

# Math 21b: Linear Algebra and Differential Equations

## Spring 2024 Course Information and Syllabus

Course head	Matt Demers	SC 420	mdemers@math.harvard.edu
Course preceptor	Erica Dinkins	SC 223	edinkins@math.harvard.edu
Course preceptor	Jill Guerra	SC 423	guerra@math.harvard.edu

Welcome to Math 21b! Linear algebra is one of the most widely used areas of mathematics, with applications in fields as varied as computer science, economics, engineering, epidemiology, ecology, physics, psychology, archeology, and statistics. It is also an elegant subject in its own right, known for being self-contained and logically cohesive.

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### General Information

In linear algebra, we study linear transformations, an important group of functions that have many applications. We will start from the concrete perspective of solving linear systems, but we will quickly see that we can also adopt a more geometric perspective. Linear transformations enable us to deal with higher-dimensional phenomena as well as vast amounts of data.

As we progress through the course, the level of abstraction will increase. This is key, because it allows us to generalize our knowledge to broader contexts. In the last third of the semester, we will see how the linear algebra we have learned can be applied to analyze and solve differential equations, which are used to model processes throughout the natural and social sciences.

### Inclusion

Mathematics is the result of people from diverse backgrounds, interests, and opinions sharing their insights and perspective, learning from each other and offering an alternative point of view. We believe in forming a learning environment where your identities are respected and valued, and where you can flourish.

### Prerequisites

Before beginning Math 21b, you are expected to have had the equivalent of a second semester course in calculus such as Math 1b. It's helpful, but not necessary, to have taken a course in multivariable calculus (e.g., Math 21a or Math 18a).

### Course Goals

As in any college course, we aim for you to gain much more from Math 21b than simply factual knowledge. By the end of this course, we intend for you to:

- Have a mathematical foundation in linear algebra and differential equations that will serve you well in your future studies, whether they are in mathematics, the sciences, engineering, or the social sciences.
- Develop your abstract reasoning skills.
- Improve your verbal and written communication of mathematical ideas.

### Course Materials

**Textbook and Handouts:** The required textbook for Math 21b is *Linear Algebra with Applications* by Otto Bretscher, published by Prentice Hall. You may use either the 4th edition, ISBN 978-0-13-600926-9, or

the 5th edition, ISBN 978-0-321-79697-4. Copies of the textbook are available through the COOP (<https://harvardcoopbooks.bncollege.com/shop/harvard-coop-books/home>). We will provide supplemental handouts that better address specific topics or are otherwise not covered in the textbook.

Math 21b is a fast-paced course which covers a lot of material. We will usually cover one section of the text per class; your TF will highlight the key concepts in class, but there will not be enough time to cover all the details. You will be expected to study the text to fill in the details; we recommend skimming each section before it is covered in class and then reading it again after it has been covered.

**Worksheets and Problem Sets:** All worksheets and problem sets will be available for download through the course Canvas page. Some homework problems refer to specific problems in the textbook and these problems are not reproduced in the assignment.

A calculator will not be necessary for this course. Throughout the semester, you will be asked to use the software package *Mathematica* but you are not expected nor required to use a calculator. A license and download are freely available to all Harvard students on the Harvard University Information Technology website (<https://downloads.fas.harvard.edu/download>) after logging in with your Harvard Key, it is also installed in the computer labs in the Science Center. You are not expected to be familiar with *Mathematica* at the outset of the course.

### Course Topics

Gauss–Jordan elimination Solutions of linear systems Linear transformations Matrix algebra Determinants Image and kernel of a linear transformation Subspaces of $\mathbb{R}^n$ Bases and dimension	The Rank-Nullity Theorem Coordinates Orthogonal projections The Gram–Schmidt Process Least squares Eigenvalues and eigenvectors Diagonalization The Spectral Theorem	Discrete and continuous dynamical systems Abstract linear spaces and transformations Linear differential equations Inner product spaces Fourier series Partial differential equations
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### Quantitative Reasoning with Data

One reason linear algebra is so widely used is that it is a powerful tool for making sense of data. Many of the problems you will encounter during Math 21b will use actual numerical data, so that you can get practical experience with the messiness that inevitably arises when applying mathematics to real data. Because of this, Math 21b fulfills the QRD general education requirement.

## Course Structure

Math 21b is taught in small synchronous sections to ensure that you have ample opportunity to ask questions, receive immediate feedback, and to interact with your teaching fellow (TF), course assistant (CA), and fellow students. Take advantage of this by being an active participant!

