$$f(x + td^*) - f(x) \geq f(x + td) - f(x), \quad d^* \neq d, \quad t > 0$$

claim $d^* = \nabla f(x)$.

proof: $f'(x; d^*) > f'(x; d)$

$$\lceil f'(x; d) \overset{\Delta}{=} \lim_{t \to 0^+} \frac{f(x + td) - f(x)}{t} \rceil$$

$\Rightarrow \quad \langle \nabla f(x), d^* \rangle > \langle \nabla f(x), d \rangle$

$\Rightarrow \quad \langle \nabla f(x), d \rangle \overset{\leq}{} |\nabla f(x)| \cdot |d|$

Take equality only if $d /\!/ \nabla f(x)$

$\Rightarrow \quad d^* /\!/ \nabla f(x)$