ECE-606: Key Equations (for exam 5)
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Physical constants:

$\hbar = 1.055\times10^{-34}$ [J-s]

$m_0 = 9.109\times10^{-31}$ [kg]

$k_B = 1.380\times10^{-23}$ [J/K]

$q = 1.602\times10^{-19}$ [C]

$\varepsilon_0 = 8.854\times10^{-14}$ [F/cm]

Silicon (T = 300K)

$N_C = 3.23\times10^{19}$ cm^{-3}

$N_V = 1.83\times10^{19}$ cm^{-3}

$n_i = 1\times10^{10}$ cm^{-3}

Miller Indices: (hkl) {hkl} [hkl] <hkl>

Angle between two planes: $\cos\theta = \dfrac{h_1h_2 + k_1k_2 + l_1l_2}{\sqrt{h_1^2+k_1^2+l_1^2}\sqrt{h_2^2+k_2^2+l_2^2}}$

Spacing between two planes: $d = 1\big/\left|\vec{N}\right| = a\big/\sqrt{h^2+k^2+l^2}$

Time independent wave eq.: $\dfrac{-\hbar^2}{2m_0}\dfrac{\partial^2\psi(x)}{\partial x^2} + U(x)\psi(x) = E\psi(x)$ $\quad\Psi(x,t) = \psi(x)e^{-i\omega t}$

$\dfrac{d^2\psi(x)}{dx^2} + k^2\psi(x) = 0 \qquad k^2 = \dfrac{2m_0}{\hbar^2}\left[E - U_0\right] \qquad \psi(x) = Ae^{\pm ikx}$

Infinite quantum well of width, W: $\varepsilon_n = \dfrac{\hbar^2k_n^2}{2m^*} = \dfrac{\hbar^2n^2\pi^2}{2m^*W^2}$ $\qquad n$ = 1, 2, 3, …

Momentum operator: $\hat{p} \equiv \dfrac{\hbar}{i}\dfrac{\partial}{\partial x}$ $\qquad$ Energy operator: $\hat{E} = -\dfrac{\hbar}{i}\dfrac{\partial}{\partial t}$

Plane wave: $\Psi(x,t) = e^{i(\pm kx - \omega t)}$ $\qquad$ Momentum of plane wave: $p = \hbar k$

Density of states in k-space:

1D: $N_k dk = 2\times(L/2\pi)dk = (L/\pi)dk$

2D: $N_k d^2k = 2\times\left[A\big/(2\pi)^2\right]d^2k = \left(A/2\pi^2\right)d^2k$

3D: $N_k d^3k = 2\times\left(\Omega/8\pi^3\right)d^3k = \left(\Omega/4\pi^3\right)d^3k$

Density of states in energy (above the bottom of the conduction band):

1D: $D_{1D}(E) = \dfrac{1}{\pi\hbar}\sqrt{\dfrac{2m_D^*}{E-E_C}}$ $\qquad$ 2D: $D_{2D}(E) = \dfrac{m_D^*}{\pi\hbar^2}$ $\qquad$ 3D: $D_{3D}(E) = \dfrac{\left(m_D^*\right)^{3/2}\sqrt{2(E-E_C)}}{\pi^2\hbar^3}$

Fermi function: $f(E)=\dfrac{1}{1+e^{(E-E_F)/k_BT}}$

Electron densities: ($\eta_F=(E_F-E_C)/k_BT$):

1D: $n_L=N_C\mathcal{F}_{-1/2}(\eta_F)$ m^{-1} $\quad N_C=\dfrac{1}{\hbar}\sqrt{\dfrac{2m_D^*k_BT}{\pi}}$ m^{-1}

2D: $n_S=N_C\mathcal{F}_0(\eta_F)$ m^{-2} $\quad N_C=\left(\dfrac{m_D^*k_BT}{\pi\hbar^2}\right)$ m^{-2}

3D: $n=N_C\mathcal{F}_{1/2}(\eta_F)$ m^{-3} $\quad N_C=\dfrac{1}{4}\left(\dfrac{2m_D^*k_BT}{\pi\hbar^2}\right)^{3/2}$ m^{-3}

FD Integral: $\mathcal{F}_j(\eta_F)=\dfrac{1}{\Gamma(j+1)}\displaystyle\int_0^\infty\dfrac{\eta^j d\eta}{1+e^{\eta-\eta_F}}$ For n an integer: $\Gamma(n)=(n-1)!$

For non-integer n: $\Gamma(1/2)=\sqrt{\pi}$ and $\Gamma(p+1)=p\Gamma(p)$

Space charge neutrality: $p-n+N_D^+-N_A^-=0$

$$\frac{N_D^+}{N_D}=\frac{1}{1+g_De^{(E_F-E_D)/k_BT}} \qquad \frac{N_A^-}{N_A}=\frac{1}{1+g_Ae^{(E_A-E_F)/k_BT}}$$

Recombination:

Radiative: $R_{Rad}=B(np-n_i^2)$ Auger: $R_{Aug}=C_nn(np-n_i^2)+C_pp(np-n_i^2)$

SRH: $-\left.\dfrac{\partial n}{\partial t}\right|_{SRH}=-\left.\dfrac{\partial p}{\partial t}\right|_{SRH}=R_{SRH}=\dfrac{(np-n_i^2)}{\tau_p(n+n_1)+\tau_n(p+p_1)}$

