Tuesday and Thursday 3:00 pm-4:15 pm

Location: Science Centre 110

Class notes: here

Office Hours: Wednesday 6 pm-7:30 pm, Science Centre 105, Friday, 1:30 pm-3:00 pm, Sctr 411

If you have taken MATH136 or some other differential geometry class, you should be good for this class. If you haven't done so and are interested in 166r, go through the notes from previous years posted below and you should be fine. 166r is essentially an undergrad version of 230B.

Potential Syllabus: In the first couple of weeks, we will review the concepts covered towards the end of MATH136: manifolds, tensors, metrics, geodesics, curvature, Fibre bundles, connections on fiber bundles, etc. For this take a look at the class notes from last semester here. Next, we will introduce semi-Riemannian metrics that would naturally lead us to the famous "Minkowski-Spacetime" (which is analogous to Euclidean space in the context when the metric is no longer positive definite). We will study calculus on Minkowski space: integral curves, vector fields, flows, isometry (Poincare group), conservation theorems, etc. Then we will introduce curvature in the semi-riemannian setting, leading us to the concept of `geodesic deviation' (if two particles start close enough, do they remain close enough for a long time or do they diverge rapidly: think of some physical situation where this is important) and subsequently Einstein's general theory of relativity. From a mathematical perspective (now that you've gone through 136), these concepts will be simple (but fancy names nonetheless). First, we will prove the existence of the solutions (proof of the local well-posedness theorem similar to the integral curves that I promised you but never did in 136, now you'll see how this is done using the contraction mapping theorem). We will introduce the function spaces (note that in 136 we always worked with C^{\\infty} and mentioned L^2 spaces a few times especially if you remember relating energy and length of a curve using Holder's inequality). Next, we will obtain special solutions of Einstein's equations such as black-hole solutions you will see why introducing tensor in 136 was so important. We will describe in a mathematically rigorous way what a black hole is. Using the simple concepts of vector fields, flows, and curvature, we will show how black holes form (we will prove a more interesting theorem: black holes can form in a vacuum i.e., even if you do not have any mass or anything, a black hole can still form: this is where magic happens if you understand curvature good enough). We will end with some real-life applications such as how these concepts can help you understand the shape and size of manifolds and can help you do high-dimensional data analysis. More things such as Gauge theory (Yang-Mills equations), hyperbolic geometry, Teichmuller theory, and spin geometry might also be included if time permits. We might study a bit of **Ricci Flow** as well.

Course goals:

We will aim to understand better the application of differential geometry that we learned in MATH136. In particular, we will try to understand more about diffeos, flows, vectors, and tensor fields, and how these are connected to the geometry and topology of the manifolds in question. We will see how these basis notions that we developed extend to semi-Riemannian manifolds where the metric is no longer positive definite. We will see how this simple modification in the metric has profound consequences such as the existence of crazy objects such as black holes in nature (and possible wormholes which are not yet observed). This will take us back to those sci-fi movies but we will try to understand (prove and dispute a lot of common beliefs that even persevere in the physics community) in a mathematically rigorous way.

Course format:

The course format is the same as MATH136. Those of you who took 136 with me in Fall 2022 or 2023 should feel right at home. But if you haven't that's alright too.

Assignments and grading:

There is no final exam for this class. Homework: 70%, Final paper (<=10 pages)-30%.

1 pset in every two Totals. Tota of 5 psets in the semester.

Each problem will consist of 3-4 lines of calculations maximum. Once you figure out the idea, it'll be trivial. There will ***not*** be problems where you'll need to perform large calculations. **Use of ChatGPT (or any other AI) is strictly prohibited for solving homework problems.**

Office hours: there will be two office hour sessions every week (most likely Wednesdays and Fridays). These are meant for HW and to clarify concepts from the class if need be. In addition, TF will hold one office hour every week. Help is always given to those who ask for it.

Text Book: **Semi-Riemannian Geometry by Barrett O'Neil. We will also try to make the daily class notes available to everyone (just like 136). As you know if you've taken a class with me before, we will *never* follow the textbook line by line (otherwise there's no point in teaching) but the content of the lectures will be in the textbook.**