Generic interest distribution for a pool

Context

- We have a pool holding some assets. There are “owners” who contribute proportionally to this pool of assets, and get back “pool ownership tokens”
    - Let’s say at time t, the total amount of “pool ownership tokens” is S_t^{total}
- Overtime, the assets in the pool will generate “yield tokens”. These “yield tokens” might or might not generate more “yield tokens” overtime.
    - Let’s call the amount of yield tokens at time t to be n_t^{yield}
    - From time t to t+1, an existing amountn_t^{yield} of yield tokens will grow to $n_t^{yield} \times m_t$. In the case where yield tokens don’t grow, $m_t$is always 1
- At any time, a pool owner can claim back his **rightful** portion of yield tokens that were generated. We need to distribute the yield tokens to the owners such that:
    - From time t to t+1, the yield tokens generated **from the assets in the pool** should be **equally distributed** to the current pool owners, proportionally to their share of the pool
    - If a certain portion of yield tokens in the pool is entitled to a user A, any interest generated from this portion is also entitled to A.
- Basically, this scenario can be applied to:
    - A market, where the assets consists of the XYTs and base tokens, and the “pool ownership tokens” are the LP tokens
        - For an Aave-based market (XYTs are derived from aTokens), the “yield tokens” which is aTokens will generate even more yield tokens
        - For a Compound-based market (XYTs are derived from cTokens), the “yield tokens” do not generate more yield tokens. As such, $m_t$is always 1
    - A liquidity mining pool’s sub-pool that consists of LPs for a certain market
        - The assets are basically the LP tokens staked
        - The “pool ownership tokens” are also the LP tokens being staked
        - The “yield tokens” could be aTokens or cTokens, depending on the type of market of the LP tokens.
        - Each of these sub-pool is basically a PendleLpHolder contract

Formula:

At t:

- User A deposits s_A new ownership tokens
- The pool now has $s_t^{total} = s_{t-1}^{total} + s_A$ ownership tokens
- A now owns $\frac{s_A}{s_t^{total}}$ of the pool.
- We now have n_t^{yield}
- E.g 1000 aUSDT

At t+1: something happened that changed n^{yield}, s^{total}

- We now have n_{t+1}^{yield} . E.g 1100, m(t) = 1.01
- The interest generated by existing yield tokens was: $n_t^{yield} \times m_t$
- As such, the interest generated by the assets in the pool is:
  $R_t = n_{t+1}^{yield} - n_t^{yield} \times m_t$
  E.g = 1100 - 1000*1.01

Let I_{t-t+n}^A be the interests for A (who deposited at t) at t+n, then:

$$I_{t-t+1}^A = \frac{s_A}{s_t^{total}} \times R_t$$

At t+2: something else happened that changed n^{yield}, s^{total}

- We now have n_{t+2}^{yield}
- The interest generated by existing yield tokens was: $n_{t+1}^{yield} \times m_{t+1}$
- As such, the interest generated by the assets in the pool is:
  $R_{t+1} = n_{t+2}^{yield} - n_{t+1}^{yield} \times m_{t+1}$
- Hence, from t to t+2, A is entitled to:

$$I_{t-t+2}^A = \frac{s_A}{s_{t+1}^{total}} \times R_{t+1} + I_{t-t+1}^A \times m_{t+1}$$

$$= \frac{s_A}{s^{total}_{t+1}} R_{t+1} + \frac{s_A}{s^{total}_{t}} R_t \times m_{t+1}$$

<u>At t+3:</u> Similarly, from t to t+3, A is entitled to receive:

$$I^A_{t-t+3} = \frac{s_A}{s^{total}_{t+2}} \times R_{t+2} + I^A_{t-t+2} \times m_{t+2}$$

$$= \frac{s_A}{s^{total}_{t+2}} R_{t+2} + \left(\frac{s_A}{s^{total}_{t+1}} R_{t+1} + \frac{s_A}{s^{total}_{t}} R_t \times m_{t+1} \right) \times m_{t+2}$$

$$= \frac{s_A}{s^{total}_{t+2}} R_{t+2} +$$

$$\frac{s_A}{s^{total}_{t+1}} R_{t+1} \times m_{t+2} + \frac{s_A}{s^{total}_{t}} R_t \times m_{t+1} \times m_{t+2}$$

$$= s_A \left(\frac{R_{t+2}}{s^{total}_{t+2}} + \frac{R_{t+1}}{s^{total}_{t+1}} \times m_{t+2} + \frac{R_t}{s^{total}_{t}} \times m_{t+1} \times m_{t+2} \right)$$

