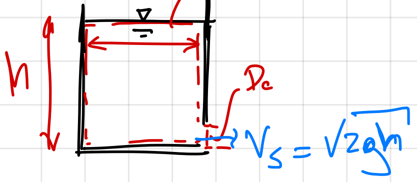
Ejercicio 1.



$$0 = \frac{d}{dt}\int \rho\, d\forall + \int \rho \vec{V}\cdot\hat{n}\, dA$$

$$= \rho \frac{d}{dt}\int d\forall + \sum_{sale} \rho V A - \sum_{entra} \rho V A$$

Supuestos

(i) Fluido incompresible

(ii) Valores promedio entrada/salida

$$0 = \rho \frac{d\forall}{dt} + \sum_{sale} \rho V A - \cancel{\sum_{entra} \rho V A}^{\,0}$$

$$\forall = \frac{\pi D^2}{4} \cdot h(t) \longrightarrow$$

$$0 = \cancel{\rho} \frac{\cancel{\pi} D^2}{\cancel{4}} \frac{dh}{dt} + \cancel{\rho} V_s \frac{\cancel{\pi} D_c^2}{\cancel{4}}$$

$$\frac{dh}{dt} = -\left(\frac{D_c}{D}\right)^2 V_s = -\left(\frac{D_c}{D}\right)^2 \sqrt{2gh}$$

$$-\int_{h_1}^{h_2} \frac{dh}{\sqrt{h}} = \left(\frac{D_c}{D}\right)^2 \sqrt{2g}\, \Delta t$$

$$\boxed{\Delta t = \frac{2}{\sqrt{2g}} \left(\frac{D}{D_c}\right)^2 \left(\frac{h_1^{3/2} - h_2^{3/2}}{3}\right)}$$

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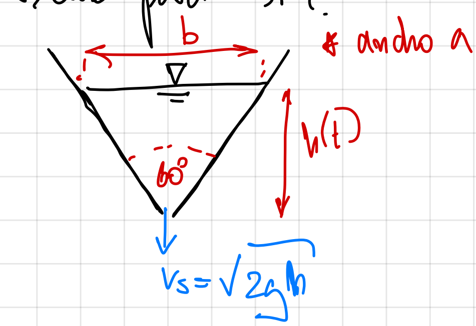
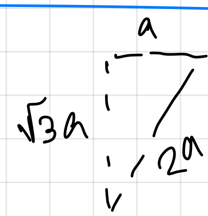
* ¿Qué pasa si?



$$\forall = \frac{1}{2} b \cdot h a = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} h \cdot h a$$

$$= \frac{1}{\sqrt{3}} h^2 \cdot a$$

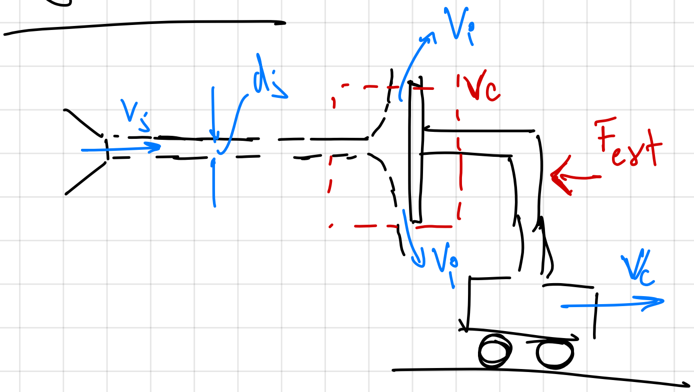


$$0 = \rho \frac{d\forall}{dt} + \rho V_s \cdot A_s \Rightarrow \rho \frac{d}{dt}\left(\frac{1}{\sqrt{3}} a h^2\right) = -\rho A_s \sqrt{2gh}$$

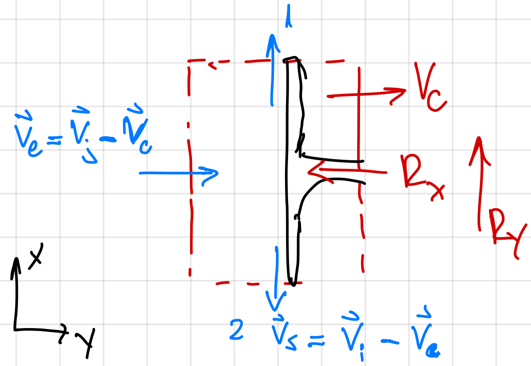
$$= \frac{\rho}{\sqrt{3}} a \cdot 2h \frac{dh}{dt} = -\rho A_s \sqrt{2gh}$$

$\vdots$

Ejercicio 2.



