Course goals:

Real analysis is the axiomatic study of the real number line. The axiomatic method, as opposed to the "physically intuitive" approach taken in high-school algebra and college calculus, allows us to resolve the "paradoxes" that already start to appear in high-school mathematics. For instance,

1) What does it mean to "multiply" numbers when those numbers have infinite decimal expansions, causing the multiplication algorithm to never halt?

2) Are 0.999... and 1 the same or different, and why?

3) How can a shape have infinite perimeter but finite area?

4) In what sense are there different kinds or sizes of infinity?

5) How can we work with infinity as a number? What is 0 x infinity, infinity/infinity, 0 + infinity?

6) How do calculators compute with transcendental functions such as sin x?

7) What's the maximum of all the numbers less than the square root of 2?

8) What is the "factorial" of a fraction like 1/2?

We will work very carefully to introduce definitions that, though delicate, allow us to avoid paradox. We will axiomatize notions like *real number*, *distance*, *closed set, open set, limit,* and *integral.* Eventually these tools will allow us to go beyond our physical intuition to reason about mathematical concepts from calculus without brushing any ideas under the rug.

In order to pursue these concepts rigorously, it is very helpful to already possess the physical intuition behind them, so we require Math 19a/b or 21a/b as prerequisites. It is highly recommended to know how to write proofs or be in an intro to proofs class concurrently (101).

I do best when students are asking a lot of questions in class, so please do that! I am a postdoc working in dynamical systems, a field that uses some real analysis. Ask me about the math of billiards.

Course format:

Conversational lecture. Ask a lot of questions!

Office hours:

Room 530, Tuesday 4 pm - 5 pm, Thursday 11:30 am - 12:30 pm.

If you are coming to office hours to discuss homework, do try the problems on your own first!

Typical enrollees:

Math concentrators, applied math concentrators, and students preparing for graduate work in math or economics. This course is a standard part of the math major at most universities, on par with abstract algebra.

When is course typically offered?

Fall or Spring. Please note that this course is also being offered this spring, with a different instructor.

Grading:

60% homework, assigned weekly, 10% midterm (in-class), 30% final (in-person).

Assignments:

Homework will be assigned after class on Tuesdays and is due on the following Tuesdays at 1 PM.

Collaboration with your peers is encouraged for thinking through problems, but problem set solutions must be written individually. If you collaborated on coming up with solutions, please give the names of the students you worked with on your assignment. No AI allowed for anything related to this course.

Work outside of class should take a typical undergraduate mathematics student 6-8 hours per week to complete. If you have not taken Math 101 or a similar introduction to proof, it is reasonable to anticipate spending considerably more time on the homework, as there is a bit of a learning curve to master the basics of proof.

Exams:

This course has an in person final (2 hours) during finals week and an in-class midterm (75 minutes), see calendar below. The problem set due before the in-class midterm is essentially an untimed, take-home version of the midterm; this is to structure your review.

Textbook:

Walter Rudin, "Principles of Mathematical Analysis" (3rd edition).

As a supplement in the first few weeks, we'll look at Tao's Analysis I, which you can find in online form under "Library Reserves."

Absence and late work policies:

1) Attendance at lecture is important and expected, but will not figure in your grade. There is no need to notify me in advance if you need to miss class.

2) All students are entitled to have their lowest problem set grade dropped. This is intended to cover situations in which an emergency prevents a student from being able to complete an assignment.

3) For more serious situations which might call for multiple assignments to be dropped, students must have their residential dean contact me.

4) No late homework will be graded, including in situations where an email fails to go through, etc. This policy is to reduce the grading burden on the course assistants.

Calendar:

9.3	Paradoxes of the real line	9.5	Natural numbers, integers, rational numbers, field operations, ordering
9.10	Sequences, limits, and convergence; PS1 due	9.12	From the rationals to the reals
9.17	Bolzano-Weierstrass Theorem and the least upper bound property; PS2 due	9.19	Exponentials, e, and roots; the gamma function
9.24	Metric spaces and R^n; PS3 due	9.26	Compactness, convergence, and completeness
10.1	Heine-Borel Theorem; PS4 due	10.3	Sequential compactness
10.8	Countability, Cardinality, Cantor's argument; PS5 due	10.10	The Cantor Set
10.15	**Midterm**; PS6 due	10.17	Series
10.22	Root and ratio tests; PS7 due	10.24	Power Series
10.29	Sum and product series, art of estimation; PS8 due	10.31	Continuity and limits
11.5	Epsilon-delta continuity; PS9 due	11.7	Interplay of continuity, compactness, connectedness; intermediate value theorem
11.12	Differentiation; PS10 due	11.14	Mean value theorem, Taylor series
11.19	Weierstrass Approximation Theorem; PS11 due	11.21	Riemann integral
11.26	Fundamental Theorem of Calculus; PS12	11.28	**no class**

	due		
12.3	Uniform convergence; PS13 due		