

Math 155R will be an introduction to enumerative and algebraic combinatorics. The course will introduce several classes of combinatorial objects (permutations, Dyck paths, trees) as well as some classical methods used to enumerate them. The course will then cover topics related to the combinatorics of the representation theory of the symmetric group such as Young tableaux, the hook-length formula and the Robinson-Schensted-Knuth correspondence.

The main references will be:

- * Richard Stanley: Enumerative Combinatorics I & II
- * Bruce Sagan: The Symmetric Group
- * Philippe Flajolet, Robert Sedgewick: Analytic Combinatorics.

Prerequisites: Basic linear algebra and also basic abstract algebra (such as from Math 122).

The course professor is Houcine Ben Dali (bendali@math.harvard.edu). Contact Prof. Ben Dali for more information about Math 155R.

The course will meet on Wednesdays and Fridays during the Fall 2024 semester from 9am to 10:15am at Science Center 228.

Grades: There will be 5 problem sets, each worth 10% of your grade, as well as a midterm worth 20%. The remaining 30% will be based on a final in-class presentation or a final written project.

Course website: <https://sites.google.com/view/houcine-ben-dali/teaching>

Tentative schedule :

Lecture 1: Introduction

Lecture 2: Combinatorial classes, generating functions

Lecture 3: Some classical combinatorial objects

Lecture 4: Tree structures

Lecture 5: Lagrange Inversion

Lecture 6: Cartier Foata monoids

Lecture 7: The Transfer-matrix Method

Lecture 8: Lindström's Gessel-Viennot lemma

Lecture 9: Labelled structures , exponential generating functions

Lecture 10: Cayley's formula

Lecture 11: Planar maps and Tutte recursion

Lecture 12: Representation theory of finite groups

Lecture 13: Characters

Lecture 14: Frobenius formula, generating function of bipartite maps

Midterm

Lecture 15: Specht Modules

Lecture 16: RS correspondence

Lecture 17: Some properties of the RS correspondence

Lecture 18: Jeu de Taquin

Lecture 19: Hook-length formula

Lecture 20: The space of symmetric functions

Lecture 21: Schur functions

Lecture 22: RSK correspondence for semistandard tableaux and Cauchy identity

Lecture 23: Jacobi-Trudi identity