About Dark Bubbles

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Seems that I need a general introduction here. I do not know what to say, to be honest.

1 How to induce dark energy on a bubble surface: A simple example

ADD REFERENCES!

Let us just start with the most basic set-up: A non-supersymmetric Anti-de Sitter vacuum, with cosmological constant given by:

$$\Lambda_D = -\tfrac{1}{2}(D-1)(D-2)L^{3-D}. \tag{1.1}$$

The parameter L represents the curvature radius of the AdS$_D$ vacuum. For computational convenience, it will be easier to refer to this parameter with its inverse, the vacuum scale $k = \frac{1}{L}$.

As we discussed in section [NON-SUSY], the aforementioned AdS$_D$ vacuum is unstable and will eventually decay to a more metastable solution, via D_p-brane nucleation. This D_p-brane, which is a Coleman-de Luccia/ BT instanton [add ref to CdL and or section], will mediate such decay, separating two different AdS$_D$ vacua. From now on, we will refer to the vacua living inside the hyper-volume encoded by the D_p-brane as the *inside* vacuum, denoted by a minus (-) subscript sign. The whole higher-dimensional space receives the name of *Bulk* and the tandem brane-inside will be colloquially called *Bubble*. On the other hand, the region *outside* the bubble has not yet decayed, and will be denoted by a positive (+) subscript sign.

Finally, as the aim of this thesis is to discuss phenomenological aspects in four dimensions[1], and given the discussion in [ref to GAUSS-CODAZZI] about co-dimension one hyper-surfaces, we will fix $D=5$. This implies that the boundary ∂B of the nucleated five-dimensional bubble will have $d = D-1 = 4$ dimensions.

In this dimensionality, the simplest geometry inside and outside the bubble can be described by the following line invariant:

$$\mathrm{ds}^2_\pm = g^\pm_{\mu\nu} dx^\mu dx^\nu = -f_\pm(r) dt^2_\pm + f^{-1}_\pm(r) dr^2 + r^2 d\Omega_3^2, \tag{1.2}$$

where

$$f_\pm(r) = 1 + k^2_\pm r^2 + \chi(r, t, k_\pm, q_1, ..., q_m). \tag{1.3}$$

Here, r will denote the radial coordinate of the AdS$_5$ vacuum, i.e. the throat direction and the line-invariant $d\Omega_3^2 = \zeta_{ij} dx^i dx^j$ is the metric on S^3, which corresponds to the three-dimensional solid angle for the usual spatial sections. From now on, Greek indices will represent bulk coordinates, while Latin ones will be used to describe quantities associated with the *induced* geometry on the boundary. Finally, it is important to mention that the function $f_\pm(r)$ represents the AdS$_5$ geometry plus some posible extra features encoded in $\chi(r, t, k_\pm, q_1, ..., q_m)$. These will be relevant in the following sections, but for now on, let us fix $\chi(r, t, k_\pm, q_1, ..., q_m) = 0$.

[1] Other bubble dimensionalities have been explored in [IVANO STUFF]

Before we start computing the Israel's junction conditions described in [ref to APPENDIX], it is important to realise the parametrical dependence of the expanding bubble. As described in [ref to INSTANTONS], the nucleated bubble can be identified with a Coleman-de Luccia instanton, which is equipped with an $O(4)$-symmetry.[2] The two relevant coordinates for this derivation are the global time t and the radial coordinate r, which also depends on t, as it is expanding, hence evolving in time. Both will be related to the proper time τ an observer living on the boundary ∂B experiments. The coordinate choices to describe the bulk geometry and the induced one are:

$$x^\mu = \{t(\tau), r(\tau), \alpha, \beta, \gamma\}, \qquad\qquad y^a = \{\tau, \alpha, \beta, \gamma\}. \tag{1.4}$$

