### Course Information

**Harvard College/GSAS: 80719**

**Fall 2017-18**

**Brian F. Farrell**

**Location: Geological Museum 418 (FAS)**
**Meeting Time: Tuesday, Thursday 10:00am - 11:29am**
**Exam Group:**

An introduction to the ideas and approaches to dynamics of generalized stability theory. Topics include autonomous and non-autonomous operator stability, stochastic turbulence models and linear inverse models. Students will learn the concepts behind non-normal thinking and how to apply these ideas in their daily intellectual life.

**Note:** Given in alternate years.

**Prerequisite:** Applied Math 105

### Topic Descriptions and Links to Lecture Notes:

Lecture 1

Overview of the concepts of stability theory and generalized stability theory.

Lecture 2

GST for autonomous operators: the initial value problem, SVD of the propagator, solution for the optimal excitation at initial time and the evolved optimal, relation between eigenfunction-based and SVD-based analysis of stability, use of penalty functions to constrain an optimization, the numerical and spectral abscissa and the global optimal, the pseudospectrum and the issue of the ill-conditioned nature of an eigenfunction-based representation of the stability of non-normal dynamical systems.

Lecture 3

Reynolds matrix example of optimal excitation at initial time.

Lecture 4

GST for autonomous operators that are continuously excited. Frequency domain analysis, the resolvent and the optimal structure and response for excitation at a chosen frequency. The distinction between resonant response in a normal system and the response of a non-normal system - the concept of the equivalent normal system. Using stochastic excitation white in space and time to study the dynamical operator. Time domain analysis of a continuously forced system - the optimal excitation and response functions for a continuously excited operator.

Lecture 5

Example of GST time and frequency based analysis applied to the Reynolds matrix
*A* = [ âˆ’ 1 âˆ’ cot â¡ (Î¸ ); 0 âˆ’ 2]*A* = [ âˆ’ 1 âˆ’ cot â¡ (Î¸ ); 0 âˆ’ 2]*A* = [ âˆ’ 1 âˆ’ cot â¡ (Î¸ ); 0 âˆ’ 2]
*A* = [ âˆ’ 1 âˆ’ cot â¡ (Î¸ ); 0 âˆ’ 2]*A* = [ âˆ’ 1 âˆ’ cot â¡ (Î¸ ); 0 âˆ’ 2]*A* = [ âˆ’ 1 âˆ’ cot â¡ (Î¸ ); 0 âˆ’ 2]
*A* = [ âˆ’ 1 âˆ’ cot â¡ (Î¸ ); 0 âˆ’ 2]*A* = [ âˆ’ 1 âˆ’ cot â¡ (Î¸ ); 0 âˆ’ 2]*A* = [ âˆ’ 1 âˆ’ cot â¡ (Î¸ ); 0 âˆ’ 2]
*A* = [ âˆ’ 1 âˆ’ cot â¡ (Î¸ ); 0 âˆ’ 2] in which the non-normality of the matrix is controlled by the parameter Î¸ Î¸ Î¸ Î¸ Î¸ Î¸ Î¸ Î¸ Î¸ Î¸ .

Lecture 6

GST time and frequency based analysis applied to baroclinic wave dynamics using the Eady model of baroclinc wave dynamics.

Lecture 7

GST applied to non-autonomous operators and the stability of linear time-dependent dynamical systems.

Lecture 8

Parametric instability of time dependent systems with harmonic and stochastic fluctuation.

Lecture 9

Parametric instability of atmospheric flows. The Eady model example.

Lecture 10

Linear inverse theory.

References