

# Orbifolds

How do we make string theory have physics that look like the standard model? We have been using two tools to do achieve this: “editing” the degrees of freedom of the embedding, and modifying the spacetime the string is embedded in. For example adding fermionic degrees of freedom allowed us to find spacetime Fermions in certain settings. But what really unlocks the door to realistic looking physics is changing the target space in various fun ways.

This is where orbifolds come in. Studying the toolkit of orbifolds will allow us to write down theories in nontrivial target space configurations that lead to interesting physics. In these notes, we begin with an intuitive description of orbifolds through the example of a single compact boson and its  $\mathbb{Z}_2$  orbifold, then highlight some generalizations, and finally apply this to the heterotic string compactification that looks a lot like a standard model unification.

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## 1 Orbifolding the Compact Boson

- 1.1 Mathematical Orbifolds
- 1.2 Untwisted Sector
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- 1.4 Cheating via Modularity

## 2 General Lessons from the Compact Boson

## 3 Heterotic String Orbifolds

The  $E_8 \times E_8$  heterotic string has 10 free bosons  $X$  with periodic boundary conditions, 32 left moving positive chirality Majorana spinors  $\lambda$  half of them with periodic and the other half with antiperiodic boundary conditions, and 10 right moving positive chirality Majorana Fermions and either periodic or antiperiodic boundary conditions.