Aczel ineq. $(a^2-b^2)(c^2-d^2) \le (ac-bd)^2$ only if $ad=bc$

proof: $(a^2-b^2)(c^2-d^2) = (ac-bd)^2 - (ad-bc)^2$

$$= [a(d+c) - b(c+d)][a(c-d) + b(c-d)]$$

$$= (a+b)(a-b)(c+d)(c-d)$$

eg. $f(x) = \sqrt{5x-4} - \sqrt{x-4}$, min $f(x)$.

$$f(x) \ge \sqrt5\,\sqrt{x-\frac45} - 1\cdot\sqrt{x-4}$$

$$\ge \sqrt{5-1}\,\sqrt{\left(x-\frac45\right)-(x-4)} = 2\cdot\frac{4}{\sqrt5}$$

also by derivative: $f'(x) = \dfrac{5}{2\sqrt{5x-4}} - \dfrac{1}{2\sqrt{x-4}}$

$$= \frac{5\sqrt{x-4} - \sqrt{5x-4}}{2\sqrt{(5x-4)(x-4)}} = 0$$

$$\Rightarrow 25(x-4) = 5x-4 \Rightarrow x^* = \frac{96}{20} = \frac{24}{5}$$

$$f(x^*) = 2\sqrt5 - \frac{2}{\sqrt5} = \frac{8}{\sqrt5}$$

and $f''(x) = 5\times\frac52\times\left(-\frac12\right)(5x-4)^{-\frac32} - \frac12\times\left(-\frac12\right)(x-4)^{-\frac32}$

$f''(x^*) > 0$

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