Distortion in elementary transistor circuits

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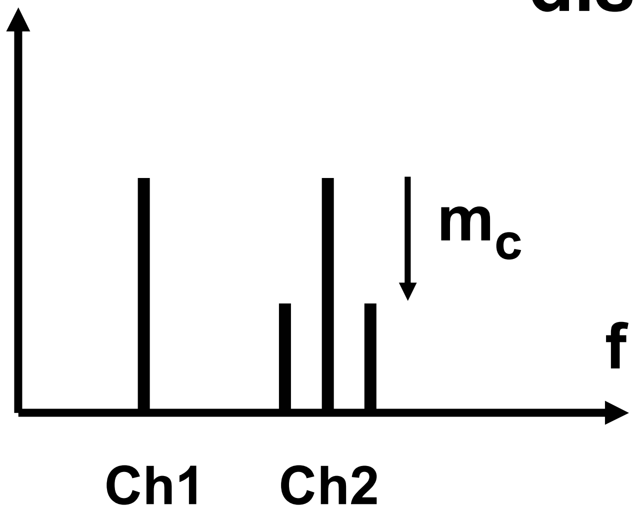
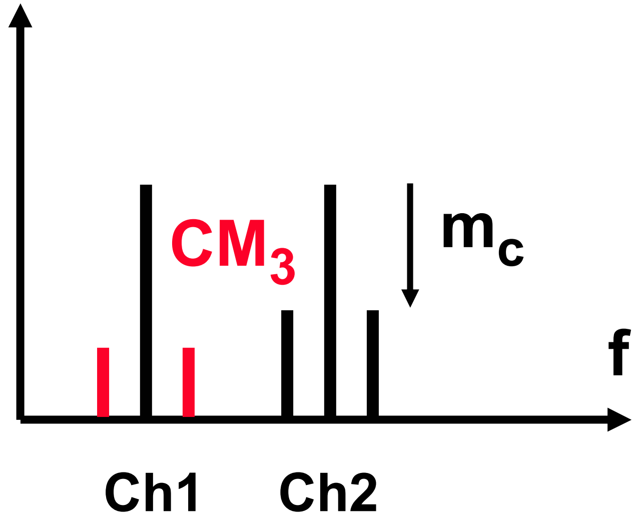
Why distortion ?

Non-linearity : distortion





Mixing up channels !!!

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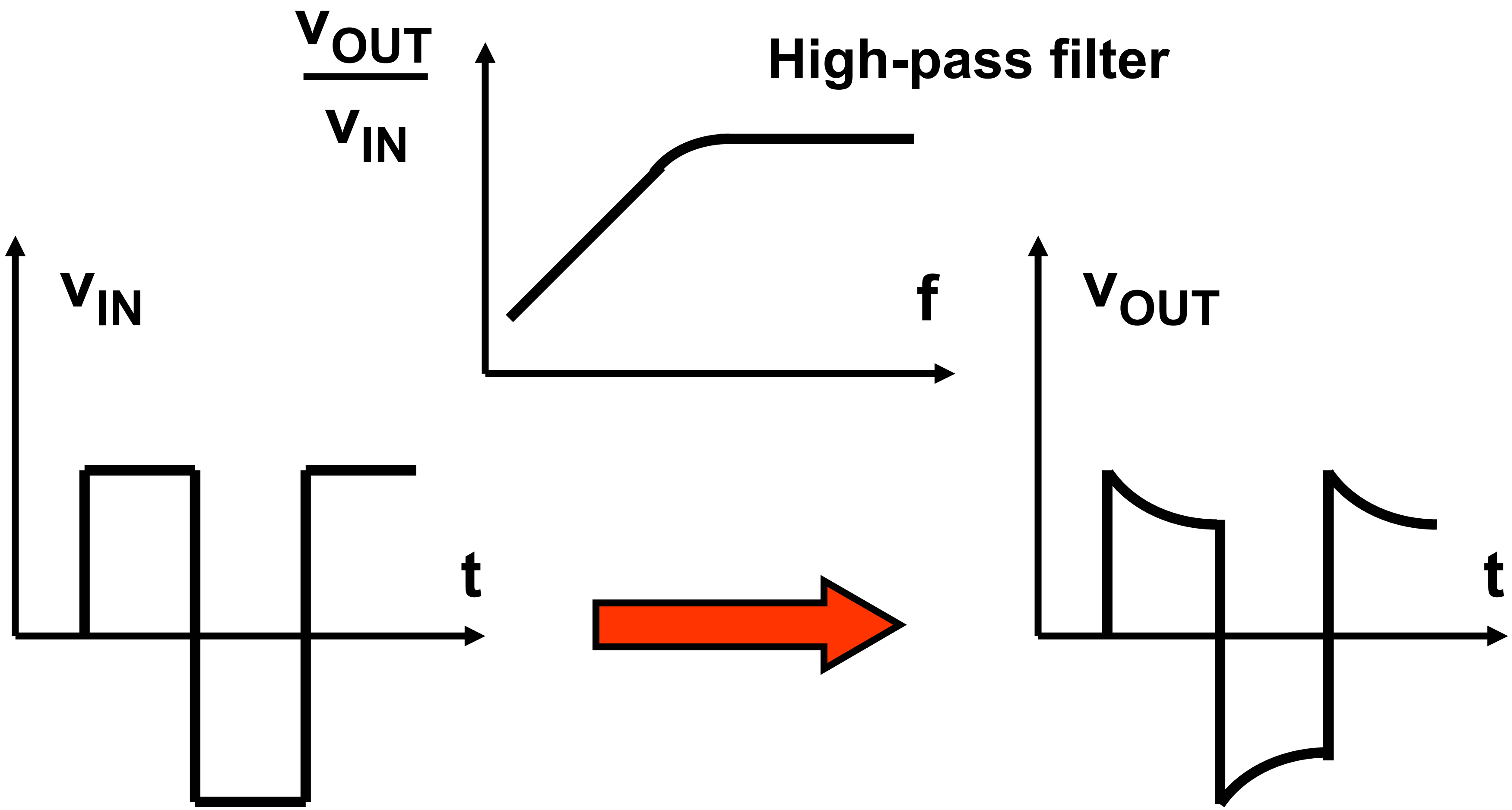
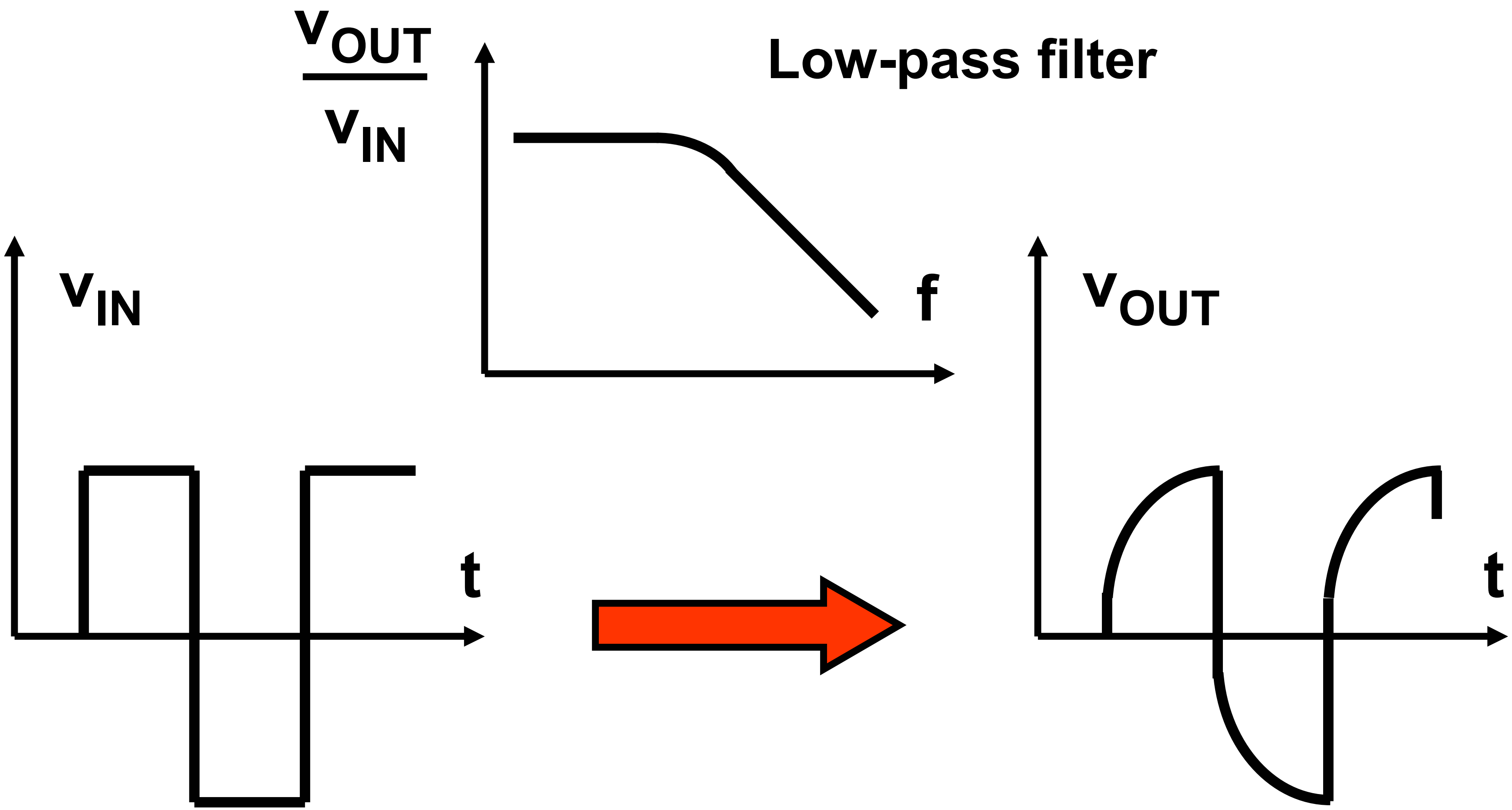
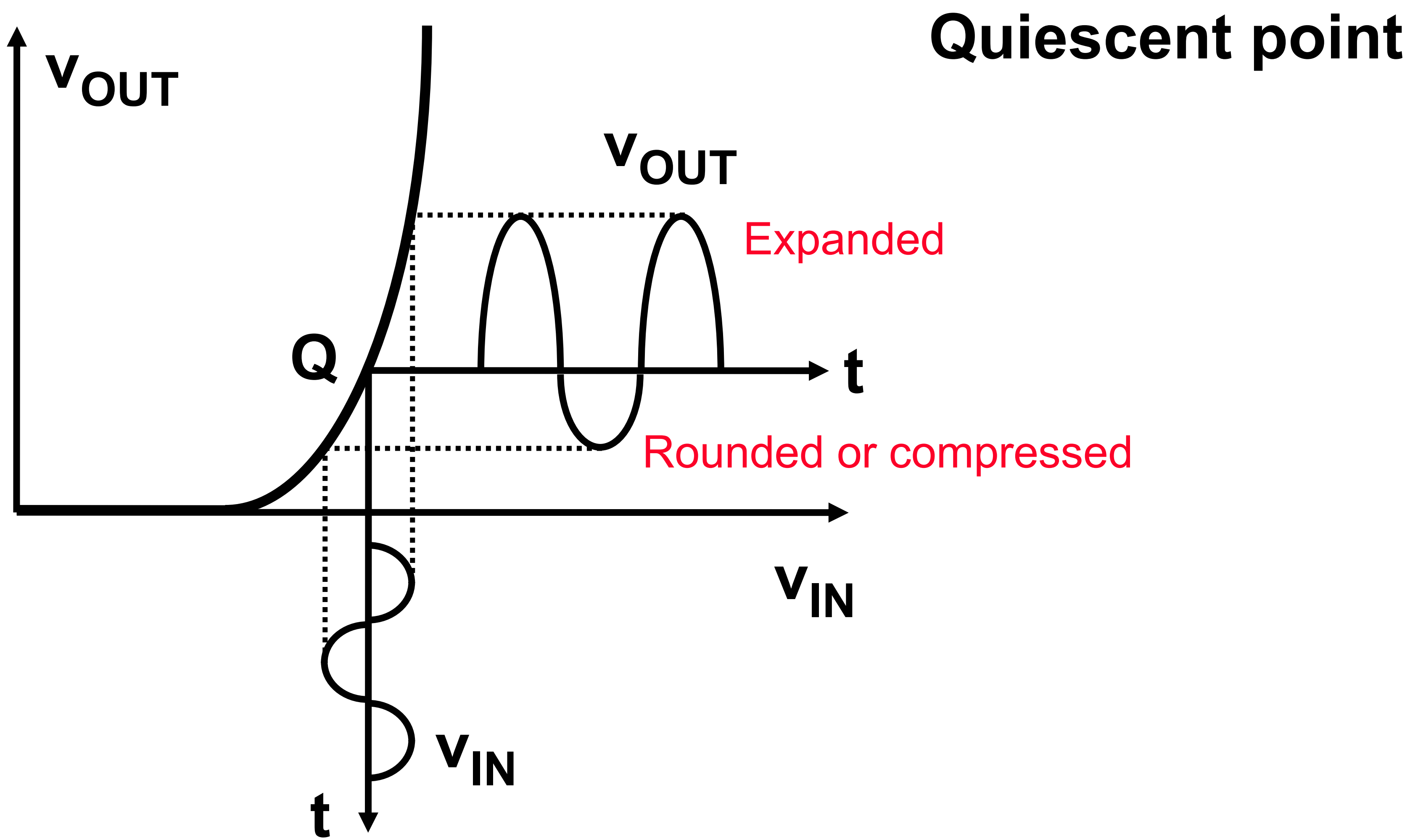
Linear distortion



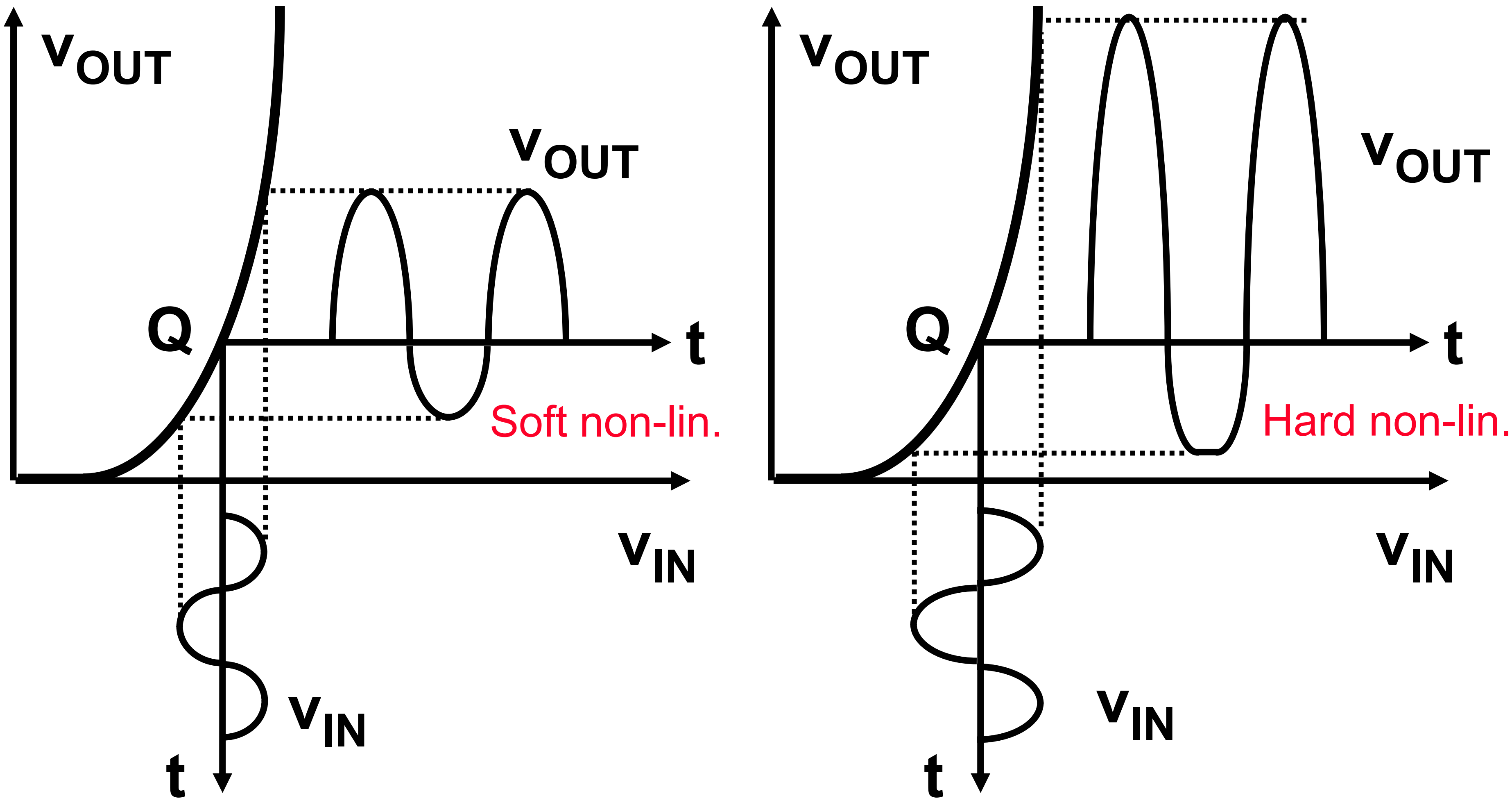
Linear distortion



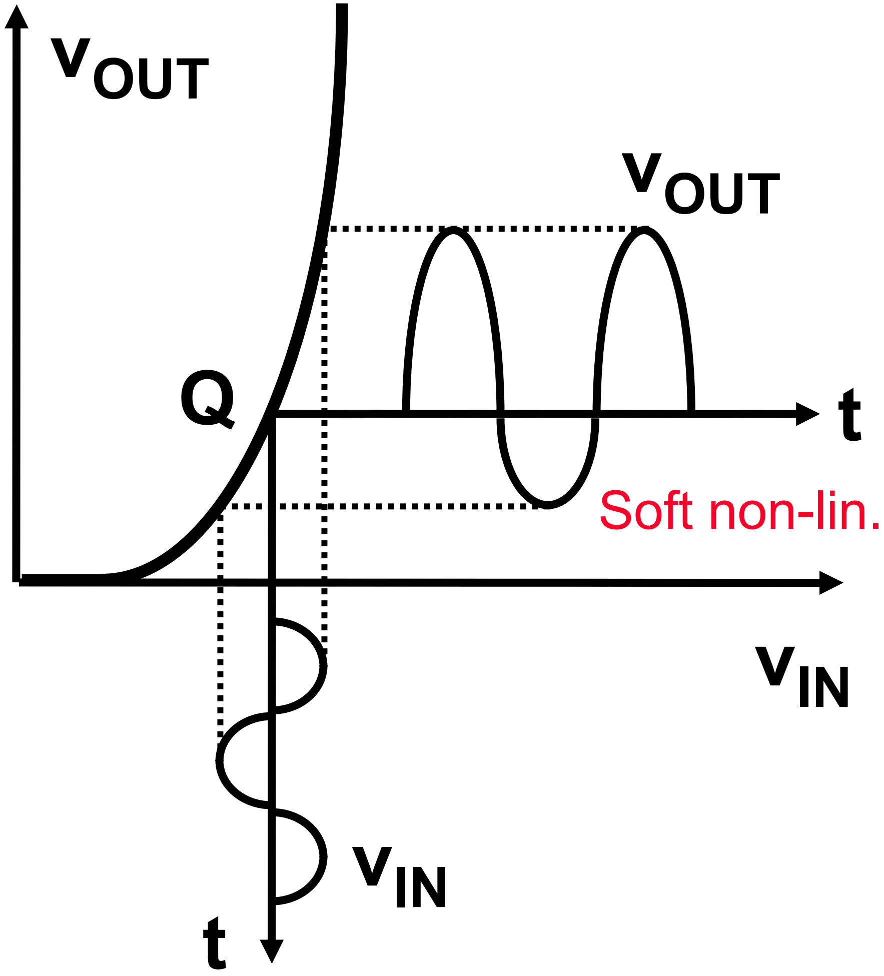
Non-linear distortion



Soft and hard non-linearity



Non-linearity : by power series



$v_{IN}(t) \Rightarrow v_{OUT}(t)$

$$v_{OUT} = a_0 + a_1 v_{IN} + a_2 v_{IN}^2 + a_3 v_{IN}^3 + \ldots$$

How to find $a_0, a_1, a_2, a_3, \ldots$

$$y = a_0 + a_1 u + a_2 u^2 + a_3 u^3 + \ldots$$

$$a_0 = y\Big|_{u=0} \qquad a_1 = \frac{dy}{du}\Big|_{u=0}$$

$$a_2 = \frac{1}{2}\,\frac{d^2y}{du^2}\Big|_{u=0} \qquad a_3 = \frac{1}{6}\,\frac{d^3y}{du^3}\Big|_{u=0}$$

