Machine Learning's Slides

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Date: August 29, 2020

Je reviendrai et je serai des millions. «Spartacus»

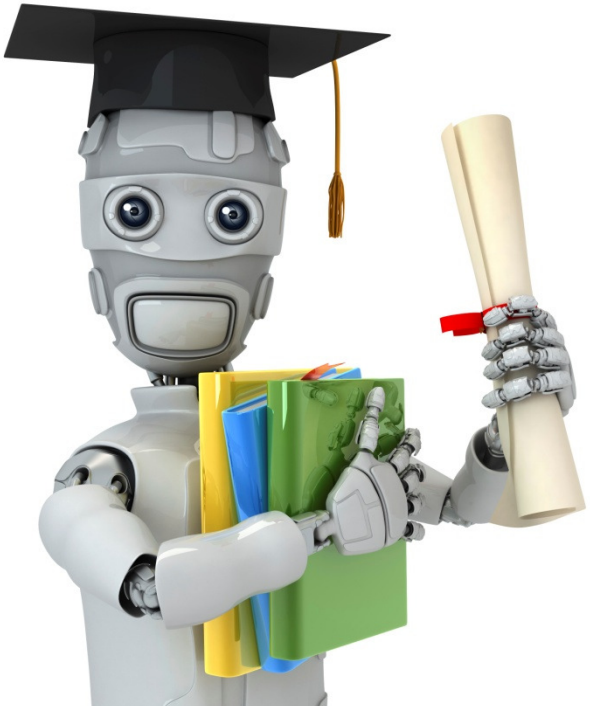
Contents

Part I

part

Chapter 1 Week1

1.1 Introduction



Machine Learning

Introduction

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What is iPhoto?

Andrew Ng

Andrew Ng

Machine Learning

- Grew out of work in AI
- New capability for computers

Examples:

- Database mining

  Large datasets from growth of automation/web.

  E.g., Web click data, medical records, biology, engineering

- Applications can't program by hand.

  E.g., Autonomous helicopter, handwriting recognition, most of Natural Language Processing (NLP), Computer Vision.

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- Self-customizing programs

  E.g., Amazon, Netflix product recommendations

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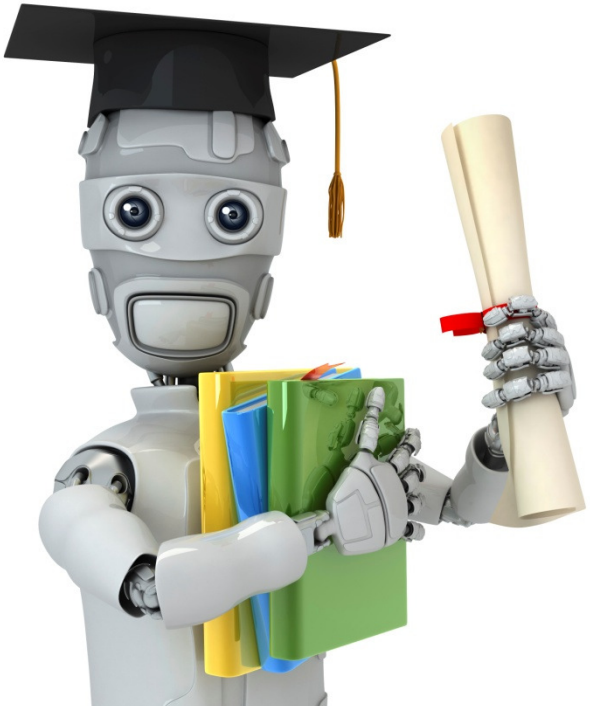
- Applications can't program by hand.
  E.g., Autonomous helicopter, handwriting recognition, most of Natural Language Processing (NLP), Computer Vision.
- Self-customizing programs
  E.g., Amazon, Netflix product recommendations
- Understanding human learning (brain, real AI).



Machine Learning

Introduction

What is machine learning

Machine Learning definition

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- Arthur Samuel (1959). Machine Learning: Field of study that gives computers the ability to learn without being explicitly programmed.

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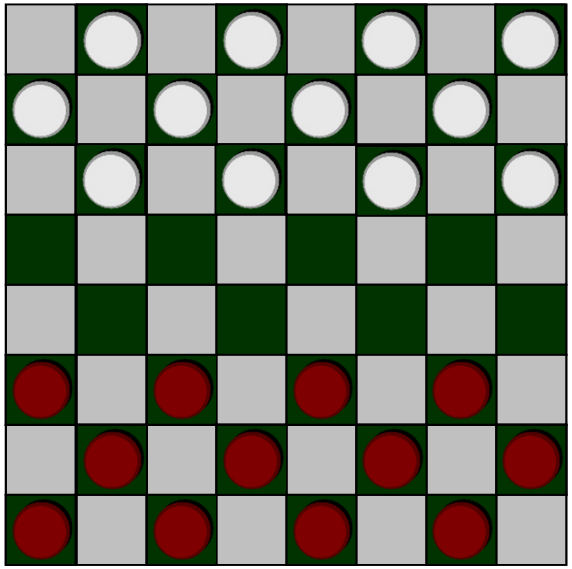


Machine Learning definition

- Arthur Samuel (1959). Machine Learning: Field of study that gives computers the ability to learn without being explicitly programmed.
- Tom Mitchell (1998) Well-posed Learning Problem: A computer program is said to *learn* from experience E with respect to some task T and some performance measure P, if its performance on T, as measured by P, improves with experience E.

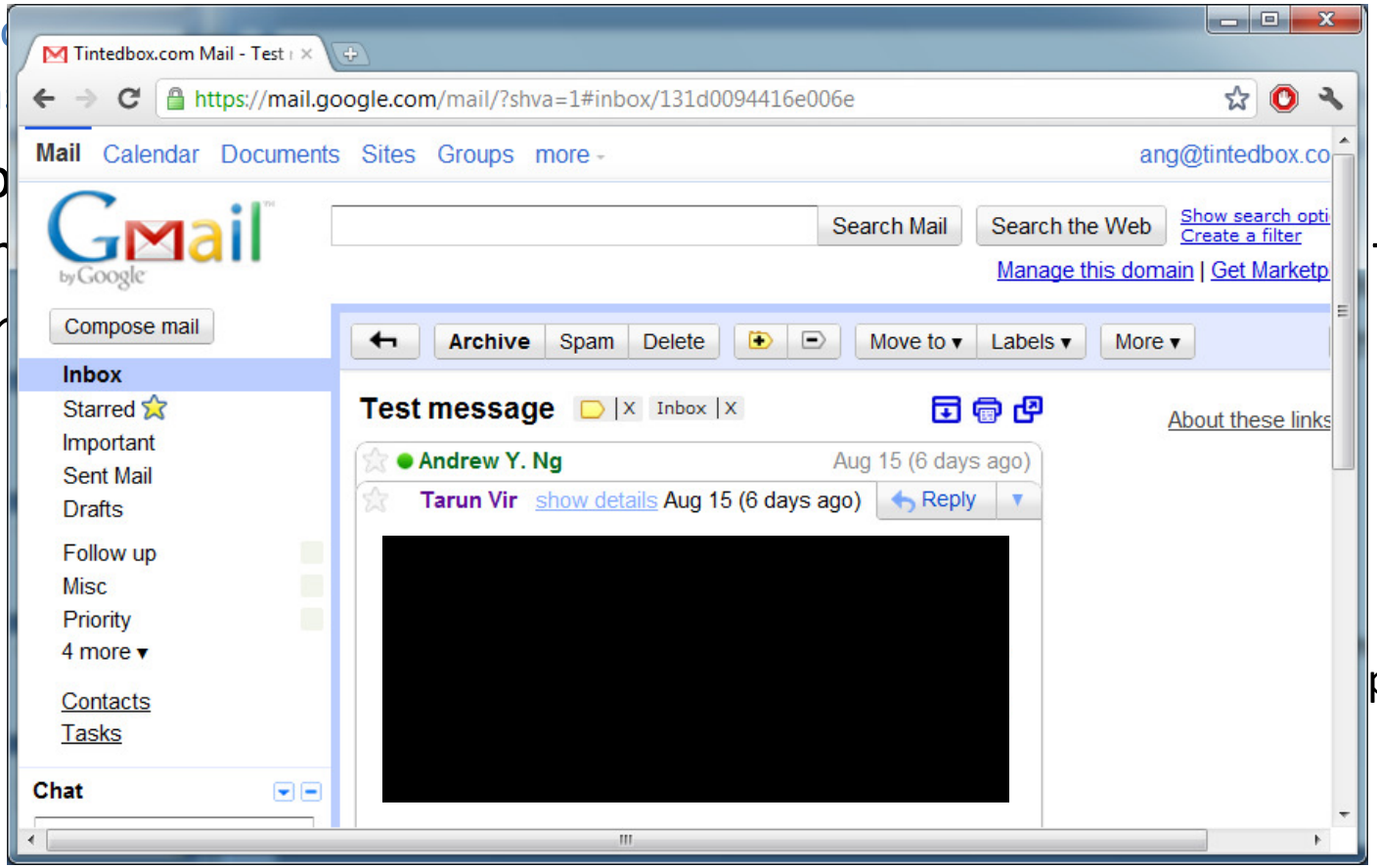
"A computer program is said to *learn* from experience E with respect to some task T and some performance measure P, if its performance on T, as measured by P, improves with experience E."

Suppose your email program watches which emails you do or do not mark as spam, and based on that learns how to better filter spam. What is the task T in this setting?

- ○ Classifying emails as spam or not spam.
- ○ Watching you label emails as spam or not spam.
- ○ The number (or fraction) of emails correctly classified as spam/not spam.
- ○ None of the above—this is not a machine learning problem.

"A computer program is said to *learn* from experience E with respect to s on T, a

Supp or do not r filter spam



pam.

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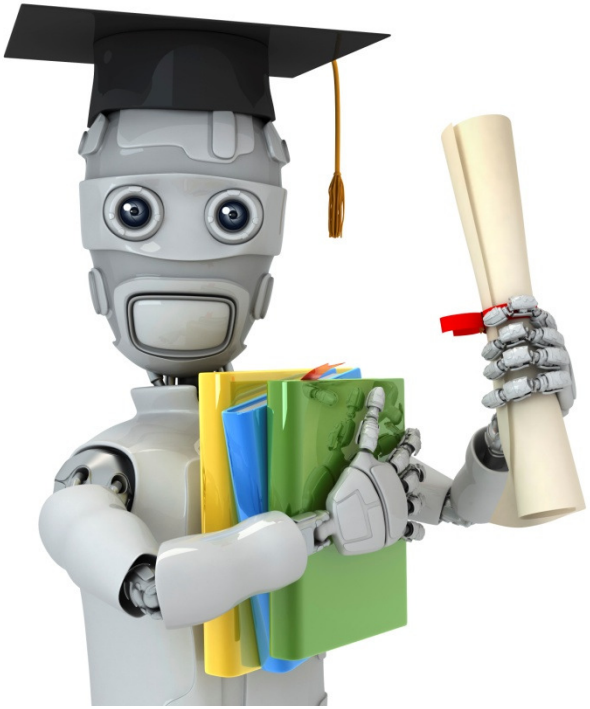
Machine learning algorithms:

- Supervised learning
- Unsupervised learning

Others: Reinforcement learning, recommender systems.

Also talk about: Practical advice for applying learning algorithms.

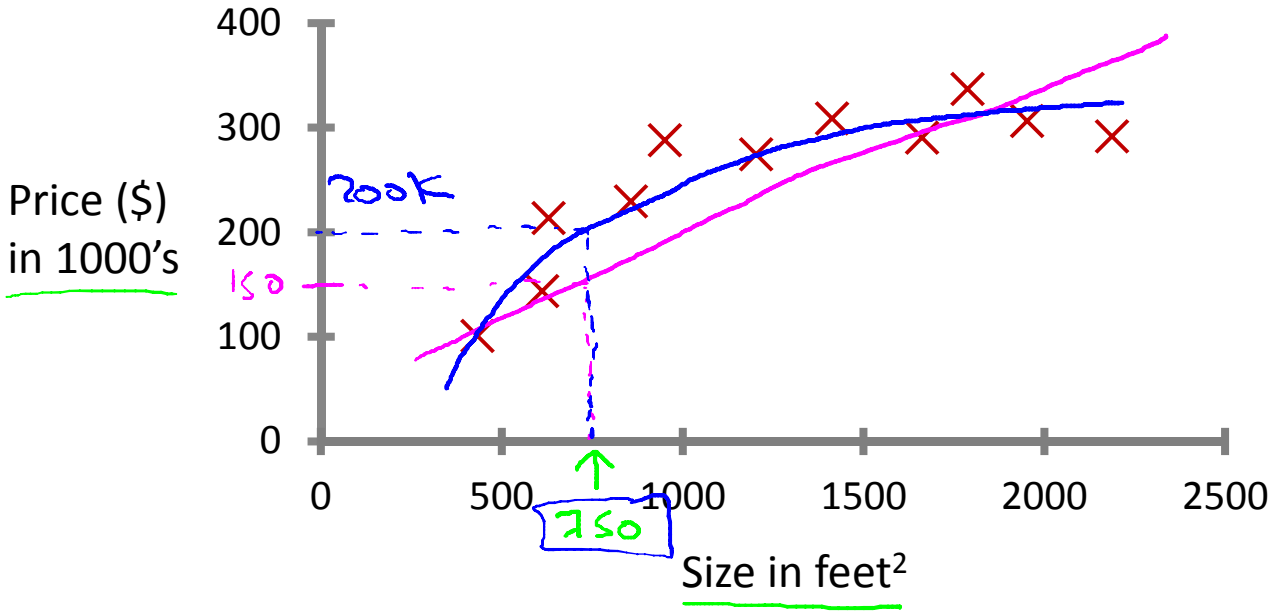


Machine Learning

Introduction

Supervised Learning

Housing price prediction.

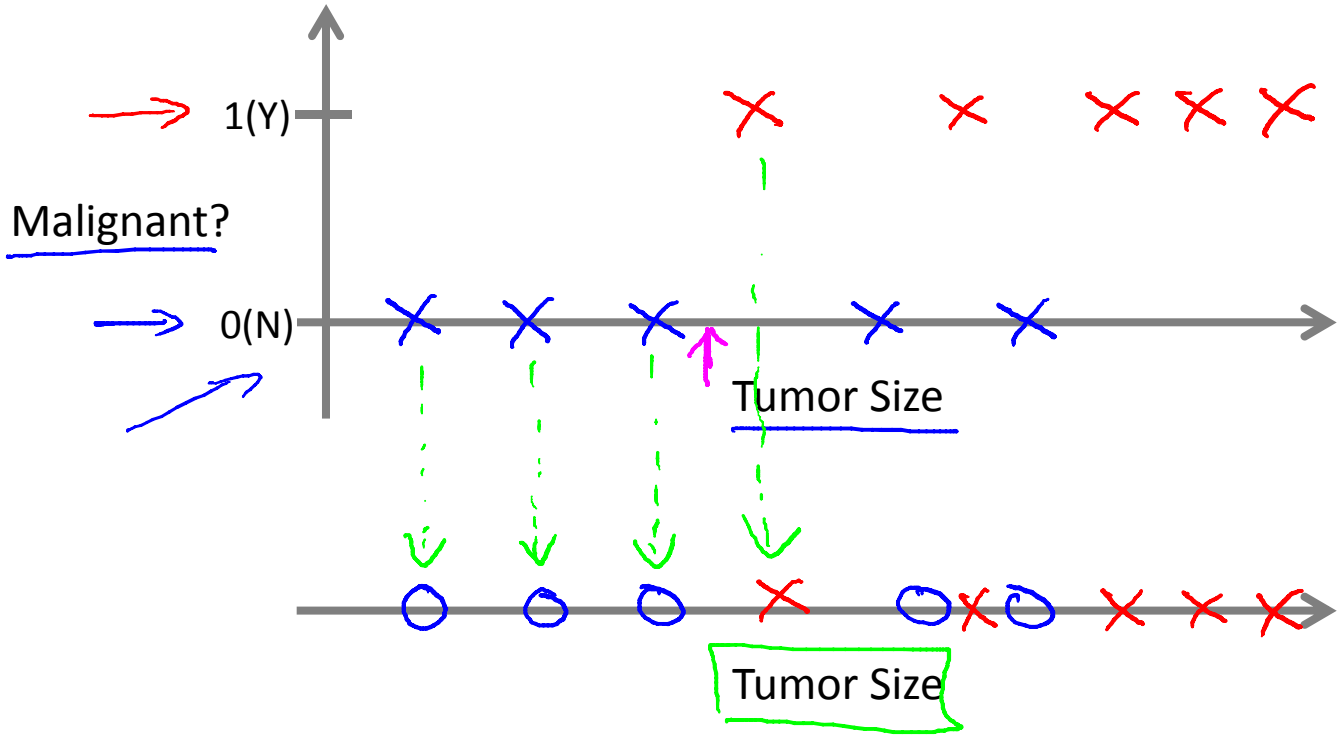


Supervised Learning

"right answers" given

Regression: Predict continuous valued output (price)

Breast cancer (malignant, benign)



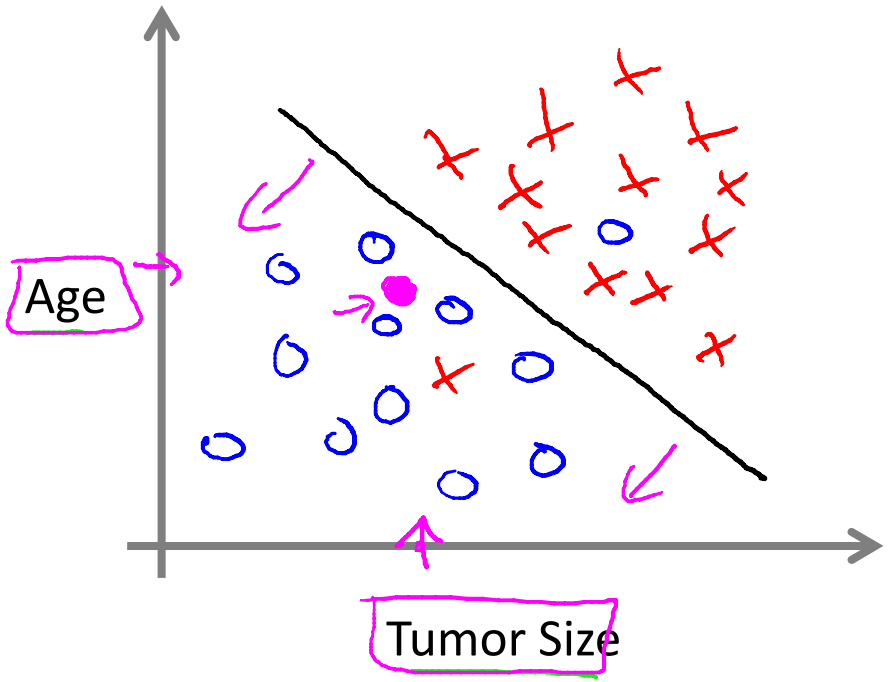
Classification

Discrete valued output (0 or 1)

0, 1, 2, 3

benign

type 1 Cancer



- Clump Thickness
- Uniformity of Cell Size
- Uniformity of Cell Shape

...

You're running a company, and you want to develop learning algorithms to address each of two problems.

1000's

Problem 1: You have a large inventory of identical items. You want to predict how many of these items will sell over the next 3 months.

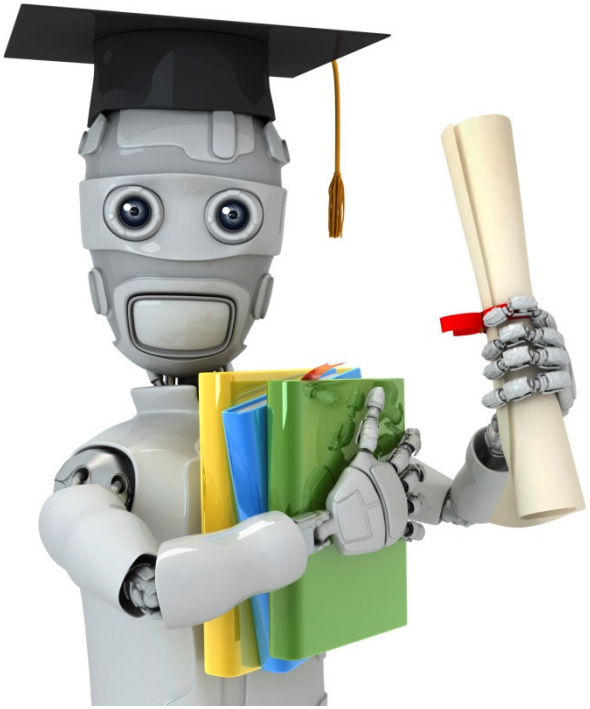
Problem 2: You'd like software to examine individual customer accounts, and for each account decide if it has been hacked/compromised.

0 – not hacked

1 – hacked

Should you treat these as classification or as regression problems?

- ○ Treat both as classification problems.
- ○ Treat problem 1 as a classification problem, problem 2 as a regression problem.
- → ○ Treat problem 1 as a regression problem, problem 2 as a classification problem.
- ○ Treat both as regression problems.

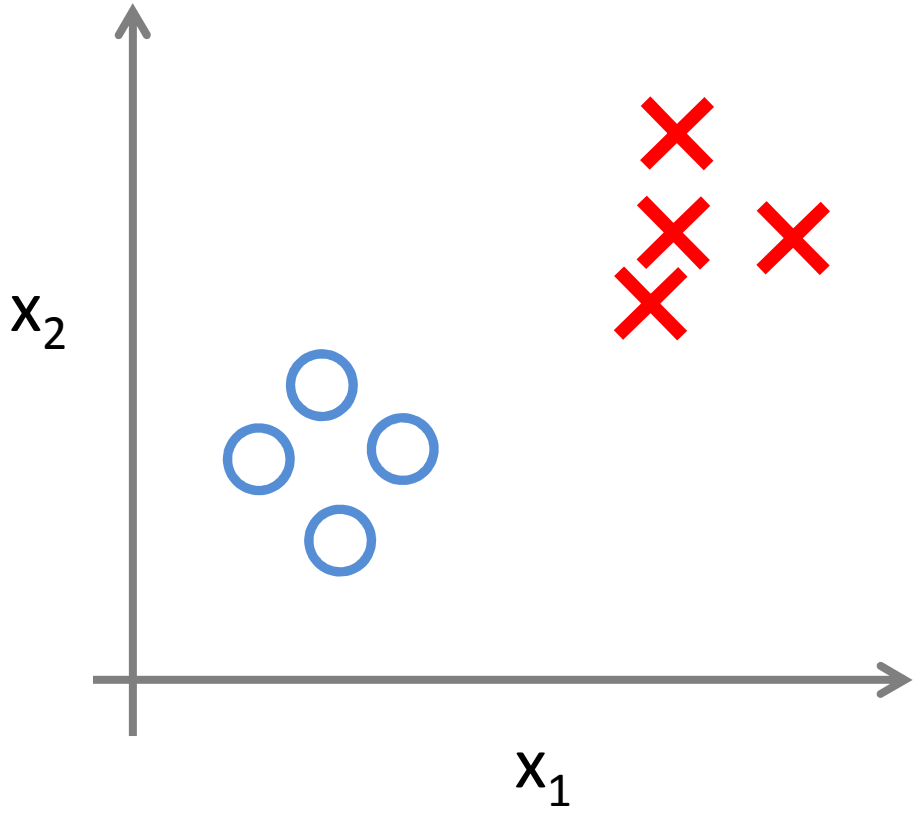


Machine Learning

Introduction

Unsupervised Learning

Supervised Learning



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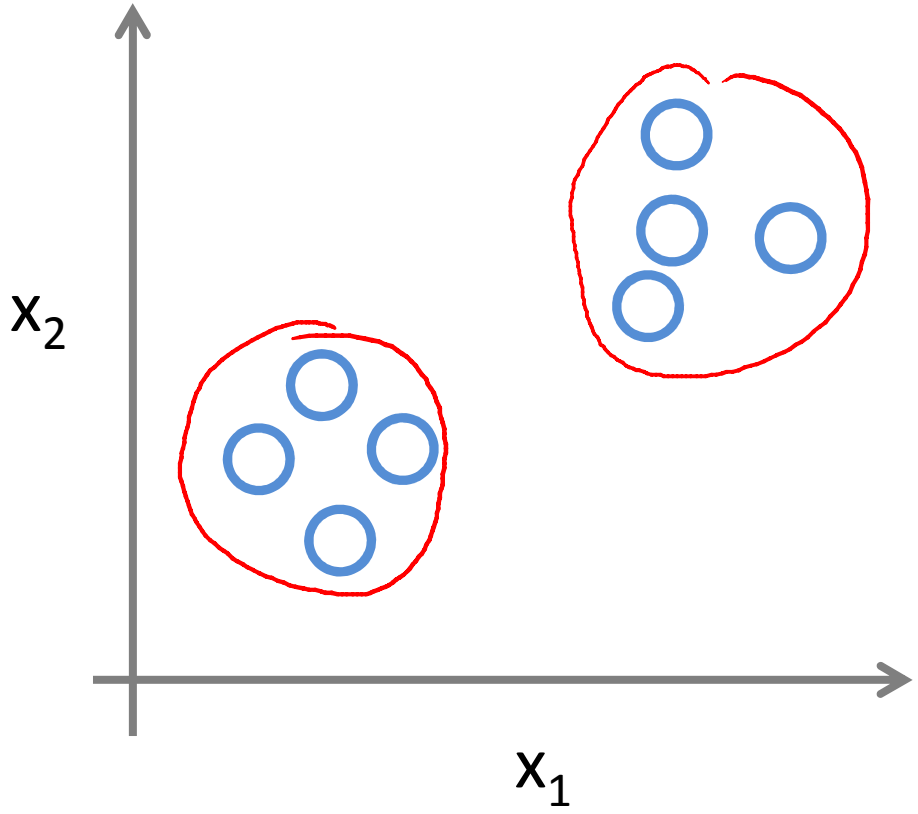
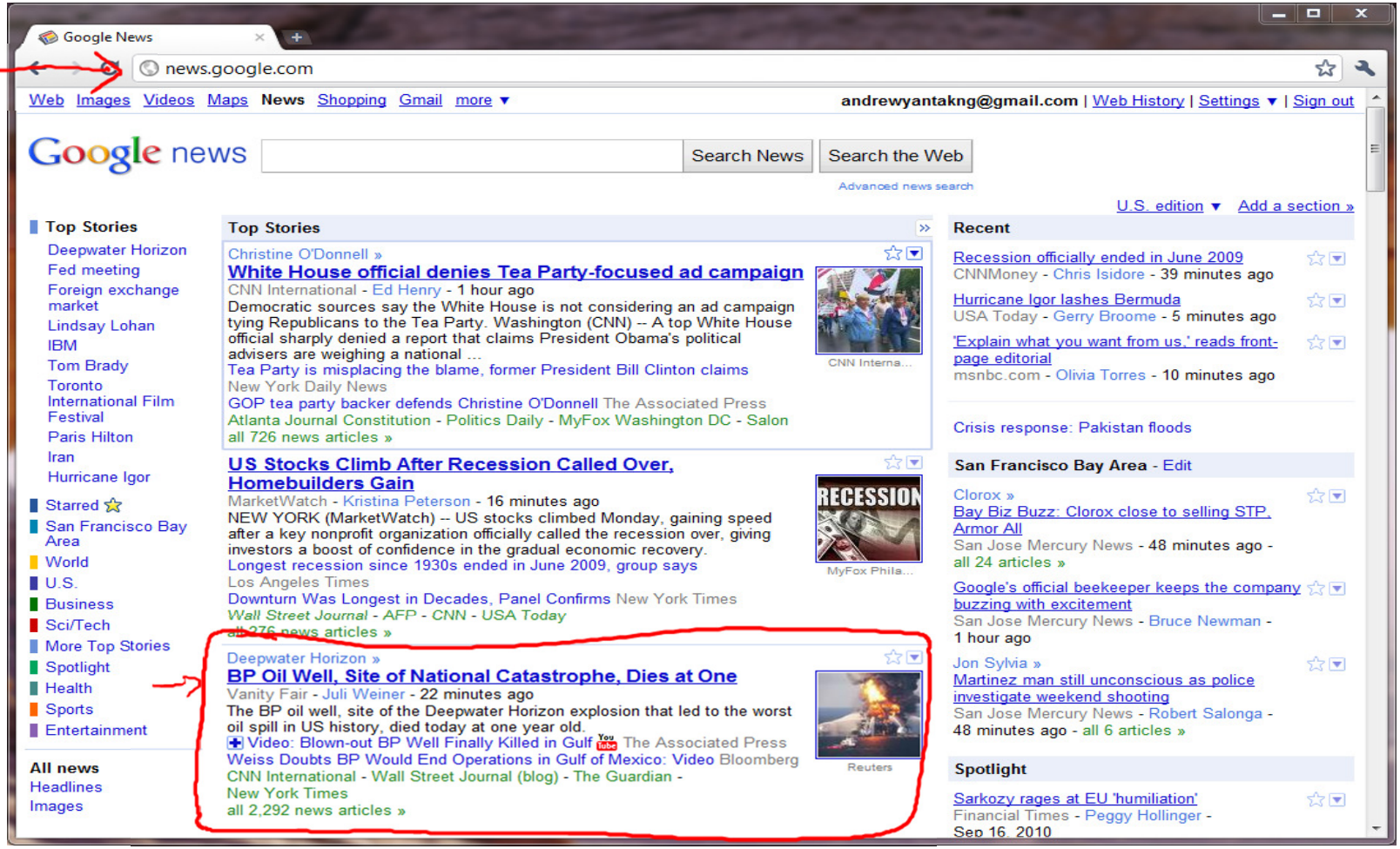
Unsupervised Learning

