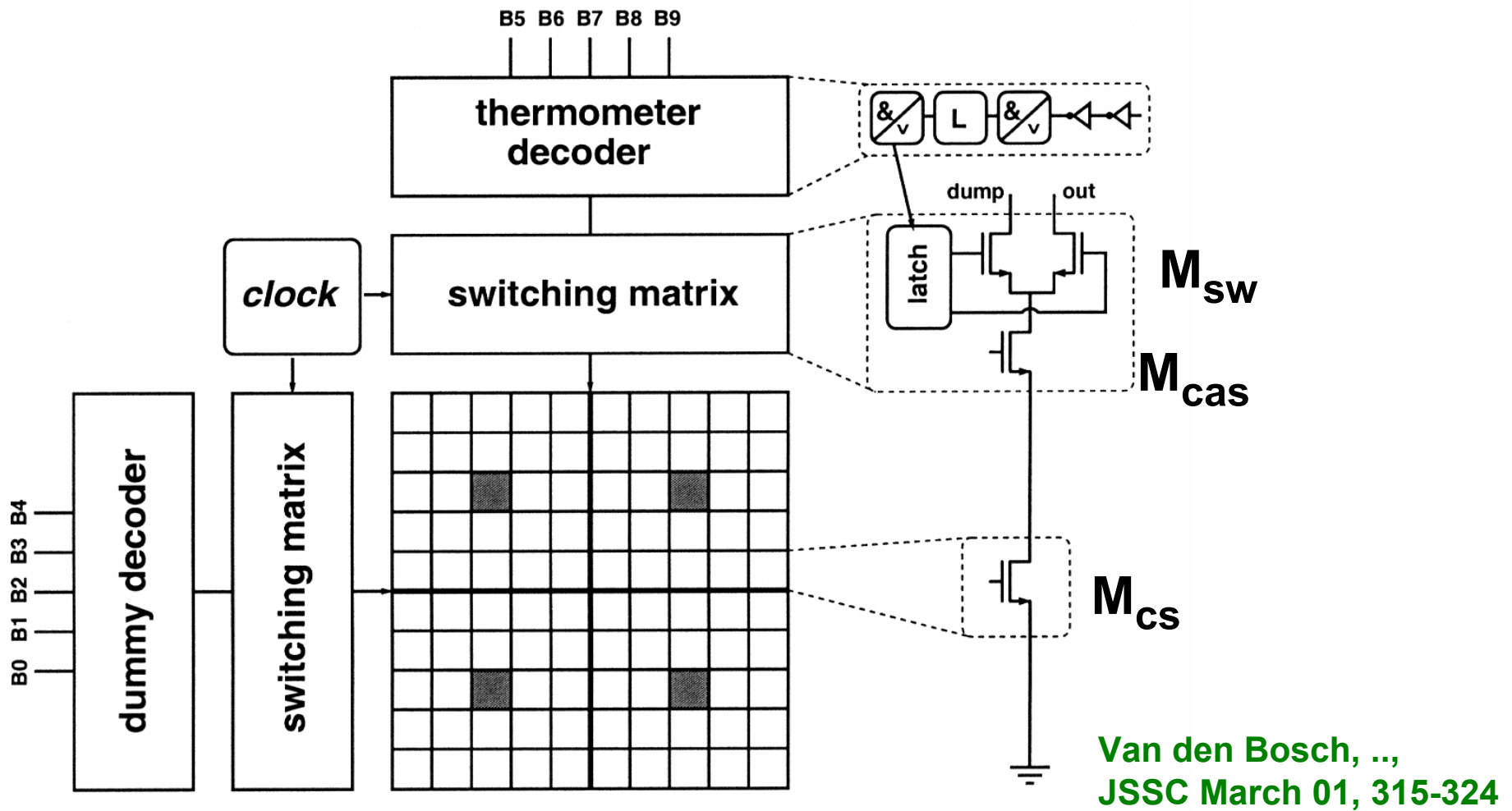
$$V_{\text{GS}} - V_{\text{T}} = 1 \text{ V}$$

$$\text{For } 0.35 \text{ } \mu\text{m CMOS } A_{\beta} \approx 2 \% \mu\text{m}$$

$$A_{V_{\text{T}}} \approx 7 \text{ mV} \mu\text{m}$$

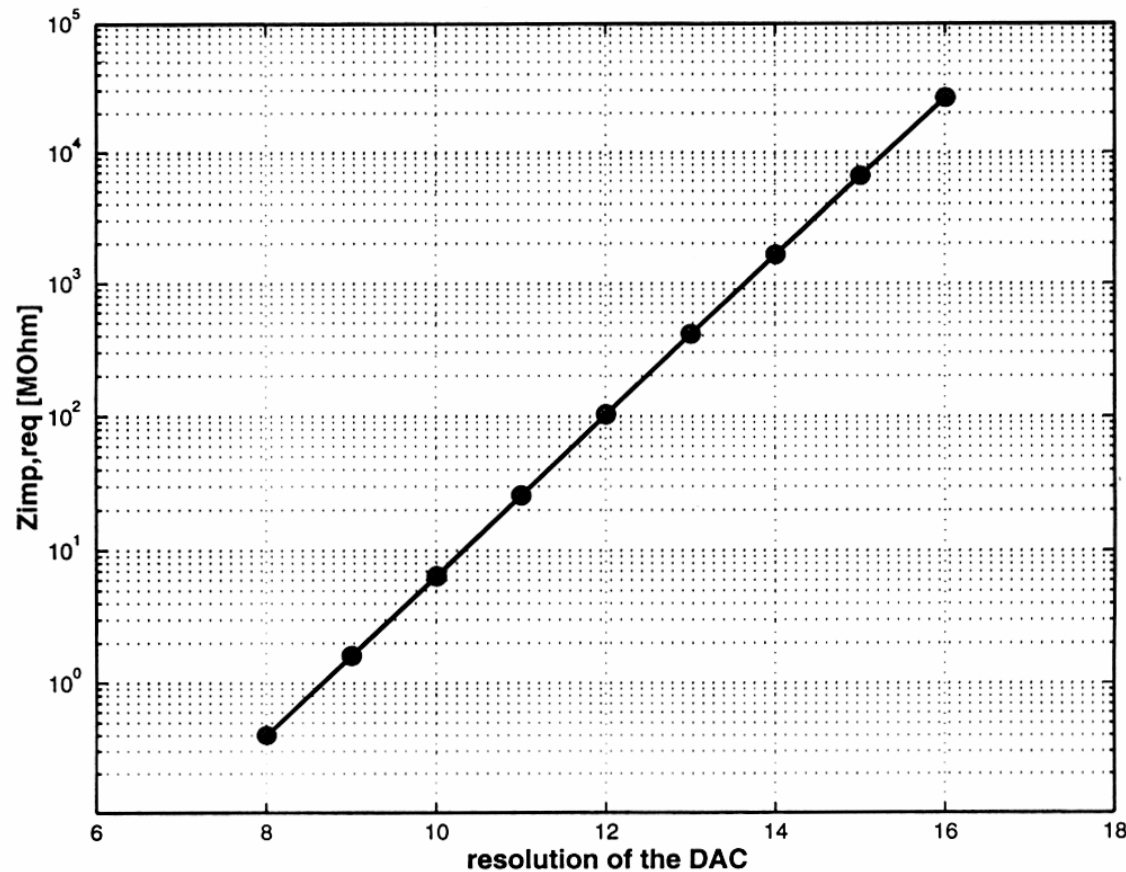
$$\begin{aligned} W &= 1.8 \text{ } \mu\text{m} \\ L &= 30 \text{ } \mu\text{m} \end{aligned}$$

# Floorplan 10-bit segmented DAC





# Required output impedance versus resolution



Van den Bosch, ..., Kluwer 2004  
JSSC March 01, 315-324

$A_{VT}$	8.94 mV $\mu$ m
$A_{\beta}$	1.9 % $\mu$ m
$\sigma(I)/I$	0.5 %
$(V_{GS} - V_T)_{cs}$	1 V
$I_{FS}$	20 mA
segmentation	5-5
$(W/L)_{cs}$	2 $\mu$ m/8 $\mu$ m
$(W/L)_{sw}$	1 $\mu$ m/0.7 $\mu$ m
$(W/L)_{cas}$	0.5 $\mu$ m/0.35 $\mu$ m

