Classes: Wednesdays and Fridays 3-4:15 pm Eastern time.
Zoom link: https://harvard.zoom.us/j/91470578955?pwd=TjZkQWZrb2UwRUpqWVpFNVZqNVkrUT09

Office hours: Fridays 4:15-5:15, or by appointment.

Some resources:

- Notes available online:
  - Notes on Descriptive Set Theory by Anush Tserunyan, available on her webpage.
  - Notes on Descriptive Set Theory by Dave Marker, available on his webpage.

- Books:
  - Invariant Descriptive Set Theory by Su Gao. (We will cover some of the material from there, especially about Polish groups.)
  - Classical Descriptive Set Theory by Alexander Kechris. (For more background on Classical Descriptive Set Theory.)
  - Set Theory by Thomas Jech. (For more background on Set Theory.)
  - An Introduction To Independence Proofs by Kenneth Kunen. (For more background on Set Theory.)

Please contact me at shani@g.harvard.edu if you want to talk about the course, discuss the topics, or if you have any questions.

The course will focus on the general study and classification of definable equivalence relations. Equivalence relations arise naturally in mathematics:

1. Most prominently, equivalence relations arise as the relation of isomorphism between some mathematical structures. For example, isomorphism between groups, fields, rings, topological spaces, and so on...
2. Given a group G acting on a space X, it induces an orbit equivalence relation defined by xEy âŸº (âˆƒg âˆˆ G)g â‹… $x = y$.

We will want to understand such equivalence relation, as much as possible, and compare the complexity of different equivalence relations. In example 1, this corresponds to classifying objects up to isomorphism, and finding the best possible complete invariants for such classification. In example 2, we want to understand what kind of actions the group G can have. The space X will be a Polish space (separable complete metric space), and the group G a Polish group. (For example, groups such as the unit circle, R, product groups such as Z^N and R^N, or the symmetry group $S_{\text{âˆž}}$ of all permutations of N.)

We may also discuss many applications of set theory, and forcing in particular, to the study of equivalence relations.

The topics will be flexible, and may include:

- Some basics of Descriptive Set Theory: Polish spaces, Borel and analytic sets, Standard Borel spaces.
- Polish groups, continuous and Borel actions of Polish groups. Universal actions of Polish groups.
- Borel equivalence relations and Borel reducibility (comparing the complexity of definable equivalence relations).
- The permutation group $S_{\text{âˆž}}$, logic actions, the Scott analysis, classifications by countable structures.
- Actions of compact groups are "concretely classifiable".
- Actions of locally compact groups are reducible to actions of countable groups.
- Hjorth's theory of turbulence (when classification by countable structures is not possible), and its connections to forcing.
- Pinned equivalence relations (this is a forcing-theoretic condition allowing, for example, to distinguish the actions of groups such as $S_{\text{âˆž}}$ vs abelian groups). Pinned cardinals and their uses for Borel reducibility.
- The use of symmetric models (models of ZF where the axiom of choice fails) to study Borel reducibility and classification problems.
- Applications to concerete classification problems.

- **Course Instructor:** Assaf Shani
- **Meeting Time:** Wednesday 03:00 PM - 04:15 PM; Friday 03:00 PM - 04:15 PM
- **Exam Group:** FAS04_F
- **Course Description:** A problem in mathematics is classifying objects up to some notion of isomorphism. Famous examples include: the classification of compact orientable surfaces up to

homeomorphism by their genus and classification of Bemoulli shifts up to isomorphism by their entropy. Descriptive set theory allows for a precise study of the complexity of various classification problems and the possible invariants which they admit. Topics: Polish groups and their actions on Polish spaces, definable equivalence relations, classifications problems and invariants, and interactions between these topics and forcing. For example, we will develp Hjorth's theory of turbulence, which provides a method for showing that certain isomophism problems cannot be classified by any "reasonable invatiants", and give an equivalent condition in terms of forcing, recently introduced by Larson-Zapletal. The topics will be flexible depending on students' interests.