Content: This course provides a rigorous introduction to abstract algebra, including group theory and linear algebra. The formal prerequisites for Math 55 are minimal, but this class does require a commitment to a demanding course, strong interest in mathematics, and some familiarity with proofs and abstract reasoning.

Important: Due to the course's place in the first-year undergraduate mathematics experience at Harvard and its role in helping our future math concentrators get to know each other and form a community, Math 55 is only open to first-year Harvard College students.

Lectures: will be held **Mondays, Wednesdays, Fridays from 10:30 to 11:45**, starting Wednesday September 4, in **Science Center 507** (to be confirmed)**.**

What can I do ahead of the semester to be ready for the class?

- Read the detailed course information below!
- Brush up on your math skills. Review basics about sets, functions, etc. (e.g. the beginning of Halmos' *Naive Set Theory*, sections 4 to 10) or getting familiar with the style of writing in the textbooks (Axler and Artin) by reading the beginning of the first chapter of each. (Don't worry yet about mastering any of the content, we'll restart completely from scratch).
- Optional review/warm-up problems can be found here, in case you want to review background on sets and functions, assess your readiness, develop your problem-solving skills, or just can't wait. Attempt them on your own, or even more fun, wait till mid-August, join the Slack workspace, and meet up with other prospective students to work on these collaboratively.
- If time permits, start learning how to use LaTeX (for example, using Overleaf, which saves you the trouble of installing it on your own computer), so that you can submit beautifully typeset assignments -- a useful skill for your subsequent math classes! (Typesetting Math 55 assignments is strongly recommended, but not mandatory; we will accept scanned handwritten work as long as it is readable).

Textbooks: there are two required texts, Axler's *Linear Algebra Done Right* and Artin's *Algebra*. (We will aim to cover most of Axler, and most of chapters 2-10 in Artin).
Other books that may (or may not) be helpful at various points in the course include Dummit and Foote's *Abstract Algebra*, Halmos' *Naive Set Theory* , Halmos' *Finite-Dimensional Vector Spaces* , Fulton and Harris' *Representation Theory: A First Course* , and Serre's *Linear Representations of Finite Groups* . Electronic versions of all of these books except for Artin and Dummit & Foote are freely available to Harvard students from the Springer-Verlag website (by clicking on the links above).

Course staff:

- Prof. Denis Auroux (auroux@math.harvard.edu)
- Course Assistants: Jinho Park, Cyrus Hamlin, Rohan Nambiar, Sophie Zhu, Stella Li, Enrico Yao-Bate, Jacob Paltrowitz, Madeleine de Belloy, Halyna Bowey, Alvan Arulandu, Stephen Yang

Lectures will be held MWF 10:30-11:45am. Students are expected to attend all lectures. Handwritten lecture notes will be provided.

Discussion sections will be held by the CAs weekly at various times, in order to go over material from lecture and homework. These are highly recommended for everyone. Even if you don't need help with specific problems on the current assignment, section is a chance to review and ask general questions about course material (or related issues that may not be dealt with in the lecture), including any content not thoroughly covered in the lecture. It's also a chance to get to know the CAs and your fellow students in smaller groups.

Homework will be assigned weekly, and is due on Wednesday each week. Assignments will be posted on this site, and should be submitted electronically via this website.
Doing the homework in a timely fashion is essential to learning the material properly; given the pace, it is extremely hard to catch up if you fall behind in this class.
Extensions will be granted for illness or other serious circumstances (having overcommitted yourself does not count), but should be requested ahead of the deadline; outside of these cases, late submissions will incur a penalty to be computed at the end of the semester once your cumulative lateness across all assignments exceeds 4 days.

Exams: there will be a **midterm** (take-home, will be posted Sept 27, due Oct 2) and a take-home **final exam** (will be posted Dec 5, due Dec 11).

Course grades will be based on your homework (65%), the midterm (10%), and the final exam (25%).

One homework score will be dropped for everyone, so you may miss one assignment without penalty, but you are still responsible for working through the material.

Community: One of the best features of Math 55 is the sense of community that most students get out of it. Getting to know the CAs and your fellow students early in the semester, and forming study groups, is an important part of the experience -- they'll be your support network when the math gets rough. Drop by office hours to introduce yourself and meet others (even if you don't have any specific questions). Participate in the Slack discussions. And please remember: it is up to you to make this community inclusive, welcoming, and supportive of all of its members -- and of all the other people around you, including those who aren't in Math 55.

Academic integrity policy: You are encouraged to discuss and collaborate with each other on the homework assignments. However, make sure that you can work through the problems yourself, and write up your answers on your own. This is not only a matter of academic integrity, but also crucial for properly learning the material and the problem-solving skills that this course aims to cover. For exams, collaboration or consultation of sources other than those explicitly permitted is not allowed.

