Math 1b: Calculus, Series and Differential Equations

Syllabus Spring 2021

https://canvas.harvard.edu/courses/79086

Course Information and Syllabus:

Welcome to Math 1b! Our goal is to help you gain a solid, deep, and portable understanding of single variable calculus as well as as sampling of its applications in other fields. In general terms, calculus is the study of change, how to measure, model, and predict changes in quantities that we observe in our universe. Change is ubiquitous: we really know that in a very visceral way at this particular moment! Calculus has become an essential tool not only in physics and astronomy, in mathematics, statistics, chemistry, biology and engineering science, but also in economics and other social sciences. More generally, the habits of mind that you practice in problem solving and the central tools that pervade the course are useful in interpreting the world. Calculus allows us to quantify change and thus helps us interpolate and extrapolate. The ideas within calculus have helped to create and shape modern human civilization as we know it.

For many of the problems that you'll encounter inside and outside of the course it will be difficult, or even impossible, to model a real-life situation mathematically without surpressing some amount of complexity. Even in a completely mathematical problem it may sometimes be difficult or impossible to find an exact solution. In these cases, we focus on finding approximate solutions, improving our approximations, and, in mathematical problems, transitioning from approximate solutions to exact solutions, if possible. Every approximation comes with an associated error; we will try to estimate or bound the errors in our approximations or figure out how much work we need to do in order to keep our error within the bounds we require.

The Teaching Team:

Section Leaders:

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Contents

Mathematical Content and Course Goals

We will be putting together a problem-solving toolkit that will enable us to to tackle a wide array of questions. Here is a sampling:

- How can we model the evolution of a population? Interactions between populations? (predator/prey, symbiotic and competitive)
  Why, during WWI, when fishing in the Adriatic Sea was reduced, did the percentage of edible fish in the fishermen's nets decrease?
  Why is overfishing a problem and how can we determine the level at which fishing becomes 'overfishing'?
- How can we model the spread of a disease? How can we adapt our model to reflect the characteristics of the epidemic we are modeling?
  What is the mathematics behind the expression "flattening the curve"?
- How can we quantify the term "middle income"? How can we measure the variability in an income distribution?
- The speed of blood through a blood vessel varies with the distance from the vessel walls. How can we calculate the flow of blood through a vessel per unit time?
- Why is the normal distribution important?

We will be employing three basic mathematical tools over the course of the semester: integration; infinite series; and differential equations. While using these tools, students will grapple with the major themes of Math 1b

- modeling
- analysis of error
- the strategy of successive approximation

Integration

In your previous math courses you studied differential calculus and were introduced to integral calculus. You studied the Fundamental Theorem of Calculus which illuminates the connection between differentiation and integration. You should already be familiar with the definite integral, its definition as the limit of Riemann sums, and its calculation using antiderivatives and perhaps simple u-substitution. (If you need to brush up on this, you can review using a calculus text of your choice or visit our website and look at the "Background" page.) In Math 1b our emphasis will be the use the definition of the integral as a limit of a Riemann sum to more broadly apply integration to different situations.

In particular, you will learn a powerful technique of slicing large, complicated problems into smaller, simpler ones. The definite integral enables us to tackle a multitude of problems in a wide array of fields; we will use the strategy of slicing, approximating, and summing in various contexts. More important than any one particular application is the ability to apply integration as appropriate in problem solving. You will learn to figure out the flow of blood through a blood vessel, the length of a curve, and how one might measure middle income.

In order to compute integrals you will study some techniques of integration, such as the integration analogues of both the Product Rule and Chain Rule for differentiation. Techniques of integration will be handled in a "flipped classroom" manner. We have a page on the course website with an online tutorial so you can get started on this as soon as you like, even before you have your section assignment! There is a page of videos made by our colleague Jameel Al-Aidroos especially for you, along with worksheets and solutions to those work sheets and this page is available for you to start work on as soon as you decide to take this course. There will be some class time devoted to mastering these techniques, but we do expect you to do some initial study before class. Despite the fact that you'll become more skilled at antidifferentiation, sometimes you'll arrive at an integral that is difficult or impossible to compute. In these instances, we will rely on numerical integration to approximate a value we seek. Sometimes we won't be dealing with an explicit function at all, but simply a set of data points capturing some information about a function. We will study a variety of methods for approximating the numerical value of an integral using data points. We'll investigate when and how we can estimate the accuracy of these approximation methods, and which aspects of our data and what we do with it contribute to error in our approximations.

