

Math 274

Complex and Arithmetic Dynamical Systems

Monday and Wednesday 12-1:15pm

The focus of the semester will be on complex-analytic techniques for studying dynamical systems and arithmetic geometry, especially in dimensions bigger than one. Specifically, we will work through the basics of plurisubharmonic functions, currents on complex manifolds, and pluripotential theory. We will see how these tools are used in dynamical applications. If time permits, we will also study their p-adic (i.e., Berkovich-analytic) analogs. The topics covered in this course are motivated by connections between complex-algebraic dynamical systems and arithmetic geometry.

Expectations: Students will be asked to work on **projects**, on topics of their choosing but related to the themes of the course. There will be presentations at the end of the semester. If you plan to take this course for credit or for a grade, then the presentation and a write-up of what you learned will be required. I will suggest readings and exercises, but these will **not** be graded. I will not take attendance, but it is important to me that you attend lectures and engage actively with the material.

Prerequisites: Standard graduate-level courses in Analysis, Algebra, and Geometry. Background in dynamical systems and algebraic number theory are helpful but not required.

Lecture notes: [Math274_LectureNotes_draft.pdf](#) (updated April 24, 2023)

[Math295_LectureNotes_Spring2021.pdf](#) (last edited on April 28, 2021)

[AWS_LectureNotes.pdf](#) (for the Arizona Winter School -- still a work in progress! updated March 5, 2023)

Useful references

Dinh-Sibony course notes from 2005 [Introduction to the theory of currents](#)

Demailly [Complex Analytic and Differential Geometry](#)

Klimek **Pluripotential Theory**, London Math Society Monographs, published by Oxford University Press. A [scan of Chapter 2](#).

Krantz **Function Theory of Several Complex Variables**, available electronically through the Harvard library.

Siu's article in the Bulletin of the AMS (1978): [Siu_LeviProblem1978.pdf](#)

Gauthier-Vigny [Geometry dynamical Northcott and Bogomolov properties](#)

Suggested homework:

Monday, Jan 23. Skim through the first 5 chapters of the Dinh-Sibony notes, and work through ALL of the exercises in Chapters 1 and 2. You'll observe that Dinh and Sibony have not included any proofs. During the next few lectures, I will highlight what I consider most important and prove some of the key results that we will need and provide additional references.

Monday, Jan 30. Read Sections 1.3 and 2.1 from Krantz and Sections 2.1-2.8 of Klimek on subharmonic functions. On currents: Read Sections 1 and 2 from Chapter I in Demailly. Work through the exercises from Chapter 3 of Dinh-Sibony.

Monday, Feb 6. Read Sections 3-5 of Chapter I in Demailly, read Section 2.9 of Klimek, read and do the exercises from Sections 6.1 and 6.2 of Dinh-Sibony.

Thursday, Feb 16, from Jit Wu Yap: [Complex Dynamics Problem Sheet 1.pdf](#)

Thursday Feb 23, from Jit Wu: [Complex Dynamics Problem Sheet 2.pdf](#)

Wednesday, March 1, from Jit Wu Yap and Yan Sheng Ang, prepared for the Arizona Winter School: [AWS Complex Dynamics and Unlikely Intersection Problem Session.pdf](#)