

Math 221

Commutative Algebra

Harvard University, Fall 2024

Tu-Th 10:30-11:45 SC 310

First Meeting Tu, September, 3, 10:30

— Instructor: Yuriy Drozd, SC 340

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— Office hours: Tu 1:30-2:30 & Th 12:00-1:00 or by appointment

This class is a graduate level course in Commutative Algebra, also aimed at undergraduates.

— Contents:

Commutative Algebra lies at the foundations of Number Theory and Algebraic Geometry. It plays an important role in Algebraic Topology, Geometry and other fields. We will cover the main topics of Commutative Algebra and give a taste of its applications. Starting from generalities on rings, modules and ideals, localization and primary decomposition in Noetherian rings and modules, we then move to integral extensions, going-up and going-down, Noether normalization and Hilbert's Nullstellensatz, dimension theory. The final part of the class will cover graded rings, Hilbert polynomials, valuations and homological methods including regular local rings.

Prerequisites: Basic courses in algebra (Math 122 and 123).

Recommended Literature (all books are available in pdf format):

- D. Eisenbud. *Commutative Algebra with a View Towards Algebraic Geometry*. (Most recommended)
- H. Matsumura. *Commutative Ring Theory*. (More elementary than the preceding one but covering all topics and containing a lot of examples and exercises)

Additional books:

- H. Matsumura. *Commutative Algebra*. (Perhaps the most considerable monograph in the subject)
- F. Atiyah, I.G. Macdonald. *Introduction to Commutative Algebra*. (Most elementary)
- M. Greuel, G. Pfister. *A Singular Introduction to Commutative Algebra*. (Containing the theory and applications of Gr bner basis and the related software package **Singular**)

— Office hours:

There will be weekly office hours. You are all encouraged to attend, whether you have a question or no. I will answer any questions about the class and discuss more topics in commutative algebra following any requests of the students, including further development and research topics in Commutative Algebra.

— Grading:

Homework will be assigned weekly on Thursday and will be due the following Thursday. The solutions can either be scanned or typed and uploaded on Canvas. Homework will count for 50% of the final grade. Late homework can only be accepted under special circumstances. There will be no final exam but rather a final project (a reading on a topic related to the class), which together with participation will count for 50% of the grade. Collaborative work on homework and final projects is accepted but you must write your own text as well as include the names of the collaborators and any other references.