

Computer Architecture (计算机体系结构)

Lecture 23 – Combinational Logic Blocks

2020-10-12

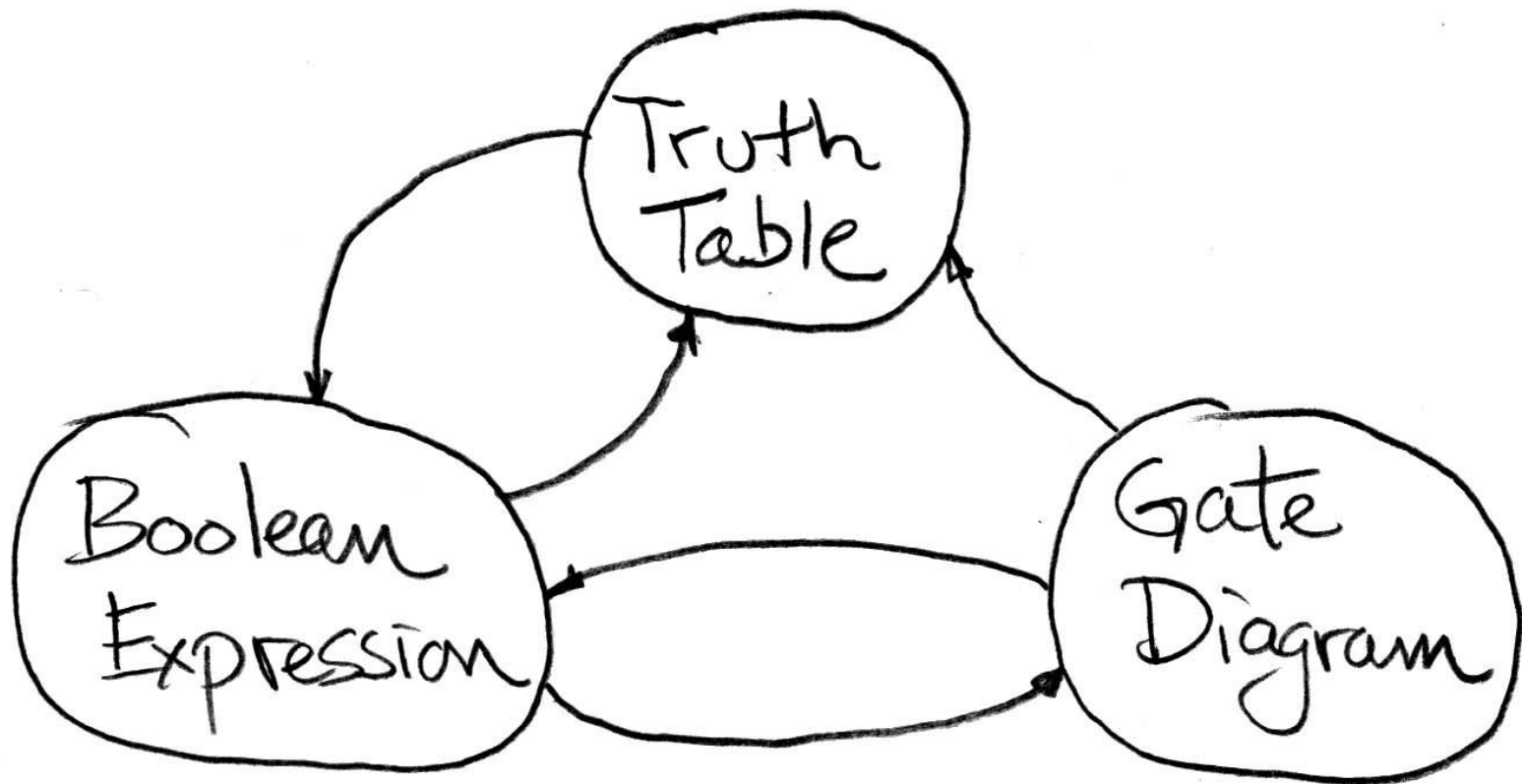


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Review

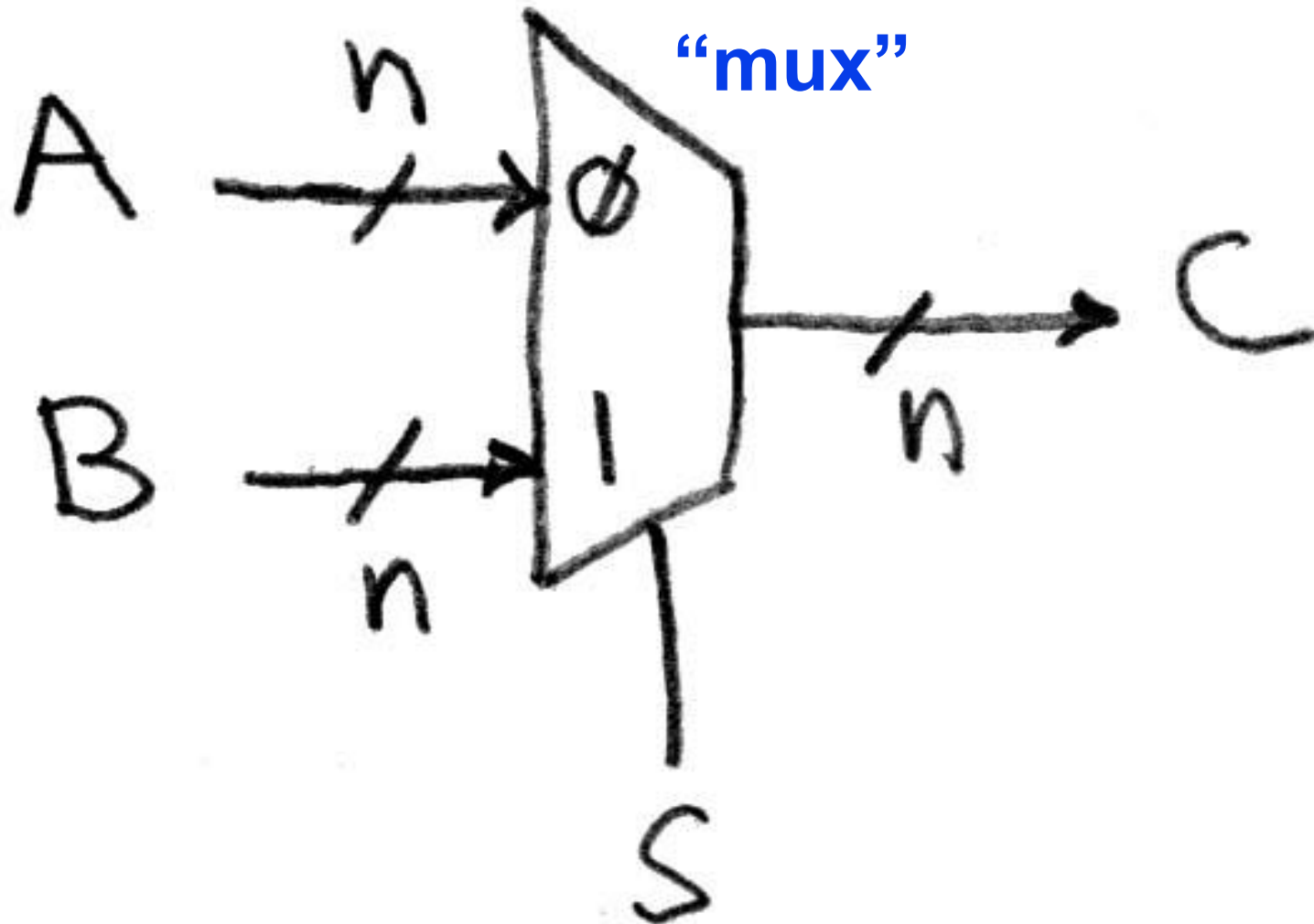
