SYLLABUS FOR MATH 287: GEOMETRIC AND TOPOLOGICAL COMBINATORICS

PROFESSOR LAUREN WILLIAMS

1. Course Description

The goal of this course is to give an introduction to geometric and topological combinatorics, focusing on four main topics: posets, simplicial complexes, matroids, and polytopes. One theme we will revisit periodically will be the question: to what extent do combinatorial properties of an object determine its topology? For example, to what extent does the face lattice of a simplicial or cell complex determine its homotopy type? To what extent does the graph of a polytope determine the polytope? And what kinds of combinatorial techniques can we use to then understand the topology of the object in question? Along the way, we will discuss interesting examples coming from posets, polytopes, matroids, Coxeter groups, the Grassmannian and flag varieties, etc.

Prerequisites: The course is aimed at graduate students and advanced undergraduates. I will expect that students have a strong background in abstract algebra, some background in combinatorics, and some exposure to basics of algebraic topology, including topics such as homotopy and homology.

2. Grading

There will be weekly homework assignments, as well as a final project. The final project will be a short (5 to 10 page) expository paper. Depending on the size of the class, students may also give final presentations.

3. Getting the most out of this class

I would like this class to be as interactive as possible. So: please:

- Feel free to interrupt me if there is something you don't understand or have questions about.
- Provide feedback if my explanations are too fast or slow, confusing, etc.
- Come to office hours.

4. Collaboration policy

Collaboration with other students in the class on homework is permitted, but you must write up your solutions independently: you cannot just copy someone else's work. If you work with other people, you must mention on your problem sets who you worked with. *Uploading course materials, problem sets, or solutions to a third-party website or soliciting solutions on the internet (stack exchange, etc) is strictly prohibited.*

5. Policy on the use of AI

It is specifically forbidden to use ChatGPT or any other generative artificial intelligence (AI) tools at all stages of the work process, including preliminary ones. Violations of this policy will be considered academic misconduct. Note that different classes at Harvard could implement different AI policies, and it is the student's responsibility to conform to expectations for each course.

6. Disability statement

Harvard University values inclusive excellence and providing equal educational opportunities for all students. Our goal is to remove barriers for disabled students related to inaccessible elements of instruction or design in this course. If reasonable accommodations are necessary to provide access, please contact the Disability Access Office (DAO). Accommodations do not alter fundamental requirements of the course and are not retroactive. Students should request accommodations as early as possible, since they may take time to implement. Students should notify DAO at any time during the semester if adjustments to their communicated accommodation plan are needed.