Current equations: $J_n=n\mu_n\dfrac{dF_n}{dx}$ $\quad J_n=nq\mu_n\mathcal{E}_x+qD_n\dfrac{dn}{dx}$ $\quad D_n/\mu_n=k_BT/q$

$J_p=p\mu_p\dfrac{dF_p}{dx}$ $\quad J_p=pq\mu_p\mathcal{E}_x-qD_p\dfrac{dp}{dx}$ $\quad D_p/\mu_p=k_BT/q$

Semiconductor Equations:

$$\frac{\partial n}{\partial t}=-\nabla\bullet\left(\frac{\vec{J}_n}{-q}\right)+G_n-R_n$$

$$\frac{\partial p}{\partial t}=-\nabla\bullet\left(\frac{\vec{J}_p}{q}\right)+G_p-R_p$$

$$0=-\nabla\bullet\left(\varepsilon\vec{\mathcal{E}}\right)+\rho$$

Minority Carrier Diffusion Eqaution

$$\frac{\partial\Delta p}{\partial t}=D_p\frac{\partial^2\Delta p}{dx^2}-\frac{\Delta p}{\tau_p}+G_L$$

$$L_p=\sqrt{D_p\tau_p}$$

Diode Equation: $I_D=qA\left(\dfrac{n_i^2D_n}{N_AL_n}+\dfrac{n_i^2D_p}{N_DL_p}\right)\left(e^{qV_A/k_BT}-1\right)$ (long)

$$I_D = qA\left(\frac{n_i^2 D_n}{N_A W_P} + \frac{n_i^2 D_p}{N_D W_N}\right)\left(e^{qV_A/k_BT} - 1\right) \text{ (short)}$$

Bipolar transistors:

$$\beta = \frac{I_C}{I_B} = \frac{N_{DE}}{N_{AB}}\frac{D_n}{D_p}\frac{W_E}{W_B} \qquad g_m = \frac{\partial I_C}{\partial V_{BE}} = \frac{I_C}{\left(k_BT/q\right)}$$

$$\gamma = \frac{I_{En}}{I_{Ep} + I_{En}} = \frac{1}{1 + \frac{W_B}{W_E}\frac{n_{iE}^2}{n_{iB}^2}\frac{N_{AB}}{N_{DE}}} \qquad I_{Cn} = \alpha_T I_{En} \approx I_C \qquad \alpha_T \approx 1 - \frac{1}{2}\left(\frac{W_B}{L_n}\right)^2$$

$$I_C = \alpha_{dc} I_E \qquad \alpha_{dc} = \alpha_T \gamma \qquad \beta_{dc} = \frac{\alpha_{dc}}{1 - \alpha_{dc}}$$

$$t_t = \frac{W_B^2}{2D_n} \qquad f_T\big|_{\max} = \frac{1}{2\pi t_t}$$

MOS Electrostatics:

Depletion conditions (p-type substrate): $0 < \phi_S < 2\phi_F \qquad \phi_F = \frac{k_BT}{q}\ln\left(\frac{N_A}{n_i}\right)$

$$W_D = \sqrt{\frac{2\kappa_S \varepsilon_0 \phi_S}{qN_A}} \qquad \mathcal{E}_S = \sqrt{\frac{2qN_A\phi_S}{\kappa_s \varepsilon_0}} \qquad Q_B = -qN_A W_D\left(\phi_S\right) \qquad Q_B\left(\phi_S\right) = -\sqrt{2q\kappa_s\varepsilon_0 N_A \phi_S}$$

Gate voltage vs. surface potential relation:

$$V_G' = \phi_S - \frac{Q_S\left(\phi_S\right)}{C_{ox}} \qquad C_{ox} = \kappa_{ox}\varepsilon_0/t_{ox} \qquad Q_S\left(\phi_S\right) = Q_n\left(\phi_S\right) + Q_B\left(\phi_S\right)$$

$$Q_n\left(\phi_S\right) = -\sqrt{\varepsilon_{Si}\, k_BT\, n_B}\, e^{q\phi_S/2k_BT}$$

Inversion conditions:

$$W_T = \left[\frac{2K_S\varepsilon_0}{qN_A}2\phi_F\right]^{1/2} \qquad V_G' = 2\phi_F - \frac{Q_B\left(2\phi_F\right) + Q_n}{C_{ox}}$$

$$V_T' = 2\phi_F - \frac{Q_B\left(2\phi_F\right)}{C_{ox}} \qquad Q_n = -C_{ox}\left(V_G - V_T\right)$$

MOS IV characteristics (square law):

$$I_D = \frac{W\bar{\mu}_n C_{ox}}{L}\left[\left(V_G - V_T\right)V_D - \frac{V_D^2}{2}\right] \qquad 0 \le V_D < V_{Dsat} \quad V_G \ge V_T$$

$$I_D = \frac{W\bar{\mu}_n C_{ox}}{2L}\left(V_G - V_T\right)^2 \qquad V_D \ge V_{Dsat} \qquad V_G \ge V_T$$