Noise performance of elementary transistor stages

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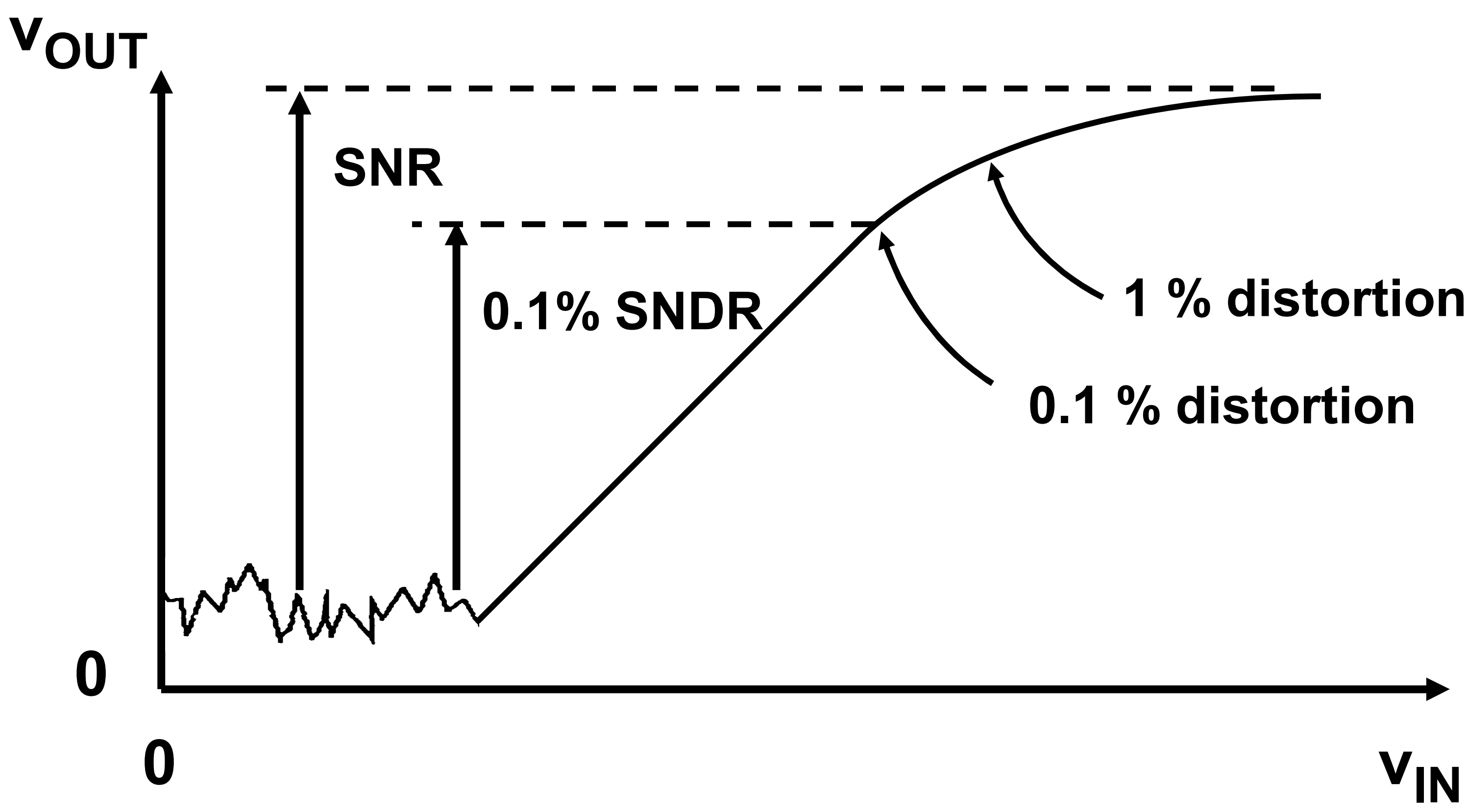
SNR and SNDR



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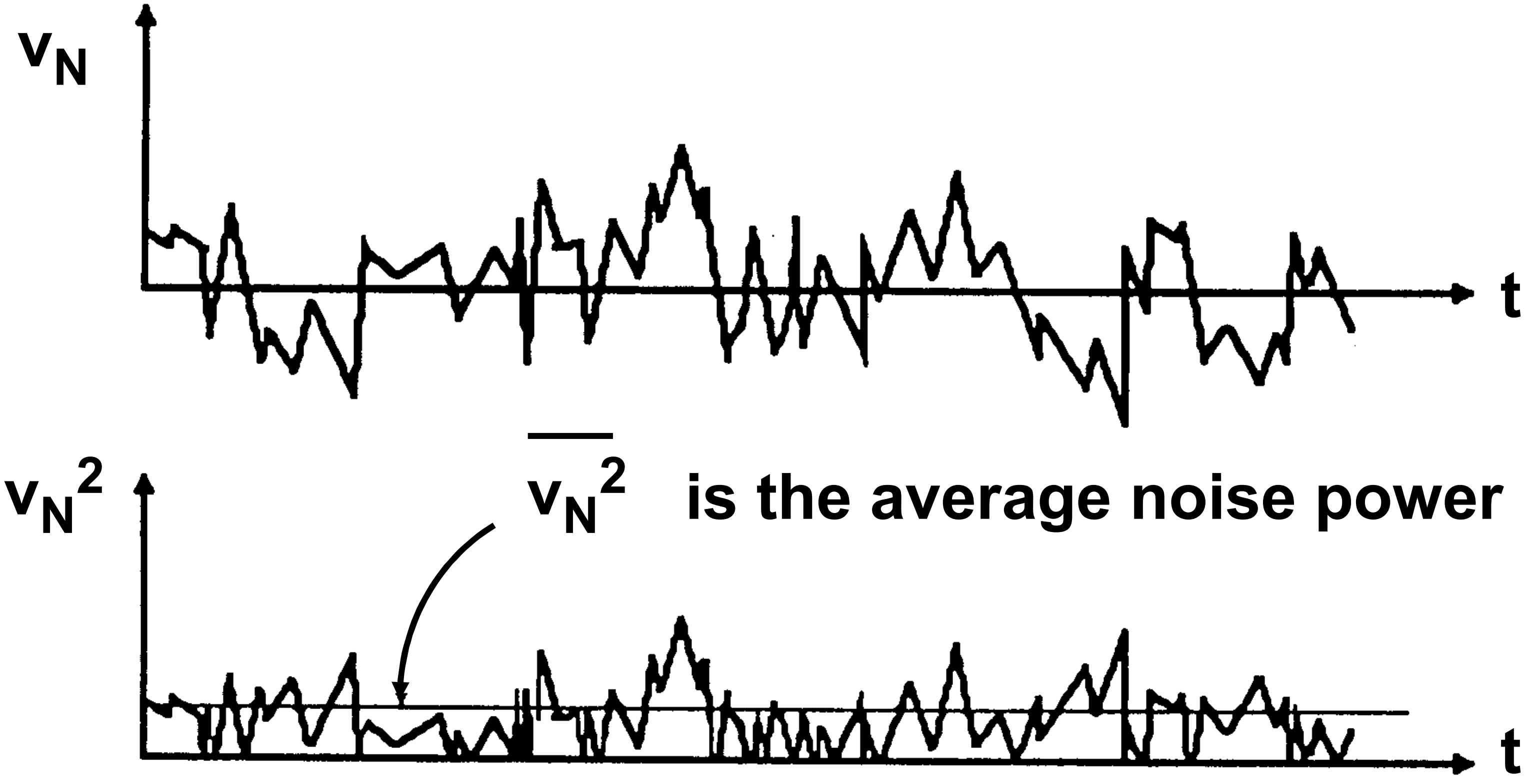
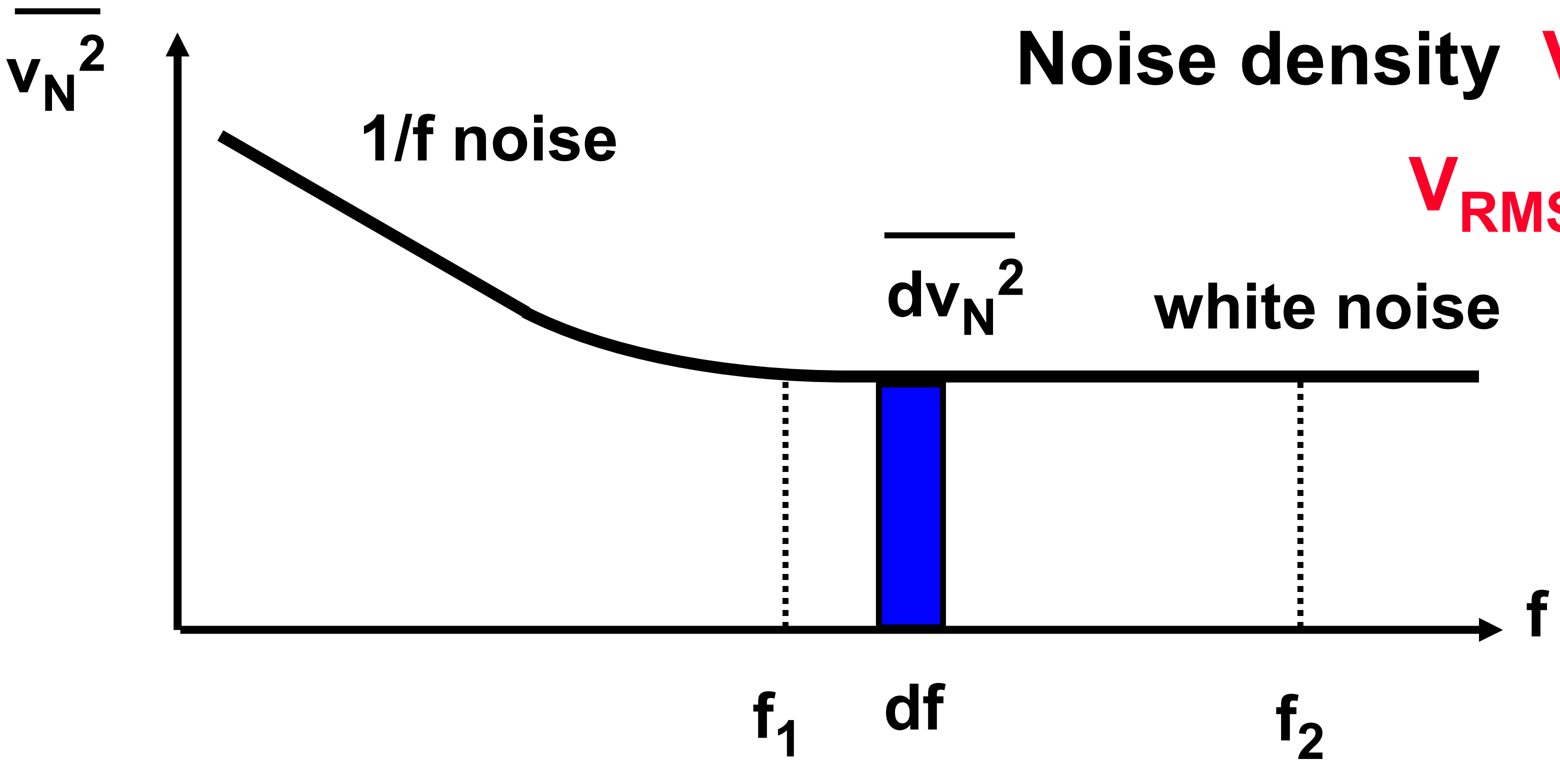
Noise versus time



$\overline{v_N^2}$ is the average noise power

Ref. Van der Ziel (Prentice Hall 1954, Wiley 1986), Ott (Wiley 1988)

Noise versus frequency



Noise density V^2/Hz

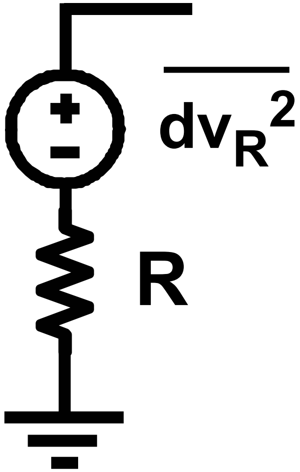
$V_{RMS}/\sqrt{Hz}$

Integrated noise

V_{RMS}

$$\overline{v_{12}} = \sqrt{\overline{v_N^2}} = \sqrt{\int_{f_1}^{f_2} \overline{dv_N^2}\, df} = \sqrt{(f_2 - f_1)\, \overline{dv_N^2}}$$

Noise of a resistor is thermal noise

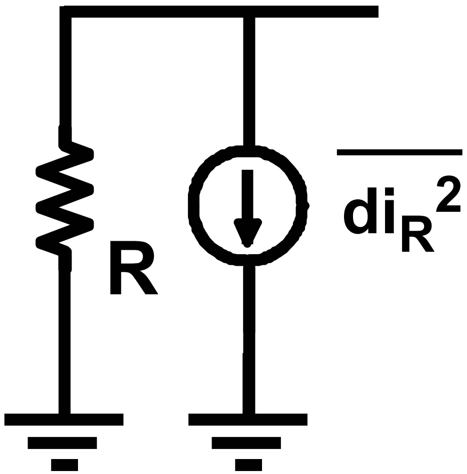


$$\overline{dv_R^2} = 4kT\,R\,df$$ is white

depends on T, not on I_R

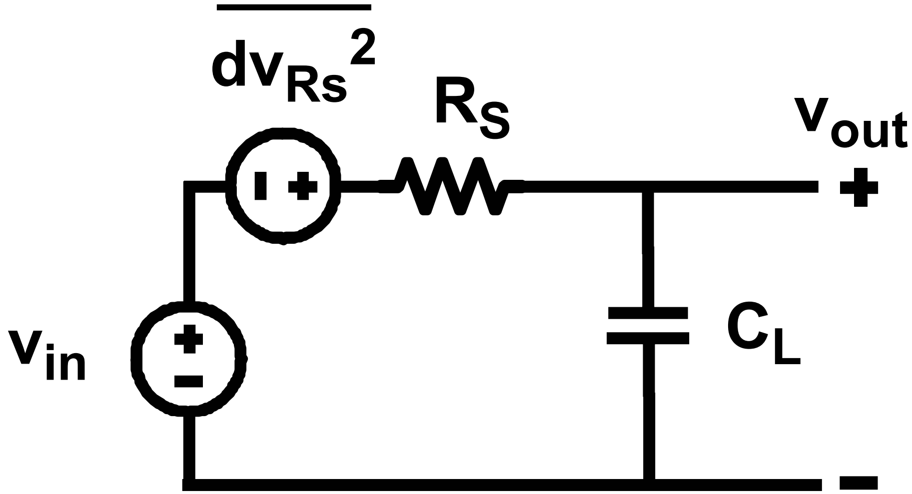
for R = 1 kΩ $\sqrt{\overline{dv_R^2}} = 4\ nV_{RMS} / \sqrt{Hz}$

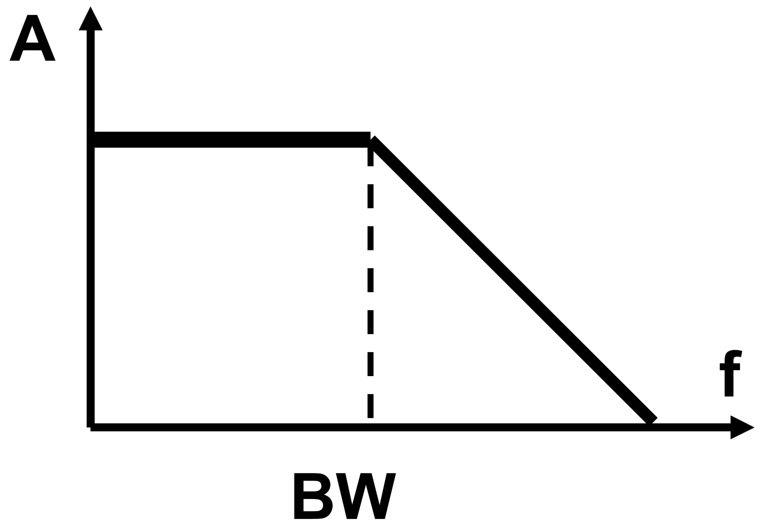
at T = 300 K or 27°C



$$\overline{di_R^2} = \frac{\overline{dv_R^2}}{R^2} = \frac{4kT}{R}\,df$$ is white

Integrated Noise of Resistor - 1



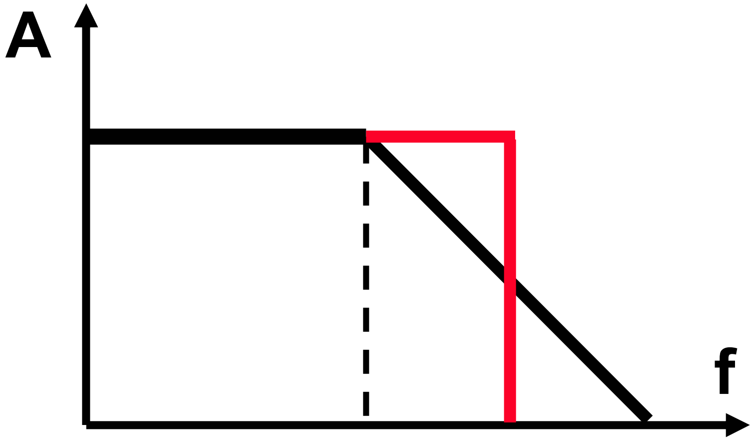


$$\overline{dv_{Rs}^2} = 4kT\,R_S\,df$$

$$BW = \frac{1}{2\pi\,R_S C_L}$$

$$\overline{v_{Rs}^2} = \int_0^{\infty} \frac{\overline{dv_{Rs}^2}}{1 + (f/\,BW)^2}$$

Integrated Noise of Resistor - 2



BW $\quad BW_n = \frac{\pi}{2} BW$

$$\overline{v_{Rs}^2} = \int_0^\infty \frac{\overline{dv_{Rs}^2}}{1 + (f/\,BW)^2}$$

