Computer Architecture
（计算机体系结构）

Lecture #2 – Number Representation

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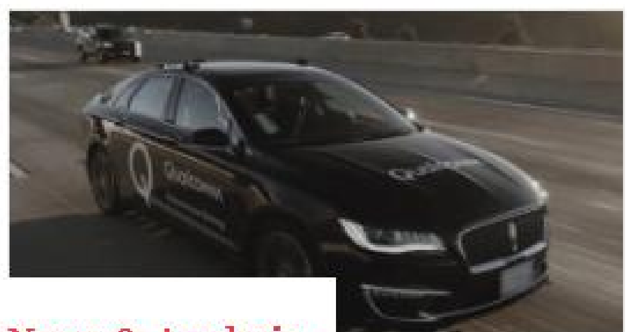


News & Analysis

Qualcomm Prepares to Take Nvidia for a Ride

Review

- **Continued rapid improvement in computing**
  - 2X every 2.0 years in memory size;
    every 1.5 years in processor speed;
    every 1.0 year in disk capacity;
  - Moore's Law enables processor
    (2X transistors/chip every 2 yrs)
- **5 classic components of all computers**

a	b	c	d	e
Control	Datapath	Memory	Input	Output

Processor

What'll be the most important part of a computer in the future?

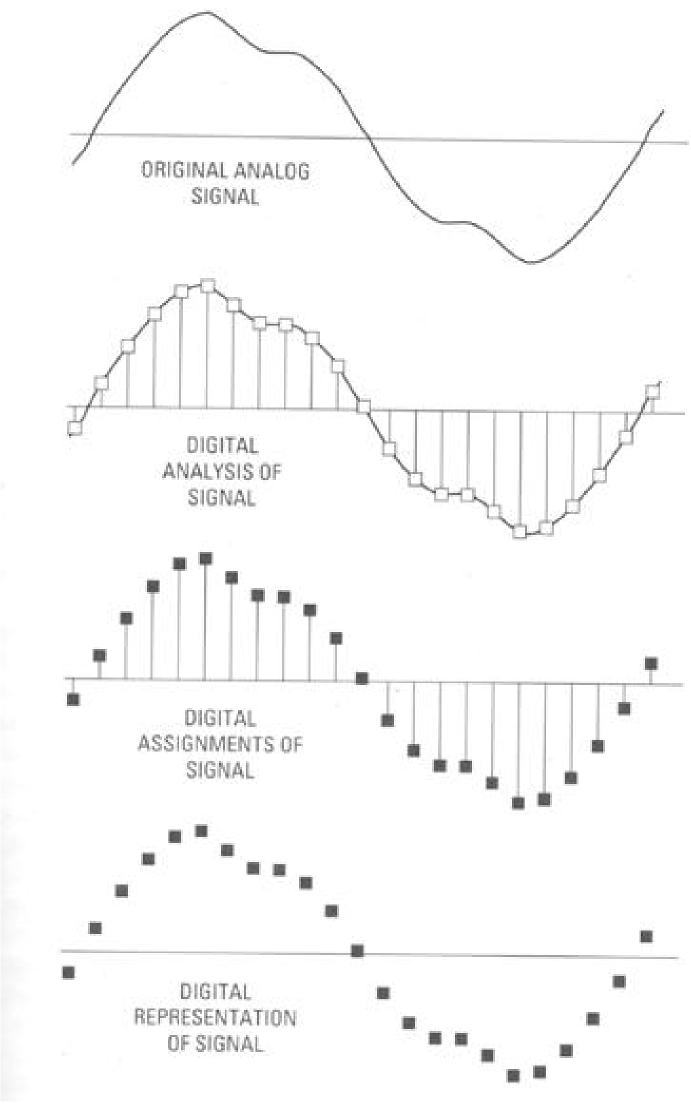
Putting it all in perspective…

“If the automobile had followed the same development cycle as the computer,

– Robert X. Cringely

Data input: Analog → Digital

- Real world is analog!
- To import analog information, we must do two things
  - Sample
    - E.g., for a CD, every 44,100ths of a second, we ask a music signal how loud it is.
  - Quantize
    - For every one of these samples, we figure out where, on a 16-bit (65,536 tic-mark) "yardstick", it lies.