Definition of harmonic distortion HD

$$y = a_0 + a_1 u + a_2 u^2 + a_3 u^3 + \ldots$$

With $u = U \cos \omega t$

$\cos^2 x = 1/2\ (1 + \cos 2x)$

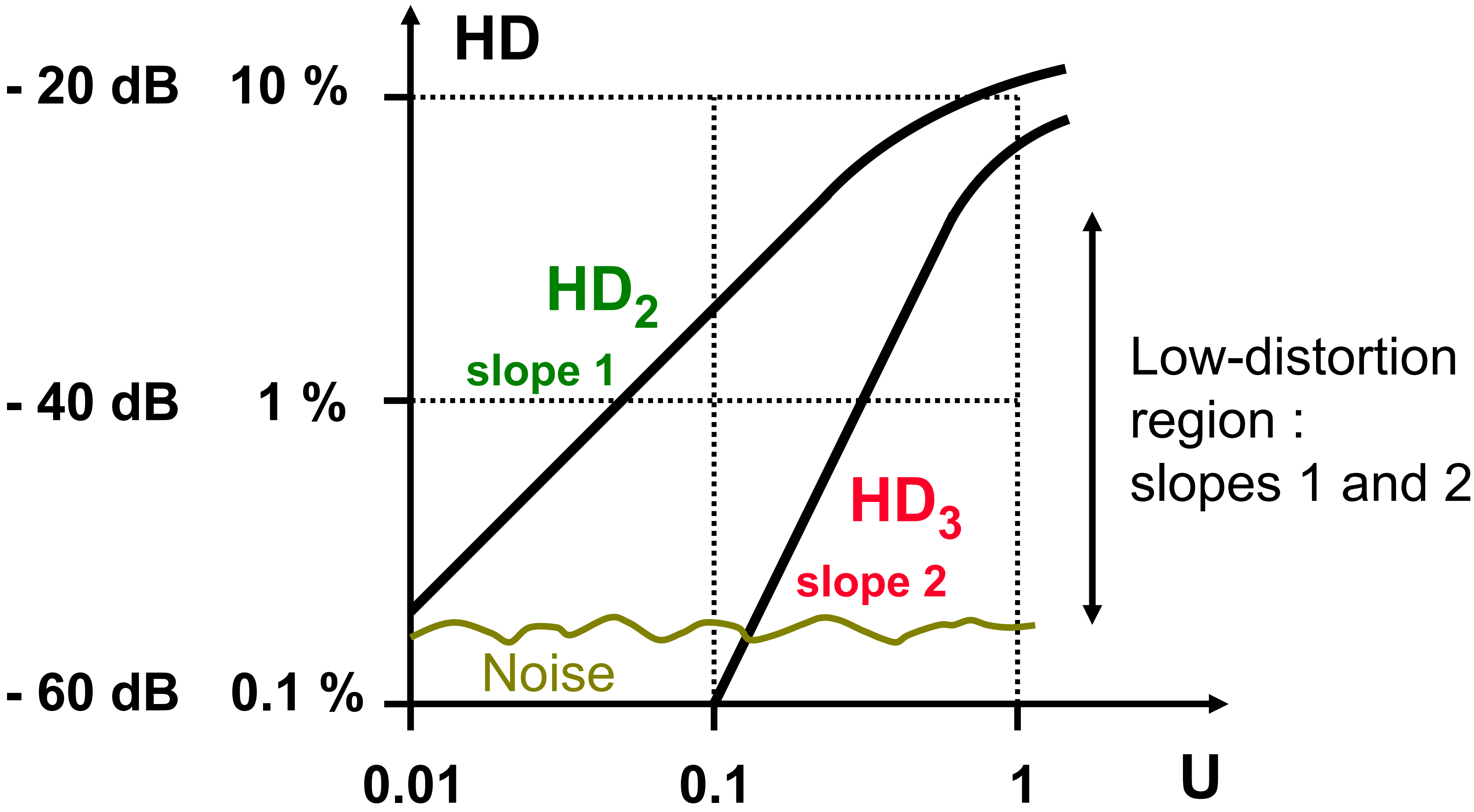
$\cos^3 x = 1/4\ (3 \cos x + \cos 3x)$

$$y = a_0 + a_1 u + a_2 u^2 + a_3 u^3 + \ldots \quad = a_0 +$$

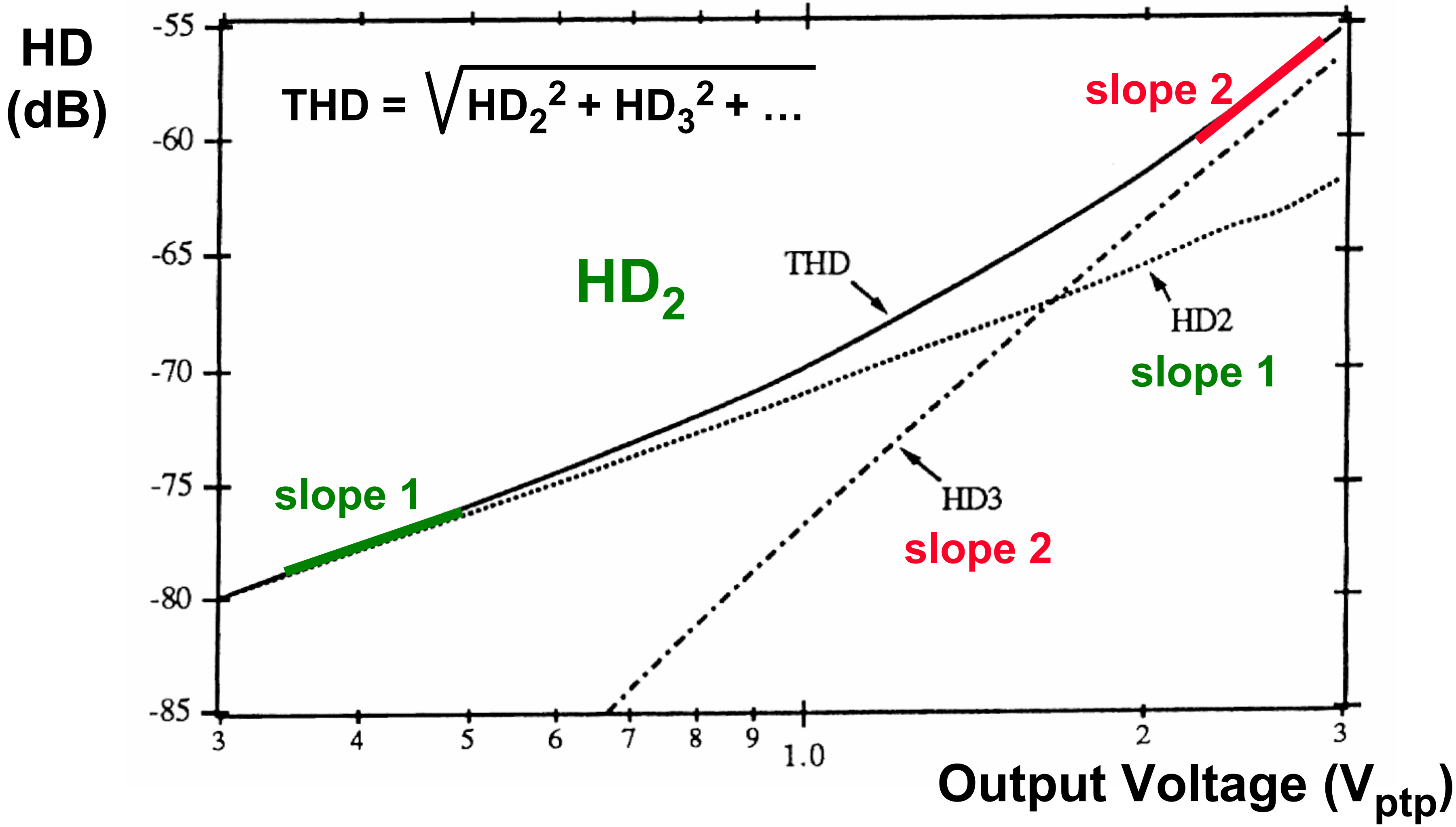
$$(a_1 + \frac{3}{4} a_3 U^2)\, U \cos \omega t + \frac{a_2}{2} U^2 \cos 2\omega t + \frac{a_3}{4} U^3 \cos 3\omega t$$

$$\boxed{HD_2 = \frac{1}{2} \frac{a_2}{a_1} U} \qquad \boxed{HD_3 = \frac{1}{4} \frac{a_3}{a_1} U^2}$$

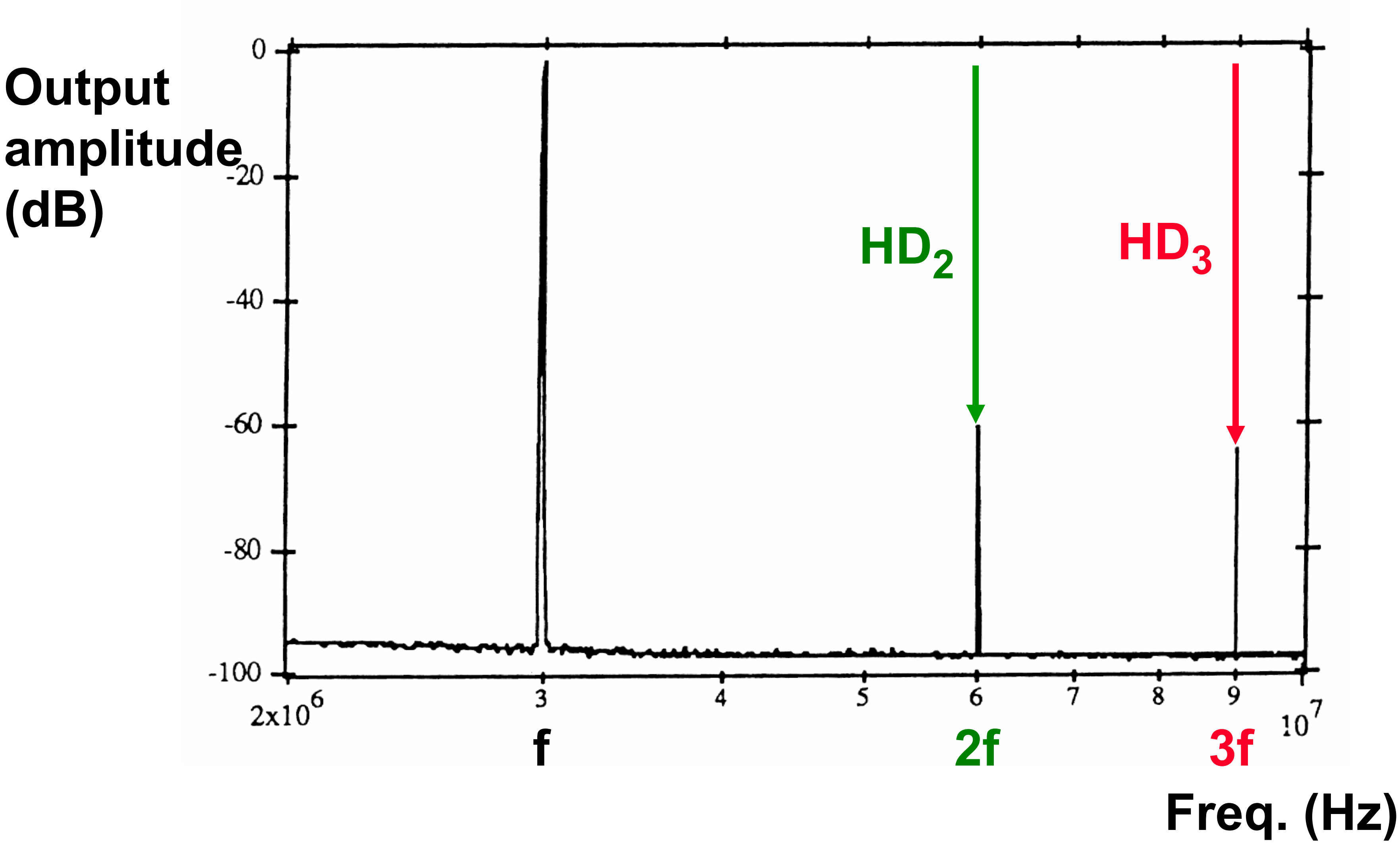
Amplitude HD versus input signal



HD of a resistor



Output spectrum



Definition of intermodulation distortion IM

$$y = a_0 + a_1 u + a_2 u^2 + a_3 u^3 + \dots$$

with $u = U\,(\cos \omega_1 t + \cos \omega_2 t)$

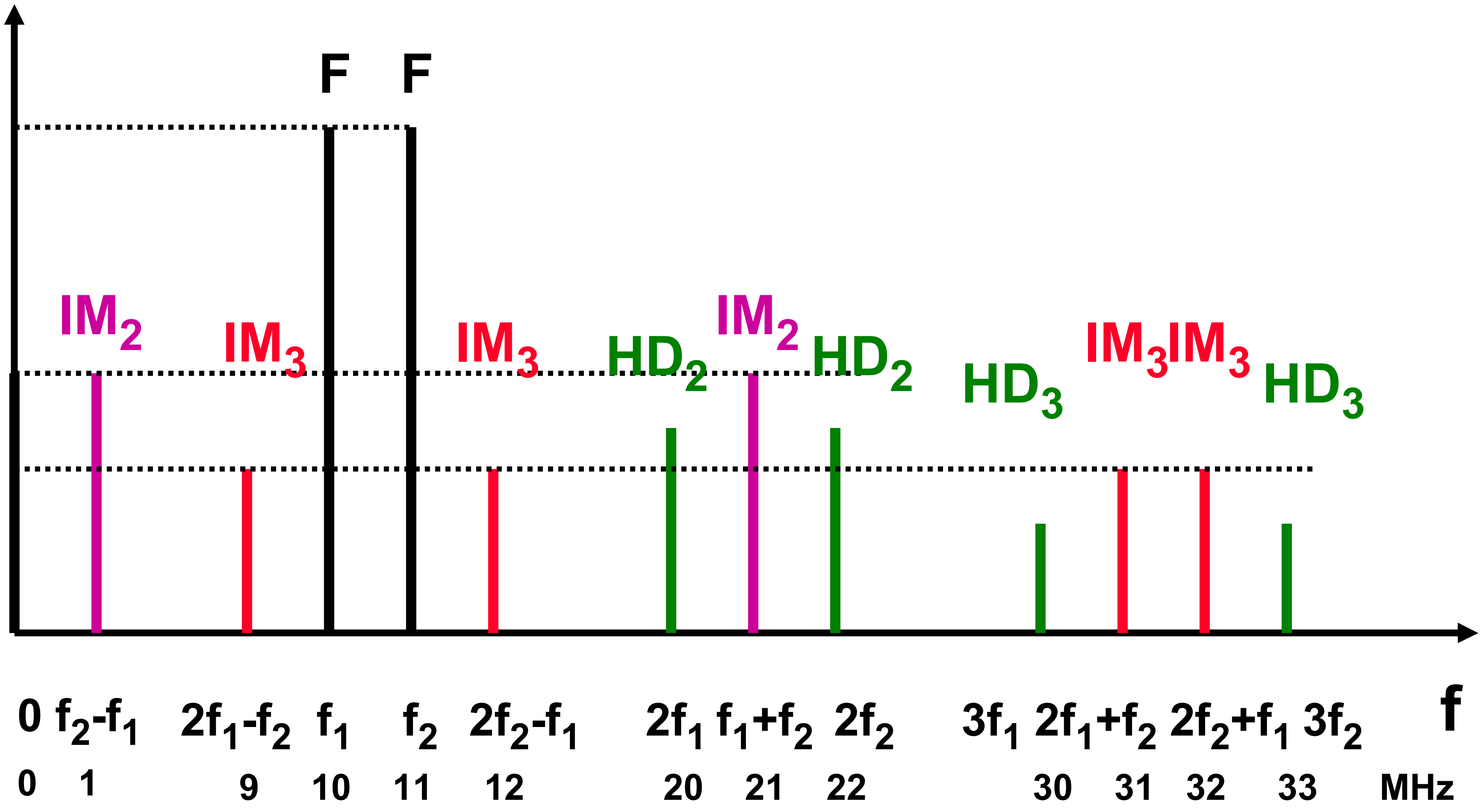
$$y = a_0 + \dots$$

IM_2 at $\omega_1 \pm \omega_2$

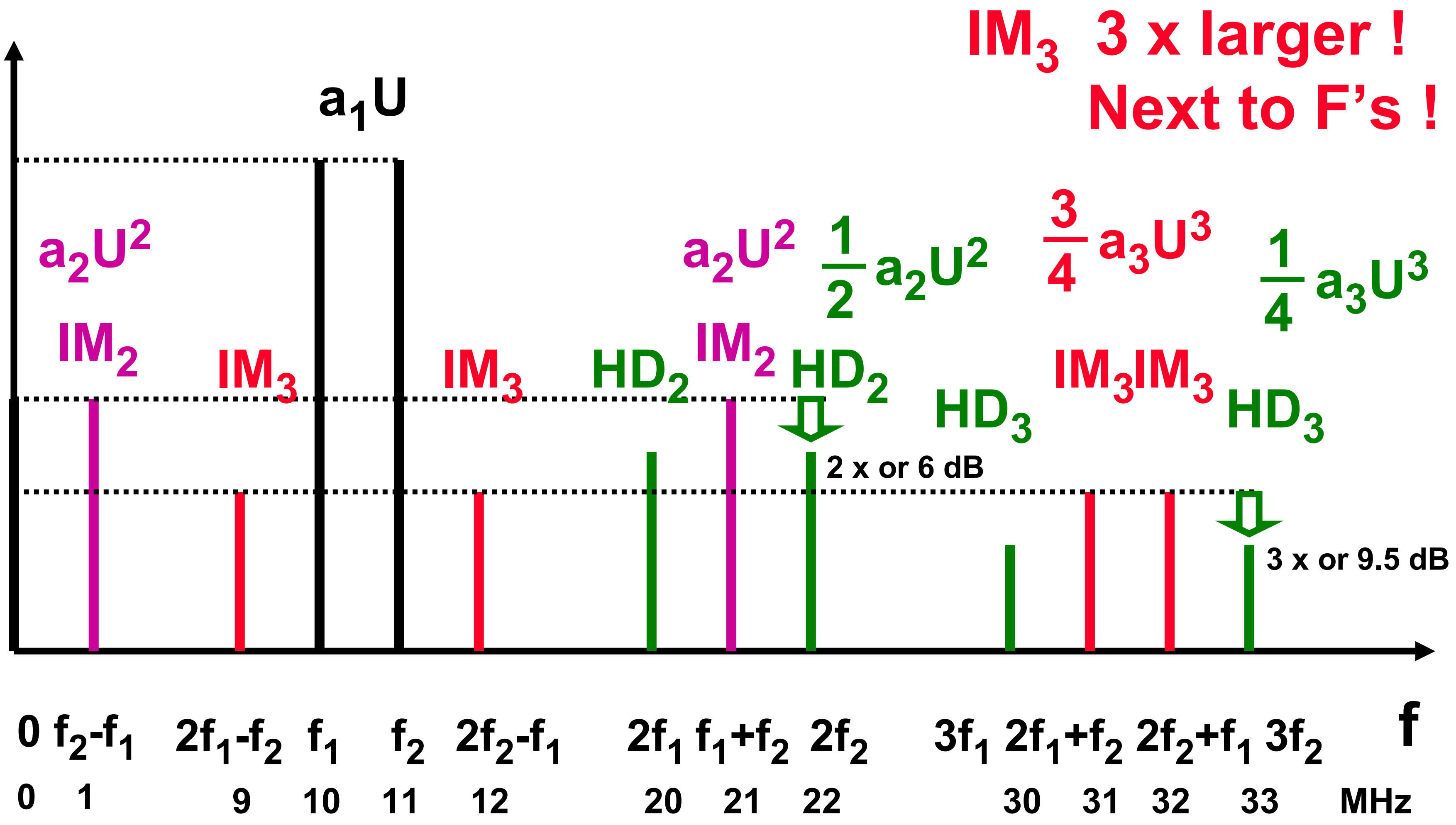
IM_3 at $2\omega_1 \pm \omega_2$ and $\omega_1 \pm 2\omega_2$

$$\boxed{IM_2 = 2\,HD_2 = \frac{a_2}{a_1}\,U} \qquad \boxed{IM_3 = 3\,HD_3 = \frac{3}{4}\,\frac{a_3}{a_1}\,U^2}$$

