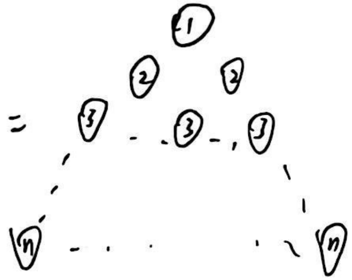
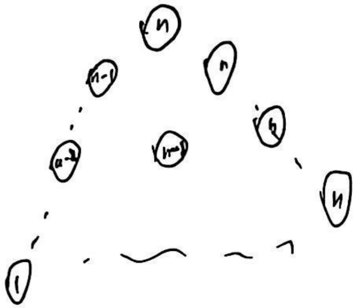
$1^2+\dots+n^2 = ?$ Consider $n = \underbrace{n+\dots+n}_{n \text{ times}}$, $1^2+\dots+n^2 =$



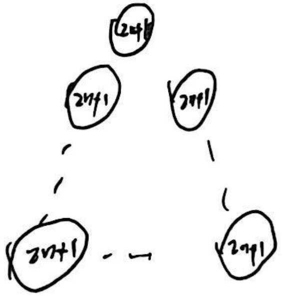
$\doteq S_0$

$S_{\frac{\pi}{3}} =$



, $S_{-\frac{\pi}{3}} =$

One can show: $3S_0 = S_0 + S_{\frac{\pi}{3}} + S_{-\frac{\pi}{3}} =$



$= \frac{n(n+1)}{2}(2n+1)$

$$\Rightarrow S_0 = \frac{n(n+1)(2n+1)}{6}$$

Furthermore, set ① is S_0 S_{00}, ② S_{10}, S_{01}, ③ S_{20} S_{11} S_{02}

WTS: $S^{(i,j)}_{\forall i,j}\left(S_0+S_{\frac{\pi}{3}}+S_{-\frac{\pi}{3}}\right)_{i,j} = 2n+1$

$\underbrace{S_0^{(i,j)} = i+j+1}_{\downarrow}$, $S_{\frac{\pi}{3}}^{(i,j)} = n-i$, $S_{-\frac{\pi}{3}}^{(i,j)} = n-j$ $\Rightarrow S_0^{(i,j)} + S_{\frac{\pi}{3}}^{(i,j)} + S_{-\frac{\pi}{3}}^{(i,j)} = 2n+1$

2-degree freedom means only this triangle can deal with $n \to +\infty$.