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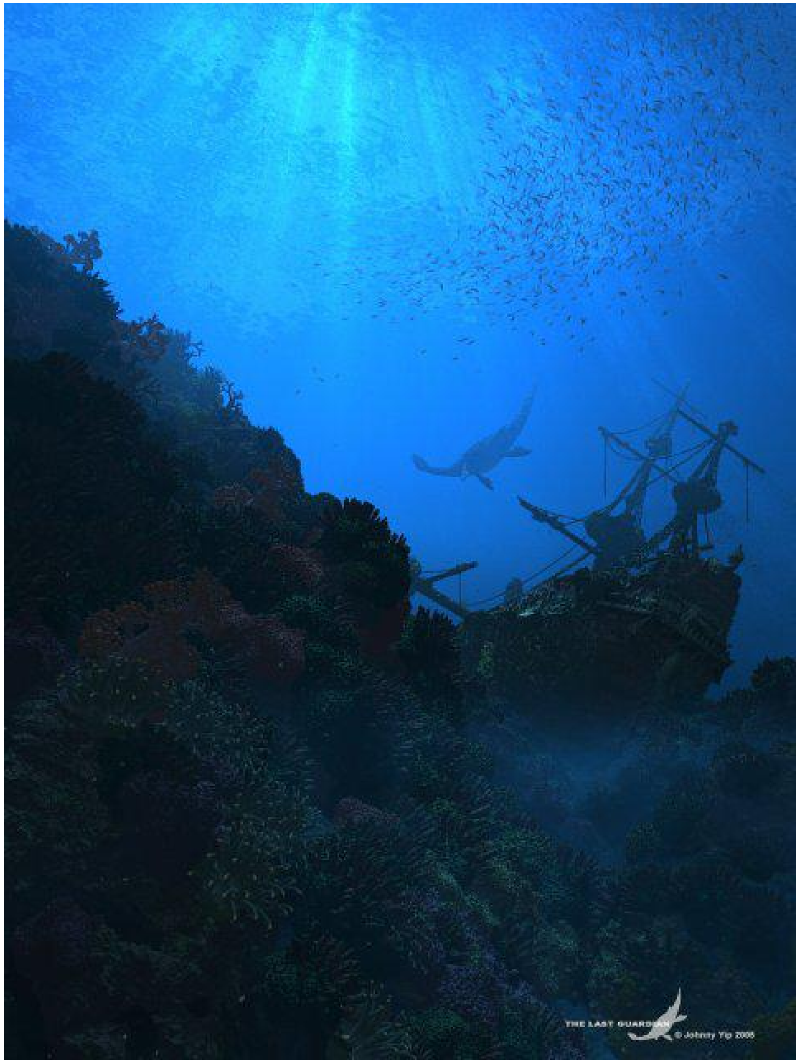
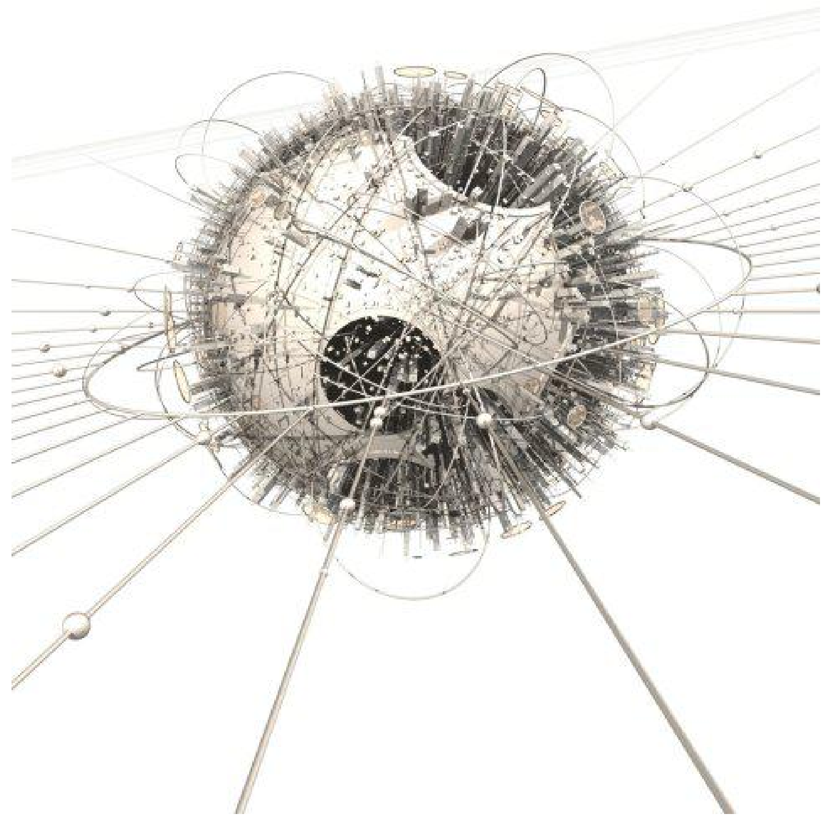
Digital data not nec born Analog…





