$g(t+s) = g(t)g(s)$

WTS $\forall s, t \geq 0 \Rightarrow g(t) = e^{-\lambda t}$

$\Downarrow$

Same as

② $h(t) = \log g(t) \Rightarrow h(s+t) = h(t) + h(s) = -\lambda t$

① $g(t) = \left[g\left(\frac{t}{2}\right)\right]^2 \geq 0$

Assume $g(t_0) = 0$, $\left[g\left(\frac{t_0}{2}\right)\right]^2 = 0$, $g\left(\frac{t_0}{2}\right) = 0$

Then $\left[g\left(\frac{t_0}{2^n}\right)\right]^{2^n} = 0$. Since g continuous,

$n \to \infty$, $g(0) = 0$, then $\forall t \geq 0$ $g(t) = g(t)\, g(0) = 0$

For ①: $g(t) > 0$

② (1) $t, s \in \mathbb{N}$, $h(n) = h(n-1) + h(1) = h(1) + (n-1)h(1)$

$= n \cdot h(1)$

$= cn$

(2) $t, s \in \mathbb{Z}$, $\cancel{h(-n) = -cn}$

For $h(0) = 2h(0) \Rightarrow h(0) = 0$

$0 = h(n-n) = h(n) + h(-n)$, $h(-n) = -cn$

(3) $t, s \in \mathbb{Q}$, $h(1) = h(n \cdot \frac{1}{n}) = n h(\frac{1}{n})$

$h(\frac{1}{n}) = \frac{c}{n}$; $h(x) = h(\lim_{n\to\infty} q_n) = \lim_{n\to\infty} h(q_n) = \lim_{n\to\infty} c q_n$

$= cx$

(4) $t, s \in \mathbb{R}$, $\exists \{q_n\} \in \mathbb{Q}$, s.t. $\lim_{n\to\infty} q_n = x$,