Parameter estimation for a model describing the human digits*

P. Tsakonas, M. J. Chappell, N. D. Evans, J. Hardwicke

I. INTRODUCTION

Partial or complete amputation of any limb is one of the most life-changing events that can happen to an individual. It has been observed that partial amputation of the hand is about 3-6 times greater in men than in women and annually there are a total of 1.9 partial amputations for every 100,000 persons aged between 25 and 60 years old [1]. The above data came from a 10-year period study, in three Norwegian cities. For the hand specifically, amputation of even a very small part of a digit can cause physiological and functional loss alongside reduction of the grip and pinch forces while performing everyday activities [1]–[3]. It is quite common for amputees to be provided with a prosthetic device in order to acquire some functionality over the lost limb. A common issue among recipients of prosthetic devices is the cosmetic dissatisfaction from the prosthetic, alongside its limited hand motion [4]. These issues frequently cause amputees to either limit or completely abandon the use of their prosthetic [4]. In [4] of the 45 people who underwent an upper limb amputation only nine continued to use the prosthetic device provided. Thus, the goal of this project is to develop a mathematical model that provides a realistic movement of the digits of the hand and which scales with anthropometry. This model will ultimately be used to support prosthetic digit design.

II. MODEL DEVELOPMENT

In the literature there has been a multitude of different models describing the movement of either the index finger and/or the thumb [5]–[7]. These models have either been developed from studying cadavers or have used data taken from cadaveric studies. These models provide little to no scalability and thus they cannot be used to study people with different anthropometry from those used in these models. To that extent the model developed in this project takes as inputs the subject's anthropometry (phalanx length and radius) making the results from the simulations subject specific. The digit model is derived using Lagrangian mechanics assuming that each segment of the digit is a uniform cylindrical rod with constant density $\rho = 1.1\ g/cm^3$ [8], [9]. The model has 4 Degrees of Freedom (DOF) and those are 3 DOF in flexion/extension for the Metacarpophalangeal (MCP), Proximal Interphalangeal (PIP) and Distal Interphalangeal (DIP) joints and 1 DOF in abduction/adduction for the MCP joint. Figure (1) shows the afore mentioned digit movements and figure (2) the anatomy of the human hand [10], [11].

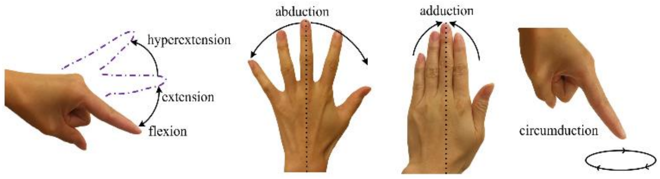

Figure 1: Digit movements [10].

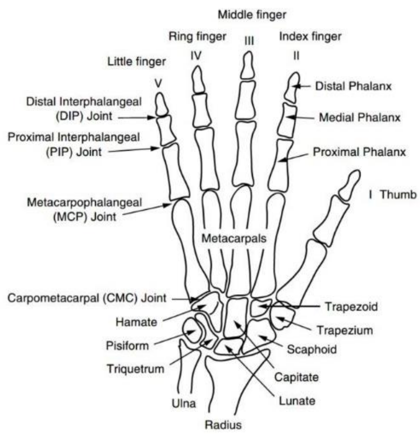

Figure 2: Bones and Joints of the human hand [11].

The kinematic chain for the index through to the little finger is described only by the proximal, medial and distal phalanges since motion of the metacarpals of these digits is limited [12]. It is also assumed that for each DOF there is a passive moment generated at each joint which can be described via linear torsional spring and damper effects [13]. A spherical coordinate system is defined as lying at the centre of rotation of each joint and the resulting equations of motion are derived for the centre of mass, which is located at the middle of each phalanx. The resulting equations are non-linear and estimating the linear torsional spring and damper parameters is not possible analytically. To that extent we hypothesised that for each DOF a linear 2nd order differential equation is sufficient in order to characterise the corresponding motion. Let $\theta_i(t), I_i, B_i, K_i, \theta_{eq,i}$ denote respectively the flexion/extension (or abduction/adduction angle for the MCP joint) of each joint and its first and second derivatives with respect to time, the moment of inertia of each phalanx, the torsional damper and spring constants and the equilibrium angle where the total net moment at the joint

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P. Tsakonas is with the University of Warwick, School of Engineering, Coventry, CV4 7AL, United Kingdom (corresponding author email: Panagiotis.Tsakonas@warwick.ac.uk).

M. J. Chappell is with the University of Warwick, School of Engineering, School of Engineering, Coventry, CV4 7AL, United Kingdom

N.D. Evans is with the University of Warwick, School of Engineering, Coventry, CV4 7AL, United Kingdom.

