

Machine Learning's Slides

Author: Pannenets.F

Date: September 2, 2020

Je reviendrai et je serai des millions. «Spartacus»

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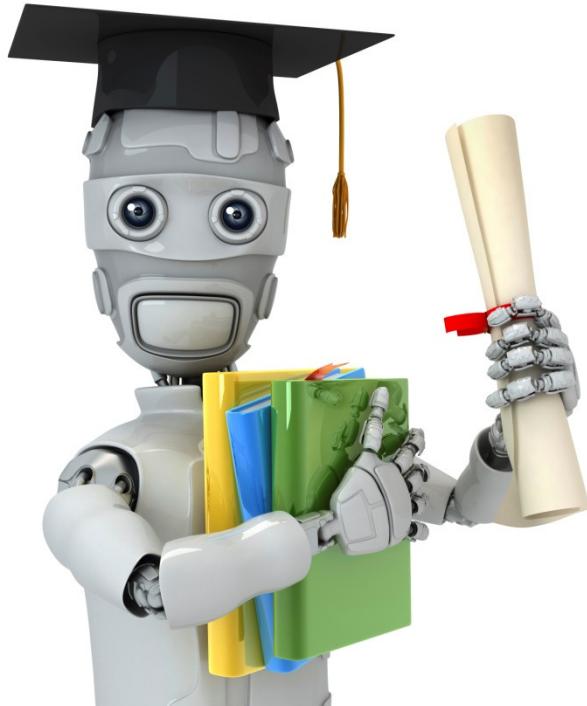
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Part I

part

Chapter 1 Week1

1.1 Introduction

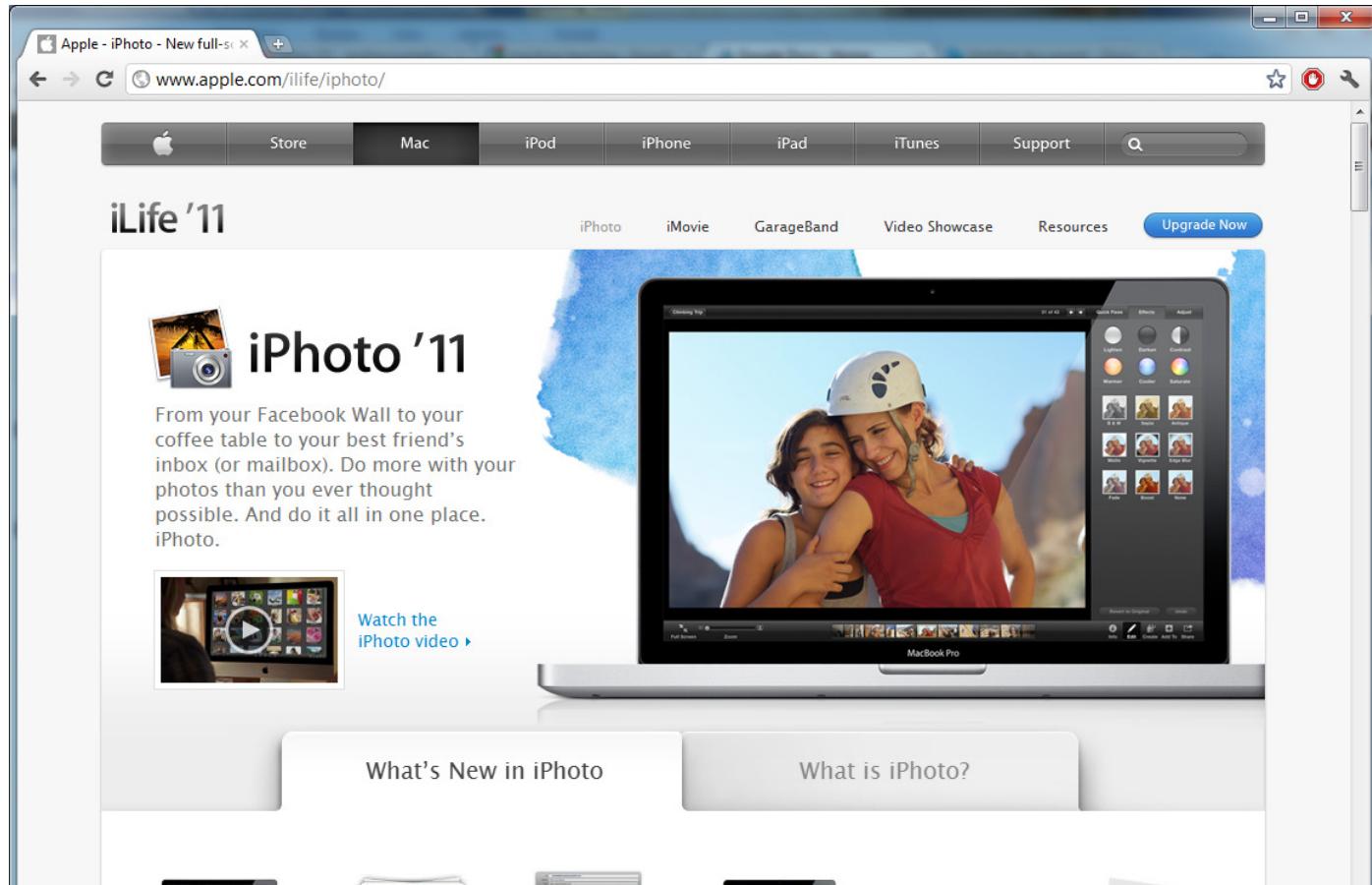


Machine Learning

Introduction

Welcome

Andrew Ng



Andrew Ng



Machine Learning

- Grew out of work in AI
- New capability for computers

Examples:

- Database mining
 - Large datasets from growth of automation/web.
E.g., Web click data, medical records, biology, engineering
- Applications can't program by hand.
 - E.g., Autonomous helicopter, handwriting recognition, most of Natural Language Processing (NLP), Computer Vision.

Machine Learning

- Grew out of work in AI

Exam

-

-



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host of

Machine Learning

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- Self-customizing programs
 - E.g., Amazon, Netflix product recommendations

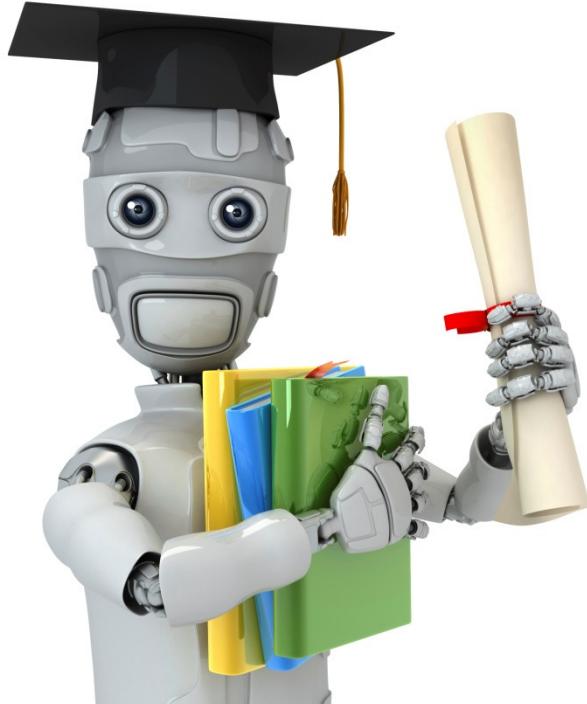
Machine Learning

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- New capability for computers

Examples:

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 - Large datasets from growth of automation/web.
E.g., Web click data, medical records, biology, engineering
- Applications can't program by hand.
 - E.g., Autonomous helicopter, handwriting recognition, most of Natural Language Processing (NLP), Computer Vision.
- Self-customizing programs
 - E.g., Amazon, Netflix product recommendations
- Understanding human learning (brain, real AI).

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Machine Learning

Introduction

What is machine learning

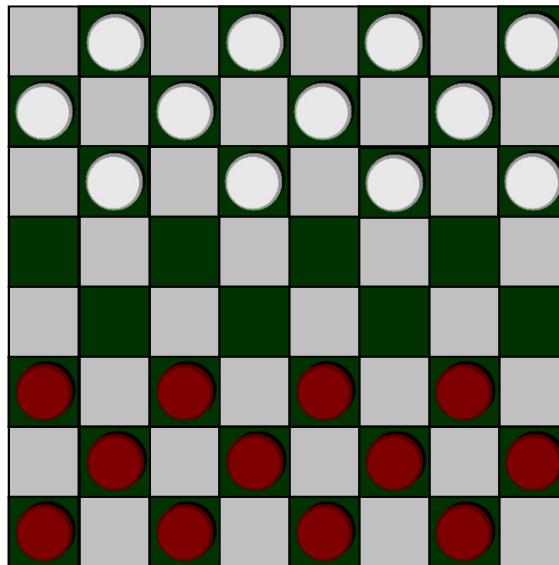
Machine Learning definition

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- Arthur Samuel (1959). Machine Learning: Field of study that gives computers the ability to learn without being explicitly programmed.

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Machine Learning definition

- Arthur Samuel (1959). Machine Learning: Field of study that gives computers the ability to learn without being explicitly programmed.
- Tom Mitchell (1998) Well-posed Learning Problem: A computer program is said to *learn* from experience E with respect to some task T and some performance measure P, if its performance on T, as measured by P, improves with experience E.

“A computer program is said to *learn* from experience E with respect to some task T and some performance measure P, if its performance on T, as measured by P, improves with experience E.”

Suppose your email program watches which emails you do or do not mark as spam, and based on that learns how to better filter spam. What is the task T in this setting?

- Classifying emails as spam or not spam. $T \leftarrow$
- Watching you label emails as spam or not spam. $E \leftarrow$
- The number (or fraction) of emails correctly classified as spam/not spam.
- None of the above—this is not a machine learning problem. $P \leftarrow$

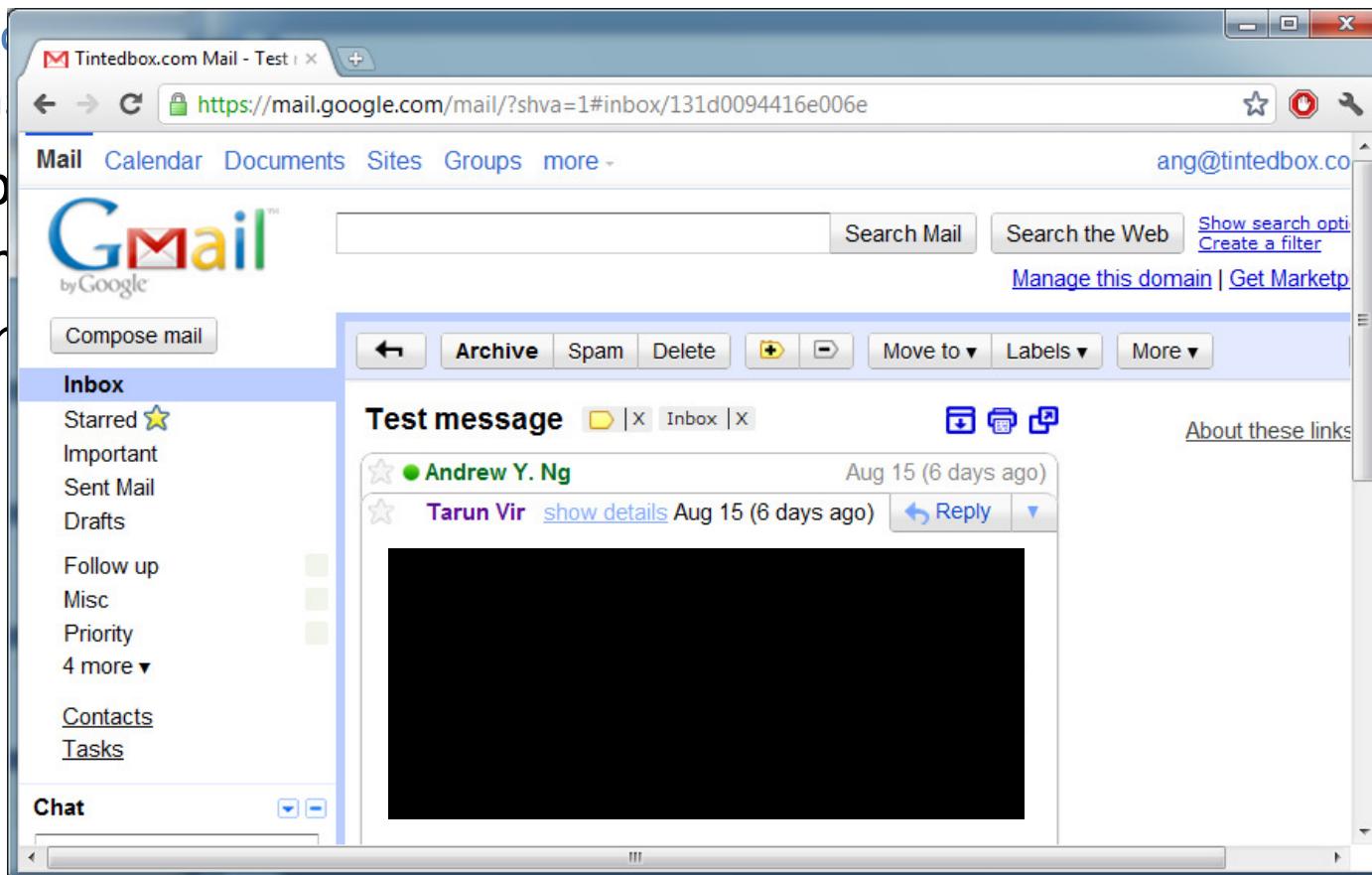
“A computer program is said to *learn* from experience E with respect to some class of tasks T if its performance improves with experience.”

Support
not n
spam

on T,

or do
filter

spam.



“A computer program is said to *learn* from experience E with respect to some task T and some performance measure P, if its performance on T, as measured by P, improves with experience E.”

Suppose your email program watches which emails you do or do not mark as spam, and based on that learns how to better filter spam. What is the task T in this setting?

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Machine learning algorithms:

- Supervised learning
- Unsupervised learning

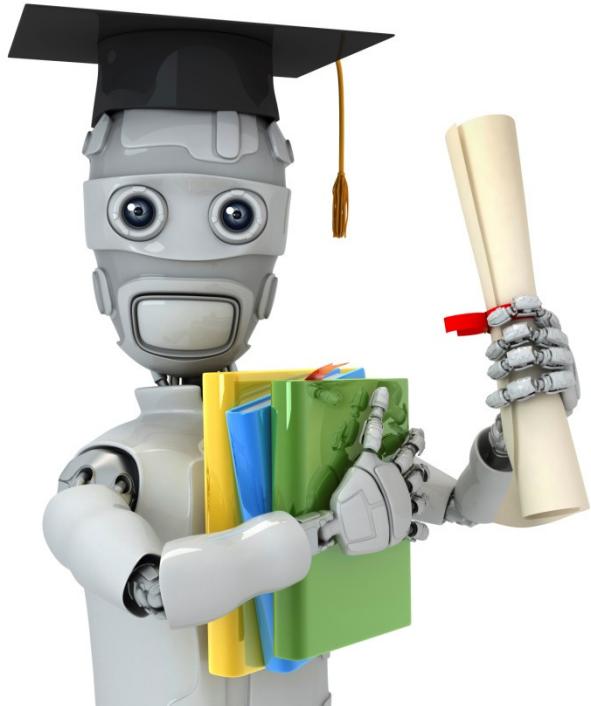


Others: Reinforcement learning, recommender systems.

Also talk about: Practical advice for applying learning algorithms.



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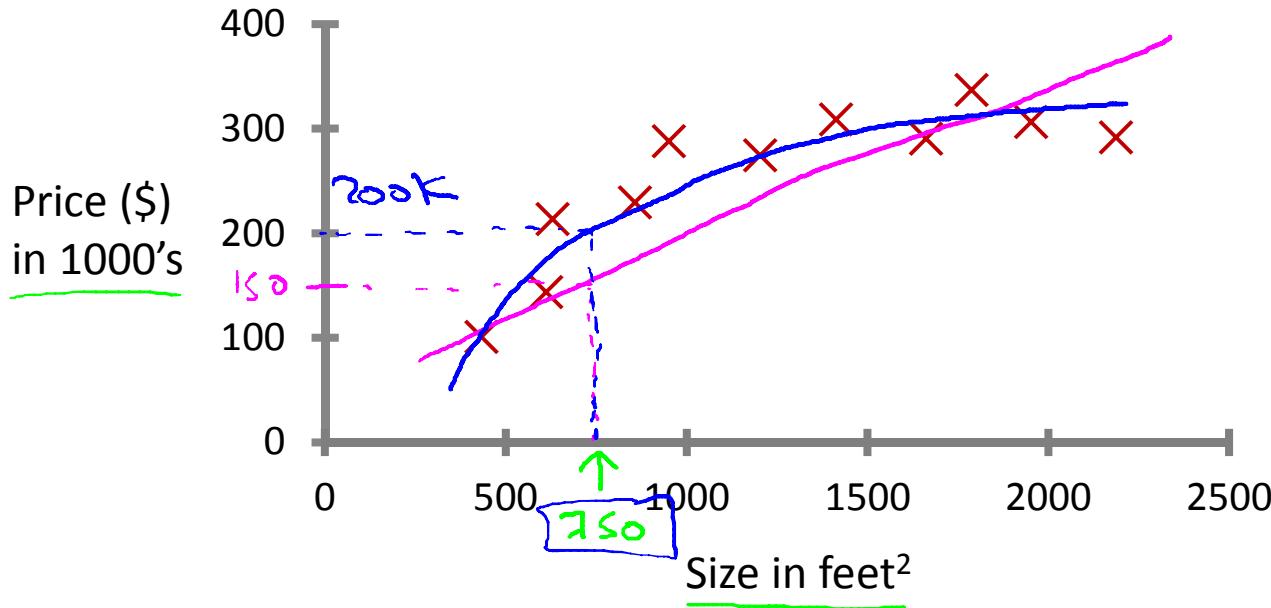


Machine Learning

Introduction

Supervised Learning

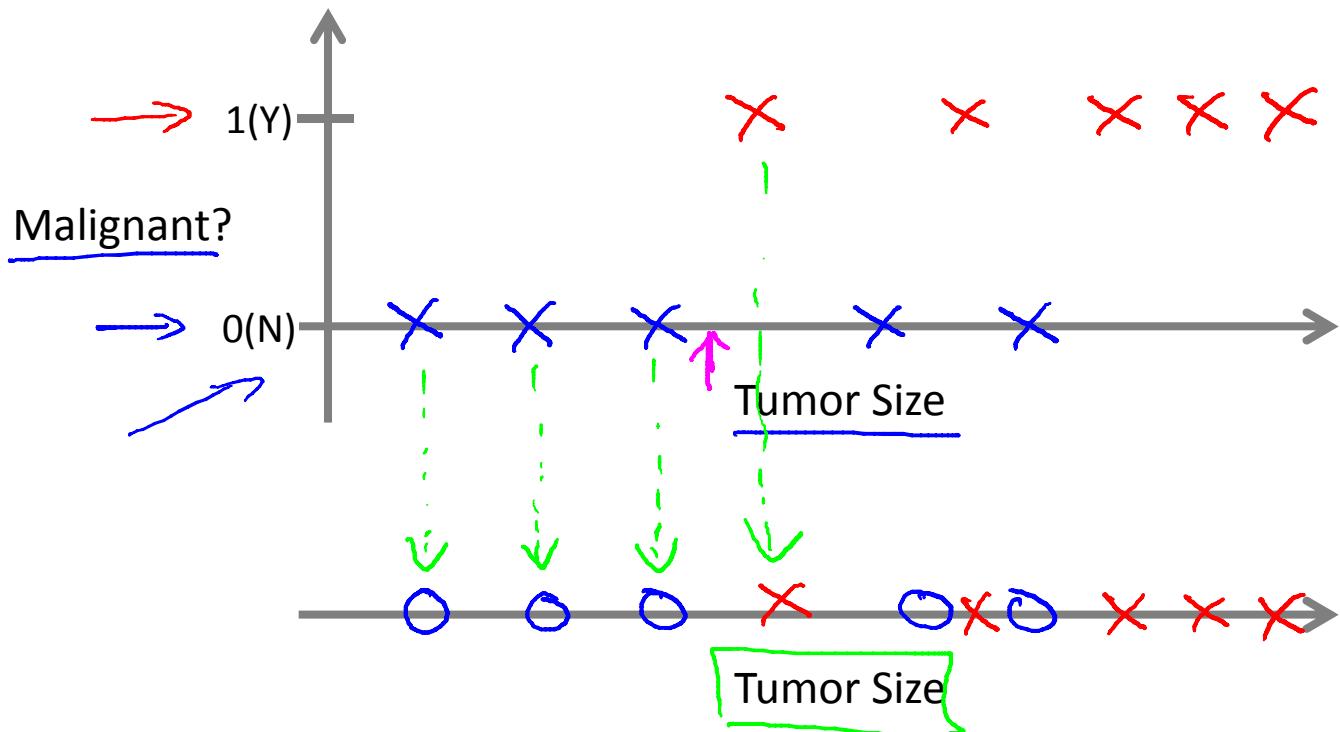
Housing price prediction.



Supervised Learning
‘right answers’ given

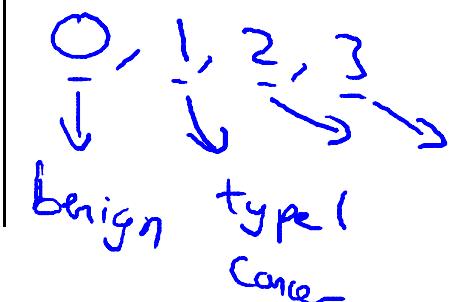
Regression: Predict continuous valued output (price)

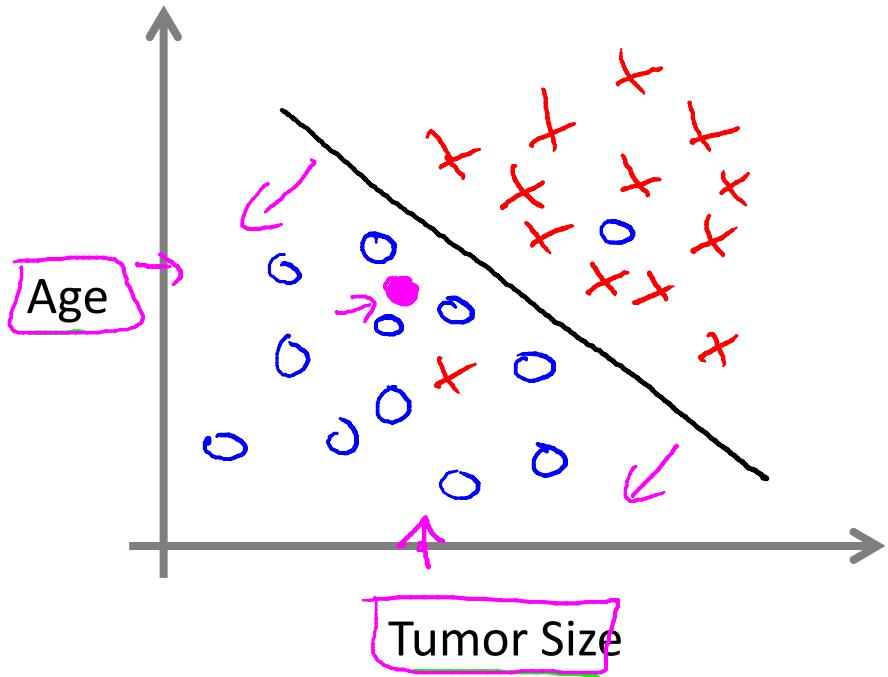
Breast cancer (malignant, benign)



Classification

Discrete valued output (0 or 1)





- Clump Thickness
- Uniformity of Cell Size
- Uniformity of Cell Shape

...

You're running a company, and you want to develop learning algorithms to address each of two problems.

1000's

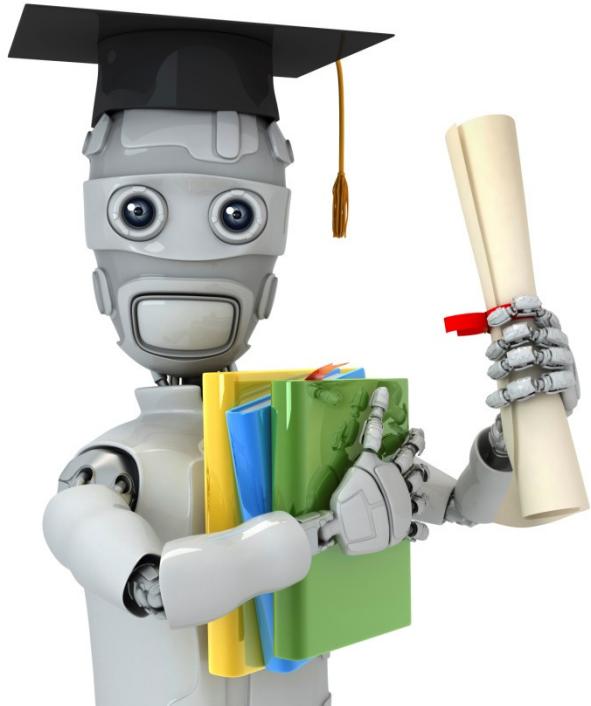
- Problem 1: You have a large inventory of identical items. You want to predict how many of these items will sell over the next 3 months.
- Problem 2: You'd like software to examine individual customer accounts, and for each account decide if it has been hacked/compromised.

→ 0 - not hacked
→ 1 - hacked

Should you treat these as classification or as regression problems?

- Treat both as classification problems.
- Treat problem 1 as a classification problem, problem 2 as a regression problem.
- Treat problem 1 as a regression problem, problem 2 as a classification problem.
- Treat both as regression problems.

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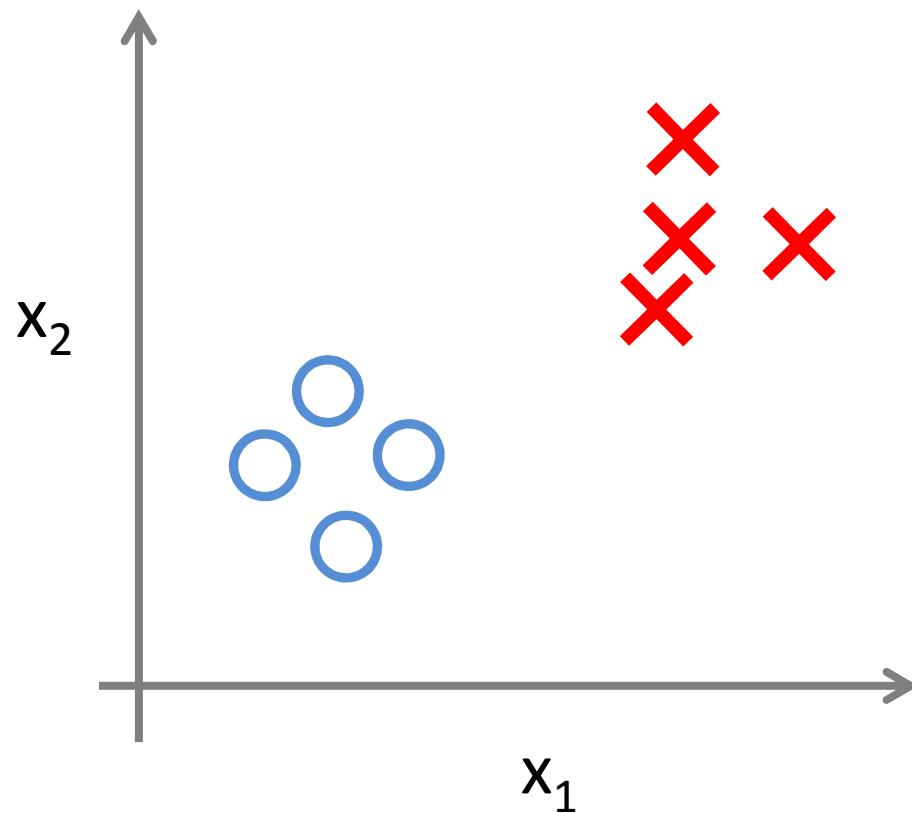


Machine Learning

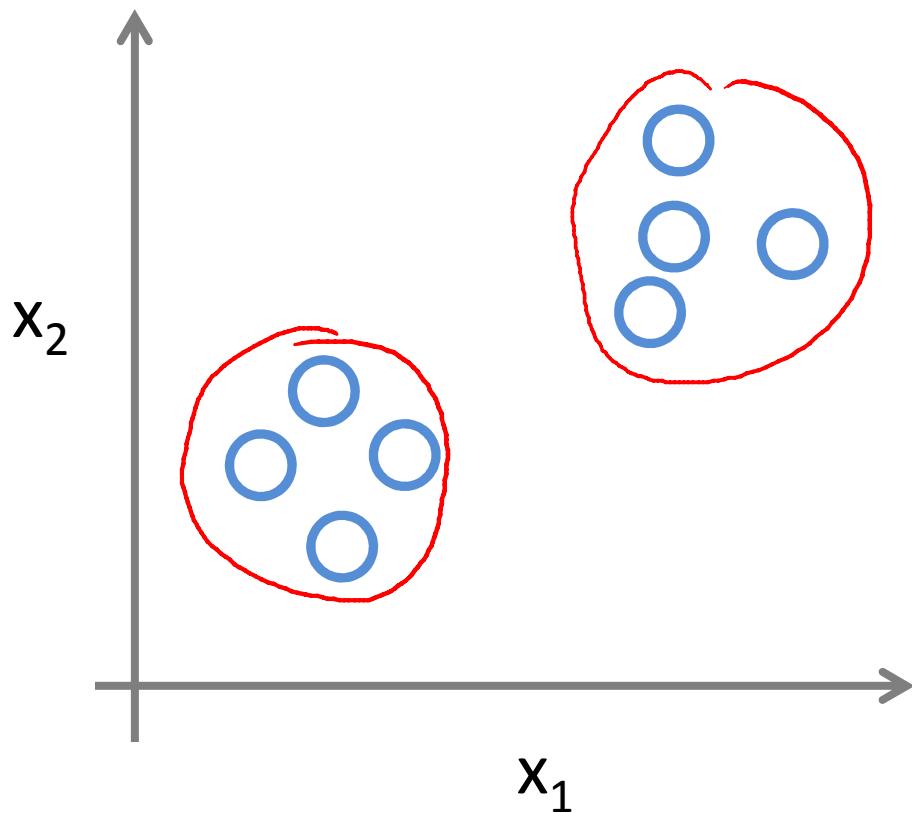
Introduction

Unsupervised Learning

Supervised Learning



Unsupervised Learning



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- World
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- Business
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Top Stories

Christine O'Donnell » [White House official denies Tea Party-focused ad campaign](#) CNN International - Ed Henry - 1 hour ago Democratic sources say the White House is not considering an ad campaign tying Republicans to the Tea Party. Washington (CNN) -- A top White House official sharply denied a report that claims President Obama's political advisers are weighing a national ...
Tea Party is misplaced the blame, former President Bill Clinton claims New York Daily News
GOP tea party backer defends Christine O'Donnell The Associated Press Atlanta Journal Constitution - Politics Daily - MyFox Washington DC - Salon all 726 news articles »

[US Stocks Climb After Recession Called Over, Homebuilders Gain](#) MarketWatch - Kristina Peterson - 16 minutes ago NEW YORK (MarketWatch) -- US stocks climbed Monday, gaining speed after a key nonprofit organization officially called the recession over, giving investors a boost of confidence in the gradual economic recovery.
Longest recession since 1930s ended in June 2009, group says Los Angeles Times
Downturn Was Longest in Decades, Panel Confirms New York Times Wall Street Journal - AFP - CNN - USA Today all 276 news articles »

Deepwater Horizon » [BP Oil Well, Site of National Catastrophe, Dies at One](#) Vanity Fair - Juli Weiner - 22 minutes ago The BP oil well, site of the Deepwater Horizon explosion that led to the worst oil spill in US history, died today at one year old.
+ Video: Blown-out BP Well Finally Killed in Gulf  The Associated Press Weiss Doubts BP Would End Operations in Gulf of Mexico: Video Bloomberg CNN International - Wall Street Journal (blog) - The Guardian - New York Times all 2,292 news articles »

Recent

Recession officially ended in June 2009 CNNMoney - Chris Isidore - 39 minutes ago Hurricane Igor lashes Bermuda USA Today - Gerry Broome - 5 minutes ago 'Explain what you want from us,' reads front-page editorial msnbc.com - Olivia Torres - 10 minutes ago

Crisis response: Pakistan floods

San Francisco Bay Area - Edit

Clorox » Bay Biz Buzz: Clorox close to selling STP, Armor All San Jose Mercury News - 48 minutes ago - all 24 articles » Google's official beekeeper keeps the company buzzing with excitement San Jose Mercury News - Bruce Newman - 1 hour ago Jon Sylvia » Martinez man still unconscious as police investigate weekend shooting San Jose Mercury News - Robert Salonga - 48 minutes ago - all 6 articles »

Spotlight

Sarkozy rages at EU 'humiliation' Financial Times - Peggy Hollinger - Sep 16, 2010

The screenshot shows a Google search results page for the query "Google news". The top result is a link to "news.google.com". Below it, the "Top Stories" section features several headlines, each with a thumbnail image and a brief description. A red arrow points from the top-left towards the "Top Stories" section. Another red arrow points from the bottom-left towards the "Deepwater Horizon" story. The right side of the screen displays a sidebar with news categories like "U.S.", "World", and "Business", along with a "Spotlight" section for "Sarkozy rages at EU summation".

Google News

news.google.com

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Google news

Search News Search the Web Advanced search

Top Stories

Deepwater Horizon
Fed meeting
Forex exchange market
Lindsay Lohan IBM
Tom Brady Toronto
International Film Festival
Paris Hilton Iran
Hurricane Igor

Starred

San Francisco Bay Area

World U.S.

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More Top Stories

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Top Stories

Christine O'Donnell : **White House official denies Tea Party-focused ad campaign**
John Hockenberry : **1 hour ago**
Democratic sources say the White House is not considering an ad campaign targeting Republicans to say the Tea Party. Washington (CNN) — A top White House official says the president claims President Obama's political advisers are weighing a national Tea Party — misplacing the blame, former President Bill Clinton claims. (Video) [Read more] [Comments]

GOP tea party backer demands Christine O'Donnell The Associated Press
Atlanta Journal-Constitution - Politics Daily - MyFox Washington DC - Salon
all 728 news articles »

US Stocks Climb After Recession Called Over, Homebuilders Gain

MarketWatch - Kristina Peterson : **10 minutes ago**
NEW YORK (MarketWatch) — US stocks climbed Monday, gaining speed after a key nonprofit organization officially called the recession over, giving investors a boost of confidence in the gradual economic recovery. Long-term interest rates, which rose from 1930 levels ended in June 2009, group says Los Angeles Times
Dowtown Was Longest in Decades, Panel Confirms New York Times
Wall Street Journal - AFP - USA Today

Deepwater Horizon

BP Oil Well, Site of National Catastrophe, Dies at One
AP : **1 minute ago**
The BP oil well, site of the Deepwater Horizon explosion that led to the worst oil spill in US history, died today at one year old.
+ See more about BP oil well - AP news stories | The Associated Press
Weiss Doubts BP Would End Operations in Gulf of Mexico - Video - Bloomberg CNN International - Wall Street Journal (blog) - The Guardian - New York Times - AP - Reuters - all 2,292 news articles »

RECESSION

MyFox Phila...
Mercury News - 48 minutes ago
all 24 articles »

Crisis response: Pakistan floods

San Francisco Bay Area - Edit

Claros - Bay Buzz - Cloxen close to selling STP...
Armor All - Mercury News - 48 minutes ago
all 24 articles »

Google's official buzzword is keeping the company buzzing in its latest statement - Mercury News - Bruce Newman - 1 hour ago

Jen Sylka - Martinez man still unconscious as police investigate weekend shooting - Jose Mercury News - Robert Salonga - 48 minutes ago - all 6 articles »

Spotlight

Sarkozy rages at EU summation - Pigeon Hollinger - 2010

A screenshot of a CNN news article. The URL in the address bar is edition.cnn.com/2010/US/09/20/gulf.oil.disaster/. The page title is "Allen: Well is dead, but much Gulf Coast work remains". Below the title is a video player with a play button labeled "Click to play". A red arrow points from the top of the image down to the video player area. At the bottom left, there is a caption: "What next for Gulf oil spill?".

BP Kills Macondo, But Its Legacy Lives On

By James Herron

BP confirmed late Sunday that the Macondo well that leaked nearly five million barrels of oil into the Gulf of Mexico has been permanently sealed, but the well will continue to affect BP and the wider oil industry for many years.

The most immediate worry for BP and its shareholders is how the authorities will apportion blame for the spill. BP's own investigation spread responsibility across

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3. Timing of Ratings Award Intriguing

4. BP Kills Macondo, But Its Legacy Lives On

5. We Already Need a Sequel to Basel III!

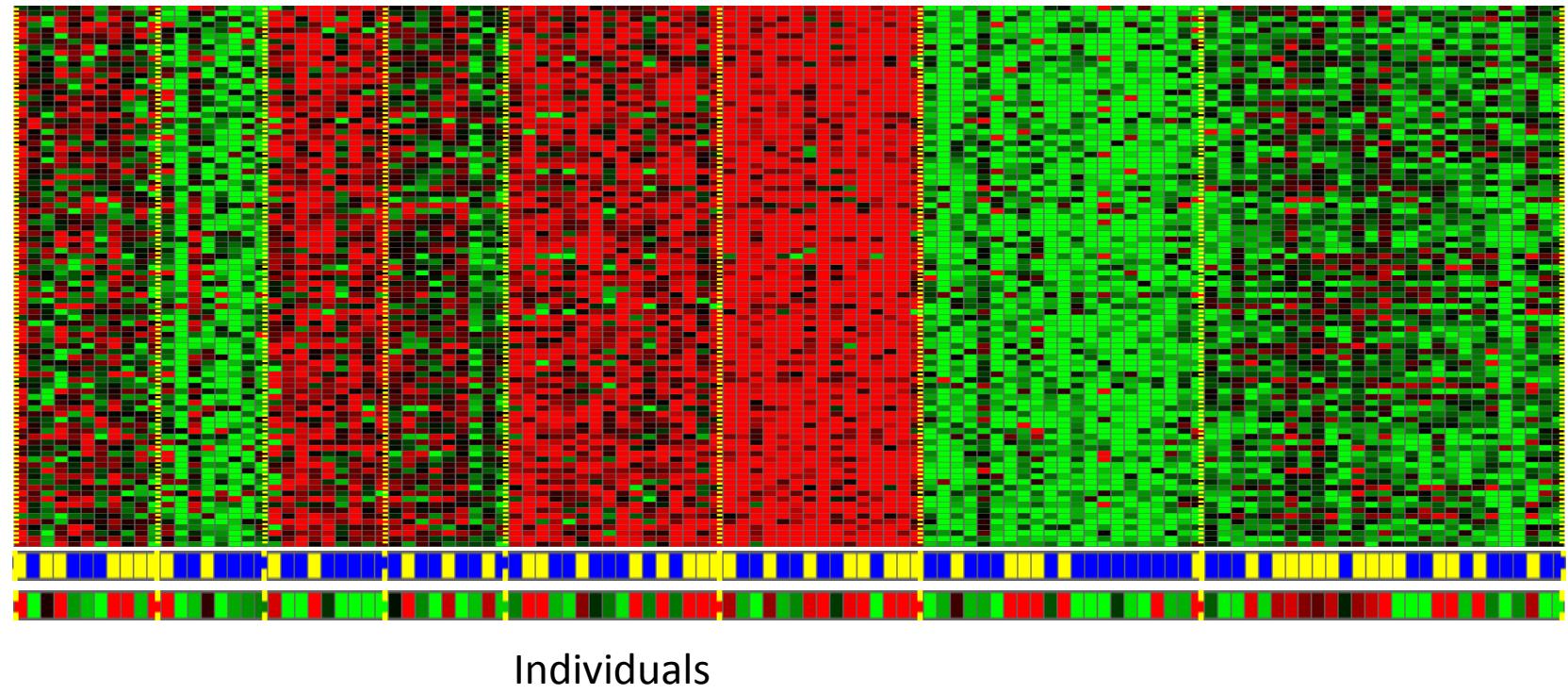


Associated Press

Fire boat response crews battled the blazing remnants of the off shore oil platform Deepwater Horizon on April 21, 2010.