- Use this table and techniques we learned to transform from 1 to another



Today

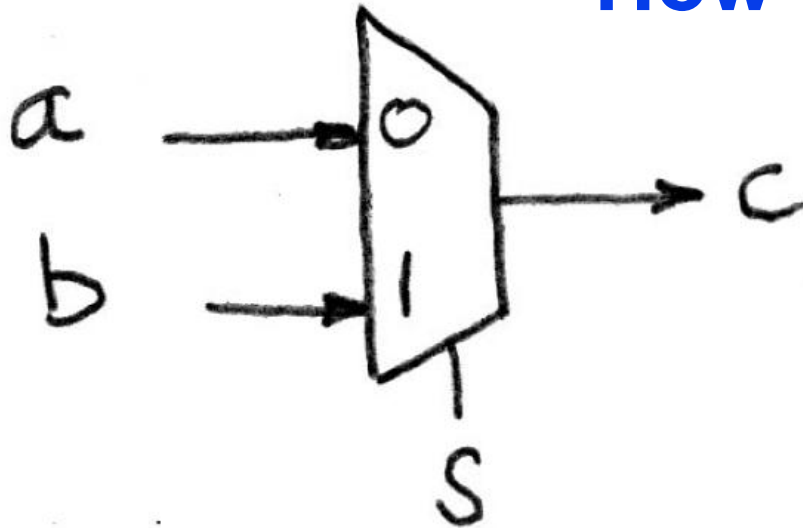
- **Data Multiplexors**
- **Arithmetic and Logic Unit**
- **Adder/Subtractor**

Data Multiplexor (here 2-to-1, n-bit-wide)



N instances of 1-bit-wide mux

How many rows in TT?

