PUMA 560 Manipulator Project

Velocity Kinematics

Saran Konala Aryan Mehta
Pakki Sai Kaushik Panshul Jindal

Indian Institute of Technology, Hyderabad

November 11th, 2025

Outline

Schematic and Coordinate Frames

Coordinate frames are assigned according to the **Denavit–Hartenberg** convention.

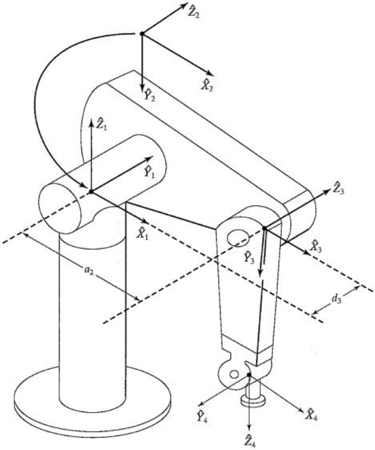

Figure 1: Assigned frames for the PUMA 560 arm

Denavit-Hartenberg Parameters

- The DH convention defines each link using:

$$(a_i, \alpha_i, d_i, \theta_i)$$

- The following table shows the DH parameters for the PUMA 560 manipulator.

Table 1: Denavit–Hartenberg parameters of PUMA 560

Link i	α_i (deg)	a_i (m)	d_i (m)	θ_i (rad)
1	90	a_1	d_1	θ_1
2	0	a_2	d_2	θ_2
3	-90	0	0	θ_3
4	90	0	d_4	θ_4
5	-90	0	0	θ_5
6	0	0	d_6	θ_6

Transformation Matrices (1-3)

The homogeneous transformation matrices are derived from the DH parameters.

$$A_1 = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 0 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad A_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & a_2\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & a_2\sin\theta_2 \\ 0 & 0 & 1 & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} \cos\theta_3 & 0 & -\sin\theta_3 & a_3\cos\theta_3 \\ \sin\theta_3 & 0 & \cos\theta_3 & a_3\sin\theta_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation Matrices (4-6)

$$A_4 = \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & 0 \\ \sin\theta_4 & 0 & -\cos\theta_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_5 = \begin{bmatrix} \cos\theta_5 & 0 & -\sin\theta_5 & 0 \\ \sin\theta_5 & 0 & \cos\theta_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_6 = \begin{bmatrix} \cos\theta_6 & -\sin\theta_6 & 0 & 0 \\ \sin\theta_6 & \cos\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Resulting Forward Kinematics

$$^{0}T_6 = T_1\, T_2\, T_3\, T_4\, T_5\, T_6$$

Final Transformation

$$^{0}T_6 = \begin{bmatrix} R_0^6 & p_0^6 \\ 0 & 1 \end{bmatrix}$$

where R_0^6 is the rotation matrix (orientation) and p_0^6 is the position vector.

Workspace of PUMA 560

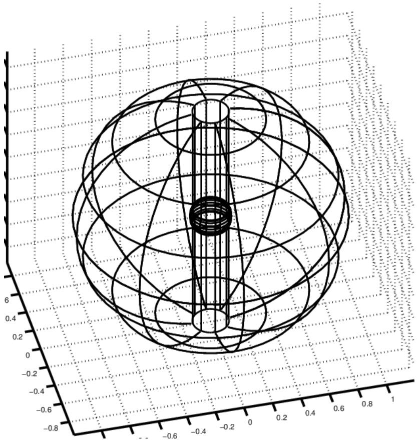

Figure 2: Workspace of the wrist point of the PUMA 560

Animated Workspace of wrist point

- Use **Okular in Linux** or **Adobe Acrobat in Windows** to view

Kinematics: Joint Axes ($^0\mathbf{z}_i$)

Axes in Base Frame

$$^0\mathbf{z}_0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \qquad ^0\mathbf{z}_1 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix}, \qquad ^0\mathbf{z}_2 = \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix},$$

$$^0\mathbf{z}_3 = \begin{bmatrix} -\, c_1\, s_{23} \\ -\, s_1\, s_{23} \\ c_{23} \end{bmatrix},$$

$$^0\mathbf{z}_4 = \begin{bmatrix} -\, s_1 c_4 + c_1 c_{23} s_4 \\ s_1 c_{23} s_4 + c_1 c_4 \\ -\, s_{23} s_4 \end{bmatrix},$$

$$^0\mathbf{z}_5 = \begin{bmatrix} -(s_1 s_4 + c_1 c_4 c_{23}) s_5 - c_1 s_{23} c_5 \\ -(c_1 s_4 - s_1 c_4 c_{23}) s_5 - s_1 s_{23} c_5 \\ s_{23} s_5 c_4 - c_{23} c_5 \end{bmatrix},$$

$$^0\mathbf{z}_6 = {}^0\mathbf{z}_5.$$

Kinematics: Frame Origins ($^0\mathbf{o}_i$)