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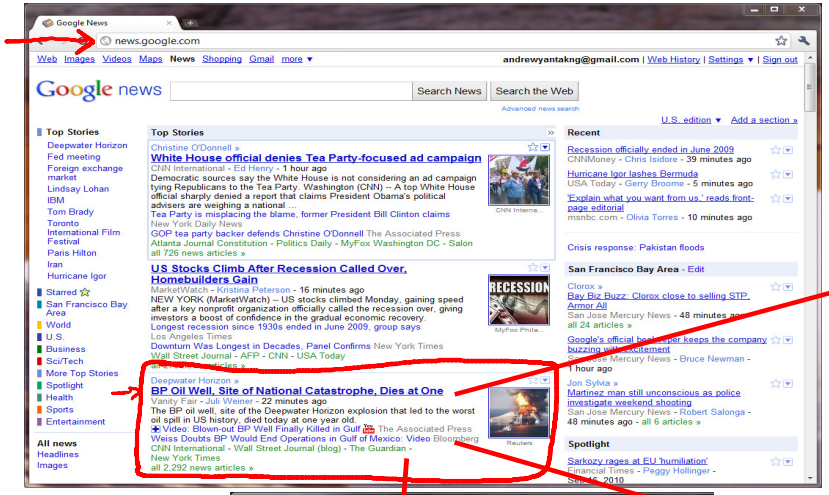
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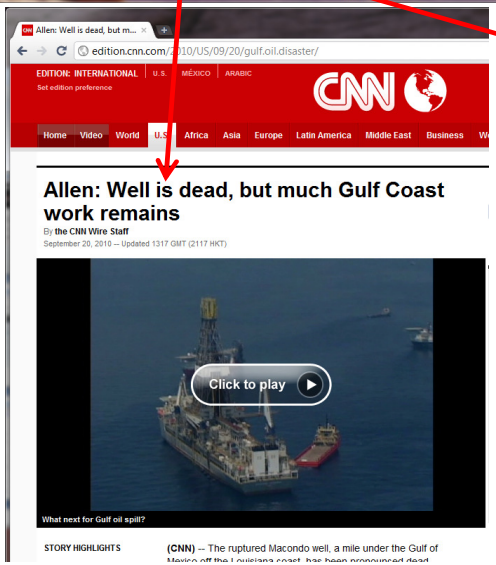
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Allen: Well is dead, but much Gulf Coast work remains
By the CNN Wire Staff
September 20, 2010 -- Updated 1317 GMT (2117 HKT)
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Business > BP
BP oil spill cost hits nearly $10bn
BP has set up a $20bn compensation fund after the Deepwater Horizon disaster, which has so far paid out 19,000 claims totalling more than $240m
Julia Kollewe
guardian.co.uk, Monday 20 September 2010 08.33 BST
Article history
BP's costs for the Deepwater Horizon disaster have hit $10bn. Photograph: Ho/Reuters

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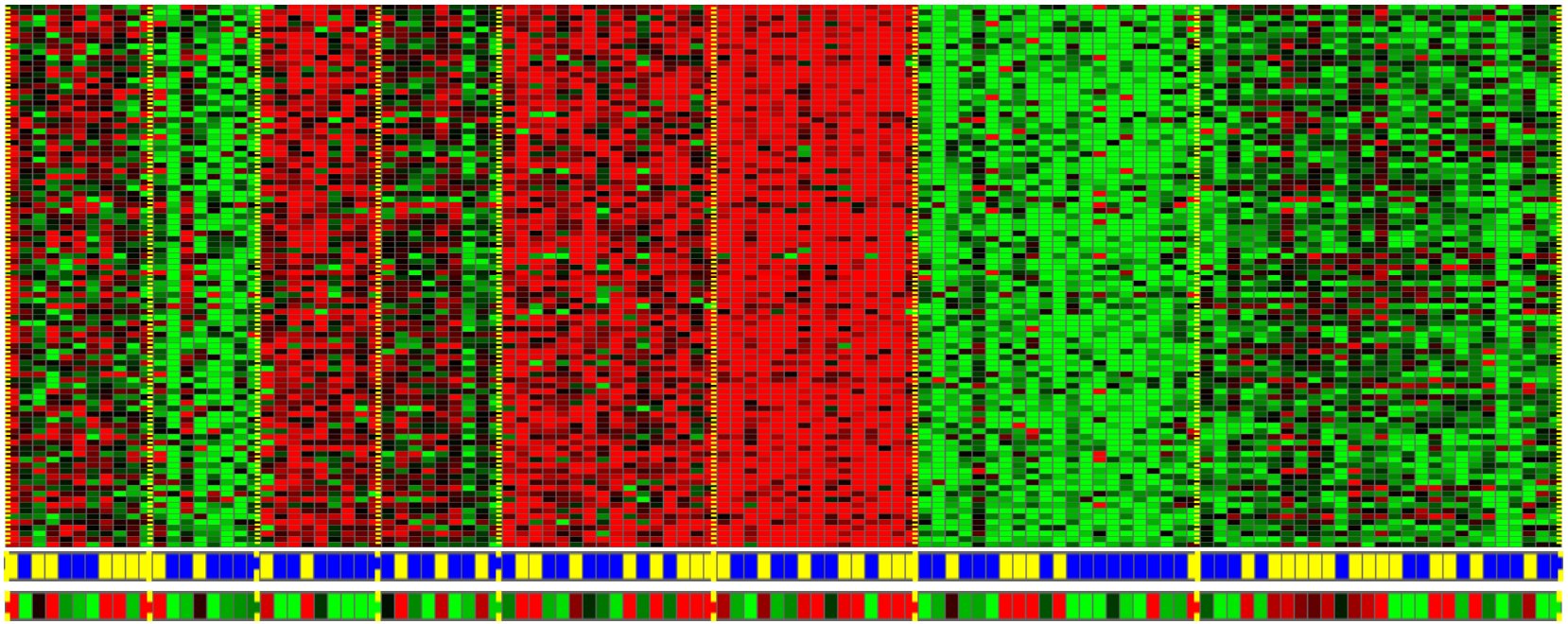
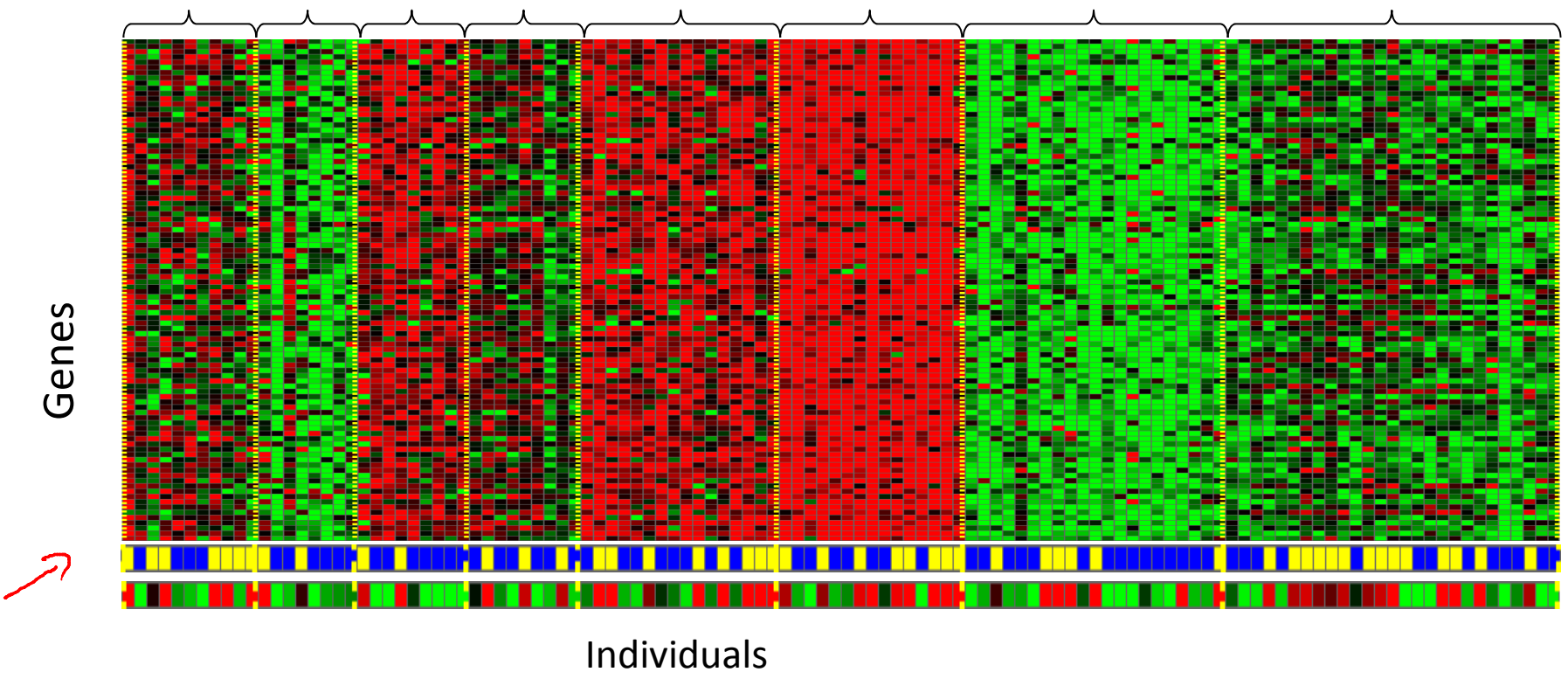
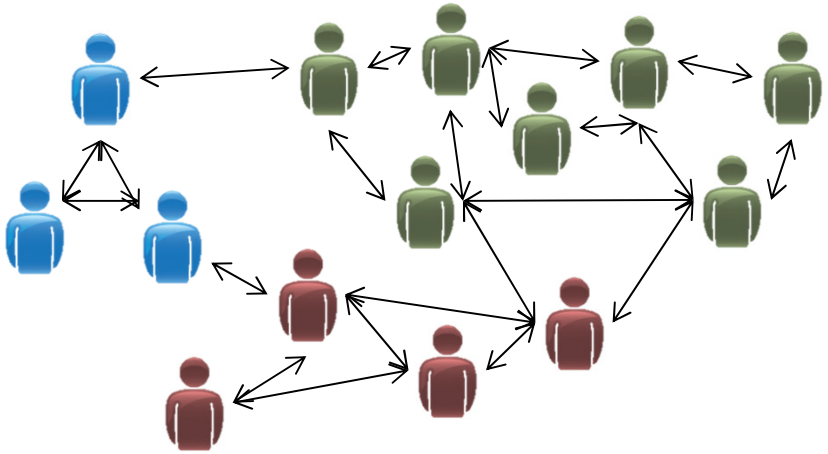


[Source: Daphne Koller]

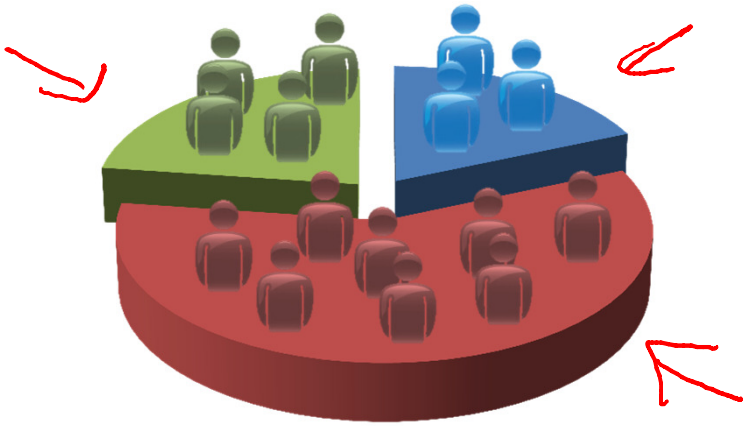


[Source: Daphne Koller]

Organize computing clusters



Social network analysis



Market segmentation

Astronomical data analysis

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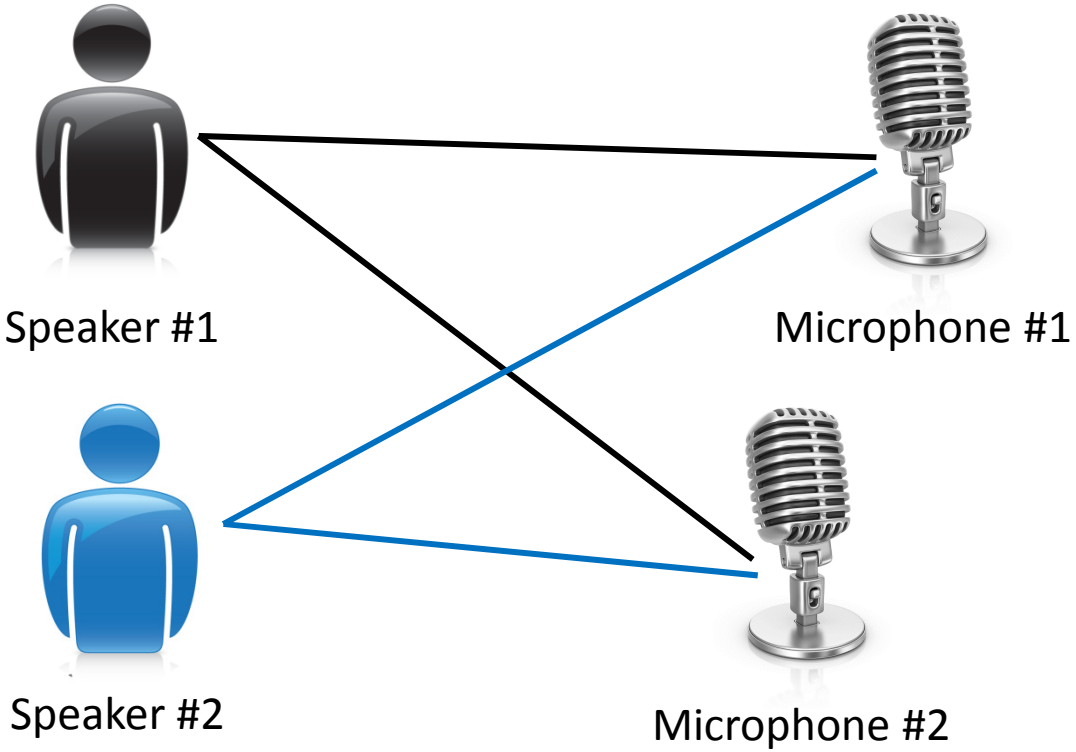
Cocktail party problem



Microphone #1:  Output #1: 

Microphone #2:  Output #2: 

Microphone #1:  Output #1: 

Microphone #2:  Output #2: 

[Audio clips courtesy of Te-Won Lee.]

Cocktail party problem algorithm

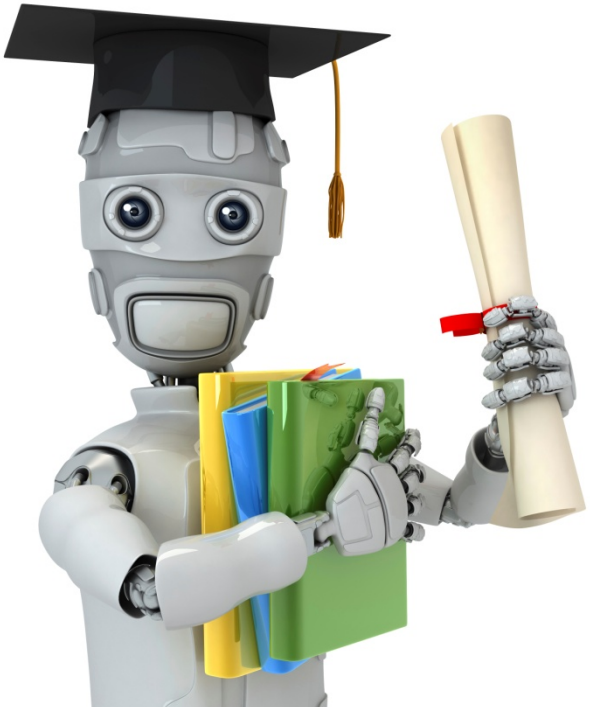
```
[W,s,v] = svd((repmat(sum(x.*x,1),size(x,1),1).*x)*x');
```

[Source: Sam Roweis, Yair Weiss & Eero Simoncelli]

Of the following examples, which would you address using an unsupervised learning algorithm? (Check all that apply.)

- [ ] Given email labeled as spam/not spam, learn a spam filter.
- [ ] Given a set of news articles found on the web, group them into set of articles about the same story.
- [ ] Given a database of customer data, automatically discover market segments and group customers into different market segments.
- [ ] Given a dataset of patients diagnosed as either having diabetes or not, learn to classify new patients as having diabetes or not.

1.2 Gradient Descent

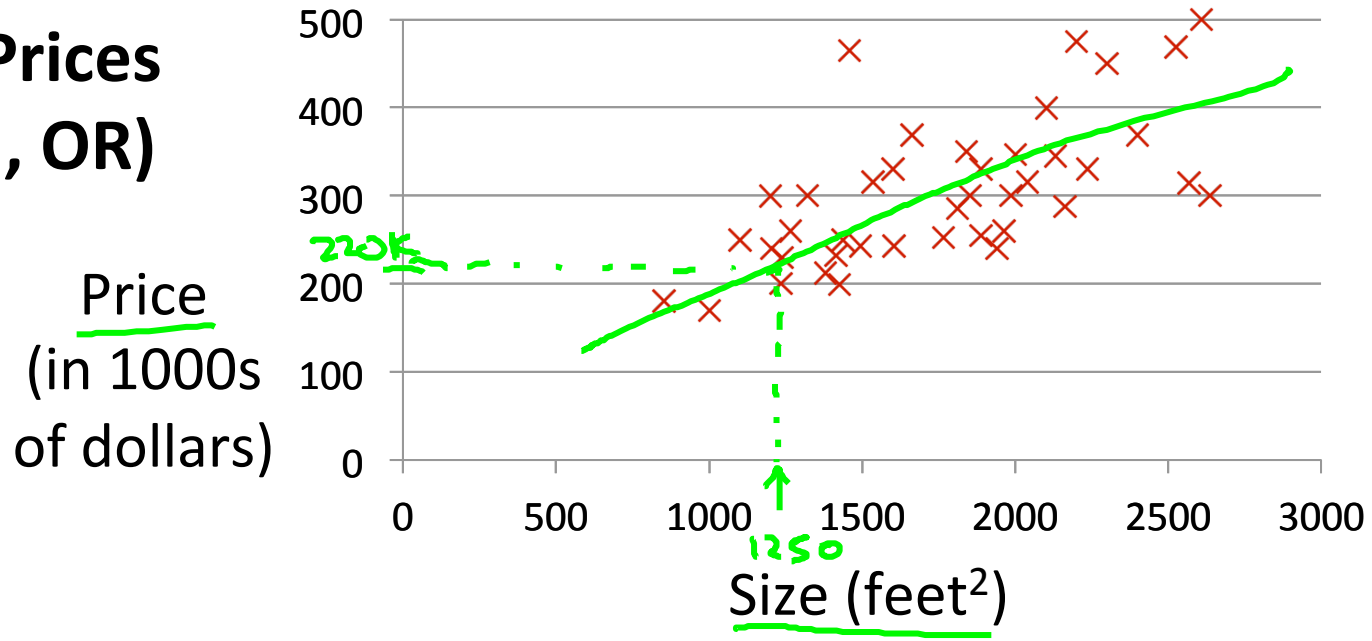


Machine Learning

Linear regression with one variable

Model representation

Housing Prices (Portland, OR)



Supervised Learning

Given the "right answer" for each example in the data.

Regression Problem

Predict real-valued output

Classification: Discrete-valued output

Training set of housing prices (Portland, OR)

Size in feet^2 (x)	Price ($) in 1000's (y)
2104	460
1416	232
1534	315
852	178
…	…

m = 47

Notation:

- **m** = Number of training examples
- **x**'s = "input" variable / features
- **y**'s = "output" variable / "target" variable

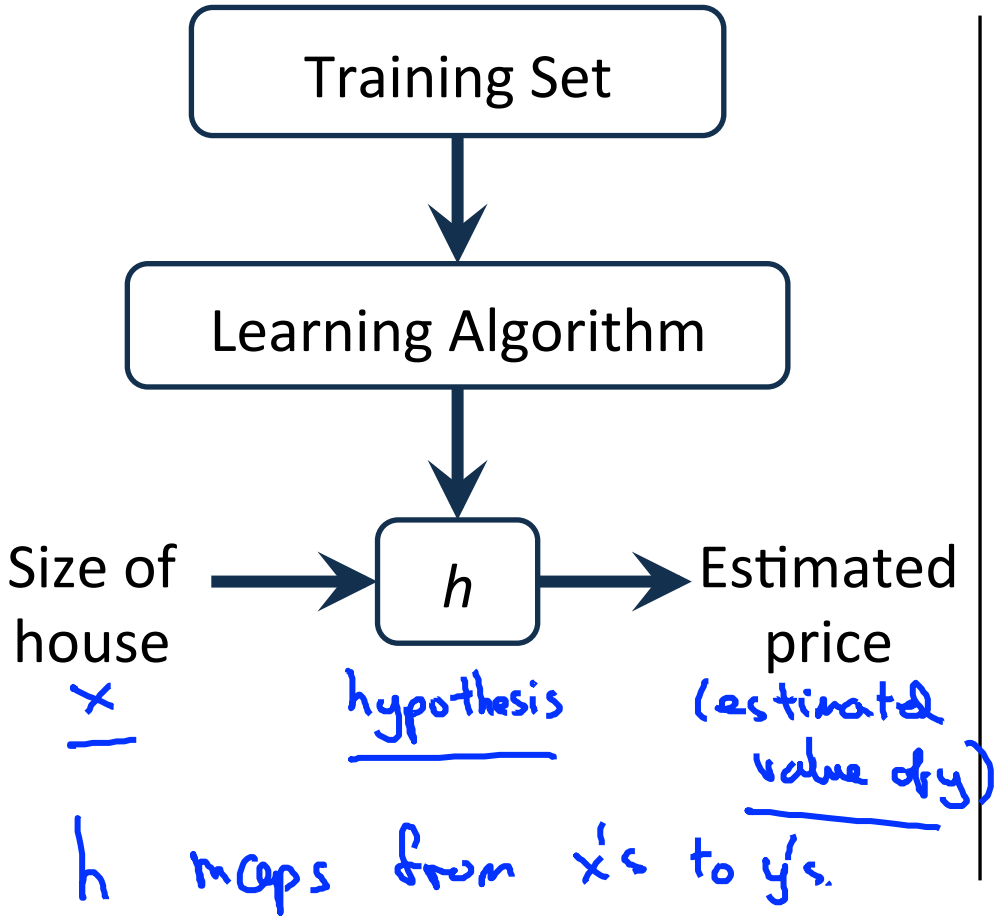
$x^{(1)} = 2104$

$x^{(2)} = 1416$

$y^{(1)} = 460$

(x, y) - one training example

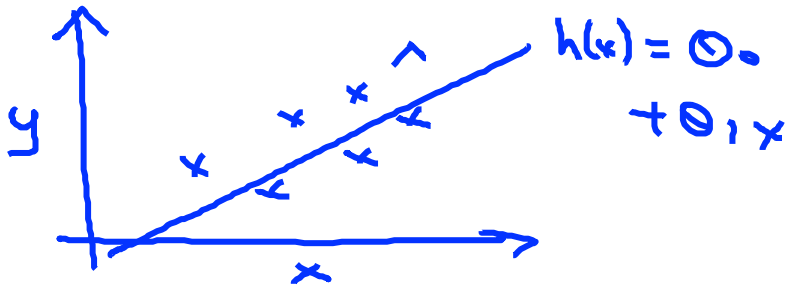
$(x^{(i)}, y^{(i)})$ - i^{th} training example



How do we represent *h* ?

$h_\theta(x) = \theta_0 + \theta_1 x$

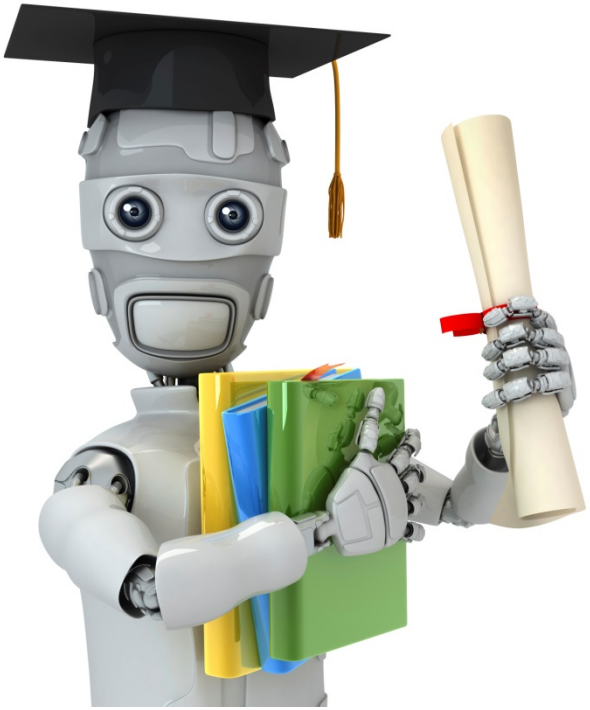
Shorthand: $h(x)$



Linear regression with one variable. (x)

Univariate linear regression.

└ one variable



Machine Learning

Linear regression with one variable

Cost function

Training Set

Size in feet² (x)	Price ($) in 1000's (y)
2104	460
1416	232
1534	315
852	178
…	…

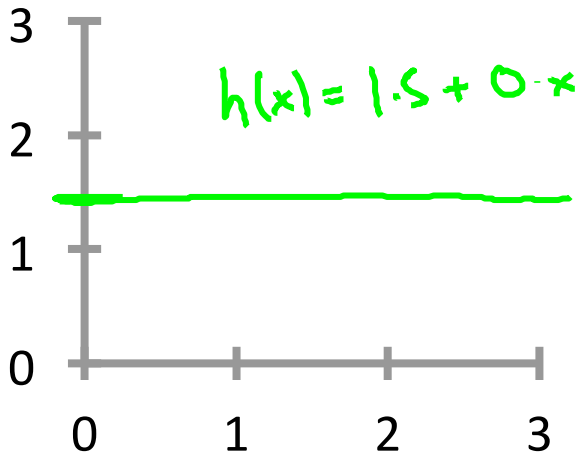
$m = 47$

Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

θ_i's: Parameters

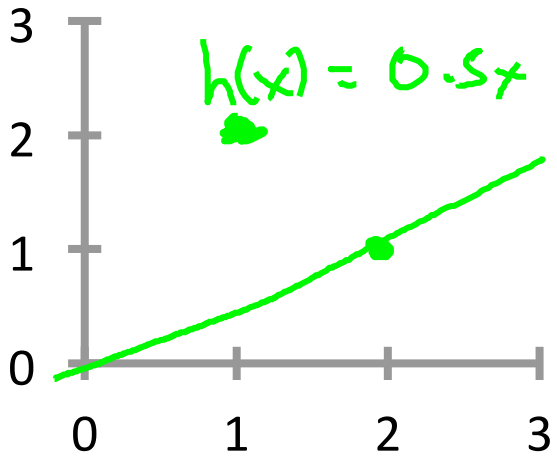
How to choose θ_i's ?

