

数字信号处理 第六周作业

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作业内容: 2.32, 2.33, 2.42, 3.40, 3.41, 5.1, 5.4, 5.12;

Problem 2.32

$$\begin{aligned}x[n] &= \cos \frac{\pi}{2} n = \frac{1}{2}(e^{j\frac{n\pi}{2}} + e^{-j\frac{n\pi}{2}}) \\H(e^{j\frac{\pi}{2}}) &= e^{-j\frac{\pi}{4}} \frac{1 + e^{-j\pi} + 4e^{-2j\pi}}{1 + \frac{1}{2}e^{-j\pi}} = e^{-j\frac{\pi}{4}} 8 \\H(e^{-j\frac{\pi}{2}}) &= e^{j\frac{3\pi}{4}} \frac{1 + e^{j\pi} + 4e^{2j\pi}}{1 + \frac{1}{2}e^{j\pi}} = -e^{-j\frac{\pi}{4}} 8\end{aligned}$$

$$y[n] = 4(e^{j\frac{\pi}{2}n-j\frac{\pi}{4}} - e^{-j\frac{\pi}{2}n-j\frac{\pi}{4}})$$

Problem 2.33

$$\begin{aligned}x[n] &= \cos\left(\frac{3\pi n}{2} + \frac{\pi}{4}\right) \\&= \frac{1}{2}(e^{j(\frac{3\pi}{2}n+\frac{\pi}{4})} + e^{-j(\frac{3\pi}{2}n+\frac{\pi}{4})})\end{aligned}$$

幅度响应恒为 1, 仅考虑相位响应:

$$\arg H(e^{j\frac{3\pi}{2}}) = \frac{2\pi}{3} = -\arg H(e^{-j\frac{3\pi}{2}})$$

$$\begin{aligned}y[n] &= \frac{1}{2}(e^{j(\frac{3\pi}{2}n+\frac{11\pi}{12})} + e^{-j(\frac{3\pi}{2}n+\frac{11\pi}{12})}) \\&= \cos\left(\frac{3\pi n}{2} + \frac{11\pi}{12}\right)\end{aligned}$$

Problem 2.42

SubProblem a

$$\begin{aligned}y[n] &= x[n] \otimes (h_2[n] + h_1[n] \otimes h_2[n]) \\h[n] &= h_2[n] + h_1[n] \otimes h_2[n] \\&= \alpha^n u[n] + \beta \alpha^{n-1} u[n-1]\end{aligned}$$

SubProblem b

转换到变换域:

$$H(z) = H_1(z)H_2(z) + H_2(z)$$

$$H_1(z) = \beta \frac{1}{z}$$

$$H_2(z) = \frac{z}{z - \alpha}$$

$$\therefore H(z) = \frac{z + \beta}{z - \alpha}$$

SubProblem c

展开得到:

$$(1 - \alpha/z)Y(z) = (1 + \beta/z)X(z)$$

进行逆变换

$$y[n] - \alpha y[n-1] = x[n] + \beta x[n-1]$$

SubProblem d

因果; 在 $|\alpha| < 1$ 时稳定, 此时极点在单位圆内。

Problem 3.40

SubProblem a

变换域:

$$H(z)(X(z) - W(z)) + E(z) = W(z)$$

$$W(z) = X(z) \frac{H(z)}{1 + H(z)} + E(z) \frac{1}{1 + H(z)}$$

SubProblem b

$$H_1(z) = \frac{1/(z-1)}{1 + 1/(z-1)} = \frac{1}{z}$$

$$H_2(z) = \frac{1}{1 + 1/(z-1)} = \frac{z-1}{z}$$

SubProblem c

由于是因果系统, z 向外扩展。 $H(z)$ 在单位圆上存在极点, 因此不稳定; $H_1(z), H_2(z)$ 极点在单位圆内部, 因此稳定。

Problem 3.41

SubProblem a 稳定, $r_{min} < 1 < r_{max}$

SubProblem b $v[n] = a^{-n}x[n], w[n] = y[n]a^{-n}$

变换域:

$$V(z) = X(az), W(z) = Y(az)$$

$$\therefore G(az) = H(z), G(z) = H(z/a)$$

$$\therefore g[n] = a^n h[n]$$

SubProblem c

收敛域带入: $0 < r_{min} < |z/a| < r_{max} < \infty$, 即: $0 < |ar_{min}| < |z| < |ar_{max}| < \infty$ 。

Problem 5.1

$$Y(e^{j\omega}) = \sum_{n=0}^{10} e^{-j\omega n} = \frac{\sin(11\omega/2)}{\omega/2} e^{-5\omega}$$

对于 $X(e^{j\omega})$ 的每一频率取值, 都有 $X(e^{j\omega})H(e^{j\omega}) = Y(e^{j\omega})$, 由于 ω 在此区间连续, 因此需要满足 $y[n] = x[n]$ 且截止频率覆盖整个频段, $\omega_c = \pi$ 。

Problem 5.4

SubProblem a

$$X(z) = \frac{z}{z-1/2} - \frac{z}{z-2}, 1/2 < |z| < 2$$

$$Y(z) = 6 \frac{z}{z-1/2}, |z| > \frac{3}{4}$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{z-2}{z-3/4}, |z| > \frac{3}{4}$$

零极点如图 1

SubProblem b

$$H(z) = \frac{z}{z-3/4} - \frac{2}{z-3/4}$$

$$h[n] = (\frac{3}{4})^n u[n] - 2(\frac{3}{4})^{n-1} u[n-1]$$

SubProblem c

$$\frac{Y(z)}{X(z)} = \frac{1-2/z}{1-3/(4z)}$$

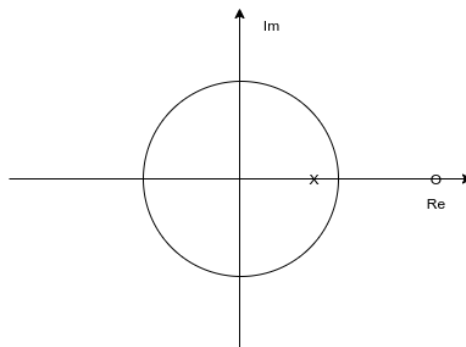


图 1: 零极点图

$$y[n] - \frac{3}{4}y[n-1] = x[n] - 2x[n-1]$$

Problem 5.12

SubProblem a

极点: $z = \pm 0.9j$ 在单位圆内, 因此稳定。

SubProblem b

单位圆外的因子: $z = \pm 3$, 那么

$$H_{ap} = \frac{1-9z^{-2}}{1-z^{-2}/9}$$

$$H_1(z) = \frac{1+0.2z^{-1}}{1+0.81z^{-2}}(1-\frac{1}{9}z^{-2})$$

z