Infinite Series

One of the primary ways in which social scientists, scientists and applied mathematicians use calculus is to approximate a complicated function near a certain input value by using a linear approximation: the tangent line approximation. The idea behind this is that a differentiable function is locally linear: from a bug's eye perspective, a differentiable function looks like a straight line. In other words, If we zoom in enough at the point of tangency, the graph of the function and the tangent line look very similar! The derivative allows us to figure out the equation of the tangent line, and locally we might simplify things by approximating the more complicated function by its tangent line. (You might have seen this in reports of CoVid 19 cases. For instance, the New York Times reported rates of infection complete with tangent line approximations drawn in to help with local extrapolation.)

Our study of series will begin with tangent line approximations. Away from the point of tangency the linear approximation may not be reasonable. We begin this unit by extending the notion of linear approximation to higher degree polynomial approximations of a function about a point. We will approximate familiar functions like e^x, e^{-x^2}, $\sin x$, and $\cos(x^2)$ by polynomials. In many cases, increasing the degree of the polynomial gives gives better and better approximations. The functions e^x, e^{-x^2}, $\sin x$, and $\cos(x^2)$ are challenging to evaluate and the latter two are challenging to integrate. We will find that some functions, such as e^x and $\sin x$ have exact representations as infinite polynomials known as power series. We'll see how useful this 'infinite polynomial' representation can be and why you might want to work with the power series (infinite polynomial) representation.

In order to obtain these alternative representations of familiar functions, we will first study infinite sums (called infinite series). Without necessarily realizing it, you already are familiar with some infinite series. For instance, you know that a rational number such as $\frac{1}{3}$ can be approximated by a decimal, such as 0.33 or, more closely, by 0.3333 and can be represented as a repeating decimal. The repeating decimal, $0.33333\ldots$ is another way of writing the infinite sum $\frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \frac{3}{10000} + \cdots$, so this infinite series is a way of representing $\frac{1}{3}$. We will need to take a closer look at infinite sums (series) to determine when and how it makes sense to talk about the 'sum' of an infinite number of terms.

While investigating infinite series, we loop back around to considering infinite polynomials, known as power series.

Just as $\frac{3}{10} + \frac{3}{100} + \frac{3}{1000} + \frac{3}{10000} + \cdots$ is an alternative representation of $1/3$, we will find that $1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \cdots$ is an alternative representation of e^x.

In fact, these power series representations are philosophically very intriguing. We can see an interplay between local and global information about a function. This tends to be the most abstract part of the course;

however, polynomial approximations based on power series representations are widely used by engineers, physicists, and many other scientists and the type of logical reasoning from the unit can be applied to a varied range of problems.

Differential Equations

We will end the course with differential equations, equations modeling rates of change. A differential equation is an equation involving one or more derivatives. Differential equations permeate quantitative analysis throughout the sciences (in physics, chemistry, biology, epidemiology environmental science, astronomy) and social sciences (economics, psychology, linguistics) because they give us the tools to model rates of change. Our focus will be to understand how one might model a scenario with a differential equation and how to use the differential equation to predict how that scenario will evolve over time. For example, we will focus on changes in populations (modeling population growth and analyzing interactions between different populations), and will give you the tools, for example, to answer the question "Viral infections and rumors are two things that can spread through a college population; in what ways are those *spreads* similar or different?" Differential equations give us a way of very succinctly and beautifully encapsulating a great deal of information, which makes them one of the most useful modeling tools. By the end of the course you will appreciate just how powerful they are, and you will see how the work we have done earlier in the course comes into play in analyzing their solutions.

Course Goals

By the end of this course we would like you to

- Appreciate mathematics as a practical sense-making process for posing and solving problems, as opposed to a box of algorithms or a list of rules and formulas
- Be able to apply the concepts, logical tools, and problem solving strategies from this course to problems that may be encountered in the future.
- Become more skillful in making a mathematical argument and communicating it in a clear and logical way. Become skilled in evaluating the validity of an argument.
- Acquire a new perspective on familiar functions and mathematical objects and to appreciate the utility of this new perspective.
- Critically analyze and model real data in order to address important questions about the world in which we live.
- Build a mathematical foundation in calculus that will serve you well in future studies, whether it be in mathematics, the sciences or the social sciences.
- Work collaboratively with peers to discover mathematical ideas and improve your understanding of them.