Let us now compute the first Israel junction condition described in [ref to APPENDIX]. This is no more than:

$$\mathrm{d}s^2_{\text{ind}} = h_{ab} dy^a dy^b = -N(\tau)^2 d\tau^2 + a^2(\tau) d\Omega_3^2, \tag{1.5}$$

where

$$N^2 = f(a(\tau))\dot{t}^2 - \frac{\dot{a}(\tau)^2}{f(a(\tau))}. \tag{1.6}$$

The metric h_{ab} represents the four-dimensional induced geometry on the hyper-surface, i.e. the D_3-brane, given by the parametrisation of $\{y^a\}$-coordinates. The lapse function $N(\tau)$ has been introduced to make time reparametrisation invariance manifest. Observe that a dot represents a proper-time derivative (i.e. the time an observer living on the hypersurface will experiment). One can always choose the right parametrical relation between $f(a(\tau))$, $\dot{t}(\tau)$ and $a(\tau)$ such that $N = 1$. In that case, it is always easy to see that Eq. (1.5) represents an expanding four-dimensional cosmology, described by a Friedmann-Robertson-Lemaître-Walker metric with closed spatial sections.

What about the second Israel junction condition? It is important to notice that the presence of the expanding D_3-brane, which divides the bulk space in two: *inside* and *outside*, will generate a discontinuity of the bulk metric across itself. This forces the presence of a non-zero induced energy-stress tensor on the brane, such that the whole configuration remains a solution to the bulk Einstein equation. This is what the second Israel junction condition accounts for. For simplicity in this example, let us assume we have an empty brane. This implies that its energy-momenum tensor only contains information about its tension σ, which can be denoted by:

$$S_{ab} = -\sigma h_{ab}. \tag{1.7}$$

COMMENT ON THE MINUS SIGN? ChECK CRITICAL SIGMA VALUE FOR POINCARE

As discussed in [APPENDIX], the second Israel's condition relates the *jump* in the extrinsic curvature between the inside and the outside geometries that the brane separates. To be as pedagogical as possible in this introductory section, we will compute the most important steps to obtain the induced energy-momentum tensor.[3]

First, we need to identify the normal and tangent vectors $\{n^\mu, e^\mu_a\}$ with respect to the embed-

[2] Both in Euclidean and Lorentzian time by analytic continuation.
[3] The whole derivation is left as an exercise to the reader.

ding. Using eqs (APPENDIX) and (APPENDIX), these result to be:

$$e^{\mu}_{a} = \begin{pmatrix} \dot{t}(\tau) & 0 & 0 & 0 & 0 \\ \dot{f}(a) & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}, \qquad n^{\mu} = \left(\frac{-\dot{a}/\dot{t}}{f(a)\sqrt{f(a) - \frac{1}{f(a)}\left(\frac{\dot{a}}{\dot{t}}\right)^2}}, \frac{f(a)}{\sqrt{f(a) - \frac{1}{f(a)}\left(\frac{\dot{a}}{\dot{t}}\right)^2}}, 0, 0, 0 \right). \tag{1.8}$$

This implies an extrensic curvature K_{ab} of the form:

$$K_{ab} = \nabla_{\beta} n_{\alpha} e^{\alpha}_{a} e^{\beta}_{b} = \tag{1.9}$$

ADD THE RIGHT BRIDGE WHEN MATHEMATICA WANTS TO WORK.

Given the previous results, if one computes the second junction condition Eq. [APPENDIX] assuming a pure constant tension brane, described by Eq. (1.7), it yields:

$$\sigma = \frac{3}{\kappa_5}\left(\sqrt{\frac{f_{-}(a)}{a^2} + \frac{\dot{a}^2}{N^2 a^2}} - \sqrt{\frac{f_{+}(a)}{a^2} + \frac{\dot{a}^2}{N^2 a^2}} \right), \tag{1.10}$$

where $\kappa_5 = 8\pi G_5$ and σ corresponds to the tension of the bubble wall. Observe how the tension of the shell is governed by the four-dimensional Hubble parameter $H = \dot{a}/a$ and the difference in the AdS_5 scales of the five-dimensional bulk. This already points to a limit situation; when the five dimensional scales $k_{\pm}$ are large compared to the four-dimensional H parameter, the tension of the shell approaches the *extremal* tension[4]

$$\sigma_{\rm cr} = \frac{3}{\kappa_5}\left(k_{-} - k_{+}\right). \tag{1.11}$$

This critical value of the tension $\sigma_{\rm cr}$ corresponds to the extremal value of the radius that maximises the bounce action of the instanton described in [APPENDIX]. This also implies that all the energy obtained in the decay process has been invested in generating a shell that will remain static, as $H \ll k_{\pm}$ in that limit situation.[5]