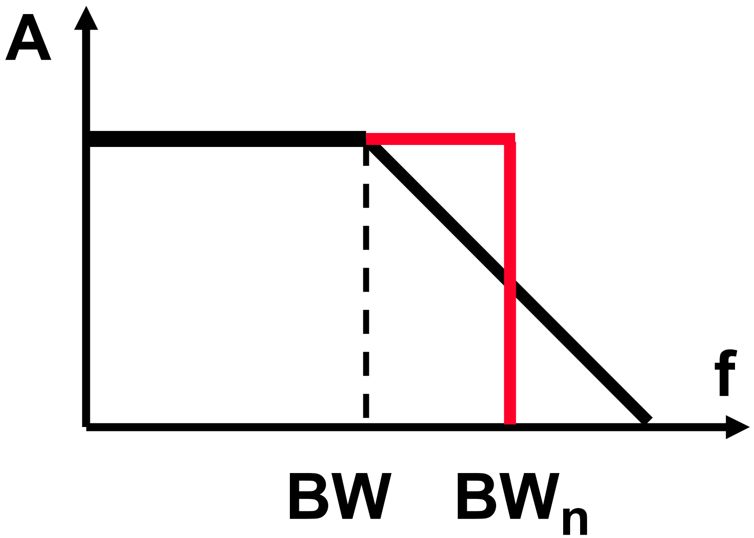
$$\boxed{\int_0^\infty \frac{dx}{1 + x^2} = \frac{\pi}{2}}$$

$$\overline{v_{Rs}^2} = 4kT\,R_S BW \frac{\pi}{2}\,df$$

$$\overline{v_{Rs}^2} = \frac{kT}{C_L}$$

$\boxed{C_L = 1pF \quad v_{Rs} = 65\ \mu V_{RMS}}$

Noise density vs integrated noise



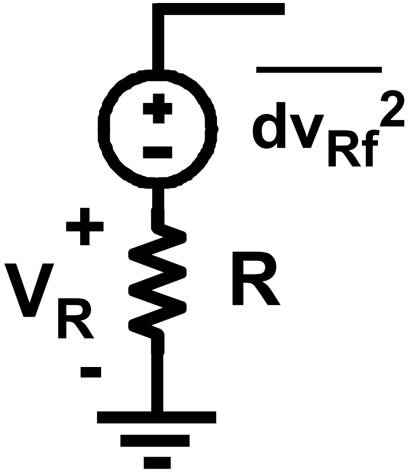
$$\overline{dv_{Rs}^{2}} = 4kT\,R_S\,df$$

$$\overline{v_{Rs}^{2}} = \int_0^{\infty} \frac{\overline{dv_{Rs}^{2}}}{1 + (f/\,BW)^{2}} = \frac{kT}{C_L}$$

Noise density (V^2/Hz) ~ R_S (or $1/g_m$)

Integrated noise (V_{RMS}) ~ $1/C_L$

A resistor also has 1/f noise



$$\overline{dv_{Rf}^2} = V_R^2 \, \frac{KF_R R_{\square}}{A_R} \, \frac{df}{f}$$ is 1/f

$KF_{RSi} \approx 2\ 10^{-21}\ \mathrm{Scm}^2$

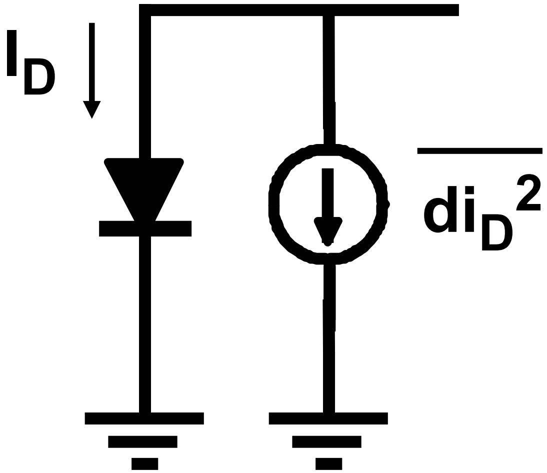
$KF_{Rpoly} \approx 10\ KF_{RSi}$

for R = 1 kΩ with 20 □'s of 50 Ω/□ and 1 μm wide and V_R= 0.1 V

$\sqrt{\overline{dv_{Rf}^2}} = 16\ nV_{RMS}/\sqrt{Hz}$ at 1 Hz

Ref. Vandamme, ESSDERC '04

Noise of a diode is shot noise



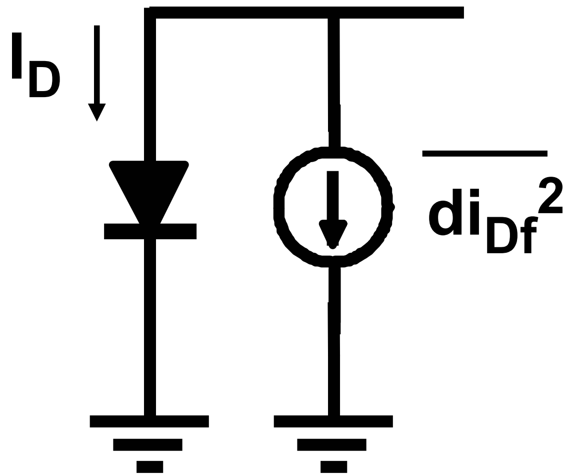
$\overline{di_D^2} = 2q\, I_D\, df$ is white

$q = 1.6\ 10^{-19}$ C

depends on I_D, not on T

for $I_D = 50\ \mu A$ $\sqrt{\overline{di_D^2}} = 4\ pA_{RMS} / \sqrt{Hz}$

A diode also has 1/f noise



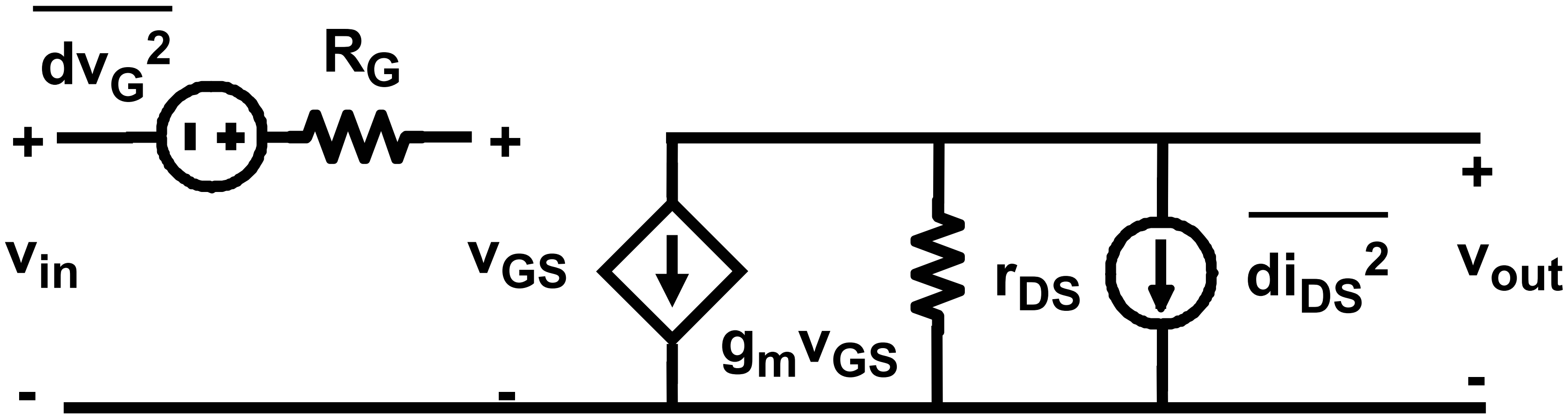
$$\overline{di_{Df}^2} = I_D \frac{KF_D}{A_D} \frac{df}{f} \qquad \text{is } 1/f$$

$$KF_D \approx 10^{-21}\ \mathrm{Acm}^2$$

For a diode of $A_D = 5 \times 2\ \mu m = 10\ \mu m^2$ and $I_D = 0.1$ mA

$$\sqrt{\overline{di_{Df}^2}} = 1\ nA_{RMS} / \sqrt{Hz} \text{ at 1 Hz}$$

Noise of a MOST

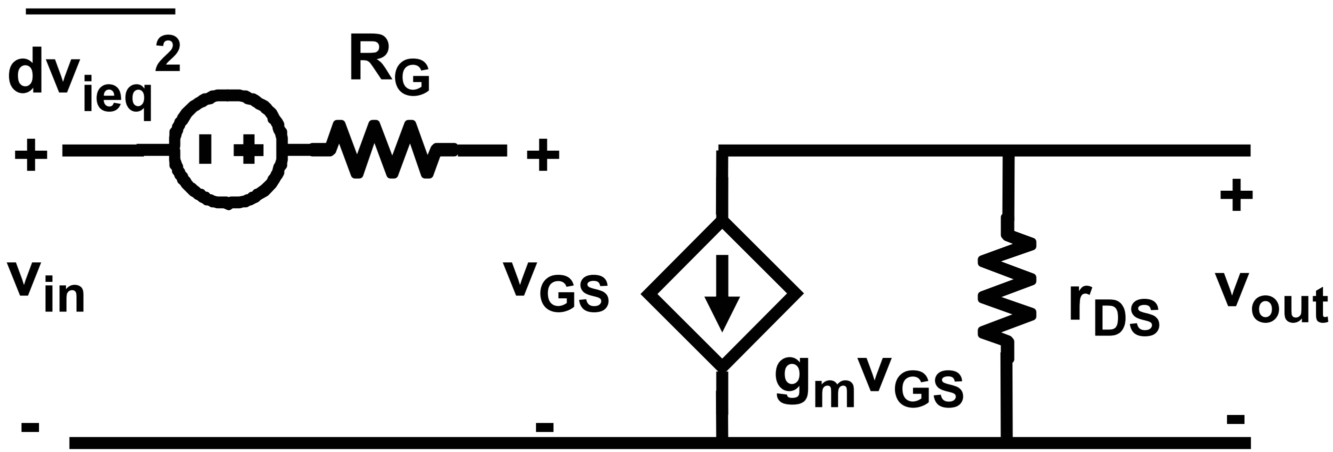


$$\overline{dv_G^2} = 4kT\, R_G\, df$$

$$\overline{di_{DS}^2} = \frac{4kT}{R_{CH}}\, df = 4kT\,\frac{2}{3}\, g_m\, df$$

Ref. Van der Ziel, Prentice Hall 1954, Wiley 1986.

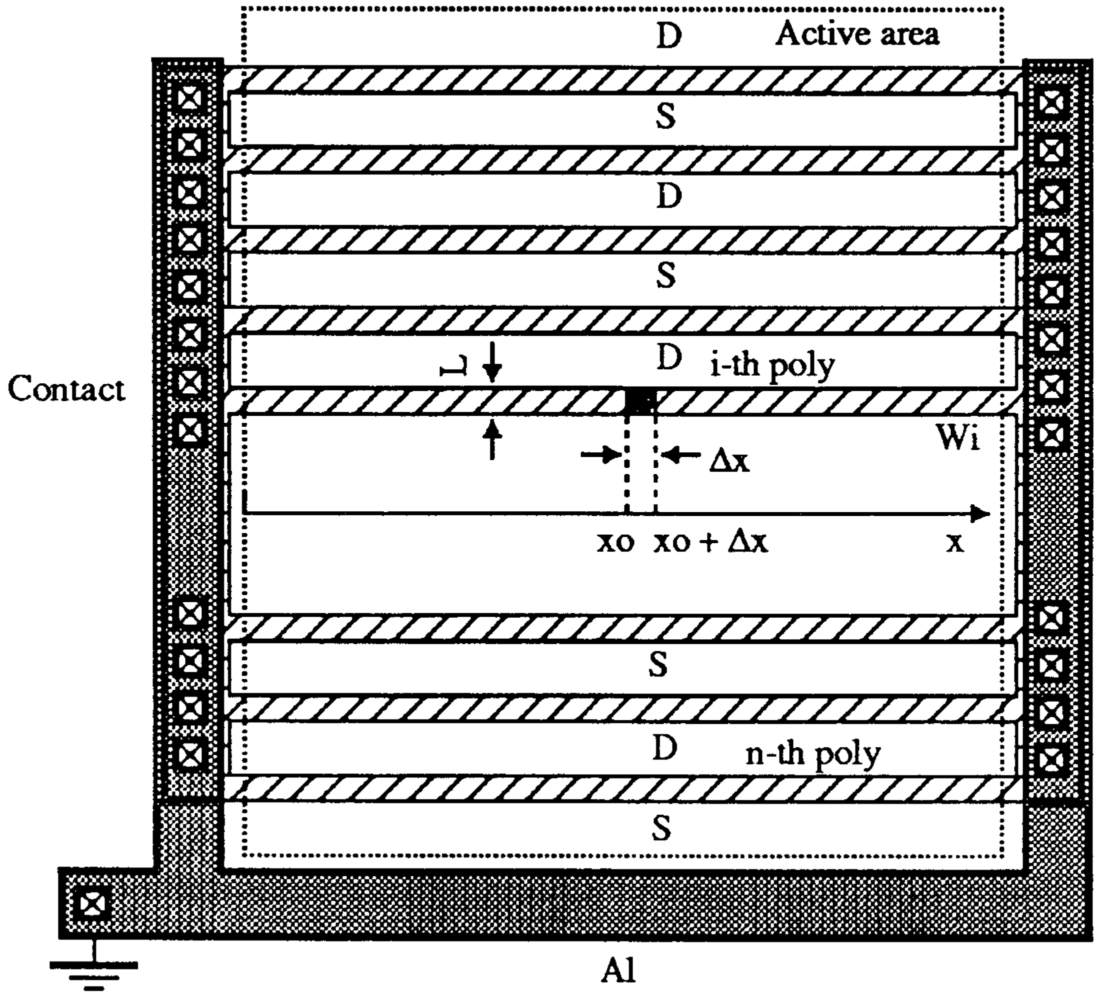
MOST: equivalent input noise : white



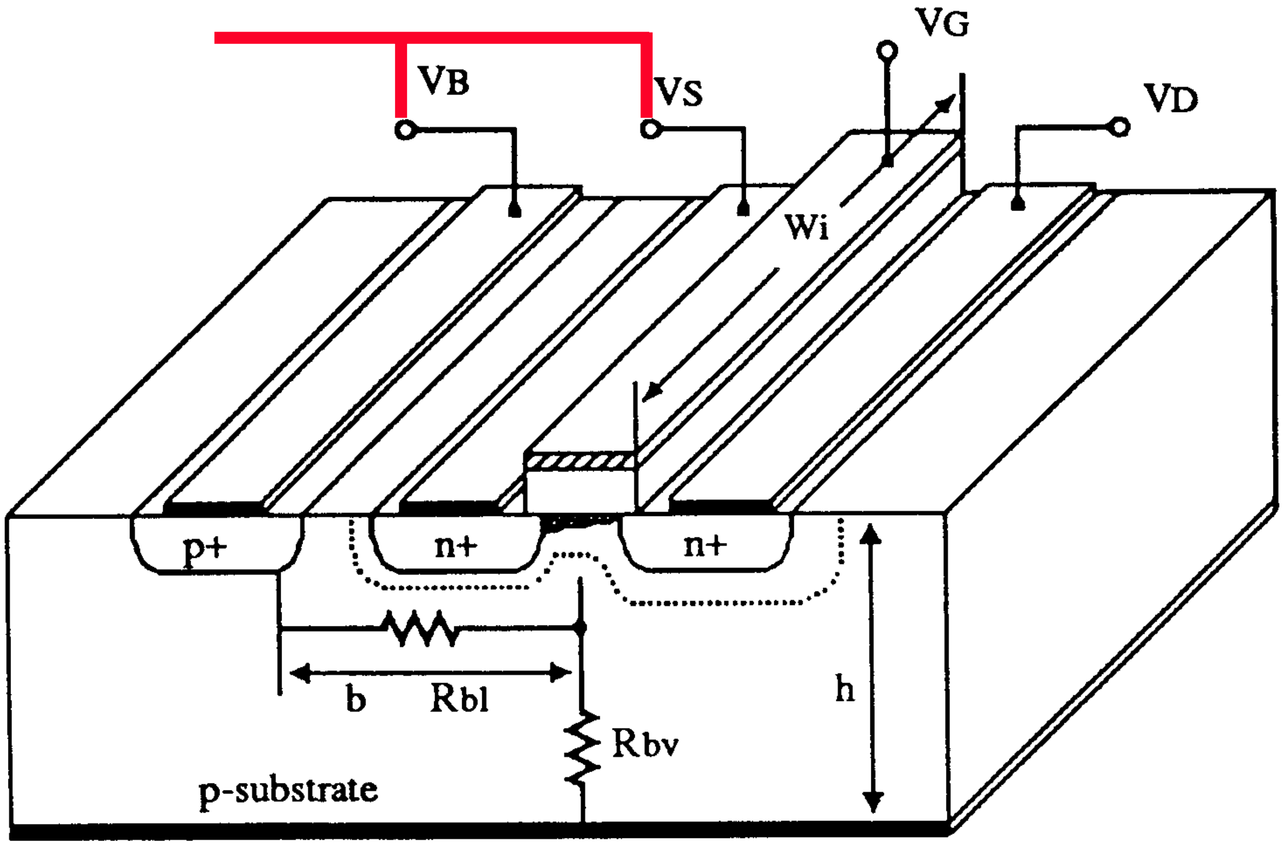
$$\overline{dv_{ieq}^2} = 4kT\,(R_{eff})\,df \qquad R_{eff} = \frac{2/3}{g_m} + R_G$$

Hi Freq.: $\overline{di_{ieq}^2} = (C_{GS}\,\omega)^2\,\overline{dv_{ieq}^2}$ is correlated

