Random High-Dimensional Optimization: Landscapes and Algorithmic Barriers

STAT 291, Spring 2024

Instructor: Mark Sellke Email: msellke@fas.harvard.edu

Class Time: Tuesday/Thursday 10:30AM-11:45AM Classroom: TBA

Office Hours TBA Office Location: Science Center 711

Teaching Fellow: Yufan Li Email: yufan_li@g.harvard.edu

COURSE DESCRIPTION

This course will focus on paradigmatic optimization problems with random objective functions. We will develop the tools needed to understand the geometric behavior of complex random landscapes, the different phase transitions that can occur, and how these transitions are linked to the success and failure of efficient algorithms. Possible topics will include:

- Random constraint satisfaction problems, spin glasses, and tensor PCA
- Landscape complexity via critical point counting
- The overlap gap property as a geometric barrier to optimization
- Implications for Markov chain sampling
- Combining these ideas to prove the spherical Parisi formula

PREREQUISITES

Students should be comfortable with probability at the level of STAT 210/212. Familiarity with high-dimensional probability/statistics and concentration of measure will be beneficial. If you are uncertain about your background, feel free to email the instructor.

COURSE PAGE

The course page is here: https://canvas.harvard.edu/courses/126859

HOMEWORK

There will be about 4 problem sets designed to reinforce the methods from class, and to highlight ideas that do not fit into lectures. Students will have the option to choose which problems to complete. One late day will be allowed during the semester; send an email to the course staff when using your late day.

Working together is permitted and encouraged, but students are expected to identify any collaborators and references consulted (including AI language models), and to write solutions in their own words.

LECTURE NOTES

Students will sign up to scribe lectures, and these notes will be posted on the course website after editing. A Latex template will be posted on the course website.

GRADING POLICY

Homework assignments (60%), Project (35%), Scribing (5%). There will be no exams.

SECTIONS

Yufan will lead a weekly section. The timing for the sections will be decided during the first week of class.

REFERENCES

There is no single main reference for this course. Relevant materials will be posted throughout the term.

PROJECT

Students will form groups of size 1-3 and complete a project over the course of the semester, consisting of a report (about 10 pages) and an end of term presentation. Groups may aim to read and understand one or multiple modern research papers and write a summary. Alternatively, they may attempt to make progress on a research problem. A list of suggested topics will be posted early in the semester, and students may also propose their own topic.

IMPORTANT DATES (SUBJECT TO CHANGE)

Deadline to choose a project topic Feb 29 Project progress report April 2

Final Report TBA (near end of term)
Final Presentation TBA (near end of term)