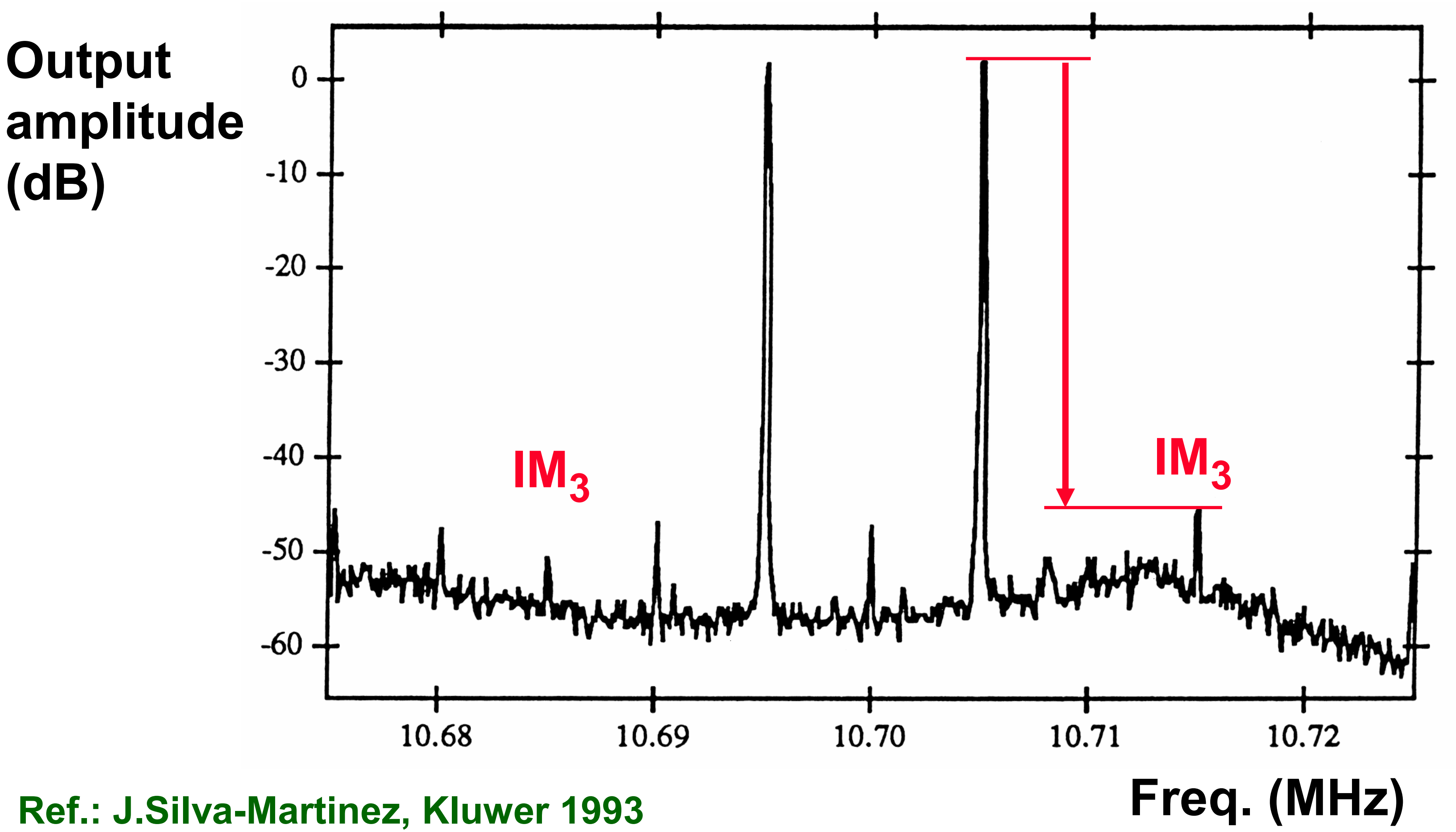
IM components



IM components

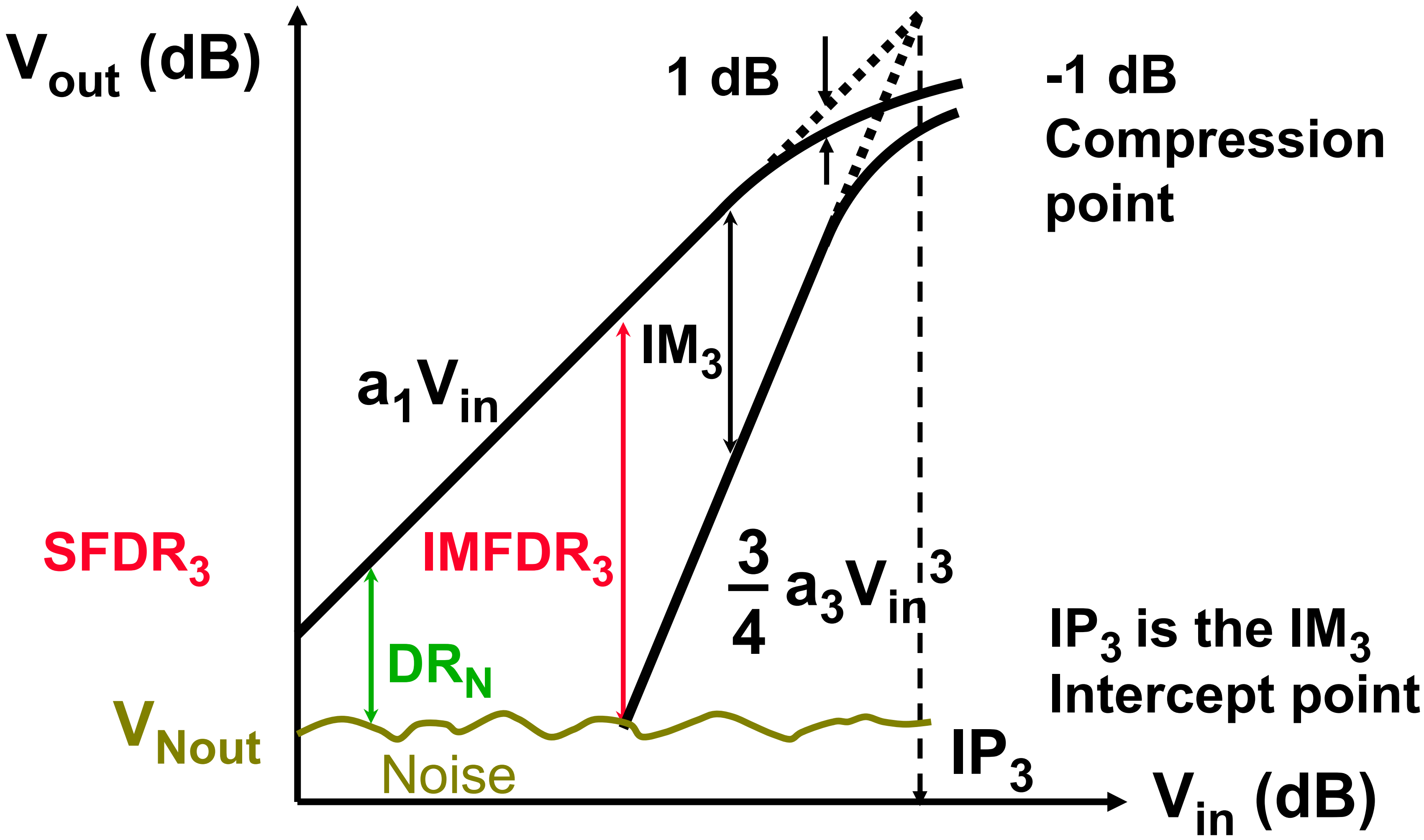


Output spectrum of amplifier for IM

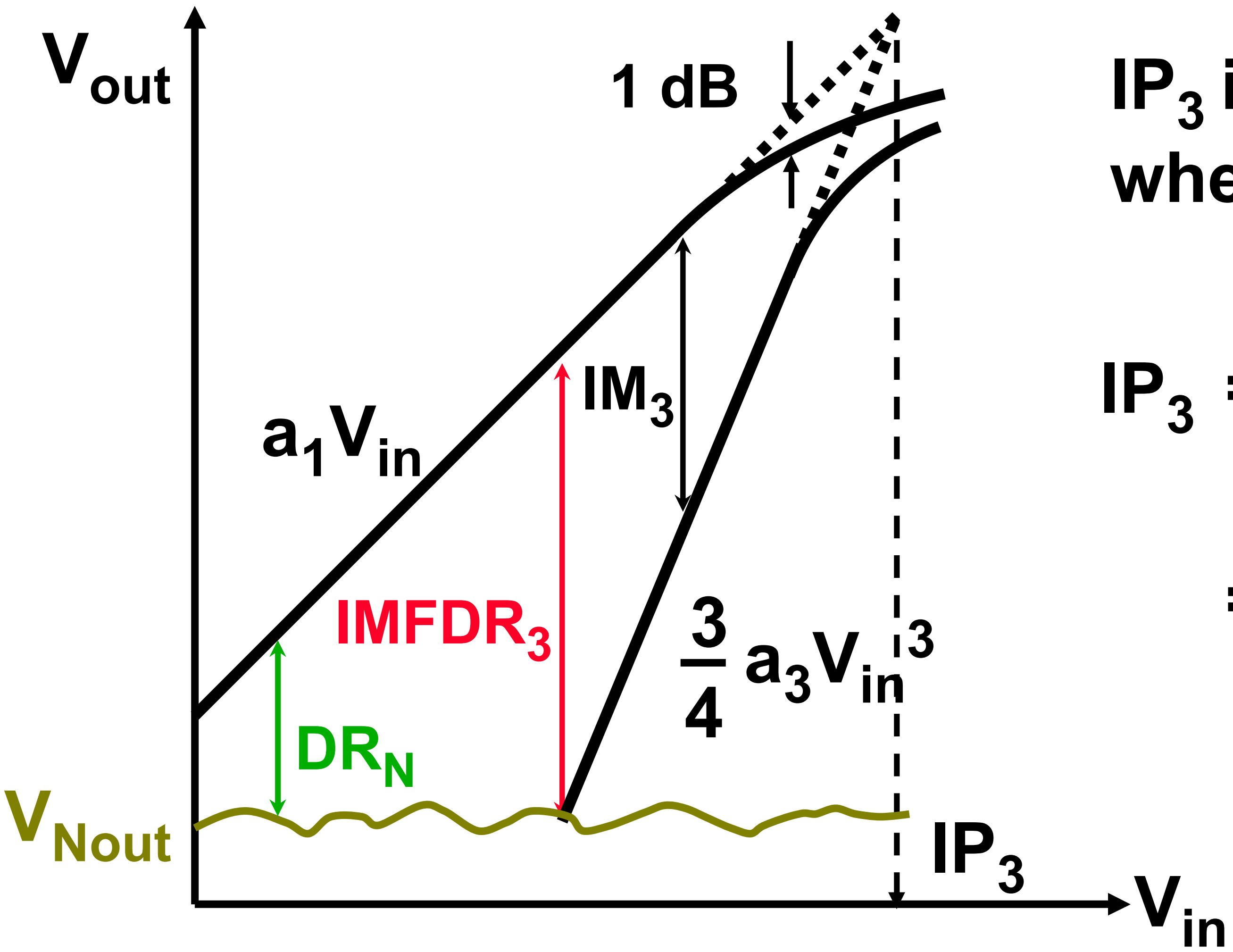


Ref.: J.Silva-Martinez, Kluwer 1993

Amplitude IM3 versus input signal



Relation IP_3 and IM_3



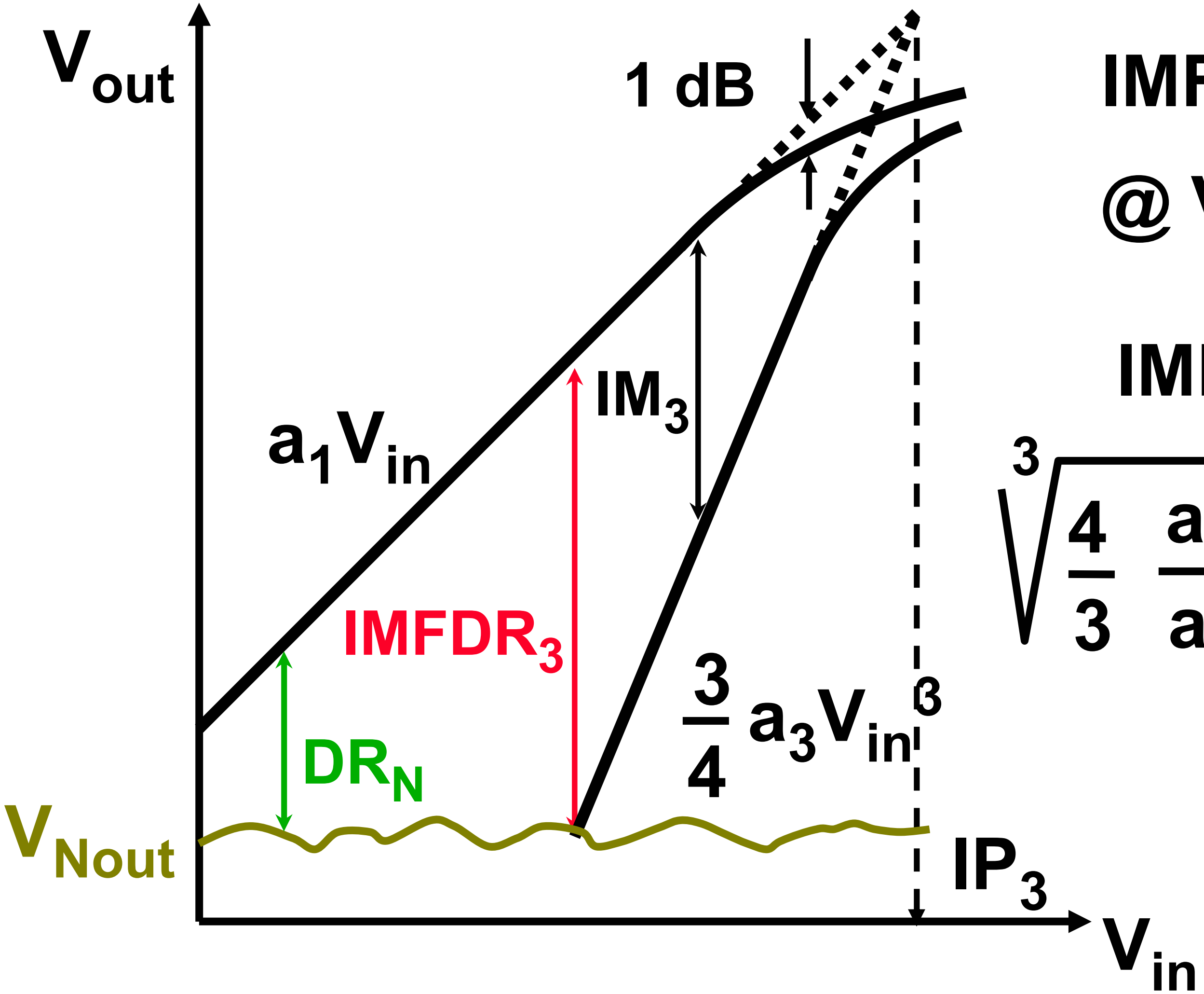
IP_3 is V_{in}
where IM_3 = F

$$IP_3 = \sqrt{\frac{4}{3}\,\frac{a_1}{a_3}}$$

$$= V_{in}\,\frac{1}{\sqrt{IM_3}}$$

$$= V_{indB} - \frac{1}{2}IM_{3dB}$$

Relation IMFDR$_3$ and IP$_3$



$$IMFDR_3 = \text{max DR}$$

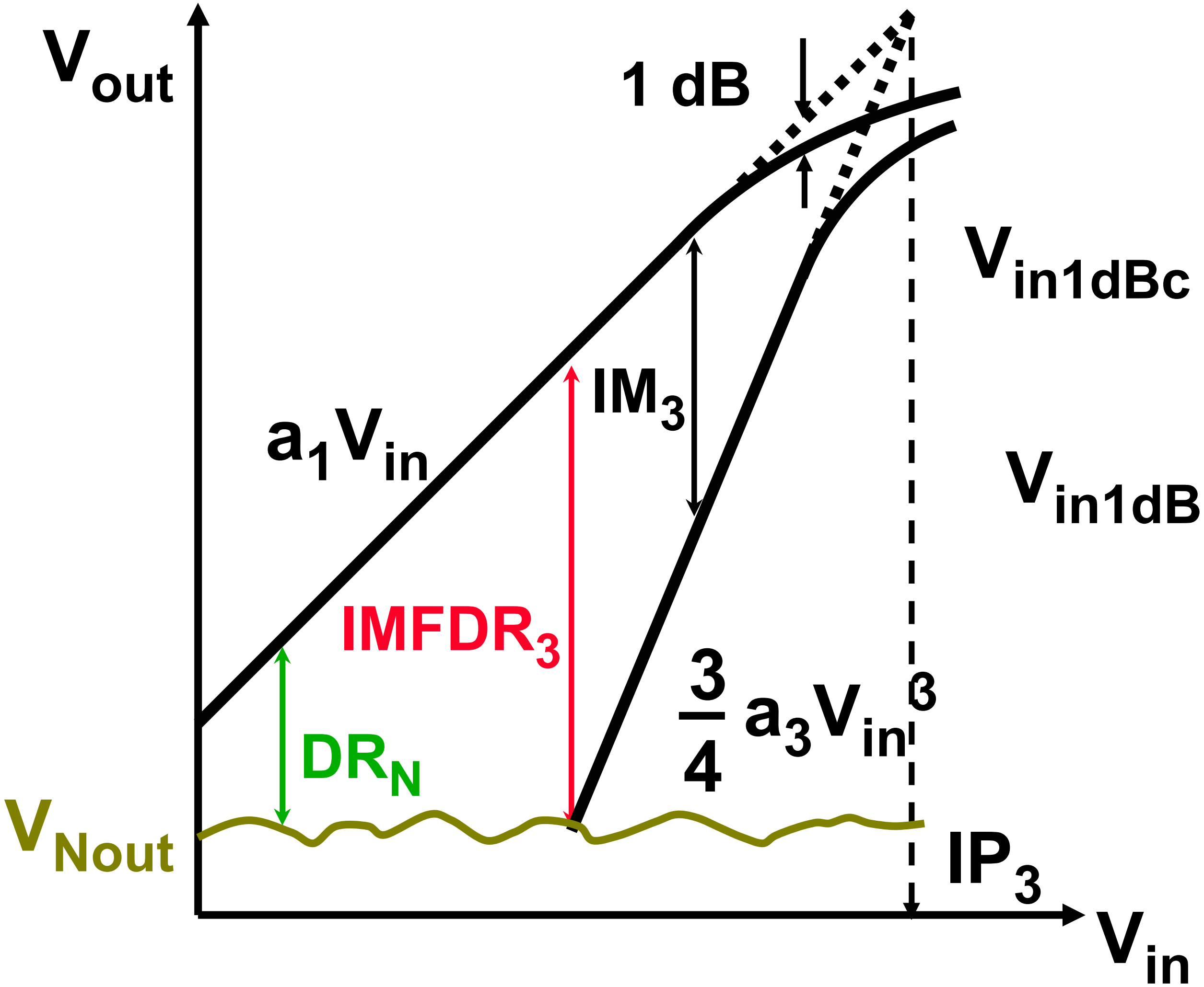
$$@\ V_{Nout} = \frac{3}{4}a_3V_{in}^3$$

$$IMFDR_3 =$$

$$\sqrt[3]{\frac{4}{3}\frac{a_1}{a_3}\frac{1}{V_{Nin}^2}} = \left(\frac{IP_3}{V_{Nin}}\right)^{2/3}$$

$$= \frac{2}{3}\left(IP_{3dB} - V_{NindB}\right)$$

The IP$_3$ and -1 dB compression point



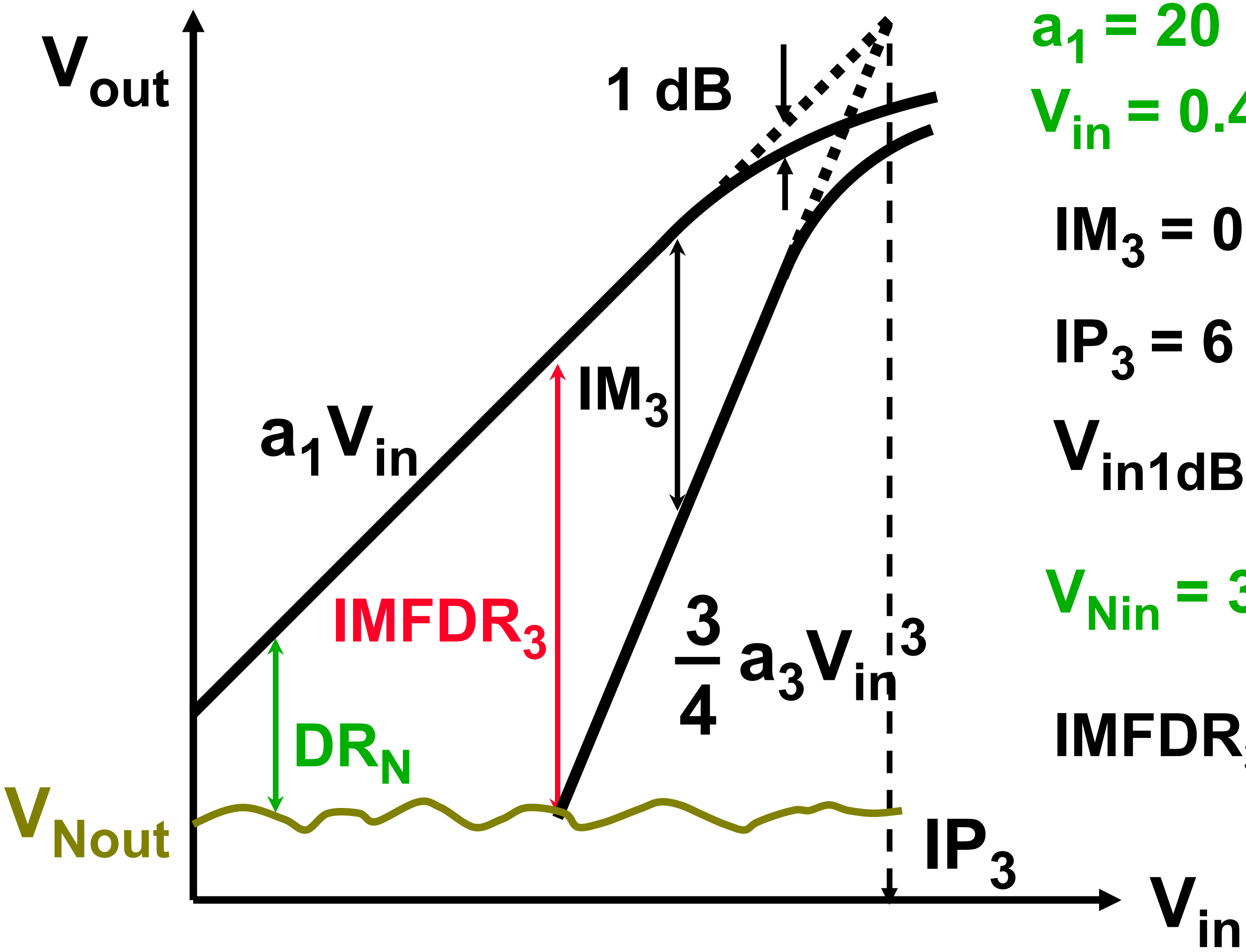
-1 dB is x 0.891

$$V_{in1dBc} = \sqrt{0.109 \frac{4}{3} \frac{a_1}{a_3}}$$

$$V_{in1dBc} = \sqrt{0.109} \; IP_3$$

$$= IP_{3dB} - 9.6 \text{ dB}$$

Relationship exercise



$a_1 = 20 \quad a_3 = 0.4$

$V_{in} = 0.45\ V_{RMS}$ or 6 dBm

$IM_3 = 0.3$ % or -50 dB

$IP_3 = 6 + 25 = 31$ dBm

$V_{in1dBc} = 21$ dBm

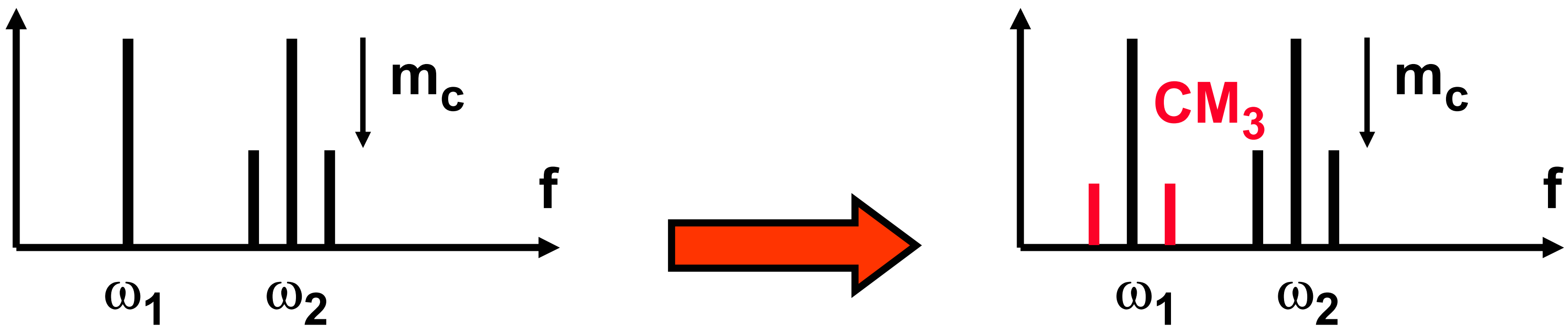
$V_{Nin} = 30\ \mu V_{RMS}$ (-78 dBm)

$$IMFDR_3 = \frac{2}{3}\, 119 = 73 \text{ dB}$$

Definition of crossmodulation distortion CM

$$y = a_0 + a_1 u + a_2 u^2 + a_3 u^3 + \ldots$$

with $u = U \cos \omega_1 t + U\,(1 + m_c \cos \omega_c t)\cos \omega_2 t$



$$CM_3 = \frac{3}{4}\, m_c\, \frac{a_3}{a_1}\, U^2 = m_c\, IM_3$$

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Distortion in a single-MOST amplifier

$$i_{DS} = K\,(v_{GS} - V_T)^2 \qquad\qquad K = K'\,\frac{W}{L}$$

$$I_{DS} + i_{ds} = K\,(V_{GS} + v_{gs} - V_T)^2$$

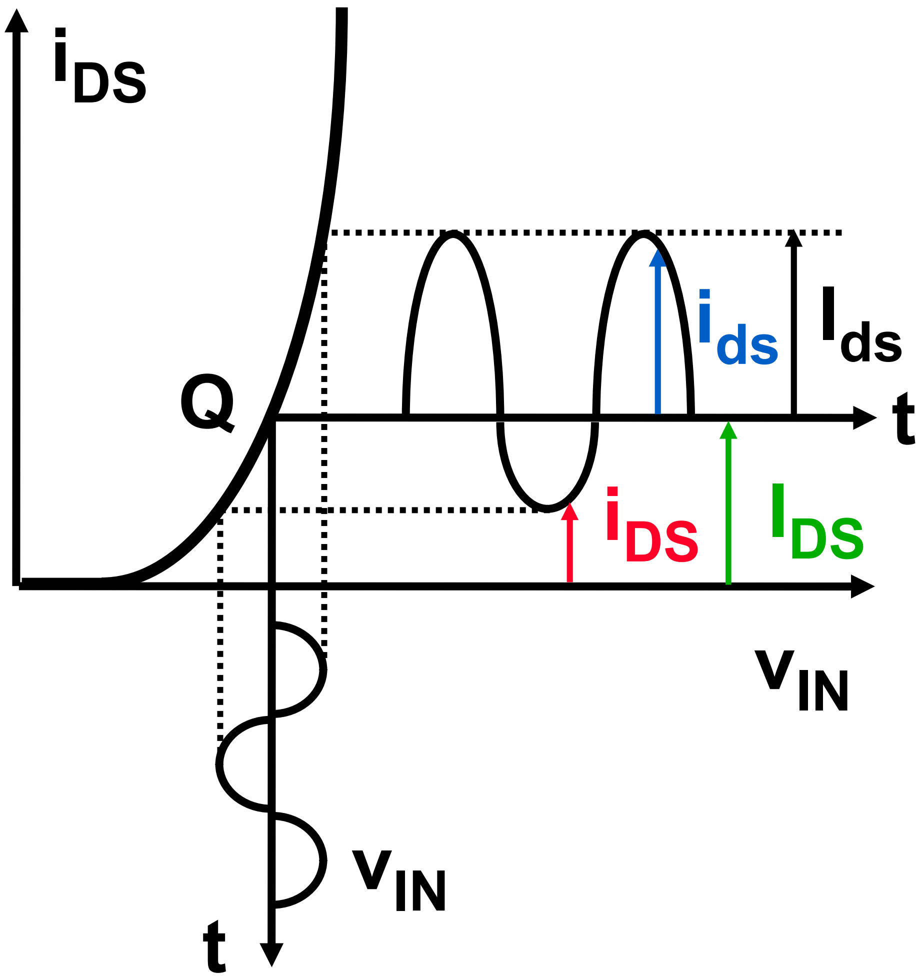
I_{DS} is the DC component

i_{DS} is the DC + ac component

i_{ds} is the ac component

I_{ds} is the amplitude of the ac component

DC and ac components



I_{DS} : DC component

i_{DS} : DC + ac component

i_{ds} : ac component

I_{ds} : amplitude of the ac component

Distortion in a single-MOST amplifier

$$I_{DS} = K\,(V_{GS} - V_T)^2 \qquad K = K' \frac{W}{L}$$

$$I_{DS} + i_{ds} = K\,(V_{GS} + v_{gs} - V_T)^2$$

$$i_{ds} = K\,(V_{GS} + v_{gs} - V_T)^2 - K\,(V_{GS} - V_T)^2$$

$$i_{ds} = 2K\,(V_{GS} - V_T)\,v_{gs} + K\,v_{gs}^2$$

Coefficients a_1, a_2, a_3 by comparison

$$i_{ds} = 2K\,(V_{GS} - V_T)\,v_{gs} + K\,v_{gs}^2$$

$$\text{or} \quad i_{ds} = g_1 v_{gs} + g_2 v_{gs}^2 + g_3 v_{gs}^3 + \ldots$$

$$g_1 = 2K\,(V_{GS} - V_T)$$

$$g_2 = K \qquad\qquad K = K' \frac{W}{L}$$

$$g_3 = 0$$

$$IM_2 = \frac{g_2}{g_1} V_{gs} = \frac{V_{gs}}{2(V_{GS}-V_T)} \qquad \& \qquad IM_3 = 0$$

Normalized current swing

$$i_{ds} = 2K\,(V_{GS} - V_T)\,v_{gs} + K\,v_{gs}^2 \qquad i_{DS} = K\,(v_{GS} - V_T)^2$$

or $y = a_1 u + a_2 u^2 + a_3 u^3 + ..$

$$y = \frac{i_{ds}}{I_{DS}} = \frac{2\,v_{gs}}{V_{GS} - V_T} + \frac{1}{4}\left(\frac{2\,v_{gs}}{V_{GS} - V_T}\right)^2$$

$$y = \frac{I_{ds}}{I_{DS}} = u + \frac{1}{4}u^2 \qquad U = \frac{V_{gs}}{(V_{GS} - V_T)/2}$$

y is the relative current swing !

Numerical example

The peak value of V_{gs} is V_{gsp} = 100 mV

(then $V_{gsRMS} = 100/\sqrt{2} = 71\ mV_{RMS}$)

if $V_{GS}-V_T = 0.5$ V then $V_{gsp}/[2(V_{GS}-V_T] = 0.1$

gives $IM_2 = 10$ % ($HD_2 = 5$ %) & $IM_3 = 0$

The relative current swing U = 0.1/0.25 = 0.4 !

More coefficients a_1, a_2, a_3 ...

