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Problem Set #3
Asset Allocation: Modern Portfolio Theory

You may work on this problem set in groups of up to four people. Hand in one solution per group. You do not need to maintain the same group from previous problem sets. You may discuss the problems only with members of your group. Answers should be typed (or printed legibly) and are due **at the beginning of the week 5 class. No late assignments will be accepted.**

For the last two problems you will need to obtain excel spreadsheets from Chalk.

Problem 1. The Triad family of mutual funds allows investors to split their money between several portfolios managed by Triad (none of the portfolios can be shorted).

- Portfolio A consists entirely of risk-free securities, and has a certain return of 4%.

- Portfolio B has an expected return of 19% and a standard deviation of 25%.

- Portfolio C has an expected return of 10% and a standard deviation of 15%.

Your client is leaning towards investing his money entirely in portfolio C, since he is unwilling to take the higher risk associated with portfolio B, but wants a higher return than offered by portfolio A.

(a) In your role as a Triad investment advisor, you suggest to him an alternative portfolio (consisting of a combination of A and B) that has the same standard deviation as portfolio C but a higher expected return. Assume he has $200,000 to invest.

- How much should he invest in A and how much in B?
- What is his expected return in this case?

(Note that since you work for Triad you cannot recommend investing in a market index fund, since this is not sold by Triad!)

(b) Sketch a risk/return diagram to help explain to him why his combination of risk and return is improved by this option, and why his risk has not increased. Show all four portfolios on the diagram (the original three plus your suggestion).

(c) Now assume that the correlation between portfolios B and C is 0. Compute the weights of the portfolio T formed with B and C that has the highest Sharpe ratio. Find also the portfolio G formed with B and C that has the smallest variance.

(d) Assume again that the correlation between portfolios B and C is 0. Draw a risk/return diagram which displays the investment opportunity set in the risky assets B and C. Then indicate the set of combinations that investors should prefer when they can also invest in the riskless portfolio A (e.g. those which minimize risk for a given level of return).

Problem 2. For the following set of questions you will need to use the returns data contained in the Excel spreadsheet ps3_prob2_data.xls. In this file, the first column contains the date. Subsequent columns contain the returns to the *a)* Value Weighted (VW) index of exchange listed stocks, *b)* IBM, *c)* USX, and *d)* GM common stocks. Column 6 contains the return on the one month T-Bill. In all cases, the return is the simple return (one month) over the period from the previous date up to the date in column 1. (Note: the return for the first row (date 30JAN1926) is from 12/31/25 to 1/30/26.) In all cases, the returns properly account for dividends, splits and stock dividends.

(a) Find the average returns on the VW Index, the T-Bill and each of the three stocks over the entire period from the beginning of 1926 through the end of 1999.

(b) Find the variance of each of these returns over the entire period.

(c) Calculate the covariance between the return on IBM and the return on USX. Calculate the covariance between the return on USX and the return on GM. Calculate the covariance between the return on IBM and the return on GM.

(d) Consider the following two portfolios:

- Portfolio A had 60% invested in IBM, 20% in USX and 20% in GM at the beginning of each month

- Portfolio B had 60% invested in USX, 20% in GM and 20% in IBM at the beginning of each month.

  i. Using just the average returns and variances of the three stocks, plus the covariances between the stocks (calculated in parts a, b and c), calculate the average returns and variances of these two portfolios, and the covariance between the two portfolios.

  ii. Now on a spreadsheet calculate the realized returns of each the portfolios for each month. Using these realized returns calculate the portfolio average returns, variances and the covariance directly, confirming your answers from above.

Problem 3. The file ps3_prob3_data.xls contains the monthly returns on

- A Salomon Brothers portfolio of non-US bonds (supplied by Ibbotson Associates).

- The Morgan Stanley Capital International (MSCI) EAFE (Europe, Asia, and the Far East) index of world stock market performance.

- The Value-Weighted portfolio of US Common Stocks

- A portfolio of 100 US Corporate Bonds, from Ibbotson Associates

- The Money Market rate.

These data series all start in January of 1985 and run through the end of 1992.

(a) For the period 1/1/1985 through 12/31/1992, calculate

- The average **excess return** (i.e. the returns above the money market return) for each of the **first four "assets".**
- Calculate the **standard deviation** for each of these excess returns.
- Calculate the **correlation matrix.**

(b) Using the values calculated above (we will do all the calculation in *excess returns;* adding or subtracting a constant doesn't impact our calculations, as it is just a "vertical shift" in our risk-reward diagrams), and using the spreadsheet MVE_4Assets.xls (you will need to change the inputs),

- Calculate the weights of each of the four assets in the mean-variance efficient portfolio (remember, the "excess risk-free rate" here will be *zero*)
- Plot the ex-post minimum-variance frontier for this set of assets. Please include each of the four assets on the plot.

(c) What do you think the optimal mix in these four assets is *right now?* (There is no "correct" answer here). Explain your answer briefly.