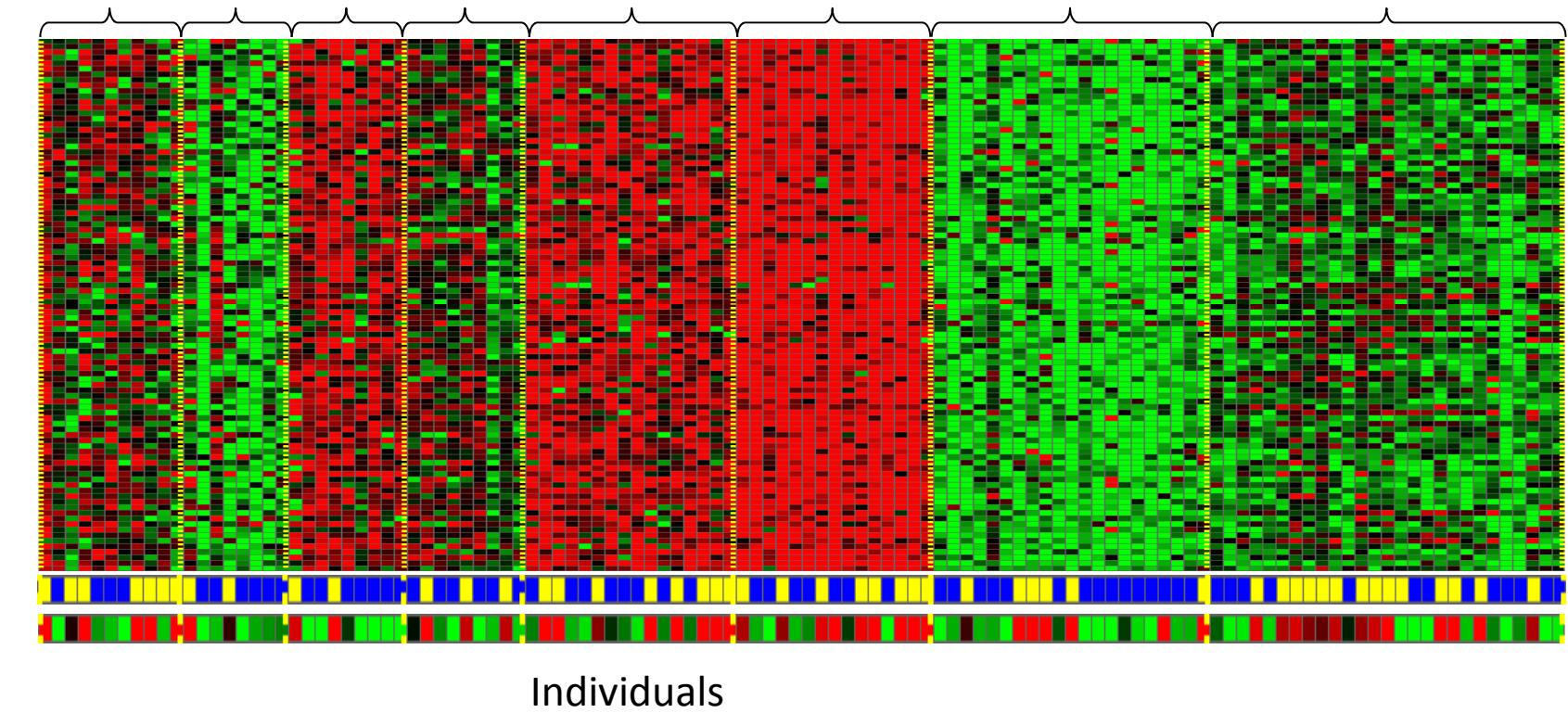
An aerial photograph capturing the Deepwater Horizon oil rig engulfed in a massive fire. The intense orange and yellow flames are visible against the dark night sky. Several boats are positioned around the burning platform, spraying streams of water onto the inferno. The surrounding ocean is visible, with some surface activity from the firefighting efforts.

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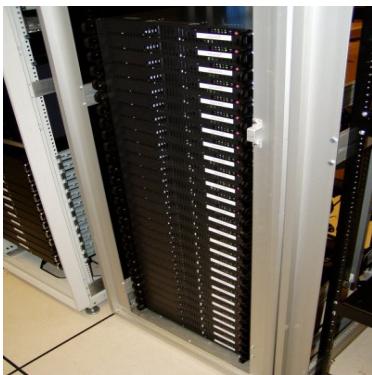
[Source: Daphne Koller]

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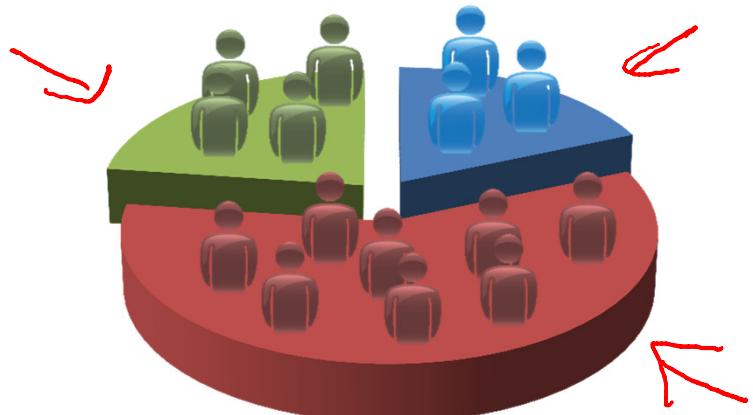


[Source: Daphne Koller]

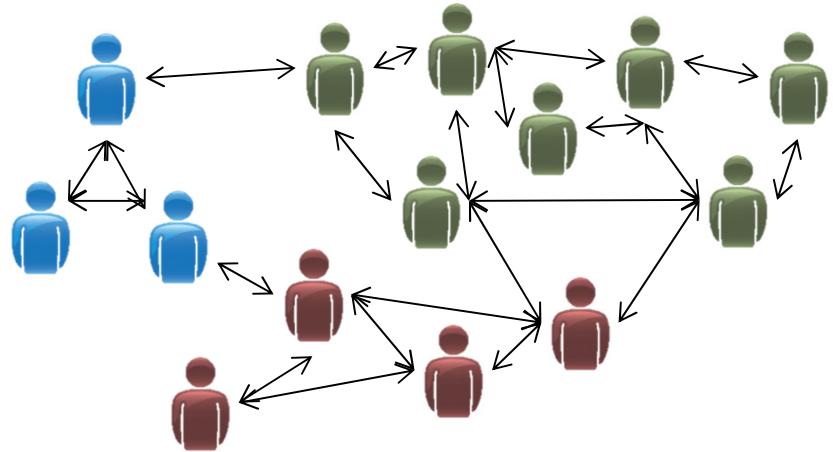
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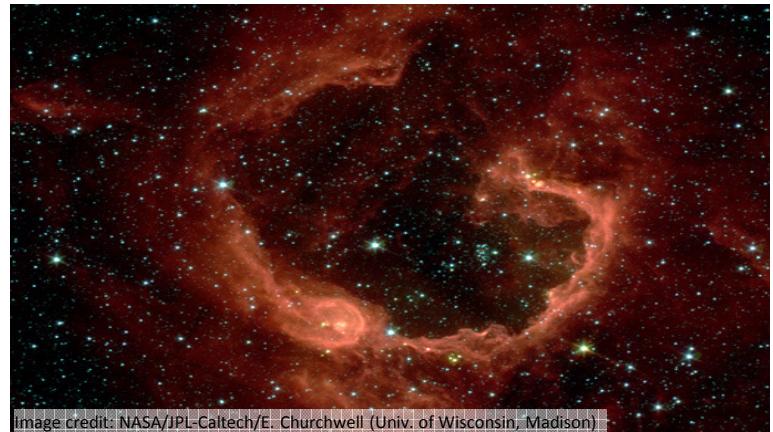
Organize computing clusters



Market segmentation



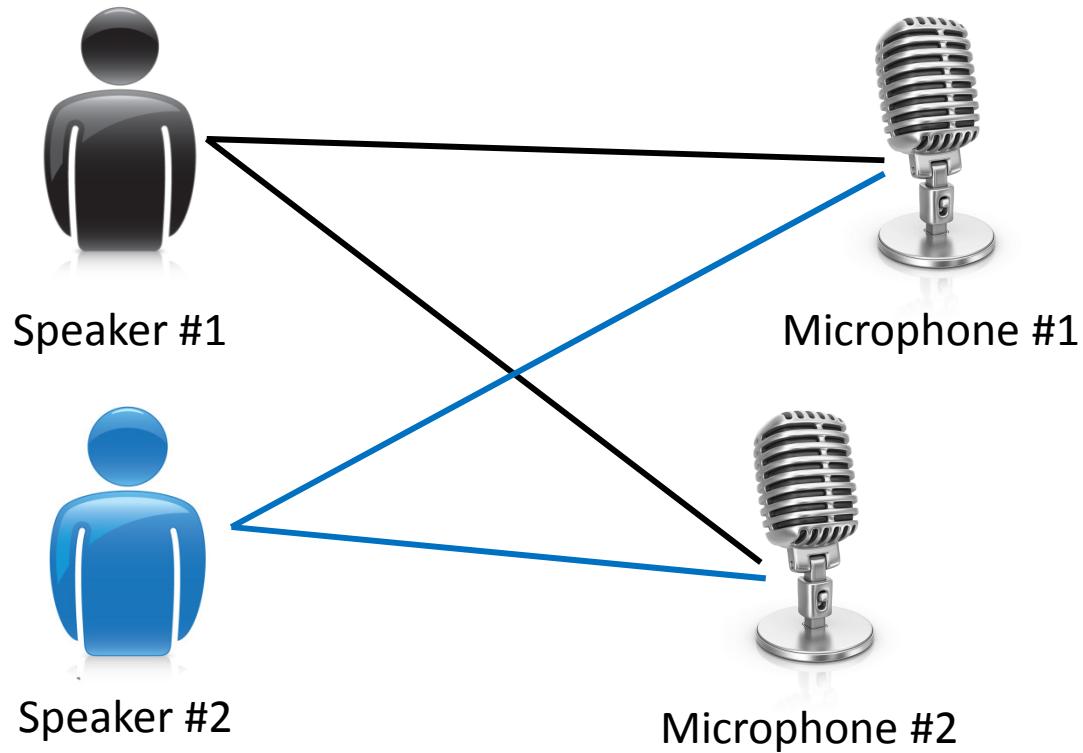
Social network analysis



Astronomical data analysis

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Cocktail party problem



Microphone #1: 

Output #1: 

Microphone #2: 

Output #2: 

Microphone #1: 

Output #1: 

Microphone #2: 

Output #2: 

[Audio clips courtesy of Te-Won Lee.]

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Cocktail party problem algorithm

```
[W,s,v] = svd((repmat(sum(x.*x,1),size(x,1),1).*x)*x');
```

[Source: Sam Roweis, Yair Weiss & Eero Simoncelli]

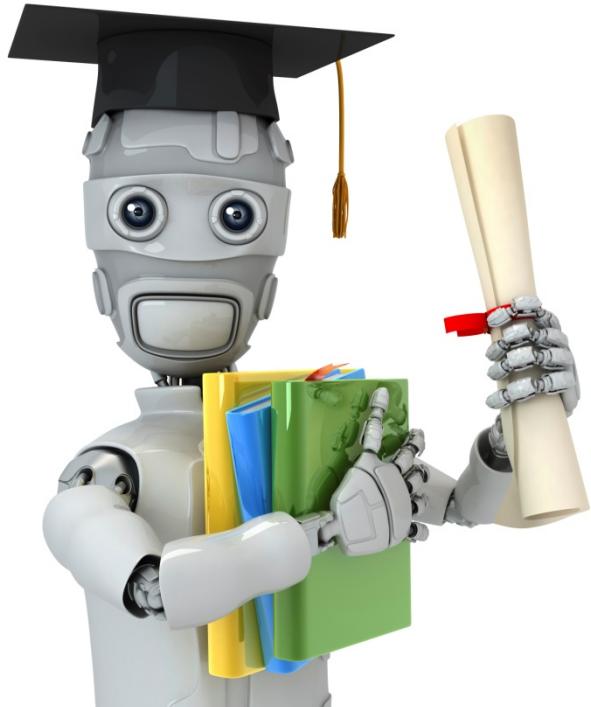
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Of the following examples, which would you address using an unsupervised learning algorithm? (Check all that apply.)

- Given email labeled as spam/not spam, learn a spam filter.
- Given a set of news articles found on the web, group them into set of articles about the same story.
- Given a database of customer data, automatically discover market segments and group customers into different market segments.
- Given a dataset of patients diagnosed as either having diabetes or not, learn to classify new patients as having diabetes or not.

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1.2 Gradient Descent



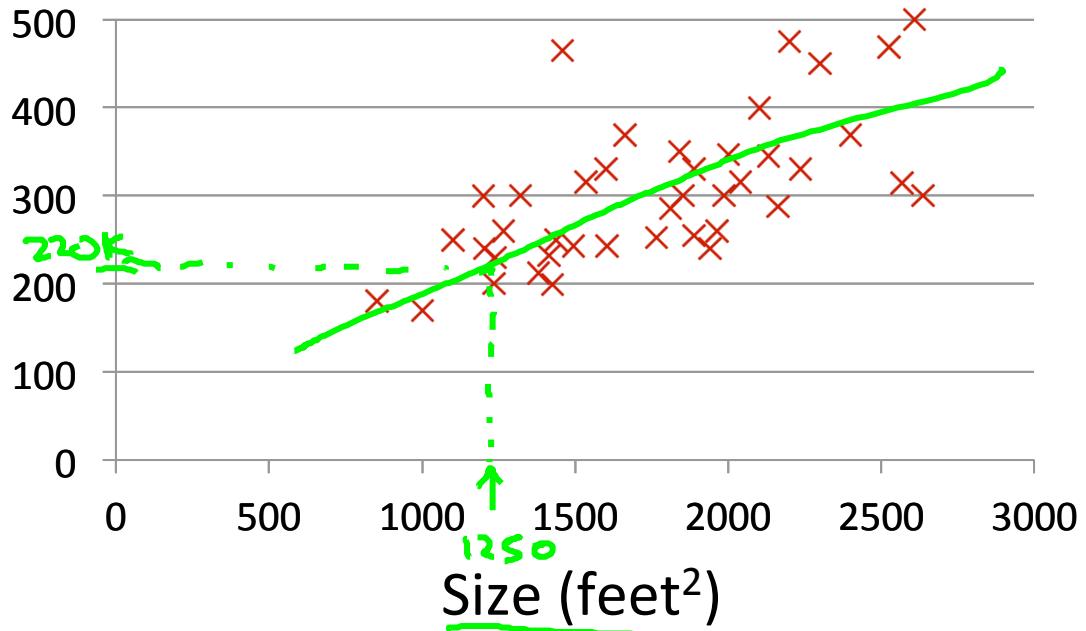
Machine Learning

Linear regression with one variable

Model representation

Housing Prices (Portland, OR)

Price
(in 1000s
of dollars)



Supervised Learning

Given the "right answer" for each example in the data.

Regression Problem

Predict real-valued output

Classification: Discrete-valued output

Training set of housing prices (Portland, OR)

Size in feet ² (x)	Price (\$) in 1000's (y)
→ 2104	460
→ 1416	232
→ 1534	315
852	178
...	...
i	i

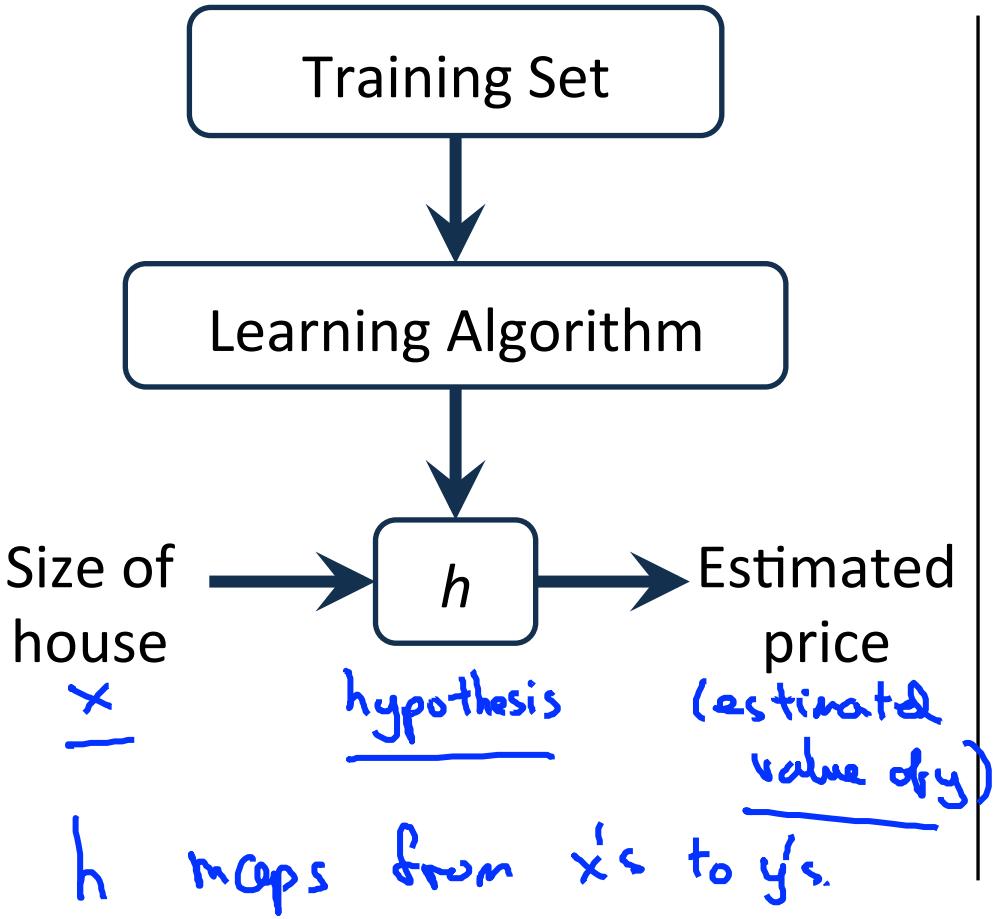
Notation:

- m = Number of training examples
- x 's = "input" variable / features
- y 's = "output" variable / "target" variable

(x, y) - one training example

$(x^{(i)}, y^{(i)})$ - ith training example

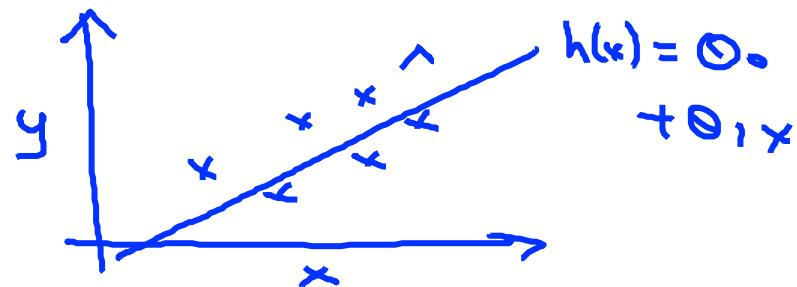
$$\begin{cases} x^{(1)} = 2104 \\ x^{(2)} = 1416 \\ \vdots \\ y^{(1)} = 460 \end{cases}$$



How do we represent h ?

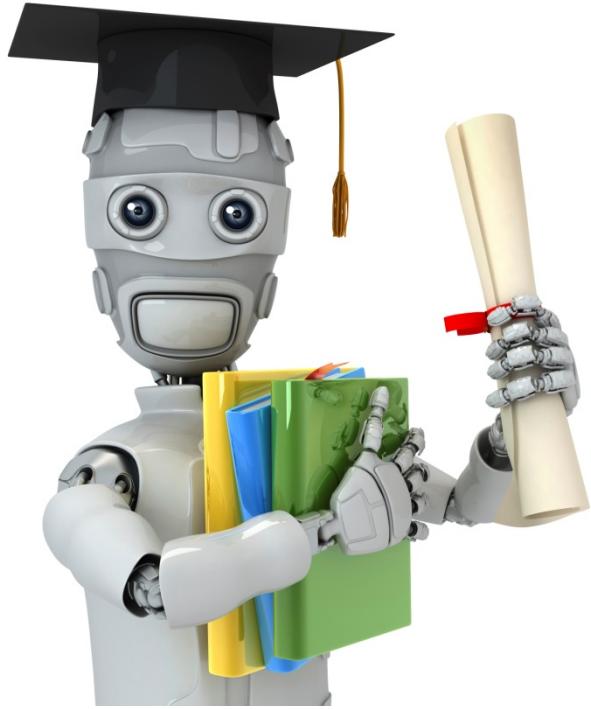
$$h_{\theta}(x) = \underline{\theta_0 + \theta_1 x}$$

Shorthand: $h(x)$



Linear regression with one variable.
Univariate linear regression.

one variable



Machine Learning

Linear regression with one variable

Cost function

Training Set

	Size in feet ² (x)	Price (\$) in 1000's (y)
	2104	460
	1416	232
	1534	315
	852	178

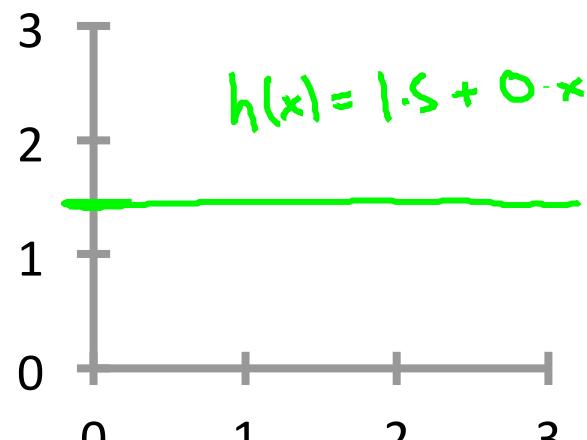
$$m = 47$$

Hypothesis:
$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

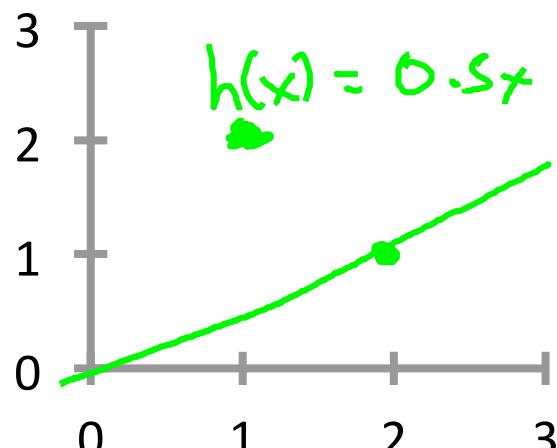
θ_i 's: Parameters

How to choose θ_i 's ?

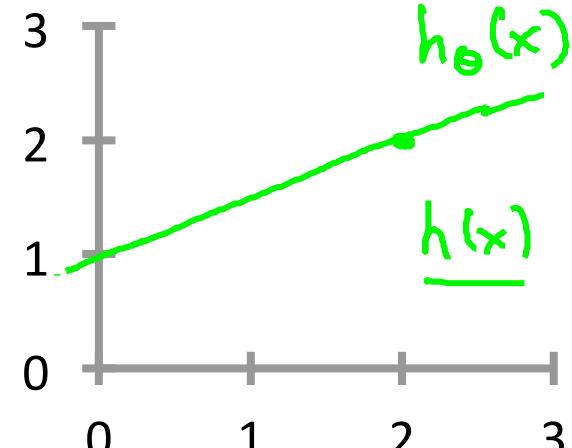
$$\underline{h_{\theta}(x) = \theta_0 + \theta_1 x}$$



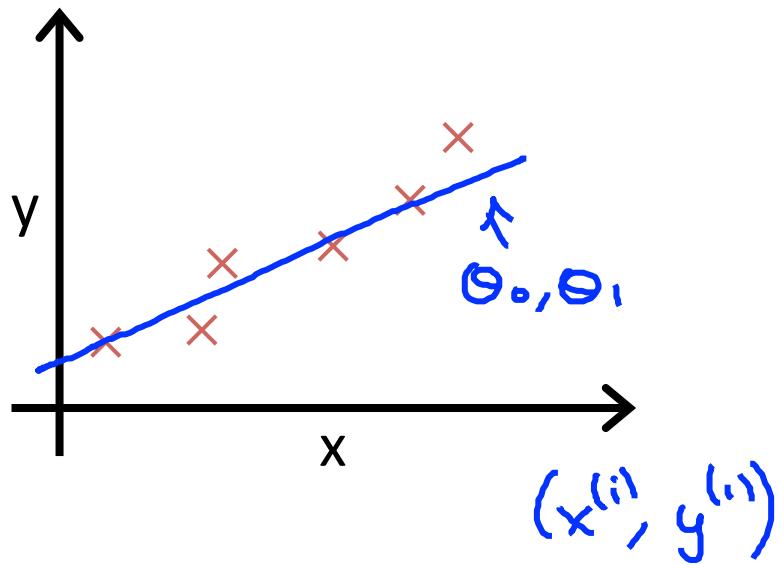
$$\rightarrow \theta_0 = 1.5$$
$$\rightarrow \theta_1 = 0$$



$$\rightarrow \theta_0 = 0$$
$$\rightarrow \theta_1 = 0.5$$



$$\rightarrow \theta_0 = 1$$
$$\rightarrow \theta_1 = 0.5$$



Idea: Choose $\underline{\theta_0}, \underline{\theta_1}$ so that
 $\underline{h_\theta(x)}$ is close to \underline{y} for our
 training examples $\underline{(x, y)}$

x, y

minimize $\underline{\theta_0, \theta_1}$

$$\frac{1}{2m} \sum_{i=1}^m (h_\theta(\underline{x^{(i)}}) - \underline{y^{(i)}})^2$$

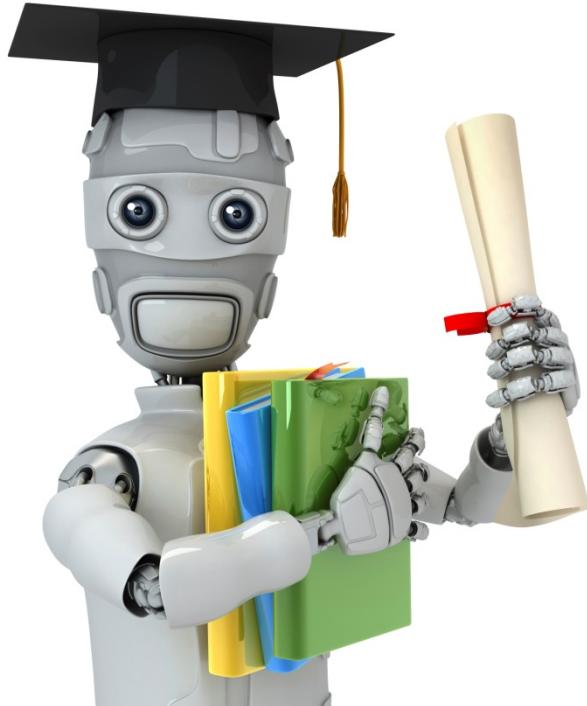
\uparrow

$$h_\theta(\underline{x^{(i)}}) = \underline{\theta_0 + \theta_1 x^{(i)}}$$

$$J(\underline{\theta_0, \theta_1}) = \frac{1}{2m} \sum_{i=1}^m (h_\theta(\underline{x^{(i)}}) - \underline{y^{(i)}})^2$$

minimize $\underline{\theta_0, \theta_1}$ $\underbrace{J(\underline{\theta_0, \theta_1})}$

Squared error function



Machine Learning

Linear regression with one variable

Cost function intuition I

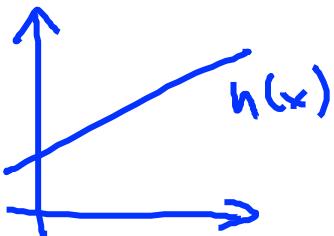
Simplified

Hypothesis:

$$\underline{h_{\theta}(x) = \theta_0 + \theta_1 x}$$

Parameters:

$$\underline{\theta_0, \theta_1}$$



Cost Function:

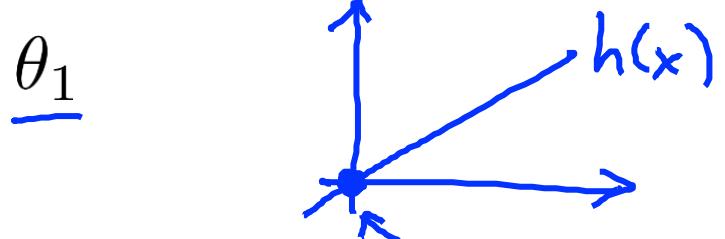
$$\rightarrow J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

Goal: minimize $J(\theta_0, \theta_1)$

$$\nearrow \theta_0, \theta_1$$

$$h_{\theta}(x) = \underline{\theta_1 x}$$

$$\underline{\theta_0 = 0}$$



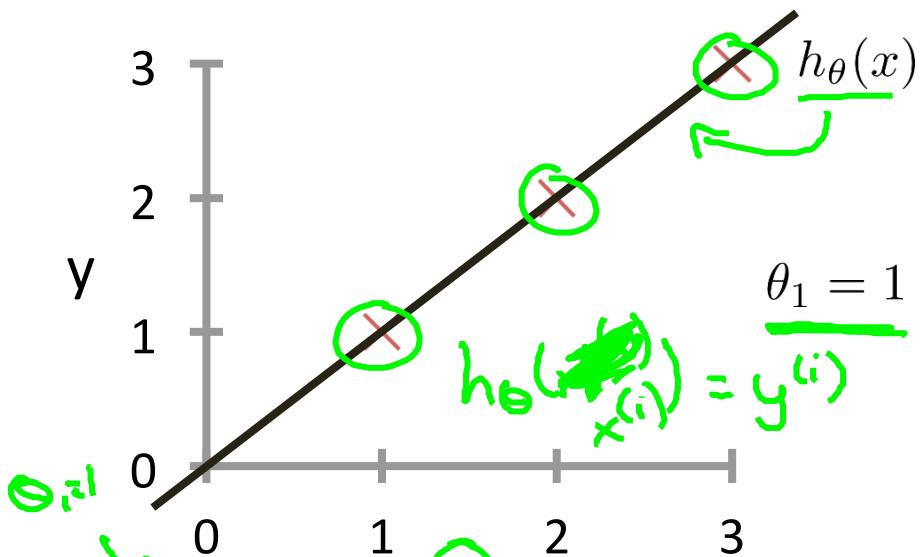
$$J(\theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

$$\text{minimize } \underline{J(\theta_1)}$$

$$\underline{\theta_0, x^{(i)}}$$

$\rightarrow h_{\theta}(x)$

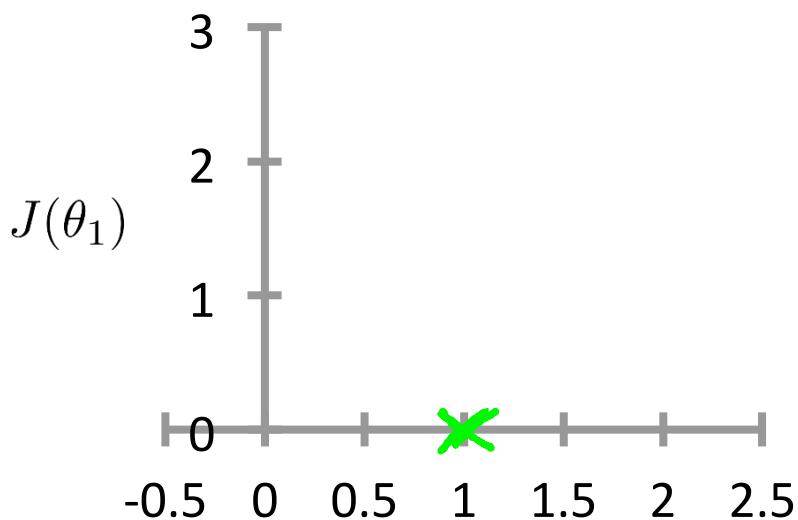
(for fixed θ_1 , this is a function of x)



$$\begin{aligned} J(\theta_1) &= \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 \\ &= \frac{1}{2m} \sum_{i=1}^m (\theta_0 + \theta_1 x^{(i)} - y^{(i)})^2 = \frac{1}{2m} (0^2 + 0^2 + 0^2) = 0^2 \end{aligned}$$

$\rightarrow J(\theta_1)$

(function of the parameter θ_1)

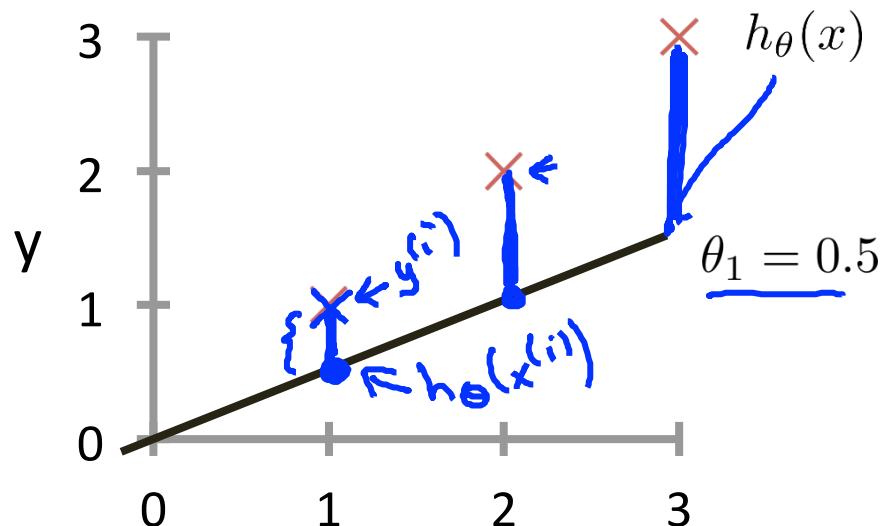


$$\theta_1 = 0.5 ?$$

$$(0^2 + 0^2 + 0^2) = 0^2$$

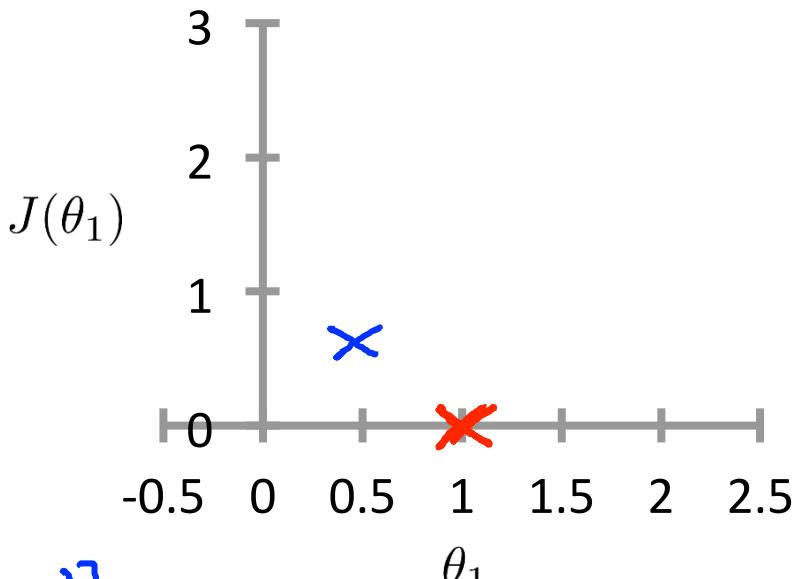
$$J(1) = 0$$

$h_\theta(x)$
(for fixed θ_1 , this is a function of x)

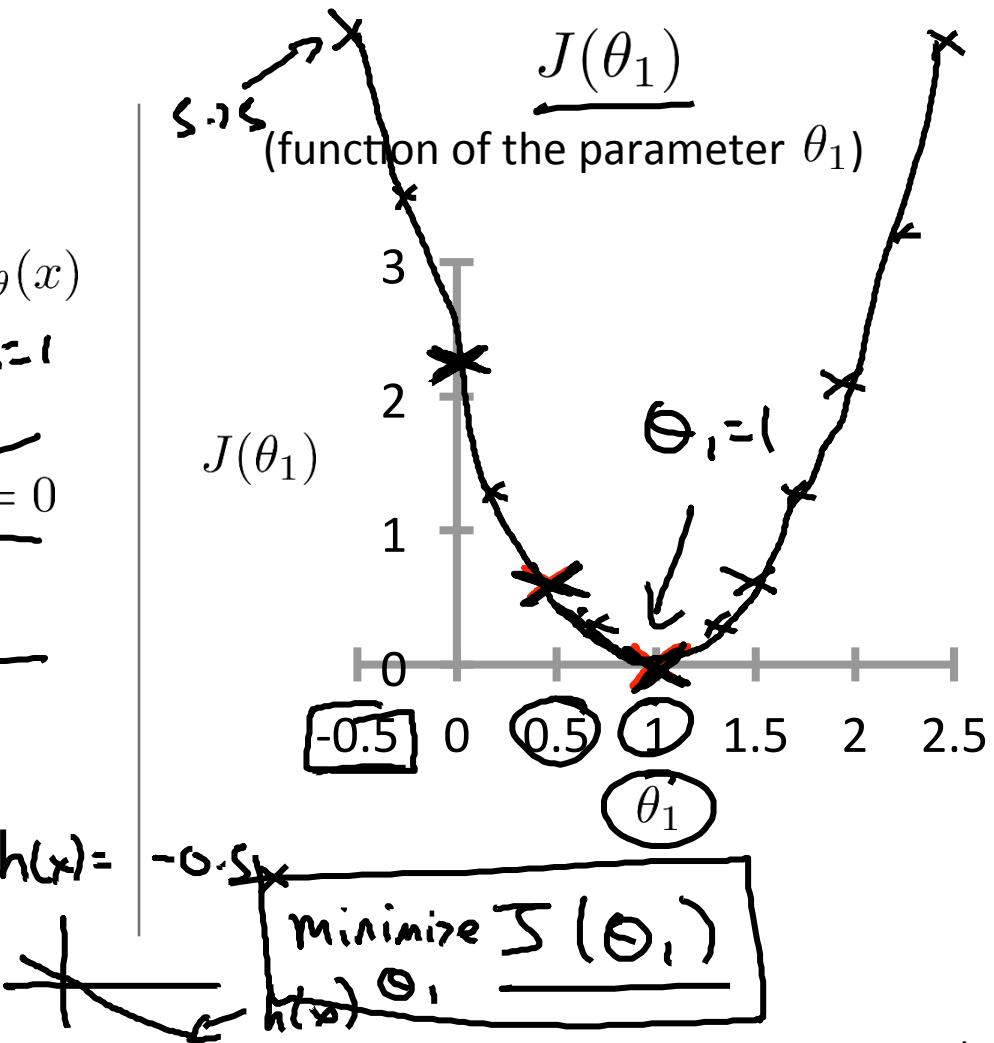
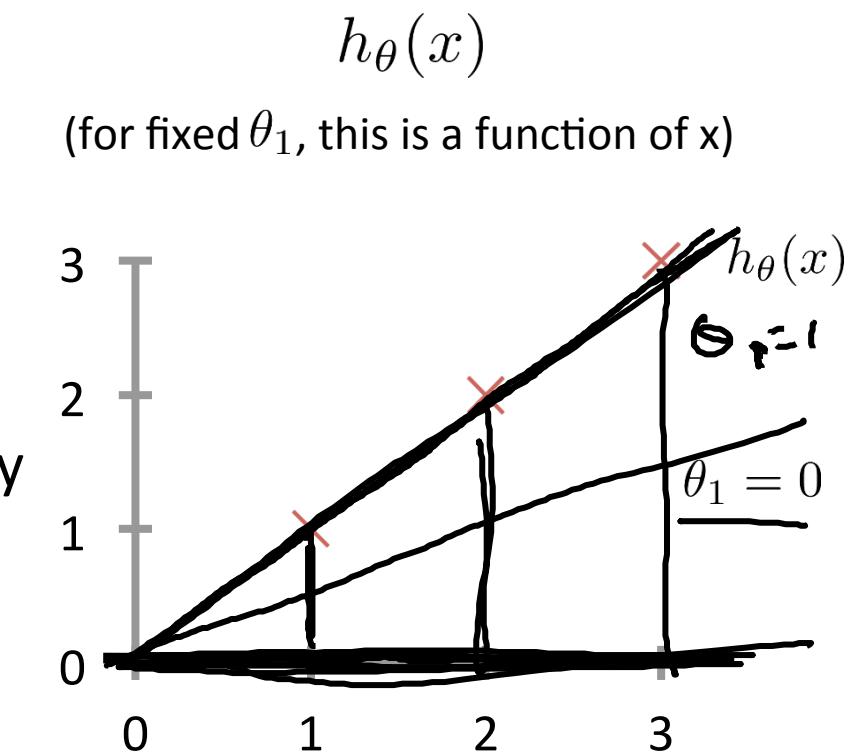