$h_\theta(x) = \theta_0 + \theta_1 x$



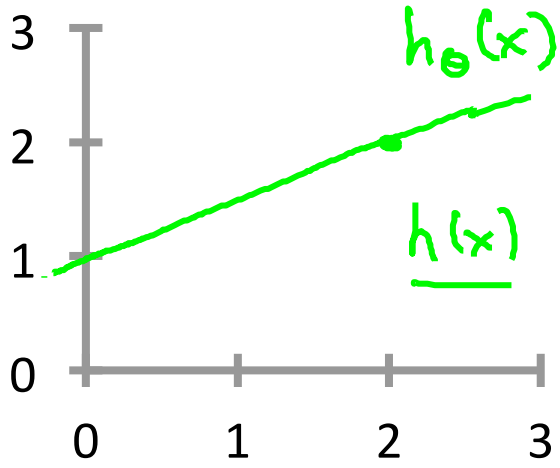
→ $\theta_0 = 1.5$

→ $\theta_1 = 0$



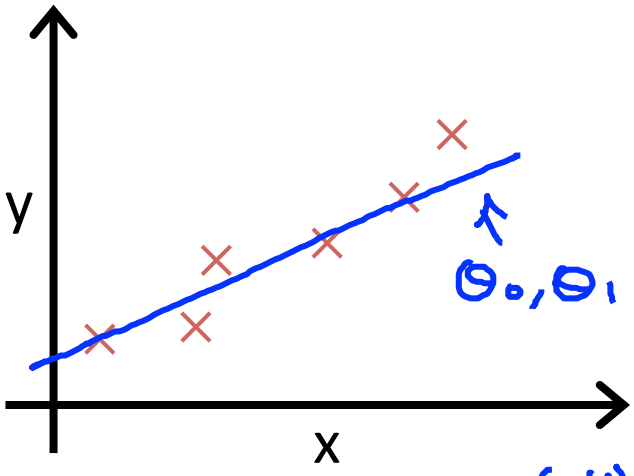
→ $\theta_0 = 0$

→ $\theta_1 = 0.5$



→ $\theta_0 = 1$

→ $\theta_1 = 0.5$



θ_0, θ_1

$(x^{(i)}, y^{(i)})$

#training examples

$$\text{minimize}_{\theta_0,\theta_1} \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

$$h_\theta(x^{(i)}) = \theta_0 + \theta_1 x^{(i)}$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

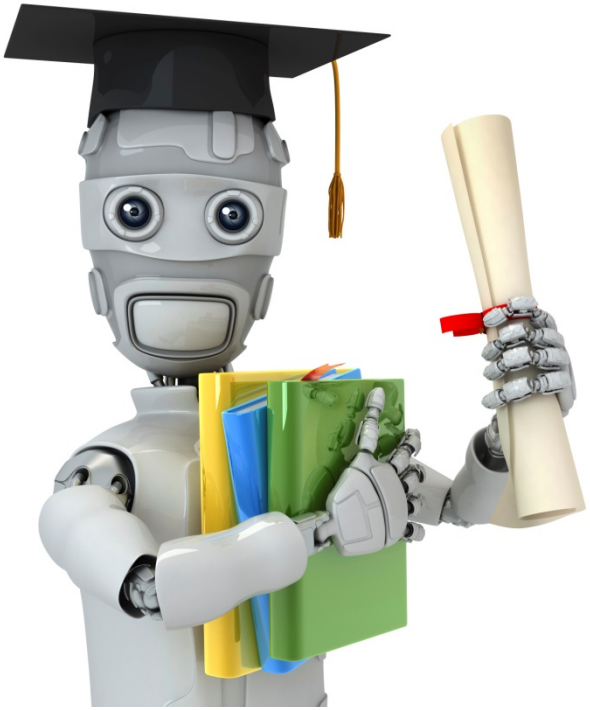
Idea: Choose θ_0, θ_1 so that $h_\theta(x)$ is close to y for our training examples (x, y)

x, y

Minimize $J(\theta_0, \theta_1)$
θ_0, θ_1

Cost function

Squared error function



Machine Learning

Linear regression with one variable

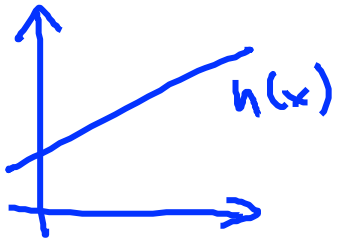
Cost function intuition I

Hypothesis:

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Parameters:

θ_0, θ_1



Cost Function:

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

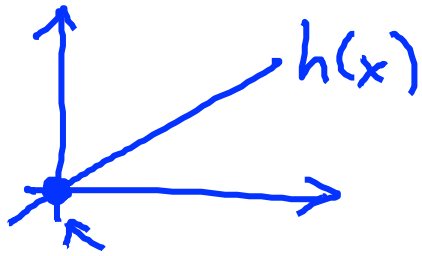
Goal: $\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$

Simplified

$$h_\theta(x) = \theta_1 x$$

$\theta_0 = 0$

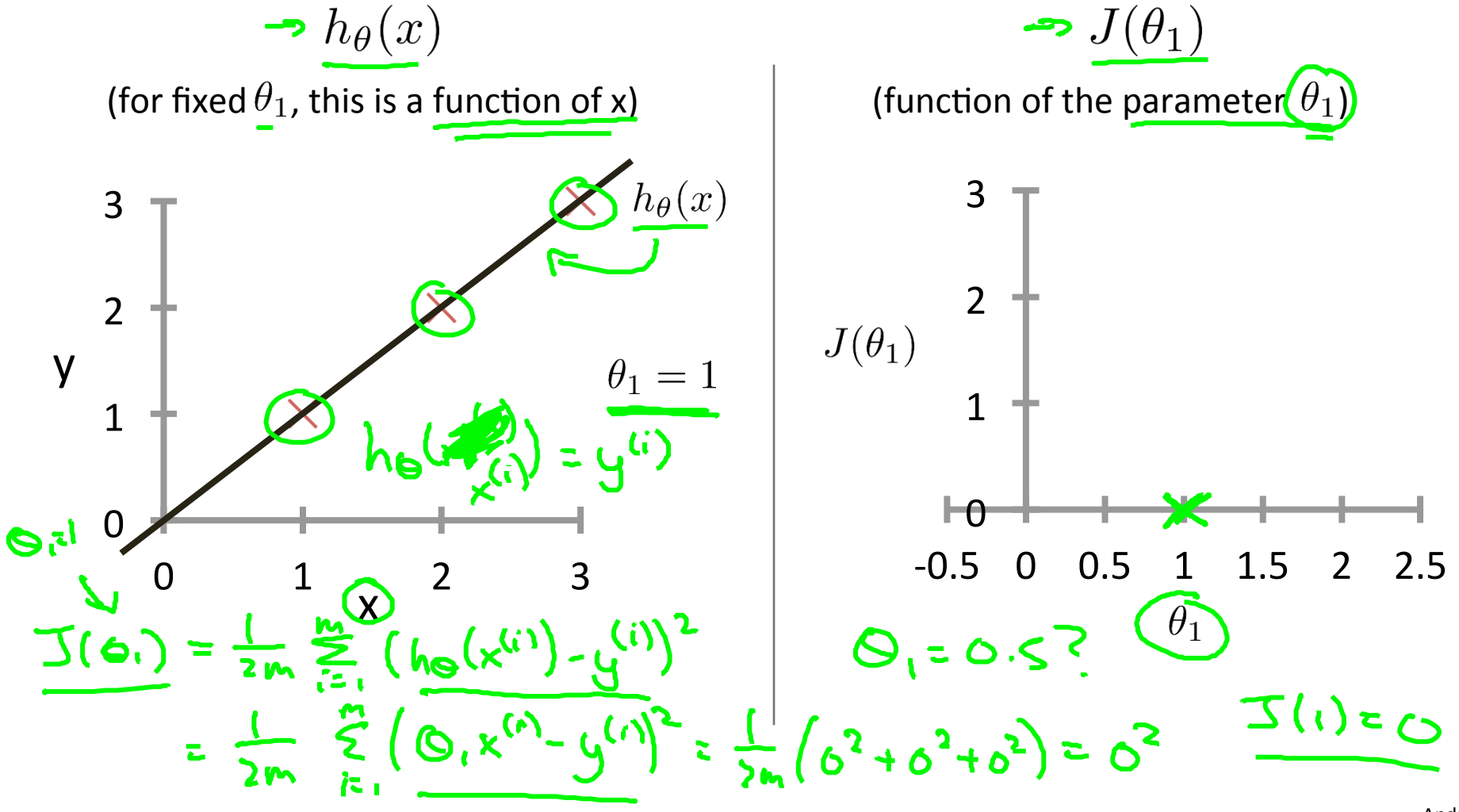
θ_1



$$J(\theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

$\theta_1 x^{(i)}$

$\underset{\theta_1}{\text{minimize}}\ J(\theta_1)$


$h_\theta(x)$
(for fixed θ_1, this is a function of x)
$J(\theta_1)$
(function of the parameter θ_1)
$h_\theta(x)$
$\theta_1 = 1$
y
x
$J(\theta_1)$
θ_1
$h_\theta(x^{(i)}) = y^{(i)}$
$J(\theta_1) = \frac{1}{2m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})^2$
$= \frac{1}{2m}\sum_{i=1}^{m}(\theta_1 x^{(i)} - y^{(i)})^2 = \frac{1}{2m}(0^2+0^2+0^2) = 0^2$
$\theta_1 = 0.5$?
$J(1) = 0$

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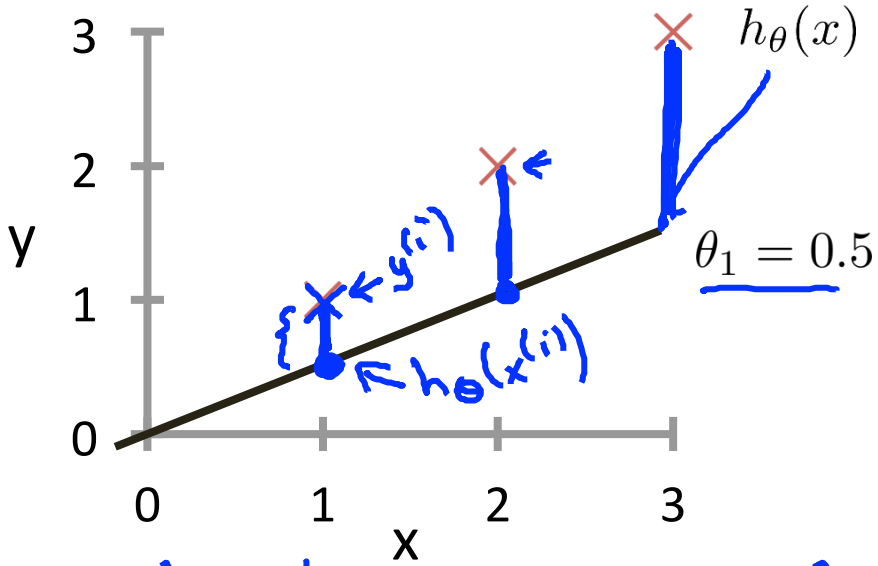
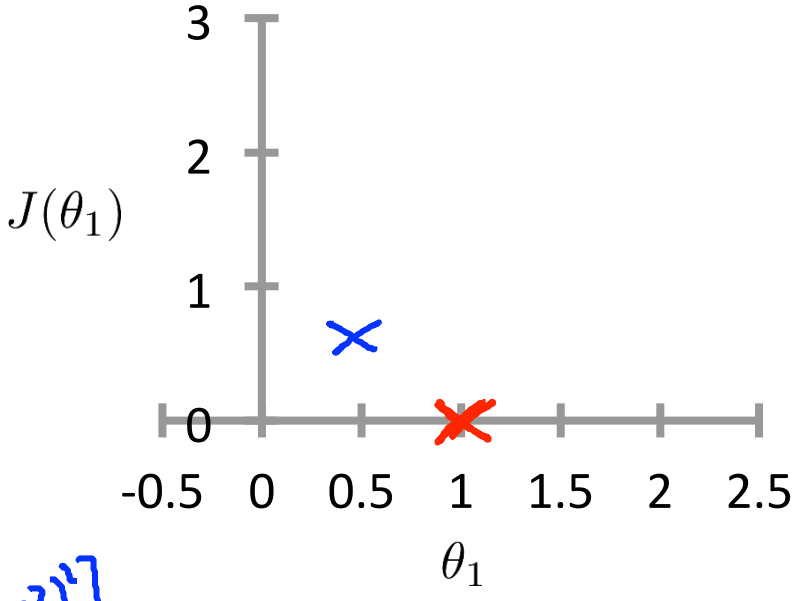
$h_\theta(x)$

(for fixed θ_1, this is a function of x)



$J(\theta_1)$

(function of the parameter θ_1)



$J(0.5) = \frac{1}{2m}\left[(0.5-1)^2 + (1-2)^2 + (1.5-3)^2\right]$

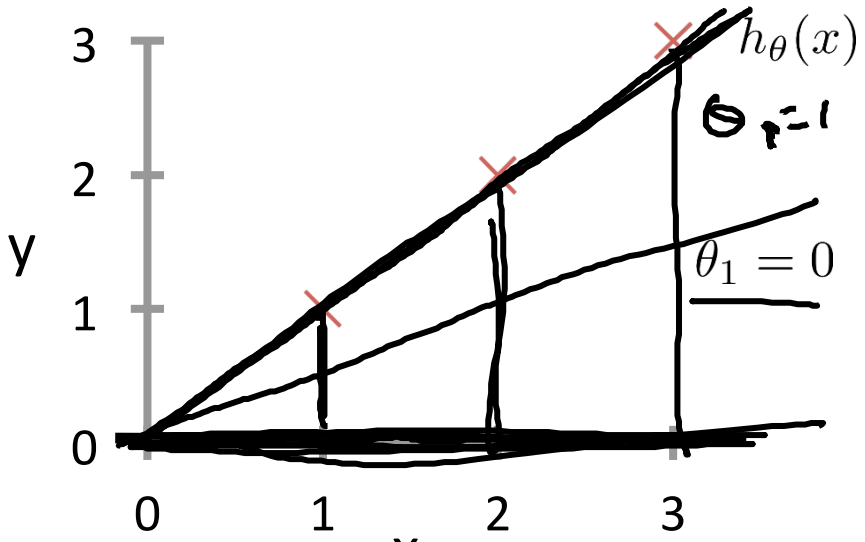
$= \frac{1}{2\times 3}(3.5) = \frac{3.5}{6} \approx 0.58$

$\theta_1 = 0$?

$J(0) = ?$

$h_\theta(x)$

(for fixed θ_1, this is a function of x)

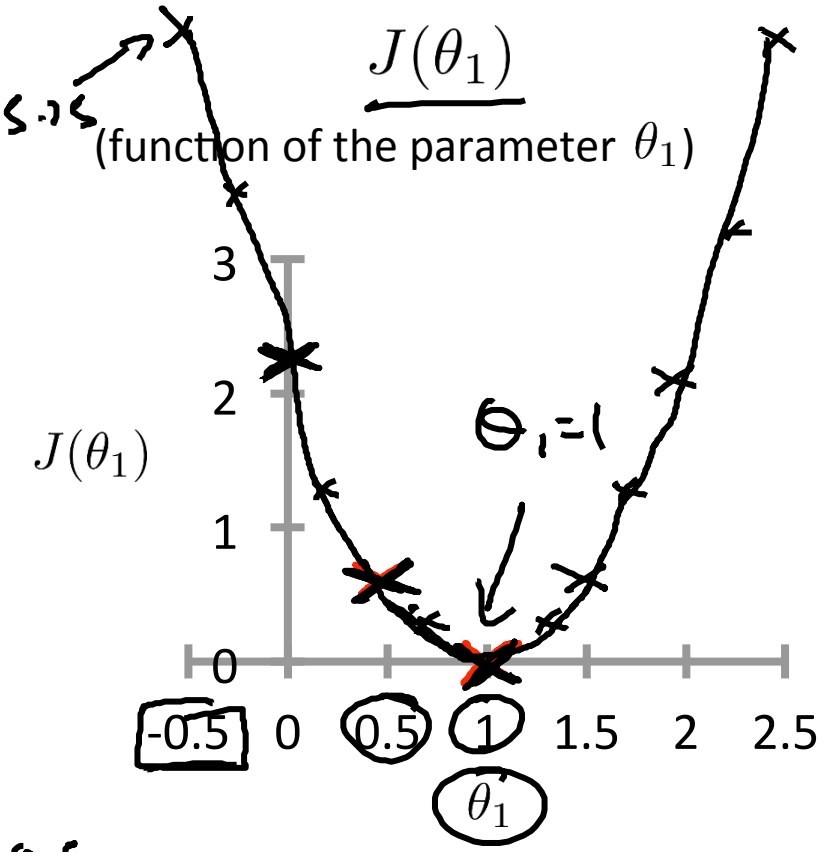


$J(0) = \frac{1}{2m}(1^2 + 2^2 + 3^2)$

$= \frac{1}{6}(14) \approx 2.3$

$J(\theta_1)$

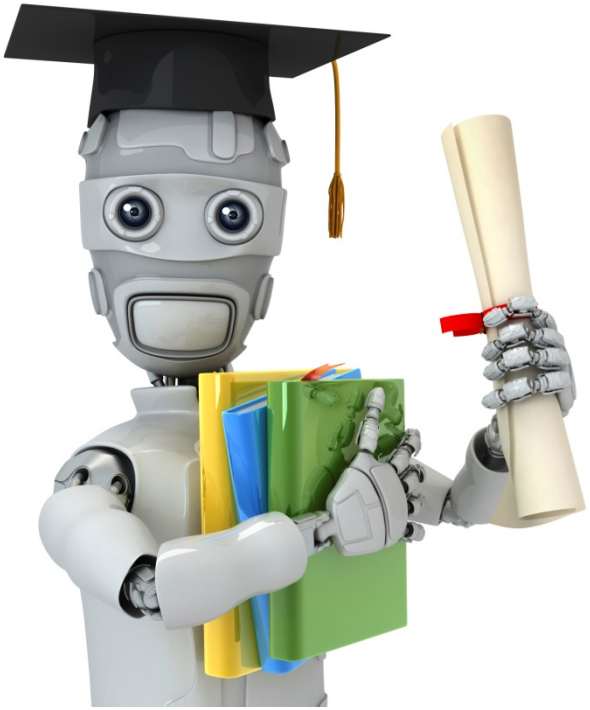
(function of the parameter θ_1)



$h(x) = -0.5 \times$

minimize $J(\theta_1)$

θ_1



Machine Learning

Linear regression with one variable

Cost function intuition II

Hypothesis: $h_\theta(x) = \theta_0 + \theta_1 x$

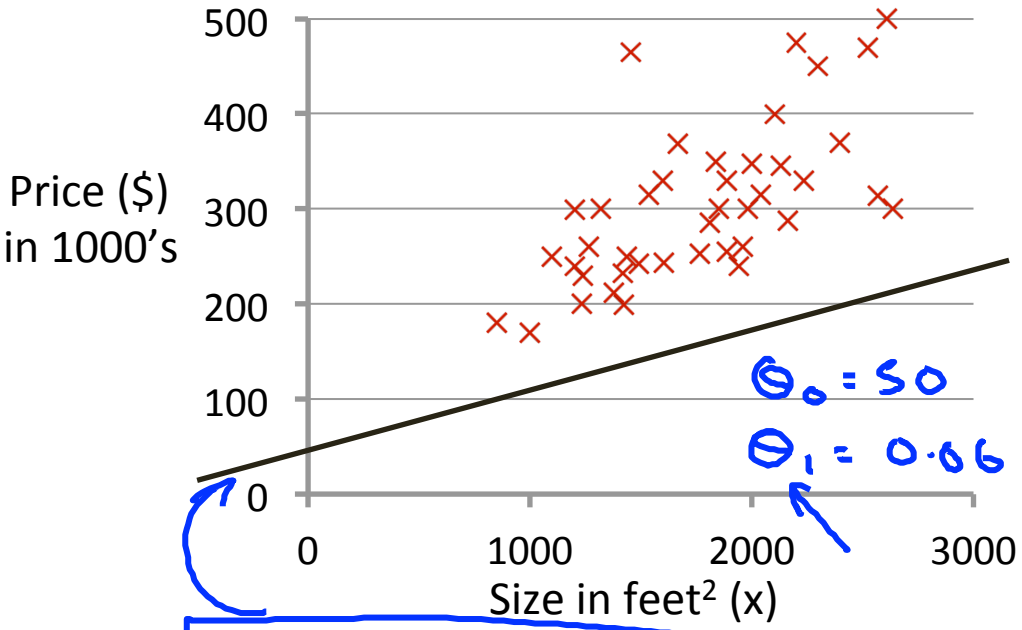
Parameters: θ_0, θ_1

Cost Function: $J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$

Goal: $\underset{\theta_0, \theta_1}{\text{minimize}}\ J(\theta_0, \theta_1)$

$h_\theta(x)$

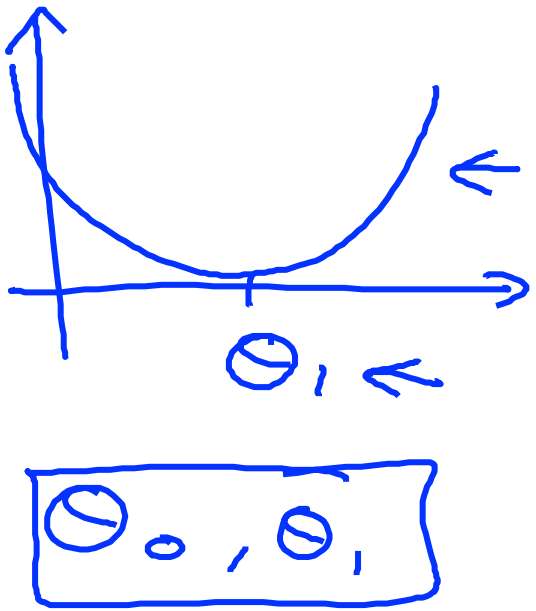
(for fixed θ_0, θ_1, this is a function of x)



$$h_\theta(x) = 50 + 0.06x$$

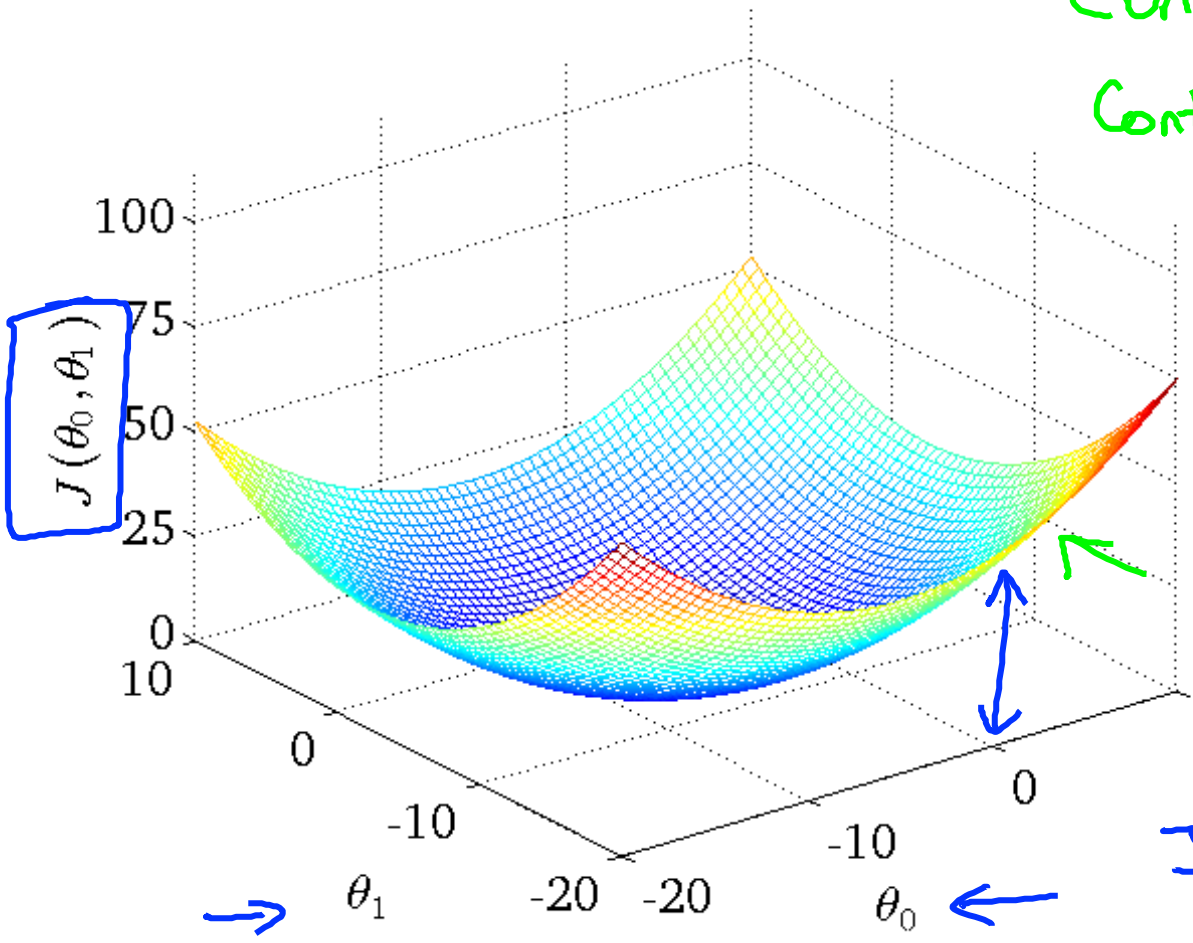
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



Contour plots

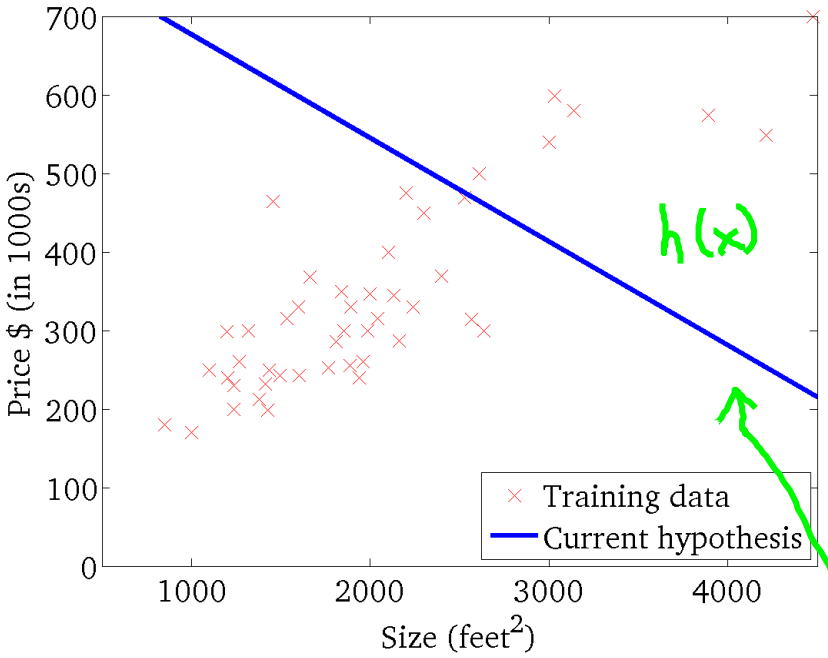
Contour figures -



$J(\theta_0, \theta_1)$

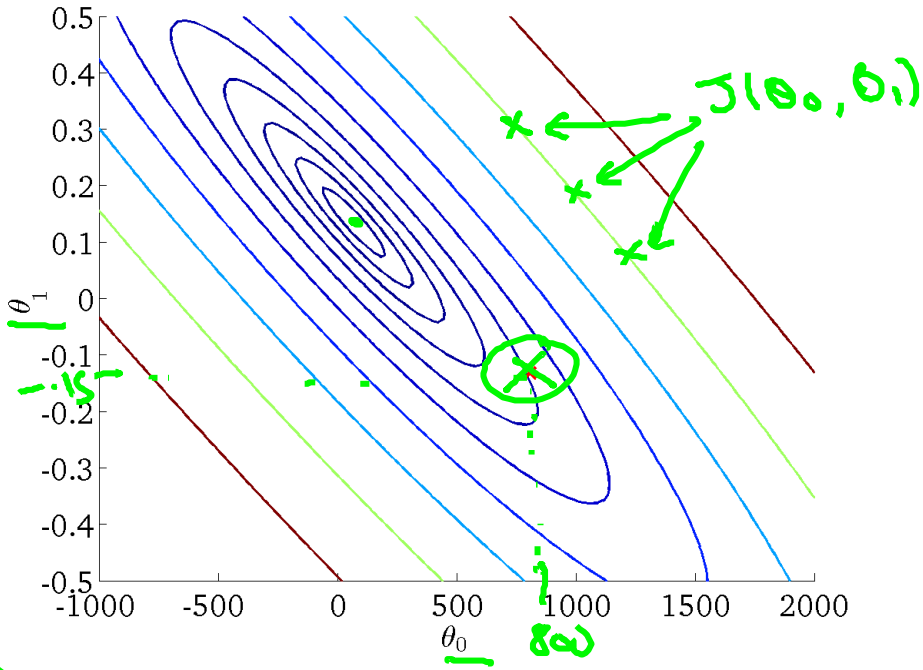
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



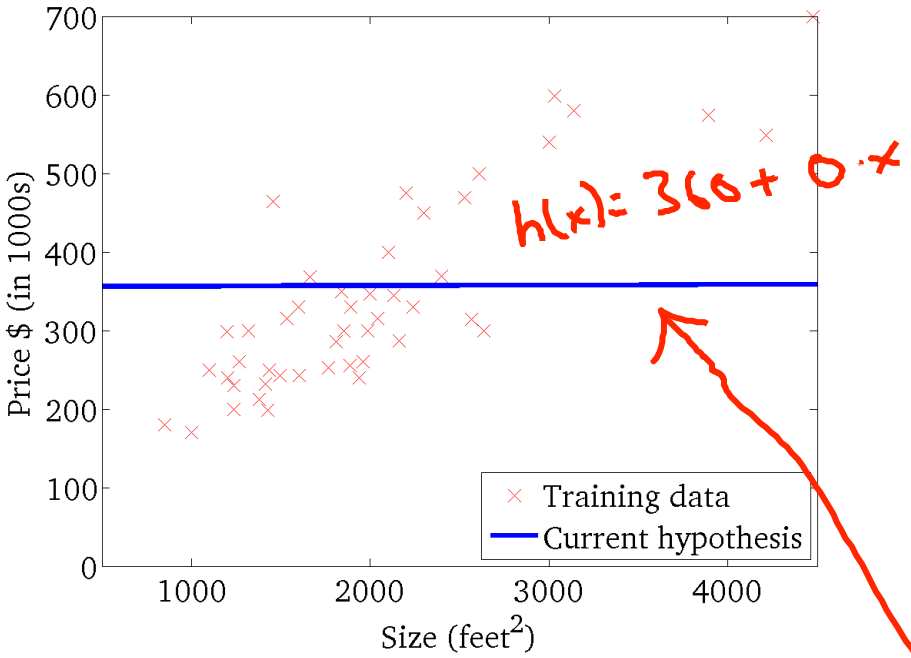
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



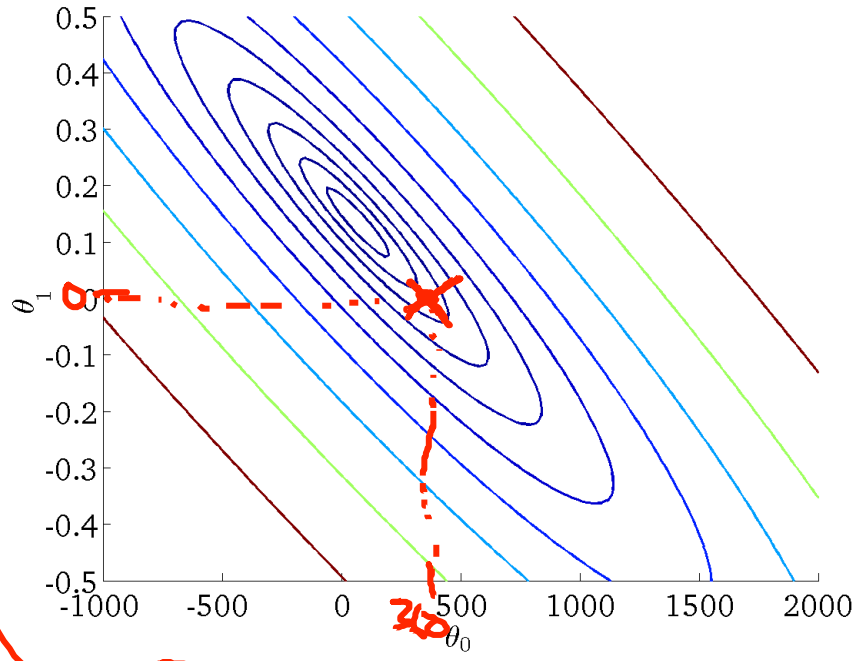
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)

