Week 2:
Fixed Income

Prof. Michael Weber
University of Chicago Booth School of Business

BUS 35000

Lecture Plan

- Overview of Bond Markets
- Pricing Fixed Income Securities
- Yield to Maturity
- Yield Curve
- Forward Rates
- Expectations Hypothesis
- Quantifying Interest Rate Risk
- Duration
- Hedging (or Duration Matching)

Learning Objectives

- Understand different types of fixed income instruments
- What is the arbitrage-free price of a bond?
- What is the yield to maturity and how do I calculate it?
- Understand yield curves and forward rates
- How are bonds exposed to interest rate risk and how can I use duration to immunize my portfolio?

Overview of Fixed Income Markets: Treasuries (I)

- Treasury Notes and Treasury Bonds
  - The U.S. government borrows money from you
  - Maturities and naming conventions:
    - T-Bills: Maturity less than one year (no coupons)
    - T-Notes: From one to ten years
    - T-Bonds: Maturity greater than ten years, up to 30 years
  - Notes and bonds pay semiannual coupons and pay the face value (normally \$1,000) back at the maturity
  - Coupons are expressed as percentages of the face value. If the face value is \$1,000
    - ...and the 10% coupon is annual, each coupon is \$100
    - ...and the 10% coupon is semiannual, each coupon is $\$100/2 = \$50$
  - If the bond's market price = face value, a bond is trading at par (or call it a par bond)

Overview of Fixed Income Markets: Treasuries (II)

Example: Treasury Note Cash Flows

A three year T-Note with semiannual coupons and a 5% annual coupon rate makes the following payments:

- \$25 in 6 months
- \$25 in 12 months
- \$25 in 18 months
- \$25 in 24 months
- \$25 in 30 months
- \$1,025 in 36 months (coupon + face value)

- Treasury Bonds are safest of all fixed income instruments
- (Practically) only interest rate risk, no default risk

Corporate Bonds (I)

- Corporate bonds enable firms to borrow money directly from the public
  - Issued by large corporations $\rightarrow$ cheap alternative to bank financing
  - Structure is similar to T-Bonds: semi-annual coupons
  - Issued via an investment bank through an underwriting process
  - Traded primarily through a dealer market (over the counter)
- How risky are corporate bonds?
  - Credit risk makes corporate bonds riskier than treasuries
  - Corporate bonds rated for credit worthiness
  - Two big ratings agencies: Moody's and Standard & Poor's
  - Moody's ratings: Aaa, Aa, A, Baa, Ba, B, Caa, Ca, C, D[1,2,3]
  - S&P's ratings: AAA, AA, A, BBB, BB, B, CCC, CC, C, D[+,-]

Corporate Bonds (II)

Riskiness of corporate bonds

- **Investment grade bond**: rated at least Baa (Moody's) or BBB (S&P)
- **Junk bond**: rated lower than Baa (Moody's) or BBB (S&P)

- Some financial institutions are not allowed to invest in junk bonds
- **Historical default rates:** AAA-rated corporate bonds almost never default whereas the ten-year default rate for CCC-rated bonds is almost 50%

US Credit Ratings

Bond Ratings									
	Very High Quality		High Quality		Speculative		Very Poor	
Standard & Poor's	AAA	AA	A	BBB	BB	B	CCC	D
Moody's	Aaa	Aa	A	Baa	Ba	B	Caa	C

At times both Moody's and Standard & Poor's have used adjustments to these ratings: S&P uses plus and minus signs: A+ is the strongest A rating and A– the weakest. Moody's uses a 1, 2, or 3 designation, with 1 indicating the strongest.

Moody's	S&P	
Aaa	AAA	Debt rated Aaa and AAA has the highest rating. Capacity to pay interest and principal is extremely strong.
Aa	AA	Debt rated Aa and AA has a very strong capacity to pay interest and repay principal. Together with the highest rating, this group comprises the high-grade bond class.
A	A	Debt rated A has a strong capacity to pay interest and repay principal, although it is somewhat more susceptible to the adverse effects of changes in circumstances and economic conditions than debt in higher-rated categories.
Baa	BBB	Debt rated Baa and BBB is regarded as having an adequate capacity to pay interest and repay principal. Whereas it normally exhibits adequate protection parameters, adverse economic conditions or changing circumstances are more likely to lead to a weakened capacity to pay interest and repay principal for debt in this category than in higher-rated categories. These bonds are medium-grade obligations.
Ba B Caa Ca	BB B CCC CC	Debt rated in these categories is regarded, on balance, as predominantly speculative with respect to capacity to pay interest and repay principal in accordance with the terms of the obligation. BB and Ba indicate the lowest degree of speculation, and CC and Ca the highest degree of speculation. Although such debt will likely have some quality and protective characteristics, these are outweighed by large uncertainties or major risk exposures to adverse conditions. Some issues may be in default.
C	C	This rating is reserved for income bonds on which no interest is being paid.
D	D	Debt rated D is in default, and payment of interest and/or repayment of principal is in arrears.

