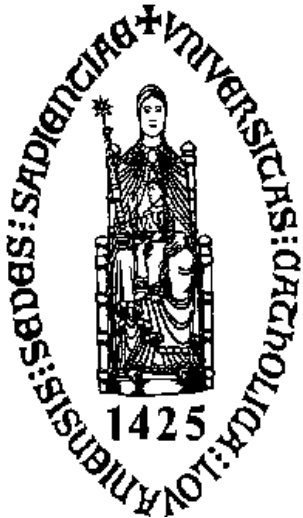

Amplifiers, Source followers & Cascodes



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Operational amplifier

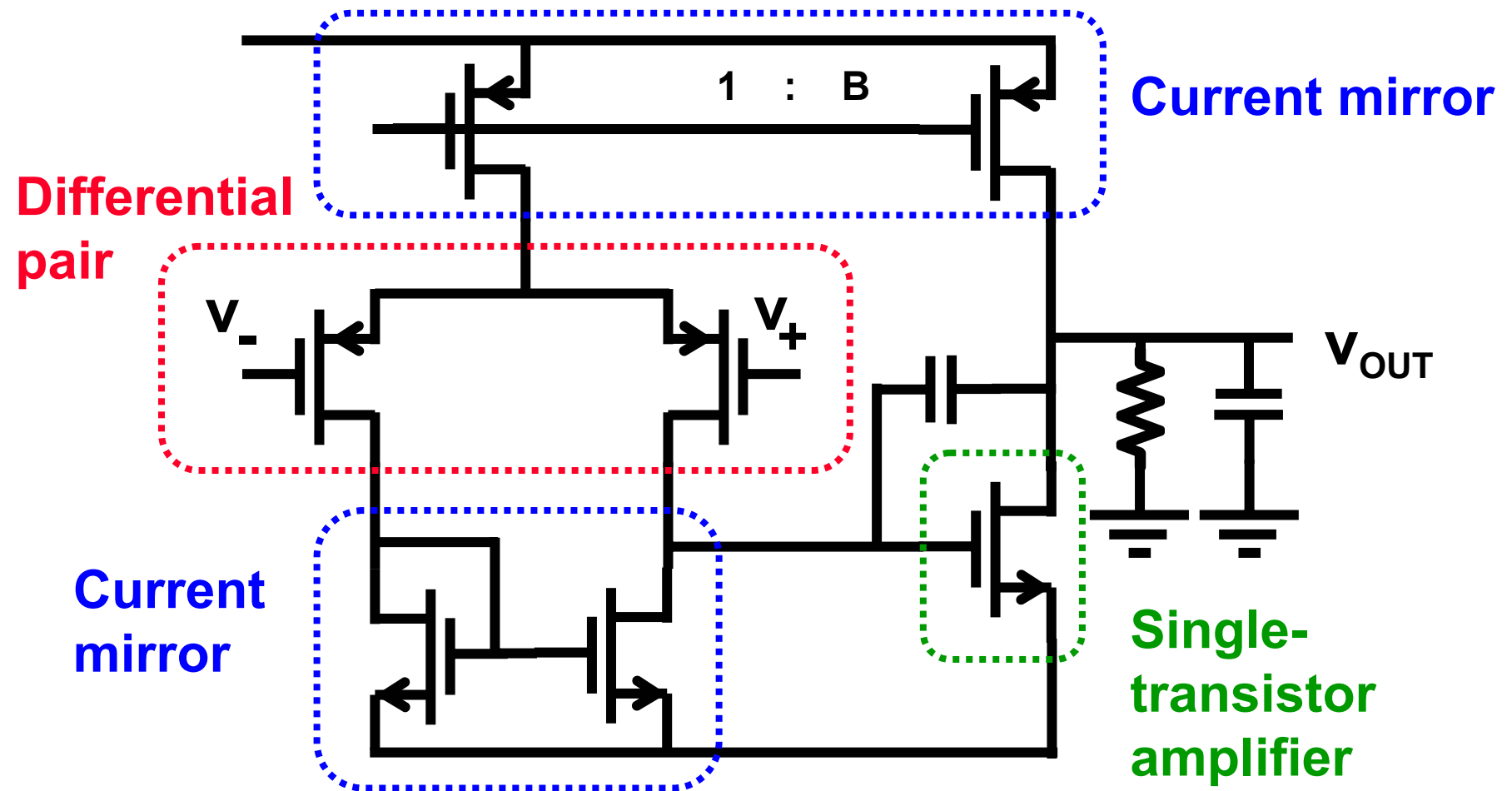


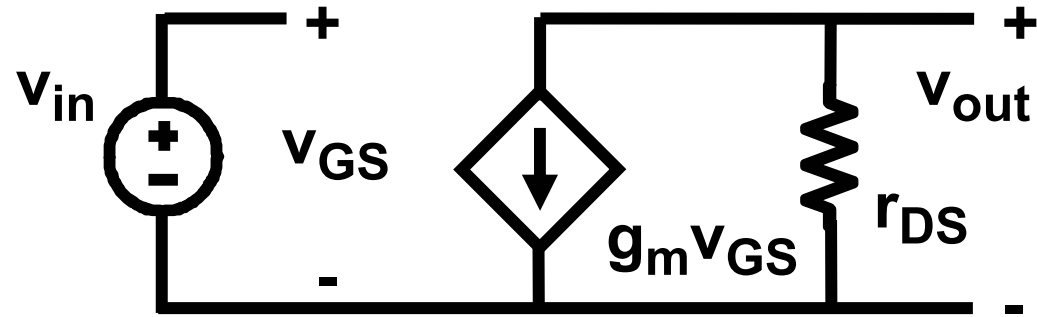
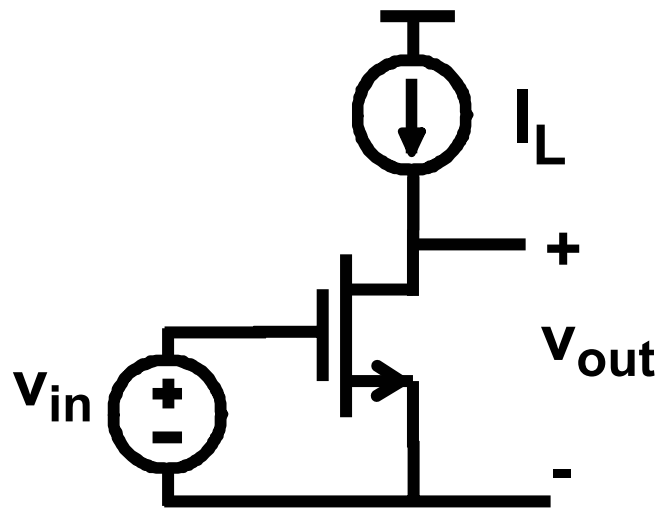
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- ☐ **Single-transistor amplifiers**

- ☐ **Source followers**

- ☐ **Cascodes**

Single-transistor amplifier - 1



$$A_v = g_m r_{DS} = \frac{2 I_{DS}}{V_{GS} - V_T} \frac{V_E L}{I_{DS}} = \frac{2 V_E L}{V_{GS} - V_T}$$

$$A_v \approx 100 \quad \text{if } V_E L \approx 10 \text{ V and } V_{GS} - V_T \approx 0.2 \text{ V}$$

Single-transistor amplifier - 2

High gain ?

Low $V_{GS} - V_T$ and large L !!!



MOST or bipolar amplifier ?

MOST $A_v = \frac{V_E L}{(V_{GS} - V_T)/2}$

$A_v \approx 100$ if $V_E L \approx 10 \text{ V}$ and $V_{GS} - V_T \approx 0.2 \text{ V}$

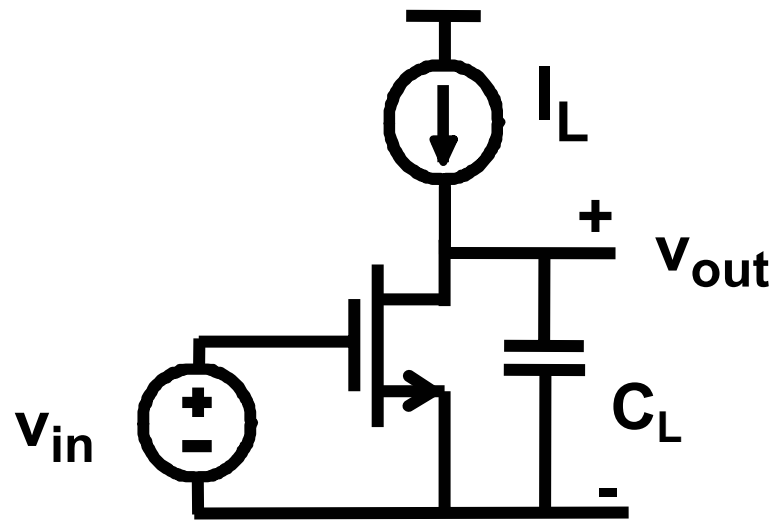
Bipolar $A_v = \frac{V_E}{kT/q}$

3 vs 2 stages for 10^6

$A_v \approx 1000$ if $V_E \approx 26 \text{ V}$ since $kT/q = 26 \text{ mV}$



Gain, Bandwidth and Gain-bandwidth



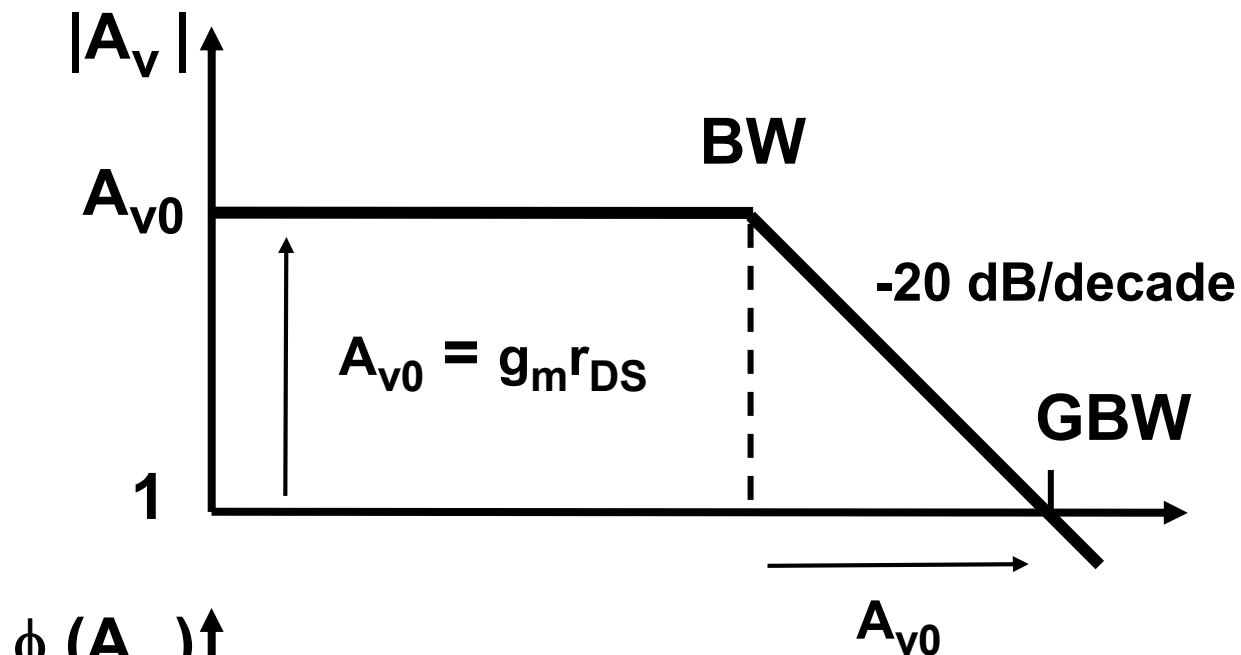
**For all single-stage
Operational amplifiers**

$$A_{v0} = g_m r_{DS}$$

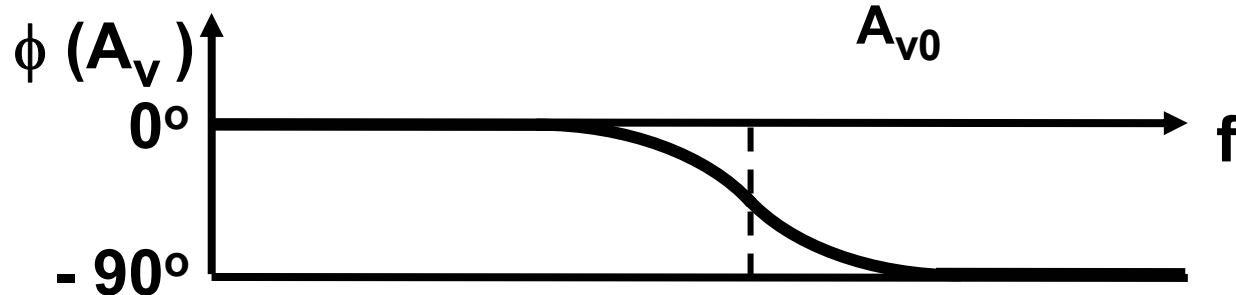
$$BW = \frac{1}{2\pi r_{DS} C_L}$$

$$GBW = \frac{g_m}{2\pi C_L}$$

Gain A_v , BW and GBW



$$GBW = \frac{g_m}{2\pi C_L}$$



$$\phi(A_v) = -45^\circ \text{ at BW}$$

Single-transistor amplifier : Exercise

$$\text{GBW} = 100 \text{ MHz for } C_L = 3 \text{ pF}$$

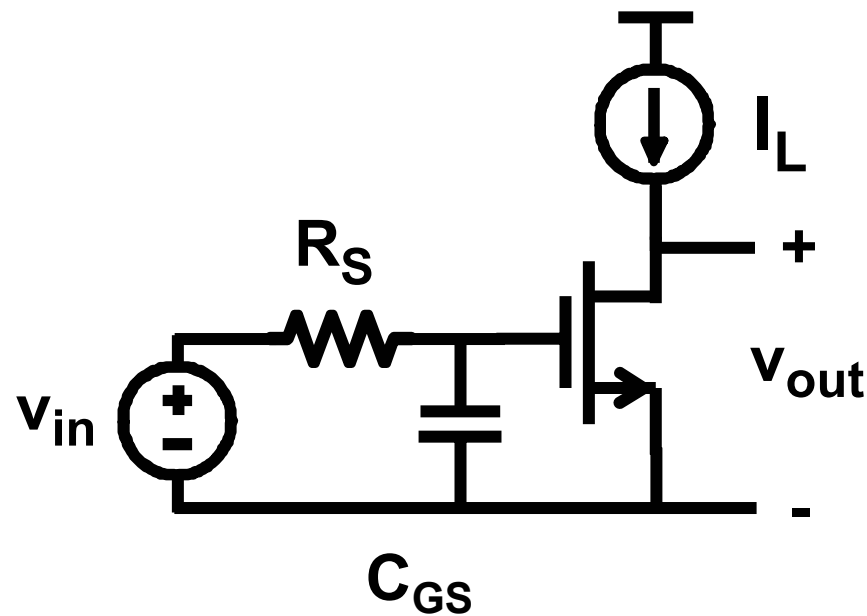
$$\text{Techno.: } K'_n \approx 50 \mu\text{A/V}^2$$

$$L_{\min} = 0.5 \mu\text{m}$$

$$I_{DS} ? \quad L ? \quad W ?$$

$$\frac{\text{GBW} \cdot C_L}{I_{DS}} ?$$

Gain, Bandwidth and Gain-bandwidth



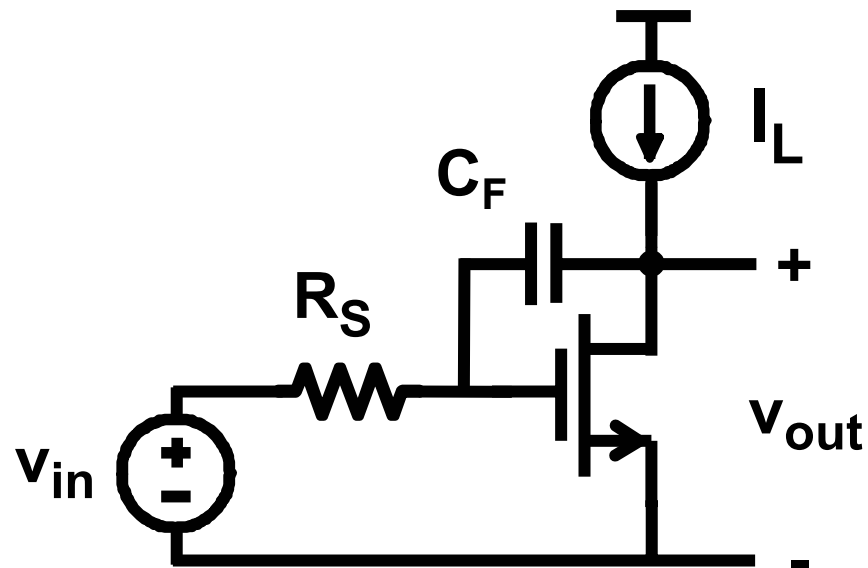
$$A_{v0} = g_m r_{DS}$$

$$BW = \frac{1}{2\pi R_S C_{GS}}$$

$$GBW = \frac{g_m}{2\pi C_{GS}} \frac{r_{DS}}{R_S} = f_T \frac{r_{DS}}{R_S} \sim \frac{1}{WC_{ox}} \frac{1}{V_{GS} - V_T}$$

$$W ? \quad L ? \quad V_{GS} - V_T ?$$

Gain, Bandwidth and Gain-bandwidth

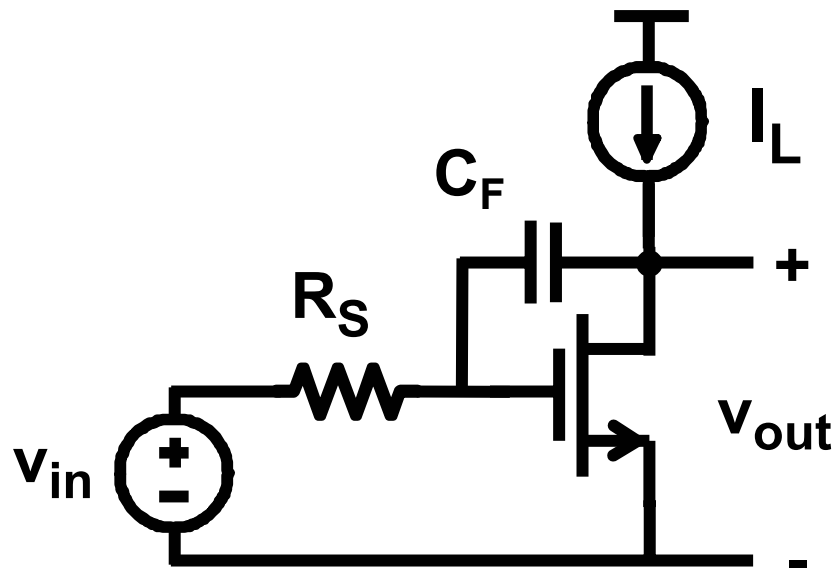


$$A_{v0} = g_m r_{DS}$$

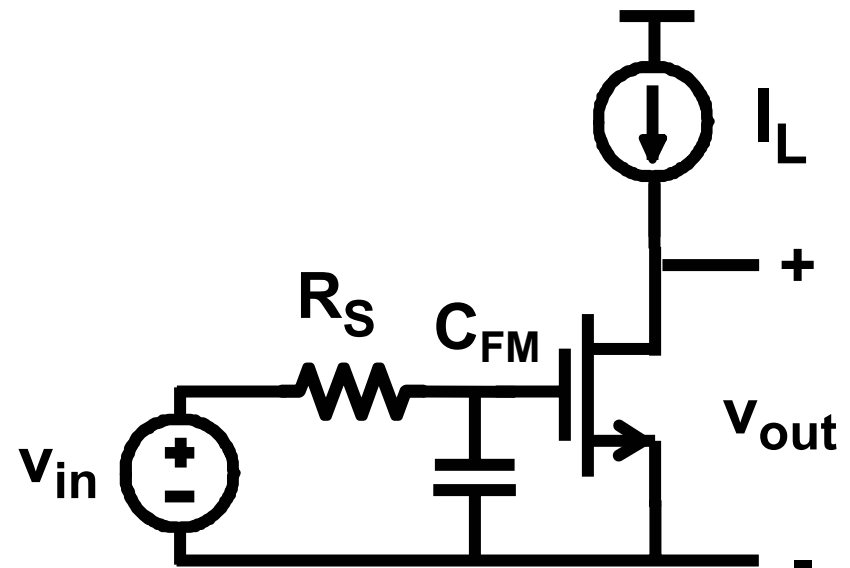
$$BW = \frac{1}{2\pi R_S A_{v0} C_F}$$

$$GBW = \frac{1}{2\pi R_S C_F}$$

Miller effect



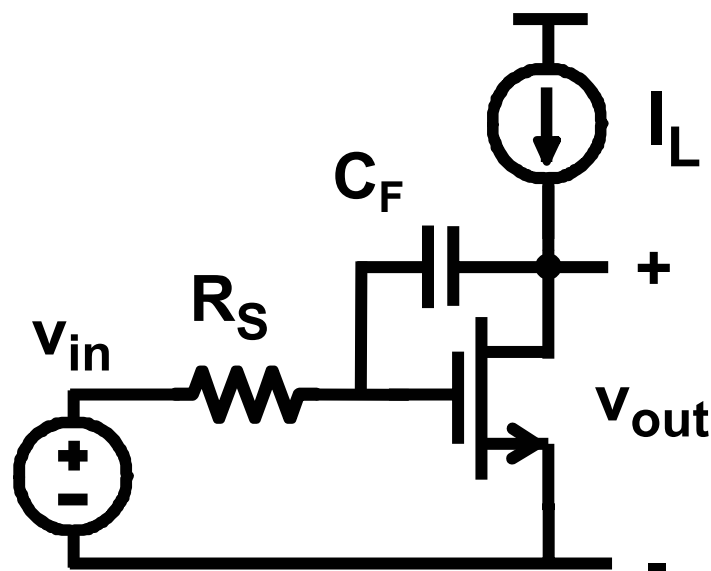
$$A_{v0} = g_m r_{DS}$$



$$C_{FM} = (1 + A_{v0}) C_F$$

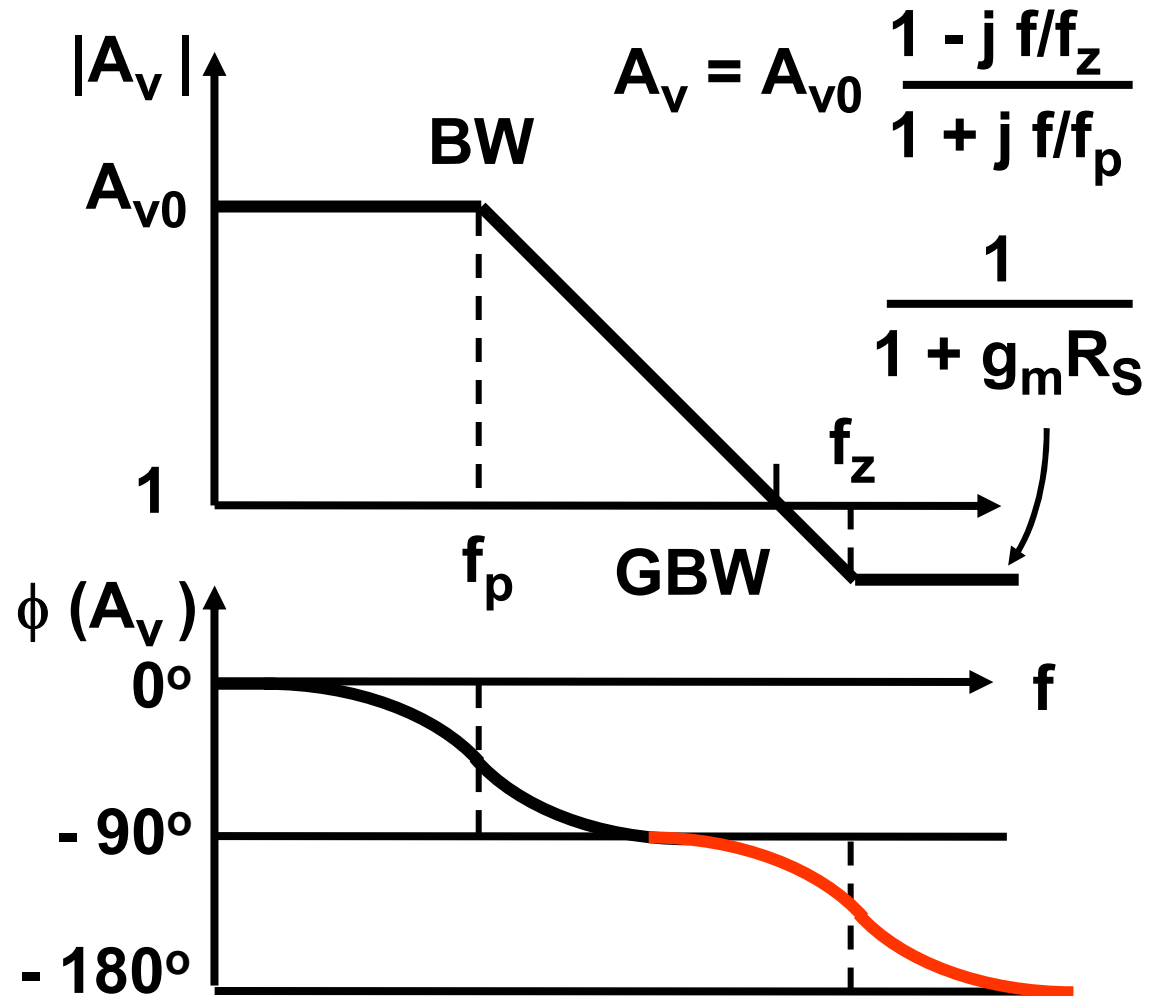
Miller, Dependence of the input impedance of a three-electrode vacuum tube upon the load in the plate circuit, *Scient. Papers Bur. Standards*, 1920, 367-385.

