Math 121: Linear Algebra

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Welcome to Math 121! Math 121 is a second course in linear algebra. In your first linear algebra course, you approached much of the material by doing calculations. Math 121 instead emphasizes understanding linear algebra at a more abstract level, so this course will help you develop your skill at thinking about theoretical mathematics. We'll also focus heavily on communicating mathematics rigorously, through definitions and proofs. You will develop your ability to use mathematical language carefully; become familiar with standard mathematical objects and types of proof; and be able to read, understand, and write proofs.

Many of the topics we'll study are ones that you've already been introduced to in your first linear algebra course, but we'll take them a bit further. These include real and complex vector spaces, linear transformations, and eigenvalues and eigenvectors. We'll also investigate some questions that you probably didn't look at in your prior linear algebra course, like how close we can get to diagonalizing a non-diagonalizable matrix. Along the way, we'll encounter a wide variety of fundamental structures that appear in many other areas of higher mathematics.

Prerequisites

You should have taken a linear algebra course such as Math 21b.

Course Meetings

Math 121 is designed based on the idea that **you will learn better by actively doing math**. This isn't just an opinion; a meta-analysis of 225 studies found that "Active learning increases student performance in science, engineering, and mathematics". Our class meetings will be a mix of interactive lessons and small group problem solving. Your participation in class is crucial; to appreciate the mathematical process, you must actively participate in it! You will be asked to suggest ideas, share your mathematical arguments, and give constructive criticism of your classmates' arguments.

Problem sessions

Our course assistants will hold a weekly problem session, in which you will have the chance to ask questions on course material and to work on challenging problems. These are optional but highly recommended, especially if this is your first proof-based course.

Homework

As we all know, the best way to learn mathematics is by doing mathematics, so homework is the most important component of this course. You will have two kinds of problem sets, described below.

Weekly problem sets

Most of your homework will be in the form of weekly problem sets, due Tuesdays at 10:30 am (except for a last short assignment, which will probably be due Friday 4/26).

Mathematical communication is a key goal of Math 121, so you should write clearly and coherently, using complete sentences. Unlike in computational math classes, you will almost certainly need to first figure out how to solve a problem and then, as a separate step, figure out how to communicate your proof clearly.

A good rule of thumb for deciding the level of detail in your written work is that another student who has taken Math 121 should be able to easily understand your solution.

In order to solve problems creatively, you need time to puzzle over them, so be sure to **start the problem sets early**!

Late days (for weekly problem sets only)

We understand that occasionally things may come up that prevent you from giving full attention to homework. Therefore, we'll give you 3 "late days" for the semester; each late day allows you to extend a due date by 24 hours. You will not be permitted to use late days for assignments due right before an exam, so that we can post solutions before the exam. You may use at most two late days on a single assignment; to be fair to your CAs, who are grading your work, and to enable us to post solutions in a timely manner, **no work will be accepted more than 48 hours after the deadline**. You don't need to let us know when you use a late day; simply submit the assignment as usual. If you use more than 3 late days over the course of the semester, each extra late day you use will reduce your semester homework average by 0.75 percentage points.

Late days are meant to give you some flexibility. For example, if you forget to upload your homework or realize that you accidentally uploaded the wrong file, or you're very busy with other courses, use a late day. But if you're dealing with more extenuating circumstances, please don't hesitate to email me so that we can work out appropriate accommodations; please either include supporting documentation or cc your resident dean.

Preparatory problem sets

You will have a short assignment due each Thursday at 10:30 am that is intended to prepare you for the material we'll be discussing in class that day. This will enable us to make the best use of our limited class time together. Because the goal of these preparatory problem sets is to prepare you for the class, there are no late days, and preparatory problem sets will be graded on completion; you will receive full credit as long as you show evidence of genuine thought and effort.