Analizamos el volúmen de control



$\vec{V}_e = (V_j - V_c)\,\hat{x}$ $\qquad$ $\vec{V}_s^{(1)} = V_i\,\hat{y}$

$\vec{V}_s^{(2)} = V_i\,(-\hat{y})$

Supuestos
- Flujo incompresible
- Prop. constantes
- Valores promedio entrada/salida
- Problema estacionario

Cons de masa

$$0 = \cancel{\frac{d}{dt}\int \rho\, d\forall} + \int \rho\, \vec{V}\cdot\hat{n}\, dA \implies 0 = \underbrace{2\rho V_i A_s}_{\dot{m}_s} - \underbrace{\rho (V_j - V_c)\pi \frac{d_e^2}{4}}_{\dot{m}_e}$$

$$\dot{m}_s = \frac{1}{2}\dot{m}_e = \frac{1}{2}\dot{m}$$

Cons momentum lineal

$$\sum F_{ext} = \cancel{\frac{d}{dt}\int \rho \vec{V}\, d\forall} + \int \rho \vec{V}\vec{V}\cdot\hat{n}\, dA = \sum_{sale} \dot{m}\vec{V} - \sum_{entra} \dot{m}\vec{V}$$

$$= \frac{1}{2}\dot{m}\vec{V}_s^{(1)} + \frac{1}{2}\dot{m}\vec{V}_s^{(2)} - \dot{m}\vec{V}_e$$

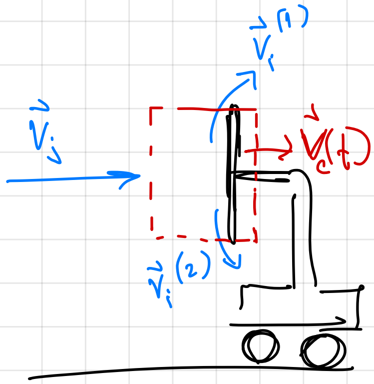
$$\vec{F}_{ext} = \cancel{\frac{1}{2}\dot{m}V_i\,\hat{y}} + \cancel{\frac{1}{2}\dot{m}V_i(-\hat{y})} - \dot{m}(V_j - V_c)(\hat{x})$$

$$\implies R_y = 0$$

$$R_x = \dot{m}(V_j - V_c) = \rho (V_j - V_c)^2 \pi d_e^2/4 = 7{,}5\,N$$

extra

¿Qué sucede si el carro no tiene una fuerza que lo frene?
Analicemos el movimiento del carro desde el reposo $V_c(t=0)=0$



$$\vec{F}_{ext} = 0 = \frac{d}{dt}\int \rho \vec{V} d\forall + \int \rho \vec{V}\vec{V}\cdot\hat{n}\, dA$$

$$0 = \underbrace{\frac{d}{dt}\left(\rho \vec{V}\forall\right)}_{\text{propiedades cte dentro del V.C.}} + \underbrace{\sum_{sale} \rho \vec{V} Q - \sum_{entra} \rho \vec{V} Q}_{\text{valores promedio entrada/salida}}$$

$$0 = \rho\forall \frac{d\vec{V}_c}{dt} + \cancel{\frac{1}{2}\dot{m}\vec{V}_s^{(1)}} + \cancel{\frac{1}{2}\dot{m}\vec{V}_s^{(2)}} - \dot{m}\vec{V}_e$$

$$\Rightarrow \rho\forall \frac{dV_c}{dt} = \dot{m}\left(V_j - V_c\right)$$

$$\frac{dV_c}{dt} = \frac{Q}{\forall}\left(V_j - V_c\right)$$

$$-\ln\left(V_j - V_c\right)\Big|_0^t = \frac{Q}{\forall}t$$

$$\ln\left(\frac{V_j - V_c}{V_j - V_0}\right) = -\frac{Q}{\forall}t$$

$$\Rightarrow V_c = V_j - \left(V_j - V_0\right)e^{-Q/\forall \cdot t}$$

Cond inicial $V_c(0) = V_0 = 0$

$$V_c = V_j\left(1 - e^{-Q/\forall\, t}\right)$$

¿Cómo acelera el carro?