hof.povray.org

BIG IDEA: Bits can represent anything!!

- **Characters?**
  - 26 letters $\Rightarrow$ 5 bits ($2^5 = 32$)
  - upper/lower case + punctuation
    $\Rightarrow$ 7 bits (in 8) ("ASCII")
  - standard code to cover all the world's languages $\Rightarrow$ 8,16,32 bits ("Unicode")
    `www.unicode.com`
- **Logical values?**
  - 0 $\Rightarrow$ False, 1 $\Rightarrow$ True
- **colors ? Ex:** *Red (00)* *Green (01)* *Blue (11)*
- **locations / addresses? commands?**
- **MEMORIZE: N bits $\Leftrightarrow$ at most 2^N things**

How many bits to represent π ?

a) 1

b) 9 (π = 3.14, so that's 011 "." 001 100)

c) 64 (Since Macs are 64-bit machines)

d) Every bit the machine has!

e) ∞

What to do with representations of numbers?

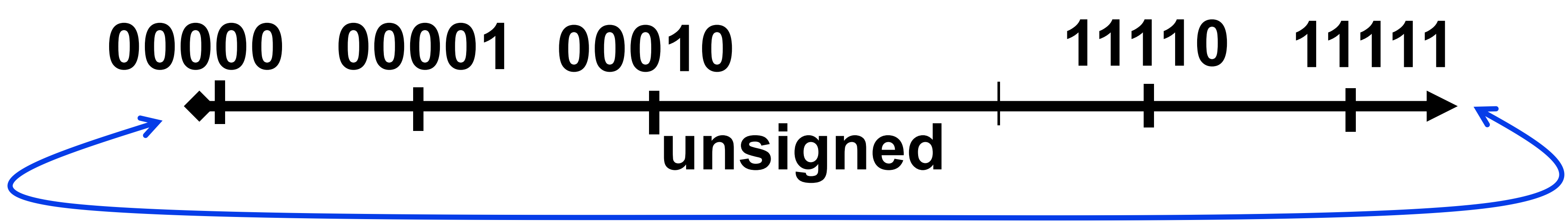
- Just what we do with numbers!
  - Add them
  - Subtract them
  - Multiply them
  - Divide them
  - Compare them

	1	1			
	1	0	1	0	
+	0	1	1	1	
1	0	0	0	1	

- Example: 10 + 7 = 17
  - …so simple to add in binary that we can build circuits to do it!
  - subtraction just as you would in decimal
  - Comparison: How do you tell if X > Y ?

What if too big?

- Binary bit patterns above are simply representatives of numbers. Strictly speaking they are called “numerals”.
- Numbers really have an ∞ number of digits
  - with almost all being same (00…0 or 11…1) except for a few of the rightmost digits
  - Just don’t normally show leading digits
- If result of add (or -, *, /) cannot be represented by these rightmost HW bits, overflow is said to have occurred.



How to Represent Negative Numbers?

