PUMA 560 Manipulator Project

Denavit-Hartenberg Parameters, Workspace, Forward Kinematics

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Outline

Introduction

- The **PUMA 560 (Programmable Universal Machine for Assembly)** is a six-degree-of-freedom industrial manipulator.
- Goal: Develop a complete mathematical model including:
  - Forward and inverse kinematics
  - Dynamics
  - Trajectory and path planning
- The manipulator will execute a simple pick-and-place task in a physics-based simulator (e.g., MuJoCo).
- This presentation focuses on the **Forward Kinematics** derivation using the Denavit-Hartenberg (DH) convention.

The PUMA 560 Manipulator

- A six-DOF serial manipulator with revolute joints.
- Composed of:
  - Base, shoulder, and elbow - responsible for positioning.
  - Wrist - controls end-effector orientation.

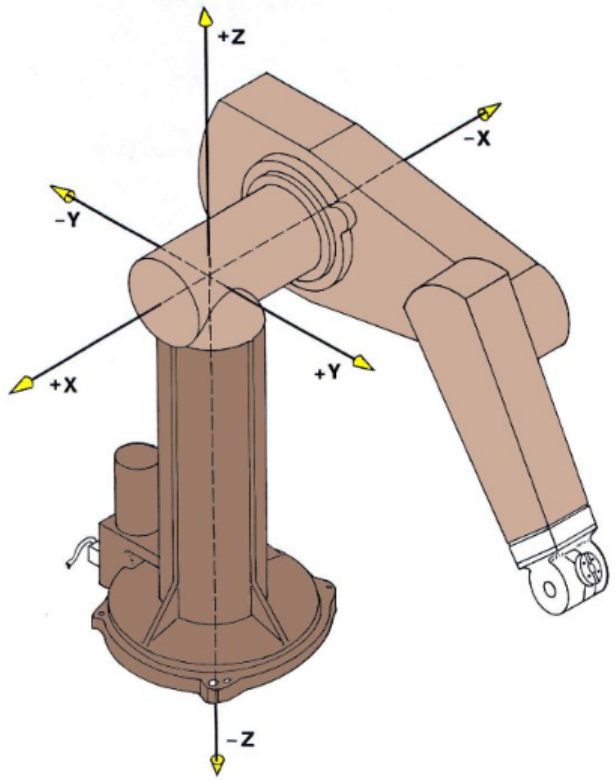

Figure 1: PUMA 560 manipulator

Schematic and Coordinate Frames

Coordinate frames are assigned according to the **Denavit–Hartenberg** convention.

