We will begin the first several weeks with the study of bilinear complexity and fast algorithms for matrix multiplication, including developing the language of linear algebra we need, Strassen's algorithm, the reduction of circuit complexity of matrix multiplication to rank of the matrix multiplication tensor, Bini's theorem and border rank, Schonhage's asymptotic sum inequality, Strassen's laser method, Coppersmith and Winograd's application of the laser method, and improvements to Coppersmith and Winograd's approach. We will continue onto other topics as there is interest, such as the lower bound problem for rank and border rank, circuit complexity of polynomials and the determinant vs permanent problem, etc.

I will assume familiarity with vector spaces, linear maps, determinants, polynomials, formal power series. We will introduce other notions as they are needed, such as the tensor product of vector spaces, vector space duals, symmetric tensors and their correspondence under polarization with polynomials, and actions of the general linear group by changes of basis on tensors and polynomials. Some familiarity with classical algebraic geometry is helpful but not necessary (affine/projective algebraic sets and varieties).

There is no text for the course. We will consult a number of sources, such as P. BÃ¼rgisser, M. Clausen, M. A. Shokrollahi "Algebraic Complexity Theory," J.M. Landsberg "Geometry and Complexity Theory" and J.M. Landsberg "Tensors: Geometry and Applications." I may occasionally cite results without proof from I. Shafarevich's "Basic Algebraic Geometry I"

Regular attendance is expected. There will be weekly homework, posted on Canvas and on this page every week, due the following week. Homework counts for 100% of the final grade. You are encouraged to collaborate on the homework problems, but you must write your own solutions and properly acknowledge any collaboration, or help you receive from others.