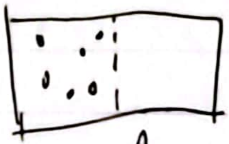
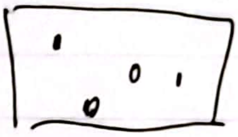
entropy: $S = k_b \log W$, W: 当前状态微观粒子排列种

eg:



A



B

$W_A = C_N^{n} \cdot C_N^{N}$ ← N 里 N 左 粒

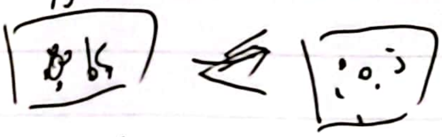
$W_B = C_N^{N_a} \cdot C_N^{N_b} C_N^{N-N_a} C_N^{N-N_b}$

$W_A < W_b$

$S_a < S_b \to$ chaos

(position, velosity can be regarded as information)

Shannon entropy: 当前系统状态下individual组合方式多样性程度



Stirling approximation:

$\log N! \approx N\log N - N$

$\approx \int_0^N \log x \, dx$

math deduction: imagine space partite to $N_1, \dots, N_M$, $W = \binom{N}{N_1}\binom{N-N_1}{N_2}\cdots\binom{N-\cdots-N_{m-1}}{N_m} = \frac{N!}{N_1!\cdots N_m!}$

$S = k_b \log W = k_b(\log N! - \log \prod_{k=1}^{m} N_k!) = k_b(\log N! - \sum_{k=1}^{m} \log N_k!)$

$\approx k_b[N\log N - N - \sum_{k=1}^{m}(N_k \log N_k - N_k)]$

$= k_b(N\log N - \sum_{k=1}^{m} N_k \log N_k)$ $\quad \underline{\underline{P_k = \frac{N_k}{N}}}$

$= k_b(N\log N - \sum_{k=1}^{m} NP_k \log NP_k)$

$= -k_b N \sum_{k=1}^{m} P_k \log P_k$, choose $k_b N = 1$

$S \approx -\sum_{k=1}^{m} P_k \log P_k$