Math262a: Quantum theory from a geometric viewpoint I

Fall, 2023

Dan Freed

Overview: This is the first semester of a year-long course centered around concepts in quantum mechanics and quantum field theory. We treat the subject from a mathematical perspective that emphasizes structural aspects and eventually applications to mathematics. Some lectures will be devoted to background mathematics. Physics topics in the first semester include: quantum mechanics, bosons and fermions, flat spacetimes, relativistic quantum mechanics, relativistic quantum field theory, Segal's axioms for Wick-rotated theories. I anticipate mathematical lectures on Clifford algebras and the spin group, symmetry types in geometry, the spectral theorem, sheaves, . . .

While I will review many mathematical ideas, I will not develop them in detail in this course. I will give references for you to learn more, and I hope that you will delve in at some point. My goal is that you emerge from this course with a deeper perspective on how different parts of mathematics work together. My focus is the mathematical structure of quantum theory, not physics applications per se, though there will be many of those as well. Bringing quantum field theory into mathematics is an ongoing longterm project; this course is a snapshot of aspects of my current understanding.

Prerequisites: This is a lecture course on the application of mathematical ideas, techniques, and viewpoints to a subject outside of mathematics. Such applications draw freely on whatever mathematics is needed with no walls between its subdisciplines (analysis, algebra, geometry, representation theory, etc.). As such the course places large demands on your mathematical knowledge and sophistication. I will assume exposure to material at least at the level of basic graduate courses, but in many cases beyond. That said, you can get lots out of the course without knowing this mathematics in advance.

Course Assistants: Yuhan Jiang, Science Center 426i
                  Clair Dai, Science Center 431h

Lectures: Monday, Wednesday 12:00–1:15 in Science Center 507. There is no lecture on September 25 or November 15. I will make up those lectures on two Fridays: September 29 and November 10.

Office Hours: I will hold office hours Mondays 1:30–2:30 and Thursdays 2:00–3:00 in Science Center 503. (My office is in a construction zone for a few weeks, so I will announce an alternate location.) I strongly encourage you to come to office hours (individually or in groups) to discuss lectures and problems, ask more general questions, explore ideas, etc.

CA Office Hours/Problem Session: Yuhan and Clair will hold a weekly problem session Tuesdays 5:30–7:00 in Science Center 232.

Class Website: Canvas. Lecture notes, problem sets, and additional readings will be posted there.

Texts: Nothing official. I plan to write lecture notes to go along with the lectures; they will be posted on Canvas. But you should take notes in lecture to generate a primary source for yourself. I will also post supplementary readings on Canvas.

Problem sets: I will post problem sets on Wednesdays. Problems are often open-ended and provide a gateway into the material. I don't expect you to solve all problems, but the more you engage the more you'll learn. Undergraduates registered for the course should hand in writeups of some of the problems on the following Wednesday. I strongly suggest you work on the problems in groups of 3 ± 1.5 (rounded up). I also encourage the groups to exchange solutions to problems

to get feedback. I do not encourage the use of Chat GPT for the problems, but if you do succeed in getting AI to solve some problems then please do come and show me. That said, I hope you'll focus on using NI unaided by computers. Discord (see below) is a good platform to communicate about the class.

Projects: Undergraduates registered for the course will write a short paper (5–10 pages). I will suggest topics as we go and you may propose your own. There is broad latitude in the choice of topic. Papers are due December 11. Please choose your topic and get my approval by November 10.

Discussion forum: Yuhan set up a Discord Server that you can join using the following link:
<div align="center">

https://discord.gg/DHAJpJsj
</div>

This is an excellent place to discuss material in the class, problems on the problem sets, to form discussion groups, etc. Please check it out.

Remark: Above all I hope you have fun in this course. I will challenge you in lectures, and I hope to give you interesting and challenging problems. When you get frustrated please seek help—from classmates, from me, or from whomever you have around to help. If you never get frustrated, give it a bit of time, but if the problem persists then please come see me; I'll arrange for something more challenging. (I'm serious.)