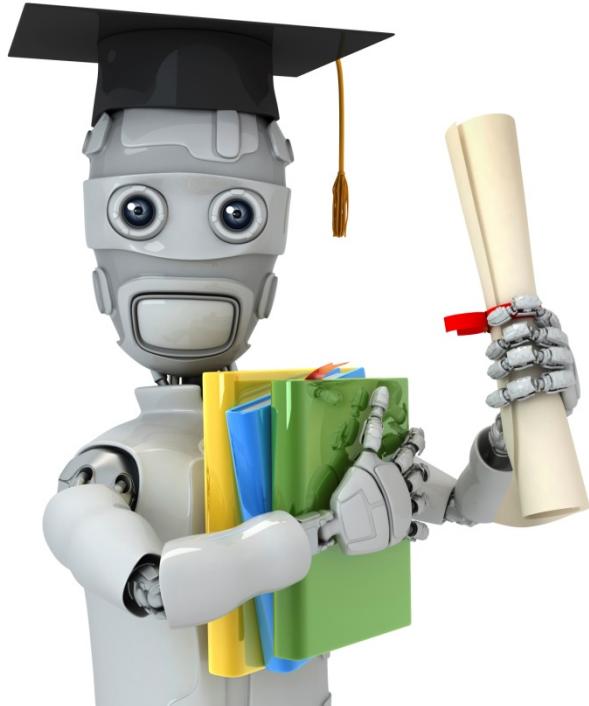
$$\begin{aligned} J(0.5) &= \frac{1}{2m} \sum_{i=1}^m [(0.5 - 1)^2 + (1 - 2)^2 + (1.5 - 3)^2] \\ &= \frac{1}{2 \times 3} (3.5) = \frac{3.5}{6} \approx \underline{0.58} \end{aligned}$$

$J(\theta_1)$
(function of the parameter θ_1)



$$\begin{aligned} \theta_1 &=? \\ J(0) &=? \end{aligned}$$





Machine Learning

Linear regression with one variable

Cost function intuition II

Hypothesis: $h_{\theta}(x) = \theta_0 + \theta_1 x$

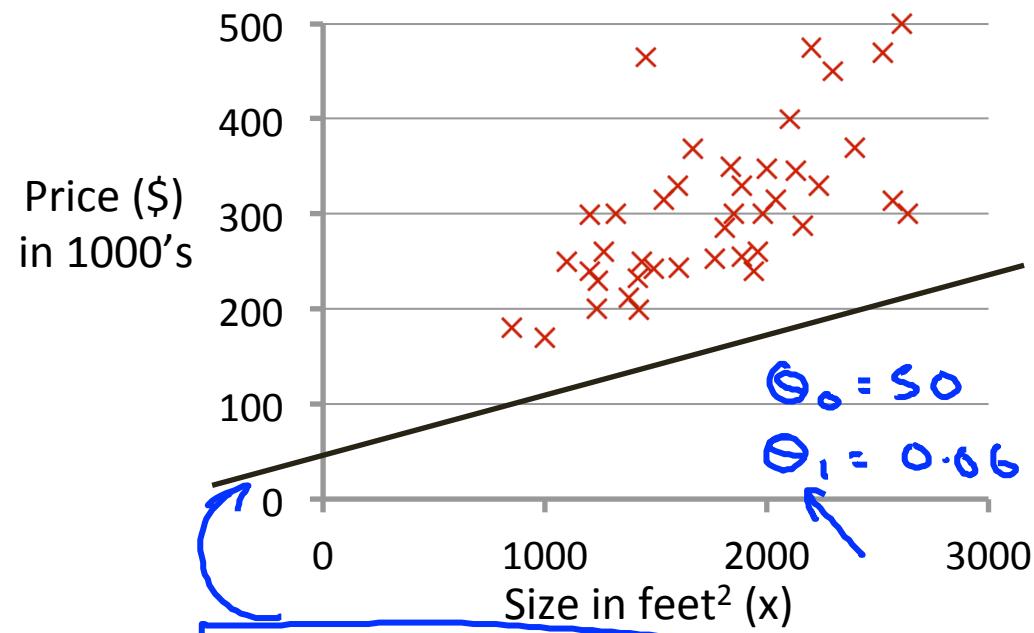
Parameters: θ_0, θ_1

Cost Function: $J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$

Goal: $\underset{\theta_0, \theta_1}{\text{minimize}} J(\theta_0, \theta_1)$

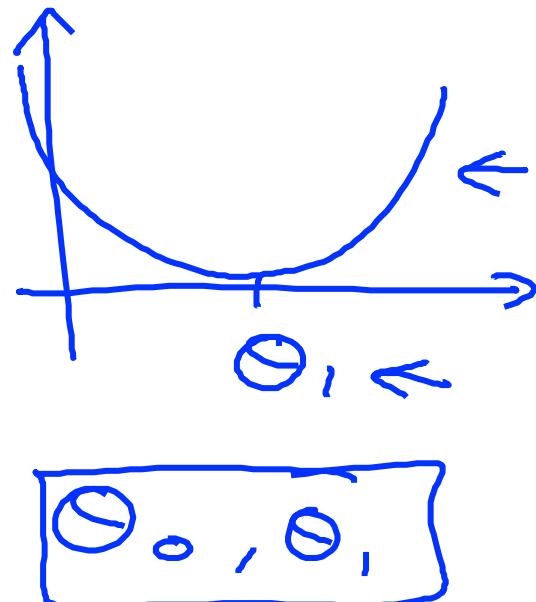
.

$h_{\theta}(x)$
(for fixed θ_0, θ_1 , this is a function of x)

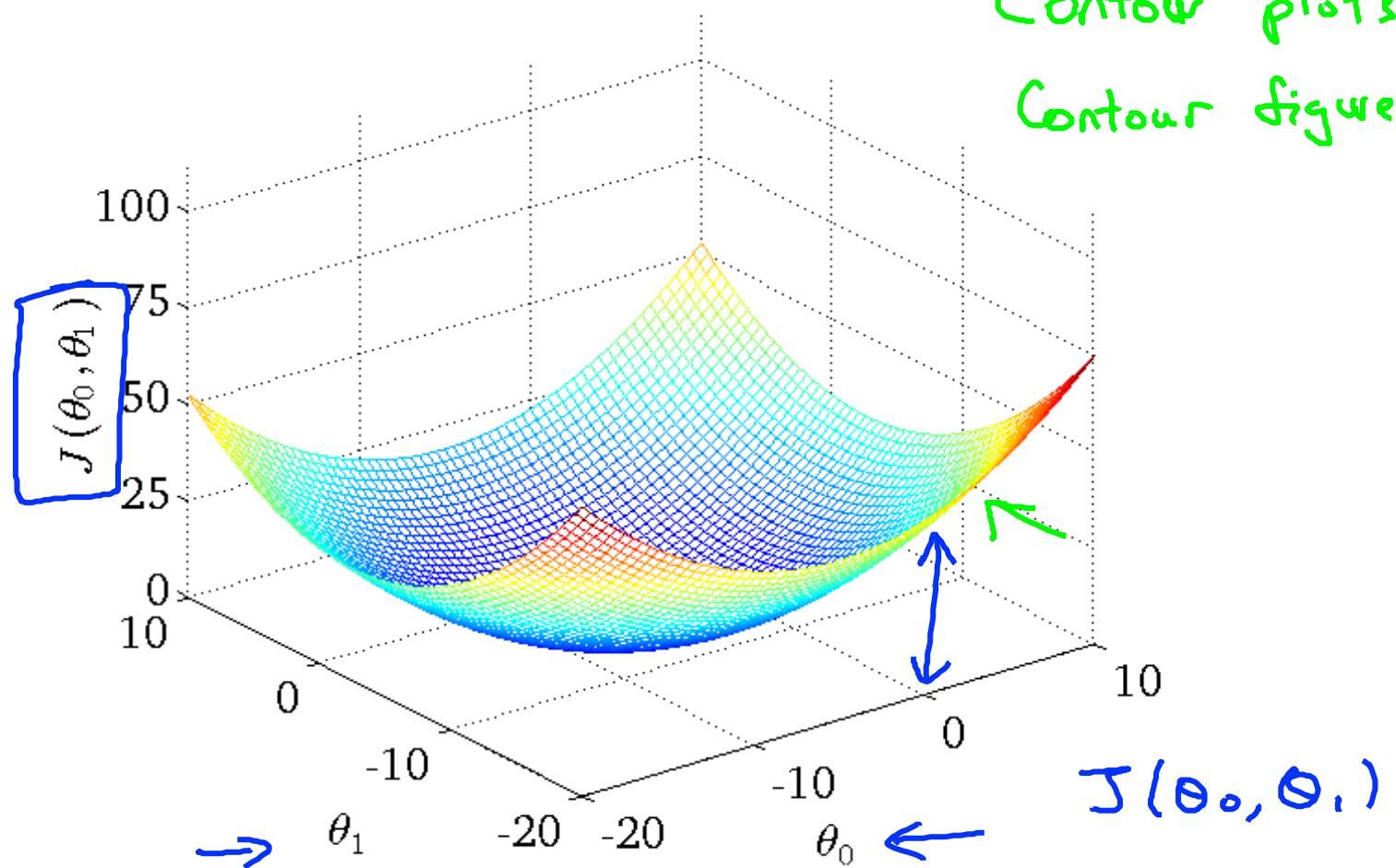


$$h_{\theta}(x) = 50 + 0.06x$$

$J(\theta_0, \theta_1)$
(function of the parameters θ_0, θ_1)

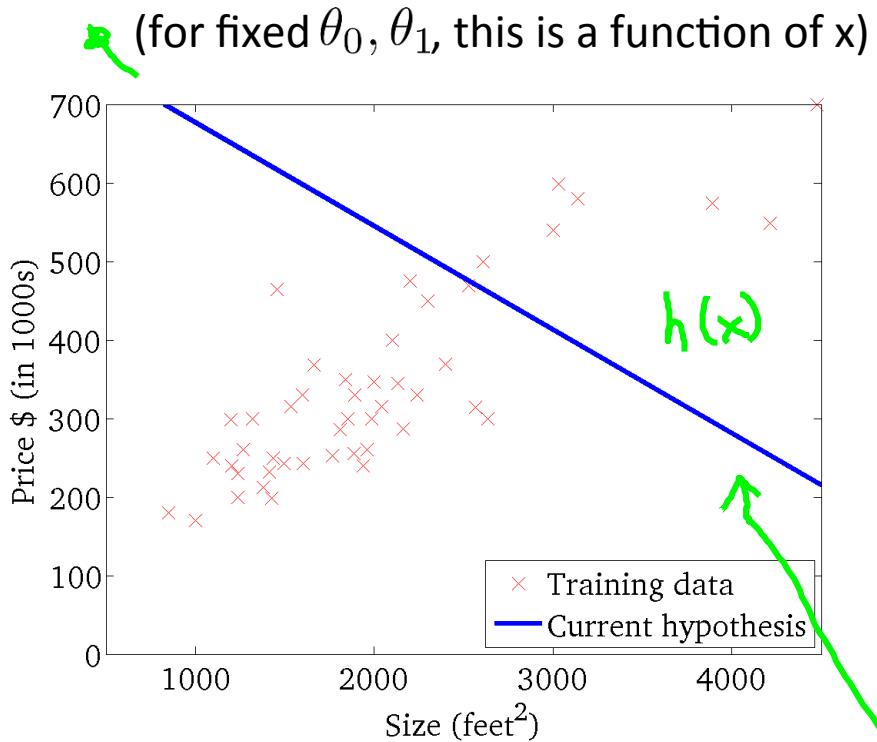


Contour plots
Contour figures -

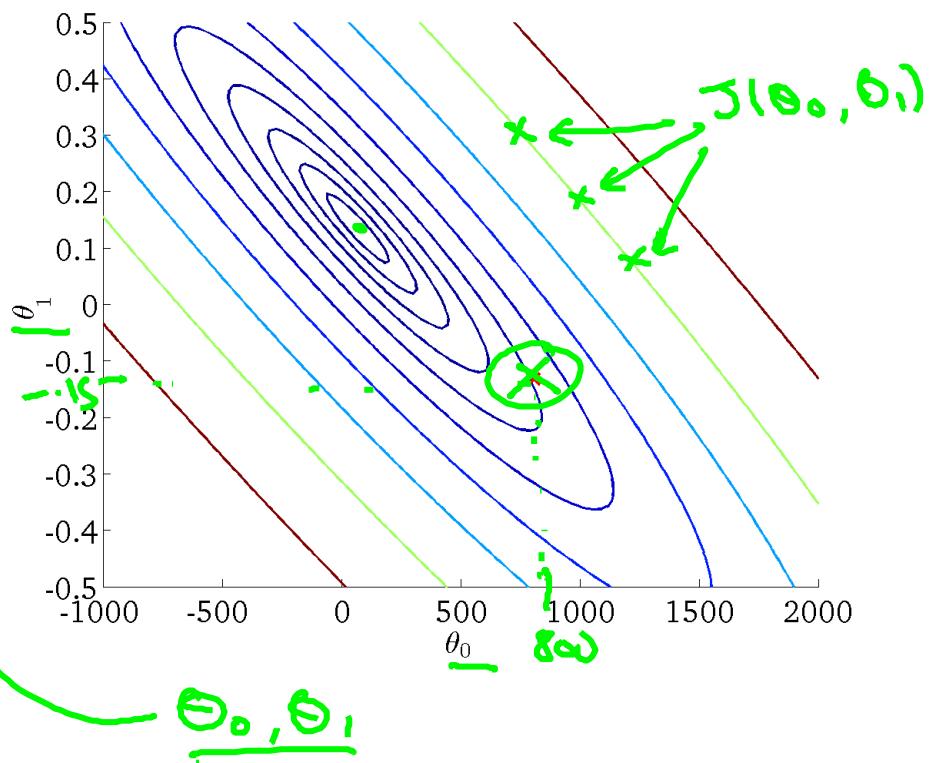


$$h_{\theta}(x)$$

$$J(\theta_0, \theta_1)$$

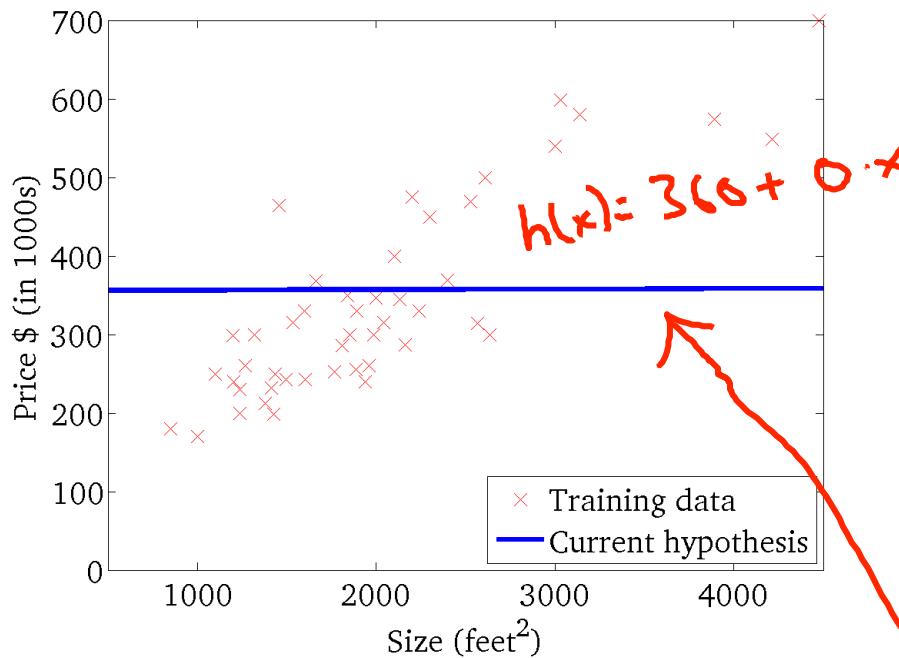


(function of the parameters θ_0, θ_1)



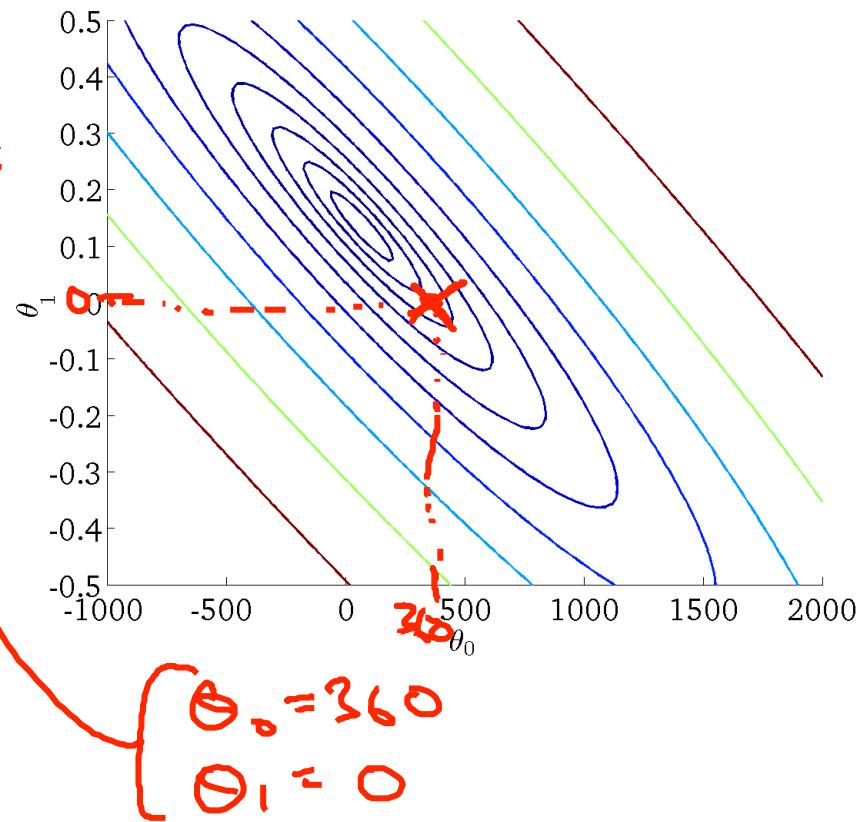
$$h_{\theta}(x)$$

(for fixed θ_0, θ_1 , this is a function of x)



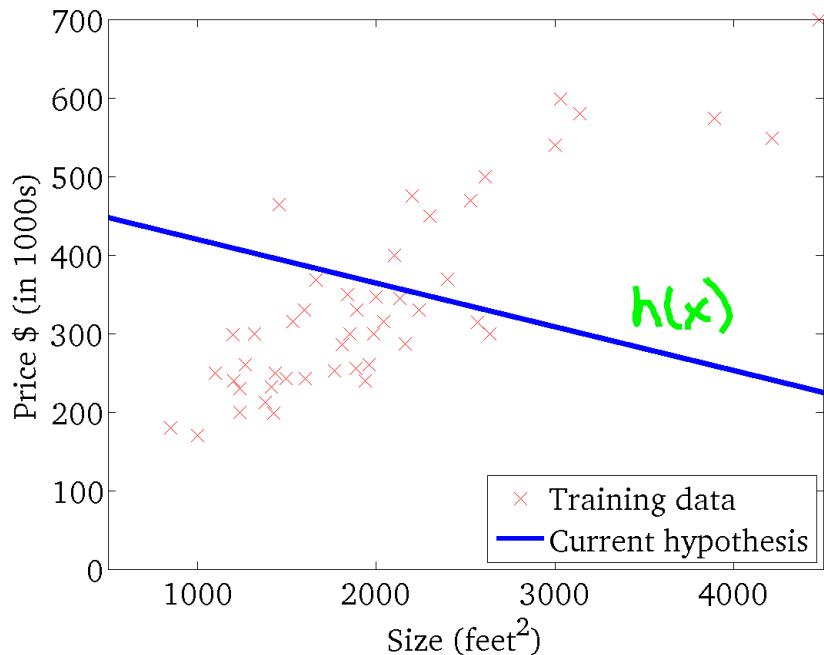
$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)



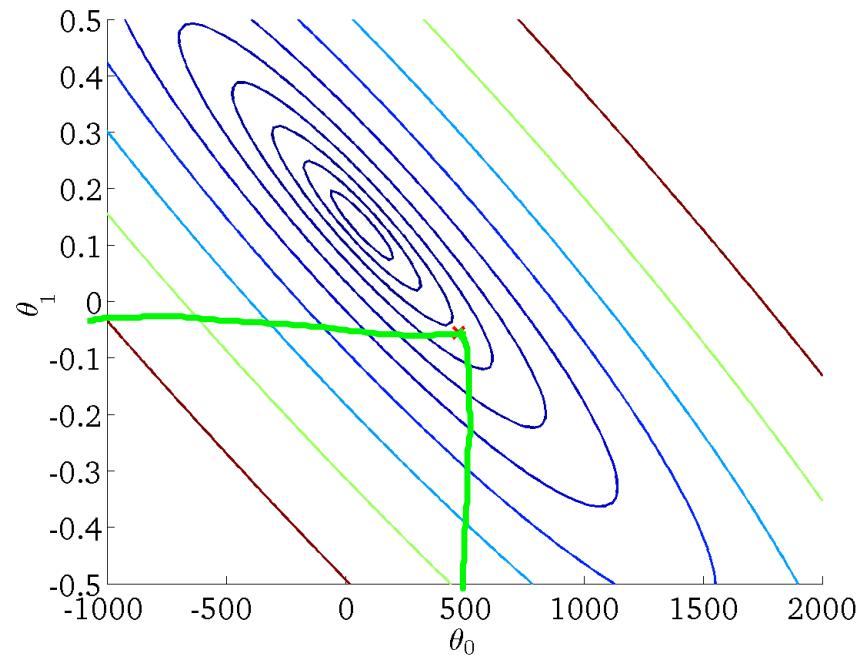
$$h_{\theta}(x)$$

(for fixed θ_0, θ_1 , this is a function of x)



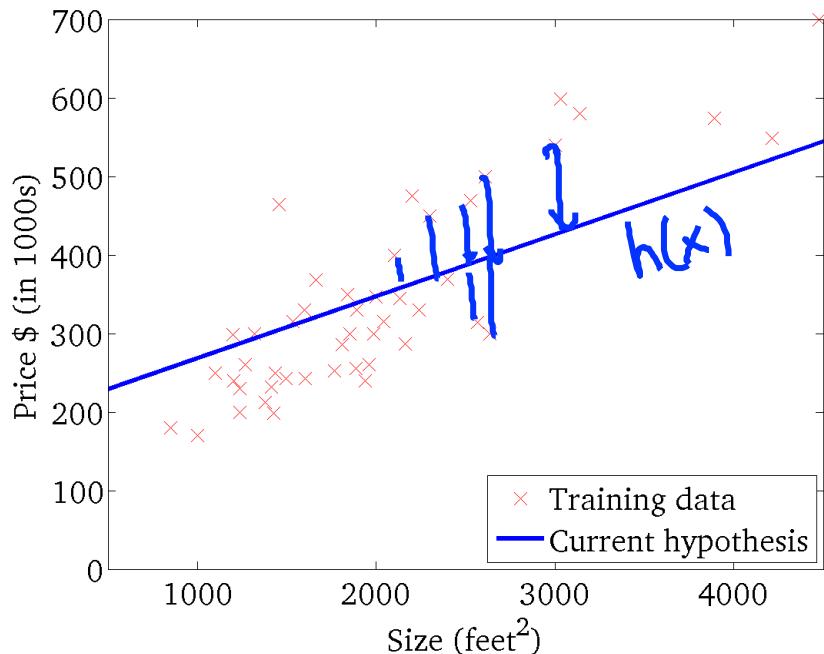
$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)



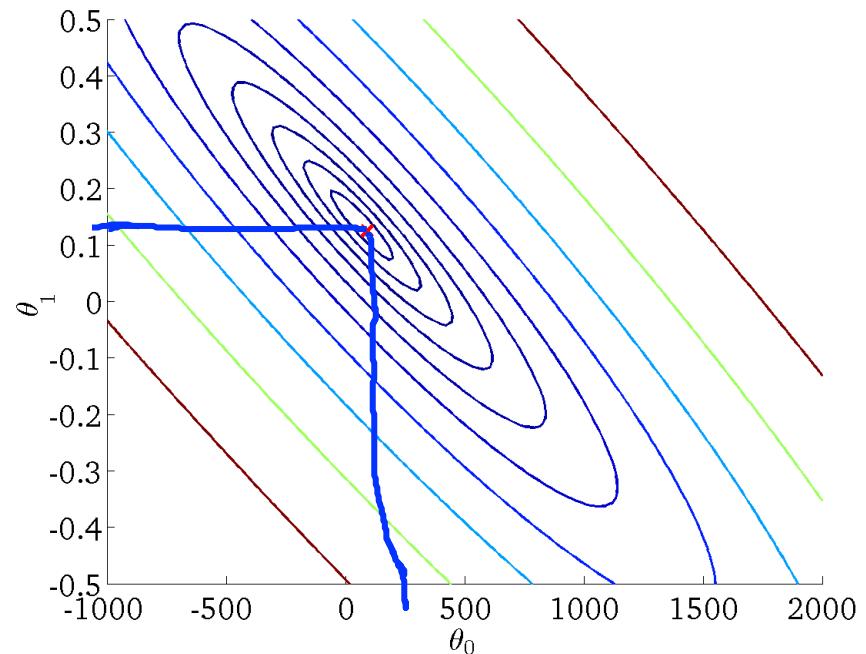
$$h_{\theta}(x)$$

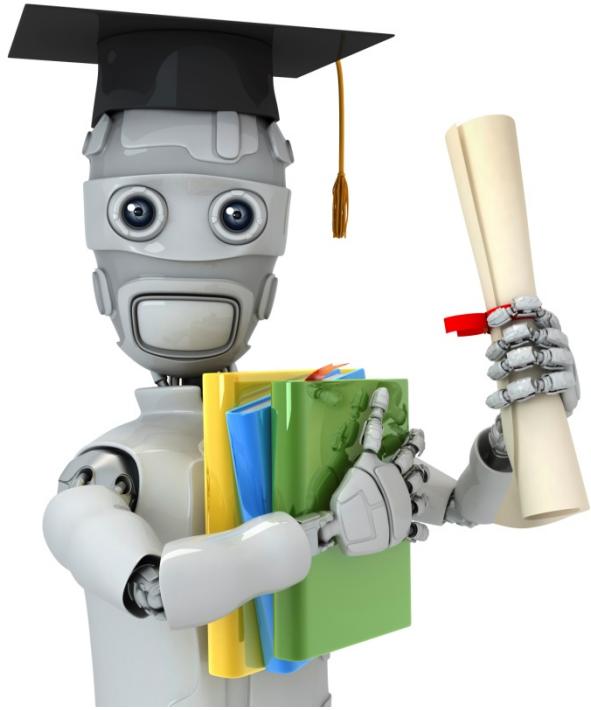
(for fixed θ_0, θ_1 , this is a function of x)



$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)





Machine Learning

Linear regression
with one variable

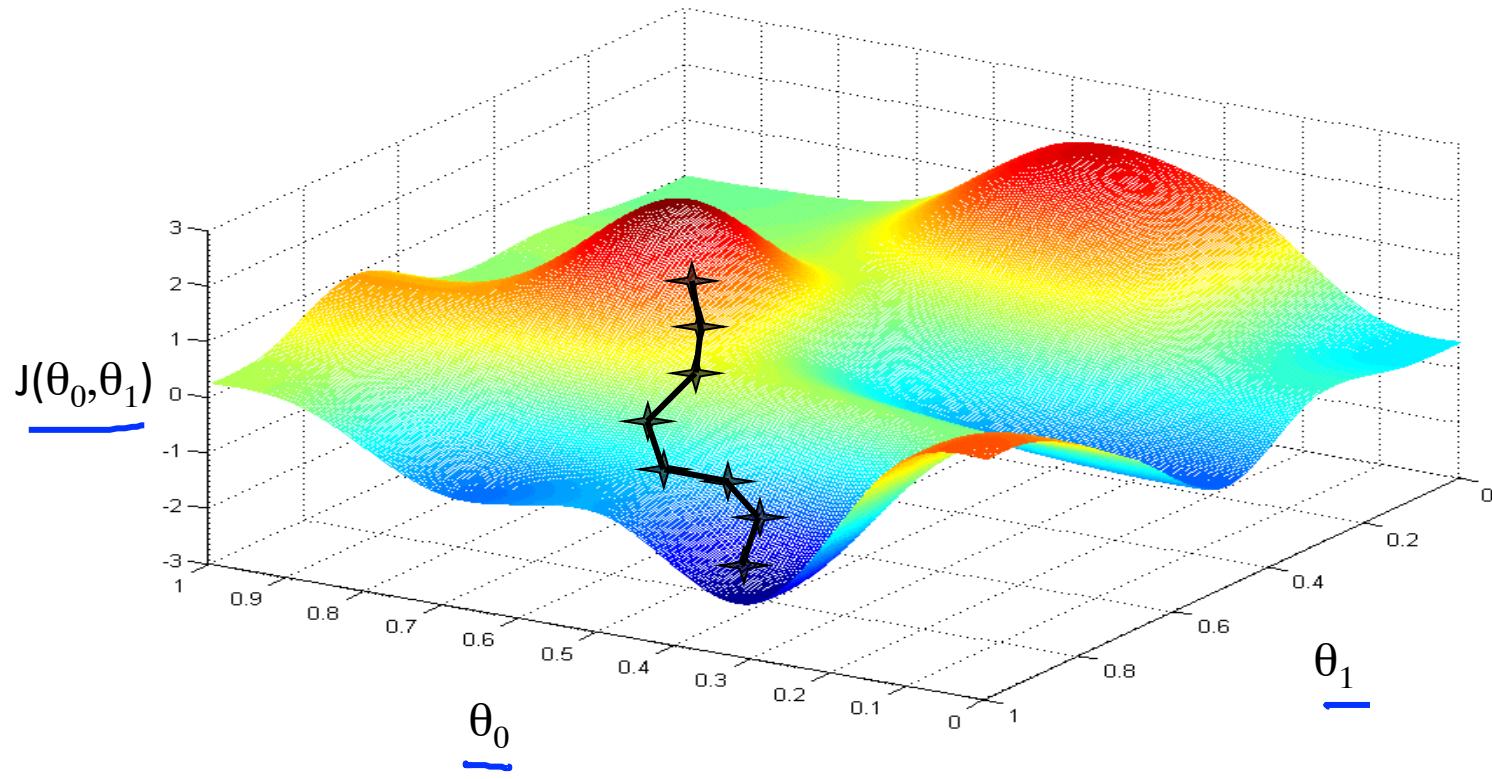
Gradient
descent

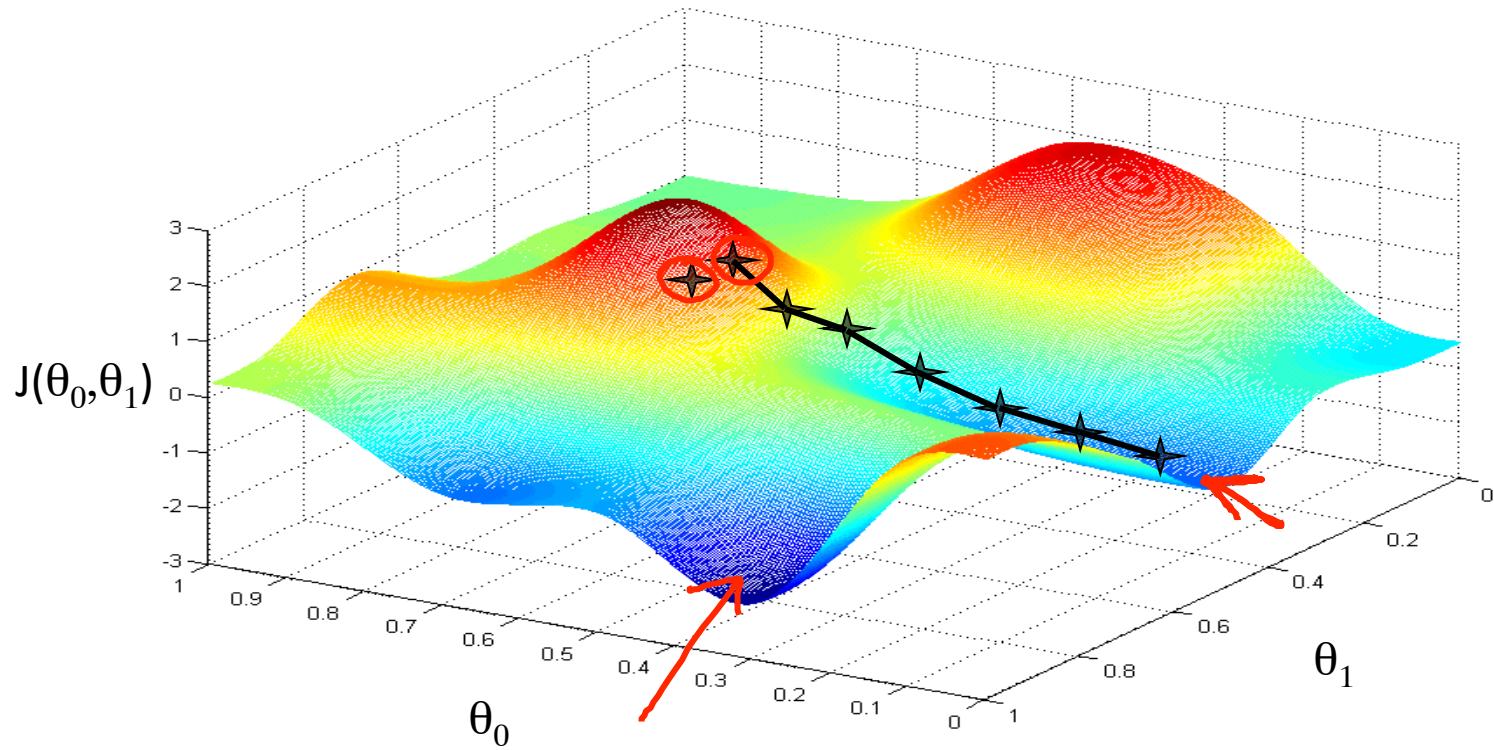
Have some function $\underline{J(\theta_0, \theta_1)}$ $J(\theta_0, \theta_1, \theta_2, \dots, \theta_n)$

Want $\min_{\theta_0, \theta_1} \underline{J(\theta_0, \theta_1)}$ $\min_{\theta_0, \dots, \theta_n} \underline{J(\theta_0, \dots, \theta_n)}$

Outline:

- Start with some $\underline{\theta_0, \theta_1}$ (say $\theta_0 = 0, \theta_1 = 0$)
- Keep changing $\underline{\theta_0, \theta_1}$ to reduce $\underline{J(\theta_0, \theta_1)}$
until we hopefully end up at a minimum





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Gradient descent algorithm

repeat until convergence {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$$

learning rate

θ_0, θ_1

Assignment

$$\begin{array}{l} a := b \\ \quad \uparrow \\ a := a + 1 \end{array}$$

Truth assertion

$$a = b \leftarrow$$

$$a = a + 1 \times$$

(for $j = 0$ and $j = 1$)

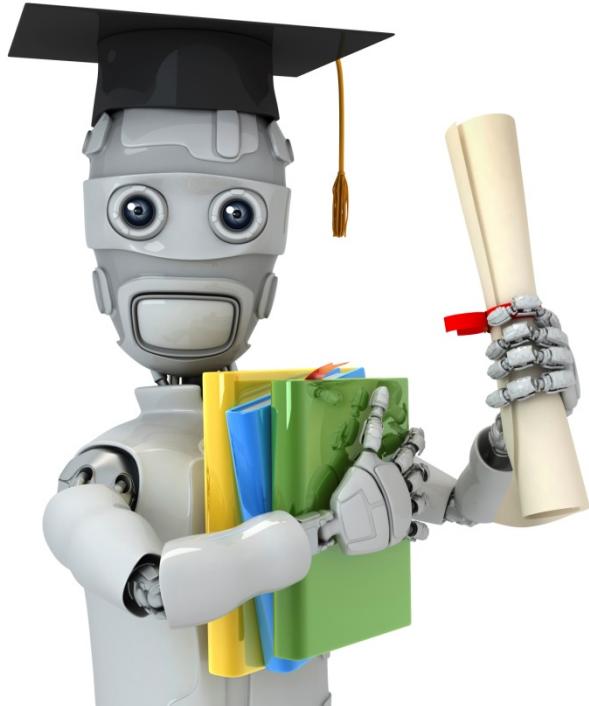
Simultaneously update
 θ_0 and θ_1

Correct: Simultaneous update

- $\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$
- $\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$
- $\theta_0 := \text{temp0}$
- $\theta_1 := \text{temp1}$

Incorrect:

- $\text{temp0} := \theta_0 - \alpha \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$
- $\theta_0 := \text{temp0}$
- $\text{temp1} := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$
- $\theta_1 := \text{temp1}$