The Role of Data in this Course

The original motivation for the discovery of calculus in the 17th century was to explain observed physical phenomena in the universe. Since then, many theoretical and philosophical advancements have been made within the discipline in order to make calculus an area of abstract mathematical study in its own right. However, calculus has endured for hundreds of years as a crucial area of study primarily because of its ability to extract practical information about our world. In the 21st century, the ability to analyze data in a meaningful way and with the appropriate skepticism is important. One of the goals of Math 1b is to use calculus techniques to analyze and model real-world data, and conversely, to use real-world data to inform our calculus methods and strategies. To that end, many of the problems that you will encounter throughout this course will involve actual numerical data . As such, some of these data will be "messy" and impractical to work with by hand calculations, so it will be beneficial to have a calculator or computer ready for such problems. We will ask you to not only perform calculations with the data, but also to critically analyze the data itself. For example, you will be expected to understand and question the sources of the data, consider the possible sources of error within the data, decide which assumptions you can and cannot reasonably make about the data, and which conclusions you can and cannot reasonably draw from such data. Thus, by completing Math 1b, you will satisfy the Quantitative Reasoning with Data (QRD) general education requirement.

Class and Problem Sessions

Math 1b is taught in small classes (sections) rather than in a large lecture in order to promote active engagement with the material and to give you lots of opportunities to ask and answer questions and interact with your peers, your undergraduate Course Assistant (CA) and your section leader. Take advantage of this by being an active participant, and by engaging with the material and the people involved in the course in class and outside of class. Sections meet three times per week on Monday, Wednesday and Friday for 75 minutes. If your timezone precludes coming to class we will try to set up "watching groups."

Before each class you will be asked to watch a short video and/or answer a few short questions in order to allow us to devote a large portion of class time to group problem-solving. During class we will develop concepts and strategies, many of which will be the product of your small group work. The idea is that we want you to spend a large portion of class time actively doing math in small groups. There is research that backs up this methodology: a meta-analysis of 225 studies found that "Active learning increases student performance in science, engineering, and mathematics", and another meta-analysis of undergraduate STEM courses found that "students who learn in small groups generally demonstrate greater academic achievement . . . than their more traditionally taught counterparts."

To facilitate communication of mathematics and work on a jamboard, we will require that students have a tablet and stylus. Students who do not have this equipment can get loaners from the University.

Participation in class is critical. To emphasize this, participation will be part of the course grade, counting for 5% of your final grade. Here is what it means to be a good participant:

- Contribute to the discussion, both in whole class discussions and small group discussions. To contribute means not only to contribute your ideas but to actively listen to others and to make space for them to contribute. We if you can work constructively together.
- Share your successes and confusions. We learn a lot from making mistakes and seeing what goes wrong, so sharing things that didn't work can be helpful. Making mistakes is part of learning.
- Check in with other people in your group to make sure they are understanding. If someone is struggling, remember that teaching is the best way to solidify your own understanding.

On the homepage of our course website there is a link to instructions to get into a section. You must fill out the Qualtrics form no later than January 21st! It is available starting on January 15th.

Homework

Daily Homework

Problem solving and problem sets are an essential part of the course; it is virtually impossible to learn the material and to do well in the course without working through the homework problems in a thoughtful manner. The only way to learn math is to do math. Don't just crank through computations and write down answers; *think* about the problems posed, the strategy you employ, the meaning of the computations you perform, and the answers you get. *It is often in this reflection that the greatest learning takes place.*

Your homework assignments in Math 1b will probably look different from the math homework you have had in the past. For starters, you will find that many of the problems on the homework look different from the problems you solved in class. This is intentional: as noted before, your goal after taking Math 1b is to be able to apply the concepts from class in many different scenarios, and to appreciate that mathematics is not a box of algorithms for solving many copies of the same problem, but rather a way of approaching and thinking about new problems that you may come across in your future. You can't build this ability and appreciation by solving the same problem over and over. When you are solving a problem, it may be helpful to ask yourself, "What other problems have we solved that are similar?" More often, however, it is more useful to ask "What are the general ideas, problem solving strategies, and concepts that we have learned that might be useful here?"

