PUMA 560 Manipulator Project

AI4000 Robotics
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Outline

Schematic and Coordinate Frames

Coordinate frames are assigned according to the **Denavit–Hartenberg** convention.

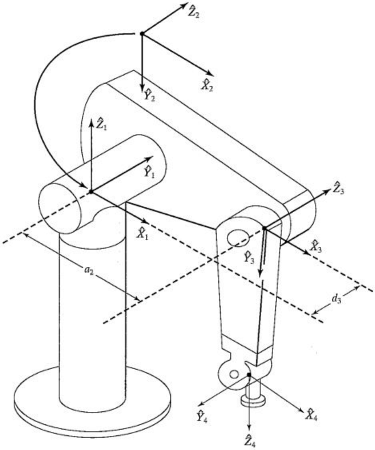

Figure 1: Assigned frames for the PUMA 560 arm

Denavit-Hartenberg Parameters

- The DH convention defines each link using:

$$(a_i, \alpha_i, d_i, \theta_i)$$

- The following table shows the DH parameters for the PUMA 560 manipulator.

Table 1: Denavit–Hartenberg parameters of PUMA 560

Link i	α_i (deg)	a_i (m)	d_i (m)	θ_i (rad)
1	-90	0	0	θ_1
2	0	a_2	0	θ_2
3	-90	a_3	d_3	θ_3
4	90	0	d_4	θ_4
5	-90	0	0	θ_5
6	0	0	0	θ_6

Transformation Matrices (1-3)

The homogeneous transformation matrices are derived from the DH parameters.

$$T_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & 0 \\ \sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\theta_2 & -\cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3 = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & a_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation Matrices (4-6)

$$T_4 = \begin{bmatrix} \cos\theta_4 & -\sin\theta_4 & 0 & a_4 \\ 0 & 0 & 1 & d_4 \\ -\sin\theta_4 & -\cos\theta_4 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_5 = \begin{bmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ \sin\theta_5 & \cos\theta_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_6 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\sin\theta_6 & -\cos\theta_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Resulting Forward Kinematics

$$^0T_6 = T_1\, T_2\, T_3\, T_4\, T_5\, T_6$$

Final Transformation

$$^0T_6 = \begin{bmatrix} R_0^6 & p_0^6 \\ 0 & 1 \end{bmatrix}$$

where R_0^6 is the rotation matrix (orientation) and p_0^6 is the position vector.

Workspace of PUMA 560

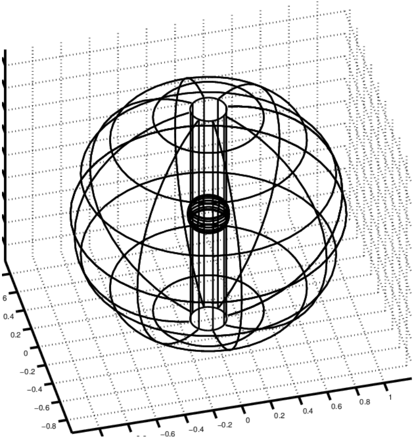

Figure 2: Workspace of the wrist point of the PUMA 560

Animated Workspace of wrist point

- Use **Okular in Linux** or **Adobe Acrobat in Windows** to view

Kinematics: Z-Axes (${}^0\mathbf{z}_i$)

Joint Axes in Base Frame

$$
{}^0\mathbf{z}_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad {}^0\mathbf{z}_1 = \begin{bmatrix} -\sin(\theta_1) \\ \cos(\theta_1) \\ 0 \end{bmatrix}, \quad {}^0\mathbf{z}_2 = \begin{bmatrix} -\sin(\theta_1) \\ \cos(\theta_1) \\ 0 \end{bmatrix},
$$

$$
{}^0\mathbf{z}_3 = \begin{bmatrix} -\sin(\theta_2+\theta_3)\cos(\theta_1) \\ -\sin(\theta_1)\sin(\theta_2+\theta_3) \\ -\cos(\theta_2+\theta_3) \end{bmatrix},
$$

$$
{}^0\mathbf{z}_4 = \begin{bmatrix} -\sin(\theta_1)\cos(\theta_4) + \sin(\theta_4)\cos(\theta_1)\cos(\theta_2+\theta_3) \\ \sin(\theta_1)\sin(\theta_4)\cos(\theta_2+\theta_3) + \cos(\theta_1)\cos(\theta_4) \\ -\sin(\theta_4)\sin(\theta_2+\theta_3) \end{bmatrix},
$$

$$
{}^0\mathbf{z}_5 = \begin{bmatrix} -(\sin(\theta_1)\sin(\theta_4) + \cos(\theta_1)\cos(\theta_4)\cos(\theta_2+\theta_3))\sin(\theta_5) - \sin(\theta_2+\theta_3)\cos(\theta_1)\cos(\theta_5) \\ (-\sin(\theta_1)\cos(\theta_4)\cos(\theta_2+\theta_3) + \sin(\theta_4)\cos(\theta_1))\sin(\theta_5) - \sin(\theta_1)\sin(\theta_2+\theta_3)\cos(\theta_5) \\ \sin(\theta_5)\sin(\theta_2+\theta_3)\cos(\theta_4) - \cos(\theta_5)\cos(\theta_2+\theta_3) \end{bmatrix}.
$$

Kinematics: Origins ($^0\mathbf{o}_i$)