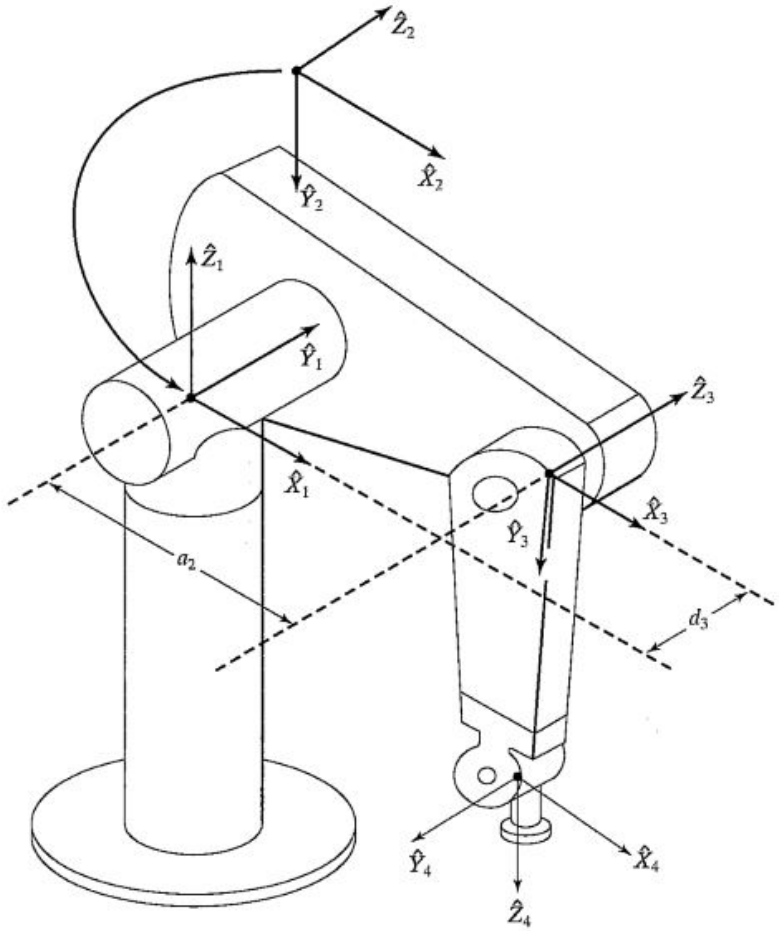

Figure 2: Assigned frames for the PUMA 560 arm

Schematic (Wrist Section)

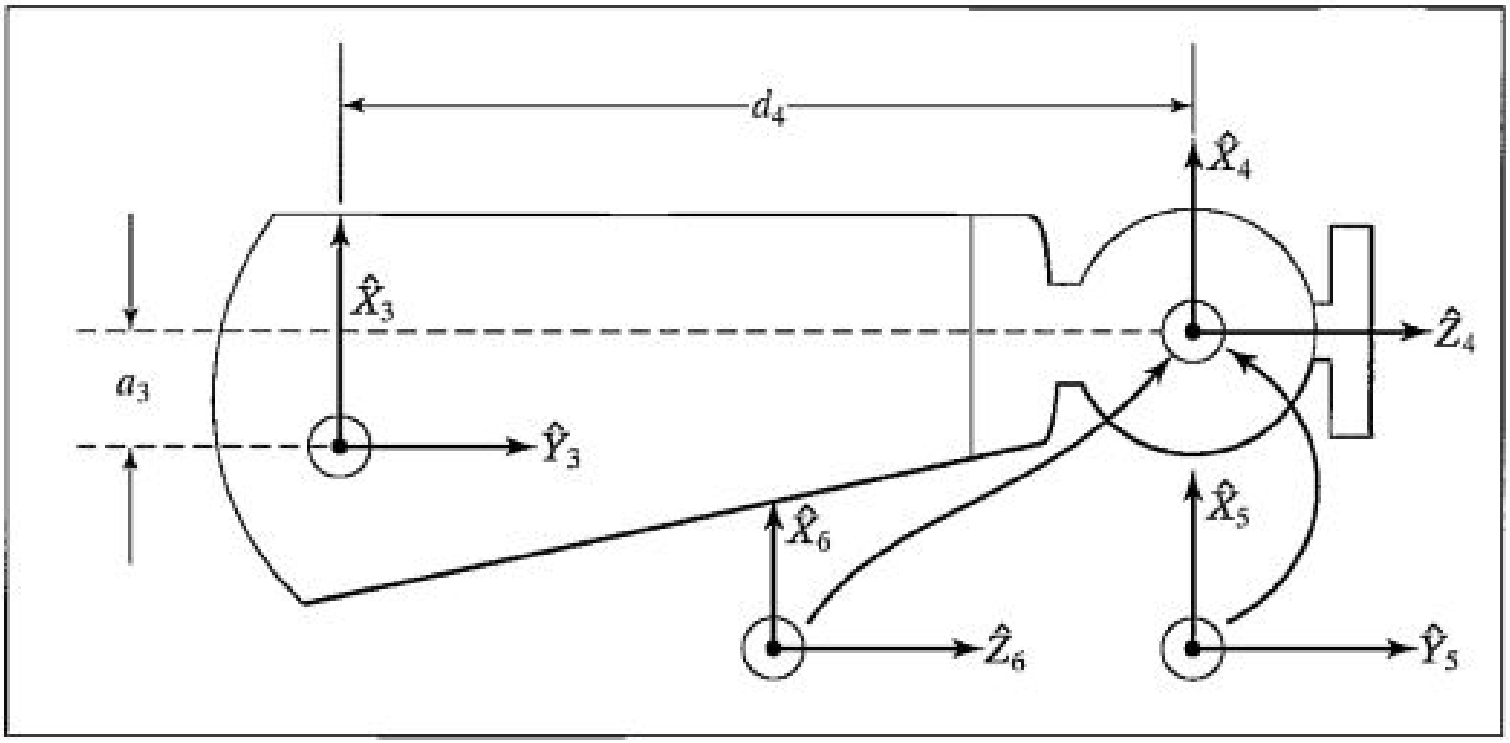

Figure 3: Assigned coordinate frames for the wrist joints

Schematic (Wrist Section)