In general

$$i_{ds} = g_m v_{gs} + K_{2gm} v_{gs}^2 + K_{3gm} v_{gs}^3 +$$
$$g_o v_{ds} + K_{2go} v_{ds}^2 + K_{3go} v_{ds}^3 +$$
$$g_{mb} v_{bs} + K_{2gmb} v_{bs}^2 + K_{3gmb} v_{bs}^3 +$$
$$K_{2gm\&gmb} v_{gs} v_{bs} + K_{3,2gm\&gmb} v_{gs}^2 v_{bs}$$
$$+ K_{3,gm\&2gmb} v_{gs} v_{bs}^2 +$$
$$\ldots\ldots\ldots\ldots +$$
$$K_{3gm\&gmb\&go} v_{gs} v_{ds} v_{bs}$$

Distortion of a MOST diode

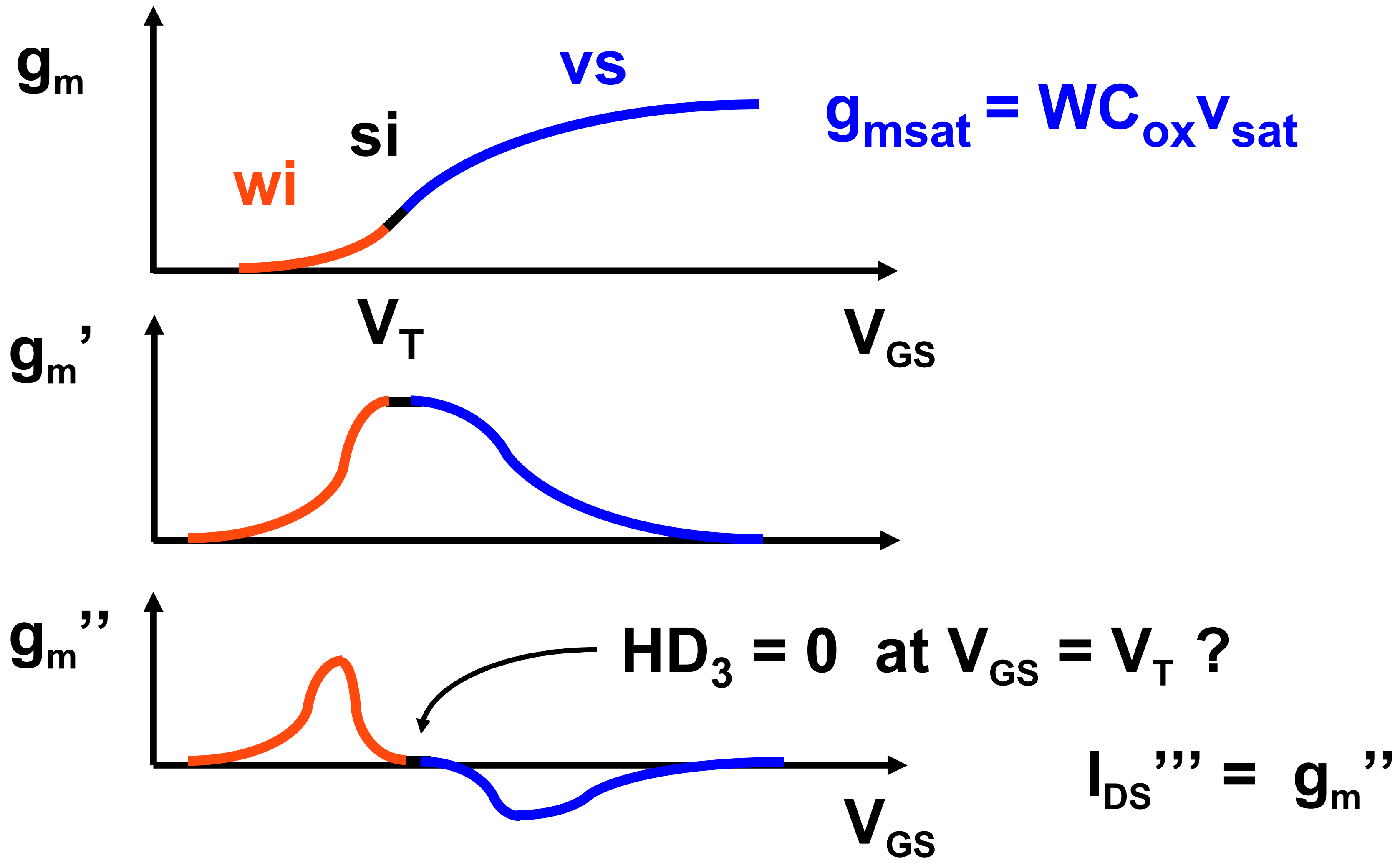
$$i_{DS} = K\,(v_{DS} - V_T)^2$$

$$y = \frac{i_{ds}}{I_{DS}} = \frac{2\,v_{ds}}{V_{DS} - V_T} + \frac{1}{4}\left(\frac{2\,v_{ds}}{V_{DS} - V_T}\right)^2$$

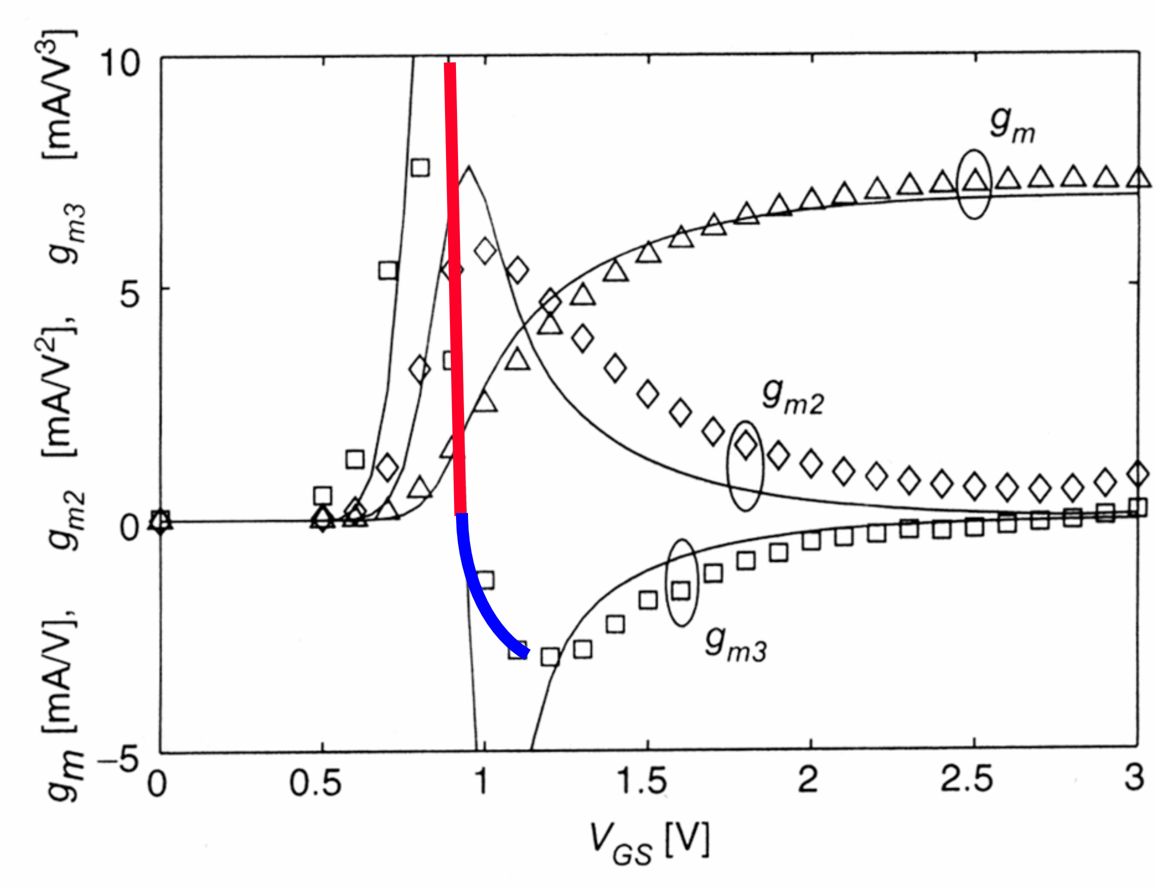
$$y = \frac{I_{ds}}{I_{DS}} = u + \frac{1}{4}u^2 \qquad U = \frac{V_{ds}}{(V_{DS} - V_T)/2}$$

Same as for a MOST transistor amplifier !

The zero HD3 point for smaller L



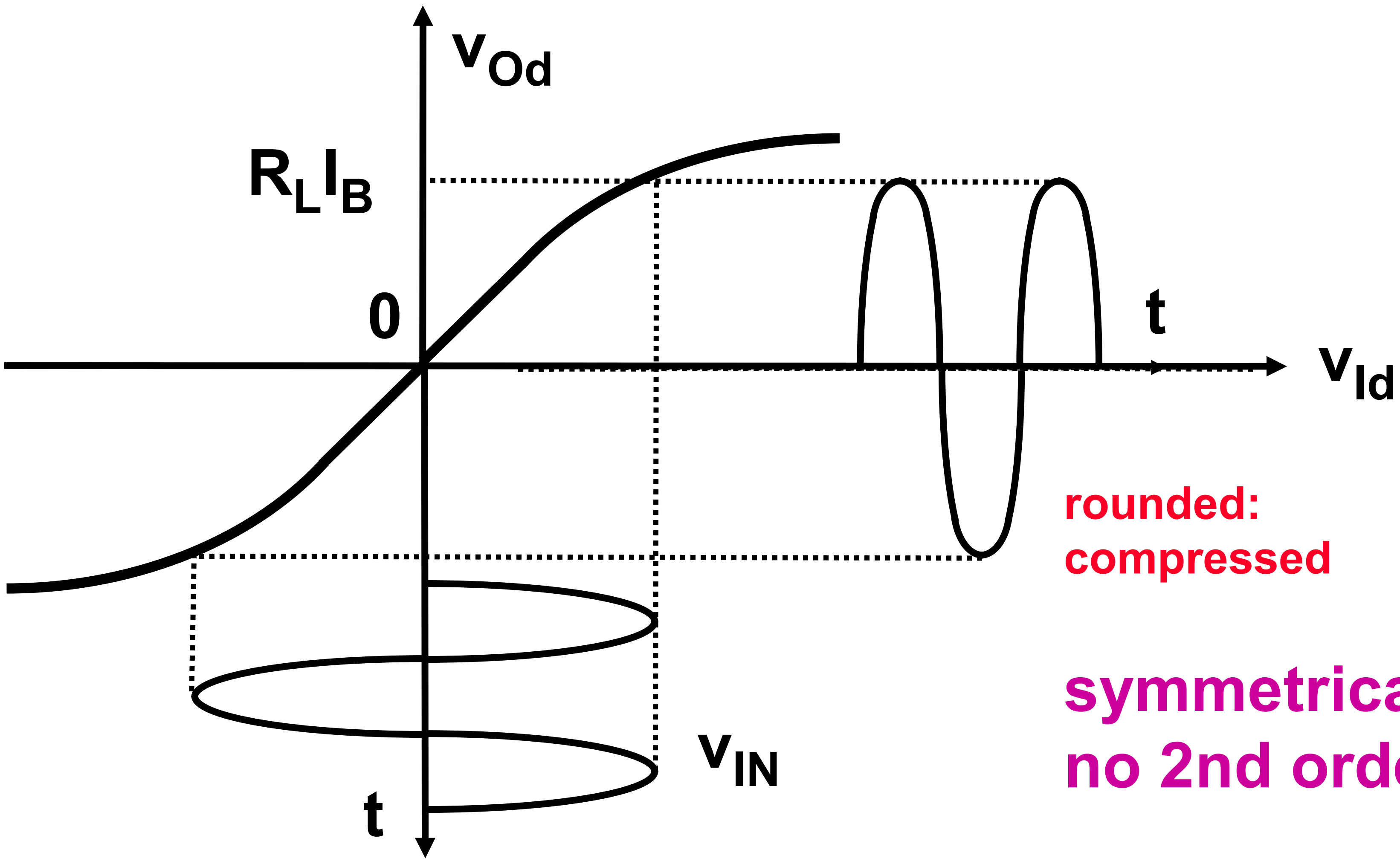
Derivatives of g_m



W = 60 μm
L = 0.6 μm
V_{DS} = 2 V

Ref. Fager JSSC Jan. 2004, 24-33

A differential pair is symmetrical



rounded:
compressed

symmetrical:
no 2nd order

Distortion in MOST differential pair

$$y = \frac{i_{Od}}{I_B} = \frac{v_{Id}}{V_{GS}-V_T}\sqrt{1 - \frac{1}{4}\left(\frac{v_{Id}}{V_{GS}-V_T}\right)^2}$$

v_{Id} is the differential input voltage

i_{Od} is the differential output current ($g_m v_{Id}$) or
twice the circular current $g_m v_{Id}$ /2

I_B is the total DC current in the pair

Note that $g_m = \dfrac{I_B}{V_{GS} - V_T} = K' \dfrac{W}{L}(V_{GS} - V_T)$

Distortion in MOST differential amplifier

$$y = \frac{i_{Od}}{I_B} = \frac{v_{Id}}{V_{GS}-V_T}\sqrt{1 - \frac{1}{4}\left(\frac{v_{Id}}{V_{GS}-V_T}\right)^2}$$

$$\sqrt{1-x} \approx 1 - \frac{x}{2}$$

$$y = \frac{I_{Od}}{I_B} = U\sqrt{1 - \frac{1}{4}U^2} \approx U - \frac{1}{8}U^3$$

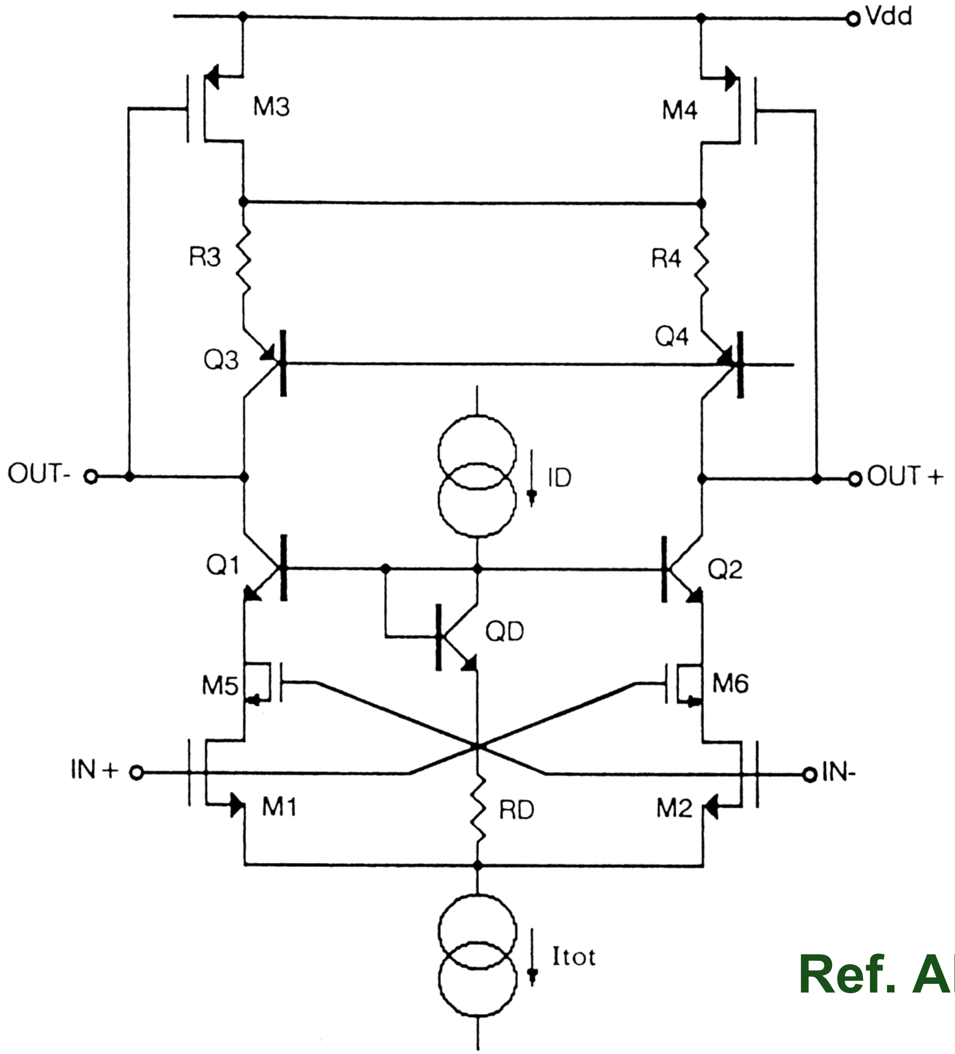
$$U = \frac{V_{Id}}{V_{GS} - V_T}$$

$$IM_2 = 0 \qquad \boxed{IM_3 = \frac{3}{32}U^2}$$

U is the relative current swing

$$IP_3 = 4\sqrt{\frac{2}{3}}(V_{GS} - V_T) \approx 3.3\,(V_{GS} - V_T)$$

Distortion in linear region



$V_{DS1} = R_D I_D \approx 0.2$ V

$I_{DS1} = \beta_1 V_{DS1}(V_{GS1} - V_T)$

$g_{m1} = \beta_1 V_{DS1}$ is constant

Low distortion !

Ref. Alini,JSSC, Dec.92, pp.1905-1915

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Distortion in a bipolar transistor amplifier

$$I_{CE} = I_S \exp\left(\frac{V_{BE}}{kT_e/q}\right)$$

$$I_{CE} + i_{ce} = I_S \exp\left(\frac{V_{BE} + v_{be}}{kT_e/q}\right)$$

$$1 + y = \exp\left(\frac{v_{be}}{kT_e/q}\right)$$

$$\approx \exp(u) = 1 + u + \frac{u^2}{2} + \frac{u^3}{6} + \ldots \quad \text{if } u << 1$$

I_{CE} DC component

i_{CE} DC + ac component

i_{ce} ac component

I_{ce} amplitude of the ac component

Distortion in a bipolar transistor amplifier

$$y \approx u + \frac{u^2}{2} + \frac{u^3}{6} + \dots \qquad U = \frac{V_{be}}{kT_e/q}$$

is the non-linear equation

y is the relative current swing !

$a_1 = 1$

$a_2 = 1/2$

$a_3 = 1/6$

$$IM_2 = \frac{a_2}{a_1} U = \frac{1}{2} \frac{V_{be}}{kT_e/q}$$

$$IM_3 = \frac{3}{4} \frac{a_3}{a_1} U^2 = \frac{1}{8} \left(\frac{V_{be}}{kT_e/q} \right)^2$$

Numerical example

1. Relative current swing is 10 %

$y_p = 0.1$ gives $IM_2 = 5\ \%$ ($HD_2 = 2.5\%$)

$IM_3 = 0.125\ \%$ ($HD_3 = 0.04\ \%$)

As a result $V_{bep} = y_p(kT_e/q) = 2.6\ mV_p$ ($1.8\ mV_{RMS}$)

$IP_3 = \sqrt{8}\,(kT_e/q) = 74\ mV_p$ or $50\ mV_{RMS}$ or -13 dBm

2. $V_{bep} = 100$ mV

then $y_p = 0.1/0.026 \approx 4$ (must be << 1 !!)

gives $IM_2 =$?? Too high distortion !!

Distortion in a diode

$$i_D = I_S \exp\left(\frac{v_D}{kT_e/q}\right) \qquad y \approx u + \frac{u^2}{2} + \frac{u^3}{6} + \ldots$$

$$y = \frac{I_d}{I_D} = u + \frac{u^2}{2} + \frac{u^3}{6} \qquad U = \frac{V_d}{kT_e/q}$$

Same as for a Bipolar transistor amplifier !

Distortion in bipolar differential amplifier

$$y = \frac{i_{Od}}{I_B} = \tanh \frac{V_{Id}}{2kT_e/q}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$\approx x - \frac{1}{3}x^3$$

$$y = \frac{I_{Od}}{I_B} \approx U - \frac{1}{3}U^3$$

$$U = \frac{V_{Id}}{2kT_e/q}$$

U is the relative current swing

$$IM_2 = 0$$

$$\boxed{IM_3 = \frac{1}{4}U^2}$$

$$IP_3 = 4\ kT_e/q$$

Distortion in a resistor or capacitor

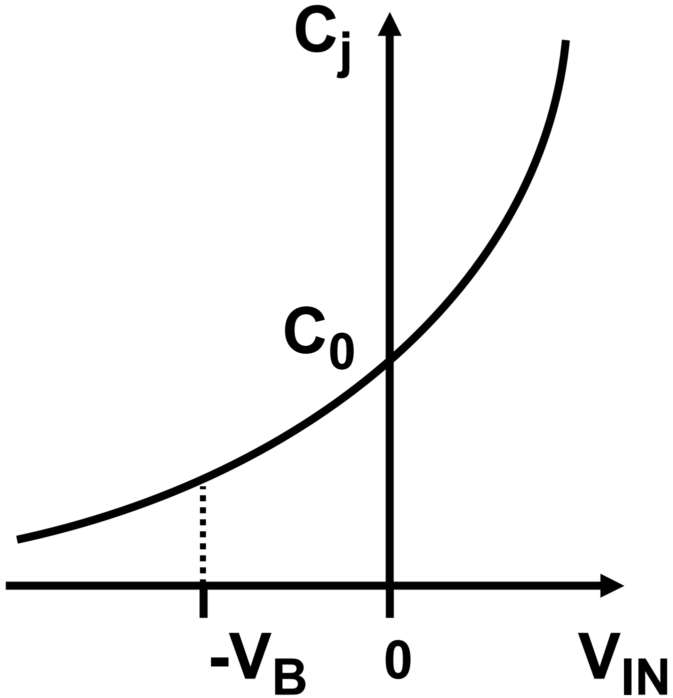
$R = R_0 (1 + a_1V + a_2V^2 + ...)$ [≈ JFET with large V_P]

For diffused resistors : $a_1 \approx 5$ ppm/V
$a_2 \approx 1$ ppm/V^2

$C = C_0 (1 + a_1V + a_2V^2 + ...)$

For poly-poly caps : $a_1 \approx 20$ ppm/V
$a_2 \approx 2$ ppm/V^2

Non-linearity depletion capacitance



$$C_j = \frac{C_0}{\sqrt{1 - \frac{v_{IN}}{\Phi}}}$$

$$v_{IN} = V_B + v_{in}$$

$$C_j = \frac{C_0}{\sqrt{1 + \frac{V_B}{\Phi}}} \frac{1}{\sqrt{1 + \underbrace{\frac{v_{in}}{V_B + \Phi}}_{x}}}$$

$$C_j = C_{0B}\,(1 + x)^{-1/2} = C_{0B}\,(1 - 1/2\,x + 3/8\,x^2 - 5/16\,x^3 + ..)$$

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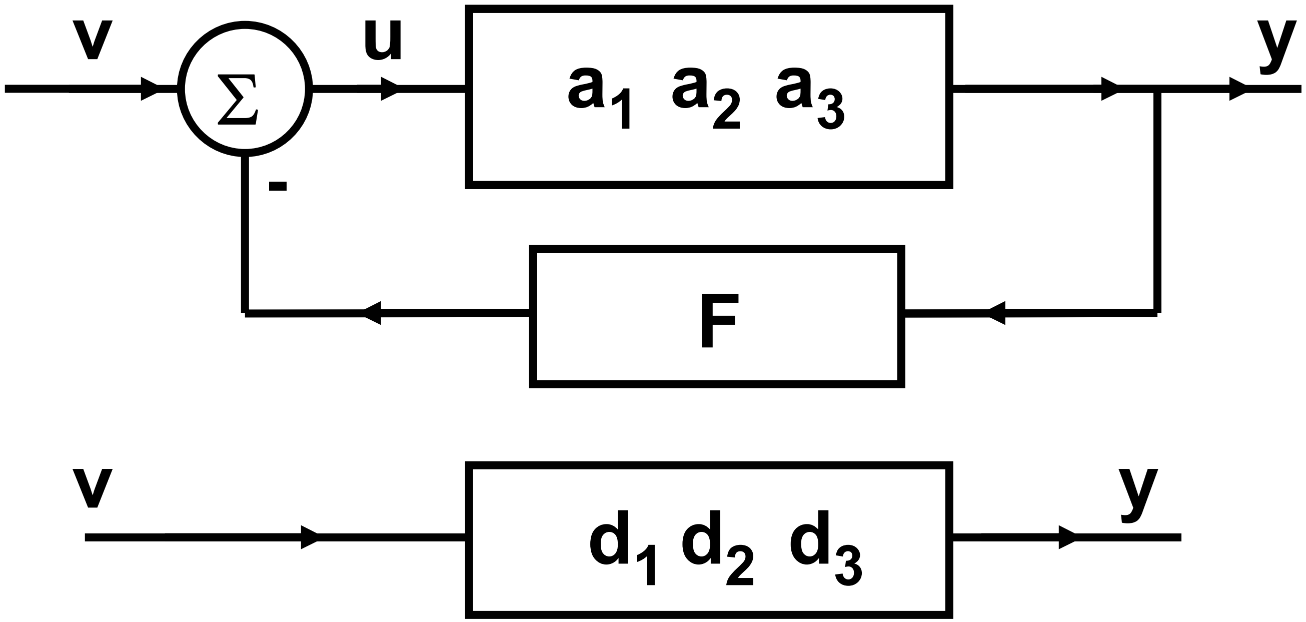
Distortion reduction by feedback



$$u = v - Fy$$

$$y = a_1 u + a_2\, u^2 + a_3\, u^3$$

$\Big\}$ elim. u

$$y = d_1 v + d_2\, v^2 + d_3\, v^3$$

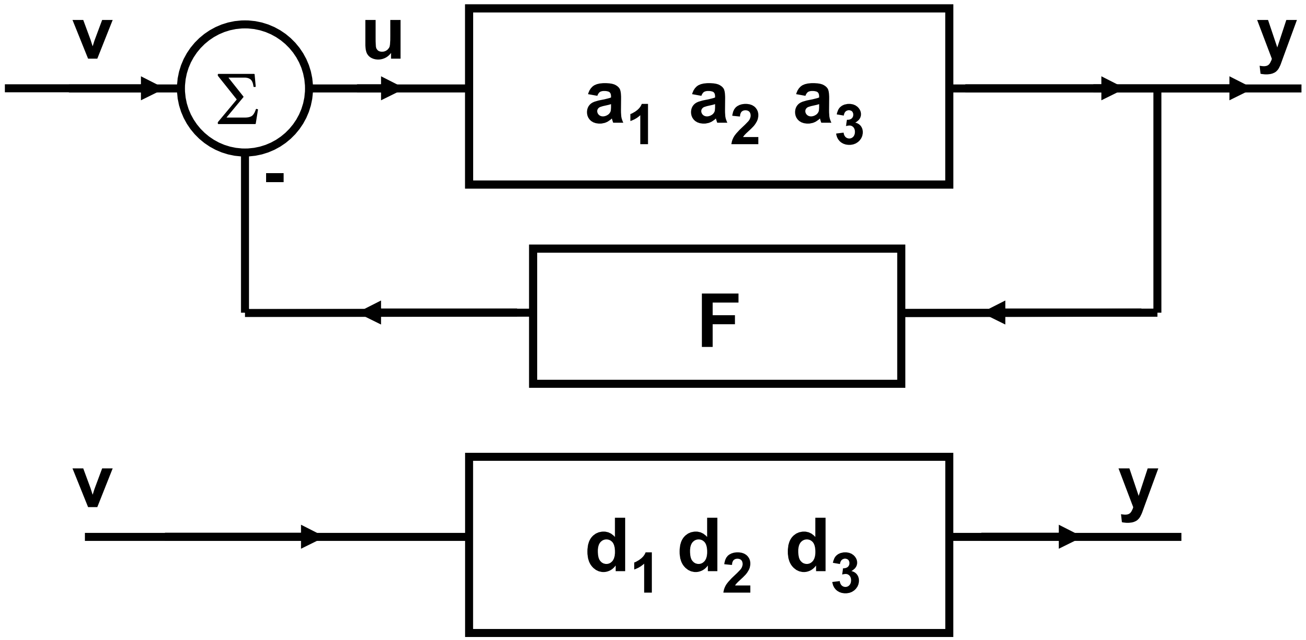
$\Big\}$ elim. y $\Bigg\{$

coeff v : d_1

coeff v^2 : d_2

coeff v^3 : d_3

Distortion reduction by feedback



$$u = v - Fy$$

$$d_1 = \frac{a_1}{1 + T} \approx \frac{1}{F}$$

$$d_2 = \frac{a_2}{(1 + T)^3}$$

$$d_3 = \frac{a_3 (1 + T) - 2F\, a_2^2}{(1 + T)^5}$$

Loop gain $1+T = 1+a_1F$

u is (1+T) times smaller than v :

v is reduced by loop gain (1+T)

