Math 279Z

Cohen-Macaulay Modules and Vector Bundles

Tu-Th 9:00-10:15 SC 309

Harvard University, Fall 2022

url: *https://canvas.harvard.edu/courses/110729*

â—Š **Instructor:** Yuriy Drozd, SC 340, email: *drozd@math.harvard.edu*, *y.a.drozd@gmail.com*

personal webpage: *https://www.imath.kiev.ua/~drozd*

â—Š **Office hours: Th 2-3:30pm** by **zoom** or by assignment.

â—Š **Contents:** A survey of the results and methods in the classification of vector bundles over singular projective curves and Cohen-Macaulay modules over curve and surface singularities. If time allows, related questions on derived categories and Yang-Baxter equations (marked with *).

1. Vector bundles over projective curves.

Generalities on vector bundles. Vector bundles over the projective line and over elliptic curves. Higher genera and the idea of *wild problems.* Vector bundles over nodal cubic and over cycles of projective lines. Technical tool: bunches of chains. Tame/wild dichotomy. Stable vector bundles and bricks. Stable vector bundles over nodal and cuspidal cubics. *Applications to Yang-Baxter equations. *Derived categories of coherent sheaves on singular curves.

2. Cohen-Macaulay modules over curve singularities.

Generalities on Cohen-Macaulay modules over one-dimesional commutative rings. Auslander-Reiten quivers. Examples. ADE singularities and finite CM-type. Hypersurface singularities and KnÃ¶rrer periodicity. Semi-continuity and its applications. T_{pq}-singularities and tame type. Tame/wild dichotomy.

3. Cohen-Macaulay modules over surface singularities.

A survey on normal surface singularities and their resolutions. Cohen-Macaulay modules over surface singularities. Du Val (ADE) singularities, quotient singularities, McKey correspondence and finite CM-type (by Esnault and Auslander). Relations with vector bundles over projective curves (Kahnâ€™s correspondence). Minimal elliptic singularities. T_{pqr}-singularities, their resolutions and criterion of tameness for Gorenstein surface singularities. *Some results on non-isolated singularities.

Prerequisites. Basic algebraic geometry (e.g. Chapters 2-3 of Hartschorn â€œAlgebraic Geometryâ€) and commutative algebra (e.g. Atiyah-Macdonalds â€œIntroduction to Commutative Algebraâ€ or Chapters 1-6 of Matsumura â€œCommutative Algberaâ€). More details will be discussed and explained during lectures.

Recommended Literature:

- Yu. Yoshino. Cohen-Macaulay Modules over Cohen-Macaulay Rings. Cambridge University Press, 1990.
- J. Le Potier. Lectures on Vector Bundles. Cambridge University Press, 1997.
- Yu. Drozd. Vector bundles over projective curves. Rio de Janeiro, 2008. (available at *www.imath.kiev.ua/~drozd/textbooks.html*)
- M. Atiyah. Vector bundles over an elliptic curve. Proc. London Math. Soc. 7, 414-452, 1957.
- C. Kahn. Reflexive modules on minimally elliptic singularities. Math. Ann. 285, 141-160 (1989).
- D.Eisenbud. Homological algebra on a complete intersection, with an application to group representations. Trans. Amer. Math. Soc. 260, 35-64, 1980.
- H. KnÃ¶rrer. Cohen-Macaulay modules on hypersurface singularities. I. Invent. Math. 88, 153-164, 1987.
- H. Esnault. Reflexive modules on quotient singularities. J. Reine Angew. Math. 362, 63-71, 1985.

- Yu. Drozd. Vector bundles and Cohen-Macaulay modules. Representations of Finite Dimensional Algebras and Related Topics in Lie Theory and Geometry. Field Institute Communications 40. AMS, 2004, 189--222. (arXiv:math.AG/0310368)
- L. Bodnarchuk et al. Vector Bundles and Torsion Free Sheaves on Degenerations of Elliptic Curves. Global Aspects of Complex Geometry. Springer, 2006, 83-128. (available at *www.imath.kiev.ua/~drozd/KLsurvey.pdf*)
- G. J. Leuschke, R. Wiegand. Cohen-Macaulay Representations. Math. Surveys and Monographs 181, AMS, 2012.
- I. Burban, B. Kreussler. Vector bundles on degenerations of elliptic curves and Yang-Baxter equations. Mem. Aner. Math. Soc. 220, no. 1035 (2012).

The course is intended for advanced graduate students. Undergraduate enrollment requires the instructorâ€™s permission. Enrolled students should attend the course regularly. For those requiring a letter grade assignments will be provided.