数字信号处理　第二周作业

范云潜 18373486

微电子学院 184111 班

日期：2020 年 9 月 22 日

作业内容：作业：2.8, 2.11, 2.17, 2.44; 3.1(b)、(g), 3.3, 3.4, 3.6(c), 3.9, 3.28;

Problem 2.8

$$h(n) = 5(-\frac{1}{2})^n u(n), x(n) = (\frac{1}{3})^n u(n)$$

对应

$$H(z) = 5\frac{z}{z+1/2}, |z| > \frac{1}{2}$$

$$X(z) = \frac{z}{z-1/3}, |z| > \frac{1}{3}$$

那么

$$\frac{Y(z)}{z} = 5\frac{z}{(z+1/2)(z-1/3)}$$

解得

$$Y(z) = 2\frac{z}{z-1/3} + 3\frac{z}{z+1/2}, |z| > \frac{1}{2}$$

$$y(n) = 3(-\frac{1}{2})^n u(n) + 2(\frac{1}{3})^n u(n)$$

Problem 2.11

由于 LTI 不会改变信号的频率，分解

$$x(n) = \frac{1}{2j}\left(e^{j\frac{\pi}{4}n} - e^{-j\frac{\pi}{4}n}\right)$$

那么

$$y(n) = \frac{1}{2j}\left(H(e^{j\pi/4})e^{j\pi n/4} - H(e^{-j\pi/4})e^{-j\pi n/4}\right)$$

带入公式

$$H(e^{j\pi/4}) = \frac{1-(-j)}{1-1/2} = 2\sqrt{2}e^{j\pi/4}$$

$$H(e^{-j\pi/4}) = \frac{1-j}{1-1/2} = 2\sqrt{2}e^{-j\pi/4}$$

代回

$$y(n) = 2\sqrt{2}\sin(\frac{\pi}{4}(n+1))$$

Problem 2.17

SubProblem a

$$r(n) = G_{M+1}(n)$$

那么

$$R(e^{j\omega}) = \frac{\sin(1/2\cdot\omega(M+1))}{\sin(1/2\cdot\omega)}e^{-j\frac{M}{2}\omega}$$

SubProblem b

如图 1

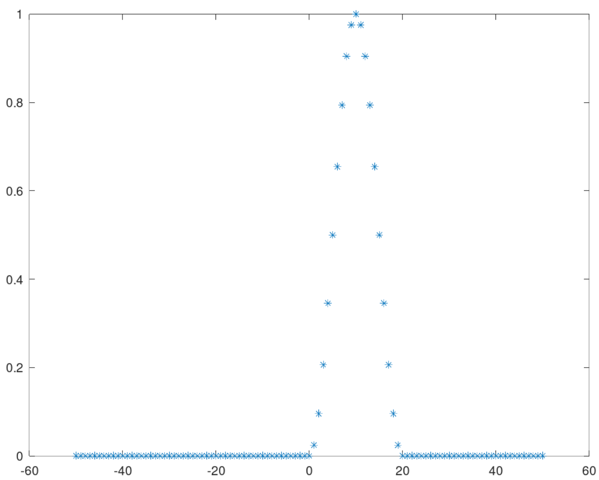

图 1: $w(n)$ 在 $M=20$ 的示意图

SubProblem c

$$
\begin{aligned}
W(e^{j\omega}) &= R(e^{j\omega}) \otimes (\sum_{n=-\infty}^{\infty} \frac{1}{2}(1-\cos\frac{2\pi n}{M})e^{-j\omega n}) \\
&= R(e^{j\omega}) \otimes (\sum_{n=-\infty}^{\infty} \frac{1}{2}(1-\frac{e^{j2\pi n/M}+e^{-j2\pi n/M}}{4})e^{-j\omega n}) \\
&= R(e^{j\omega}) \otimes (\frac{1}{2}\delta(\omega)-\frac{1}{4}\delta(\omega+\frac{2\pi}{M})-\frac{1}{4}\delta(\omega-\frac{2\pi}{M})) \\
&= \frac{R(e^{j\omega})}{2}+(-\frac{1}{4})(R(e^{j(\omega+2\pi/M)})+R(e^{j(\omega-2\pi/M)}))
\end{aligned}
$$