Distortion components with feedback

$$IM_{2f} = \frac{d_2}{d_1}V = \frac{a_2}{a_1}\,\frac{V}{(1+T)^2} = \frac{a_2}{a_1}\,\underbrace{\frac{1}{(1+T)}}_{\text{reduction by loop gain}}\;\underbrace{\frac{V}{(1+T)}}_{\text{reduction in current swing}}$$

$$IM_{3f} = \frac{3}{4}\,\frac{d_3}{d_1}V^2 = \frac{3}{4}\left[\frac{a_3}{a_1}\,\frac{1}{(1+T)} - \left(\frac{a_2}{a_1}\right)^2\frac{2T}{(1+T)^2}\right]\underbrace{\frac{V^2}{(1+T)^2}}_{\text{reduction in current swing}}$$

compression   expansion

Distortion components with feedback : examples

$$IM_{3f} = \frac{3}{4}\frac{d_3}{d_1}V^2 = \frac{3}{4}\underbrace{\left[\frac{a_3}{a_1}\frac{1}{(1+T)} - \left(\frac{a_2}{a_1}\right)^2\frac{2T}{(1+T)^2}\right]}\frac{V^2}{(1+T)^2}$$

For large T : $\dfrac{a_3 a_1 - 2 a_2^2}{a_1^2}\dfrac{1}{T} = \dfrac{a_3}{a_1}\left(1 - \dfrac{2 a_2^2}{a_1 a_3}\right)\dfrac{1}{T}$

MOST : $a_3 = 0$: a_2 dominant

Bipolar : $a_1 = 1$ $a_2 = 1/2$ $a_3 = 1/6$: a_2 dominant

Diff. pair : $a_2 = 0$: a_3 dominant

Emitter resistor to reduce distortion IM_{2f}

$$T = g_m R_E = \frac{V_{RE}}{kT_e/q} \qquad \frac{a_2}{a_1} = \frac{1}{2}$$

$$IM_{2f} = \frac{1}{2} \frac{1}{(1+T)^2} \frac{V_{in}}{kT_e/q} = \frac{1}{(1+T)} \frac{U}{2}$$

$$U = \frac{1}{(1+T)} \frac{V_{in}}{kT_e/q}$$ is the relative current swing

IM_{2f} decreases linearly with T for constant U !

Emitter resistor to reduce distortion IM_{3f}

$$IM_{3f} = \frac{1 - 2T}{(1 + T)^2} \frac{U^2}{8} \qquad \frac{a_2}{a_1} = \frac{1}{2} \qquad \frac{a_3}{a_1} = \frac{1}{6}$$

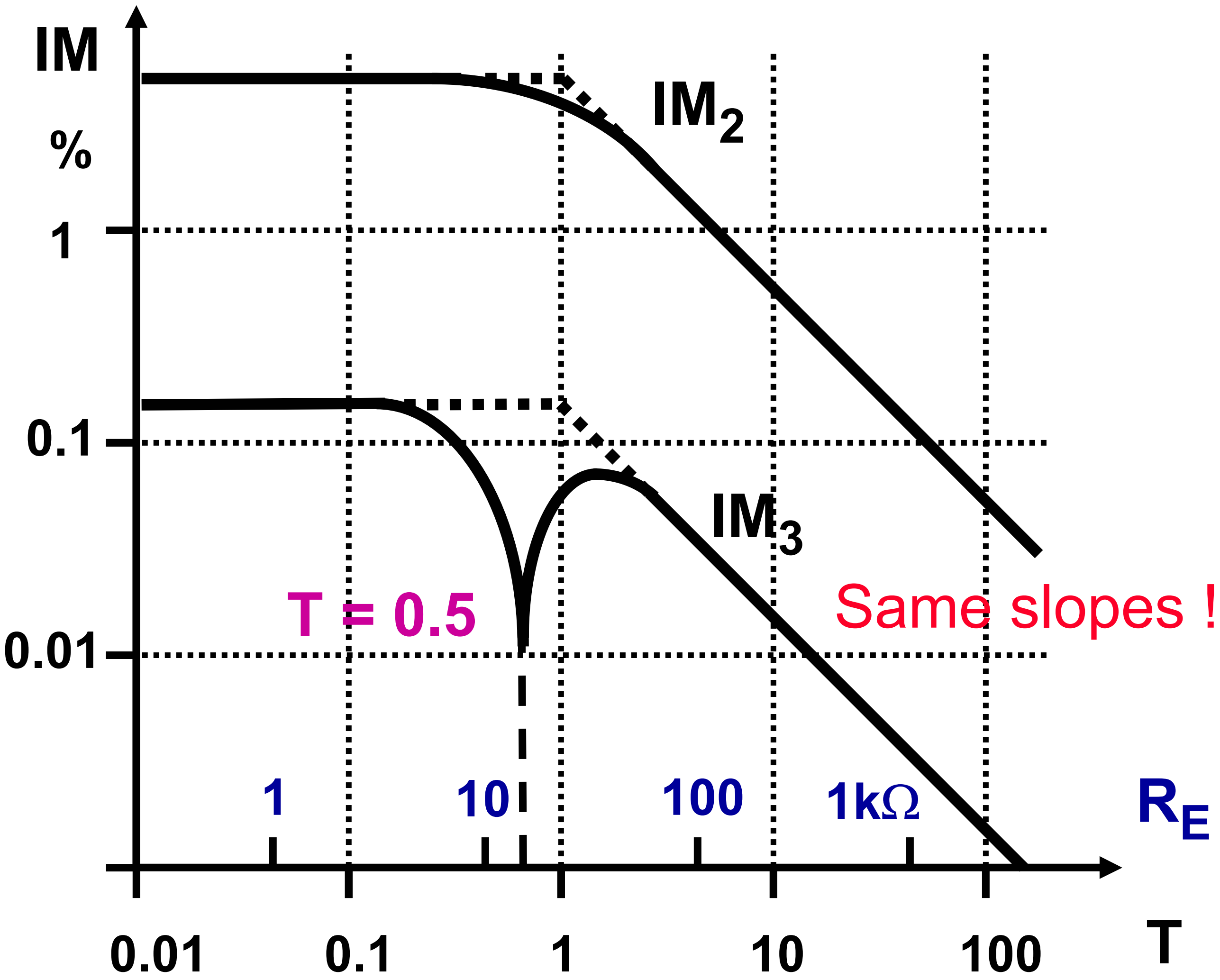
$$U = \frac{1}{(1 + T)} \frac{V_{in}}{kT_e/q}$$ is the relative current swing

Null for T = 0.5

IM_{3f} also decreases with T for constant U

for large T !!

Null in IM_3 by R_E (Bipolar trans. I_{CE} = 1 mA)



Null in IM_3 if

$$a_3\,(1 + T) = 2f\,a_2^2$$

$$a_3\,(1 + T) = 2T\,\frac{a_2^2}{a_1}$$

$$T = \frac{1}{\dfrac{2a_2^2}{a_1 a_3} - 1}$$

Emitter resistor R_E reduces distortion for large T

$$U = \frac{1}{T}\,\frac{V_{in}}{kT_e/q} = \frac{V_{in}}{R_E I_{CE}}$$

$$IM_{2fT} = \frac{U}{2T} = \frac{V_{in}}{kT_e/q}\,\frac{1}{2\,T^2} = \frac{V_{in}\,kT_e/q}{2\,(R_E I_{CE})^2}$$

$$IM_{3fT} = \frac{U^2}{4T} = \left(\frac{V_{in}}{kT_e/q}\right)^2 \frac{1}{4\,T^3} = \frac{V_{in}^2\,kT_e/q}{4\,(R_E I_{CE})^3}$$

Source resistor R_S to reduce distortion

$$T = g_m R_S = \frac{V_{RS}}{(V_{GS}-V_T)/2} \qquad \frac{a_2}{a_1} = \frac{1}{4} \qquad a_3 = 0$$

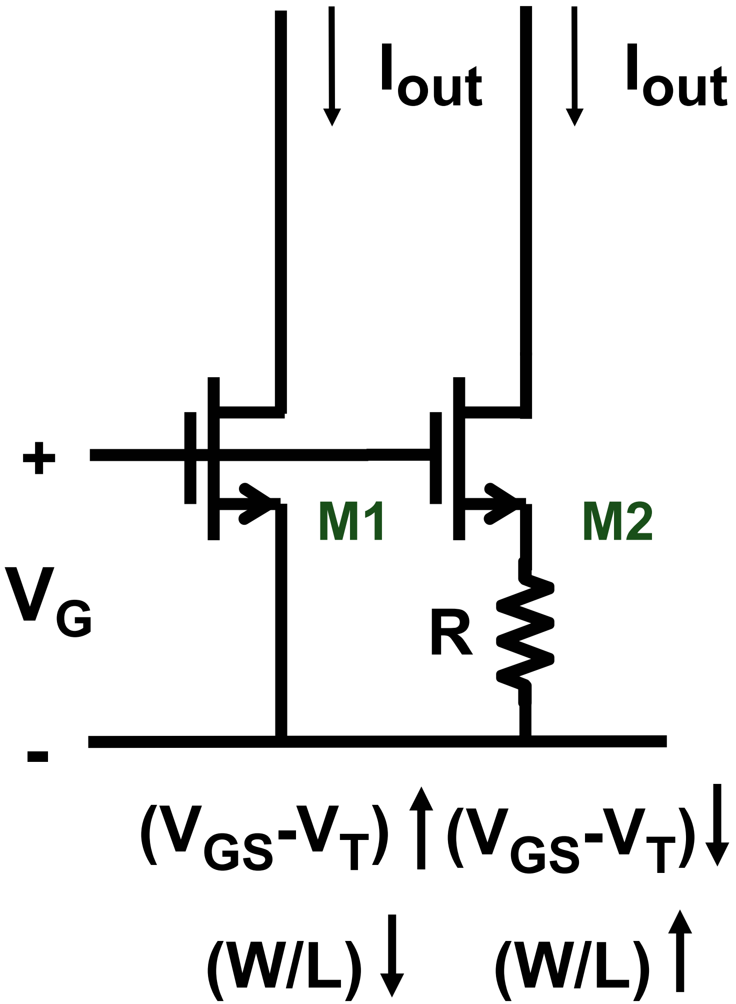
$$U = \frac{1}{(1+T)} \frac{V_{in}}{(V_{GS}-V_T)/2}$$

is the relative current swing

$$IM_{2f} = \frac{1}{(1+T)} \frac{U}{4} \approx \frac{V_{in}}{(V_{GS}-V_T)/2} \frac{1}{4\,T^2} = \frac{V_{in}\,(V_{GS}-V_T)/2}{4\,(R_S I_{DS})^2}$$

$$IM_{3f} = \frac{T}{(1+T)^2} \frac{3U^2}{32} \approx \frac{V_{in}^2}{(V_{GS}-V_T)^2/4} \frac{3}{32T^3} = \frac{3V_{in}^2\,(V_{GS}-V_T)/2}{32\,(R_S I_{DS})^3}$$

Current source with series R



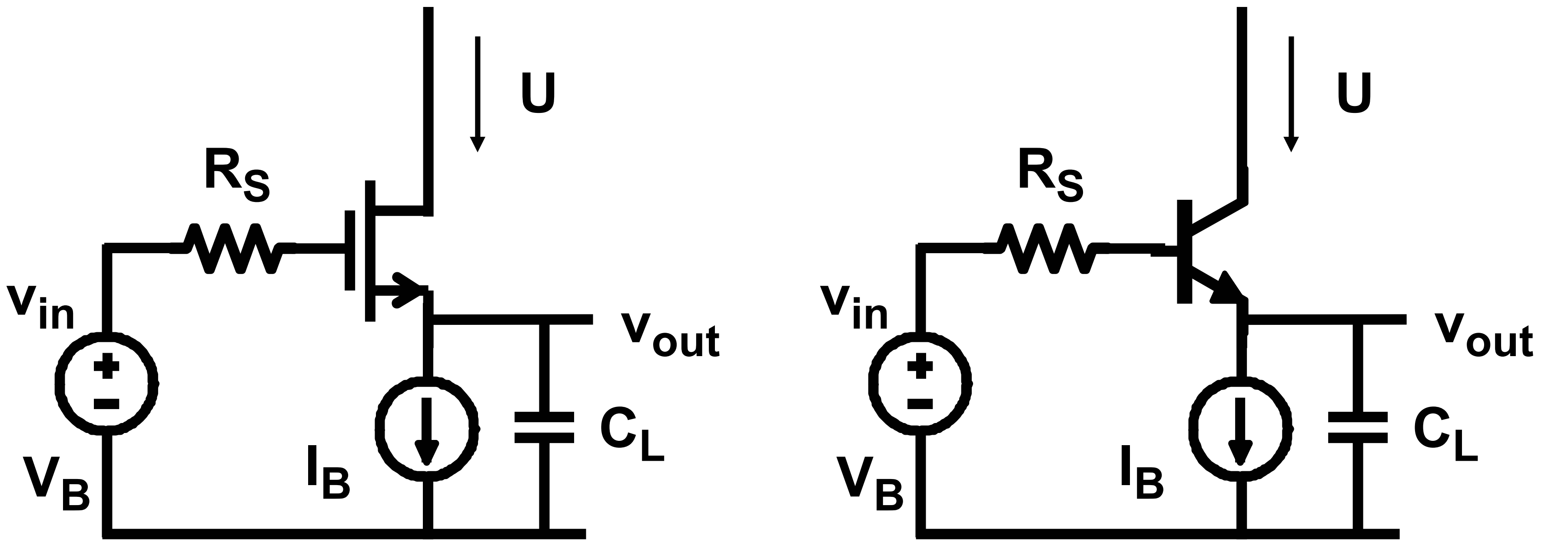
Same I_{out} & same V_G :

Same gain !
Same output noise !

Same distortion ?

$$\frac{IM_{2f}}{IM_2} = \frac{1 - \dfrac{V_R}{V_{GST1}}}{\left(1 + \dfrac{V_R}{V_{GST1}}\right)^2}$$

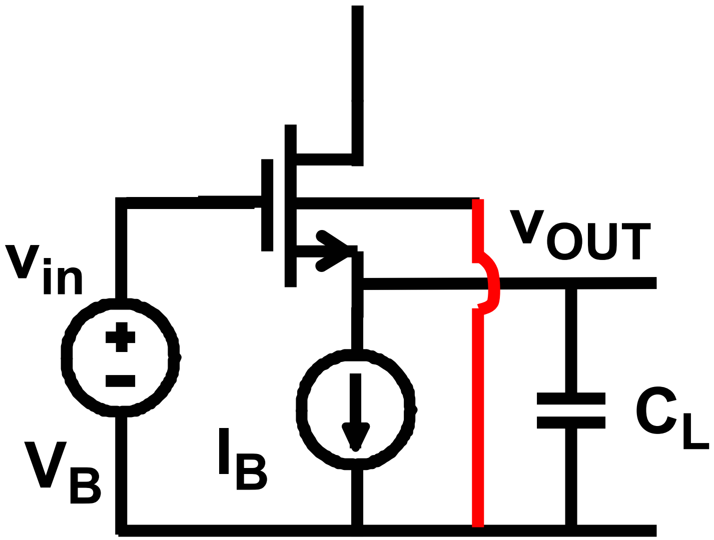
Source & Emitter Follower



$$U = \frac{1}{g_m r_{DS}} \frac{V_{in}}{(V_{GS} - V_T)/2} = \frac{V_{in}}{V_{En} L} \qquad U = \frac{1}{g_m\, r_o} \frac{V_{in}}{kT_e/q} = \frac{V_{in}}{V_E}$$

If $v_{BS} = 0$!!

Distortion Source follower with substrate effect



$$v_{OUT} = v_{IN} - v_{GS}$$

$$v_{GS} = V_T + \sqrt{\frac{I_B}{K'W/L}}$$

$$V_T = V_{T0} + \gamma \left[v_{OUTF} \right]$$

$$v_{OUTF} = \sqrt{|2\Phi_F| + v_{OUT}} - \sqrt{|2\Phi_F|}$$

$$v_{IN} = v_{OUT} + V_{T0} + \gamma \left[v_{OUTF} \right] + \sqrt{\frac{I_B}{K'W/L}}$$

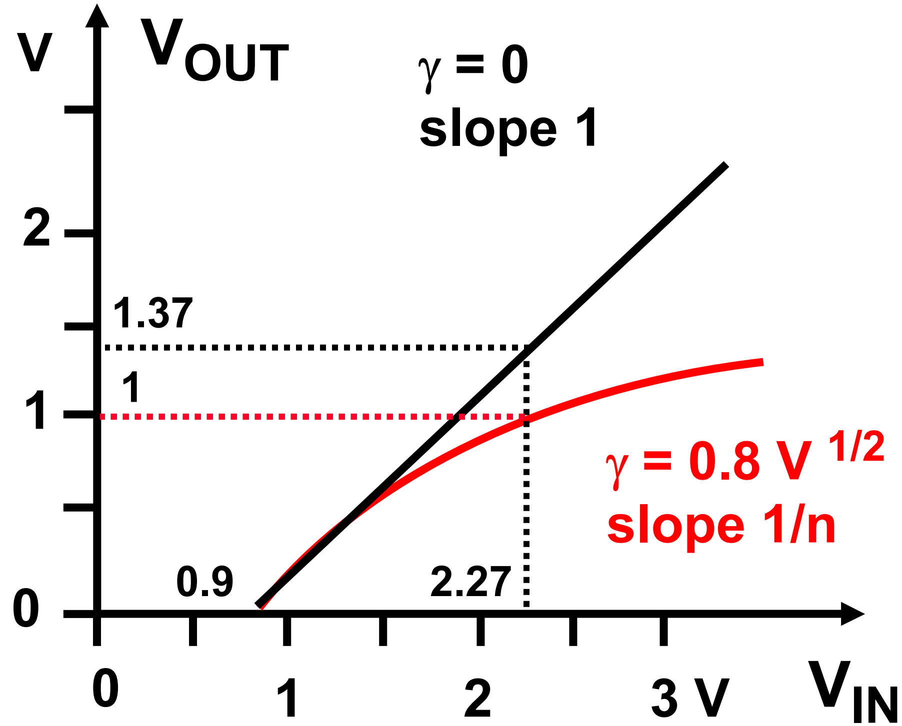
Distortion Source follower - Example

$$v_{IN} = u^2 + \gamma\ u + B$$

$$u^2 = v_{OUT} + |2\Phi_F|$$

$$B = V_{GS0} - |2\Phi_F| - \gamma \sqrt{|2\Phi_F|}$$

$$V_{GS0} = V_{T0} + \sqrt{\frac{I_B}{K'W/L}}$$

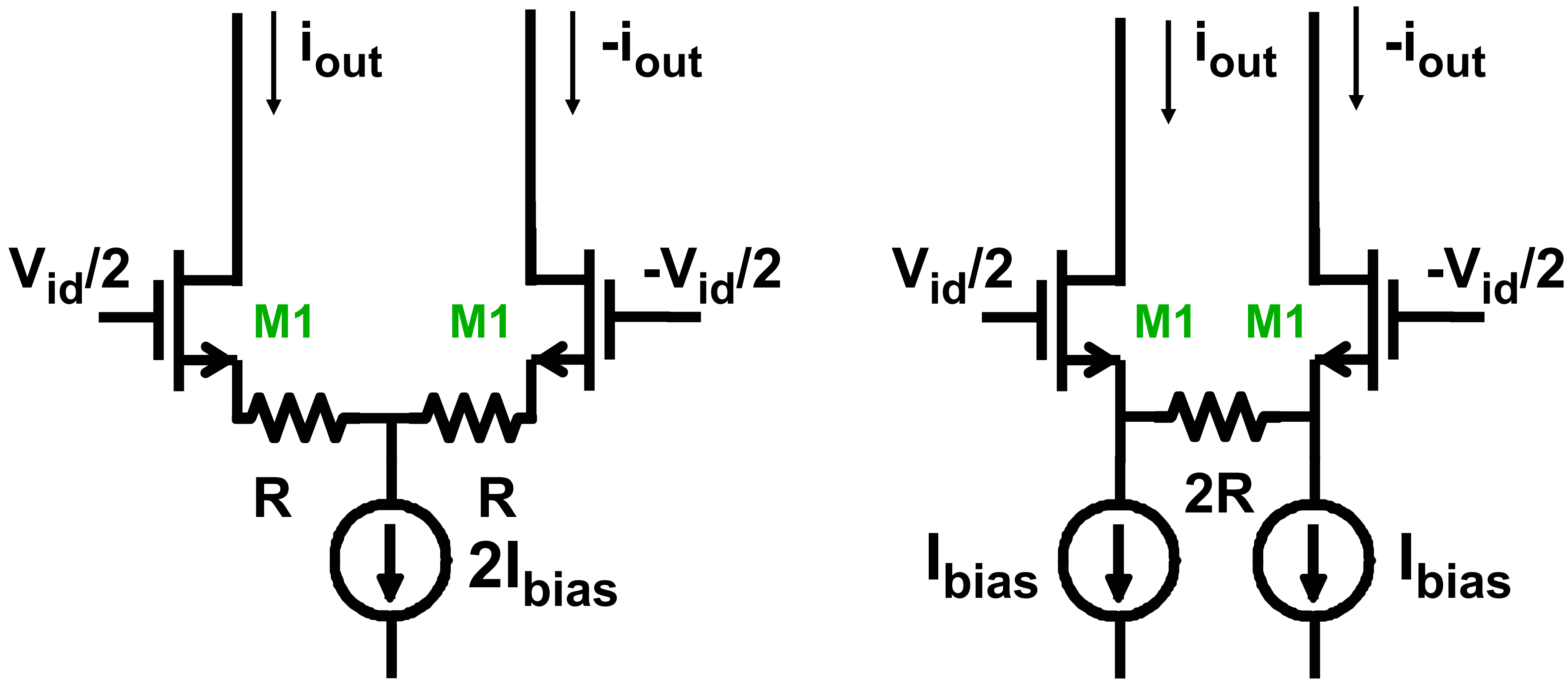


V_{T0} = 0.6 V ; V_{GS0} = 0.9 V; $2\Phi_F$ = 0.7 V; B = -0.47 V; 1/n = 0.73

a_1 = 0.765; a_2 = 0.02; a_3 = -0.0035

V_{INp} = 1 V_p; HD_2 = 1.32 %; HD_3 = -0.114 %

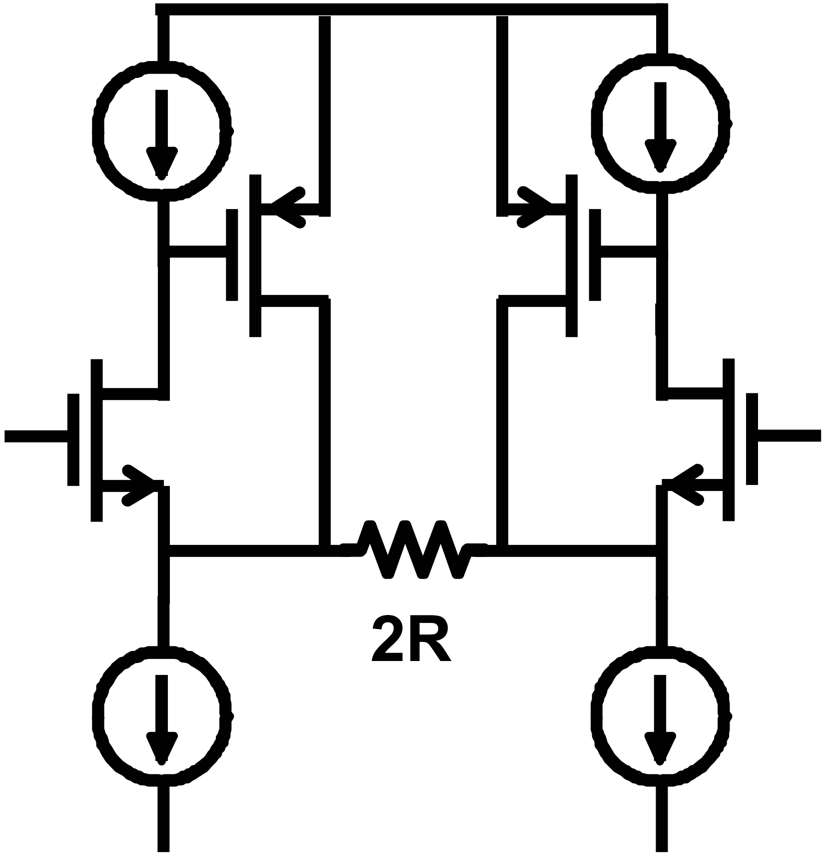
Increasing the IP_3 by feedback



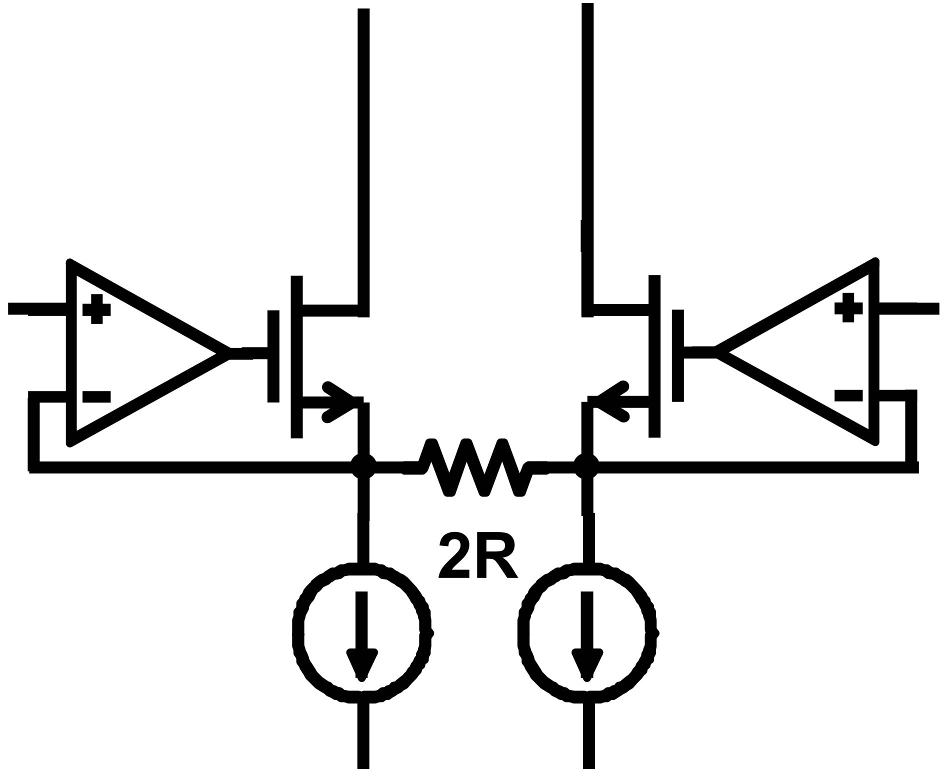
$IP_3 \approx 3.3\,(V_{GS}-V_T)(1+g_{m1}R)^2 \quad HD_3/n^2 \quad n = 1+g_{m1}R$

HD_3 = - 60 dB for V_{id} = 1 V requires V_{GS}-V_T = 0.38 V and $g_{m1}R$ = 3 !!!

