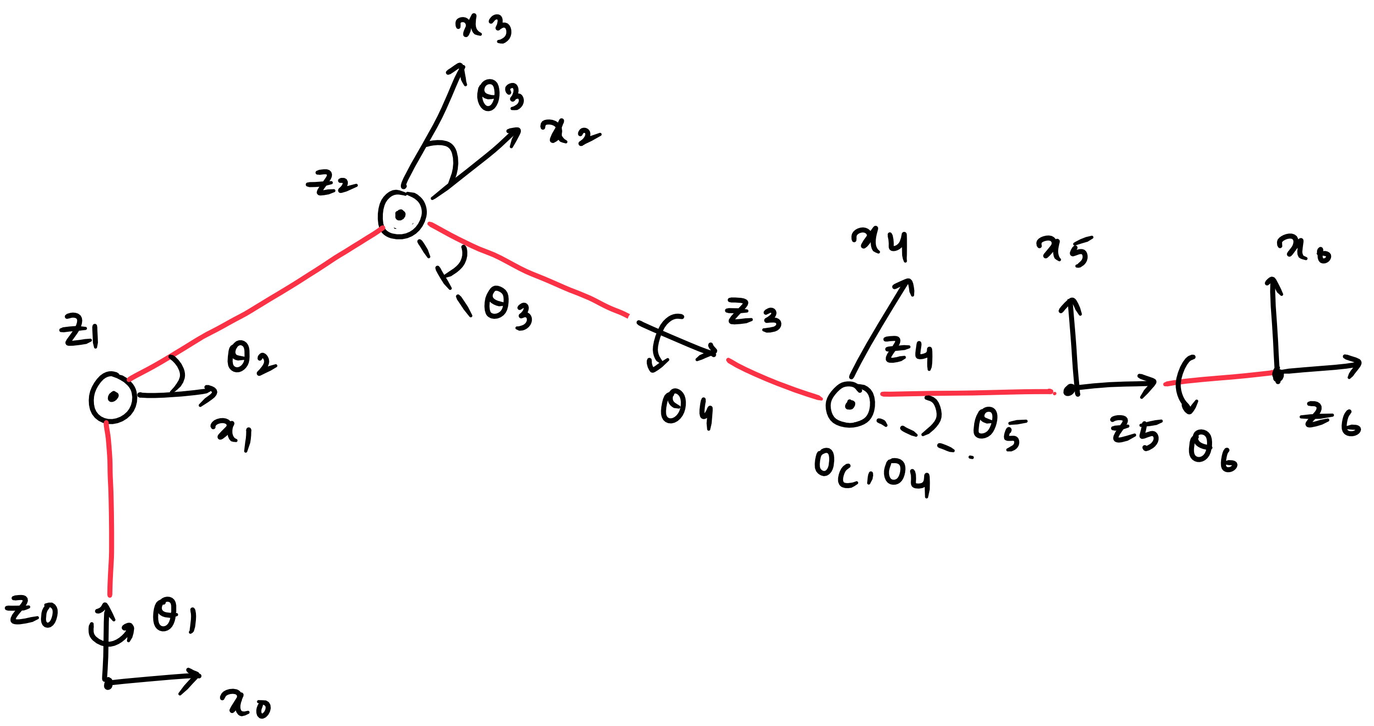


Kinematic Decoupling:

$$O_4^0 = \begin{bmatrix} c_1(a_2c_2 + a_3c_{23}) + d_2s_1 - d_4c_1s_{23} \\ s_1(a_2c_2 + a_3c_{23}) - d_2c_1 - d_4s_1s_{23} \\ d_1 + a_2s_2 + a_3s_{23} + d_4c_{23} \end{bmatrix} = \begin{bmatrix} P_x \\ P_y \\ P_z \end{bmatrix}$$