Frame Origins in Base Frame

$$^0\mathbf{o}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad ^0\mathbf{o}_2 = \begin{bmatrix} a_2 \cos(\theta_1) \cos(\theta_2) \\ a_2 \sin(\theta_1) \cos(\theta_2) \\ -a_2 \sin(\theta_2) \end{bmatrix},$$

$$^0\mathbf{o}_3 = \begin{bmatrix} a_2 \cos(\theta_1) \cos(\theta_2) + a_3 \cos(\theta_1) \cos(\theta_2 + \theta_3) - d_3 \sin(\theta_1) \\ a_2 \sin(\theta_1) \cos(\theta_2) + a_3 \sin(\theta_1) \cos(\theta_2 + \theta_3) + d_3 \cos(\theta_1) \\ -a_2 \sin(\theta_2) - a_3 \sin(\theta_2 + \theta_3) \end{bmatrix},$$

$$^0\mathbf{o}_4 = \begin{bmatrix} a_2 \cos(\theta_1) \cos(\theta_2) + a_3 \cos(\theta_1) \cos(\theta_2 + \theta_3) - d_3 \sin(\theta_1) - d_4 \sin(\theta_2 + \theta_3) \cos(\theta_1) \\ a_2 \sin(\theta_1) \cos(\theta_2) + a_3 \sin(\theta_1) \cos(\theta_2 + \theta_3) + d_3 \cos(\theta_1) - d_4 \sin(\theta_1) \sin(\theta_2 + \theta_3) \\ -a_2 \sin(\theta_2) - a_3 \sin(\theta_2 + \theta_3) - d_4 \cos(\theta_2 + \theta_3) \end{bmatrix},$$

$$^0\mathbf{o}_5 = \begin{bmatrix} a_2 \cos(\theta_1) \cos(\theta_2) + a_3 \cos(\theta_1) \cos(\theta_2 + \theta_3) - d_3 \sin(\theta_1) - d_4 \sin(\theta_2 + \theta_3) \cos(\theta_1) \\ a_2 \sin(\theta_1) \cos(\theta_2) + a_3 \sin(\theta_1) \cos(\theta_2 + \theta_3) + d_3 \cos(\theta_1) - d_4 \sin(\theta_1) \sin(\theta_2 + \theta_3) \\ -a_2 \sin(\theta_2) - a_3 \sin(\theta_2 + \theta_3) - d_4 \cos(\theta_2 + \theta_3) \end{bmatrix},$$

$$^0\mathbf{o}_6 = \begin{bmatrix} a_2 \cos(\theta_1) \cos(\theta_2) + a_3 \cos(\theta_1) \cos(\theta_2 + \theta_3) - d_3 \sin(\theta_1) - d_4 \sin(\theta_2 + \theta_3) \cos(\theta_1) \\ a_2 \sin(\theta_1) \cos(\theta_2) + a_3 \sin(\theta_1) \cos(\theta_2 + \theta_3) + d_3 \cos(\theta_1) - d_4 \sin(\theta_1) \sin(\theta_2 + \theta_3) \\ -a_2 \sin(\theta_2) - a_3 \sin(\theta_2 + \theta_3) - d_4 \cos(\theta_2 + \theta_3) \end{bmatrix}.$$

Spatial Manipulator Jacobian

Jacobian Structure

The spatial velocity Jacobian $\mathbf{J}$ is a 6×6 matrix composed of:

- **Linear Velocity $\mathbf{J}_v$**: Contribution of joint i to end-effector's linear velocity
- **Angular Velocity $\mathbf{J}_\omega$**: Contribution of joint i to end-effector's angular velocity

Column Formulas for Revolute Joint i

$$\mathbf{J}_{v_i} = {}^0\mathbf{z}_{i-1} \times \left({}^0\mathbf{o}_6 - {}^0\mathbf{o}_{i-1}\right)$$
$$\mathbf{J}_{\omega_i} = {}^0\mathbf{z}_{i-1}$$

Full 6×6 Spatial Jacobian $\mathbf{J}$

Structure

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_v \\ \mathbf{J}_\omega \end{bmatrix}$$

Linear Velocity Component $\mathbf{J}_v$

$$\mathbf{J}_v = \begin{bmatrix} -a_2 s_1 c_2 - a_3 s_1 c_{23} - d_3 c_1 + d_4 s_1 s_{23} & -(a_2 s_2 + a_3 s_{23} + d_4 c_{23}) c_1 & -(a_3 s_{23} + d_4 c_{23}) c_1 & 0 & 0 & 0 \\ a_2 c_1 c_2 + a_3 c_1 c_{23} - d_3 s_1 - d_4 s_{23} c_1 & -(a_2 s_2 + a_3 s_{23} + d_4 c_{23}) s_1 & -(a_3 s_{23} + d_4 c_{23}) s_1 & 0 & 0 & 0 \\ 0 & -a_2 c_2 - a_3 c_{23} + d_4 s_{23} & -a_3 c_{23} + d_4 s_{23} & 0 & 0 & 0 \end{bmatrix}$$

Angular Velocity Component $\mathbf{J}_\omega$

$$\mathbf{J}_\omega = \begin{bmatrix} 0 & -s_1 & -s_1 & -s_{23} c_1 & -s_1 c_4 + s_4 c_1 c_{23} & -(s_1 s_4 + c_1 c_4 c_{23}) s_5 - s_{23} c_1 c_5 \\ 0 & c_1 & c_1 & -s_1 s_{23} & s_1 s_4 c_{23} + c_1 c_4 & -(s_1 c_4 c_{23} - s_4 c_1) s_5 - s_1 s_{23} c_5 \\ 1 & 0 & 0 & -c_{23} & -s_4 s_{23} & s_5 s_{23} c_4 - c_5 c_{23} \end{bmatrix}$$

Notation: $s_i = \sin\theta_i,\ c_i = \cos\theta_i,\ s_{23} = \sin(\theta_2 + \theta_3),\ c_{23} = \cos(\theta_2 + \theta_3)$

Conclusion

Future work:

- Inverse kinematics
- Dynamics modeling
- Trajectory and path planning in simulation

Thank You!