Submitting homework

You should submit each assignment as a PDF to Gradescope. You may type your solutions or write them by hand. If you're typing, I encourage you to use LaTeX to typeset your work; this is commonly used in math, science, and engineering. You can find more information about getting started with LaTeX on the course website.

I understand that technology issue do arise. So, we'll have a 15-minute grace period for homework deadlines. That means that preparatory problem sets that are less than 15 minutes late will still be accepted, and problem sets that are less than 15 minutes late won't use a late day.

Collaboration & Academic Integrity

I encourage you to collaborate with your classmates on homework assignments and to discuss your arguments and approaches. However, **you must write up your work independently and in your own words, and you should only submit work that you feel comfortable explaining clearly to another student and to the course staff**. In addition, you should cite any outside source (including fellow students) to which you refer. Under no circumstances should you turn in a solution that you have copied from a classmate, a textbook, the internet, or any other source.

In general, I discourage you from trying to get help on the internet or from students who have taken 121 or related courses. The point of the problem sets is for you to build your own mathematical muscles; asking for help from outside experts is like asking someone else to lift weights for you — it doesn't build

your skills at all!

Generative artificial intelligence (GenAI) tools such as ChatGPT can be a useful tool for developing your understanding of a concept, but you should not ask such tools to solve your homework problems.

Here are some examples of acceptable and unacceptable things to ask a source (whether that's a GenAI tool, person, website, etc.):

✓ Examples of acceptable questions:

- What's the difference between "for some" and "for all"?
- How do I visualize linearly independent vectors?
- What does it mean to show a function is well-defined? When do we need to actually prove this?

✗ Examples of unacceptable uses of support:

- Asking a person, GenAI tool, or other source to solve a specific homework problem for you.
- Asking another student to show you their homework
- Copying a solution from any source

Finally, you may not share any course materials (worksheets, homework, and so on) with anyone not enrolled in Math 121, with the exception of a peer tutor.

Exams

Taking a test on material strengthens your ability to use that material; therefore, exams are a crucial part of the learning process. Tentative dates are:

Exam 1	Thursday 2/22, in class
Exam 2	Wednesday 3/27, 5:30 - 7:30 pm
Final Exam	scheduled by registrar

Textbooks

There is no required textbook for Math 121. I will post recommended reading from multiple textbooks that are freely available online. All of the relevant links will be on Canvas.

Grading Policy

Your participation in each class will be graded on this scale:

Full credit	You arrive on time and prepared, and you stay on task and engaged in class activities for the entire class period.
Half credit	You are on task and engaged for most but not all of the class (for example, because you arrive late, leave early, or aren't focused on class material for part of the time).
No credit	You are on task and engaged for less than half of the class period.

We will excuse two class absences for the semester. If you need to miss class because of a Harvard-sponsored event like athletic travel, a serious illness, or an extenuating circumstance, we can excuse additional absences; please email me with supporting documentation or cc'ing your resident dean.

Your course grade will be determined by taking the higher of:

- 35% homework + 5% class participation + 15% Exam 1 + 20% Exam 2 + 25% final exam

- 35% homework + 5% class participation + 10% Exam 1 + 17.5% Exam 2 + 32.5% final exam

We will use the following table to convert your numerical score to a letter grade, with the caveat that you must score over 50% on the final exam to guarantee yourself a C- or above.

You will guarantee yourself a grade of at least...	...by earning a numerical score of at least...
A	93
A-	90
B+	87
B	83
B-	80
C+	77
C	73
C-	70
D-	60

Inclusion

We are all responsible for creating and sustaining a learning environment that is respectful and welcoming for everyone. We all bring different perspectives, experiences, and mathematical backgrounds, and I ask that you engage in discussion with care and empathy for each other.

If you ever have concerns, please don't hesitate to come and talk with me.

Course Accessibility

I am committed to providing an accessible and welcoming academic community. If you need accommodation or assistance for a documented disability, please get in touch with the Disability Access Office as soon as possible so that they can arrange accommodations for you in all of your courses.