Welcome to Math 19b!
Linear Algebra, Probability and Statistics for the Life Sciences

Instructor / course head: Rosalie Bélanger-Rioux (rbr@math.harvard.edu)
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Course meeting time and place: MWF 10:30-11:45am, in room SC 109.

The subject: linear algebra, probability and statistics, with a focus on its applications to the life sciences.

The course covers essentially all of the linear algebra covered in Math 21b or Applied Math 21b (meaning we will not cover the differential equations – you might be interested in taking Math 19a to get your differential equations fix). Meanwhile, the probability and statistics taught here has overlap with the contents of Stat 110 and Stat 111. The course teaches the subjects of linear algebra, probability and statistics hand in hand with applications to the life sciences.

More specifically, from linear algebra we will focus on matrices, eigenvalues, eigenvectors, determinants, and applications to probability, statistics, dynamical systems; in the probability and statistics part of the course we'll cover standard models and techniques, and their uses including the central limit theorem, Markov chains, curve fitting, and regression. Aside from that, successful completion of this course will provide you with a level of sophistication in mathematics that will serve you in your future science courses, and in your future scientific career.

Who should take this course: This course is recommended if you are considering Math 21b or Applied Math 21b and are also thinking of concentrating in one of the life sciences, chemistry, or ESPP. You should also take this course if you have taken Math 19a, but it may also be taken with Math 21a. (If you are coming from Math 19a you will have seen already much of the differential equations that are taught in Math 21b, which is why 19b would be better for you.)

Students who have seen some multivariable calculus (especially vectors, lines and planes in three dimensions) can take Math 19b before Math 19a. This course, when taken for a letter grade, meets the General Education requirement for Empirical and Mathematical Reasoning or the Core area requirement for Quantitative Reasoning.

Prerequisites: Either Math 1b with a satisfactory grade, or an AP Calculus BC score of at least 4, or appropriate scores on the Harvard University Math Placement Tests.

Resources for help: Something that we emphasize a great deal in this course is the ability to apply concepts to *new* situations. Therefore, you should expect homework problems to look different from the problems we discuss in class. We offer many resources outside of class, and I highly encourage you to take advantage of them:

- *Matlab Bootcamp:* We will use Matlab for assignments. You can attend a bootcamp if you like (info online).
- *Office hours:* Come and ask anything! Those are times when I'm available to answer your questions about the course or the homework or the project. No need to make an appointment, just come by! If you can't make it to my scheduled office hours, you are always welcome to email me, and we can set up another time to meet. I'll announce the times of my regular office hours soon, after I've had a chance to look over your schedules.
- *CA office hours and problem session:* Our undergraduate Course Assistant (CA) will hold office hours and a problem session. Times will be posted on the course website soon. The CA also grades your homework and tallies your Daily Problems.
- *Notes, Handouts, References:* You'll be going to the course website often for your daily problem and assignments. I will be posting solutions and handouts used in class. Our reference for the linear algebra part of the course is *Linear Algebra with Applications* by Otto Bretscher. As for the probability, statistics, and applications parts of the course, we will use course notes written and modified by previous instructors and by me, posted on the course website: `https://canvas.harvard.edu/courses/49495`
- Each other! Please write down contact info of other students in the class:

Class meetings and problem sessions: The class meeting time is Mondays, Wednesdays and Fridays from 10:30-11:45am in Science Center Room 109. There will also be (optional) problem sessions and office hours run by the course staff, to be arranged during the first week of class. The schedule and rooms for these will be posted on the Math 19b web site.

Homework: Problems are posted on the course web site. These are due on the Wednesday of the subsequent week unless noted otherwise. Solutions will be posted to the website as well. You are strongly urged to work the homework problems that are assigned on any given day before the next course meeting. These problems are designed to help you understand the material so as to be ready for its applications in subsequent classes. In particular, any given class will seem easier (and you will learn more efficiently) if you have already done the homework from the previous day. In fact, learning math is (even though this might not be the impression you have had from previous math classes) very much based in discovering, in asking questions, in following leads, in talking about the math with others. This is the attitude you want to strive to have as you are working on the material, in and out of the classroom. You are very much encouraged to discuss the homework with your fellow students and form study groups to work on these assignments. However, you must write up the solutions by yourself, and note the names of your collaborators. See the Academic Integrity Policy (link in the left menu) for more details.

Your lowest weekly homework score *that is above 50%* will be dropped when computing your average homework grade. Because the course builds on itself, it is important you stay on top of homework and that I be able to post the solutions in a timely fashion. For this reason, late homework assignments will *not* be be accepted, except in two circumstances: if you have a doctor's or a dean's note; or if you use a "grace day." I will allow each student to use up to 2 grace days total during the semester, to be used for assignments only, allowing you to submit them late. So you can use 2 grace days (48 hours) on one assignment or twice one grace day (24 hours each time).

Daily Problems: In addition to the weekly homework, there will be a daily problem assigned each day and due the following class. The idea is two-fold: First, I'd like to encourage you to attend class. The class will be run interactively. Your presence and participation is important, not only for you, but for the other students in the class as well (did I mention I encourage collaboration?). The second reason is that I'd like you to get the most out of class, so the daily problem often will be a transition from one day's material to the next day's.