Math is not just a list of rules to be followed in order to solve certain types of problems, and one of the major goals of college-level mathematics education is to move students from computational processes to conceptual thinking. Your instructor will *prepare* you to do the homework but may not necessarily *show you how* to do your homework. The learning occurs when you can move yourself into the unknown territory. Giving you problems different from those done in class is consistent with our goal of teaching you the art of applying ideas of integration and differentiation to different contexts. See the Course Resources section of this syllabus to learn about the wide and varied support mechanism in place to help you succeed.

After class you will have homework which will generally be a combination of some the following:

- Problems that you will write up and turn in on Gradescope online.

  These will be graded by a course assistant. These will sometimes push you beyond the techniques that you learned in class. These problems will also be important in terms of your write-up and presentation. We expect you to explain your thought process; the reasoning you used to arrive at your final answer is of prime importance. As a rule of thumb, if you find that you're not using many English words when writing up your solution, you are probably not giving enough explanation. When writing your solutions, it may help to imagine that you are writing a concise note to a fellow student to explain how to solve the problem. If you are ever in doubt about whether you are giving sufficient explanations, please consult with a section leader or a CA.

- A short video for you to watch and questions about the pre-class video or pre-class problems.

  These are meant to confirm that you've engaged with the video and to prepare yourself to be a productive participant in class. These questions may include exploratory problems. Exploratory problems introduce you to ideas or issues that we will study more in future classes. If a problem is exploratory, we won't expect you to be able to answer it perfectly; therefore you'll get full credit on exploratory problems as long as we can tell from your solutions that you have given them real effort and consideration. Both preview and exploratory problems are very important in laying the groundwork for understanding the upcoming class material, and we'll often count on you to come to class prepared to discuss your ideas and solutions to these problems as we introduce the material.

- A part or all of a canvas module.

  Several times throughout the term there will be a module on canvas that you will work your way through. Most of the time these modules will be focused on applying the techniques we learn in class to real world problems involving data. These pieces of the course were introduced in order to satisfy the QRD requirement.

If all goes as planned, our classes will be capped at around 32 students. Each class will be split in two, with each group of 16 forming a unit with a dedicated course assistant. Group work will consist of permutations of these 16 students so that you have a small community within our class.

Homework Policy/ Flexibility Policy

You will typically have three homework assignments each week, and these will be posted on the course web page. Your CAs will, whenever possible, return your graded homework to you the following class via Gradescope,, and we expect that you will look over his/her comments and compare your solutions with the solutions on the course Canvas site. The posted solutions are carefully written out and are a good model for explaining your reasoning completely. You must turn your written homework online at the beginning of the day on MWF. We realize that sometimes things may come up that prevent you from giving your full attention to your homework, and we will accommodate you by dropping your three lowest homework scores. In addition, three problem sets can be turn up to 24 hours later without alerting course staff. (Please realize that you are fully responsible for the material on these dropped problem sets, so you should not simply 'skip' them!) We are operating during a pandemic; we understand that and will work with you should you get ill or become a primary caretaker. Please alert your TF and cc- the course head so your situation can be accommodated.

Collaboration Policy

You are encouraged to collaborate with other Math 1b students on solving homework problems. Talking about your ideas and solutions is a great way to improve your understanding and doing mathematics is really about communicating ideas and questions. However, **all work you submit must be written up individually in your own words, and you shouldn't ever submit work that you wouldn't be comfortable explaining clearly to another student or to a TF.** At the end of each assignment, please acknowledge any help that you received from others by writing the names of your collaborators; doing so will not affect your homework score in any way. Of course, you should not under any circumstances turn in work that you have copied from and answer sheet, the internet, from a solution manual, from another student or from any other source. Please see the Harvard College Student Handbook for Students for University policy on academic dishonesty.

Assignments, as well as solutions, will be available on the course website. When your homework assignments are returned to you, consult the solutions for help with any mistakes you might have made. The solutions have been rather carefully put together. Check them out; you may find them helpful whether or not you got a problem correct. Questions you have on the homework can be tackled in office hours and at the Math

Question Center, where there will be fellow students and during certain hours, course staff, available.