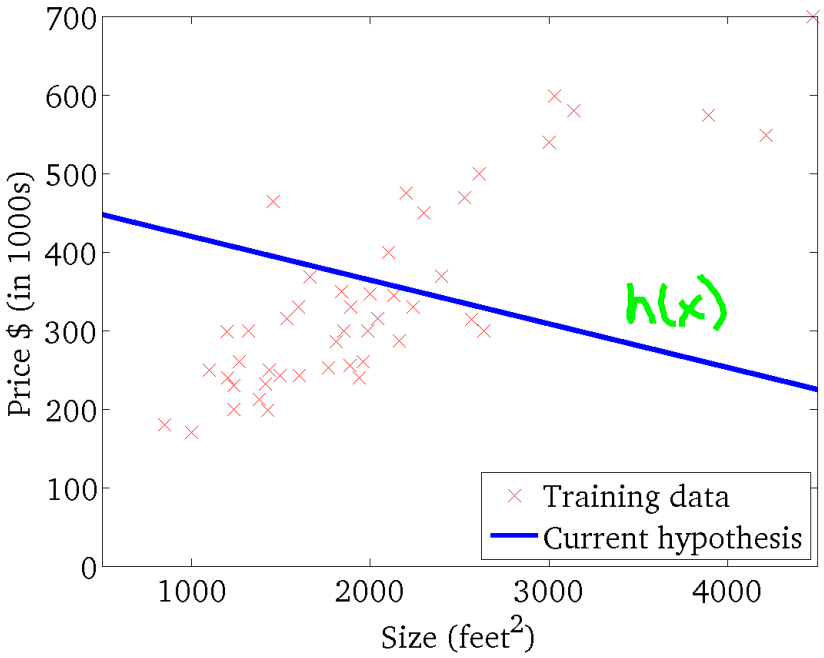


$\theta_0 = 360$

$\theta_1 = 0$

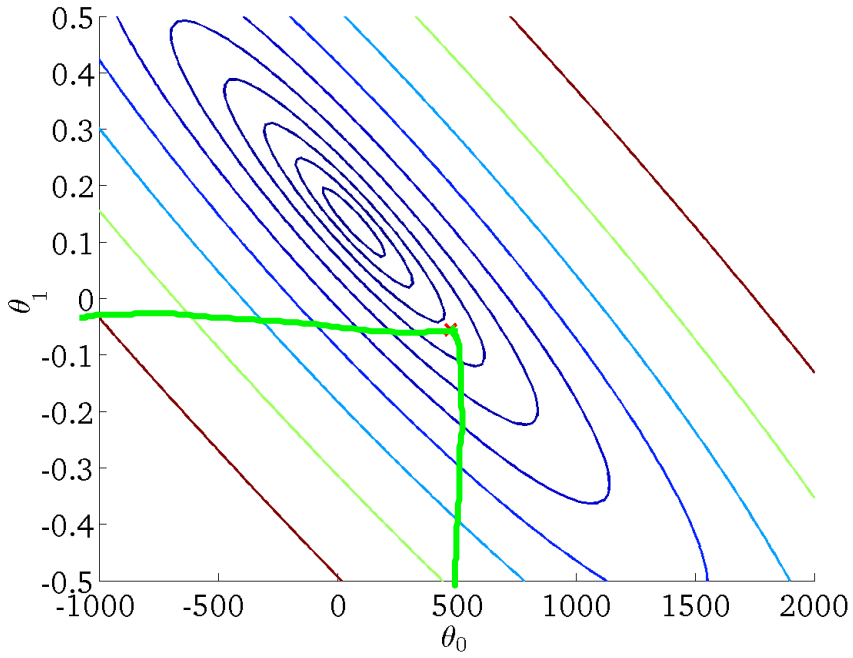
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



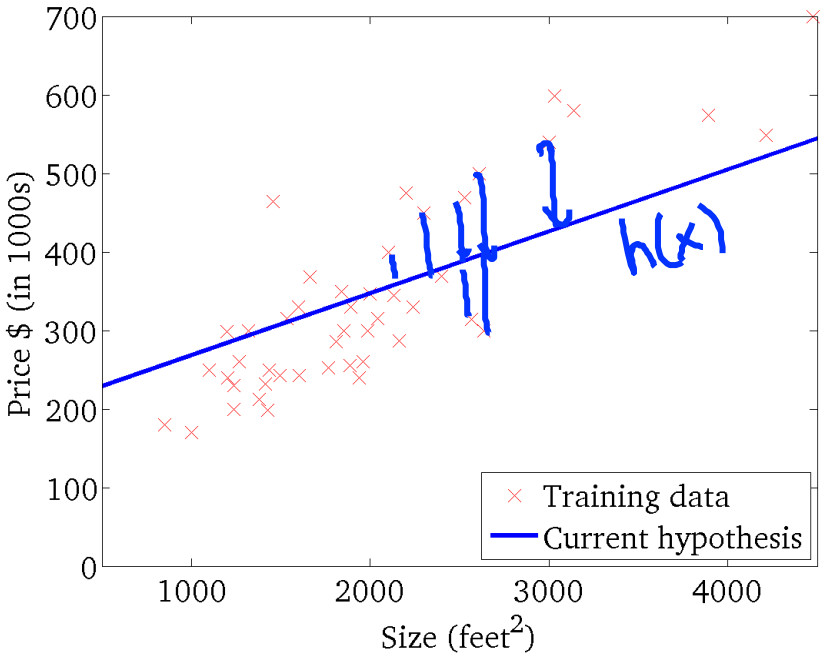
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



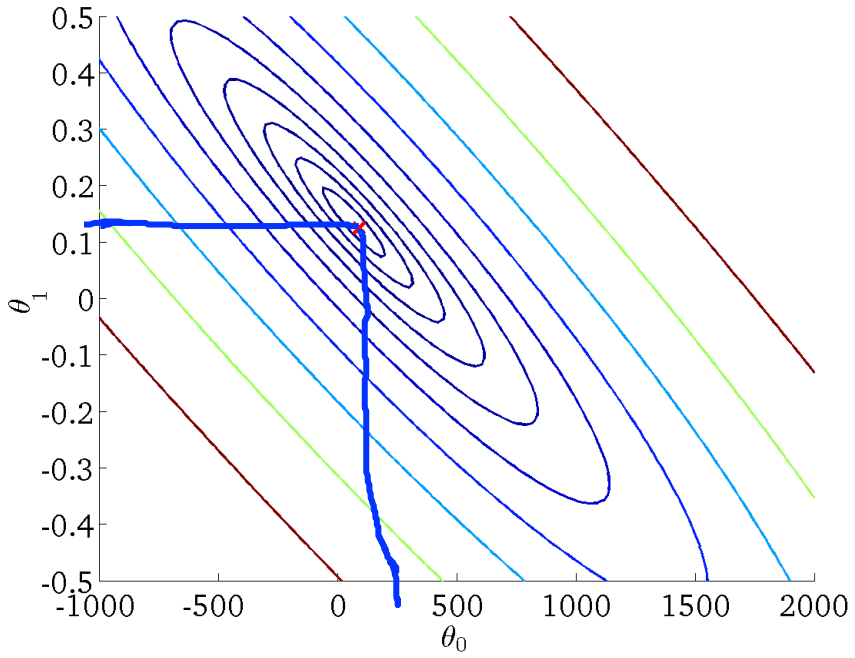
$h_\theta(x)$

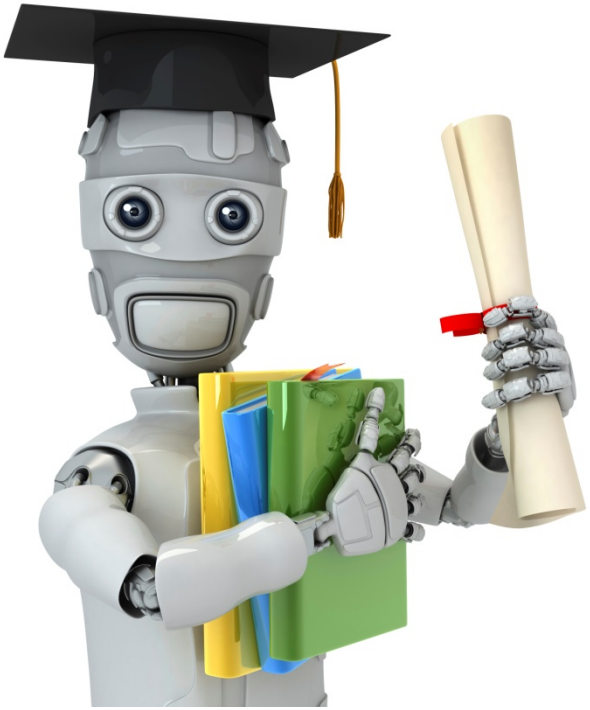
(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)





Machine Learning

Linear regression with one variable

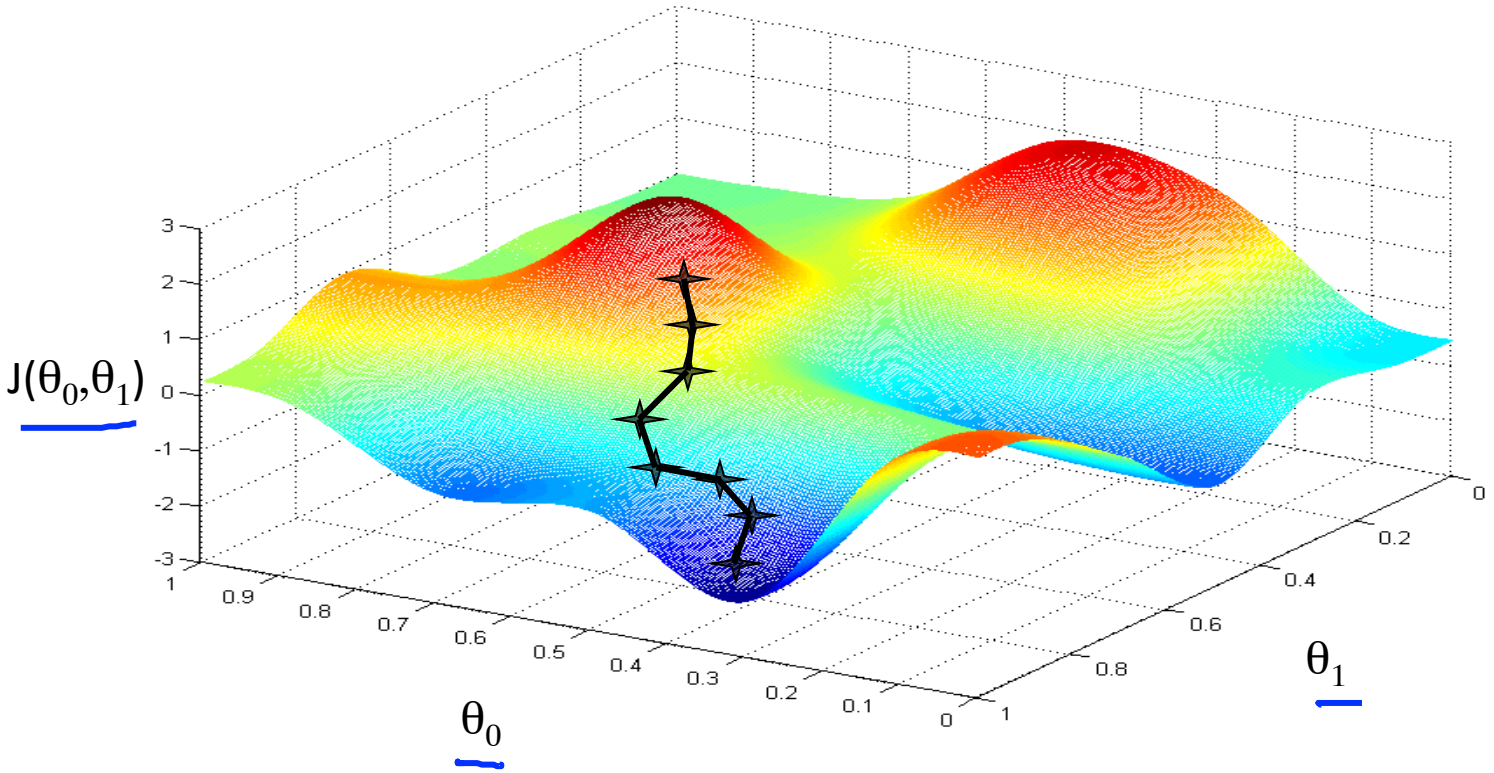
Gradient descent

Have some function $J(\theta_0, \theta_1)$

Want $\min_{\theta_0, \theta_1} J(\theta_0, \theta_1)$

Outline:

- Start with some θ_0, θ_1
- Keep changing θ_0, θ_1 to reduce $J(\theta_0, \theta_1)$ until we hopefully end up at a minimum


$J(\theta_0,\theta_1)$
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ_0
0
0.2
0.4
0.6
0.8
1
θ_1

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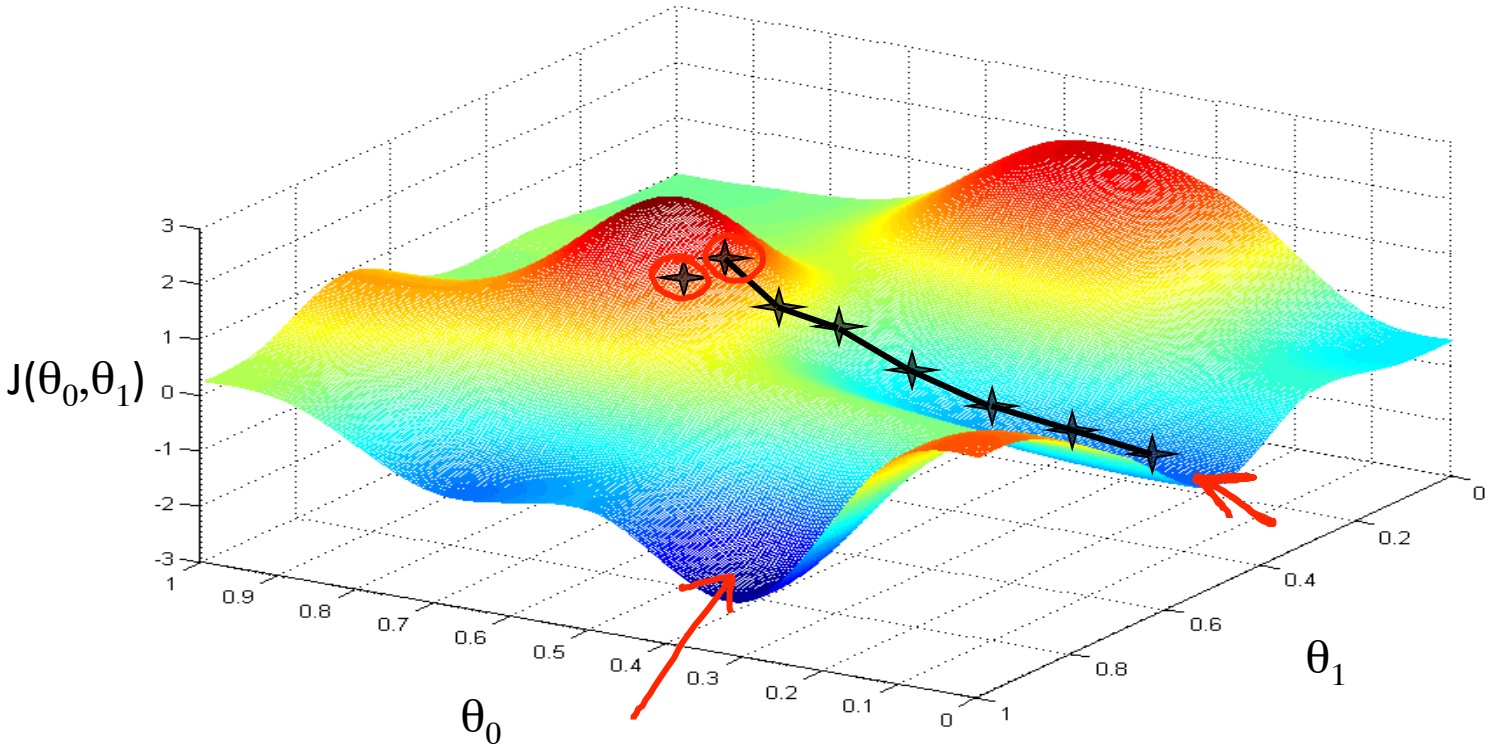
$J(\theta_0,\theta_1)$
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ_0
0
0.2
0.4
0.6
0.8
1
θ_1

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Gradient descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) \quad \text{(for } j = 0 \text{ and } j = 1\text{)}$$

}

learning rate

Simultaneously update θ_0 and θ_1

Assignment: $a := b$, $a := a+1$

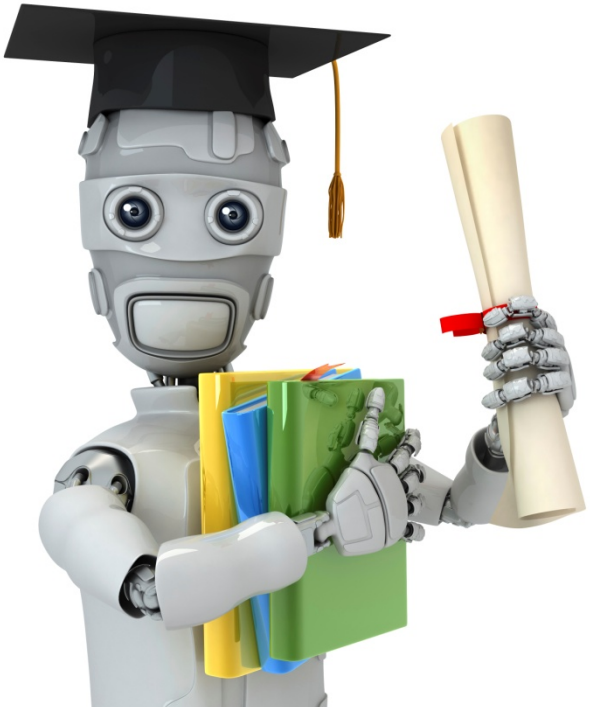
Truth assertion: $a = b$, $a = a+1$ ✗

Correct: Simultaneous update

$$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$
$$\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$
$$\theta_0 := \text{temp0}$$
$$\theta_1 := \text{temp1}$$

Incorrect:

$$\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$
$$\theta_0 := \text{temp0}$$
$$\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$
$$\theta_1 := \text{temp1}$$



Machine Learning

Linear regression with one variable

Gradient descent intuition

Gradient descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

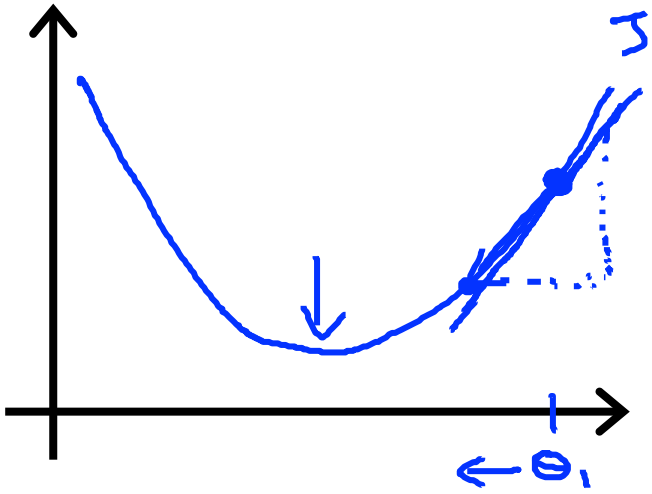
(simultaneously update $j = 0$ and $j = 1$)

}

learning rate

derivative

$\min_{\theta_1} J(\theta_1)$ $\quad \theta_1 \in \mathbb{R}$.

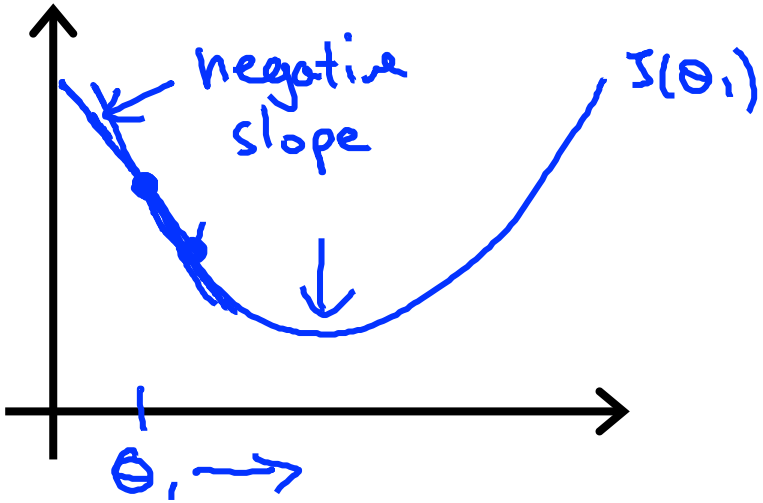


$(\theta_1 \in \mathbb{R})$

$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$

$\frac{d}{d\theta_1} J(\theta_1) \geq 0$

$\theta_1 := \theta_1 - \alpha \cdot$ (positive number)



$\frac{d}{d\theta_1} J(\theta_1) \leq 0$

$\theta_1 := \theta_1 - \alpha$ (negative number)

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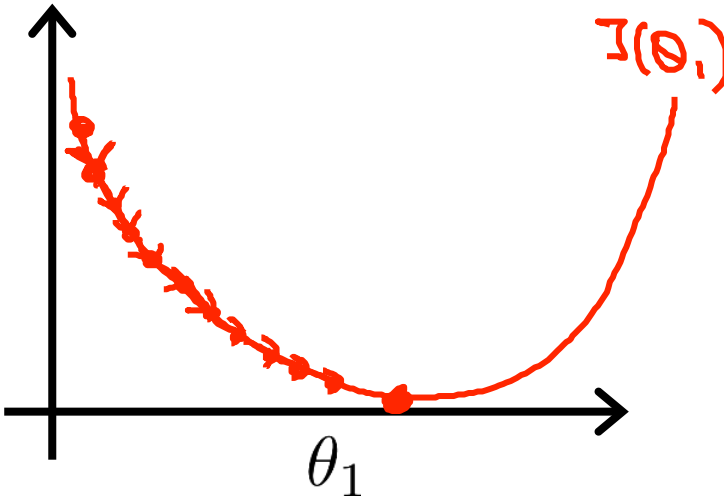
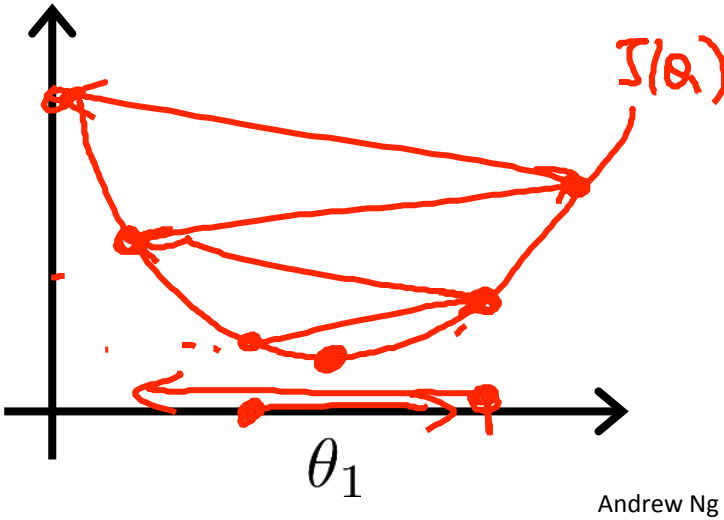
$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

If α is too small, gradient descent can be slow.



If α is too large, gradient descent can overshoot the minimum. It may fail to converge, or even diverge.



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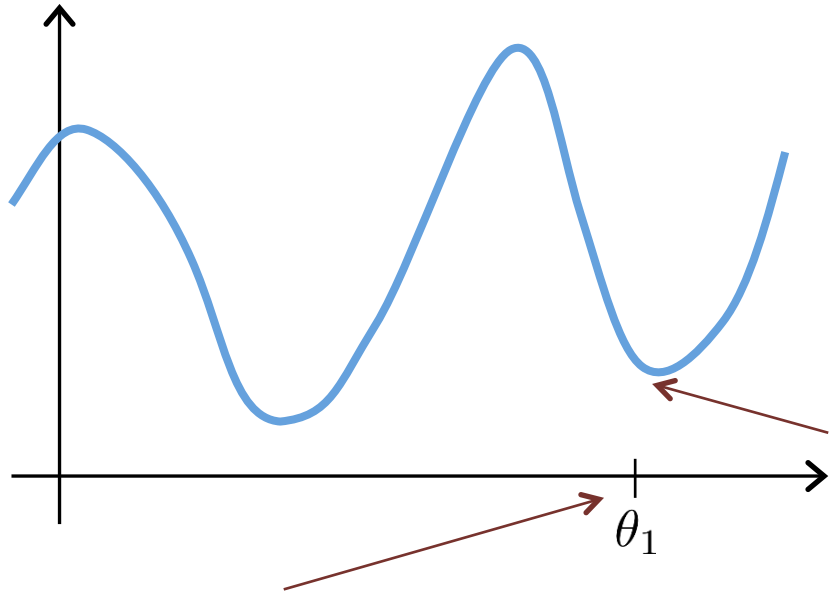
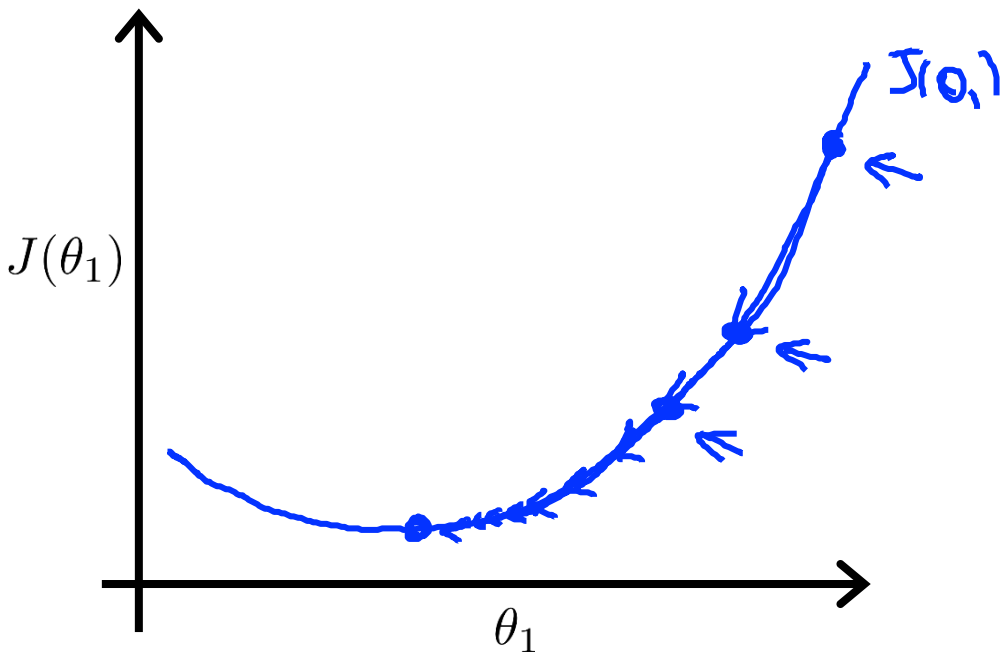


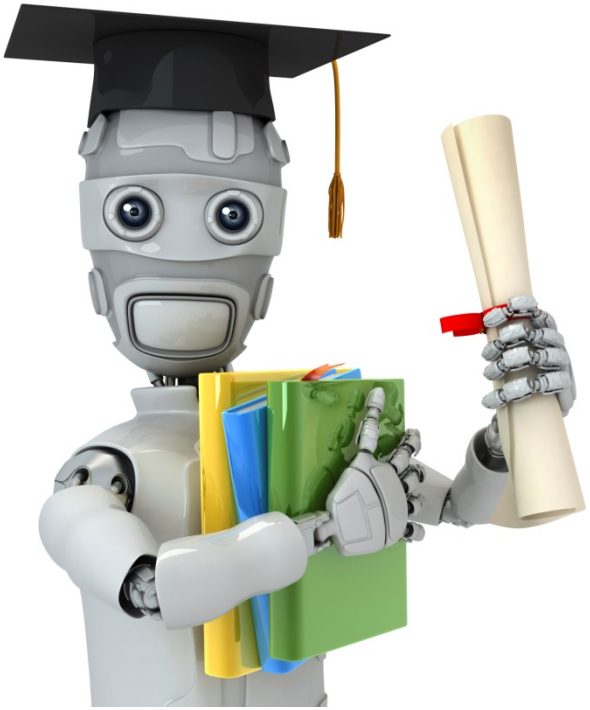
$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$$

Gradient descent can converge to a local minimum, even with the learning rate α fixed.

$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$$

As we approach a local minimum, gradient descent will automatically take smaller steps. So, no need to decrease α over time.





