Math212b: Real Analysis

Instructor: Christian Brennecke (brennecke@math.harvard.edu), Science Ctr. 239
Office Hours: Monday & Wednesday 5:00pm – 6.00pm (tentative)

Class: Monday & Wednesday 9:00am – 10:15am in Science Ctr. 310

Prerequisites: Basic knowledge in measure & integration theory, Sobolev spaces and functional analysis; in particular, Math 114 and Math 212a

Course Website: https://canvas.harvard.edu/courses/49408 On the course website, you find announcements, homework assignments, information on the project, and so on. Please check the course website regularly.

Course Objectives and Content: As a continuation of Math 212a, we discuss further topics in real analysis, harmonic analysis and functional analysis. As a warm-up, we recall in the first week some basics about L^p- and Sobolev spaces and discuss the extension of Sobolev functions as well as the trace of Sobolev functions on smooth domains. After that, I am planning to discuss:

- **Calculus of Variations:** basic functionals and Euler-Lagrange equations, null Lagrangians, existence and uniqueness of minimizers, variational problems with constraints and applications, critical points, Palais-Smale condition with applications to nonlinear PDE, Mountain Pass Lemma, introduction to minimax theory and closed geodesics on spheres
- **Oscillatory Integrals in Fourier Analysis:** Riesz interpolation theorem, oscillatory integrals, Fourier transform of surface carried measures, averaging operators and Fourier restriction theorems, TT^* lemma, application to the linear homogeneous, inhomogeneous and a nonlinear Schrödinger equation
- **Spectral Theory of Self-Adjoint Operators:** unbounded operators in Hilbert spaces, characterization of self-adjoint operators, basic examples and properties of self-adjoint operators, Riesz-Markov Theorem, Spectral Theorem for bounded and unbounded self-adjoint operators, applications of the Spectral theorem: solution to the Schrödinger equation, min-max principle, existence and uniqueness of ground states

Textbooks:

Main references:

- Evans, *Partial Differential Equations*
- Stein & Shakarchi, *Functional Analysis*
- Reed & Simon, *Methods of Modern Mathematical Physics*, vol. I, II

Additional references:

- Lieb & Loss, *Analysis*
- Struwe, *Variational Methods*
- Dacorogna, *Introduction to the Calculus of Variations*

- Giaquinta & Hildebrandt, *Calculus of Variations I*
- Reed & Simon, *Methods of Modern Mathematical Physics*, vol. IV

Problem Sets: The problem sets should complement the lecture and provide you with a possibility to test your understanding of its content. There will be several problem sets, each posted on Wednesdays every two weeks on the course website. You will have two weeks to hand in your solutions, typeset in LaTeX. Also if you do not need a grade for this course, you are very welcome to hand in your solutions if you wish to have some feedback.

Project: Instead of having a final exam, you will prepare a project on a topic of your choice (subject to my approval) and present it in class at the end of the term. The project is an opportunity for you to explore an advanced topic in real analysis of your interest and related to the general course content of Math 212a and Math 212b. At the end of the term, you will give a short presentation in class and produce a written report on the topic. More details, a list of possible topics as well as the due dates for proposal, first draft and final draft of the project will be posted on the course website during the semester.

Grading: If you need a grade for this course, the grade will be based on your solutions of the problem sets and on your final project.