1. An insurance company has an obligation to pay the medical costs for a claimant. Annual claim costs today are 5000, and medical inflation is expected to be 7% per year. The claimant will receive 20 payments.

Claim payments are made at yearly intervals, with the first claim payment to be made one year from today.

Calculate the present value of the obligation using an annual effective interest rate of 5%.

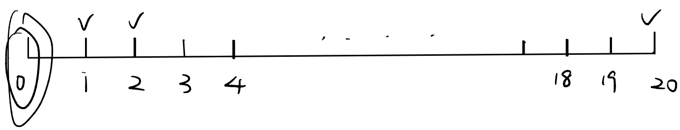
A) 87,900

B) 102,500

C) 114,600

✓ D) 122,600

E) Cannot be determined



支付年份. 1 $5000 \cdot 1.07$ 贴现率. $1/1.05$

2   $5000 \cdot 1.07^2$ $1/1.05^2$

⋮

20   $5000 \cdot 1.07^{20}$ $1/1.05^{20}$

$$PV = \frac{5000 \cdot 1.07}{1.05} + \frac{5000 \cdot 1.07^2}{1.05^2} + \cdots + \frac{5000 \cdot 1.07^{20}}{1.05^{20}}$$

$\frac{1.07}{1.05}$

$$= 5000 \, \frac{1.07}{1.05} \, \frac{1 - \left(\frac{1.07}{1.05}\right)^{20}}{1 - \frac{1.07}{1.05}} = 122633$$

或者

$i = 5\%$. $i' = \frac{1.05}{1.07} - 1 = -0.01869$. $PV = 5000 \, a_{\overline{20|}\,-0.01869}$

Solution: The present value is

$$5000[1.07v + 1.07^2v^2 + ... + 1.07^{20}v^{20}] = 5000\frac{1.07v - 1.07^{21}v^{21}}{1 - 1.07v} = 122,617$$

2. Annuity A pays 1 at the beginning of each year for three years. Annuity B pays 1 at the beginning of each year for four years. The Macaulay duration of Annuity A at the time of purchase is 0.93. Both annuities offer the same yield rate.

Calculate the Macaulay duration of Annuity B at the time of purchase.

A) 1.240

✓ B) 1.369

C) 1.500

D) 1.930

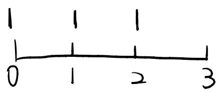
E) Cannot be determined

$$\text{Macaulay duration} = \frac{\sum_{t=0}^{n-1} t v^{t-1}}{\sum_{t=0}^{n-1} v^t}$$

— 加权到期价值 ~~+ Σ t v~~

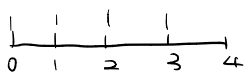
— 年金现值

对 A



$$\frac{0\cdot 1 + 1\cdot(1\cdot v) + 2\cdot(1\cdot v^2)}{1+v+v^2} = 0.93$$

$$\hookrightarrow 1.07v^2 + 0.07v - 0.93 = 0 \Rightarrow v = 0.90015 \quad \cancel{v = -0.96557}$$



对 B

$$\frac{0\cdot 1 + 1\cdot v + 2\cdot v^2 + 3\cdot v^3}{1+v+v^2+v^3} = 1.3689$$

Solution: The Macaulay duration of Annuity A is

$$0.93 = \frac{0(1) + 1(v) + 2(v^2)}{1+v+v^2} = \frac{v + 2v^2}{1+v+v^2}$$

The unique positive solution is $v = 0.9$

The Macaulay duration of Annuity B is

$$\frac{0(1) + 1(v) + 2(v^2) + 3(v^3)}{1+v+v^2+v^3} = 1.369$$

3. You are given the following information about two bonds, Bond A and Bond B:

i) Each Bond is a 10-year bond with semiannual coupons redeemable at its par value of 10,000, and is bought to yield an annual nominal interest rate of i, convertible semiannually.

ii) Bond A has an annual coupon rate of $\boxed{(i + 0.04)}$, paid semiannually.

iii) Bond B has an annual coupon rate of $\boxed{(i - 0.04)}$, paid semiannually.

iv) The price of Bond A is $\boxed{5,341.12}$ greater than the price of Bond B.

- 贴现 $\frac{i}{2}$

Calculate i.

A) 0.042

B) 0.043

C) 0.081

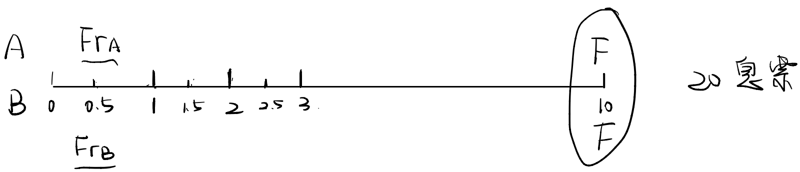
✓ D) 0.084

E) 0.086

息票 coupon

$r_A = \frac{i+0.04}{2}$ $\qquad r_B = \frac{i-0.04}{2}$

$Fr_A = 5000 \cdot (i+0.04)$ $\qquad Fr_B = 5000(i-0.04)$



$$PV_A = Fr_A \cdot a_{\overline{20}|\frac{i}{2}} + \cancel{\frac{F}{(1+\frac{i}{2})^{20}}} \quad (1)$$

$$PV_B = Fr_B \, a_{\overline{20}|\frac{i}{2}} + \frac{F}{(1+\frac{i}{2})^{20}} \quad (2)$$

$(1) - (2) = PV_A - PV_B$

$= F(r_A - r_B)\, a_{\overline{20}|\frac{i}{2}}$

$= 10000\,\Big(\underbrace{\frac{i+0.04}{2} - \frac{i-0.04}{2}}_{0.04}\Big)\, a_{\overline{20}|\frac{i}{2}}$

$= 400\, a_{\overline{20}|\frac{i}{2}} = 5341.12$

$1 \cdot a_{\overline{20}|\frac{i}{2}} = \frac{5341.12}{400} = 13.3528$

$\frac{i}{2} = 4.2\%$

$\hookrightarrow i = 8.4\%$

$= 0.084$

Solution: Throughout the solution, let $j = i/2$

For bond A, the coupon rate is $(i + 0.04)/2 = j + 0.02$

For bond B, the coupon rate is $(i - 0.04)/2 = j - 0.02$

The price of bond A is $P_A = 10,000(j + 0.02)a_{\overline{20}|j} + 10,000(1 + j^{-20})$

The price of bond B is $P_B = 10,000(j - 0.02)a_{\overline{20}|j} + 10,000(1 + j^{-20})$

Thus,

$$P_A - P_B = 5,341.12 = 400a_{\overline{20}|j}$$

$$a_{\overline{20}|j} = 5341.12/400 = 13.3528$$

Using the financial calculator, $j = 0.042$ and $i = 2j = 0.084$