Poly Gate resistance r_G in a MOST

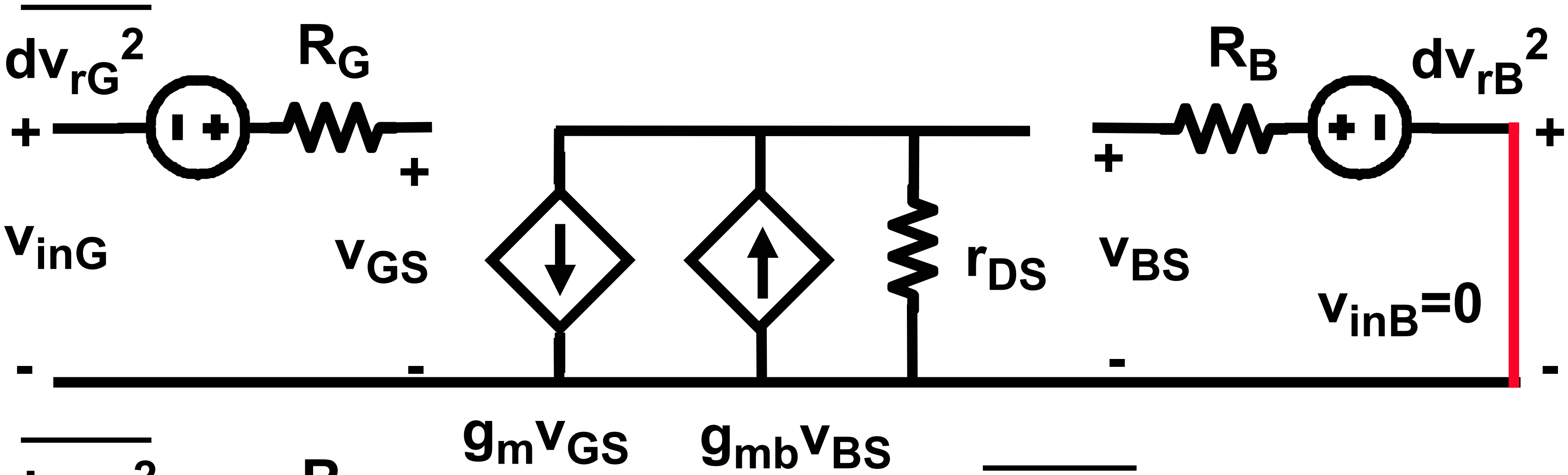


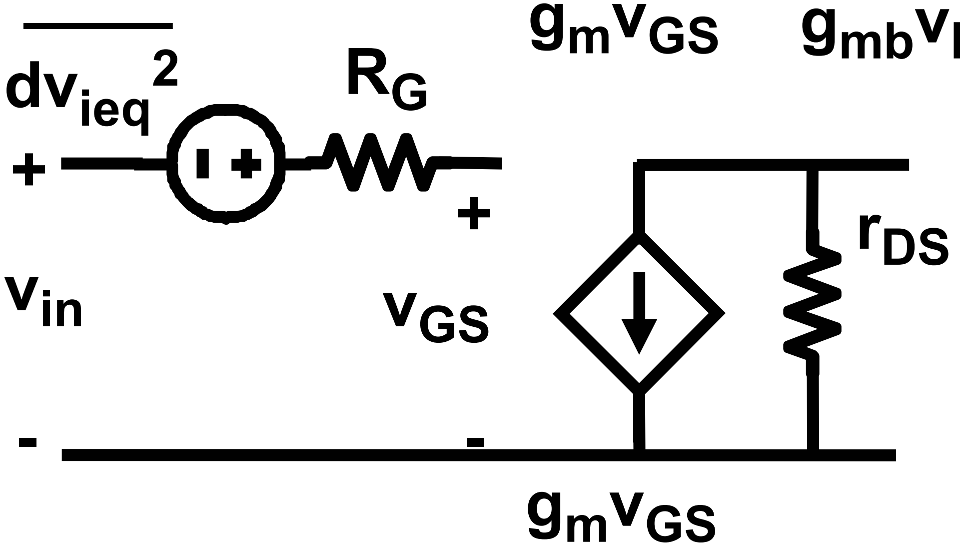
Substrate resistances r_B in a MOST



Ref. Chang, Kluwer 1991

Noise by the Bulk resistance



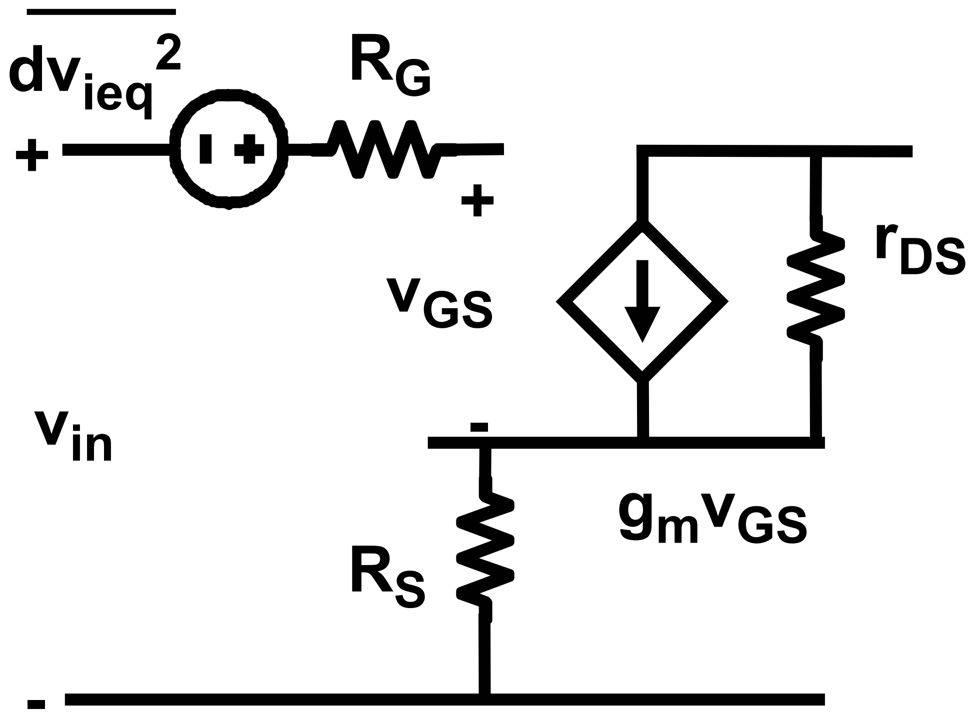


$$\overline{dv_{ieq}^2} = 4kT\,(R_{eff})\,df$$

$$R_{eff} = \frac{2/3}{g_m} + R_G + R_B\,(n-1)^2$$

$$(n-1) = C_D/C_{ox} = g_{mb}/g_m$$

Noise by the Source resistance

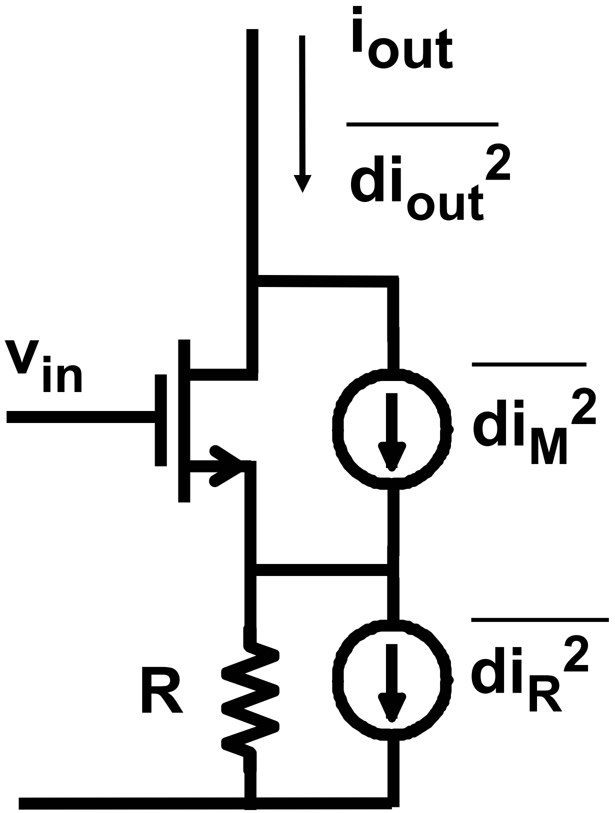


Noise of R_S
= noise R_G

$$\overline{dv_{ieq}^{2}} = 4kT\,(R_{eff})\,df$$

$$R_{eff} = \frac{2/3}{g_m} + R_G + R_S + R_B\,(n-1)^2$$

Noise by Source resistor R



$$g_m R \gg 1$$

$$i_{out} = \frac{v_{in}}{R}$$

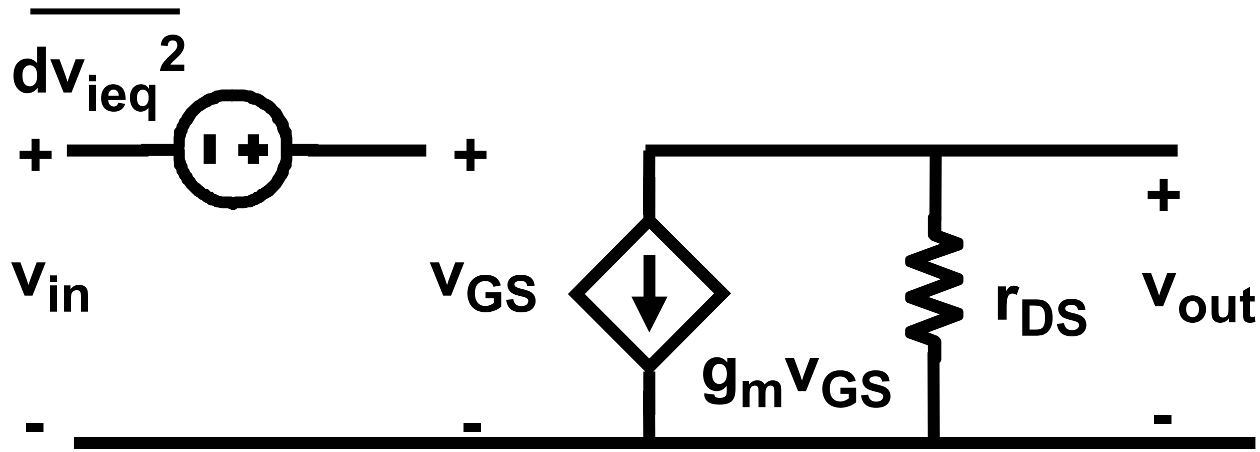
$$\overline{di_M^2} = 4kT\, 2/3\, g_m\, df \qquad \overline{di_{outM}^2} = \frac{\overline{di_M^2}}{(g_m R)^2}$$

$$\overline{di_R^2} = \frac{4kT}{R}\, df \qquad \overline{di_{outR}^2} = \overline{di_R^2}$$

$$\overline{di_{out}^2} = \frac{4kT}{R}\left(\frac{2/3}{g_m R} + 1\right) df \approx \frac{4kT}{R}\, df$$

$$\overline{dv_{in}^2} = 4kT\, R\, df$$

MOST: equivalent input noise : Exercise

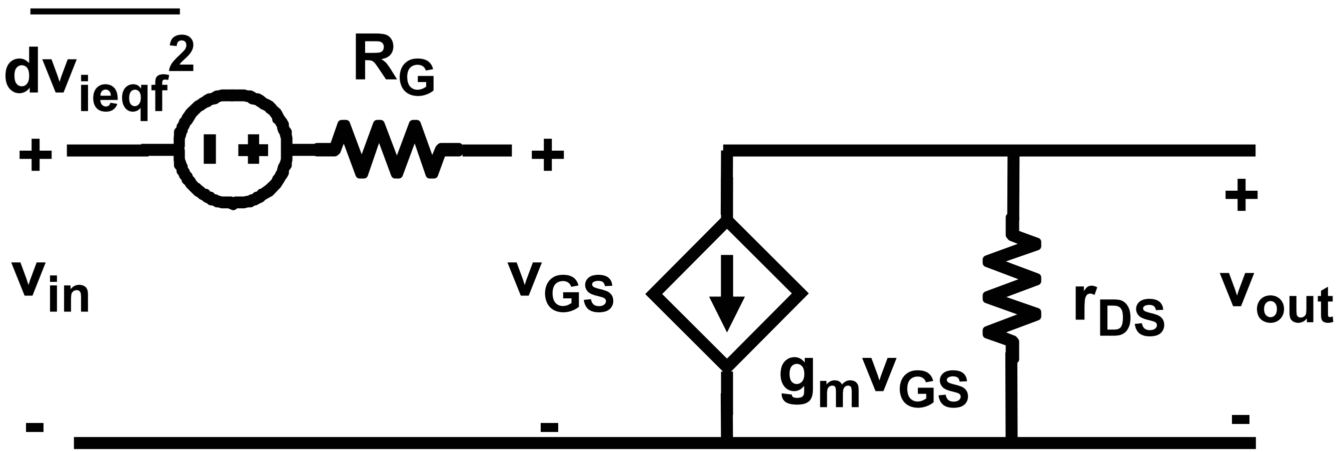


$$\overline{dv_{ieq}^2} \approx 4kT\left(\frac{2/3}{g_m}\right) df$$

$$\overline{dv_{ieq}^2} \approx ?$$

for $I_{DS} = 65\ \mu A$

MOST: equivalent input noise : 1/f noise



$$\overline{dv_{ieqf}^2} = \frac{KF_F}{WL\, C_{ox}^2} \frac{df}{f}$$

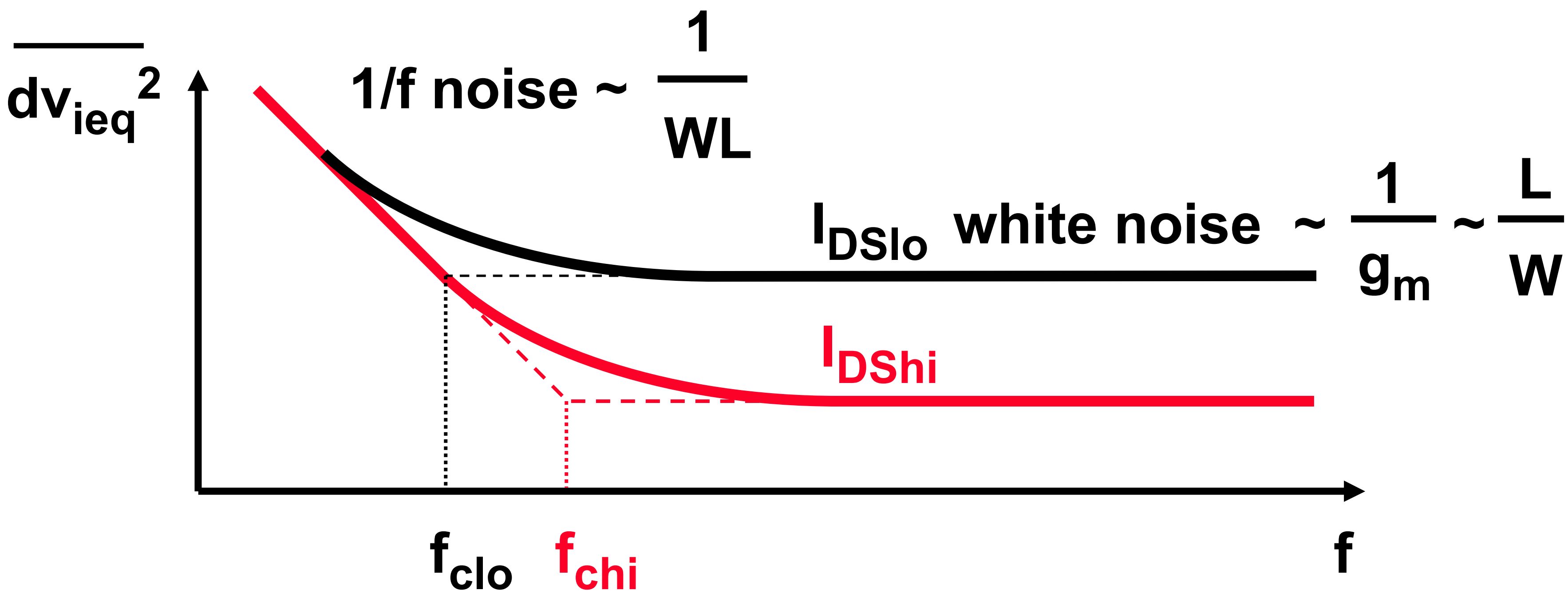
pMOST $KF_F \approx 10^{-32}\ C^2/cm^2$

nMOST $KF_F \approx 4\ 10^{-31}\ C^2/cm^2$

pJFET $KF_F \approx 10^{-33}\ C^2/cm^2$

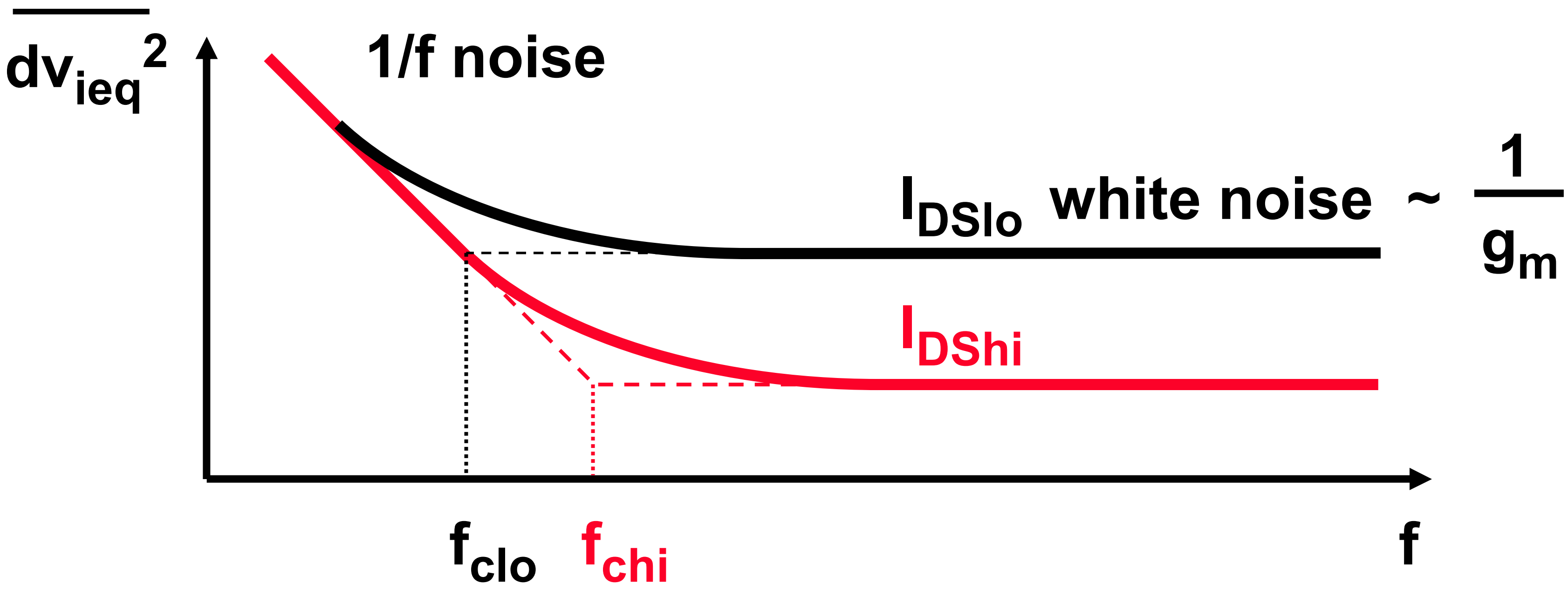
W & L in cm; C_{ox} in F/cm^2

Noise vs current : corner frequency



Corner frequency ~ g_m

Noise vs current : exercise f_c

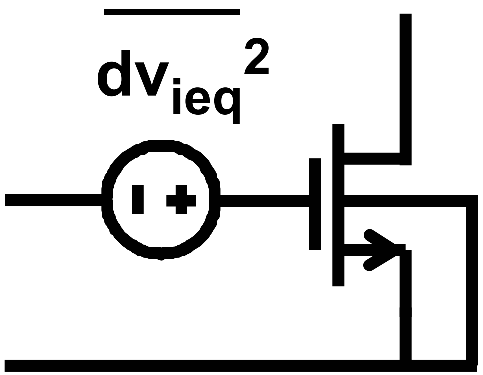


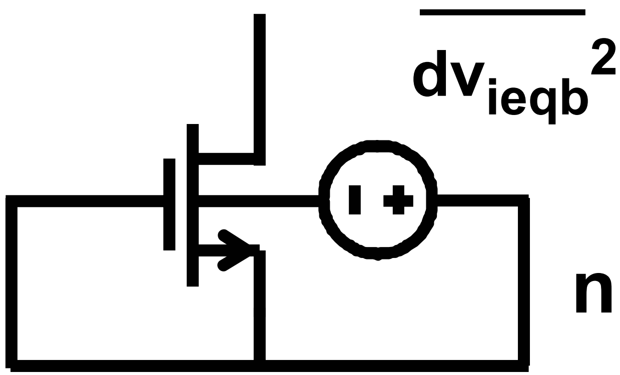
Ex. : f_c ? For I_{DS} = 65 μA;

K'$_n$ = 60 μA/V^2 and L = 1 μm (0.35 μm process)

