SVM:

[*] lead in: WTF hyperplane seperate data points into 2 classes

In math, set 2 points (x_1, y_1) ← support vector
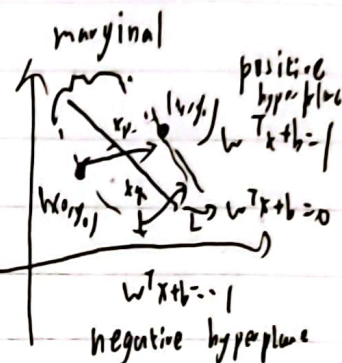
(x_0, y_0), max L

Then: $\vec{w}^T \cdot (\vec{x}_1 - \vec{x}_0) = 2 \cdots \cdots$ ①

$\vec{x}_1 = (x_1, y_1) - (0, 0)$

$\vec{x}_0 = (x_0, y_0) - (0, 0)$

marginal / positive hyperplane $w^T x + b = 1$

$w(0, y_i)$ → $w^T x + b = 0$

$w^T x + b = -1$ negative hyperplane

Find 2 points $\vec{x}_p, \vec{x}_{qc}, \vec{w}^T (\vec{x}_p - \vec{x}_{qc}) = 0 \cdots$ ②
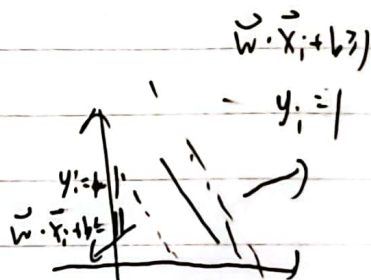
①: $|\vec{x}_1 - \vec{x}_2| \cos\theta = L$

$|\vec{x}_1 - \vec{x}_2| \cdot |\vec{w}| \cos\theta = 2 \Rightarrow L = \frac{2}{|\vec{w}|}$

$\Rightarrow \max L (\Leftrightarrow) \min |\vec{w}|$ ← $\vec{w} \cdot \vec{x}_i + b \geqslant 1$

In figure right: let $y_i = \pm 1$ — $y_i = 1$

indicate the points position in plane

$y_i = +1$
$\vec{w} \cdot \vec{x}_i + b = 1$

optimal problem:

$\min |\vec{w}|$, s.t. $y_i \cdot (\vec{w} \cdot \vec{x}_i + b) \geqslant 1$, i count out all datas s

make inequality to equality

$\Rightarrow$ $\min f(w) = \frac{|\vec{w}|^2}{2}$ $(f: \mathbb{R}^n \to \mathbb{R})$

s.t. $g_i(w, b) = y_i * (\vec{w} \cdot \vec{x}_i + b) - 1 = P_i^2, P_i \geqslant 1$

2nd calculate L.

$\Rightarrow L(w, b, \lambda_i, P_i) = \frac{|\vec{w}|^2}{2} - \sum_{i=1}^{s} \lambda_i * [y_i * (\vec{w} \cdot \vec{x}_i + b) - 1 - P_i^2]$

$\begin{cases} \frac{\partial L}{\partial(w, b, \lambda_i, P_i)} = 0 \Rightarrow \\ \Rightarrow L = 0 \end{cases}$

$\begin{cases} \vec{w} - \sum_{i=1}^{s} \lambda_i y_i \vec{x}_i = 0 \cdots ③ \\ -\sum_{i=1}^{s} \lambda_i y_i = 0 \cdots ④ \\ y_i * (\vec{w} \cdot \vec{x}_i + b) - 1 - P_i^2 = 0 \cdots ⑤ \\ 2\lambda_i P_i = 0 \Rightarrow \lambda_i P_i = 0 \cdots ⑥ \end{cases}$

⑤→⑥: $\lambda_i [y_i (\vec{w} \cdot \vec{x}_i + b) - 1] = 0$

Since $y_i (\vec{w} \cdot \vec{x}_i + b) \geqslant 1$

1° $\lambda_i = 0$, $y_i * (\vec{w} \cdot \vec{x}_i + b) - 1 = 0$

lemma : $\lambda_i \geq 0$

If $\lambda_i < 0 \Rightarrow -\sum_{i=1}^{s} \lambda_i [y_i(\vec{w}\cdot\vec{x}_i + b) - 1 - p_i^2] < 0$

Then larger the $y_i \cdot (\vec{w}\cdot\vec{x}_i + b)$, smaller the $||\vec{w}||$

which violate the constraint. ∇

Simpler: $\nabla f \in Span\{\nabla g_i\}$, and $\nabla g_i \nearrow \nabla f$ $\nabla f_j \Rightarrow \lambda_i > 0$

3^* translate to dual problem; set optimal w^*, b^*

$q(\lambda_i) \stackrel{\Delta}{=} \min [L(w,b,\lambda_i)] \leq f(\vec{w}) - \sum_{i=1}^{s} \lambda_i g_i(\vec{w}^*, b^*)$