(C's `unsigned int`, C99's `uintN_t`)

- So far, unsigned numbers

Binary odometer

00000 00001 ... 01111 10000 ... 11111

- Obvious solution: define leftmost bit to be sign!
  - 0 → + 1 → –
  - Rest of bits can be numerical value of number
- Representation called sign and magnitude

Binary odometer

00000 00001 ... 01111

11111 ... 10001 10000

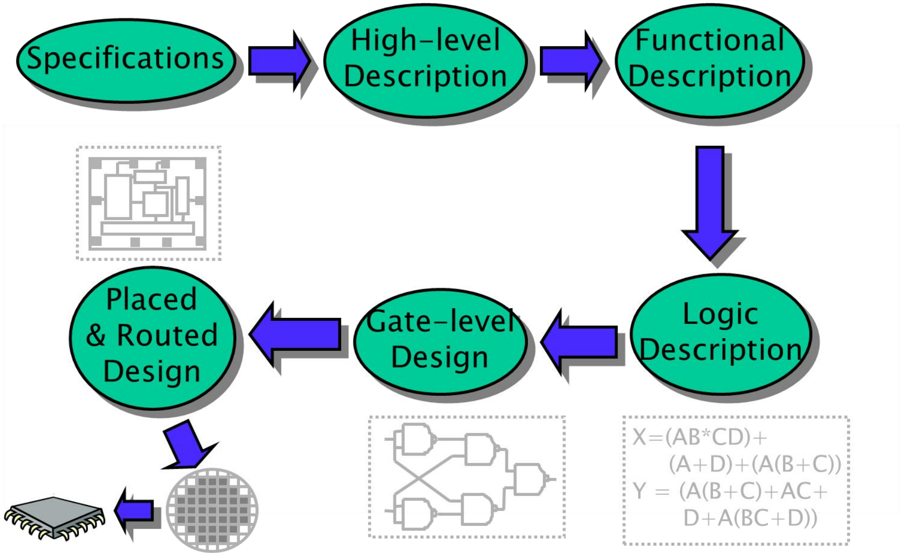
META: Ain't no free lunch

Shortcomings of sign and magnitude?

- **Arithmetic circuit complicated**
  - Special steps depending whether signs are the same or not
- **Also, two zeros**
  - $0x00000000 = +0_{ten}$
  - $0x80000000 = -0_{ten}$
  - What would two 0s mean for programming?
- **Also, incrementing "binary odometer", sometimes increases values, and sometimes decreases!**
- **Therefore sign and magnitude abandoned**

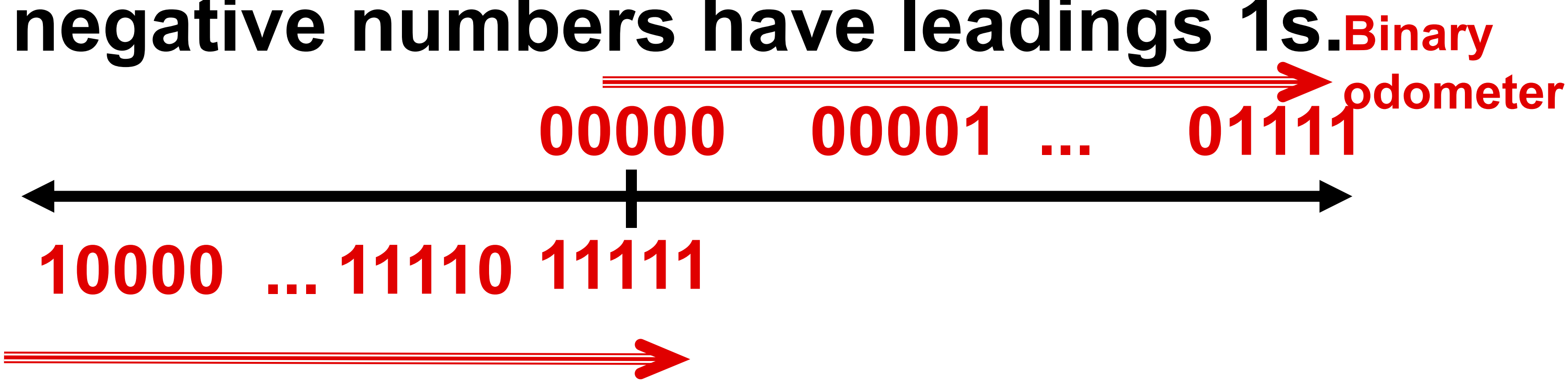
Great EDA course I supervise

- **Introduction to VLSI Design Automation**
  - The first EDA course in Beihang University
  - Learn physical design or design automation of ICs
  - Prereqs (data structures, programming language, algorithms, VLSI design)
  - `http://www.cadetlab.cn/courses`



Another try: complement the bits

- Example: $7_{10} = 00111_2$ $-7_{10} = 11000_2$
- Called <u>One's Complement</u>
- Note: positive numbers have leading 0s, negative numbers have leadings 1s.



