Riemann surfaces

- **MATH 213BR**
- **Term:** 2024 Spring / Full Term
- **Course Instructor:** Peter Kronheimer
- **Course assistant:** Jonas Iskander (jonasiskander@college.harvard.edu)
- **Meeting Time:** Monday 1:30 PM - 2:45 PM; Wednesday 1:30 PM - 2:45 PM

The exploration of Riemann surfaces grew naturally out of the 19th century world of classical complex analysis, motivated by the desire to understand definite integrals involving multi-valued algebraic functions. Today the subject stands as a beautiful marker for the junction where analysis, geometry and algebra meet.

In this course we will explore Riemann surfaces with a mixture of classical and modern viewpoints. We will see the proofs of the classical theorems of Abel, Jacobi and Riemann on periods of analytic differentials, the Riemann-Roch theorem, and the uniformization theorem, which connects Riemann surfaces to geometry via real analysis. We will uncover the close relationship between Riemann surfaces and projective algebraic curves on the one hand, and hyperbolic geometry on the other.

Prerequisites for this course include a good grounding in analysis of a single complex variable, some topology of the sort covered here in Math 131 (including the fundamental group and covering spaces), and a familiarity with smooth manifolds, differential forms and de Rham cohomology. We will also make light use of the idea of singular or simplicial homology, including particularly the first homology group of a space and its connection to the fundamental group. Math 213a is not a prerequisite.

Evaluation of this course will be based on weekly homework assignments, submitted through Canvas, and a final paper on a topic chosen from a provided menu of ideas.

Materials for the course will be a few notes of my own, but mostly we will draw on two contrasting texts. The first is the book ***Lectures on Riemann Surfaces*** by Otto Forster and second is the book ***Riemann Surfaces*** by Simon Donaldson. Both of these texts are freely available in digital form to Harvard affiliates through Hollis.

There are many other good sources for much of this material. I originally learned about Riemann surfaces from a text by George Springer - *Introduction to Riemann Surfaces* - which was first published in 1957 and is still available through AMS Chelsea Publishing. It has beautiful mathematical illustrations. It seemed a little old-fashioned even when I was a graduate student, but still has a lot to offer. I mention the book here because it is the reason I love this subject.

I will have weekly **office hours** and will be glad to meet you there. My office is SC 343, which is at the back of the Birkhoff Library on the third floor. Our course assistant is TBA, at this time.