Machine Learning

Linear regression with one variable

Gradient descent for linear regression

Gradient descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

(for $j = 1$ and $j = 0$)

}

Linear Regression Model

$$h_\theta(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

$$\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) = \frac{\partial}{\partial \theta_j} \frac{1}{2m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)^2$$

$$= \frac{\partial}{\partial \theta_j} \frac{1}{2m} \sum_{i=1}^{m} \left(\theta_0 + \theta_1 x^{(i)} - y^{(i)} \right)^2$$

$$j = 0 : \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)$$

$$j = 1 : \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) \cdot x^{(i)}$$

Gradient descent algorithm

repeat until convergence {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)$$

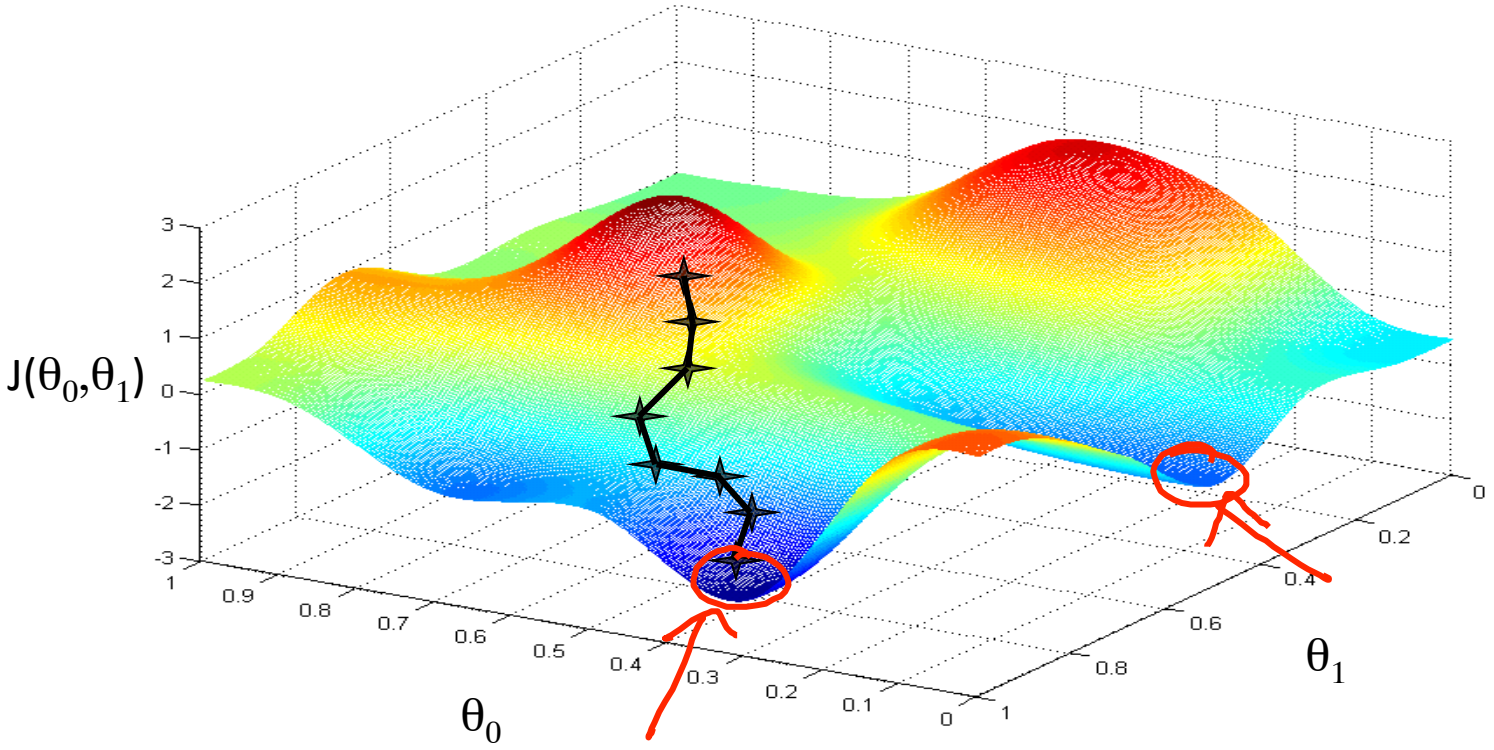
$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right) \cdot x^{(i)}$$

}

$\frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$

$\frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$

update θ_0 and θ_1 simultaneously


$J(\theta_0,\theta_1)$
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ_0
0
0.2
0.4
0.6
0.8
1
θ_1

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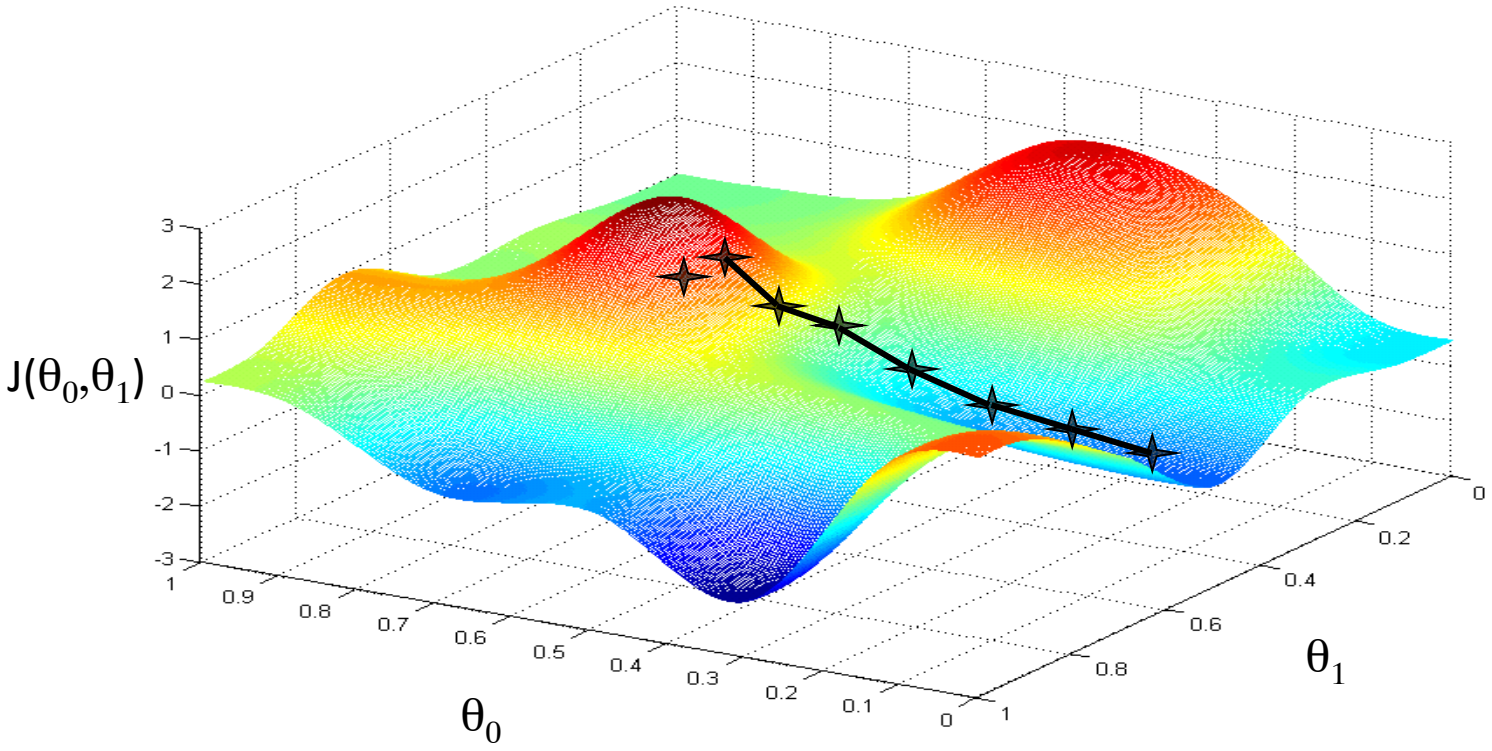
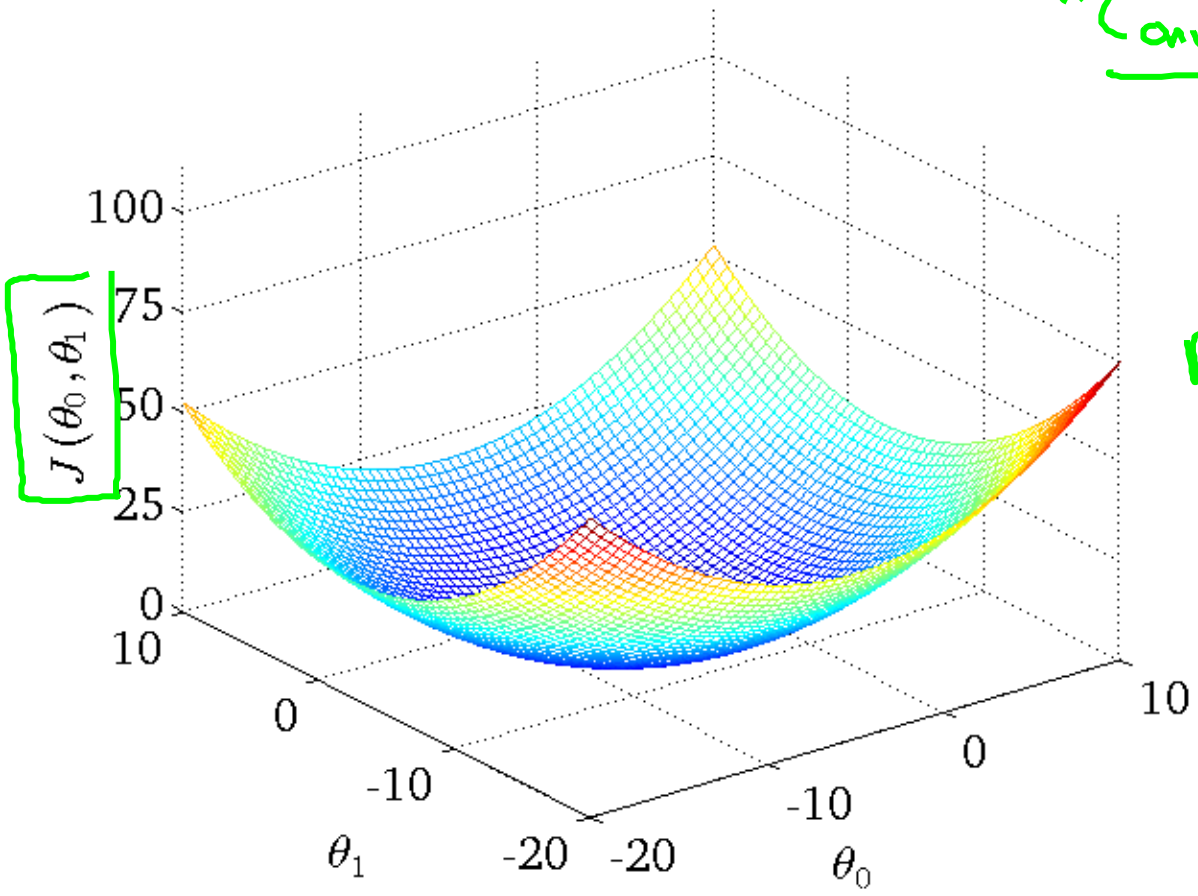
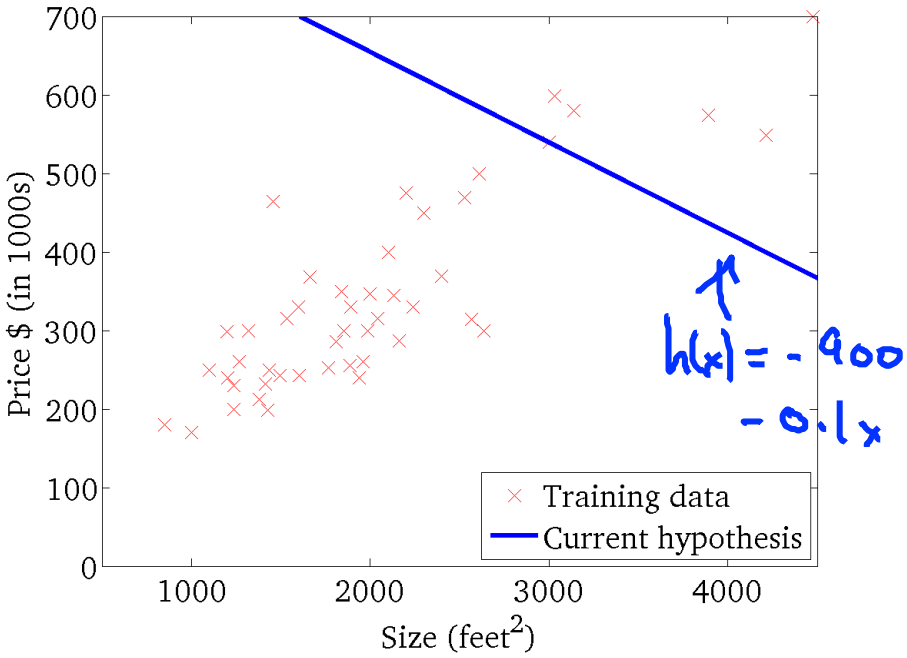

$J(\theta_0,\theta_1)$
3
2
1
0
-1
-2
-3
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
θ_0
0
0.2
0.4
0.6
0.8
1
θ_1

Andrew Ng


"Convex function"
Bowl-shaped
$J(\theta_0, \theta_1)$
100
75
50
25
0
10
0
-10
-20
θ_1
-20
-10
0
10
θ_0

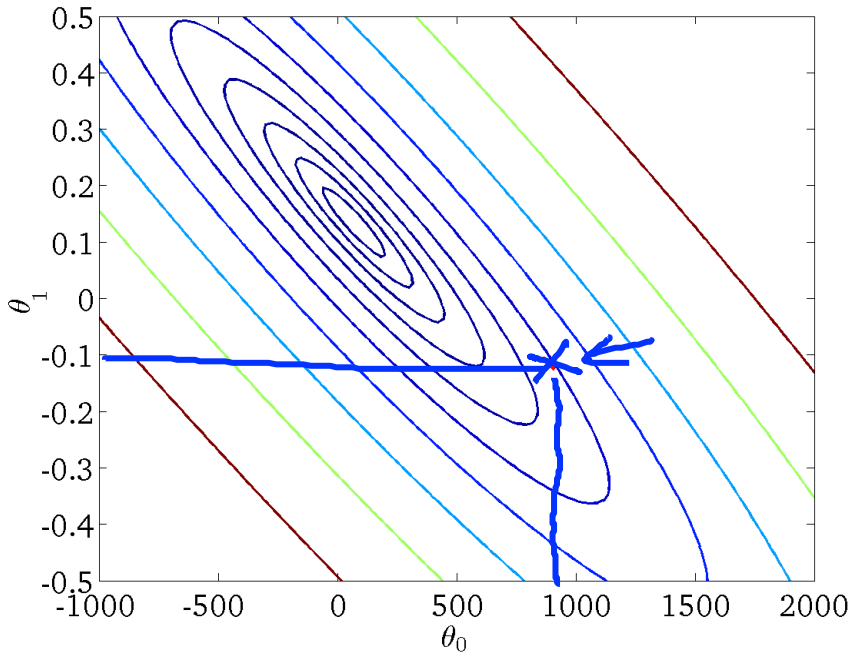
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



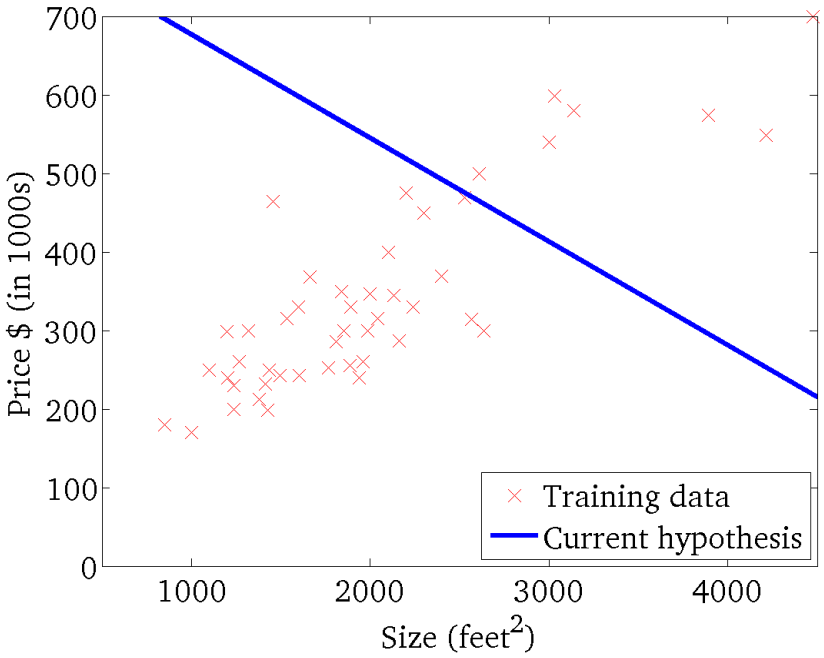
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



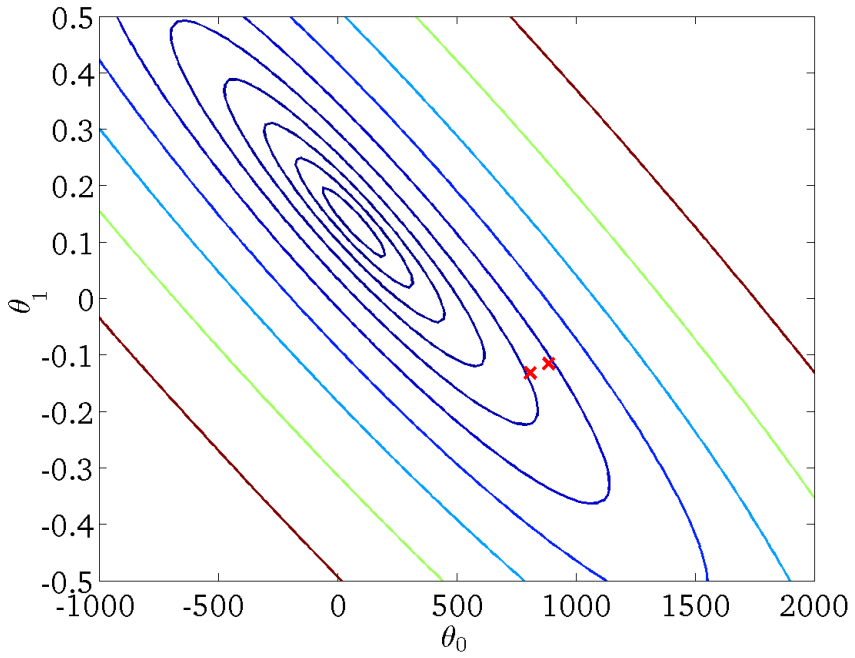
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



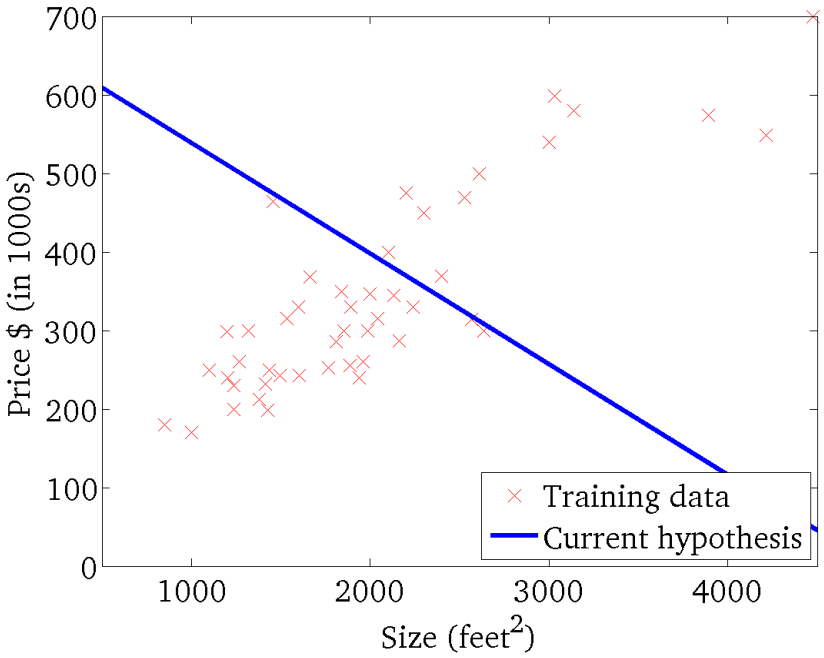
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



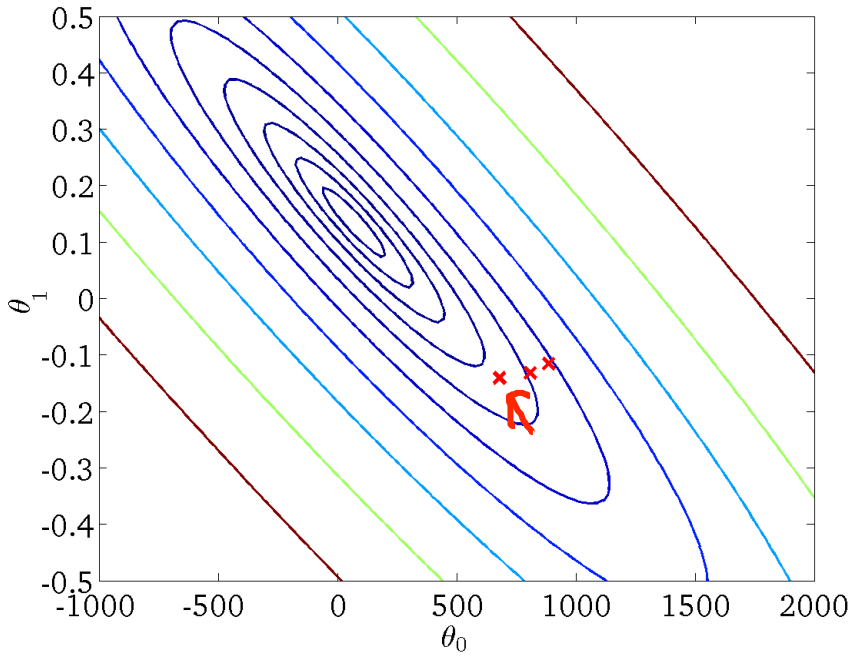
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



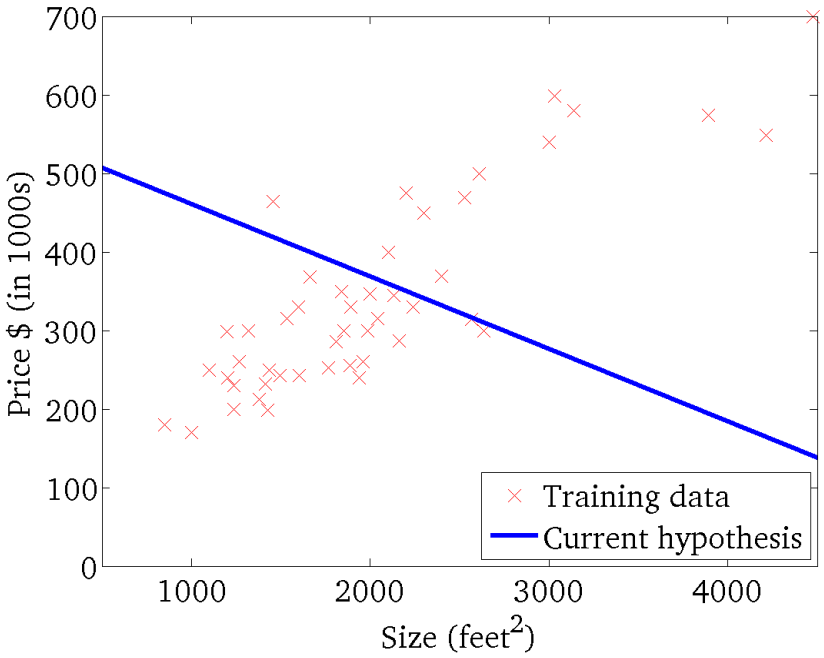
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



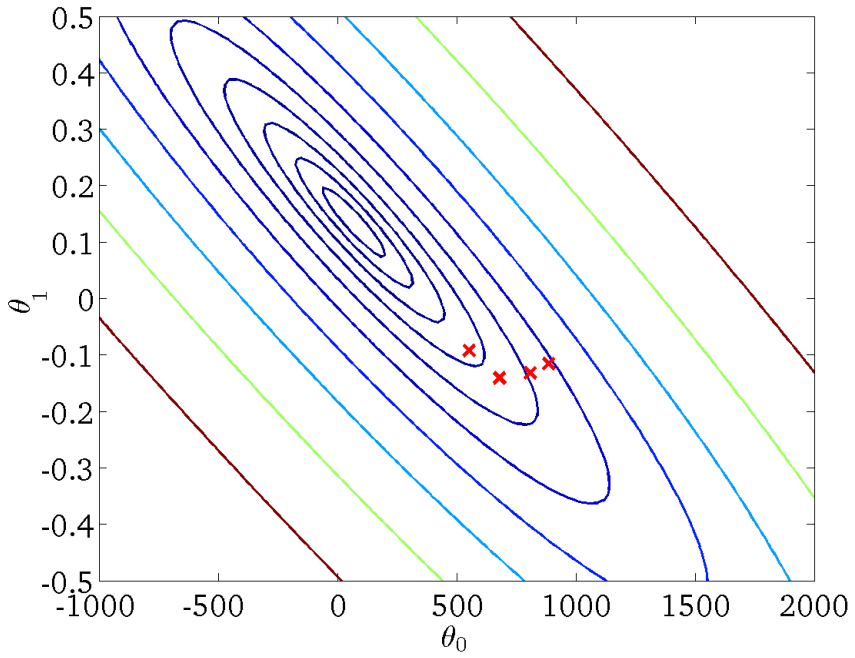
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



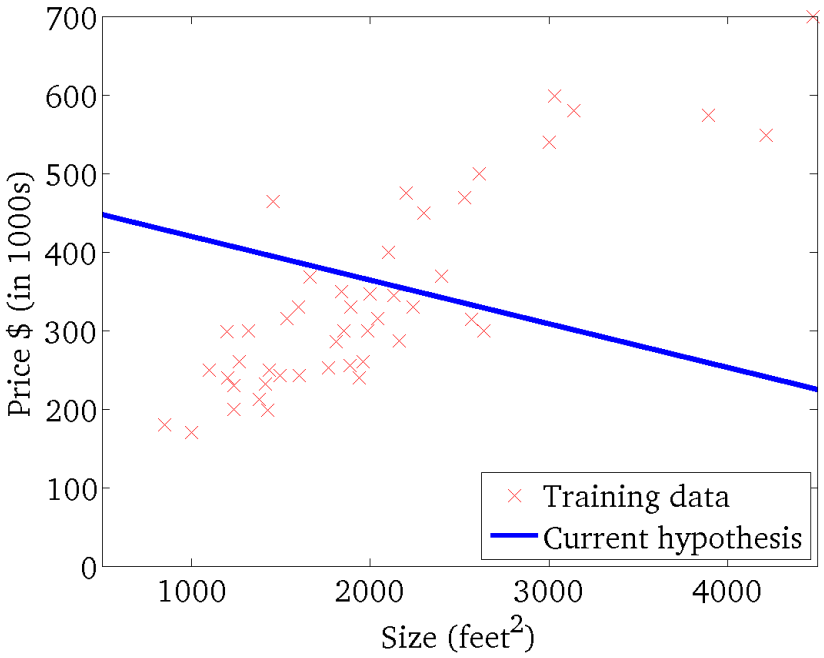
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



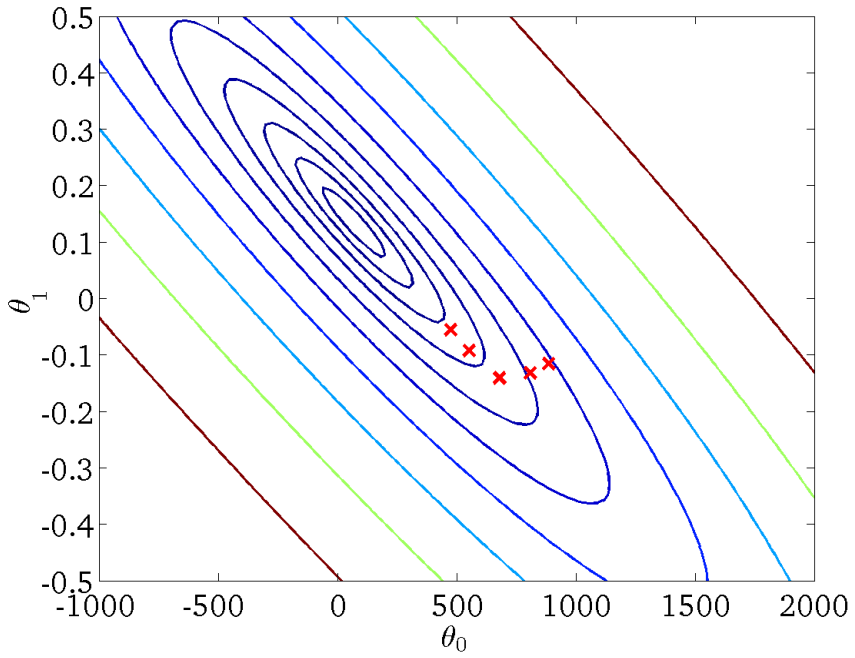
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



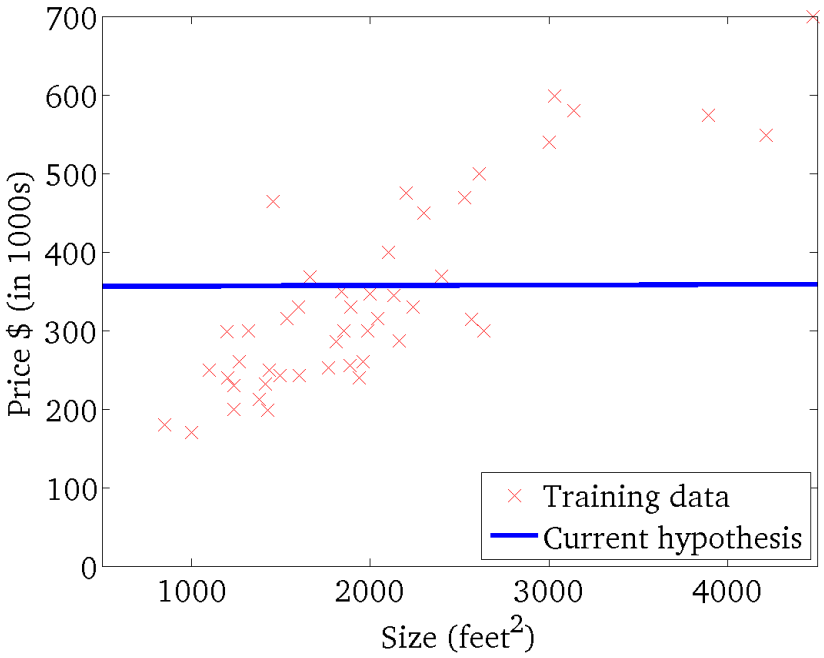
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



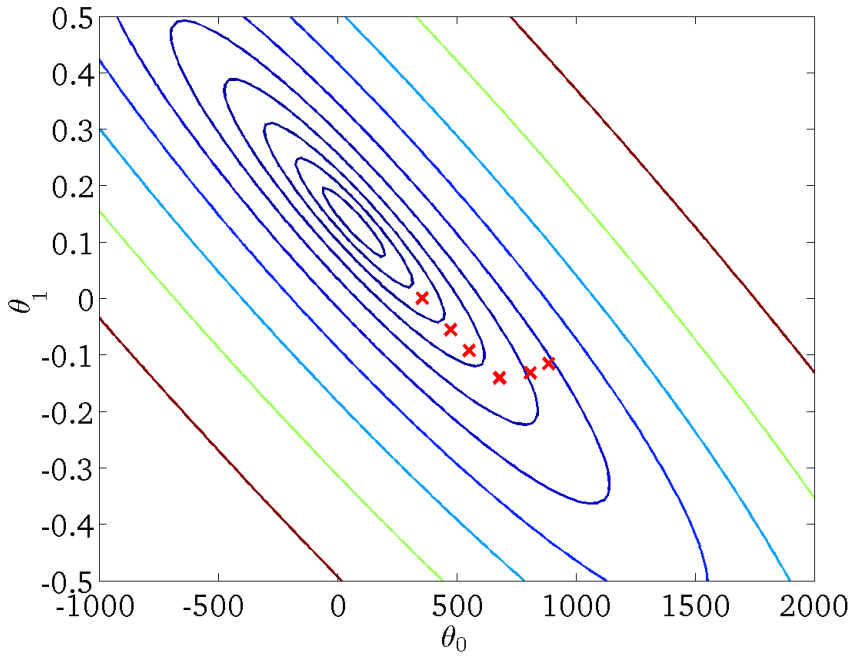
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



