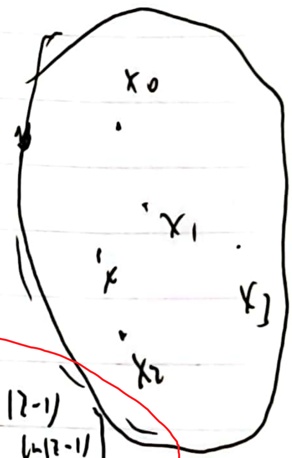
# Runge's Phenomenon:

$f(x) = \frac{1}{1+25x^2}$, $x \in [-1,1]$ oscillating near $-1, 1$ when equi-distant interpolation. → reason: $f$ should be analyzied in $\Gamma$:


on $\mathbb{C}$ but has poles $\pm 0.2i$.

Why: $|f(x) - p_n(x)| = \frac{1}{2\pi i}\int_\Gamma \frac{\ell(x) f(t)\,dt}{\ell(t)(t-x)}$, $\Gamma: t \to \mathbb{C}$, $\ell(x) := (x-x_0)\cdots(x-x_n)$ → Hermite integral formula.

want $|f(x) - p(x)| \to 0$, then $\left|\frac{\ell(x)}{\ell(t)}\right| \to 0$ ⇒ $\Gamma$ sufficient large to make $t$ away from $x_0, \dots, x_n$. $\left(\sqrt[n]{\frac{\ell(x)}{\ell(t)}} < 1\right)$ $\forall n$.




count $\sqrt[n+1]{\frac{|x-x_0|\cdots|x-x_n|}{|t-x_0|\cdots|t-x_n|}} < 1$ ⇒

$u_n(z) = \frac{1}{n+1}\sum_{j=0}^{n} \ln|z - x_j|$, want $u_n(t) > u_n(x)$ as $n \to \infty$. then $\lim_{n\to\infty} u_n(z) = -1 + \frac{1}{2}\mathrm{Re}\left[(z+1)\ln(z+1) - (z-1)\ln(z-1)\right]$

by $u_n(t) > u_n(x)$ ⇒ $t$ outside the $\Gamma$:

Since $f$ needs be analyzed in $\Gamma$ ⇒ $f$ more smooth, error lower

obs1 n increasing, u_n(x) becomes larger

obs2: |x| increasing, u increasing