**Mathematics115ï¼ŒFall 2019**

**Methods of Analysis**

Instructor: Tianqi Wu (email: mike890505@gmail.com)

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Office Hours (tentative): Mon., Fri., 1:00-5:00pmin Room 115, CMSA; or send me an email to schedule a meeting.

**General information:**

This is a course on mathematical methods in physics. The purpose of this course is to expose students to the type of mathematics that is used in intermediate and advanced physics classes. For examples: Functional analysis, Complex analysis, variation principle, Fourier transform and classification of partial differential equations.

The textbookfor this course is, â€œMathematical Methods in Physics and Engineeringâ€ By John W. Dettman.

Prerequisite: Calculus, Linear algebra, ODE, real analysis

Note:Mathematics 115 is especially for students interested in physics.

**Homework:**

Homework will be assigned weekly. Each assignment will be posted on Harvard Canvas, usually about a week before it is due. Assignments are due weekly at the start of class on each Tuesday. You are encouraged to discuss the homework with other students, but you must always write your homework solutions out yourself in your own words.

**Grades:**

Your course grade will be determined as follows:

1. Problem sets, 50 points.
2. Midterm exam, 20 points.
3. Final exam, 30 points

**Lecture Schedule (Outline):**

- 5 lectures: Hilbert Space
- 4 lectures: Calculus of Variations
- 4 lectures: Boundary Problems
- 8 lectures: Analytic Function Theory
- 3 lectures: Integral Transform Methods

**Lecture Schedule (detailed, tentative**).

- Lecture (9/3)

Introduction.

- Lecture (9/5) Chapter 1.5-1.7

Vector space, Basis and Scalar product

- Lecture (9/10) Chapter 1.8

Quadratic form, Hermitian form, Eigen value, Eigen vectors

- Lecture (9/12) Chapters 2.1-2.2

Hilbert space $l^2$, $L^2([a, b])$, completeness. Parseval relation and Theorem 1.

- Lecture (9/17) Chapters 2.2-2.4:

Corollary 1.1, Theorem 2 and subspace.

- Lecture (9/19) Chapters 2.5, 2.7:

Linear functional and continuity Theorem 1 on page 89.

- Lecture (9/24) Chapters 3.1, 3.2:

Lagrange multipliers, min-max definition of eigenvalues, Eulerâ€™s equation.

- Lecture (9/26) Chapters 3.2-3.3:

Eulerâ€™s equation, examples, Hamiltonâ€™s principle, Lagrangeâ€™s equations

- Lecture (10/1) Chapters 3.6, 3,7:

Wave equation, electro-magnetism, heat equation Schrodingerâ€™s equation

- Lecture (10/3) Chapters 3.8, 4.1:

Laplacian in different coordinates, coordinate transformations

- Lecture (10/8) Chapters 4.1-4.2 1 & pgs 202-209

Separation of variables, Legendreâ€™s equation, Helmholtz equation, associated Legendre equation and Besselâ€™s equation.

- Lecture (10/10) Chapters 5.1, 5.2:

Greenâ€™s function, Dirac delta function, Sturm-Louiville Greenâ€™s function.

- Lecture (10/15) Chapters 5.3, 5.4:

Dirac delta function, test function, generalized function and using the Greenâ€™s function to solve the Dirichelet problem.

- Lecture (10/17) Chapters 7.1-7.3:

Complex number, continuity, differentiability,*analytic*function and examples

- Lecture (10/22) Chapter 7.4, 7.5:

Length of a curve. Integrals over a curve, Theorem 1,2

- Lecture (10/24) Chapter 7.5:

Cauchy integral formula, Theorem 1,2, Liouvilleâ€™s theorem, Theorem 4 and examples.

- Lecture (10/29) Chapters 7.6:

Convergence, point wise and uniform convergence, Theorems 1,2, 3, several tests and series.

- Lecture (10/31) Chapter 7.7:

Theorem 1-3, analytic continuation, Laurent expansion, principle part, different singularities and definition of residue

- Lecture (11/5) Chapter 7.8:

Residue theorem, infinite cover and Laplace transform.

- Lecture (11/7) Chapter 7.9:

Theorems 1, 2, harmonic function, conformal mappings and examples

- Lecture (11/12) Chapter 7.10:

Poisson integral formula, Riemann mapping theorem, Greenâ€™s function for the disk, using the conformal map to solve the Neumann

- Lecture (11/14) Chapter 8.1:

Integrality, Plancherelâ€™s theorem, pointwise convergence, Lemma 2 and properties of Fourier transform

- Lecture (11/19) Chapters 8.1, 8.3:

Convolution, complex Fourier transform, application of Fourier transform on wave equation and the heat equation.

- Lecture (11/21) Chapter 8.5:

Laplace transform, properties and examples

- Catch up or review lectures: (11/26) and (12/3).