**10 bit 1GB/s**

**INL = 99.7 %**

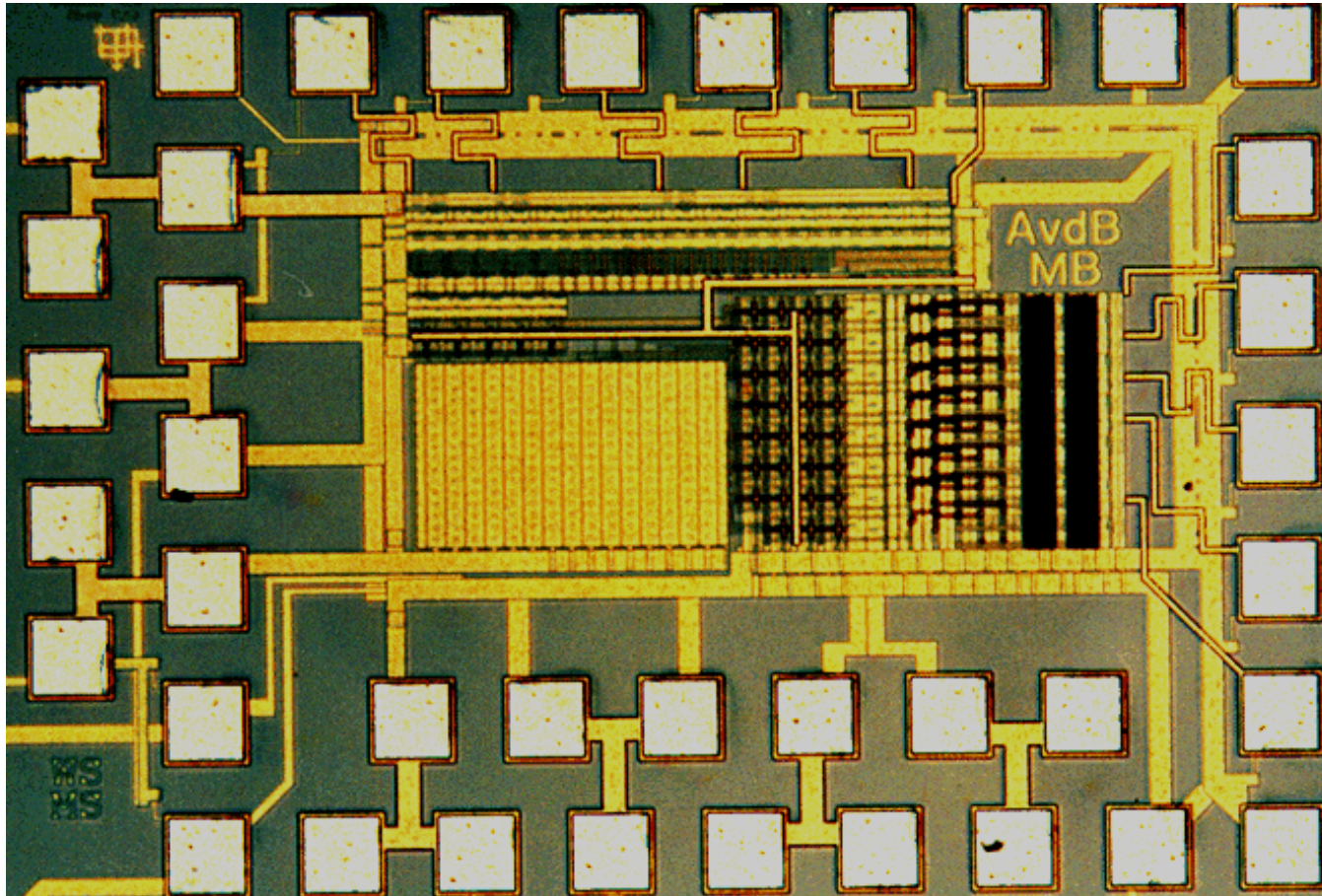
**requires  $\sigma(I)/I < 0.5$  % :**

**$W = 2 \mu$ m &  $L = 8 \mu$ m**

---

# 10-bit 1 GS/s Nyquist Current steering CMOS DAC

---



**Current steering  
DAC**

**10-bit**

**1 GS/s**

**0.35  $\mu\text{m}$  CMOS**

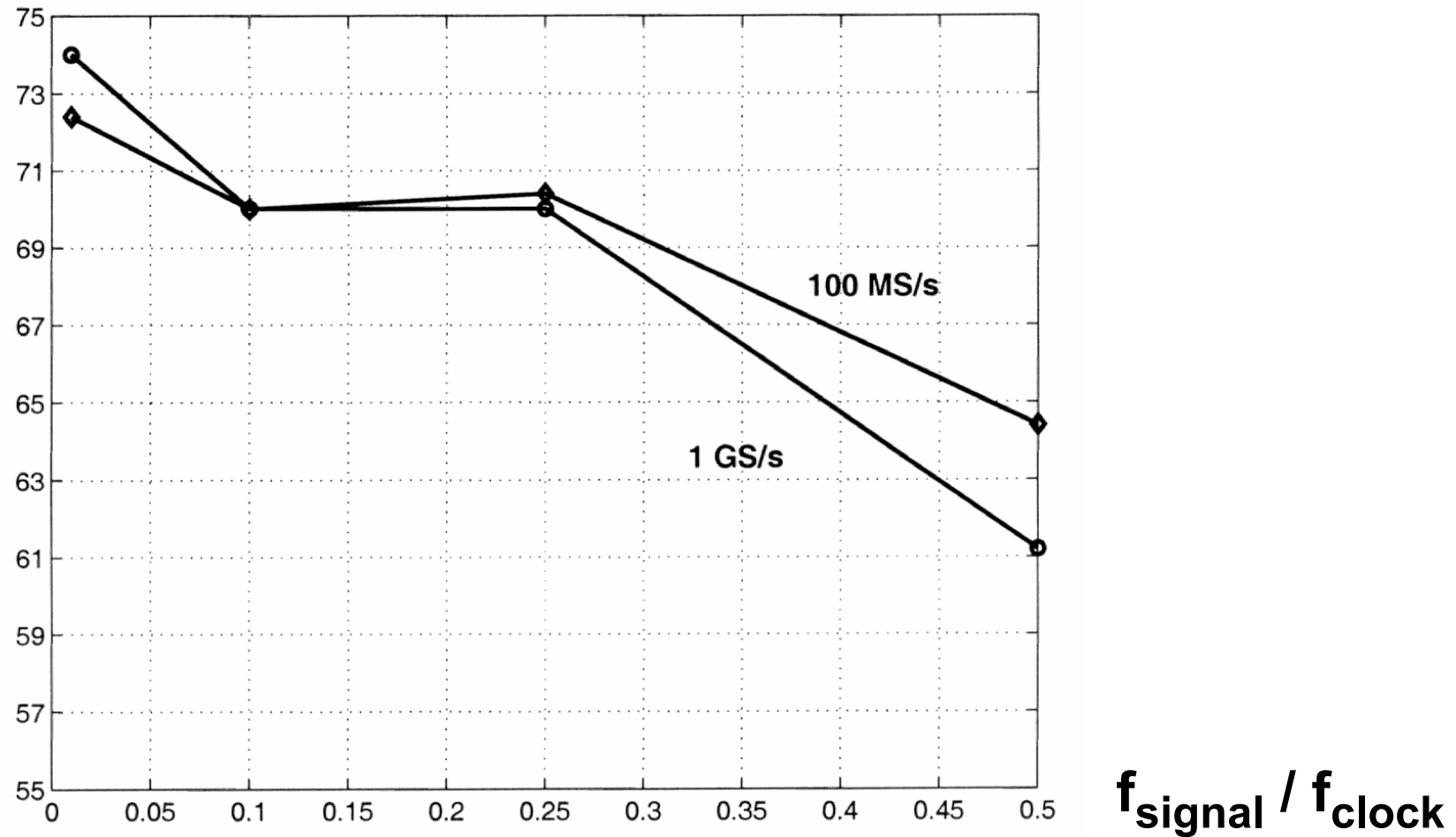
**110 mW**

Van den Bosch, .., JSSC, March 01, 315-324

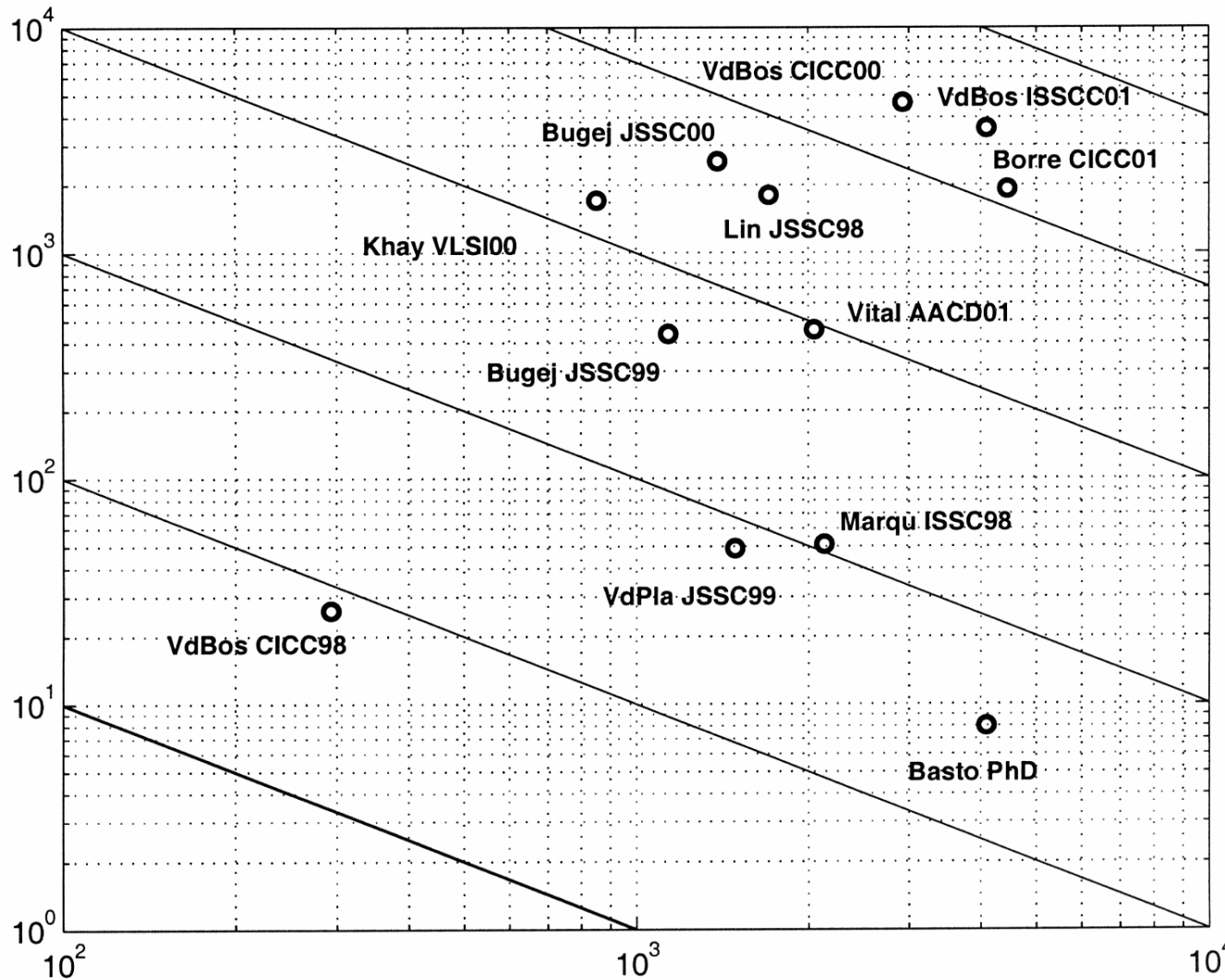
---

# SFDR (dB) versus relative signal frequency

---



# FOM (MHz/mW) vs inverse area



$$\text{FOM} = \frac{2^N f_s(-6\text{dB})}{P}$$

$2^N/\text{area (mm}^2\text{)}$

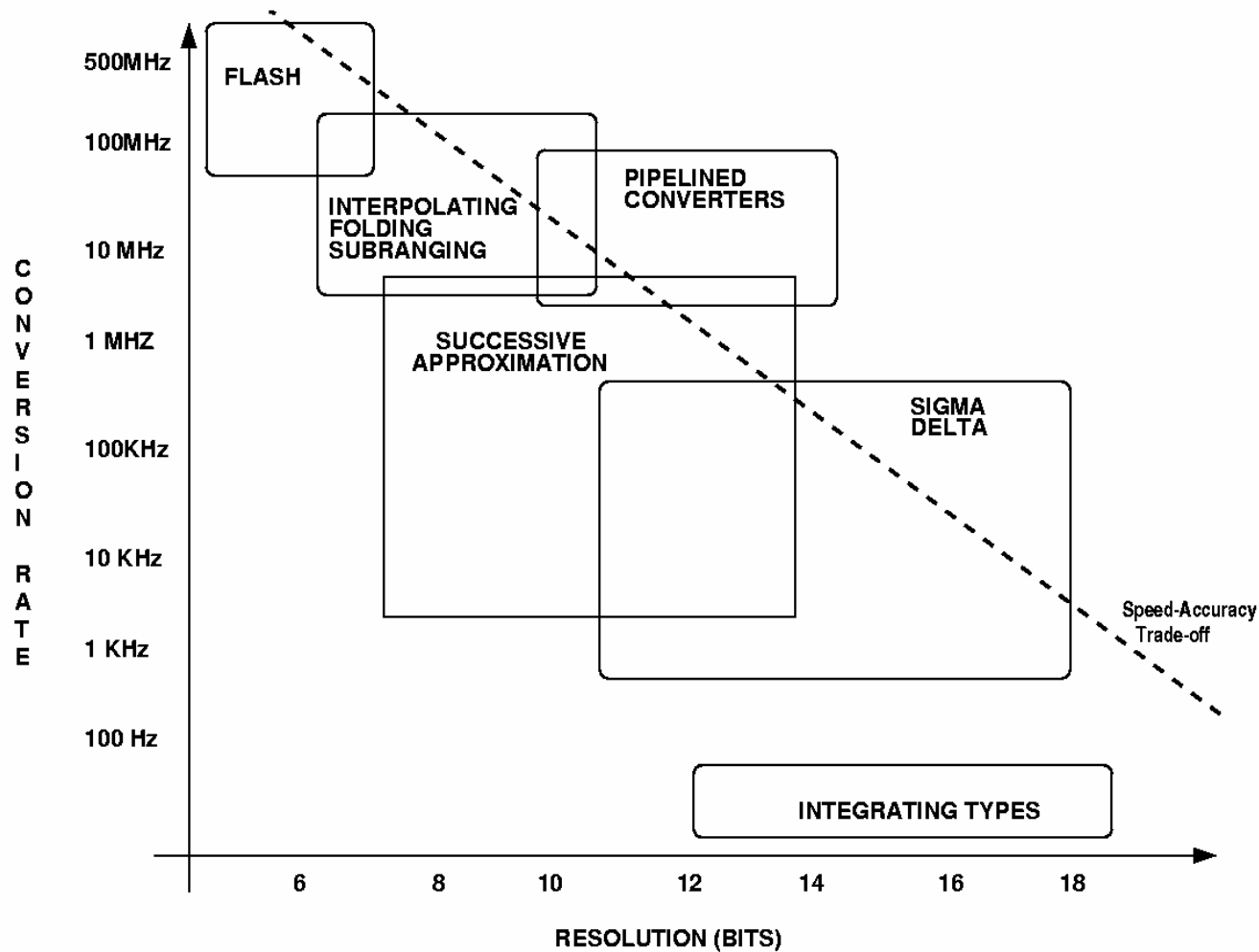
---

# Table of contents

---

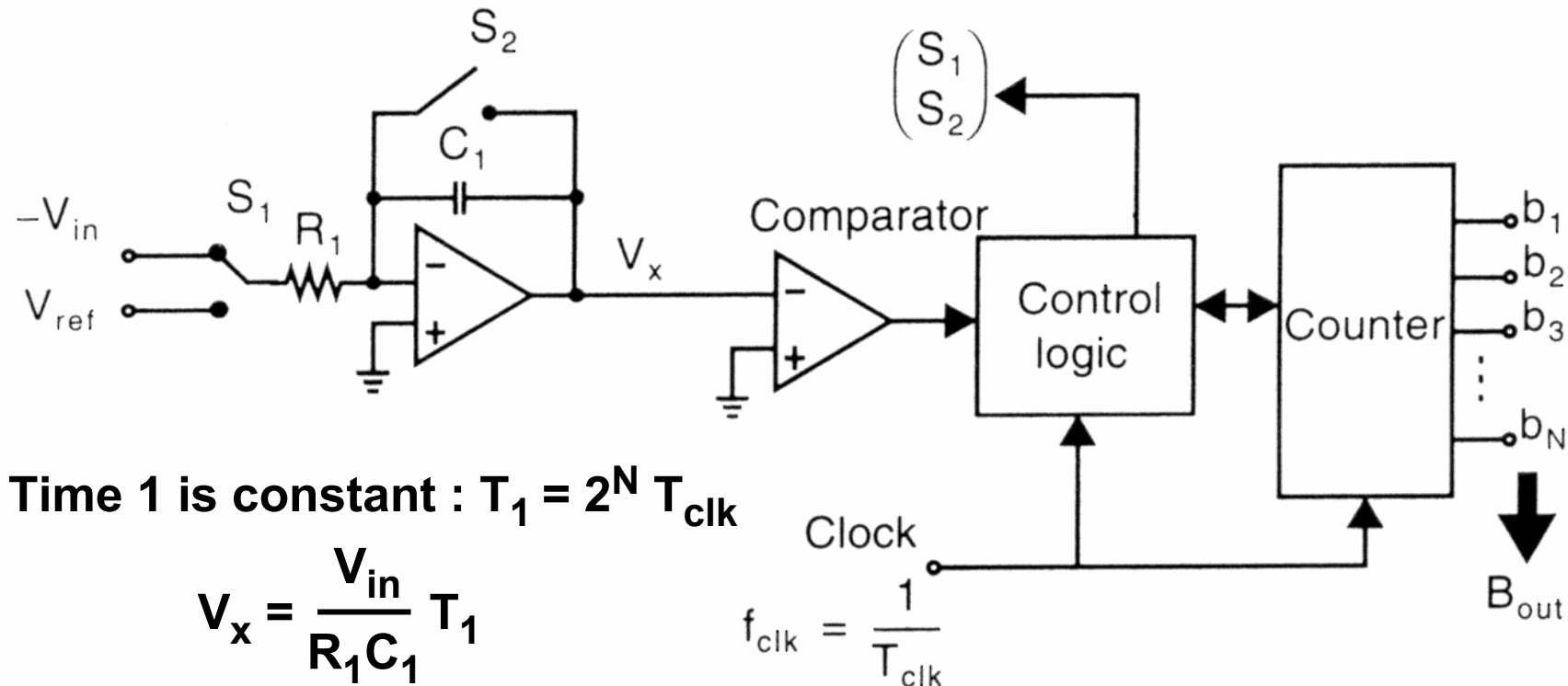
- **Definitions**
- **Digital-to-analog converters**
  - Resistive
  - Capacitive
  - Current steering
- **Analog-to-digital converters**
  - Integrating
  - Successive approximation
  - Algorithmic
  - Flash / Two-step
  - Interpolating / Folding
  - Pipeline

# Speed Resolution Limits ADC



$$\text{FOM} = \frac{4kT \text{ BW DR}}{P}$$
$$\frac{2^N 2\text{BW}}{P}$$

# Dual-slope (integrating) ADC



Time 1 is constant :  $T_1 = 2^N T_{clk}$

$$V_x = \frac{V_{in}}{R_1 C_1} T_1$$

Time 2 :  $V_x$  decreases with constant slope :

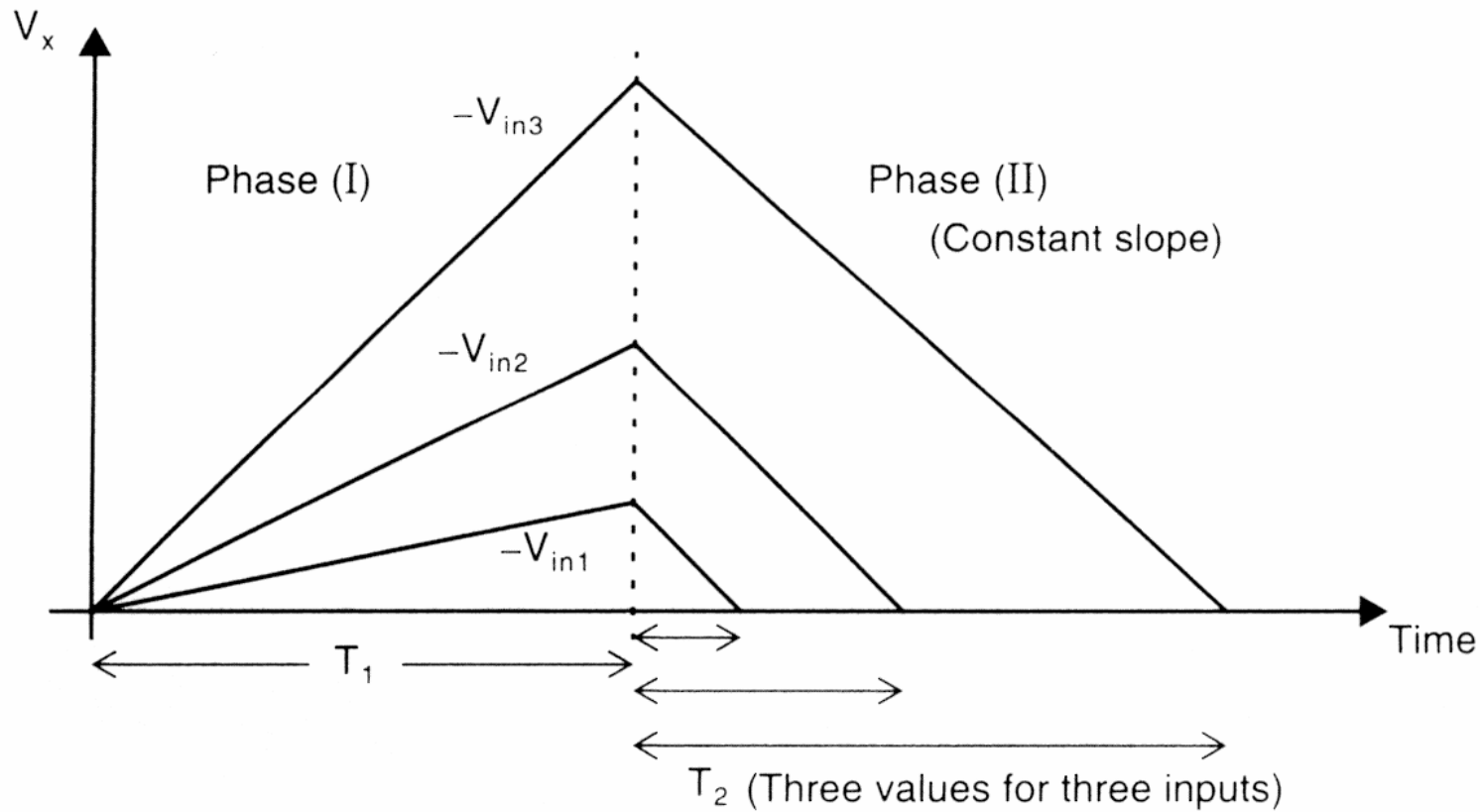
$$V_x = \frac{V_{ref}}{R_1 C_1} T_2$$

$$T_2 = T_1 \frac{V_{in}}{V_{ref}}$$

$$\Rightarrow B_{out} = \frac{V_{in}}{V_{ref}}$$

Johns, Martin, Wiley 1997

# Operation of integrating ADC



Time 1 :  $V_x = \frac{V_{in}}{R_1 C_1} T_1$

Time 2 :  $V_x = \frac{V_{ref}}{R_1 C_1} T_2$

$T_2 = T_1 \frac{V_{in}}{V_{ref}}$



---

# Integrating ADC

---

## Advantages:

High resolution

High linearity

Low circuit complexity

Mainly for voltmeters, ...

Eliminates mains supply 50 Hz if T1 is  $n \times 20$  ms

## Disadvantages:

Very slow :

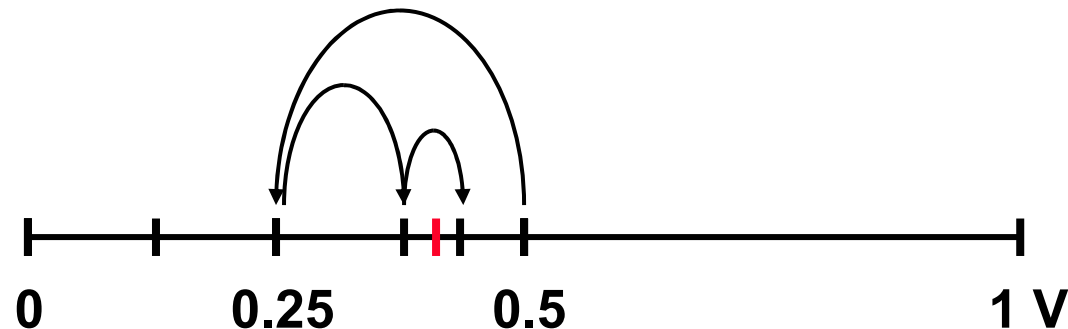
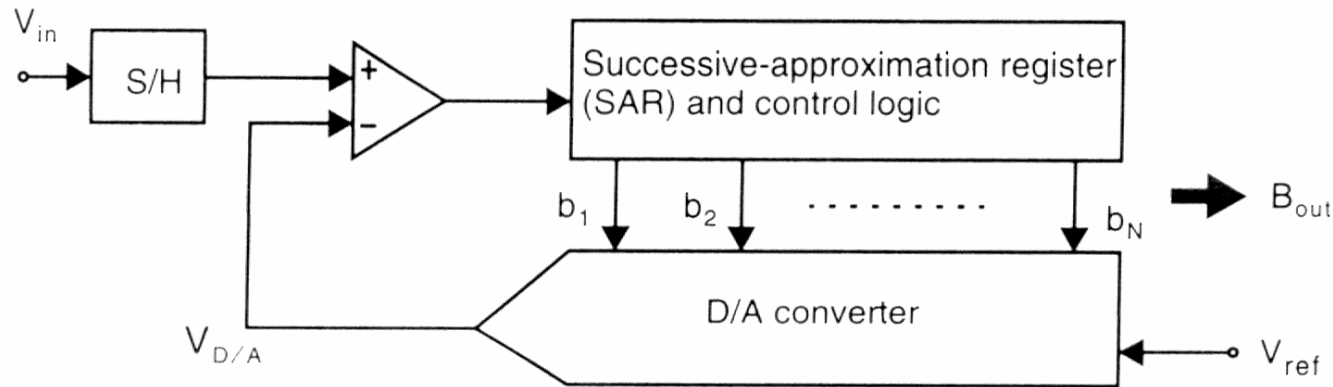
Worst case for  $V_{in} = V_{ref}$  :  $2^{2n+1}$  clock cycles required !