Machine Learning

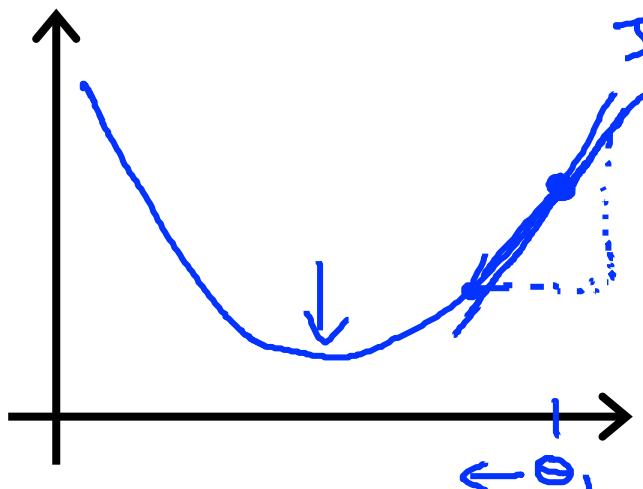
Linear regression with one variable

Gradient descent intuition

Gradient descent algorithm

repeat until convergence {
→ $\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$ (simultaneously update
}
learning rate derivative
 $j = 0$ and $j = 1$)

$$\min_{\theta_1} J(\theta_1) \quad \theta_1 \in \mathbb{R}$$

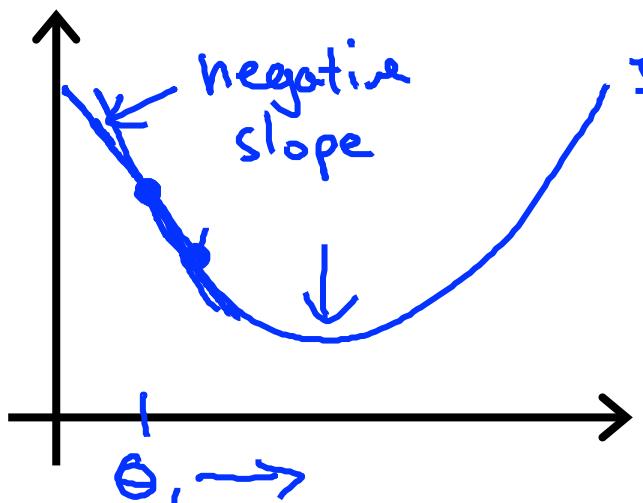


$J(\theta_1)$ ($\theta_1 \in \mathbb{R}$)

$$\theta_1 := \theta_1 - \frac{\alpha}{\frac{\partial}{\partial \theta_1} J(\theta_1)} \geq 0$$

Diagram illustrating the update rule: A blue box contains the formula $\theta_1 := \theta_1 - \frac{\alpha}{\frac{\partial}{\partial \theta_1} J(\theta_1)}$. Above the box, a small diagram shows a blue rectangle with a horizontal arrow labeled $\frac{\partial}{\partial \theta_1}$ pointing to its right side. A blue circle with a minus sign inside is placed over the term $\frac{\alpha}{\frac{\partial}{\partial \theta_1}}$.

$$\theta_1 := \theta_1 - \frac{\alpha}{\frac{\partial}{\partial \theta_1} J(\theta_1)} \cdot (\text{positive number})$$

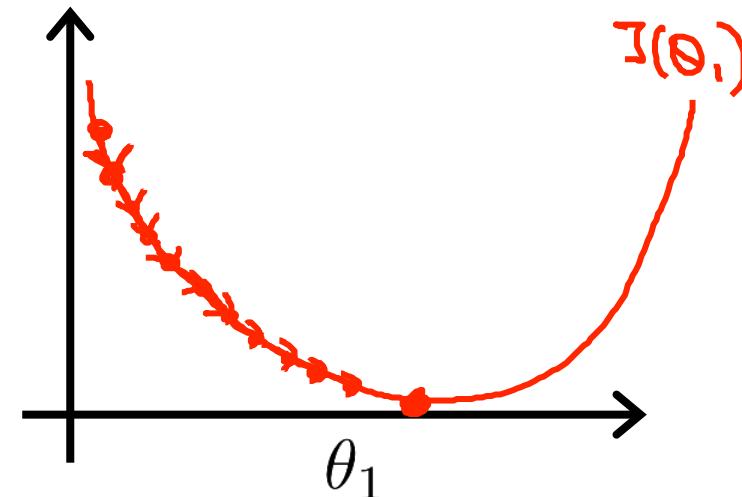


$$\frac{\frac{\partial}{\partial \theta_1} J(\theta_1)}{\leq 0}$$

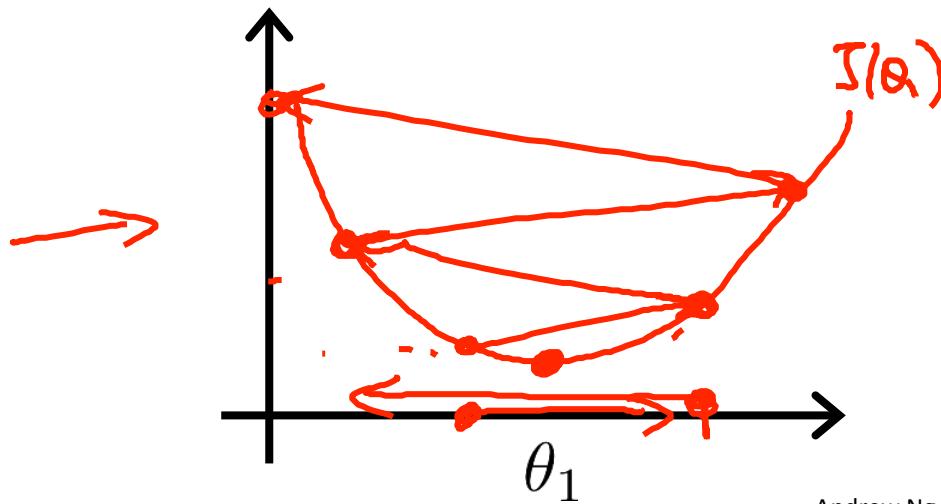
$$\theta_1 := \theta_1 - \frac{\alpha}{\frac{\partial}{\partial \theta_1} J(\theta_1)} \cdot (\text{negative number})$$

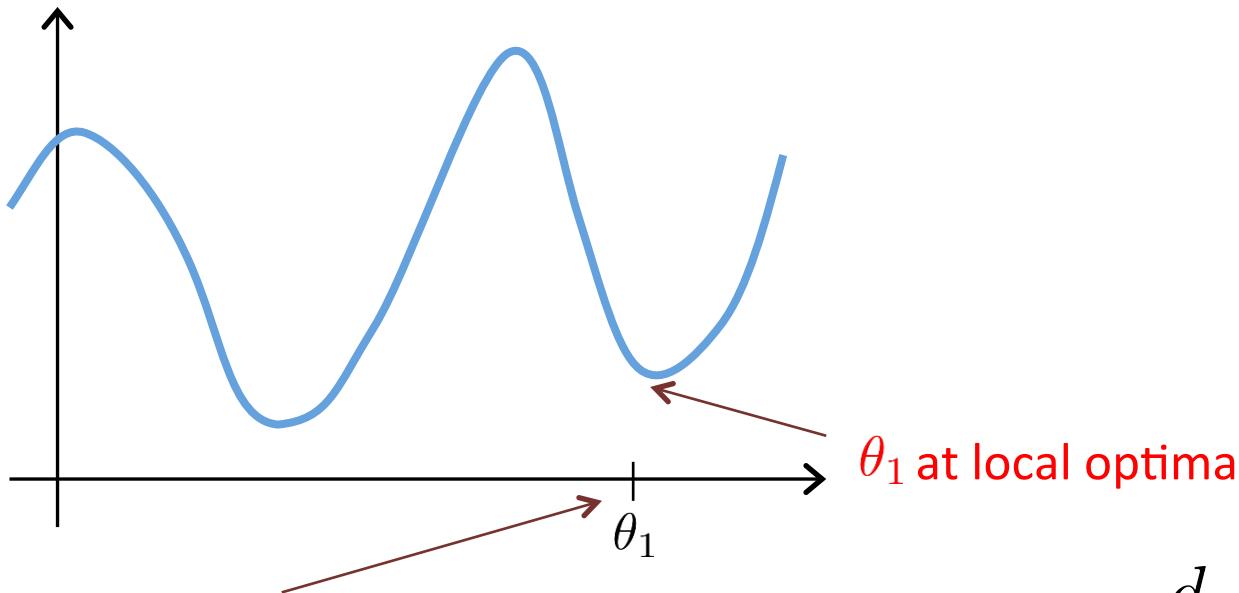
$$\theta_1 := \theta_1 - \alpha \frac{\partial}{\partial \theta_1} J(\theta_1)$$

If α is too small, gradient descent can be slow.



If α is too large, gradient descent can overshoot the minimum. It may fail to converge, or even diverge.



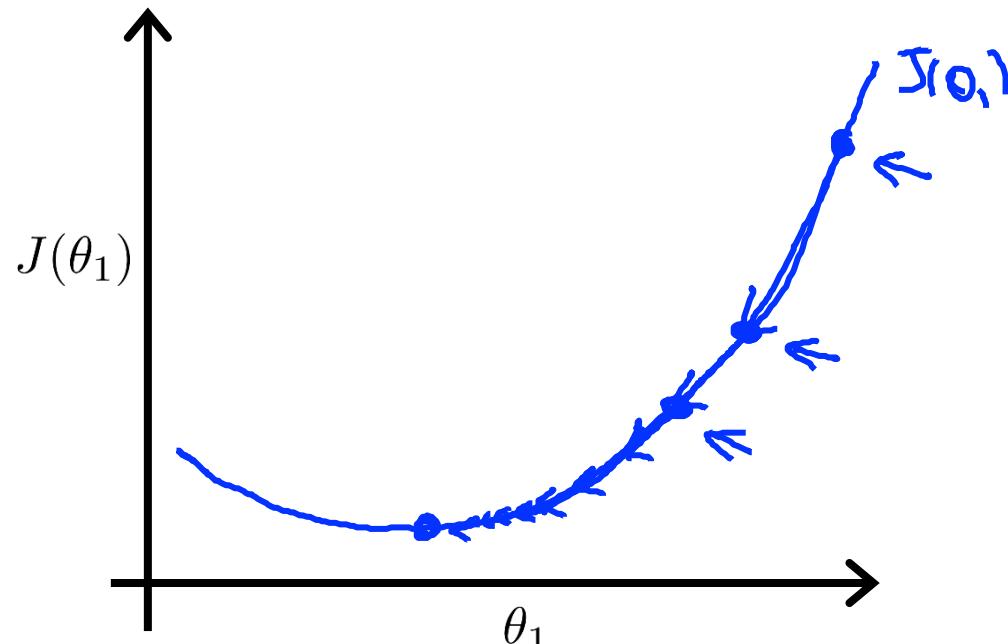


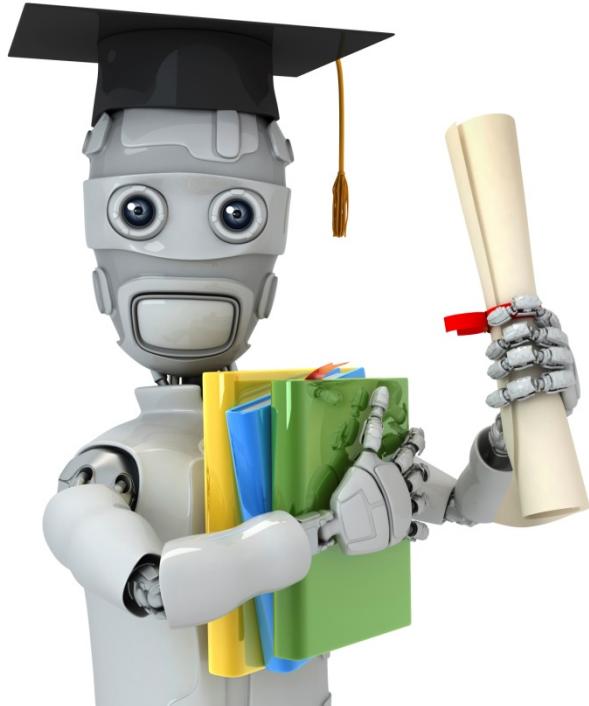
$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$$

Gradient descent can converge to a local minimum, even with the learning rate α fixed.

$$\theta_1 := \theta_1 - \alpha \frac{d}{d\theta_1} J(\theta_1)$$

As we approach a local minimum, gradient descent will automatically take smaller steps. So, no need to decrease α over time.





Machine Learning

Linear regression with one variable

Gradient descent for linear regression

Gradient descent algorithm

```
repeat until convergence {  
     $\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1)$   
    (for  $j = 1$  and  $j = 0$ )  
}
```

Linear Regression Model

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

$$\frac{\partial}{\partial \theta_j} J(\theta_0, \theta_1) = \frac{2}{m} \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

$$= \frac{2}{m} \frac{1}{m} \sum_{i=1}^m (\underline{\theta_0 + \theta_1 x^{(i)}} - y^{(i)})^2$$

$$j = 0 : \frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})$$

$$j = 1 : \frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1) = \frac{1}{m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)}) \cdot x^{(i)}$$

Gradient descent algorithm

repeat until convergence {

$$\theta_0 := \theta_0 - \alpha \left[\frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) \right]$$

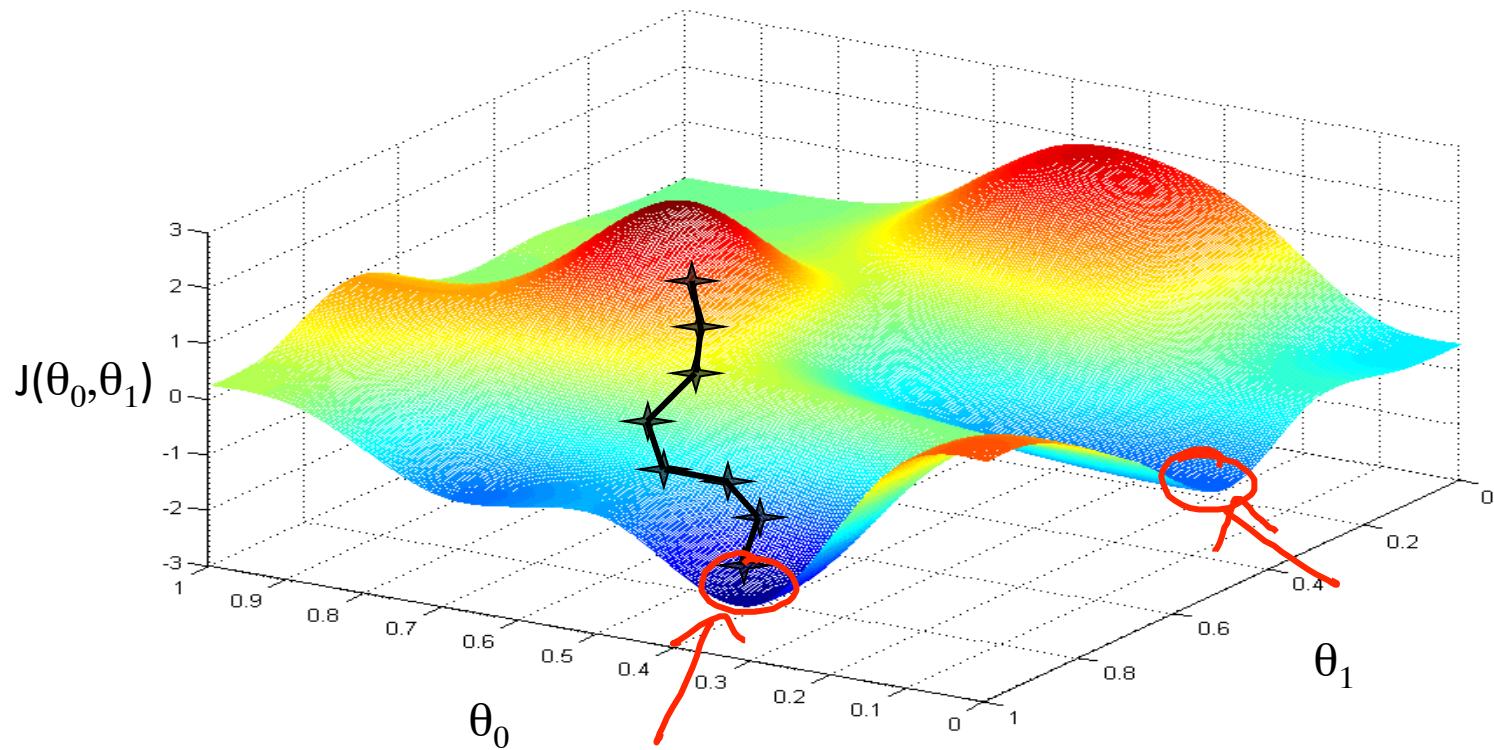
$$\theta_1 := \theta_1 - \alpha \left[\frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) \cdot x^{(i)} \right]$$

}

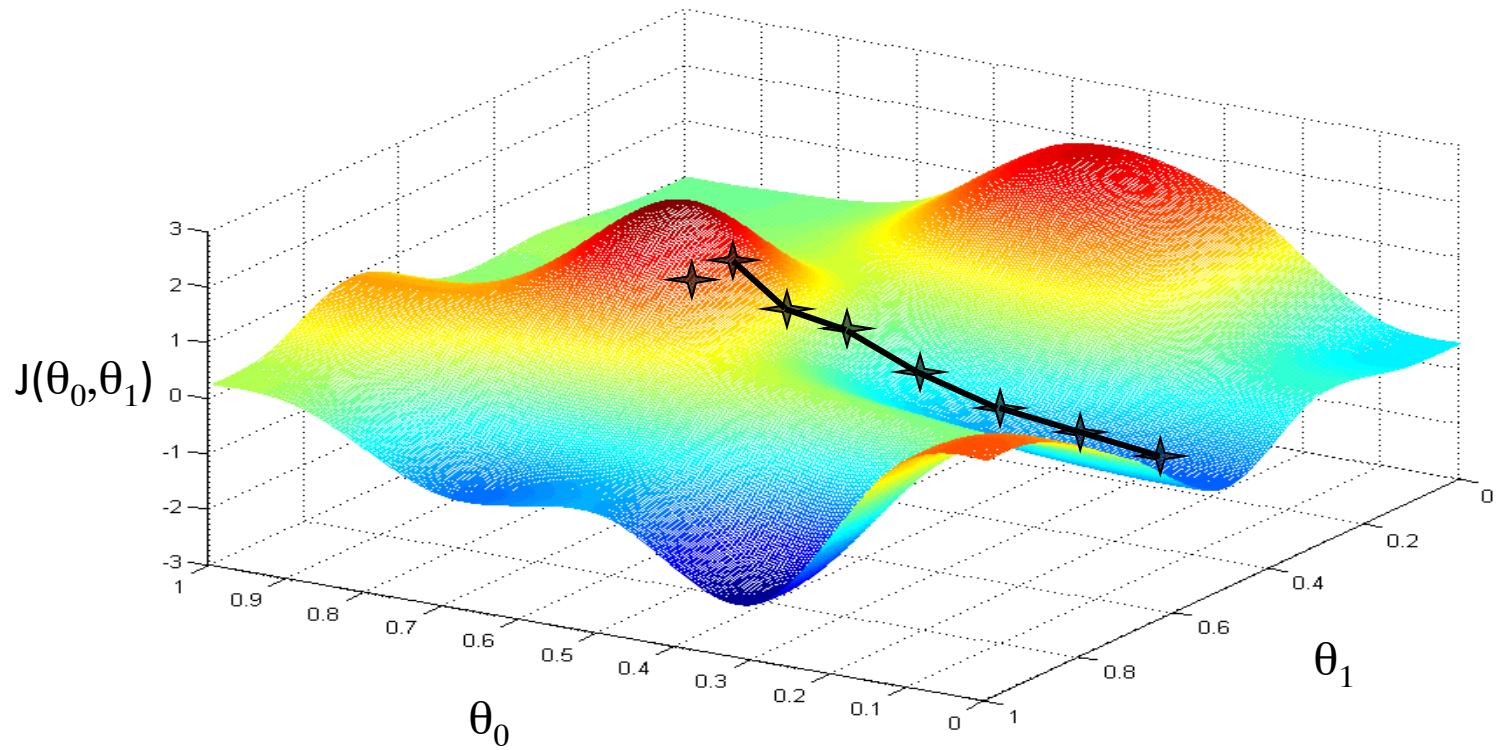
$$\frac{\partial}{\partial \theta_0} J(\theta_0, \theta_1)$$

update
 θ_0 and θ_1
simultaneously

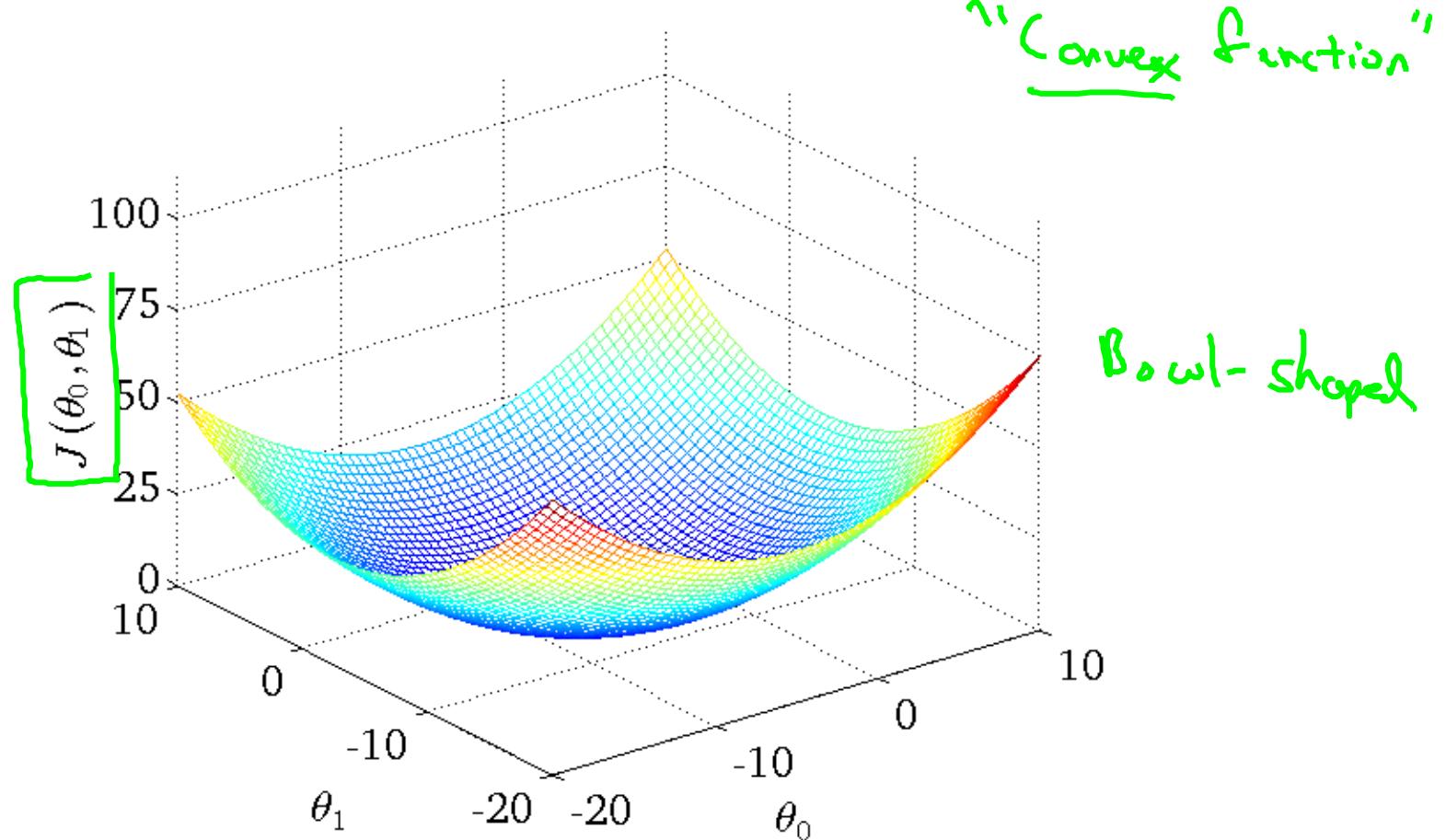
$$\frac{\partial}{\partial \theta_1} J(\theta_0, \theta_1)$$



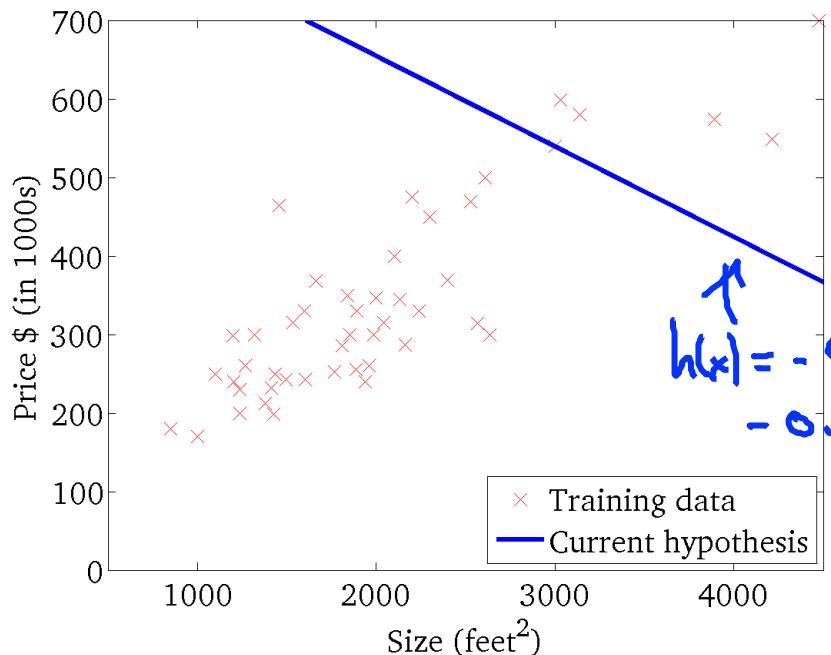
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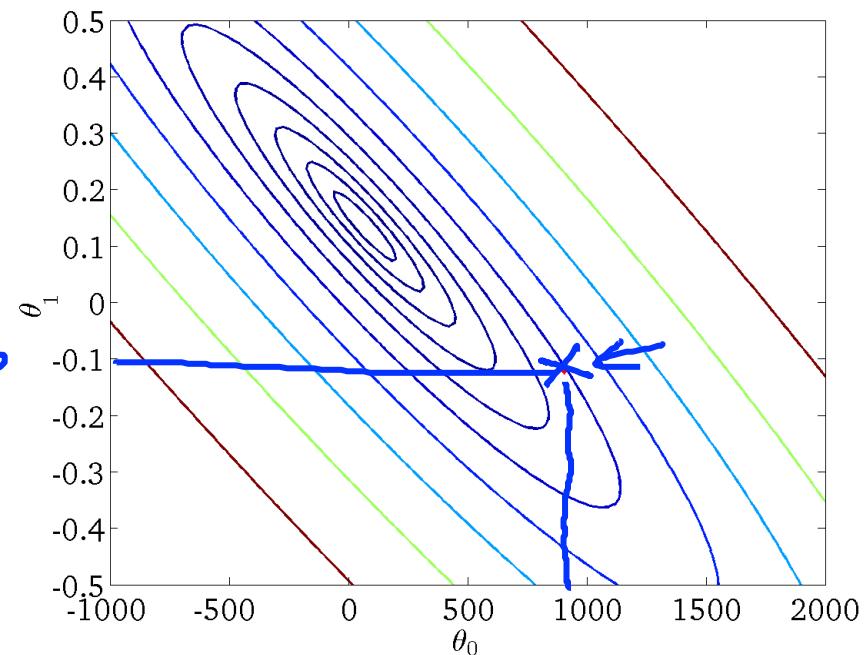
Andrew Ng



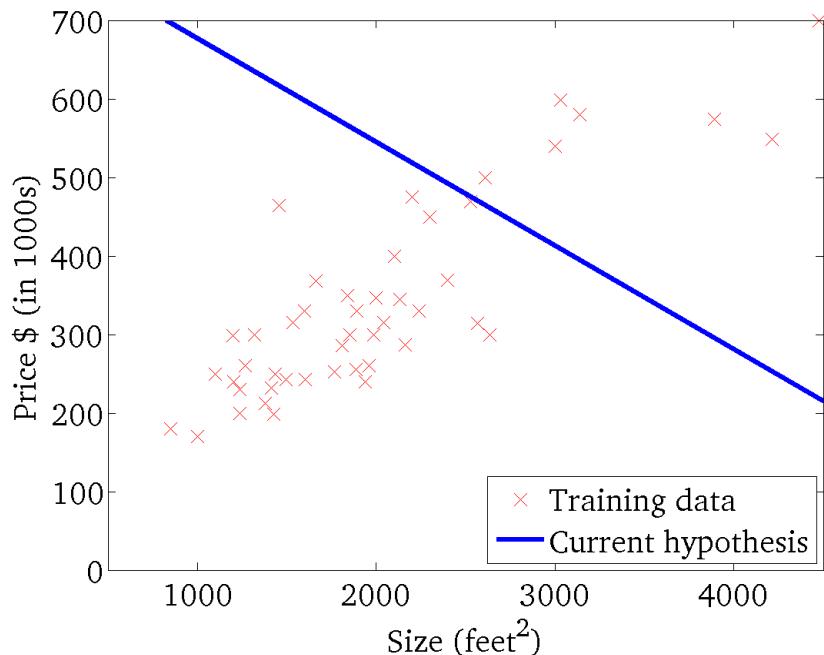
$\underline{h_{\theta}(x)}$
 (for fixed θ_0, θ_1 , this is a function of x)



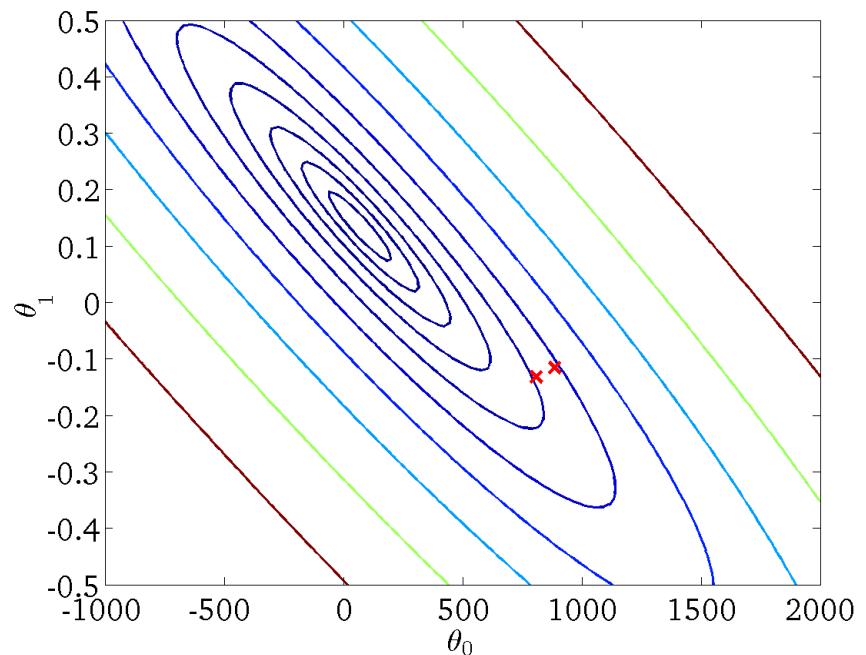
$\underline{J(\theta_0, \theta_1)}$
 (function of the parameters θ_0, θ_1)



$h_{\theta}(x)$
(for fixed θ_0, θ_1 , this is a function of x)

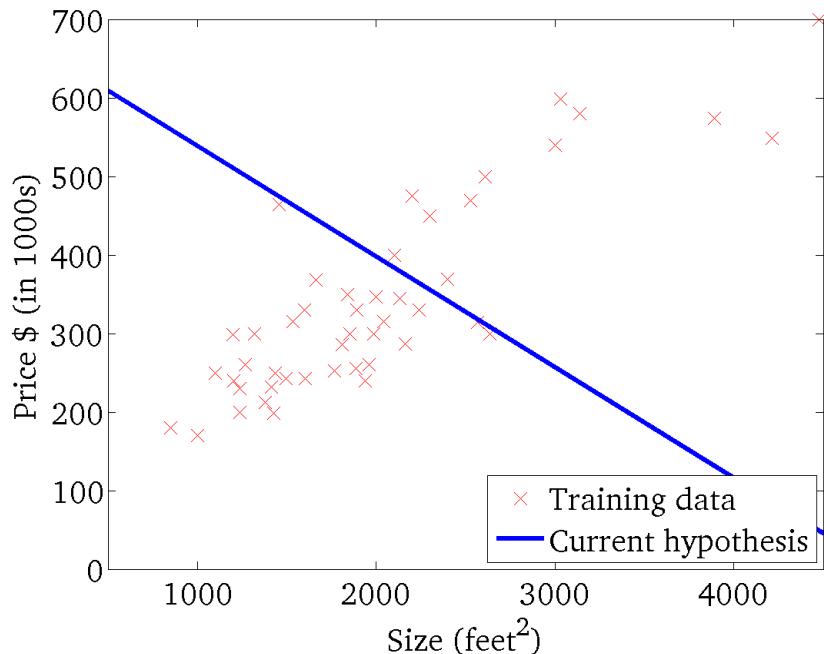


$J(\theta_0, \theta_1)$
(function of the parameters θ_0, θ_1)



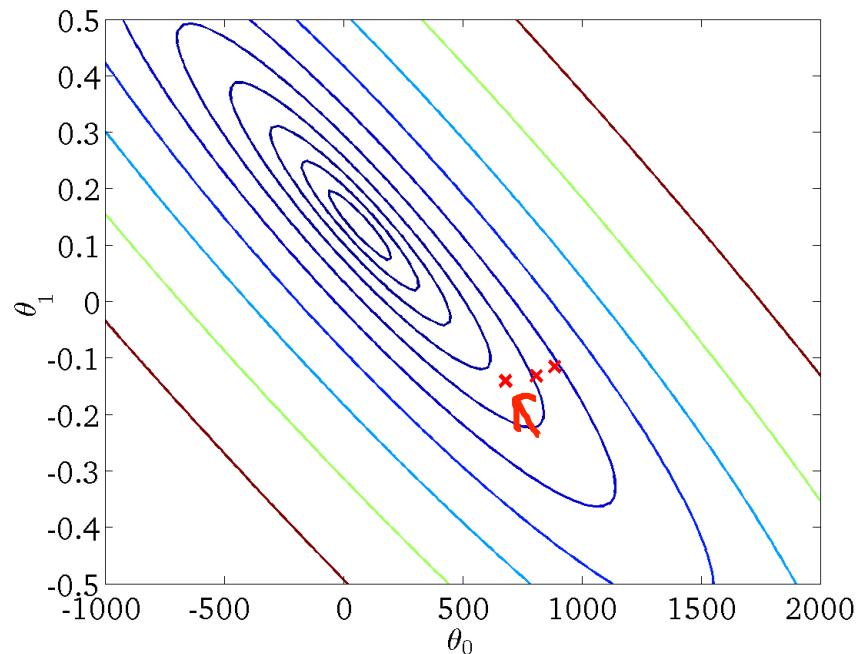
$$h_{\theta}(x)$$

(for fixed θ_0, θ_1 , this is a function of x)



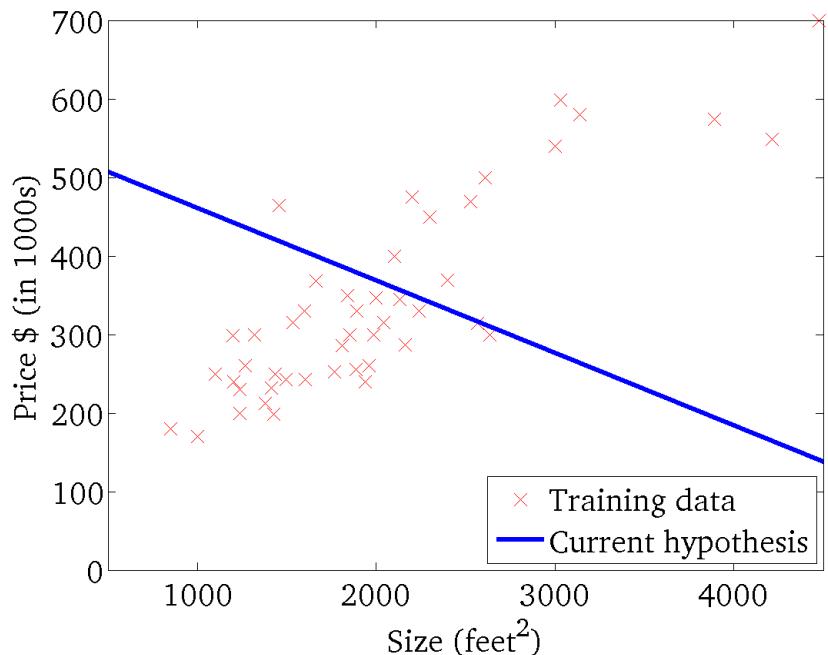
$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)



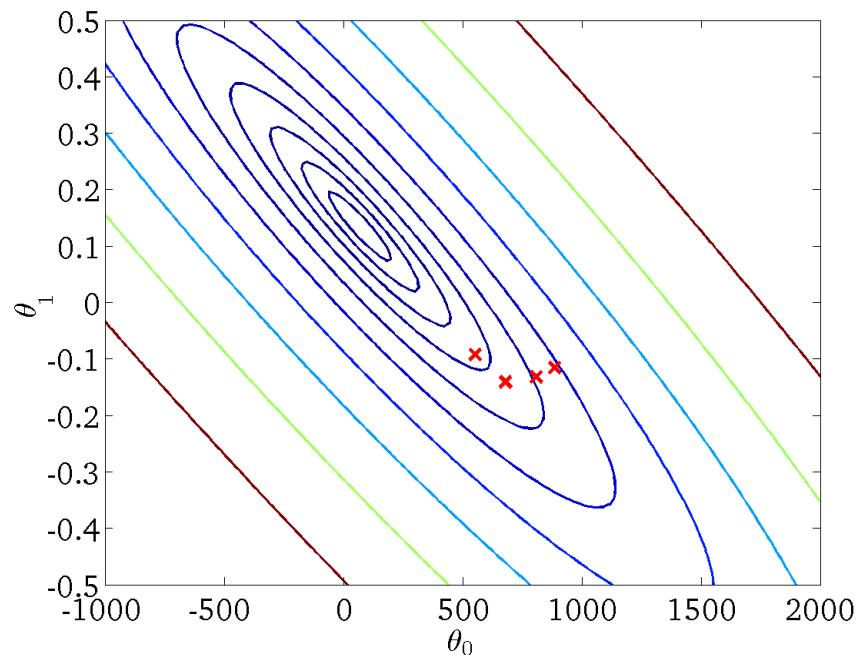
$$h_{\theta}(x)$$

(for fixed θ_0, θ_1 , this is a function of x)