- What is -00000 ? Answer: 11111
- How many positive numbers in N bits?
- How many negative numbers?

Shortcomings of One's complement?

- Arithmetic still a somewhat complicated.
- Still two zeros
  - `0x00000000` = $+0_{ten}$
  - `0xFFFFFFFF` = -0_{ten}
- Although used for a while on some computer products, one's complement was eventually abandoned because another solution was better.

Standard Negative # Representation

- **Problem is the negative mappings "overlap" with the positive ones (the two 0s). Want to shift the negative mappings left by one.**
  - Solution! For negative numbers, complement, then add 1 to the result
- **As with sign and magnitude, & one's compl. leading 0s is positive, leading 1s is negative**
  - 000000...xxx is ≥ 0, 111111...xxx is < 0
  - except 1…1111 is -1, not -0 (as in sign & mag.)
- **This representation is Two's Complement**
  - This makes the hardware simple!

(C's `int`, aka a "signed integer")

(Also C's `short`, `long long`, …, C99's `intN_t`)

Two's Complement Formula

- Can represent positive and negative numbers in terms of the bit value times a power of 2:

$$d_{31} \times -(2^{31}) + d_{30} \times 2^{30} + \ldots + d_2 \times 2^2 + d_1 \times 2^1 + d_0 \times 2^0$$

- Example: 1101_{two} in a nibble?

$= 1x-(2^3) + 1x2^2 + 0x2^1 + 1x2^0$

$= -2^3 + 2^2 + 0 + 2^0$

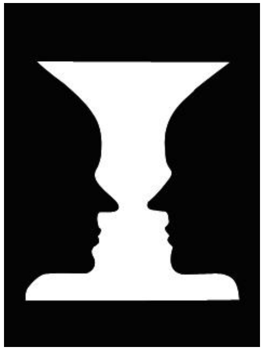
$= -8 + 4 + 0 + 1$

$= -8 + 5$

$= -3_{ten}$

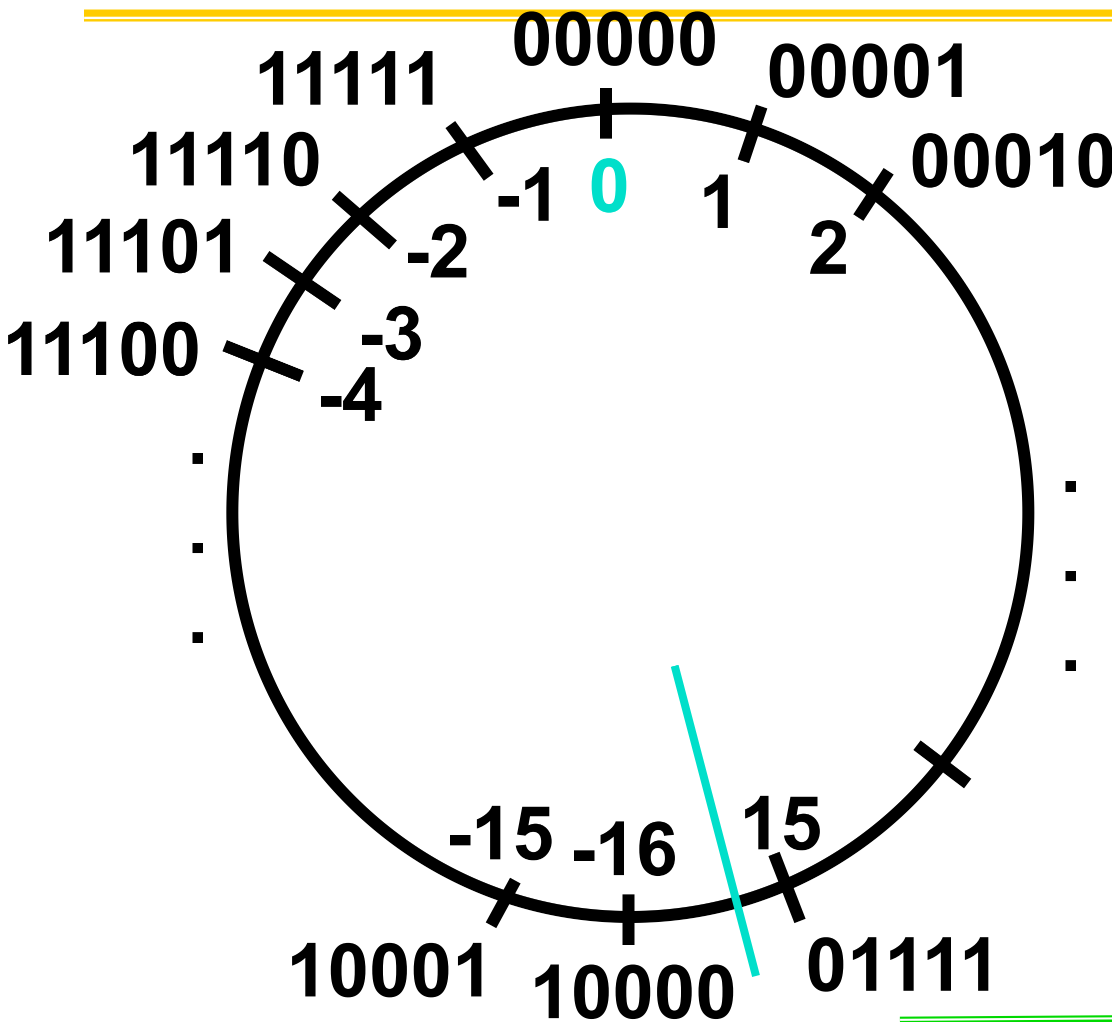
Example: -3 to +3 to -3 (again, in a nibble):