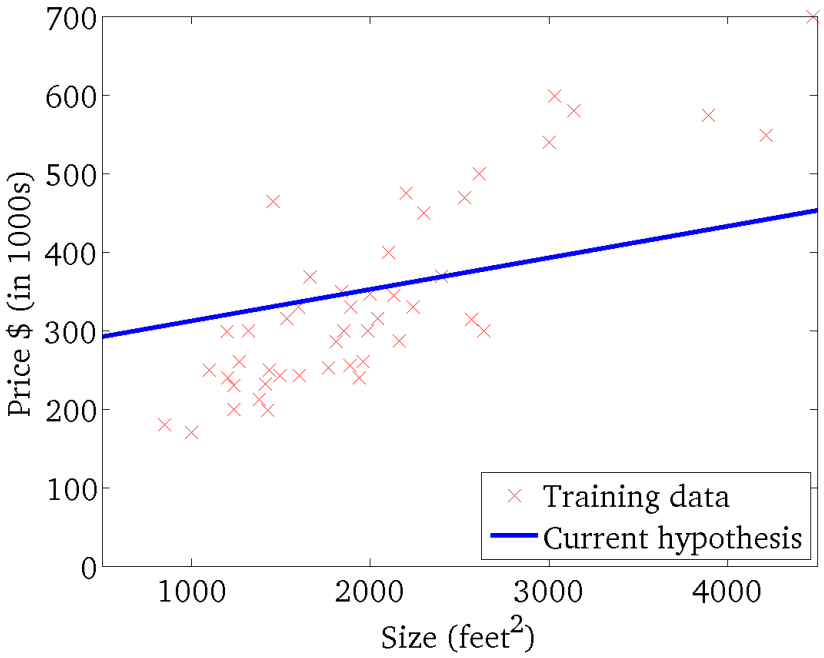
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



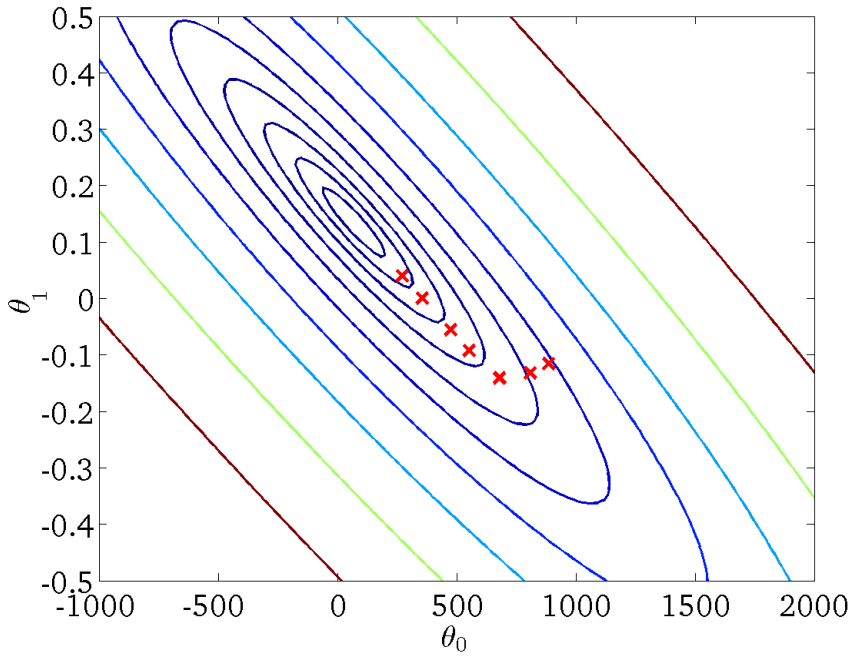
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



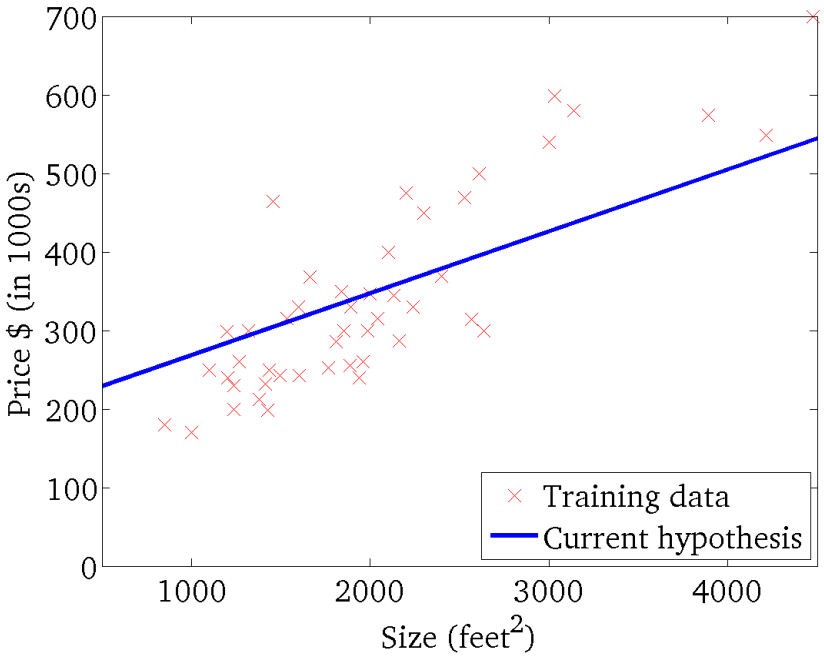
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



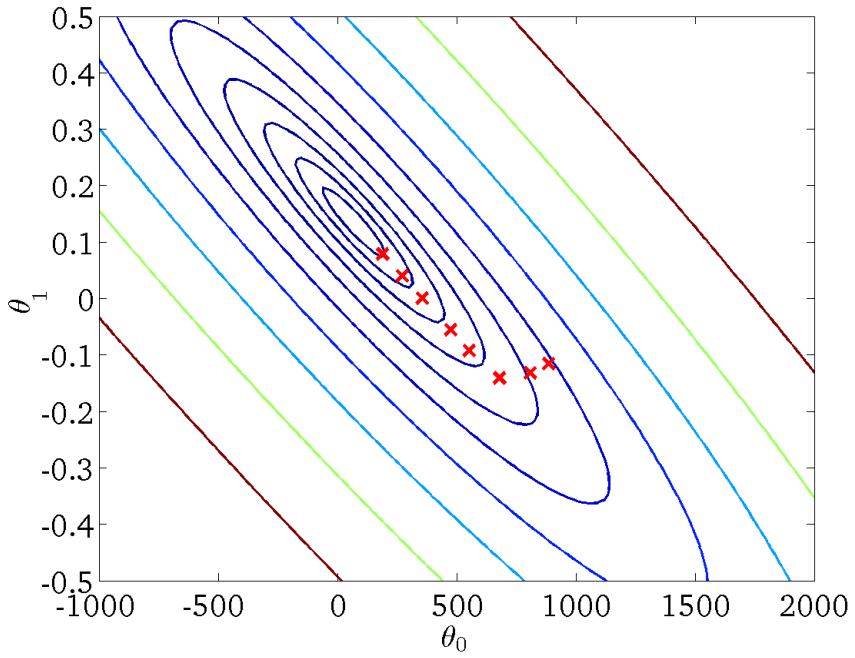
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



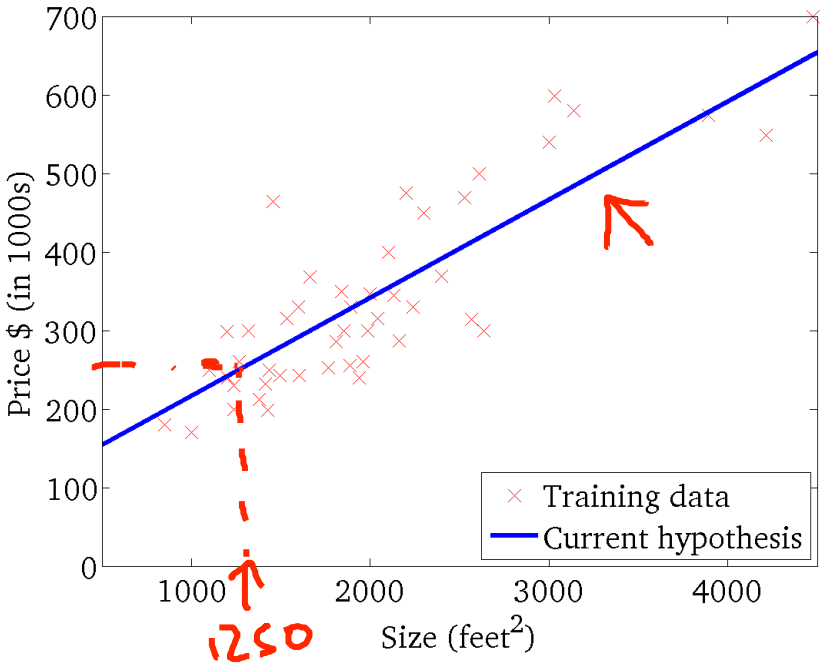
$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)



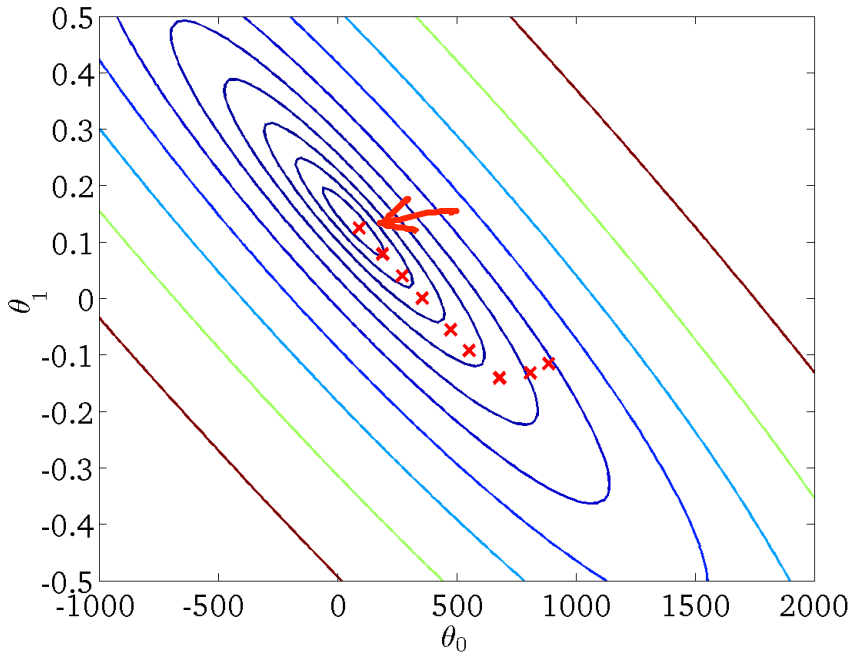
$h_\theta(x)$

(for fixed θ_0, θ_1, this is a function of x)



$J(\theta_0, \theta_1)$

(function of the parameters θ_0, θ_1)

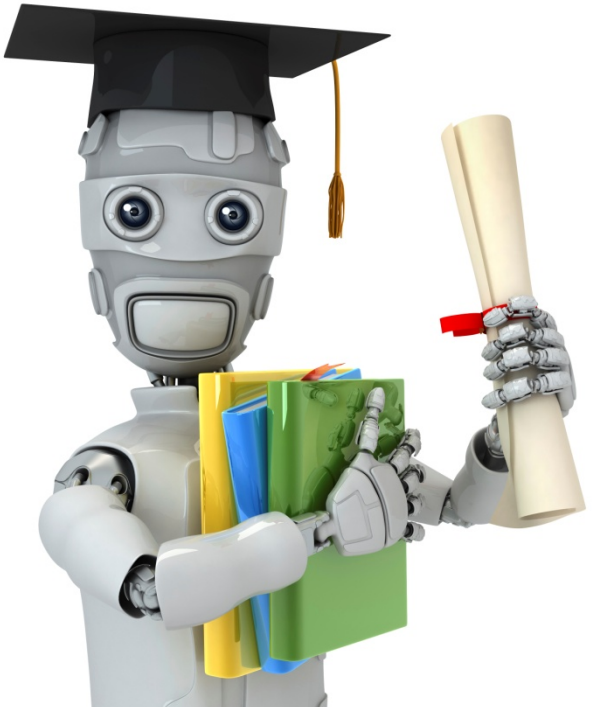


“Batch” Gradient Descent

“Batch”: Each step of gradient descent uses all the training examples.

$$\sum_{i=1}^{m} \left(h_\theta(x^{(i)}) - y^{(i)} \right)$$

1.3 Linear Algebra

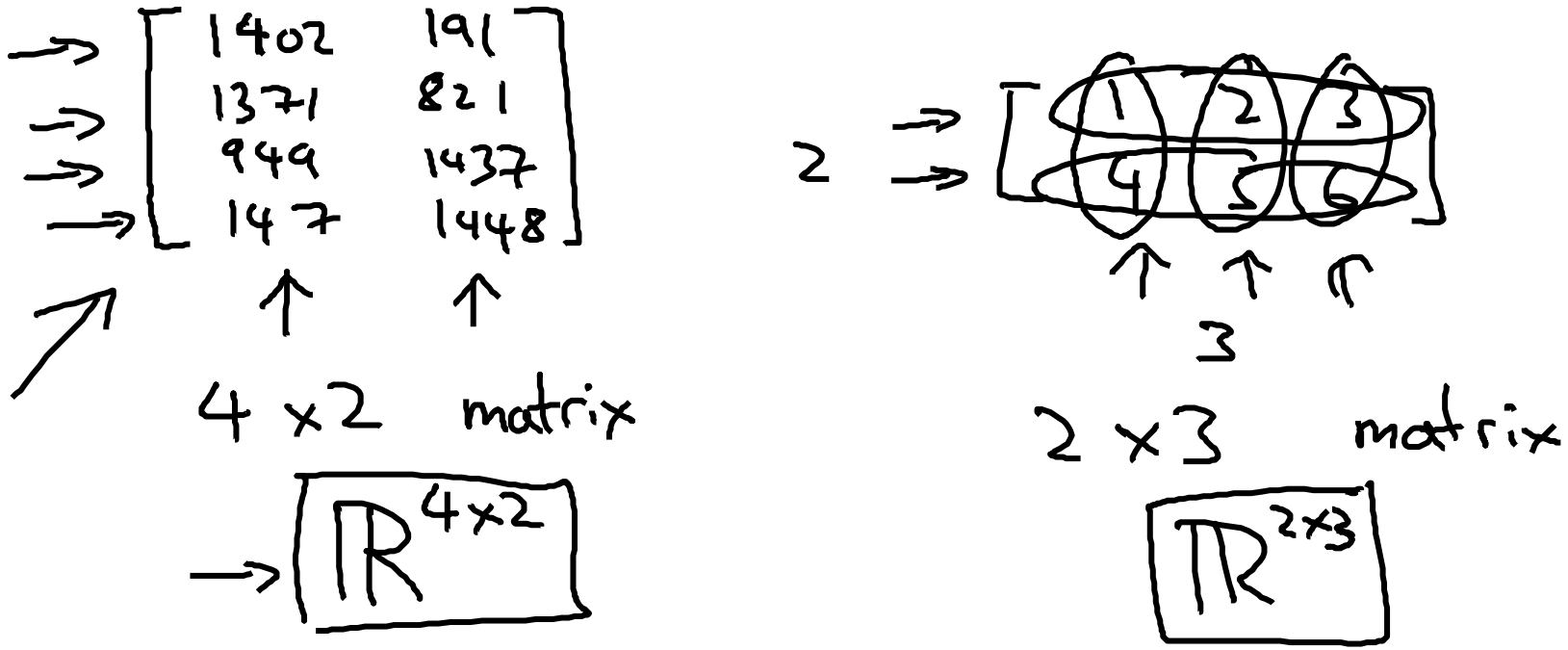


Machine Learning

Linear Algebra review (optional)

Matrices and vectors

Matrix: Rectangular array of numbers:



Dimension of matrix: number of rows x number of columns

Matrix Elements (entries of matrix)

$$A = \begin{bmatrix} 1402 & 191 \\ 1371 & 821 \\ 949 & 1437 \\ 147 & 1448 \end{bmatrix}$$

A_{ij} = "i, j entry" in the i^{th} row, j^{th} column.

$A_{11} = 1402$

$A_{12} = 191$

$A_{32} = 1437$

$A_{41} = 147$

A_{43} = undefined (error)

Vector: An n x 1 matrix.

$$y = \begin{bmatrix} 460 \\ 232 \\ 315 \\ 178 \end{bmatrix}$$

$n = 4$

← 4-dimensional vector. $\mathbb{R}^4$

$y_i = i^{th}$ element

$y_1 = 460$

$y_2 = 232$

$y_3 = 315$

A, B, C, X

a, b, x, y

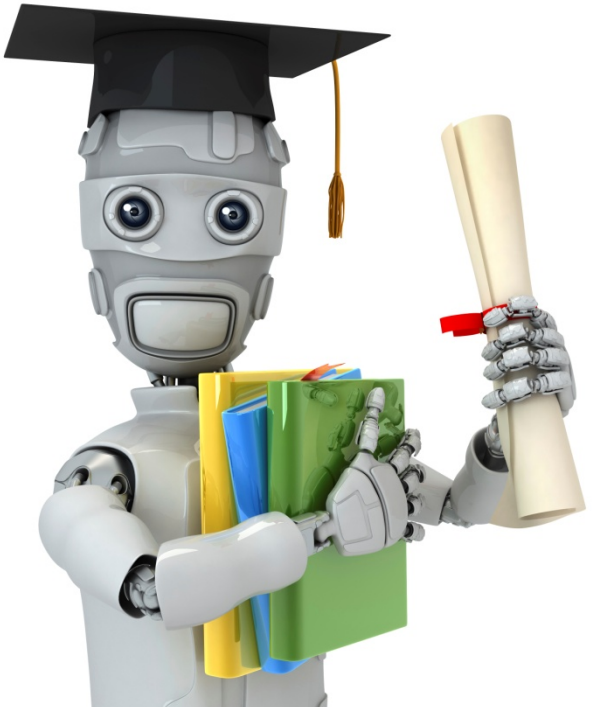
1-indexed vs 0-indexed:

$$y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

1-indexed

$$y = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

0-indexed

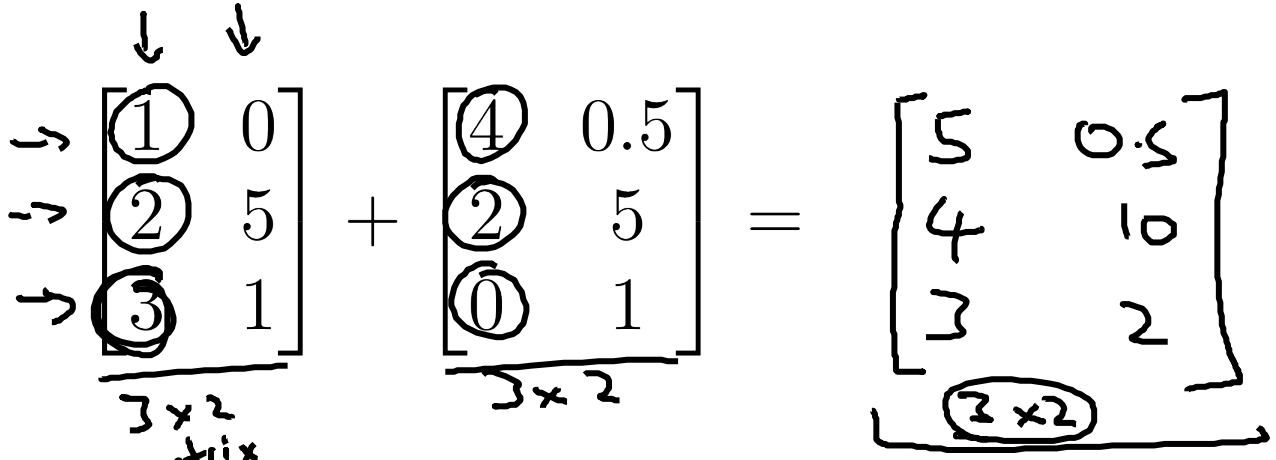


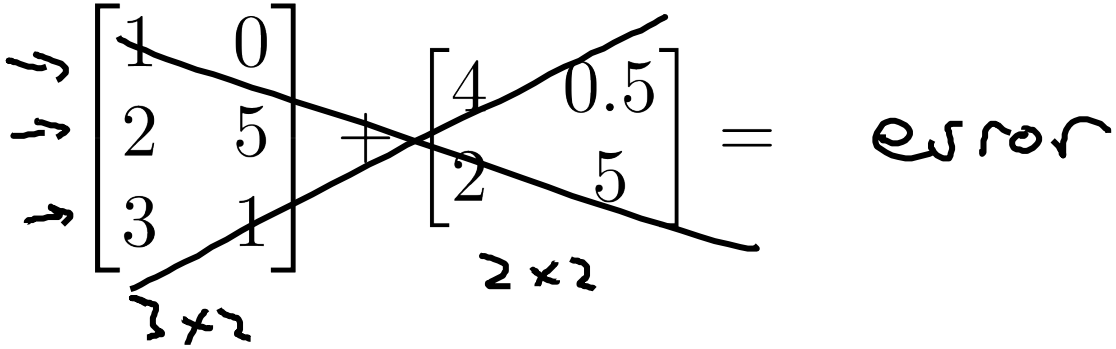
Machine Learning

Linear Algebra review (optional)

Addition and scalar multiplication

Matrix Addition





Scalar Multiplication

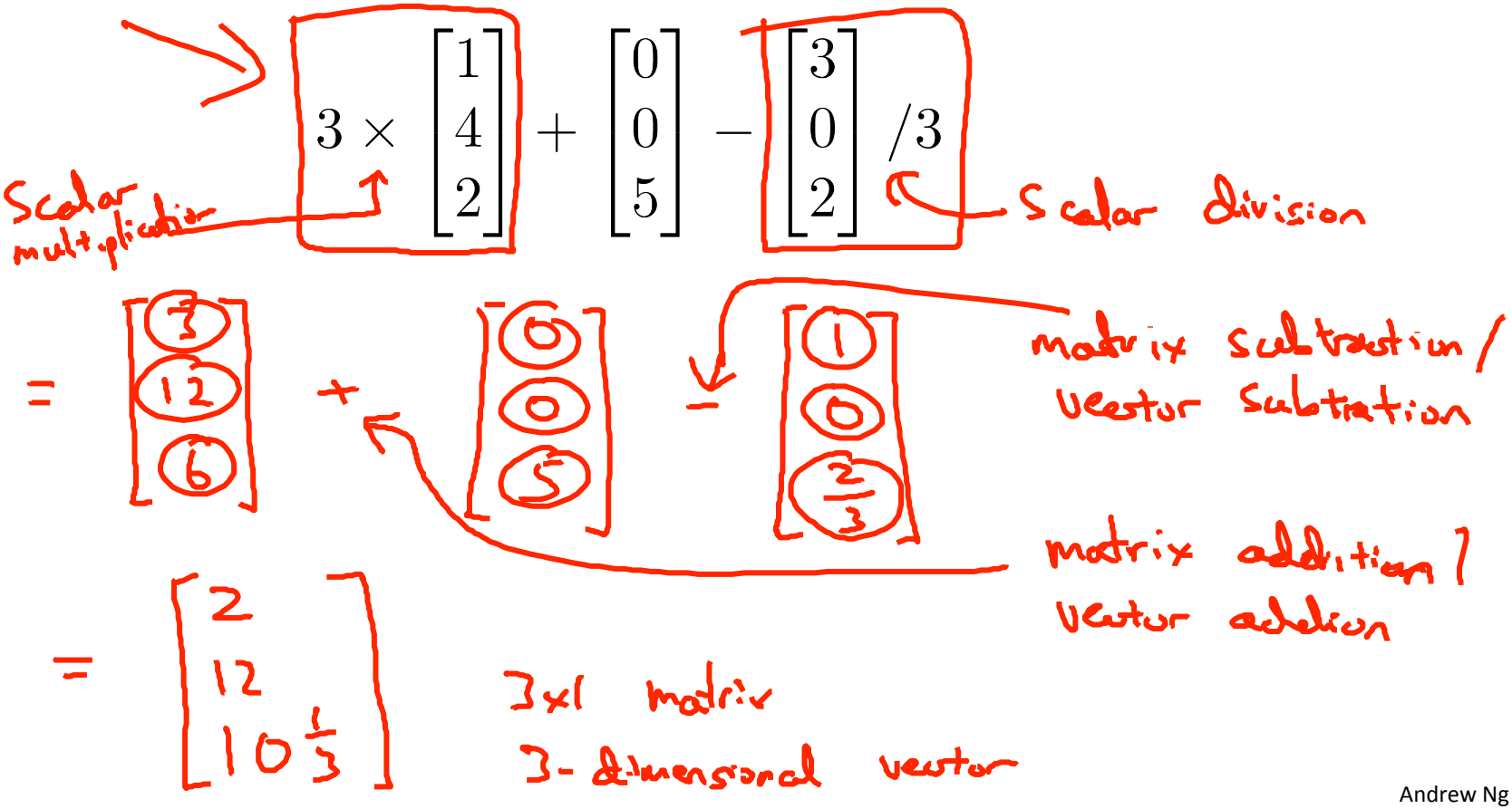
real number

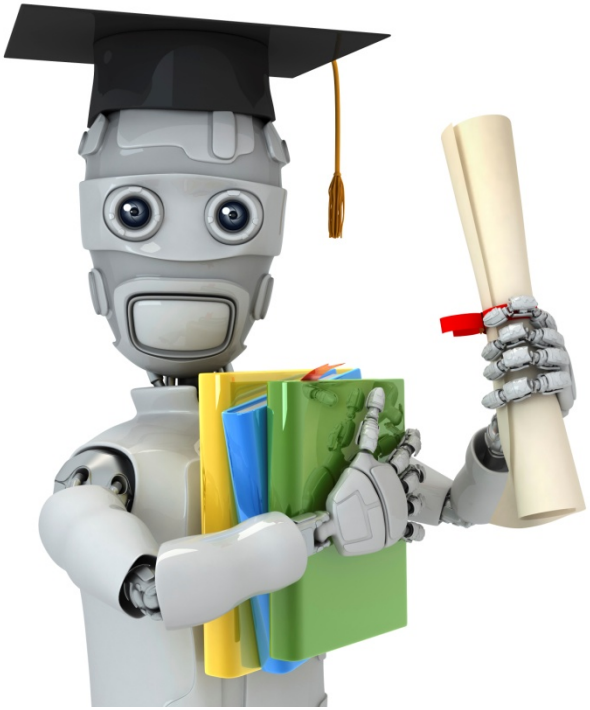
$$3 \times \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 6 & 15 \\ 9 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} \times 3$$

3×2 $\qquad$ 3×2

$$\begin{bmatrix} 4 & 0 \\ 6 & 3 \end{bmatrix} /4 = \frac{1}{4} \begin{bmatrix} 4 & 0 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{3}{2} & \frac{3}{4} \end{bmatrix}$$

Combination of Operands





Machine Learning

Linear Algebra review (optional)

Matrix-vector multiplication

Example

$$\begin{bmatrix} 1 & 3 \\ 4 & 0 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \end{bmatrix} = \begin{bmatrix} 16 \\ 4 \\ 7 \end{bmatrix}$$

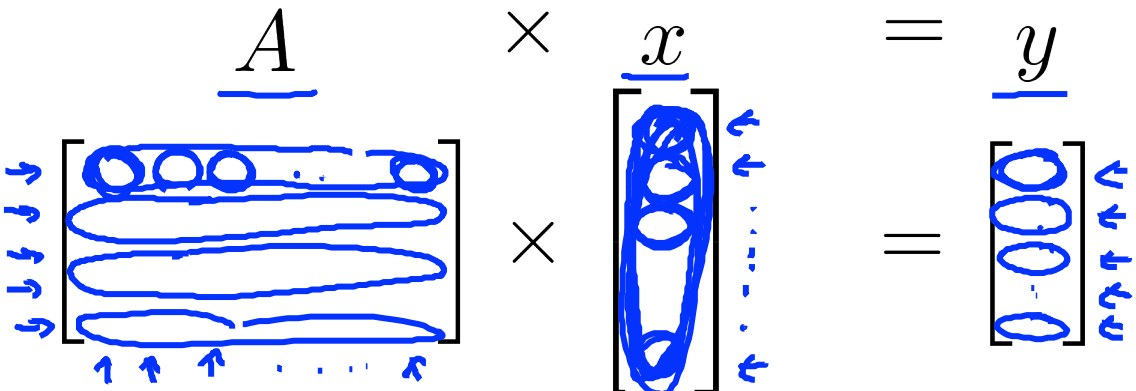
3×2 $\quad 2 \times 1$ $\quad 3 \times 1$ matrix

$1 \times 1 + 3 \times 5 = 16$

$4 \times 1 + 0 \times 5 = 4$

$2 \times 1 + 1 \times 5 = 7$

Details:



To get y_i, multiply A's i^{th} row with elements of vector x, and add them up.

