报告摘要

凯莱公式有多种来自天成之书的证明。Chapuy 与 Perarnau 在研究同步化自动机的过程中顺带找到了一个可能是凯莱公式目前最短的证明Chapuy and Perarnau 2023。本报告从讨论该证明开始，接着说明其与英年早逝的俄罗斯数学家 Burtin 的一个小引理[1]以及 the forest volume formula[2]的关系，再进一步说明这些结论都是关于马氏链提升的马尔科夫链树定理[3]的特殊情形。如果时间允许，我们会再简短介绍对生成树做均匀采样的 Propp-Wilson 算法与 Aldous-Broder 算法[4]。

[1]Yuri D. Burtin (1980). "On a simple formula for random mappings and its applications". In: *J. Appl. Probab.* 17.2, pp. 403–414. DOI: 10.2307/3213029.

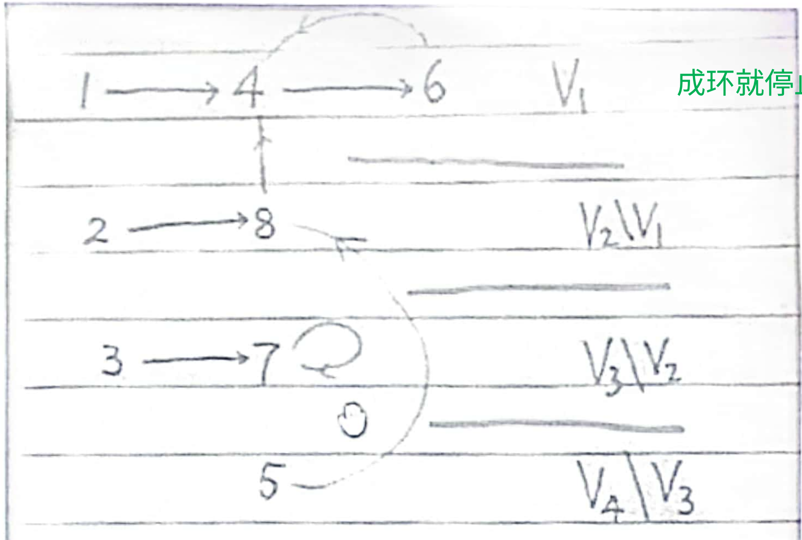
[2]Jim Pitman (2002). "Forest volume decompositions and Abel-Cayley-Hurwitz multinomial expansions". In: *J. Combin. Theory Ser. A* 98.1, pp. 175–191. DOI: 10.1006/jcta.2001.3238.

[3]F.T. Leighton and R.L. Rivest (1983). "The Markov chain tree theorem". M.I.T. Laboratory for Computer Science Technical Report, MIT/LCS/TM-249. URL: https://dspace.mit.edu/bitstream/handle/1721.1/149059/MIT-LCS-TM-249.pdf?sequence=1.

[4]Luis Fredes and Jean-François Marckert (2 rial proof of Aldous-Broder theorem for general Markov ch *Structures Algorithms* 62.2, pp. 430–449. DOI: 10.1002/rsa.21101; Russell Lyons and Yuval Peres (2016). *Probability on trees and networks*. Vol. 42. Cambridge Series in Statistical and Probabilistic Mathematics. Cambridge University Press, New York,

Chapuy-Perarnau：用随机数来数树

随机选择点1



成环就停止操作重新随机选择点

$$P(f \text{ is a tree} \mid V_1, V_2, V_3, V_4)$$

$$= \frac{1}{|V_1|} \cdot \frac{|V_1|}{|V_2|} \cdot \frac{|V_2|}{|V_3|} \cdot \frac{|V_3|}{|V_4|} = \frac{1}{|V_4|}$$

$$= \frac{1}{|V|}$$

$P(f \text{ is a rooted tree}) = \sum P(f \text{ is a rooted tree} \mid X_1, \ldots, X_T) P(X_1, \ldots, X_T) = \frac{1}{|V|}$

Markov chain tree theorem

Theorem

Assign the weight $\Phi_f \doteq \prod_{u \in V \setminus \{root\}} P(u, f(u))$ *to each in-tree* f. *Let* $\pi_v \doteq \sum_{f:\text{rooted trees at } v} \Phi_{\cancel{v} f}$. *Then* $\pi P = P$.

markov chain tree theorem