Miller capacitance feedback effects

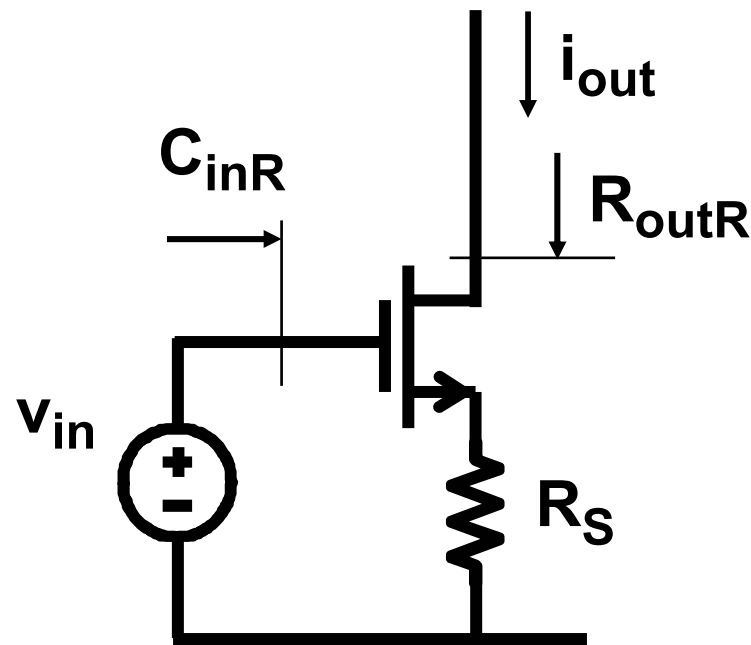


$$f_z = \frac{g_m}{2\pi C_F}$$

For phase, a positive zero is like a negative pole !!!



Amplifier with local R- (series) feedback



$$g_{mR} = \frac{g_m}{1 + g_m R_S} \sim \frac{1}{R_S}$$

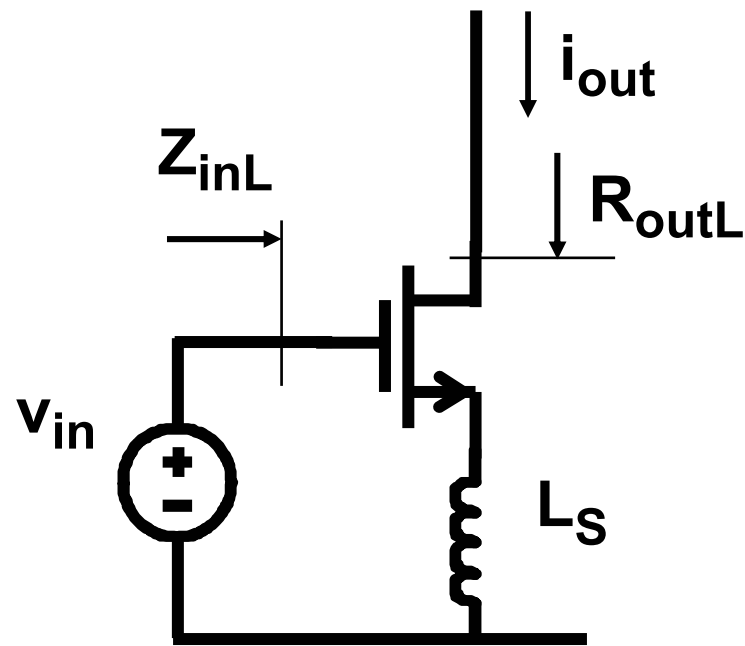
$$R_{outR} = r_{DS} (1 + g_m R_S) \\ \approx (g_m r_{DS}) R_S$$

$$C_{inR} = \frac{C_{GS}}{1 + g_m R_S}$$

But R_S gives extra noise !



Amplifier with local L- feedback



$$g_{mL} = \frac{g_m}{1 + g_m L_S s}$$

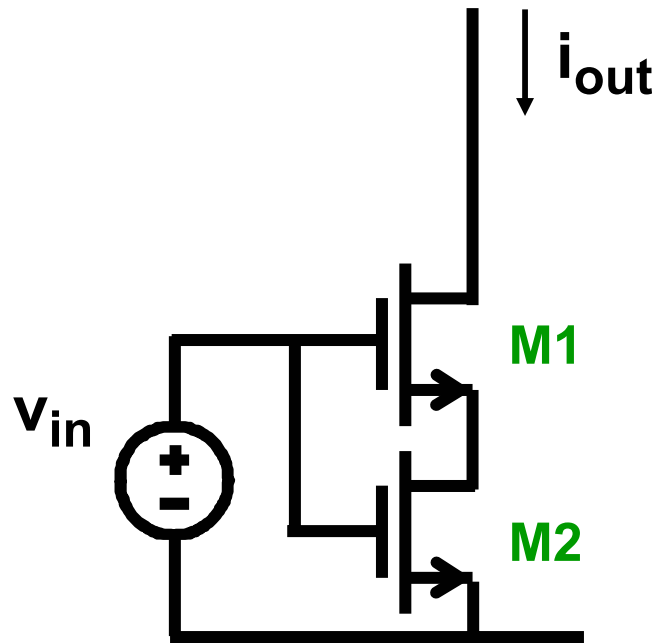
$$R_{outL} = r_{DS} (1 + g_m L_S s)$$

$$Z_{inL} = g_m \frac{L_S}{C_{GS}} + \frac{1 + L_S C_{GS} s^2}{s C_{GS}}$$

No extra noise !

$$Z_{inL} = L_S \omega_T + L_S s + \frac{1}{s C_{GS}}$$

Amplifier with local MOST-R- Feedback



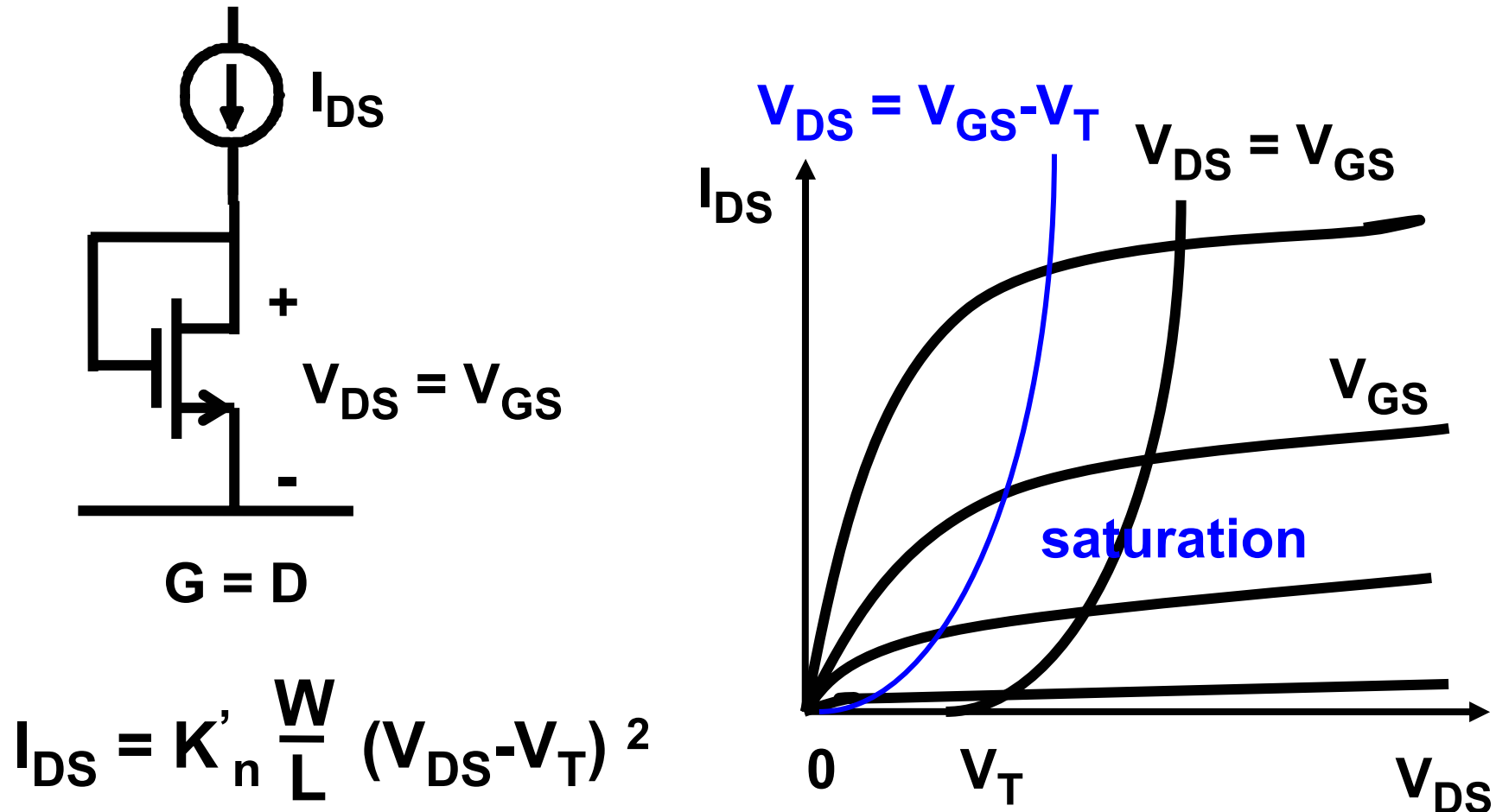
$$V_{DS2} = V_{GS2} - V_{GS1} \approx 0.2 \text{ V}$$

$$r_{DS2} = \frac{1}{K_P W_2/L_2 (V_{GS2} - V_T)}$$

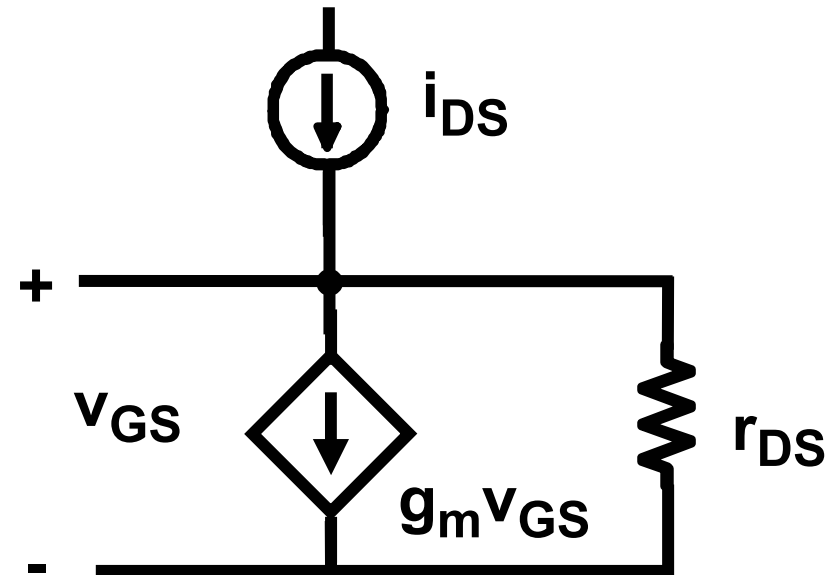
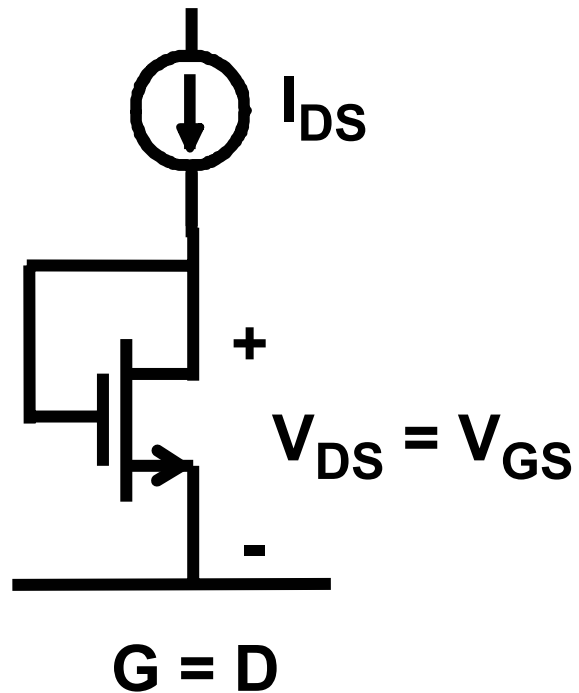
$$R_{outR} = r_{DS1} (1 + g_{m1} r_{DS2})$$

$$C_{inR} = \frac{C_{GS1} + C_{GS2}}{1 + g_{m1} r_{DS2}}$$

Diode-connected MOST : parallel Feedback

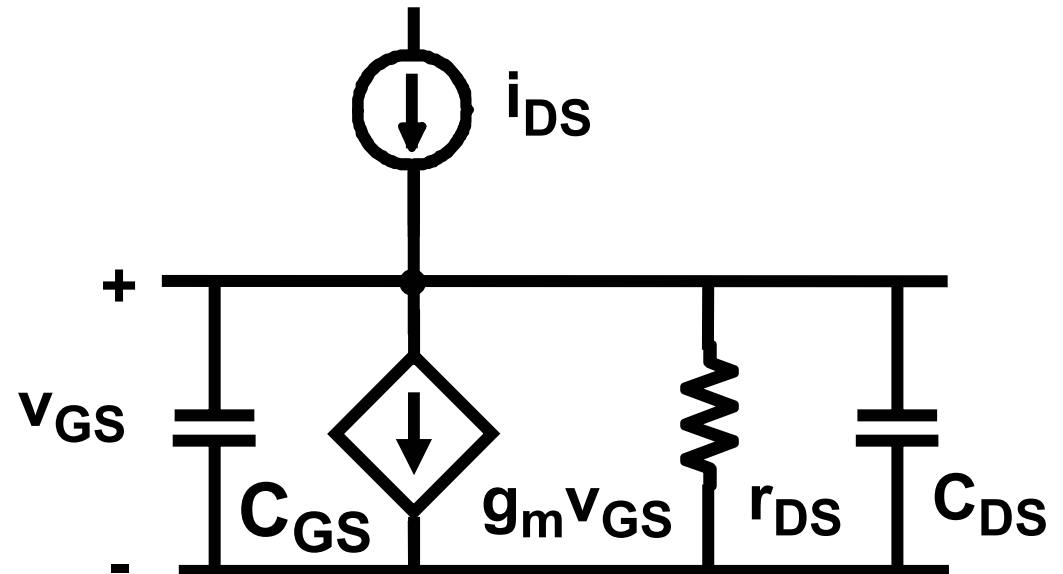
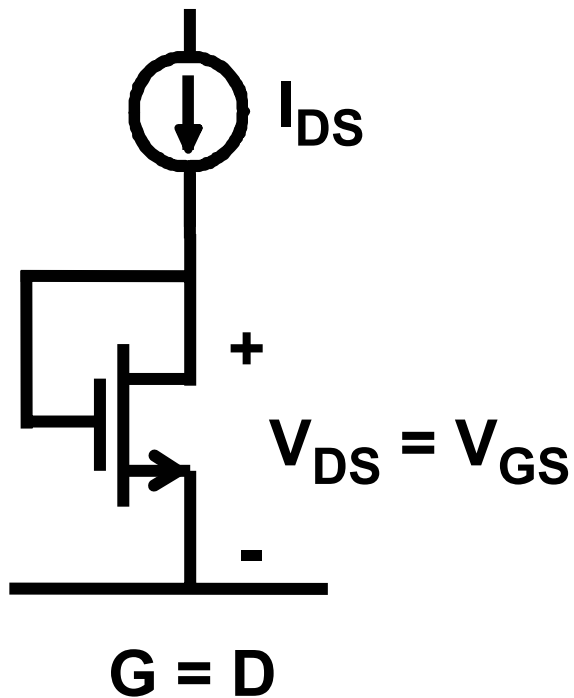


Diode-connected MOST: small-signal



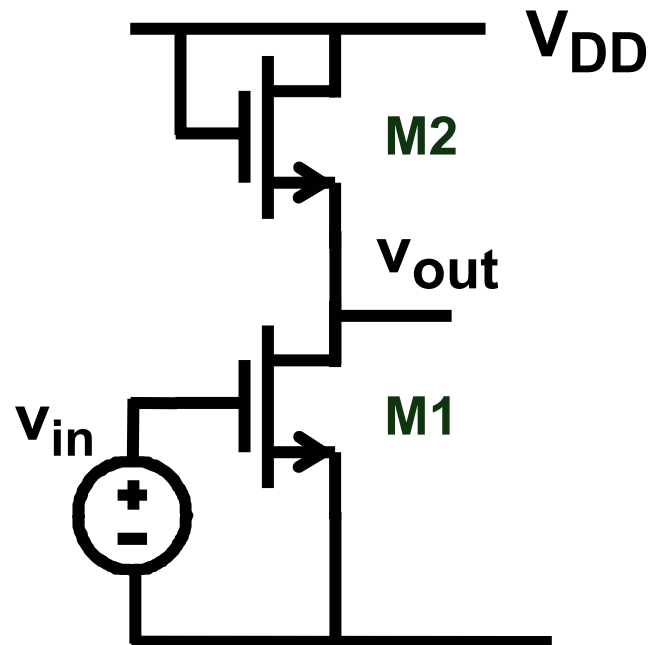
$$r_{ds} = 1/g_m \parallel r_{DS} \approx 1/g_m$$

Diode-connected MOST at high frequencies



$$BW = \frac{g_m}{2\pi (C_{GS} + C_{DS})} \approx \frac{f_T}{2}$$

Wideband amplifier

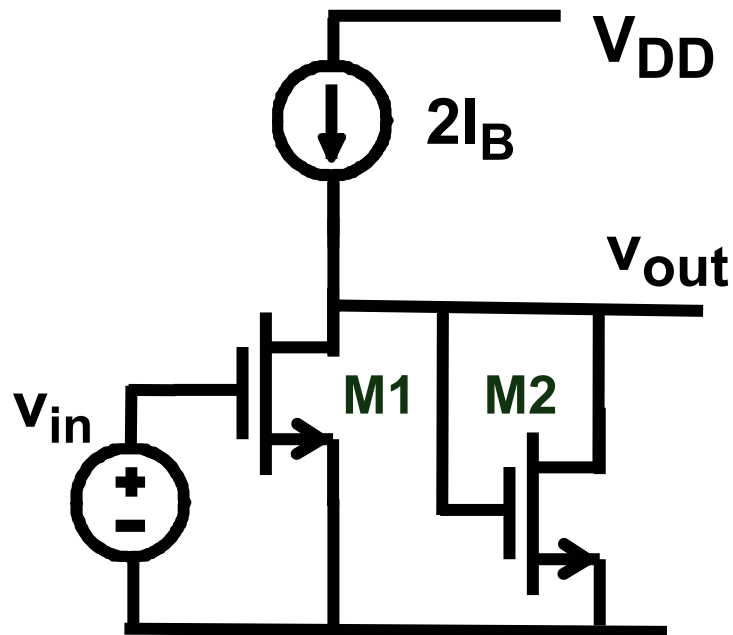


$$V_{OUT} = V_{DD} - V_{GS2}(V_{OUT})$$

$$A_{v0} = \frac{g_{m1}}{g_{m2}} = \sqrt{\frac{(W/L)_1}{(W/L)_2}} = \frac{V_{GS2} - V_T}{V_{GS1} - V_T}$$

$$R_{OUT} = 1/g_{m2}$$

Linear wideband amplifier



$$V_{OUT} = V_{GS2}$$

$$A_{v0} = \frac{g_{m1}}{g_{m2}} = \sqrt{\frac{(W/L)_1}{(W/L)_2}} = \frac{V_{GS2} - V_T}{V_{GS1} - V_T}$$

$$R_{OUT} = 1/g_{m2}$$

Current mirror with only nMOSTs

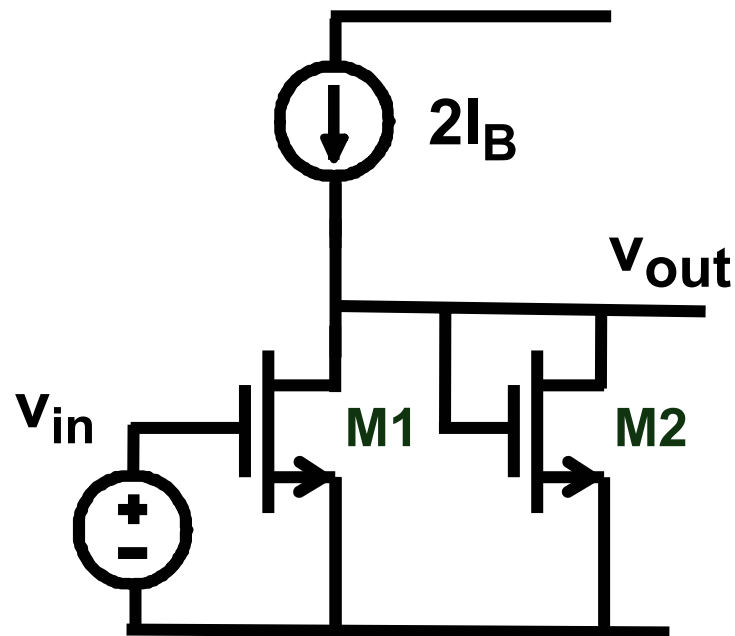
Same V_{OUTDC} as V_{INDC}

No body bias effect

Good PSRR

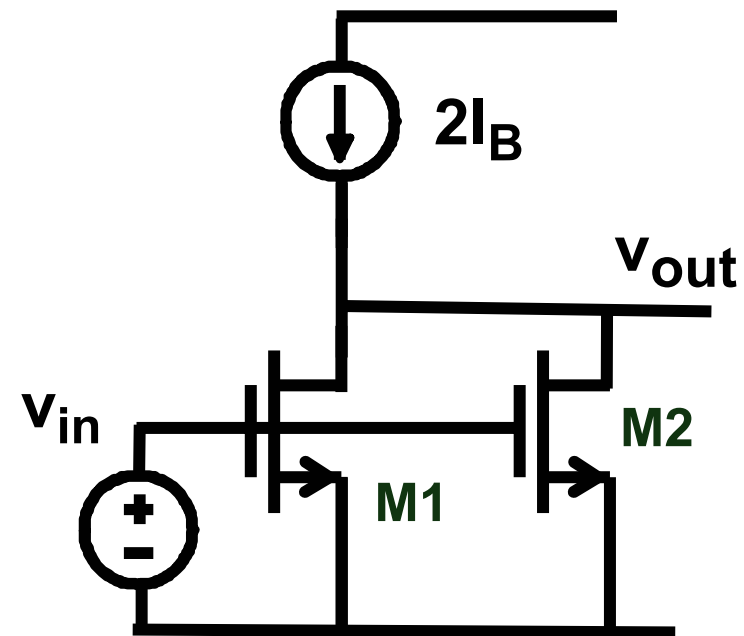
Double power consumption

Wideband amplifiers



$$A_{v0} = \frac{g_{m1}}{g_{m2}} = \sqrt{\frac{(W/L)_1}{(W/L)_2}} = \frac{V_{GS2} - V_T}{V_{GS1} - V_T}$$