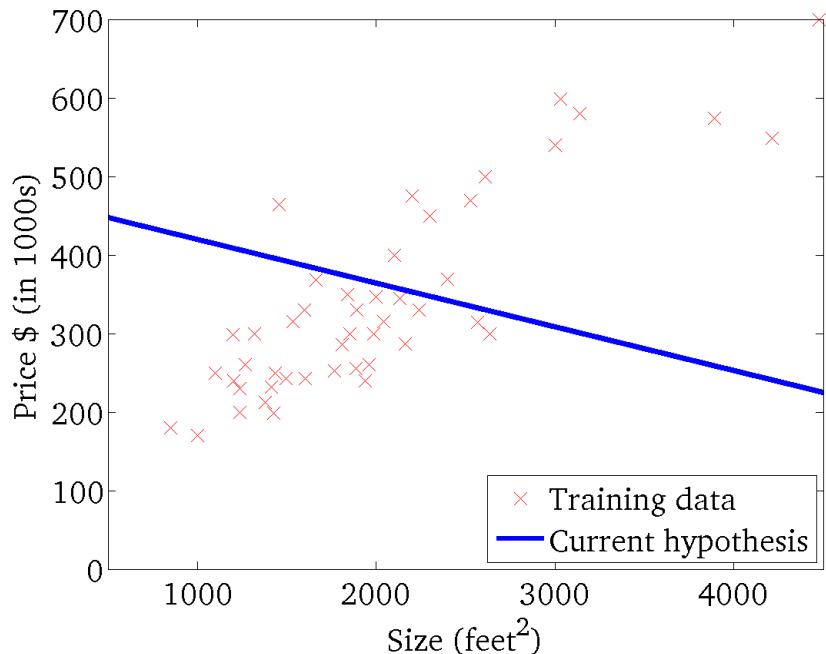
$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)



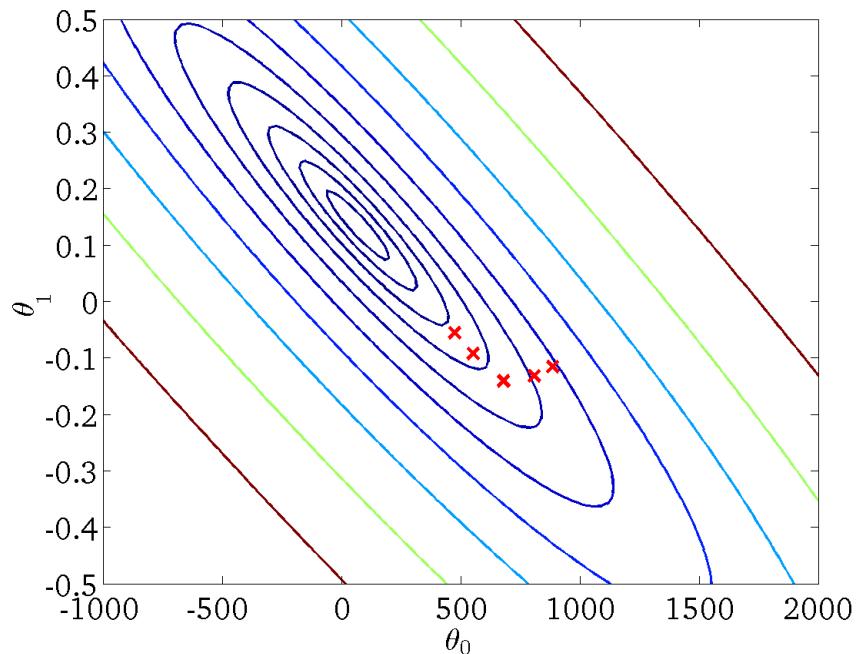
$$h_{\theta}(x)$$

(for fixed θ_0, θ_1 , this is a function of x)



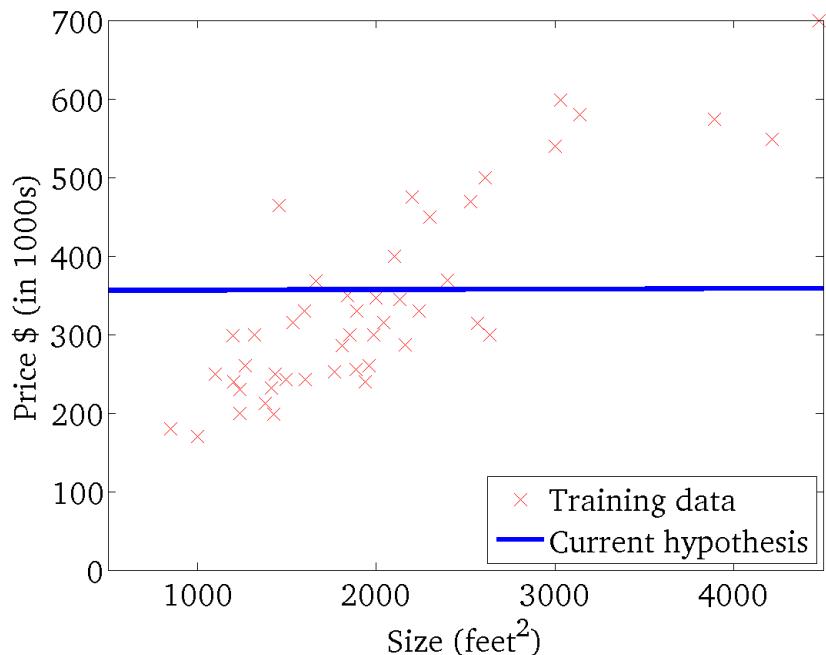
$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)



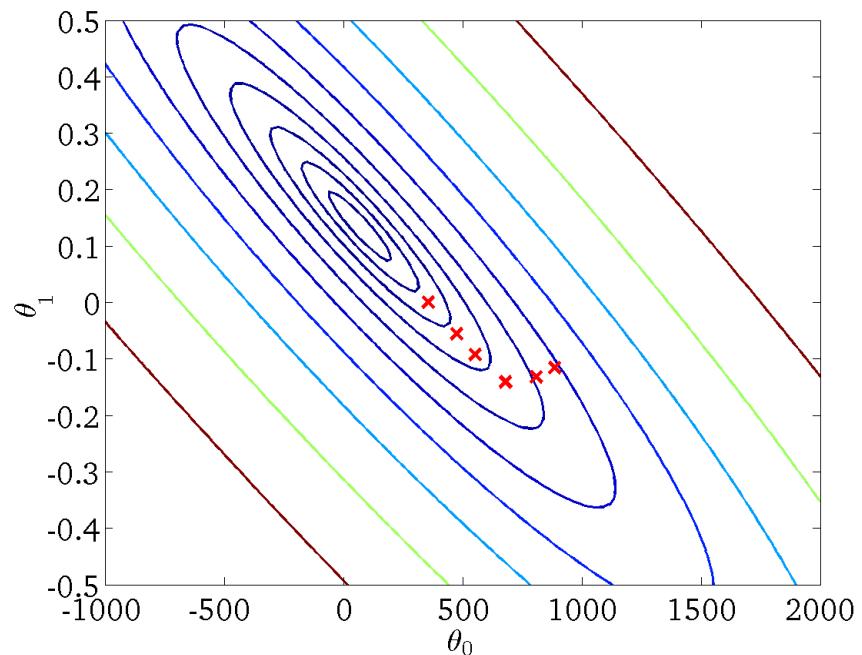
$$h_{\theta}(x)$$

(for fixed θ_0, θ_1 , this is a function of x)



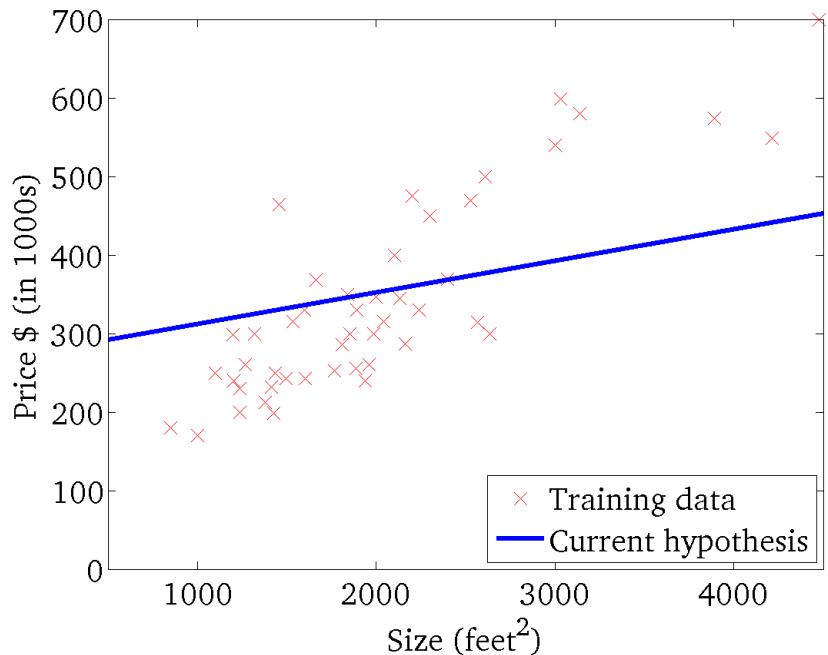
$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)



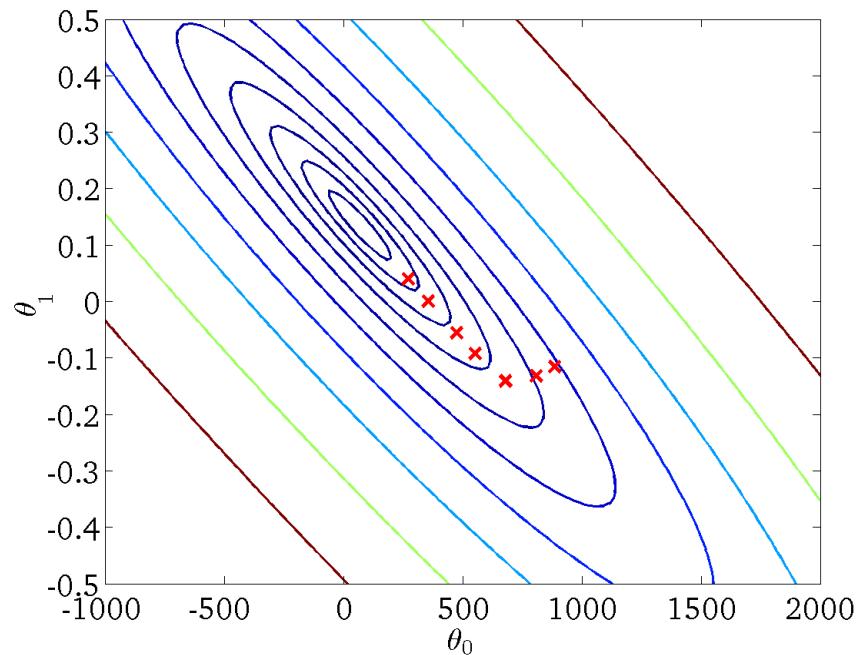
$$h_{\theta}(x)$$

(for fixed θ_0, θ_1 , this is a function of x)

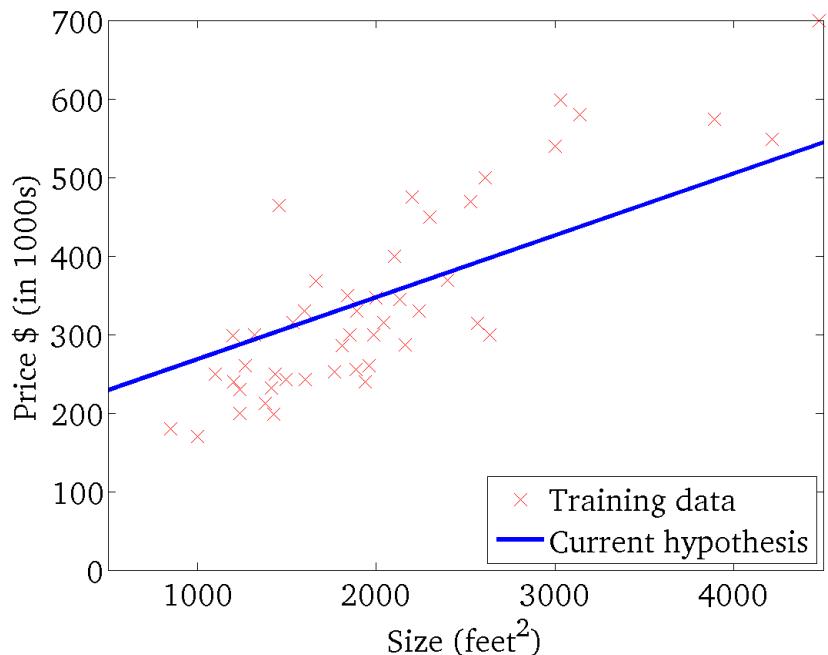


$$J(\theta_0, \theta_1)$$

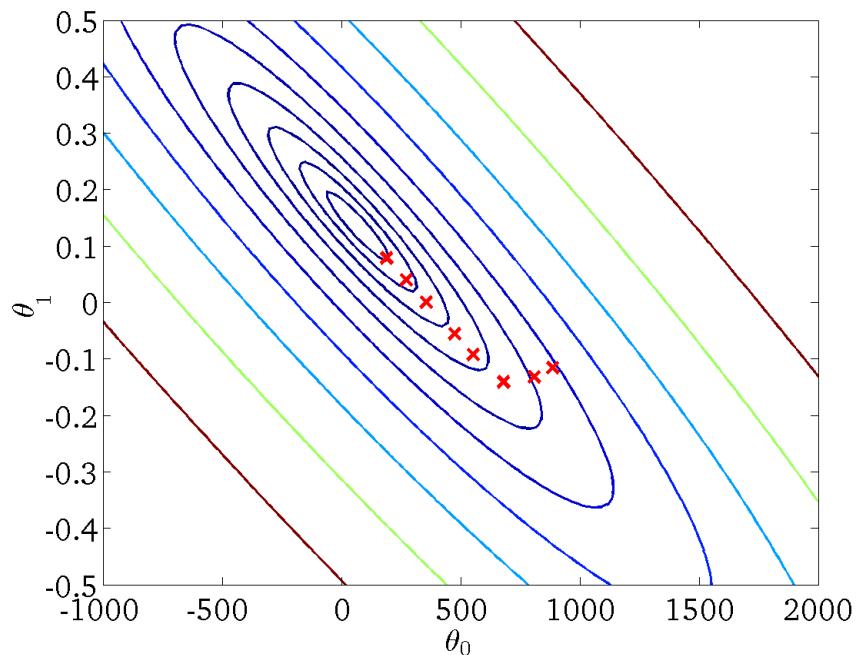
(function of the parameters θ_0, θ_1)



$h_{\theta}(x)$
(for fixed θ_0, θ_1 , this is a function of x)

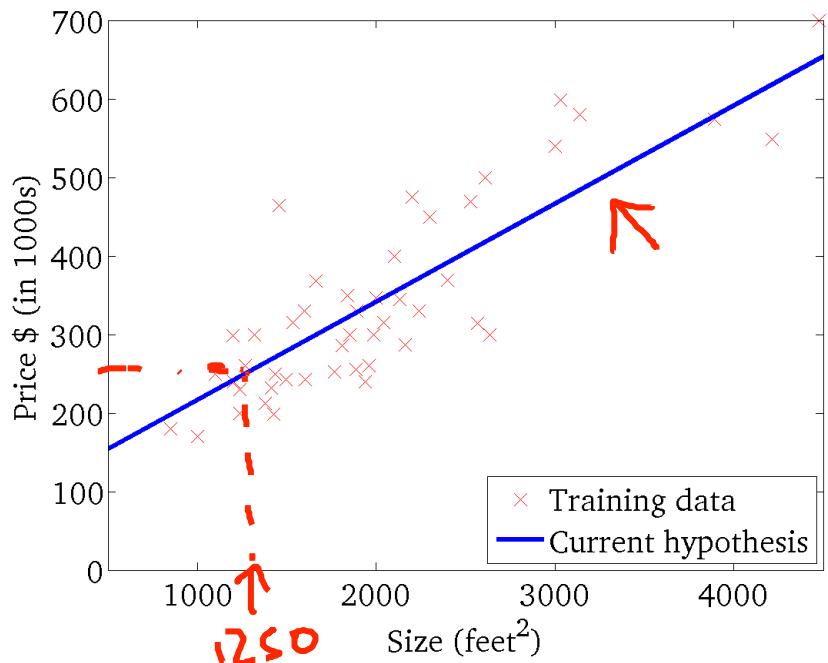


$J(\theta_0, \theta_1)$
(function of the parameters θ_0, θ_1)



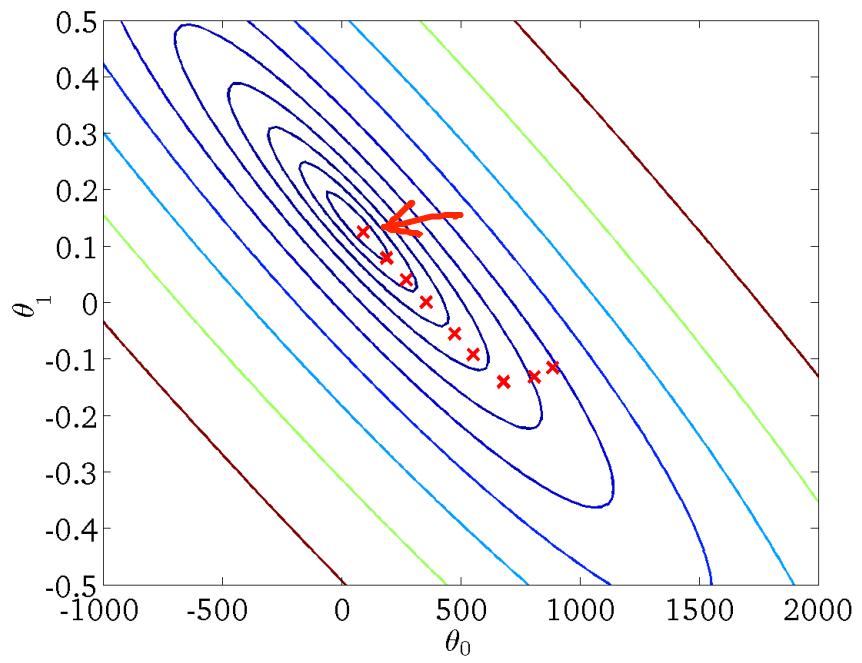
$$h_{\theta}(x)$$

(for fixed θ_0, θ_1 , this is a function of x)



$$J(\theta_0, \theta_1)$$

(function of the parameters θ_0, θ_1)

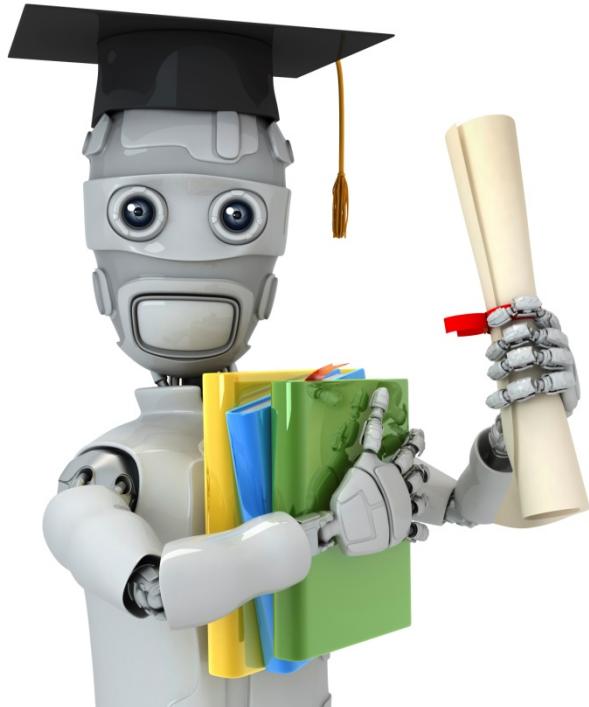


“Batch” Gradient Descent

“Batch”: Each step of gradient descent uses all the training examples.

$$\xrightarrow{\text{all}} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})$$

1.3 Linear Algebra



Machine Learning

Linear Algebra review (optional)

Matrices and vectors

Matrix: Rectangular array of numbers:

$$\begin{array}{c} \rightarrow \\ \rightarrow \\ \rightarrow \\ \rightarrow \\ \nearrow \quad \uparrow \quad \uparrow \end{array} \left[\begin{array}{cc} 1402 & 191 \\ 1371 & 821 \\ 949 & 1437 \\ 147 & 1448 \end{array} \right]$$

4 × 2 matrix

$$\rightarrow \boxed{\mathbb{R}^{4 \times 2}}$$

$$2 \rightarrow \left[\begin{array}{ccc} 1 & 2 & 3 \\ 4 & 5 & 6 \end{array} \right] \quad \begin{matrix} \uparrow & \uparrow & \uparrow \\ 3 & & c \end{matrix}$$

2 × 3 matrix

$$\boxed{\mathbb{R}^{2 \times 3}}$$

Dimension of matrix: number of rows × number of columns

Matrix Elements (entries of matrix)

$$A = \begin{bmatrix} 1402 & 191 \\ 1371 & 821 \\ 949 & 1437 \\ 147 & 1448 \end{bmatrix}$$

A_{ij} = " i, j entry" in the i^{th} row, j^{th} column.

$$A_{11} = 1402$$

$$A_{12} = 191$$

$$A_{\underline{3}\underline{2}} = 1437$$

$$A_{41} = 147$$

~~A_{43}~~ = Undefined (error)

Vector: An $n \times 1$ matrix.

$$y = \begin{bmatrix} 460 \\ 232 \\ 315 \\ 178 \end{bmatrix} \quad \begin{matrix} \nearrow & \searrow \\ n=4 \end{matrix} \quad \leftarrow \text{4-dimensional vector}$$

~~$\mathbb{R}^{3 \times 2}$~~

\mathbb{R}^4

$y_i = i^{th}$ element

$$y_1 = 460$$

$$y_2 = 232$$

$$y_3 = 315$$

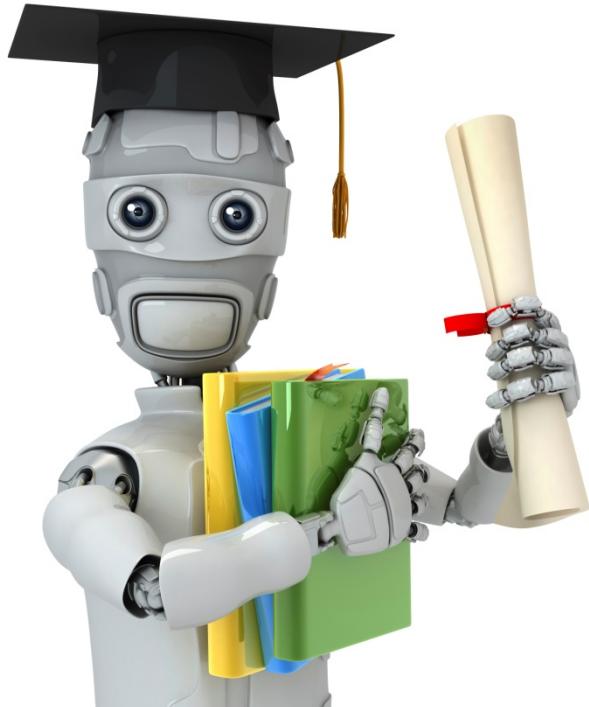
$\rightarrow [A, B, C, X]$

a, b, x, y

1-indexed vs 0-indexed:

$$y[1] \quad y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} \quad \begin{matrix} \leftarrow \\ \leftarrow \\ \leftarrow \\ \leftarrow \end{matrix} \quad \boxed{\text{1-indexed}}$$

$$y[0] \quad y = \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad \begin{matrix} \leftarrow \\ \leftarrow \\ \leftarrow \\ \leftarrow \end{matrix} \quad \boxed{\text{0-indexed}}$$



Machine Learning

Linear Algebra review (optional)

Addition and scalar
multiplication

Matrix Addition

$$\begin{matrix} & \downarrow & \downarrow \\ \rightarrow & \circled{1} & 0 \\ \rightarrow & \circled{2} & 5 \\ \rightarrow & \circled{3} & 1 \\ \hline & 3 \times 2 & \text{matrix} \end{matrix} + \begin{matrix} & \circled{4} & 0.5 \\ & \circled{2} & 5 \\ & \circled{0} & 1 \\ \hline & 3 \times 2 & \end{matrix} = \begin{matrix} & 5 & 0.5 \\ & 4 & 10 \\ & 3 & 2 \\ \hline & 3 \times 2 & \end{matrix}$$

$$\begin{matrix} & \downarrow & \downarrow \\ \rightarrow & 1 & 0 \\ \rightarrow & 2 & 5 \\ \rightarrow & 3 & 1 \\ \hline & 3 \times 2 & \end{matrix} + \begin{matrix} & 4 & 0.5 \\ & 2 & 5 \\ \hline & 2 \times 2 & \end{matrix} = \text{error}$$

Scalar Multiplication

↙ real number

$$3 \times \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 3 & 0 \\ 6 & 15 \\ 9 & 3 \end{bmatrix}} = \begin{bmatrix} 1 & 0 \\ 2 & 5 \\ 3 & 1 \end{bmatrix} \times 3$$
$$\frac{1}{4} \begin{bmatrix} 4 & 0 \\ 6 & 3 \end{bmatrix} / 4 = \frac{1}{4} \begin{bmatrix} 4 & 0 \\ 6 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{3}{2} & \frac{3}{4} \end{bmatrix}$$

Combination of Operands

$$3 \times \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} - \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix} / 3$$

Scalar multiplication

Scalar division

$$= \begin{bmatrix} 3 \\ 12 \\ 6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ \frac{2}{3} \end{bmatrix}$$

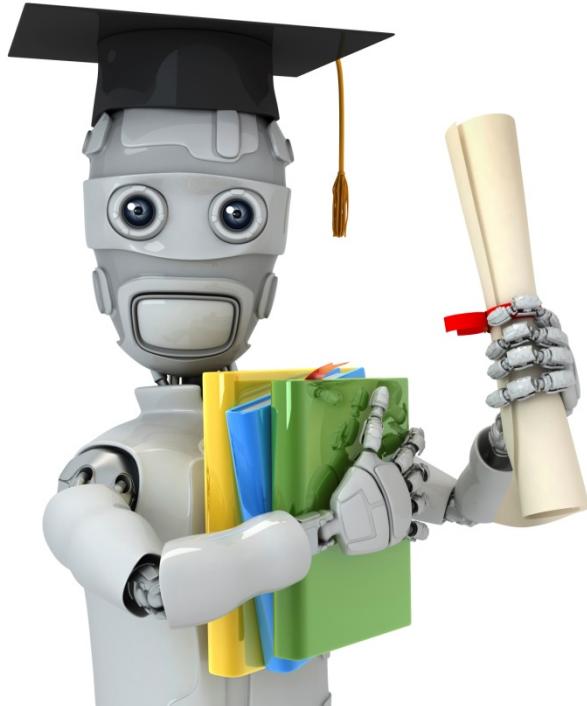
matrix subtraction / vector subtraction

matrix addition / vector addition

$$= \begin{bmatrix} 2 \\ 12 \\ 10 \frac{1}{3} \end{bmatrix}$$

3x1 matrix

3-dimensional vector



Machine Learning

Linear Algebra review (optional)

Matrix-vector multiplication

Example

$$\begin{matrix} & \begin{matrix} 1 & 3 \\ 4 & 0 \\ 2 & 1 \end{matrix} \\ \underbrace{\quad\quad}_{3 \times 2} & \end{matrix} \begin{matrix} & \begin{matrix} 1 \\ 5 \end{matrix} \\ \underbrace{\quad\quad}_{2 \times 1} & \end{matrix} = \begin{bmatrix} 16 \\ 4 \\ 7 \end{bmatrix}$$

3x1 matrix

$$1 \times 1 + 3 \times 5 = 16$$

$$4 \times 1 + 0 \times 5 = 4$$

$$2 \times 1 + 1 \times 5 = 7$$

Details:

$$\underbrace{A}_{\substack{\text{m} \times \text{n} \text{ matrix} \\ (\text{m rows}, \\ \text{n columns})}} \times \underbrace{x}_{\substack{\text{n} \times 1 \text{ matrix} \\ (\text{n-dimensional} \\ \text{vector})}} = \underbrace{y}_{\substack{\text{m-dimensional} \\ \text{vector}}}$$

The diagram illustrates the multiplication of a matrix A by a vector x to produce a vector y . Matrix A is shown as a stack of n horizontal rows, each represented by a blue oval. Vector x is shown as a vertical column of n elements, also represented by blue ovals. The result y is a vector with m elements, represented by blue ovals. Blue arrows indicate the mapping from the rows of A to the elements of x , and from the elements of x to the elements of y .

To get y_i , multiply A 's i^{th} row with elements of vector x , and add them up.

Example

$$\begin{bmatrix} 1 & 2 & 1 & 5 \\ 0 & 3 & 0 & 4 \\ -1 & -2 & 0 & 0 \end{bmatrix}$$

3×4

$$\begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ -7 \end{bmatrix} = \begin{bmatrix} 14 \\ 13 \\ -7 \end{bmatrix}$$

$$1 \times 1 + 2 \times 3 + 1 \times 2 + 5 \times 1 = 14$$
$$0 \times 1 + 3 \times 3 + 0 \times 2 + 4 \times 1 = 13$$
$$-1 \times 1 + (-2) \times 3 + 0 \times 2 + 0 \times 1 = -7$$

House sizes:

→ 2104

→ 1416

→ 1534

→ 852

Matrix x

1	2104
1	1416
1	1534
1	852

{

4x2

$$h_{\theta}(x) = -40 + 0.25x$$

$h_{\theta}(x)$

2x1

Vector

-40
0.25

X

4x1 matrix

$-40 \times 1 + 0.25 \times 2104$
$-40 \times 1 + 0.25 \times 1416$

$h_{\theta}(2104)$

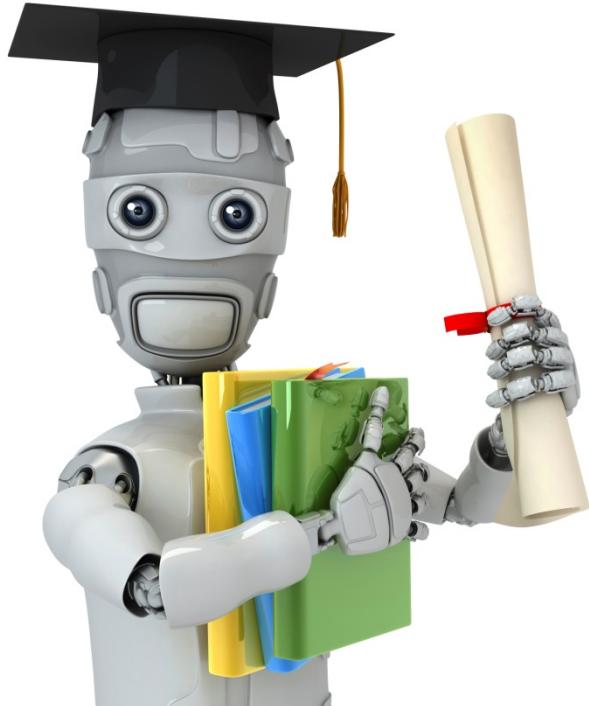
$h_{\theta}(1416)$

$\boxed{\text{prediction}} = \boxed{\text{Data Matrix}} \times \boxed{\text{Parameters}}$

4x1

n

for $i = 1: 1000$,
 $\text{prediction}(i) := \dots$



Machine Learning

Linear Algebra review (optional)

Matrix-matrix multiplication

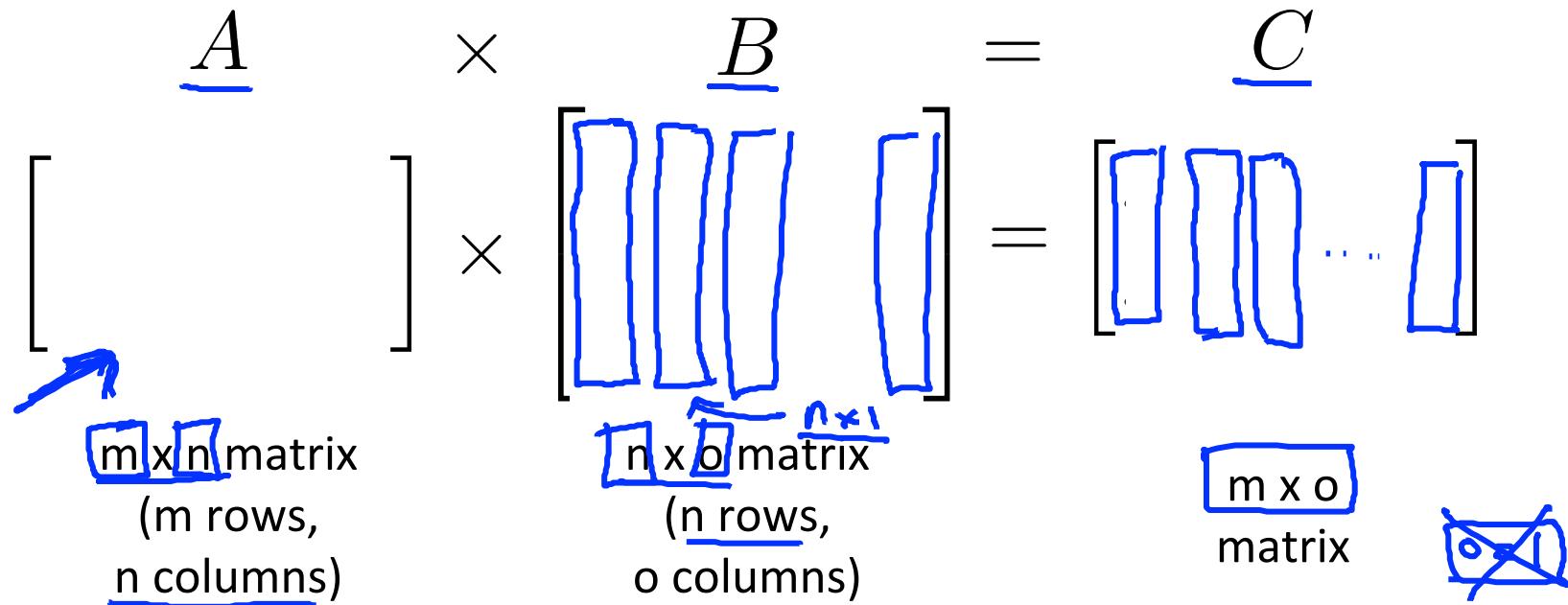
Example

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 11 & 10 \\ 9 & 14 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 0 \\ 5 \end{bmatrix} = \begin{bmatrix} 11 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 \\ 14 \end{bmatrix}$$

Details:



The i^{th} column of the matrix C is obtained by multiplying A with the i^{th} column of B. (for $i = 1, 2, \dots, o$)

Example

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 9 & 7 \\ 15 & 12 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 1 \times 0 + 3 \times 3 \\ 2 \times 0 + 5 \times 3 \end{bmatrix} = \begin{bmatrix} 9 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \times 1 + 3 \times 2 \\ 2 \times 1 + 5 \times 2 \end{bmatrix} = \begin{bmatrix} 7 \\ 12 \end{bmatrix}$$

House sizes:

$$\left\{ \begin{array}{r} 2104 \\ 1416 \\ 1534 \\ \hline 852 \end{array} \right.$$

Have 3 competing hypotheses:

$$1. h_{\theta}(x) = -40 + 0.25x$$

$$2. h_{\theta}(x) = 200 + 0.1x$$

$$3. h_{\theta}(x) = -150 + 0.4x$$

Matrix

$$\begin{bmatrix} 1 & 2104 \\ 1 & 1416 \\ 1 & 1534 \\ 1 & 852 \end{bmatrix}$$

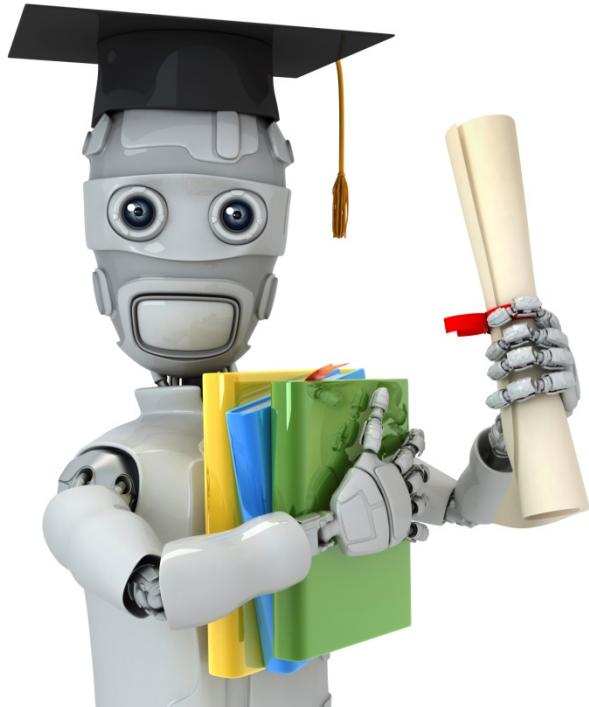
Matrix

$$\begin{bmatrix} -40 \\ 200 \\ -150 \\ 0.25 \\ 0.1 \\ 0.4 \end{bmatrix}$$

$$\begin{bmatrix} 486 \\ 410 \\ 692 \\ 314 \\ 342 \\ 416 \\ 344 \\ 353 \\ 464 \\ 173 \\ 285 \\ 191 \end{bmatrix}$$

↑
Prediction
of 1st
 h_{θ}

↑
Predictions
of 2nd
 h_{θ}



Machine Learning

Linear Algebra review (optional)

Matrix multiplication properties

$$\begin{matrix} 3 \times 5 \\ \curvearrowleft \end{matrix} = 5 \times 3$$

"Commutative"

Let A and B be matrices. Then in general,
 $A \times B \neq B \times A$. (not commutative.)

E.g.

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\begin{matrix} A \times B \\ m \times n \end{matrix}$$

$$\begin{bmatrix} 0 & 0 \\ 2 & 0 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 2 & 2 \end{bmatrix}$$

$$\begin{matrix} \frac{A \times B}{\text{is}} & m \times m \\ \frac{B \times A}{\text{is}} & n \times n \end{matrix}$$



$$\underline{3 \times 5 \times 2}$$

$$3 \times 10 = 30 = 15 \times 2$$

$$3 \times (5+2) = (3 \times 5) + 2$$

"Associative"

$$\begin{array}{c} A \times (B \times C) \\ (A \times B) \times C \end{array}$$

$$A \times B \times C.$$

Let $D = B \times C$.

Compute $A \times D$.

Let $E = A \times B$.

Compute $E \times C$.

$\begin{array}{c} A \times (B \times C) \\ (A \times B) \times C \end{array}$

Some answer.

Identity Matrix

1 is identity

Denoted I (or $I_{n \times n}$).