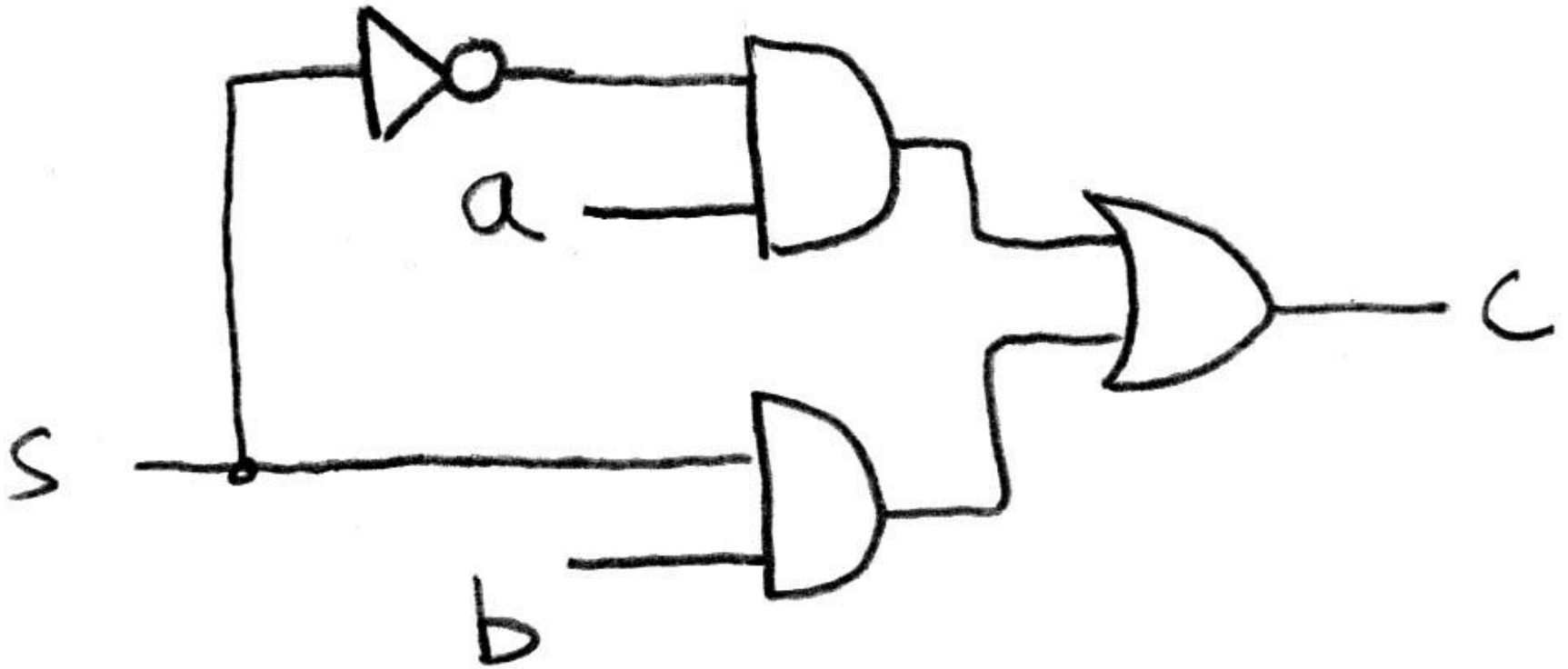


$$\begin{aligned}c &= \bar{s}a\bar{b} + \bar{s}ab + s\bar{a}b + sab \\&= \bar{s}(a\bar{b} + ab) + s(\bar{a}b + ab) \\&= \bar{s}(a(\bar{b} + b)) + s((\bar{a} + a)b) \\&= \bar{s}(a(1) + s((1)b) \\&= \bar{s}a + sb\end{aligned}$$



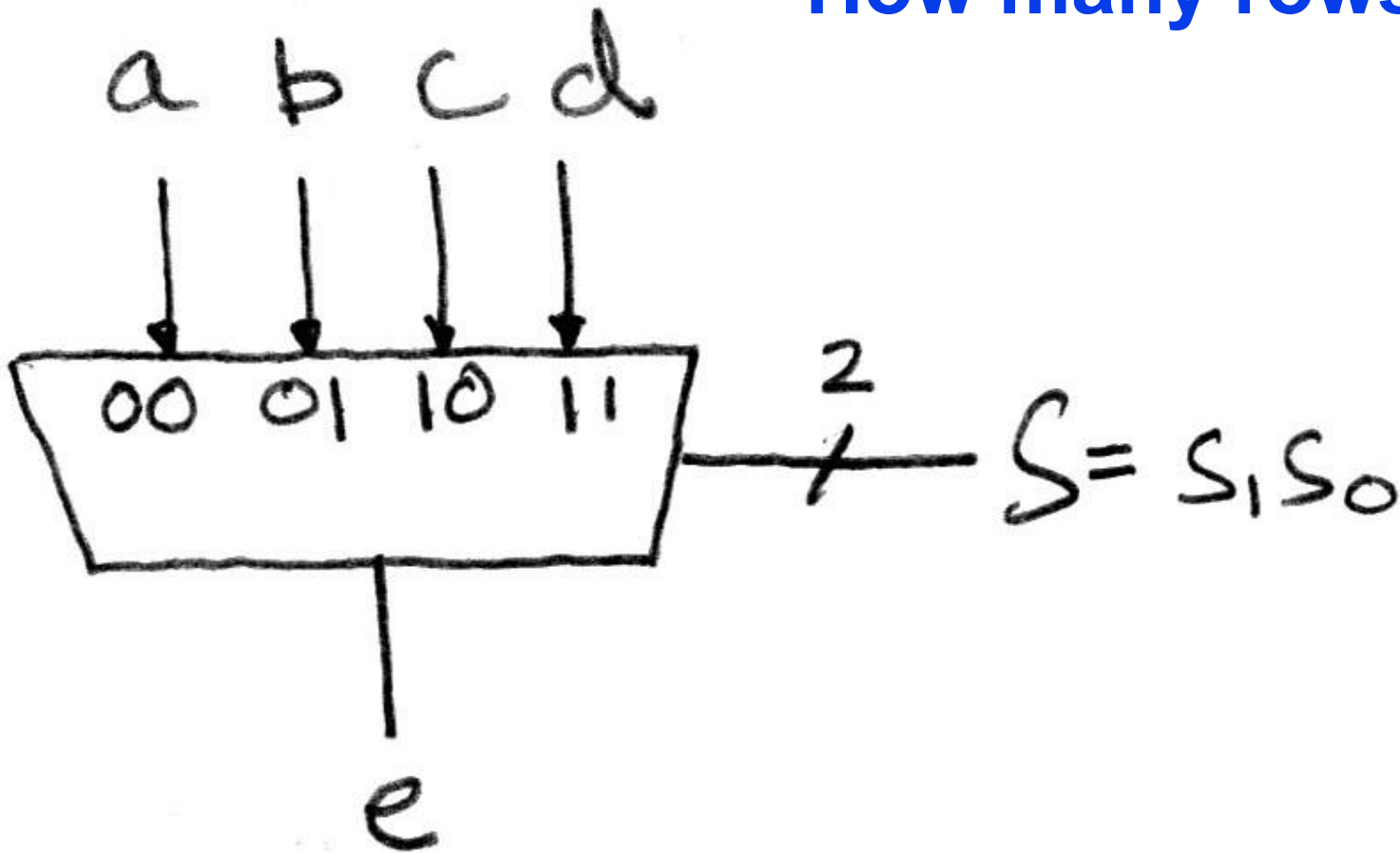
How do we build a 1-bit-wide mux?

