$E(f) = \frac{h^{n+3} f^{(n+2)}(\xi)}{(n+2)!}\int_0^n t^2(t-1)\cdots(t-n)\,dt$, $f\in C^{n+2}[a,b]$, n is even

proof: $E(f) = \int_a^b f[x, x_0, \dots, x_n]\, W_{n+1}(x)\,dx$

$= f[x_0,\dots,x_n,x]\,W(x)\Big|_a^b - \int_a^b \frac{d}{dx}\left(f[x,x_0,\dots,x_n]\right) W_{n+1}(x)\,dx \cdots ①$

where $W(x) \triangleq \int_a^x W_{n+1}(s)\,ds$. Since $\frac{d}{dx} f[x,x_0,\dots,x_n] \triangleq \frac{d\varphi(x)}{dx}$

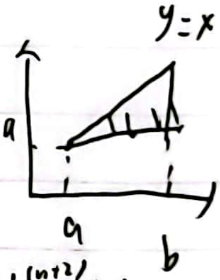
$= \lim_{\tau\to 0}\frac{\varphi(x+\tau)-\varphi(x)}{\tau} \overset{\exists \xi\in(x+\xi,\,x+\tau)}{=\!=} \lim_{\tau\to 0}\varphi'(\xi) = f[x_0,\dots,x_n,\xi] = \frac{f^{(n+2)}(\xi_1(x))}{(n+2)!}$

$\exists \xi_1\in(x_0,\, x+\tau)$

and $W(b)=\int_a^b W_{n+1}(s)\,ds = 0$ by $(s-x_0)\cdots(s-x_n)$ can be transform to odd-function integral on symmetric interval at 0 ; and $W(a)=\int_a^a W_{n+1}(s)\,ds=0$

$\Rightarrow$ ①: $-\int_a^b \frac{f^{(n+2)}(\xi_1)}{(n+2)!}\int_a^x W_{n+1}(s)\,ds\,dx$

Since $\int_a^b\int_a^x W_{n+1}(s)\,ds\,dx = \int_a^b\int_s^b W_{n+1}(s)\,dx\,ds$;



①: $-\frac{f^{(n+2)}(\xi_1)}{(n+2)!}\int_a^b (s-x_0)\cdots(s-x_n)(x_n-s)\,ds \triangleq -\frac{f^{(n+2)}(\xi_1)}{(n+2)!}\, I$

$b=x_n$

$s=x_0+th$

$I = \int_0^n th\cdots((t-n)h)[(n-t)h]\,h\,dt = -h^{n+3}\int_0^n t(t-1)(t-n+1)(t-n)^2\,dt$

$= -h^{n+3}\int_0^n t^2(t-1)\cdots(t-n)\,dt$

$\Rightarrow E(f) = \frac{h^{n+3} f^{(n+2)}(\xi_1)}{(n+2)!}\int_0^n t^2(t-1)\cdots(t-n)\,dt$