Increasing the IP_3 by feedback

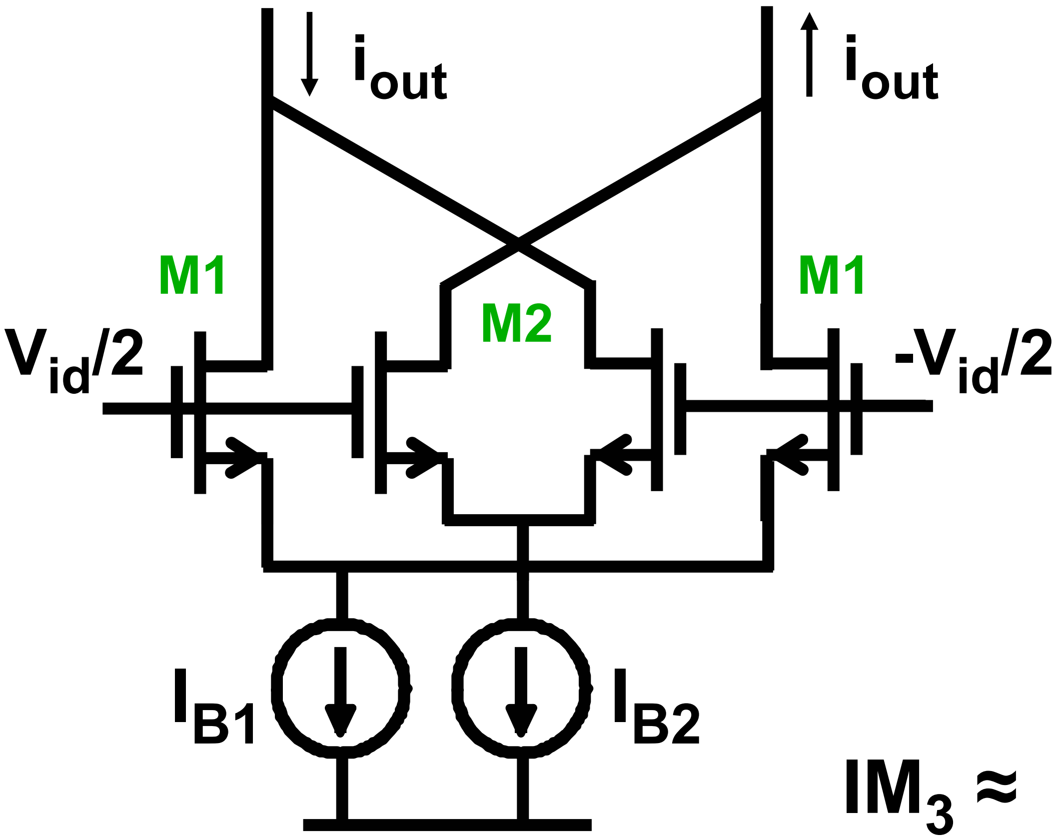


Additional local FB



More FB with opamps

Distortion cancellation



Parameters :

$\alpha = I_{B2} / I_{B1}$

≈ 0.25

$v = V_{GST1} / V_{GST2}$

≈ 1.6

$V_{GST} = V_{GS} - V_T$

$IM_3 \approx 0$ if $v = \alpha^{-1/3}$

then $i_{out} = g_{m1} V_{id} (1 - \alpha^{2/3})$

Distortion cancellation

$$i_{out} = 2\,(i_{DS1} - i_{DS2})$$

$$\frac{i_{DS}}{I_B} = U - \frac{1}{8}U^3 \qquad U = \frac{V_{id}}{V_{GS} - V_T} \qquad IM_3 = \frac{3}{32}U^2$$

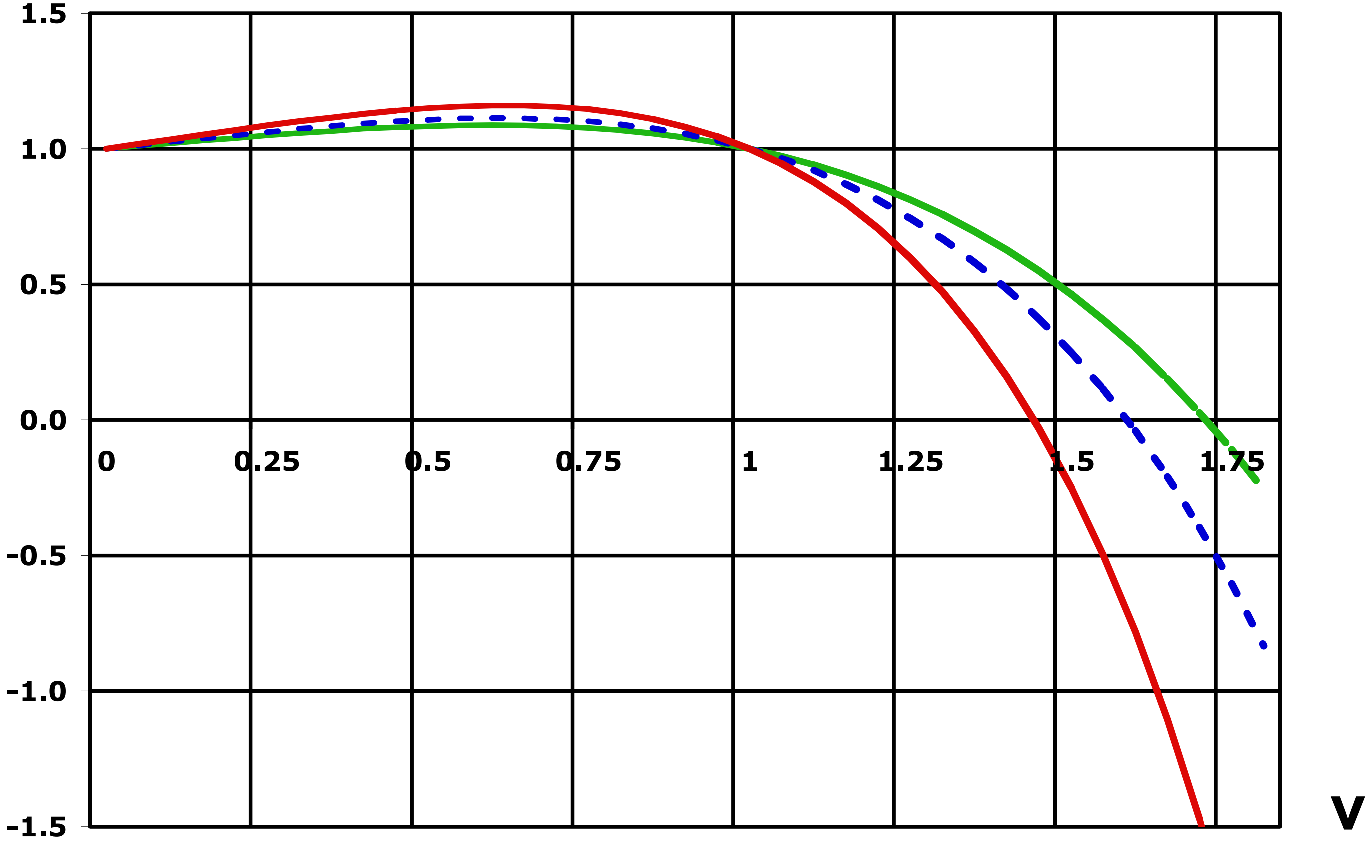
$$IM_3 \approx \frac{3}{32}\left(\frac{V_{id}}{V_{GS1} - V_T}\right)^2 \frac{1 - \alpha\, v^3}{1 - \alpha\, v}$$

$$IM_3 \approx 0 \quad \text{if} \quad v_{00} = \alpha^{-1/3}$$

at which point $\quad i_{out} = g_{m1}\, V_{id}\, (1 - \alpha^{2/3})$

Compensation of IM3

$$\frac{1 - \alpha v^3}{1 - \alpha v}$$

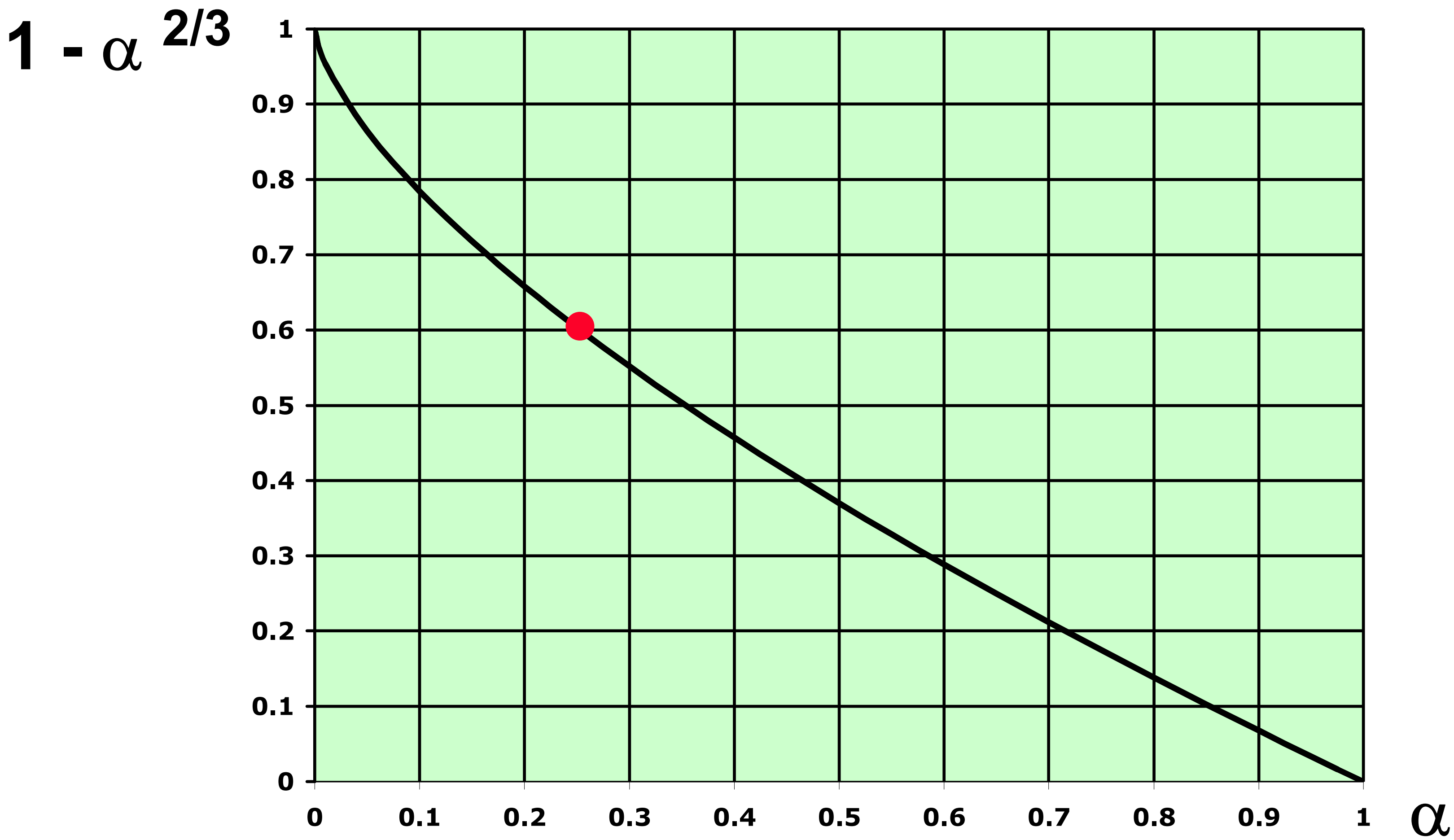


$\alpha = 0.20$
$v_{00} = 1.71$

$\alpha = 0.25$
$v_{00} = 1.6$

$\alpha = 0.33$
$v_{00} = 1.44$

Output signal vs current ratio



$\alpha \approx 0.25$

$v_{00} = 1.6$

x 0.6

Table of contents

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  - Single-ended amplifier
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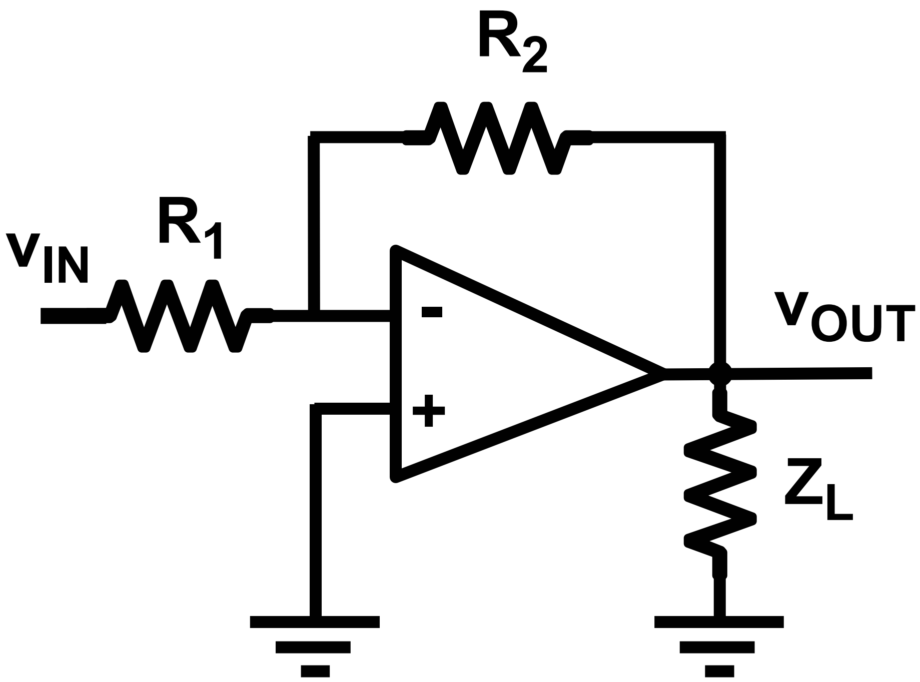
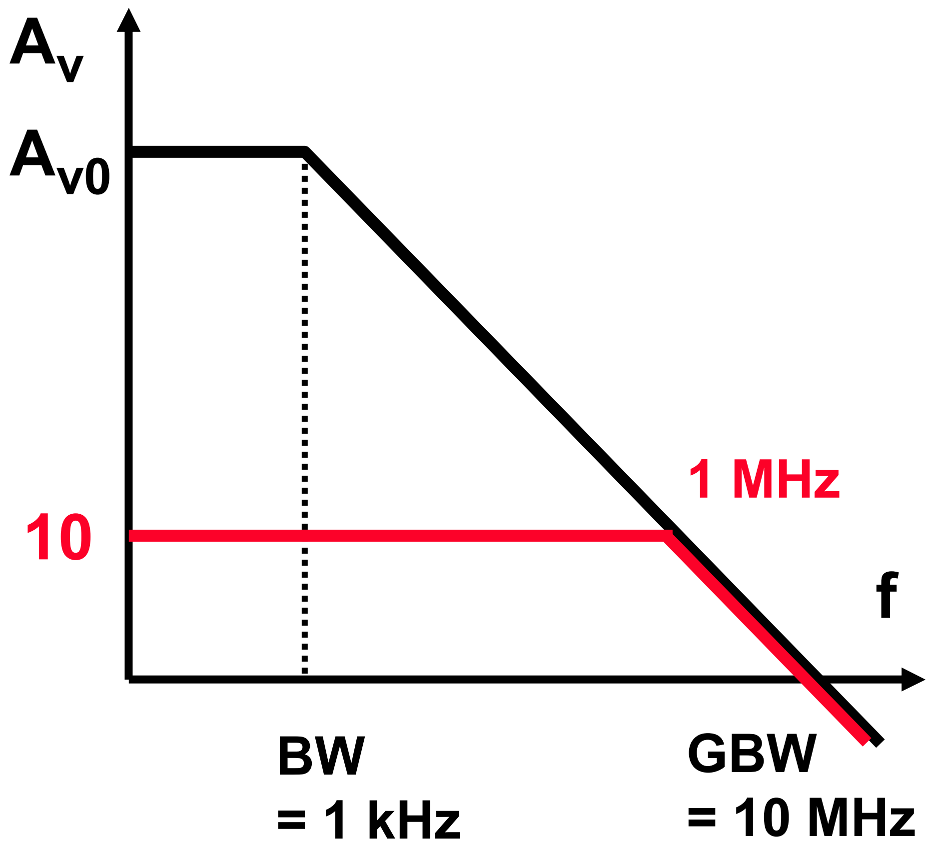
- ❑ Distortion in a bipolar transistor
- ❑ Reduction of distortion by feedback
- ❑ Distortion in an opamp
- ❑ Other cases of distortion and guide lines

Miller CMOS opamp with Feedback

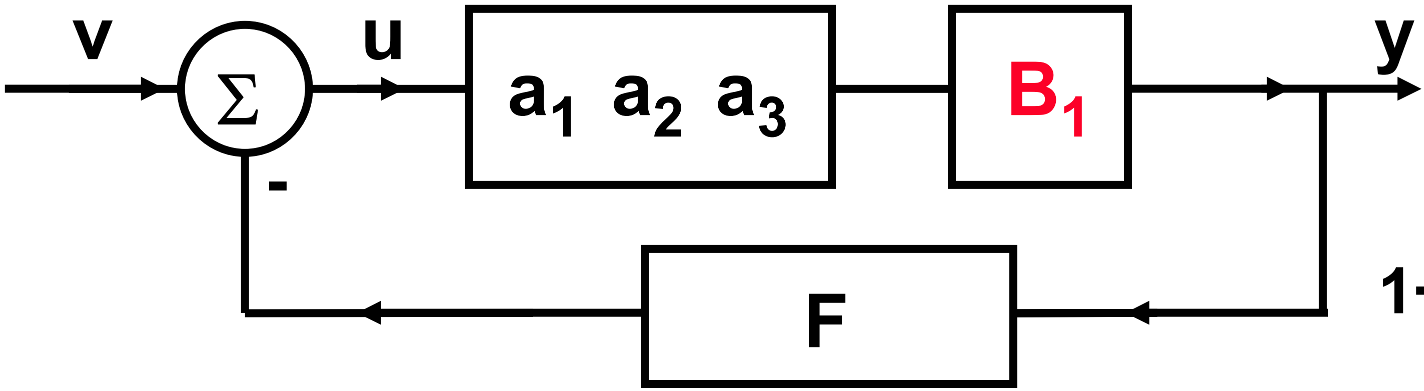




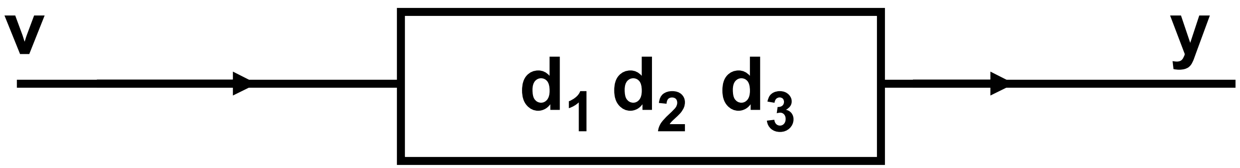
GBW = 10 MHz & A_{vc} = 10

Z_L = 100 kΩ//5pF

Distortion in input stage



$1+T = 1 + B_1 a_1 F$



$u = v - Fy$

$y = B_1(a_1 u + a_2 u^2 + a_3 u^3)$

} elim. u

$y = d_1 v + d_2 v^2 + d_3 v^3$

} elim. y

coeff v : d_1

coeff v^2 : d_2

coeff v^3 : d_3

Distortion in input stage

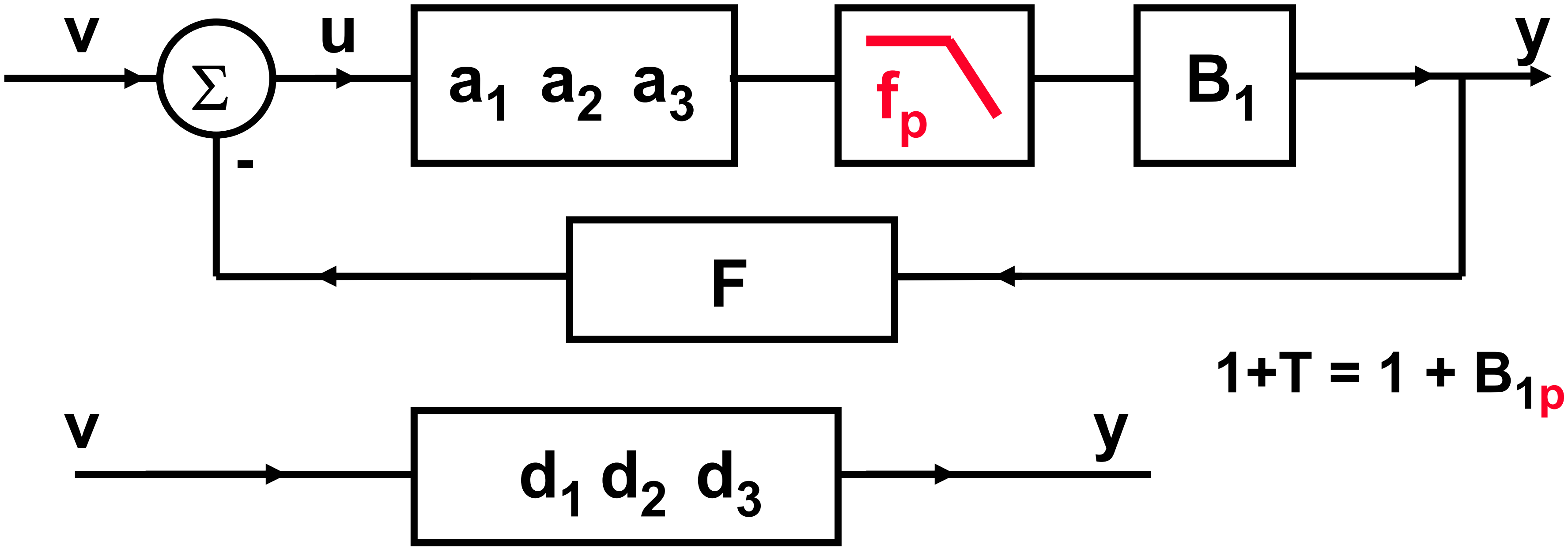
$$IM_{2f} = \frac{d_2}{d_1} V = \frac{a_2}{a_1} \frac{V}{(1+T)^2} = \frac{a_2}{a_1} \frac{1}{(1+T)} \frac{V}{(1+T)}$$

$$IM_{3f} = \frac{3}{4} \frac{d_3}{d_1} V^2 = \frac{3}{4} \left[\frac{a_3}{a_1} \frac{1}{(1+T)} - \left(\frac{a_2}{a_1} \right)^2 \frac{2T}{(1+T)^2} \right] \frac{V^2}{(1+T)^2}$$