$$\bar{s}a + sb$$



4-to-1 Multiplexor?

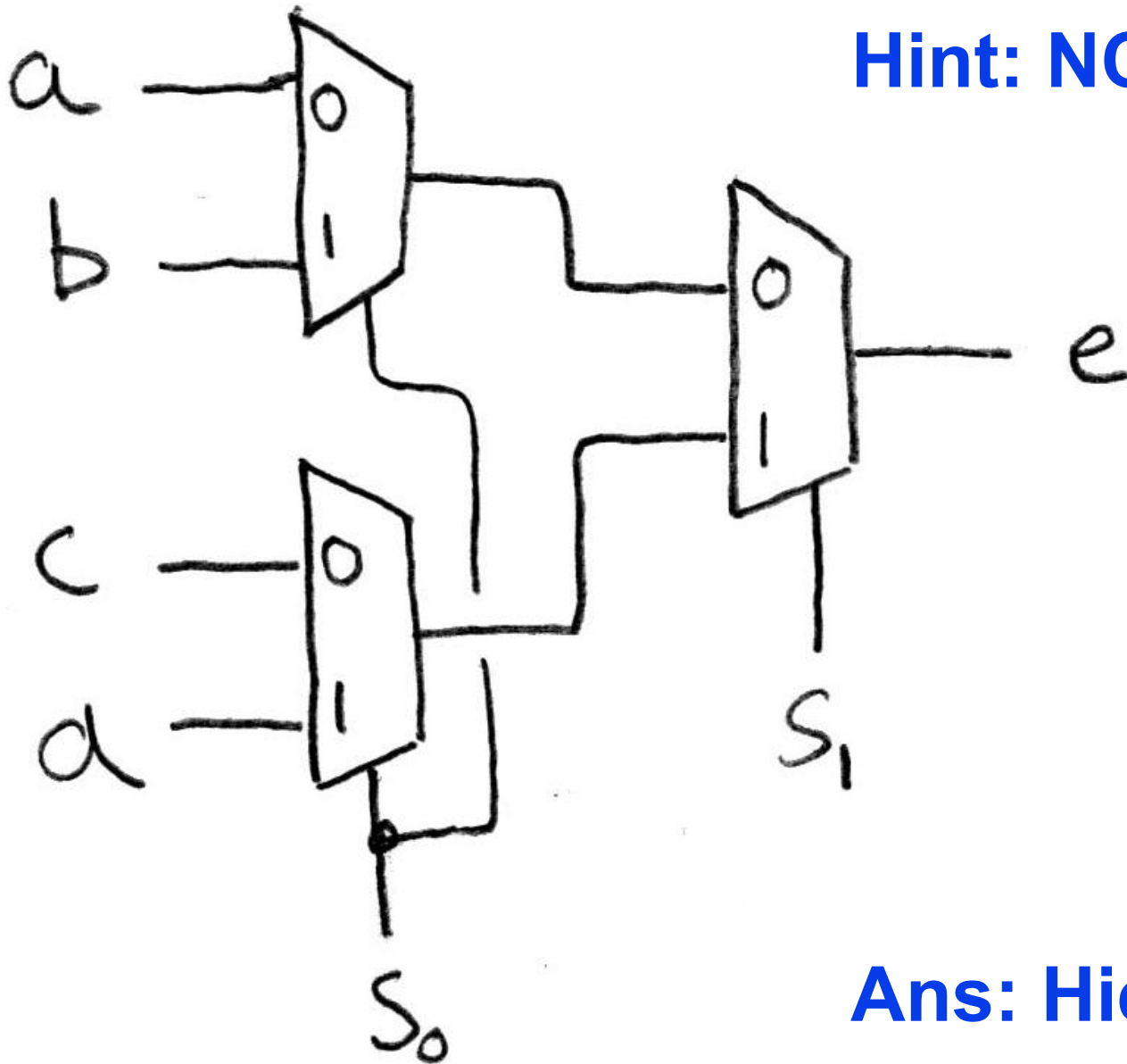
How many rows in TT?



$$e = \overline{s_1} \overline{s_0} a + \overline{s_1} s_0 b + s_1 \overline{s_0} c + s_1 s_0 d$$

Is there any other way to do it?

