
About Dark Bubbles

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1 HOW TO INDUCE DARK ENERGY ON A BUBBLE SURFACE: A SIMPLE EXAMPLE

Let us just start with the most basic set-up: A non-supersymmetric Anti-de Sitter vacuum, with cosmological constant $\Lambda_D = -\frac{1}{2}(D-1)(D-2)L^{3-D}$. The parameter L represents the curvature radius of the AdS_D vacuum. For computational convenience, it will be easier to refer to this parameter with its inverse, the vacuum scale $k = \frac{1}{L}$.

As we discussed in section [NON-SUSY], the aforementioned AdS_D vacuum is unstable and will eventually decay to a more metastable solution, via D_p -brane nucleation. This D_p -brane, which is a Coleman-de Luccia/ BT instanton [add ref to CdL and or section], will mediate such decay, separating two different AdS_D vacua. From now on, we will refer to the vacua living inside the hyper-volume encoded by the D_p -brane as the *inside* vacuum, denoted by a minus (-) subscript sign. The whole higher-dimensional space receives the name of *Bulk* and the tandem brane-inside will be colloquially called *Bubble*. On the other hand, the region *outside* the bubble has not yet decayed, and will be denoted by a positive (+) subscript sign.

I THINK THAT A SKETCH WITH NAMES AND K VALUES HERE COULD BE NICE.

Finally, as the aim of this thesis is to discuss phenomenological aspects in four dimensions¹, and given the discussion in [ref to GAUSS-CODAZZI] about co-dimension one hyper-surfaces, we will fix $D = 5$. This implies that the boundary ∂B of the nucleated five-dimensional bubble will have $d = D - 1 = 4$ dimensions.

In this dimensionality, the simplest geometry inside and outside the bubble can be described by the following line invariant:

$$ds_{\pm}^2 = g_{\mu\nu}^{\pm} dx^{\mu} dx^{\nu} = -f_{\pm}(r) dt_{\pm}^2 + f_{\pm}^{-1}(r) dr^2 + r^2 d\Omega_3^2, \quad (1)$$

where

$$f_{\pm}(r) = 1 + k_{\pm}^2 r^2 + \chi(r, t, k_{\pm}, q_1, \dots, q_m). \quad (2)$$

Here, r will denote the radial coordinate of the AdS_5 vacuum, i.e. the throat direction and the line-invariant $d\Omega_3^2 = \gamma_{ij} dx^i dx^j$ is the metric on S^3 , which corresponds to the three-dimensional solid angle for the usual spatial sections. The function $f_{\pm}(r)$ represents the AdS_5 geometry plus some possible extra features encoded in $\chi(r, t, k_{\pm}, q_1, \dots, q_m)$. These will be relevant in the following sections, but for now on, let us fix $\chi(r, t, k_{\pm}, q_1, \dots, q_m) = 0$.

Before we start computing the Israel's junction conditions described in [ref to APPENDIX], it is important to realise the parametrical dependence of the expanding bubble. As described in [ref to INSTANTONS], the nucleated bubble can be identified with a Coleman-de Luccia instanton, which is equipped with an $O(4)$ -symmetry.² The two relevant coordinates for this derivation are the global time t and the radial coordinate r , which also depends on t , as it is expanding, hence evolving in time. Both will be related to the proper time τ an observer living on the boundary ∂B experiments. The coordinate choices to describe the bulk geometry and the induced one are:

$$x^{\mu} = \{t(\tau), r(\tau), \theta, \phi, \psi\}, \quad y^a = \{\tau, \theta, \phi, \psi\}. \quad (3)$$

¹Other bubble dimensionalities have been explored in [IVANO STUFF]

²Both in Euclidean and Lorentzian time by analytic continuation.

