

1st Introduction of Convolution:

Take f, g on $\mathbb{Z}/n\mathbb{Z}$, let $F \triangleq \sum_{i=0}^{n-1} a_i f(i)$ to be generation fun., $G \triangleq \sum_{j=0}^{n-1} a_j g(j)$ let $g \cdot h = (f(0)g(0) + f(1)g(n-1) + \dots + f(n-1)g(1))a_0 + \dots$
 find co-efficient of a_r : $\sum_{y \in \mathbb{Z}/n\mathbb{Z}} f(r-y)g(y) \triangleq f * g(r)$
 which is convolution

$$2^{\text{nd}} \quad X+Y \xrightarrow{\text{D}} f_X * g_Y \xrightarrow{\text{Z}} M_X \cdot M_Y$$

$$1. \text{ For } P(X+Y \leq r) = P(X \leq k, Y \leq r-k) = P(X \leq k)P(Y \leq r-k)$$

If X, Y only takes the value on $\mathbb{Z}/n\mathbb{Z}$, $X+Y \equiv r \pmod{n}$

$$\text{which is } P(X=0)P(Y=r) + P(X=1)P(Y=r-1) + \dots + P(X=r)P(Y=0) \\ \triangleq \sum_{i=0}^r P(X=i)P(Y=r-i) = f_X * g_Y$$

2. Choose $a_i = z^i$ in 1st, F is z -transformation of pdf of X
 So $F \cdot h = M_X(t) \cdot M_Y(t)$, where $M_X(t) = \sum_{i=0}^{n-1} f_X(i) z^i$
 and co-efficient of t : $f_X * g_Y(t)$
 in $F \cdot h$

3rd General: let G is a finite group, denote

$$\mathbb{C}[G] \triangleq \{ (\sum c_i g_i) (\sum d_i g_i), g_i \in G \} \cong \{ f: G \rightarrow \mathbb{C} \}$$

multiplication $f * g$.