Math 281Z

Homological Algebra

Harvard University, Spring 2023

Tu-Th 9:00-10:15 SC 111

â—Š **Instructor:** Yuriy Drozd, SC 340

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â—Š **Office hours: Th 10:30-11:30** or by appointment

â—Š **Contents:** The course will cover the main topics in homological algebra:

Complexes and homology, examples from analysis and topology. Homotopical category, its triangular structure. Projective and injective modules, projective and injective resolutions. Derived functors, functors Ext and Tor. Homological dimensions, Hilbert syzygies theorem, dimensions of local rings. Koszul complexes and their use. Homologies of groups, Tate cohomologies. Homologies of Lie algebras. Cohomologies of sheaves. Spectral sequences and their use. The spectral sequence of Lyndon/Hochschield-Serre. The spectral sequence of KÃ¼nneth. Eilenberg-Zilber theorem. Derived categories, their construction. Derived functors, their properties and use.

Prerequisites. Basic courses in algebra (Math 122 and 123). Elements of the theory of categories are desirable but not required. Some topics and especially examples will use some knowledge of analysis (e.g. differential forms) and topology.

Recommended Literature:

- A. Weibel. An Introduction to Homological Algebra. Cambridge University Press, 1997.
- H. Cartan, S. Eilenberg. Homological Algebra. Princeton University Press, 1956.
- S. Gelfand, Yu. Manin. Methods of Homological Algebra. Springer, 1996.
- N. Bourbaki. AlgÃ¨bre, Ch. X. AlgÃ¨bre homologique. Springer, 2006.
- J. Rotman. An Introduction to Homological Algebra. Springer, 2009.
- S. MacLane. Homology. Springer, 1995.

The course is intended for graduate students and advanced undergraduate students. Enrolled students should attend the course regularly. For those requiring a letter grade assignments will be provided.