Math 277x: Quantum geometry

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Fall 2020

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Course description

The end goal of this course is a discussion of applications of the theory of differential equations with irregular singularities to nonperturbative phenomena in quantum field theory. But how (or even whether) we get there depends on the preferences and background of students enrolled in the course. In my current vision for the course, it is split into three parts:

Introduction: "What we talk about when we talk about QFT"

We begin the course with a modern introduction to perturbative quantum field theory and BV quantization. These notions are best formulated in the language of derived algebraic geometry; we do not require any prior background in this area, although a working knowledge of homological algebra and chain complexes will be very helpful.

Quantization

We will have seen in the first part of our course a disjuncture between the "classical" information of the space of solutions to the Euler-Lagrange equations, and the "quantum" information of integrals over that space. In this portion of the course, we will systematize some aspects of that process via the theory of BV quantization, and we will relate it to other procedures that go by the name of "quantization," especially geometric quantization and deformation quantization, which we will spend some time explaining.

Applications

In the final part of the course, we will address current research directions and applications of the topics discussed earlier. These will depend on the available time and student interest, but they will likely revolve around questions regarding quantization of holomorphic Lagrangians in a hyperkähler manifold.

References

- For background in QFT:
  - Hori et al., *Mirror Symmetry*, Clay Math.
  - Witten, lectures from *Quantum Fields and Strings, Vol. 2*, AMS.
- On BV quantization:
  - Mnev, *Lectures on Batalin-Vilkovisky formalism and its applications in topological field theory*.
- On geometric quantization:
  - Bates-Weinstein, *Lectures on the Geometry of Quantization*.
  - Gukov & Witten, "Branes and quantization."

  On deformation quantization:
  - Fedosov, "A simple geometrical construction of deformation quantization."
  - Kontsevich, "Deformation quantization of Poisson manifolds."
  - Kontsevich, "Deformation quantization of algebraic varieties."
  - Calaque & Halbout: "Weak quantization of Poisson structures."
- Possible additional topics:
  - Malgrange, *Equations différentielles à coefficients polynomiaux*.
  - Iwaki-Nakanishi, "Exact WKB analysis and cluster algebras."
  - Kuwagaki, "Sheaf quantization from exact WKB analysis."
  - Gukov-Mariño-Putrov, "Resurgence in complex Chern-Simons theory."

Course policies

Office hours

Although Zoom is useful for lectures or seminars, it is less useful for a less structured environment like office hours, where splitting into groups or having informal discussions are important, and I think it is more useful to hold office hours in a sofrware environment with more functionality. Therefore all office hours for the course will be held in Runescape. During office hour times I will be available in Draynor village.

Grading policy

Any student taking this course for a grade must complete a final presentation, on a topic to be agreed upon with the instructor.

Disclaimer

This syllabus is a statement of my intentions for the structure of the course but is not a binding document, and I reserve the right to modify it at any time. Also the Runescape thing is just a joke, my office hours are on Zoom.