Theorem: set $P, Q \in \mathbb{R}^n$, the straight line satisfies the shortest curve. straight line: $l = P + t(Q-P), t \in \mathbb{R}$.
If parameterized curve $l: \mathbb{R}^m \to \mathbb{R}^n$ satisfies $\cancel{l(P)=0, l(Q)=1}$, $l(0)=P, l(1)=Q$
length of curve: $\int_0^1 |l'(t)| dt$.

proof 1: For curve $r(t) = (x^{(1)}(t), \dots, x^{(n)}(t))$
geodesic formula is $\frac{d^2x^i}{dt^2} + \Gamma^i_{jk} \frac{dx^j}{dt} \frac{dx^k}{dt} = 0$ 1#

In $\mathbb{R}^n$, $\Gamma^i_{jk} = 0$ 2# $\Rightarrow \frac{d^2x^i}{dt^2} = 0$, solve this formula $\Rightarrow$
$x^i = a^i t + b^i$, by boundary condition: $x^i(0) = P$, $x^i(1) = Q$.