Examples of identity matrices:

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad 1 \times 1$$
$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad 2 \times 2$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad 3 \times 3$$

~~$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad 4 \times 4$$~~

For any matrix A ,

$$A \cdot I = I \cdot A = A$$

$\begin{matrix} \nearrow mxn & \nearrow nxn & \nearrow mxm & \nearrow mxn & \nearrow mxn \end{matrix}$

$$1 \times z = z \times 1 = z$$

for any z

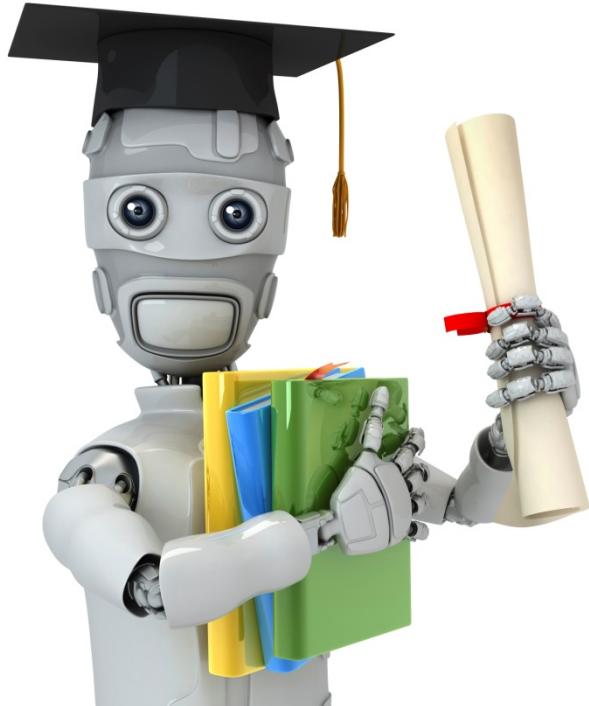
Informally:

$$\begin{bmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & \dots \end{bmatrix} \leftarrow$$

Note:

$$AB \neq BA \text{ in general}$$

$$AI = IA \quad \checkmark$$



Machine Learning

Linear Algebra review (optional)

Inverse and transpose

I = "identity."

$$3 \begin{pmatrix} 3^{-1} \\ \frac{1}{3} \end{pmatrix} = 1$$

$$\frac{12 \times (12^{-1})}{\frac{1}{12}} = 1$$

$$0 \begin{pmatrix} 0^{-1} \\ \underline{\quad} \end{pmatrix}$$

undefined

Not all numbers have an inverse.

Matrix inverse:

If A is an $m \times m$ matrix, and if it has an inverse,

$$\rightarrow A(A^{-1}) = A^{-1}A = I.$$

Square matrix
(#rows = #columns)

A^{-1}

$$A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

E.g.

$$\begin{bmatrix} 3 & 4 \\ 2 & 16 \end{bmatrix} \underbrace{\qquad}_{2 \times 2} \qquad A$$

$$\begin{bmatrix} 0.4 & -0.1 \\ -0.05 & 0.075 \end{bmatrix} \underbrace{\qquad}_{A^{-1}}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I_{2 \times 2}$$

$A^{-1}A$

Matrices that don't have an inverse are "singular" or "degenerate"

Matrix Transpose

Example:

$$\underline{A} = \begin{bmatrix} 1 & 2 & 0 \\ 3 & 5 & 9 \end{bmatrix}_{2 \times 3}$$

$$\underline{B} = \underline{A^T} = \begin{bmatrix} 1 & 3 \\ 2 & 5 \\ 0 & 9 \end{bmatrix}_{3 \times 2}$$

Let A be an $\underline{m \times n}$ matrix, and let $B = A^T$.

Then B is an $\underline{n \times m}$ matrix, and

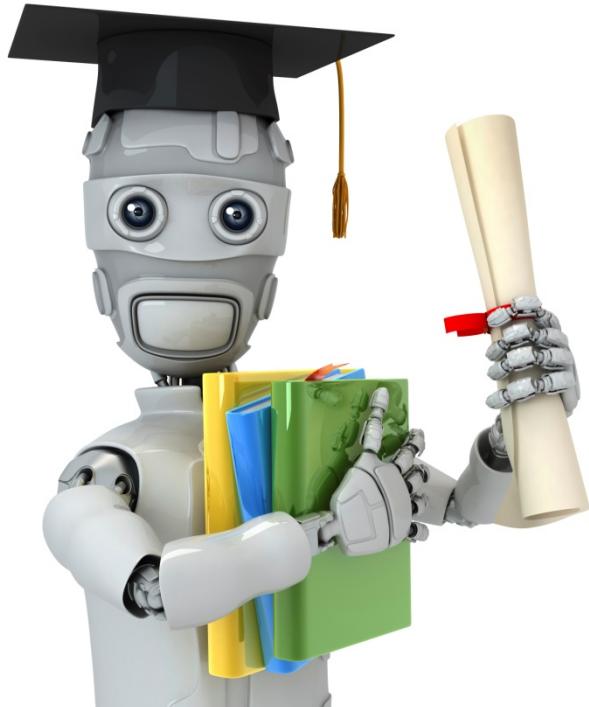
$$B_{ij} = A_{ji}.$$

$$B_{12} = A_{21} = 2$$

$$B_{32} = 9 \quad A_{23} = 9.$$

Chapter 2 Week2

2.1 Multiple Features



Machine Learning

Linear Regression with multiple variables

Multiple features

Multiple features (variables).

Size (feet ²)	Price (\$1000)
\xrightarrow{x}	$y \xleftarrow{ }$
2104	460
1416	232
1534	315
852	178
...	...

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

Multiple features (variables).

<u>Size (feet²)</u>	<u>Number of bedrooms</u>	<u>Number of floors</u>	<u>Age of home (years)</u>	Price (\$1000)
x_1	x_2	x_3	x_4	y
2104	5	1	45	460
1416	3	2	40	232
1534	3	2	30	315
852	2	1	36	178
...

Notation:

- n = number of features $n=4$
- $x^{(i)}$ = input (features) of i^{th} training example.
- $x_j^{(i)}$ = value of feature j in i^{th} training example.

$\underline{x}^{(2)} = \begin{bmatrix} 1416 \\ 3 \\ 2 \\ 40 \end{bmatrix}$

$x_3^{(2)} = 2$

Hypothesis:

Previously: $h_{\theta}(x) = \theta_0 + \theta_1 x$

$$h_{\theta}(x) = \theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_3 + \theta_4 x_4$$

E.g. $\underline{h_{\theta}(x)} = \underline{80} + \underline{0.1x_1} + \underline{0.01x_2} + \underline{3x_3} - \underline{2x_4}$

$$\rightarrow h_{\theta}(x) = \underline{\theta_0} + \underline{\theta_1}x_1 + \underline{\theta_2}x_2 + \cdots + \underline{\theta_n}x_n$$

For convenience of notation, define $x_0 = 1.$ ($x_0^{(i)} = 1$)

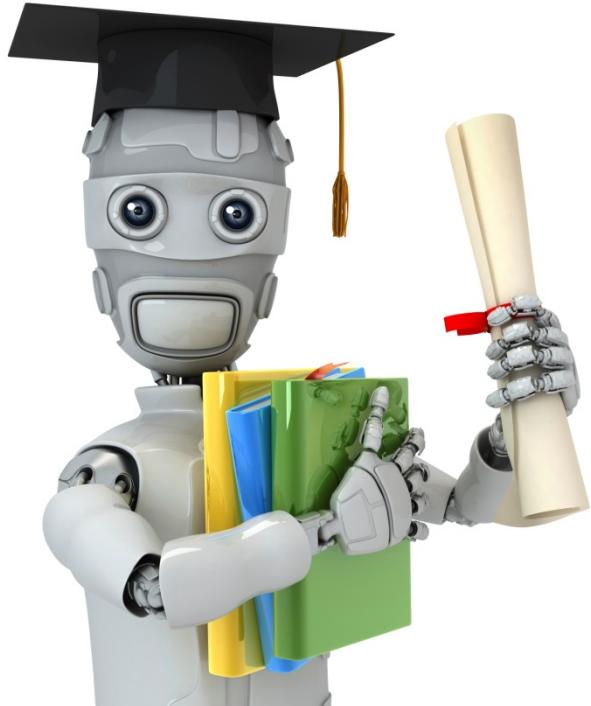
$$x = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \in \mathbb{R}^{n+1}$$

$$\Theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \in \mathbb{R}^{n+1}$$

$$\begin{aligned} h_{\theta}(x) &= \underline{\theta_0}x_0 + \underline{\theta_1}x_1 + \cdots + \underline{\theta_n}x_n \\ &= \boxed{\Theta^T x} \end{aligned}$$

$$\begin{bmatrix} \theta_0 & \theta_1 & \cdots & \theta_n \end{bmatrix} \Theta^T \quad (n+1) \times 1 \text{ matrix} \quad \Theta^T x$$

Multivariate linear regression. 



Machine Learning

Linear Regression with multiple variables

Gradient descent for multiple variables

Hypothesis: $h_{\theta}(x) = \theta^T x = \theta_0 x_0 + \theta_1 x_1 + \theta_2 x_2 + \cdots + \theta_n x_n$

$\xrightarrow{x_0 = 1}$

Parameters: $\theta_0, \theta_1, \dots, \theta_n$ Θ $n+1$ -dimensional vector

Cost function:

$$J(\theta_0, \theta_1, \dots, \theta_n) = J(\Theta) = \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2$$

Gradient descent:

Repeat {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta_0, \dots, \theta_n) \quad J(\Theta)$$

(simultaneously update for every $j = 0, \dots, n$)

Gradient Descent

Previously (n=1):

Repeat {

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) \underbrace{\frac{\partial}{\partial \theta_0} J(\theta)}$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_1^{(i)}$$

(simultaneously update θ_0, θ_1)

}

New algorithm ($n \geq 1$):

Repeat {

$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

(simultaneously update θ_j for $j = 0, \dots, n$)

}

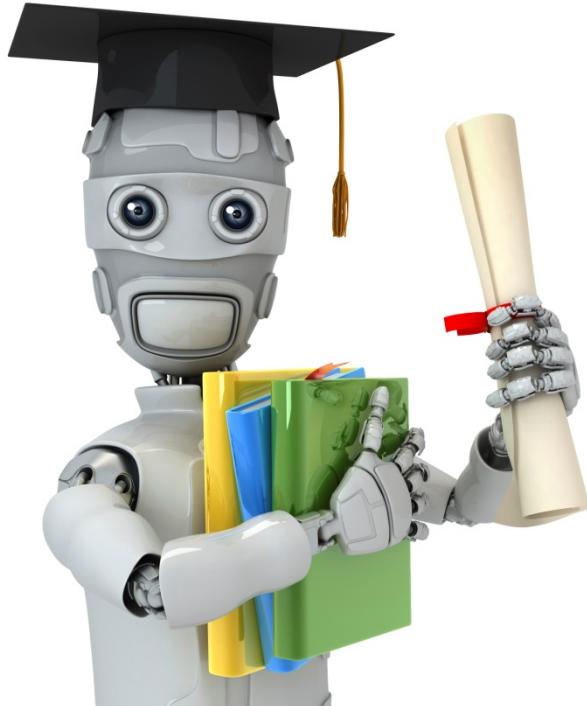
$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_0^{(i)}$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_1^{(i)}$$

$$\theta_2 := \theta_2 - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_2^{(i)}$$

...

Andrew Ng



Machine Learning

Linear Regression with multiple variables

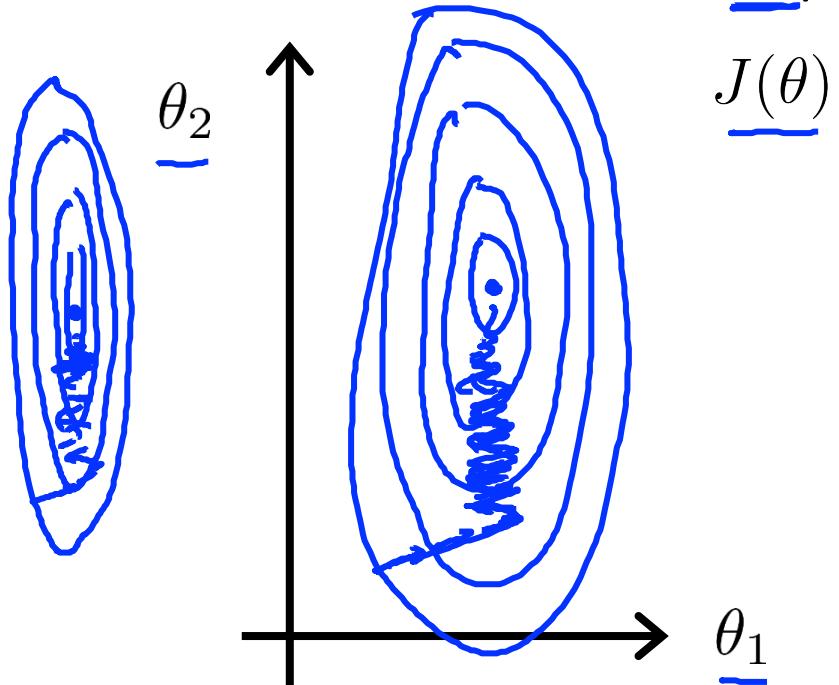
Gradient descent in practice I: Feature Scaling

Feature Scaling

Idea: Make sure features are on a similar scale.

E.g. $x_1 = \text{size } (\underline{0-2000} \text{ feet}^2)$ ←

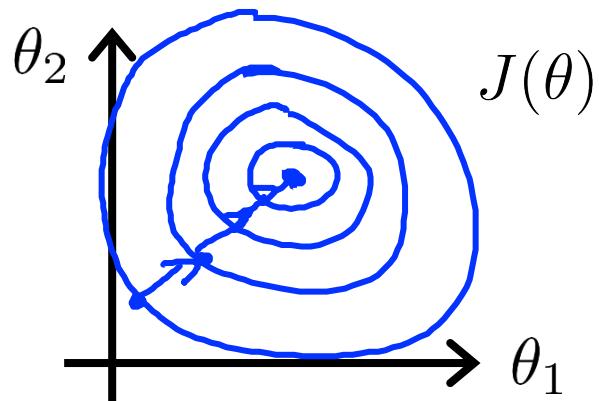
$x_2 = \text{number of bedrooms } (\underline{1-5})$ ←



$$\rightarrow x_1 = \frac{\text{size (feet}^2)}{2000} \quad \leftarrow$$

$$\rightarrow x_2 = \frac{\text{number of bedrooms}}{5}$$

$$0 \leq x_1 \leq 1 \quad 0 \leq x_2 \leq 1$$



Feature Scaling

Get every feature into approximately a $-1 \leq x_i \leq 1$ range.

$$x_0 = 1$$

$$6 \leq x_1 \leq 3 \quad \checkmark$$

$$-2 \leq x_2 \leq 0.5 \quad \checkmark$$

$$-100 \leq x_3 \leq 100 \quad \times$$

$$-0.0001 \leq x_4 \leq 0.0001 \quad \times$$

$$\boxed{-1 \leq x_i \leq 1}$$

$$-3 \text{ to } 3 \quad \checkmark$$

$$-\frac{1}{3} \text{ to } \frac{1}{3} \quad \checkmark$$

Mean normalization

Replace x_i with $\frac{x_i - \mu_i}{\sigma_i}$ to make features have approximately zero mean
(Do not apply to $x_0 = 1$).

E.g. $x_1 = \frac{\text{size} - 1000}{2000}$

Average size = 100

$$x_2 = \frac{\#\text{bedrooms} - 2}{5 - 4}$$

1 - 5 bedrooms

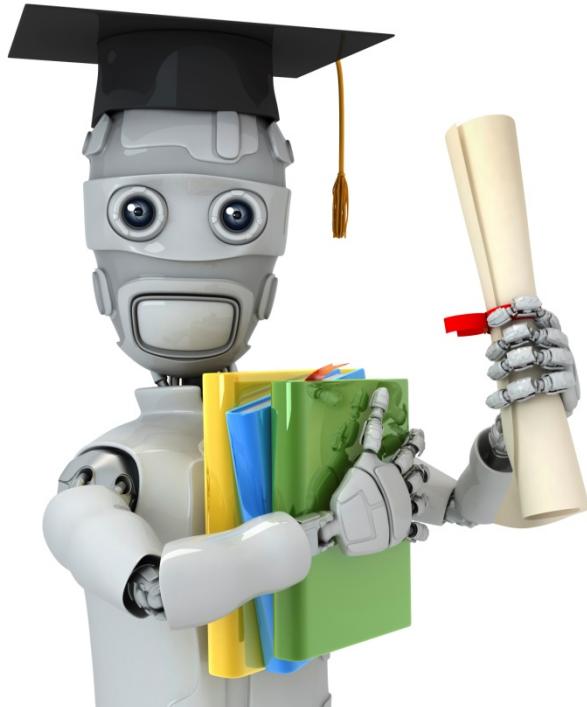
$$\rightarrow [-0.5 \leq x_1 \leq 0.5, -0.5 \leq x_2 \leq 0.5]$$

$$x_1 \leftarrow \frac{x_1 - \mu_1}{\sigma_1}$$

avg value
of x_1
in training
set

range ($\frac{\max - \min}{\sigma}$)
(or standard deviation)

$$x_2 \leftarrow \frac{x_2 - \mu_2}{\sigma_2}$$



Machine Learning

Linear Regression with multiple variables

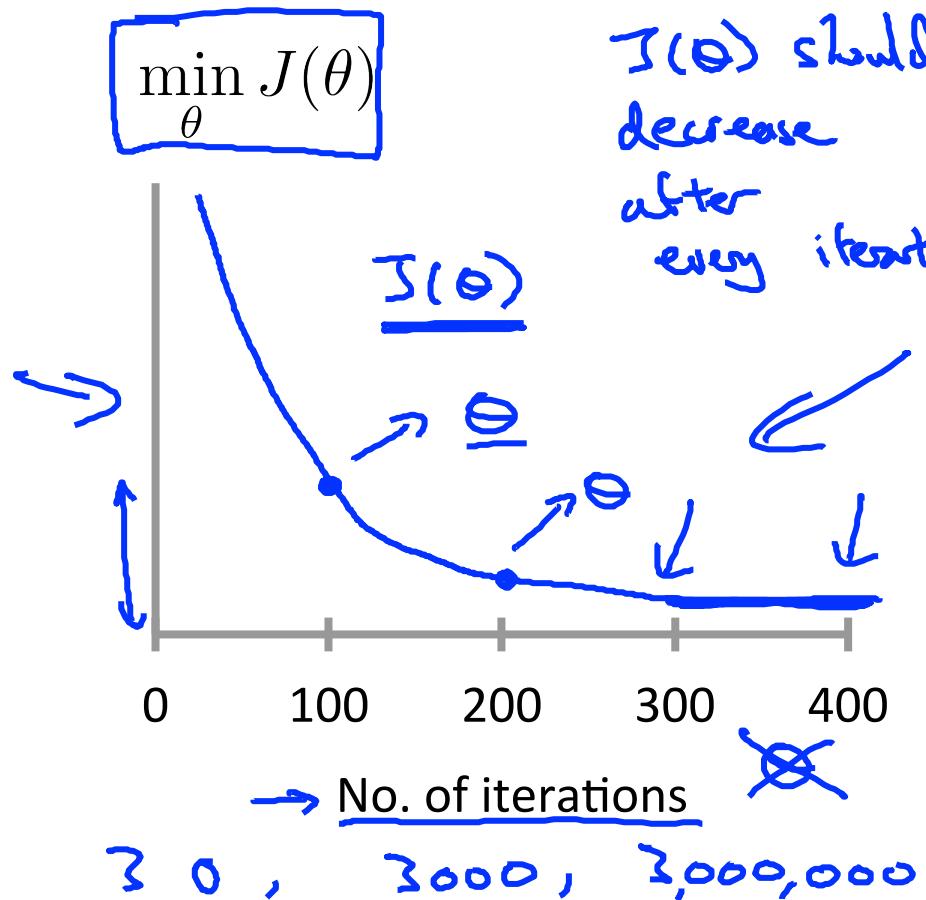
Gradient descent in
practice II: Learning rate

Gradient descent

$$\rightarrow \theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

- “Debugging”: How to make sure gradient descent is working correctly.
- How to choose learning rate α .

Making sure gradient descent is working correctly.

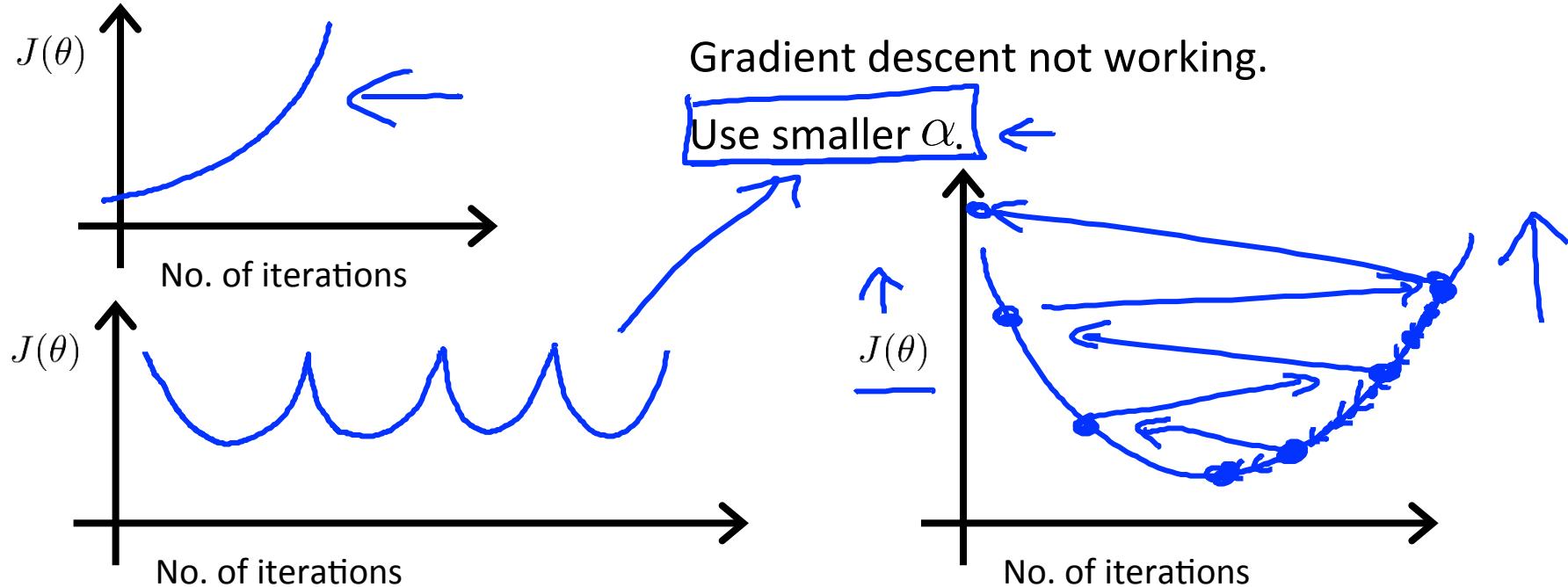


$J(\theta)$ should decrease after every iteration.

→ Example automatic convergence test:

→ Declare convergence if $J(\theta)$ decreases by less than 10^{-3} in one iteration.

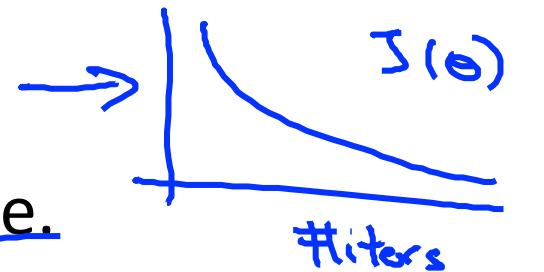
Making sure gradient descent is working correctly.



- For sufficiently small α , $J(\theta)$ should decrease on every iteration.
- But if α is too small, gradient descent can be slow to converge.

Summary:

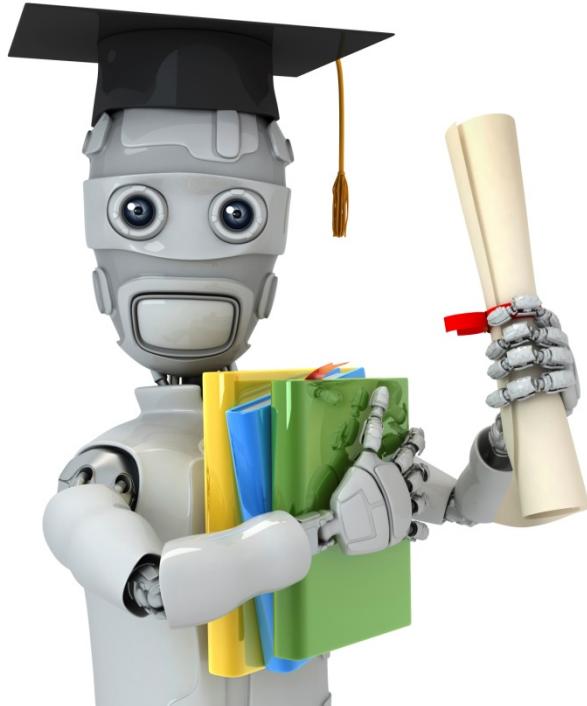
- If α is too small: slow convergence.
- If α is too large: $J(\theta)$ may not decrease on every iteration; may not converge. (Slow converge also possible)



To choose α , try

$$\dots, \underline{0.001}, \underline{0.003}, \underline{0.01}, \underline{0.03}, \underline{0.1}, \underline{0.3}, \underline{1}, \dots$$

$\nearrow 2x$ $\nwarrow 2x$ $\nearrow 3x$ $\nwarrow 3x$ \nearrow



Machine Learning

Linear Regression with multiple variables

Features and
polynomial regression

Housing prices prediction

$$h_{\theta}(x) = \theta_0 + \theta_1 \times \boxed{\text{frontage}} + \theta_2 \times \boxed{\text{depth}}$$

x_1 x_2



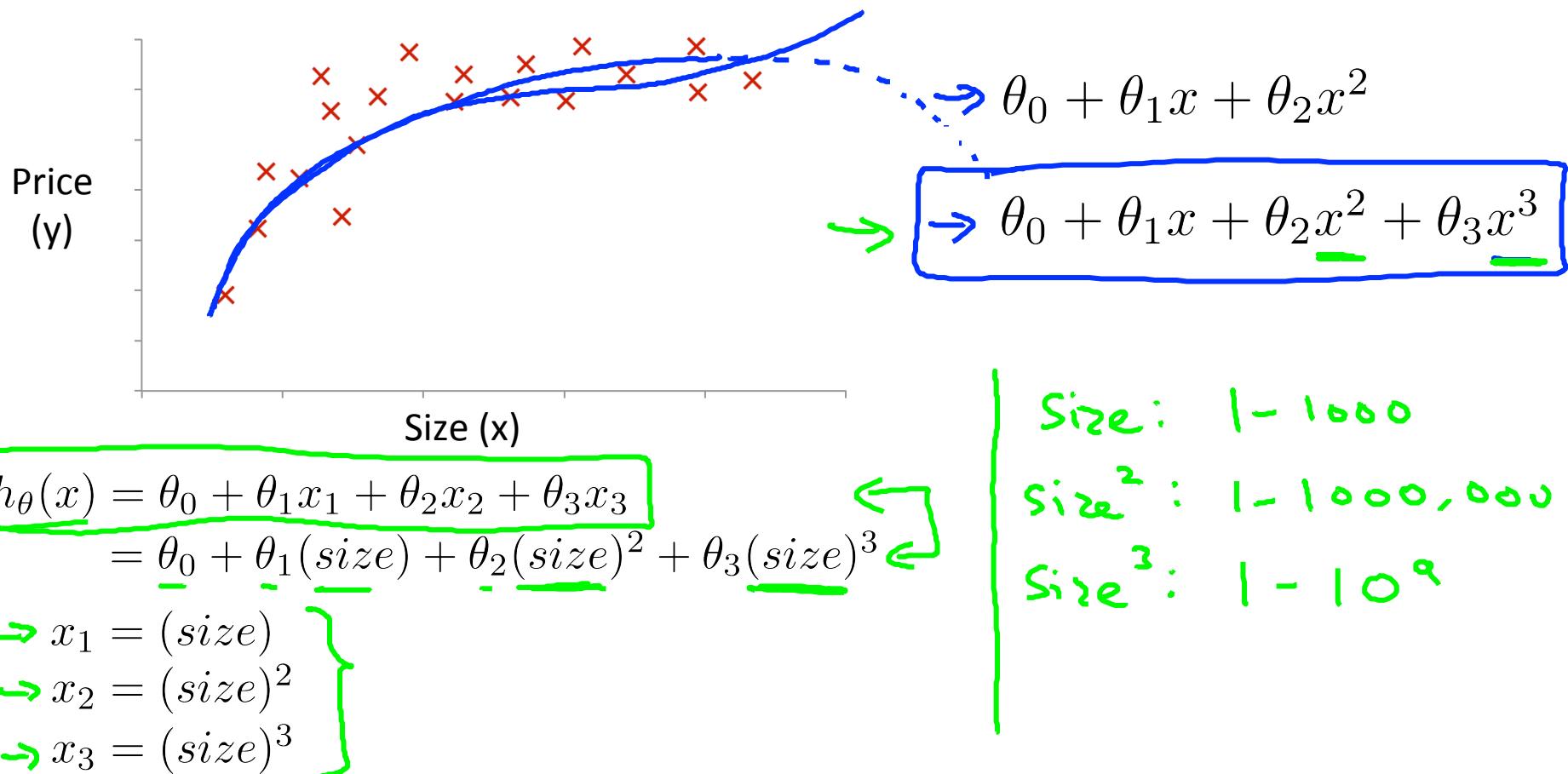
Area

$$x = \underline{\text{frontage} \times \text{depth}}$$

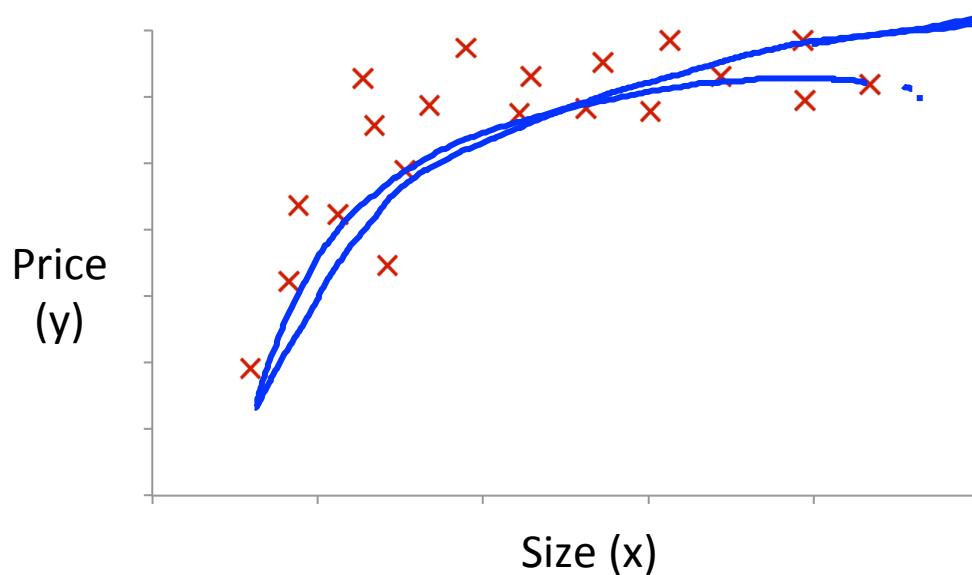
$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

↑ land area

Polynomial regression

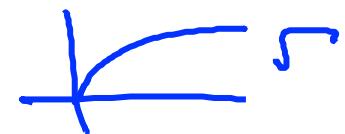


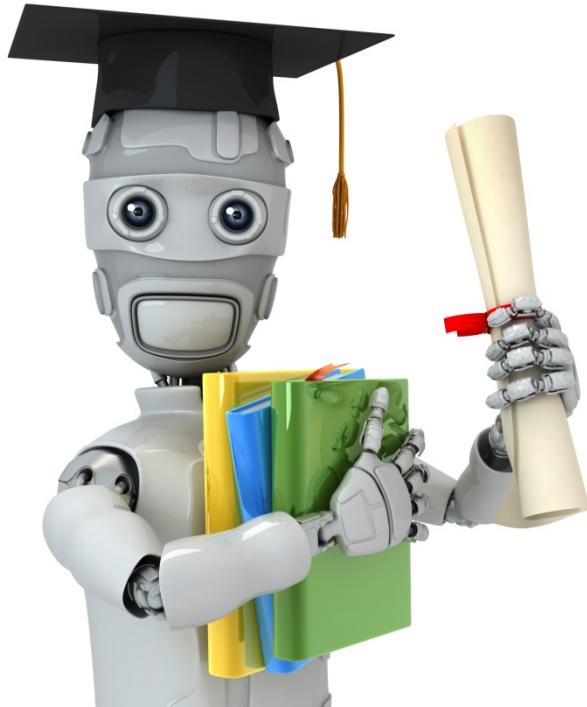
Choice of features



$$\rightarrow h_{\theta}(x) = \theta_0 + \theta_1(\text{size}) + \theta_2(\text{size})^2$$

$$\rightarrow h_{\theta}(x) = \theta_0 + \theta_1(\text{size}) + \theta_2 \sqrt{(\text{size})}$$



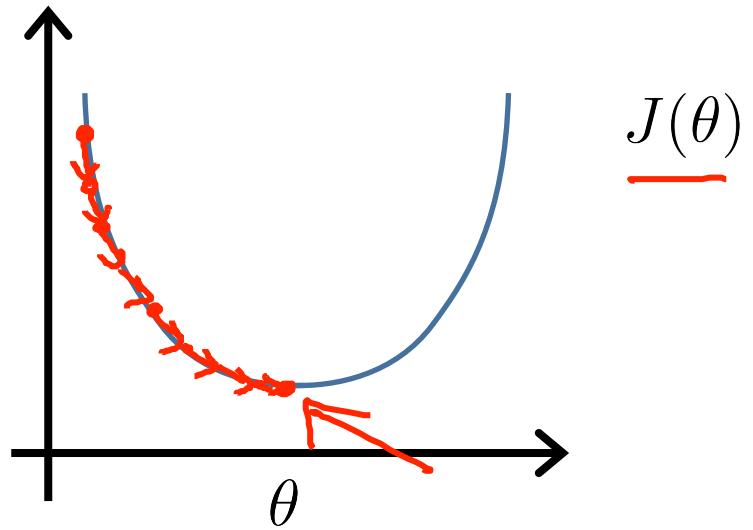


Machine Learning

Linear Regression with multiple variables

Normal equation

Gradient Descent



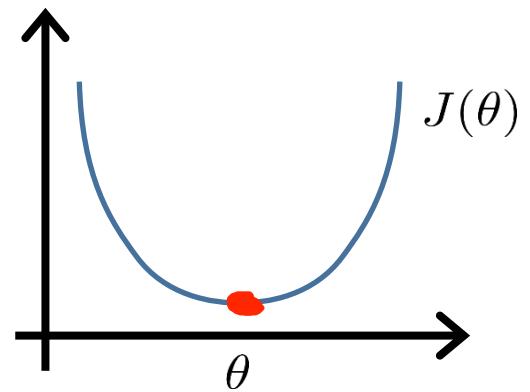
Normal equation: Method to solve for θ
analytically.

Intuition: If 1D ($\theta \in \mathbb{R}$)

$\rightarrow J(\theta) = a\theta^2 + b\theta + c$

$\frac{\partial}{\partial \theta} J(\theta) = \dots \stackrel{\text{set}}{=} 0$

Solve for θ



$\theta \in \mathbb{R}^{n+1}$

$J(\theta_0, \theta_1, \dots, \theta_m) = \frac{1}{2m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})^2$

$\frac{\partial}{\partial \theta_j} J(\theta) = \dots \stackrel{\text{set}}{=} 0 \quad (\text{for every } j)$

Solve for $\underline{\theta_0, \theta_1, \dots, \theta_n}$

Examples: $m = 4$.

	Size (feet ²)	Number of bedrooms	Number of floors	Age of home (years)	Price (\$1000)
x_0	x_1	x_2	x_3	x_4	y
1	2104	5	1	45	460
1	1416	3	2	40	232
1	1534	3	2	30	315
1	852	2	1	36	178

$X = \begin{bmatrix} 1 & 2104 & 5 & 1 & 45 \\ 1 & 1416 & 3 & 2 & 40 \\ 1 & 1534 & 3 & 2 & 30 \\ 1 & 852 & 2 & 1 & 36 \end{bmatrix}$
 $y = \begin{bmatrix} 460 \\ 232 \\ 315 \\ 178 \end{bmatrix}$

$m \times (n+1)$

$\theta = (X^T X)^{-1} X^T y$

m examples $(x^{(1)}, y^{(1)}), \dots, (x^{(m)}, y^{(m)})$; n features.

$$x^{(i)} = \begin{bmatrix} x_0^{(i)} \\ x_1^{(i)} \\ x_2^{(i)} \\ \vdots \\ x_n^{(i)} \end{bmatrix} \in \mathbb{R}^{n+1}$$

\times (design matrix)

$$X = \begin{bmatrix} \cdots & (x^{(1)})^\top & \cdots \\ \cdots & (x^{(1)})^\top & \cdots \\ \vdots & \vdots & \vdots \\ \cdots & (x^{(m)})^\top & \cdots \end{bmatrix}$$

E.g. If $x^{(i)} = \begin{bmatrix} 1 \\ x_1^{(i)} \end{bmatrix}$

$$\mathcal{O} = (X^\top X)^{-1} X^\top y$$

$$X = \begin{bmatrix} 1 & x_1^{(1)} \\ 1 & x_1^{(2)} \\ \vdots & \vdots \\ 1 & x_1^{(m)} \end{bmatrix}_{m \times 2}$$

$$y = \begin{bmatrix} y^{(1)} \\ y^{(2)} \\ \vdots \\ y^{(m)} \end{bmatrix}_{m \times 1}$$

$$\theta = \boxed{(X^T X)^{-1} X^T y}$$

$(X^T X)^{-1}$ is inverse of matrix $X^T X$.

Set A = $X^T X$

$$\boxed{(X^T X)^{-1}} = A^{-1}$$

Octave: $\text{pinv}(X' * X) * X' * y$

$$\boxed{\text{pinv}(X^T * X) * X^T * y}$$

$$\Theta = \boxed{(X^T X)^{-1} X^T y}$$

$$\min_{\Theta} J(\Theta)$$

$$\left| \begin{array}{l} X' \quad X^T \\ \cancel{\text{Feature Scaling}} \\ 0 \leq x_1 \leq 1 \\ 0 \leq x_2 \leq 1000 \\ 0 \leq x_3 \leq 10^{-5} \end{array} \right| \checkmark$$

m training examples, n features.

Gradient Descent

- • Need to choose α .
- • Needs many iterations.
- Works well even when n is large.

$$\overbrace{n = 10^6}^{}$$

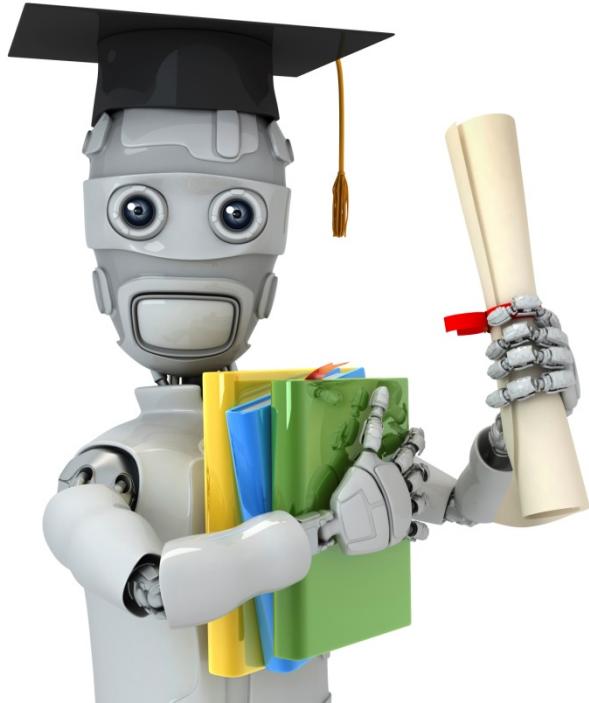
Normal Equation

- • No need to choose α .
- • Don't need to iterate.
- Need to compute $(X^T X)^{-1}$ $\underset{n \times n}{}$ $\underline{O(n^3)}$
- $(X^T X)^{-1}$
- Slow if n is very large.

$$n = 100$$

$$n = 1000$$

$$\dots - \overbrace{n = 10000}^{}$$



Machine Learning

Linear Regression with multiple variables

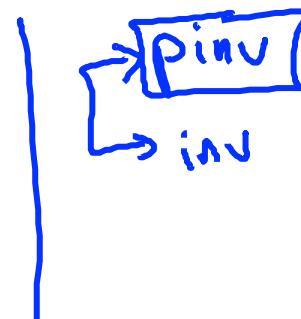
Normal equation
and non-invertibility
(optional)

Normal equation

$$\theta = \underline{(X^T X)^{-1} X^T y}$$

$$\underline{X^T X}$$

- What if $X^T X$ is non-invertible? (singular/
degenerate)
- Octave: $\text{pinv}(X' * X) * X' * y$



What if $\boxed{X^T X}$ is non-invertible?



