Math 129: Number Fields

Place: Science Center Room 507
Time: Tues, Thurs, 10:30 am - 11:45 am
Instructor: Barry Mazur, Sci Ctr Room 512 mazur@g.harvard.edu

Final Exam: Tuesday April 30 at 10:30 am in SC 507. Bring a writing utensil.

Office Hours: Wednesdays 2-3 pm. This hour will be devoted to this class (Math 129). You are all invited to come to my office (Room 512) during this time whether or not you have a question to ask. In each office hour session I will begin by discussing the questions that people explicitly ask about, and in the remaining time we can talk about number theory following the topics of interests and the general requests of the students present at the session. **Do come!**

Course Assistants	Emails	Office Hours
Hari Iyer	hiyer@college	Wednesdays 8 pm, Quincy Dining Hall, and by appointment
Eric Shen	ericshen1@college	Mondays 8-10 PM, Leverett Dining Hall (math night)
Eliot Hodges	eliothodges@college	Tuesdays 4:30-5:45 pm, SC309 (SC111 on 2/13)

In addition to Professor Mazur's notes, the CAs are also live-TeXing notes each class. You can find them here. Here are homework expectations, please refer to these as you write your problem sets. Here are notes for the 1/30 review session on Galois theory.

Now that we're in the second half of the semester, student presentations are in full swing! You can find the schedule for presentations here, and here is a list of items that should be completed for your presentation.

Late Homework Policy: Each student is allotted with 10 late days to use at their discretion throughout the semester. Work that is submitted later than two weeks after its deadline will not be accepted.

What we will cover

Math 129 is an advanced undergraduate course in algebraic number theory. We will cover:

1. The fundamentals of number fields. Rings of integers, ideals. Localization and completion. Local fields. Fractional ideals, unique factorization of ideals. Finiteness of the ideal class group, and Dirichletâ€™s theorem giving the structure of unit group.

2. Related Computations and introduction to the use of sites containing databases; specifically: https://www.lmfdb.org/NumberField/ .

3. The fine structure of the Galois group of (Galois) extensions of global and local fields: Frobe- nius elements, tame and wild ramification.

4. An introduction to analytic methods; specifically the Dedekind zeta-function and the class number formula.

5. We will most likely not have time for this, but we might include the briefest introduction to adeles, idelesâ€”and some mention of Class Field Theory.

Texts:

- â€“ **Principal text:** [Marcus] â€œNumber Fieldsâ€ by Daniel A. Marcus (Springerâ€”Second Edition (2018)

- â€“ **Suggested reading:** [Samuel] â€œAlgebraic Theory of Numbersâ€ by Pierre Samuel (Hermann 1970). http://wstein.org/edu/2010/581b/books/samuel-algebraic_ theory_of_numbers.pdf

Some historical sources

- â€“ (Hilbertâ€™s Zahlbericht) **â€œThe Theory of Algebraic Number Fieldsâ€** by David Hilbert (Springer 1998). This hugely influential treatise onâ€”essentiallyâ€”the material we will cover on our course has been a mainstay, introducing generations of mathematicians to the subject, but also has been the target of criticism by Andr Èe Weil (see http: //www.fen.bilkent.edu.tr/~franz/publ/hil.pdf).

- â€“ **â€œLectures on the Theory of Algebraic Numbersâ€** by Eric Hecke (Springer 1981). To mention Andr Èe Weil again: Weil wrote â€œTo improve on Hecke in a treatise along classical lines of the theory of algebraic numbers, would be a futile and impossible task.â€

The Computational side of our subject

- Take a look at

  - â€“ â€œA Course in Computational Algebraic Number Theoryâ€ by Henri Cohen (Springer 1993)

  - â€“ Algebraic Number Theory: A Computational Approachâ€ by William Stein (2012) \\http: //wstein.org/books/ant/ant.pdf

  - â€“ (And itâ€™s instructive just to explore this useful site that offers concrete data for all the relevant features of number fields: LMFDB (Number Fields) http://www.lmfdb.org/ NumberField/)

Prerequisites

Familiarity with the material in Mathematics 123 (which may be taken concurrently) or Math 124 or equivalent.

Format of our Course

There will be a weekly homework assignment, and two (one-and-a-half-hour) exams. Both of these exams are in class. The second one will take place at the end of the semester (instead of an official `final exam'). Also required is participation in the presentation of some specific topic to the class. Such a presentation may be twenty minutes to half-an-hour long and may be given solo, or could be prepared by a team of two or three of students, with one member of the team presenting the prepared material to the rest of the class. Very roughly, homework will count for about 50% of the grade for the course; participation 20%; and the exams 30%.