

(a) Calculate the no-arbitrage price of the coupon bond today.

For the six-month zero-coupon bond, the yield to maturity $r_1 = \frac{\$100}{\$99.46} - 1 = 0.0054$. Similarly, the yield to maturity of the one-year zero-coupon bond $r_2 = \left(\frac{\$100}{\$97.23}\right)^{\frac{1}{2}} - 1 = 0.0141$ and the yield to maturity of the 18-month zero-coupon bond $r_3 = \left(\frac{\$100}{\$90.50}\right)^{\frac{1}{3}} - 1 = 0.0338$.

So we have the non-arbitrage price of the new coupon

$$P = \frac{4.5}{1 + 0.0054} + \frac{4.5}{(1 + 0.0141)^2} + \frac{100 + 4.5}{(1 + 0.0338)^3} = \$103.4332$$

(b) Calculate the implied forward rates in this economy.

$$\text{Forward rate after one year } f_{1,2} = \frac{(1+r_2)^2}{1+r_1} - 1 = \frac{(1+0.0141)^2}{1+0.0054} - 1 = 0.0229.$$

$$\text{Forward rate after 18 months } f_{2,3} = \frac{(1+r_3)^3}{(1+r_2)^2} - 1 = \frac{(1+0.0338)^3}{(1+0.0141)^2} - 1 = 0.0744$$

(c) If the liquidity preference theory is correct and there exists a liquidity premium of 0.5% per period, what is the market's expectation of the price the bond will sell for in one year? 1 year = 2 periods here.

If the liquidity preference theory holds and the liquidity premium is 0.5%, the expectation of the interest rate of each period is:

$$E(r_2) = 0.0744 - 0.005 = 0.0694$$

$$E(r_1) = 0.0229 - 0.005 = 0.0179$$

So the market price is:

$$\text{MarketPrice} = \frac{FV + CP}{1 + E(r_2)} = \$97.71$$