Chapter 1

DFSDFT

1.1

CTFS CTFT

$$X\left(e^{j\Omega T}\right)=\sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$$

$$x(n)=\frac{1}{2\pi}\int_{-\pi}^{\pi} X\left(e^{j\omega}\right) e^{j\omega n} d\omega$$

1.2

DTFT

N

$$\tilde{x}(n)=\tilde{x}(n+rN)$$

$$\tilde{x}(n)=\frac{1}{N}\sum_{k=0}^{N-1}\tilde{X}(k) e^{j\frac{2\pi}{N}kn}$$

$$\begin{aligned}\sum_{n=0}^{N-1}\tilde{x}(n) e^{-j\frac{2\pi}{N}rn} &= \frac{1}{N}\sum_{n=0}^{N-1}\sum_{k=0}^{N-1}\tilde{X}(k) e^{j\frac{2\pi}{N}(k-r)n} \\ &= \sum_{k=0}^{N-1}\tilde{X}(k)\left[\frac{1}{N}\sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}(k-r)n}\right]\end{aligned}$$

$$\frac{1}{N}\sum_{n=0}^{N-1} e^{j\frac{2\pi}{N}rn}=\frac{1}{N}\frac{1-e^{j\frac{2\pi}{N}rN}}{1-e^{j\frac{2\pi}{N}}}=1,\ \text{when } r=mN, 0,\ \text{else}$$

$W_N=e^{-j\frac{2\pi}{N}}$

$$\tilde{X}(k)=DFS[\tilde{x}(n)]=\sum_{n=0}^{N-1}\tilde{x}(n) e^{-j\frac{2\pi}{N}nk}=\sum_{n=0}^{N-1}\tilde{x}(n) W_N^{nk}$$

$$\tilde{x}(n)=IDFS[\tilde{X}(k)]=\frac{1}{N}\sum_{k=0}^{N-1}\tilde{X}(k) e^{j\frac{2\pi}{N}nk}=\frac{1}{N}\sum_{k=0}^{N-1}\tilde{X}(k) W_N^{-nk}$$

DFS Z

$$\tilde{X}(k)=X(e^{j\omega})|_{\omega=w\pi k/N}$$

1.2.1

N N $X(z)$ $X(e^{j\omega})$

1.2.2

$$\frac{1-\exp{-j\omega N}}{1-\exp{-j\omega}}=\exp{-j\omega\frac{N-1}{2}}\frac{\sin{\omega N/2}}{\sin{\omega/2}}$$

DFS

DTFT N

1.3

DFT

$$\tilde{X}(k)=DFT[\tilde{x}(n)]=\sum_{n=0}^{N-1}\tilde{x}(n)e^{-j\frac{2\pi}{N}nk}=\sum_{n=0}^{N-1}\tilde{x}(n)W_N^{nk}$$

$$\tilde{x}(n)=IDFT[\tilde{X}(k)]=\frac{1}{N}\sum_{k=0}^{N-1}\tilde{X}(k)e^{j\frac{2\pi}{N}nk}=\frac{1}{N}\sum_{k=0}^{N-1}\tilde{X}(k)W_N^{-nk}$$

$x[n]=\tilde{x}[n]R_N[n]$ $\tilde{x}[n]=x[n \bmod N]=x[[n]]_N$

1.3.1

$$DFT[X(n)]=Nx\tilde{((-k))}_NR_N(k)$$

$$IDFT[X(k)]=\frac{1}{N}\overline{DFT\left[X^*(k)\right]}$$

1.3.2

-
- $x_m(n)=x((n+m))_NR_N(n)=W_N^{-mk}\tilde{X}[k]2$
-
-
-

1.4 DFT

- $W_N^{nk}*=W_N^{-nk}$
- $W_N^{nk}=W_N^{(n+N)k}=W_N^{n(k+N)}$
- $W_N^{nk}=W_{mN}^{mnk}$

1.4.1

$$X(k)=\sum_{r=0}^{N-1}x(r)W_N=W_N^{-kN}\sum_{r=0}^{N-1}x(r)W_N=\sum_{r=0}^{N-1}x(r)W_N^{-k(N-r)}$$

$$y_k(n)=x(n)*W_N{}^{-kn}u(n)=\sum_{r=0}^{N-1}x(r)W_N^{-k(n-r)}u(n-r)$$

$$y_k(n)|_{n=N}=\sum_{r=0}^{N-1}x(r)W_N^{-k(N-r)}u(N-r)=\sum_{r=0}^{N-1}x(r)W_N^{-k(N-r)}=X(k)$$

W

1.4.2 2-FFT

1.4.3 Z

1.5 DFT

DFT CTFT

$$x(t)|_{t=nT}=x(nT)=x(n)$$

CTFT

$$X(j\Omega)\approx\sum_{n=-\infty}^{\infty}x(nT)e^{-j\Omega nT}\cdot T$$

N

$$X(j\Omega)\approx T\sum_{n=0}^{N-1}x(nT)e^{-j\Omega nT}$$

$$X\left(jk\Omega_0\right)\approx T\sum_{n=0}^{N-1}x(nT)e^{-jk\Omega_0 nT}$$

$$=T\sum_{n=0}^{N-1}x(nT)e^{-jnk\frac{2\pi F_0}{f_s}}=T\sum_{n=0}^{N-1}x(nT)e^{-jnk\frac{2\pi}{N}}=T\left\{DFT[x(n)]|_{x(n)=x(nT)}\right\}$$

CTFS N

$$x(t) = \sum_{k=-\infty}^{\infty} X\left(jk\Omega_0\right) e^{jk\Omega_0 t}$$

$T_0 = NT$

$$X\left(jk\Omega_0\right) \approx \frac{T}{T_0} \sum_{n=0}^{N-1} x(nT) e^{-jk\Omega_0 nT} = \frac{1}{N} \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}nk} = \frac{1}{N} DFT[x(n)]$$

1.5.1

$2\pi/N$

$$\Delta\Omega = \frac{2\pi}{\mathrm{N}T} = \frac{2\pi}{\mathrm{L}}$$

HZ

$$\Delta = \frac{1}{L}$$

1.6 FFT FIR