Ex. For  $n = 16$  bit (64000) and  $F_{clock} = 1$  MHz :

7.6 s conversion time

Mainly for voltmeters, ...

# Successive-approximation ADC



**Divide interval by 2 ;**

**Determine bit :**

**< 1 : 0  $b_1$  = MSB**

**< 0.5 : 0  $b_2$**

**> 0.25 : 1  $b_3$**

**> 0.375 : 1  $b_4$**

**< 0.4375 : 0  $b_5$**

.....

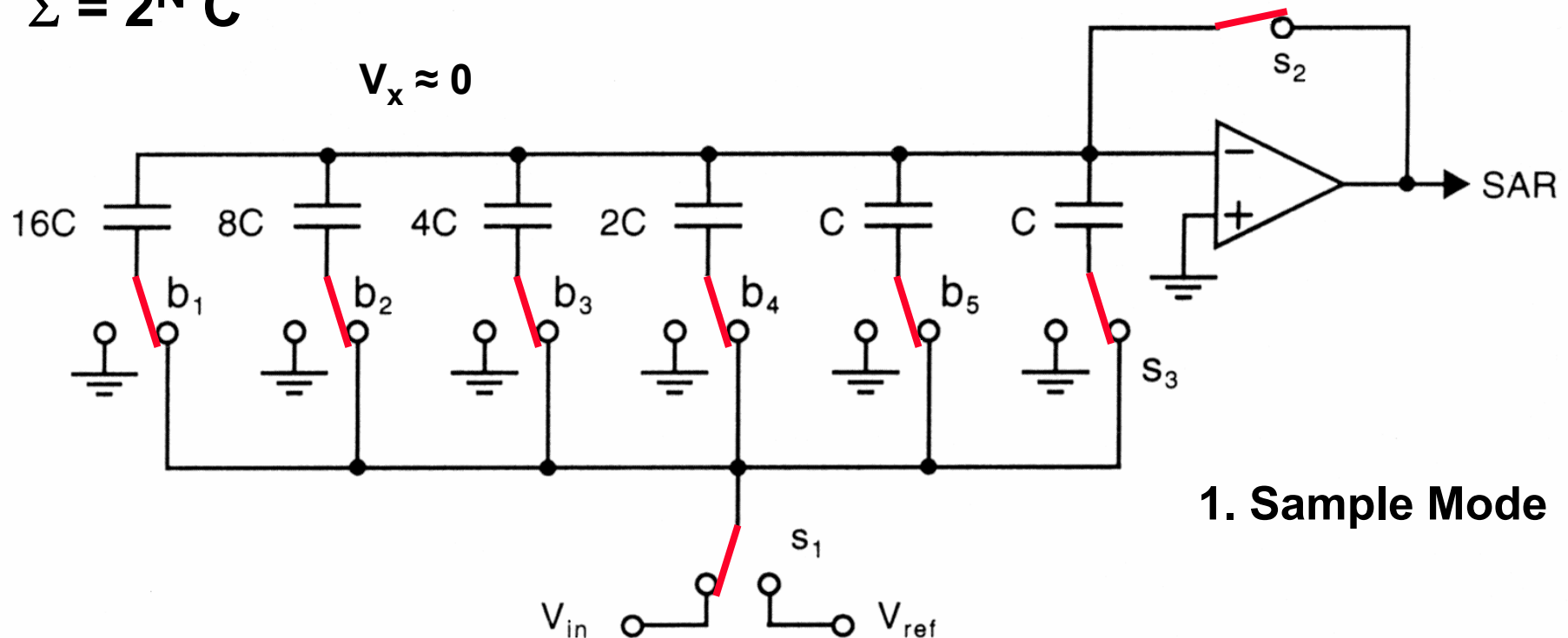
**Johns, Martin, Wiley 1997**

---

# 5-bit Charge redistribution ADC

---

$$\Sigma = 2^N C$$



Accuracy limited by capacitive matching to 10-12 bit

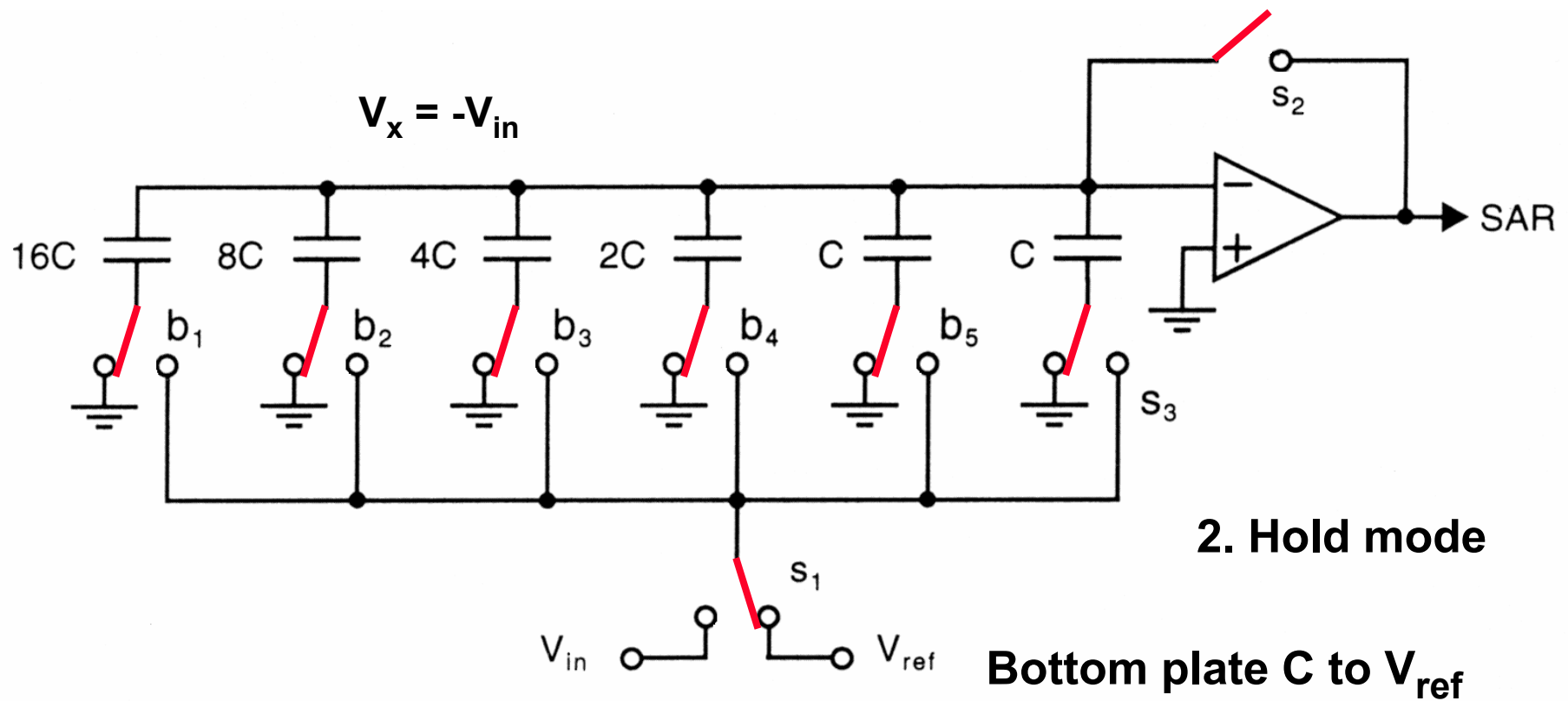
Speed limited by  $R_{\text{switch}} C$  time constants

McCreary, JSSC

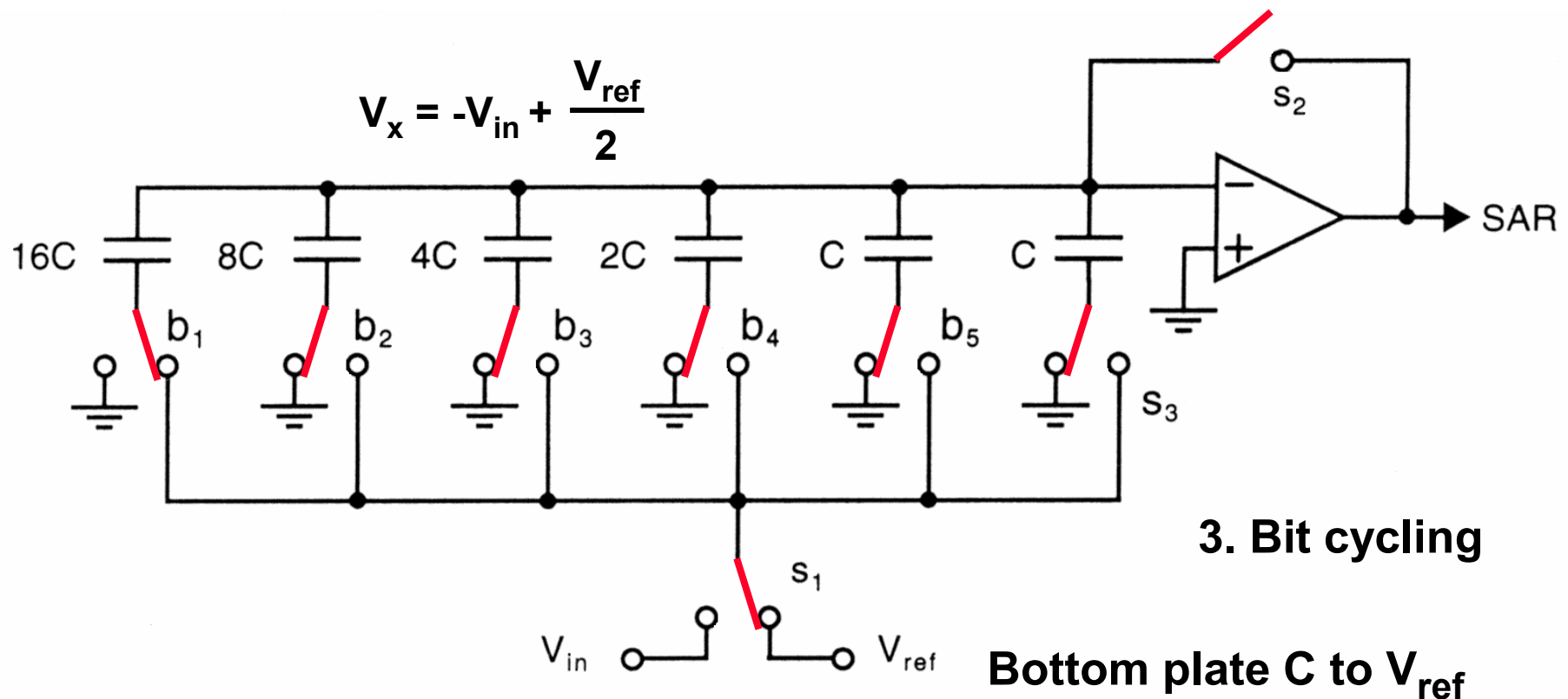
Dec 75, 371-379

Johns, Martin, Wiley 1997

# 5-bit Charge redistribution ADC



# 5-bit Charge redistribution ADC

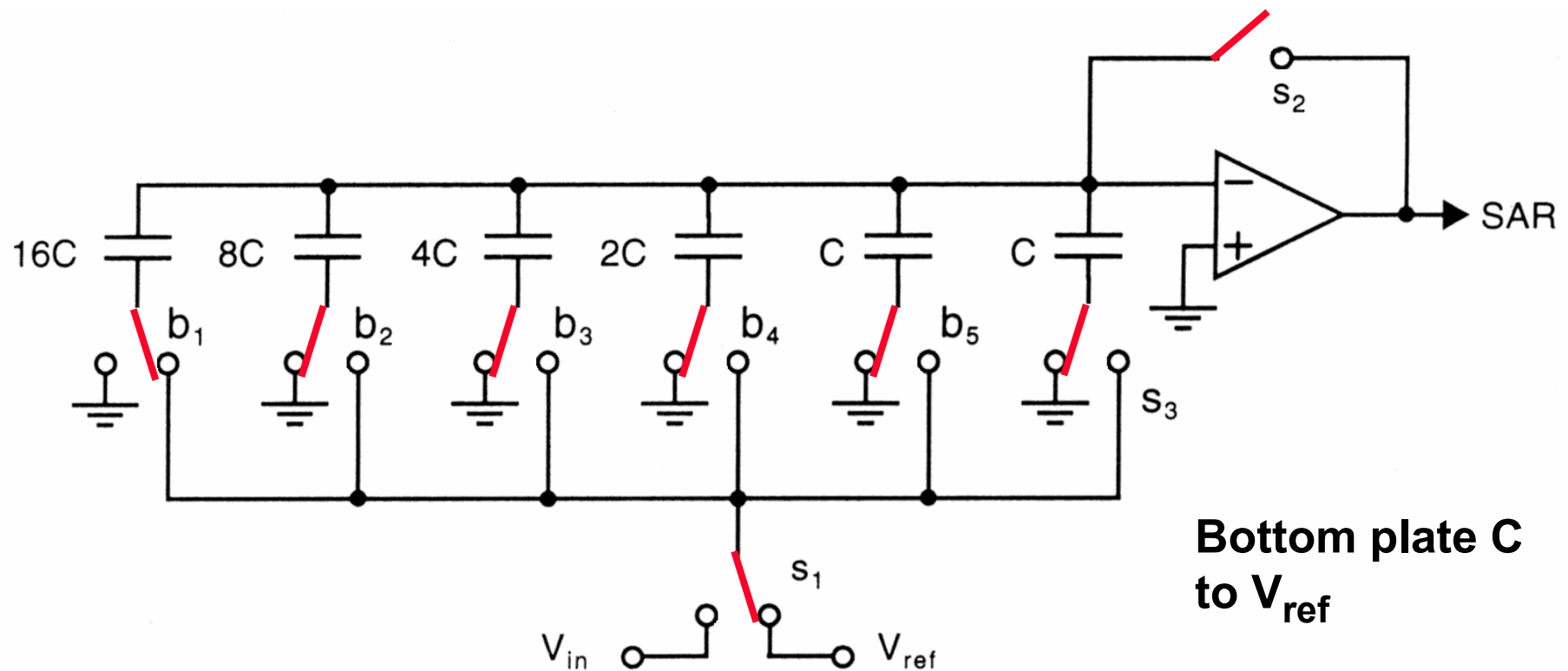


if $V_{in} > V_{ref}/2$	$SAR \Rightarrow 1$	leave $C_{b1}$ to $V_{ref}$	: try $C_{b2}$
if $V_{in} < V_{ref}/2$	$SAR \Rightarrow 0$	leave $C_{b1}$ to Gnd	: try $C_{b2}$

---

# Charge redistribution ADC

---

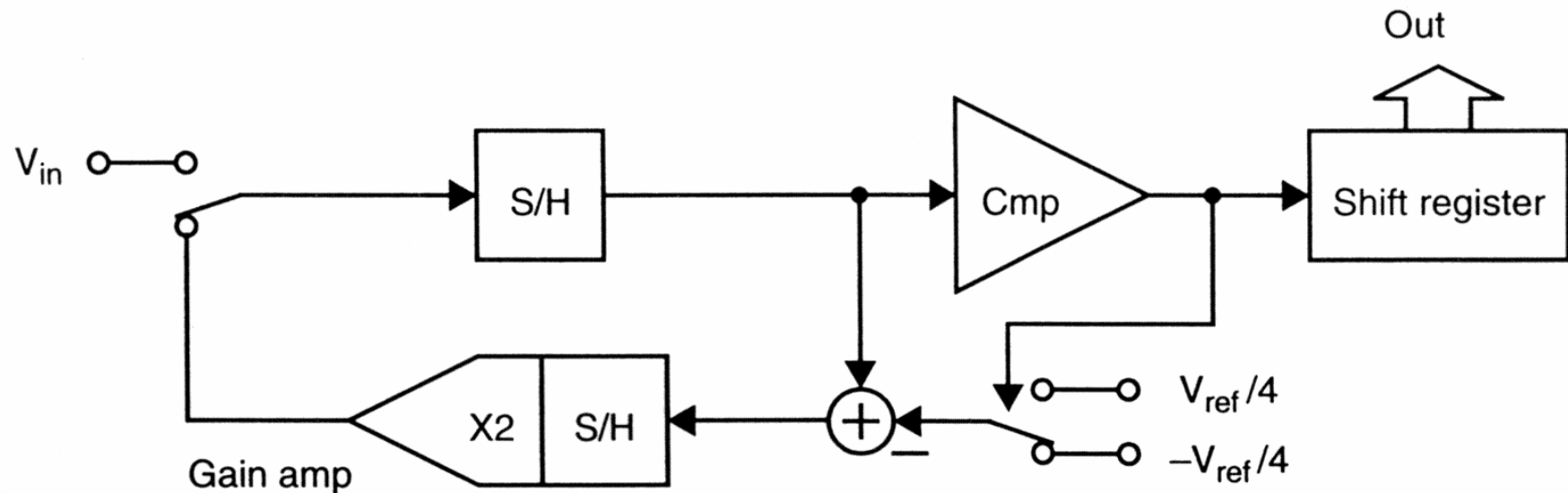


Charge redistribution ADC halves  $V_{ref}$  in each cycle  
Algorithmic ADC doubles  $V_{error}$  in each cycle

