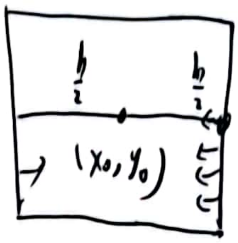
$u(x,y,t)$: temperature at (x,y) on time t.

Amount of heat energy at t:



$$H(t) = 6\int_S u(x,y,t)\,dx \quad (6>0)$$

Heat flow: $\frac{\partial H}{\partial t} = 6\int_S \frac{\partial u}{\partial t}\,dx \approx 6h^2 \frac{\partial u}{\partial t}$ (h is sufficient small).

By Newton cooling law: $\frac{dT}{dt} = -T[u(x_0+\frac{h}{2},y) - u(x_0,y)]$

Take continuous (change a little). $-k\frac{\partial u}{\partial x}(x_0+\frac{h}{2},t) \rightarrow$ all from $x = x_0+\frac{h}{2}$
$-kh\frac{\partial u}{\partial x}(x_0+\frac{h}{2},t)$

$$\frac{\partial H}{\partial t} = kh[u_x(x_0+\tfrac{h}{2}) - u_x(x_0-\tfrac{h}{2}) + u_y(y_0+\tfrac{h}{2}) - u_y(y_0-\tfrac{h}{2})]$$

(+- for opposite direction, details are $x_0+\frac{h}{2}$ hard to get but easy for following analysis)

since $\frac{u_x(x_0+\frac{h}{2}) - u_x(x_0-\frac{h}{2})}{h} \xrightarrow{h\to 0} u_{xx}$

$$\Rightarrow \frac{6}{k}\frac{\partial u}{\partial t} = \Delta u$$