Since $\lambda_i \geq 0 \Rightarrow g_i(\vec{w}^*, b^*) \geq 0 \Rightarrow \lambda_i \cdot g_i(\vec{w}^*, b^*) \geq 0$

So $q(\lambda_i) \leq f(\vec{w}^*) - \sum_{i=1}^{s} \lambda_i \cdot g_i(\vec{w}^*, b^*) \leq f(\vec{w}^*) \leq f(w)$

$q(\lambda_i^*) \leq f(\vec{w})$ weak dual. WTF λ_i^* s.t. $q(\lambda_i) \leq q(\lambda_i^*) \leq f(\vec{w}^*) \leq f(\vec{w})$

$q(\lambda_i^*) \leq f(\vec{w})$ strong dual $[q(\lambda_i^*) \rightarrow f(\vec{w}^*)]$

dual problem: $\max q(\lambda_i) = \max[\min L(w,b,\lambda_i)]$

s.t. $\lambda_i \geq 0$, $i \in [1, s]$.

Lemma: $q(\lambda_i^*) = f(\vec{w}^*)$, proorblea and dual probler optimal is amoterialsly

proof: $f(w) \geq q(\lambda_i^*) = f(\vec{w}^*) \Rightarrow f(\vec{w}^*) \leq f(w)$

$f(w) \geq q(\lambda_i^*) = f(\vec{w}^*) \geq q(\lambda_i) \Rightarrow q(\lambda_i^*) \geq q(\lambda_i)$ ∇

e.g.: original problem: $\min f(x) = x^2$ s.t. $x - 1 \geq 0$

construct $q(\lambda) = \min[L(x,\lambda)] = \min[x^2 - \lambda(x-1)]$, s.t. $\lambda \geq 0$

$\frac{\partial L}{\partial x} = 0 \Rightarrow x = \frac{\lambda}{2} \Rightarrow q(\lambda) = -\frac{\lambda^2}{4} + \lambda$
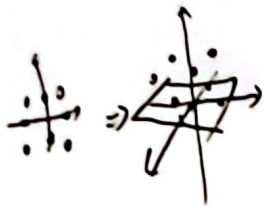
$\max[q(\lambda)] = 1$ when $\lambda = 2$, $x = 1$

use dual problem to simplify original problem

max $q(\lambda) = \max\left(\sum\limits_{i=1}^{s}\lambda_i - \frac{1}{2}\sum\limits_{i=1}^{s}\sum\limits_{j=1}^{s}\lambda_i\lambda_j y_i y_j (\vec{x}_i \cdot \vec{x}_j)\right)$

s.t. $\lambda_i \geq 0,\ i \in [1, s]$

Solve $\lambda_i \xRightarrow{\text{(1)}} \vec{w} \Rightarrow y_i(\vec{w}\cdot\vec{x}_i + b) - 1 = 0$, sol b.

4^{*} Kernel trick: some problem can solve in higher dimension by SVM. : $x_i \to T_s(x_i)$

max $q(\lambda_i) = \max\left(\sum\limits_{i=1}^{s}\lambda_i - \frac{1}{2}\sum\limits_{i=1}^{s}\sum\limits_{j=1}^{s}\lambda_i\lambda_j y_i y_j\, T(\vec{x}_i)\cdot T(\vec{x}_j)\right)$

s.t. $\lambda_i \geq 0$; $i \in [1, s]$

$1°$ $T(\vec{x}_i),\ T(\vec{x}_j)$ first, $\sum\limits_{h=1}^{n} T(x_i)\, T(x_j)$

$2°$ $T(\vec{x}_i)\cdot T(\vec{x}_j) = K(\vec{x}_i, \vec{x}_j)$, $\sum\limits_{i=1}^{n}\sum\limits_{j=1}^{n} K(\vec{x}_i, \vec{x}_j)$

normally : $K(\vec{x}_i, \vec{x}_j) = (c + \vec{x}_i\cdot\vec{x}_j)^d$

use T, s.t. $\vec{x} \in (x_1, x_2) \xrightarrow{T} T(\vec{x}) = (a_1 x_1, a_1 x_2, a_1 x_1 x_2, a_2 x_1^2, a_2 x_2^2, -$

$T: \mathbb{R}^2 \to \mathbb{R}^\infty$ $\qquad a_n x_1^n, a_n x_2^n, \cdots \alpha)$

$1°$ can't be done

$2°$, RBF function : $K(\vec{x}_i, \vec{x}_j) = e^{-r\|\vec{x}_i - \vec{x}_j\|^2}$

$K(\vec{x}_i, \vec{x}_j) = c\, e^{\langle \vec{x}_i, \vec{x}_j\rangle}$ $\qquad c = e^{\exp[-\frac{1}{2}(\|x_i\|^2 + \|x_j\|^2)]}$

$K(\vec{x}_i, \vec{x}_j) = c\sum\limits_{n=0}^{\infty}\frac{\vec{x}_i\cdot\vec{x}_j^{\,n}}{n!} = c\sum\limits_{n=0}^{\infty}\frac{K_{polynomial}(\vec{x}_i, \vec{x}_j)}{n!}$