---

# Algorithmic (or cyclic) ADC

---

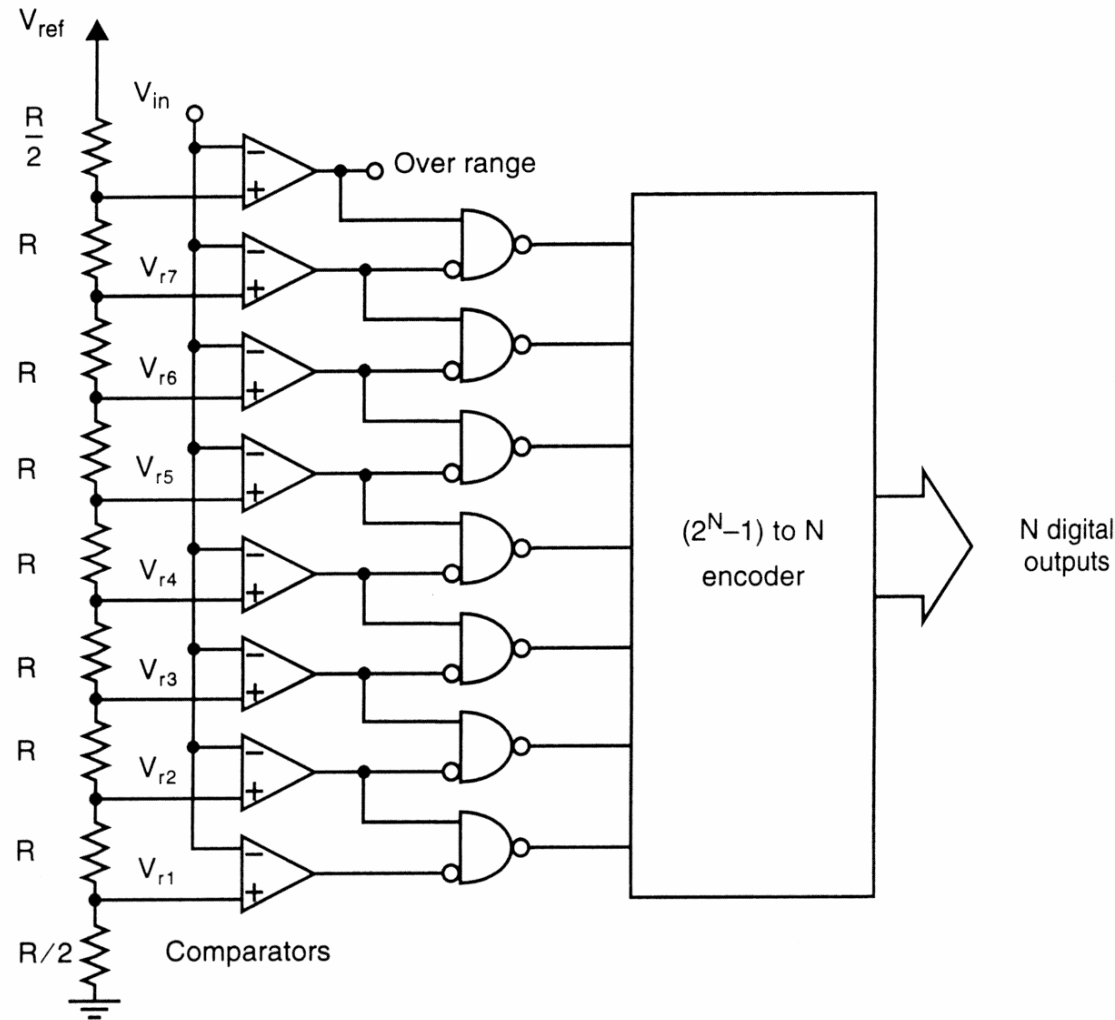


**Advantage :** small amount of analog circuitry

**Difficulty :** accuracy  $\times 2$  Gain amplifier  
(fully diff. ;  $C_{par}$  insensitive)

Johns, Martin, Wiley, 2003

# Flash converter



**3-bit flash ADC :**

**fastest**  
 **$2^3$  comparators**  
**input cap.  $\sim 2^3$**

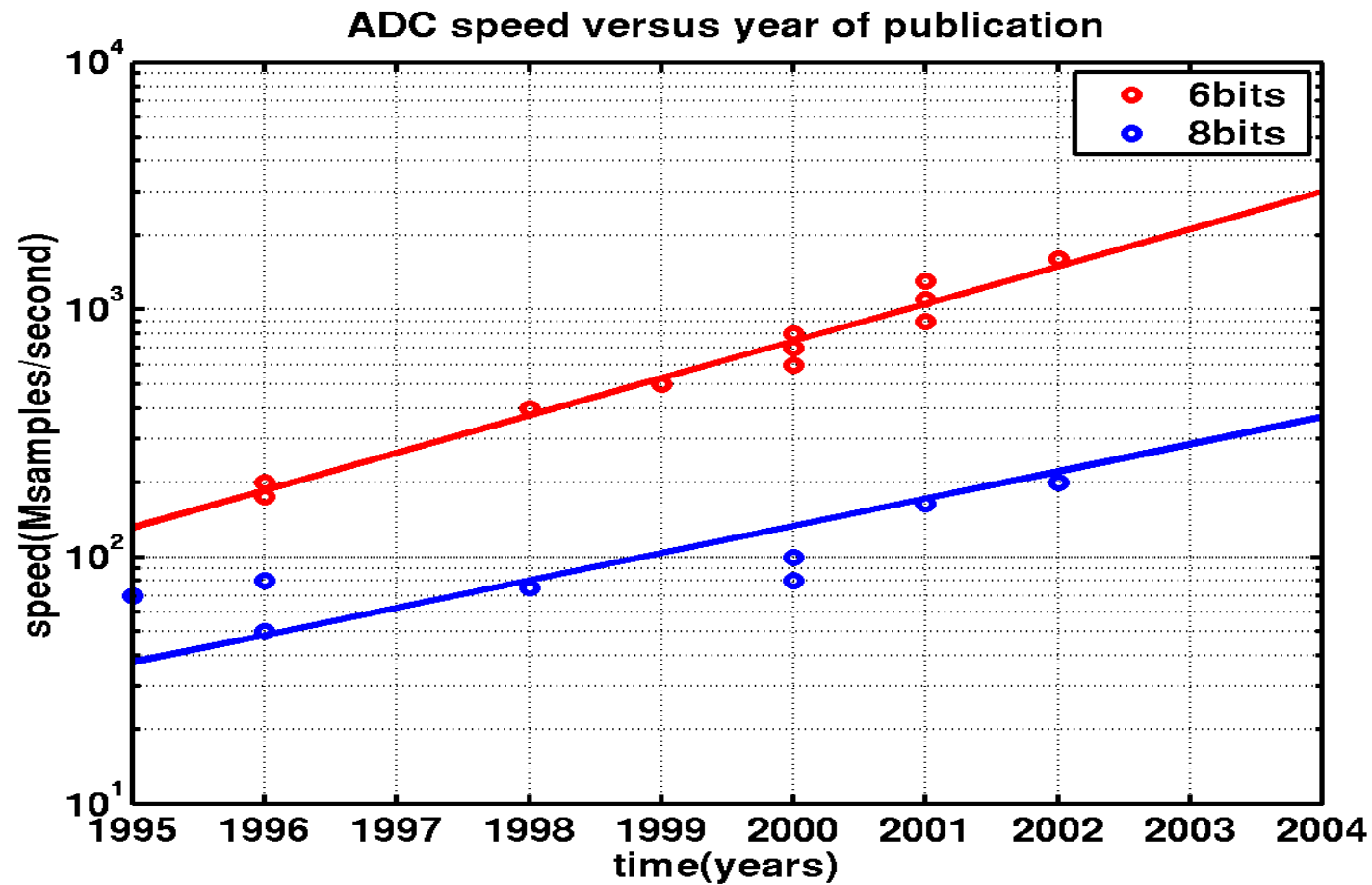
**limited to 6 bit**  
**(1 ... 2 GS/s)**



---

# Evolution in ADC's

---

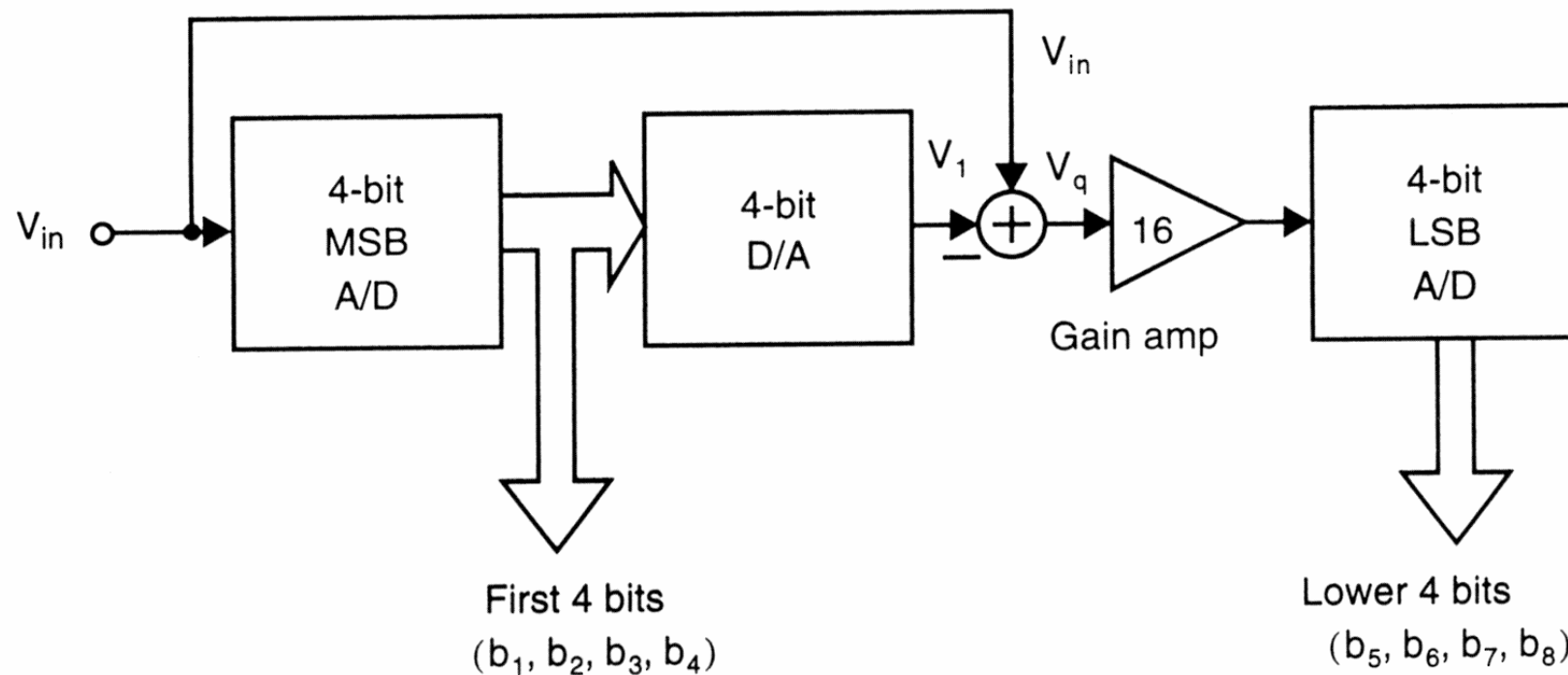


Uyttenhove, KULeuven, 2003

---

# Subranging (or two-step) ADC

---



**8-bit two-step ADC :**  
less comparators  
introduces latency

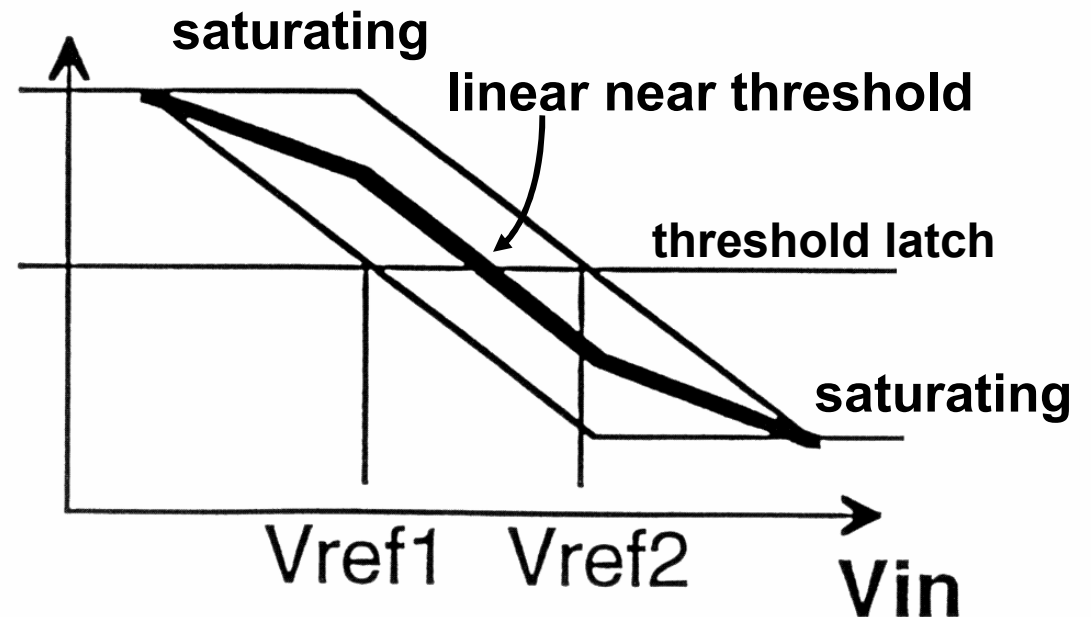
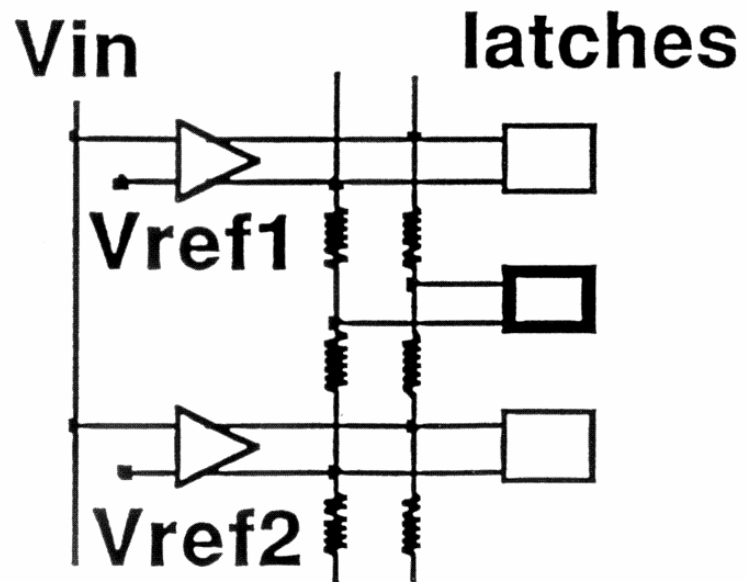
**$2^8 = 256$  comp.  $\Rightarrow$  now 32 !**  
**All circuits : 8b accurate**  
**Digital correction required !**

