$\min_{\bar{x}} f(\bar{x})$ $\quad f: \mathbb{R}^N \to \mathbb{R}$

$\bar{x} = \alpha_1 \hat{x}_1 + \cdots + \alpha_N \hat{x}_N$ $\quad \underline{f: \mathbb{B}^N \to \mathbb{R}}$ , $\mathbb{B} = \{0,1\}$

① $f(\bar{x}) = \|\bar{x} - \bar{x}^*\|$ w.r.t. $\|*\|$ — norm, e.g., $\|*\|_2$ Euclidean-norm

$\bar{x}^* = \min_{\bar{x}} f(\bar{x})$

e.g., $f(\bar{x}) = \|\bar{x}\|$ since $\bar{x}^* = \bar{0}$

② Separability (why cooperative CEA used for Divide-and-conquer)

$f(\bar{x}) = \sum_{i=1}^{N} \|\hat{x}_i\|$ - fully additive separable

In Rudolph's argument, Evol : $\mathbb{R}^N \to \mathbb{R}^N$

$G \in$ Evol

$\bar{x}^{t+1} = G(\bar{x}^t)$

Elitist and make strong claim

$G(\bar{x}^*) = \bar{x}^*$ i.e. $\bar{x}^*$ is a fixed-point of $G$

for any initial $\bar{x} \neq \bar{x}^*$ can find

$f(G(\bar{x}^0)) > f(G(\bar{x}^1)) \geq \cdots \geq f(G(\bar{x}^*))$

③ What is cooperative coevolutionary algorithm? (CEA)

Assume separability as in ②

Define $G = H_1^T \circ H_2^T \circ \cdots \circ H_N^T$ $\quad H_i : \mathbb{R} \to \mathbb{R}$

$H_i^T = \underbrace{H_i \circ \cdots \circ H_i}_{T \text{ times}}$

Cooperative CEA apply $G$ as before

Note: if ② holds, then such a $G$ action lead to convergence

④ Real-world problems

* Not fully but additive separable

$f(\bar{x}) = \sum_{m=1}^{M} f_m(\bar{x}_m)$, $\bar{x} = \bar{x}_1 + \cdots + \bar{x}_M$ note $\mathbb{R}^m$ differs

* If a priori $\bar{x}_1, \ldots, \bar{x}_m$ known, then

$G = H_1^T \circ \cdots \circ H_M^T$, $\bar{x}_m^{t+1} = H_m(\bar{x}_m^t)$

So $G$ converges

* Question

(i) $\bar{x}_1, \ldots, \bar{x}_m$ unknown, then do random grouping in each $G$ action, e.g., in paper 2

Fix $K$, ensure all $\mathbb{R}^k$ same dimensionality

$G^1 = H_1^{T,1} \circ \cdots \circ H_K^{T,1}$

$\vdots$

$G^q = H_1^{T,q} \circ \cdots \circ H_K^{T,q}$

$\vdots$

But in each cycle $q = 1, 2, \ldots$

selection of $\bar{x}_k = X$ where $|X| = N/K$

$\bigcup_{k=1}^{M} \{\bar{x}_k\} = \{\bar{x}\}$

$\bigcap_{k=1}^{M} \{\bar{x}_k\} = \emptyset$

Possibly no elitist effect, i.e., cannot guarantee

$f(H_k^{t,q}(\bar{x}_k)) \geq f(H_k^{t+1,q}(\bar{x}_k))$