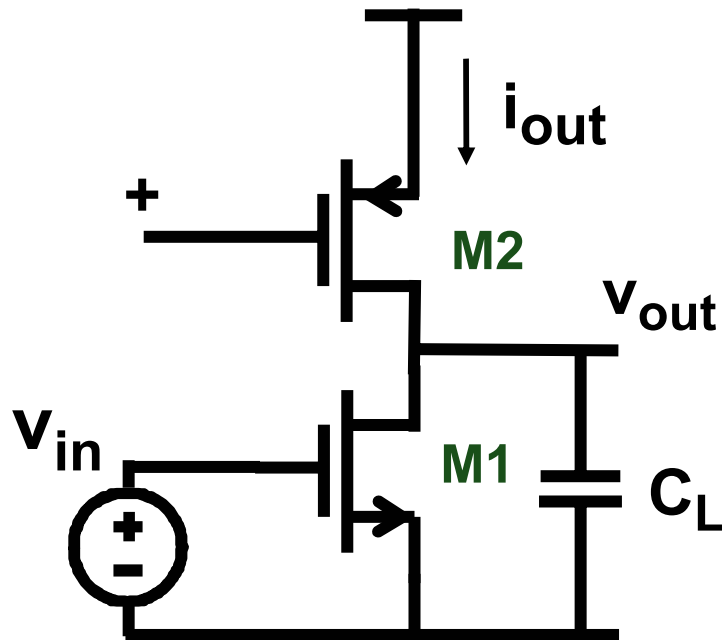
$$R_{out} = 1/g_{m2}$$



$$A_{v0} = g_m R_{out}$$

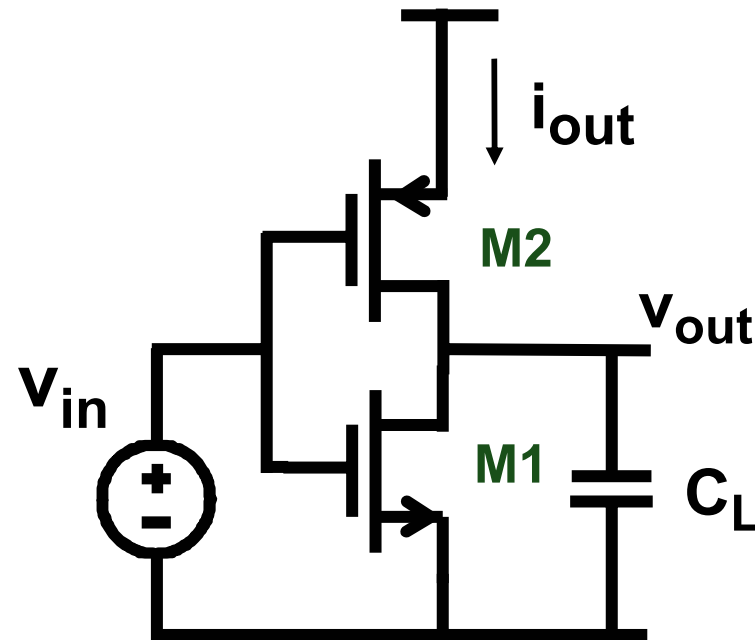
$$R_{out} = r_{DS1} // r_{DS2}$$

Class A versus class AB amplifier



$$v_{out} = A_v v_{in}$$

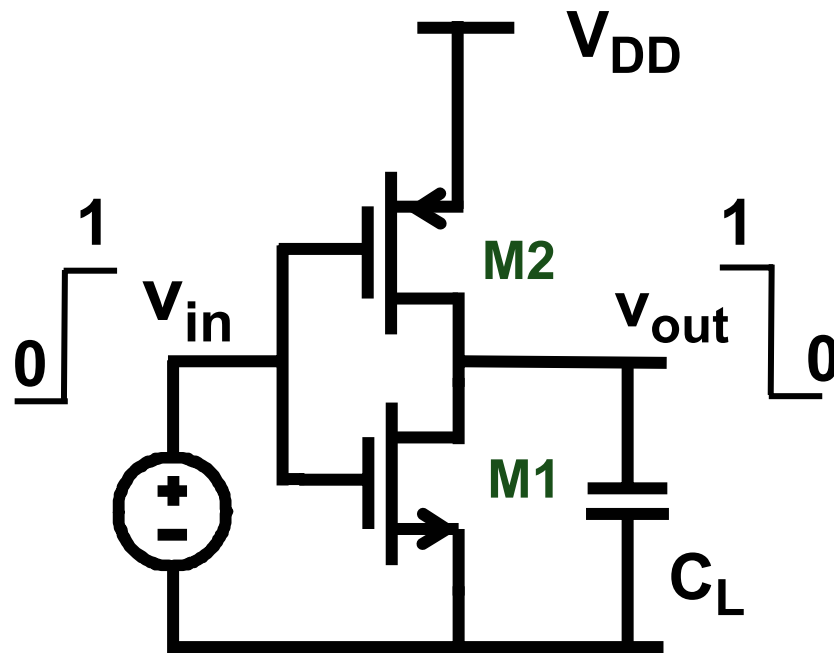
Class A stage



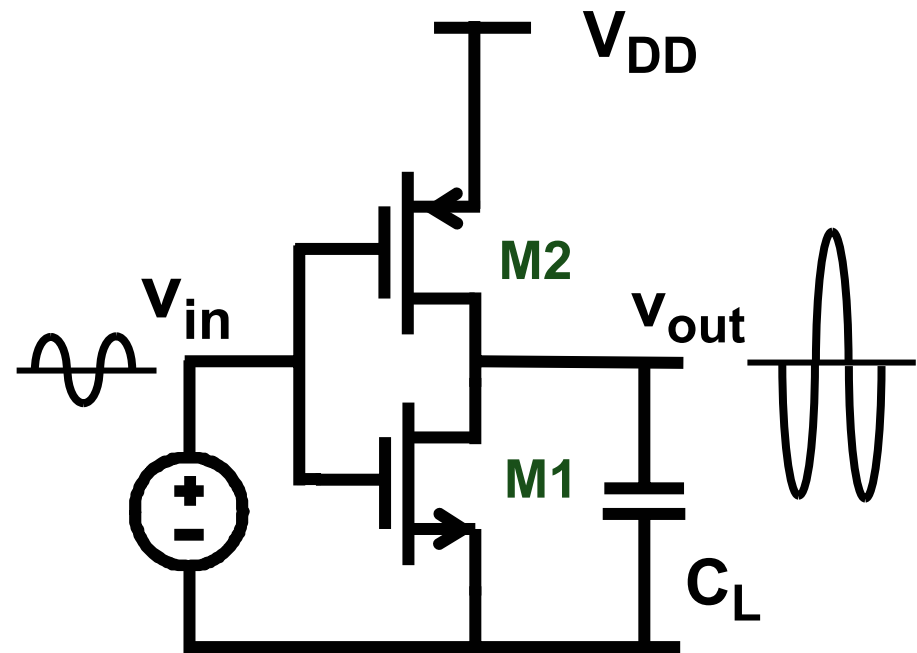
$$v_{out} = A_v v_{in}$$

Class AB stage

CMOS inverter-amplifier

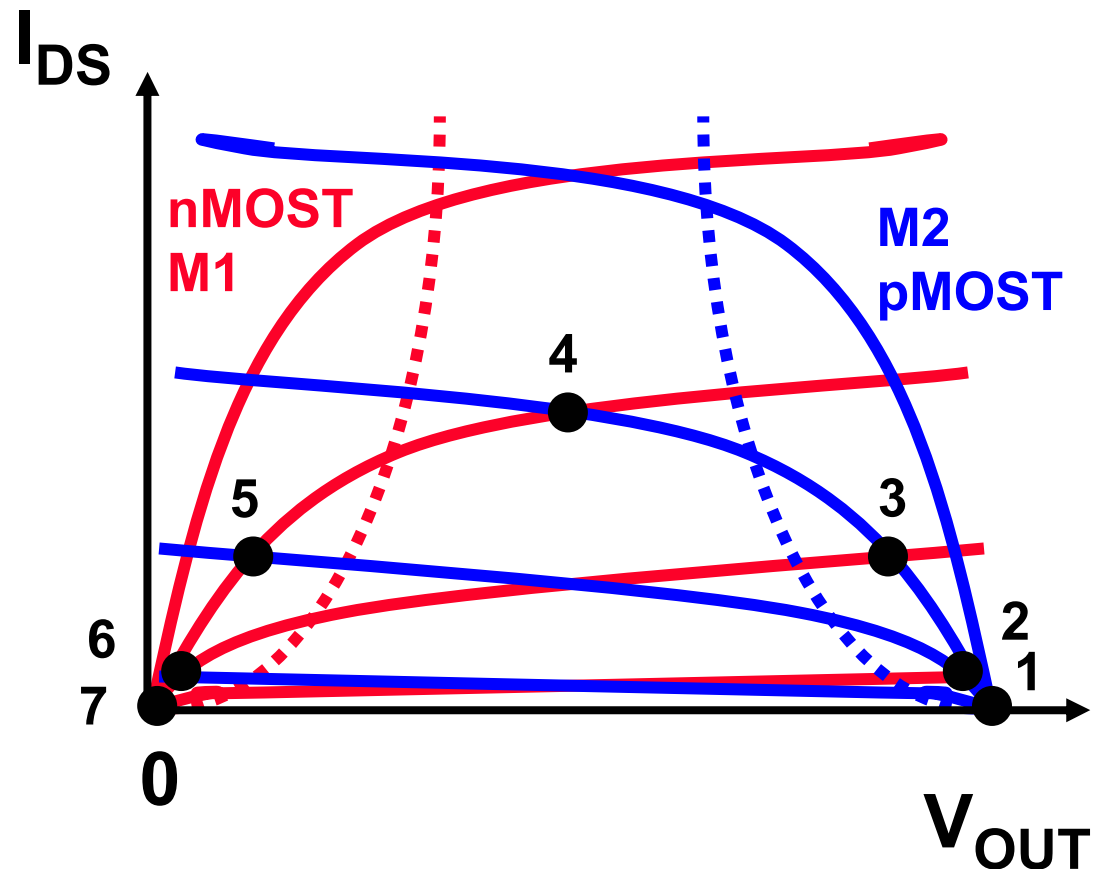
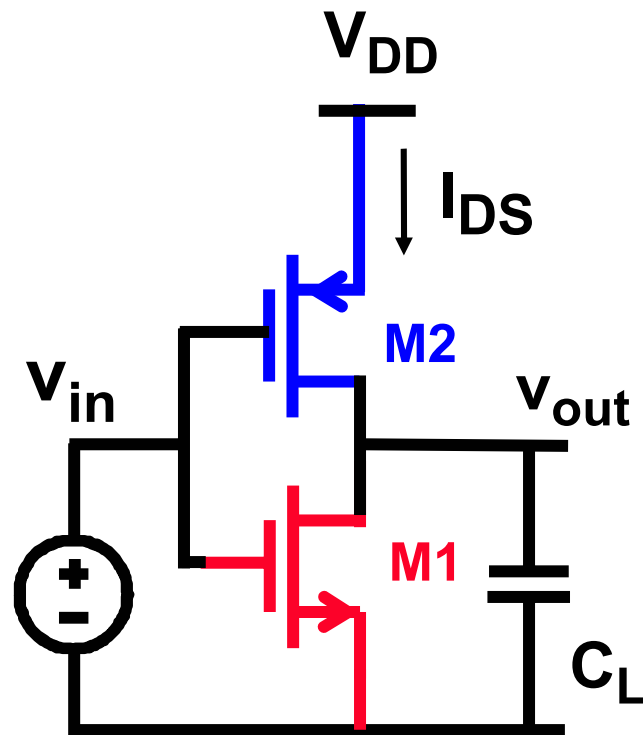


Digital inverter



Analog amplifier

Operating points nMOST & pMOST

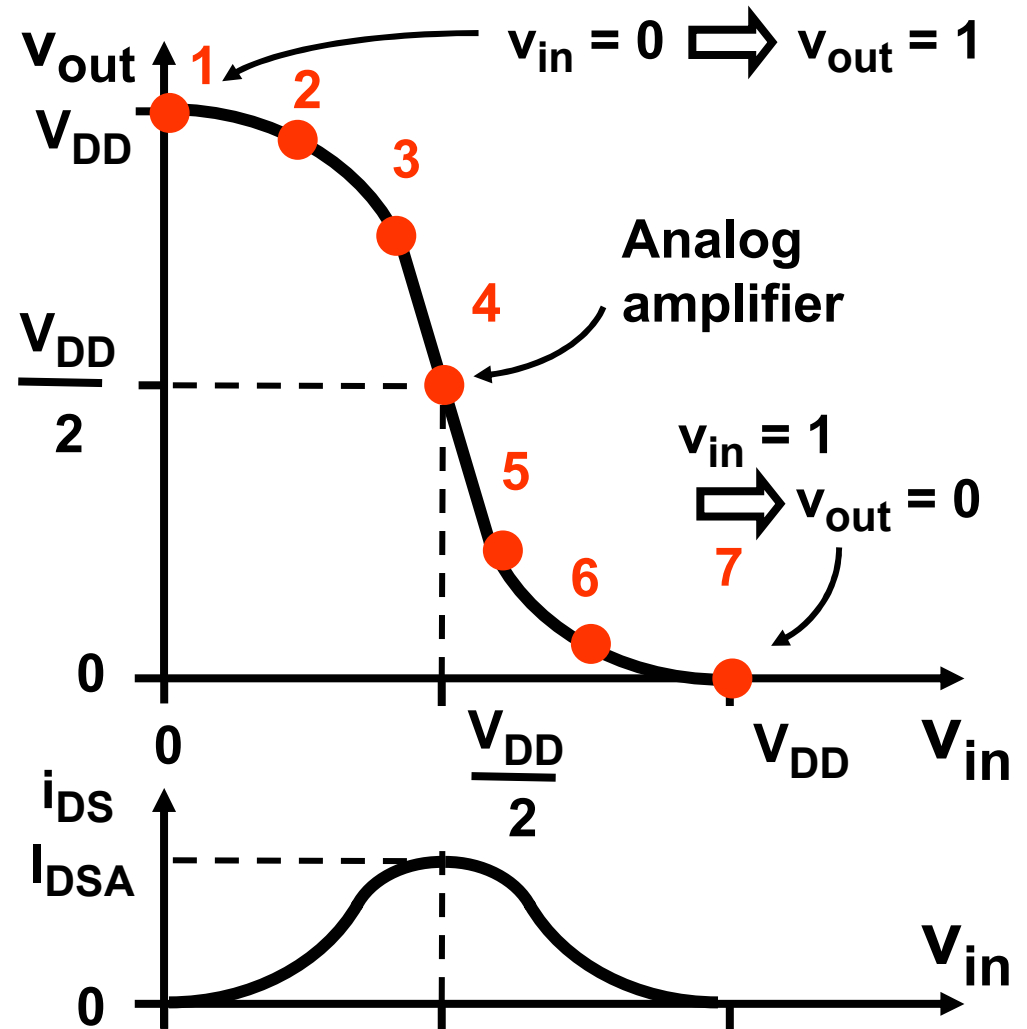
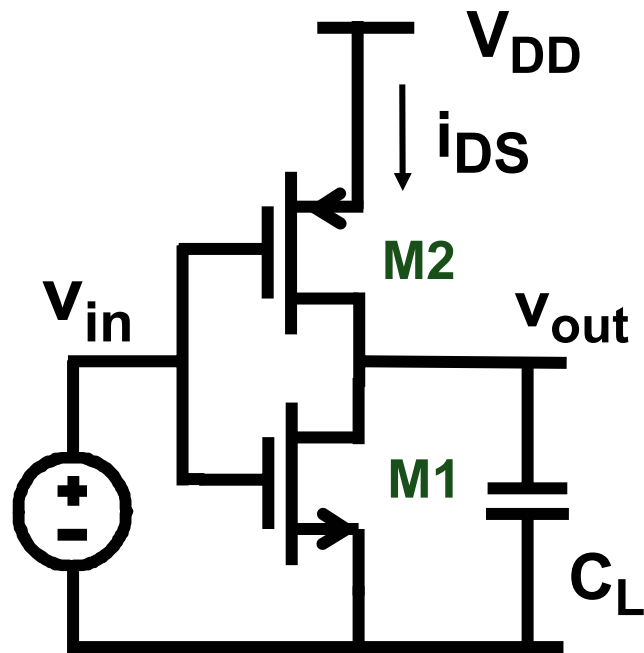


$$\begin{aligned} V_{DD} &= V_{DSn} + V_{DSp} \\ &= V_{GSn} + V_{GSp} \end{aligned}$$

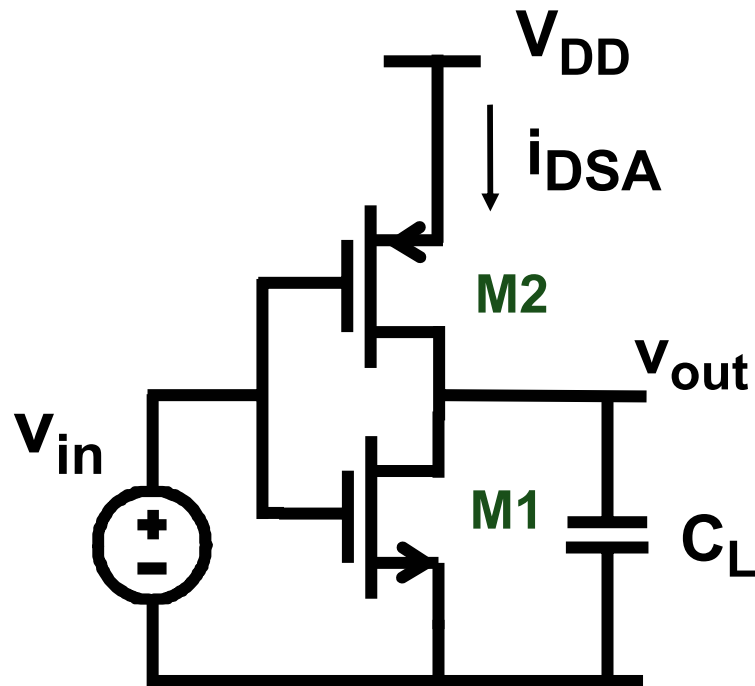
$$\begin{aligned} V_{DSn} &= V_{OUT} \\ V_{GSn} &= V_{IN} \end{aligned}$$

$$\begin{aligned} V_{DSp} &= V_{DD} - V_{OUT} \\ V_{GSp} &= V_{DD} - V_{IN} \end{aligned}$$

Transfer characteristic



Analog amplifier : DC



$$V_{in} = \frac{V_{DD}}{2} \Rightarrow V_{out} = \frac{V_{DD}}{2}$$

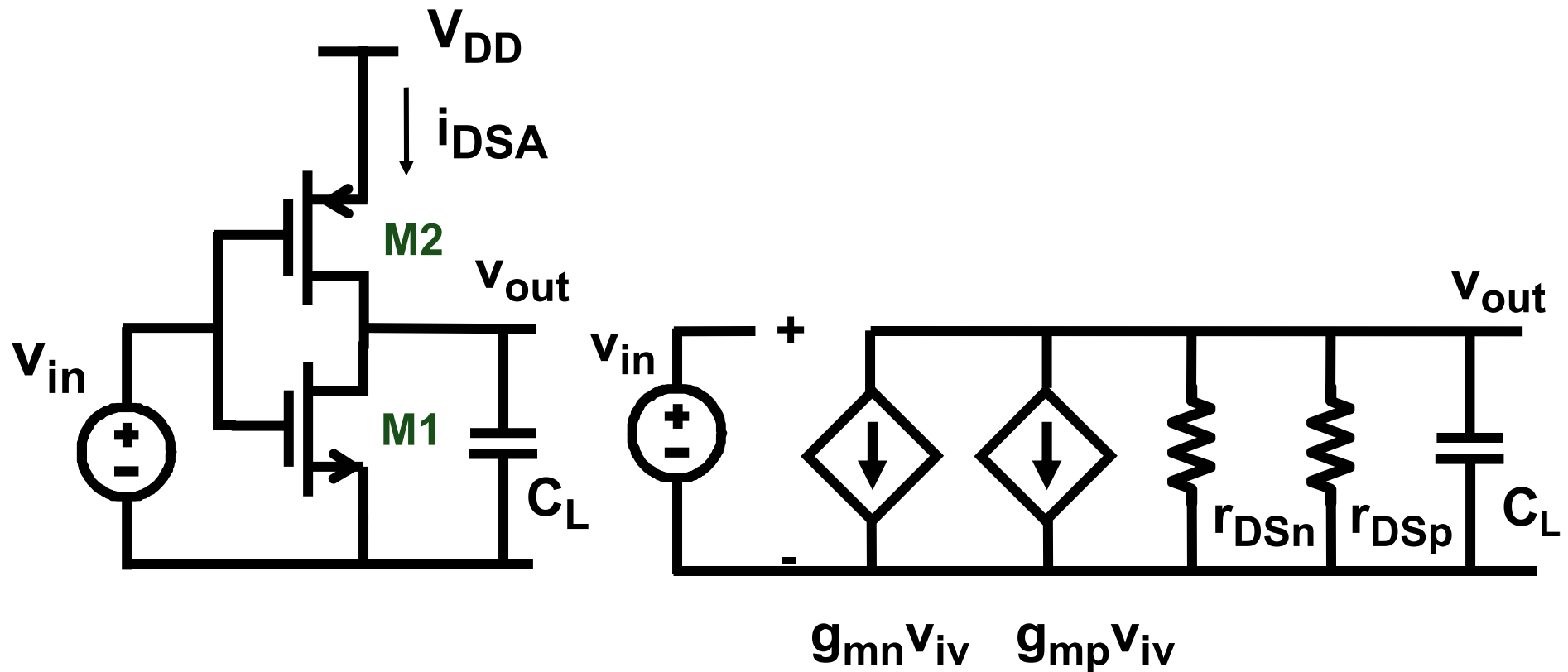
$$I_{DSn} = K'_n \frac{W_n}{L_n} (V_{in} - V_T)^2$$

$$I_{DSp} = K'_p \frac{W_p}{L_p} (V_{DD} - V_{in} - V_T)^2$$

$$\Rightarrow K'_n \frac{W_n}{L_n} = K'_p \frac{W_p}{L_p}$$

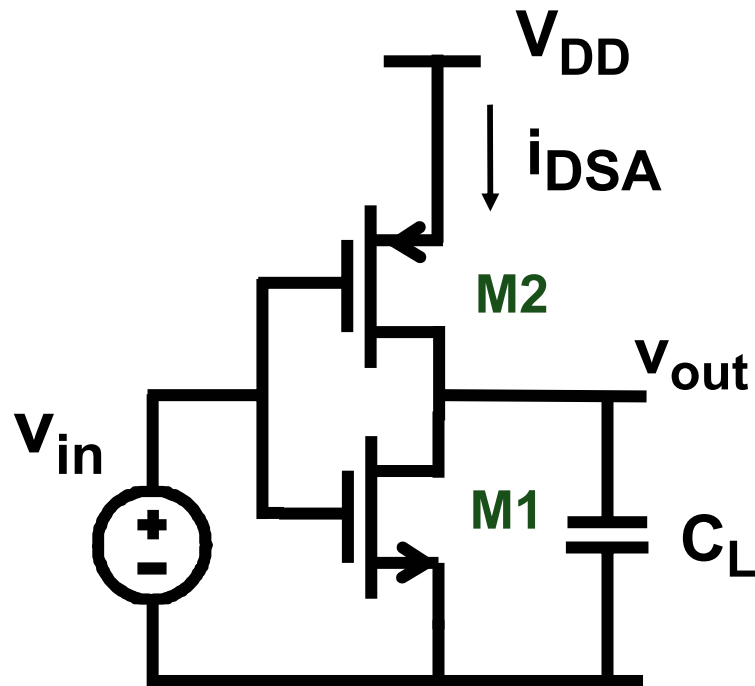
$$I_{DS} = K'_n \frac{W_n}{L_n} \left(\frac{V_{DD}}{2} - V_T \right)^2$$

Analog amplifier : AC model



For the same I_{DS} en $V_{GS}-V_T$: $g_{mn} = g_{mp} = g_m$

Analog amplifier: AC gain A_v



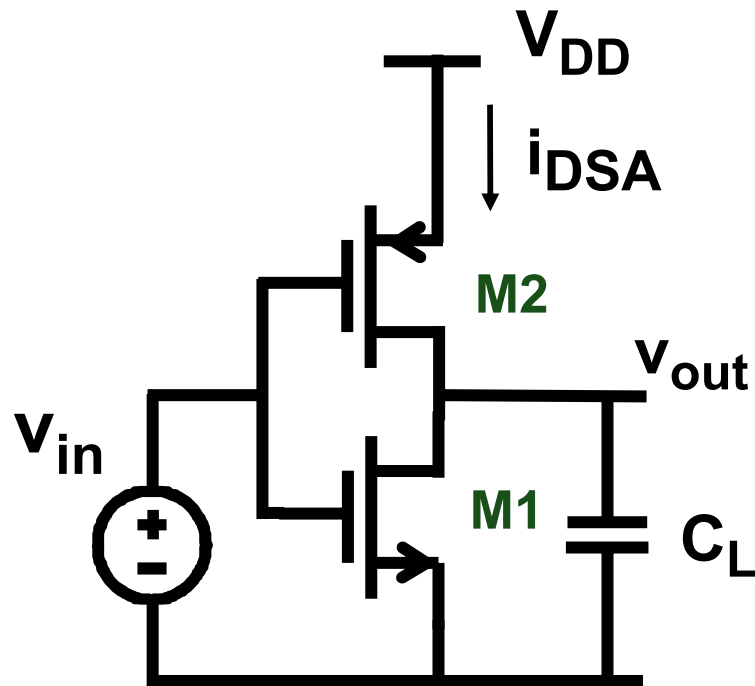
$$\text{If } V_{En}L_n = V_{Ep}L_p = V_E$$

$$g_{DSn} = g_{DSp} = g_{DS}$$

$$(g_{DS} = 1/r_{DS})$$

$$A_{v0} = - \frac{2g_m}{2g_{DS}} = - \frac{2V_E}{\frac{V_{DD}}{2} - V_T}$$

Analog amplifier : BW & GBW



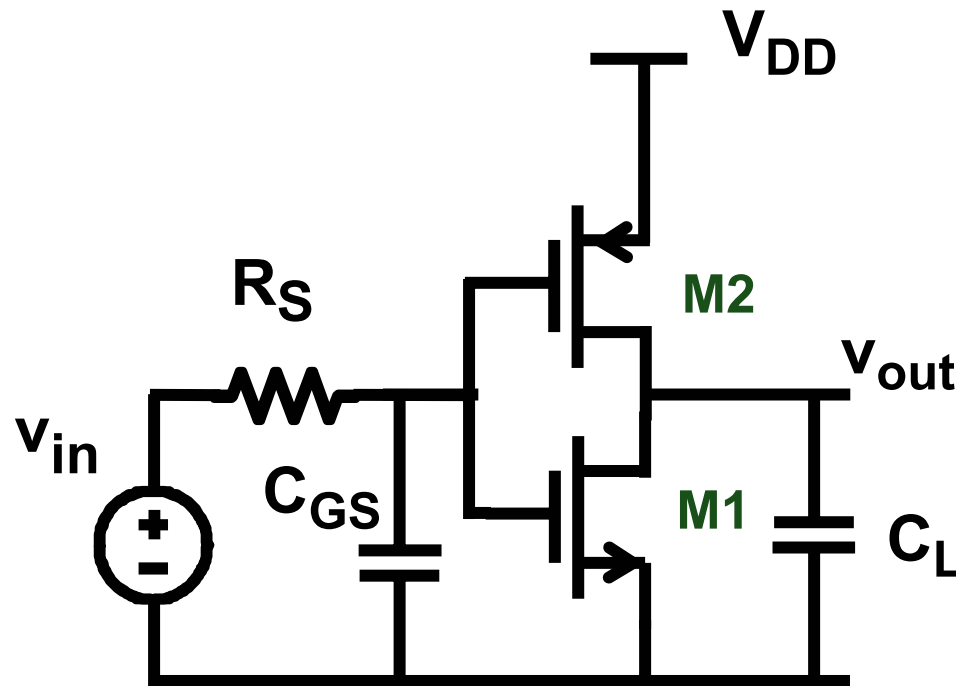
$$A_{v0} = 2g_m R_{out}$$

$$R_{out} = \frac{r_{DS}}{2}$$

$$BW = \frac{1}{2\pi R_{out} C_L}$$

$$GBW = \frac{2g_m}{2\pi C_L}$$

Analog amplifier: poles due to C_{GS}



$$A_{v0} = 2g_m R_{out}$$

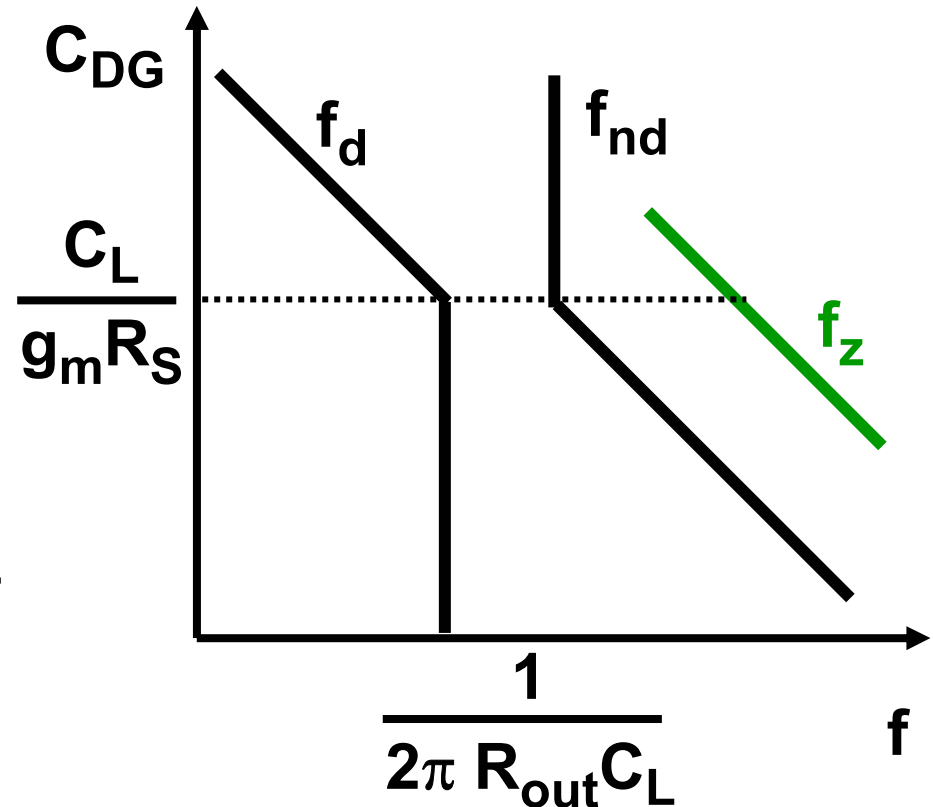
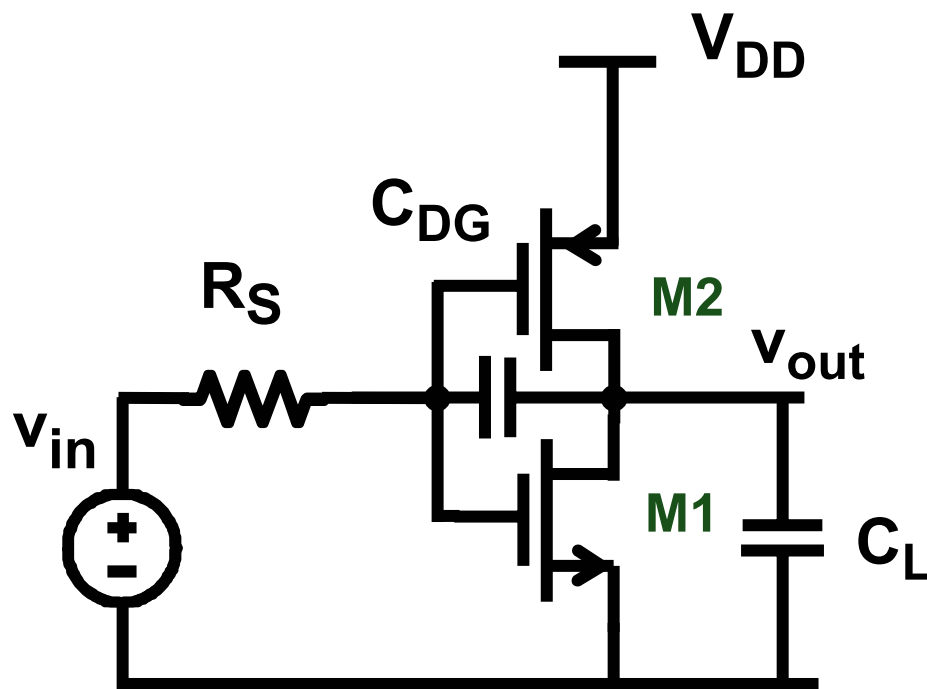
$$GBW = \frac{2g_m}{2\pi C_L}$$

$$C_{GSst} = C_{GS1} + C_{GS2}$$

But if $R_S C_{GSst} > r_{DS} C_L$:

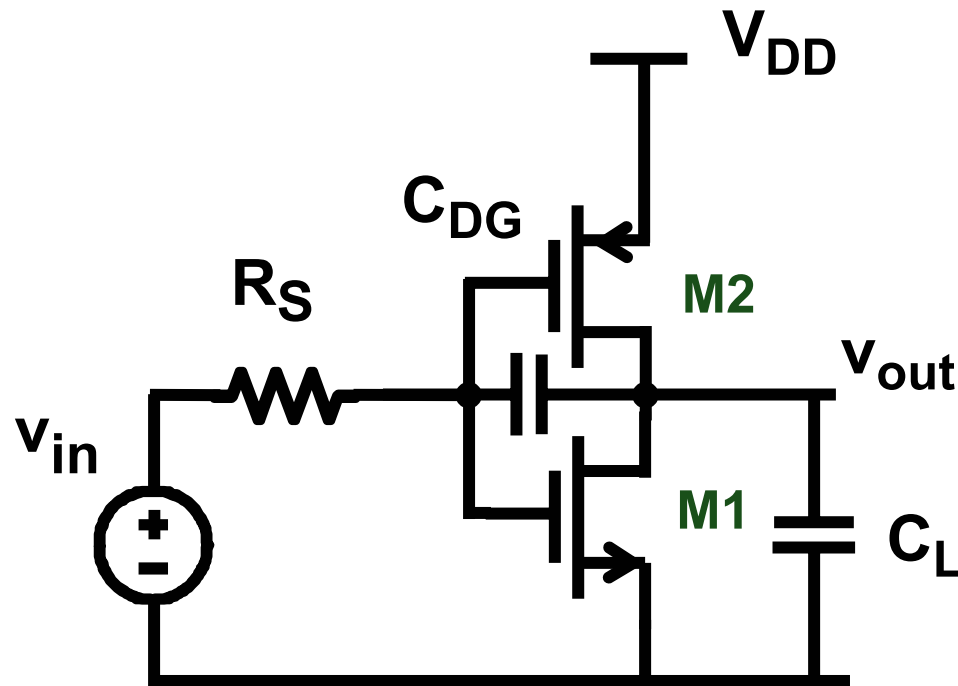
$$GBW = f_T \frac{r_{DS}}{R_S}$$

Analog amplifier: poles due to C_{DG}



$$\frac{v_{out}}{v_{in}} = \frac{A_{v0} (1 - sC_{DGt}/g_m)}{1 + s (R_{out}C_L + A_{v0}R_S C_{DGt}) + s^2 R_S R_{out} C_{DGt} C_L}$$

Analog amplifier: other poles



$$A_{v0} = 2g_m R_{out}$$

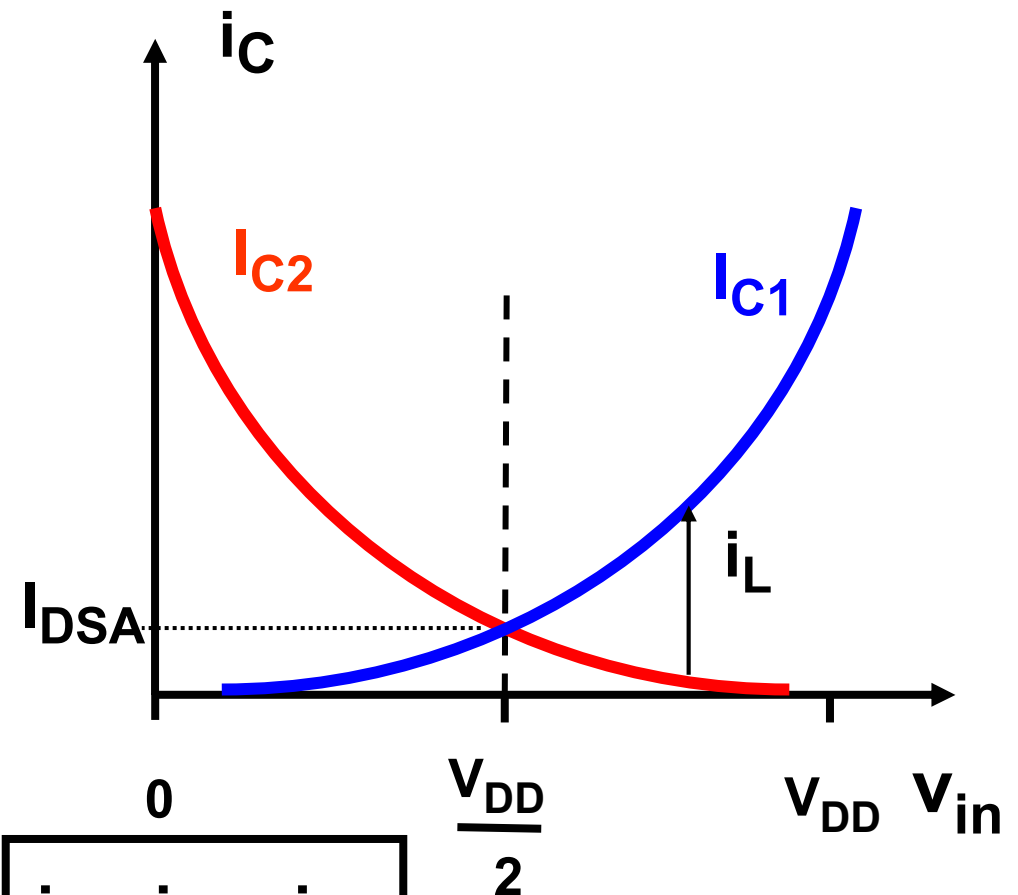
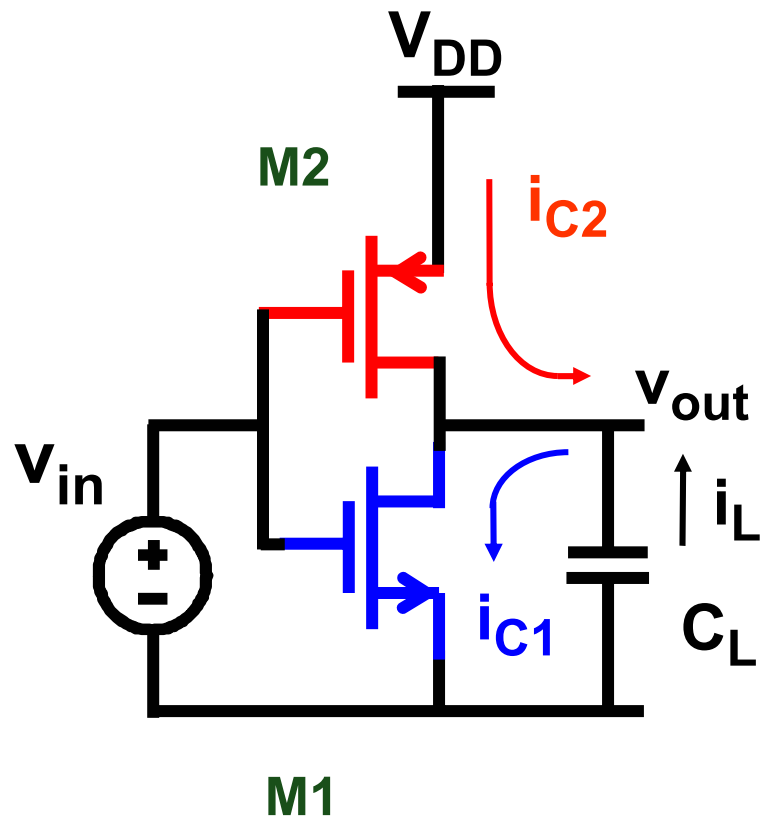
$$GBW = \frac{2g_m}{2\pi C_L}$$

$$C_{DGt} = C_{DG1} + C_{DG2}$$

$$\text{But if } R_S C_{DGt} > \frac{1}{2\pi GBW} :$$

$$GBW = \frac{1}{2\pi R_S C_{DGt}}$$

Class AB operation



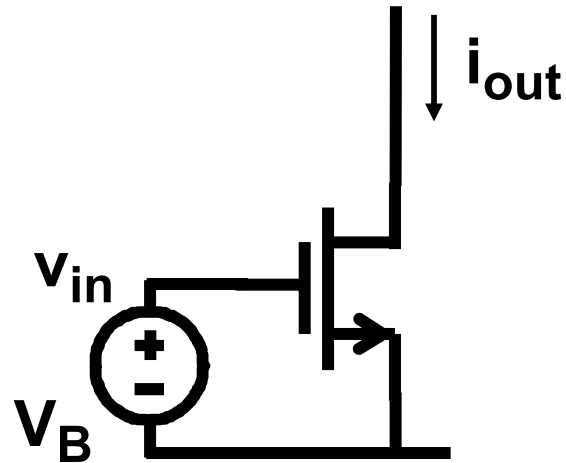
$$i_L = i_{C2} - i_{C1}$$

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- ☐ **Cascodes**

Single-transistor stages

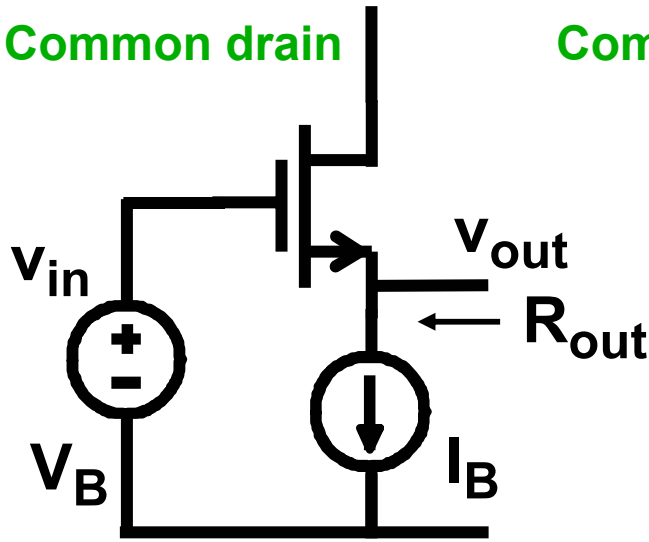
Common source



$$i_{out} = g_m v_{in}$$

Amplifier

Common drain



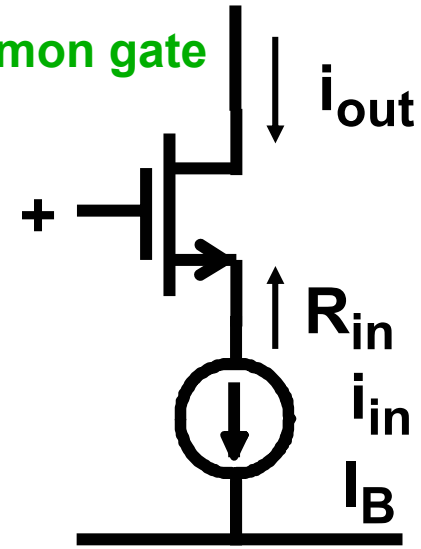
$$v_{out} = v_{in}$$

$$R_{out} \approx 1/g_m$$

Source follower

Voltage buffer

Common gate



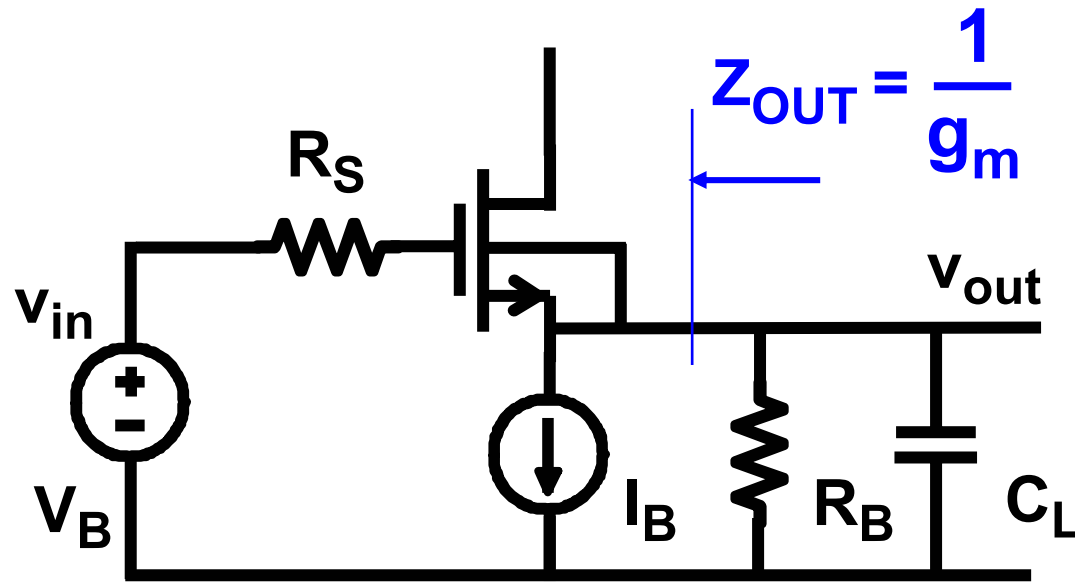
$$i_{out} = i_{in}$$

$$R_{in} \approx 1/g_m$$

Cascode

Current buffer

Source follower with $V_{BS} = 0$ (p-well)



$$V_{GS} = V_{T0} + \sqrt{\frac{I_B}{K'W/L}}$$

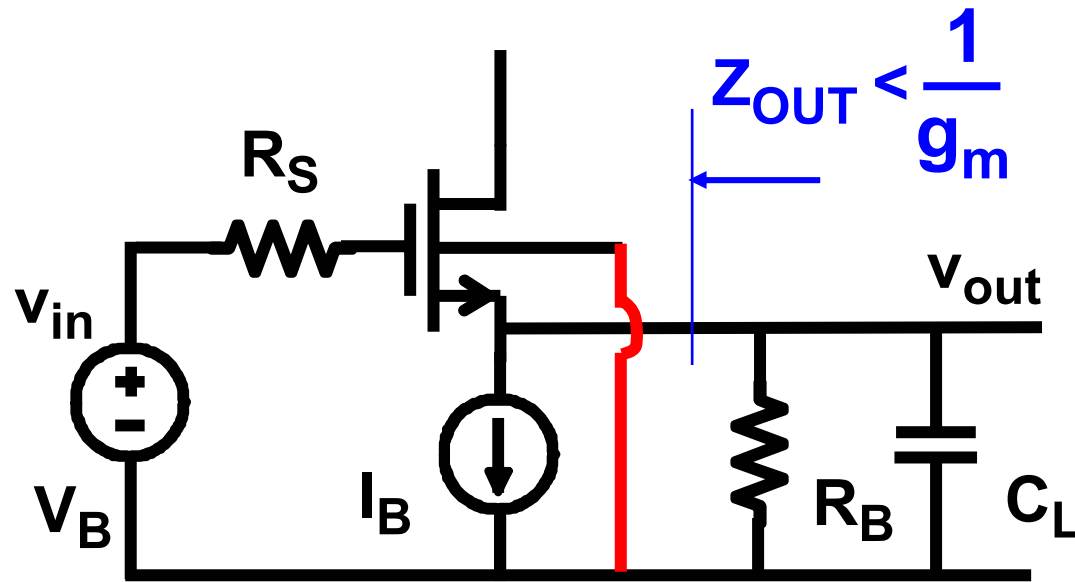
$$V_{GS} = ct \text{ if } I_B = ct$$

$$V_{OUT} = V_{IN} - V_{GS}$$

$$\Delta V_{OUT} = \Delta V_{IN}$$

$$A_v = 1$$

Source follower with $V_{BS} \neq 0$ (n-well)



$$V_{GS} = V_T + \sqrt{\frac{I_B}{K'W/L}}$$

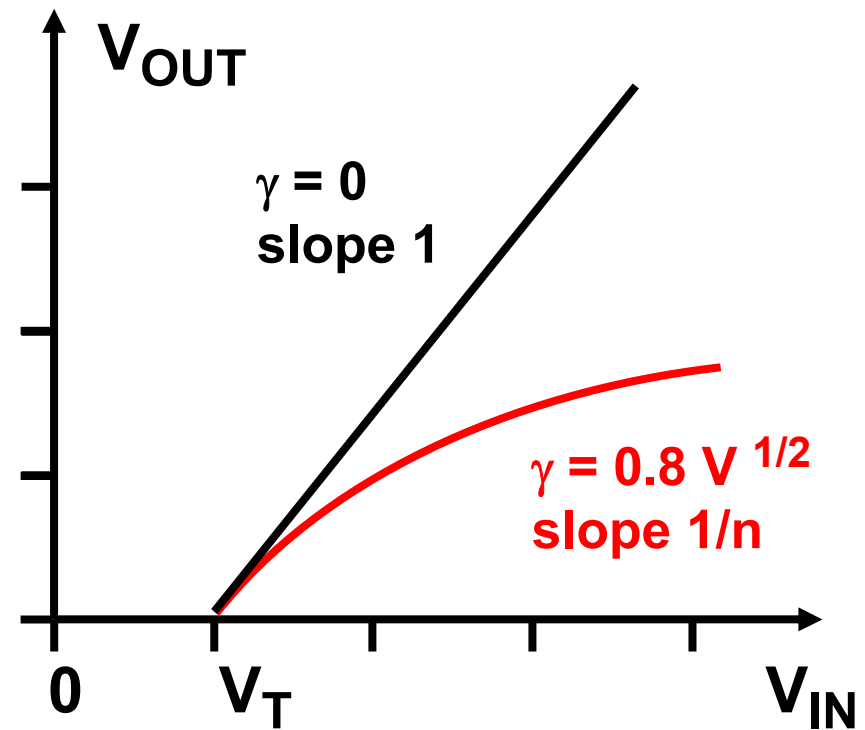
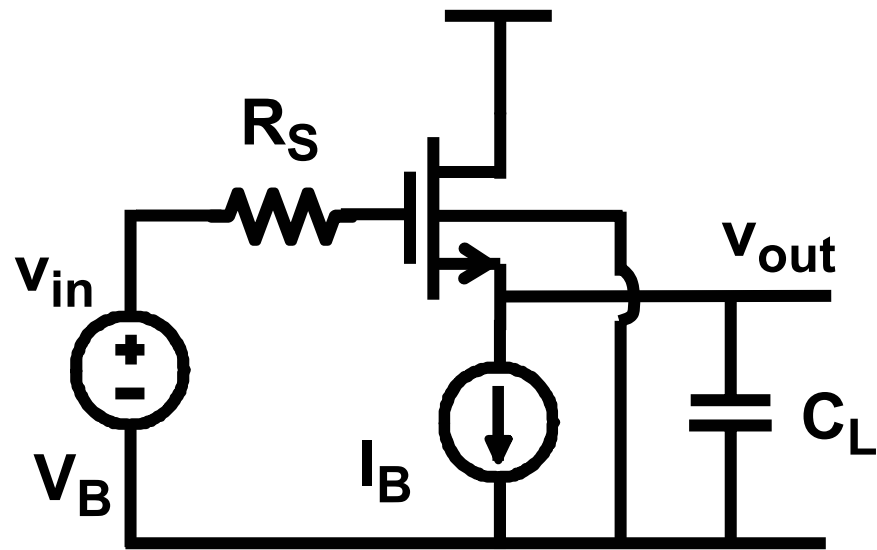
$$V_{GS} \neq \text{ct}$$

$$V_{OUT} = V_{IN} - V_{GS}$$

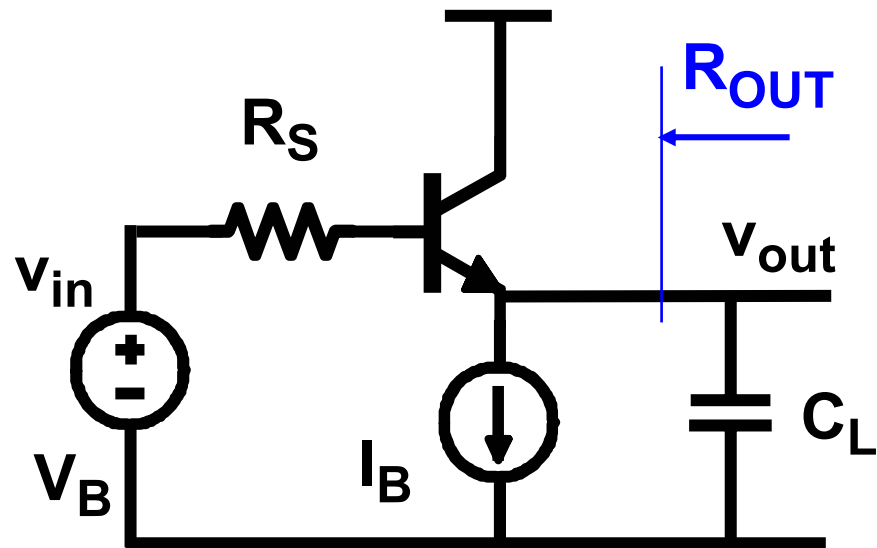
$$V_T = V_{T0} + \gamma \left[\sqrt{|2\Phi_F| + V_{OUT}} - \sqrt{|2\Phi_F|} \right]$$

$$A_v = \frac{1}{n}$$

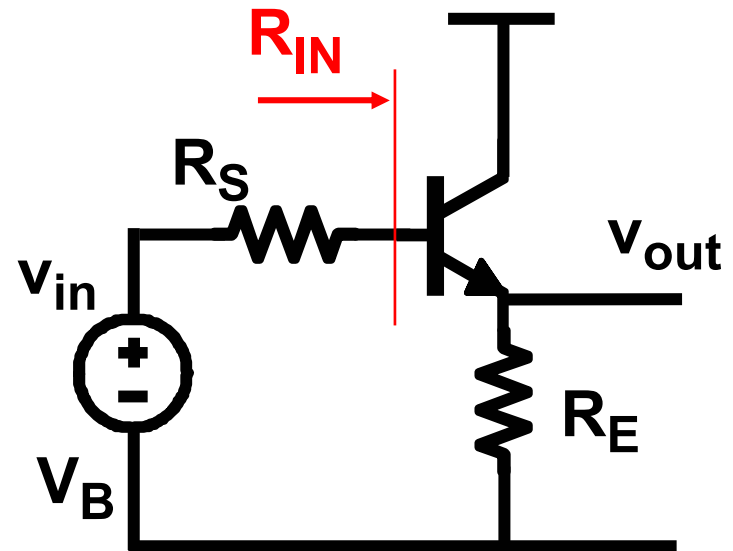
Source follower non-linearity



Emitter follower



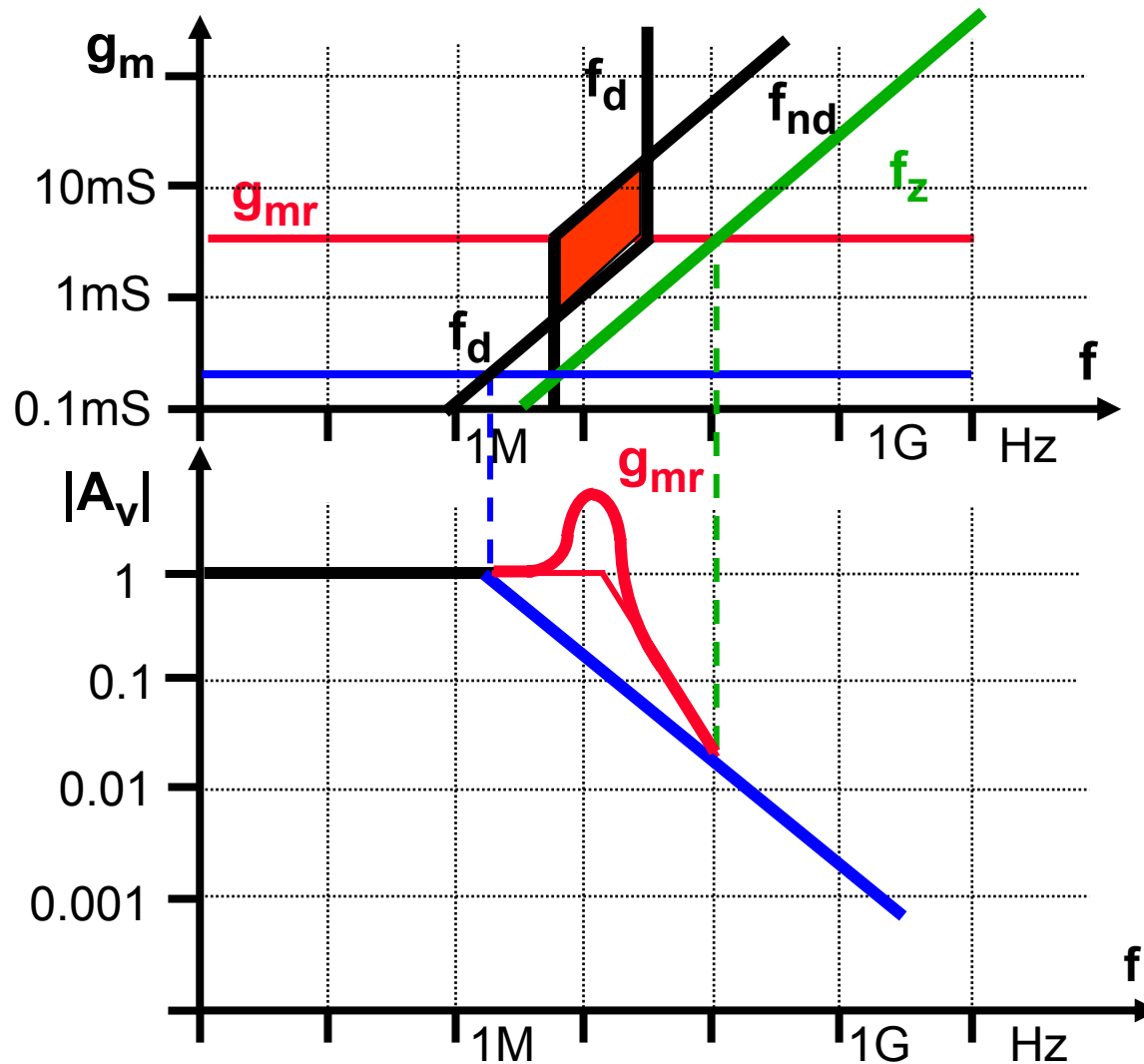
$$\boxed{A_v = 1} \quad R_{OUT} = \frac{1}{g_m} + \frac{R_S + r_B}{\beta + 1}$$



$$R_{IN} = r_{\pi} + r_B + (\beta + 1)R_E$$

Limited isolation !