Johns, Martin, Wiley 1997

---

# Interpolating saves amplifiers

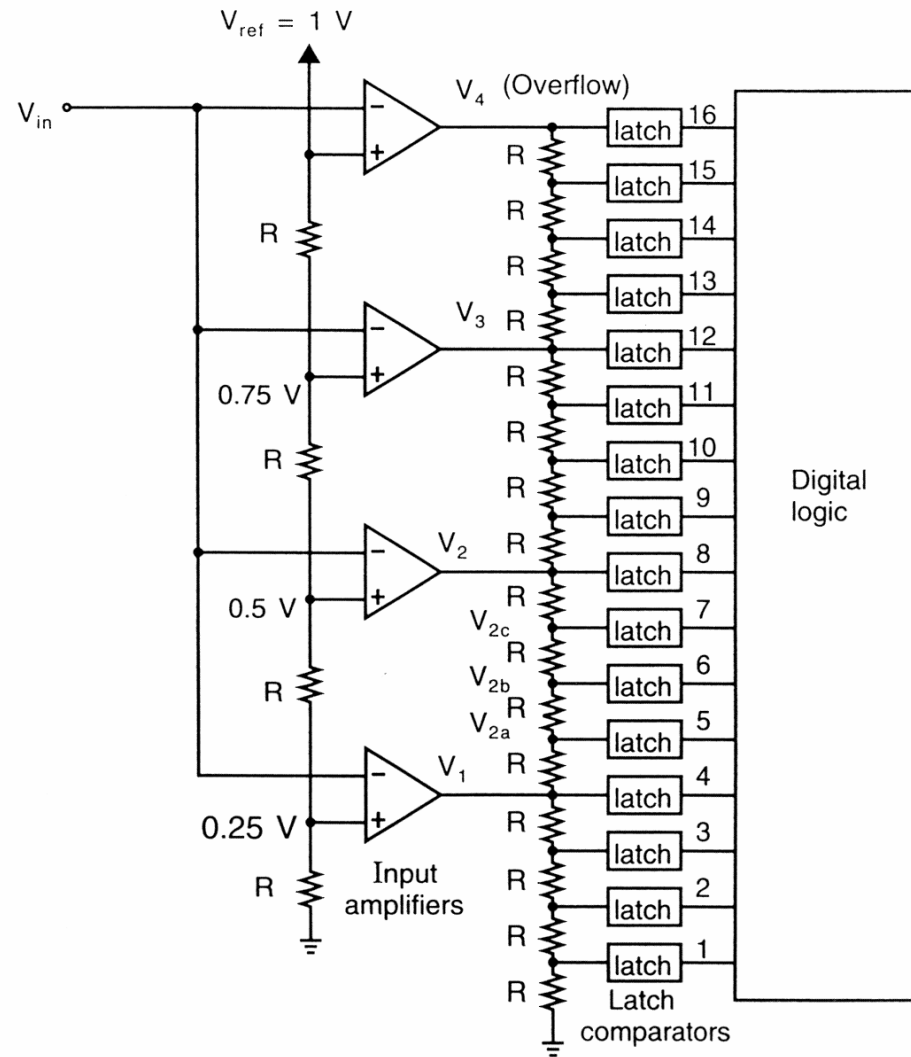
---



Input amplifiers  
which saturate

Van de Grift, JSSC Dec. 87, 944-953; Steyaert CICC 1993

# Interpolating ADC

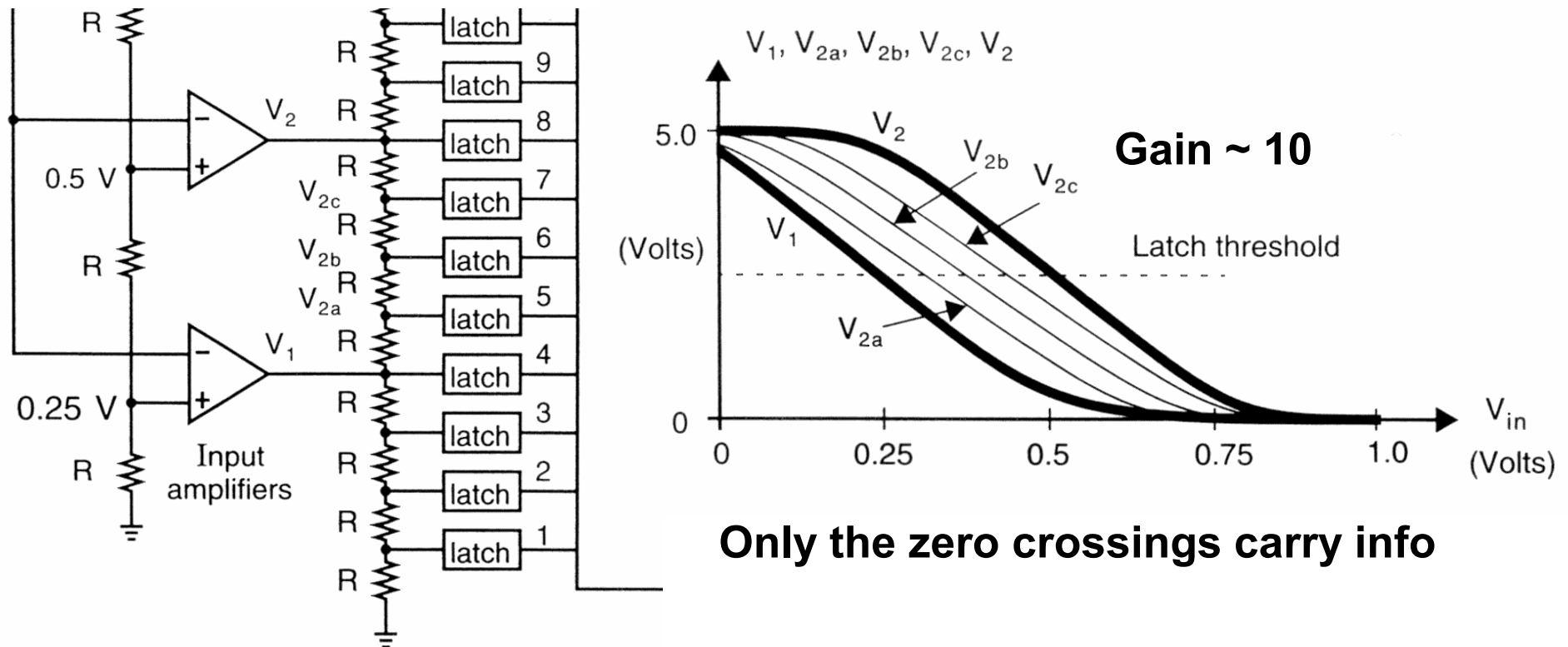


**4-bit interpolating ADC**  
**Resistive interpolation**

**leave out 3 out of 4 amps**  
**Less power consumption**  
**Less input capacitance**

Johns, Martin, Wiley 1997

# Transfer curves



**Resistors generate the intermediate outputs**

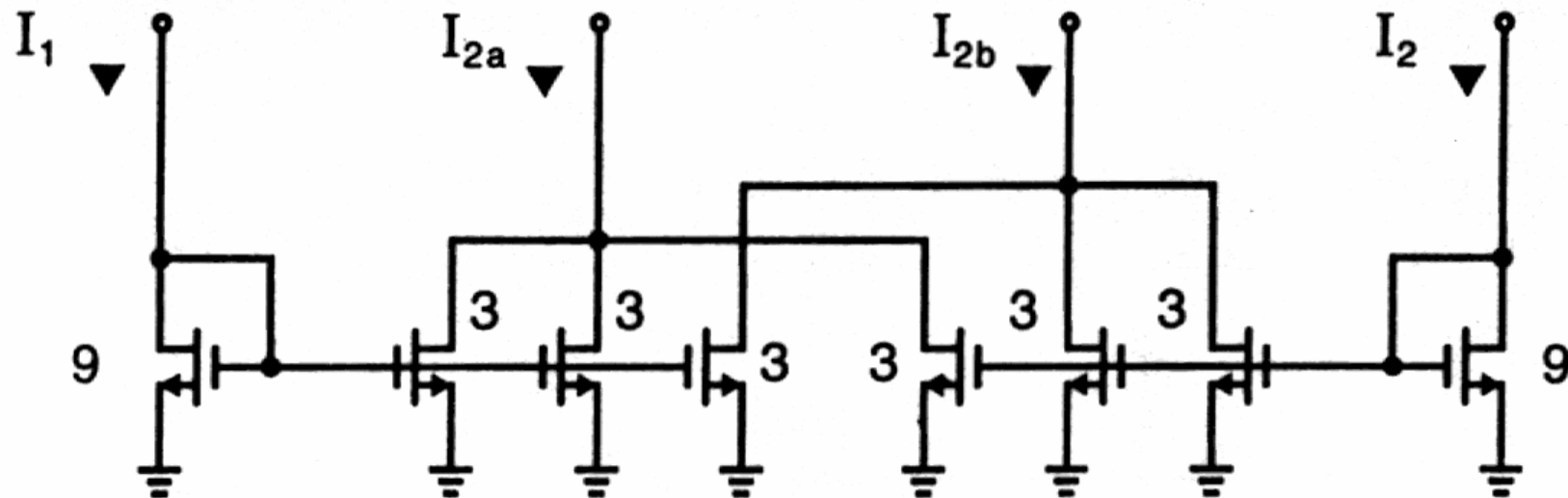
**Resistors average out offsets, etc.**

**Add series resistors to latch inputs to equalize delay times**

---

# Averaging with output currents

---



Relative width sizing shown  
All lengths same

$$I_{2a} = \frac{2}{3} I_1 + \frac{1}{3} I_2$$

$$I_{2b} = \frac{1}{3} I_1 + \frac{2}{3} I_2$$

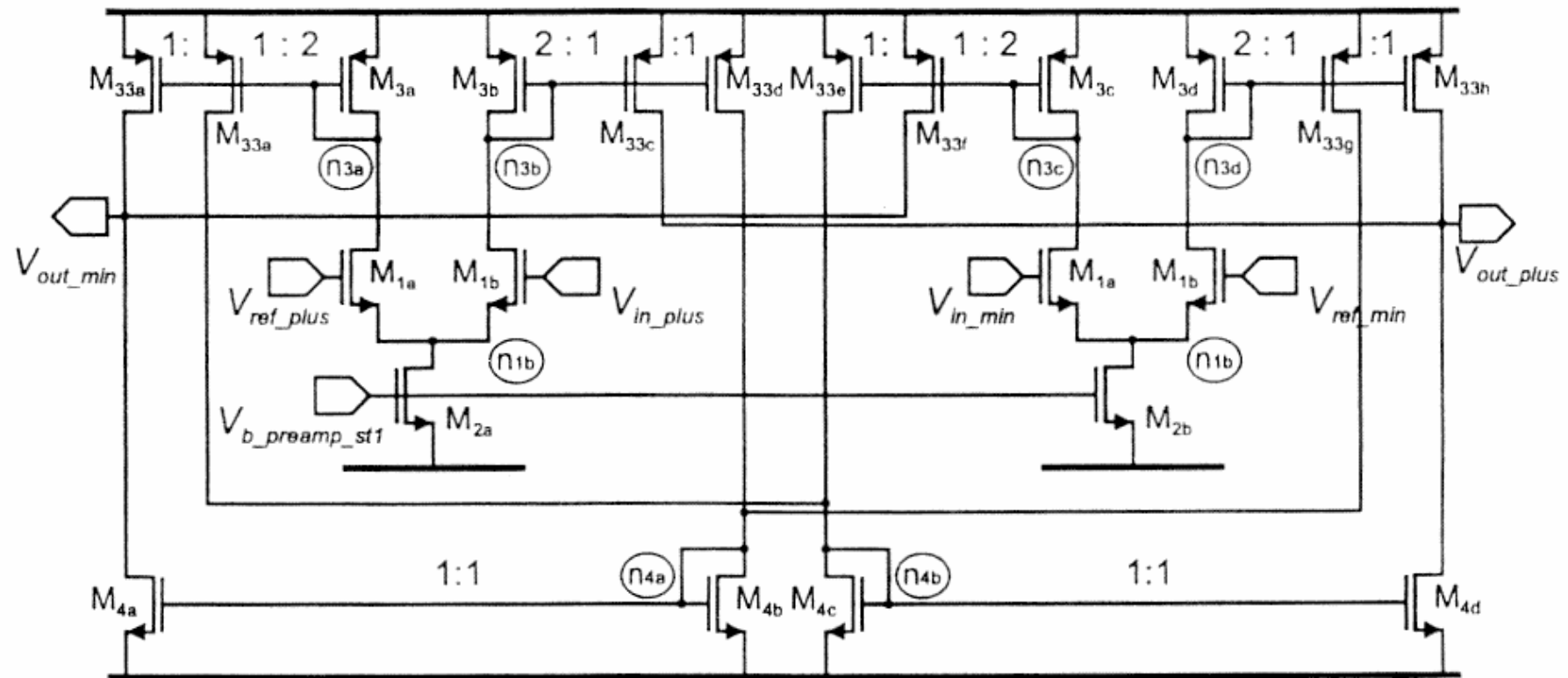
Interpolating by 3

between output currents  $I_1$  &  $I_2$

Requires 1/3 input amps.:  $C_{in}/3$

Steyaert CICC 93

# Interpolating/Averaging ADC - 1st amp



Current mirror interpolator

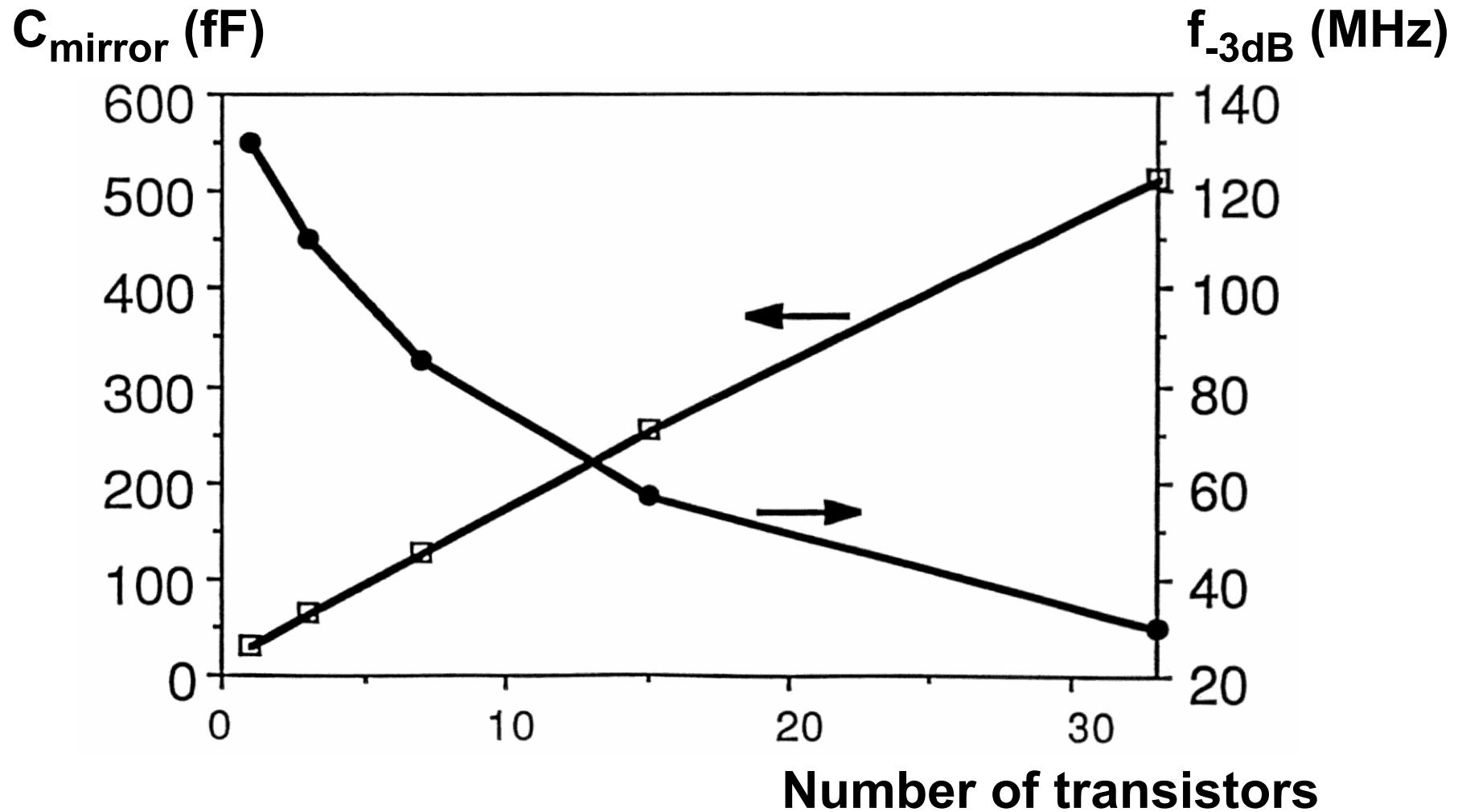
Steyaert CICC 93

Roovers JSSC July 96, 938-944

---

# Interpolating : limitations

---

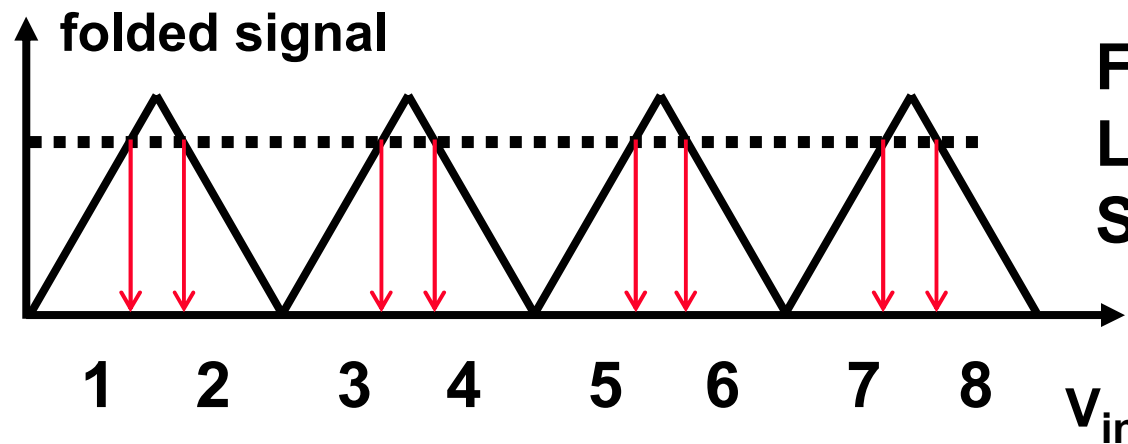
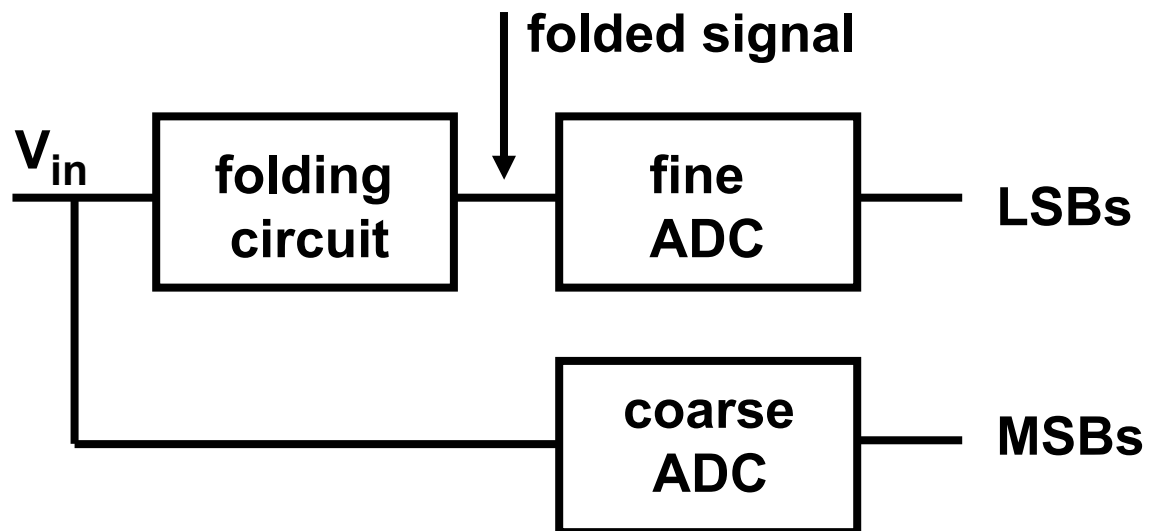




