Harvard College/Graduate School of Arts and Sciences: 111463

Term: Fall 2018-2019; first class on September 5, last on December 5.

Time: Wednesday, 9am to 10:30am

Room: SC 705 (Science Center, just north of Harvard yard, 7th floor)

Course instructor: Pierre Jacob

Link to the book: https://arxiv.org/abs/1803.00567

Course description:

Optimal transport (OT) is a branch of mathematics, related to probability, geometry and linear programming. It was first formalized by Gaspard Monge in the 18th century who considered the problem of optimally transporting resources from a number of sources to a number of destinations. It has been a very active and exciting subfield of mathematics over the last few decades, with Fields medals awarded to contributors Cedric Villani in 2010 and Alessio Figalli in 2018. Books on the topic include:

- Villani. (2008). *Optimal transport: old and new. Springer Science & Business Media.*
- Ambrosio (2003). *Lecture notes on optimal transport problems. Mathematical aspects of evolving interfaces. Springer, Berlin, Heidelberg. 1-52.*
- Ambrosio, Gigli (2013). *A userâ€™s guide to optimal transport.* In *Modelling and optimisation of flows on networks, pp. 1-155. Springer, Berlin, Heidelberg.*
- Santambrogio (2015). *Optimal transport for applied mathematicians. BirkÃ¤user, NY,* 99-102*.*
- *PeyrÃ©, Marco Cuturi (2018), Computational Optimal Transport, ArXiv:1803.00567.*

The field has started to inspire methods for machine learning and statistics, with e.g. the articles (in no particular order)

- El Moselhy, Marzouk. *Bayesian inference with optimal maps*. *Journal of Computational Physics* 231.23 (2012): 7815-7850.
- Cuturi. *Sinkhorn distances: Lightspeed computation of optimal transport*. In *Advances in neural information processing systems*, pp. 2292-2300. 2013.
- Tabak, Trigila. *Data-driven optimal transport*. *Commun. Pure. Appl. Math. doi* 10 (2014): 1002.
- Papadakis, PeyrÃ©, Oudet. *Optimal transport with proximal splitting*. *SIAM Journal on Imaging Sciences* 7, no. 1 (2014): 212-238.
- Solomon, De Goes, PeyrÃ©, Cuturi, Butscher, Nguyen, Du, Guibas. *Convolutional Wasserstein distances: efficient optimal transportation on geometric domains*. *ACM Transactions on Graphics (TOG)* 34, no. 4 (2015): 66.
- Alvarez-Melis, Jaakkola, Jegelka. *Structured Optimal Transport*. In *International Conference on Artificial Intelligence and Statistics*, pp. 1771-1780. 2018.
- Courty, Flamary, Tuia. *Domain adaptation with regularized optimal transport*. In *Joint European Conference on Machine Learning and Knowledge Discovery in Databases*, pp. 274-289. 2014.
- Heng, Doucet, Pokern, *Gibbs Flow for Approximate Transport with Applications to Bayesian Computation,* https://arxiv.org/abs/1509.08787
- Arjovsky, Chintala, Bottou. *Wasserstein GAN*. https://arxiv.org/abs/1701.07875
- Bernton, Jacob, Gerber, Robert. *Inference in generative models using the Wasserstein distance*. https://arxiv.org/abs/1701.05146

Some of these methods have been made possible by advances in the computational side of OT. The goal of this course is to learn about these computational aspects by going through the recent book:

Gabriel PeyrÃ© and Marco Cuturi, Computational Optimal Transport, ArXiv:1803.00567, 2018.

We will aim at reading all chapters, approximately one per week. A volunteer will lead the discussion every week. All participants are expected to have read the material beforehand so that the discussion can focus on the more challenging and interesting questions.

The abstract of the book is below.

> This book reviews OT with a bias toward numerical methods and their applications in data sciences, and sheds lights on the theoretical properties of OT that make it particularly useful for some of these applications. Our focus is on the recent wave of efficient algorithms that have helped translate attractive theoretical properties onto elegant and scalable tools for a wide variety of applications. We

also give a prominent place to the many generalizations of OT that have been proposed in but a few years, and connect them with related approaches originating from statistical inference, kernel methods and information theory.

Course requirements:

It is strongly recommended to read the first two chapters of the book before deciding whether to join the course. We welcome participants from different backgrounds, and different fields, but the goal is to cover the entire book, so the intended level is that of advanced graduate students in statistics, probability or related fields. A useful reference for some of the methods is the book:

- Bertsimas, Dimitris, and John N. Tsitsiklis. *Introduction to linear optimization*. Vol. 6. Belmont, MA: Athena Scientific, 1997.

Schedule

- Week 1 (September 5): we have stopped around pages 17-18.
- Week 2 (September 12): Espen presented, we have stopped just before Section 2.5.
- Week 3 (September 19): Stephane and Han presented, we have stopped at the end of Chapter 2.
- Week 4 (September 26): Wenshuo and Yucong presented, we have stopped at the end of Section 3.5.
- Week 5 (October 3): Feicheng Wang presented (from Section 3.5 onwards).
- Week 6 (October 10): Tiancheng Yu and Renbo Zhao presented (Chapter 4).
- Week 7 (October 17): Tiancheng Yu and Renbo Zhao presented (Chapter 4); we're done with up to 4.4.
- Week 8 (October 24): Jeremy Heng presented (end Chapter 4 and some of Chapter 7).
- Week 9 (October 31): Jeremy might take 10 minutes to discuss some elements of Chapter 7, and Chenguang Dai will discuss Chapter 8.
- Week 10 (November 7): Jonathan Weed presented some of his research on "Mirror descent, the entropic penalty, and near-linear time optimal transport".
- Week 11 (November 14): David Alvarez Melis presented some of his research on "Transporting with additional information: invariance and structure in machine learning applications of optimal transport".
- Week 12 (November 21): Thanksgiving (no class).
- Week 13 (November 28): Last session of the term: Eric Dunipace presents last part of Chapter 8, and then we will all discuss open problems.
- Week 14 (December 5): no session.

Typos

If you spot any typos in the Computational Optimal Transport book, feel free to add them to the collaborative list by following this link: https://v2.overleaf.com/7869625363jtghcbcsvmsd
(you do need an Overleaf account).

This is a shared .tex document, so please make sure it compiles properly after making any changes (i.e. be considerate of others and do not leave the document with compiling errors, the preview on the right panel should display properly with no error message after you're done).

If you are not familiar with LaTeX, you can simply follow the lead and mimic the formatting of the already listed typos. All you need to know is that:
>>> typos are listed within the "begin{itemize} ... end{itemize}" environments, with one environment for each chapter.
>>> each typo is introduced via the following command

%
\\p{x}{y} comment
%

where your inputs consist in replacing:
- "x" by the page number
- "y" by an approximate location on the page (e.g. top, middle, bottom, Remark + number, etc...)
- "comment" by a description of the typo.
(the "%" is just here to separate items so that the .tex file is more readable).

Thanks to Stephane Shao for this shared document!