如图 2

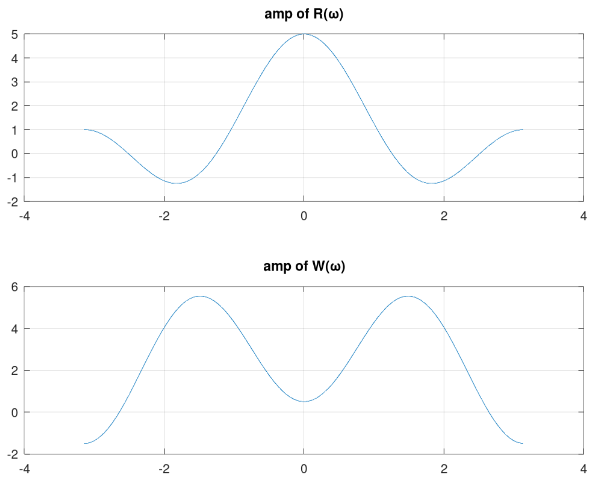

图 2: $w(n)$ 在 $M=20$ 的示意图

Problem 2.44

SubProblem a

$$\mathrm{DTFT}[x(n)]|_{\omega=0} = \sum_n x(n) = 6$$

SubProblem b

$$\mathrm{DTFT}[x(n)]|_{\omega=\pi} = -\sum_n x(n) = -6$$

SubProblem c

序列关于 $n=2$ 对称，得到实数

$$\mathrm{DTFT}[x(n)] = \sum_n x(n)e^{-j\omega(n-2)}e^{-2j\omega}$$

那么辐角为 -2ω

SubProblem d

$$\int_{-\pi}^{\pi} X(e^{j\omega})\mathrm{d}\omega = 2\pi \mathrm{DTFT}^{-1}[X(e^{j\omega})]\mid_{n=0} = 2\pi$$

SubProblem e

$$X(e^{-j\omega}) = \sum_n x(n)e^{j\omega n} = \sum_n e^{-j\omega n} = \mathrm{DTFT}[x(-n)]$$

如图 3

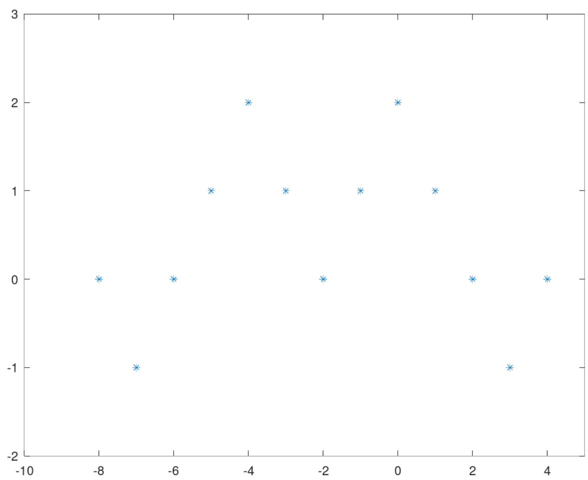

图 3: (e) 题解

SubProblem f

$$\mathrm{Re}(X(e^{j\omega})) = \frac{1}{2}(X(e^{j\omega}) + X^*(e^{-j\omega})) = \mathrm{DTFT}[X_e(n)]$$

对于实序列

$$X_e(n) = \frac{1}{2}(x(n) + x(-n))$$

计算得到

$$\mathrm{Re}(X(e^{j\omega})) = -\cos 7\omega + \cos 5\omega + 2\cos 4\omega + 2\cos\omega + 2$$

如图 4

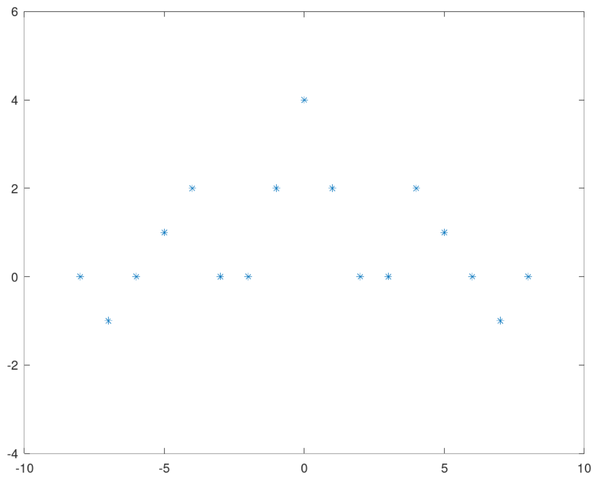

图 4: (f) 题解

Problem 3.1

SubProblem b

$$\mathscr{Z}[-(\frac{1}{2})^n u(-n-1)] = -\frac{z}{z-1/2}, \text{ where } |z| < \frac{1}{2}$$

SubProblem g

$$\mathscr{Z}[(\frac{1}{2})^n G_9(n)] = \frac{1-1/(2z)^{10}}{1-1/2z}, \text{ where } |z| \neq 0$$

Problem 3.3

SubProblem a

$$X_a(z) = \frac{z}{z-a} + \frac{z}{z-1/a}, \text{ where } a < |z| < \frac{1}{a}$$

SubProblem b

$$X_b(z) = \frac{1-1/z^N}{1-1/z} = \frac{z^N - 1}{z^{N-1}(z-1)}$$

SubProblem c

三角形状的函数一般是由门函数卷积而得，由于起点为 0 ，需要移位

$$x_c(n) = x_b(n) \otimes x_b(n-1)$$

$$X_c(z) = X_b(z) \cdot \frac{X_b(z)}{z} = \frac{1}{z^{2N-1}}(\frac{z^N - 1}{z - 1})^2$$