Source follower with C_L load



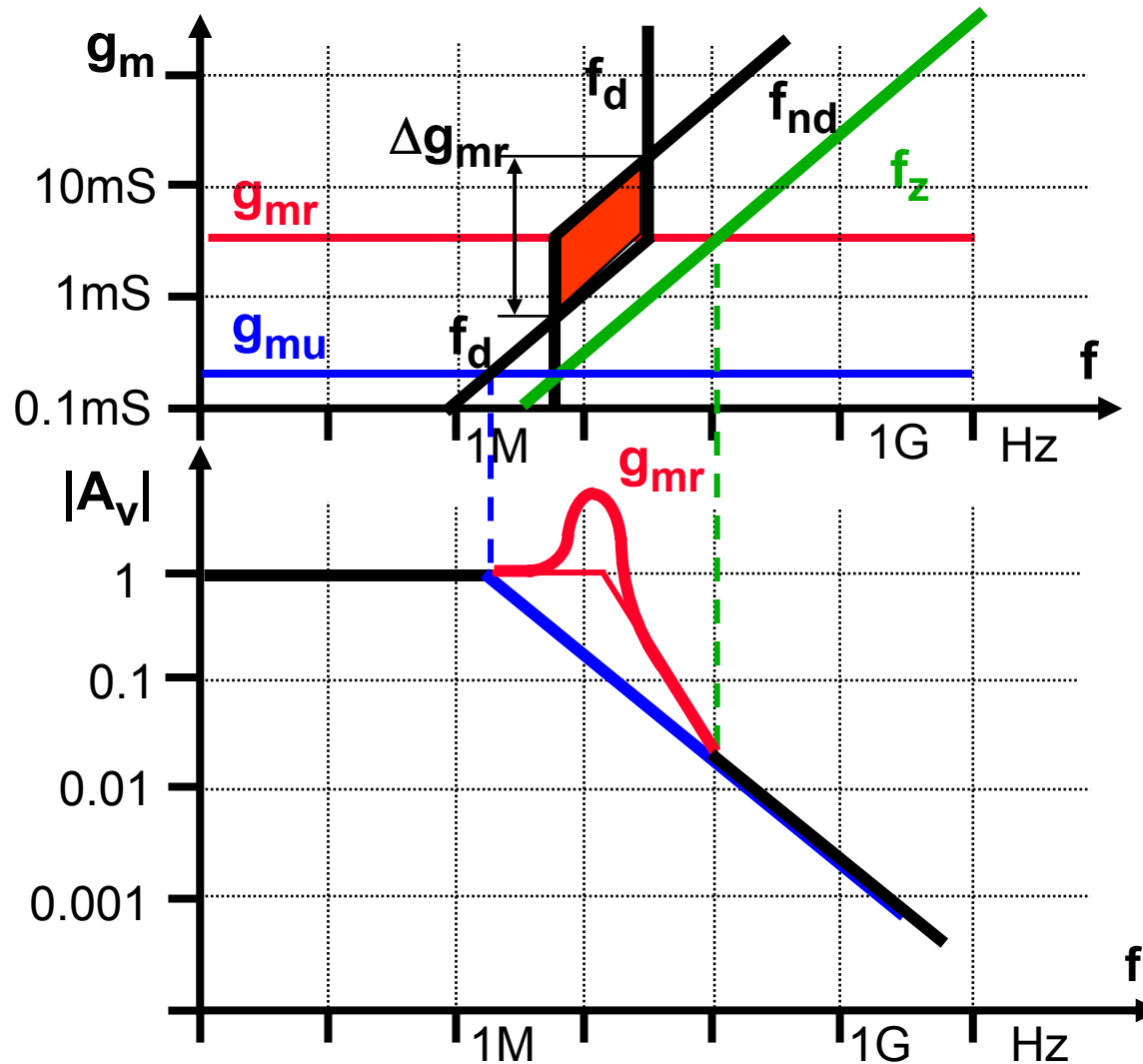
$$A_v = \frac{(1 + s C_{GS} / g_m)}{1 + s B + s^2 C^2 R_S / g_m}$$

$$B = R_S C_{DG} + \frac{C'_{DS}}{g_m} + \frac{C_{GS}}{g_m} \left(1 + \frac{R_S}{r_{DS}}\right)$$

$$C^2 = C'_{DS} C_{DG} + C'_{DS} C_{GS} + C_{DG} C_{GS}$$

$$C'_{DS} = C_L + C_{DS}$$

Source follower with C_L load



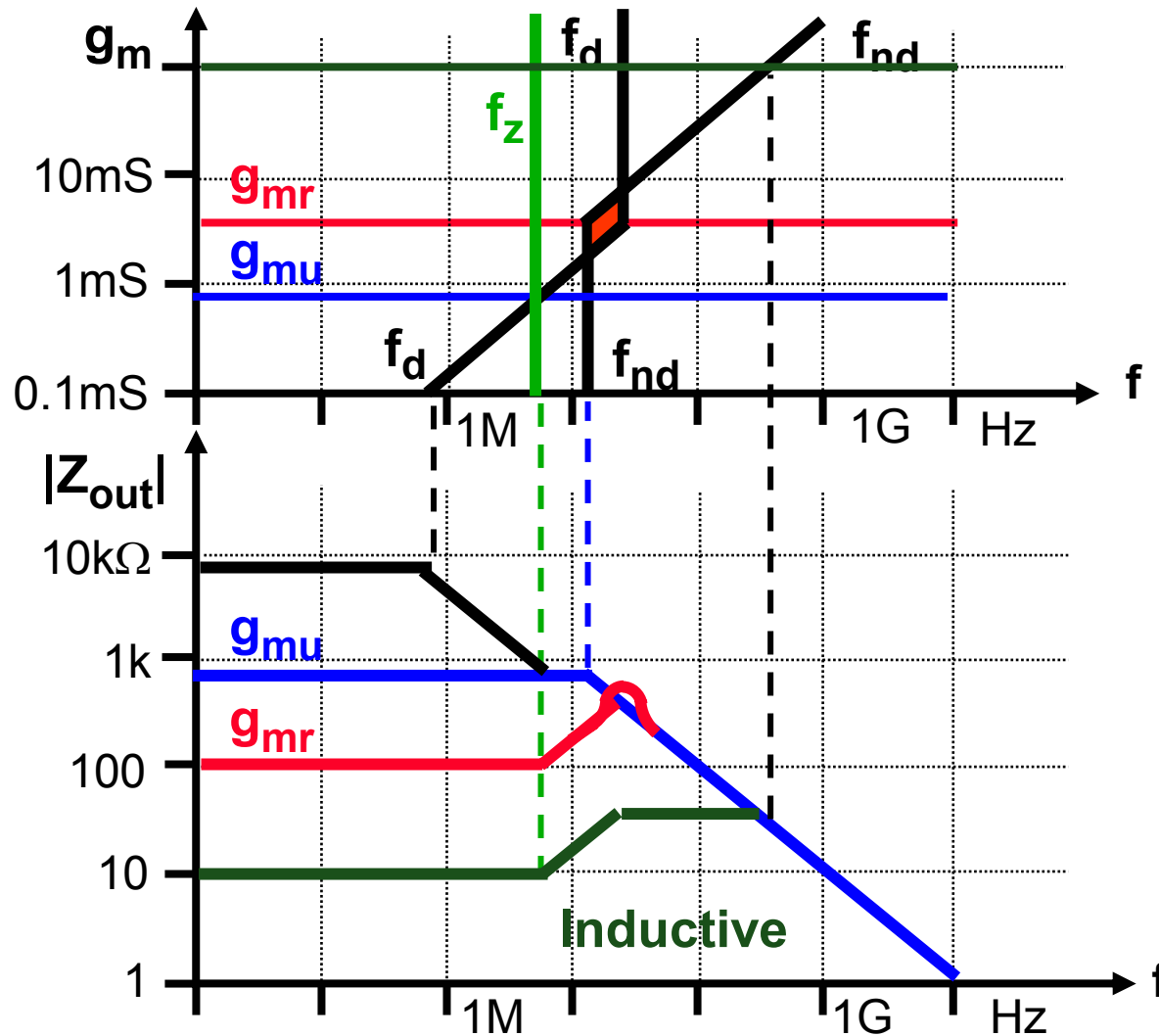
$$g_{mr} = \frac{1}{R_S} \frac{C_L + C_{DS} + C_{GS}}{C_{DG}}$$

$$\Delta g_{mr} = \frac{C_{DGt}}{C_{DG}}$$

$$C_{DGt} = \frac{C'_{DS} C_{GS}}{C'_{DS} + C_{GS}}$$

$$g_{mu} = \frac{1}{R_S}$$

Source follower : Output impedance



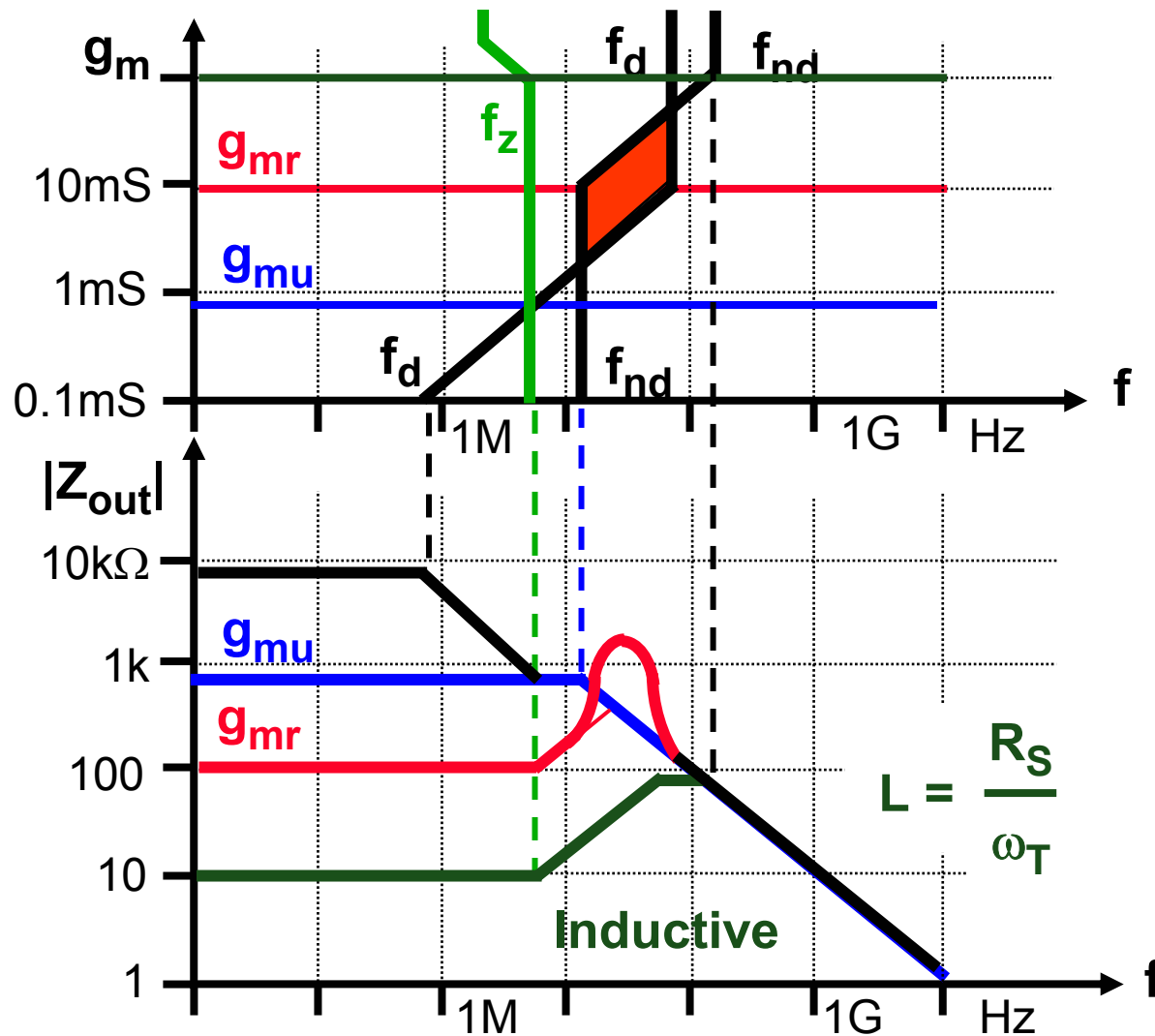
$$g_{mr} = \frac{1}{R_S} \frac{C_{GS} + C_{DS}}{C_{DG}}$$

$$g_{\mu} \approx \frac{1}{R_S} \frac{C_{GS} + C_{DS}}{C_{GS} + C_{DG}}$$

$$f_z = \frac{1}{2\pi R_S C_{GS}}$$

$$f_{d,higm} = \frac{1}{2\pi R_S C_{DG}}$$

Emitter follower : Output impedance

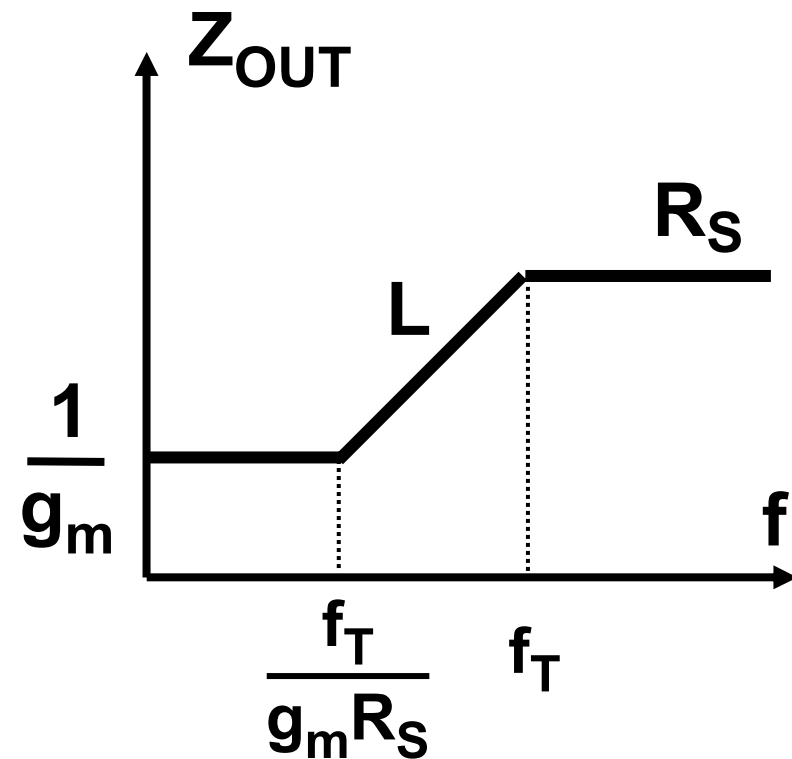
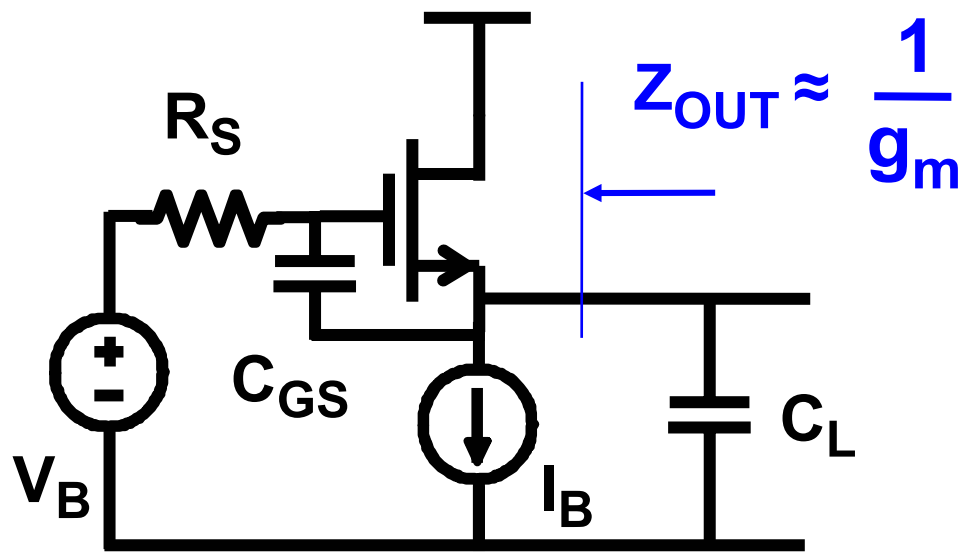


$$g_{mr} = \frac{1}{R_S} \frac{C_\pi + C_{CE}}{C_\pi + C_\mu}$$

$$g_{mu} \approx \frac{1}{R_S} \frac{C_{jE} + C_{CE}}{C_{jE} + C_\mu}$$

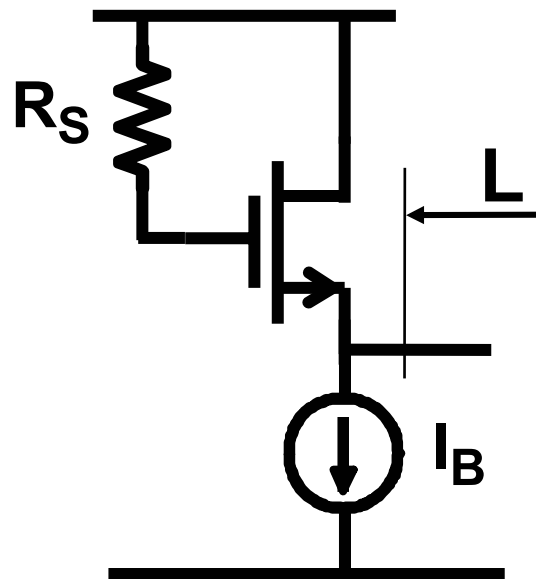
$$f_z = \frac{1}{2\pi R_S // r_\pi (C_\pi + C_\mu)}$$

Source follower as active L



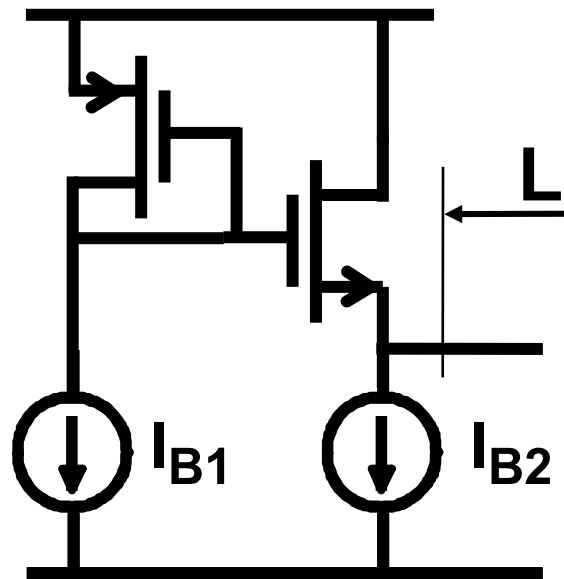
$$Z_{OUT} \approx \frac{1}{g_m} (1 + R_S C_{GS} s) \quad L \approx \frac{R_S}{2\pi f_T} \quad \text{up to } f_T = \frac{g_m}{2\pi C_{GS}}$$

Source follower as active L



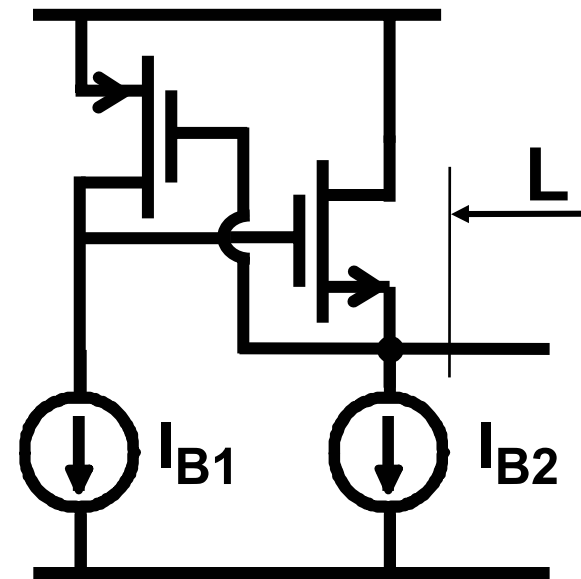
$$L \approx \frac{R_S}{2\pi f_T}$$

$$V_{DSn} = V_{GSn}$$



$$L \approx \frac{1/g_{mp}}{2\pi f_{Tn}}$$

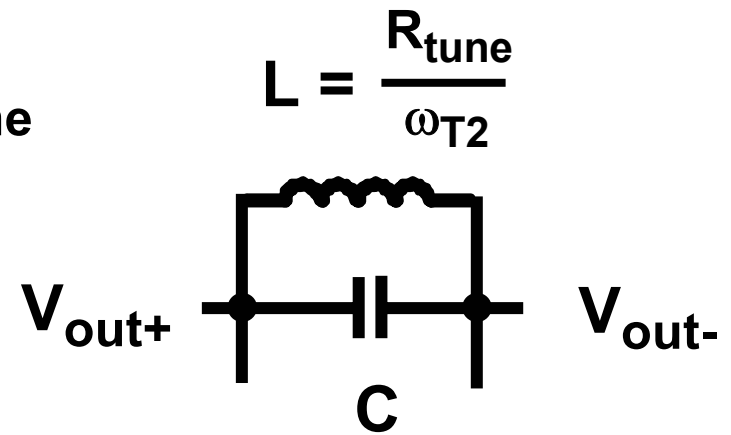
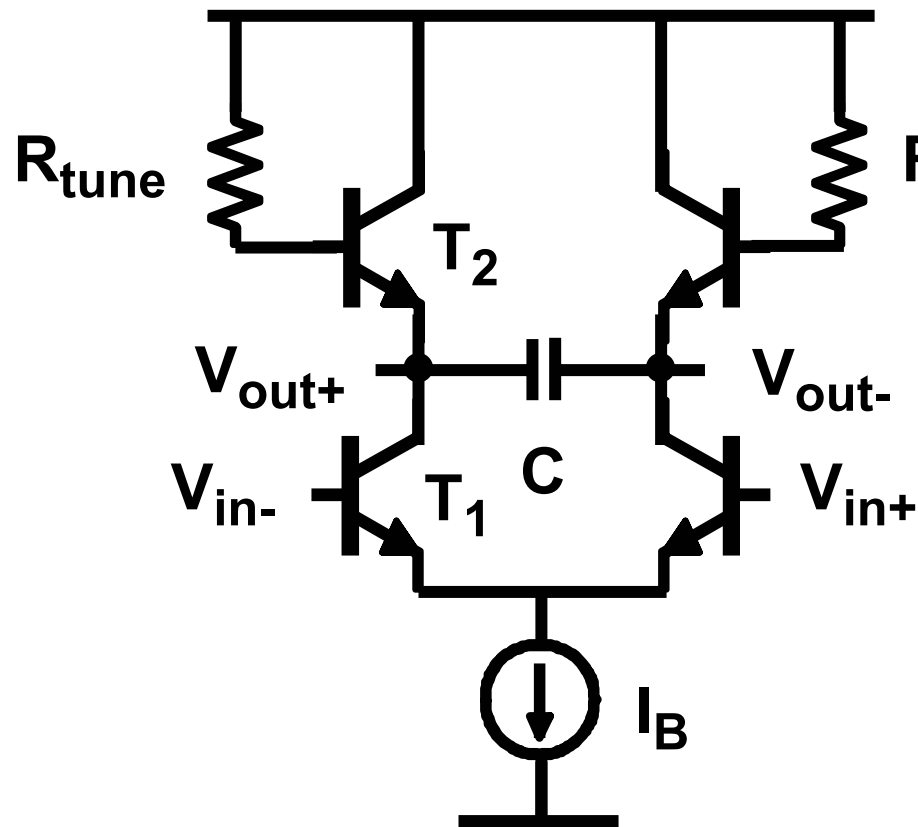
$$V_{DSn} = V_{GSn} + V_{GSp}$$



$$L \approx \frac{1/g_{mp}}{2\pi f_{Tn}}$$

$$V_{DSn} = V_{GSp}$$

Floating inductor with parallel C



$$A_v = \frac{g_{m1}}{g_{m2}}$$

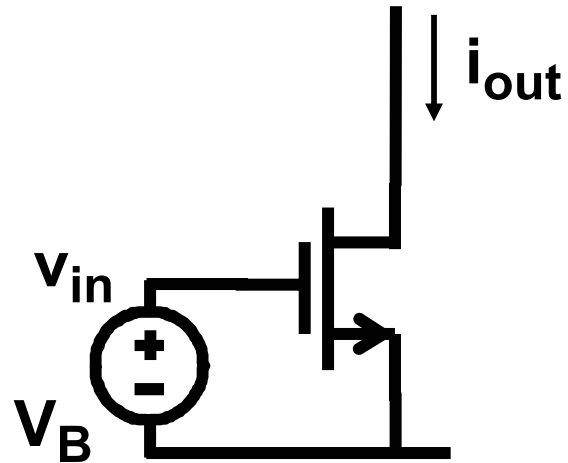
with HF peaking !