$f_c \approx$ 370 kHz

Noise seen at the Bulk



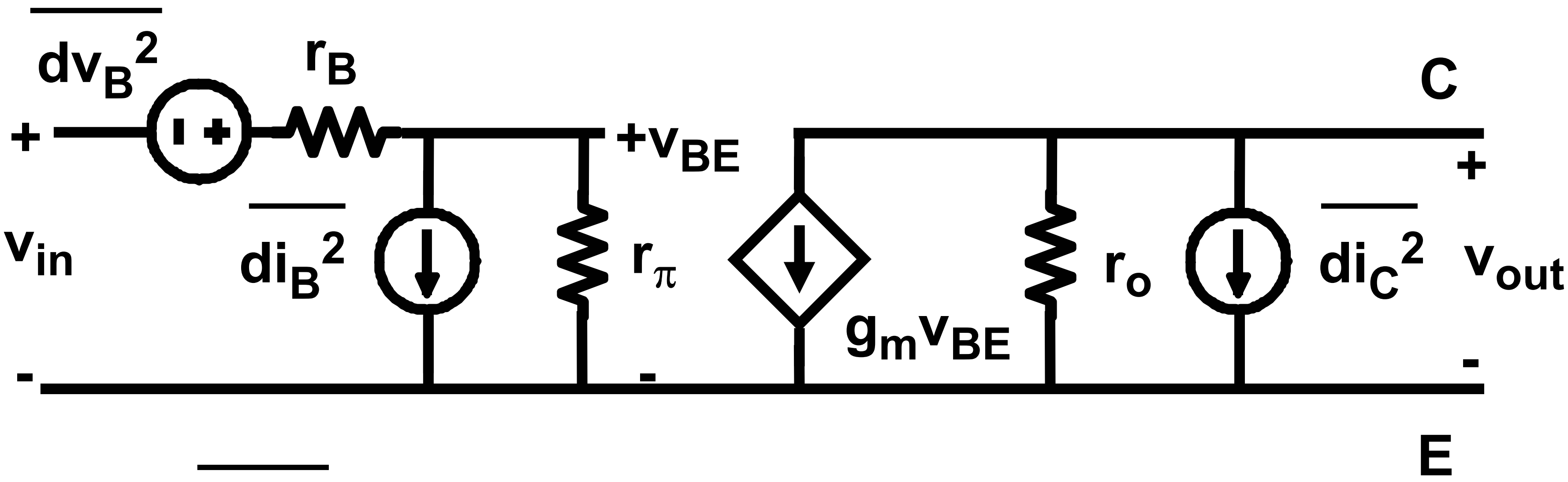


$$n-1 = \frac{g_{mb}}{g_m}$$

$$\overline{dv_{ieq}^2} = 4kT\,(\frac{2/3}{g_m})\,df \qquad \overline{dv_{ieqb}^2} = 4kT\,(\frac{2/3\;g_m}{g_{mb}^2})\,df$$

$$\overline{dv_{ieqf}^2} = \frac{KF_F}{WL\,C_{ox}^2}\,\frac{df}{f} \qquad \overline{dv_{ieqfb}^2} = \frac{KF_F}{WL\,C_{ox}^2}\,\frac{g_m^2}{g_{mb}^2}\,\frac{df}{f}$$

Noise of a Bipolar transistor



$$\overline{dv_B^2} = 4kT\, r_B\, df$$

$$\overline{di_B^2} = 2q\, I_B\, df$$

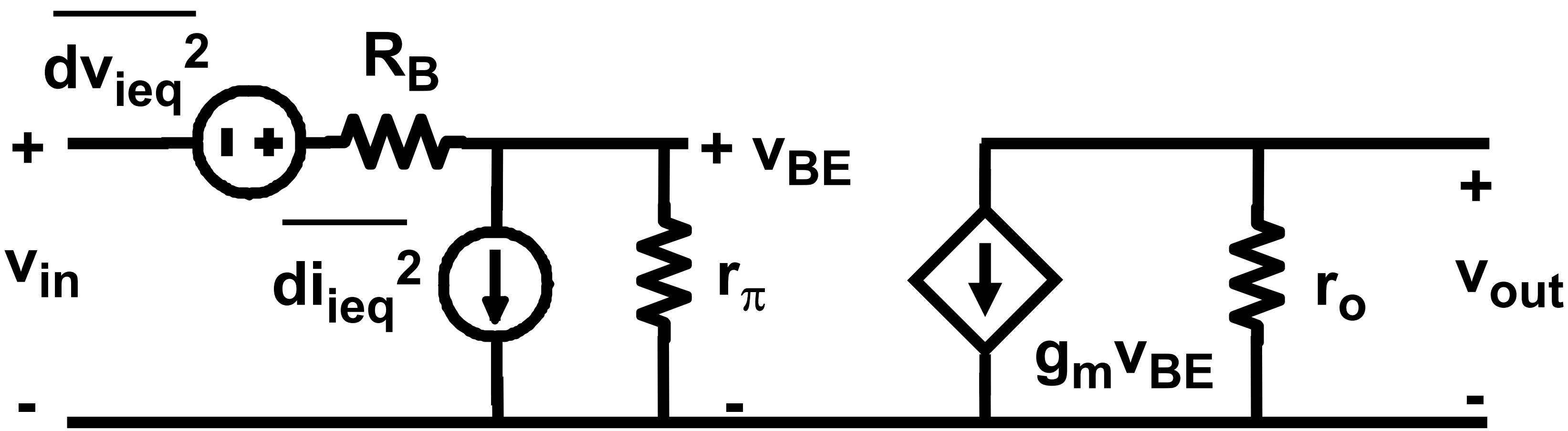
$$\overline{di_C^2} = 2q\, I_C\, df$$

$$\overline{di_{Bf}^2} = \frac{KF_B\, I_B}{A_{EB}} \frac{df}{f}$$

$$KF_B \approx 10^{-21}\ \mathrm{Acm^2}$$

Ref. Van der Ziel (Prentice Hall 1954)

Bipolar trans.: equivalent input noise



$$\overline{dv_{ieq}^2} = 4kT\,(R_{eff})\,df \qquad R_{eff} = \frac{1/2}{g_m} + R_B + R_E$$

$$\overline{di_{ieq}^2} = \overline{di_B^2} = 2q\,I_B\,df$$

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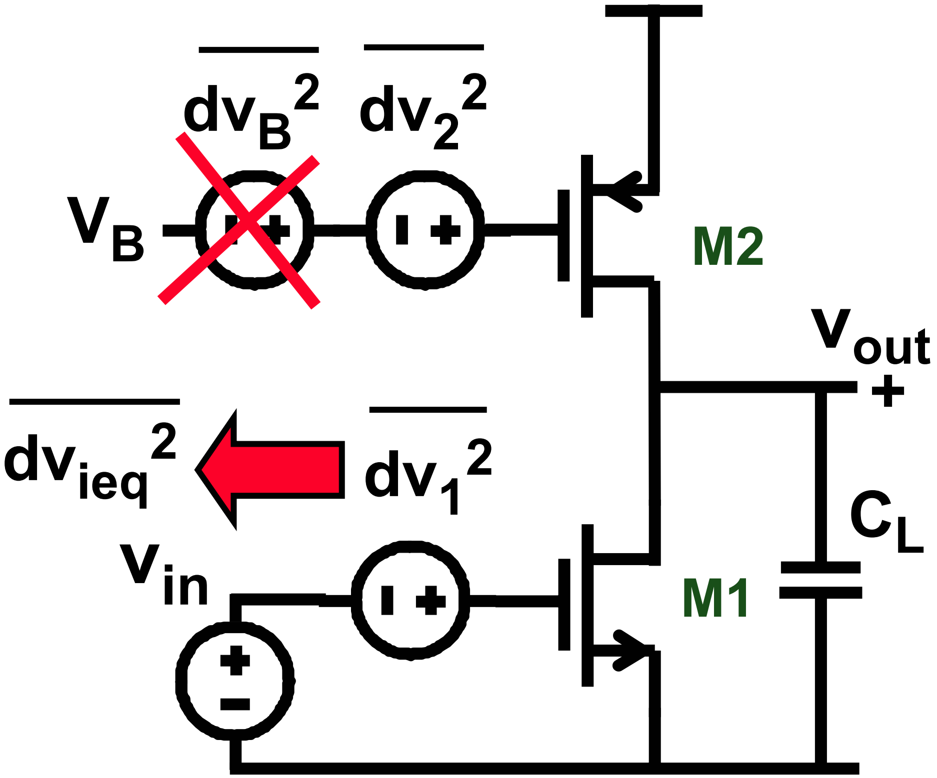
Noise of an amplifier with active load



If $\overline{dv_B^2}$ is negligible :

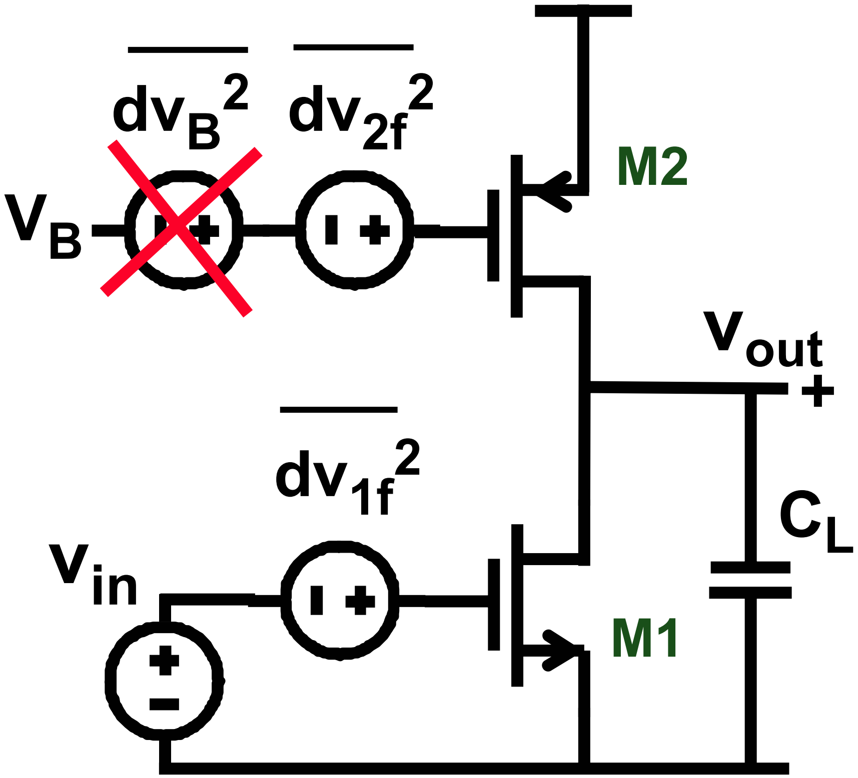
$$\overline{di_{out}^2} = g_{m1}^2\,\overline{dv_1^2} + g_{m2}^2\,\overline{dv_2^2}$$

$$\overline{dv_{ieq}^2} = \overline{dv_1^2} + \overline{dv_2^2}\left(\frac{g_{m2}}{g_{m1}}\right)^2$$

$$\overline{dv_{ieq}^2} = \overline{dv_1^2}\left(1 + \frac{g_{m2}}{g_{m1}}\right)$$

Small g_{m2} : small $(W/L)_2$ or large $(V_{GS} - V_T)_2$

1/f Noise of amplifier with active load



If $\overline{dv_B^2}$ is negligible :

$$\overline{dv_{if}^2} = \overline{dv_{1f}^2} + \overline{dv_{2f}^2}\left(\frac{g_{m2}}{g_{m1}}\right)^2$$

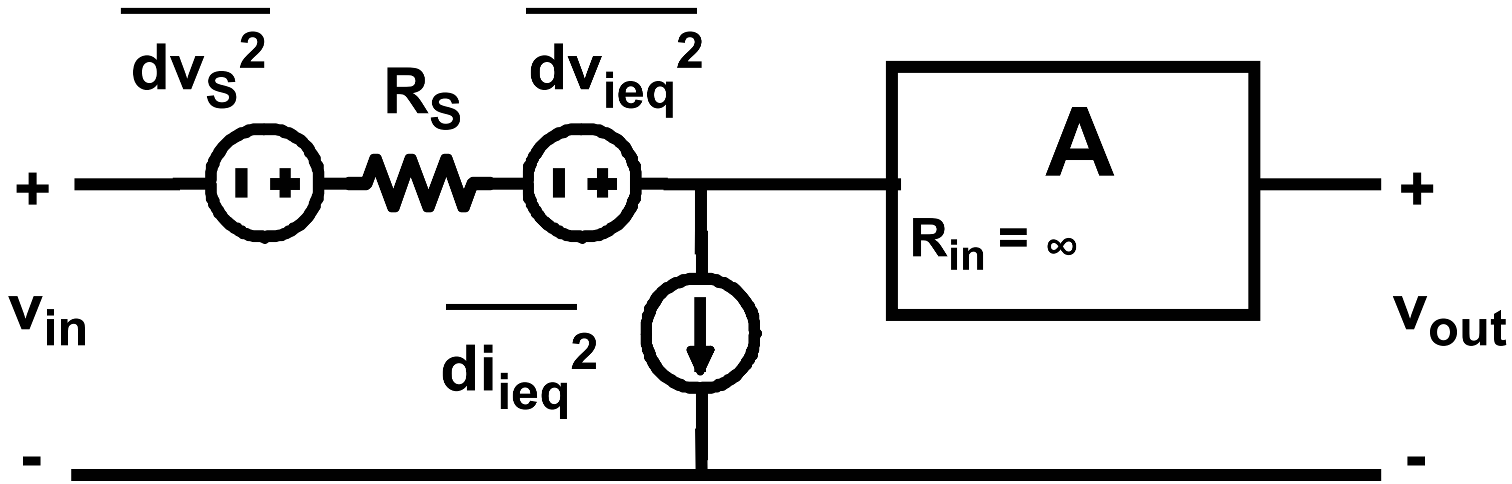
$$\overline{dv_{if}^2} = \overline{dv_{1f}^2}\left[1 + \left(\frac{g_{m2}}{g_{m1}}\right)^2\left(\frac{\overline{dv_{2f}}}{\overline{dv_{1f}}}\right)^2\right]$$

$$\overline{dv_{if}^2} = \overline{dv_{1f}^2}\left[1 + \frac{KF_2}{KF_1}\frac{K'_2}{K'_1}\left(\frac{L_1}{L_2}\right)^2\right]$$

$\overline{dv_{if}^2}$ has minimum at

$$L_{1opt} = L_2\sqrt{\frac{KF_1}{KF_2}\frac{K'_1}{K'_2}} \approx 10\ L_2 \quad \text{then} \quad \overline{dv_{if}^2} = 2\ \overline{dv_{1f}^2}$$

Noise figure of an amplifier



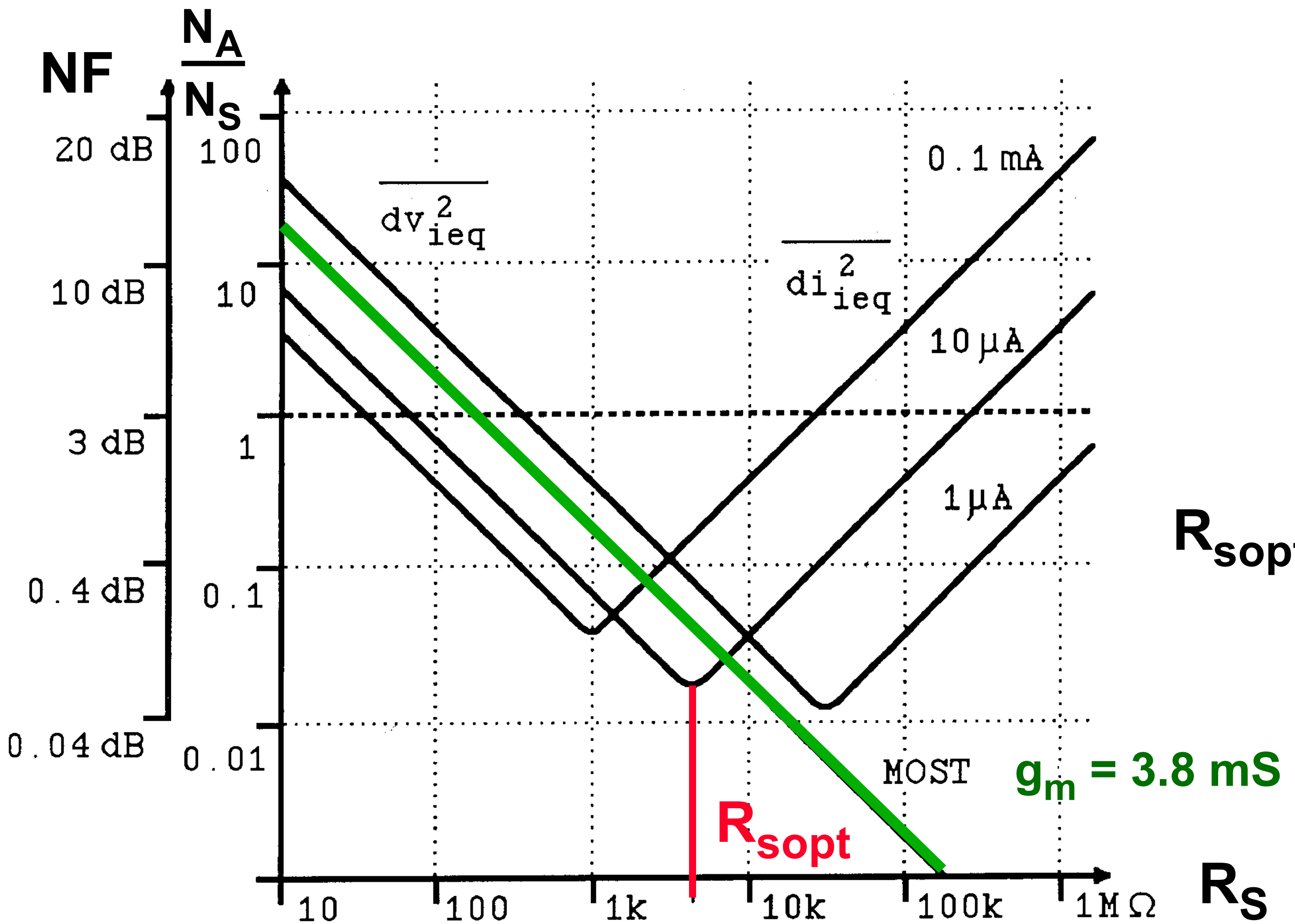
$$NF = \frac{N_S + N_A}{N_S} = 1 + \frac{N_A}{N_S}$$

$$NF = 1 + \frac{\overline{dv_{ieq}^2} + R_S^2\,\overline{di_{ieq}^2}}{4kT\,R_S\,df}$$

