

$$15 + 6 + 5 + 1$$

No.

Date / /

$$\frac{9}{10} = \frac{1}{2} + \frac{1}{3} + \frac{1}{15}, \Rightarrow \frac{a}{b} = \sum_{i=1}^n \frac{1}{n_i} \quad (\text{decomposition})$$

proof: $\int_0^1 \frac{1}{n} = \frac{1}{n+1} + \frac{1}{n(n+1)} = \frac{1}{n+1} \left(\frac{n+1}{n} \right) = \frac{1}{n} \quad \text{--- ①}$

Let $\frac{a}{b} = \underbrace{\frac{1}{b} + \dots + \frac{1}{b}}_{a \text{ times}} \stackrel{\text{①}}{=} \frac{1}{b} + \left[\frac{1}{b+1} + \frac{1}{b(b+1)} \right] + \dots + \left[\frac{1}{b+1} + \frac{1}{b(b+1)} \right]$

$$= \frac{1}{b} + \frac{a-1}{b+1} + \frac{a-1}{b(b+1)} \quad \begin{matrix} I_1: \frac{a-1}{b+1} = \frac{1}{b+1} + \left[\frac{1}{b+2} + \frac{1}{(b+1)(b+2)} \right] (a-2) \\ I_2: \frac{a-1}{b_0} = \frac{1}{b_0+1} + \left[\frac{1}{b_0+1} + \frac{1}{b_0(b_0+1)} \right] (a-2) \end{matrix}$$

So at last $(a-1)$ steps can we get $\frac{a}{b} = \sum_{i=1}^n \frac{1}{n_i}$

□

For $\frac{9}{10} = \frac{1}{10} + \left(\frac{1}{11} + \frac{1}{10 \times 11} \right) \times 8 = \frac{9}{10} = \frac{1}{2} + \frac{2}{5}, \quad \frac{2}{5} = \frac{1}{5} + \left(\frac{1}{6} + \frac{1}{30} \right)$

$$\frac{9}{10} = \frac{1}{2} + \frac{1}{5} + \frac{1}{6} + \frac{1}{30}$$