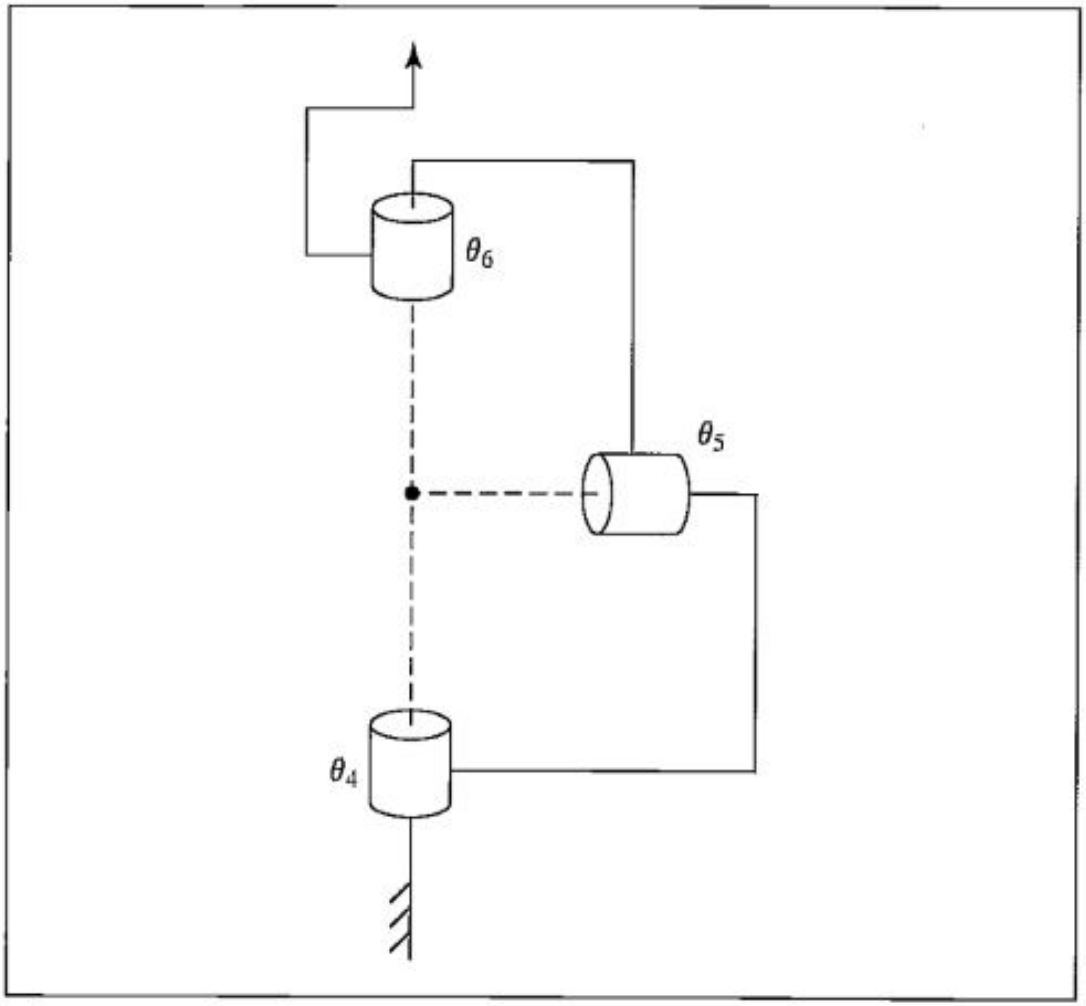

Figure 4: Simplified schematic of wrist joint arrangement

Denavit-Hartenberg Parameters

- The DH convention defines each link using:

$$(a_i, \alpha_i, d_i, \theta_i)$$

- The following table shows the DH parameters for the PUMA 560 manipulator.