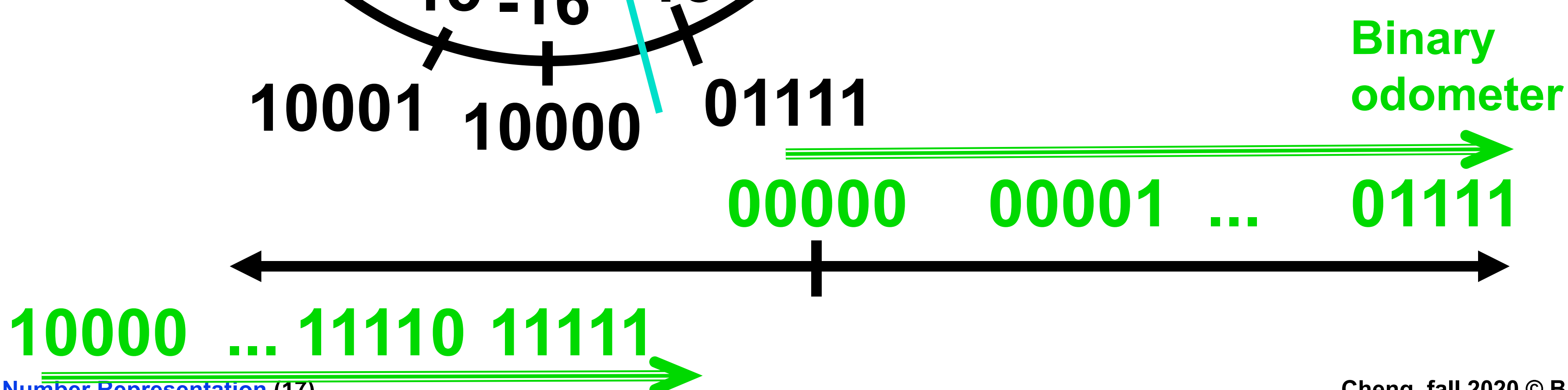
x : 1101_{two}
x': 0010_{two}
+1: 0011_{two}
()': 1100_{two}
+1: 1101_{two}