J. Hardwicke is with the University Hospitals Coventry and Warwickshire NHS Trust, Coventry, CV22DX, United Kingdom.

is zero. Then the original non-linear system can be approximated by:

$$\sum_j M_{j,i}(t) = I_i\, \ddot{\theta}_i(t) + B_i\, \dot{\theta}(t) + K_i\big(\theta_i - \theta_{eq,i}\big) \qquad (1)$$

where $M_{j,i}(t)$ is the muscle moment generated by muscle j that spans joint i. The free response of (1) is explored for underdamped, critically damped and overdamped cases. For these simulations $\theta_{eq,i}$ was set to zero due to the lack of real-life data. The Laplace transform of (1) was performed with the given initial conditions $\theta(0) = \theta_0,\ \dot{\theta}(0) = 0$, to give:

$$\Theta(s) = \frac{\theta(0)(s + \frac{B_i}{I_i})}{(s^2 + \frac{B_i}{I_i}s + \frac{K_i}{I_i})} \qquad (2)$$

One important aspect of parameter estimation is a structural identifiability analysis of the model, which determines whether, or not, the output of a model contains adequate information in order to determine the unknown parameters uniquely, or otherwise [14]. The Laplace transform approach is used to determine the structural identifiability of the model via (2) in which the coefficients of powers of s in both the numerator and denominator are uniquely determined by the input-output relationship [14]. From (2) the parameters that are uniquely identifiable are $\theta(0), \frac{B_i}{I_i}, \frac{K_i}{I_i}$. It becomes apparent that the system is not structurally globally identifiable since the parameters B_i, I_i, K_i cannot be determined uniquely without *a-priori* knowledge of at least one of these parameters. To solve this problem, the cylindrical approximation of the digits will be used to determine the moment of inertia of each phalanx to facilitate and aid in parameter estimation.

III. Simulation

A set of simulations was performed that corresponded to the solutions of the linear 2nd order differential equation (underdamped, critically damped, and overdamped cases) for the MCP, PIP and DIP joints respectively. The free response of the Lagrangian system was simulated and the data were exported into MATLAB. The inputted parameter values to the Lagrangian system for each joint are summarised in table 1.

Table 1: Values of the parameters used in the free response of the Lagrangian system.

Joint	$K_i\left(\frac{Nm}{rad}\right)$	$B_i\ (\frac{Nms}{rad})$	$I_i\ (Kg \cdot m^2)$
MCP	0.17	0.003	$6.64 * 10^{-5}$
PIP	153	0.1	$1.64 * 10^{-5}$
DIP	10	0.1	$1.21 * 10^{-6}$

Equation (2) has been solved symbolically for each of the 3 distinct cases. The analytical solutions obtained were used as fitting functions in the MATLAB curve fitting toolbox. The parameters determined from fitting the responses and assuming that the parameter I_i is known are summarised in table 2.

Table 2: Parameters estimated from fitting the responses of the Lagrangian system to the corresponding analytic expression derived from solving (2).

Estimates	MCP	PIP	DIP
$K_i\left(\frac{Nm}{rad}\right)$	0.1652	183	12.85
$B_i\ (\frac{Nms}{rad})$	0.0025	0.11	0.13

IV. Discussion

From the estimated parameters there is a good correlation between the values used in the Lagrangian system and those estimated from (2). The highest percentage error found between the Lagrangian system and the analytical solution of (2) is for the B_i constant of the DIP joint at 30%. This error is attributed to the simplistic nature of (1) compared to the non-linear Lagrangian system, and to the error propagation from the uncertainties of the fitted values. Even with this relatively high level of error, the highest absolute difference between the Lagrangian system and equation (2) for the DIP joint is less than 0.01 degrees as seen in figure (3).

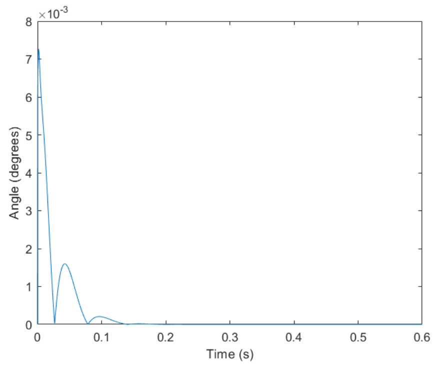

Figure 3: Absolute difference between the Lagrangian system response and (2) for the DIP joint.

V. Conclusion

The linear 2nd order differential equation described by (1) can be used as an approximation of the non-linear Lagrangian system to determine the parameters B_i, K_i for the free response case for the digit motion model. This approximation will be used in future experiments as a means of determining these parameters from motion-capture data from able-bodied participants. The parameters extracted from this process will be used to support the design of novel body powered hand prostheses, tailored to the anthropometry of the individual.

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