

Math262b: Quantum theory from a geometric viewpoint II

Spring, 2024

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Overview: This is the second semester of a year-long course centered around concepts in quantum mechanics and quantum field theory. We treat the subject from a mathematical perspective that emphasizes structural aspects and eventually applications to mathematics. Some lectures will be devoted to background mathematics. In the second semester we will work almost exclusively in Wick-rotated field theory on compact manifolds. There will be extensive units on topological field theory, invertible field theory, and supersymmetric field theory. We will give applications to both physics and mathematics. The latter will include discussions of mirror symmetry and low-dimensional topology.

While I will review many mathematical ideas, I will not develop them in detail in this course. I will give references for you to learn more, and I hope that you will delve in at some point. My goal is that you emerge from this course with a deeper perspective on how different parts of mathematics work together. My focus is the mathematical structure of quantum theory, not physics applications per se, though there will be many of those as well. Bringing quantum field theory into mathematics is an ongoing longterm project; this course is a snapshot of aspects of my current understanding.

Prerequisites: This is a lecture course on the application of mathematical ideas, techniques, and viewpoints to a subject outside of mathematics. Such applications draw freely on whatever mathematics is needed with no walls between its subdisciplines (analysis, algebra, geometry, representation theory, etc.). As such the course places large demands on your mathematical knowledge and sophistication. I will assume exposure to material at least at the level of basic graduate courses, but in many cases beyond. That said, you can get lots out of the course without knowing this mathematics in advance.

Course Assistants: To be determined

Lectures: Monday, Wednesday 12:00–1:15 in Science Center 507.

Office Hours: I will hold office hours Mondays 1:30–2:30 in Science Center 503. I strongly encourage you to come to office hours (individually or in groups) to discuss lectures and problems, ask more general questions, explore ideas, etc.

CA Problem Session: To be determined

Class Website: Canvas. Lecture notes, problem sets, and additional readings will be posted there.

Texts: Nothing official. I plan to write lecture notes to go along with the lectures; they will be posted on Canvas. But you should take notes in lecture to generate a primary source for yourself. I will also post supplementary readings on Canvas.

Problem sets: I will post problem sets on Fridays. Problems are often open-ended and provide a gateway into the material. I don't expect you to solve all problems, but the more you engage the more you'll learn. Undergraduates registered for the course should hand in writeups of some of the problems. I strongly suggest you work on the problems in groups of 3 ± 1.5 (rounded up). I also encourage the groups to exchange solutions to problems to get feedback. I do not encourage the use of Chat GPT for the problems, but if you do succeed in getting AI to solve some problems then please do come and show me. That said, I hope you'll focus on using NI unaided by computers. Discord is a good platform to communicate about the class.

Remark: Above all I hope you have fun in this course. I will challenge you in lectures, and I hope to give you interesting and challenging problems. When you get frustrated please seek help—from classmates, from me, or from whomever you have around to help. If you never get frustrated, give it a bit of time, but if the problem persists then please come see me; I'll arrange for something more challenging. (I'm serious.)