You can download the syllabus for the course here: Math 273X Fall 2020 Syllabus.pdf or read it below.

MATH 273X: Distributions of Class Groups of Global Fields
Fall 2020-21
Wednesday and Friday 1:30-2:45 PM Eastern time
Course webpage on Canvas

Instructor

Melanie Matchett Wood
Office Hours Monday and Thursday 10am-11am, Zoom link on canvas webpage
Questions for the professor are best asked in class or during office hours
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Problem sets at below link

http://people.math.harvard.edu/~mmwood/Fall2020_273X_HW/

Course description

As we vary over number fields, what distribution of class groups do we see? More generally, as we vary over number fields, how many (and how large) unramified extensions do the number fields have? Starting with the case of quadratic fields, we will learn about what was classically known about class groups of quadratic fields, and the conjectures of Cohen and Lenstra on their distribution, including many different motivations for the conjectures. We will learn about the relationship between class group distributions and counting number fields, and learn about the heuristics on counting number fields. We will learn about probability distributions on abelian groups that arise in these conjectures, and how they are determined by their moments. We will then learn what is known about how this picture generalizes to arbitrary number fields. We will then learn about two major recent research directions in this area: (1) Bounding class groups of number fields, and (2) Applications of topological component counting to class groups of function fields.

The lectures will reference many recent papers and also include a significant amount of material that is considered ``known by experts'' but is not in the literature. There will be weekly problem sets to give students a chance to work with the concepts introduced. Project ideas relating to the course topics will be suggested, and a final project will be required of undergraduates, which can include any blend of exposition, original computations, and computer programming. Some project topics could lead to original research by graduate students beyond the course.

Students in this class should have a strong, general undergraduate level math background plus familiarity with algebraic number theory and aspects of class field theory and algebraic geometry. People with questions about whether their math background prepares them for this class should contact Professor Wood.

Course Objectives

At the end of this course, students will know the current conjectures on distributions of class groups and unramified extensions of number fields, be able to make specific computations and predictions using those conjectures, and know the problems with and gaps in the current conjectures. Students will be able to work with probability distributions on isomorphism classes of groups, will understand their moments and be able to determine distributions from matrix or generator-relation models. Students will understand the main new ideas behind recent research on bounding class groups of number fields, and applications of topological component counting to class groups of function fields, and be able to understand the proofs of this recent work in small examples.

Course Policies and Expectations

Students are expected to attend twice weekly live lectures on zoom, share video and ask and answer questions during the zoom meetings. Feedback will be requested during all lectures regarding whether students are comfortable with the material or need more examples or practice, and students are expected to give this feedback. Students are expected to be singly focused on the lecture during lectures and to ask questions when they donâ€™t understand material presented. The professor will tailor the course to the understanding of the students. The professor will be available twice weekly for office hours to answer further questions. All students will be expected to come at least once to office hours during the first three weeks of class, and are encouraged to come often throughout the semester. We will all be understanding of one another if challenges arise during these difficult times.

Course Topics

- Dedekindâ€™s correspondence between class groups of quadratic fields and equivalence classes of binary quadratic forms
- Genus theory of quadratic fields
- Growth of size of class groups
- Cohen-Lenstra heuristics and 1/Aut philosophy
- Modern motivations for the Cohen-Lenstra heuristics: matrix models (including universality), function field analogs
- Davenport-Heilbronn Theorem
- Moments of random groups and the moment problem
- Non-discrete probability distributions on groups
- Relationship between class groups distributions and bounding l-torsion in class groups
- Relationship of counting number fields to class group distributions
- Malle-Bhargava principle for predicting number field counts
- Cohen-Martinet conjectures for class groups of higher degree fields
- Different ways of ordering fields and effects on statistics
- Recent work on bounding l-torsion in class groups
- Recent work on applications of topological component counting to class groups of function fields

Materials and Technology

The sources for the material in this course are recent papers, or in many cases there is no source in the literature. When possible, references to papers will be given, but reading them will not be required.

All students will be required to have a tablet and stylus compatible with zoom screensharing, in order to enable group discussion of problems and examples. You may borrow a tablet and stylus from Harvard if you do not already have them, and if you need to borrow the equipment , please contact ipadrequest@fas.harvard.edu

Assignments, Grading Procedures, and Collaboration

Students who are receiving grades will receive a grade that is based 25% on the final project and 75% on weekly problem sets. The final project should be at least 8 pages (or at least 4 pages if including a significant computer programming component) and typed in LaTex. The final project will be due **December 17 and should be emailed to the professor**. Throughout the course, project ideas will be suggested, and students should confirm with the professor when they pick a project topic, which should happen at latest a month before the due date (and earlier is preferred). There will be fewer problem sets at the end of the course to give students more time to work on their projects. Grades will be determined by showing mastery of the material and not curved.

Students are encouraged to work together on problem sets, but should write up solutions on their own, and write at the top of their problem set which other students they collaborated with. There will be a Slack workspace available through Canvas for students to discuss problems, ask questions of one another, and find others to work with. The final projects should be the studentsâ€™ own work, include citations to any sources, and should acknowledge any helpful advice or suggestions in an acknowledgements section.

Academic Integrity

Students are expected to follow the Harvard Honor Code (below). When in doubt, err of the side of being generous in attributions of sources and acknowledging advice.

Harvard Honor Code: Members of the Harvard College community commit themselves to producing academic work of integrity â€“ that is, work that adheres to the scholarly and intellectual standards of accurate attribution of sources, appropriate collection and use of data, and transparent acknowledgement of the contribution of others to their ideas, discoveries, interpretations, and conclusions. Cheating on exams or problem sets, plagiarizing or misrepresenting the ideas or language of someone else as oneâ€™s own, falsifying data, or any other instance of academic dishonesty violates the standards of our community, as well as the standards of the wider world of learning and affairs.

Accommodations for students with disabilities

Students needing academic adjustments or accommodations because of a documented disability must present their Faculty Letter from the Accessible Education Office (AEO) and speak with the professor by the end of the second week of the term, September 11. Failure to do so may result in the professor's inability to respond in a timely manner. All discussions will remain confidential, although Faculty are invited to contact AEO to discuss appropriate implementation.

Official information from the registrar