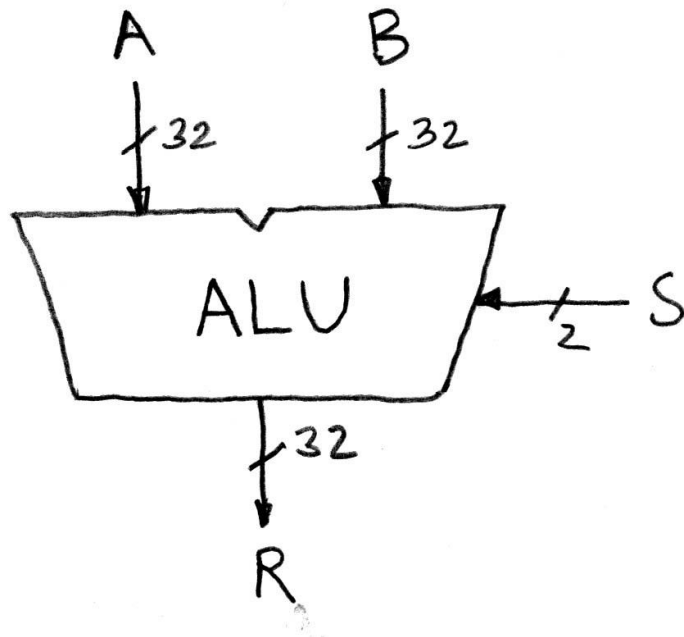
Hint: NCAA tourney!



Ans: Hierarchically!

Arithmetic and Logic Unit

- Most processors contain a special logic block called “Arithmetic and Logic Unit” (ALU)
- We’ll show you an easy one that does ADD, SUB, bitwise AND, bitwise OR



