Course Description and Goals: Stochastic processes collectively describe various random phenomena arising in nature and our daily life. We will study some important class of probabilistic models and deeply understand their properties. A recurring theme will be the use of stochastic processes for modeling in applied sciences. Computer simulations (in a language of your choice) will also play a major role in understanding the concepts. Use of the R language is encouraged, but not required. We will run R bootcamps early in the semester to introduce the language, in case you are unfamiliar. R syntax (or any other programming language) will not be tested in the course. Mostly, the course is mathematical at core. We will place key emphasis on concrete calculations for building intuition. Thus a solid background in Stat 110 (or equivalent course in undergraduate probability with solid footing on expectations, conditional expectations, familiarity with various distributions etc.) is a prerequisite. If you do not have the necessary background, please come talk to me.

**Lectures:** Monday, Wednesday 1:30PM - 2:45PM (ET), SC Hall E. Live participation in lectures is highly encouraged. I welcome interruptions for clarifications and other questions during lectures.

Prerequisites: STAT 110, MATH 21a, 21b, or equivalent

Instructor: Subhabrata Sen, email: subhabratasen@fas.harvard.edu.
Teaching Fellows: Benjamin Schiffer, email: bschiffer1@g.harvard.edu

Course Assistants:

Max Li, email: maxli@college.harvard.edu;

Rushil Mallarapu, email: rushil\_mallarapu@college.harvard.edu; Matthew Scott Tan, email: matthew.scott.tan@college.harvard.edu;

Eric W Tang, email: etang@college.harvard.edu; Simon Sun, email: simonsun@college.harvard.edu;

Sections:

Monday, 6:00-7:00 PM (ET), SC 706. (Rushil, Eric, Simon) Thursday, 6:00-7:00 PM (ET), SC 706. (Matthew, Max)

Friday, 10:30-11:30 AM (ET), SC 706. (Ben) Friday, 4:30-5:30 PM (ET), SC 706. (Ben)

Course Webpage: https://canvas.harvard.edu/courses/126551

Office Hours:

Subhabrata Tuesday 4:30 - 6:30 PM (ET), or by appointment, SC 713.

Rushil Monday 7:00-9:00 PM (ET), SC 706. Matthew Tuesday 3:00-5:00 PM (ET), Math lounge.

Max Wednesday 9:00-11:00 PM (ET), Winthrop dining hall. Simon Thursday 8:00-10:00 PM (ET), Cabot dining hall.

Ben Friday 12:00-2:00 PM (ET), SC 316.

Eric Saturday 2:00-4:00 PM (ET), Quincy dining hall.

**Textbook:** The **only** required text book is *Introduction to Stochastic processes with R* by Robert P. Dobrow. We will try to follow the chapters of Dobrow pretty closely, with some supplementary material. Students can download it for free from Harvard Library. Additional references include:

Introduction to Stochastic modeling, Fourth Edition by By Mark A. Pinsky and Samuel Karlin.

Markov Chains by J.R. Norris

Introduction to Stochastic Processes, Second Edition by Greg Lawler

*Probability with Martingales* by David Williams

**Grading:** Problem Sets (25%), Midterm ( $2 \times 20\%$ ), Final Exam (35%).

Problem Set Policies: There will be a total of ten problem sets. Solutions will be due on Fridays at 11:59 PM and must be submitted through Gradescope as a single pdf file. Submissions on paper or through email will not be accepted. Solutions may be typeset or handwritten and scanned but must be clear, legible, and correctly oriented. Otherwise, the assignment will receive no credit. If multiple files are submitted, only the final pdf submitted prior to the deadline will be graded. Each problem set will contain a few computational problems in addition to theoretical calculations. Use of the R language is encouraged for these problems, but it is not required. Collaboration with other students is encouraged, but students must write their own solutions in their own words. In addition, students must acknowledge their collaborators on the submitted assignments. The two lowest homework scores will be dropped. In addition, to cover unforeseen circumstances, every student will be given two Saturday extensions—homework due Friday may be submitted by Saturday 5:00 PM (Eastern). You do not need to send an email to avail this extension.

Midterm Exams: Two closed-book, closed-note, in-class midterms will be given on February 26th and April 1st. A single (one-sided) page of notes is allowed for the midterms. *Make-up exams will not be given*. Rather, students with an excused absence will have their score on the final exam used as replacement. *Students without an excused absence will receive no credit*.

Revised: January 29, 2024

**Final Exam:** A closed-book, closed-note final exam will be given on TBA (by Registrar). A single (two-sided) page of notes is allowed for the finals. *Make-up final examinations will not be given outside of extreme situations and at the sole discretion of the instructor.* Please plan travel accordingly.

## **Important Dates:**

**Problem Sets:** February 2, 9, 16, 23; March 8, 22, 29; April 12, 19, 24.

First Midterm: Monday February 26, in class, Second Midterm: Monday April 1, in class,

Final Exam: TBA

## Tentative Schedule:

- Week 1: January 22 26
  - Topics: Motivation, Review of probability, Introduction to random walk; Markov Chain First steps. (Chapters 1-2)
- Week 2: January 29 February 2
  - Topics: Markov chains long time behavior, Reversibility. (Chapter 3)
  - Assignments: **pset #1 due** (Friday 2/2, 11:59 PM)
- $\bullet$  Week 3: February 5 9
  - Topics: Absorbing chains, Branching Processes (Chapter 4)
  - Assignments: **pset #2 due** (Friday 2/9, 11:59 PM)
- Week 4: February 12 16
  - Topics: Markov Chain Algorithms. Metropolis-Hastings, Gibbs Samplers. (Chapter 5)
  - Assignments: **pset #3 due** (Friday 2/16, 11:59 PM)
- Week 5: February 19 23
  - Topics: Markov Chain Algorithms & Review
  - Assignments: **pset #4 due** (Friday 2/23, 11:59 PM)
- Week 6: February 26 March 1
  - Topics: Poisson process (Chapter 6)
  - Assignments: Midterm I (2/26)
- Week 7: March 4 8
  - Topics: Continuous time Markov chains, Birth-death Processes, Queues, Renewal processes (Chapter 7)
  - Assignments: **pset #5 due** (Friday 3/8, 11:59 PM)
- Week 8: March 9 17
  - Spring Break
  - No Assignments
- $\bullet$  Week 9: March 18 22
  - Topics: Long term behavior, Queues, Birth-Death chains, Martingales
  - Assignments: **pset #6 due** (Friday 3/22, 11:59 PM)
- Week 10: March 25 March 29
  - Topics: Brownian motion. Introduction, Computations (Chapter 8)
  - Assignments: **pset** #7 **due** (Friday 3/29, 11:59 PM)
- Week 11: April 1 5
  - Topics: Brownian Motion & Gaussian processes (Chapter 9)
  - Assignments: **Midterm II** (4/1)

- Week 12: April 8- 12
  - Topics: Stochastic calculus (Chapter 9)
  - Assignments: **pset #8 due** (Friday 4/12, 11:59 PM)
- Week 13: April 15 19
  - Topics: Stochastic Calculus, Ito's formula (Chapter 9)
  - Assignments:  $\mathbf{pset}~\#\mathbf{9}~\mathbf{due}~(\mathrm{Friday}~4/19~,~11:59~\mathrm{PM})$
- $\bullet$  Week 14: April 22 26
  - Topics: Review, Open Questions
  - Assignments: **pset #10 due** (Wednesday 4/24, 11:59 PM)
- Final Exam: TBA

REVISED: JANUARY 29, 2024