1° Jacobi's method: $Ax=b$, $\boxed{a_{ii}x_i = b_i - \sum_{j=1, j\neq i}^{n} a_{ij}x_j}$, $i\in[1,n]$.

$\Rightarrow$ $x_i^{(k+1)} = \left(b_i - \sum_{j=1, j\neq i}^{n} a_{ij}x_j^{(k)}\right)/a_{ii}$

Matrix version: $x^k = Tx^{k-1}+c$, $\begin{cases} T = D^{-1}(L+U) \\ c = D^{-1}b \end{cases}$

$A = D-L-U$

2° Gauss-Seided iteration: $x_i^{(k+1)} = \left(b_i - \sum_{j=1}^{i-1} a_{ij}x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}x_j^{(k)}\right)/a_{ii}$

Matrix version: $T=(D-L)^{-1}U$, $c=(D-L)^{-1}b$

3° Convergence:

Thm: $x^k = Tx^{k-1}+c$ converge $\iff \rho(T)<1$

Thm: (1) $\|x - x^k\| \le \|T\|^k\|x-x^0\|$

By $\|Tx - Tx^{k-1}\| \le \|T\|\|x-x^{k-1}\|$

(2) $\|x^m - x^k\| \le \sum_{t=m}^{k-1}\|x^{t+1}-x^t\| \le \sum_{t=m}^{k-1}\|T\|^t\|x^1-x^0\|$

$\le \dfrac{\|T\|^k}{1-\|T\|}\|x^1-x^0\|$