如图 **5**

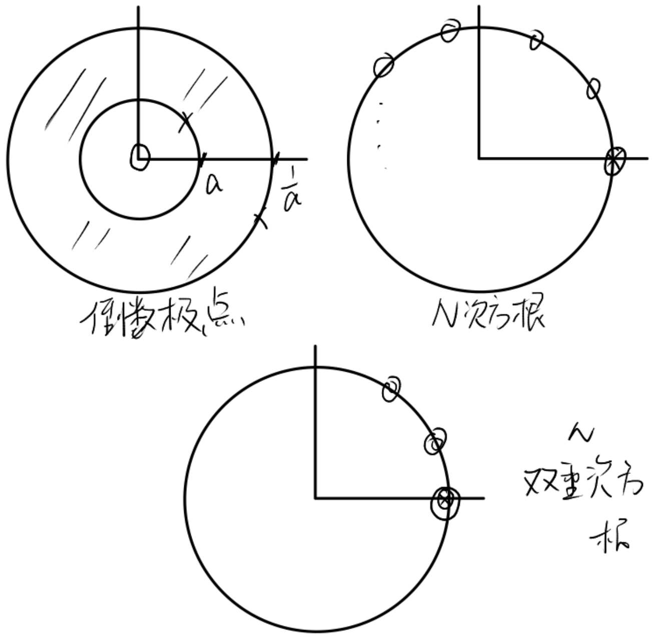

图 5: 收敛域

Problem 3.6

SubProblem c 部分分式

$$\frac{X(z)}{z} = \frac{z - 1/2}{z^2 + \frac{3}{4}z + \frac{1}{8}} = \frac{4}{z + 1/2} - \frac{3}{z + 1/4}, \text{ where } |z| > \frac{1}{2}$$

$$x(n) = 4(-\frac{1}{2})^n u(n) - 3(-\frac{1}{4})^n u(n)$$

SubProblem c 展开法

$$\frac{1 - \frac{1}{2}z^{-1}}{1 + \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} = 1 - \frac{5}{4}z^{-1} + \frac{13}{16}z^{-2} - \frac{29}{64}z^{-3} \cdots$$

观察可得，存在较大较小的两项使得其在正负之间变化，并且均为 2 的幂次，最终得到

$$x(n) = 4(-\frac{1}{2})^n u(n) - 3(-\frac{1}{4})^n u(n)$$

Problem 3.9

SubProblem a

$$H(z)=\frac{2z}{z-1/2}-\frac{z}{z+1/4}$$

又因为是因果系统，收敛域为 $|z|>1/2$

SubProblem b

稳定，$|z|=1$ 在收敛域内

SubProblem c

$$y(n)\xrightarrow{\mathscr{Z}}-\frac{1}{3}\frac{z}{z+1/4}+\frac{4}{3}\frac{z}{z-2}$$

那么

$$X(z)=\frac{Y(z)}{H(z)}=\frac{z-1/2}{z-2}=\frac{1}{4}+\frac{3}{4}\frac{z}{z-2}$$

所以

$$x(n)=\frac{1}{4}\delta(n)-\frac{3}{4}2^{n}u(-n-1)$$

Problem 3.28

SubProblem a

设

$$x_0(n)\xrightarrow{\mathscr{Z}}\frac{3}{(1-\frac{1}{4}z^{-1})^2}=X_0(z)$$

那么

$$X_0(z)=\frac{3z}{z-1/4}+\frac{3}{4}z\cdot\frac{1}{(z-1/4)^2}$$

设

$$\frac{\mathrm{d}X_1(z)}{\mathrm{d}z}=-\frac{1}{(z-1/4)^2}$$

那么

$$X_1(z)=\frac{1}{z-1/4}=4(-1+\frac{z}{z-1/4})$$

那么

$$\frac{3}{4}z\cdot\frac{1}{(z-1/4)^2}=\frac{3}{4}(-z)\frac{\mathrm{d}X_1(z)}{\mathrm{d}z}\xrightarrow{\mathscr{Z}^{-1}}\frac{3}{4}nx_1(n)$$

$$x_0(n) = 3(-1)\frac{1}{4^n}u(-n-1)+\frac{3}{4}n(-4\delta(n)+(-1)\frac{1}{4^n}u(-n-1)) = -3n\delta(n)+(-3-\frac{3}{4}n)\frac{n}{4}u(-n-1)$$

那么

$$x(n) = x_0(n-3) = -3(n-3)\delta(n-3) + (-3-\frac{3}{4}(n-3))\frac{n-3}{4}u(-n+2)$$

SubProblem b

$$X(z) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}z^{2k+1}$$

$$x(n) = \sum_{k=0}^{\infty}\frac{(-1)^k}{(2k+1)!}\delta(n+2k+1)$$

SubProblem c

$$X(z) = z^7 + \frac{-1}{1-z^{-7}} = z^7 + (-1)\sum_{n=0}^{\infty}(z^{-7})^n$$

$$x(n) = \delta(n+7) - \sum_{k=0}^{\infty}\delta(n-7k)$$