Let us now compute the first Israel junction condition described in [ref to APPENDIX]. This is no more than:

$$ds_{\text{ind}}^2 = h_{ab} dy^a dy^b = -N(\tau)^2 d\tau^2 + a^2(\tau) d\Omega_3^2, \quad (4)$$

where

$$N^2 = f(a(\tau)) \dot{t}^2 - \frac{\dot{a}(\tau)^2}{f(a(\tau))}. \quad (5)$$

The metric h_{ab} represents the four-dimensional induced geometry on the hyper-surface, i.e. the D_3 -brane, given by the parametrisation of $\{y^a\}$ -coordinates. The lapse function $N(\tau)$ has been introduced to make time reparametrisation invariance manifest. Observe that a dot represents a proper-time derivative. One can always choose the right parametrical relation between $f(a(\tau))$, $\dot{t}(\tau)$ and $a(\tau)$ such that $N = 1$. In that case, it is always easy to see that Eq. (4) represents an expanding four-dimensional cosmology, described by a Friedmann-Robertson-Lemaître-Walker metric with closed spatial sections.

What about the second Israel junction condition? As discussed in [APPENDIX], this condition relates how the brane is embedded in the bulk geometry to the energy-momentum tensor induced on the brane, such that the whole bulk-brane geometry remains a solution to the Einstein equation. For simplicity in this example, let us assume we have an empty brane. This implies that its energy-momentum tensor only contains information about its tension σ , which can be denoted by:

$$S_{ab} = -\sigma h_{ab}. \quad (6)$$

COMMENT ON THE MINUS SIGN? CHECK CRITICAL SIGMA VALUE FOR POINCARÉ

As discussed in [APPENDIX], the second Israel's condition relates the *jump* in the extrinsic curvature between the inside and the outside geometries that the brane separates. To be as pedagogical as possible in this introductory section, we will compute the most important steps to obtain the induced energy-momentum tensor.³

First, we need to identify the normal and tangent vectors $\{n^\mu, e_a^\mu\}$ with respect to the embedding. Using eqs (APPENDIX) and (APPENDIX), these result to be:

$$e_a^\mu = \begin{pmatrix} \dot{t}(\tau) & 0 & 0 & 0 & 0 \\ \dot{f}(a) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \quad n^\mu = \begin{pmatrix} \frac{-\dot{a}/\dot{t}}{f(a)\sqrt{f(a) - \frac{1}{f(a)}\left(\frac{\dot{a}}{\dot{t}}\right)^2}}, \frac{f(a)}{\sqrt{f(a) - \frac{1}{f(a)}\left(\frac{\dot{a}}{\dot{t}}\right)^2}}, 0, 0, 0 \end{pmatrix}. \quad (7)$$

This implies an extrinsic curvature K_{ab} of the form:

$$K_{ab} = \nabla_\beta n_\alpha e_a^\alpha e_b^\beta = \quad (8)$$

ADD THE RIGHT BRIDGE WHEN MATHEMATICA WANTS TO WORK.

Given the previous results, if one computes the second junction condition Eq. [APPENDIX] assuming a pure constant tension brane, described by Eq. (6), it yields:

$$\sigma = \frac{3}{\kappa_5} \left(\sqrt{\frac{f_-(a)}{a^2} + \frac{\dot{a}^2}{N^2 a^2}} - \sqrt{\frac{f_+(a)}{a^2} + \frac{\dot{a}^2}{N^2 a^2}} \right), \quad (9)$$

³The whole derivation is left as an exercise to the reader.

where $\kappa_5 = 8\pi G_5$ and σ corresponds to the tension of the bubble wall. Observe how the tension of the shell is governed by the four-dimensional Hubble parameter $H = \dot{a}/a$ and the AdS_5 scales of the five-dimensional bulk. This already points to a limit situation; when the five dimensional scales k_{\pm} are large compared to the four-dimensional H parameter, the tension of the shell approaches the *extremal* tension

$$\sigma_{\text{cr}} = \frac{3}{\kappa_5} (k_- - k_+). \quad (10)$$

This critical value of the tension σ_{cr} corresponds to the extremal value of the radius that maximises the bounce action of the instanton described in [APPENDIX]. This also implies that all the energy obtained in the decay process has been invested in generating a shell that will remain static, as $H \ll k_{\pm}$ in that limit situation.⁴

What now if the tension results to be sub-critical, i.e. a small portion of energy of the decay is invested in accelerating the bubble growth, as described in [APPENDIX]?⁵ In this case, we can assume:

$$\sigma = (1 - \epsilon) \sigma_{\text{cr}}, \quad (11)$$

with $\epsilon > 0$. Expanding this sub-critical value σ in Eq. (9), results into:

$$H = \left(\frac{\dot{a}}{a} \right)^2 = -\frac{1}{a^2} + \frac{8\pi}{3} \frac{2k_- k_+}{k_+ - k_-} G_5 (\sigma_{\text{cr}} - \sigma) + \mathcal{O}(\epsilon^2). \quad (12)$$

This previous equation should result familiar to the reader that does not get distracted by the suppressed corrections $\mathcal{O}(\epsilon^2)$. A little bit of dimensional analysis may help those absent-minded readers.

