Math 132: Differential topology

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Spring 2024

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Office: Science Center 341 (inside the Mathematics Department's Birkhoff Library)

Canvas page: https://canvas.harvard.edu/courses/123145 Class Hours: Tuesday/Thursday 1:30pm - 2:45pm in SC507.

Student Hours: Thursdays 9:30am-11:30am in SC341, & by appointment. (Immediately after

class is another good time to talk to me.)

Course description

This is a course covering fundamental results in the topology of smooth manifolds. These are the general class of spaces on which "one can do calculus," and they lie at the heart of our modern understandings of geometry and physics. We will see how to answer questions like: What kind of shapes can manifolds take? How can we distinguish them? How do different features of a manifold's geometry and topology interact with each other? This course will draw on tools from analysis, linear algebra, and topology as we investigate these questions.

Prerequisites

This course will make extensive use of the tools of multivariable calculus, especially derivatives of functions $\mathbb{R}^m \to \mathbb{R}^n$. We will also rely heavily on the notion of open and closed subsets of \mathbb{R}^n . If you need a refresher on these concepts, I recommend consulting a basic text in real analysis or topology.

Course materials

The textbook for the course is Differential Topology, by Guillemin and Pollack. An online copy can be found at http://www.ams.org.ezp-prod1.hul.harvard.edu/books/chel/370/

¹This is what used to be called "office hours," only there's now a movement to rebrand them as "student hours" to ensure students know that they should be coming to them. You should come to my student hours!

Course policies

Discussion questions

Before each meeting of the course (except Day 1 and midterm exam days), you are required to post a question on the Discussion Board on the course's Canvas page. These questions should not be about homework exercises, but they should relate to the material we have been discussing, or are preparing to discuss, in class, or which has been assigned for readings. Your question should display evidence that you have been thinking seriously about the mathematics covered in the course.

Each question should be in a new thread (which Canvas calls a "Discussion") with a clear, descriptive subject line. (For instance, "Why do we study intersections modulo 2?") You are also encouraged to post answers or comments in response to your classmates' questions. These questions are helpful to me because they give me a sense of which concepts you do and don't understand as the course progresses, and they should be helpful to you for the same reason.

Problem sets

Problem sets will be due every week at **11am on Wednesday**. Late assignments will not be accepted, but your lowest three problem set scores will be dropped. If there are extenuating circumstances which will regularly prevent you from turning in problem sets on time, please e-mail me.

Grading policy

The grades for the course will be calculated as follows:

- 10% of your grade is based on class participation. To get a complete score on this portion, you should post a discussion question before each class, attend class consistently and be engaged during class.
- 40% of your grade is based on problem sets.
- The remaining 50% of your grade will be determined by your exam scores. There will be one midterm exam and one final exam, and your exam score will be a weighted average of these, calculated as the higher among the following possibilities:
 - 20% midterm, 30% final
 - 10% midterm, 40% final

That is, if you perform poorly on the midterm but better on the final exam, the midterm will be weighted less in the final calculation.

Student hours

On Thursdays from 10:00am - 12:00pm, I will hold "student hours" in my office, which is Room 341 in the Science Center. These are for you! I encourage you to come visit me to ask questions about mathematics or anything else.

Student work policy

All work turned in must be your own. You may not copy any portion of your submitted assignments, either from another human being or from a generative software program, and you should not search online for answers to homework exercises. You may work together with your classmates, but everything you write in your submissions must be in your own words.

Disclaimer

This syllabus is a statement of my intentions for the structure of the course but is not a binding document, and I reserve the right to modify it at any time.