Syllabus Ay140: Introduction to General Relativity

- **Course Information**

  Course number: Astron 140

  Course title: Introduction to General Relativity

  Term: Fall 2024

  Time: Monday and Wednesday, 3pm to 4:15pm

  Location: Science Ctr 300H

- **Contact Information**

  - Instructor: Xingang Chen
    Email address: Xingang.Chen@cfa.harvard.edu
    Office: CfA B-228
    Office hours: TBD, over zoom at `https://harvard.zoom.us/j/8015170608`.
  - Teaching Fellow: TBD
    Email address: TBD
    Office hours: TBD
    Students are encouraged to use these sessions to complete the course materials with help from the TF and peers.

  There is also a class Slack workspace that you can access from the Canvas website under the Slack tab. In addition to visiting the office hours, you are also encouraged to ask questions using Slack, which has the additional advantage that all students can benefit from the discussion.

- **Course Objective**

  Recent exploration of black holes and gravitational waves have opened an experimental and observational window to the deepest secrets of gravity. These physical phenomena are described by one of the most profound theories of Nature discovered by humankind – the theory of general relativity. This course will introduce students to the basics of the theory of general relativity and some of its key applications, including gravitational lensing, black holes, gravitational waves and cosmology.

  In this course, students will learn:

  - Newtonian physics versus Einstein's relativity.
  - Spacetime in special relativity.
  - The equivalence principle.
  - Gravity as geometry and curved spacetime.
  - How do objects move in curved spacetime?
  - The celebrated Einstein equations.

- Key experimental confirmations of general relativity.
- Relativistic stars.
- What is black hole? How do they form?
- The extreme properties of gravity and spacetime near black holes.
- What are gravitational waves?
- Sources and generation of gravitational waves.
- Experimental detection of gravitational waves.
- If time allows, use relativity to model the universe as a whole (cosmology).

Einstein famously spent eight years searching for the mathematics that describes his theory. Some advanced mathematics (differential geometry) plays a fundamental role in the theory of general relativity. We will introduce these mathematics gradually throughout the course in a way that is accessible to prepared undergraduate students.

General relativity in many universities is traditionally a graduate course (but nowadays increasingly popular in undergraduates worldwide for good reasons – so Harvard needs to keep up). I will explain in the first lecture why this does not have to be the case at all. While it's true that you will often find yourself struggling through the content and exercises, the reality is that even graduate students face the same challenges with the vast majority of these intellectual pursuits.

Upon completing this course with significant effort, you will not only feel a sense of pride for having acquired a whole new level of knowledge at such a young age, but you will also be better equipped to appreciate and/or further explore the cutting-edge research on the relativistic universe.

- **Prerequisites**

  Multivariable calculus (e.g. Math 21A), linear algebra and differential equations (e.g. Math 21B), college-level Mechanics including special relativity (e.g. Physics 15A), and E&M (e.g. Physics 15B).

- **Textbooks**

  There is no specific required textbook for this course. Taking notes during the lectures should be sufficient. Students are encouraged to discuss with me and TFs, as well as classmates. On the other hand, having a reference book on such a subject does help a lot. I recommend any of the following textbooks. I will describe the specialty of each book below, and you may decide which one suits you the best. My lecture will be closer to the combination of the first three books.

  - Bernard Schutz's "A first course in general relativity". The treatment of this book is concise and more mathematical. It gets to modern applications pretty fast. This book does not mention the action principle (that students learn in advanced mechanics) anywhere, so has the least prerequisites.

- James Hartle's "An introduction to Einstein's general relativity", and Ta-Pei Cheng's "Relativity, gravitation and cosmology". These two books present more physical pictures with less mathematics. At a few places, the action principle is used. Students can skip these if this concept is unfamiliar (and refer to lectures). The full machinery of the Einstein equations are slowly introduced through gradual intermediate steps, till the late stage.
- Anthony Zee's "Einstein gravity in a nutshell". This book has the slowest and most patient presentation of all four books. It is also combined with the history of physics, many vivid explanations and coverage on other branches of physics, which you may enjoy independently from the main equations. The book uses the action principle and group theory as the guiding principles, and the technical part gradually becomes quite advanced comparing to what we plan to teach. But you do not need to read all of them. This book has nearly 900 pages.
- Sean Carroll's "An introduction to general relativity". This book is more at the graduate level. It uses the Lagrangian and action principle which we do not require in this course, and introduces mathematics in a more advanced language. If you have learned advanced mechanics and are familiar with some of these concepts, this can a good reference as its treatment is clear and modern. The lecture notes that this book grew out of is publicly available at https://arxiv.org/abs/gr-qc/9712019.
- There are many other excellent graduate-level textbooks on general relativity (e.g. by Misner/Thorne/Wheeler, Weinberg, Peebles, Wald) with different emphases. It is not a bad idea to have them instead (including some books mentioned above), if you anticipate continuing to study related subjects in the future. If you are not sure, you are welcome to talk with me.

