

Syllabus of Math 212

- I: This class is an advanced class in real analysis and its application. Main topics include advanced inequalities such as (logarithmic) Sobolev inequalities and heat kernel estimates. It is assumed that you are familiar with measure theory at the level of Stein's book Chap 1, 2, 3, although a short review will be provided.
- II: The last four weeks of the class will be a fast introduction to probability theory and stochastic analysis.
- III: Textbooks: (i) Lieb and Loss: Analysis; (ii) Evans: Partial differential equations; (iii) Stein and Shakarchi: Real Analysis. The material for the probability theory will be given later on in class.
- IV: Class time: 10:30-11:45, Tu, Thu. Office hour: 11:45-12:45, Tu, Thu.

Main topics of the class

1. Reviews of measure theory: Stein's book Chap 1, 2, 3
2. absolute continuity and mutually singular of measures, Lebeqgue-Rodon-Nikodym differentiation theorems. Stein Chap 6, section 4.
3. L_p spaces. Lieb Chap 2.
4. Integral inequalities: Young inequality, Hardy-Littlewood inequality, Hausdorff-Young inequality. Lieb Chap 4.
5. Fourier transforms. Lieb Chap 5.
6. logarithmic Sobolev inequality, heat kernel estimate and concentration inequalities.
7. Distribution and Sobolev spaces. Evans, Chap 5
8. Application to Probability theory: Law of large numbers, central limit theorem and large deviation.
9. Ito's calculus.