No.
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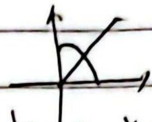
WTF $x$ s.t. $x = \cos x$

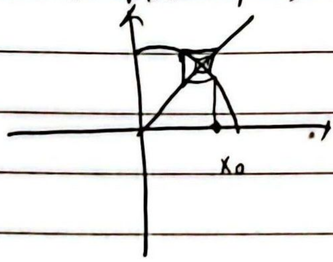
Method: Initially put $x_0 \in [0, \frac{\pi}{2}]$, $x_{n+1} = \cos x_n$.

1° $\exists$ and ! : Intuition: 

of $x = \cos x$ Analitically: $f(x) = x - \cos x$, $f'(x) = 1 - \sin x \geq 0$ on $[0, \frac{\pi}{2}] \Rightarrow f(x)$ is increasing, $f(0) = -1$, $f(\frac{\pi}{2}) = \frac{\pi}{2} - 0 > 0$
$\Rightarrow f(x)$ is continuous $\Rightarrow \exists\, x^*$ s.t. $f(x^*) = 0$, $x^* \in [0, \frac{\pi}{2}]$. Since $f(x)\uparrow \Rightarrow x^*$ is unique.

2° Robust of method: WTS: $|\cos x - x^*| \leq |x - x^*|$
Proof: suppose $\exists\, x \in [0, \frac{\pi}{2}]$ s.t. $|\cos x - x^*| > |x - x^*|$
$\Rightarrow \dfrac{|\cos x - \cos x^*|}{|x - x^*|} \geq 1$, by Lagrange mean-value Th.,
$\exists\, c \in (x, x^*)$ [or $(x^*, x)$] s.t. $\cancel{f'(c) \geq 1}$ $\cos'(c) \geq 1$, which is $\sin c \geq 1$.
contradict to the domaine of $\sin x$, $x \in [0, \frac{\pi}{2}) \Rightarrow |\cos x - x^*| \leq |x - x^*|$ □

More precisely, choose $[0, \frac{\pi}{2} - \varepsilon]$ s.t. $\sin(\frac{\pi}{2} - \varepsilon) = 0.999$.
$|x_{n+1} - x^*| = |\cos x_n - \cos x^*| = |\sin(\xi)|\,|x_n - x^*| < 0.999\,|x_n - x^*|$
$\Rightarrow |x_n - x^*| < 0.999^n\,|x_0 - x^*| \Rightarrow n \to +\infty$ $x_n \to x^*$.

In graph: 


$x^* \approx 0.73$

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