Table 1: Denavit–Hartenberg parameters of PUMA 560

Link i	α_i (deg)	a_i (m)	d_i (m)	θ_i (rad)
1	90	a_1	d_1	θ_1
2	0	a_2	d_2	θ_2
3	-90	0	0	θ_3
4	90	0	d_4	θ_4
5	-90	0	0	θ_5
6	0	0	d_6	θ_6

Transformation Matrices (1-3)

The homogeneous transformation matrices are derived from the DH parameters.

$$A_1 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} \cos\theta_3 & 0 & -\sin\theta_3 & a_3\cos\theta_3 \\ \sin\theta_3 & 0 & \cos\theta_3 & a_3\sin\theta_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation Matrices (4-6)

$$A_4 = \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & 0 \\ \sin\theta_4 & 0 & -\cos\theta_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} \cos\theta_5 & 0 & -\sin\theta_5 & 0 \\ \sin\theta_5 & 0 & \cos\theta_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Resulting Forward Kinematics

$$^0T_6 = T_1\, T_2\, T_3\, T_4\, T_5\, T_6$$

Final Transformation

$$^0T_6 = \begin{bmatrix} R_0^6 & p_0^6 \\ 0 & 1 \end{bmatrix}$$

where R_0^6 is the rotation matrix (orientation) and p_0^6 is the position vector.

Workspace of PUMA 560

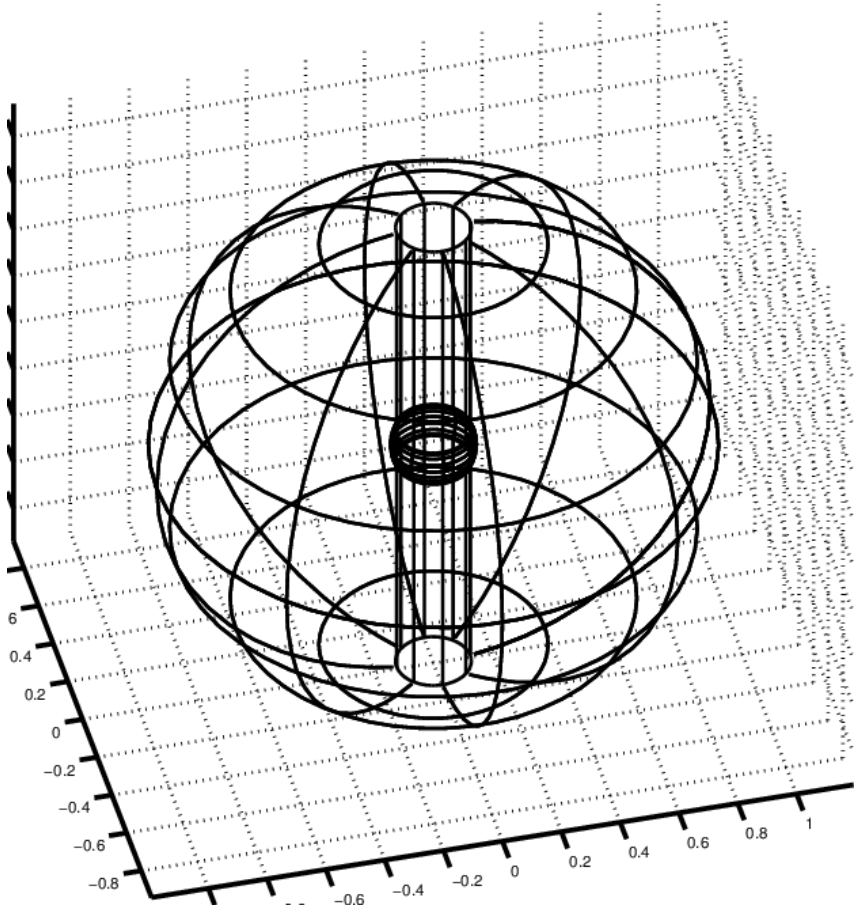

Figure 5: Workspace of the wrist point of the PUMA 560

Animated Workspace of wrist point

- Use **Okular in Linux** or **Adobe Acrobat in Windows** to view

Conclusion

- Derived the homogeneous transformation matrices for all six joints.
- Established the overall forward kinematic model ${}^0T_6$.
- This model defines the end-effector's position and orientation as functions of six joint variables.
- Future work:
  - Inverse kinematics
  - Dynamics modeling
  - Trajectory and path planning in simulation

Thank You!