What now if the tension results to be sub-critical, i.e. a small portion of energy of the decay is invested in accelerating the bubble growth, as described in [APPENDIX]?[6] In this case, we can assume:

$$\sigma = (1-\epsilon)\,\sigma_{\rm cr}, \tag{1.12}$$

with $\epsilon > 0$. Expanding this sub-critical value σ in Eq. (1.10), results into:

$$H = \left(\frac{\dot{a}}{a}\right)^2 = -\frac{1}{a^2} + \frac{8\pi}{3}\frac{2\,k_{-}k_{+}}{k_{+} - k_{-}} G_5\left(\sigma_{\rm cr} - \sigma\right) + \mathcal{O}\left(\epsilon^2\right). \tag{1.13}$$

This previous equation should be familiar to the reader that does not get distracted by the supressed corrections $\mathcal{O}\left(\epsilon^2\right)$. A little bit of dimensional analysis may help those absent-minded readers.

[4] This can be seen as a *flat* shell, as the curvature parameter a is neglegible compared to the AdS_5 scale $k_{\pm}$.

[5] We will soon see that this is always the case, as the energy hierarchy of the embedding of this construction into string theory provides $k_{\pm} \gg H$. These corrections are also independent of $a(\tau)$.

[6] This implies that the bounce [APPENDIX] is not extremised.

As observed in Eq. (1.11), $[k_{\pm}] = [\text{Length}]^{-1}$, while $\kappa_5 = 8\pi G_5 = \ell_5^3$, where ℓ_5 is the five dimensional Planck length. So $[\kappa_5] = [\text{Length}]^3$. Hence, $[\sigma_{\text{cr}}] = [\sigma] = [\text{Length}]^{-4}$. This has the same dimensionality as the dark energy density ρ_Λ in four dimensions as discussed in [SECTION COSMO]. Similarly, we can perform the same dimensional analysis to conclude that $\left[G_5 \frac{k_- k_+}{k_+ - k_-}\right] = [\text{Length}]^2$. This is the dimensionality of the Newton's constant G_4.[7] These do not seem like coincidence, pointing to the following identifications:

$$\kappa_4 = 2\frac{k_- k_+}{k_+ - k_-}\kappa_5, \qquad \rho_\Lambda = \sigma_{\text{cr}} - \sigma. \tag{1.14}$$

These allow us to "unmask" Eq. (1.13). It is no more than a regular Friedmann equation in disguise! From Eq. (1.14), we see that dark energy ρ_Λ is no more than a dynamical property of the expanding D_3-brane in the bulk. The expansion rate, and hence the induced dark energy density, are controlled by the sub-criticallity parameter ϵ.

Let us close this section of the chapter making comments on points that will be relevant and discussed in the following sections.

- de Sitter induced cosmologies can only occur when the tension of the brane is *sub-critical* as seen in Eq. (1.14). The tension of the brane σ will dictate its evolution. As discussed in [APPENDIX to CdL], the brane will always nucleate at rest, i.e. $\dot{a} = 0$. If the tension is critical $\sigma = \sigma_{\text{cr}}$, the repulsion generated by charge of the brane Q, related to the flux difference across the wall (see [SWAMPLAND SECTION] for further details), will be exactly compensated by its tension, preventing its expansion. This results in an induced flat cosmology.