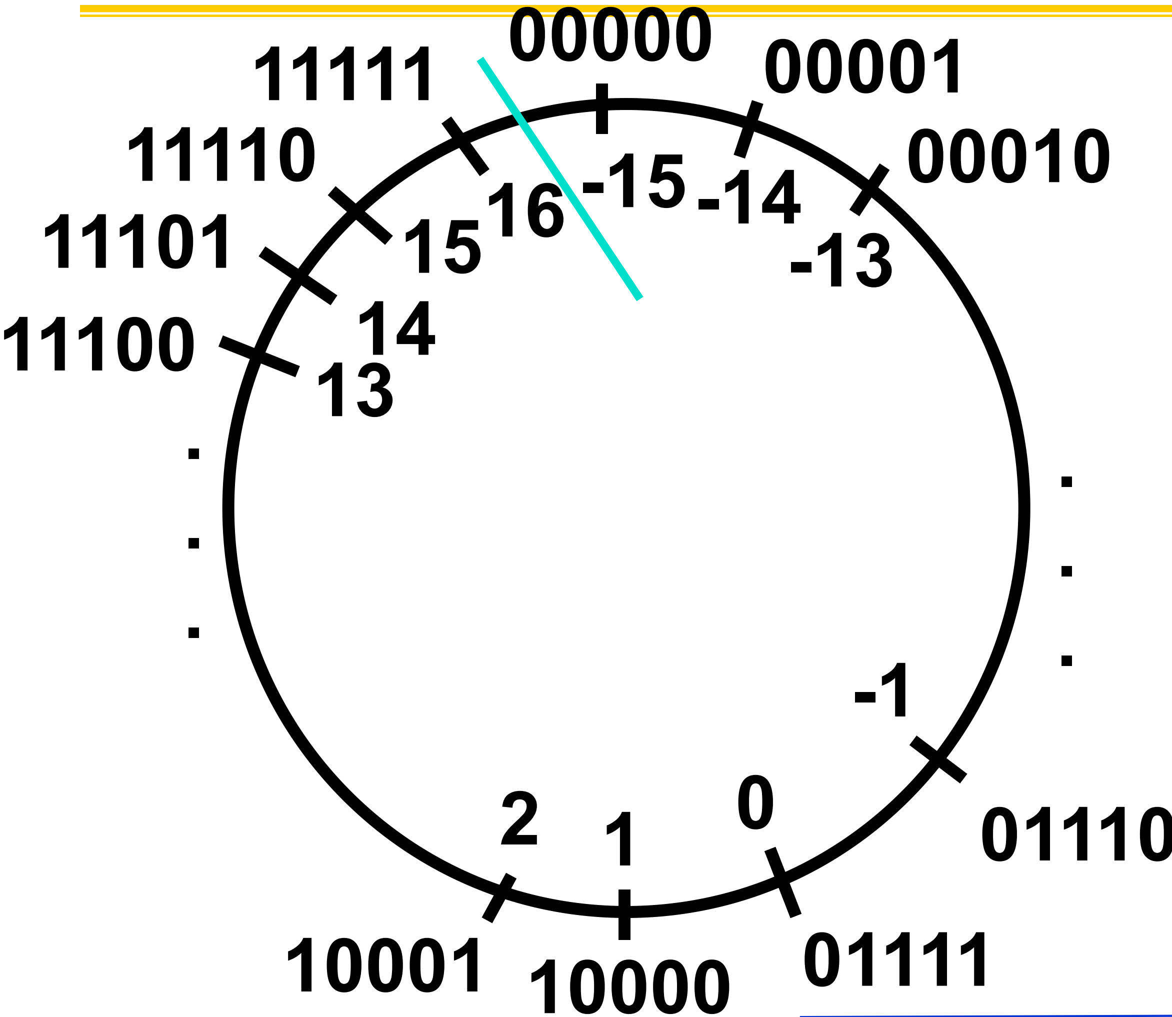
2’s Complement Number “line”: N = 5

- 2^{N-1} non-negatives
- 2^{N-1} negatives
- one zero
- how many positives?