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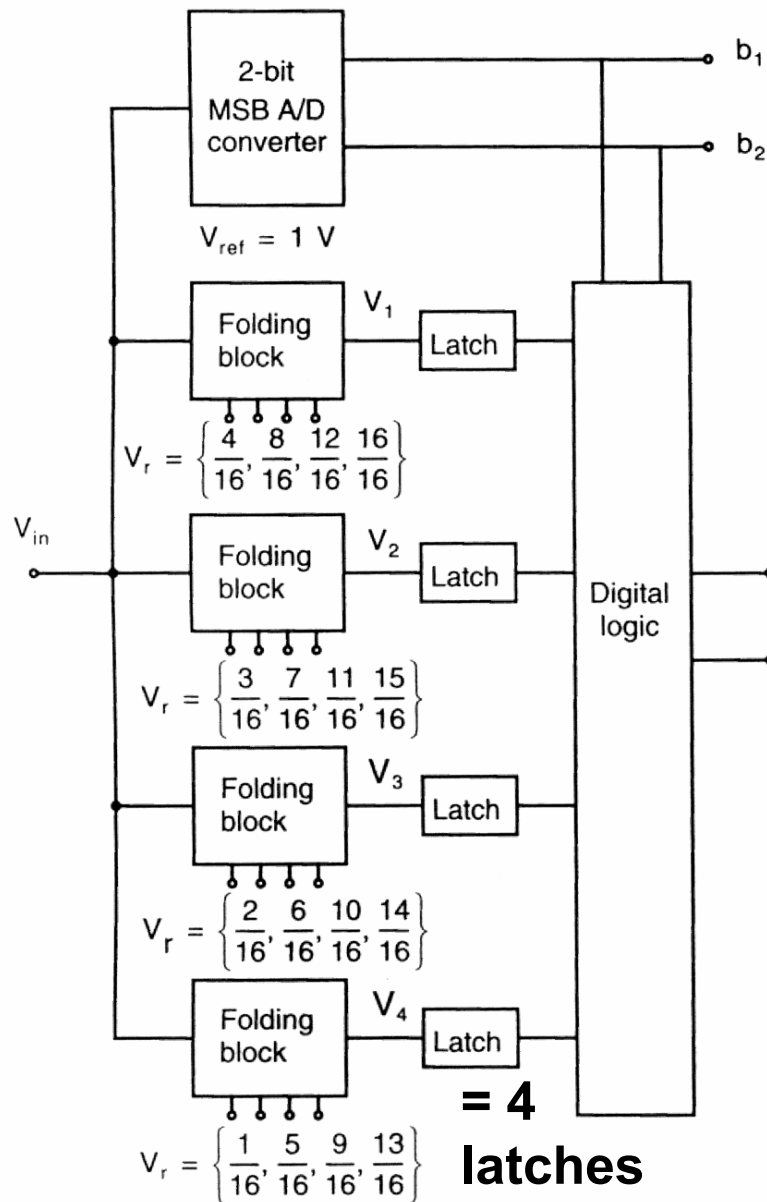
# Folding ADC Analog preprocessing

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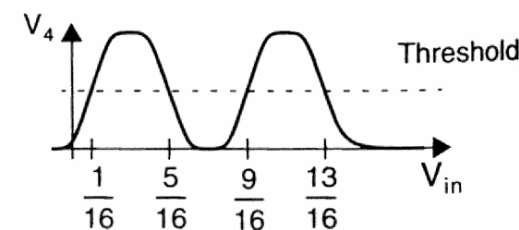
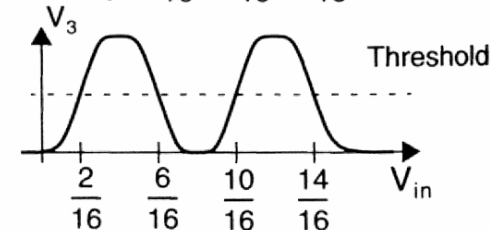
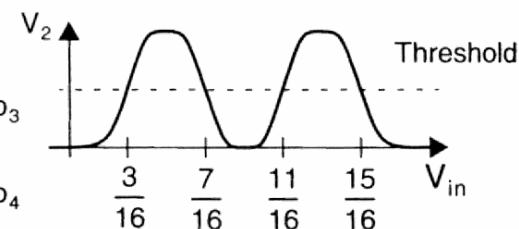
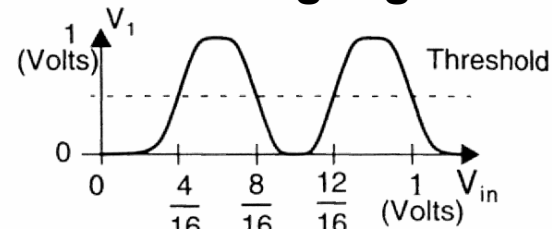
**Folding rate 8**  
**Less comparators**  
**Same input capacitance**

# 4-bit Folding ADC



Folding block responses

**4 folding regions**



$V_{in}$  starts

from 0 to  $\frac{1}{4}$  :

0001

0011

0111

1111

from  $\frac{1}{4}$  to  $\frac{1}{2}$  :

1110

1100

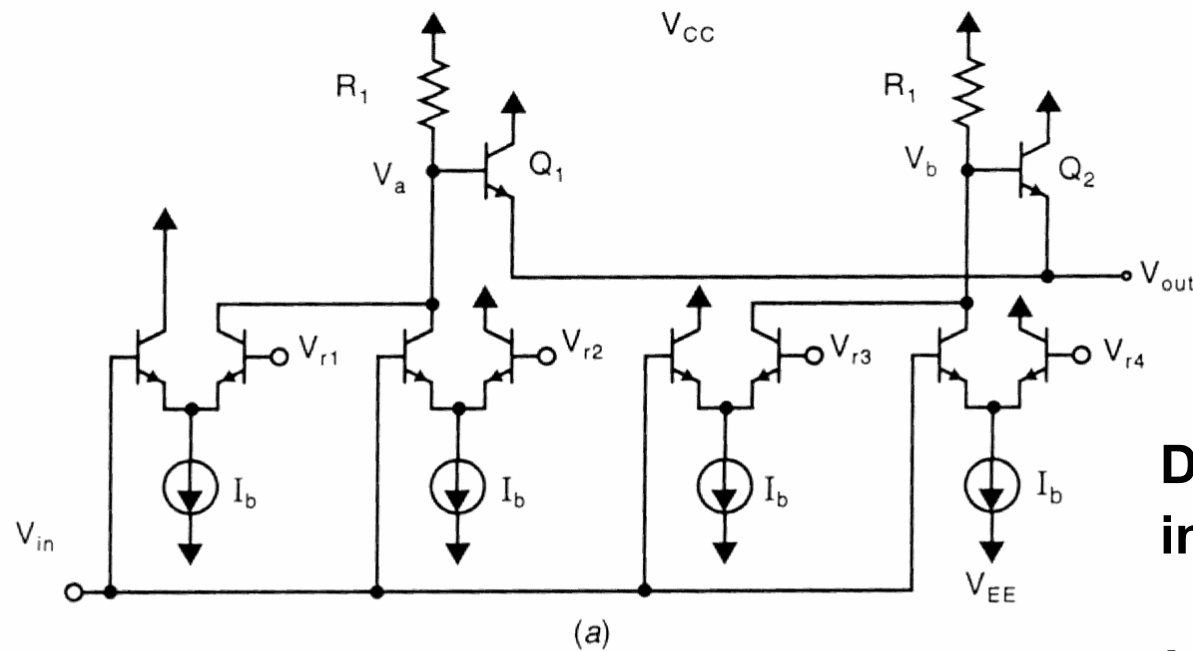
1000

0000

**4-bit flash: 16 comp.  
folding: 8 comp.**

Johns, Martin, Wiley 1997

# Folding block realization

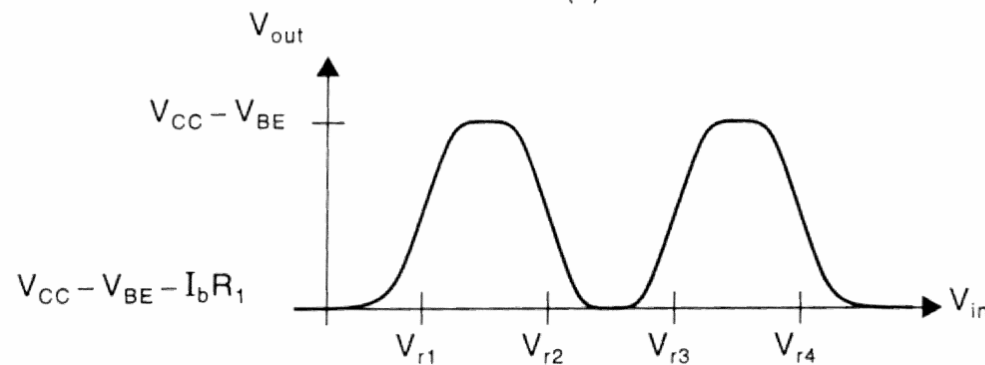


**Folding rate of 4**

**Differential pairs  
in parallel !**

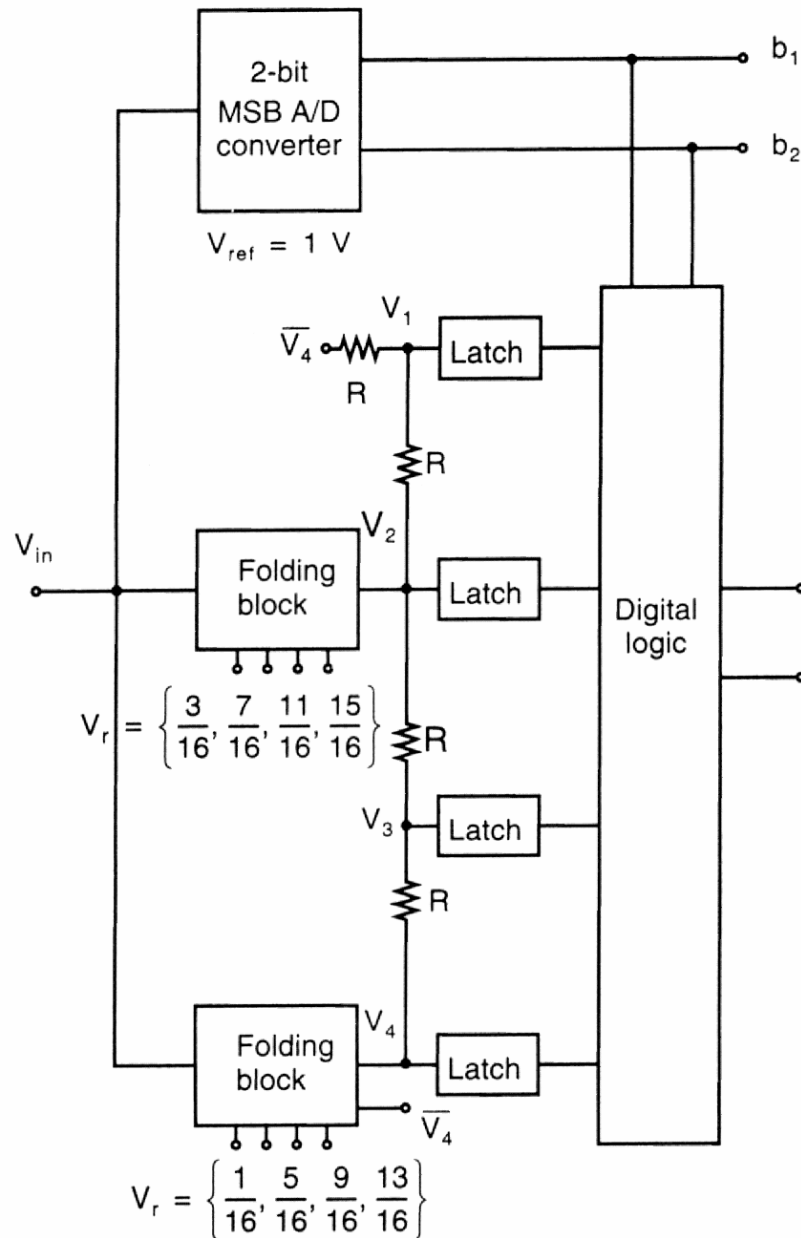
**Large  $C_{in}$  !**

**Output at higher freq. =  
 $f_{in} \times \text{folding rate}$**

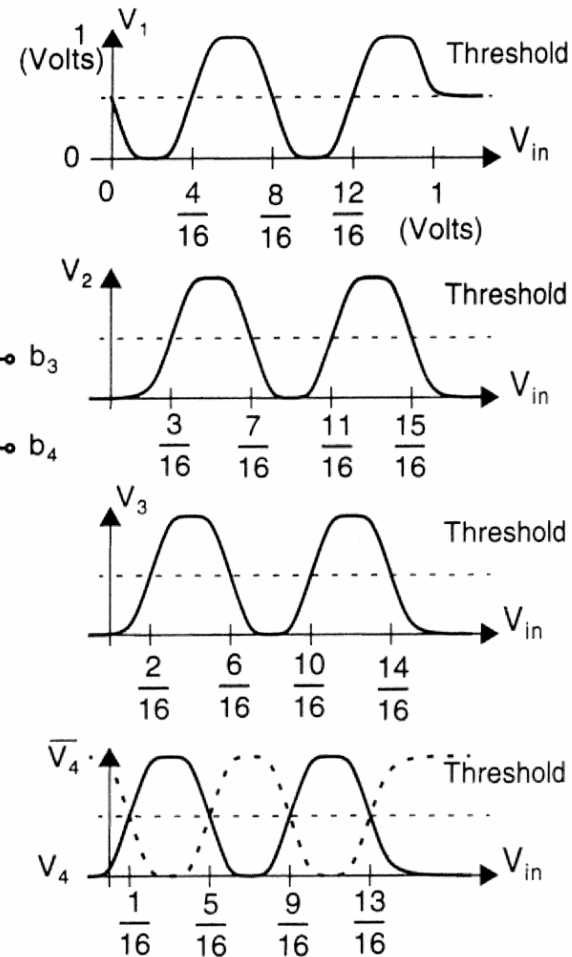


**Johns, Martin, Wiley 1997**

# Folding + interpolation



Folding-block responses



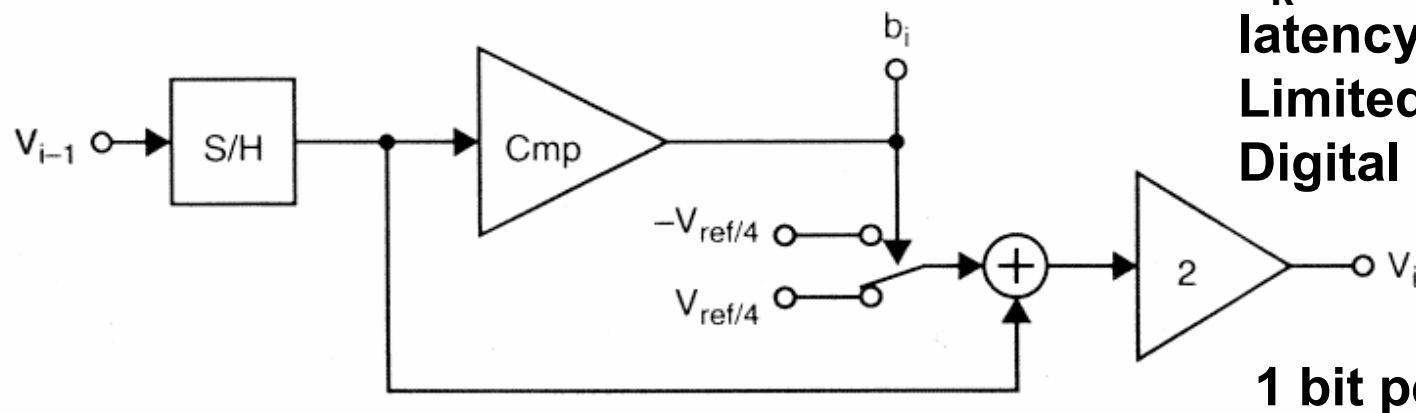
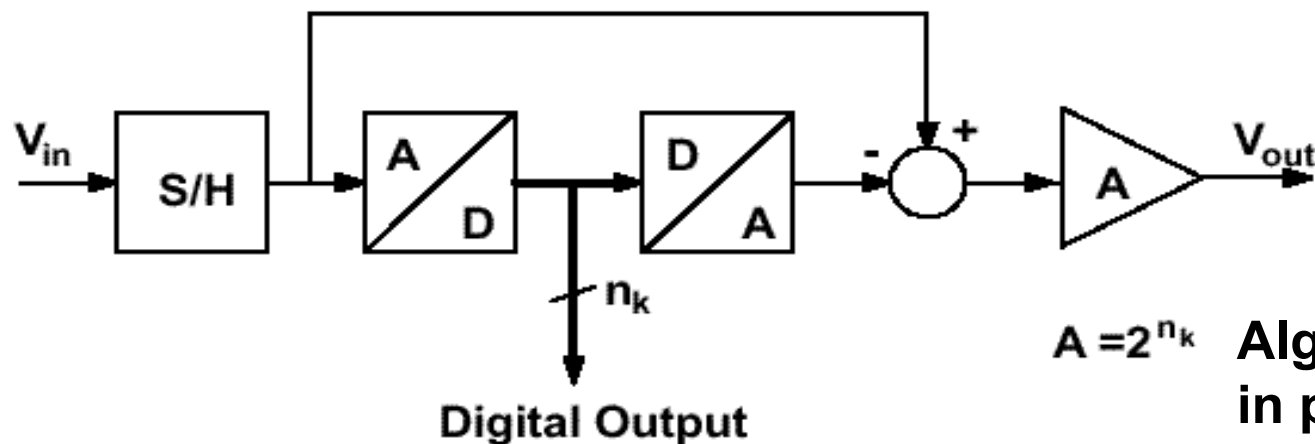
**Folding rate of 4  
Interpolate by 2**