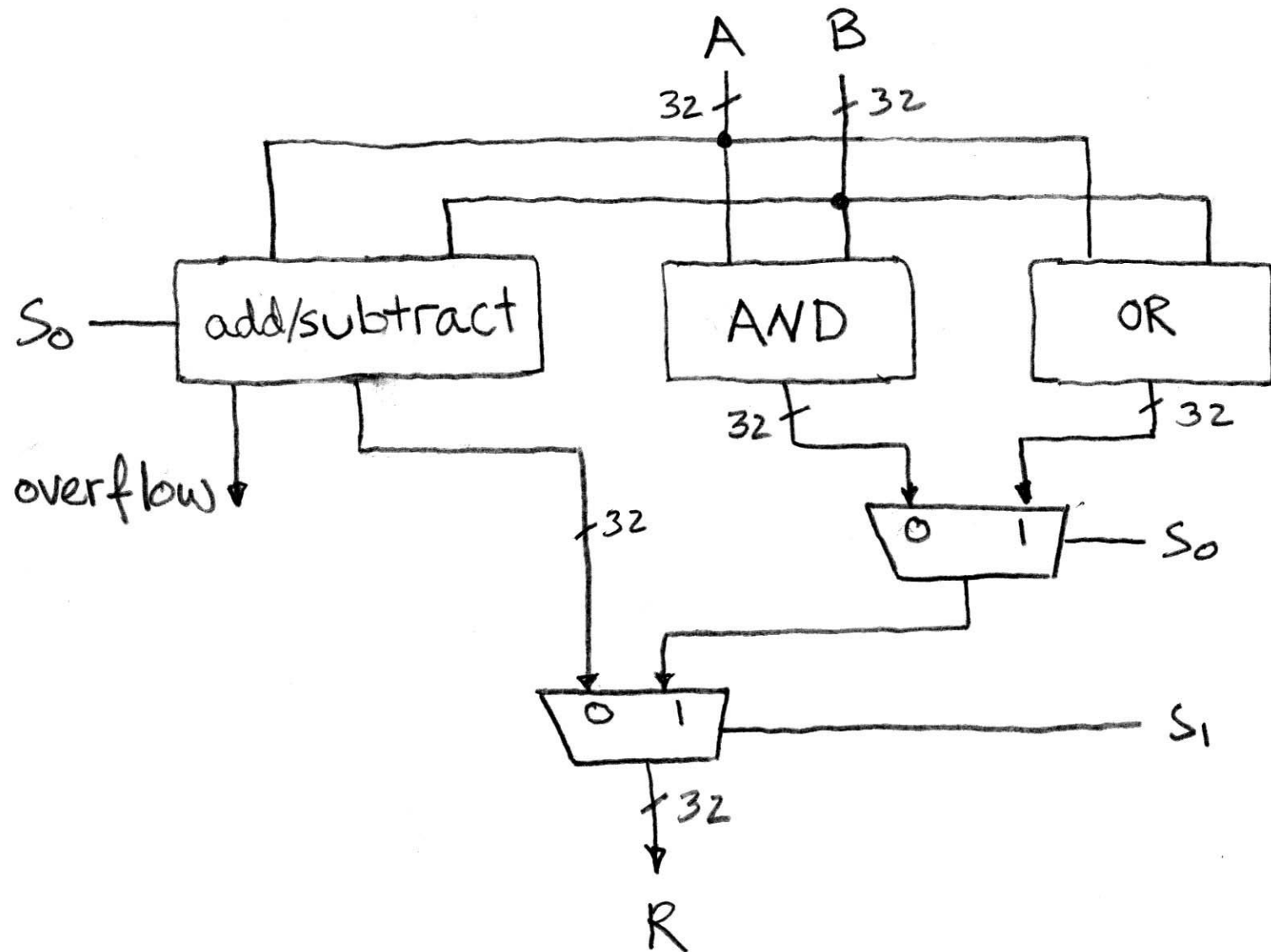
when $S=00$, $R=A+B$

when $S=01$, $R=A-B$

when $S=10$, $R=A \text{ AND } B$

when $S=11$, $R=A \text{ OR } B$

Our simple ALU



Adder/Subtractor Design -- how?

- Truth-table, then determine canonical form, then minimize and implement as we've seen before
- Look at breaking the problem down into smaller pieces that we can cascade or hierarchically layer

Adder/Subtractor – One-bit adder LSB...

$$\begin{array}{rcccc} & a_3 & a_2 & a_1 & a_0 \\ + & b_3 & b_2 & b_1 & b_0 \\ \hline s_3 & s_2 & s_1 & s_0 \end{array}$$

a_0	b_0	s_0	c_1
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

$$s_0 =$$

$$c_1 =$$

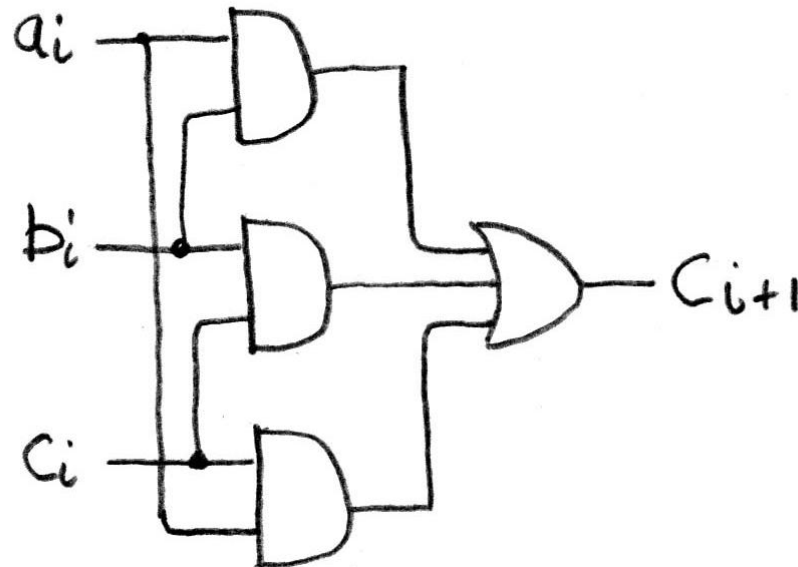
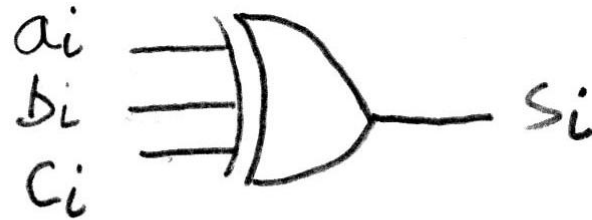
Adder/Subtractor – One-bit adder (1/2)...

$$\begin{array}{rcccc}
 & a_3 & a_2 & a_1 & a_0 \\
 + & b_3 & b_2 & b_1 & b_0 \\
 \hline
 s_3 & s_2 & s_1 & s_0
 \end{array}$$