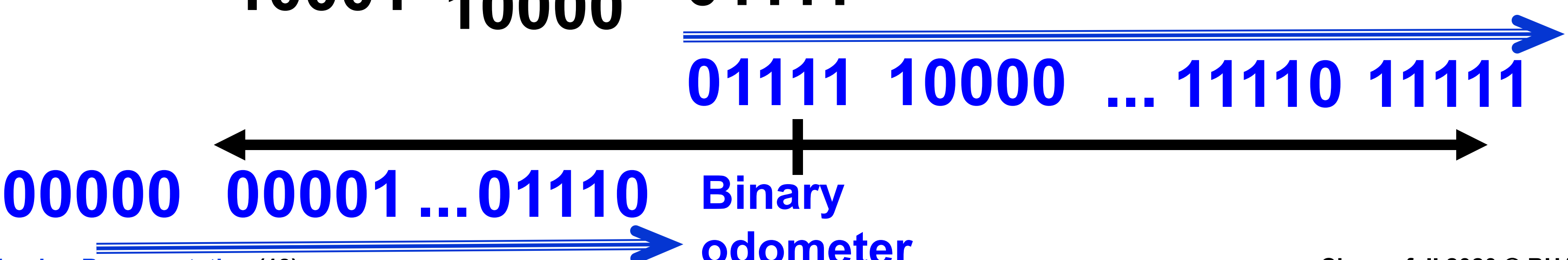




Bias Encoding: N = 5 (bias = -15)



- # = unsigned + bias
- Bias for N bits chosen as $-(2^{N-1}-1)$
- one zero
- how many positives?



How best to represent -12.75?

a) **2s Complement** (but shift binary pt)

b) **Bias** (but shift binary pt)

c) **Combination of 2 encodings**

d) **Combination of 3 encodings**

e) **We can't**

Shifting binary point means "divide number by some power of 2. E.g., $11_{10} = 1011.0_2 \rightarrow 10.110_2 = (11/4)_{10} = 2.75_{10}$

And in summary...

META: We often make design decisions to make HW simple

- **We represent "things" in computers as particular bit patterns: N bits ⇒ 2^N things**
- **These 5 integer encodings have different benefits; 1s complement and sign/mag have most problems.**
- **unsigned (C99's `uintN_t`) :**

00000 00001 ... 01111 10000 ... 11111

- **2's complement (C99's `intN_t`) universal, learn!**
-

00000 00001 ... 01111

10000 ... 11110 11111

- **Overflow: numbers ∞ ; computers finite, errors!**

META: Ain't no free lunch

REFERENCE: Which base do we use?

- **Decimal:** great for humans, especially when doing arithmetic
- **Hex:** if human looking at long strings of binary numbers, its much easier to convert to hex and look 4 bits/symbol
  - Terrible for arithmetic on paper
- **Binary:** what computers use;
  you will learn how computers do +, -, *, /
  - To a computer, numbers always binary
  - Regardless of how number is written:
  - $32_{ten} == 32_{10} == 0x20 == 100000_2 == 0b100000$
  - Use subscripts "ten", "hex", "two" in book, slides when might be confusing

Two's Complement for N=32

0000 ... 0000 0000 0000 0000_{two} =		0_{ten}
0000 ... 0000 0000 0000 0001_{two} =		1_{ten}
0000 ... 0000 0000 0000 0010_{two} =		2_{ten}
. . .		
0111 ... 1111 1111 1111 1101_{two} =		$2,147,483,645_{ten}$
0111 ... 1111 1111 1111 1110_{two} =		$2,147,483,646_{ten}$
0111 ... 1111 1111 1111 1111_{two} =		$2,147,483,647_{ten}$
1000 ... 0000 0000 0000 0000_{two} =		$-2,147,483,648_{ten}$
1000 ... 0000 0000 0000 0001_{two} =		$-2,147,483,647_{ten}$
1000 ... 0000 0000 0000 0010_{two} =		$-2,147,483,646_{ten}$
. . .		
1111 ... 1111 1111 1111 1101_{two} =		-3_{ten}
1111 ... 1111 1111 1111 1110_{two} =		-2_{ten}
1111 ... 1111 1111 1111 1111_{two} =		-1_{ten}

- One zero; 1st bit called <u>sign bit</u>
- 1 "extra" negative:no positive $2,147,483,648_{ten}$

Two's comp. shortcut: Sign extension

- Convert 2's complement number rep. using n bits to more than n bits
- Simply replicate the most significant bit (sign bit) of smaller to fill new bits
  - 2's comp. positive number has infinite 0s
  - 2's comp. negative number has infinite 1s
  - Binary representation hides leading bits; sign extension restores some of them
  - 16-bit -4_{ten} to 32-bit:

$$1111\ 1111\ 1111\ 1100_{two}$$

$$1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1111\ 1100_{two}$$