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- ☐ Single-transistor amplifiers
- ☐ Source followers
- ☐ Cascodes

Single-transistor stages

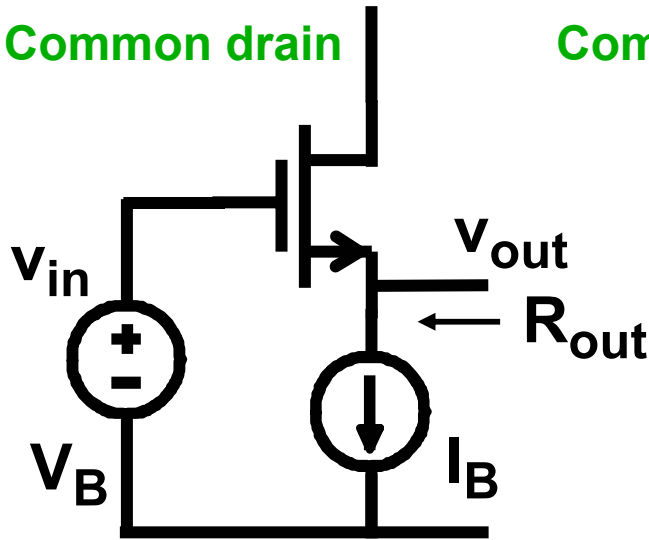
Common source



$$i_{out} = g_m v_{in}$$

Amplifier

Common drain



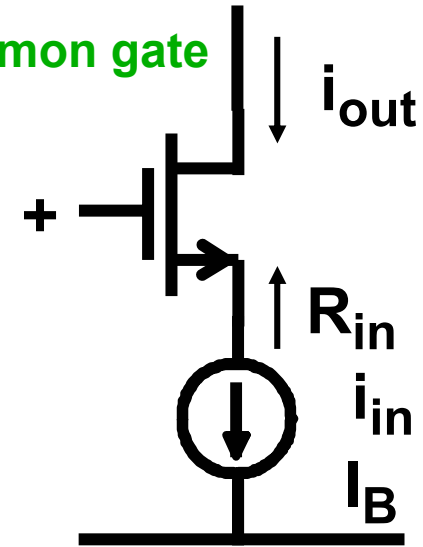
$$v_{out} = v_{in}$$

$$R_{out} \approx 1/g_m$$

Source follower

Voltage buffer

Common gate



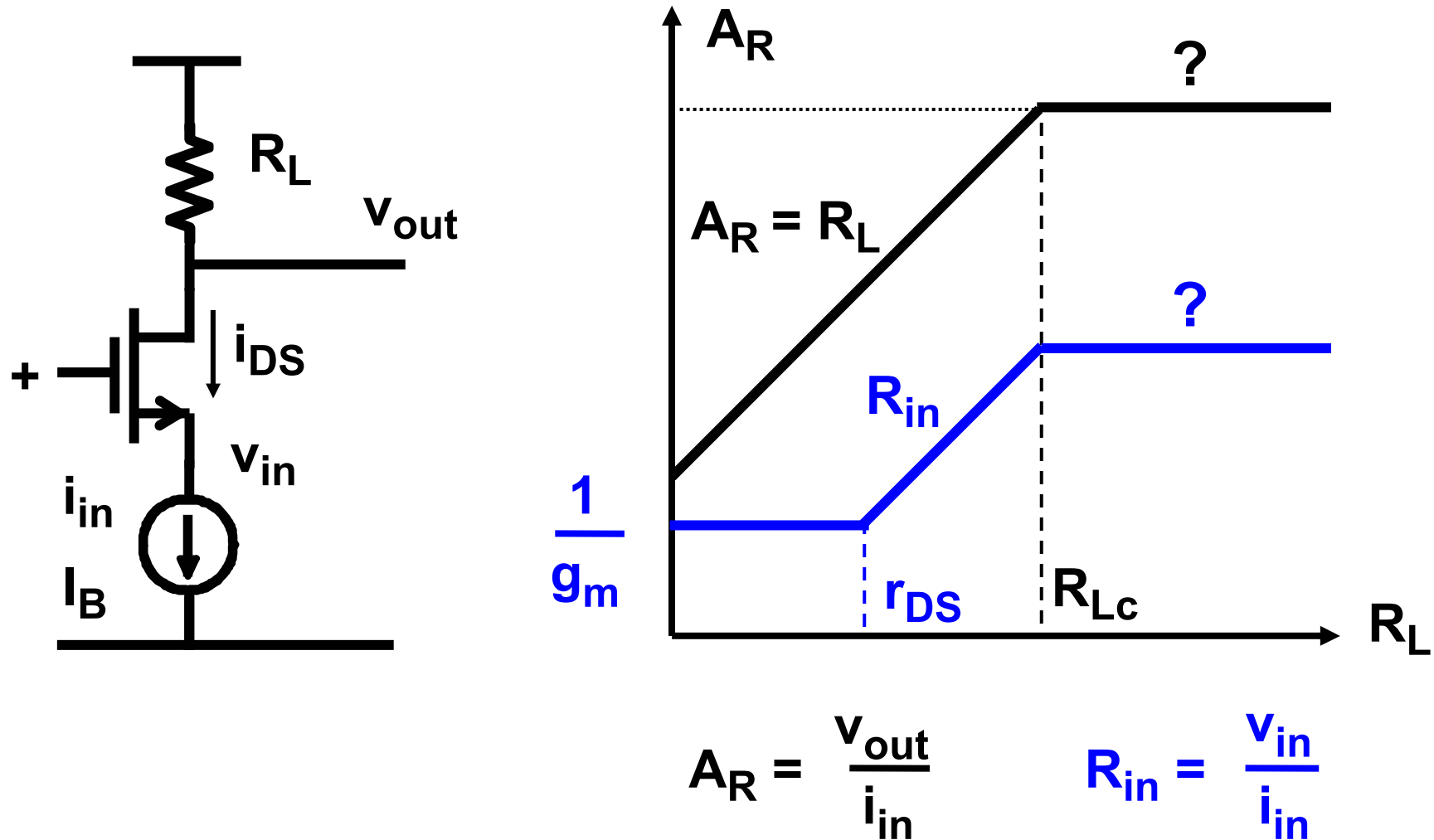
$$i_{out} = i_{in}$$

$$R_{in} \approx 1/g_m$$

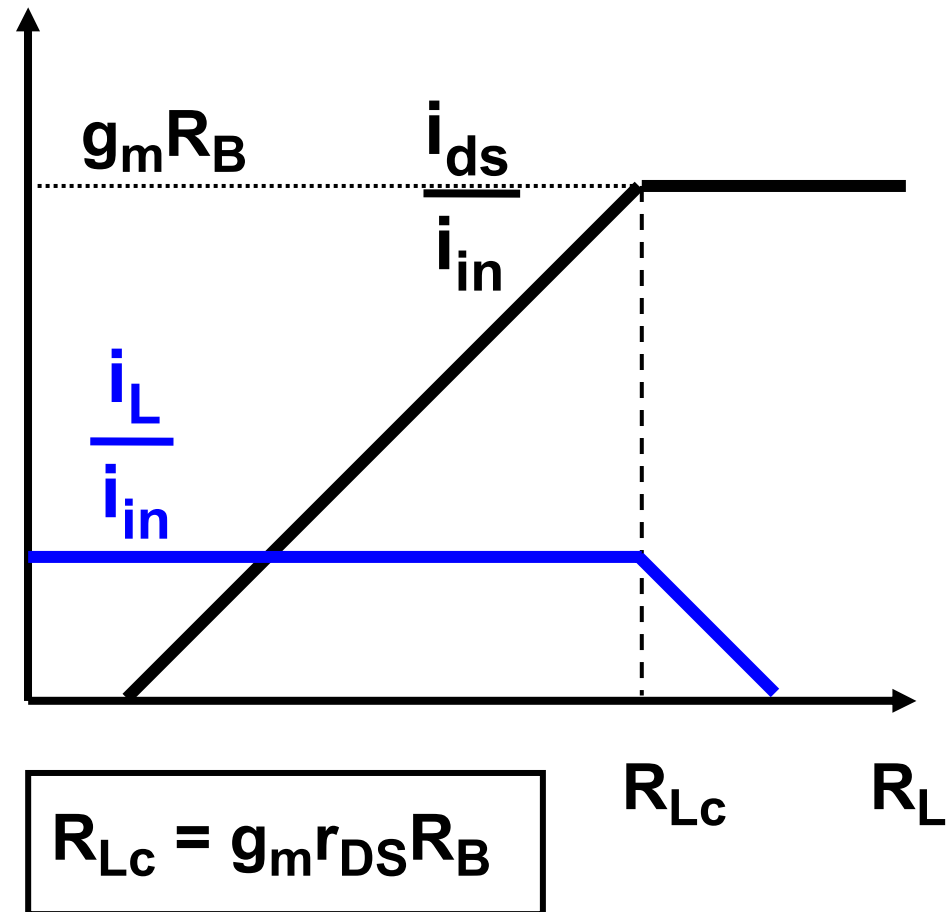
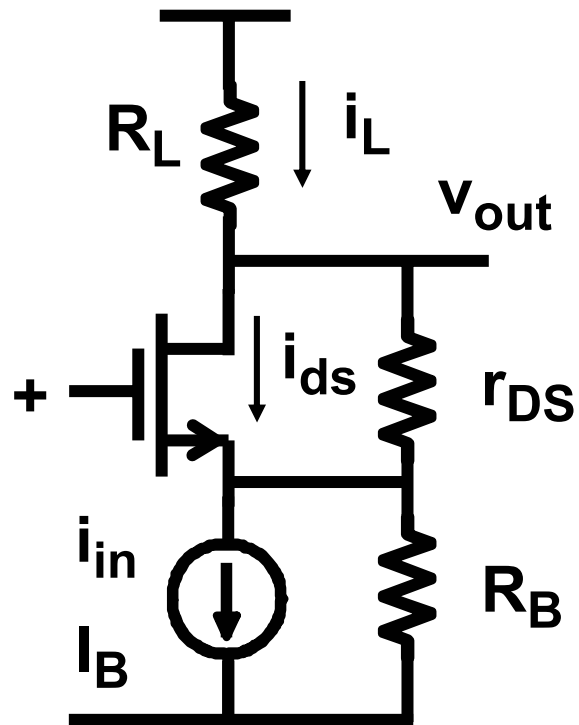
Cascode

Current buffer

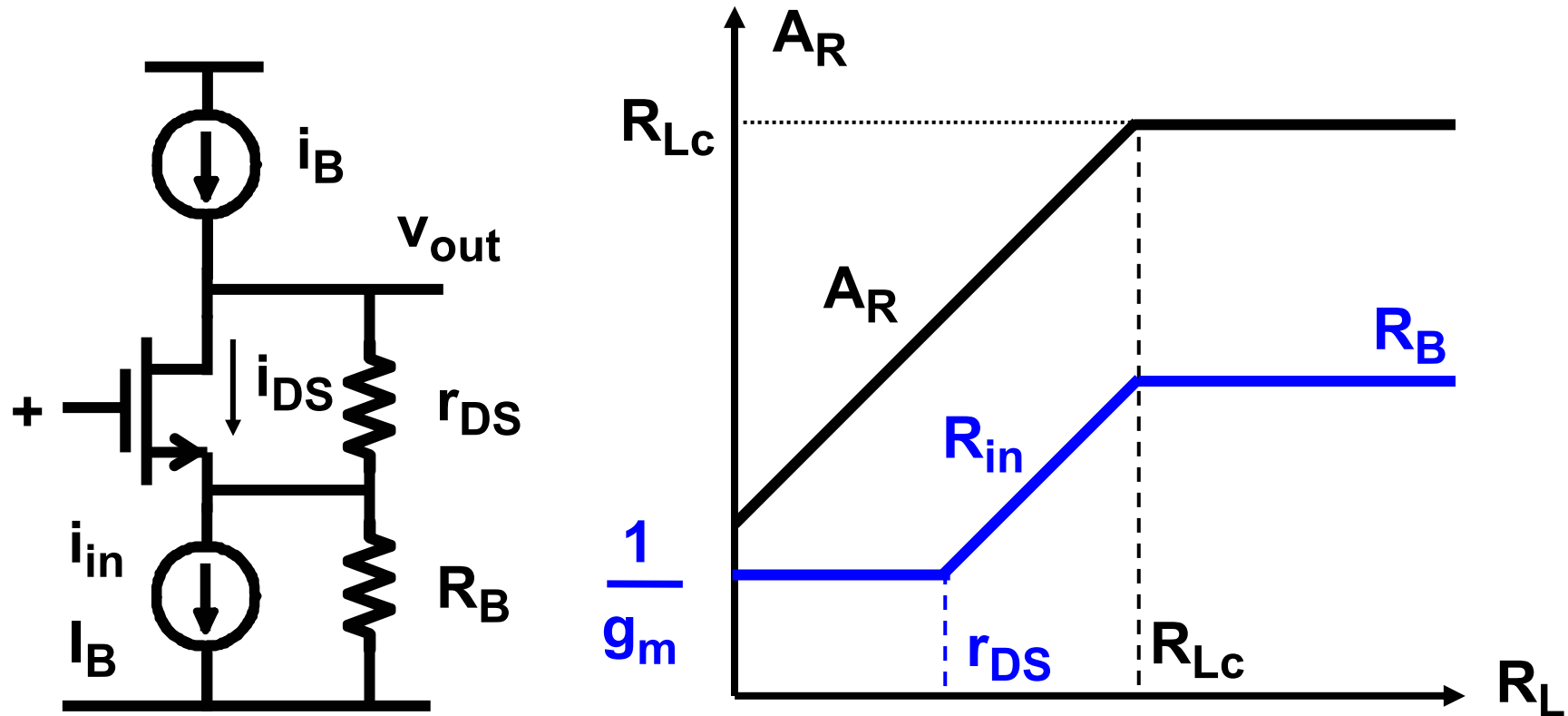
Cascode with resistive load



Cascode with resistive load



Cascode with active load

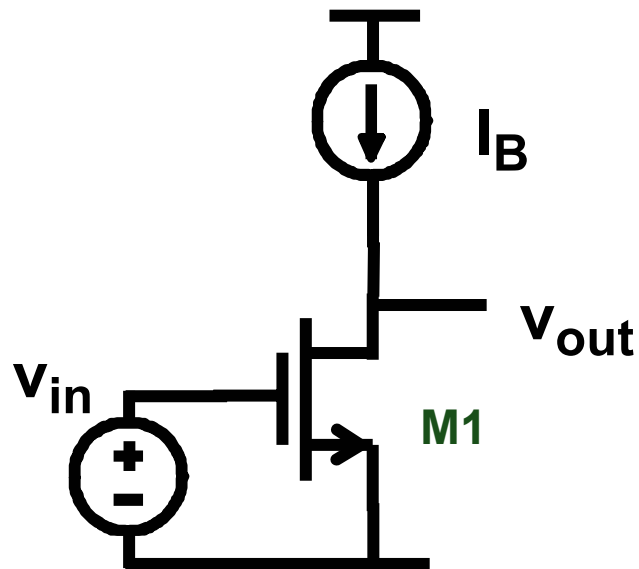


$$A_R = \frac{v_{out}}{i_{in}}$$

$$R_{in} = \frac{v_{in}}{i_{in}}$$

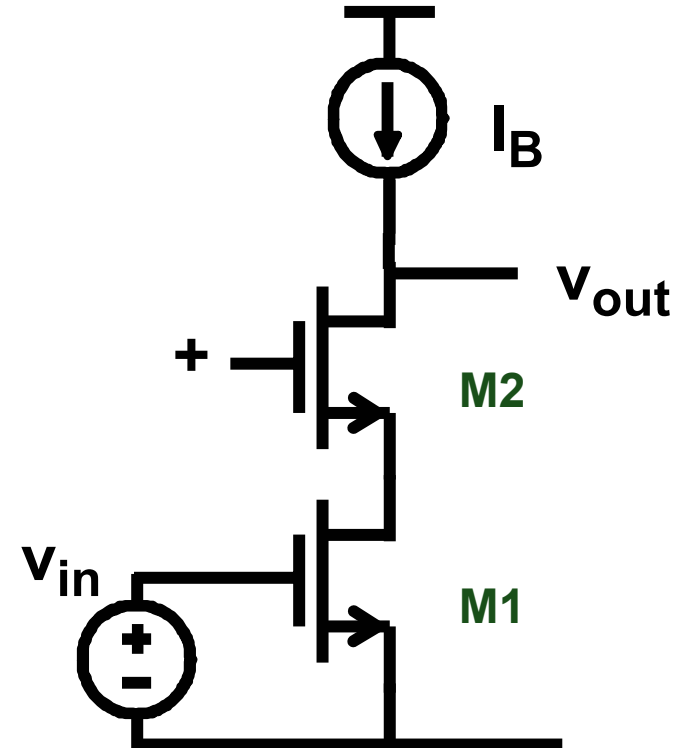
$$R_{Lc} = g_m r_{DS} R_B \approx 100 R_B$$

Cascode versus single-transistor



$$A_v = (g_m r_{DS})_1$$

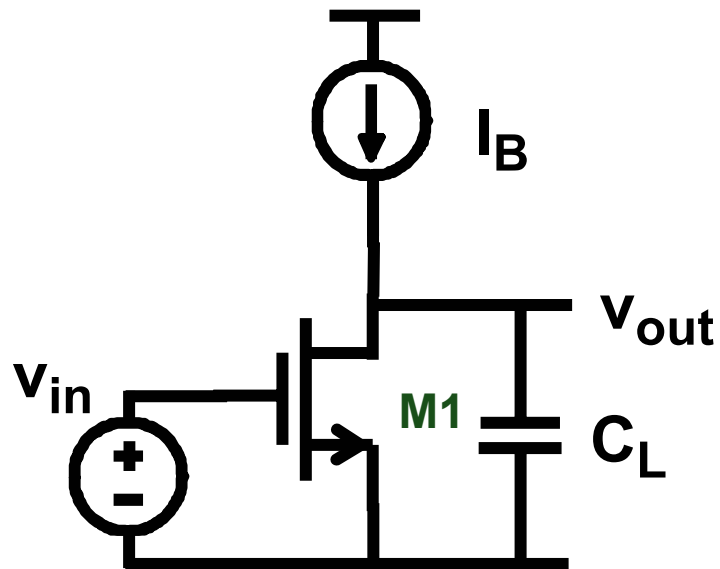
$$R_{out} = r_{DS1}$$



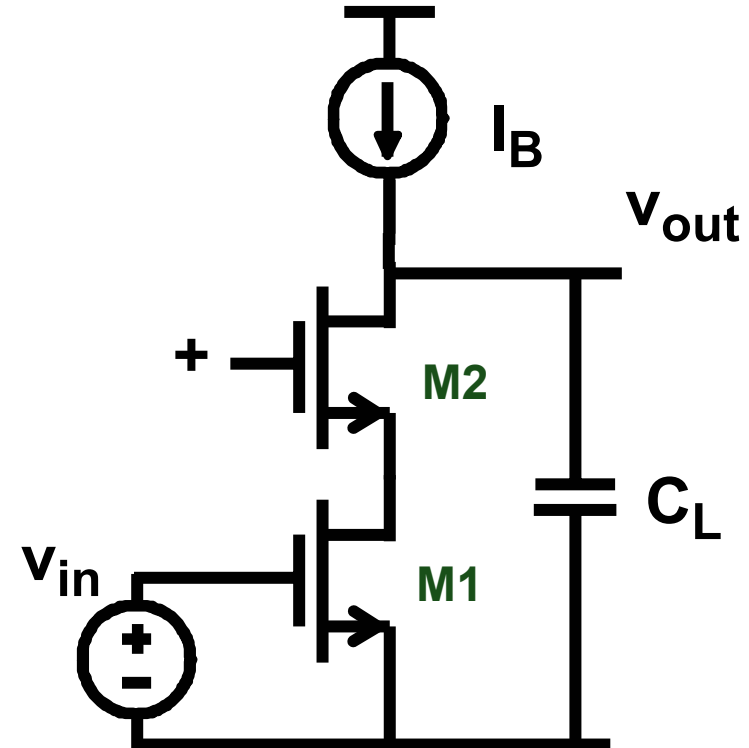
$$A_v = (g_m r_{DS})_1 (g_m r_{DS})_2$$

$$R_{out} = r_{DS1} (g_m r_{DS})_2$$

Cascode versus single-transistor

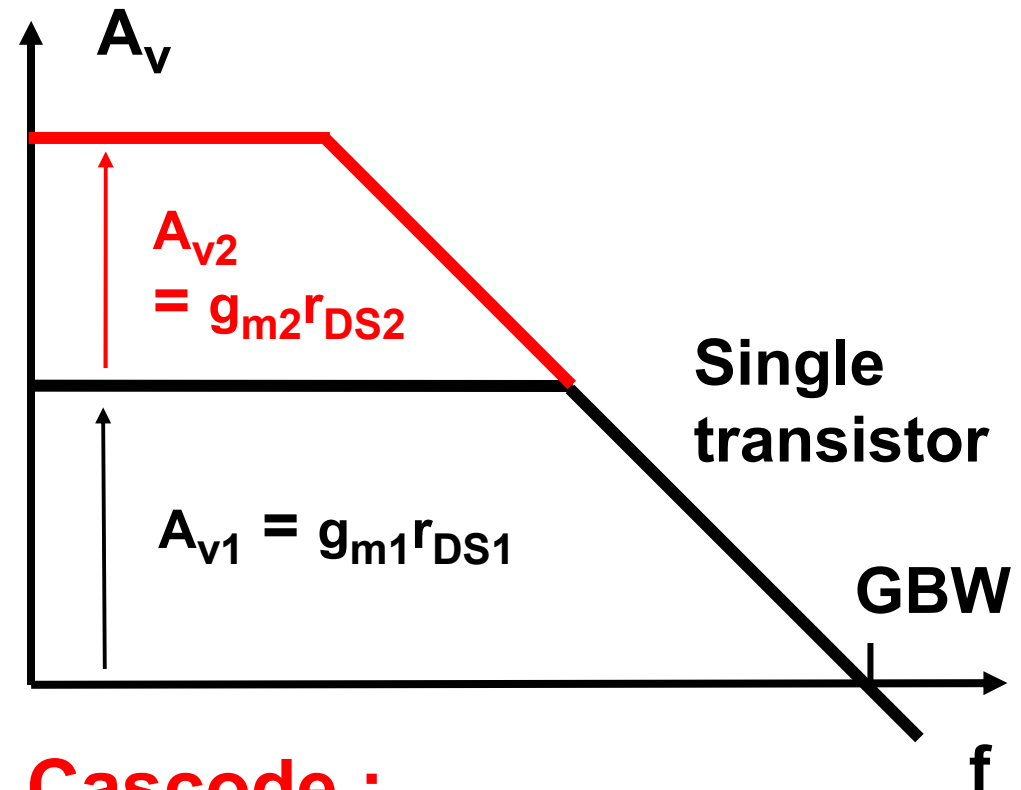
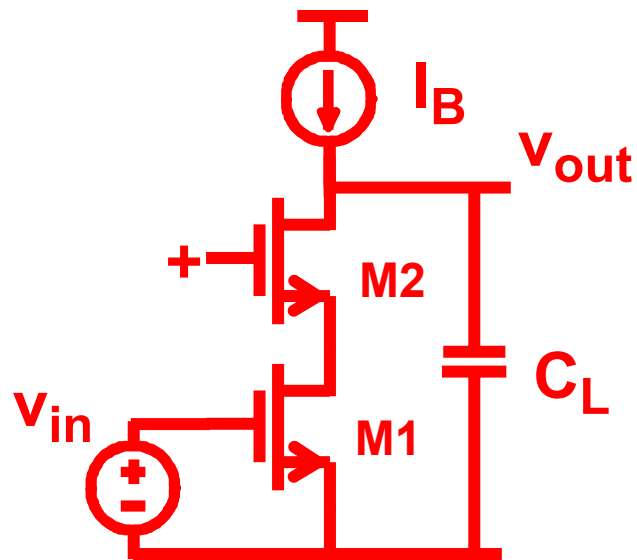
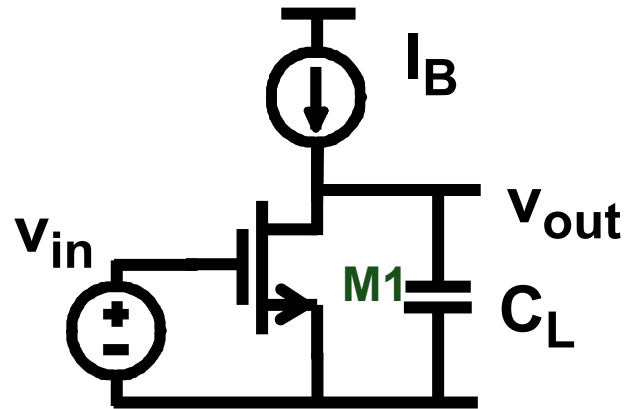


$$BW = \frac{1}{2\pi R_{out} C_L}$$



$$GBW = \frac{g_{m1}}{2\pi C_L} \quad \text{for both !}$$

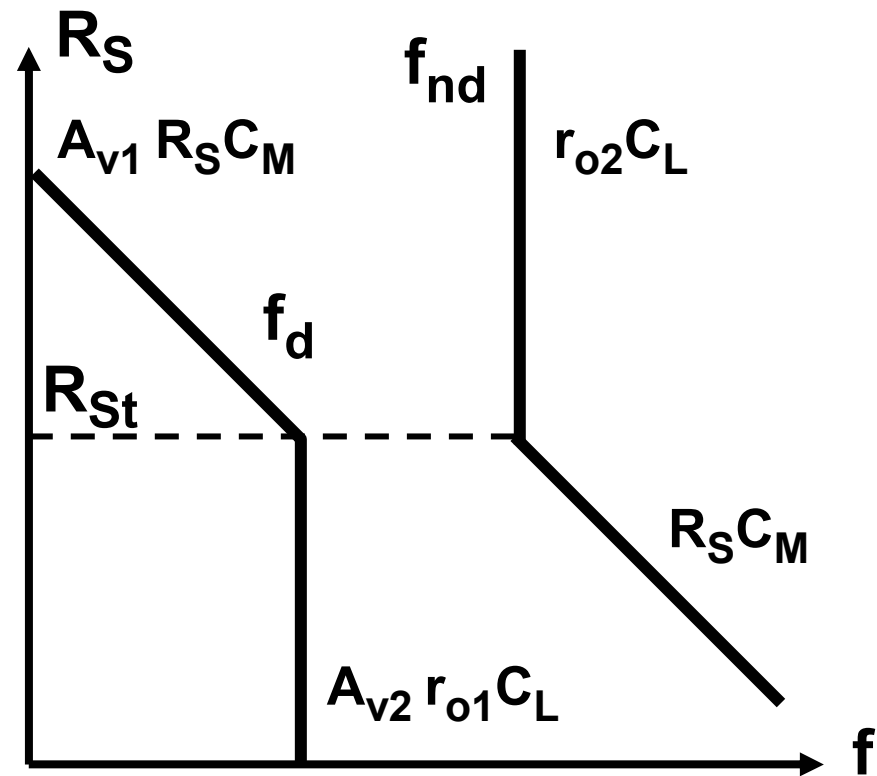
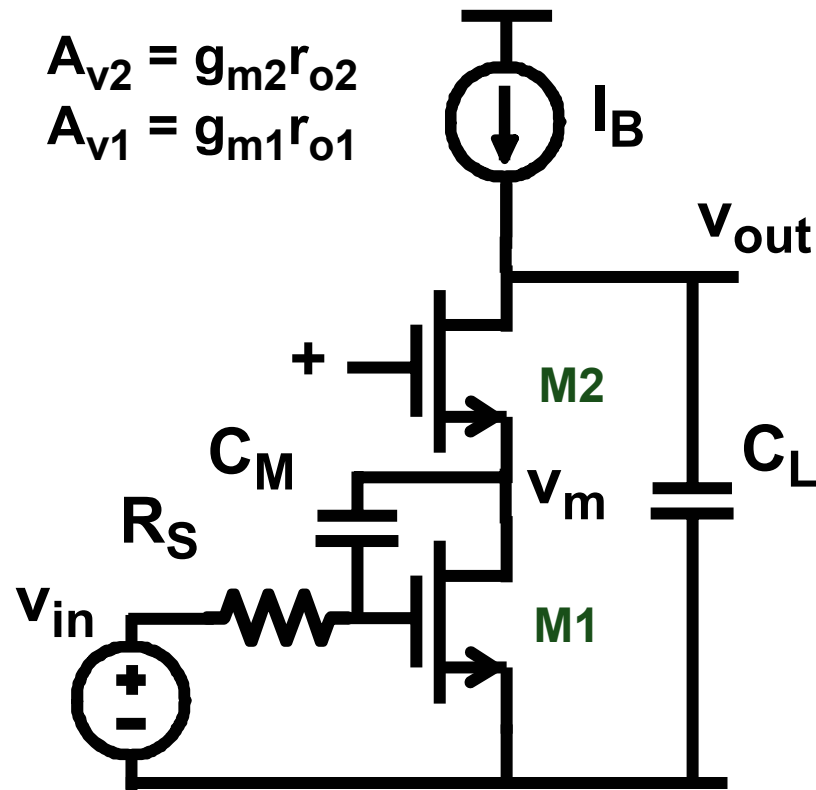
Cascode versus single-transistor