Voltage drive $NF \sim \frac{1}{R_S}$

Current drive $NF \sim R_S$

Resistive noise matching



$\beta = 100$

$r_B = 100\ \Omega$

$$R_{sopt} = \sqrt{\frac{\overline{dv_{ieq}^2}}{\overline{di_{ieq}^2}}}$$

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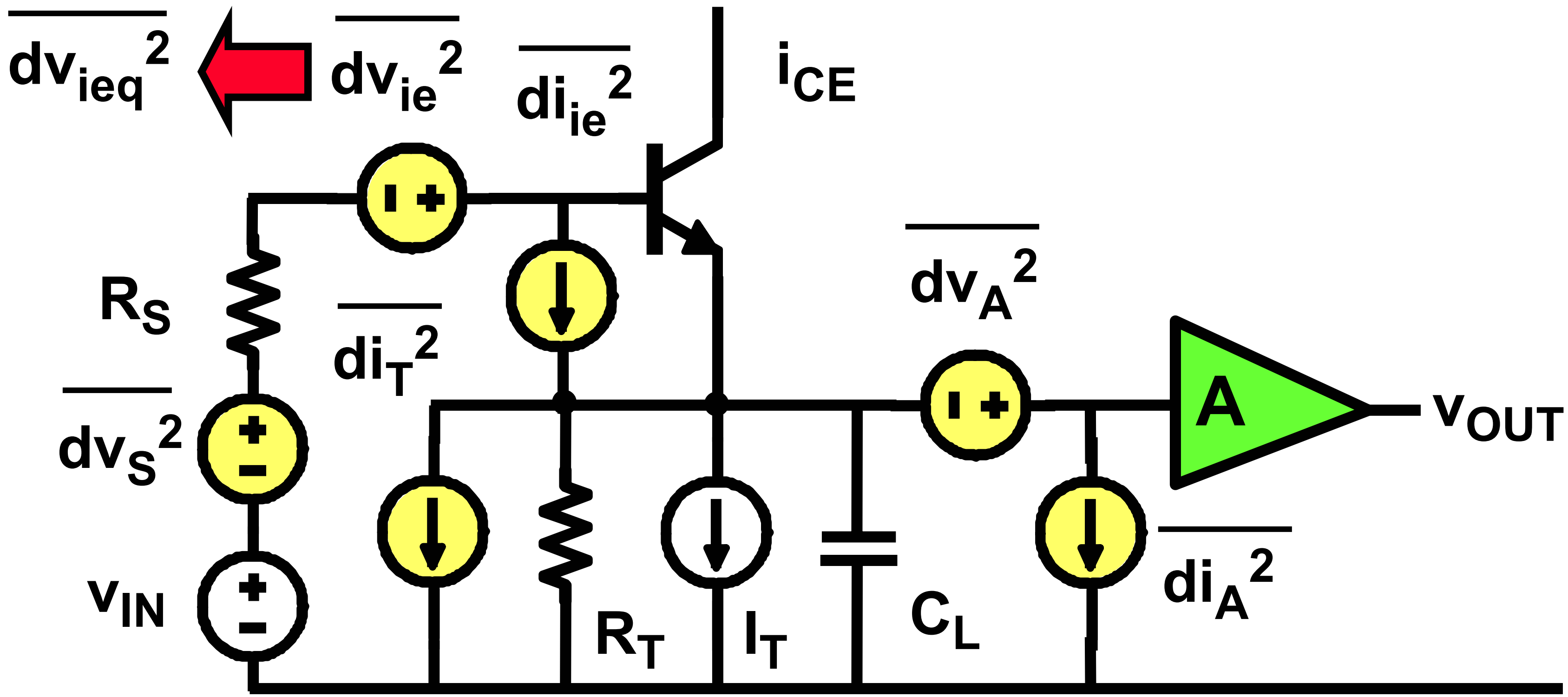
Noise of an emitter follower



$$\overline{dv_{ieq}^2} = \overline{dv_{ie}^2} + \overline{dv_A^2} + \left(R_S - \frac{1}{g_m}\right)^2 \overline{di_{ie}^2} + \frac{\overline{di_T^2} + \overline{di_A^2}}{g_m^2}$$

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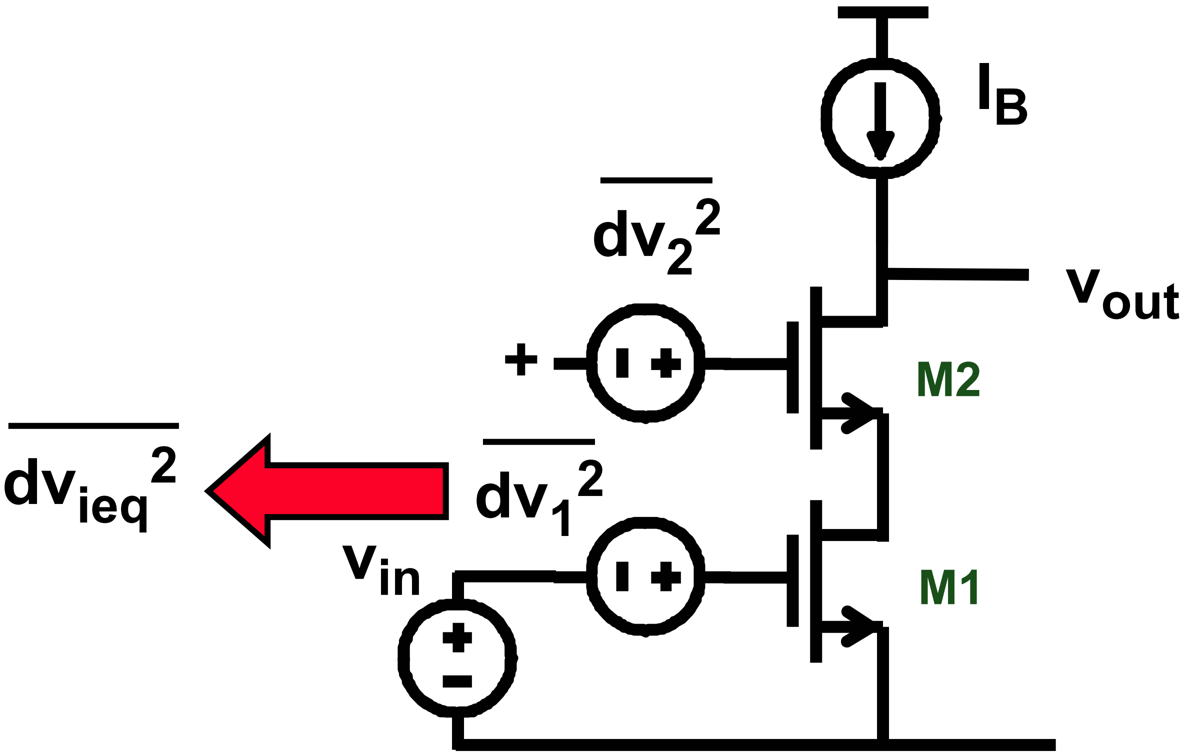
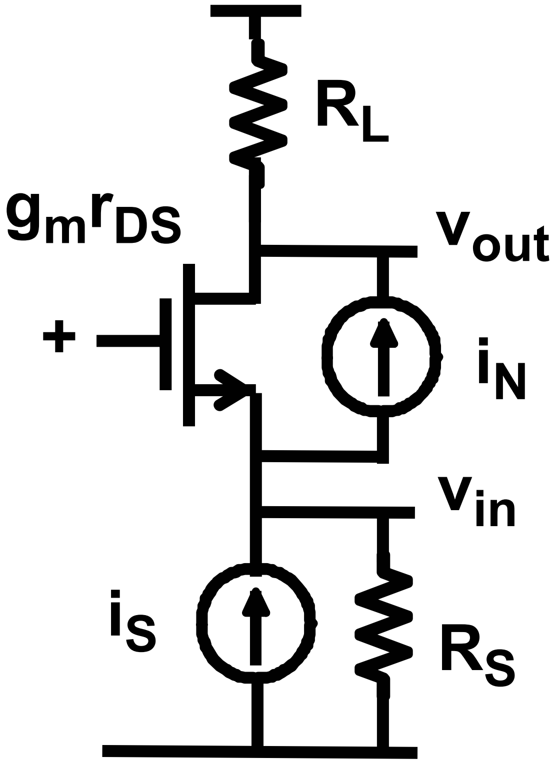
Noise of a cascode amplifier

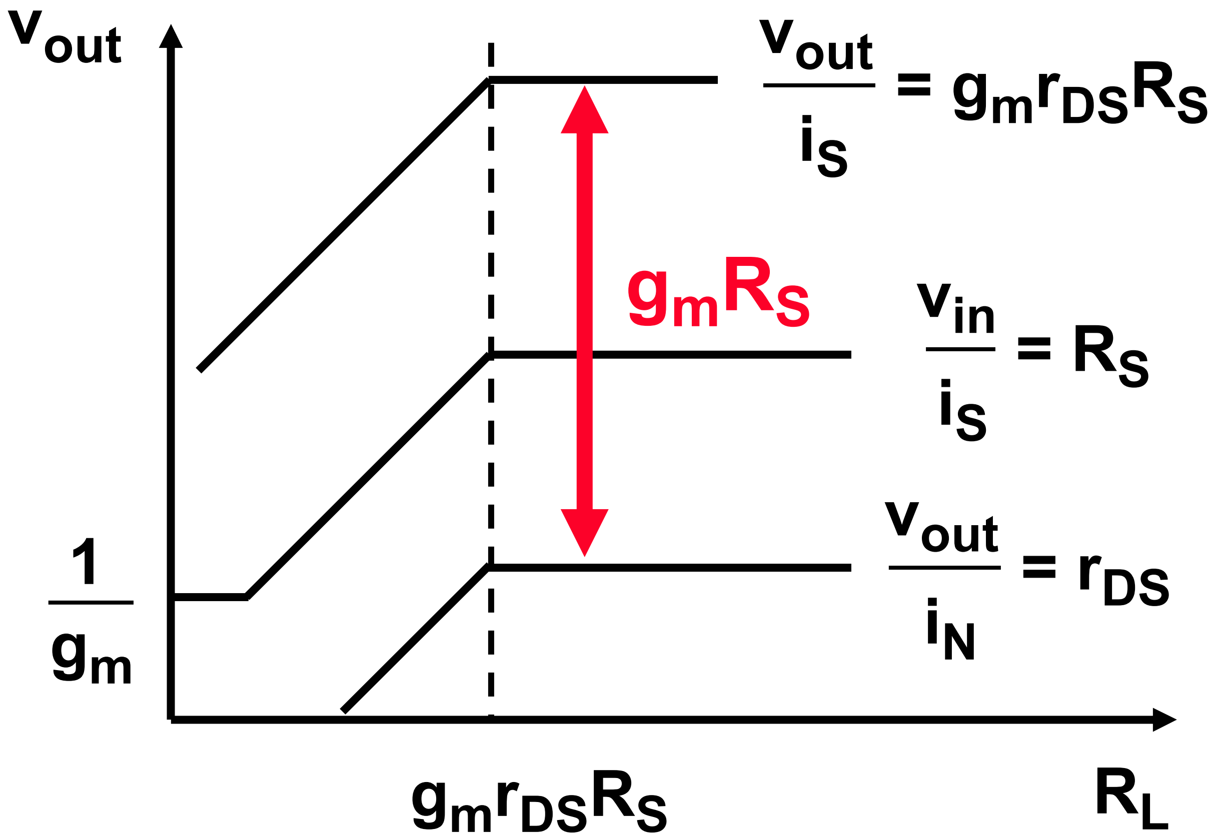


$$\overline{dv_{ieq}^2} = \overline{dv_1^2} + \overline{dv_2^2} \frac{1}{(g_{m1}\, r_{o1})^2} \approx \overline{dv_1^2}$$

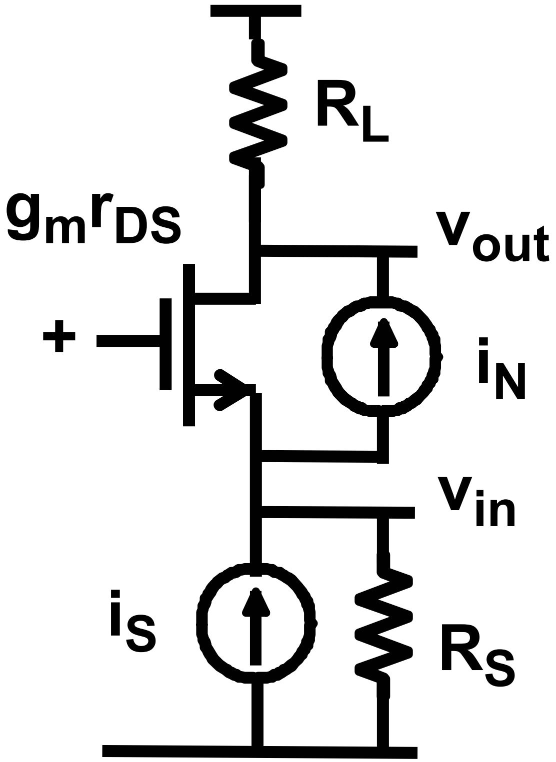
Input referred noise of a cascode

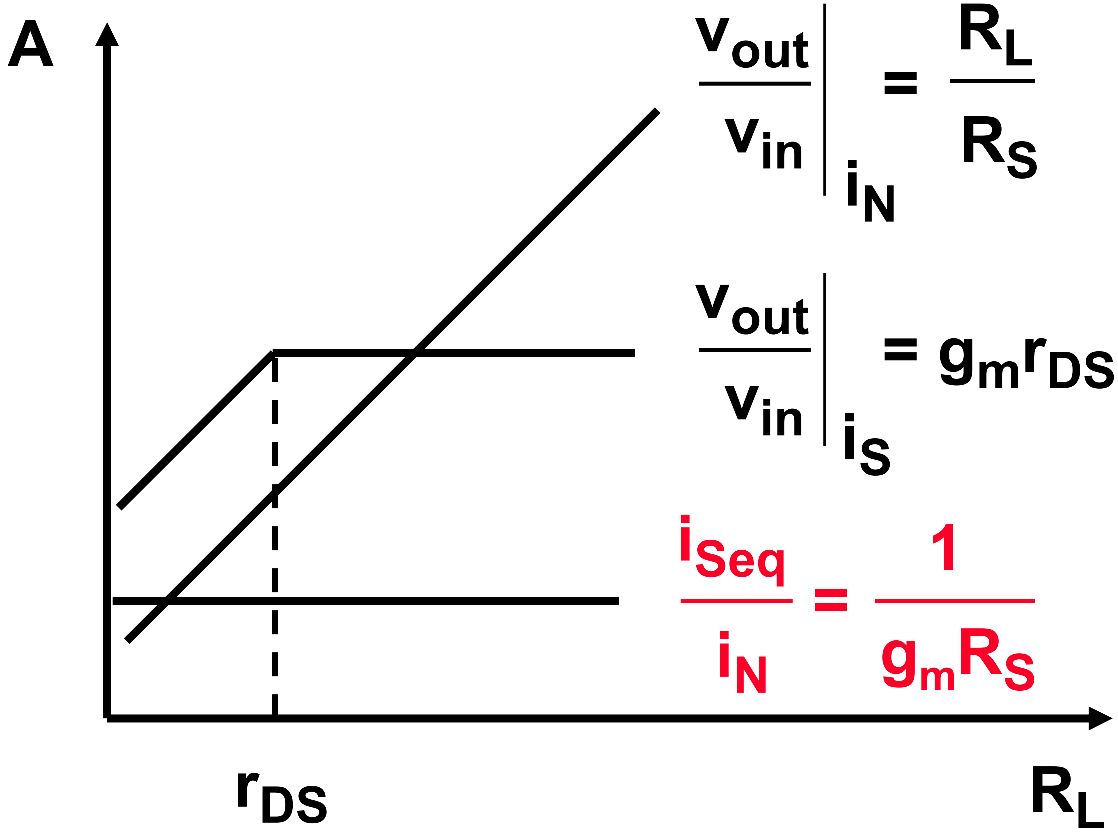


$g_m r_{DS} >> 1$



Noise gains in a cascode





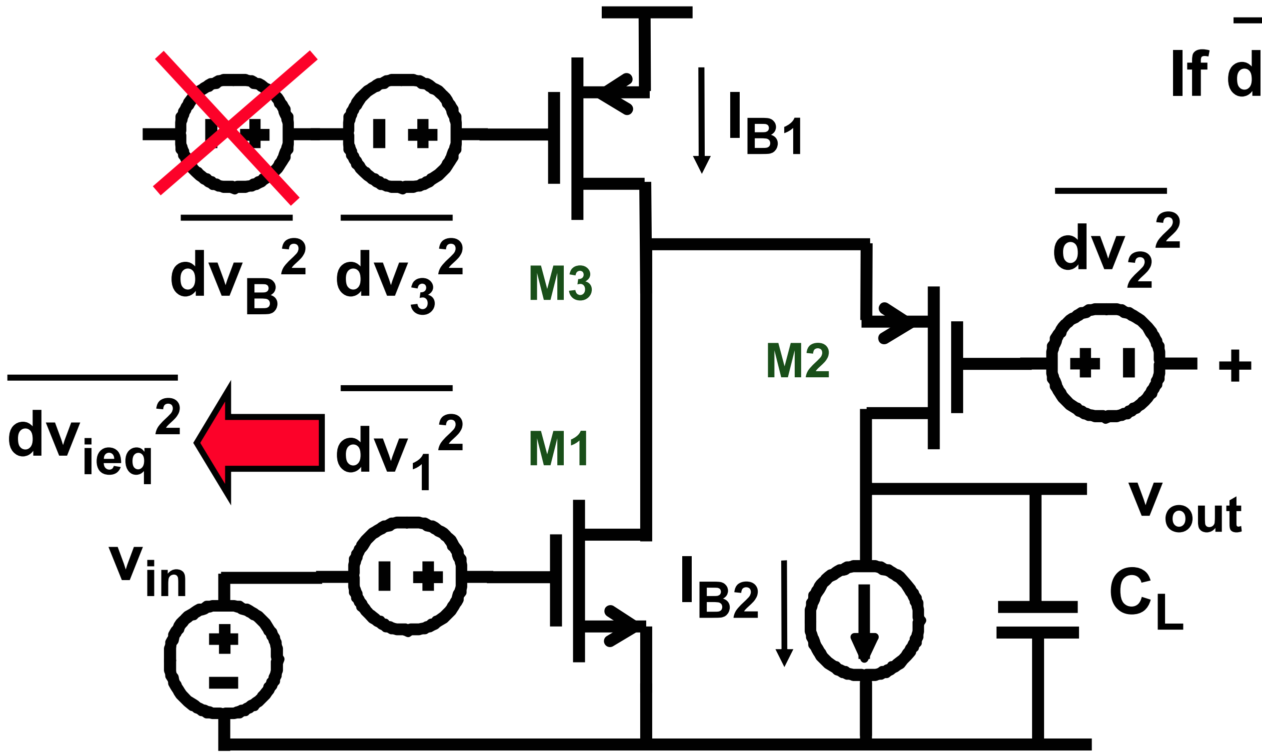
$$\left.\frac{v_{out}}{v_{in}}\right|_{i_N} = \frac{R_L}{R_S}$$

$$\left.\frac{v_{out}}{v_{in}}\right|_{i_S} = g_m r_{DS}$$

$$\frac{i_{Seq}}{i_N} = \frac{1}{g_m R_S}$$

Cascode noise i_N is only negligible if R_S is large !!!