$$P_x = c_1[a_2c_2 + a_3c_{23} - d_4s_{23}] + d_2s_1$$

$$P_y = s_1[a_2c_2 + a_3c_{23} - d_4s_{23}] - d_2c_1$$

let $r = a_2c_2 + a_3c_{23} - d_4s_{23} \longrightarrow (*)$

$$\Rightarrow P_x = c_1r + d_2s_1$$

$$P_y = s_1r - d_2c_1$$

$$\Rightarrow P_x^2 + P_y^2 = r^2 + d_2^2$$

$$\Rightarrow r = \pm\sqrt{P_x^2 + P_y^2 - d_2^2}$$

From diagram

$\phi = \operatorname{atan2}(P_y, P_x)$

$c^2 = P_x^2 + P_y^2$

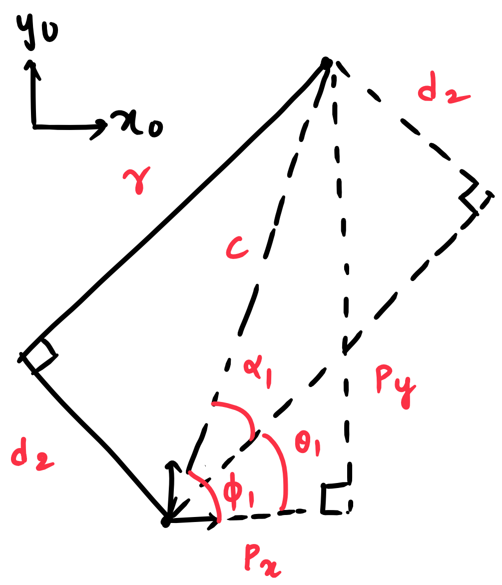
$r^2 + d_2^2 = c^2$

$\Rightarrow \alpha_1 = \operatorname{atan2}(d_2, r)$

$\theta_1 = \phi_1 - \alpha_1$

$\theta_1 = \operatorname{atan2}(P_y, P_x) - \operatorname{atan2}(d_2, r)$

TOP VIEW



r has 2 possible values

$\Rightarrow \theta_1$ has 2 possible values.

From ⊛

$$(r - a_2 c_2)^2 + (P_z - d_1 - a_2 S_2)^2 = (a_3 C_{23} - d_4 S_{23})^2 + (a_3 S_{23} + d_4 C_{23})^2$$

$$\Rightarrow r^2 + a_2^2 c_2^2 - 2 r a_2 c_2 + (P_z - d_1)^2 + a_2^2 s_2^2 - 2(P_z - d_1) a_2 s_2$$

$$= a_3^2 c_{23}^2 + d_4^2 s_{23}^2 - \cancel{2 a_3 d_4 C_{23} S_{23}} + a_3^2 s_{23}^2 + d_4^2 C_{23}^2 + \cancel{2 a_3 d_4 S_{23} C_{23}}$$

$$\Rightarrow r^2 + a_2^2 + (P_z - d_1)^2 - 2 r a_2 c_2 - 2(P_z - d_1) a_2 s_2 = a_3^2 + d_4^2$$

$$\Rightarrow \underbrace{(2 r a_2)}_{K_1} c_2 + \underbrace{2(P_z - d_1) a_2}_{K_2} s_2 = \underbrace{r^2 + a_2^2 - a_3^2 - d_4^2 + (P_z - d_1)^2}_{K_3}$$

$$\Rightarrow \quad K_1 C_2 + K_2 S_2 = K_3$$

has a standard solution

$$\theta_2 = \operatorname{atan2}(K_2, K_1) \pm \cos^{-1}\left(\frac{K_3}{\sqrt{K_1^2 + K_2^2}}\right)$$

$$\theta_2 = \operatorname{atan2}(P_z - d_1, r) \pm \cos^{-1}\left(\frac{r^2 + a_2^2 - a_3^2 - d_4^2 + (P_z - d_1)^2}{2 a_2 \sqrt{r^2 + (P_z - d_1)^2}}\right)$$

let $r - a_2 C_2 = m_1 \Rightarrow m_1 = a_3 C_{23} - d_4 S_{23}$

$P_z - d_1 - a_2 S_2 = m_2 \Rightarrow m_2 = a_3 S_{23} + d_4 C_{23}$

From solving 2. LE in C_{23}, S_{23}

$$C_{23} = \frac{a_3 m_1 + d_4 m_2}{a_3^2 + d_4^2} \qquad S_{23} = \frac{a_3 m_2 - d_4 m_1}{a_3^2 + d_4^2}$$

$$\theta_{23} = \operatorname{atan2}(S_{23}, C_{23}) \qquad \theta_3 = \theta_{23} - \theta_2$$

$$R = R_6^0 = R_3^0 R_6^3 \Rightarrow R_6^3 = (R_3^0)^{-1} R = R_3^{0\,T} R$$

substitute R^0_3, R from forward kinematics to get

$$R_6^3 = \begin{bmatrix} C_4 C_5 C_6 - S_4 S_6 & C_4 C_5 S_6 - C_6 S_4 & C_4 S_5 \\ C_4 S_6 + C_5 C_6 S_4 & C_4 C_6 + C_5 S_4 S_6 & S_4 S_5 \\ -C_6 S_5 & S_5 S_6 & C_5 \end{bmatrix}$$

$$\theta_4 = \operatorname{atan2}(T_6^3(2,3), T_6^3(1,3))$$

$$\theta_5 = \operatorname{atan2}\left(\sqrt{1 - T_6^3(3,3)^2}, T_6^3(3,3)\right)$$

$$\theta_6 = \operatorname{atan2}(T_6^3(3,2), -T_6^3(3,1))$$