p-adic number

1° Represent fraction and negative number;

Find $7x \cdots 142857143 = 1$

So $\frac{1}{7} = \cdots 142857143$ in 10-adic sense.

Find $1 + \cdots 9999 = 0$

So $\cdots 99 = -1$ in 10-adic sense

Find fraction: use multiplication, if $ad = a\frac{b}{c}$

then $d = \frac{b}{c}$

Find negative number: use addition, if $a+d = a+c$

$\Rightarrow d = c$

2° Why $x^2 = x$ has 4 sol in 10-adic sense;

cause $10 = 2 \times 5 \equiv 0 \pmod{10}$ so "$ab = 0 \not\Rightarrow a = 0$ or $b \neq 0$",

That's why we need p-adic number, cause

p-adic numbers form an integral domain ($ab = 0 \Rightarrow a = 0, b = 0$)