- Redundant features (linearly dependent).

E.g.
$$\begin{bmatrix} \underline{x_1} = \text{size in feet}^2 \\ \underline{x_2} = \text{size in m}^2 \\ \underline{x_1} = (3.28)^2 \underline{x_2} \end{bmatrix}$$

$$l_m = 3.28 \text{ feet}$$

$$\rightarrow \underline{m = 10} \leftarrow$$

$$\rightarrow \underline{n = 100} \leftarrow$$

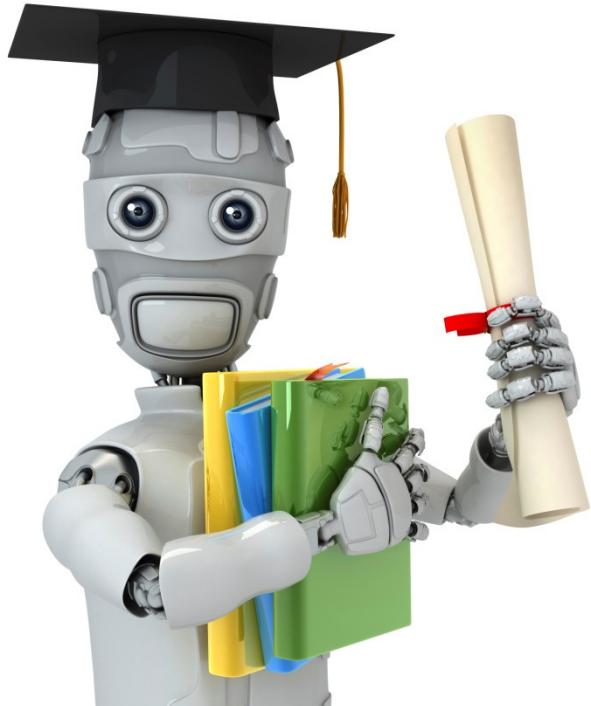
$$\mathbf{O} \in \mathbb{R}^{101}$$

- Too many features (e.g. $m \leq n$).

- Delete some features, or use regularization.

↓
later

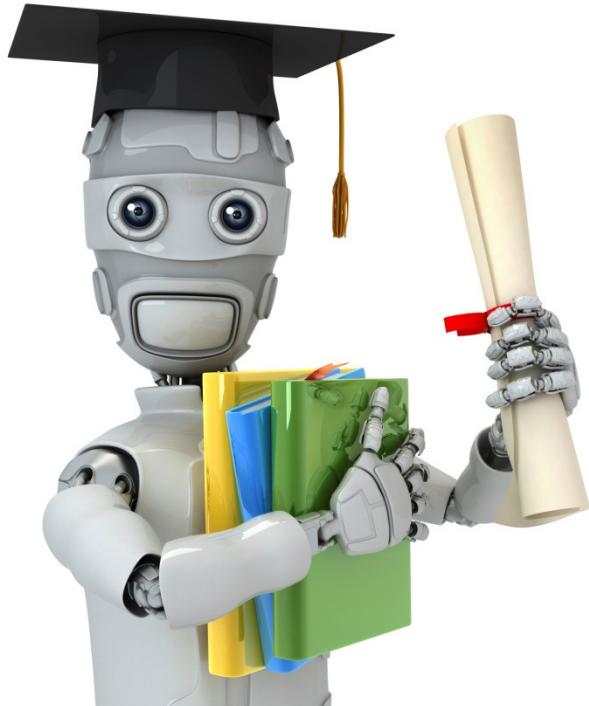
2.2 Octave



Machine Learning

Octave Tutorial

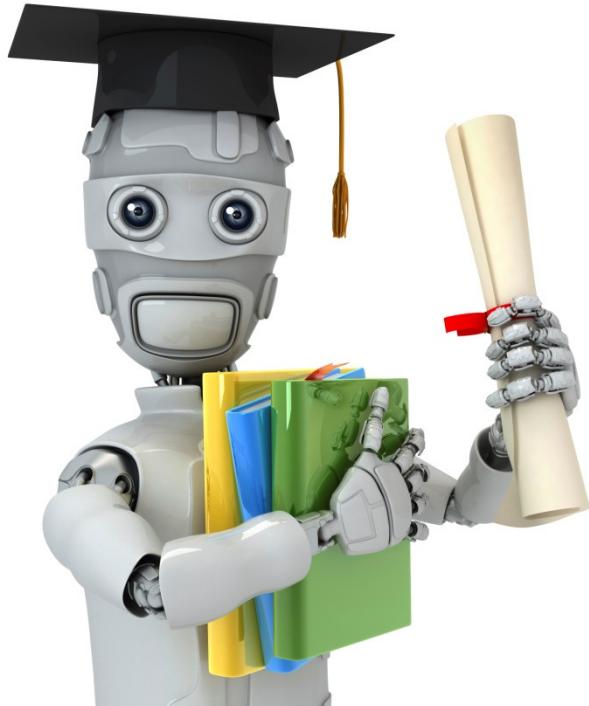
Basic operations



Machine Learning

Octave Tutorial

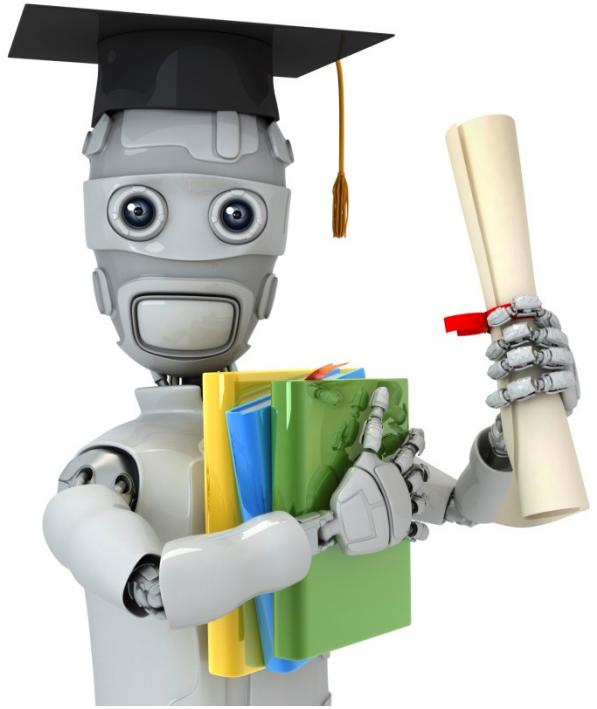
Moving data around



Machine Learning

Octave Tutorial

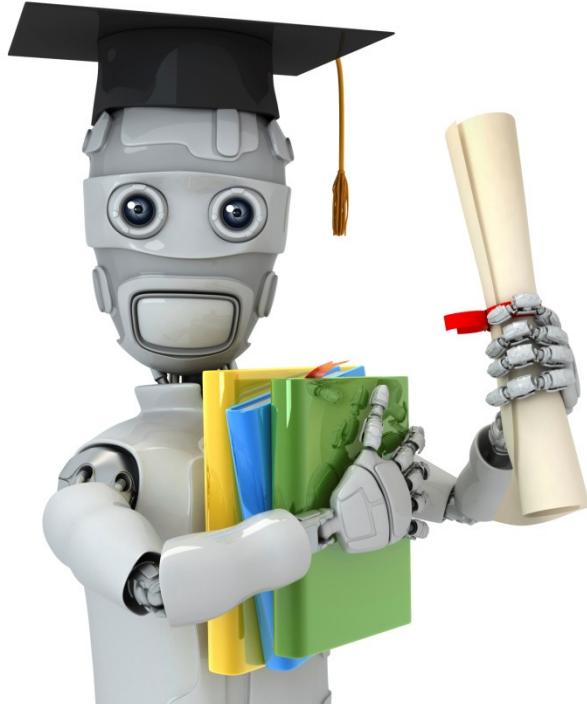
Computing on data



Machine Learning

Octave Tutorial

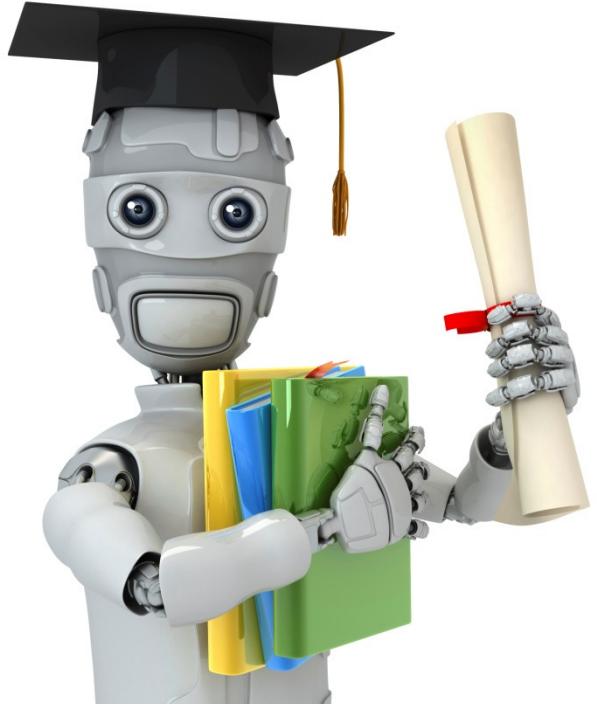
Plotting data



Machine Learning

Octave Tutorial

Control statements: for,
while, if statements



Machine Learning

Octave Tutorial

Vectorial implementation

Vectorization example.

$$h_{\theta}(x) = \sum_{j=0}^n \theta_j x_j \\ = \theta^T x$$

Unvectorized implementation

```
prediction = 0.0;  
for j = 1:n+1,  
    prediction = prediction +  
        theta(j) * x(y)  
end;
```

Vectorized implementation

```
prediction = theta' * x;
```

Vectorization example.

$$h_{\theta}(x) = \sum_{j=0}^n \theta_j x_j \\ = \theta^T x$$

Unvectorized implementation

```
double prediction = 0.0;  
for (int j = 0; j < n; j++)  
    prediction += theta[j] * x[y];
```

Vectorized implementation

```
double prediction  
= theta.transpose() * x;
```

Gradient descent

$$\theta_j := \theta_j - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)} \quad (\text{for all } j)$$

$$\theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_0^{(i)}$$

$$\theta_1 := \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_1^{(i)}$$

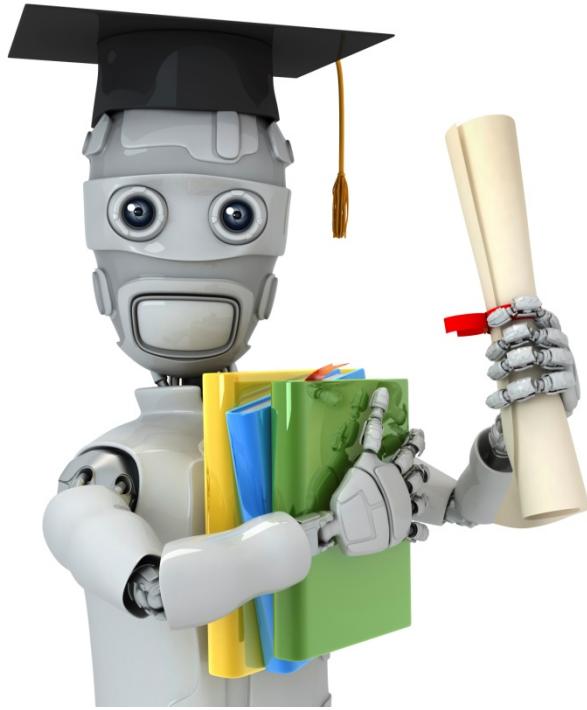
$$\theta_2 := \theta_2 - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_2^{(i)}$$

$$\begin{aligned}
\theta_0 &:= \theta_0 - \alpha \frac{1}{m} \sum_{\substack{i=1 \\ i \neq m}}^m (h_\theta(x^{(i)}) - y^{(i)}) x_0^{(i)} \\
\theta_1 &:= \theta_1 - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_1^{(i)} \\
\theta_2 &:= \theta_2 - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_2^{(i)} \\
(n = 2)
\end{aligned}$$

$$\left| \begin{array}{l} u(j) = 2v(j) + 5w(j) \quad (\text{for all } j) \\ u = 2v + 5w \end{array} \right.$$

Chapter 3 Week3

3.1 Logistic Reggression



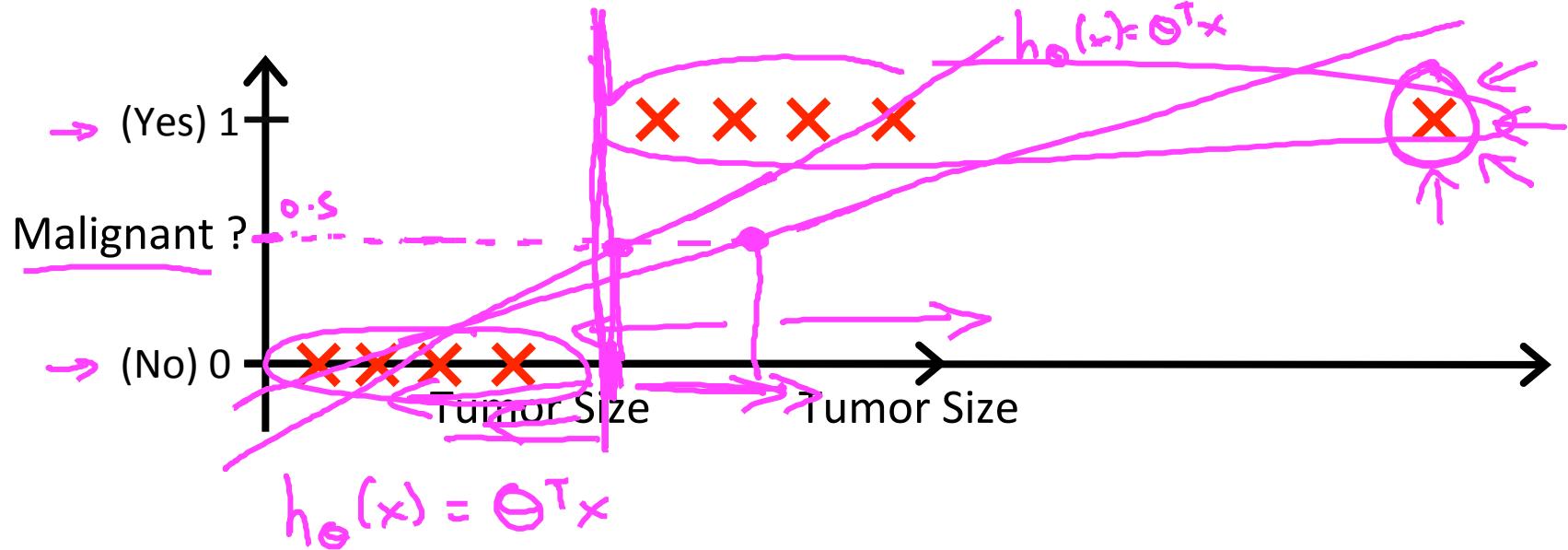
Machine Learning

Logistic Regression

Classification

Classification

- Email: Spam / Not Spam?
 - Online Transactions: Fraudulent (Yes / No)?
 - Tumor: Malignant / Benign ?
- $y \in \{0, 1\}$
- 0: “Negative Class” (e.g., benign tumor)
 - 1: “Positive Class” (e.g., malignant tumor)
- $y \in \{0, 1, 2, 3\}$



→ Threshold classifier output $h_\theta(x)$ at 0.5:

→ If $h_\theta(x) \geq 0.5$, predict "y = 1"

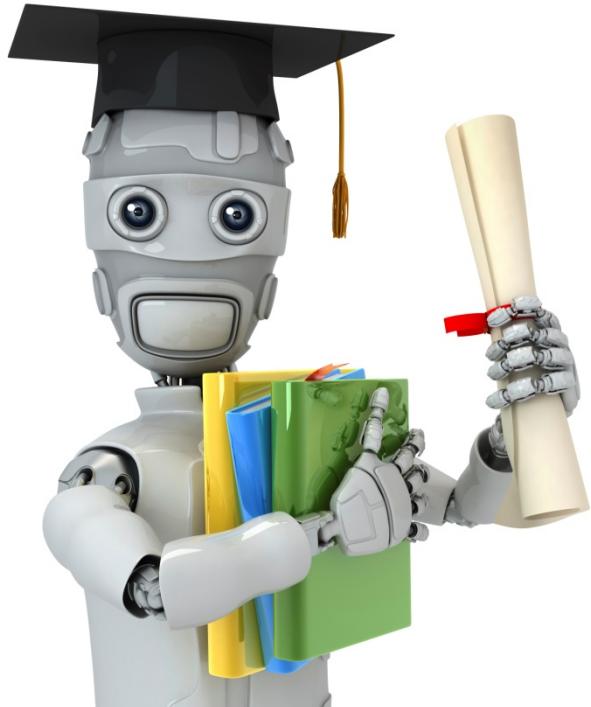
If $h_\theta(x) < 0.5$, predict "y = 0"

Classification: $y = 0 \text{ or } 1$

$h_\theta(x)$ can be $\underline{\quad}$ or $\underline{\quad}$

Logistic Regression: $0 \leq h_\theta(x) \leq 1$

Classification



Machine Learning

Logistic Regression

Hypothesis Representation

Logistic Regression Model

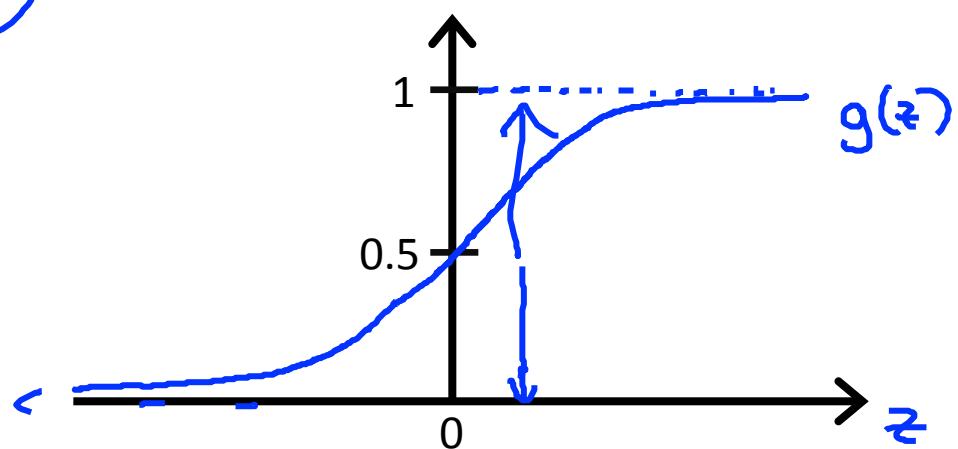
Want $0 \leq h_\theta(x) \leq 1$

$$h_\theta(x) = g(\theta^T x)$$

$$\rightarrow g(z) = \frac{1}{1 + e^{-z}}$$

$\theta^T x$

$$h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$$



- Sigmoid function
- Logistic function

Parameters $\underline{\theta}$.

Interpretation of Hypothesis Output

$$h_{\theta}(x)$$

$h_{\theta}(x)$ = estimated probability that $y = 1$ on input x

Example: If $\underline{x} = \begin{bmatrix} x_0 \\ x_1 \end{bmatrix} = \begin{bmatrix} 1 & \leftarrow \\ \text{tumorSize} & \leftarrow \end{bmatrix}$

$$\underline{h_{\theta}(x) = 0.7} \quad y=1$$

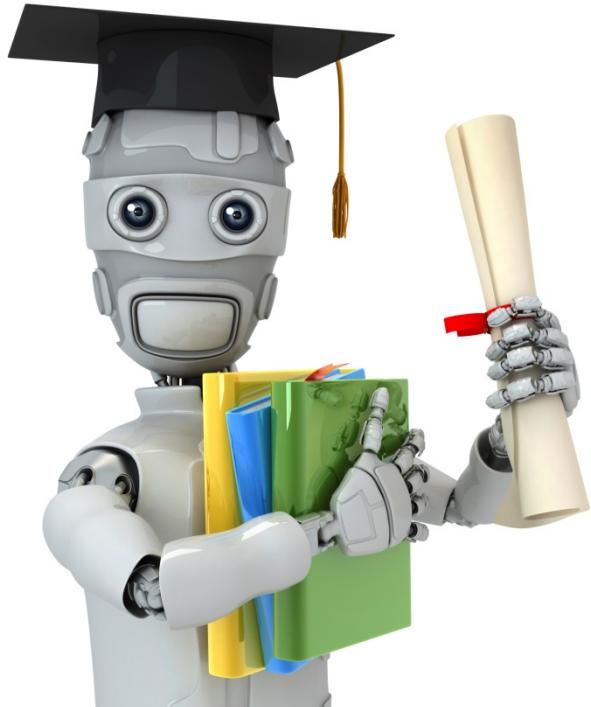
Tell patient that 70% chance of tumor being malignant

$$\underline{h_{\theta}(x) = P(y=1|x; \theta)}$$

“probability that $y = 1$, given x , parameterized by θ ”

$$\underline{y = 0 \text{ or } 1}$$

$$\begin{aligned} \rightarrow P(y = 0|x; \theta) + P(y = 1|x; \theta) &= 1 \\ \rightarrow P(y = 0|x; \theta) &= 1 - P(y = 1|x; \theta) \end{aligned}$$



Machine Learning

Logistic Regression

Decision boundary

Logistic regression

$$h_{\theta}(x) = g(\theta^T x)$$

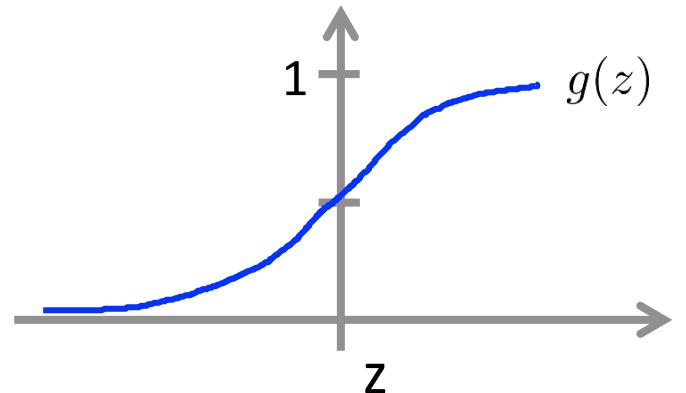
$$g(z) = \frac{1}{1+e^{-z}}$$

Suppose predict " $y = 1$ " if $h_{\theta}(x) \geq 0.5$

$$\theta^T x \geq 0$$

predict " $y = 0$ " if $h_{\theta}(x) < 0.5$

$$\theta^T x < 0$$



$$g(z) \geq 0.5$$

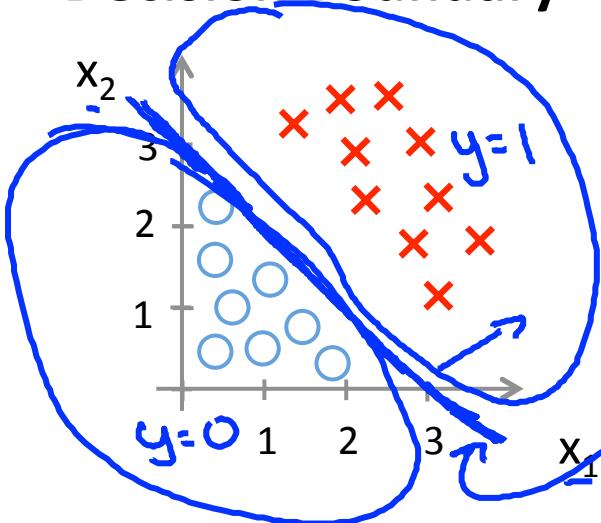
when $z \geq 0$

$$h_{\theta}(x) = g(\theta^T x)$$

$$g(z) < 0.5$$

when $z < 0$

Decision Boundary



$$\Theta = \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$$

$$h_{\theta}(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2)$$

Decision boundary

Predict " $y = 1$ " if $\underline{-3 + x_1 + x_2 \geq 0}$

$$\underline{\Theta^T x}$$

$$\underline{x_1 + x_2 \geq 3}$$

$$x_1, x_2$$

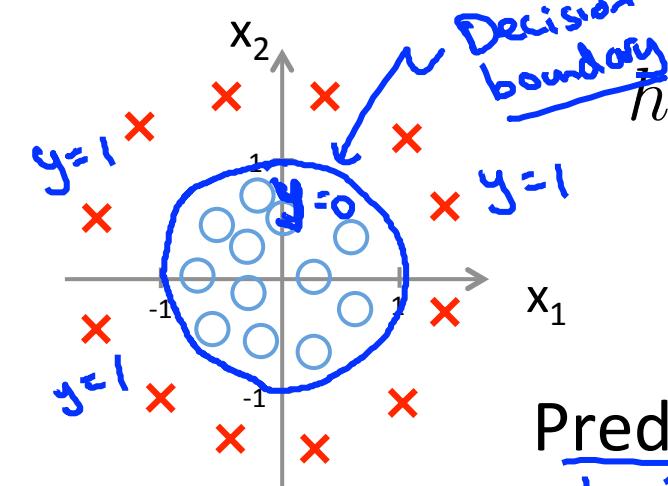
$$h_{\theta}(x) = 0.5$$

$$x_1 + x_2 = 3$$

$$x_1 + x_2 < 3$$

$$y = 0$$

Non-linear decision boundaries

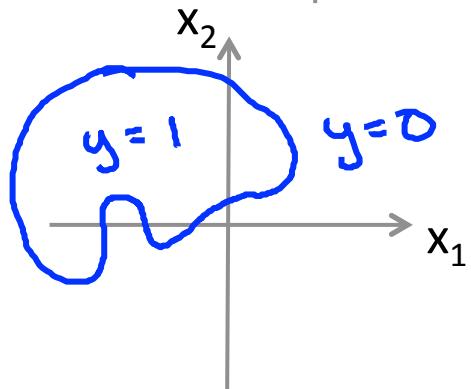


$$h_{\theta}(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 x_1^2 + \theta_4 x_2^2)$$

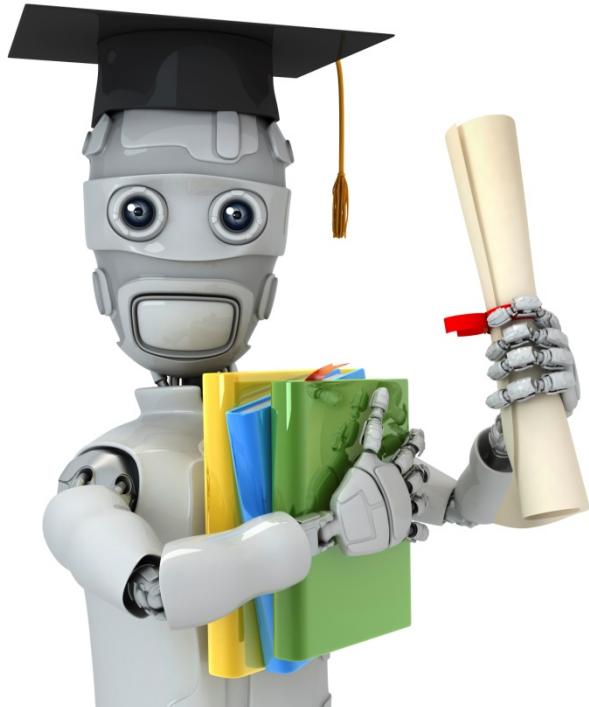
" " " "

$$\theta = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Predict " $y = 1$ " if $\frac{-1 + x_1^2 + x_2^2 \geq 0}{x_1^2 + x_2^2 \geq 1}$



$$h_{\theta}(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_2 + \theta_3 \underline{x_1^2} + \theta_4 \underline{x_1^2 x_2} + \theta_5 \underline{x_1^2 x_2^2} + \theta_6 \underline{x_1^3 x_2} + \dots)$$



Machine Learning

Logistic Regression

Cost function

Training
set:

m examples

$$x \in \begin{bmatrix} x_0 \\ x_1 \\ \dots \\ x_n \end{bmatrix} \quad \mathbb{R}^{n+1}$$

$$x_0 = 1, y \in \{0, 1\}$$

$$h_{\theta}(x) = \frac{1}{1 + e^{-\underline{\theta^T x}}}$$

How to choose parameters θ ?

Cost function

→ Linear regression: $J(\theta)$

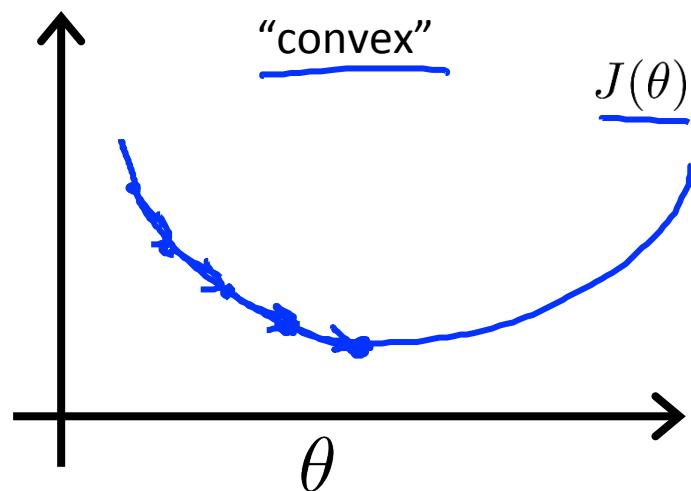
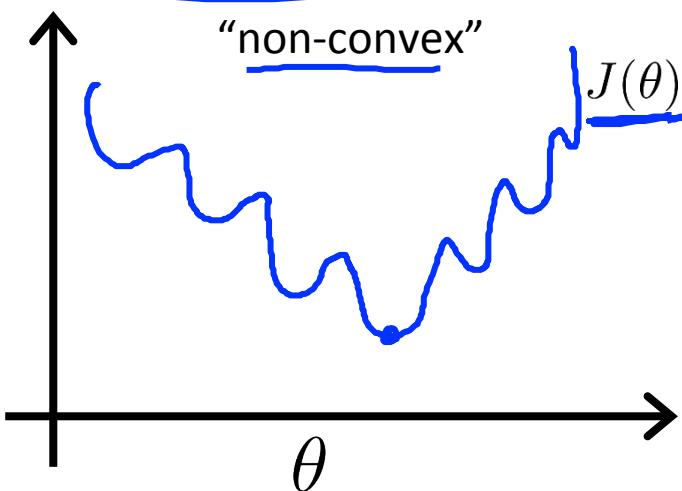
logistic

$$J(\theta) = \frac{1}{m} \sum_{i=1}^m \frac{1}{2} (h_\theta(x^{(i)}) - y^{(i)})^2$$

cost($h_\theta(x^{(i)})$, $y^{(i)}$)

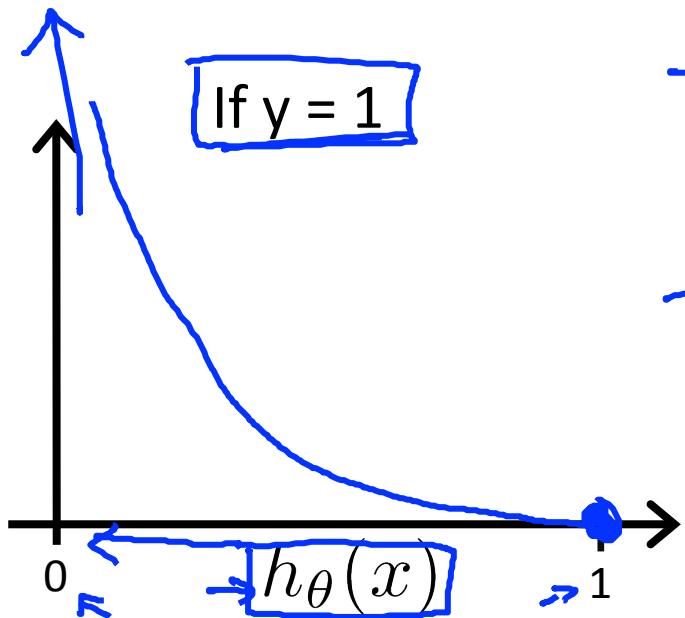
\rightarrow Cost($h_\theta(x^{(i)})$, $y^{(i)}$) = $\frac{1}{2} (h_\theta(x^{(i)}) - y^{(i)})^2$

$\frac{1}{2} e^{-\theta^T y}$



Logistic regression cost function

$$\text{Cost}(h_{\theta}(x), y) = \begin{cases} -\log(h_{\theta}(x)) & \text{if } y = 1 \\ -\log(1 - h_{\theta}(x)) & \text{if } y = 0 \end{cases}$$

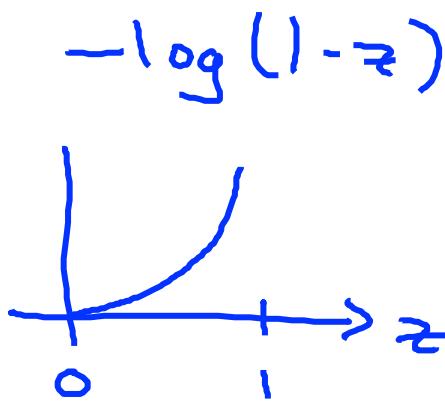
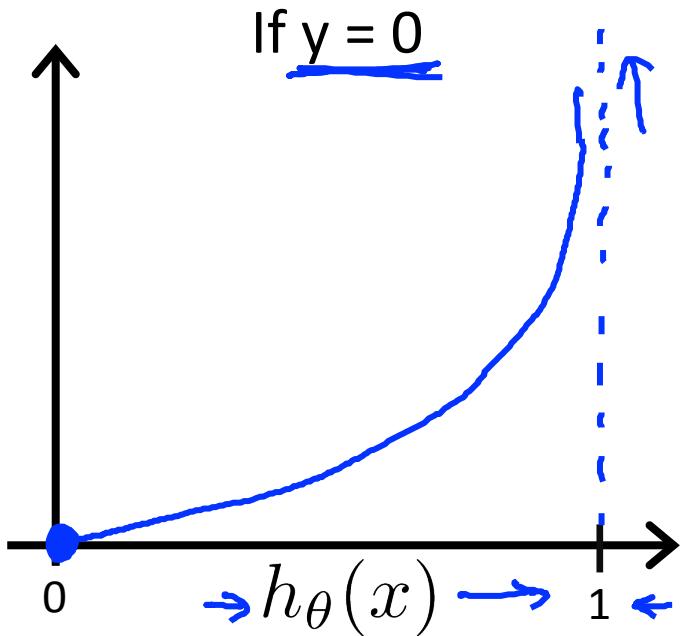


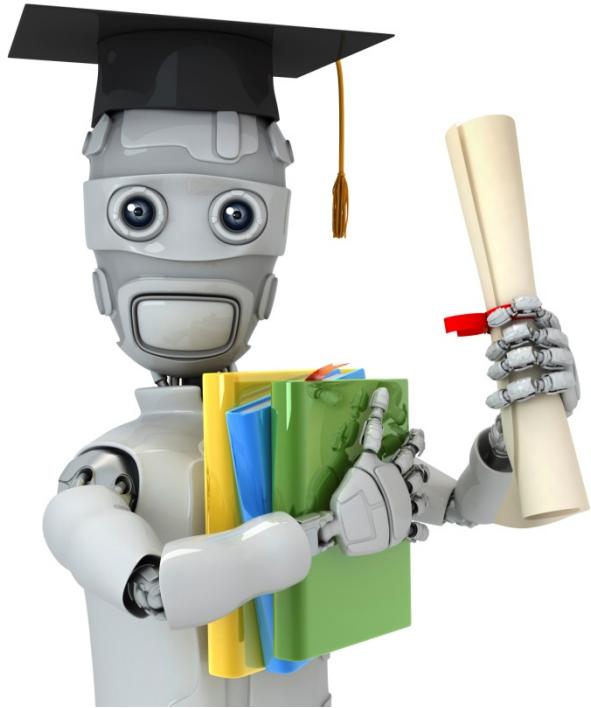
→ Cost = 0 if $y = 1, h_{\theta}(x) = 1$
But as $h_{\theta}(x) \rightarrow 0$
 $\underline{\text{Cost}} \rightarrow \infty$

→ Captures intuition that if $h_{\theta}(x) = 0$,
(predict $P(y = 1|x; \theta) = 0$), but $y = 1$,
we'll penalize learning algorithm by a very
large cost.

Logistic regression cost function

$$\text{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$





Machine Learning

Logistic Regression

Simplified cost function
and gradient descent

Logistic regression cost function

$$\rightarrow J(\theta) = \frac{1}{m} \sum_{i=1}^m \text{Cost}(h_\theta(x^{(i)}), y^{(i)})$$

$$\rightarrow \text{Cost}(h_\theta(x), y) = \begin{cases} -\log(h_\theta(x)) & \text{if } y = 1 \\ -\log(1 - h_\theta(x)) & \text{if } y = 0 \end{cases}$$

Note: $y = 0$ or 1 always

$$\rightarrow \text{Cost}(h_\theta(x), y) = -y \log(h_\theta(x)) - (1-y) \log(1 - h_\theta(x))$$

If $y=1$: $\text{Cost}(h_\theta(x), y) = -\log h_\theta(x)$

If $y=0$: $\text{Cost}(h_\theta(x), y) = -\log(1 - h_\theta(x))$

Logistic regression cost function

$$\begin{aligned} J(\theta) &= \frac{1}{m} \sum_{i=1}^m \text{Cost}(h_\theta(x^{(i)}), y^{(i)}) \\ &= -\frac{1}{m} \left[\sum_{i=1}^m y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log (1 - h_\theta(x^{(i)})) \right] \end{aligned}$$

To fit parameters θ :

$$\min_{\theta} J(\theta)$$

Get $\underline{\theta}$

To make a prediction given new \underline{x} :

$$\text{Output } \underline{h_\theta(x)} = \frac{1}{1+e^{-\theta^T x}}$$

$$\underline{p(y=1 | x; \theta)}$$

Gradient Descent

$$\rightarrow J(\theta) = -\frac{1}{m} \left[\sum_{i=1}^m y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log (1 - h_\theta(x^{(i)})) \right]$$

Want $\min_{\theta} J(\theta)$:

Repeat {

$$\theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

}

(simultaneously update all θ_j)

$$\frac{\partial}{\partial \theta_j} J(\theta) = \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

Gradient Descent

$$J(\theta) = -\frac{1}{m} \left[\sum_{i=1}^m y^{(i)} \log h_\theta(x^{(i)}) + (1 - y^{(i)}) \log (1 - h_\theta(x^{(i)})) \right]$$

Want $\min_{\theta} J(\theta)$:

Repeat {

$$\theta_j := \theta_j - \alpha \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}$$

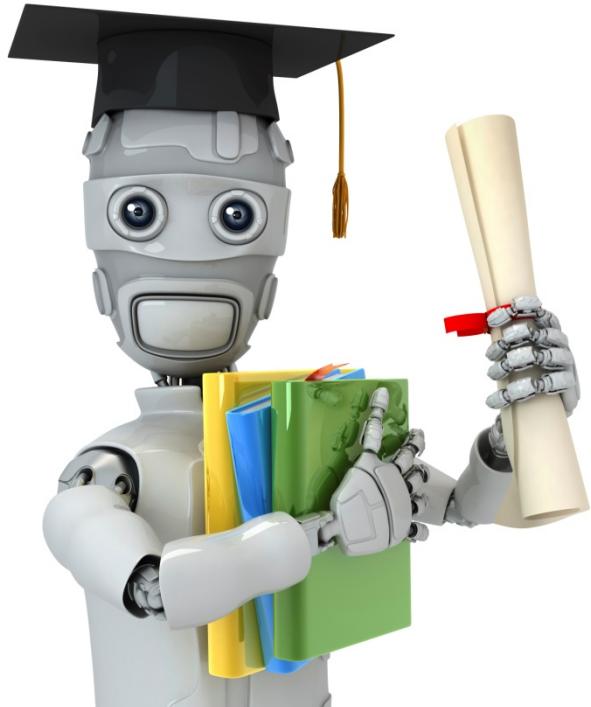
(simultaneously update all θ_j)

$$\Theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_n \end{bmatrix} \quad \text{for } i = 0 \dots n$$

$$h_\theta(x) = \theta^T x$$

$$h_\theta(x) = \frac{1}{1 + e^{-\theta^T x}}$$

Algorithm looks identical to linear regression!



