MATH 263X: Mirror Symmetry for Toric Varieties and Other GIT Quotients

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Course Description

GIT quotients of vector spaces $V//G$ and their representation theoretic subvarieties form the basis of most constructions of mirror symmetry. The focus of this course will be to explore mirror symmetry for Fano varieties of this form. We will also discuss the role these constructions are expected to play in the Fano classification program.

When G is abelian, these GIT quotients are called toric varieties. We will introduce the basics of geometric invariant theory (GIT), and then develop toric varieties via GIT. We will relate this to other constructions of toric varieties via symplectic reduction. Using this, we will introduce two mirror symmetry constructions for Calabi–Yau hypersurfaces in Fano toric varieties: Batyrev–Borisov mirror symmetry and Berglund-Hübsch mirror symmetry.

The next focus will be the mirror theorem for toric complete intersections. We will dicusss quantum cohomology and the moduli space of stable maps and define the J function. We will then define the I function of a toric complete intersection, and prove the mirror theorem for Fano complete intersections in projective space.

When G is not abelian, the geometry of $V//G$ can be more complicated. We will discuss examples of non toric GIT quotients, especially quiver flag varieties. The Abelian/non-Abelian correspondence allows one to compute to the J-function of of a non-Abelian GIT quotient from that of an Abelian GIT quotient.

The last part of the course will be on the Fano search program, which seeks to classify Fano varieties via mirror symmetry. We will introduce Laurent polynomial mirrors for Fano toric complete intersections and discuss their properties, and discuss methods of finding Laurent polynomial mirrors for other Fano varieties.

Resources

The following books may be helpful. As the course progresses, I will suggest other resources. Course notes will also be posted.

- Mumford, Fogarty, Kirwan *Geometric Invariant Theory*
- Newstead *Introduction to Moduli Problems and Orbit Spaces*
- Cox and Katz *Mirror Symmetry and Algebraic Geometry*
- Cox, Little, Schenck *Toric Varieties*

Prerequisites

Some algebraic geometry will be useful.

Course Outline

A tentative course outline is as follows:

1. Weeks 1-2: Introduction to GIT
2. Weeks 3-4: Toric Varieties as GIT quotients
3. Weeks 5-6: Batyrev–Borisov and Berglund-Hübsch mirror symmetry
4. Weeks 7-10: Hori–Vafa mirror symmetry: The mirror theorem for toric complete intersections.
5. Week 11: The Abelian/non-Abelian correspondence.
6. Week 12-13: Mirror symmetry and the classification of Fano varieties

Homework and grades

Undergraduate students and graduate students who have not passed their Quals require a grade. Grades will be based on homework assignments and a final project.