Spring 2021

D-modules in birational geometry

Professor Mihnea Popa

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Office Hours: Tuesday 1.30-2.30pm, and by appointment. Zoom link:

https://harvard.zoom.us/j/99458779129pwd=aTdnTkFTbW44bERMKytvdjZBY1hjQT09

Password: 462015

Lectures: Monday and Wednesday 10.30-11.45am, first lecture Monday Jan.25

Here is a brief overview of the topics we will cover: V-filtration and Hodge filtration on D-modules, and connections with singularities in birational geometry; vanishing and positivity theorems for Hodge modules; Hodge ideals; applications to singularities, holomorphic forms, generic vanishing, families of varieties, etc.

I will assume some familiarity with fundamental algebraic geometry, roughly at the level of Hartshorne's book, but will review various things when necessary. Some basic familiarity with D-modules, as in Chapters 1-3 in the book by Hotta-Takeuchi-Tanisaki "D-modules, perverse sheaves, and representation theory", would be very helpful. I have posted notes that I wrote for a quite different version of this course as a link on my webpage. What I will need from those are some basic facts about the Bernstein-Sato polynomial and about the V-filtration on D-modules, found in Chapters 1 and 2. (I will review those, however briefly and without proofs.) I will
indicate further sources as we go along, and will likely post lecture notes.

Course structure

Zoom meetings on Monday and Wednesday, 10.30-11.45am. I strongly encourage participation with camera on -- I understand that special circumstances might prevent this, in which case I would appreciate an email explanation. The Zoom button on the left column has links to all the lectures. In addition, I will have a set office hour plus office hours by appointment in case the time is not convenient.

I am currently not planning to have homework sets or an exam. There will be however various exercises scattered throughout the lectures, and you are certainly encouraged to do them. Moreover, if need be, for beginning graduate students or undergraduates we can find an accommodation involving some homework and/or a small reading project for a grade -- the main point is to get something out of it, regardless of the level.