Machine Learning

Logistic Regression

Advanced optimization

Optimization algorithm

Cost function $\underline{J(\theta)}$. Want $\min_{\theta} \underline{J(\theta)}$.

Given θ , we have code that can compute

- - $J(\theta)$
- - $\frac{\partial}{\partial \theta_j} J(\theta)$ (for $j = 0, 1, \dots, n$)

Gradient descent:

Repeat {

$$\rightarrow \theta_j := \theta_j - \alpha \frac{\partial}{\partial \theta_j} J(\theta)$$

}

Optimization algorithm

Given θ , we have code that can compute

- $J(\theta)$
- $\frac{\partial}{\partial \theta_j} J(\theta)$ (for $j = 0, 1, \dots, n$)

Optimization algorithms:

- - Gradient descent
- Conjugate gradient
- BFGS
- L-BFGS

Advantages:

- No need to manually pick α
- Often faster than gradient descent.

Disadvantages:

- More complex

Example:

$$\min_{\theta} J(\theta)$$

$$\rightarrow \theta = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} \quad \theta_1 = 5, \theta_2 = 5.$$

$$\rightarrow J(\theta) = (\theta_1 - 5)^2 + (\theta_2 - 5)^2$$

$$\rightarrow \frac{\partial}{\partial \theta_1} J(\theta) = 2(\theta_1 - 5)$$

$$\rightarrow \frac{\partial}{\partial \theta_2} J(\theta) = 2(\theta_2 - 5)$$

\rightarrow options = optimset('GradObj', 'on', 'MaxIter', '100');

\rightarrow initialTheta = zeros(2,1);

[optTheta, functionVal, exitFlag] ...

= fminunc(@costFunction, initialTheta, options);

↑ ↑

$\theta \in \mathbb{R}^d \quad d \geq 2$.

```
function [jVal, gradient]
    = costFunction(theta)
jVal = (theta(1)-5)^2 + ...
      (theta(2)-5)^2;
gradient = zeros(2,1);
gradient(1) = 2*(theta(1)-5);
gradient(2) = 2*(theta(2)-5);
```

$$\underline{\text{theta}} = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_n \end{bmatrix} \quad \begin{array}{l} \xrightarrow{\text{theta}(1)} \\ \xrightarrow{\text{theta}(2)} \\ \vdots \\ \xrightarrow{\text{theta}(n+1)} \end{array}$$

function [jVal, gradient] = costFunction(theta)

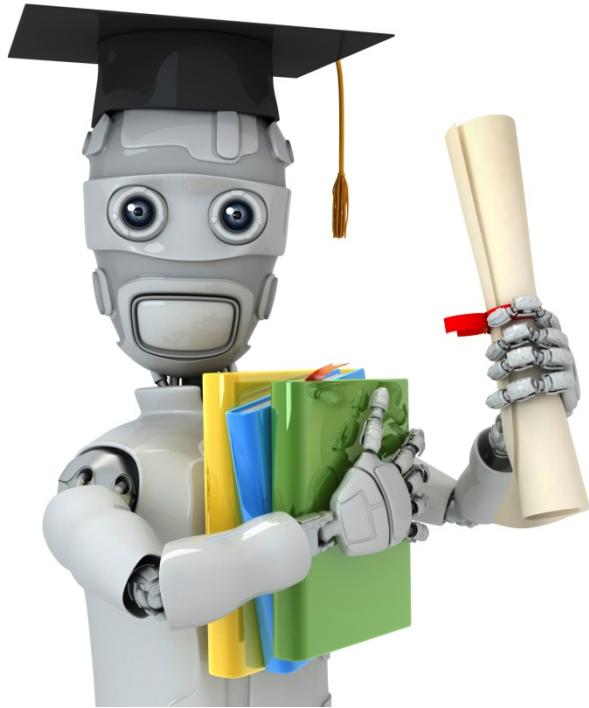
jVal = [code to compute $J(\theta)$];

gradient(1) = [code to compute $\frac{\partial}{\partial \theta_0} J(\theta)$];

gradient(2) = [code to compute $\frac{\partial}{\partial \theta_1} J(\theta)$];

⋮

gradient(n+1) = [code to compute $\frac{\partial}{\partial \theta_n} J(\theta)$];



Machine Learning

Logistic Regression

Multi-class classification:
One-vs-all

Multiclass classification

Email foldering/tagging: Work, Friends, Family, Hobby

$$y=1 \quad y=2 \quad y=3 \quad y=4$$

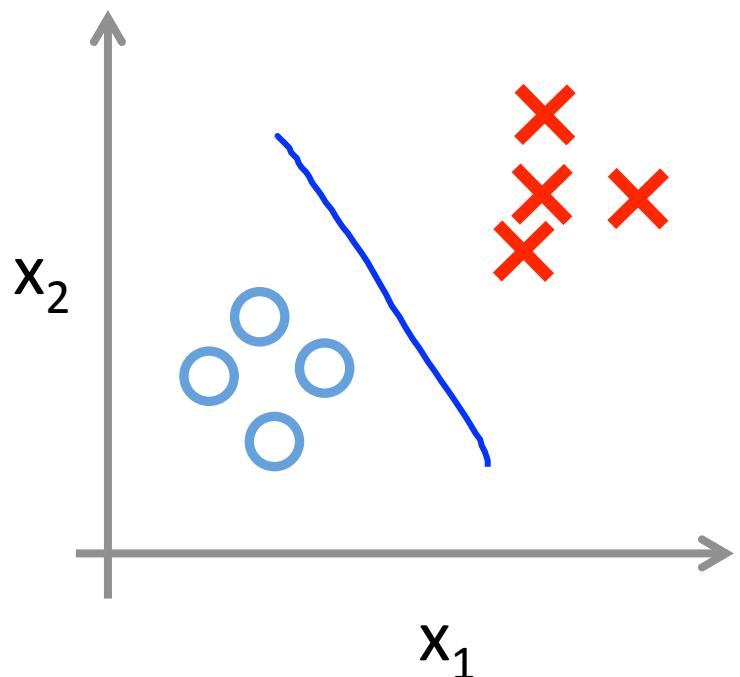
Medical diagrams: Not ill, Cold, Flu

$$y=1 \quad 2 \quad 3$$

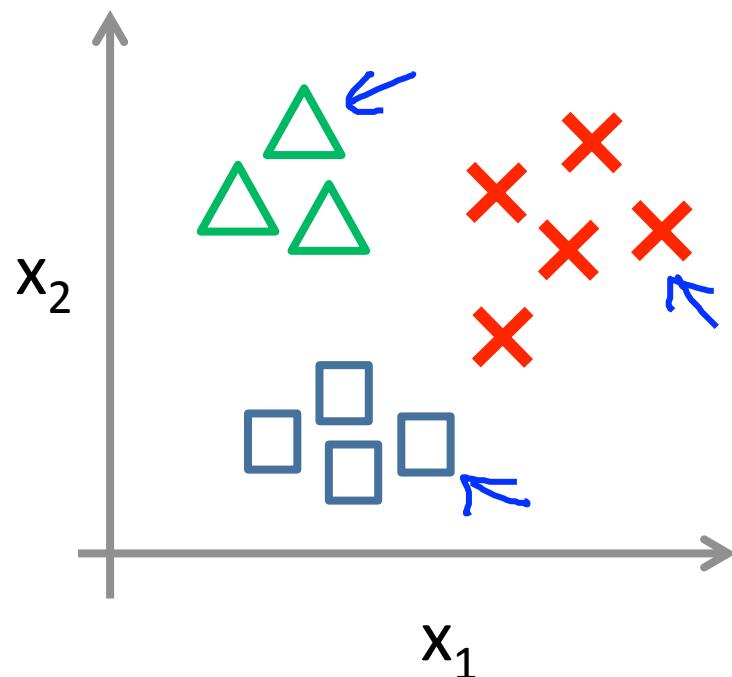
Weather: Sunny, Cloudy, Rain, Snow



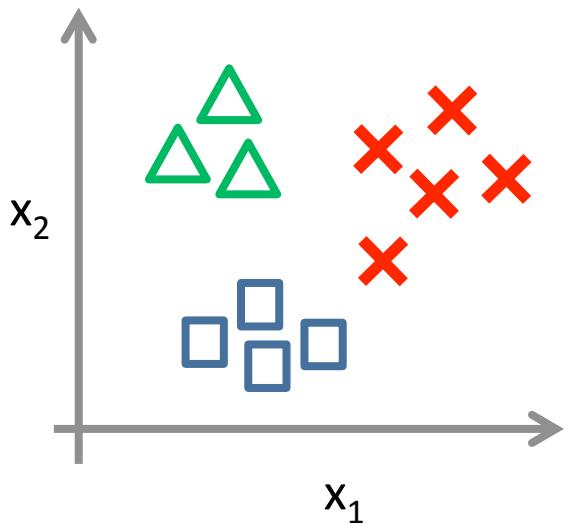
Binary classification:



Multi-class classification:

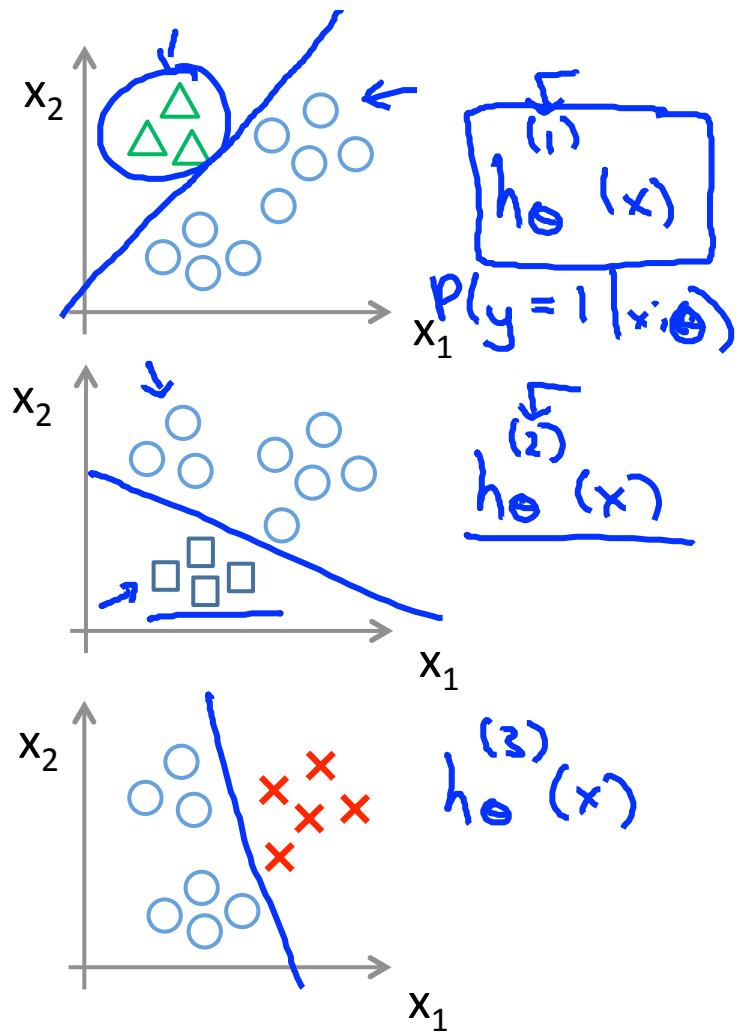


One-vs-all (one-vs-rest):



- Class 1: \leftarrow
- Class 2: \leftarrow
- Class 3: \leftarrow

$$h_{\theta}^{(i)}(x) = P(y = i|x; \theta) \quad (i = 1, 2, 3)$$

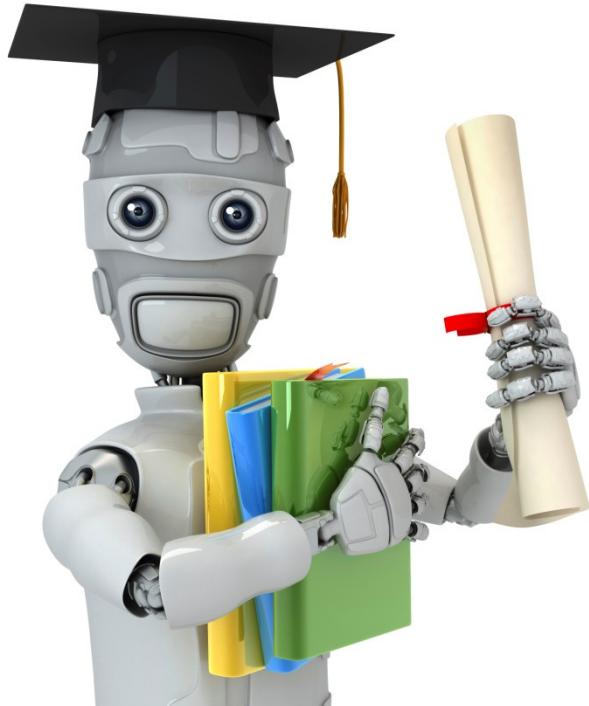


One-vs-all

Train a logistic regression classifier $\underline{h_{\theta}^{(i)}(x)}$ for each class \underline{i} to predict the probability that $\underline{y = i}$.

On a new input \underline{x} , to make a prediction, pick the class i that maximizes

$$\max_i \underline{\underline{h_{\theta}^{(i)}(x)}}$$

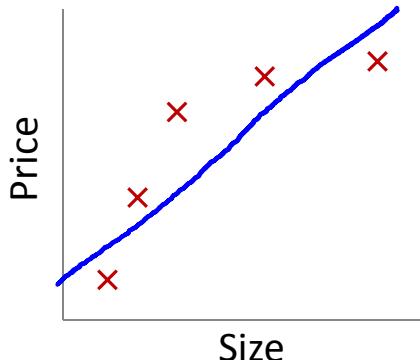


Machine Learning

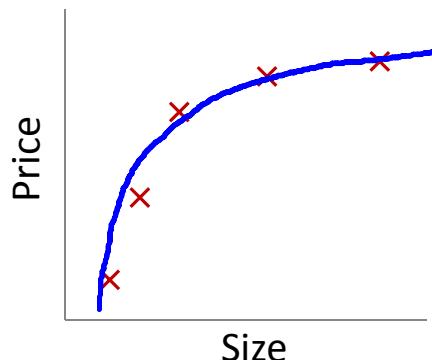
Regularization

The problem of overfitting

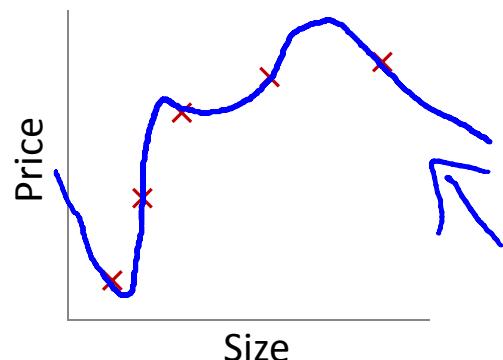
Example: Linear regression (housing prices)



$\rightarrow \theta_0 + \theta_1 x$
"Underfit" "High bias"



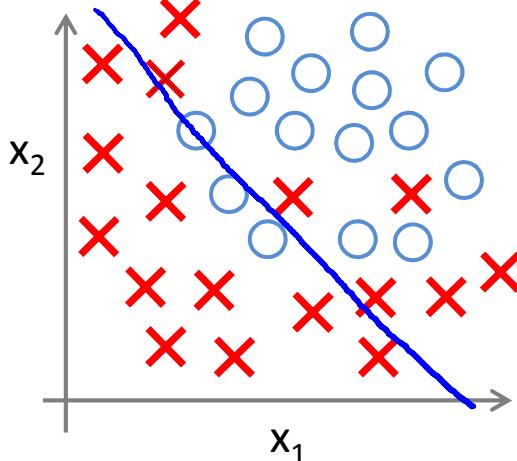
$\rightarrow \theta_0 + \theta_1 x + \theta_2 x^2$
"Just right"



$\rightarrow \theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$
"Overfit" "High variance"

Overfitting: If we have too many features, the learned hypothesis may fit the training set very well ($J(\theta) = \frac{1}{2m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})^2 \approx 0$), but fail to generalize to new examples (predict prices on new examples).

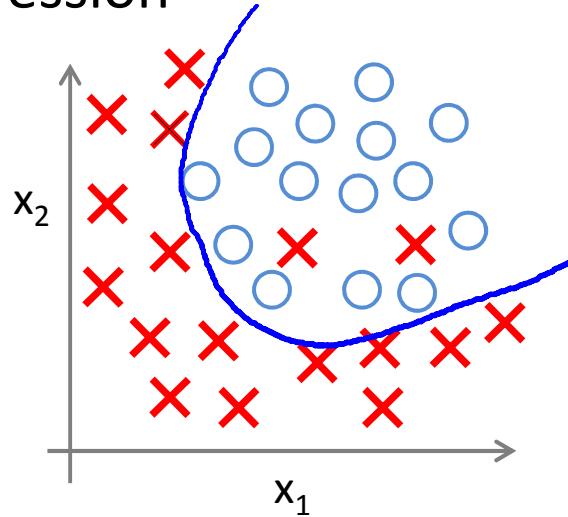
Example: Logistic regression



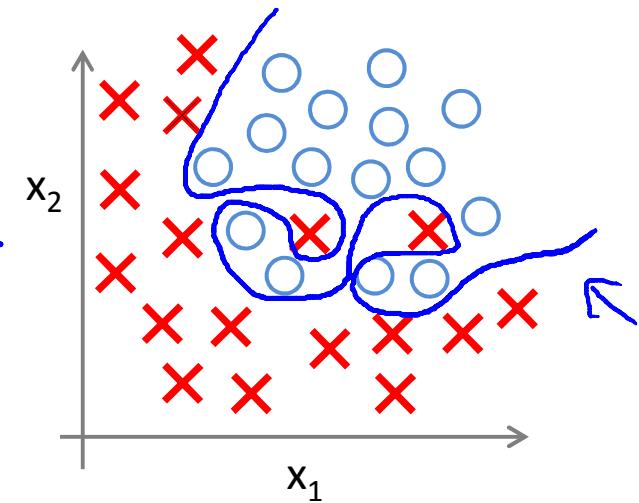
$$\rightarrow h_{\theta}(x) = g(\underline{\theta_0 + \theta_1 x_1 + \theta_2 x_2})$$

(g = sigmoid function)

\nwarrow
"Underfit"



$$g(\underline{\theta_0 + \theta_1 x_1 + \theta_2 x_2} \\ + \underline{\theta_3 x_1^2} + \underline{\theta_4 x_2^2} \\ + \underline{\theta_5 x_1 x_2}})$$

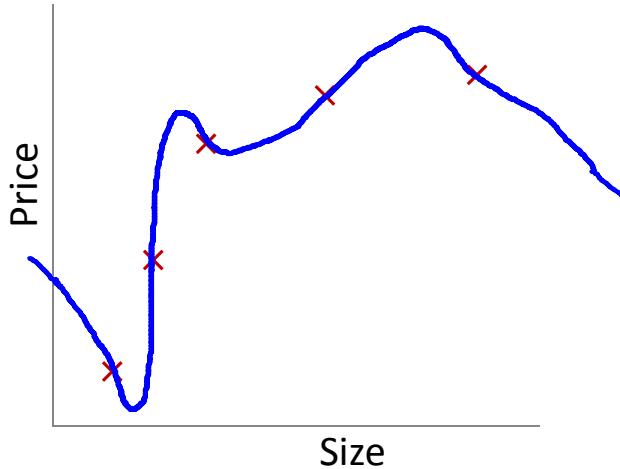


$$g(\underline{\theta_0 + \theta_1 x_1 + \theta_2 x_1^2} \\ + \underline{\theta_3 x_1^2 x_2} + \underline{\theta_4 x_1^2 x_2^2} \\ + \underline{\theta_5 x_1^2 x_2^3} + \underline{\theta_6 x_1^3 x_2} + \dots)$$

\nwarrow
"Overfit"

Addressing overfitting:

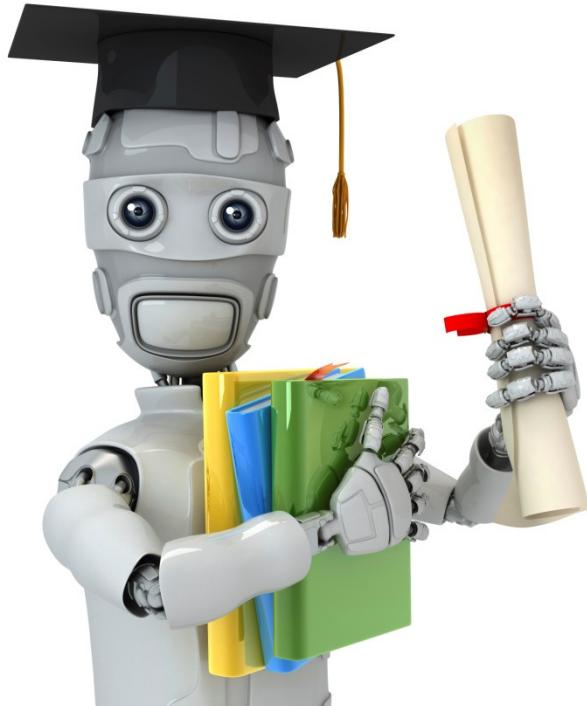
- x_1 = size of house
- x_2 = no. of bedrooms
- x_3 = no. of floors
- x_4 = age of house
- x_5 = average income in neighborhood
- x_6 = kitchen size
- :
- x_{100}



Addressing overfitting:

Options:

1. Reduce number of features.
 - — Manually select which features to keep.
 - — Model selection algorithm (later in course).
2. Regularization.
 - — Keep all the features, but reduce magnitude/values of parameters θ_j
 - Works well when we have a lot of features, each of which contributes a bit to predicting y .

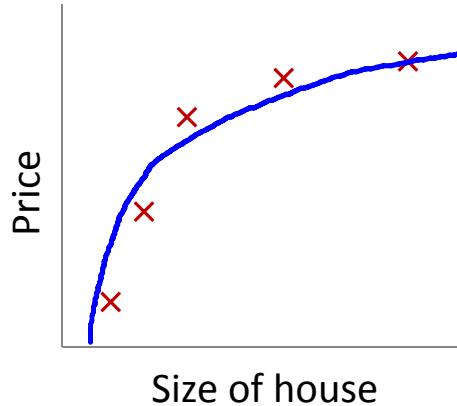


Machine Learning

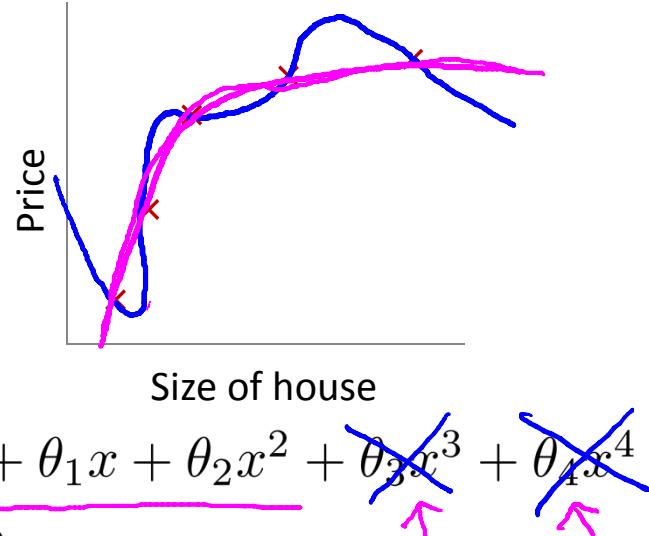
Regularization

Cost function

Intuition



$$\theta_0 + \theta_1 x + \theta_2 x^2$$



$$\theta_0 + \theta_1 x + \theta_2 x^2 + \theta_3 x^3 + \theta_4 x^4$$

Suppose we penalize and make θ_3, θ_4 really small.

$$\min_{\theta} \frac{1}{2m} \sum_{i=1}^m (h_{\theta}(x^{(i)}) - y^{(i)})^2 + 1000 \underline{\theta_3^2} + 1000 \underline{\theta_4^2}$$

$\underline{\theta_3 \approx 0}$ $\underline{\theta_4 \approx 0}$

Regularization.

Small values for parameters $\theta_0, \theta_1, \dots, \theta_n$

- “Simpler” hypothesis
- Less prone to overfitting

$$\theta_3, \theta_4 \approx 0$$

Housing:

- Features: x_1, x_2, \dots, x_{100}
- Parameters: $\theta_0, \theta_1, \theta_2, \dots, \theta_{100}$

$$J(\theta) = \frac{1}{2m} \left[\sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^n \theta_j^2 \right]$$

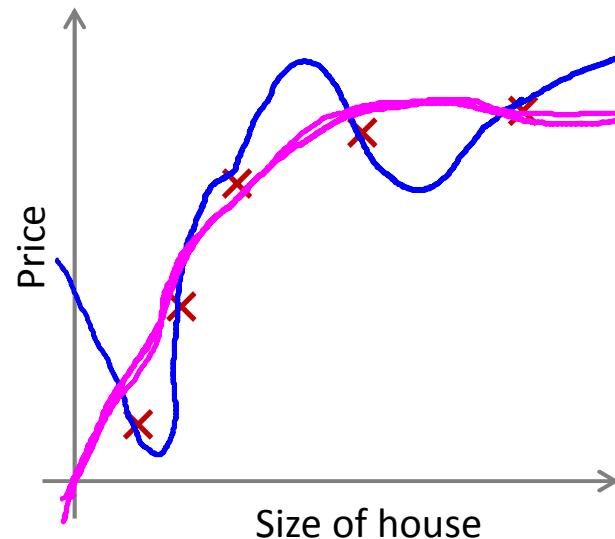
~~$\theta_1, \theta_2, \theta_3, \dots, \theta_{100}$~~

Regularization.

$$\rightarrow J(\theta) = \frac{1}{2m} \left[\sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^n \theta_j^2 \right]$$

$\min_{\theta} J(\theta)$

regularization parameter



In regularized linear regression, we choose θ to minimize

$$J(\theta) = \frac{1}{2m} \left[\sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^n \theta_j^2 \right]$$

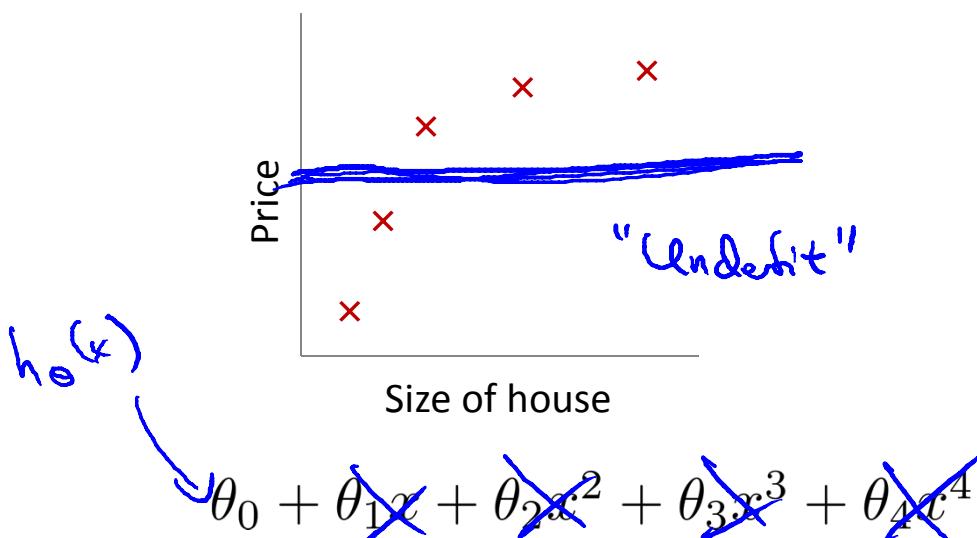
What if λ is set to an extremely large value (perhaps for too large for our problem, say $\lambda = 10^{10}$)?

- Algorithm works fine; setting λ to be very large can't hurt it
- Algorithm fails to eliminate overfitting.
- Algorithm results in underfitting. (Fails to fit even training data well).
- Gradient descent will fail to converge.

In regularized linear regression, we choose θ to minimize

$$J(\theta) = \frac{1}{2m} \left[\sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^n \theta_j^2 \right]$$

What if λ is set to an extremely large value (perhaps for too large for our problem, say $\lambda = 10^{10}$)?

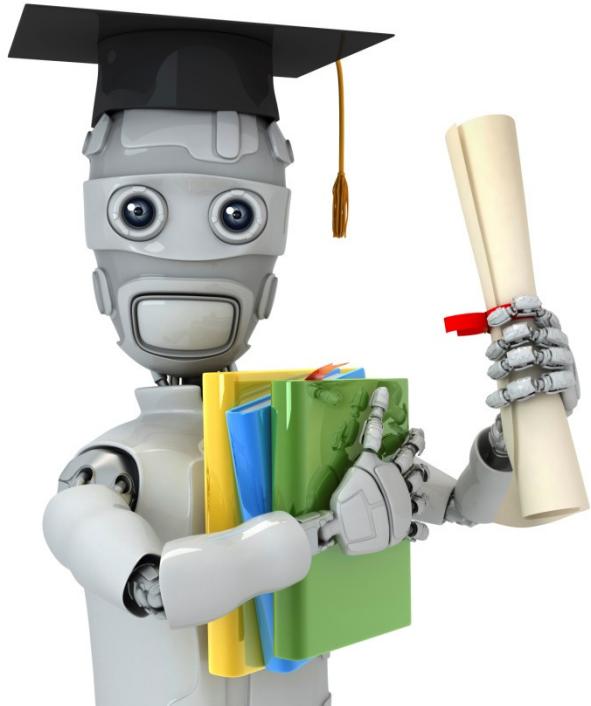


$$\underline{\theta}_1, \underline{\theta}_2, \underline{\theta}_3, \underline{\theta}_4$$

$$\theta_1 \approx 0, \theta_2 \approx 0$$

$$\theta_3 \approx 0, \theta_4 \approx 0$$

$$h_\theta(x) = \underline{\theta}_0$$



Machine Learning

Regularization

Regularized linear regression

Regularized linear regression

$$J(\theta) = \frac{1}{2m} \left[\sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})^2 + \lambda \sum_{j=1}^n \theta_j^2 \right]$$

$$\min_{\theta} J(\theta)$$



Gradient descent

Repeat {

$$\rightarrow \theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_0^{(i)}$$

$$\frac{\partial}{\partial \theta_0}$$

$$\theta_1, \theta_2, \dots, \theta_n$$

$$\frac{\partial}{\partial \theta_0} J(\theta)$$

$$\begin{aligned} \theta_j &:= \theta_j - \boxed{\alpha} \left[\frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)} - \frac{\lambda}{m} \theta_j \right] \\ &\quad (j = \cancel{0}, 1, 2, 3, \dots, n) \\ \theta_j &:= \boxed{\theta_j (1 - \alpha \frac{\lambda}{m})} - \boxed{\alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}} \rightarrow J(\theta) \end{aligned}$$

$$\boxed{\theta_j}$$

$$1 - \alpha \frac{\lambda}{m} < 1$$

$$\underline{0.99}$$

$$\theta_j \times 0.99$$

Normal equation

$$\underline{X} = \begin{bmatrix} (x^{(1)})^T \\ \vdots \\ (x^{(m)})^T \end{bmatrix} \leftarrow \text{m} \times (n+1)$$

$$y = \begin{bmatrix} y^{(1)} \\ \vdots \\ y^{(m)} \end{bmatrix} \uparrow \mathbb{R}^m$$

$$\rightarrow \min_{\theta} J(\theta)$$

$$\frac{\partial}{\partial \theta_j} J(\theta) \stackrel{\text{set}}{=} 0 \quad \rightsquigarrow$$

$$\Rightarrow \theta = (X^T X + \lambda \underbrace{\begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 1 & 0 & 1 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{bmatrix}}_{(n+1) \times (n+1)})^{-1} X^T y$$

E.g. n=2

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(n+1) × (n+1)

Non-invertibility (optional/advanced).

Suppose $m \leq n$, \leftarrow
(#examples) (#features)

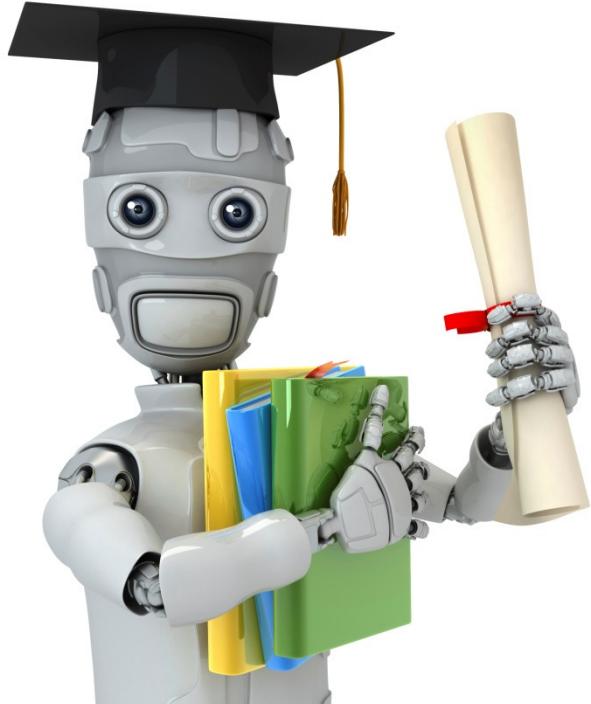
$$\theta = \underbrace{(X^T X)^{-1}}_{\text{non-invertible / singular}} X^T y$$

pinv $\frac{\text{inv}}{\kappa}$

If $\lambda > 0$,

$$\theta = \left(X^T X + \lambda \begin{bmatrix} 0 & & & \\ & 1 & & \\ & & 1 & \\ & & & \ddots \\ & & & & 1 \end{bmatrix} \right)^{-1} X^T y$$

invertible .

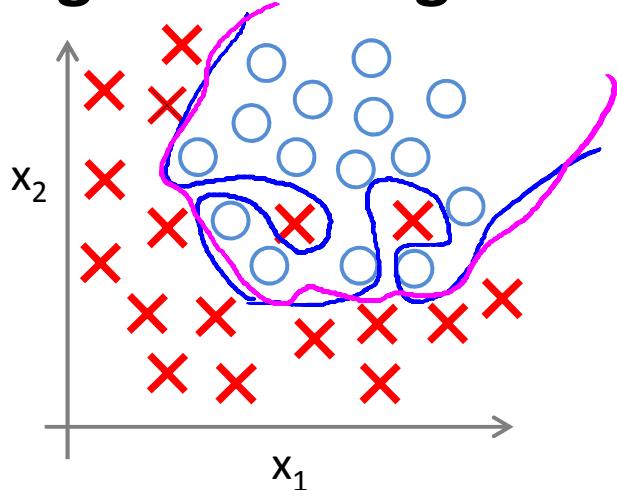


Machine Learning

Regularization

Regularized logistic regression

Regularized logistic regression.



$$h_{\theta}(x) = g(\theta_0 + \theta_1 x_1 + \theta_2 x_1^2 + \theta_3 x_1^2 x_2 + \theta_4 x_1^2 x_2^2 + \theta_5 x_1^2 x_2^3 + \dots)$$

Cost function:

$$\rightarrow J(\theta) = - \left[\frac{1}{m} \sum_{i=1}^m y^{(i)} \log h_{\theta}(x^{(i)}) + (1 - y^{(i)}) \log(1 - h_{\theta}(x^{(i)})) \right] + \frac{\lambda}{2m} \sum_{j=1}^n \theta_j^2$$

$\boxed{\theta_0, \theta_1, \dots, \theta_n}$

Gradient descent

Repeat {

$$\rightarrow \theta_0 := \theta_0 - \alpha \frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_0^{(i)}$$

$$\rightarrow \theta_j := \theta_j - \alpha \left[\underbrace{\frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_j^{(i)}}_{(j = \cancel{x}, 1, 2, 3, \dots, n)} - \frac{1}{m} \theta_j \right] \leftarrow$$

$\theta_1, \dots, \theta_n$

}

$$\frac{\partial}{\partial \theta_j} \underline{J(\theta)}$$

$$h_\theta(x) = \frac{1}{1 + e^{-\theta^\top x}}$$

Advanced optimization

f_{minimise} (a cost function) $\theta = \begin{bmatrix} \theta_0 \\ \theta_1 \\ \vdots \\ \theta_n \end{bmatrix}$ $\theta_0 \leftarrow \text{theta}(1)$
 $\theta_1 \leftarrow \text{theta}(2)$
 $\theta_n \leftarrow \text{theta}(n+1)$

→ function [jVal, gradient] = costFunction(theta)

jVal = [code to compute $J(\theta)$];

→ $J(\theta) = \left[-\frac{1}{m} \sum_{i=1}^m y^{(i)} \log(h_\theta(x^{(i)})) + (1 - y^{(i)}) \log 1 - h_\theta(x^{(i)}) \right] + \left[\frac{\lambda}{2m} \sum_{j=1}^n \theta_j^2 \right]$

→ gradient (1) = [code to compute $\frac{\partial}{\partial \theta_0} J(\theta)$];
 $\frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_0^{(i)}$

→ gradient (2) = [code to compute $\frac{\partial}{\partial \theta_1} J(\theta)$];
 $\left(\frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_1^{(i)} \right) - \frac{\lambda}{m} \theta_1$ $J(\theta)$

→ gradient (3) = [code to compute $\frac{\partial}{\partial \theta_2} J(\theta)$];
 $\vdots \quad \left(\frac{1}{m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)}) x_2^{(i)} \right) - \frac{\lambda}{m} \theta_2$

gradient (n+1) = [code to compute $\frac{\partial}{\partial \theta_n} J(\theta)$];

