Provisional Syllabus for Math 258: **L**-Functions and Arithmetic Statistics

The format of the "topics course" Math 258 will consist of some lectures from me (and possibly other faculty) but I hope it will mainly be composed of student lectures. There are no exams in this course. The only requirements will be **(a)** participation in general and (**b)** specifically: to give one or two lectures on a topic germane to one of the three the general areas related to modular symbols and arithmetic described below. How much time, we spend on any of these three areas will depend on the preference of the participants.

I. **Modular symbols, Theta-elements and L-functions.**

(a) The basics

(b) Some statistics

(c) Applications to theorems (and conjectures) regarding rational

points.

A few relevant references:

1. S. Lang, Introduction to Modular Forms, Springer-Verlag (Chapters IV, V) https://wstein.org/edu/Fall2003/252/references/lang-intro_modform/Lang-Introduction_to_modular_forms.pdf

2. J. I. Manin, Parabolic points and zeta functions of modular curves,

Izv. Akad. Nauk SSSR Ser. Mat. 36 (1972), 19-66. https://wstein.org/edu/Fall2003/252/references/Manin-Parabolic/Manin-Parabolic_points_and_zeta_functions_of_modular_curves.pdf

3. B. Mazur-K. Rubin (to be specified; various articles)

4. W. Stein, Modular Forms, a Computational Approach, AMS https://wstein.org/books/modform/stein-modform.pdf (Chapter 3, especially and Chapter 8)

5. W. Stein, Statistics of modular symbols: https://sites.mathwashington.edu/~bviray/NTS/SteinApril7.pdf

II. Selmer groups

(a) The basics

(b) Results about statistics

(c) Applications, especially to theorems regarding rational points

and (perhaps) Diophantine Stability.

A few relevant references:

1. M. Stoll, Selmer groups and Descent https://people.maths.bris.ac.uk/~matyd/Trieste2017/Stoll.pdf

2. B. Poonen, Selmer group heuristics http://math.mit.edu/~poonen/papers/aws2014.pdf

3. Z. Djabri, E. F. Schaefer, N.P. Smart, Computing the **p**-Selmer group of an elliptic curve http://www.hpl.hp.com/techreports/98/HPL-98-178R1.pdf

4. B. Mazur, K. Rubin, M. Larsen, Diophantine Stability, https://arxiv.org/abs/1503.04642

III. Bounding rational points.

(a) some basics, but on to:

(b) Chabauty's method and its refinements.

A few relevant references:

1. W. McCallum, B. Poonen, The method of Chabauty and Coleman http://wwwmath.mit.edu/~poonen/papers/chabauty.pdf

2. M. Kim, The motivic fundamental group of **P**^1-{ 0, 1}, and the theorem of Siegel, http://people.maths.ox.ac.uk/kimm/papers/siegelinv.pdf

3. M. Kim, The Unipotent Albanese Map and Selmer Varieties for Curves, http://people.maths.ox.ac.uk/kimm/papers/alb.pdf

4. J. S. Balakrishnan, N, Dogra, Quadratic Chabauty and rational points I: p-adic heights https://arxiv.org/abs/1601.00388

5. J. S. Balakrishnan, N, Dogra, Quadratic Chabauty and rational points II: Generalised height functions on Selmer varieties, https://arxiv.org/abs/1705.00401

6. J. S. Balakrishnan, N, Dogra, J.S. Muller, J. Tuitman, J. Vonk, Explicit Chabauty-Kim for the split Cartan modular curve of level 13 http://people.maths.ox.ac.uk/vonk/documents/p_cartan.pdf

7. B. Lawrence, A.Venkatesh, Diophantine problems and p-adic period mappings, arxiv.org/abs/1807.02721

8. A â€œlearning seminarâ€ for nonabelian Chabauty run by Bjorn Poonen at MIT: http://math.mit.edu/nt/old/stage_s18. html