MATH 264x W Fr 3:00 - 4:15 PM, Room: Science Center 221 (FAS)

Course Title: Algebraic Cobordism.

Instructor: Elden Elmanto, Science Center 231. elmanto@math.harvard.edu

Office Hours: By appointment only!

What is this class about? The grand tension in developing cohomology theories is between computability and sensitivity. Algebraic cobordism does a decent job balancing both. On the one hand, it satisfies a certain universal property and reproduces many known invariants (like Chow groups or the Grothendieck group of vector bundles). On the other, it is relatively amenable to explicit computations, internally and externally. One of the aims of this class is to give a modern exposition of this theory with the aid of derived algebraic geometry. Another aim is for the instructor to try (with the very possibility of failure!) and reprove Levine-Morel's moving lemmas using derived algebraic geometry. Roughly, we will proceed in the following way:

1. An introduction to derived algebraic geometry.
2. The universal property of (derived) algebraic cobordism.
3. The algebraic Spivak theorem.
4. The coefficient ring of algebraic cobordism and other basic properties.
5. The localization theorem.
6. Virtual fundamental classes in algebraic cobordism.
7. Chow groups and K-theory from algebraic cobordism.
8. Applications: counterexamples to the integral Hodge/Tate conjectures, Rost's degree formulas, the "flop ideal" in algebraic cobordism, Grothendieck-Riemann-Roch theorems.

Textbook: *Algebraic Cobordism* by Marc Levine and Fabiel Morel, Springer (2017). I will make lecture notes.

Prerequisites: A first course in algebraic geometry and algebraic topology and then some (namely, basics of simplicial homotopy theory, derived categories of coherent sheaves, the cotangent complex and the yoga of complex-oriented cohomology theories).