Noise of a folded cascode



If $\overline{dv_B^2}$ is negligible :

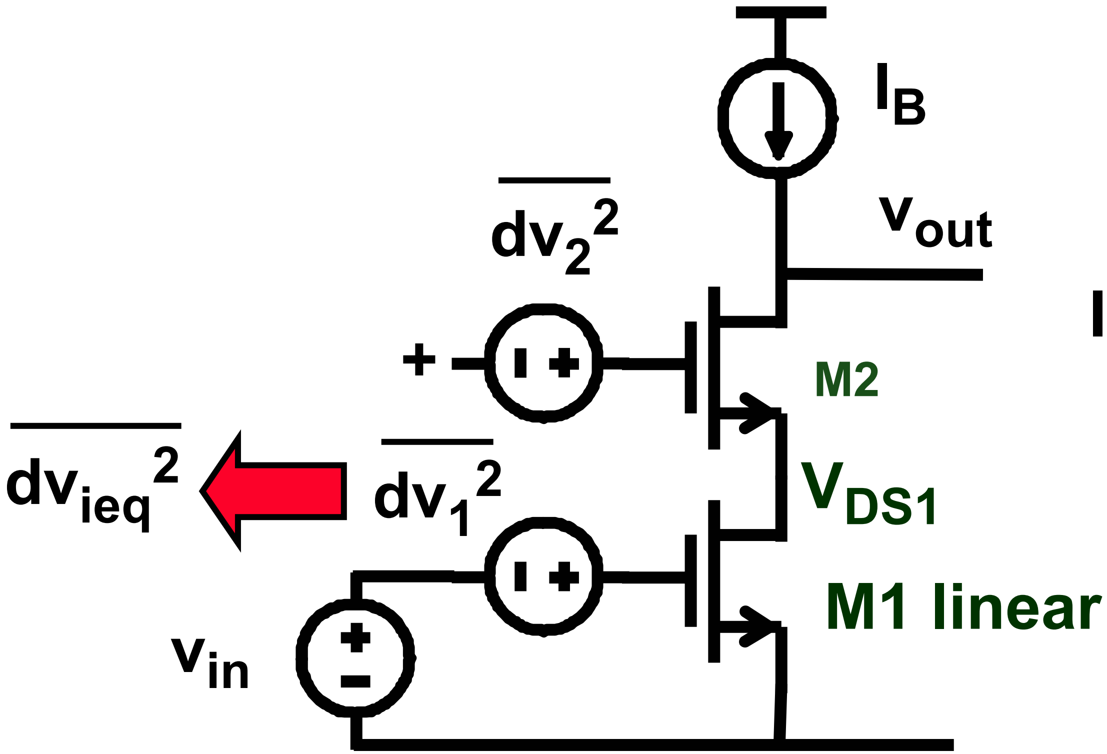
$$\overline{dv_{ieq}^2} = \overline{dv_1^2} + \overline{dv_2^2}\,\frac{1}{(g_{m1}\, r_{o1})^2} + \overline{dv_3^2}\,\frac{(g_{m3})^2}{(g_{m1})^2}$$

Small g_{m3} :

$(W/L)_3 \downarrow$

$(V_{GS}-V_T)_3 \uparrow$

Noise of a cascode with linear M1



$$\alpha_1 = \frac{V_{DS1}}{V_{GS1} - V_T} \qquad \alpha_1 < 0.5$$

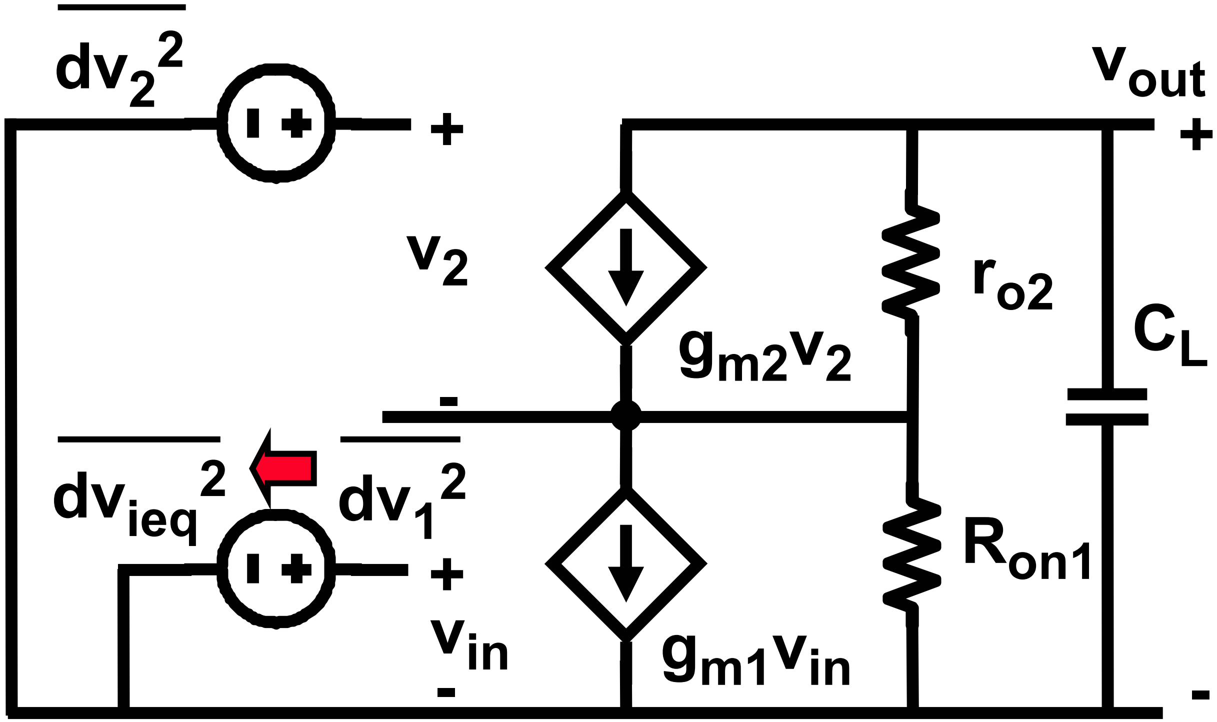
$$I_{DS1} = \beta_1 V_{DS1}(V_{GS1} - V_T)$$

$$R_{on1} = \frac{1}{\beta_1 (V_{GS1} - V_T)}$$

$$A_v = \alpha_1 g_{m2} r_{o2}$$

$$\overline{dv_{ieq}^2} = \frac{4kT}{\alpha_1^2} \left(R_{on1} + \frac{2/3}{g_{m2}}\right) df$$

Small-signal model of a cascode with linear M1



$$I_{DS1} = \beta_1 V_{DS1}(V_{GS1} - V_T)$$

$$R_{on1} = \frac{1}{\beta_1 (V_{GS1} - V_T)}$$

$$\overline{dv_1^2} = \frac{4kT\,R_{on1}}{\alpha_1^2}$$

$$\overline{dv_2^2} = \frac{4kT\,2/3}{g_{m2}}$$

$$v_{out} / dv_1 = \alpha_1\, g_{m2}\, r_{o2}$$

$$v_{out} / dv_2 = g_{m2}\, r_{o2}$$

$$\overline{dv_{ieq}^2} = \frac{4kT}{\alpha_1^2}\left(R_{on1} + \frac{2/3}{g_{m2}}\right) df$$

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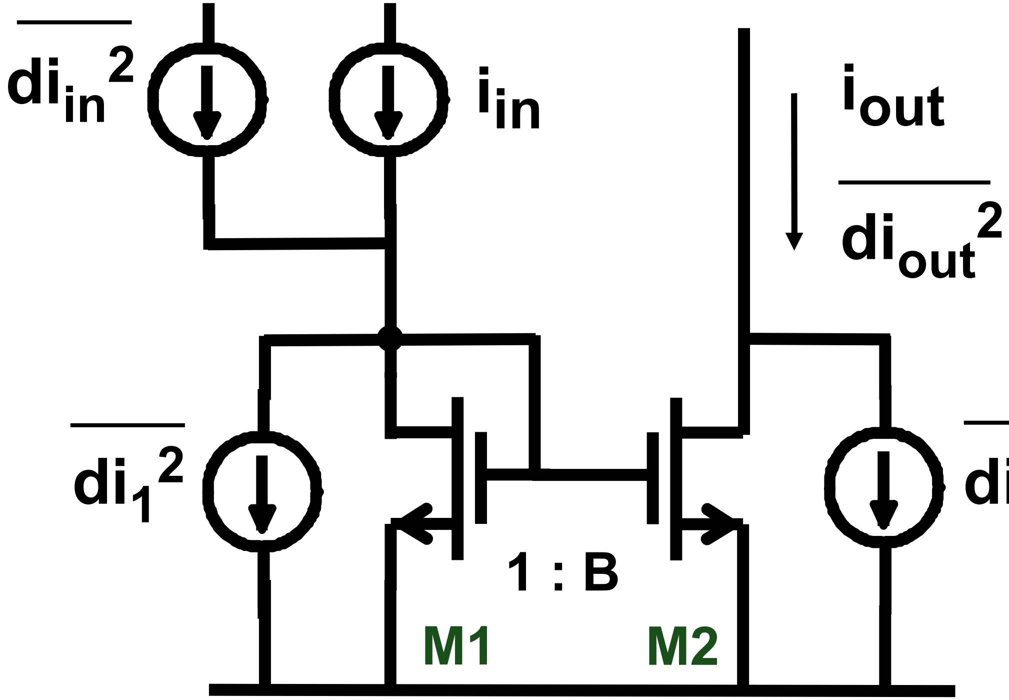
Noise of a current mirror



$$\overline{di_2^2} = \frac{2}{3}\, 4kT\, g_{m2}\, df$$

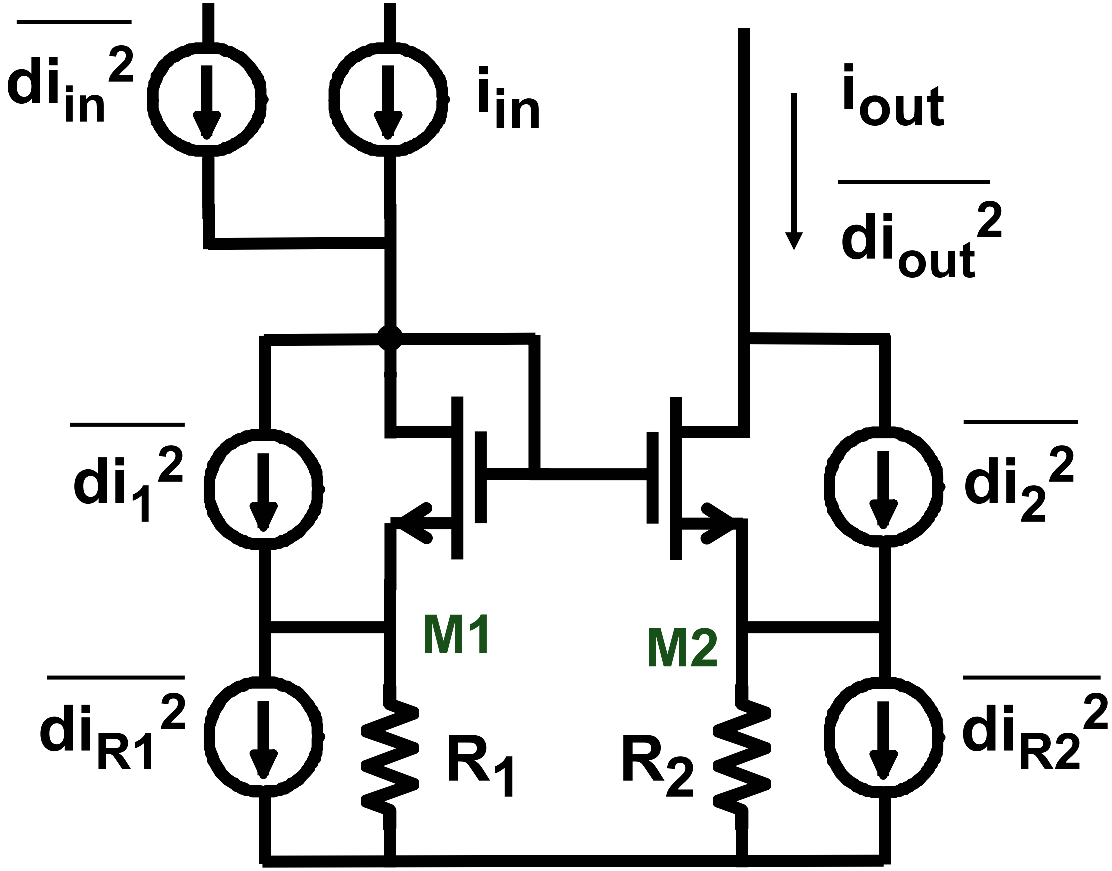
$$\overline{di_{out}^2} = \overline{di_2^2} + B^2 \left(\overline{di_{in}^2} + \overline{di_1^2}\right)$$

Small g_m :

$(W/L) \downarrow$

$(V_{GS}-V_T) \uparrow$

Noise of a current mirror with series R



$$\overline{di_{out}^2} = \overline{di_2^2} + \overline{di_{R2}^2} + \frac{(R_1)^2}{(R_2)^2}\left(\overline{di_1^2} + \overline{di_{R1}^2}\right)$$

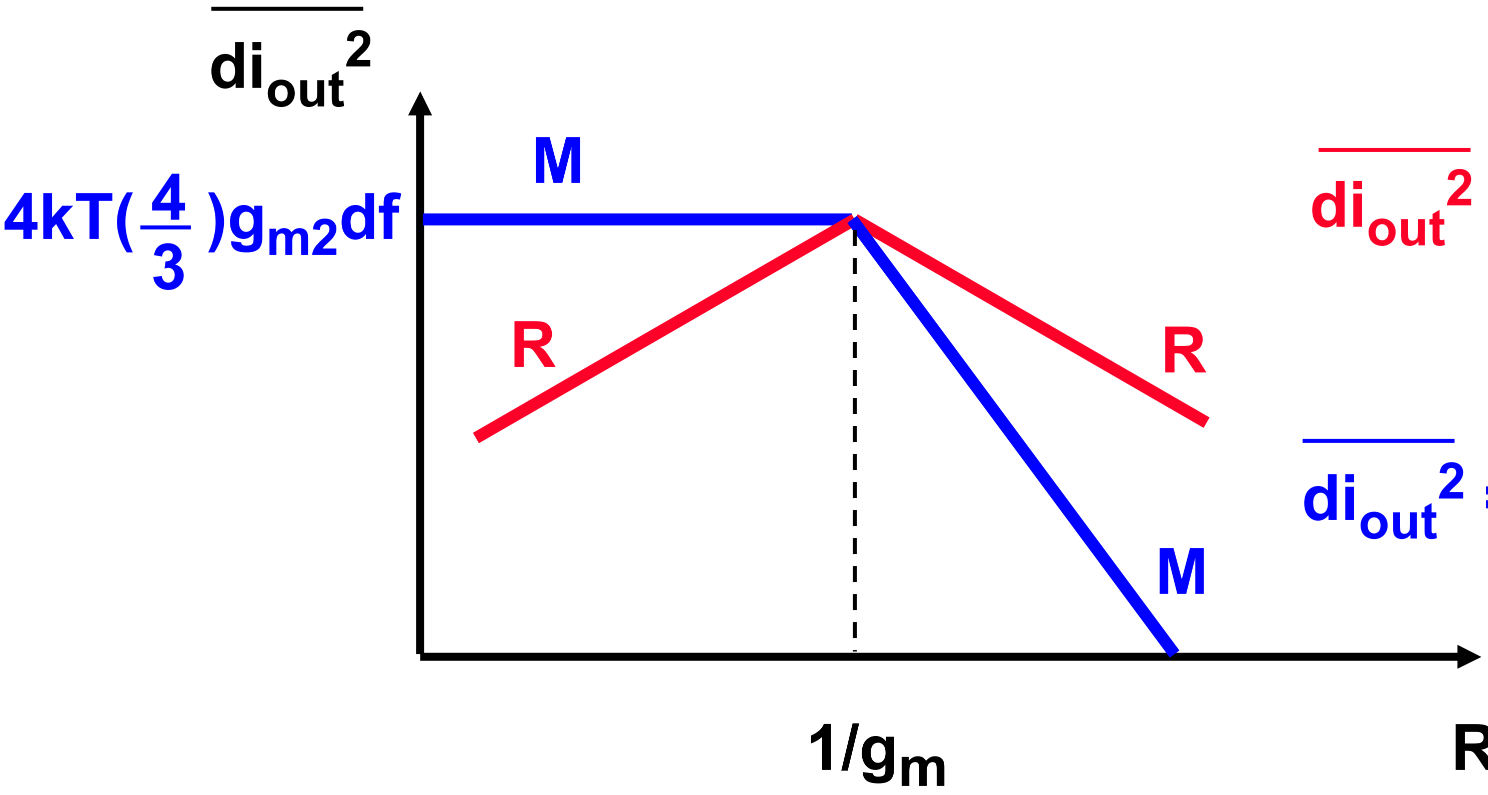
Small g_m :