a_i	b_i	c_i	s_i	c_{i+1}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

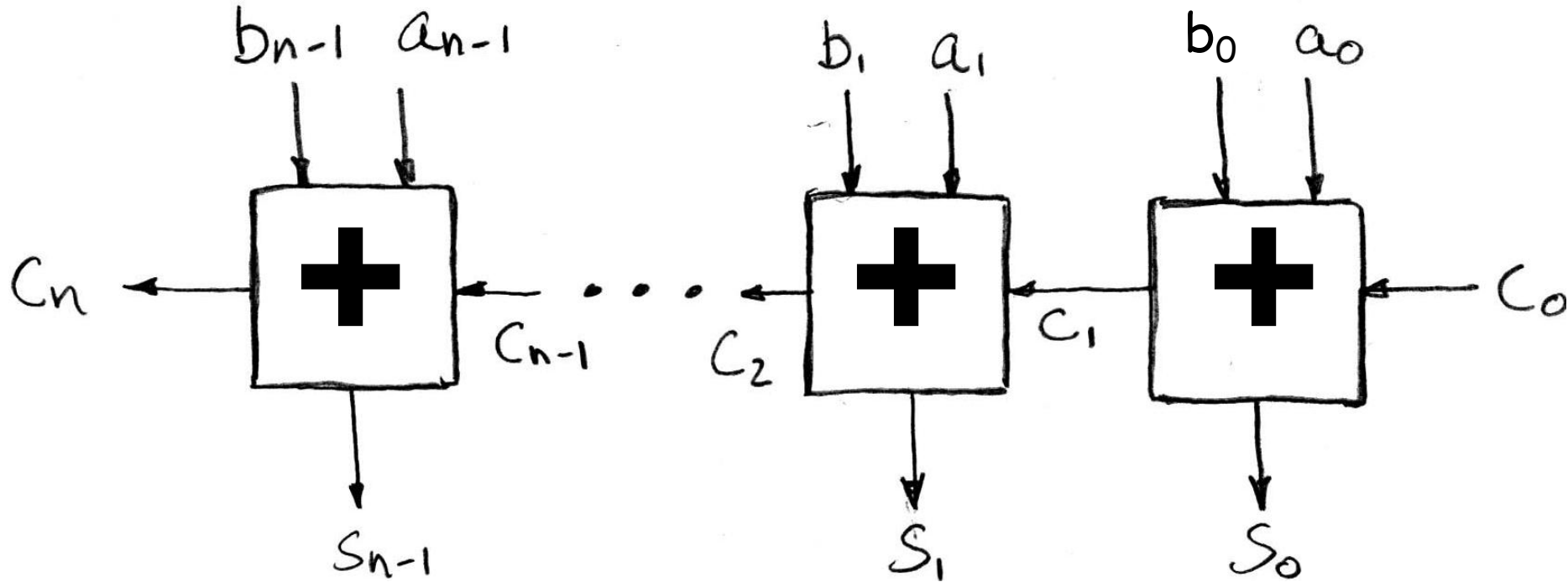
$$\begin{aligned}
 s_i &= \\
 c_{i+1} &=
 \end{aligned}$$

Adder/Subtractor – One-bit adder (2/2)...



$$S_i = \text{XOR}(a_i, b_i, c_i)$$
$$C_{i+1} = \text{MAJ}(a_i, b_i, c_i) = a_i b_i + a_i c_i + b_i c_i$$

N 1-bit adders \Rightarrow 1 N-bit adder

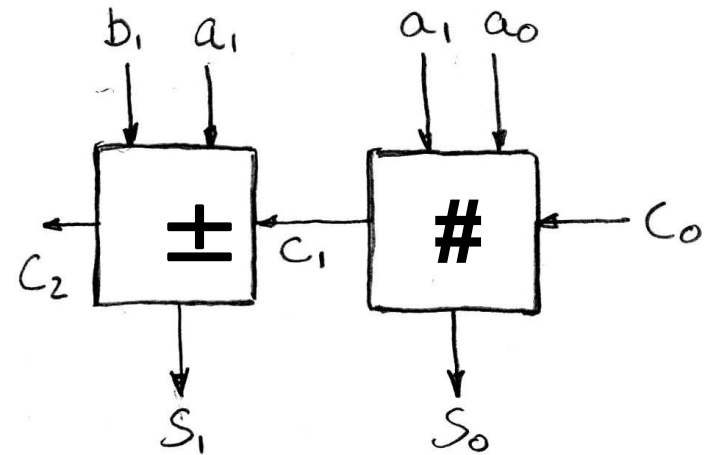


What about overflow?
Overflow = c_n ?

What about overflow?

• Consider a 2-bit signed # & overflow:

- $10 = -2 + -2$ or -1
- $11 = -1 + -2$ only
- $00 = 0$ NOTHING!
- $01 = 1 + 1$ only



• Highest adder

- $C_1 = \text{Carry-in} = C_{in}$, $C_2 = \text{Carry-out} = C_{out}$
- No C_{out} or $C_{in} \Rightarrow$ NO overflow!

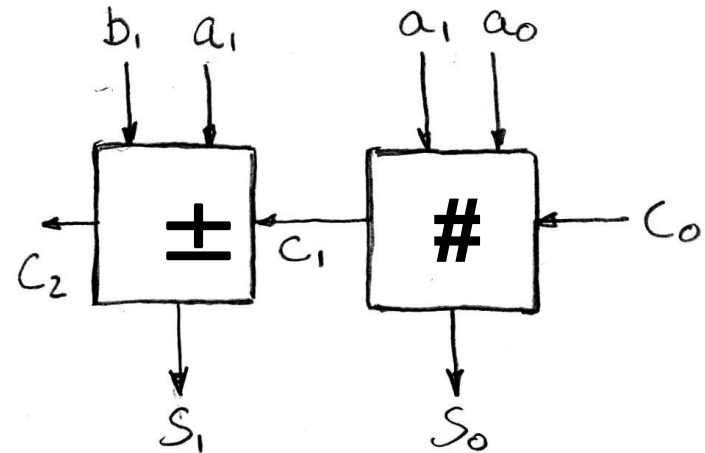
What op? • C_{in} , and $C_{out} \Rightarrow$ NO overflow!

- C_{in} , but no $C_{out} \Rightarrow A, B$ both > 0 , overflow!
- C_{out} , but no $C_{in} \Rightarrow A, B$ both < 0 , overflow!

What about overflow?

