## Assignment info:

**HW 2**: **Due: February 20**

Handout 2, Ex. 2(3,4)

Handout 3, Ex. 2, 3

Handout 4, Ex. 3 (first four bullet points), Ex. 5

+ choose 2 other exercises from Handouts 2-5 that interest you.

Please also give an estimate for the amount of time it takes, so I can calibrate later assignments.

**Homework 1, due Thursday 2/1:** If you're taking the class for a grade, the first homework assignment is Exercise 1 and 2 from Handout 1. Complete solutions to these will involve writing computer programs that visualize a dynamical system (like what I showed in class) and that estimate arithmetic entropy of some orbit. As with all coding tasks, this may be more time-consuming than it looks! Homeworks should be LaTeX'ed and for Homework 1, pictures of the program's outputs should be provided.

## Course goals:

Entropy is how mathematicians quantify chaos. We will study the natural notions of entropy in a few categories of discrete-time dynamical systems, mainly rational maps of algebraic varieties, but also metric spaces and topological spaces.

They say an example is worth a thousand theorems. The goal of this course is to learn two theorems and four examples in close detail, bringing us to a total of 4002 theorems covered. The examples will be the true heart of the course. Students will get a thorough understanding of tori, abelian varieties, rational maps of the projective plane, and Markoff surfaces. The theorems we will prove are:

1) The Gromov-Yomdin bridge between topological entropy and dynamical degrees,

2) The Kawaguchi-Silverman conjecture (or some cases of what has been proved of it) relating dynamical degrees and arithmetic entropy.

## Course format:

The course will be a mix of 75% lecture and 25% active learning. "Active learning" will happen when we introduce new definitions. This will be aimed at getting students to make simple calculations (and mistakes) with the definitions of entropy and will include worksheets and basic mathematical programming exercises.

## Typical enrollees:

This course is designed with graduate students in mind, but interested undergraduates, postdocs, and faculty are more than welcome to join us. Prior experience with varieties, regular maps, and rational maps will be very helpful.

## When is course typically offered?

One-time only!

## What can students expect from you as an instructor?

My goal as a teacher is to ensure that the whole class is following along with the material as much as possible.

## Assignments and grading:

Anyone taking the class for a grade/credit will give a final presentation, the length to be determined based on number of enrollees. The grade will also incorporate 2-4 (LaTeX'ed) assignments based on formally writing up some of the active learning exercises we do in class.

## Provisional Topics:

Algebraic entropy and dynamical degrees

Case studies: monomial maps, maps of P^2, abelian varieties, Markoff surfaces

Topological entropy

Case studies: hyperbolic toral endomorphisms, shifts, smooth maps of compact manifolds

Arithmetic entropy and arithmetic degree

Gromov-Yomdin Theory (algebra to topology)

The Kawaguchi-Silverman Conjecture (algebra to arithmetic)

## Enrollment cap, selection process, notification:

There is no formal requirement for joining our course, but undergraduates interested in the course should chat with me briefly in the first two weeks of class to make sure they have the necessary background knowledge.

## Absence and late work policies:

Attendance will not be taken, but do come to class. Late work will not be accepted. Assignments that cannot be turned in on time due to extenuating circumstances will be dropped from the grade.

## Generative AI policy:

Work composed using the assistance of generative AI is likely to be very entertaining. It's fine to try it, but I couldn't get it to solve the homework exercises for me.