As such, we can generalise it:

$$I^A_{t-t+n} =$$

$$s_A \times \left(\frac{R_{t+n-1}}{s^{total}_{t+n-1}} + \ldots + \frac{R_{t+1}}{s^{total}_{t+1}} m_{t+2} m_{t+3} \ldots m_{t+n-1} + \frac{R_t}{s^{total}_{t}} m_{t+1} m_{t+2} \ldots m_{t+n-1} \right)$$

Let's define:

$H_{t-t+n} = \frac{I^A_{t-t+n}}{s_A}$,which is the interest per ownership token from t to t+n

$$= \frac{R_{t+n-1}}{s^{total}_{t+n-1}} + \ldots + \frac{R_{t+1}}{s^{total}_{t+1}} m_{t+2} m_{t+3} \ldots m_{t+n-1} + \frac{R_t}{s^{total}_{t}} m_{t+1} m_{t+2} \ldots m_{t+n-1}$$

Let’s also define:

$L_t = H_{0-t} =$ intuitively, interest per LP for anyone staking from the very start

$N_t = m_1 m_2 ... m_{t-1}$ = current income index / initial income index

We can observe that:

$$L_{t+n} = \frac{R_{t+n-1}}{s^{total}_{t+n-1}} + ... + \frac{R_{t+1}}{s^{total}_{t+1}} m_{t+2} m_{t+3} ... m_{t+n-1} + \frac{R_0}{s^{total}_0} m_1 m_2 ... m_{t+n-1}$$

$$= L_t \times m_t m_{t+1} ... m_{t+n-1} + H_{t-t+n}$$

$$= L_t \times \frac{m_1 m_2 ... m_{t+n-1}}{m_1 m_2 ... m_{t-1}} + H_{t-t+n}$$

$$= L_t \times \frac{N_{t+n}}{N_t} + H_{t-t+n}$$

In other words,

$$H_{t-t+n} = L_{t+n} - L_t \times \frac{N_{t+n}}{N_t}$$

Another way to explain the formula:

The amount of interest for a user A staking from t to t+n is:
Amount of interest if A has staked from the start to t+n
minus "the amount of interest if A has staked from the start to t, and compounded separately from t to t+n"

Universe 1: user A stake from 0 -> t
Universe

Also, we can get L_{t+1} recursively from L_t:

$$L_{t+1} = L_t \times m_{t+1} + \frac{R_{t+1}}{s_t^{total}}, \text{ where } R_t = n_{t+1}^{yield} - n_t^{yield} \times m_t$$

Implementation

We notice that L and N are **global variables** that can be updated every time there is a change to n^{yield} or s^{total}

As such, we just need to save L and N for each user at the time when their interest was last claimed, and we will be able to get their current interest at t+n as:

$$I_{t-t+n}^{A} = s_A \times H_{t-t+n} = s_A \times \left(L_{t+n} - L_t \times \frac{N_{t+n}}{N_t}\right)$$

Further observation:

For Aave:

$normalizedIncome_t$ is the normalizedIncome of the corresponding Aave's asset at time t_0

We have:

$$m_1 = normalizedIncome_1 / normalizedIncome_0$$

$$m_2 = normalizedIncome_2 / normalizedIncome_1$$

....

$$m_t = normalizedIncome_t / normalizedIncome_{t-1}$$

$$=> m_1 \times m_2 \times .... \times m_t = \frac{normalizedIncome_t}{normalizedIncome_0}$$

Hence, $\frac{N_{t+n}}{N_t} = \frac{normalizedIncome_{t+n}}{normalizedIncome_t}$