Homework assignments

Direct links to the homework assignments will be posted here as PDF files. Go to "Assignments" to see the LaTeX source, and to submit your solutions.

- Homework 0 (warm-up, not due)
- Homework 1 (due Wed Sep 11)
- Homework 2 (due Wed Sep 18)
- Homework 3 (due Wed Sep 25)
- Homework 4 (due Wed Oct 2, along with the midterm)
- Homework 5 (due Wed Oct 9)
- Homework 6 (due Wed Oct 16)
- Homework 7 (due Wed Oct 23)
- Homework 8 (due Wed Oct 30)
- Homework 9 (due Wed Nov 6)
- Homework 10 (due Wed Nov 13)
- Homework 11 (due Wed Nov 20)
- Homework 12 (due Wed Dec 4)

List of Topics and Lecture Notes

Direct links to the lecture notes will be posted here as PDF files.

Here is a **tentative** list of topics / dates; we may deviate from these.

- Lecture 1 (Wed Sep 4): Course logistics; groups, examples
- Lecture 2 (Fri Sep 6): Set theory; more examples; subgroups; homomorphisms
- Lecture 3 (Mon Sep 9): Subgroups of $\mathbf{Z}$; cyclic groups; equivalence relations; cosets
- Lecture 4 (Wed Sep 11): Normal subgroups, quotient groups; exact sequences; the symmetric group
- Lecture 5 (Fri Sep 13): The symmetric group; center, commutators; free groups; rings and fields
- Lecture 6 (Mon Sep 16, online): More about fields; vector spaces; independence, span, basis, dimension
- Lecture 7 (Wed Sep 18, online): Bases, dimension; linear maps and matrices; direct sums; rank, dimension formula
- Lecture 8 (Fri Sep 20): Change of basis; quotient spaces; duals, transpose; linear operators
- Lecture 9 (Mon Sep 23): Linear operators; invariant subspaces, eigenvectors, eigenvalues
- Lecture 10 (Wed Sep 25): Triangular matrices; generalized kernel, generalized eigenspaces
- Lecture 11 (Fri Sep 27): Nilpotent operators; Jordan normal form; characteristic polynomial
- Lecture 12 (Mon Sep 30): Real operators; categories and functors
- Lecture 13 (Wed Oct 2): Bilinear forms; orthogonality; inner products
- Lecture 14 (Fri Oct 4): Orthonormal bases; orthogonal and self-adjoint operators
- Lecture 15 (Mon Oct 7): Spectral theorem for real operators; Hermitian inner products
- Lecture 16 (Wed Oct 9): The complex spectral theorem; classifying bilinear forms
- Lecture 17 (Fri Oct 11): Tensor product: definition and properties
- Lecture 18 (Wed Oct 16): Trace; tensor algebra; symmetric and exterior algebras

- Lecture 19 (Fri Oct 18): Exterior algebra, determinant; modules over rings
- Lecture 20 (Mon Oct 21): Modules continued; classification of finitely generated abelian groups
- Lecture 21 (Wed Oct 23): Group actions, orbits and stabilizers; Burnside's lemma; conjugacy classes
- Lecture 22 (Fri Oct 25): Finite subgroups of SO(3) and regular polyhedra
- Lecture 23 (Mon Oct 28): Conjugacy classes; groups of order p^2; the symmetric group
- Lecture 24 (Wed Oct 30): The alternating group A_n; statement of Sylow theorems
- Lecture 25 (Fri Nov 1): Statement of Sylow theorems; examples, semi-direct products
- Lecture 26 (Mon Nov 4): Proof of Sylow theorems; groups of order 12
- Lecture 27 (Wed Nov 6): Group presentations, Cayley graph, normal forms
- Lecture 28 (Fri Nov 8): Presentations and normal forms: S_n, braid group, SL(2,Z)
- Lecture 29 (Mon Nov 11): Representations; sub-representations; irreducibility
- Lecture 30 (Wed Nov 13): Irreducibility; Schur's lemma; representations of S_3
- Lecture 31 (Fri Nov 15): Symmetric polynomials; the character of a representation
- Lecture 32 (Mon Nov 18): Characters, orthogonality, and consequences; characters of S_4 and A_4
- Lecture 33 (Wed Nov 20): Irreducible characters form a basis; the representation ring
- Lecture 34 (Fri Nov 22): Irreducible characters of S_5 and A_5; restriction and induction
- Lecture 35 (Mon Nov 25): Restriction and induction, Frobenius reciprocity; real representations
- Lecture 36 (Mon Dec 2): Real and quaternionic representations
- Lecture 37 (Wed Dec 4): Wrap-up, final info, and review