.

MATH 295 - Topics in Discrete Probability: Random Structures and Algorithms

Fall 2018, MW, 1:30-2:45. Science Ctr 116 (FAS)

Lecturer: David Gamarnik

Administrative details

Class meets: MW 1:30-2:45 Science Ctr 116

Instructor: David Gamarnik Science Center 333i, [gamarnik@mit.edu]

Office hours: Upon request

Homework assignments (about 6)

Grades: Homeworks, end year projects. The projects will be distributed in the middle of the semester when the class size is settled.

Course Summary

This is a graduate-level introduction to probabilistic reasoning for discrete systems, including random graphs and random structures, graphical models and Markov Random Fields (MRF). The topics include large deviations theory and concentration inequalities. Theory of random graphs and component structure of random graphs. The first and second moment method. Combinatorial optimization on random graphs and the differential equations method. Interpolation method and scaling limits in random graphs. Planted clique problem and applications to high-dimensional statistics. Gibbs measures on finite and infinite MRF, Dobrushin's uniqueness and the correlation decay method. Introduction to statistical physics. Reconstruction of MRF from observations. Sample and computational complexity.

The course will touch on many topics of active ongoing research directions. A heavy emphasis will be placed on algorithms, algorithmic complexity, computations, and applications to high-dimensional statistics and machine learning.

Reading materials

- **Primary:** for some lectures course notes prepared by the instructor will be distributed before classes.

- Alon, Noga, and Joel H. Spencer. The probabilistic method. John Wiley & Sons, 2004.
- Simon Foucart and Holger Rauhut. A mathematical introduction to compressive sensing. Springer, 2013.
- David Asher Levin, Yuval Peres, and Elizabeth Lee Wilmer. Markov chains and mixing times. American Mathematical Soc., 2009
- Janson, Svante, Tomasz Luczak, and Andrzej Rucinski. Random graphs. Vol. 45. John Wiley & Sons, 2011.
- Amir Dembo and Ofer Zeitouni. Large Deviations Techniques and Applications (2nd Edition), Springer, 2009.

Lecture Topics

I Foundations: large deviations theory and concentration inequalities.

1. Large deviations in one and many dimensions. Change of measure technique.
2. Concentration inequalities. Azuma-Hoeffding, McDiarmid, Effron-Stein, Janson's inequality, Second Moment Method and Paley-Zigmund inequality. Applications to the theory of random graphs, random matrices and sparse regression.

II Sparse linear regression. Algorithms and analysis.

1. Restricted Isometry Property (RIP). Recovering by linear programming. The compressive sensing method.
2. Explicit construction of RIP matrices. The "square root" bottleneck.

III Theory of random graphs.

1. Random graphs. Dense and sparse. Component structure and the giant component. Connectivity and other basic properties.
2. Random regular graphs. Configuration model.
3. Combinatorial optimization on random graphs and random structures. Cliques, independent sets and a proper coloring in dense and sparse random graphs. The interpolation method and scaling limits in sparse random graphs. Counting triangles using Janson's inequality. The Differential Equations (Wormald's) method. The largest submatrix problem.
4. THe Hidden Clique Problem and applications to sparse PCA problem in statistics.

IV Markov Random Fields (MRF), Graphical Models and Elements of Statistical Physics.

1. Markov Random Fields and Gibbs measures. Partition functions. Spatial Markovian Property and Hammersley-Clifford Theorem.
2. Gibbs measures on infinite graphs and Dobrushin Uniqueness Theorem.
3. Markov Chain and mixing in MRF. Applications to computing partition functions.
4. The Correlation Decay method for counting and inference in MRF.
5. Reconstructing MRF from samples and applications to Machine Learning. The Correlation Decay method.