As observed in Eq. (10), $[k_{\pm}] = [\text{Length}]^{-1}$, while $\kappa_5 = 8\pi G_5 = \ell_5^3$, where ℓ_5 is the five dimensional Planck length. So $[\kappa_5] = [\text{Length}]^3$. Hence, $[\sigma_{\text{cr}}] = [\sigma] = [\text{Length}]^{-4}$. This has the same dimensionality as the dark energy density ρ_{Λ} in four dimensions as discussed in [SECTION COSMO]. Similarly, we can perform the same dimensional analysis to conclude that $\left[G_5 \frac{k_- k_+}{k_+ - k_-} \right] = [\text{Length}]^2$. This is the dimensionality of the Newton's constant G_4 .⁶ These do not seem like coincidence, pointing to the following identifications:

$$\kappa_4 = 2 \frac{k_- k_+}{k_+ - k_-} \kappa_5, \quad \rho_{\Lambda} = \sigma_{\text{cr}} - \sigma. \quad (13)$$

These allow us to "unmask" Eq. (12). It is no more than a regular Friedmann equation in disguise!

CONTINUE HERE. I SHOULD TALK ABOUT HOW DARK ENERGY IS NO MORE THAN DYNAMICS IN 5D. ALSO ABOUT SUPER CRITICAL TENSION AS IMPOSSIBLE. ALSO ABOUT DECORATION, BUT THAT LATER, WHEN GC INTRODUCED. FIRST 10D EMBEDDING. WHAT ABOUT RS COMPARISON? WHEN?

⁴We will soon see that this is always the case, as the energy hierarchy of the embedding of this construction into string theory provides $k_{\pm} \gg H$. These corrections are also independent of $a(\tau)$.

⁵This implies that the bounce [APPENDIX] is not extremised.

⁶Recall, as one can see in [SECTION], that $\kappa_D = \ell_D^{D-2}$.

2 DARK BUBBLES FROM STRING THEORY: AN EXPLICIT CONSTRUCTION

Here some general shity introduction about the general features of the model in 10D.

2.1 A rotating ten-dimensional black hole

Talk about whole construction that Oscar used and present all ingredients in the game. Two option: Describe the basics of the dual field theory here or create appendix about AdS/Cft?

2.2 Comparing junctions to EOM

Basically, compare as we did in the paper. Just to prove, that at some specific limit EOM = JC.

2.3 Higher curvature corrections to the rescue

Point to the fact that previous EOM has no CC. Discuss about how to obtain it and elaborate on all pieces in the game. End with the famous $1/N$ corrections that transform in CC. (First part of 5th paper)

2.4 Energy scales from Dark Bubble embedding: A new hope

Compare to observed CC and obtain N to fix all scales. Discuss and defend the size of extra dimensions and the stringy scales.

3 DECORATING THE COSMOS: GAUSS-CODAZZI EQUATION AT WORK

Argue somehow that we are going to look at things from a 5D perspective and will start to induce stuff in the 4D cosmos.

3.1 Matter

3.2 Radiation

3.3 Gravitational waves

3.4 The standard model of particle physics

3.4.1 Electromagnetism

3.4.2 Weak force: Neutrinos

4 HOLOGRAPHIC BUBBLES AND HANGING STRINGS

5 QUANTUM BUBBLES: HIGHER DIMENSIONS TO SOLVE BOUNDARY CHOICES

[1]

REFERENCES

- [1] U.H. Danielsson and T. Van Riet, *What if string theory has no de Sitter vacua?*, *Int. J. Mod. Phys. D* **27** (2018) 1830007 [[1804.01120](#)].