Example

$$\begin{bmatrix} 1 & 2 & 1 & 5 \\ 0 & 3 & 0 & 4 \\ -1 & -2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ -7 \end{bmatrix}$$

3×4 $\quad$ 4×1 $\quad$ 3×1

$1 \times 1 + 2 \times 3 + 1 \times 2 + 5 \times 1 = 14$

$0 \times 1 + 3 \times 3 + 0 \times 2 + 4 \times 1 = 13$

$-1 \times 1 + (-2) \times 3 + 0 \times 2 + 0 \times 1 = -7$

House sizes:

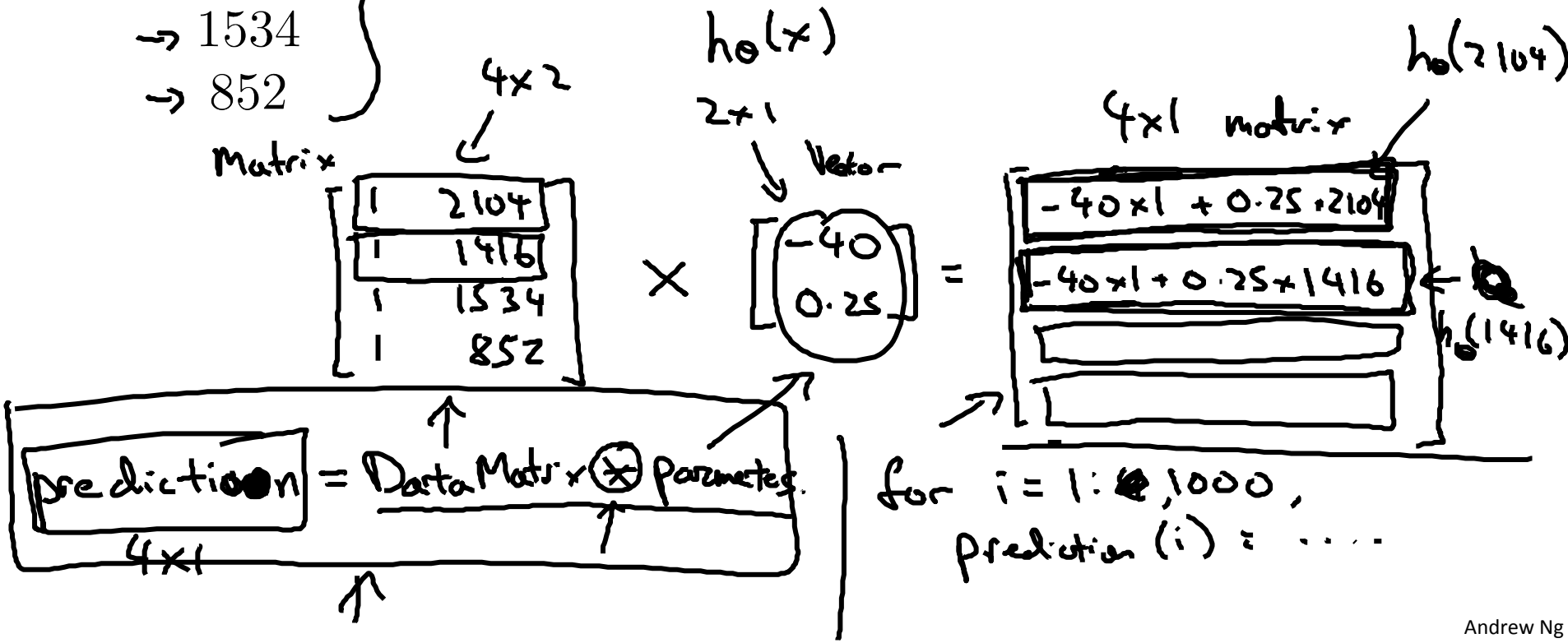
→ 2104

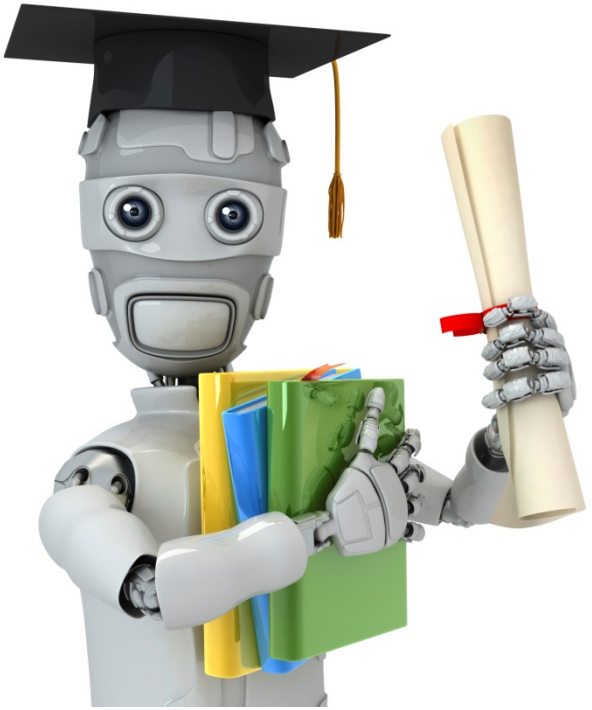
→ 1416

→ 1534

→ 852

$$h_\theta(x) = -40 + 0.25x$$





Machine Learning

Linear Algebra review (optional)

Matrix-matrix multiplication

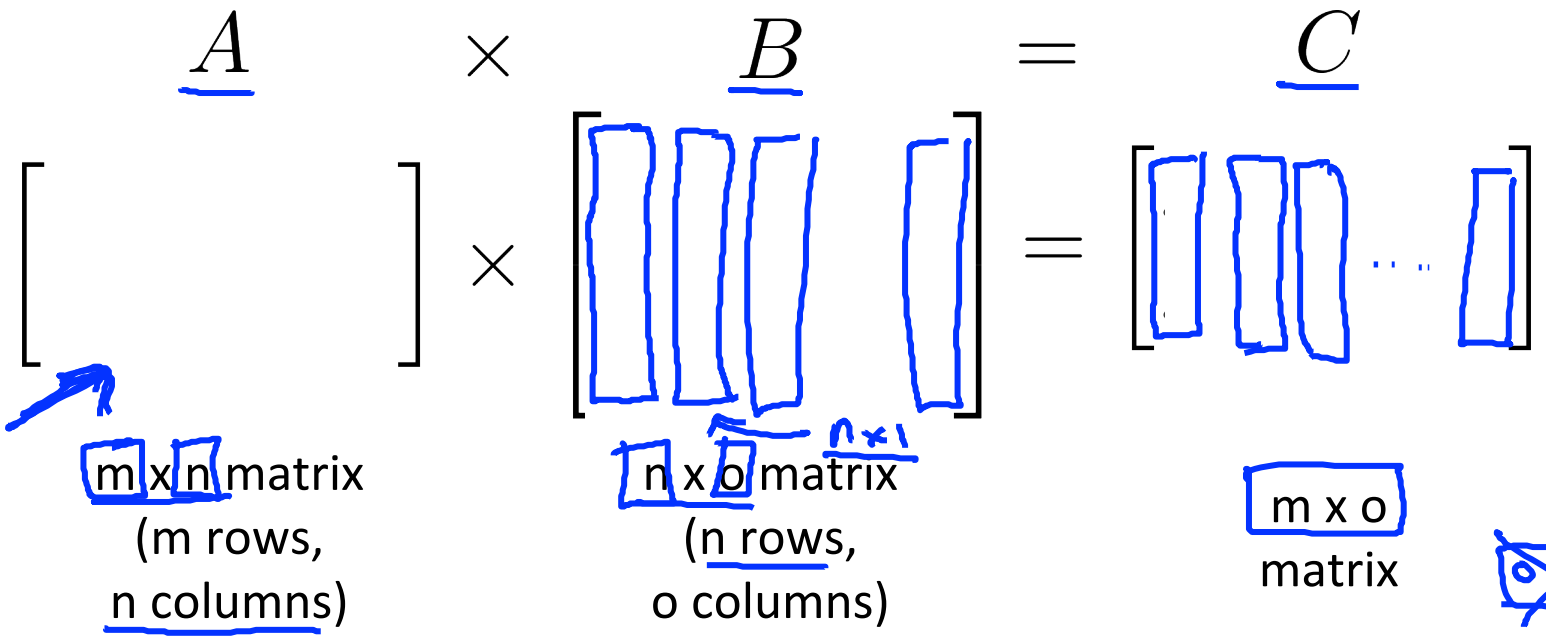
Example

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 11 & 10 \\ 9 & 14 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 11 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 \\ 14 \end{bmatrix}$$

Details:



The i^{th} column of the matrix C is obtained by multiplying A with the i^{th} column of B. (for i = 1,2,…,o)

Example

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 9 & 7 \\ 15 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 1\times 0 + 3\times 3 \\ 2\times 0 + 5\times 3 \end{bmatrix} = \begin{bmatrix} 9 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1\times 1 + 3\times 2 \\ 2\times 1 + 5\times 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \end{bmatrix}$$

House sizes:

$$\begin{Bmatrix} 2104 \\ 1416 \\ 1534 \\ 852 \end{Bmatrix}$$

Have 3 competing hypotheses:

1. $h_\theta(x) = -40 + 0.25x$
2. $h_\theta(x) = 200 + 0.1x$ ←
3. $h_\theta(x) = -150 + 0.4x$

Matrix

$$\begin{bmatrix} 1 & 2104 \\ 1 & 1416 \\ 1 & 1534 \\ 1 & 852 \end{bmatrix}$$

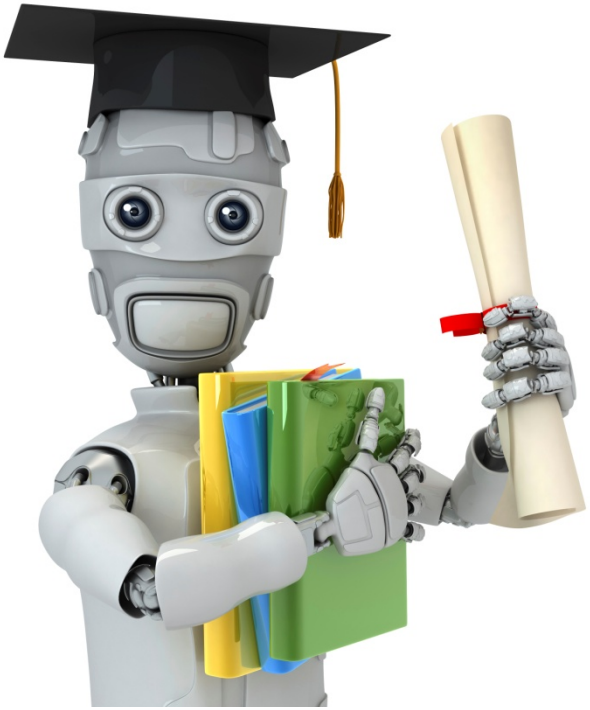
$\times$ Matrix

$$\begin{bmatrix} -40 & 200 & -150 \\ 0.25 & 0.1 & 0.4 \end{bmatrix} =$$

$$\begin{bmatrix} 486 & 410 & 692 \\ 314 & 342 & 416 \\ 344 & 353 & 464 \\ 173 & 285 & 191 \end{bmatrix}$$

Prediction of first ho

Predictions of 2nd ho



Machine Learning

Linear Algebra review (optional)

Matrix multiplication properties

$3 \times 5 = 5 \times 3$ "Commutative"

Let A and B be matrices. Then in general,

$A \times B \neq B \times A$. (not commutative.)

E.g.

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix} \neq \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}$$

$A \times B$

$m \times n \quad n \times m$

$A \times B$ is $m \times m$

$B \times A$ is $n \times n$

$3 \times 5 \times 2$

$3 \times 10 = 30 =$ $3 \times (5 \times 2) = (3 \times 5) \times 2$ 15×2

"Associative"

$A \times (B \times C)$

$(A \times B) \times C$

$A \times B \times C.$

Let $D = B \times C.$ Compute $A \times D.$

Let $E = A \times B.$ Compute $E \times C.$

$A \times (B \times C)$

$(A \times B) \times C$

Same answer.

Identity Matrix

1 is identity

$1 \times z = z \times 1 = z$ for any z

Denoted I (or $I_{n \times n}$).

Examples of identity matrices:

$[1]$ 1 x 1

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ 2 x 2

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ 3 x 3

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$ 4 x 4

Informally:

For any matrix A,

$$A \cdot I = I \cdot A = A$$

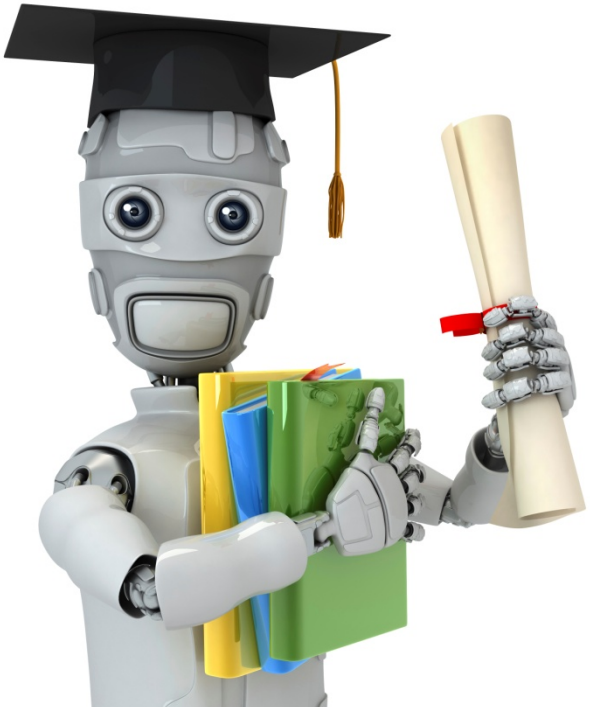
$I_{n \times n}$

$m \times n$ $\quad n \times n$ $\quad m \times m$ $\quad m \times n$ $\quad m \times n$

Note:

$AB \neq BA$ in general

$AI = IA$ ✓



Machine Learning

Linear Algebra review (optional)

Inverse and transpose

1 = "identity"

$3 \times (3^{-1}) = 1$, $3^{-1} = \frac{1}{3}$

$12 \times (12^{-1}) = 1$, $12^{-1} = \frac{1}{12}$

$0 \times (0^{-1})$ undefined

Not all numbers have an inverse.

Matrix inverse:

If A is an m x m matrix, and if it has an inverse,

square matrix (# rows = # columns)

A^{-1}

$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

$$A(A^{-1}) = A^{-1}A = I.$$

E.g. $\begin{bmatrix} 3 & 4 \\ 2 & 16 \end{bmatrix}$ $\begin{bmatrix} 0.4 & -0.1 \\ -0.05 & 0.075 \end{bmatrix}$ $= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_{2\times 2}$

2x2

A A^{-1} $A^{-1}A$

Matrices that don't have an inverse are "singular" or "degenerate"

Matrix Transpose

Example: $A = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 5 & 9 \end{bmatrix}$ $B = A^T = \begin{bmatrix} 1 & 3 \\ 2 & 5 \\ 0 & 9 \end{bmatrix}$

Let A be an m x n matrix, and let $B = A^T$.

Then B is an n x m matrix, and

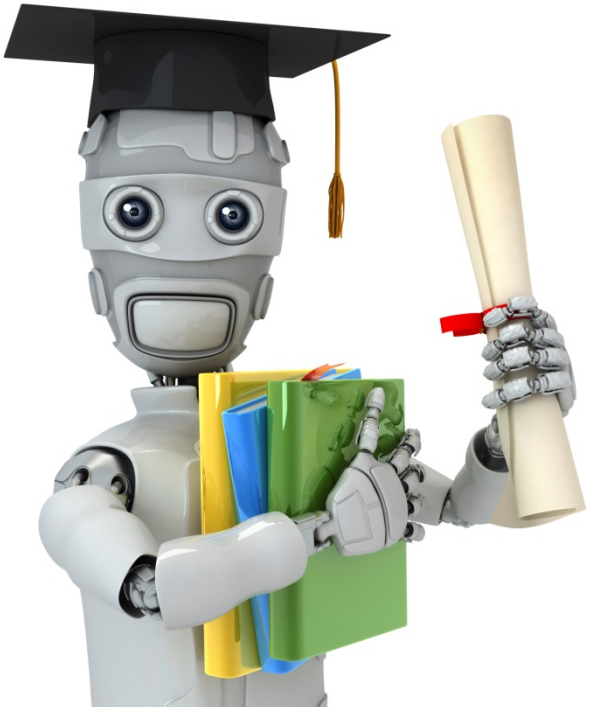
$$B_{ij} = A_{ji}.$$

$B_{12} = A_{21} = 2$

$B_{32} = 9$ $A_{23} = 9$.

Chapter 2 Week2

2.1 Multiple Featuress



Machine Learning

Linear Regression with multiple variables

Multiple features

Multiple features (variables).

Size (feet^2)	Price ($1000)
x	y
2104	460
1416	232
1534	315
852	178
…	…

$$h_\theta(x) = \theta_0 + \theta_1 x$$

Multiple features (variables).

Size (feet^2) x_1	Number of bedrooms x_2	Number of floors x_3	Age of home (years) x_4	Price ($1000) y
2104	5	1	45	460
1416	3	2	40	232
1534	3	2	30	315
852	2	1	36	178
…	…	…	…	…

$m = 47$

$x^{(2)} = \begin{bmatrix} 1416 \\ 3 \\ 2 \\ 40 \end{bmatrix}$

Notation:

- n = number of features $n = 4$
- $x^{(i)}$ = input (features) of i^{th} training example.
- $x_j^{(i)}$ = value of feature j in i^{th} training example.

$x_3^{(2)} = 2$

Hypothesis:

Previously: $h_\theta(x) = \theta_0 + \theta_1 x$

$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \theta_4 x_4$

e.g. $h_\theta(x) = 80 + 0.1 x_1 + 0.01 x_2 + 3 x_3 - 2 x_4$

x_2: blood pressure; x_3: family history; x_4: age

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$$

For convenience of notation, define $x_0 = 1$. $(x_0^{(i)} = 1)$

$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^{n+1} \qquad \theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \in \mathbb{R}^{n+1}$$

$$h_\theta(x) = \theta_0 x_0 + \theta_1 x_1 + \cdots + \theta_n x_n \quad (x_0 = 1)$$

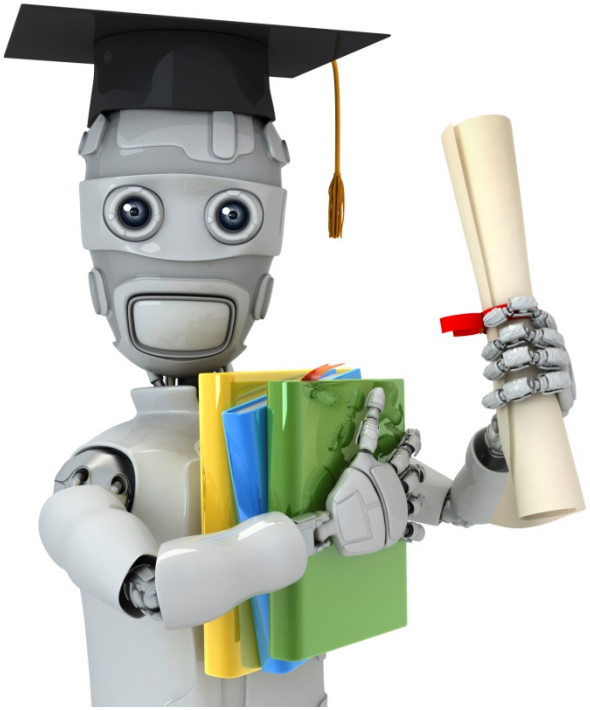
$$= \theta^T x$$

θ^T: $(n+1) \times 1$ matrix

$\theta^T x$

$$x = \begin{bmatrix} x_0 \\ x_1 \\ \vdots \\ x_n \end{bmatrix}$$

Multivariate linear regression.



Machine Learning

Linear Regression with multiple variables

Gradient descent for multiple variables

Hypothesis: $h_\theta(x) = \theta^T x = \theta_0 x_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$ ($x_0 = 1$)

Parameters: $\theta_0, \theta_1, \ldots, \theta_n$ (θ) — $n+1$-dimensional vector

Cost function:

$$J(\theta_0, \theta_1, \ldots, \theta_n) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$$

($J(\theta)$)

Gradient descent:

Repeat $\{$

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \ldots, \theta_n)$$ ($J(\theta)$)

$\}$

(simultaneously update for every $j = 0, \ldots, n$)

Gradient Descent

Previously (n=1):

Repeat $\{$

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})$$

$\frac{\partial}{\partial \theta_0} J(\theta)$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x^{(i)}$$

(simultaneously update θ_0, θ_1)

$\}$

New algorithm $(n \geq 1)$:

Repeat $\{$

$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

$\frac{\partial}{\partial \theta_j} J(\theta)$

(simultaneously update θ_j for $j = 0, \ldots, n$)

$\}$

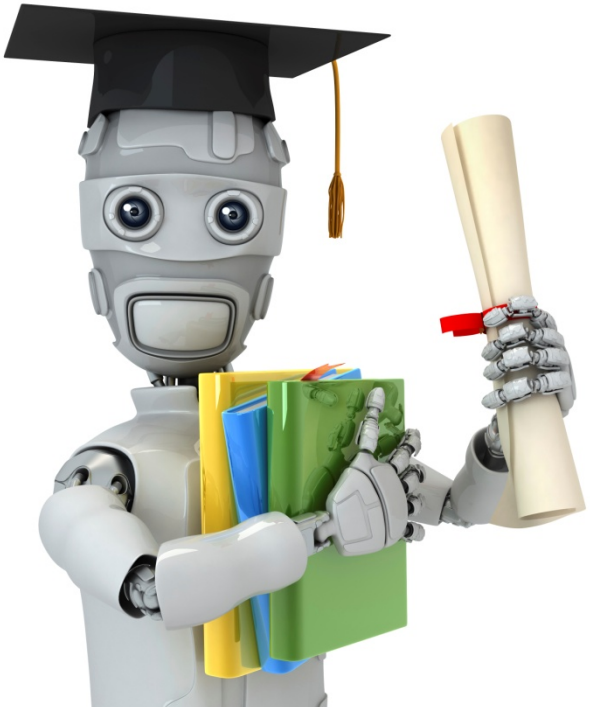
$x_0^{(i)} = 1$

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_0^{(i)}$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_1^{(i)}$$

$$\theta_2 := \theta_2 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_2^{(i)}$$

$\ldots$



Machine Learning

Linear Regression with multiple variables

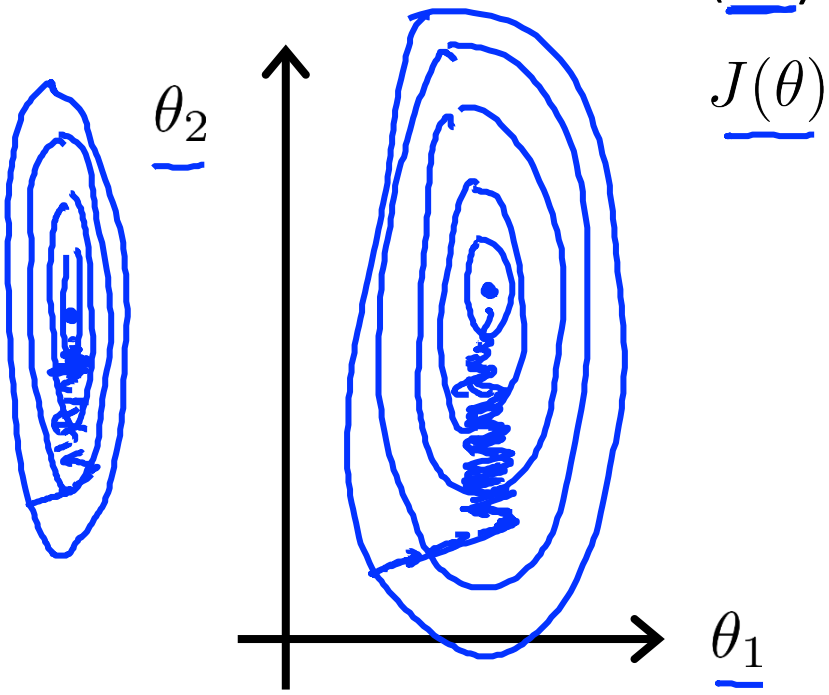
Gradient descent in practice I: Feature Scaling

Feature Scaling

Idea: Make sure features are on a similar scale.

E.g. x_1 = size (0-2000 feet^2)

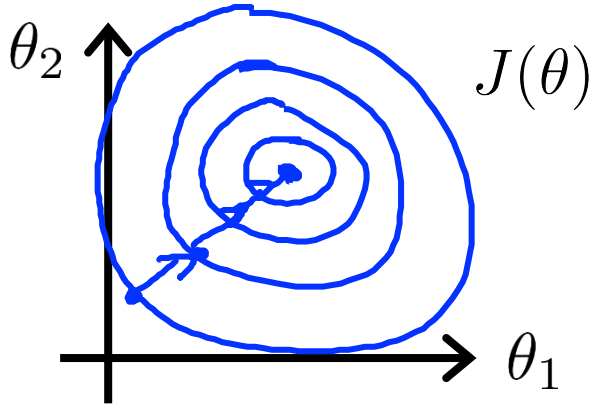
x_2 = number of bedrooms (1-5)



$$x_1 = \frac{\text{size (feet}^2\text{)}}{2000}$$

$$x_2 = \frac{\text{number of bedrooms}}{5}$$

$0 \le x_1 \le 1$ $\quad 0 \le x_2 \le 1$



Feature Scaling

Get every feature into approximately a $-1 \leq x_i \leq 1$ range.

$x_0 = 1$

$0 \leq x_1 \leq 3$ ✓

$-2 \leq x_2 \leq 0.5$ ✓

$-100 \leq x_3 \leq 100$ ✗

$-0.0001 \leq x_4 \leq 0.0001$ ✗

-3 to 3 ✓

$-\frac{1}{3}$ to $\frac{1}{3}$ ✓

Mean normalization

Replace x_i with $x_i - \mu_i$ to make features have approximately zero mean (Do not apply to $x_0 = 1$).

E.g. $x_1 = \frac{size - 1000}{2000}$

Average size = 100

$x_2 = \frac{\#bedrooms - 2}{5}$ 4

1-5 bedrooms

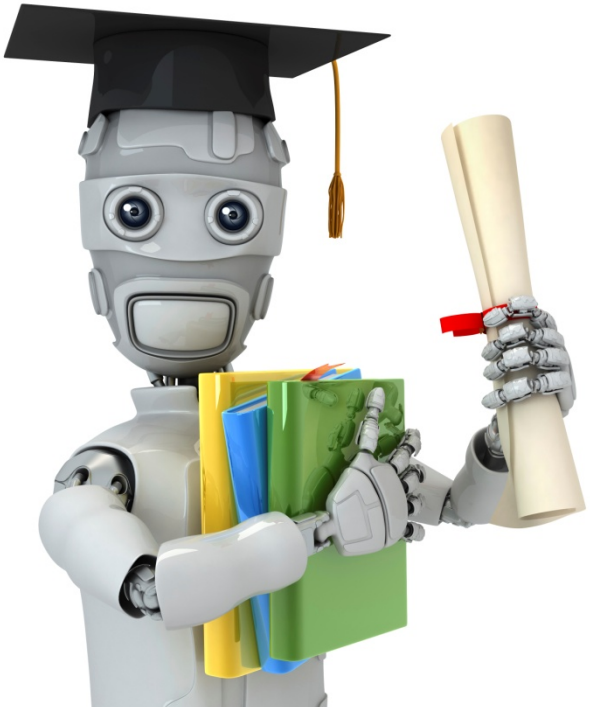
$-0.5 \leq x_1 \leq 0.5$, $-0.5 \leq x_2 \leq 0.5$

$x_1 \leftarrow \frac{x_1 - \mu_1}{s_1}$ ← avg value of x_1 in training set

range (max - min)

(or standard deviation)

$x_2 \leftarrow \frac{x_2 - \mu_2}{s_2}$



Machine Learning

Linear Regression with multiple variables

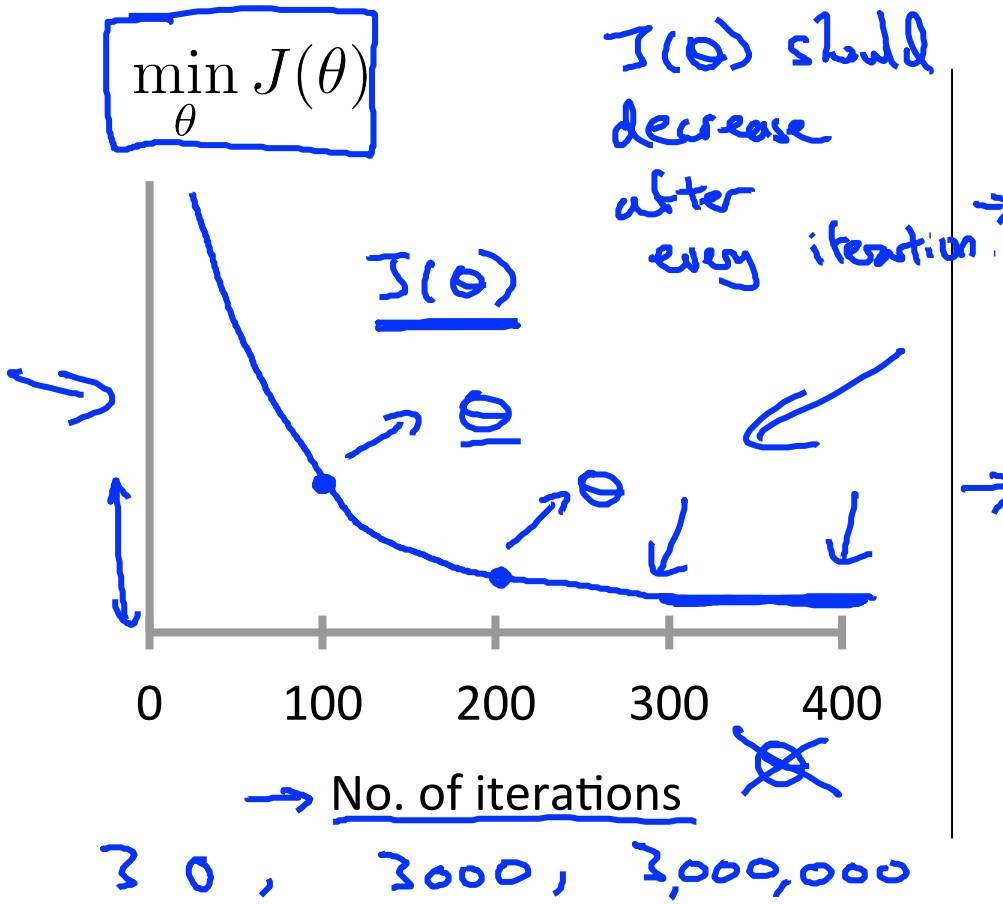
Gradient descent in practice II: Learning rate

Gradient descent

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

- "Debugging": How to make sure gradient descent is working correctly.
- How to choose learning rate α.

Making sure gradient descent is working correctly.