Same as before but with different Loop gain :

$$1+T = 1 + B_1 a_1 F$$

Distortion in input stage with LPF



$$1+T = 1 + B_{1p}a_1F$$

$$u = v - Fy$$

$$y = B_{1p}(a_1u + a_2\,u^2 + a_3\,u^3)$$

elim. u

$$y = d_1v + d_2\,v^2 + d_3\,v^3$$

elim. y

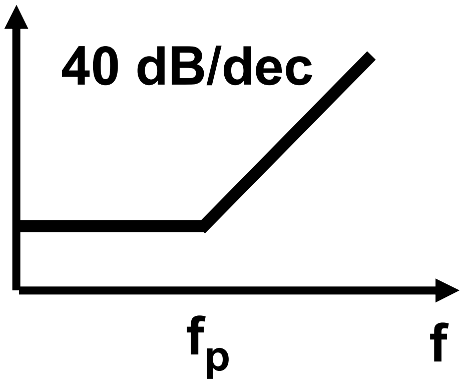
coeff v : d_1

coeff v^2 : d_2

coeff v^3 : d_3

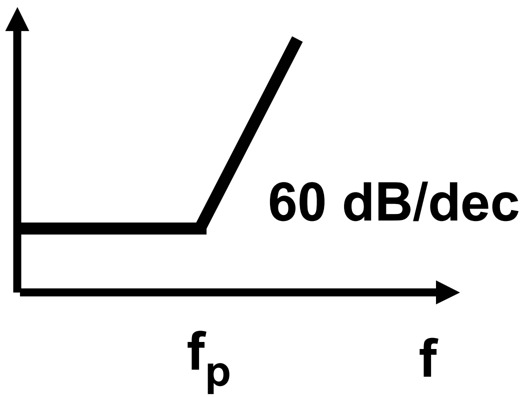
Distortion in input stage with LPF

$$IM_{2f} = \frac{a_2}{a_1}\frac{V}{(1+T)^2} = \frac{a_2}{a_1}\frac{1}{(B_{1p}a_1F)^2}V$$

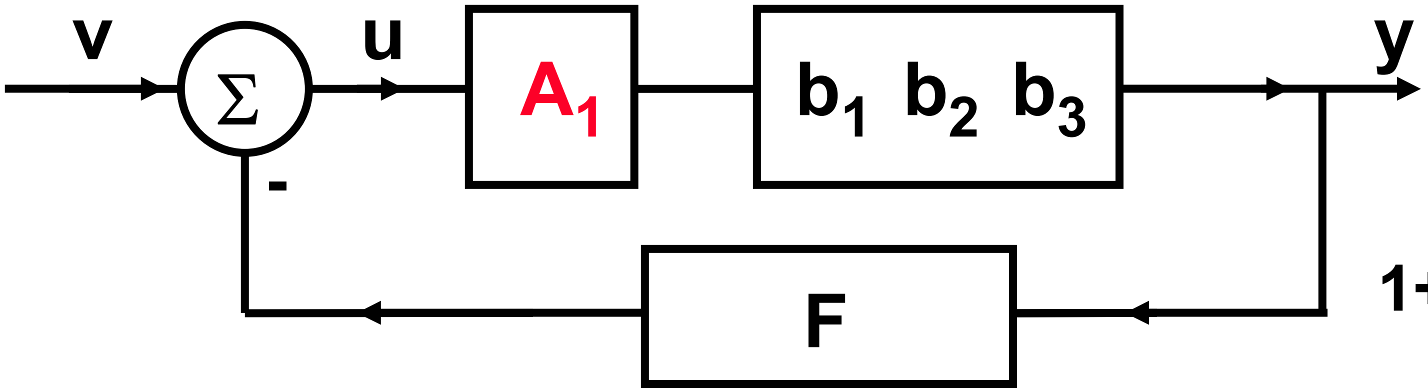


$$IM_{3f}\Big|_{\text{diff.pair}} = \frac{3}{4}\frac{a_3}{a_1}\frac{1}{(1+T)}\frac{V^2}{(1+T)^2} = \frac{3}{4}\frac{a_3}{a_1}\frac{1}{(B_{1p}a_1F)^3}V^2$$

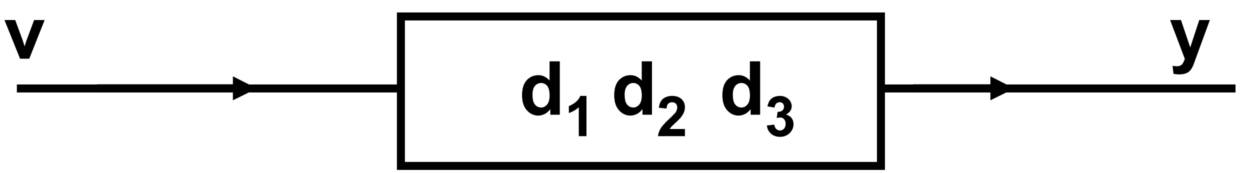
$$IM_{3f}\Big|_{\text{Single trans.}} = \frac{3}{4}\frac{a_2^2}{a_1^2}\frac{2}{(B_{1p}a_1F)^3}V^2$$



Distortion in output stage



$1+T = 1 + A_1 b_1 F$



$u = v - Fy$

$y = A_1 b_1 u + A_1^2 b_2 u^2 + A_1^3 b_3 u^3$

elim. u

$y = d_1 v + d_2 v^2 + d_3 v^3$

elim. y

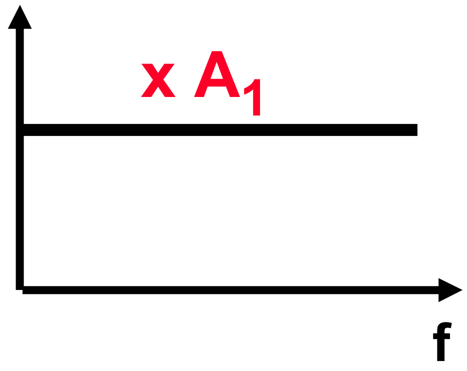
coeff v : d_1

coeff v^2 : d_2

coeff v^3 : d_3

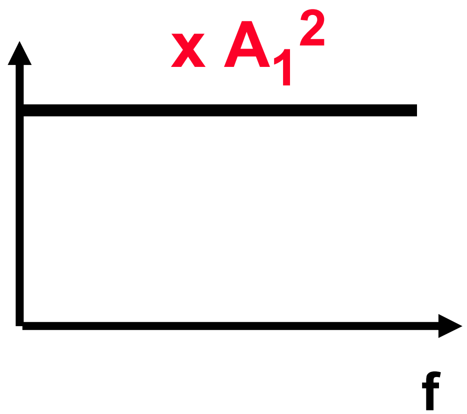
Distortion in output stage

$$IM_{2f} = \frac{b_2}{b_1}\,\frac{V}{(1+T)^2} = \frac{b_2}{b_1}\,\frac{A_1}{(A_1 b_1 F)^2}\,V$$

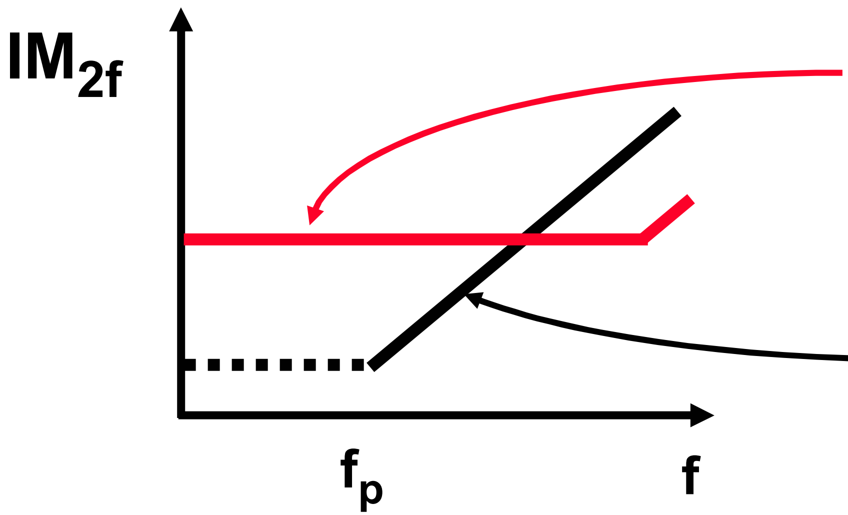


$$IM_{3f}\Big|_{\text{Single trans.}} = \frac{3}{4}\left(\frac{b_2}{b_1}\right)^2 \frac{2T}{(1+T)^2}\,\frac{V^2}{(1+T)^2}$$

$$= \frac{3}{4}\,\frac{{b_2}^2}{{b_1}^2}\,\frac{2\,{A_1}^2}{(A_1 b_1 F)^3}\,V^2$$



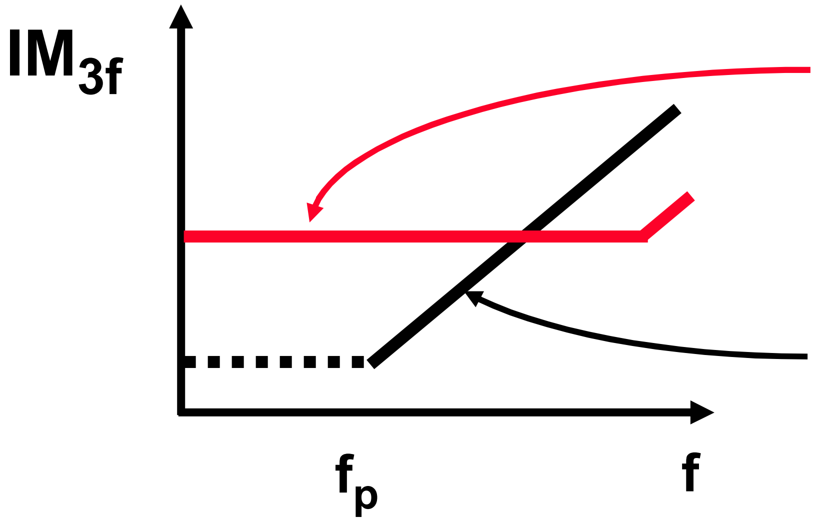
Two-stage opamp a & b



$$\frac{b_2}{b_1}\, a_1\, \frac{V}{T^2}$$

$$T = a_1 b_1 F$$

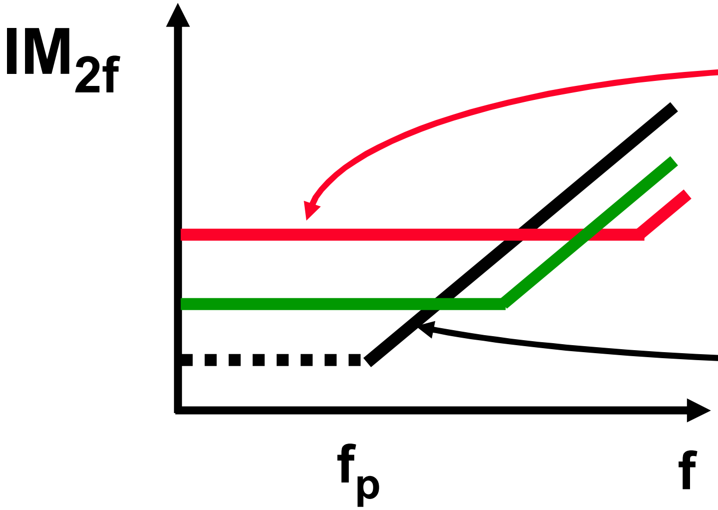
$$\frac{a_2}{a_1}\, \frac{V}{T^2} \left(\frac{f}{f_p}\right)^2$$



$$\frac{3}{4}\, \frac{b_3}{b_1}\, a_1^2\, \frac{V^2}{T^3} \left(\frac{b_1 b_3 - 2\, b_2^2}{b_1 b_3}\right)$$

$$\frac{3}{4}\, \frac{a_3}{a_1}\, \frac{V^2}{T^3} \left(\frac{f}{f_p}\right)^3 \left(\frac{a_1 a_3 - 2\, a_2^2}{a_1 a_3}\right)$$

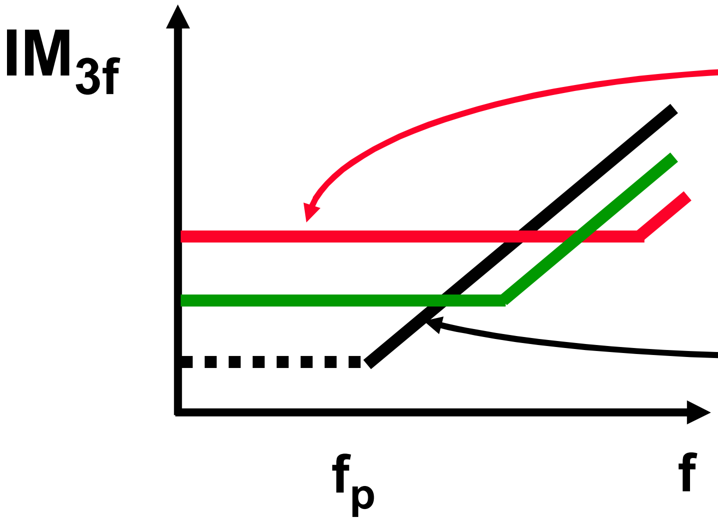
Three-stage opamp a & b & c



$$\frac{c_2}{c_1}\, a_1 b_1 \,\frac{V}{T^2}$$

$$T = a_1 b_1 c_1 F$$

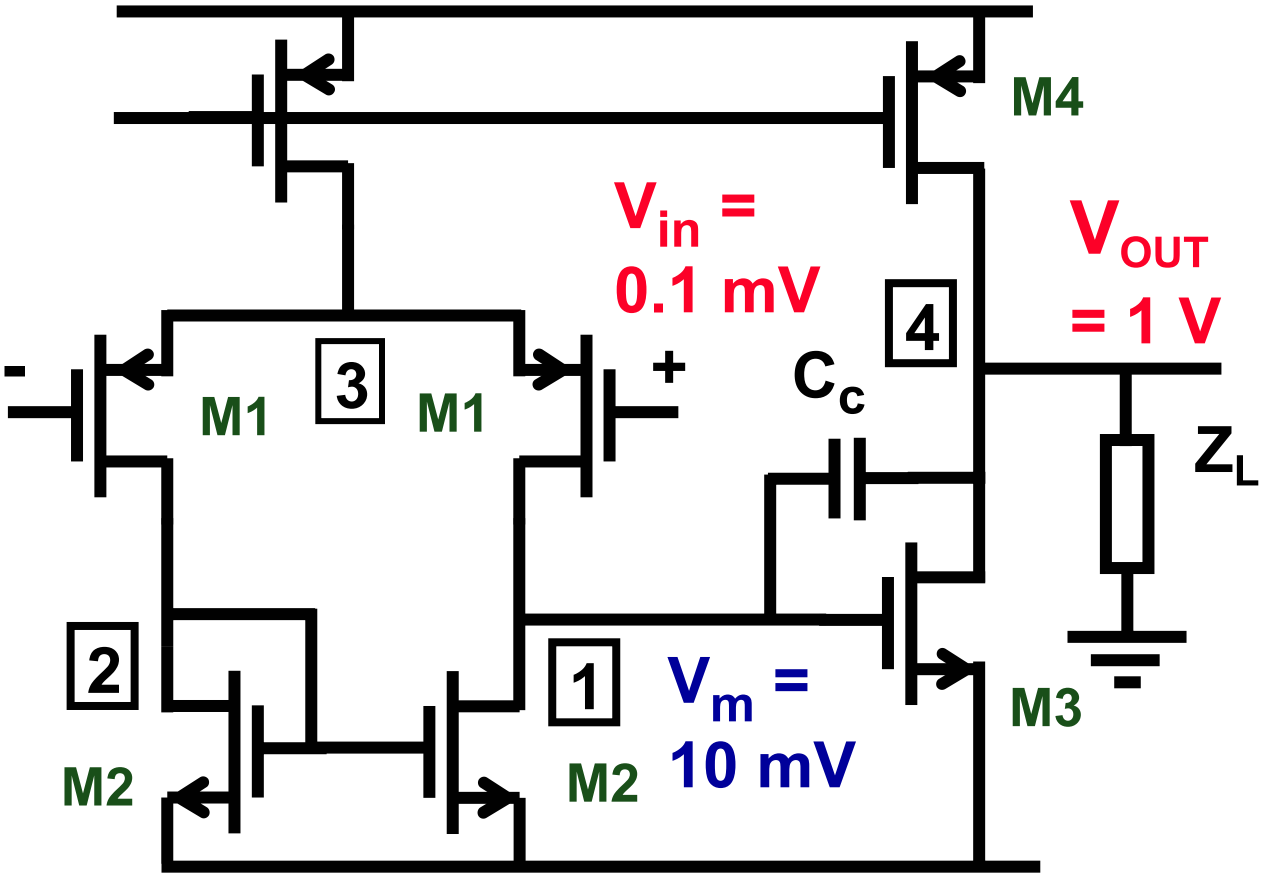
$$\frac{a_2}{a_1}\;\frac{V}{T^2}\left(\frac{f}{f_p}\right)^2$$



$$\frac{3}{4}\,\frac{c_3}{c_1}\, a_1^2 b_1^2 \,\frac{V^2}{T^3}\left(\frac{c_1 c_3 - 2\, c_2^2}{c_1 c_3}\right)$$

$$\frac{3}{4}\,\frac{a_3}{a_1}\;\frac{V^2}{T^3}\left(\frac{f}{f_p}\right)^3\left(\frac{a_1 a_3 - 2\, a_2^2}{a_1 a_3}\right)$$

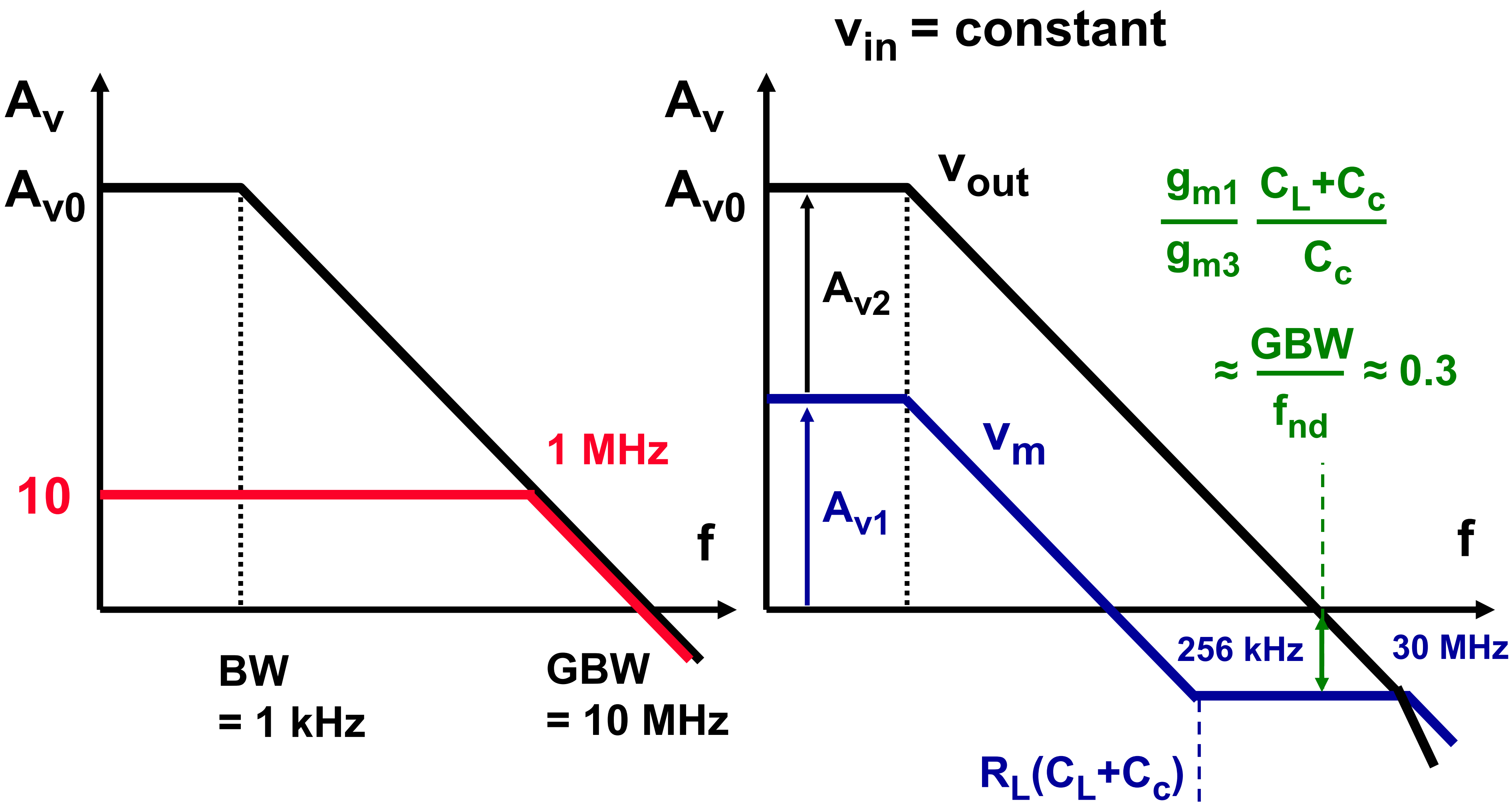
Distortion in an opamp at low frequencies



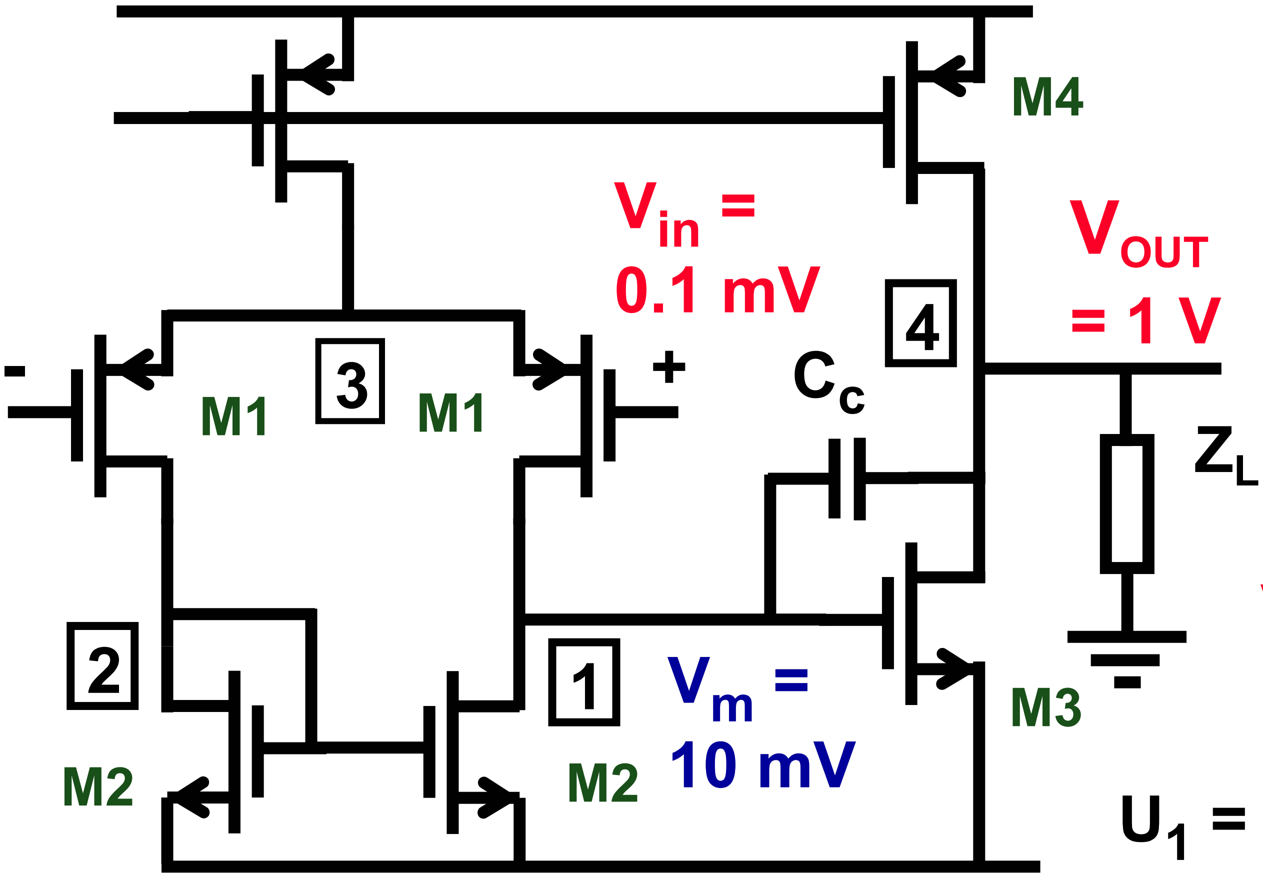
GBW = 10 MHz
A_{v0} = 10.000
BW = 1 kHz
A_{vc} = 10

I_{DS1} = 6 μA
g_{m1} = 60 μS
I_{DS3} = 120 μA
g_{m3} = 1.2 mS
R_L = 100 k Ω
C_L = 5 pF
C_c = 1 pF

Low-distortion amplifier



Distortion in an opamp at low frequencies



GBW = 10 MHz

$A_{vc} = 10$

$I_{DS1} = 6\ \mu A$

$I_{DS3} = 120\ \mu A$

V_{node} at 100 Hz ?