**Lower  $C_{in}$  !**

**Van Valburg JSSC  
Dec. 92, 1662-1666**

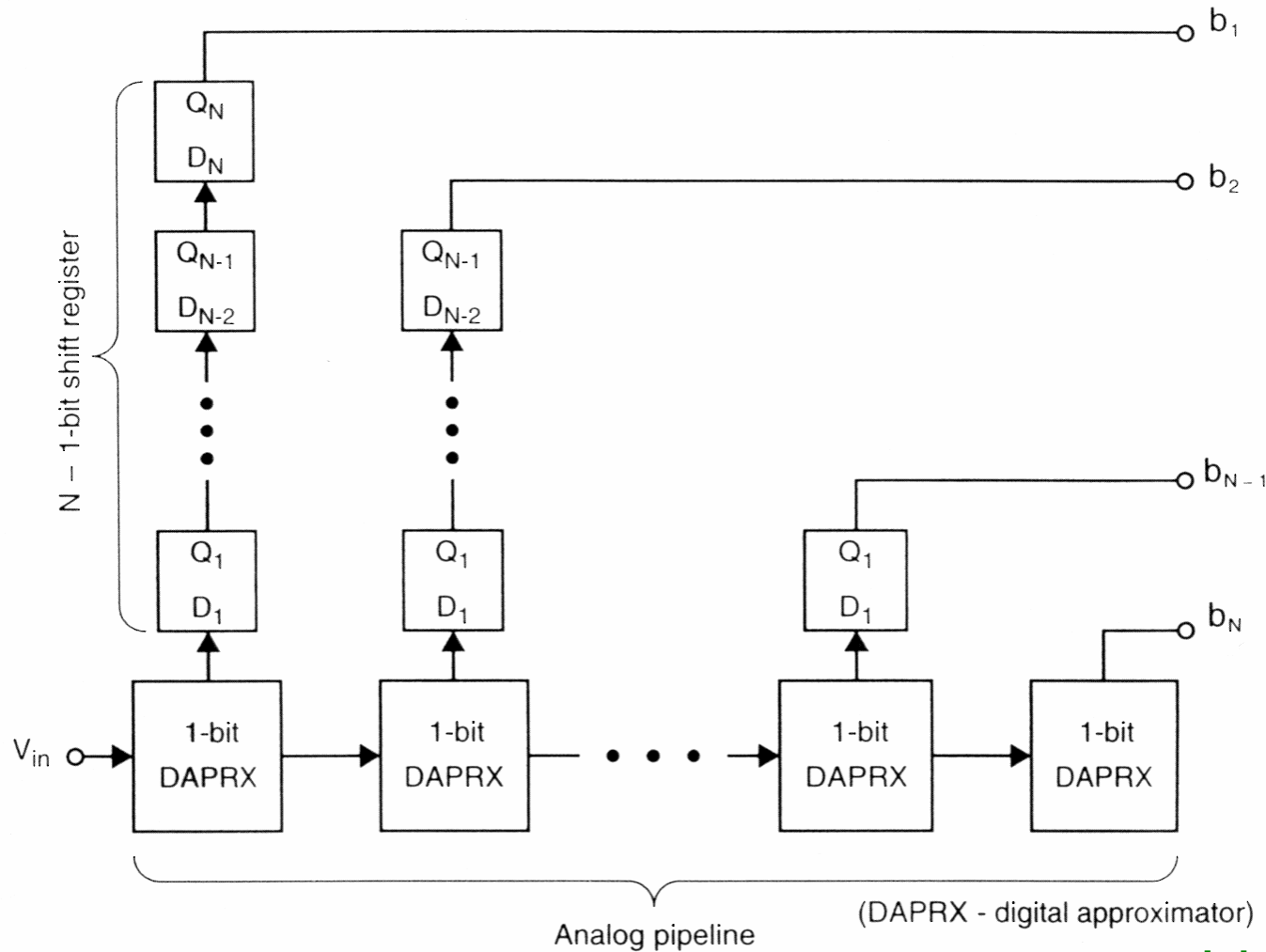
**Johns, Martin, Wiley 1997**

# Pipelined ADC : $n_k$ and single bit per stage



Johns, Martin, Wiley 1997

# Pipelined ADC block diagram



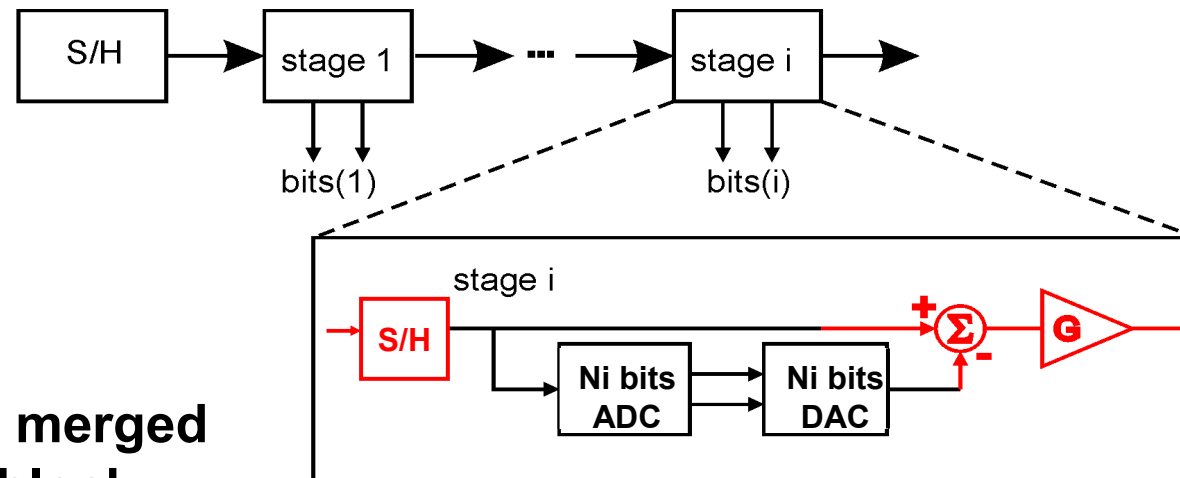
**New sample  
each clock cycle**

**Johns, Martin, Wiley 1997**

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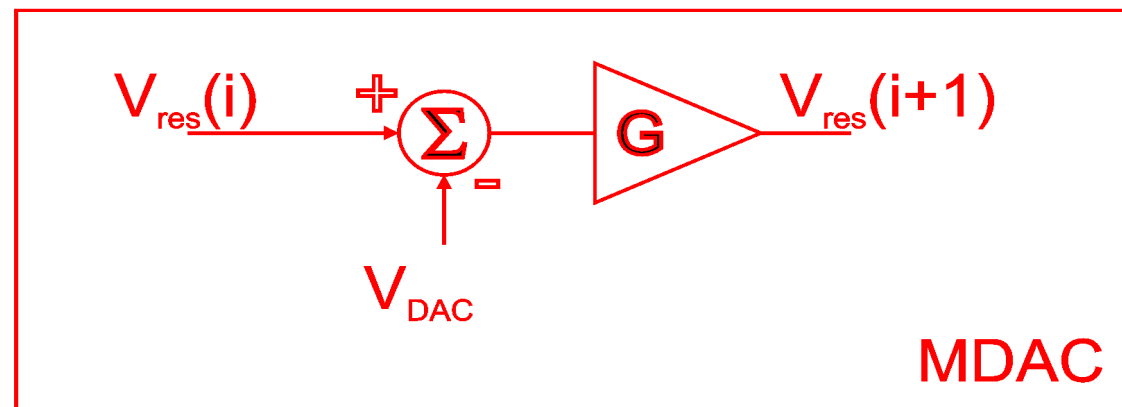
# Multiplying DAC

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**DAC + Gain Are merged  
In one building block:**

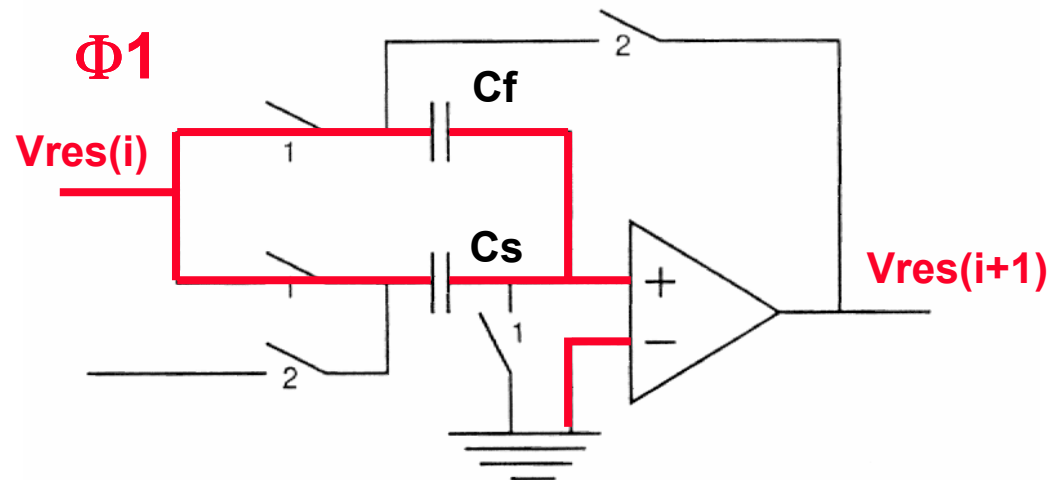
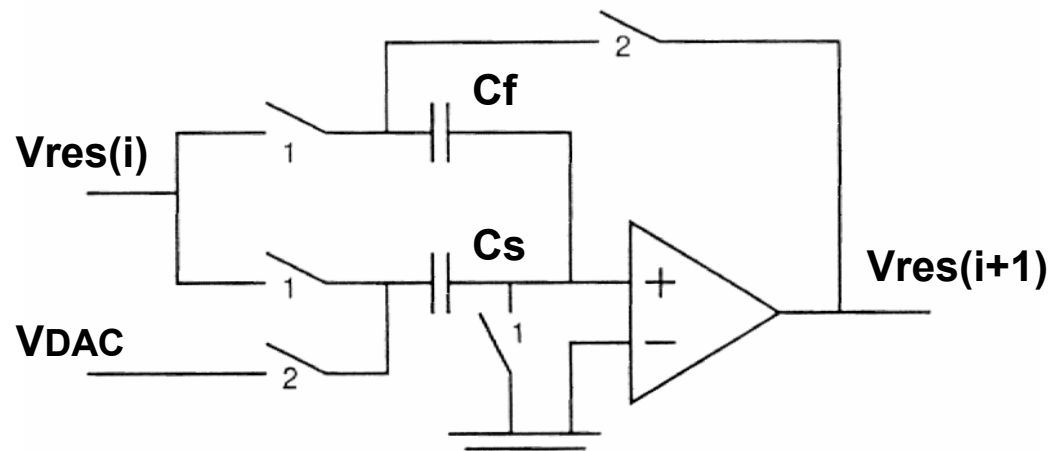
**Multiplying DAC**



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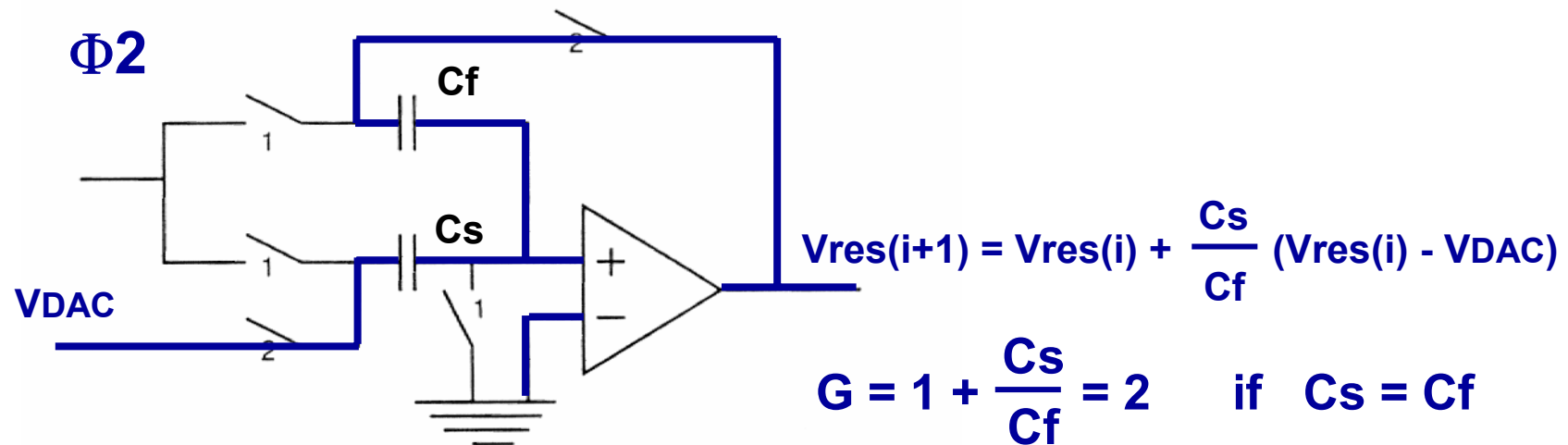
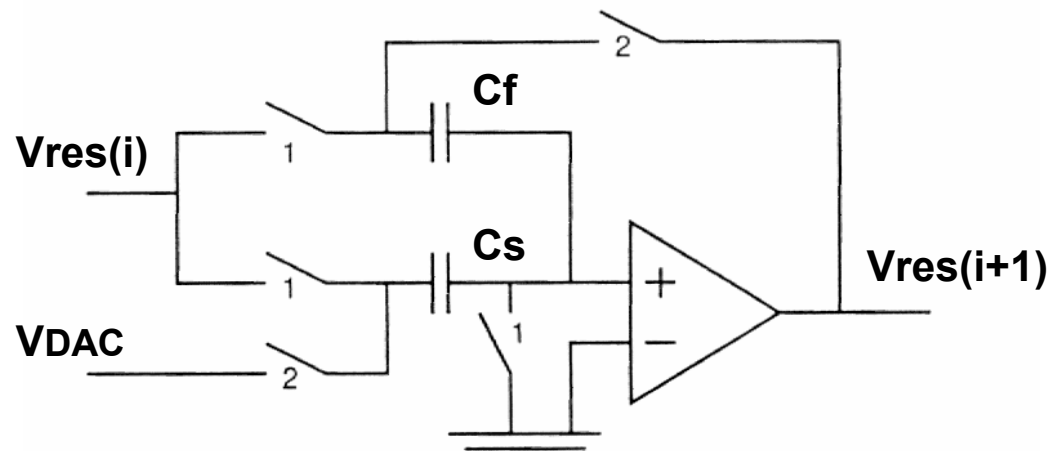
# Multiplying DAC : Phase 1