Cascode :
High gain
At low freq.

$$GBW = \frac{g_{m1}}{2\pi C_L}$$

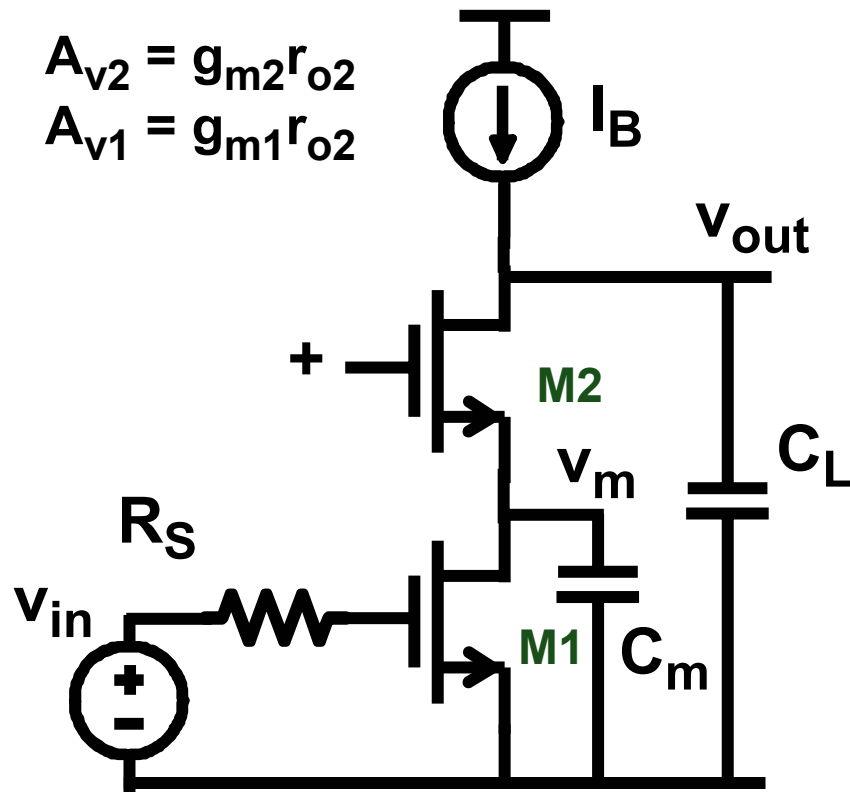
Miller effect in cascode ?



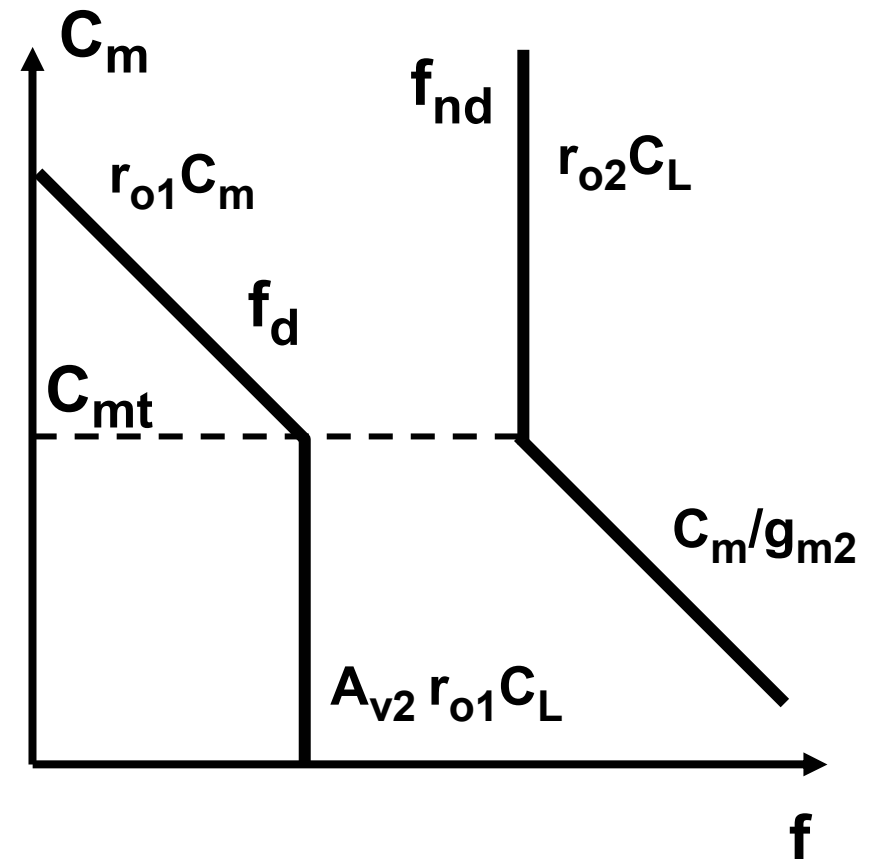
$$GBW = \frac{g_{m1}}{2\pi C_L}$$

No Miller if $R_S < R_{St} = r_{o2} \frac{C_L}{C_M} \frac{g_{m2}}{g_{m1}}$

Cascode with capacitance C_m at middle point

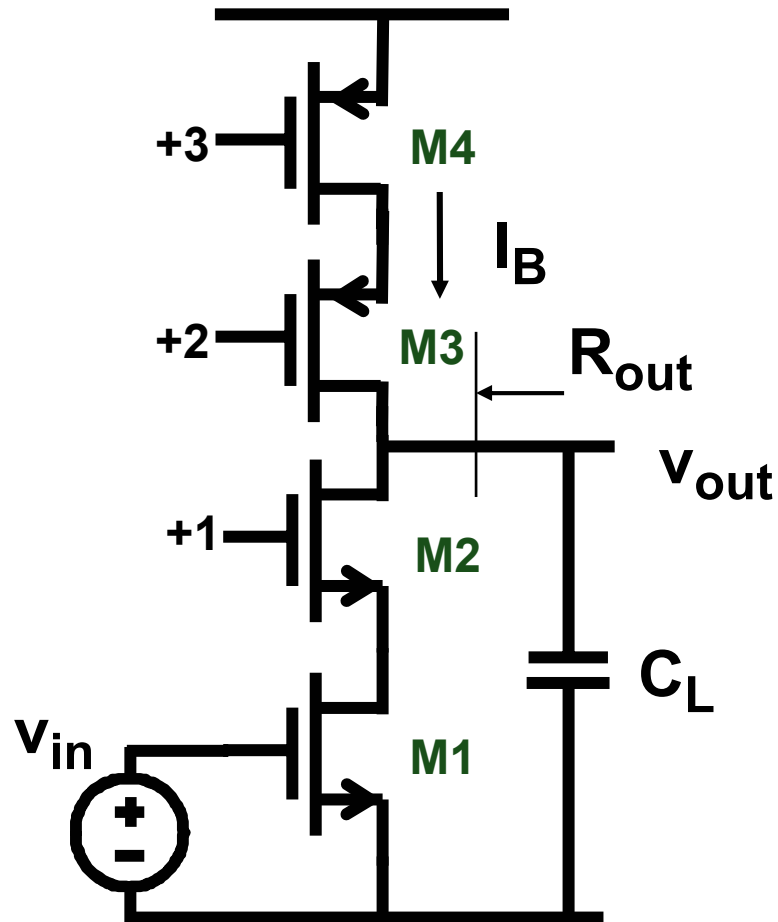


$$GBW = \frac{g_{m1}}{2\pi C_L}$$



$$C_{mt} = g_{m2}r_{o2} C_L = A_{v2} C_L$$

Telescopic Cascode



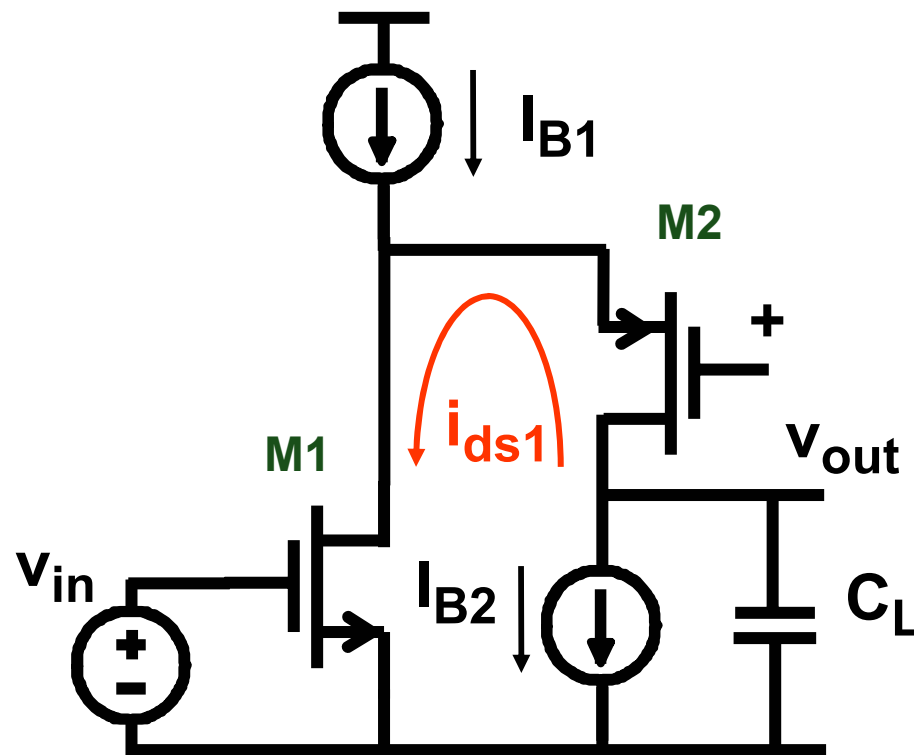
$$A_v = g_{m1} R_{out}$$

$$R_{out} = \frac{1}{2} r_{DS1} g_{m2} r_{DS2}$$

$$BW = \frac{1}{2\pi R_{out} C_L}$$

$$GBW = \frac{g_{m1}}{2\pi C_L}$$

Folded Cascode



$$I_{DS1} = I_{B1} - I_{B2} \approx I_{B1} / 2$$

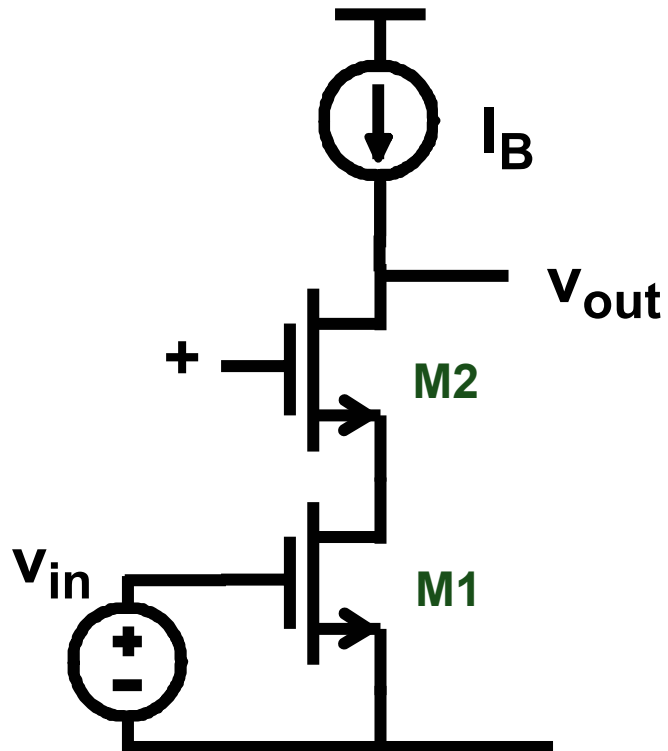
$$A_v = g_{m1} R_{out}$$

$$R_{out} = r_{DS1} g_{m2} r_{DS2}$$

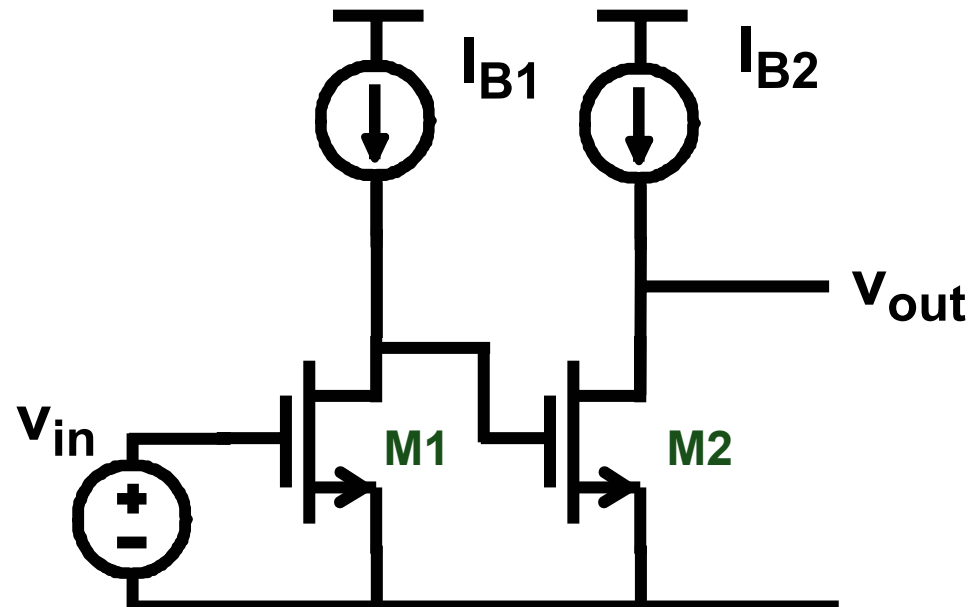
$$BW = \frac{1}{2\pi R_{out} C_L}$$

$$GBW = \frac{g_{m1}}{2\pi C_L}$$

Cascode versus cascade

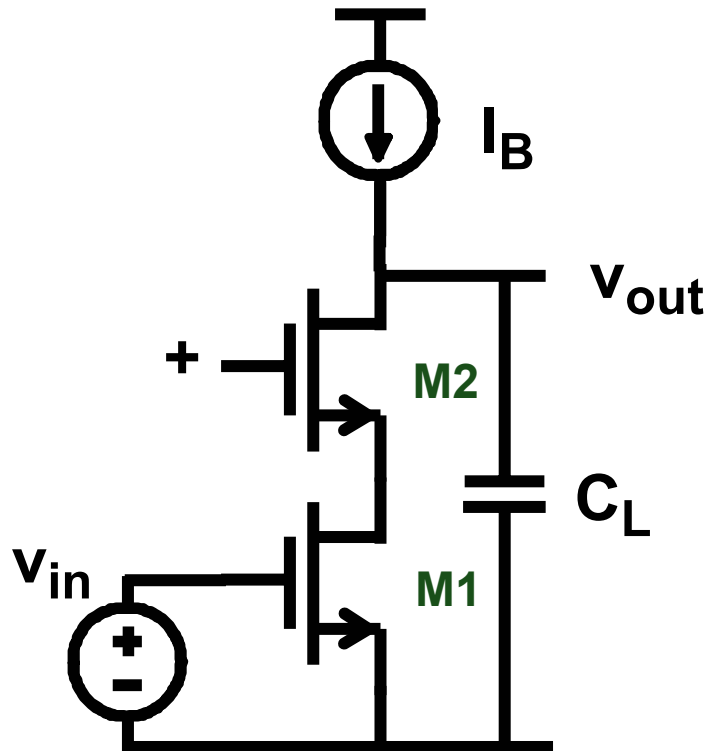


$$A_v = (g_m r_{DS})_1 (g_m r_{DS})_2$$



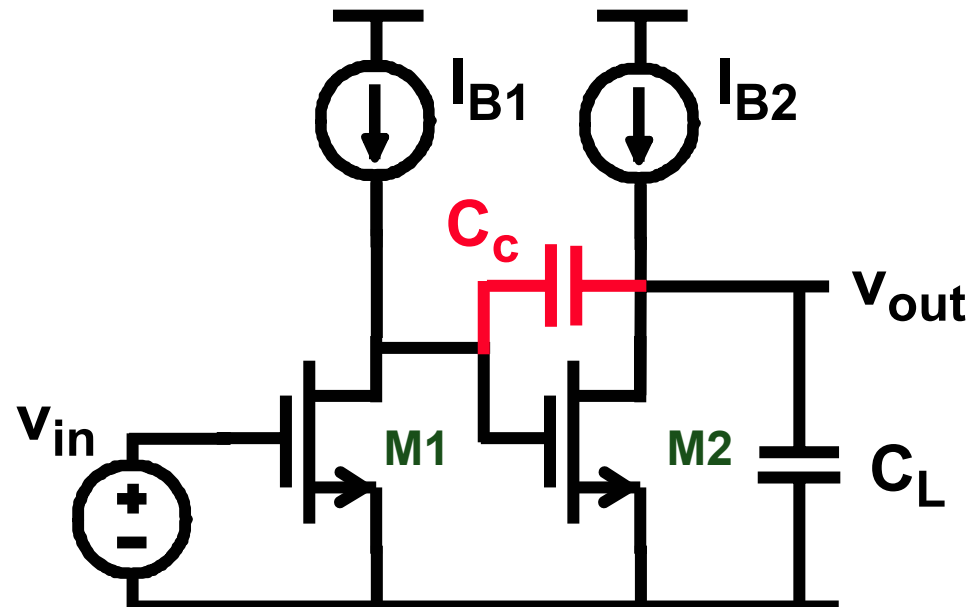
$$A_v = (g_m r_{DS})_1 (g_m r_{DS})_2$$

Cascode versus cascade



$$GBW = \frac{g_{m1}}{2\pi C_L}$$

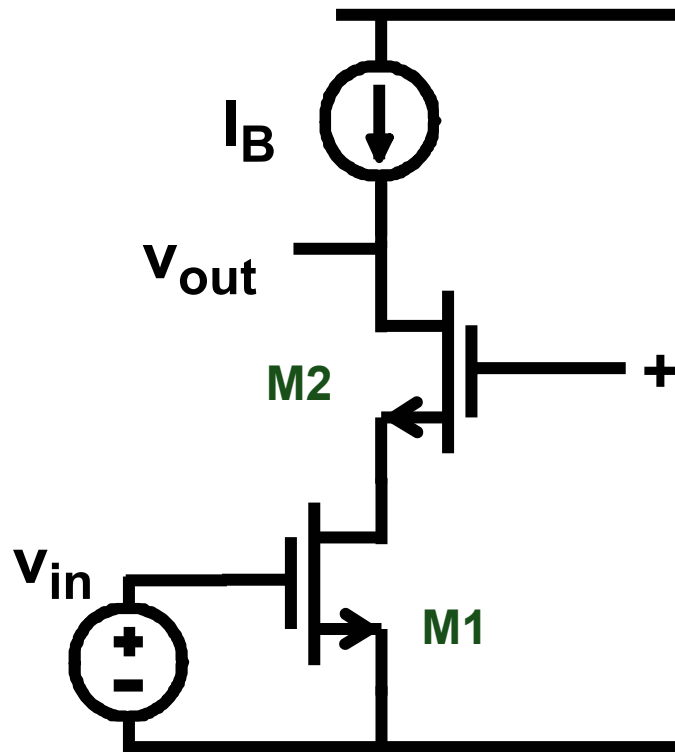
Two-stage Miller amplifier



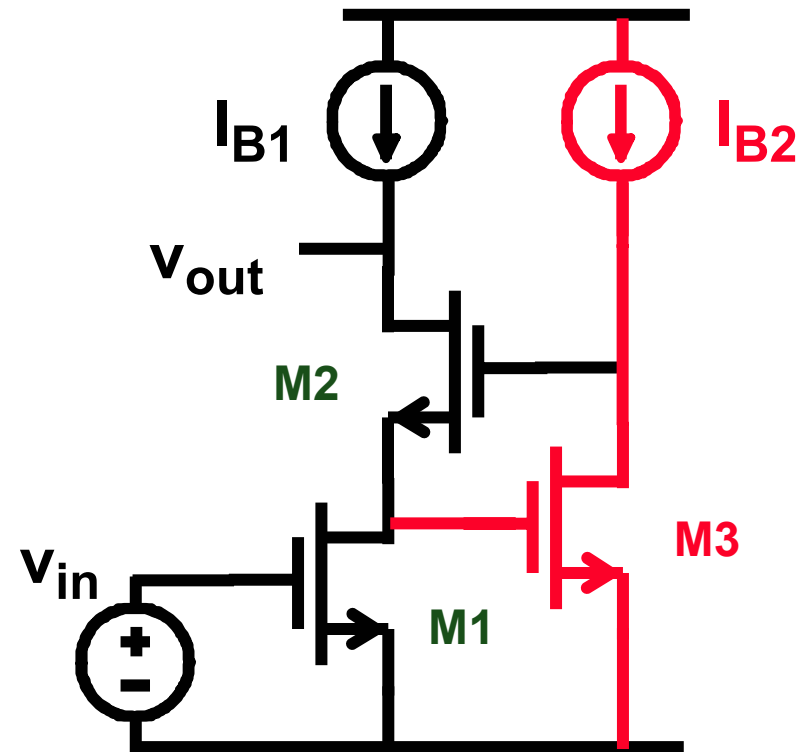
$$GBW = \frac{g_{m1}}{2\pi C_c}$$



Regulated cascode or gain boosting



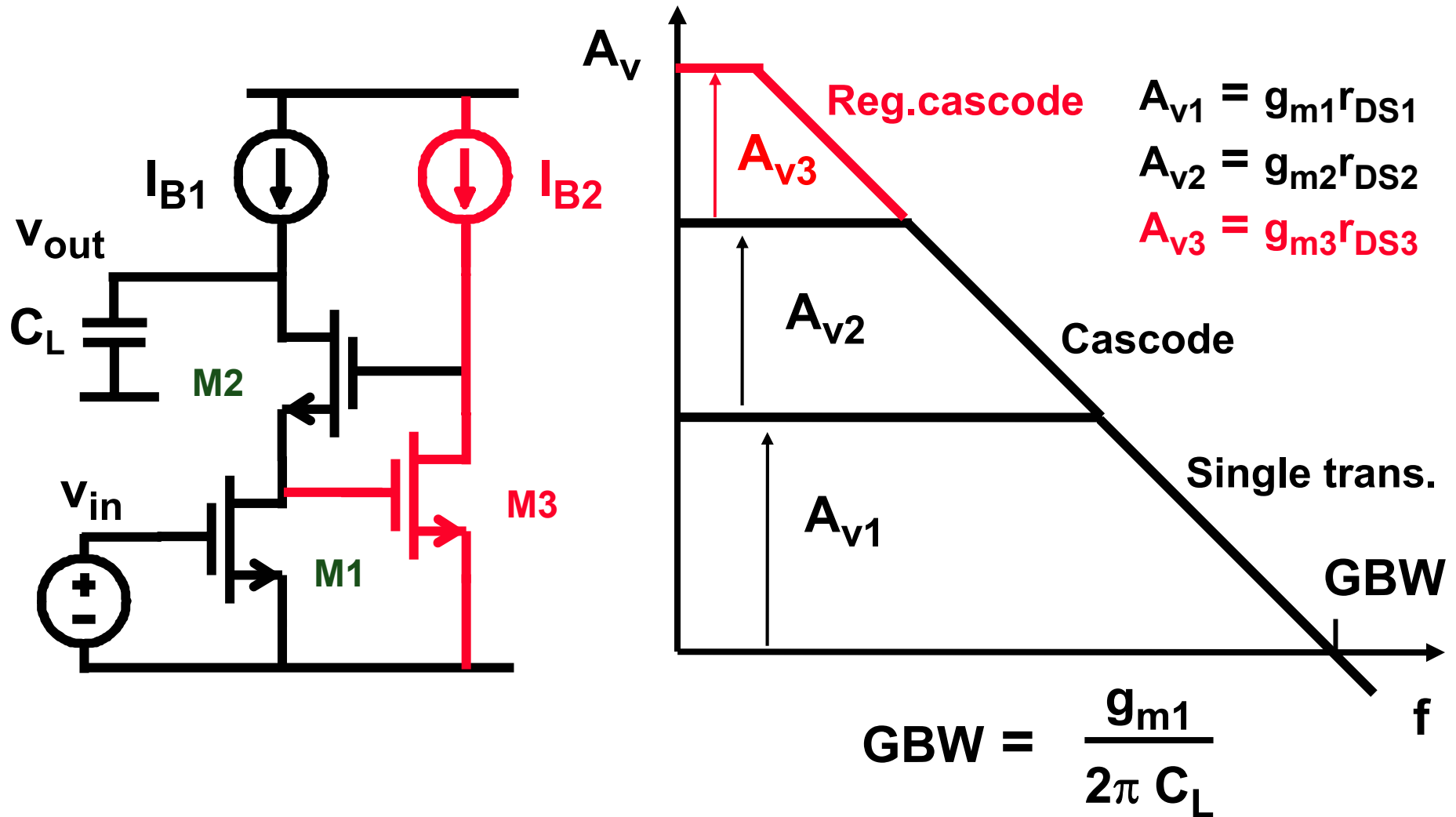
$$A_v = (g_m r_{DS})_1 (g_m r_{DS})_2$$



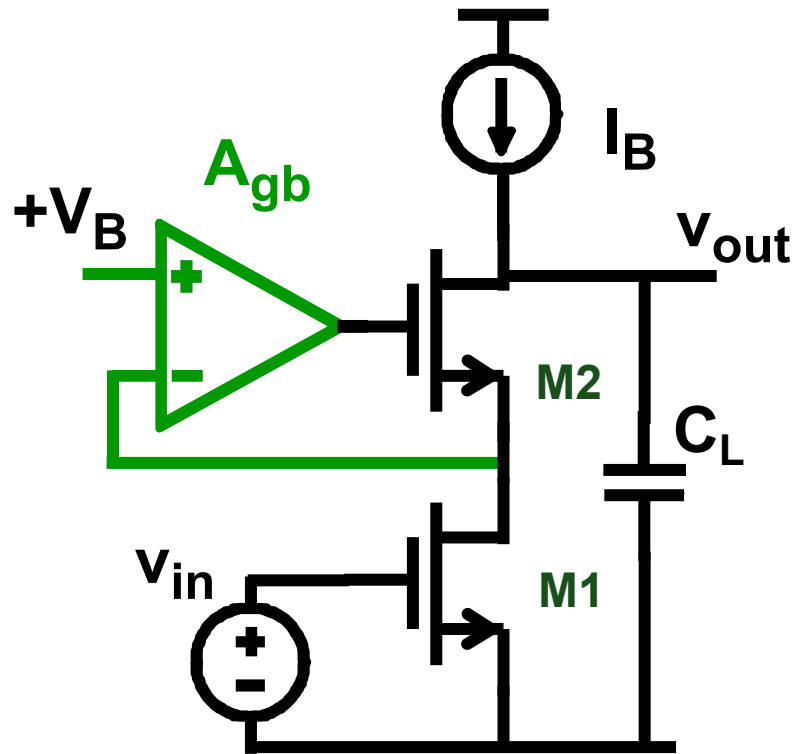
$$A_v = (g_m r_{DS})_1 (g_m r_{DS})_2 (g_m r_{DS})_3$$