$U_1 = g_{m1}V_{in}/I_{DS1} = 5\ 10^{-4}$

$U_3 = g_{m3}V_m/I_{DS3} = 0.1$

Distortion in an opamp at low frequencies

Distortion generation by nonlinear output stage :

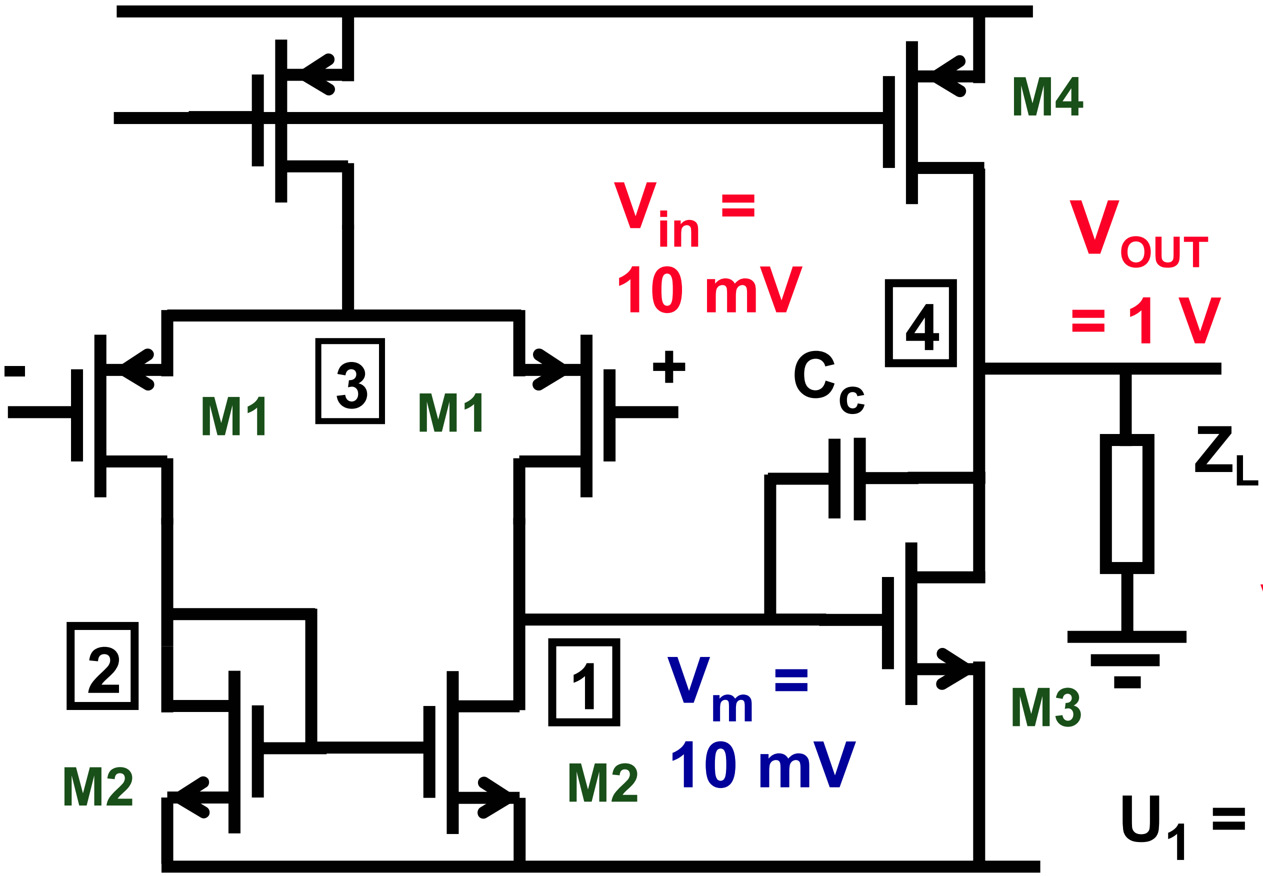
$U_3 = g_{m3}V_m/I_{DS3} = 0.1$

$IM_2 = U_3/4 = 0.25\ 0.1 = 2.5\ \%$

Distortion reduction by feedback :

$T = 1000$ $IM_{2f} = 2.5\ \%/1000 = 0.0025\ \%$ Negligible !

Distortion in an opamp at high frequencies



GBW = 10 MHz

$A_{vc} = 10$

$I_{DS1} = 6\ \mu A$

$I_{DS3} = 120\ \mu A$

V_{node} at 100 kHz ?

$U_1 = g_{m1}V_{in}/I_{DS1} = 5\ 10^{-2}$

$U_3 = g_{m3}V_m/I_{DS3} = 0.1$

Distortion in an opamp at high frequencies

Distortion generation by nonlinear output stage :

$U_3 = g_{m3}V_m/I_{DS3} = 0.1$

$IM_2 = U_3/4 = 0.25\ 0.1 = 2.5\ \%$

Distortion generation by nonlinear input stage :

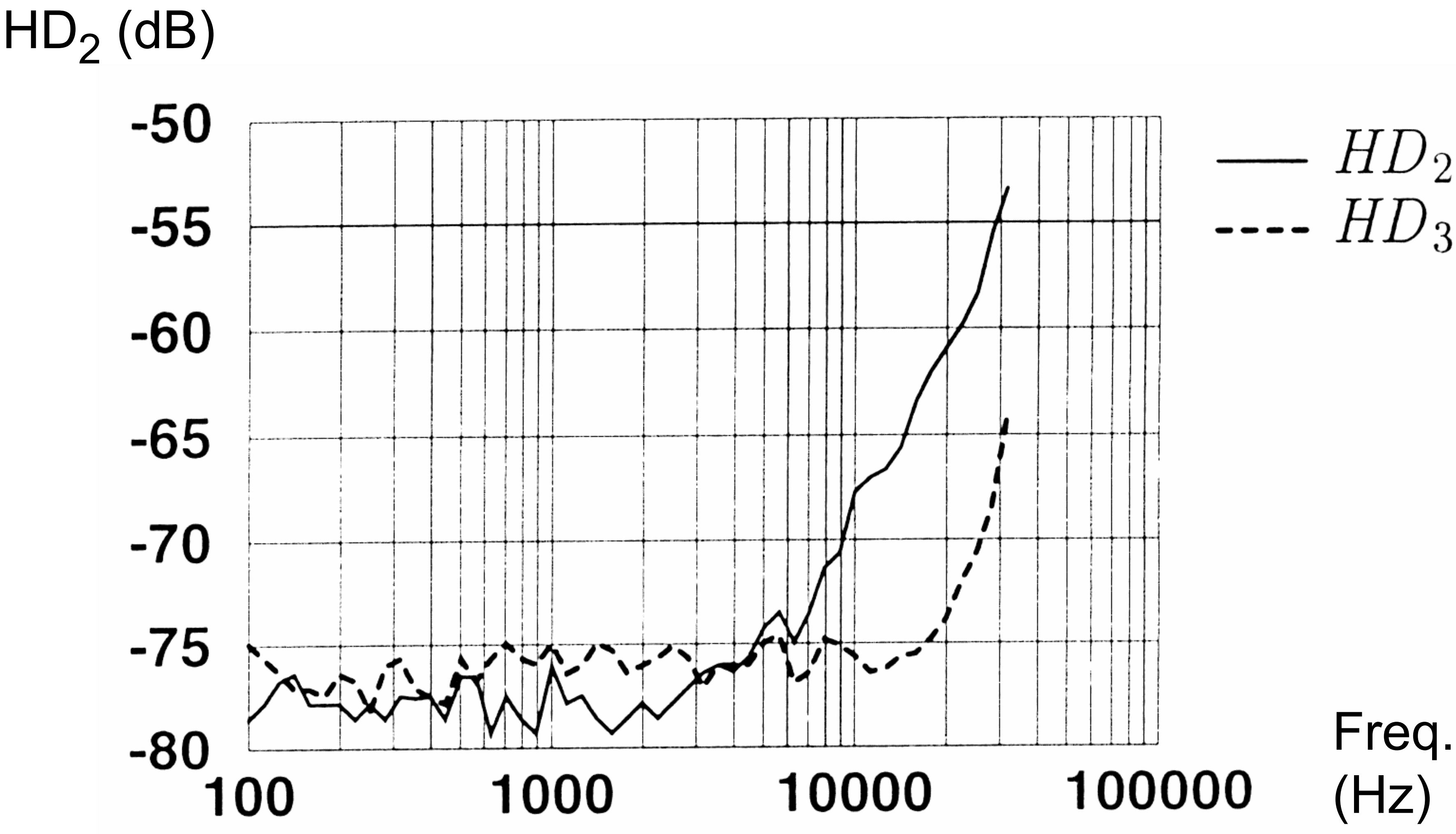
$U_1 = g_{m1}V_m/I_{DS1} = 0.05$

$IM_3 = U_1^2/10 = 0.0025/10 = 0.025\ \%$ Negligible !

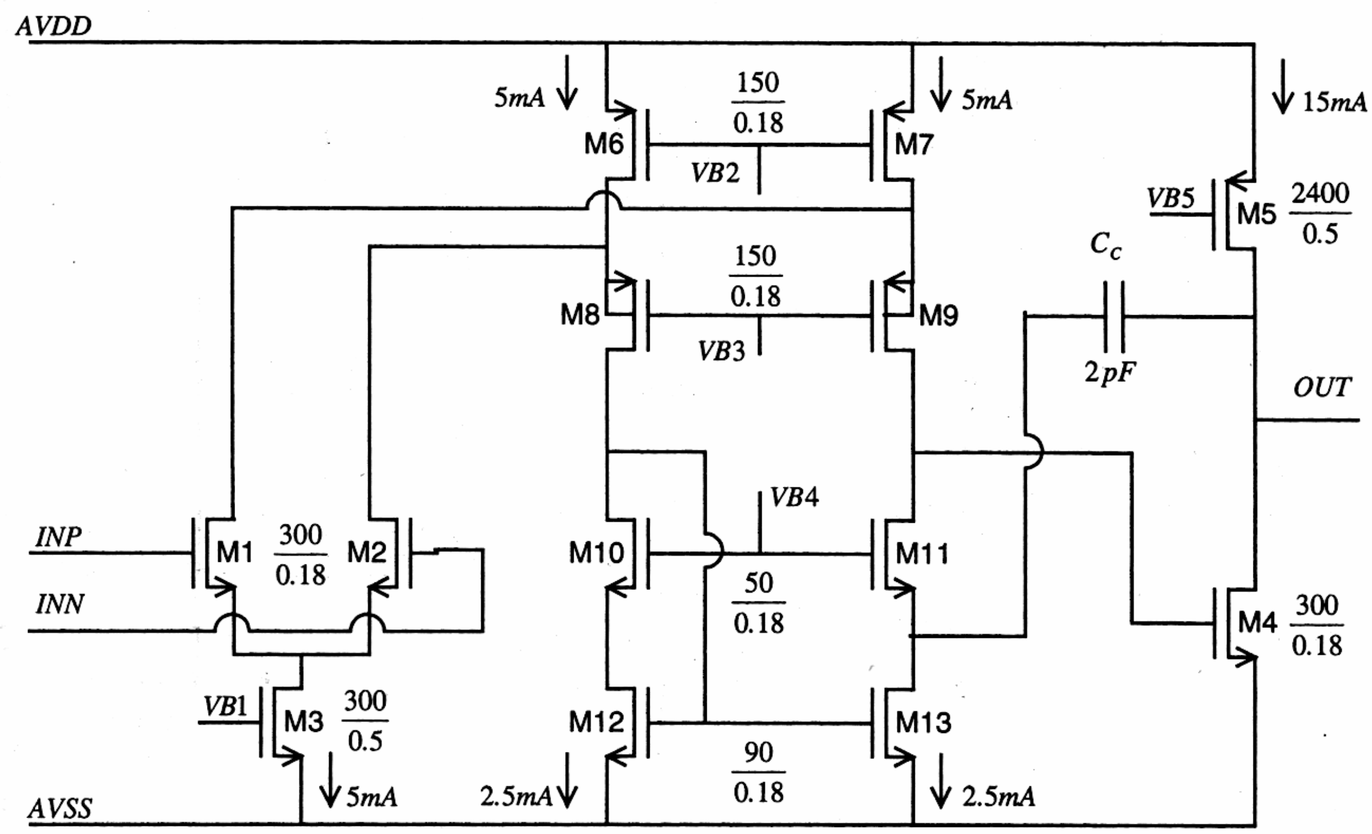
Distortion reduction by feedback :

$T = 10 \qquad IM_{2f} = 2.5\ \%/100 = 0.25\ \%$

Miller CMOS OTA Measured Distortion



1.8 V Low distortion CMOS Opamp



GBW ≈ 3 GHz

C_L = 8 pF

SR ≈ 900 V/μs

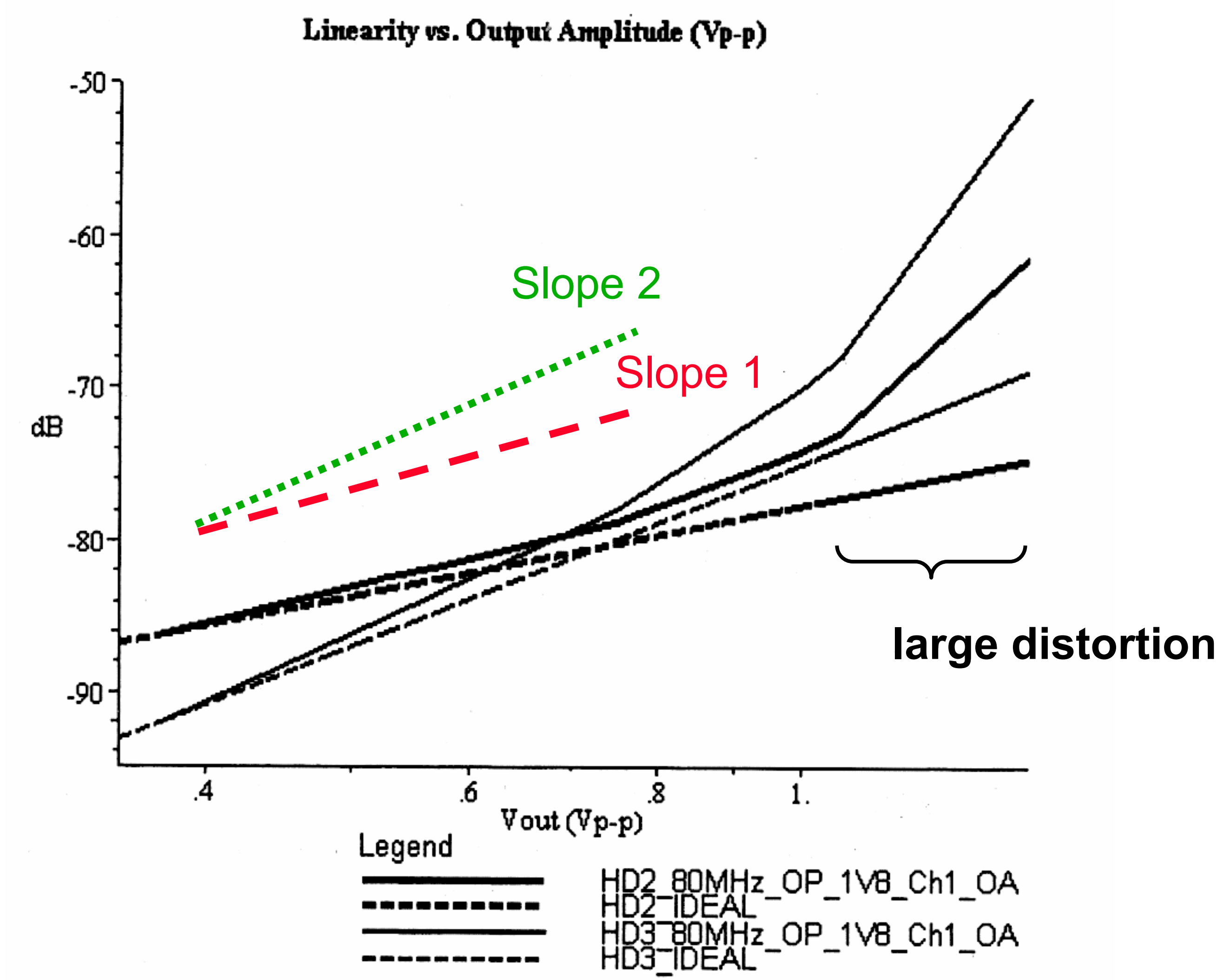
$$f_P = \frac{SR}{2\pi\ Vpeak}$$

f_P = 380 MHz
at 0.38 Vpeak

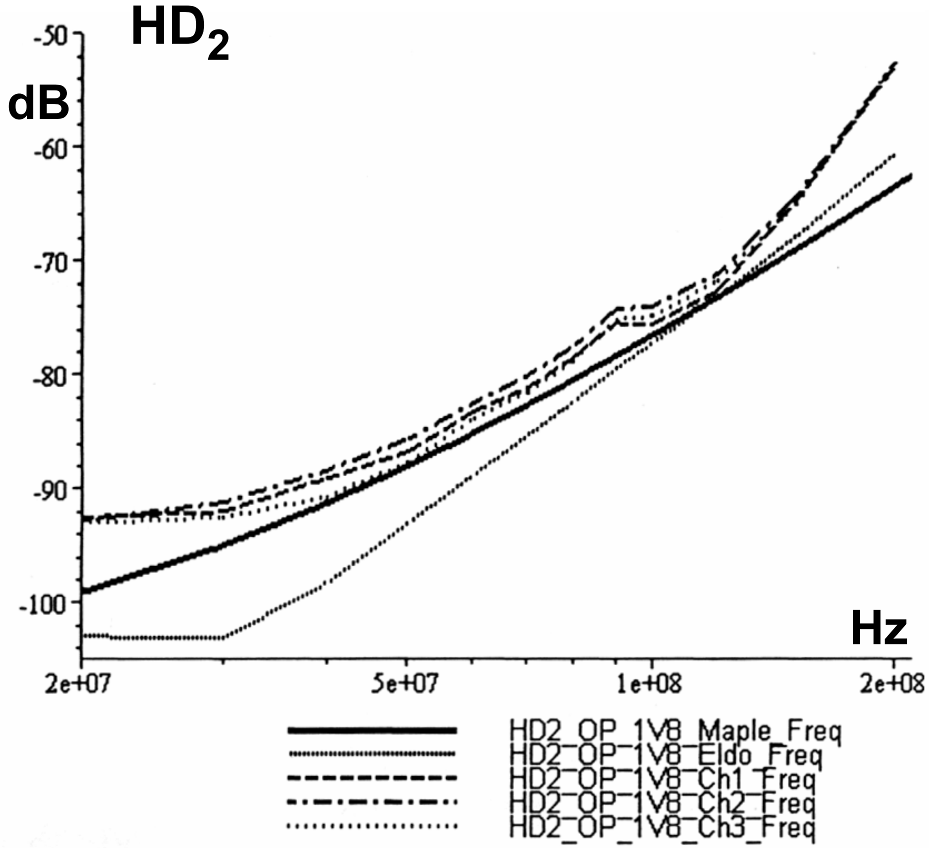
Large V_{GS4}-V_T

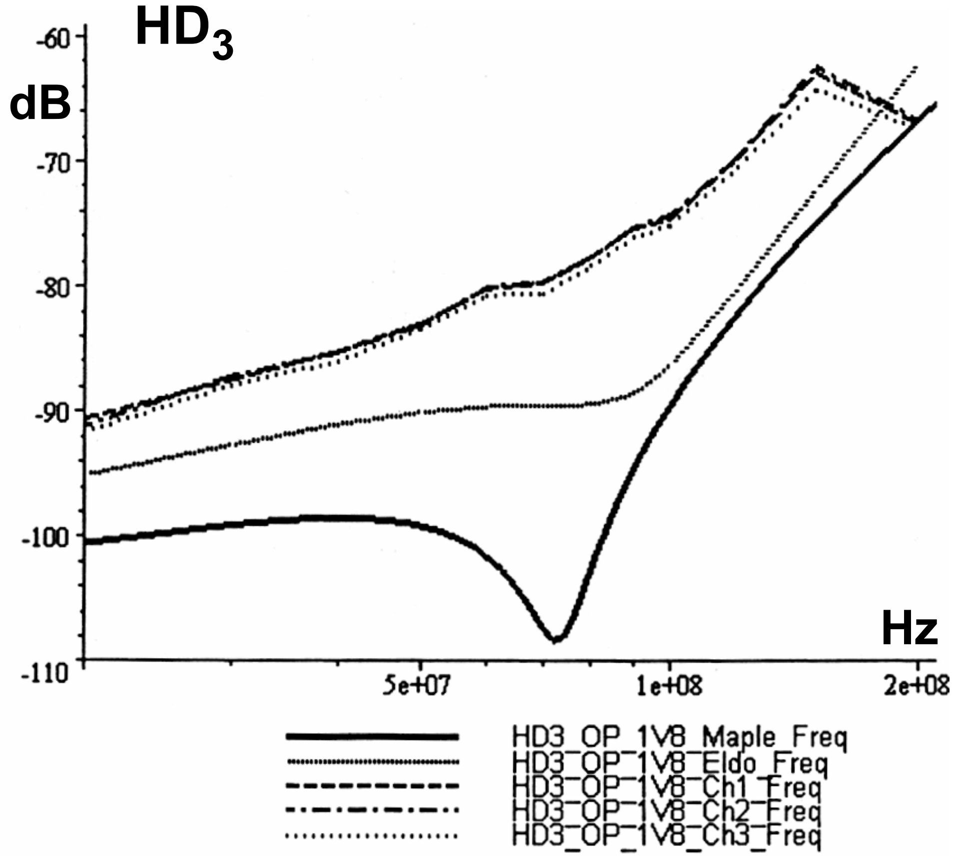
Ref.Hernes Kluwer 2003

HD2 & HD3 vs Amplitude



HD2 & HD3 vs Frequency





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Other cases of distortion and guide lines

- Distortion caused by limited SR
- Distortion of a switch
- Distortion at high frequencies :

  Volterra series instead of power series
- Distortion in continuous-time filters
- Guide lines

Guide lines for low distortion

- Scaling such that voltage amplitudes
  are limited
- Scaling such that relative current swings
  are limited
- Feedback
- All fully differential

Distortion components

Distortion comp.	IM_2 x U_p	IM_3 x U_p^2	$U_p = \frac{V_{ip}}{V_{ref}}$ V_{ref} =
Bipolar	1/2	1/8	kT_e/q
MOST	1/4	0	$(V_{GS}-V_T)/2$
Bip. diff.pair	0	1/4	$2kT_e/q$
MOST diff.pair	0	3/32	$(V_{GS}-V_T)$

Distortion components with Feedback (T > 5)

Distortion comp.	IM_2 $\times U_p$	$-IM_3$ $\times U_p^2$	$U_p = \dfrac{V_{ip}}{V_{ref}}$ $V_{ref} =$
Bipolar	1/2T	1/4T	$kT_e/q \times T$
MOST	1/4T	3/32T	$(V_{GS}-V_T)/2 \times T$
Bip. diff.pair	0	1/4T	$2kT_e/q \times T$
MOST diff.pair	0	3/32T	$(V_{GS}-V_T) \times T$

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- ❑ Definitions : HD, IM, intercept point, ..
- ❑ Distortion in a MOST
  - Single-ended amplifier
  - Differential amplifier
- ❑ Distortion in a bipolar transistor
- ❑ Reduction of distortion by feedback
- ❑ Distortion in an opamp
- ❑ Other cases of distortion and guide lines