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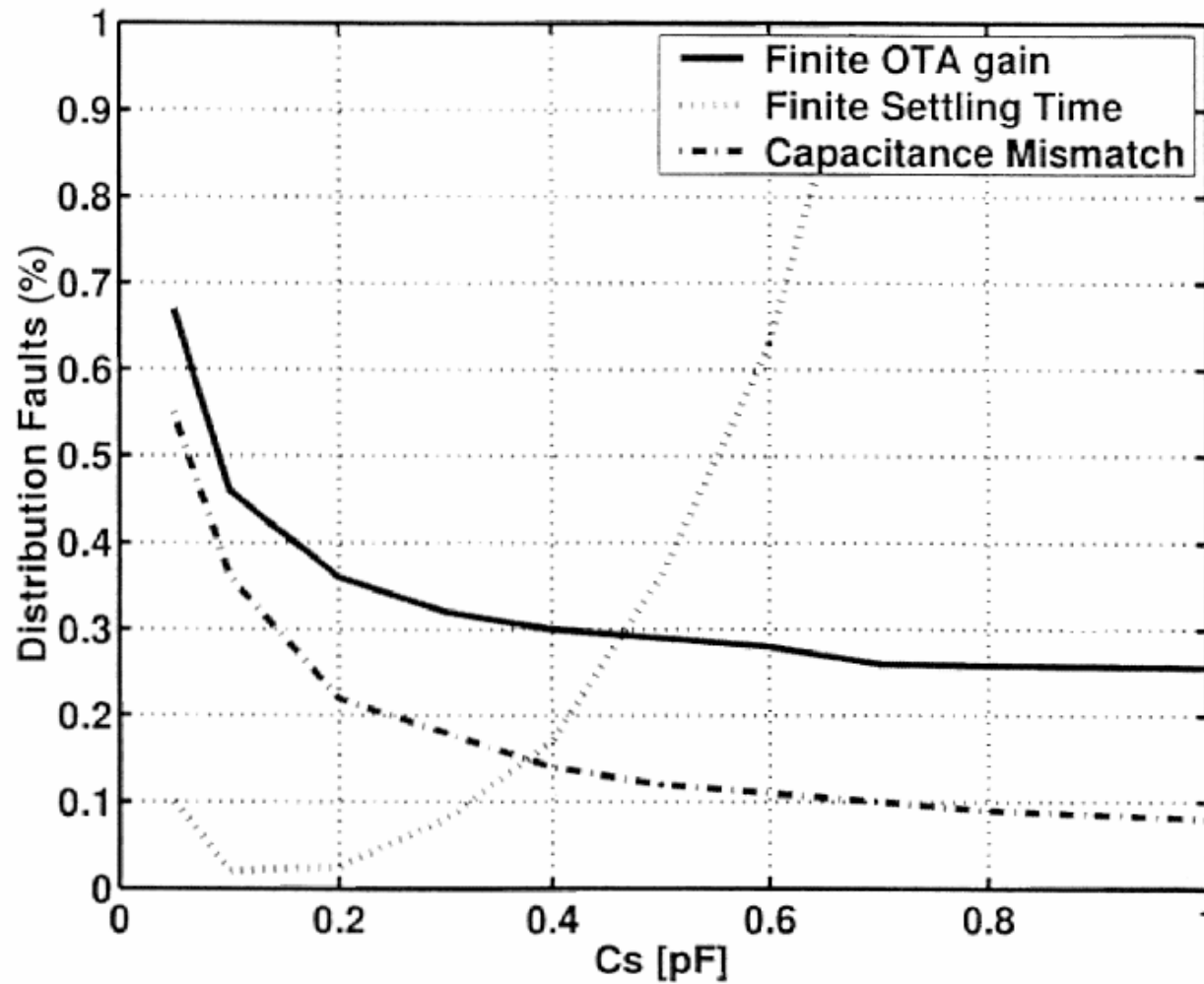
# Multiplying DAC : Phase 2



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# Non-idealities versus $C_s$

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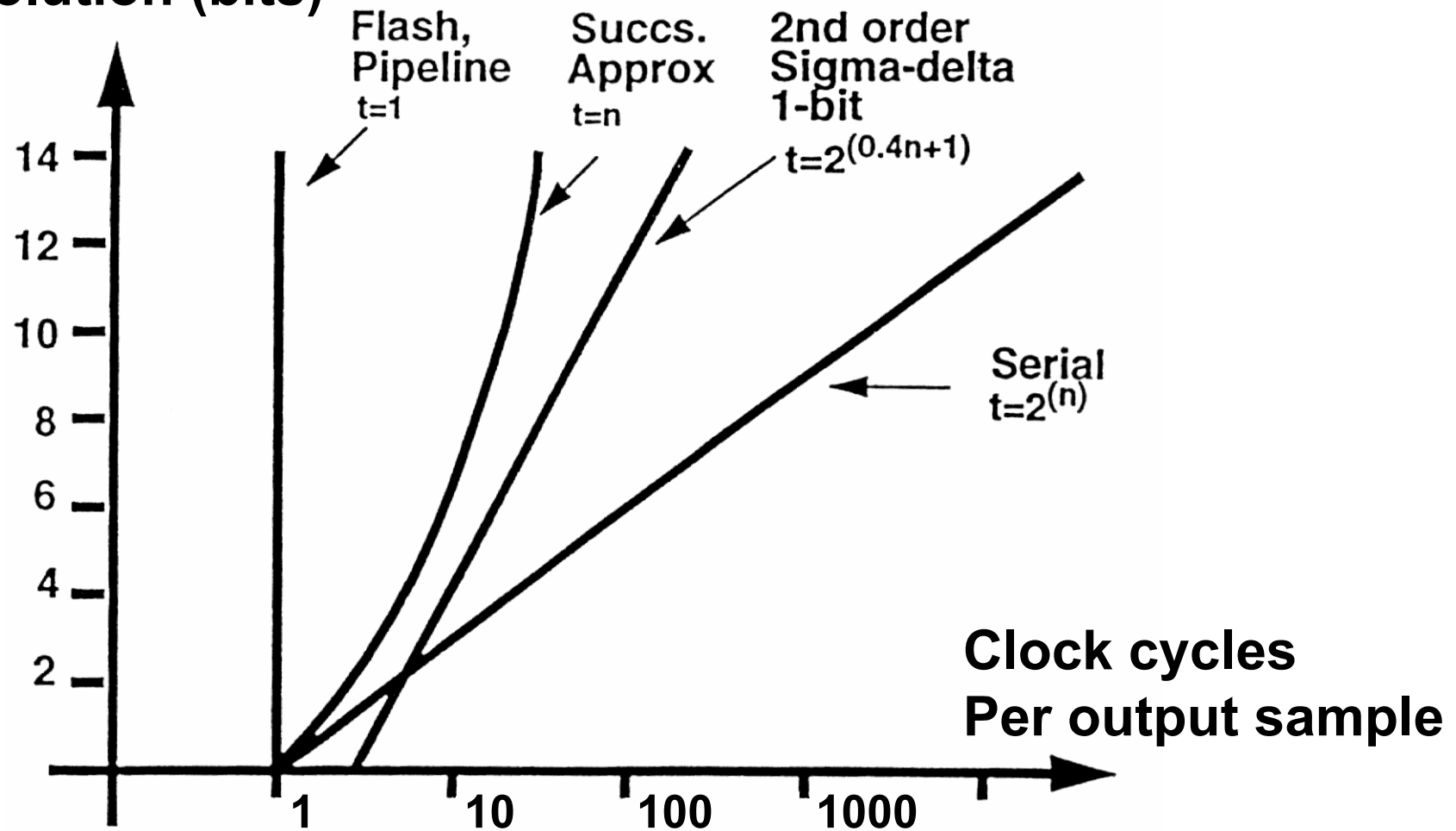


**0.25  $\mu\text{m}$  CMOS**  
 **$f_s = 400$  MHz**

Uyttenhove, Kuleuven, 2003

# Comparison ADCs

Resolution (bits)



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## Impact of device mismatch on resolution/power

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Two transistors :  $\sigma^2(\text{Error}) \sim \frac{1}{WL}$

$$\sigma_{VT} = \frac{A_{VT}}{\sqrt{WL}}$$

$(\text{Accuracy})^2 \sim WL$

By design : increasing W increases  $I_{DS}$  and Power  
decreasing L increases the speed

$\frac{\text{Speed} \times (\text{Accuracy})}{\text{Power}} = \text{Technol. constant}$

Ref. Kinget, ...“Analog VLSI ..”  
pp 67, Kluwer 1997.

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## Power and mismatch/noise

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**Accuracy**

$$1/\sigma^2(V_{os}) \sim \text{Area} / A_{VT}$$

**Dynamic range**

$$\text{DR} = V_{s\text{RMS}} / (3 \sigma(V_{os}))$$

**Capacitance**

$$C \sim C_{ox} \text{Area}$$

**Power**

$$P = 8 f C V_{s\text{RMS}}^2$$

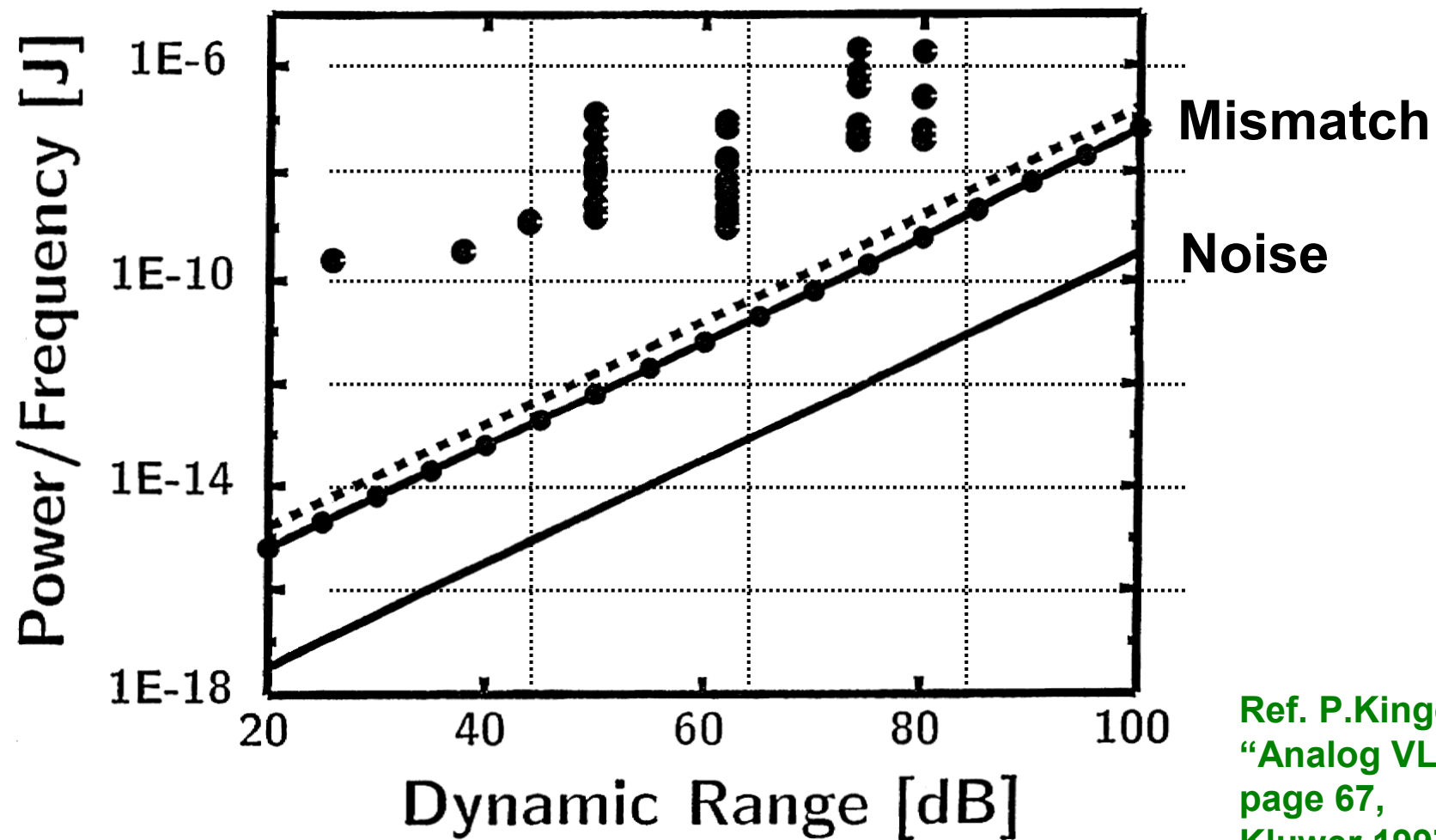
$$\text{Mismatch : } P = 24 C_{ox} A_{VT}^2 f \text{DR}^2$$

$$\text{Noise : } P = 8 kT f \text{DR}^2$$

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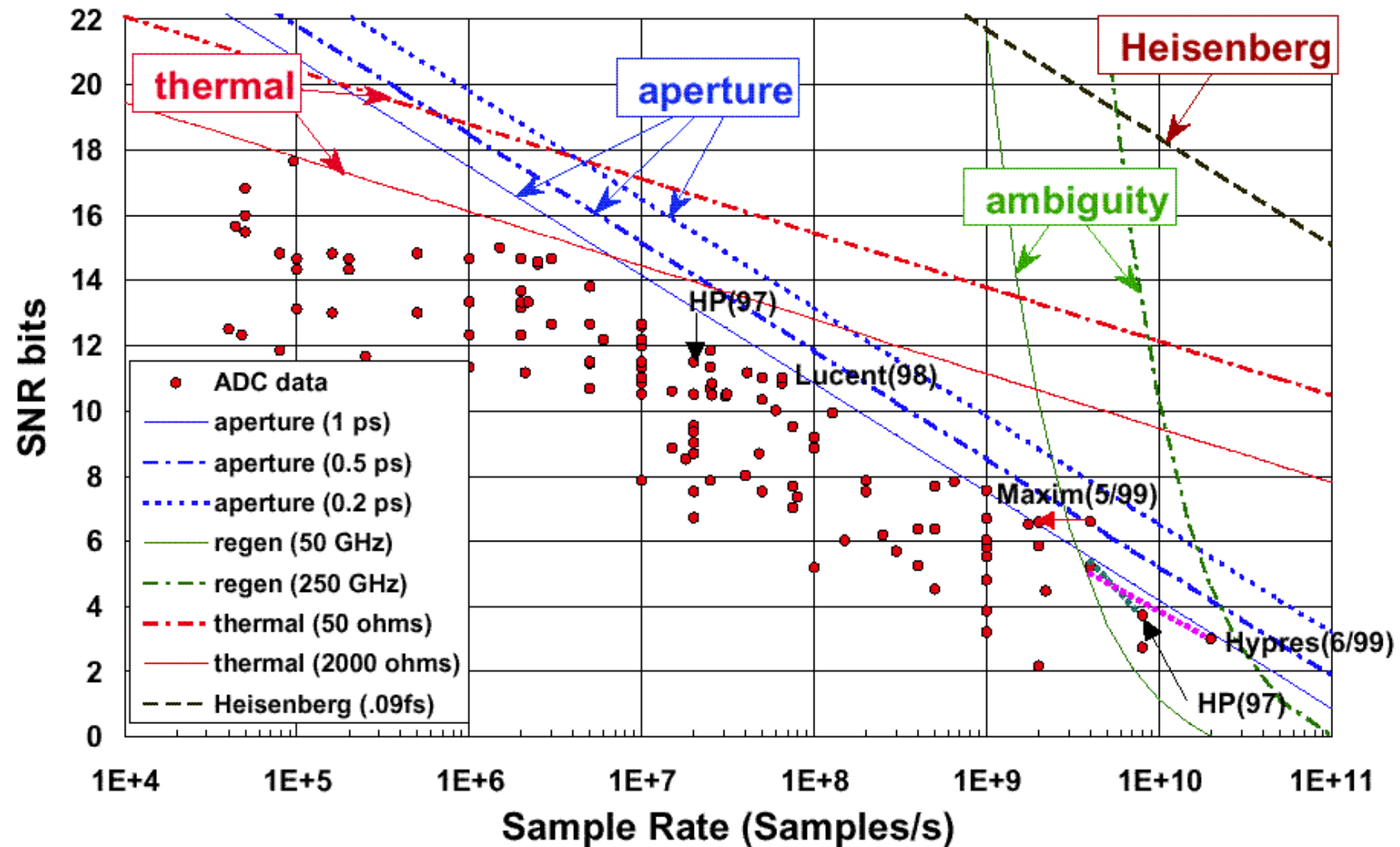
# Noise vs mismatch for DR

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Ref. P.Kinget, ...  
"Analog VLSI .."  
page 67,  
Kluwer 1997.

# ADC limitations



Ref Walden IEEE Selected Areas Comm. April 1999, 539-550; Uyttenhove 2003

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# References

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- **D. Johns & K. Martin, “Analog Integrated Circuit Design”, Wiley 1997**
- **P. Jespers, “Integrated Converters”, Oxford Univ. Press, 2001**
- **B. Razavi, “Principles of Data Conversion System Design”, IEEE Press 1995**
- **K. Uyttenhove, “High-speed CMOS Analog-to-digital converters”, PhD KULeuven, 2003**
- **A. Van den Bosch, ... “High-resolution high-speed CMOS current-steering Digital-to-Analog Converters, Kluwer Ac. Press 2004.**
- **R. Van de Plassche, “Integrated Analog-to-digital and Digital-to-Analog converters”, Kluwer Ac. Press, 1994**



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