(W/L) ↓

$(V_{GS}-V_T)$ ↑

R ↑

Noise of a current mirror with series R

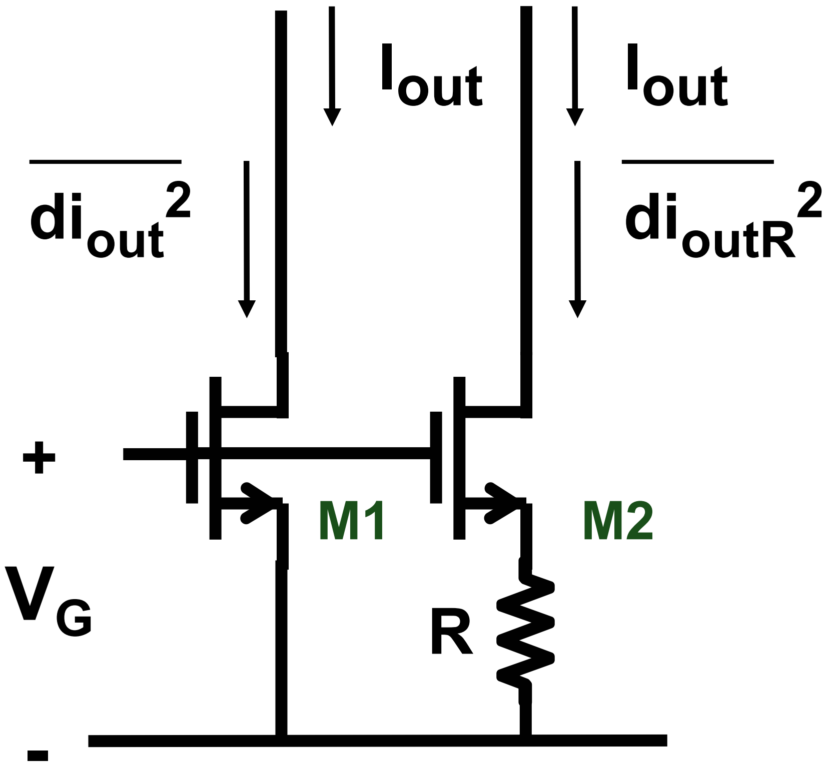


$$\overline{di_{out}^2} = \frac{8kT}{R_2}df$$

$$\overline{di_{out}^2} = \frac{\frac{4}{3}4kTg_{m2}}{(g_{m2}R_2)^2}df$$

Bilotti, JSSC Dec 75, 516-524

Current mirror with series R



$(V_{GS}-V_T)\uparrow$ $(V_{GS}-V_T)\downarrow$

$(W/L)\downarrow$ $(W/L)\uparrow$

Same I_{out} & same V_G :

$$\boxed{\overline{di_{outR}^2} = \overline{di_{out}^2}}$$

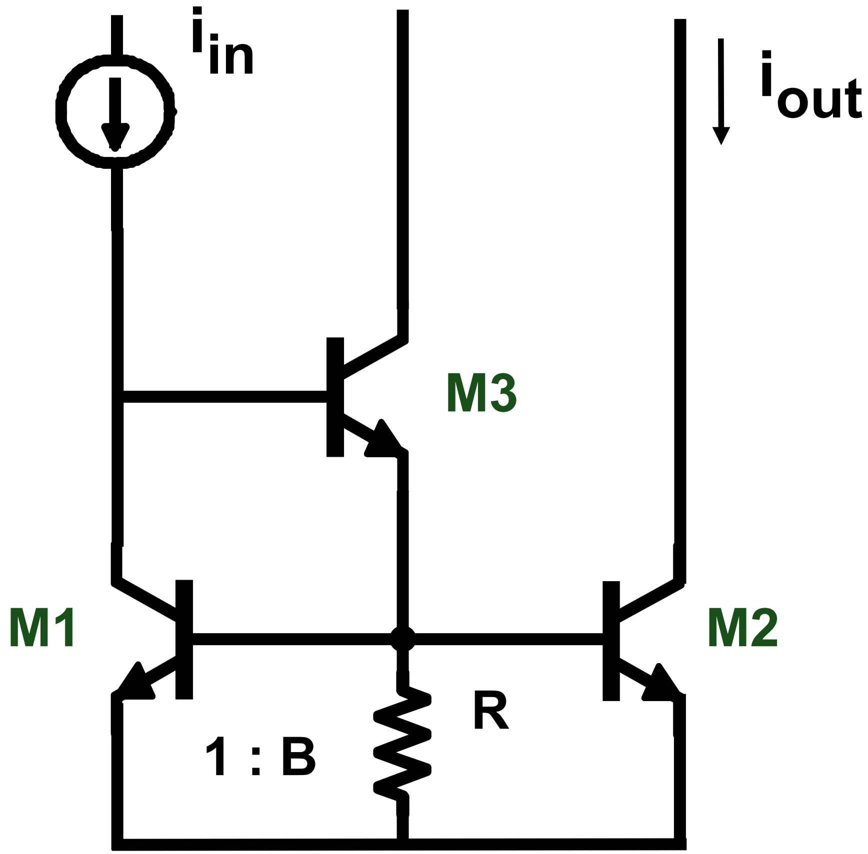
Small g_m :

$(W/L)\downarrow$

$(V_{GS}-V_T)\uparrow$

$V_G\uparrow$

Noise in bipolar current mirror



Noise added by M3 :

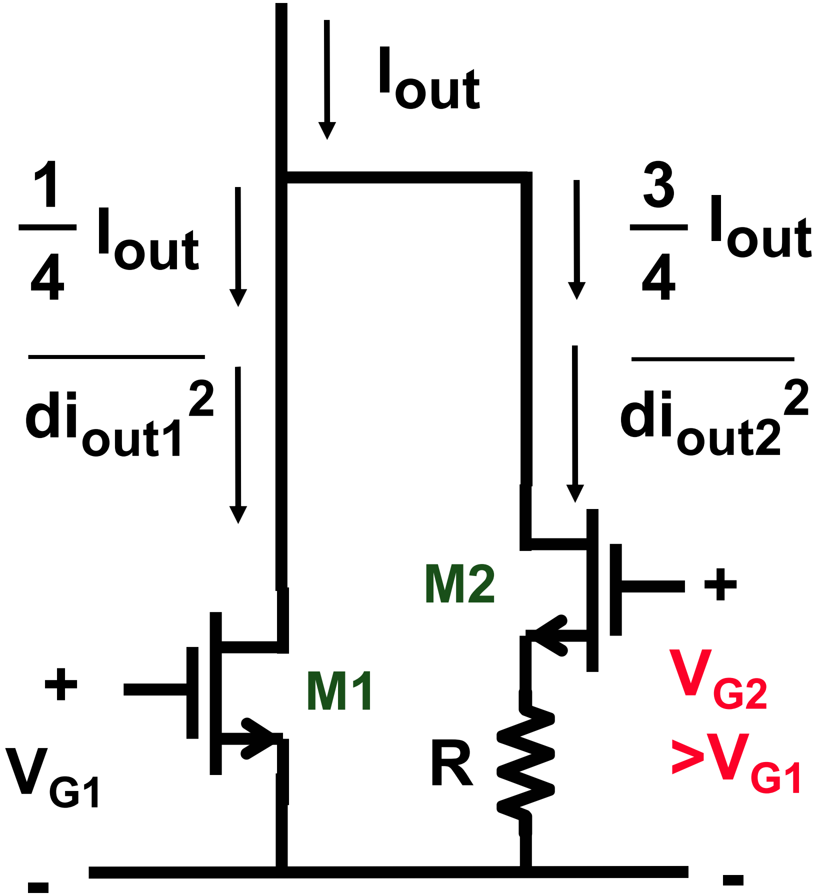
$$\overline{di_{outM3}^2} = 2qI_{C3}\, df$$

Noise added by R :

$$\overline{di_{outR}^2} = 4kT/R\ \ df$$

Both are divided by β_3^2 to be added to the output and are thus negligible !

Low-noise current mirror with series R



Large R or V_{G2}

Same I_{out} & different V_G :

1 MOST: $\overline{di_{out}^2} = \frac{8kT}{3} \frac{2I_{out}\,df}{V_{GS}-V_T}$

2 MOSTs: $V_{G2} > V_{G1}$

$$\overline{di_{out}^2} = \overline{di_{out1}^2} + \overline{di_{out2}^2} =$$

$$\frac{8kT}{3} \frac{2I_{out}\,df}{V_{G1}-V_T} \left(\frac{1}{4} + \frac{9}{16} \frac{V_{G1}-V_T}{V_{G2}-V_{G1}} \right)$$

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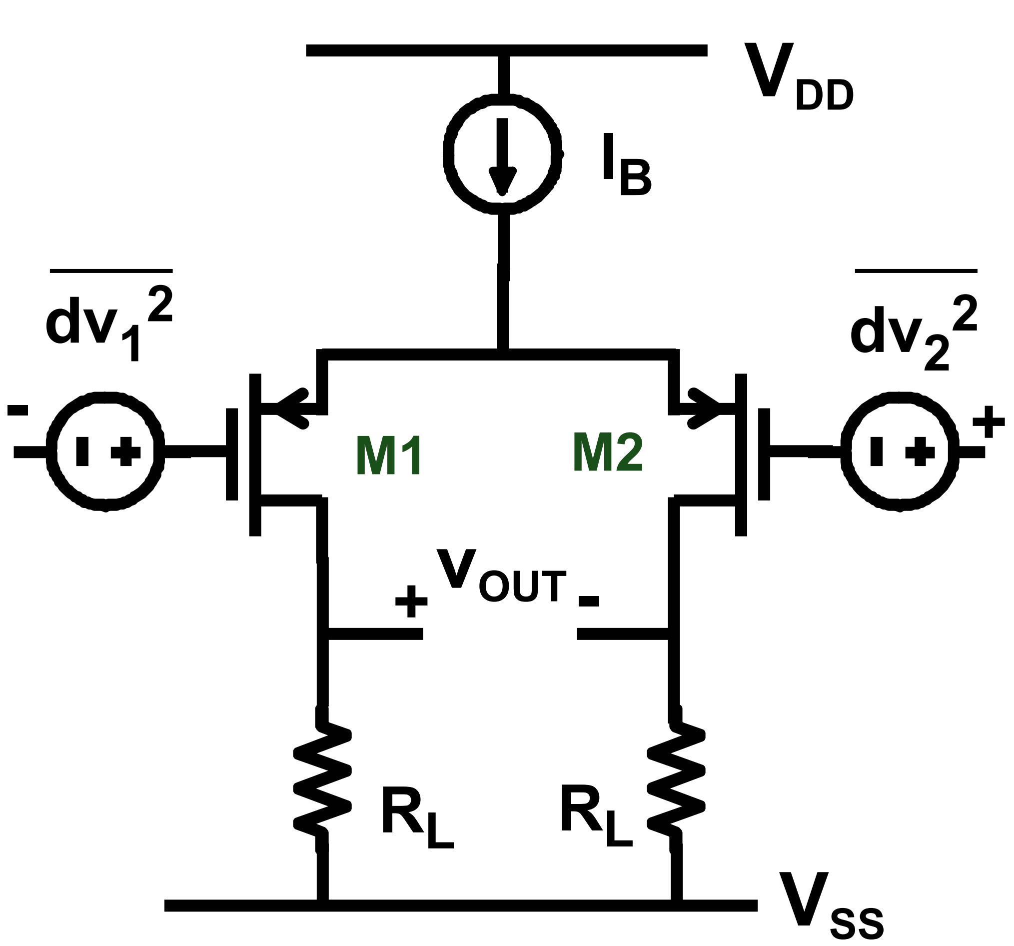
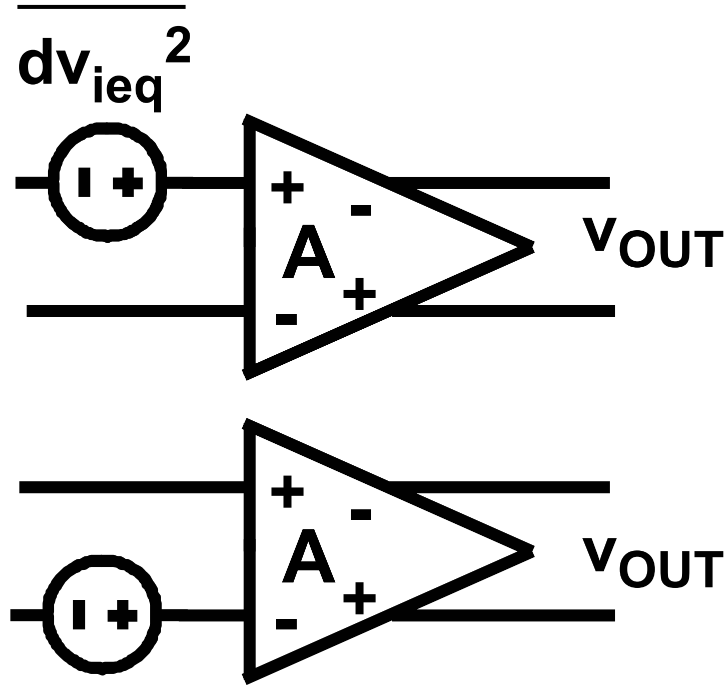
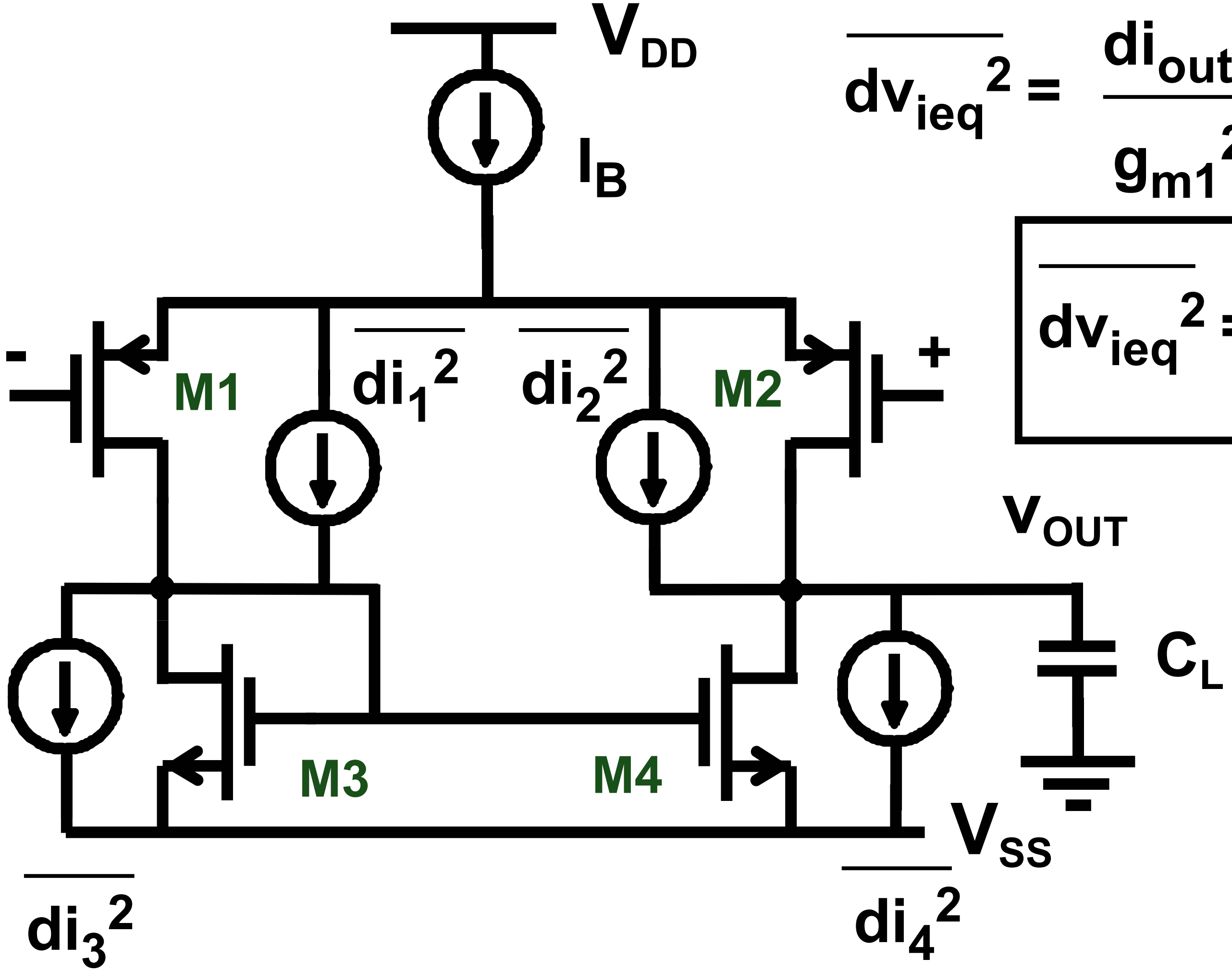
Noise of differential pair





$$\overline{dv_{ieq}^2} = 2\,\overline{dv_1^2}$$

Noise of differential pair with active load



$$\overline{dv_{ieq}^2} = \frac{\overline{di_{out}^2}}{g_{m1}^2} = \frac{2\overline{di_1^2} + 2\overline{di_3^2}}{g_{m1}^2}$$

$$\boxed{\overline{dv_{ieq}^2} = 2\overline{dv_1^2}\left(1 + \frac{g_{m3}}{g_{m1}}\right)}$$

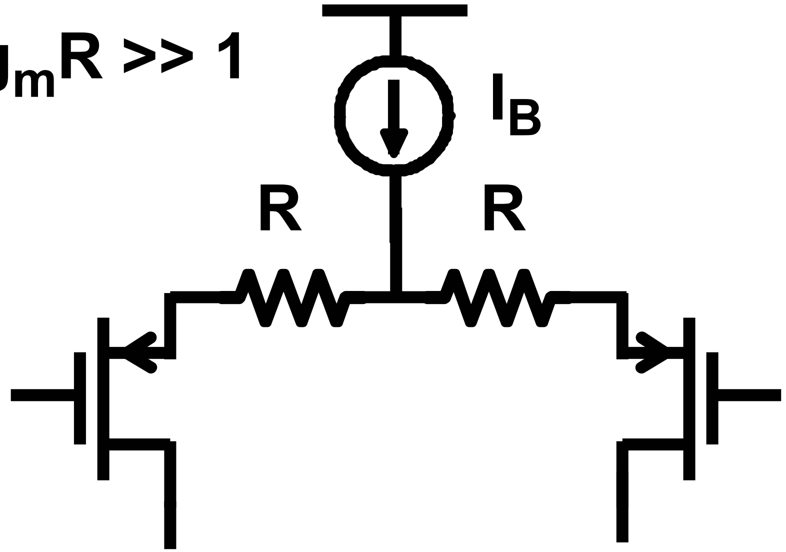
Small g_{m3} :

