Overview: This course aims to introduce the mathematical foundations and a few modern developments of Einstein's theory of general relativity. This is a very rich subject both from a physical and a mathematical perspective. As we will see throughout this course, a plethora of interesting phenomena emerges which impacts other fundamental topics of physics and mathematics such as fluid dynamics, partial differential equations, classical geometry & topology, gauge theory, etc. We now have observational evidence that black holes and gravitational waves exist in nature confirming Einstein's theory. Therefore it is prime time to study (and get involved in research) mathematical general relativity. Math and Physics graduate students and advanced undergraduates with knowledge of basic topology and geometry are welcome.

Possible Syllabus: Topics include but are not limited to the following

(1) Introduction to semi-Riemannian geometry, special relativity, null geometry

(2) The geometry of submanifolds in semi-Riemannian manifolds

(3) Global hyperbolicity, equivalence principle, and general co-variance

(4) Variational principle, Einstein's field equations, stress-energy tensor, Noether's theorem, and conservation laws,

(5) Cauchy problem for Einstein's equations

(6) Geodesic deviation, Jacobi fields, singularity theorems

(7) Characteristic initial value formulation, dynamical formation of trapped surfaces and black-holes

(8) Geometry of Schwarzschild, Kerr, and Kerr-Newman black-holes, FLRW, AdS, dS spacetimes

(9) Gravitational waves.

If time permits, we will also introduce perturbation theory, stability problems, the basic notion of mathematically rigorous quantum gravity, Connection with Teichmuller theory, and other special topics.

Prerequisite: Students should have taken topology, geometry courses at the level of MATH132,136. Knowledge of Riemannian geometry (at the level of MATH230A) will be helpful but not necessary. A few important basic concepts will be reviewed in the first few classes.

Grading: Homework will be assigned on a weekly/bi-weekly basis. Students interested in letter grades should submit their homework regularly. In addition, there will be a term paper/final project due at the end of the semester.

Recommended Textbooks: Semi-Riemannian geometry with applications to relativity by B. O'Neil, General relativity by Robert Wald. Gravitation by Misner, Thorne, and Wheeler (this is a classic textbook with excellent intuitive explanations; mainly a physics book). For curious students: General Relativity and Einstein's equations by Yvonne Choquet-Bruhat (this book provides an excellent concise introduction to several advanced topics)