Problem Set 2

Nana Li, Keqi Meng, Zhaoyang Chen

Question 1

(a) Suppose that the yield curve will remain unchanged for the following five years and you have decided to use bond A to fund the liability. That is, you want to invest in bond A and invest the coupons at the prevailing interest rates to produce a future value at the end of year five of 1 million. How much should you invest in bond A?

Yield to maturity = 10%

Suppose invest x bond A, then

$15X(1+10\%)^4 + 15X(1+10\%)^3 + 15X(1+10\%)^2 + 15X(1+10\%) + (100+15)X = 1000{,}000$

$X = 5219.85$

So I should invest 5219.85 bond A or 5219.85 * 118.95 = $620,901.16 in bond A.

(b) Now suppose that right after you invested in bond A, the yield curve makes a parallel move down by 1% to 9%. What is the future value five years from now of your investment? What is the future value if the yield curve moves up by 1% to 11%? Please explain why the future value changes differently depending on the direction of the change in the yield curve.

Yield to maturity = 9%

$FV_5 = 15 \text{ X } 5219.85 \text{ X } (1+9\%)^4 + 15 \text{ X } 5219.85 \text{ X } (1+9\%)^3 + 15 \text{ X } 5219.85 \text{ X } (1+9\%)^2 + 15 \text{ X } 5219.85 \text{ X } (1+9\%) + (100+15) \text{ X } 5219.85 = \$990{,}574.38$

$\Delta FV = \$9{,}425.62$

Yield to maturity = 11%

$FV_5 = 15 \text{ X } 5219.85 \text{ X } (1+11\%)^4 + 15 \text{ X } 5219.85 \text{ X } (1+11\%)^3 + 15 \text{ X } 5219.85 \text{ X } (1+11\%)^2 + 15 \text{ X } 5219.85 \text{ X } (1+11\%) + (100+15) \text{ X } 5219.85 = \$1{,}009{,}607.84$

$\Delta FV = \$9607.84$

With a parallel shift, when the yield curve is shifting up, the change in FV in multiple periods will be higher than the change in FV in multiple periods if the yield curve is shifting down by the same amount because of the compounding effect.

(c)

(i)

The duration of bond A is

$$D_A = \frac{15/(1+10\%)}{118.95} \times 1 + \frac{15/(1+10\%)^2}{118.95} \times 2 + \frac{15/(1+10\%)^3}{118.95} \times 3 + \frac{15/(1+1\%)^4}{118.95} \times 4 + \frac{115/(1+10\%)^5}{118.95} \times 5 = 3.95$$

Similarly, we can find the duration of bond B is

$$D_B = \frac{15/(1+10\%)}{130.72} \times 1 + \frac{15/(1+10\%)^2}{130.72} \times 2 + \frac{15/(1+10\%)^3}{130.72} \times 3 + \ldots + \frac{115/(1+10\%)^{10}}{130.72} \times 10 = 6.28$$

Suppose the proportion of bond A in the portfolio is x and the proportion of bond B is $1-x$, then we want the duration of the portfolio to be 5 years.

$$x \times 3.95 + (1-x) \times 6.28 = 5$$
$$x = 0.549$$

So the proportion of bond A in the portfolio is 54.9% and the value of bond A in the portfolio is $\$620901.16 \times 54.9\% = \$340874.7$, which is $\$340874.7/\$118.95 = 2865.7$ units of bond A. Similarly, we should buy $\$620901.16 \times (1-54.9\%)/\$130.72 = 2142.2$ units of bond B.

(ii)

Based on (i), we know the cash flow of the coupon of bond A is $2865.7 \times \$15 = \$42985.5$. Similarly, the cash flow of the coupon of bond B is $2142.2 \times \$15 = \$32133$.

We can easily calculate the accumulated value of these cash flows at the end of the fifth year as

$$(\$42985.5 + \$32133) \times (1+r)^4 + \ldots + (\$42985.5 + \$32133) \times (1+r) + (\$42985.5 + \$32133) + FV_A \times N_A + FV_B \times N_B + \frac{\$32133}{1+r} + \ldots + \frac{\$32133}{(1+r)^5}$$

When $r = 10\%$, we have the future value in the fifth year as \$1,000,000.

If $r = \%9$, we have the future value in the fifth year as \$1,000,347.

If $r = 11\%$, we have the future value in the fifth year as \$1,000,283.

Clearly, the three future values are close.

(c). iii

At the end of Year 1:

Bond A Duration is 3.35 years
Bond B Duration is 5.98 years

FV of Bond B at the end of Year 5 with 9% yield is equal to \$123.34

Assume we will purchase A bond A and B bond B, solve for the follow equations:

(15A + 15B)*1.09^3 + (15A +15B)*1.09^2 + (15A +15B)*1.09 + (15A + 15B) + 100A + 123.34B = 1,000,000
And
3.35*A/(A+B) + 5.98*B/(A+B) = 4

We get A = 4,319.07; B = 1,416.09

If yield changes to 10%, FV of Bond B will change to $118.95, and at the end of Year 5 we will receive:

(15A + 15B)*1.1^3 + (15A +15B)*1.1^2 + (15A +15B)*1.1 + (15A + 15B) + 100A + 118.95B, which equals to $999,604, approximately $1,000,000

If yield changes to 8%, FV of Bond B will change to $127.95, and at the end of Year 5 we will receive:

(15A + 15B)*1.1^3 + (15A +15B)*1.1^2 + (15A +15B)*1.1 + (15A + 15B) + 100A + 118.95B, which equals to $1,000,736, approximately $1,000,000