Each section of Math 21b has a course assistant who assists students during section, grades homework assignments, staffs [MQC](#), and facilitates weekly [quiz sessions](#). You are strongly encouraged to attend these quiz sessions. A schedule of all quiz sessions will be posted on the course website, and you are welcome to go to any quiz session that fits your schedule.

Sections meet three (3) times a week for 75 minutes. **Section attendance is a mandatory part of the course.** A worksheet accompanies each section meeting and will provide a collection of problems that address the material for that day. These **worksheets are intentionally designed so that not all of the problems will be covered in section.** The additional problems serve as practice problems for you. Since a significant amount of class time will involve time spent working collaboratively on problems, we expect you to be ready to actively participate in section.

## Homework Assignments

Homework is an essential part of the course; the only way to learn math is by doing math. In general, **homework problems will look different from problems discussed in class.** This is intentional; the only way to develop a deep and flexible understanding of the material is to constantly practice applying it in different contexts. As you work through the homework, think carefully about the problems posed, the strategies you've used, and the meaning of the computations you've performed.

There will be three problem sets per week. These will consist of different types of problems.

- Problems that deal with the material covered in class that day and are due the following class. These problems aim to give you a mastery of the material so that you will be ready to apply it in the next class and in various contexts and applications.
- Exploratory problems introduce you to ideas or issues we will study in later classes. According to *Make It Stick*, a book on learning research, "Trying to solve a problem before being taught the solution leads to better learning, even when errors are made in the attempt." Because these problems ask you to think deeply about ideas we have not yet covered in class, we do not expect you to do them perfectly; therefore, you will earn full credit for these problems as long as you show evidence of real thought and effort. Exploratory problems are very important in laying the groundwork to help you understand upcoming material.

**Problem sets are assigned for every class.** This means that, between section meetings, you will always have a problem set to complete.

## Homework Policies

When writing up your homework, **we expect you to explain your thought process**; your final answer is less important than the reasoning you used to reach it. As a rule of thumb, if you're not using any words when writing up the problems, you probably aren't giving enough explanation. When writing out your answers, it may help to imagine that you are writing to a fellow student to explain how to solve the problem; this will help you include the right level of detail.

- **All problem sets are due by 8:30am Eastern time** on the day specified on Canvas.
- You must **submit your assignment via Gradescope**, a grading platform accessible through the course Canvas page.

- Problem sets will be graded by your CA under the guidance of the course staff. An assignment submitted one class day should be returned to you by the next.
- **You are responsible for ensuring that your assignment is professionally presented** and is in accordance with the course [Collaboration Policy](#). In particular, you should check that each submission is well-organized, clearly visible, complete, and represents your understanding.
- Sample solutions will be made available on Canvas. You may find that your solution is substantially different from that presented and this reflects how flexible linear algebra can be – you should not expect there to be a single path from start to finish and your creativity will benefit you substantially.

To be fair to the other students and the graders, late homework will **not** be accepted.

We realize that circumstances may prevent you from giving your full attention to homework, so **we will drop your lowest two problem set scores when computing your final homework grade**. You will have the opportunity to earn more problem set drops by taking weekly quizzes in quiz sessions.

### Quiz Sessions

A large body of research shows that frequent testing helps people learn much more effectively than other methods of studying. Thus, most weeks, CAs from the course will lead quiz sessions which include:

- a short quiz testing your understanding of recent material, and
- a subsequent discussion of the problems.

You can earn additional problem set drops by taking these quizzes and participating in the follow-up discussion: **for every 3 quiz sessions you complete, you will earn one additional problem set drop**. In general, all quiz sessions in a given week will give the same quiz, so you may attend a session that is convenient for you.

### Exams

Preparing for and subsequently taking a test on material strengthens your ability to use that material; therefore, exams are a crucial part of the learning process. **Tentative** exam dates are:

Mini-Assessment	Thursday Feb 8	6:30 - 7:45 pm
Exam 1	Thursday, Feb 29	6:30 - 8:30 pm
Exam 2	Thursday, Apr 11	6:30 - 8:30 pm
Final Exam	TBD by registrar	

The most up-to-date information about exam timing and location can be found on the Canvas page and we will communicate any changes to the entire course.

**We do not offer late make-up exams.** Any sitting of an exam that takes place outside of the official time must be scheduled in advance. If an exam overlaps with extreme hardship and you miss an exam, we reserve the ability to modify how your course grade is calculated. What this means and what we expect from you will be determined on a case-by-case basis. Please contact the course head to discuss this option.