  On the other hand, the brane can nucleate with a sub-critical tension $\sigma < \sigma_{\text{cr}}$, and the whole previous discussion holds, inducing an accelerated expanding cosmology. This is in line with the weak gravity conjecture described in [SWAMPLAND SECTION]. What about supercritical stuff?

- As the simplest example that can be provided, we have omitted the presence of extra dimensions (up to ten or eleven) required from supergravity models. These will be the main discussion in section (2), where we will embed the dark bubble model into supegravity theories. Its connections to the swampland will be also commented in this section.

- So far, we have just discussed how to induced dark energy on the brane, but the observable universe contains more than that. This will be the topic of section (3), where the induced cosmology will be decorated with matter, radiation and waves (plenty of waves)... coming from the extra dimensions. There will be understand the true power of the second Israel condition, its relation to the Gauss-Codazzi equation describe in [APPENDIX] and how to interpret them in the right way to obtain a meaningful induced energy-momentum tensor.

- Where is gravity? Is it localised on the induced geometry or does it propagates across extra dimensions? This will be examined in section (4), where will be study how strings associated to the expanding brane are required to have a good understanding of gravity in this model.

[7] Recall, as one can see in [SECTION], that $\kappa_D = \ell_D^{D-2}$.

- We have talked about expanding branes in the bulk, but, How did those branes appear there? This will be reviewed, from a quantum cosmology perspective, in section (5)
- Finally, the dark bubble model will be compared against the Randall-Sundrum model. Although both proposals share features, there are key differences at both conceptual and computational levels thay may not be perceived at first glance. Section (6) will be devoted to unravel on these issues.

2 Dark bubbles from string theory: An explicit construction

I GUESS THAT HERE I NEED TO WRITE A GOOD SUMMARY OF THE MODEL I WILL USE (OSCAR'S). SPECIALLY WHAT IT WAS USED FOR IN THE PAST AND HOW GOOD IT MAY CONNECT TO THE KIND OF SCENARIO WE HAD IN MIND FOR BUBBLES.

EMPHASISE THAT THE STACK OF BLACK BRANES IS A BLACK HOLE IN HIGHER DIM.

2.1 A black hole in the bulk: Ten-dimensional Kerr or five-dimensional Reissner-Nordström one?

Let us start by defining the parametric coordinates that will be used to describe the 10 dimensional rotating background[8]: OBSERVE THE CHANGE OF COORDINATE DEFINITION WRT THE PAPER. SO THE RESULTING EQUATIONS ARE MORE FAMILIAR WITH THE USUAL THROAT.

$$x^{\mu} = \{t, \alpha, \beta, \gamma, z, \theta, \psi, \phi_1, \phi_2, \phi_3\}, \tag{2.1}$$

and its line invariant

$$\mathrm{d}s_{10}^2 = \mathrm{d}s_5^2 + L^2 \sum_{i=1}^{3} \left\{ \mathrm{d}\sigma_i^2 + \sigma_i^2 \left(\mathrm{d}\phi_i + \tfrac{1}{L} A(z) \right)^2 \right\}, \tag{2.2}$$

where $A(z)$ is a 10 dimensional one-form that will be later discussed and the σ_i-functions are combinations of trigonometric functions of two angles[9] $\{\theta, \psi\}$ of the S^5 as:

$$\sigma_1 = \sin\theta\,, \qquad \sigma_2 = \cos\theta \sin\psi\,, \qquad \sigma_3 = \cos\theta\cos\psi\,. \tag{2.3}$$

As introduced in section (1), L is the AdS$_5$ radius in Eq. (2.2). It is important to notice that this radius also sets the scale of the extra compact dimensions of S^5. This can already raise some eyebrows, as it points to lack of scale separation, as discussed in [SEC Landscape]. However, this will be a key signature of the dark bubble embedding into supegravity, as this model acquires the aforementioned scale separation via tension-to-charge ratio instead. We will later explore this method in section (2.6).