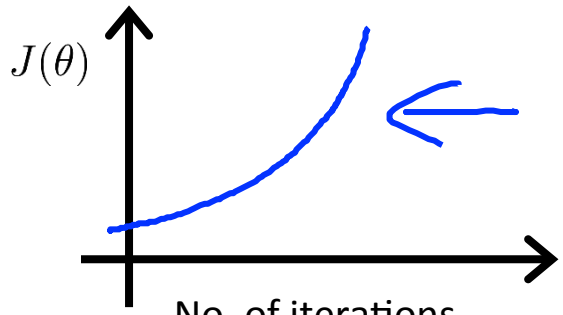


→ Example automatic convergence test:

→ Declare convergence if $J(\theta)$ decreases by less than 10^{-3} in one iteration.

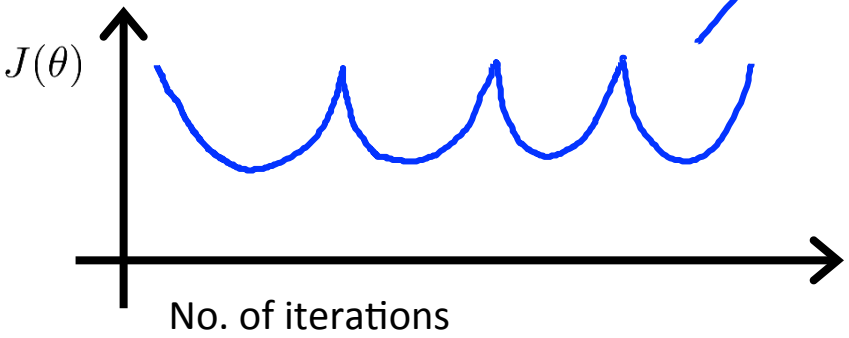
ε

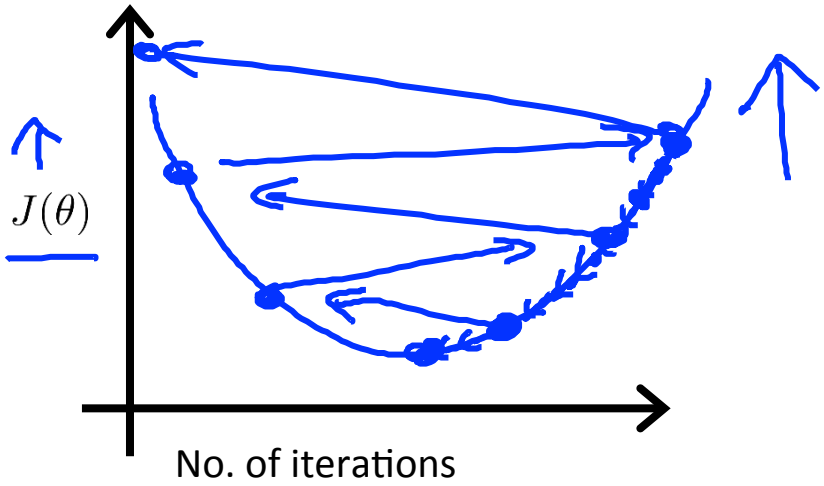
Making sure gradient descent is working correctly.



Gradient descent not working.

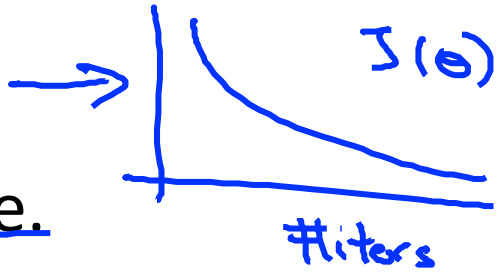
Use smaller α.





- For sufficiently small α, $J(\theta)$ should decrease on every iteration.
- But if α is too small, gradient descent can be slow to converge.

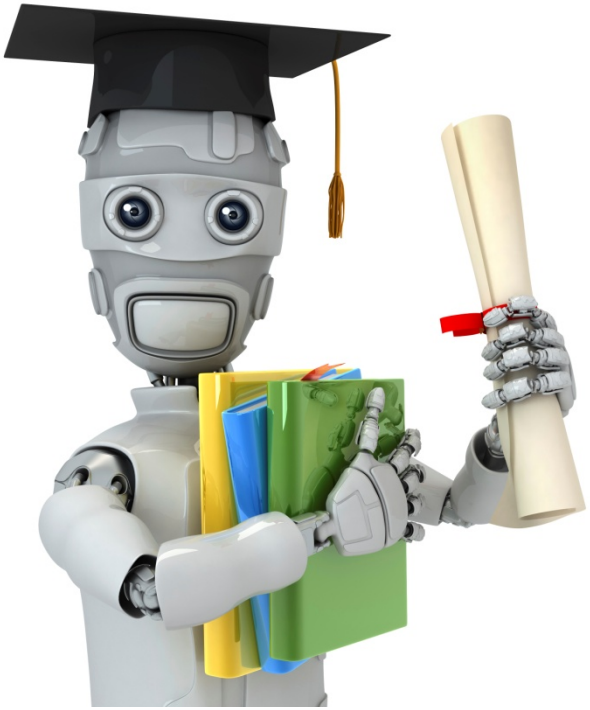
Summary:



- If α is too small: slow convergence.
- If α is too large: $J(\theta)$ may not decrease on every iteration; may not converge. (Slow converge also possible)

To choose α, try

$$\ldots, 0.001, 0.003, 0.01, 0.03, 0.1, 0.3, 1, \ldots$$



Machine Learning

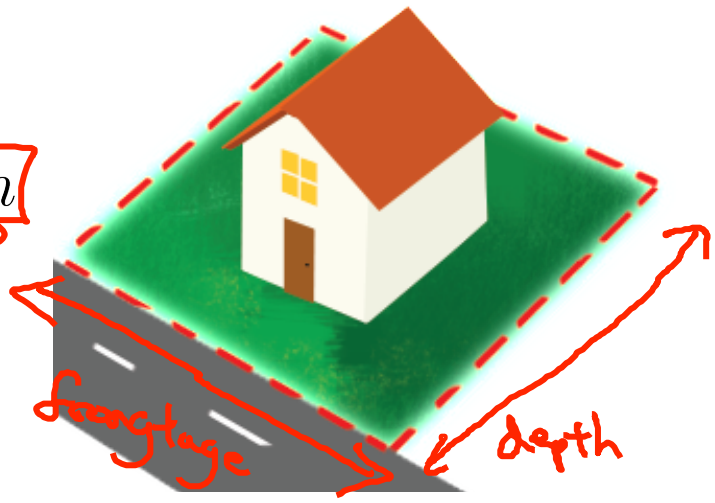
Linear Regression with multiple variables

Features and polynomial regression

Housing prices prediction

$$h_\theta(x) = \theta_0 + \theta_1 \times frontage + \theta_2 \times depth$$

x_1 x_2



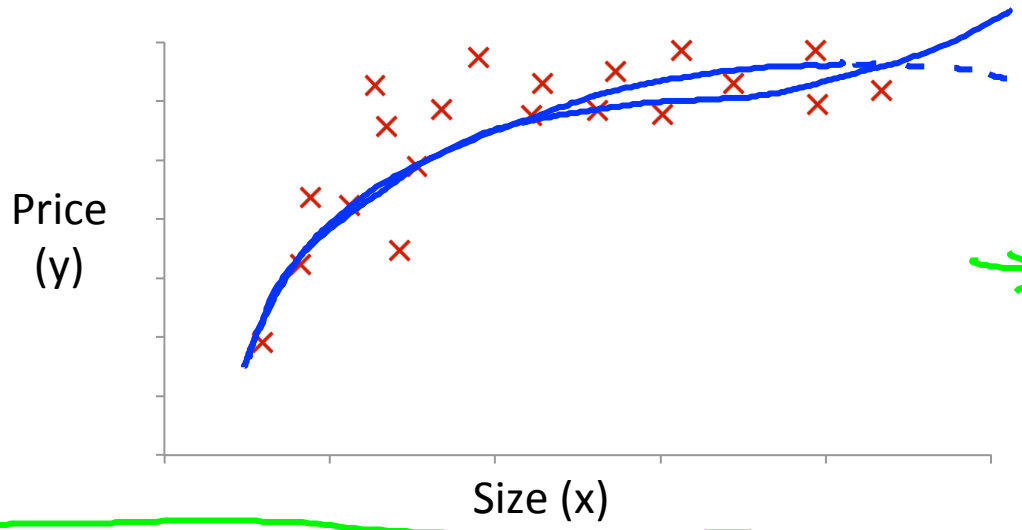
Area

x = frontage * depth

$h_\theta(x) = \theta_0 + \theta_1 x$

↑ land area

Polynomial regression



$$\theta_0 + \theta_1 x + \theta_2 x^2$$

$$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3$$

$$h_\theta(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3$$
$$= \theta_0 + \theta_1 (size) + \theta_2 (size)^2 + \theta_3 (size)^3$$

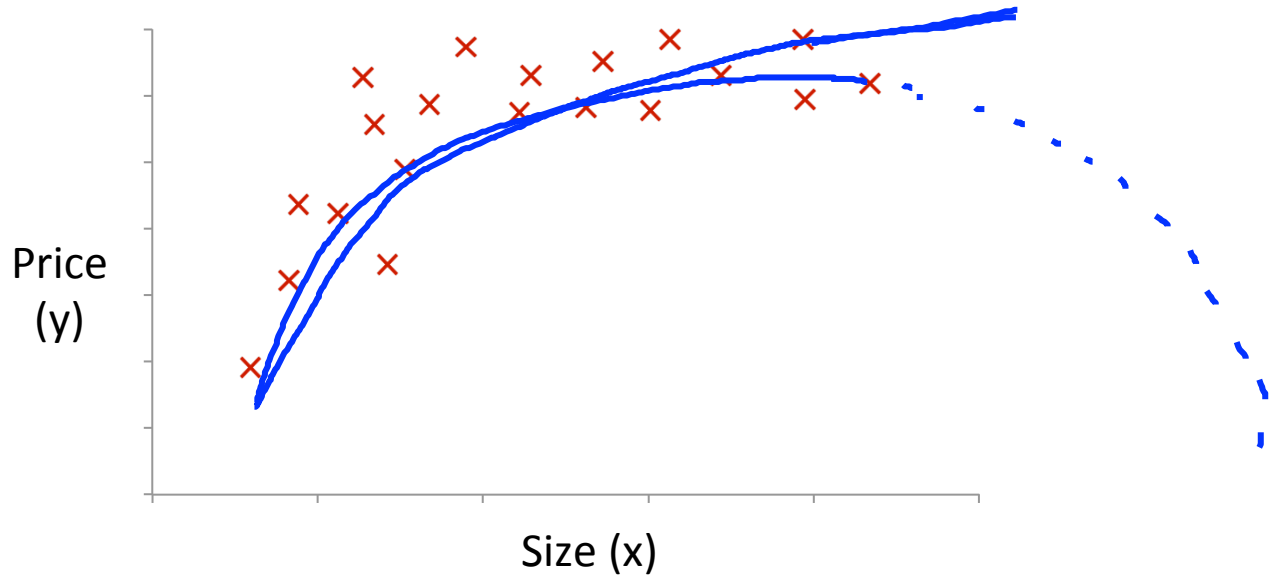
$$x_1 = (size)$$
$$x_2 = (size)^2$$
$$x_3 = (size)^3$$

Size: 1 - 1000

$Size^2$: 1 - 1000,000

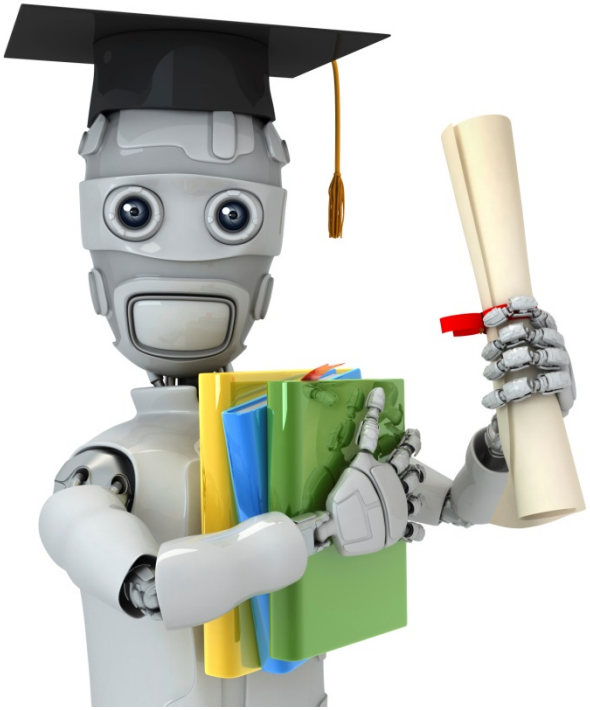
$Size^3$: 1 - 10^9

Choice of features



$$h_\theta(x) = \theta_0 + \theta_1(size) + \theta_2(size)^2$$

$$h_\theta(x) = \theta_0 + \theta_1(size) + \theta_2\sqrt{(size)}$$

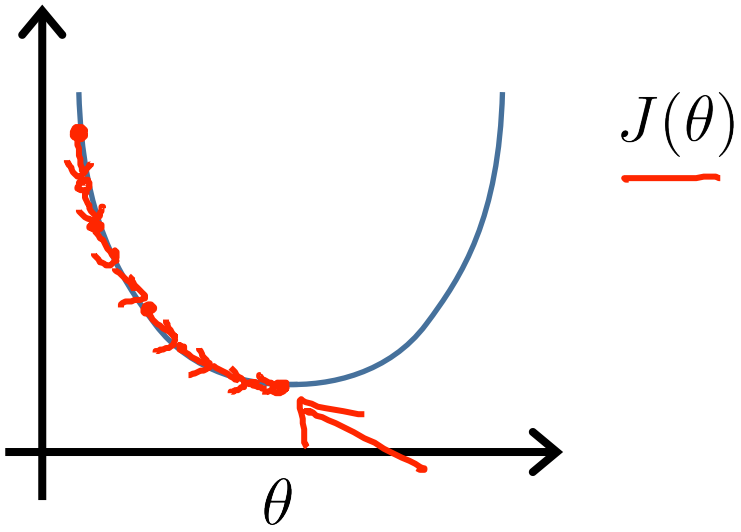


Machine Learning

Linear Regression with multiple variables

Normal equation

Gradient Descent



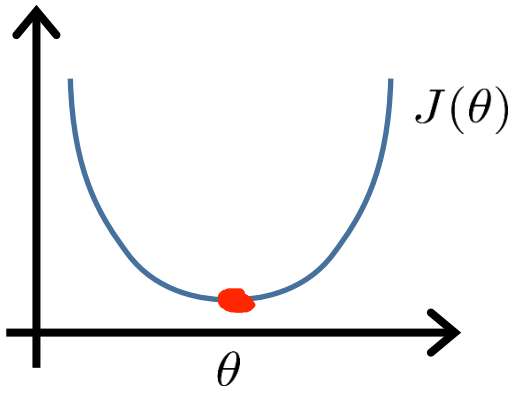
Normal equation: Method to solve for θ analytically.

Intuition: If 1D $(\theta \in \mathbb{R})$

$J(\theta) = a\theta^2 + b\theta + c$

$\frac{d}{d\theta} J(\theta) = \cdots$ set 0

Solve for θ



$\theta \in \mathbb{R}^{n+1}$ $\quad J(\theta_0, \theta_1, \ldots, \theta_m) = \frac{1}{2m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)})^2$

$\frac{\partial}{\partial \theta_j} J(\theta) = \cdots \overset{\text{set}}{=} 0$ (for every j)

Solve for $\theta_0, \theta_1, \ldots, \theta_n$

Examples: $m = 4$.

x_0	Size (feet^2) x_1	Number of bedrooms x_2	Number of floors x_3	Age of home (years) x_4	Price ($1000) y
1	2104	5	1	45	460
1	1416	3	2	40	232
1	1534	3	2	30	315
1	852	2	1	36	178

$$X = \begin{bmatrix} 1 & 2104 & 5 & 1 & 45 \\ 1 & 1416 & 3 & 2 & 40 \\ 1 & 1534 & 3 & 2 & 30 \\ 1 & 852 & 2 & 1 & 36 \end{bmatrix}$$

$m \times (n+1)$

$$y = \begin{bmatrix} 460 \\ 232 \\ 315 \\ 178 \end{bmatrix}$$

m-dimensional vector

$$\theta = (X^T X)^{-1} X^T y$$

m **examples** $(x^{(1)}, y^{(1)}), \ldots, (x^{(m)}, y^{(m)})$ **;** n **features.**

$$x^{(i)} = \begin{bmatrix} x_0^{(i)} \\ x_1^{(i)} \\ x_2^{(i)} \\ \vdots \\ x_n^{(i)} \end{bmatrix} \in \mathbb{R}^{n+1}$$

X (design matrix) $=$

$$\begin{bmatrix} \text{—} & (x^{(1)})^T & \text{—} \\ \text{—} & (x^{(2)})^T & \text{—} \\ & \vdots & \\ \text{—} & (x^{(m)})^T & \text{—} \end{bmatrix}$$

$m \times (n+1)$

E.g. If $x^{(i)} = \begin{bmatrix} 1 \\ x_1^{(i)} \end{bmatrix}$

$$X = \begin{bmatrix} 1 & x_1^{(1)} \\ 1 & x_1^{(2)} \\ \vdots & \\ 1 & x_1^{(m)} \end{bmatrix} \quad m \times 2 \qquad y = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(m)} \end{bmatrix}$$

$\theta = (X^T X)^{-1} X^T y$

$$\theta = (X^T X)^{-1} X^T y$$

$(X^T X)^{-1}$ is inverse of matrix $X^T X$.

Set $A = X^T X$

$(X^T X)^{-1} = A^{-1}$

Octave: `pinv(X'*X)*X'*y`

`pinv(X'*X)*X'*y`

$\theta = (X^T X)^{-1} X^T y$

$\min_\theta J(\theta)$

X' X^T

~~Feature Scaling~~

$0 \le x_1 \le 1$

$0 \le x_2 \le 1000$

$0 \le x_3 \le 10^{-5}$ ✓

m **training examples,** n **features.**

Gradient Descent	Normal Equation
• Need to choose α.	• No need to choose α.
• Needs many iterations.	• Don't need to iterate.
• Works well even when n is large.	• Need to compute $(X^T X)^{-1}$
	• Slow if n is very large.

$n \times n$

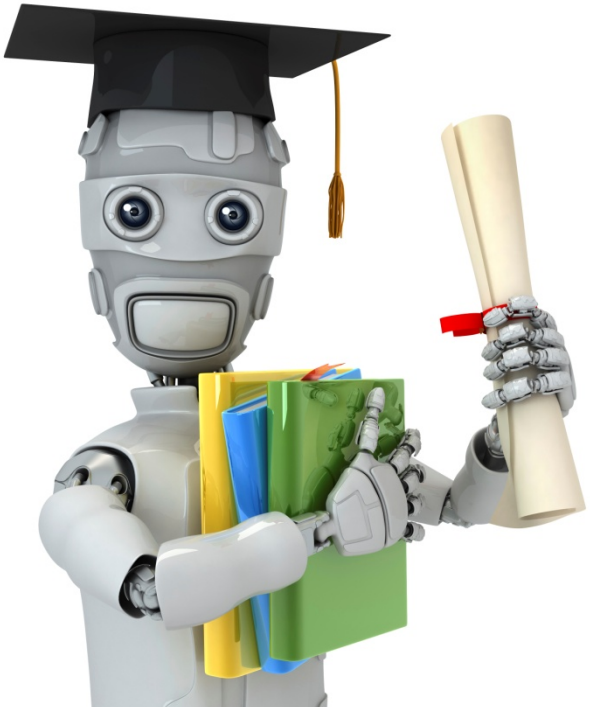
$O(n^3)$

$n = 10^6$

$n = 100$

$n = 1000$

$n = 10000$



Machine Learning

Linear Regression with multiple variables

Normal equation and non-invertibility (optional)

Normal equation

$$\theta = (X^T X)^{-1} X^T y$$

- What if $X^T X$ is non-invertible? (singular/ degenerate)
- Octave: `pinv(X'*X)*X'*y`

What if $X^T X$ is non-invertible?

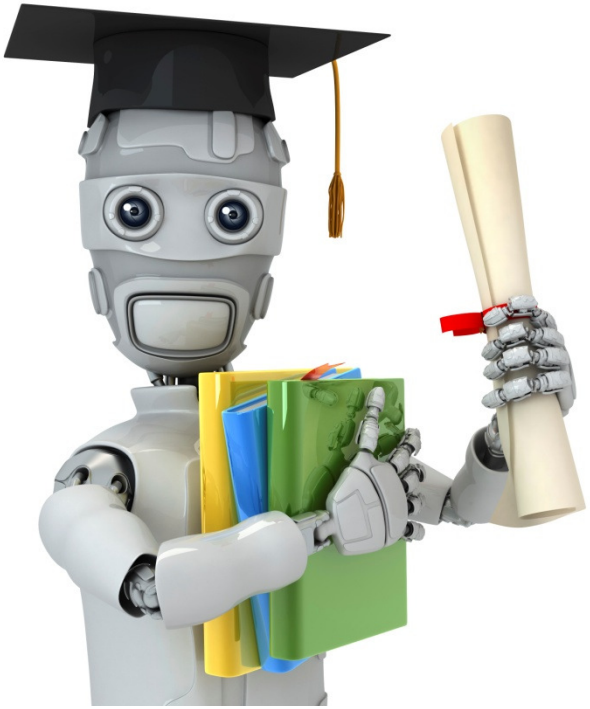
- Redundant features (linearly dependent).
  E.g. x_1 = size in feet^2
  ~~x_2 = size in m^2~~
  $x_1 = (3.28)^2 x_2$

  $1\text{m} = 3.28$ feet

  $\rightarrow m = 10$
  $\rightarrow n = 100$
  $\Rightarrow \in \mathbb{R}^{101}$

- Too many features (e.g. $m \leq n$).
  - Delete some features, or use regularization.
    ↓ later

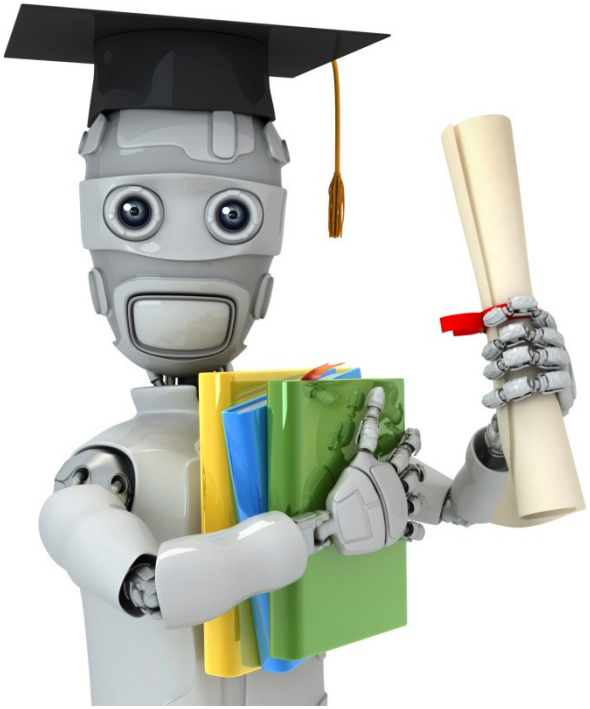
2.2 Octave



Machine Learning

Octave Tutorial

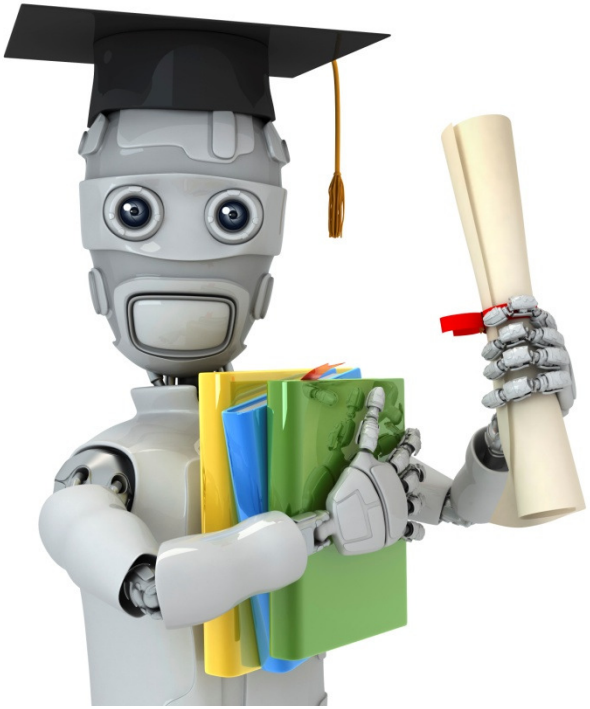
Basic operations



Machine Learning

Octave Tutorial

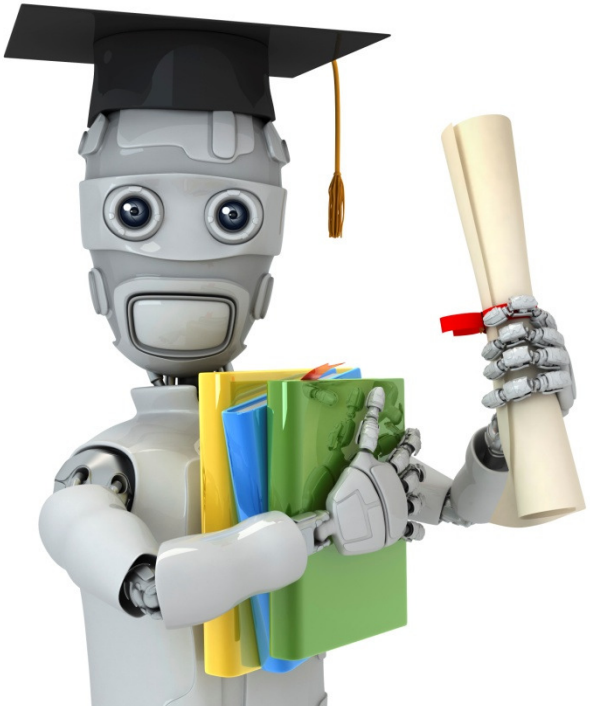
Moving data around



Machine Learning

Octave Tutorial

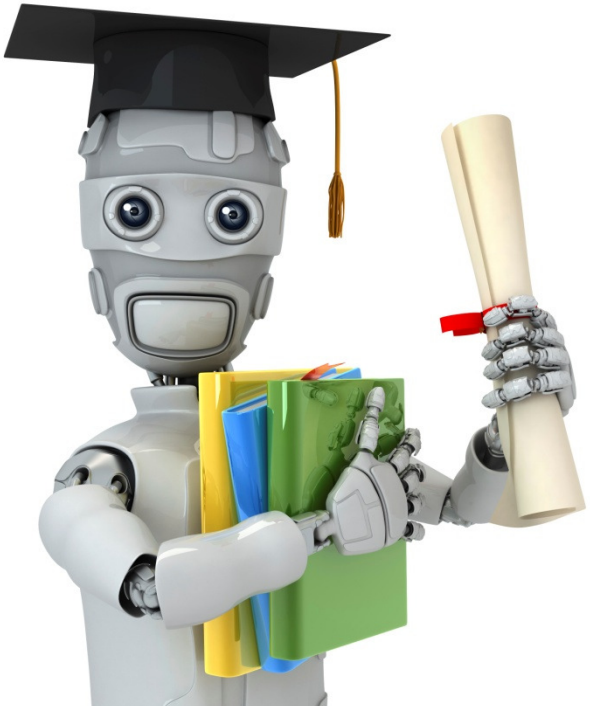
Computing on data



Machine Learning

Octave Tutorial

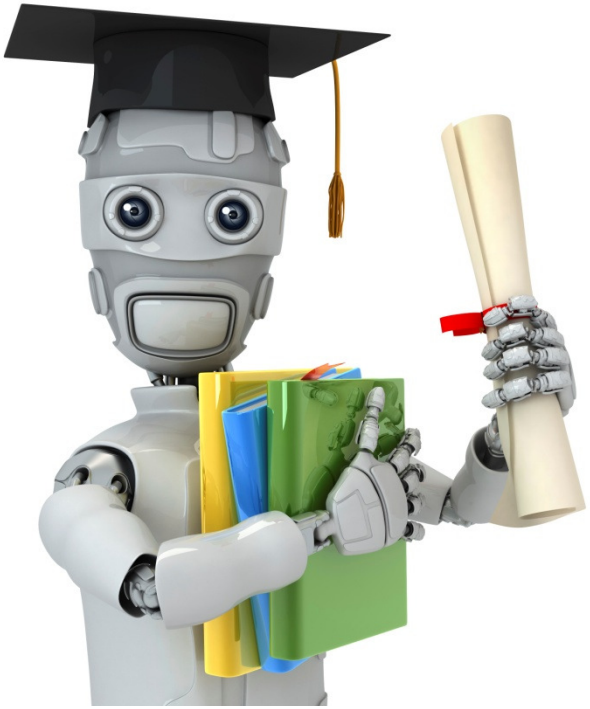
Plotting data



Machine Learning

Octave Tutorial

Control statements: for, while, if statements



Machine Learning

Octave Tutorial

Vectorial implementation

Vectorization example.

$$h_\theta(x) = \sum_{j=\theta}^{n} \theta_j x_j$$

$$= \theta^T x$$

Unvectorized implementation

```
prediction = 0.0;
for j = 1:n+1,
  prediction = prediction +
               theta(j) * x(y)
end;
```

Vectorized implementation

```
prediction = theta' * x;
```

Vectorization example.

$$h_\theta(x) = \sum_{j=\theta}^{n} \theta_j x_j$$

$$= \theta^T x$$

Unvectorized implementation

```
double prediction = 0.0;
for (int j = 0; j < n; j++)
  prediction += theta[j] * x[y];
```

Vectorized implementation

```
double prediction
  = theta.transpose() * x;
```

Gradient descent

$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)} \qquad \text{(for all } j\text{)}$$

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_0^{(i)}$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_1^{(i)}$$

$$\theta_2 := \theta_2 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_2^{(i)}$$

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_0^{(i)}$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_1^{(i)}$$

$$\theta_2 := \theta_2 - \alpha \frac{1}{m} \sum_{i=1}^{m} (h_\theta(x^{(i)}) - y^{(i)}) x_2^{(i)}$$

$(n = 2)$

$$u(j) = 2v(j) + 5w(j) \quad \text{(for all } j\text{)}$$

$$u = 2v + 5w$$