
Course goals:

Math 116 is a course in real analysis and functional analysis (theory of normed infinite-dimensional vector spaces) with the mathematics motivated by optimization problems. The basic strategy is to find approaches that can be visualized in two or three dimensions and to use them as the inspiration for theorems that apply even in the infinite-dimensional case. This makes it possible to solve problems where the solution is a countably infinite set of values or a function. Classical calculus of variations is a well-known example of this approach.

The emphasis in Math 116 is on mathematics, not on algorithms for solving optimization problems, but there will be plenty of concrete examples.

Course format:

Before the pandemic, all lectures were captured on video. This frees up class time for interactive activities.

The class meets once weekly, on Fridays. Everyone is expected to watch two lecture videos, perhaps at 1.5 times normal speed, before coming to class.

The first half of class is devoted to a set of seven student presentations, mostly of proofs that were introduced in the videos. Generally these proofs are challenging enough that you need to see them at least twice! Everyone is expected to volunteer to present a proof almost every week.

For the second half of class, I introduce three problems based on the week's material, and the class breaks up into groups of three or four students to solve the problems on whiteboards, usually with lots of help from the course staff.

Typical enrollees:

• Math concentrators meeting the 11x requirement.

• Undergraduates meeting the optimization requirement for Applied Math concentration.

• Graduate students in Economics taking theory courses.

• Students doing a secondary field in Mathematical Sciences.

• Economics concentrators who are planning to go on to PhD programs or to work for firms that do sophisticated research.

Prerequisites for Math 116: You need to know linear algebra and multivariable calculus and be comfortable with proofs.

Math 22ab, 25ab, or 55ab is fine.

If you took Math 21, you should also have taken another course where you did proofs. Obvious candidates are Math 101, Math 112, Math 121, Math 130, Math 152, and Math 154. The intersection of the content of these six courses is approximately the empty set, but any of them should have got you to the point where you can read and understand a textbook at the level of the one we are using.

In the first month of the course, some of you will be reviewing real analysis while others are learning it seriously for the first time. Everyone is then well prepared for the rest of the course.

When is course typically offered?

Fall only. It is by no means certain that this course will be offered in the same format next year.

What can students expect from you as an instructor?

The course content looks forbidding, but the course style is surprisingly lighthearted. Students who take on the most difficult proofs get a nice round of applause from their classmates, and collaborating on the workshop problems is good fun. Homework and exam problems frequently involve fanciful references to

current events.

Assignments and grading:

Weekly problem sets are challenging, but they are the only way to master the course material.

The midterm and final exams both include proofs that are taken verbatim from lectures and presentations. The proof lists initially look forbidding. Here are examples from recent years.

[Midterm proof list](#)

[Final proof list](#)

As a practical matter, many students get perfect scores on the proof questions that appear on exams.

Your course grade will be determined as follows:

• problem sets, 50 points. Your worst score will be converted to a perfect score.

• midterm exam, 50 points

• final exam, 70 points

• Volunteering to present a seminar topic, 1 point each week (maximum of 10)

• Participation in problem solving and uploading solutions, 10 points

• (optional) for each proof list, 1 point extra credit for submitting a pdf file of one of the proofs for posting on the Web site

• Optional term paper or R Shiny app, up to 25 points added to numerator, but 80% of your score also added to the denominator.

The grading scheme is as follows:

Points Minimum Grade

93:0% A

87:0% A-

81:0% B+

75:0% B

69:0% B-

63:0% C+

57:0% C

51:0% C-

If you are conscientious about the homework, proofs, and quizzes, you should end up with a grade between B plus and A.

Reading list:

Textbook:

Optimization by Vector Space Methods, Luenberger, Wiley, ISBN# 0471-18117-X.

The author says in the preface,

"As a text it is aimed at first or second-year graduate students (at Stanford). "

He later comments,

"The mathematician...may wish simply to scan chapters 2,3,5, and 6 for review and then concentrate on the other chapters."

We will spend a lot of time on chapters 2, 3, and 5, filling in missing background as necessary, then do chapter 7 thoroughly.

Enrollment cap, selection process, notification:

If enrollment is unexpectedly large and a second classroom is available between noon and 1:30, we can split the course into two sections for the student presentations.

Past syllabus:

Here is the [syllabus from Fall 2023](#)

Absence and late work policies:

Volunteering for presentations, participating in solving workshop problems, and contributing solutions and

proof to the course website are a component of the grade.

Late assignments are a nightmare to grade. If they look fairly complete, they will receive a default score of 50%