Conducting our attention back to the line invariant (2.2), we can define the five dimensional asymptotically-AdS metric ds_5 as:

$$\mathrm{d}s_5^2 = -h(z)^{-2} f(z)\mathrm{d}t^2 + h(z)\left[f(z)^{-1}\mathrm{d}z^2 + z^2 \mathrm{d}\Omega_3^2 \right], \tag{2.4}$$

[8] It is important to notice the choice of z to describe the AdS$_5$ throat direction. Spoiler alert: There will be a convenient change of coordinates soon to adequate r-direction. Stay alert.

[9] ADD REF AND COMMENTS ABOUT THE PROCEDENCE OF THIS ANGLES (from where to where do they run) AND PROBABLY, COMMENTS ABOUT THE SPHERE?

where $d\Omega_3^2$ is the usual unit metric of the 3-sphere [ref to COSMO SECTION]. The radial functions $h(z)$ and $f(z)$ are expressed as

$$h(z) = 1 + \frac{q^2}{z^2}\,, \qquad f(r) = 1 - \frac{m}{z^2} + \frac{z^2}{L^2}h(z)^3 \tag{2.5}$$

where m and q are respectively the mass and charge of the five dimensional black. Wait? What five-dimensional black hole? Let us then perform a "massage" to Eqs. (2.4, 2.5) to provide a more familiar black hole-ish line invariant. One can start this task by performing a simple change of coordinates, defining a new radial coordinate r as:

$$r^2 = z^2 h(z) = q^2 + z^2. \tag{2.6}$$

This change of coordinates will substantialy transform Eq. (2.4), yielding:

$$\mathrm{d}s_5^2 = -g(r)\mathrm{d}t^2 + g(r)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega_3^2, \qquad g(r) = 1 + k^2 r^2 - \frac{2\kappa_5 M}{r^2} + \frac{\kappa_5^2 Q^2}{r^4}, \tag{2.7}$$

with

$$M = \frac{1}{2\kappa_5}\left(m + 2q^2\right), \qquad Q^2 = \frac{1}{\kappa_5^2}q^2(m+q^2), \tag{2.8}$$

with $\kappa_5 = 8\pi G_5$. It is now easy to see that the change of variables described in Eq. (2.6) casts the line invariant (2.4) in the familiar form[10] of a five dimensional Reissner-Nordström black hole living in an AdS vacuum [REF]. Observe that, for a small black hole, with $r_H \ll L$, we recover a flat space description and it is required $Q < M$ to get a horizon and no naked singularity. On the other hand, if we want to study a horizon larger that the AdS$_5$ radius, this immediately implies $Q \ll M$.

Let us recap now the interpretation of both ten and five dimensional geometries. In the ten dimensional approach, we will observe a Kerr black hole which rotates in three of the compact directions of S^5 (i.e. $\{\phi_i\}$). When we zoom out and not have resolution of the five dimensional sphere, the black hole no longer rotates. It becames "static" from the point of view of a five dimensional observer, but it acquires a charge q due to its motion in the compact directions. This charge becomes effective (i.e. charge Q) when the change of coordinates (2.6) is performed to recover the more familiar description of a Reissner-Nordström black hole living in AdS.

As it is well known, this type of black holes enjoy two different horizons [REFS to book]: The *outer* horizon (i.e. event horizon) and the *inner* one (i.e. the Cauchy horizon). When these two horizon become degenerate (i.e. $r_H = r_h$), this corresponds to an extremal black hole description. We will not further discuss charged black holes and their connection to the weak gravity conjecture here (see [SWAMP] for more details), but, for computational purposes in the incoming sections, we find more convenient to express (2.7) in terms of the two aforementioned horizons: $\{r_h, r_H\}$ as the inner and outer horizons. In this way, we can express $g(r)$ in Eq. (2.7) with the following Ansatz:

$$g(r) = \frac{k^2}{r^4}\left(r^2 + c\right)\left(r^2 - r_h^2\right)\left(r^2 - r_H^2\right), \tag{2.9}$$

[10] The patch we are describing with the metric (2.4) corresponds to the requirement $r \geqslant q$. This is actually not an issue since the horizon r_H will be covered by the patch.

