

Zoom ID for class meetings: 947 3508 9259 with password 7m453w

Professor Laura DeMarco (demarco@math.harvard.edu)

Office hours: Monday 3-4pm, Tuesday 4-5pm Eastern time, or by appointment. **Zoom info for Office Hours:** 989 8875 1067 with password 607138.

[Lecture notes](#)

The goal of the semester is to study a collection of **Arithmetic Equidistribution Theorems** addressing the geometry of points with "small height" in a projective algebraic variety (defined over a number field). The case of heights on P^1 , especially those arising from dynamical systems, will play a key role. We will study topics such as:

- basics of complex dynamical systems for maps on P^1
- potential theory in C
- Berkovich spaces in dimension 1
- basics of p-adic dynamics
- potential theory on the Berkovich affine line
- height functions, adelic measures, and small points on P^1

and discuss the equidistribution theorems of many authors (Baker, Bilu, Chambert-Loir, Favre, Rivera-Letelier, Rumely, Thuillier, and Yuan), with ideas that go back to an important article of Szpiro-Ullmo-Zhang (1995) about abelian varieties.

Prerequisites. Standard graduate-level courses in Analysis, Algebra, and Geometry are expected. Background in dynamical systems and algebraic number theory are helpful but not required.

Course details. The course will run synchronously, by Zoom, on Mondays and Wednesdays at 1:30pm Eastern time. Attendance and participation are important to me, and questions are always welcome. Learning does not happen passively; all are encouraged to work on the suggested Exercises. If you will take this course for a grade, then you will need to submit solutions to (many of) these Exercises. Please schedule individual meetings with me during the first two weeks of the semester so that I can know more about you, your background, and your goals.

Useful references and textbooks

Complex:

John Milnor, Dynamics in One Complex Variable, Third Edition
Thomas Ransford, Potential Theory in the Complex Plane

Arithmetic:

Joseph Silverman, The Arithmetic of Dynamical Systems
Robert Benedetto, Dynamics in One Non-Archimedean Variable
Matthew Baker and Robert Rumely, Potential Theory and Dynamics on the Berkovich Projective Line
Enrico Bombieri and Walter Gubler, Heights in Diophantine Geometry