Course Resources and Getting Assistance

We want you to feel supported in your work in this course. It is entirely natural and to be expected that problem solving is sometimes challenging. You are not expected to be able to whiz through all the problems you are given. Be patient with yourself. Give yourself enough time to play with the problems before you have worked out a solution and enough time to reflect upon them afterwards. Your classmates are often excellent and underused resources. Other help is also available without any appointment:

- Math Question Center/Office Hours: Every TF and CA will hold "office hours" each week at various times in a Zoom room we're calling the Math Question Center (MQC). You are welcome to attend anyone's MQC hours (not just the ones held by your TF or CA), and you are strongly encouraged to do so! No appointment is necessary, and you can come and go at any point. In fact you are welcome to hang out in the MQC Zoom room even when no hours are posted if you're looking for other students to work with. The full schedule of MQC hours will be on the course website. (They may change slightly from week to week, especially in the beginning, so make sure you're looking at the most recent hours which will be posted on the Math Question Center page in canvas.)
- Academic Resource Center (ARC): The ARC is an excellent resource outside of the math department that offers peer tutoring, workshops on study skills and test-taking skills, and many other services. For more information, visit their website:
https://academicresourcecenter.harvard.edu
- Problem sessions lead by some of our course assistants: go to as many as you like: times will be posted on the course website.
- Your fellow students: There is a great wealth of knowledge and experience among your peers! We strongly encourage you to form study groups for discussing homework problems and preparing for exams. Discussion often helps solidify the ideas as well as increasing fluency in the language of mathematics. The best way to really learn something well is to teach it to someone else.

Finding the right time to ask for help is a delicate balance; you need to give yourself enough time to work on, get stuck on, and finally persevere in your own problem solving. This is only possible if you give yourself ample time to fully immerse in the problems in the course. Try a few different approaches first before you ask for help from others; often you will learn the most about a new topic from the things that you try that don't work! Our advice is to start the homework the day it is assigned, then, once you see where you're having trouble, talk with others. Then write it up on your own.

Textbooks, Calculators, and Computers

Text: We will have many notes, problems, and solutions posted on the Notes, Worksheets, and Problem Sets page of the course website. Check them out. Many students find one of these adequate for framing/review. the There are two different styles. Read whichever speaks to you. (Personally, I'd use Nathan Pflueger's notes for framing and the anonymous notes for a quick review afterwards.) We will reference reading in *Calculus: an Integrated Approach to Functions and Their Rates of Change* by Robin Gottlieb. The book is being distributed in pdf on the website to students officially in the course free of charge. Please do not redistribute. If this is helpful as a resource you are welcome to use it. (The book will be available in print for $25 on Amazon if you want a hard copy.) You are also free to use whatever calculus book you find helps you most. You are welcome to refer to other outside sources to supplement your understanding, such as other calculus textbooks (e.g., Stewart's Calculus), Khan Academy, etc.

Feel free to use a calculator or computer to check or investigate problems for homework. We have a link to a very powerful calculator, Wolfram Alpha, on our website. However, you should not rely on computers and calculators to the extent that you lose fluency with the material and do not develop your own computational skills. Some students like to use a calculator when working on their homework, but no calculators will be allowed in exams, so it is not advisable to use your calculator as a crutch for remembering plots of simple functions or simple derivatives and integrals. We will make sure that problems on the exams require minimal calculation to allow you to spend your time demonstrating your mathematical knowledge as opposed to your calculating ability.

Exams

Midterms will be timed 2.5 hour exams given over a period of at least 24 hours and will be released and submitted on gradescope. There will be two midterms, one on the integration unit on **Tuesday, March 2nd** and one on the series unit on **Tuesday April 6th**. The final exam will be 50% differential equations, 25% integration and 25% series. The date of the final exam is to be determined.

Grading Policy

- Homework: 35% (written homework, canvas modules, preclass problems, integration techniques)
- Midterms: 35% (Maximum of (60%(Exam 1) + 40%(Exam 2)) and (40%(Exam 1) + 60%(Exam 2)))
- Participation: 5%
- Final: 25%

Students need to get at least 50% on the final to get a satisfactory grade in the course.

Educational Accommodations

If you have a documented need requiring educational accommodation or assistance, you should get in touch with the Accessible Education Office (AEO) as soon as possible. Once you do, please contact the course head with a letter from the AEO detailing the accommodations you need, so that we can make the necessary arrangements for you. For more information, visit the AEO website: https://aeo.fas.harvard.edu. There is no 'curve' in Math 1b, nor are there any grade quotas: if you all do wonderful jobs, we are happy to give you all wonderful grades! Let me close all this talk of grades with the following advice: Do not be entirely focused on your grade. Find something you like about calculus, and focus on that. Try to learn calculus for its own sake; it is a beautiful subject.

Welcome again to Math 1b. Enjoy the course! We're all looking forward to working with you.