with $c \in \mathbb{R}$. A simple match between Eqs. (2.7, 2.9) shows that:

$$
\begin{aligned}
M &= \frac{1}{2\kappa_5}\left[r_h^2 + r_H^2 + k^2\left(r_h^4 + r_h^2 r_H^2 + r_H^4\right)\right], \\
Q^2 &= \frac{r_h^2\, r_H^2}{\kappa_5^2}\left[1 + k^2\left(r_h^2 + r_H^2\right)\right], \\
c &= r_h^2 + r_H^2 + \tfrac{1}{k^2}.
\end{aligned}
\tag{2.10}
$$

which simplifies when the black hole is extremal. Let us keep in mind these identifications for future section and conduct our attention back to the one-form potential $A(z)$ in Eq. (2.2). This form acts as a gauge field from the 5D point of view WHY?. In term of the old coordinates (i.e. $\{t, \alpha, \beta, \gamma, z\}$) it can be written as:

$$
A(z) = \frac{q}{z_H^2 + q^2}\sqrt{\left(q^2 + z_H^2\right) + \frac{z_H^4}{L^2} h(z_H)^3}\left(1 - \frac{z_H^2 + q^2}{z^2 + q^2}\right)\mathrm{d}t. \tag{2.11}
$$

Observe that gauge freedom has been summoned to add a constant that sets $A(z_H) = 0$. This is required as a $A(z)$ is temporal gauge potential, and must vanish at the horizon. This potential can be expressed in a more handable way in two steps:

1. Part of the argument of the root in Eq. (2.11) is the mass m of the black hole[11]. This is:

$$
m = z_H^2\left(1 + \frac{z_H^2}{L^2} h(z_H)^3\right). \tag{2.12}
$$

2. Furthermore, performing the change of variables (2.6) and making use of the identifications (2.8) to further simplify in the new coodinate system, one obtains:

$$
A(r) = \kappa_5 Q\left(\frac{1}{r_H^2} - \frac{1}{r^2}\right). \tag{2.13}
$$

With this we finish the black hole description, both from ten and five dimensional perspectives. Let us now explore the explicit embedding of the D_3-brane in the ten dimensional ambient space.

2.2 The action of the D_3-brane

The rotating D3-branes will source the self dual F_5 Ramond-Ramond (RR) field strength, which can be written in terms of the radial function $h(r)$ and the one-form A

$$
C_4 = \frac{1}{L}\left[(r^2 + q^2)^2 - (r_H^2 + q^2)^2\right] dt \wedge \epsilon_3 + L^2 q \sqrt{r_H^2 + q^2 + \frac{r_H^4}{L^2} h(r_H)^3}\, \sum_i \sigma_i^2\, d\phi_i \wedge \epsilon_3\,, \tag{2.14}
$$

2.3 Comparing junctions to EOM

Basically, compare as we did in the paper. Just to prove, that at some specific limit EOM = JC.

[11] This can be easily obtained by computing the outer horizon in Eq. (2.4)

2.4 Collapsing Cosmology: Evacuation required

2.5 Higher curvature corrections to the rescue

Point to the fact that previous EOM has no CC. Discuss about how to obtain it and elaborate on all pieces in the game. End with the famous $1/N$ corrections that transform in CC. (First part of 5th paper)

2.6 Energy scales from Dark Bubble embedding: A new hope

Compare to observed CC and obtain N to fix all scales. Discuss and defend the size of extra dimensions and the stringy scales.

3 Decorating the cosmos: Gauss-Codazzi equation at work

Argue somehow that we are going to look at things from a 5D perspective and will start to induce stuff in the 4D cosmos.

3.1 Matter

3.2 Radiation

3.3 Gravitational waves

3.4 The standard model of particle physics

3.4.1 Electromagnetism

3.4.2 Weak force: Neutrinos

4 Holographic bubbles and hanging strings

5 Quantum bubbles: Higher dimensions to solve boundary choices

6 No, this is not a Randall-Sundrum model on steroids

References