Out of sequence exams are only offered for the following reasons:

- Medical: please have your resident dean contact the course preceptor.
- A university-sponsored event such as an athletic tournament, play, or musical performance (but *not* athletic practices and rehearsals)

- Religious holiday: please contact the course preceptor.
- Extreme hardship (e.g., family emergency): please have your resident dean contact the course preceptor.

### **Class Participation**

Attending section is a great opportunity to struggle with problems, come up with new ideas, make mistakes, and help each other work through confusion. To get the most out of these meetings, it's essential to be an active participant. Here's what that means:

- Actively contribute to discussions and problem solving opportunities, both with the entire class and in small groups. Share your successes, mistakes, and confusions. Research shows that we learn more from mistakes than from seeing correct solutions, so sharing what you tried that didn't work will help everybody learn more!
- Intentionally make space for others to contribute. Actively listen to others' ideas, and try to understand and build off of them.
- When working in a small group, actively check in with your group members to make sure that everyone in the group understands the material.

Because class participation is important for your success in the course, it is part of your course grade (see below).

### **Course Grade**

The grading policy is designed so that you can represent your mastery of the material in a favorable light. Your course grade will be determined as follows:

- Midterm score: Take the higher of
  - 40% first exam + 60% second exam
  - 60% first exam + 40% second exam
- Course score: Take the higher of
  - 40% final exam + 30% midterm score + 20% homework score + 5% mini-assessment + 5% class participation
  - 30% final exam + 40% midterm score + 20% homework score + 5% mini-assessment + 5% class participation

### **Support**

In Math 21b, we offer a variety of opportunities for you to engage with members of the course staff and course to discuss course content. We encourage you to test each of them out to determine what suits your needs best.

### **Collaboration Policy**

You are welcome to collaborate with other students on solving homework problems; in fact, we encourage you to do so. Working with others provides one of the best opportunities for learning material *so long as you engage earnestly and express your concerns and ideas*. That being said:

- You must understand and write up your homework by yourself; writing up a solution gives you a chance to probe your own understanding of the strategies used. It is always unacceptable to copy a solution from any source.

- You must acknowledge help you received from any source other than the course TFs and CAs. This will not impact your homework score in any way. However, attributing appropriate credit for ideas is an important skill to develop further.
- You may not share course materials and documents with anyone not enrolled in the course (excluding peer tutors).

## Getting Help

Learning to use linear algebra is challenging and takes time; you should not expect to solve every problem immediately. Try a couple of different approaches on your own first; often you learn the most from things you try that don't work. But after that, don't hesitate to ask for help! We recognize that linear algebra is very different from calculus and there are many resources available to help you make the transition smoothly:

- Your fellow students are one of your best resources. You are encouraged to form study groups with your classmates, whether from section or people you meet through Slack or MQC, and to work together on problems (see the [Collaboration Policy](#) above).
- TF office hours. Office hours are times when you can drop in and ask a TF some questions about the class, work on your homework, or even sit and read the textbook. You're encouraged to go to any TF's office hours, and you don't need to make an appointment; just show up! Office hours are also a great place to meet other students. A schedule will be posted on the Canvas site.
- CA quiz sessions. You may go to any CA's quiz session. A schedule will be posted on the course website.
- Math Question Center (MQC). In addition, CAs will be there to facilitate your discussions. The MQC is a great place to meet other students. Many previous 21b students have used this time to complete their homework with other students in the course. A schedule will be posted on the course Canvas page.
- The **Academic Resource Center (ARC)** (<https://academicresourcecenter.harvard.edu/>) offers peer tutoring, workshops on skills such as time management, and academic coaching.

## Accommodations

We believe it is essential that everyone is able to participate fully in this course. If you have any accommodations or assistance provided through the **Disability Access Office (DAO)** (<https://aeo.fas.harvard.edu/>), please contact the course head as soon as you can. To ensure our ability to support you and make any necessary arrangements, it is best to let us know well before the first exam.

**We do not offer late make-up exams.** Any sitting of an exam that takes place outside of the official time must be scheduled in advance. If an exam overlaps with extreme hardship and you miss an exam, we reserve the ability to modify how your course grade is calculated. What this means and what we expect from you will be determined on a case-by-case basis. Please contact the course head to discuss this option.

*\*\*Any modifications to this syllabus will be announced to the course via Canvas.\*\**