Origins in Base Frame

$$^0\mathbf{o}_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \qquad ^0\mathbf{o}_1 = \begin{bmatrix} 0 \\ 0 \\ d_1 \end{bmatrix},$$

$$^0\mathbf{o}_2 = \begin{bmatrix} a_2 c_1 c_2 + d_2 s_1 \\ a_2 s_1 c_2 - d_2 c_1 \\ a_2 s_2 + d_1 \end{bmatrix},$$

$$^0\mathbf{o}_3 = \begin{bmatrix} c_1(a_2 c_2 + a_3 c_{23}) + d_2 s_1 \\ s_1(a_2 c_2 + a_3 c_{23}) - d_2 c_1 \\ d_1 + a_2 s_2 + a_3 s_{23} \end{bmatrix},$$

$$^0\mathbf{o}_4 = \begin{bmatrix} c_1(a_2 c_2 + a_3 c_{23}) + d_2 s_1 - d_4 c_1 s_{23} \\ s_1(a_2 c_2 + a_3 c_{23}) - d_2 c_1 - d_4 s_1 s_{23} \\ d_1 + a_2 s_2 + a_3 s_{23} + d_4 c_{23} \end{bmatrix},$$

$$^0\mathbf{o}_5 = {}^0\mathbf{o}_4,$$

$$^0\mathbf{o}_6 = {}^0\mathbf{o}_5 + d_6\, {}^0\mathbf{z}_5.$$

Spatial Manipulator Jacobian

Jacobian Structure

The spatial velocity Jacobian $\mathbf{J}$ is a 6×6 matrix composed of:

- **Linear Velocity $\mathbf{J}_v$**: Contribution of joint i to end-effector's linear velocity
- **Angular Velocity $\mathbf{J}_\omega$**: Contribution of joint i to end-effector's angular velocity

Column Formulas for Revolute Joint i

$$\mathbf{J}_{v_i} = {}^0\mathbf{z}_{i-1} \times \left({}^0\mathbf{o}_6 - {}^0\mathbf{o}_{i-1}\right)$$
$$\mathbf{J}_{\omega_i} = {}^0\mathbf{z}_{i-1}$$

Full 6×6 Spatial Jacobian $\mathbf{J}$

Structure

$$\mathbf{J} = \begin{bmatrix} \mathbf{J}_v \\ \mathbf{J}_\omega \end{bmatrix}$$

Linear Velocity Component $\mathbf{J}_v$

$$\mathbf{J}_v = \begin{bmatrix} d_2c_1 + d_4s_1s_{23} - (a_2c_2 + a_3c_{23})s_1 + d_6[(s_1c_4c_{23} - s_4c_1)s_5 + s_1s_{23}c_5] & c_1[-a_2s_2 - a_3s_{23} - d_4c_{23} + d_6(s_5s_{23}c_4 - c_5c_{23})] & c_1[-a_3s_{23} - d_4c_{23} + d_6(s_5s_{23}c_4 - c_5c_{23})] & d_6(-s_1c_4 + s_4c_1c_{23})s_5 & -d_6[(s_1s_4 + c_1c_4c_{23})c_5 - s_5s_{23}c_1] & 0 \\ d_2s_1 - d_4s_{23}c_1 + (a_2c_2 + a_3c_{23})c_1 + d_6[(s_1s_4 + c_1c_4c_{23})s_5 - s_{23}c_1c_5] & -s_1[a_2s_2 + a_3s_{23} + d_4c_{23} + d_6(s_5s_{23}c_4 + c_5c_{23})] & -s_1[a_3s_{23} + d_4c_{23} + d_6(s_5s_{23}c_4 + c_5c_{23})] & -d_6(s_1s_4c_{23} + c_1c_4)s_5 & d_6[(s_1c_4c_{23} - s_4c_1)c_5 + s_1s_5s_{23}] & 0 \\ 0 & a_2c_2 + a_3c_{23} - d_4s_{23} + d_6(s_5c_4c_{23} + s_{23}c_5) & a_3c_{23} - d_4s_{23} + d_6(s_5c_4c_{23} + s_{23}c_5) & -d_6s_4s_5s_{23} & d_6(s_5c_{23} + s_{23}c_4c_5) & 0 \end{bmatrix}$$

Angular Velocity Component $\mathbf{J}_\omega$

$$\mathbf{J}_\omega = \begin{bmatrix} 0 & s_1 & s_1 & -c_1s_{23} & -s_1c_4 + s_4c_1c_{23} & -(s_1s_4 + c_1c_4c_{23})s_5 - c_1s_{23}c_5 \\ 0 & -c_1 & -c_1 & -s_1s_{23} & s_1s_4c_{23} + c_1c_4 & (s_1c_4c_{23} - s_4c_1)s_5 - s_1s_{23}c_5 \\ 1 & 0 & 0 & c_{23} & -s_4s_{23} & s_5s_{23}c_4 - c_5c_{23} \end{bmatrix}$$

Notation: $s_i = \sin\theta_i,\ c_i = \cos\theta_i,\ s_{23} = \sin(\theta_2 + \theta_3),\ c_{23} = \cos(\theta_2 + \theta_3)$

Conclusion

Future work:

- Inverse kinematics
- Dynamics modeling
- Trajectory and path planning in simulation

Thank You!