- Consider a 2-bit signed # & overflow:

10	=	-2
11	=	-1
00	=	0
01	=	1

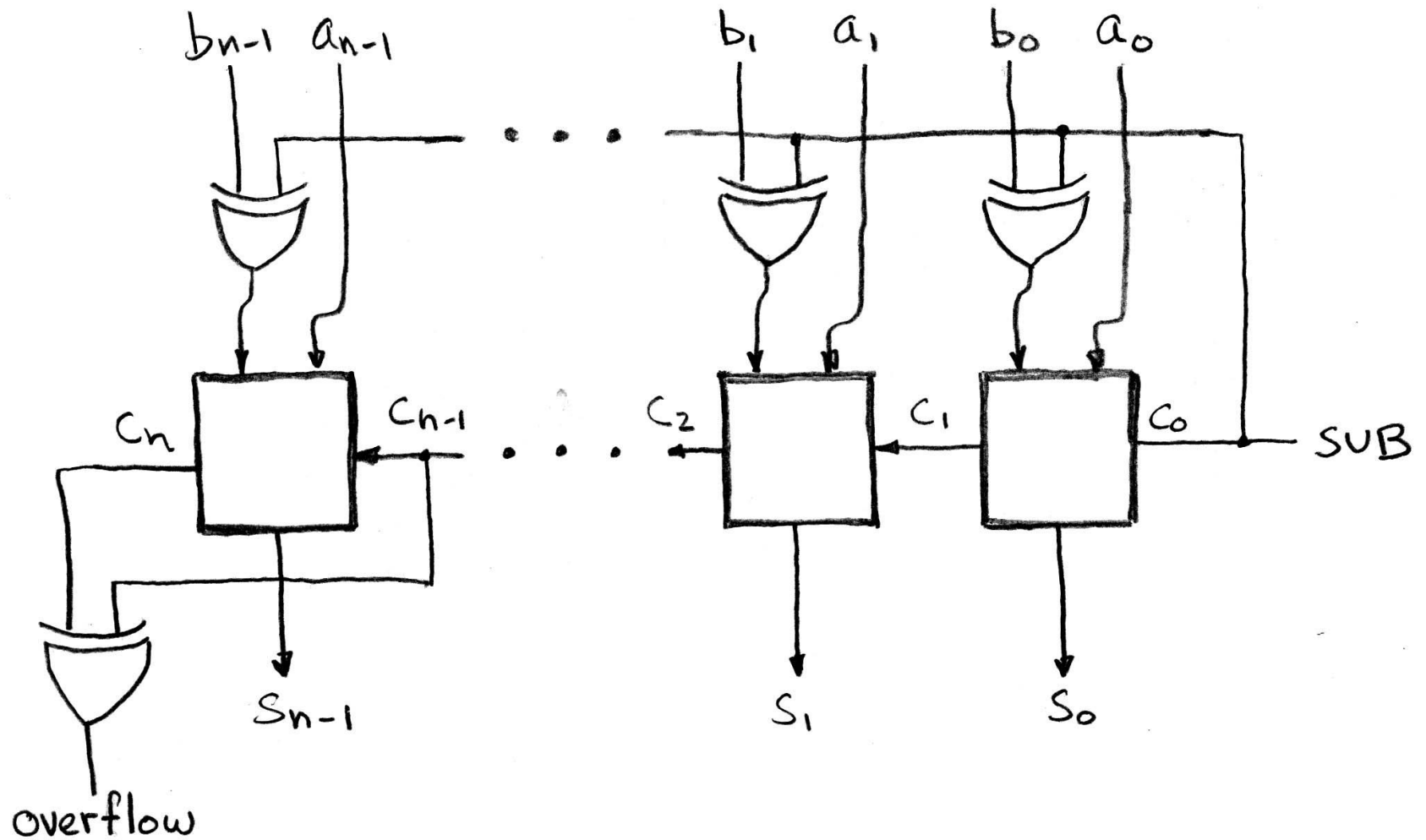


- Overflows when...

- C_{in} , but no $C_{out} \Rightarrow A, B$ both > 0 , overflow!
- C_{out} , but no $C_{in} \Rightarrow A, B$ both < 0 , overflow!

$$\text{overflow} = c_n \text{ XOR } c_{n-1}$$

Extremely Clever Subtractor



Peer Instruction

- 1) Truth table for mux with 4-bits of signals has 2^4 rows
- 2) We could cascade N 1-bit shifters to make 1 N-bit shifter for sll, srl

	12
a)	FF
b)	FT
c)	TF
d)	TT

Peer Instruction Answer

- 1) Truth table for mux with 4-bits of signals controls 16 inputs, for a total of 20 inputs, so truth table is 2^{20} rows... **FALSE**
- 2) We could cascade N 1-bit shifters to make 1 N-bit shifter for sll, srl ... **TRUE**

1) Truth table for mux with 4-bits of signals is 2^4 rows long

2) We could cascade N 1-bit shifters to make 1 N-bit shifter for sll, srl

	12
a)	FF
b)	FT
c)	TF
d)	TT

“And In conclusion...”

- **Use muxes to select among input**
 - S input bits selects 2^S inputs
 - Each input can be n-bits wide, indep of S
- **Can implement muxes hierarchically**
- **ALU can be implemented using a mux**
 - Coupled with basic block elements
- **N-bit adder-subtractor done using N 1-bit adders with XOR gates on input**
 - XOR serves as conditional inverter