Some of the above recommended books are available in Harvard Library as e-books that are accessible for Harvard community. Please check out the "Library Reserve" tab on Canvas to find out the updated listing.

- **Course Outline**

Sessions	Contents
1	Introduction
2	Special relativity 1
3	Special relativity 2
4	Manifolds
5	Vectors & tensors
6	Metric 1
7	Metric 2
8	Covariant derivatives
9	Riemann tensor
10	Geodesics
11	Gravitational lensing 1
12	Einstein equations 1
13	Einstein equations 2
14	In-class mid-term exam
15	Black hole solutions 1
16	Black hole solutions 2
17	Gravitational lensing 2
18	Black holes in astronomy
19	Hawking radiation
20	Gravitational waves 1
21	Gravitational waves 2
22	Generation of gravitational waves
23	Detection of gravitational waves
24	Relativistic cosmology (TBD)
25	Take-home final exam

- **Course and Grading Policy**

  - Homeworks (50%)

    Homework is the most important practice after lectures. Students should try to solve the homework problems as independently and carefully as possible. Collaboration and discussions with classmates, TFs and the instructor on homework is encouraged, but you must ensure that anything submitted is the result of your own work and reflects your own approach to the topic, after the discussions. See Collaboration Policies for more details.

    Students are expected to not only solve the problems, but also contemplate thoroughly about the physical pictures emerged from the computation, in connection to lectures. Relativity is counter-intuitive. It applies to realms far beyond the physical environment of our everyday life, based on which we have been building up our intuition about physics. Part of the goal of this course is to correct our normal intuitions about spacetime, mass, energy, force, and gravity. To achieve that, besides lectures, reading and discussions, a more important and irreplaceable practice is to explicitly work out the mathematics involved, think and learn about their physical implications, and then repeat this cycle at increasingly more advanced levels many times.

  - Midterm exam (20%)

    The midterm exam will be closed-book in-class exam. Two pages of "cheat sheets" are allowed to write down whatever materials that might help to remind you about the contents of the lectures. Laptops are allowed only to use computer programs to compute the Riemann tensors (to be learned in the class). Calculators (or equivalent laptop programs) are allowed.

  - Final exam with course projects (20%)

    The final exam will be take-home open-book course projects, which deal with more physical, hence more interesting and intensive, problems. Students are expected to work independently; collaboration and consultation is not allowed. Date and deadline will be announced in class.

  - Participation (10%)

    Students are expected to attend all lectures unless there are special reasons. Questions and discussions are encouraged during and after lectures. It rarely happens that someone does not have many questions and does not make many mistakes about the theory of relativity while learning, so please don't be shy. Office hours and problem sessions provide additional opportunities for such interactions.

- **Academic Integrity and Collaboration Policies**

  Any material submitted to meet course requirements – homework assignments, papers, projects, examinations – is expected to be a student's own work. Collaboration on studying and on homework assignments is encouraged, but you must ensure that anything submitted is the result of your own work and reflects your own approach to the topic. If unsure, ask the instructor or TFs. Students must make note of any collaborators, and any resources employed apart from the course textbooks and lecture notes when submitting work. This includes discussions with other students, websites, and other books and course notes.

All members of the Harvard College community are expected to abide by the Harvard College Honor Code, which states:

Members of the Harvard College community commit themselves to producing academic work of integrity – that is, work that adheres to the scholarly and intellectual standards of accurate attribution of sources, appropriate collection and use of data, and transparent acknowledgement of the contribution of others to their ideas, discoveries, interpretations, and conclusions. Cheating on exams or problem sets, plagiarizing or misrepresenting the ideas or language of someone else as ones own, falsifying data, or any other instance of academic dishonesty violates the standards of our community, as well as the standards of the wider world of learning and affairs.

- **Policy on course material distribution**

  Please refrain from distributing any of the course materials outside this class, both to specific individuals and through untargeted channels. This includes the homework assignments, exams, their solutions, and lecture notes.

  Your cooperation is greatly appreciated and contributes to a fair and enriching learning environment for all current and future students.