Midterm evaluations: 2 midterms, 1.5h each in the evening, in late February/early March and late March/early April.

Final evaluation: A final project. The final project can be done in teams of two. If you would prefer to do it alone ask me first. The final project will have a written part and an "oral" part. The "oral" part could be a presentation in front of the class, a video you make and show the class, a puppet show…whatever you are comfortable with! (Instead of doing a formal presentation in front of class, some of you might prefer to present your work at the Cambridge Science Festival, more details on that soon!) In order to give some practice and an opportunity for feedback before your final project, a mini-project (as part of a homework) will be done early on in the semester.

Extra-credit opportunity: for up to an extra 2 points on your total course grade, you may write a few paragraphs summing up an interview you did with either a professor or TF at Harvard or elsewhere in the life sciences who uses the math from 19b, or someone who works in the life sciences and uses the math from 19b. It's a great excuse for you to do some networking, reaching out to a prof you're interested in, etc etc! Your write-up should be about 50% on the math used, and 50% on the application... plus 10% on that person's story if you find it interesting.

Grading: Your final grade will be the max of either scheme below:
1) Homework 30%, Daily Problems 5%. Midterm 1 15%, Midterm 2 20%, Final Project 30%.
2) Homework 30%, Daily Problems 5%. Midterm 1 20%, Midterm 2 15%, Final Project 30%.
3) If you do the Matlab bootcamp, it will count for 5%, and I will remove 5% from the weight of your lowest score out of midterm 1 or 2.

Online solutions: Please do not use solutions to problem sets you may find on online websites, and please do no post such things yourself. You would hinder your ability to learn, or that of future students, and would be in violation of the course's academic integrity policy.

Tentative schedule and list of topics:

Week One: An Introduction to Probability, and sign up for a 10min individual meeting
M Jan 28 Thinking with data
W Jan 30 Randomness & Probability
F Feb 1st Conditional Probability and Bayes' Theorem

Week Two: Intro to Linear Algebra and Matlab beginnings
M Feb 4 Transitioning to Linear Algebra / Gaussian Elimination (Row Echelon Form)
W Jan 6 Reduced Row Echelon Form and solutions of systems
F Feb 8 Rank, Dot Product, and Matrix-Vector Multiplication

Week Three: Matrices as linear transformations, and prepare for your mini-project
M Feb 11 Image and Kernel, solutions – putting it all together!
W Feb 13 Linear Transformations
F Feb 15 Matrix Products and Inverses

Week Four: Random variables and mini-projects!
M Feb 18 No Class: Presidents' Day
W Feb 20 Mini-project presentations! Geometry and Linear Transformations
F Feb 22 Random Variables

Week Five: Probability Distributions, study for midterm 1 next week, and submit an article you might be interested in for your final project
M Feb 25 Counting and probabilities
W Feb 27 Discrete Probability Functions
F Mar 1 Chebychev's Theorem / Review for midterm

Week Six: Intro to Statistics, midterm 1 on Th the 7th (tentative) in evening (material up to Mar 1)
M Mar 4 P Values
W Mar 6 Independence, Covariance and Correlation
F Mar 8 More on Correlation, start of bases

Week Seven: Bases, Coordinates, and intro to model testing
M Mar 11 Bases and Coordinates
W Mar 13 More on Bases and Coordinates
F Mar 15 Model Testing and Maximum Likelihood

Spring Break!

Week Eight: Statistical Model Testing, Preparation for Curve Fitting, and submit a paragraph-long final project proposal
M Mar 25 The Bayesian approach to statistics
W Mar 27 Orthonormal Bases, Projections
F Mar 29 Projections and Gram-Schmidt

Week Nine: Curve Fitting
M Apr 1 Curve fitting, and the least squares method
W Apr 3 Curve Fitting, connections to probability
F Apr 5 Continuous Probability Distributions

Week Ten: Continuous Probability and the Central Limit Theorem, and study for midterm 2
M Apr 8 The Exponential Distribution / The Normal Distribution / A Standard Normal Distribution Table
W Apr 10 Central Limit Theorem and Hypothesis Testing
F Apr 12 More on the Central Limit Theorem and Hypothesis Testing

Week Eleven: Determinants and Spectral Theory, midterm 2 on Th the 18th (tentative) in evening (material up to Apr 12)
M Apr 15 Review class
W Apr 17 The Determinant
F Apr 19 Spectral Analysis / Markov Matrices

Week Twelve: Dynamical Systems and Markov Chains
M Apr 22 Discrete Dynamical Systems
W Apr 24 Continuous Dynamical Systems
F Apr 26 Complex numbers / Symmetric Matrices (The Spectral Theorem)

Week Thirteen: SVD and PCA
M Apr 29 Symmetric Matrices (The Spectral Theorem) / Singular Value Decompositions
W May 1 Applications of the SVD / Principal Component Analysis

Reading week: work on your final report, you'll get feedback on your draft.

Week of exams: Final presentations and submit final written report.