Hosticka, JSSC Dec.79, pp. 1111-1114; Sackinger, JSSC Febr.90, pp. 289-298;
Bult JSSC Dec.90, pp. 1379-1384

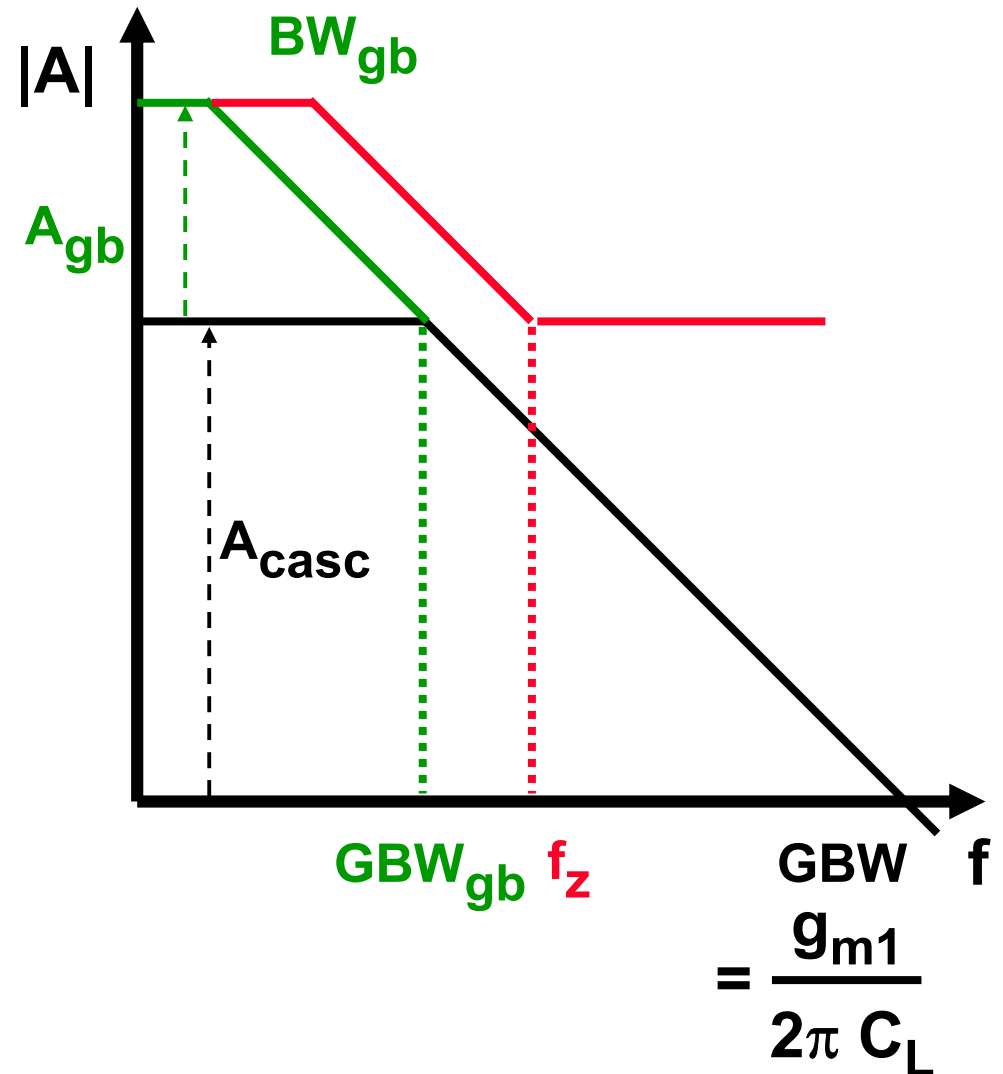
Regulated cascode, Cascode & single-transistor



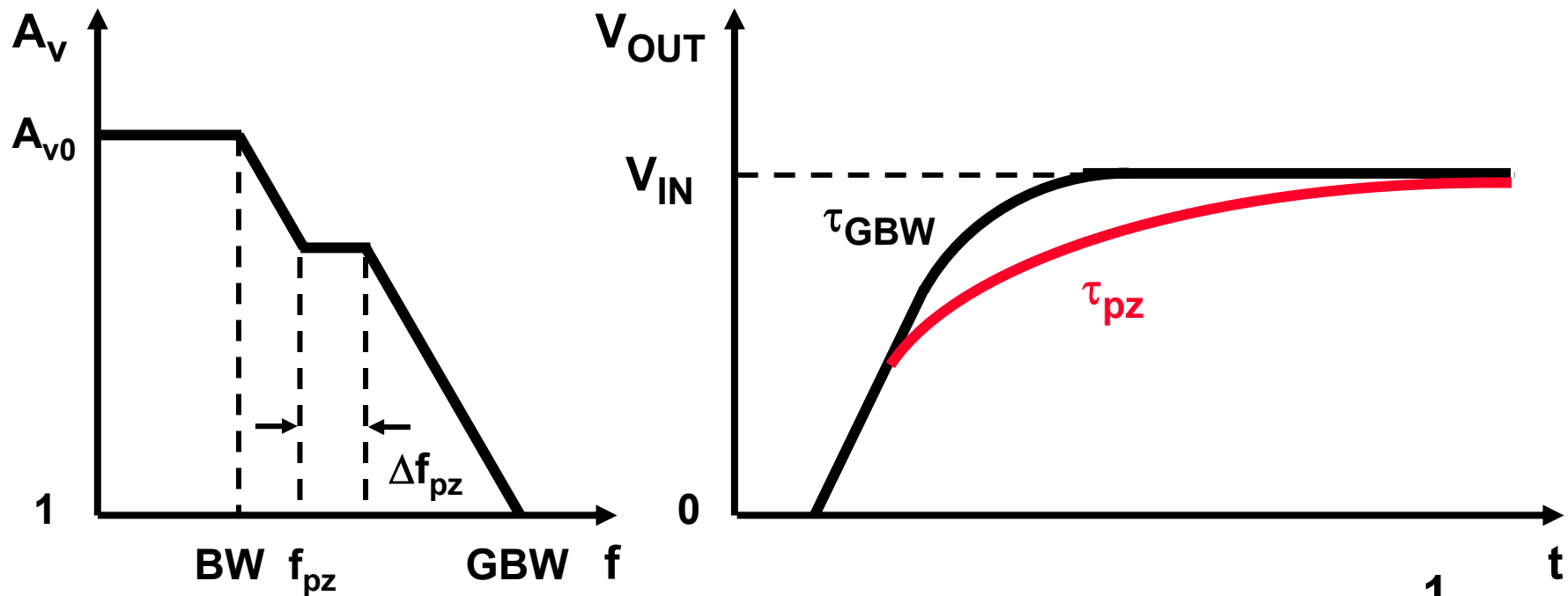
Gain boosting



$$A_v = A_{gb}(g_m r_{DS})_1 (g_m r_{DS})_2$$



Pole-zero doublet and settling time



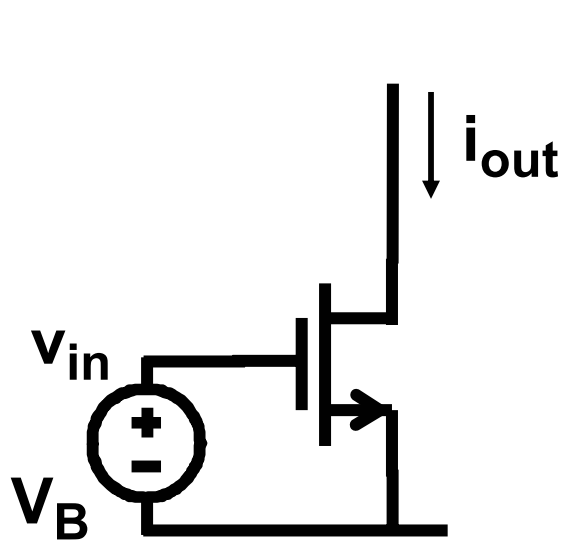
$$V_{OUT} = V_{IN} \left[1 - \exp \left(- \frac{t}{\tau_{GBW}} \right) - \frac{\Delta f_{pz}}{GBW} \exp \left(- \frac{t}{\tau_{pz}} \right) \right]$$

$$f_{pz} = \frac{1}{2\pi \tau_{pz}}$$

$$GBW = \frac{1}{2\pi \tau_{GBW}}$$

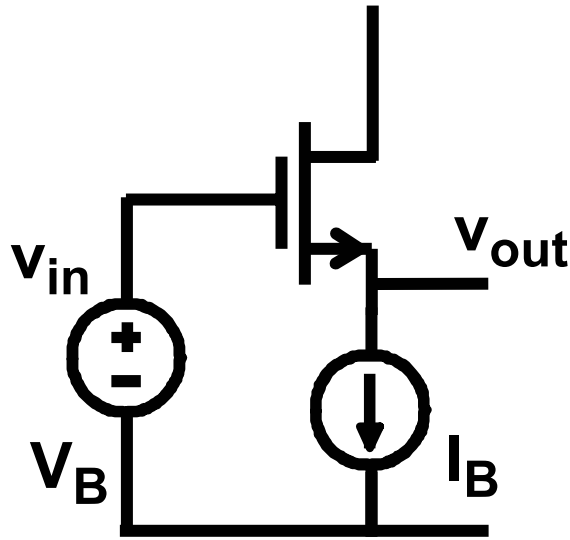
Kamath, etal, JSSC Dec.74, pp. 347-352

Single-transistor stages



$$i_{out} = g_m v_{in}$$

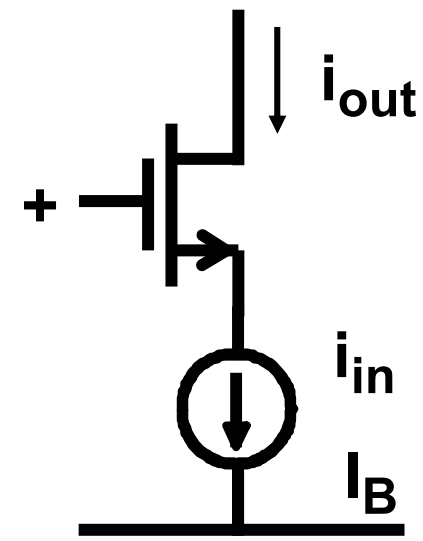
Amplifier



$$v_{out} = v_{in}$$

$$Z_{out} \approx 1/g_m$$

Source follower

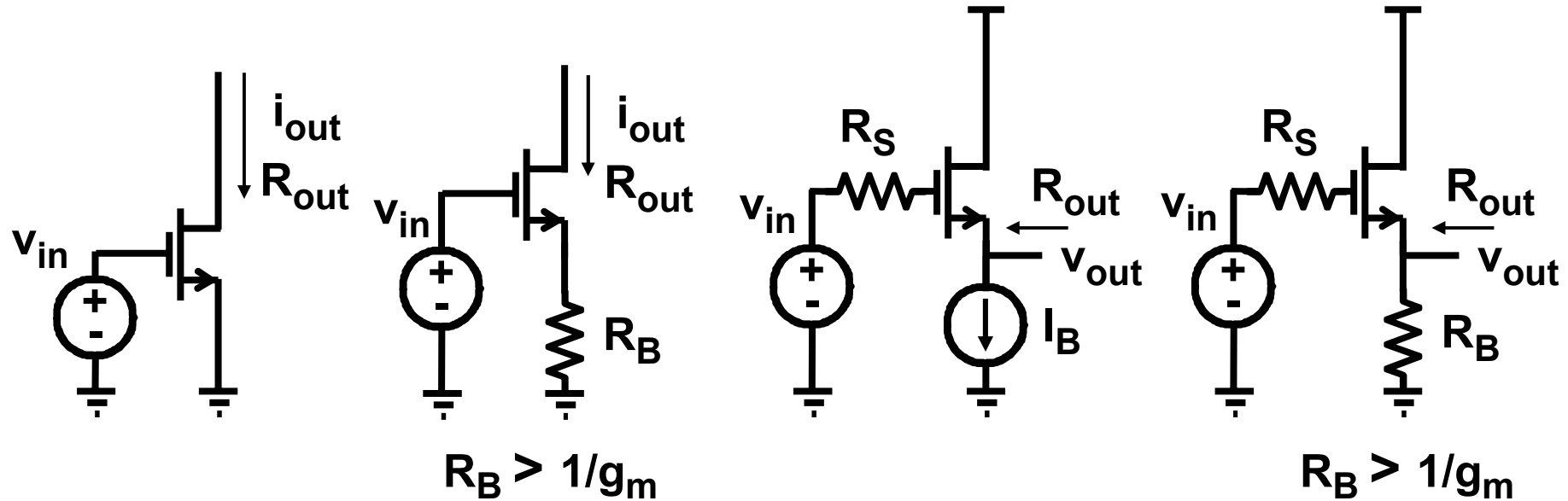


$$i_{out} = i_{in}$$

$$Z_{in} \approx 1/g_m$$

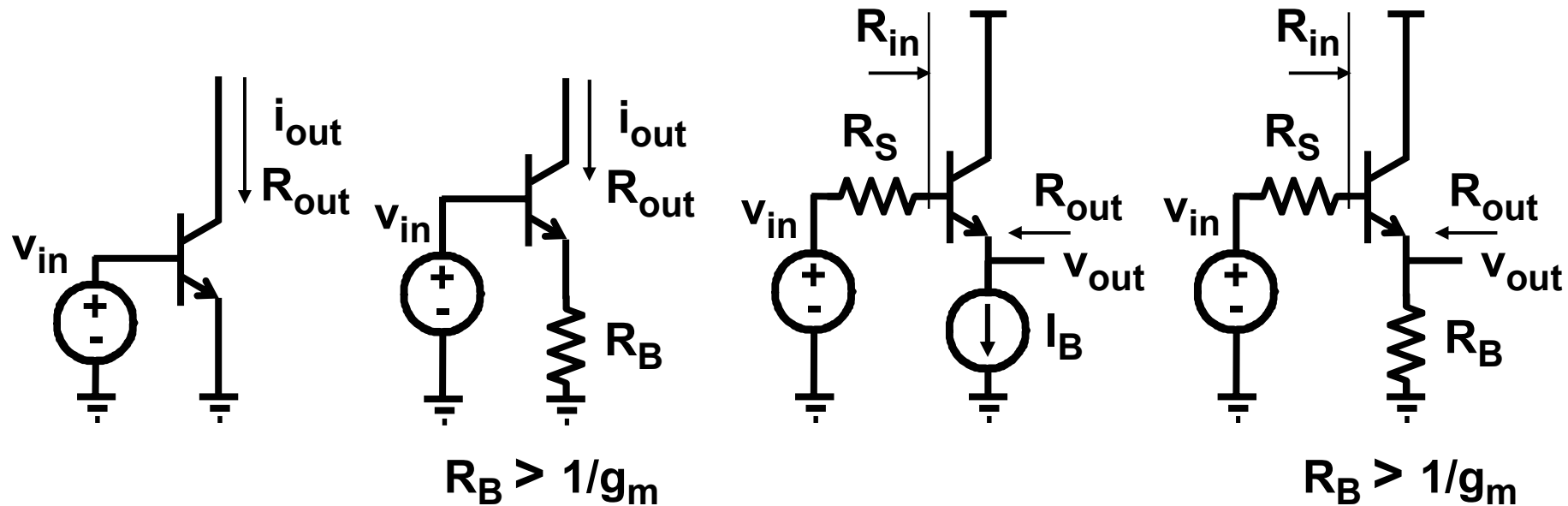
Cascode

MOST amplifier & follower



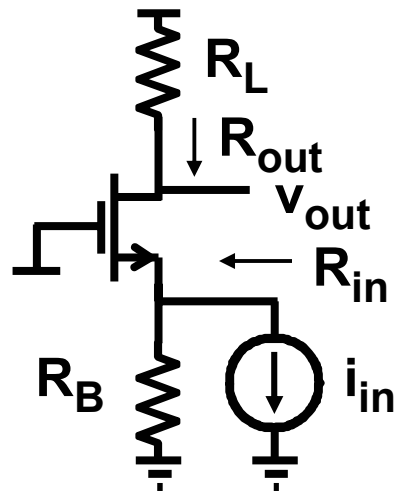
A_G	g_m	$1/R_B$	A_V	1	1
R_{in}	∞	∞		∞	∞
R_{out}	r_o	$g_m R_B r_o$		$1/g_m$	$1/g_m$

Bipolar transistor ($\beta \gg 1$)



A_G	g_m	$1/R_B$	A_V	1	1
R_{in}	$r_B + r_\pi$	$r_B + r_\pi + \beta R_B$		$r_B + r_\pi + \beta r_o$	$r_B + r_\pi + \beta R_B$
R_{out}	r_o	$g_m R_B r_o$		$1/g_m + R_S/\beta$	$1/g_m + R_S/\beta$

In- & output resistances MOST cascode

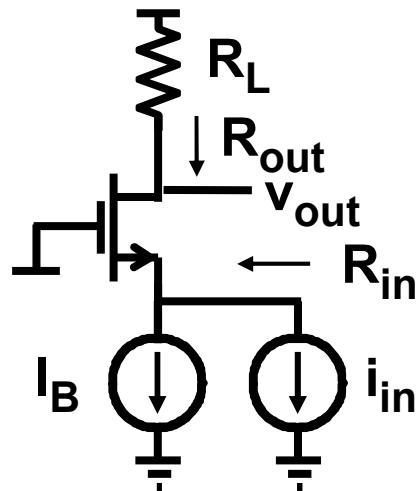


$$R_B > 1/g_m$$

$$A_R \quad R_L$$

$$R_{in} \quad 1/g_m$$

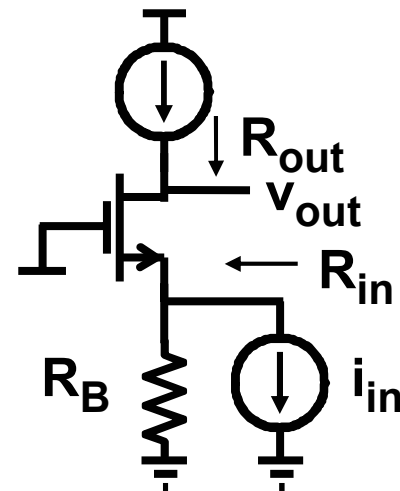
$$R_{out} \quad g_m r_o R_B$$



$$R_L$$

$$1/g_m$$

$$\infty$$

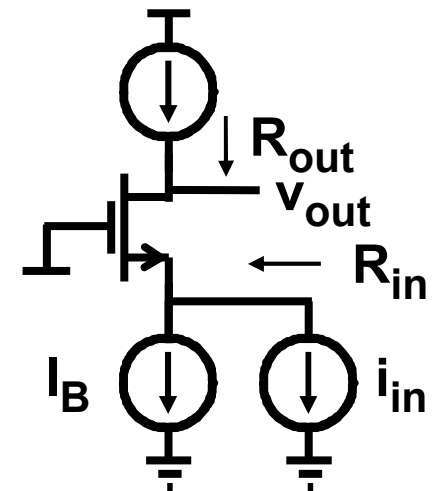


$$R_B > 1/g_m$$

$$g_m r_o R_B$$

$$R_B$$

$$g_m r_o R_B$$

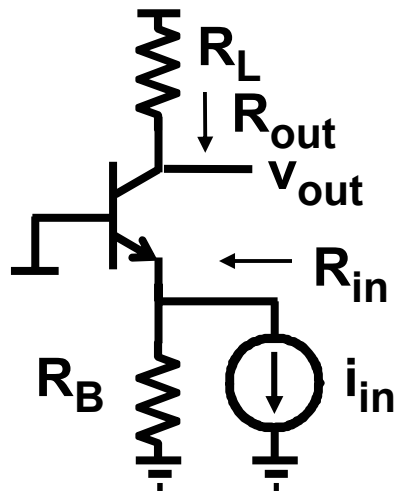


$$-$$

$$\infty$$

$$\infty$$

In- & output resistances Bipolar trans. cascode

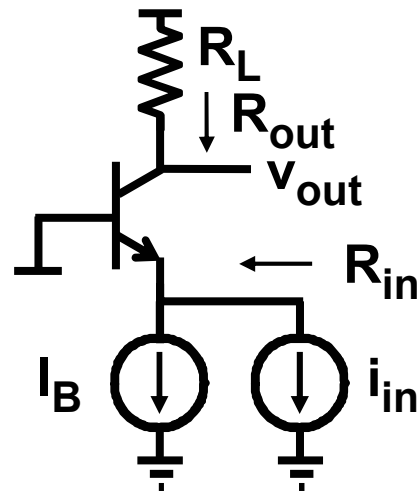


$$R_B > 1/g_m$$

$$A_R \quad R_L$$

$$R_{in} \quad 1/g_m$$

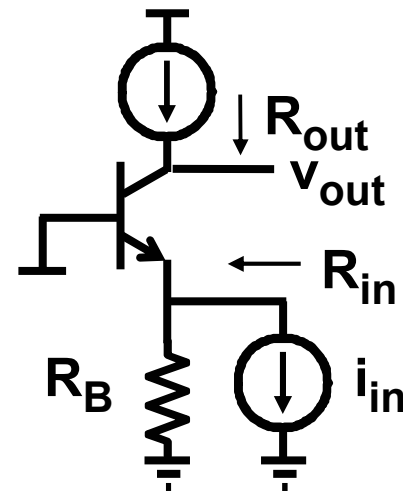
$$R_{out} \quad g_m r_o R_B$$



$$R_L$$

$$1/g_m$$

$$\approx \beta r_o$$

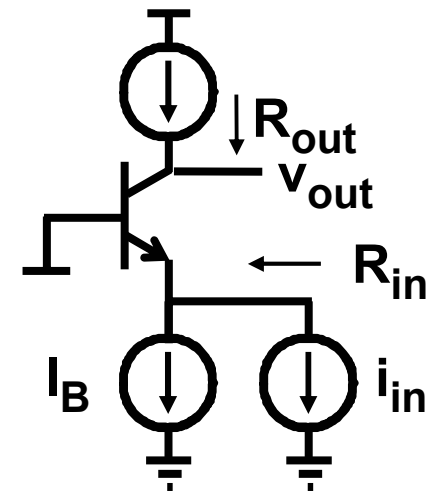


$$R_B > 1/g_m$$

$$g_m r_o R_B$$

$$R_B // (r_B + r_\pi)$$

$$g_m r_o (R_B // (r_B + r_\pi))$$

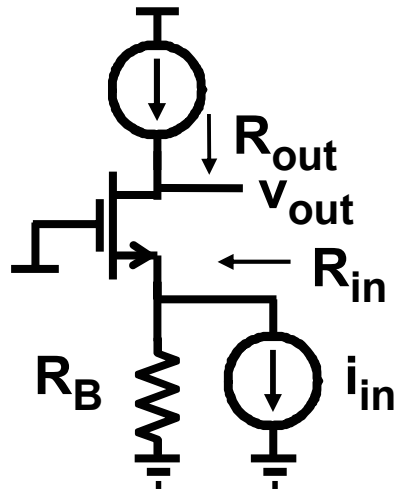


$$-$$

$$r_B + r_\pi$$

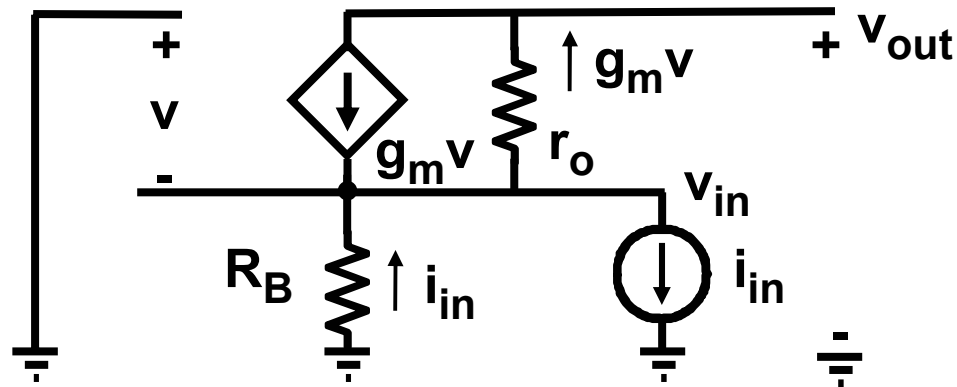
$$\approx \beta r_o$$

Calculation of AR for a MOST cascode



$$R_B > 1/g_m$$

$$A_R = g_m r_o R_B$$



yields
and

$$v = -v_{in}$$

$$v_{out} = v_{in} - g_m v r_o$$

$$v_{in} = -R_B i_{in}$$

$$v_{out} = -R_B i_{in} (1 + g_m r_o)$$

$$g_m r_o \gg 1$$

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- ☐ **Source followers**
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