# SYLLABUS FOR FRESHMAN SEMINAR ON "MATHEMATICAL PROOFS FROM THE BOOK"

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## 1. Course Description

The Hungarian mathematician Paul Erdos used to talk about "The Book," in which God keeps the most beautiful proofs for mathematical theorems. Erdos' highest praise, upon learning from someone a new proof of a theorem, was "That's one from the book." In this seminar we will survey a collection of elegant results and proofs in mathematics, ranging in topic from number theory to geometry to combinatorics and graph theory, and including some of Erdos' favorite results. We will also discuss some of the history of the results and the mathematicians involved. A main text for the seminar will be "Proofs from the Book," by Aigner and Ziegler. Students will be expected to do short readings or problem sets each week, and to give a final presentation at the end of the course.

**Prerequisites**: While some exposure to proofs in mathematics may be helpful, this seminar aims to be accessible to students without a sophisticated math background.

## 2. Grading

There will be short problem sets or readings every week. Additionally, each student will give a short presentation on one of the last few days of class.

## 3. Getting the most out of this class

I would like this class to be as interactive as possible. So: please:

- Feel free to interrupt me if there is something you don't understand or have questions about.
- Provide feedback if my explanations are too fast or slow, confusing, etc.
- Come to office hours.

## 4. Collaboration policy

Collaboration with other students in the class on homework is permitted, but you must write up your solutions independently: you cannot just copy someone else's work. If you work with other people, you must mention on your problem sets who you worked with. *Uploading course materials, problem sets, or solutions to a third-party website or soliciting solutions on the internet (stack exchange, etc) is strictly prohibited.*

## 5. Policy on the use of AI

It is specifically forbidden to use ChatGPT or any other generative artificial intelligence (AI) tools at all stages of the work process, including preliminary ones. Violations of this policy will be considered academic misconduct. Note that different classes at Harvard could implement different AI policies, and it is the student's responsibility to conform to expectations for each course.

## 6. Disability statement

Harvard University values inclusive excellence and providing equal educational opportunities for all students. Our goal is to remove barriers for disabled students related to inaccessible elements of instruction or design in this course. If reasonable accommodations are necessary to provide access, please contact the Disability Access Office (DAO). Accommodations do not alter fundamental requirements of the course and are not retroactive. Students should request accommodations as early as possible, since they may take time to implement. Students should notify DAO at any time during the semester if adjustments to their communicated accommodation plan are needed.

## 7. Weekly schedule of topics

Note that the choice of topics may be modified, based on the backgrounds of the students in the class.

- Week 1: Background on Paul Erdos and the book. What is a proof? Techniques of proof. What makes a proof beautiful? Discussion of the *infinitude of primes*
- Week 2: How to color a map: the four and five-color theorems.
- Week 3: Number theory: representing numbers as a sum of two squares.
- Week 4: The birthday paradox and card shuffling.
- Week 5: Graph theory and Sylvester's problem on lines in the plane.
- Week 6: Graph theory and Euler's formula.
- Week 7: Irrational numbers: $e$ and $\pi$.
- Week 8: Pigeonholes and double counting
- Week 9: Determinants and the Gessel-Viennot Lemma.
- Week 10: Tiling rectangles and de Bruijn's problem.
- Week 11: Final presentations
- Week 12: Final presentations