GP (Generalized-Pareto) Distribution.

CDF: $F(x) = \begin{cases} 1-(1-k\frac{x}{\alpha})^{\frac{1}{k}}, & k \neq 0. \\ 1-e^{-\frac{x}{\alpha}}, & k=0. \end{cases}$ if $k \leq 0$. $x \in [0, +\infty)$

$k>0$. $x \in [0, \frac{\alpha}{k}]$.

pdf: $f(x) = \begin{cases} \frac{1}{\alpha}(1-k\frac{x}{\alpha})^{\frac{1}{k}-1}, & k \neq 0. \\ \frac{1}{\alpha}e^{-\frac{x}{\alpha}}, & k=0. \end{cases}$ α. scale parameter.

k. shape parameter.

Properties.

1. failure rate $r(x) = \frac{f(x)}{1-F(x)} = \frac{\frac{1}{\alpha}(1-k\frac{x}{\alpha})^{\frac{1}{k}-1}}{(1-k\frac{x}{\alpha})^{1/k}} = \frac{1}{\alpha-kx}$.

for $k>0$. $r(x)\uparrow$ as $x\uparrow$; $k=0$. $r(x)$ constant; $k<0$. $r(x)\searrow$ as $x\nearrow$.

2. If X has GP distribution. $X-t$ given $X\geq t$ is also GP distribution. with the same value k.

for GP distribution. $-\frac{1}{2}<k<\frac{1}{2}$. is usually the case (near 0, which is exponential case). tail analysis.

MLE. MME. not very great. PWM. ✓

PWM. method of probability-weighted moments.

for random variable X with CDF $F(x)$. $M_{p,r,s} = E[X^p(F(x))^r(1-F(x))^s]$

let $\alpha_s = M_{1,0,s} = E[X(1-F(X))^s] = \frac{\alpha}{(s+1)(s+1+k)}$ $\alpha_0 = \frac{\alpha}{1+k}$ $\alpha_1 = \frac{\alpha}{2(k+2)}$

$\hookrightarrow k = \frac{\alpha_0}{\alpha_0-2\alpha_1}-2$ $\alpha = \frac{2\alpha_0\alpha_1}{\alpha_0-2\alpha_1}$

for n observations. $\alpha_r = \frac{1}{n}\sum_{j=1}^{n}\frac{(n-j)(n-j-1)\cdots(n-j-r+1)}{(n-1)(n-2)\cdots(n-r)}x_{j,n}$

$\hat{\alpha}_r = \frac{1}{n}\sum_{j=1}^{n}(1-P_{j,n})^r x_{j,n}$. $P_{j,n} = \frac{j+\gamma}{n+\delta}$ where γ and δ are suitable constants.

$\gamma=-0.35$. $\delta=0$.