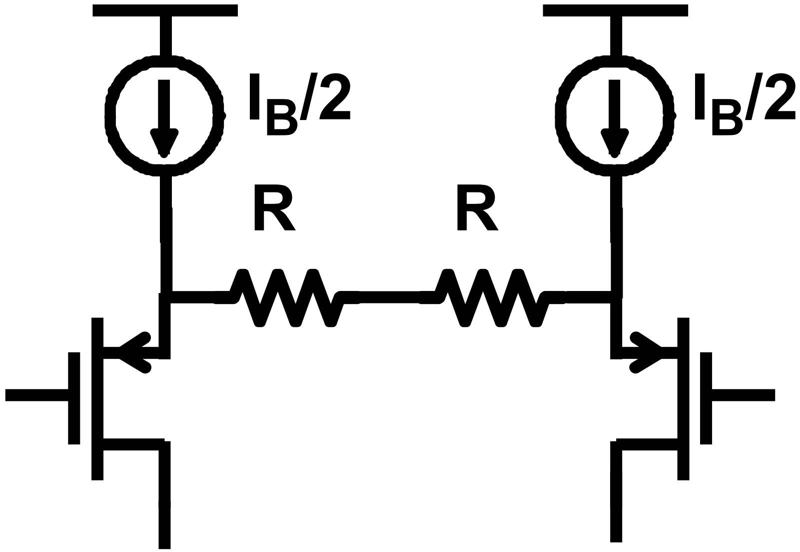
$(W/L)_3 \downarrow$

$(V_{GS}-V_T)_3 \uparrow$

Differential pair with source resistors

$g_m R >> 1$





$$\overline{di_{out}^2} = 2\ \frac{4kT}{R}\ df$$

$$\overline{dv_{in}^2} = 2\ (4kT\ R\ df)$$

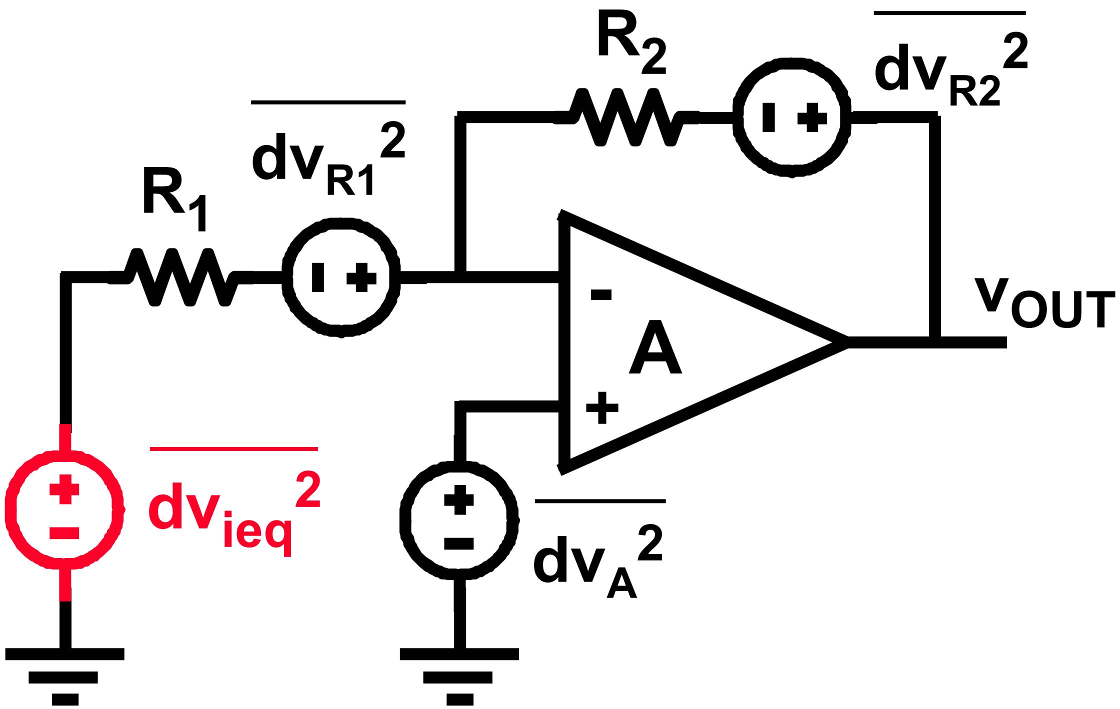
$\overline{di_B^2}$ is negligible

$$\overline{di_{out}^2} = 2\ \left(\frac{4kT}{R}\ df + \overline{di_B^2}\ \right)$$

$$\overline{di_B^2} = 4kT\ 2/3\ g_{mB}\ df$$

$$\overline{dv_{in}^2} = 2\ (4kT\ R\ df)\ (1 + 2/3\ g_{mB} R)$$

Noise of an opamp



$$\overline{dv_{ieq}^2} = \Sigma\, \overline{dv_{out}^2} \left(\frac{R_1}{R_2}\right)^2$$

$$\overline{dv_{out}^2} = \overline{dv_{R1}^2} \left(\frac{R_2}{R_1}\right)^2$$

$$\overline{dv_{out}^2} = \overline{dv_{R2}^2}$$

$$\overline{dv_{out}^2} = \overline{dv_A^2} \left(1 + \frac{R_2}{R_1}\right)^2$$

$$\overline{dv_{ieq}^2} = \overline{dv_{R1}^2} + \overline{dv_{R2}^2} \left(\frac{R_1}{R_2}\right)^2 + \overline{dv_A^2} \left(1 + \frac{R_1}{R_2}\right)^2 \approx \overline{dv_{R1}^2} + \overline{dv_A^2}$$

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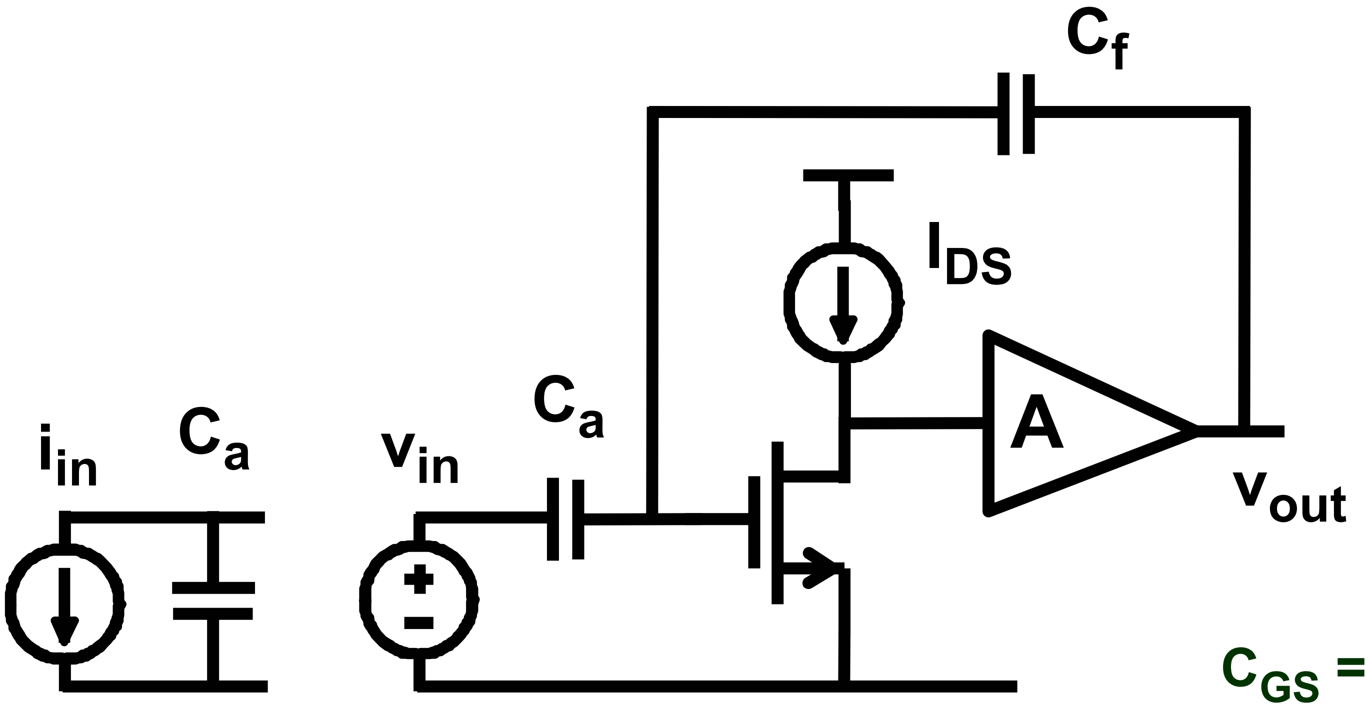
Ref.: Z.Y.Chang, W.Sansen, Low-noise wide-band amplifiers, Kluwer AP, 1991

Capacitive-source amplifier



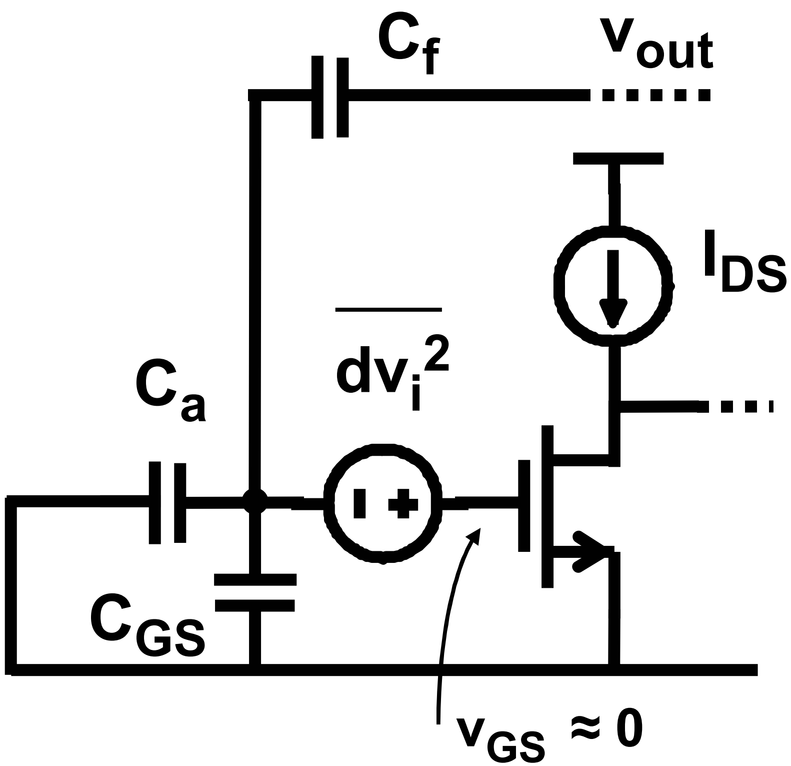
$$A_v = \frac{C_a}{C_f}$$

C_a = 5 pF
C_f = 1 pF

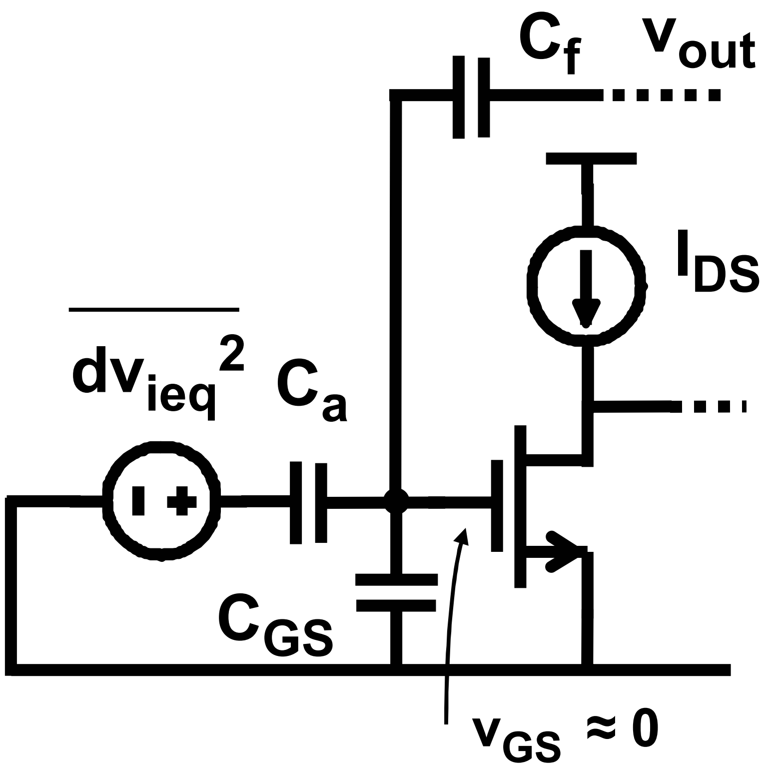
C_{GS} = kW   k≈ 2 fF/μm

W_{opt} ? I_{DSopt} ? S/N_{opt} for V_{in} = 10 mV_{RMS} ?

Capacitive noise matching - 1



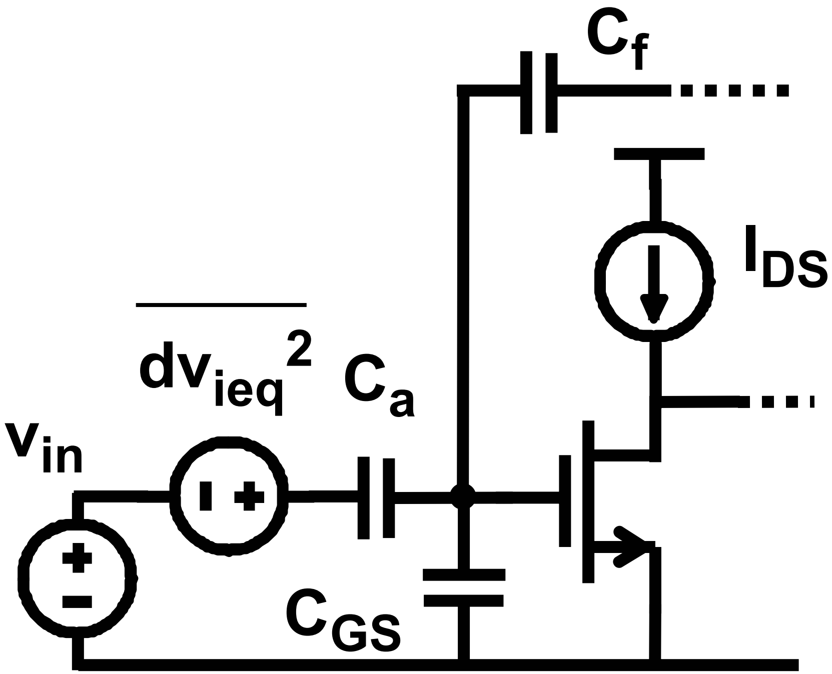
$$\frac{v_{out}}{v_i} = \frac{C_f + C_a + C_{GS}}{C_f}$$



$$\frac{v_{out}}{v_{ieq}} = \frac{C_a}{C_f}$$

No Miller with C_{DG} !!!

Capacitive noise matching - 2



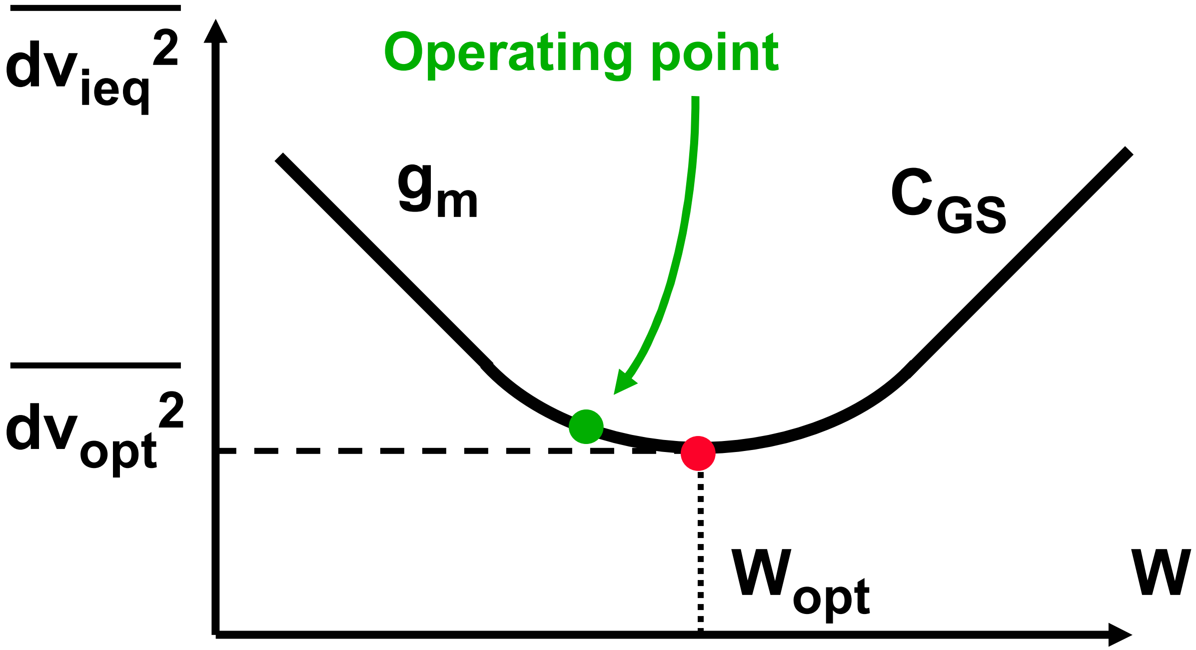
$$\overline{dv_{ieq}^2} = \frac{(C_f+C_a+C_{GS})^2}{C_a^2}\,\overline{dv_i^2}$$

$$\overline{dv_i^2} = \frac{8kT}{3}\,\frac{1}{g_m}\,df$$

$$g_m = 2\,K'_n\,\frac{W}{L}\,(V_{GS}-V_T)$$

$$\overline{dv_{ieq}^2} = \frac{(C_f + C_a + kW)^2}{C_a^2}\,\frac{L}{W}\,\frac{8kT}{3}\,\frac{1}{2\,K'_n\,(V_{GS}-V_T)}$$

Capacitive noise matching - 3

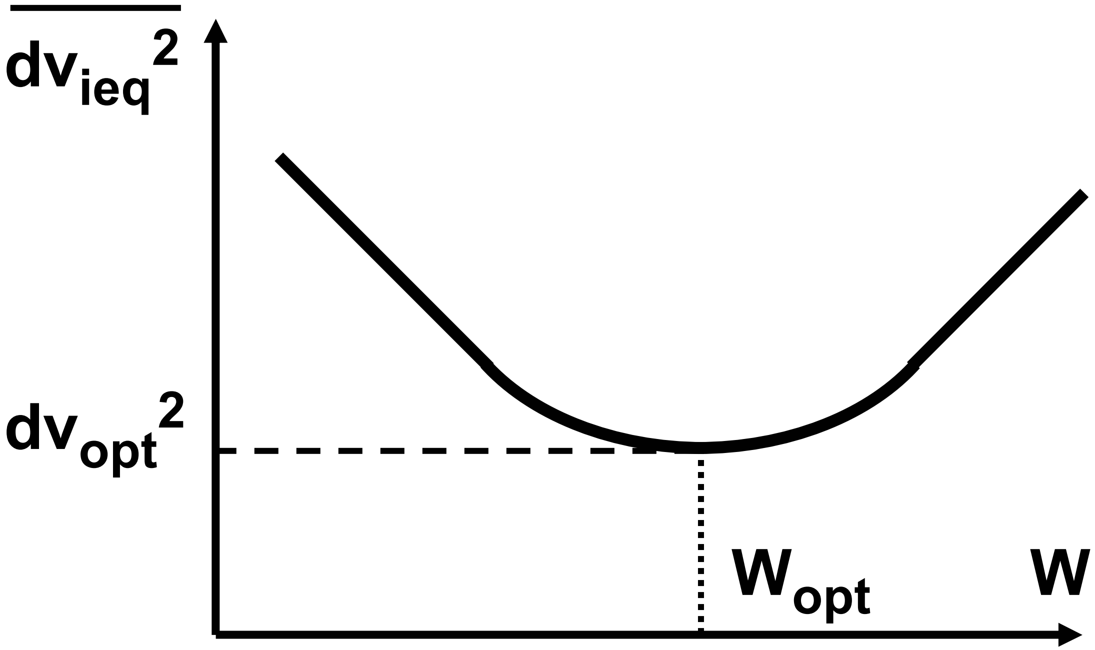


Noise matching where
$C_{GS} = C_f + C_a$

$$\boxed{W_{opt} = \frac{C_f + C_a}{k}}$$

$$\overline{dv_{ieq}^2} = \frac{(C_f + C_a + kW)^2}{C_a^2} \frac{L}{W} \frac{8kT}{3} \frac{1}{2\,K'_n\,(V_{GS}-V_T)}$$

Capacitive noise matching - 4



$$W_{opt} = \frac{C_f + C_a}{k}$$

C_{GSopt}

I_{DSopt} , g_{mopt}

$$\overline{dv_{opt}^2} = 4 \frac{8kT}{3} \frac{df}{g_{mopt}}$$

$$BW_n = \frac{\pi}{2} BW = \frac{\pi}{2} \frac{f_T}{A_v} = \frac{1}{4A_v} \frac{g_{mopt}}{C_{GSopt}}$$

$$\frac{S}{N_{opt}} = \frac{10\ mV_{RMS}}{\sqrt{\overline{dv_{opt}^2}\, BW_n}}$$

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