Source: Bodie, Kane, and Marcus, 10th Edition, p.469

Financial Ratios by Credit Class

- "Financially-healthy" firms have higher ratings
- Ratings often assume stable relationship between past ratios and future default probabilities

	3-year medians						
	AAA	AA	A	BBB	BB	B	CCC
EBIT interest coverage multiple	23.8	19.5	8.0	4.7	2.5	1.2	0.4
EBITDA interest coverage multiple	25.5	24.6	10.2	6.5	3.5	1.9	0.9
Funds from operations/total debt (%)	203.3	79.9	48.0	35.9	22.4	11.5	5.0
Free operating cash flow/total debt (%)	127.6	44.5	25.0	17.3	8.3	2.8	(2.1)
Total debt/EBITDA multiple	0.4	0.9	1.6	2.2	3.5	5.3	7.9
Return on capital (%)	27.6	27.0	17.5	13.4	11.3	8.7	3.2
Total debt/total debt + equity (%)	12.4	28.3	37.5	42.5	53.7	75.9	113.5

Source: Bodie, Kane, and Marcus, 10th Edition, p.471

Potential Issues with Ratings

- Incentives: rating paid by issuer
- Rating shopping
- Cross selling
- Repeated interaction
- Future business
- Political intervention: sovereign debt

Mortgage-Backed Securities (I)

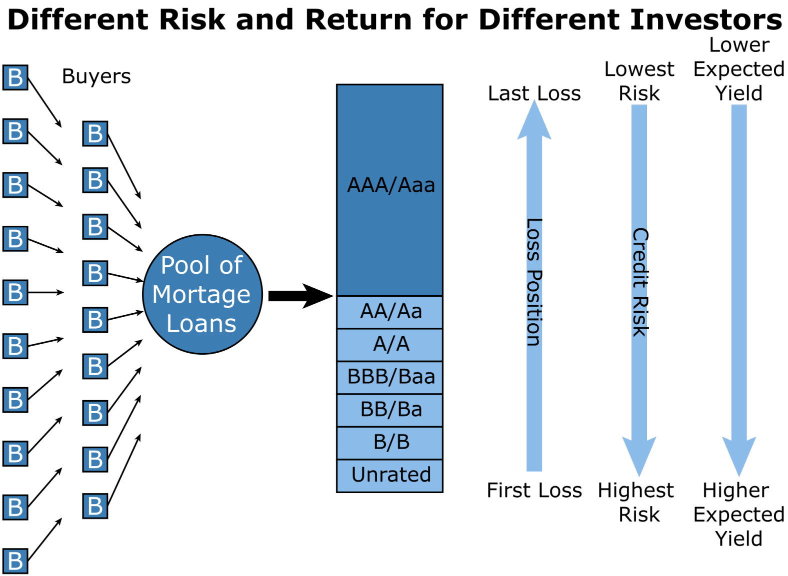
- In 1970, 70% of mortgages were originated by local banks and thrifts
  - Costs of collecting information made long-distance lending difficult
  - System constrained by local supply-demand imbalances: who wants to borrow, who wants to lend
- Government agencies introduced securitization in response to the Savings & Loans collapse

Mortgage-Backed Securities (II)

Mortgage Securitization

- Collect many mortgages into a pool
  - Why pool? In normal times, people default on their loans for idiosyncratic reasons
  - If so, default rates are predictable in pools $\rightarrow$ lower risk
- Issue securities that are claims on the pool's cash flows
  - Different "tiers" determine who loses first when defaults occur
- Converts illiquid loans into liquid securities that can be held by more investors
  - An institution may not be allowed to buy an individual mortgage, but it can buy them as investment grade bonds
  - Lower borrowing costs to consumers
  - Securitization has been applied to other loan types as well, such as car loans, student loans, etc.

Mortgage-Backed Securities (II)



Mortgage-Backed Securities (III)

- Result of Securitization
  - Mortgage-backed securities were a three-trillion dollar market (22% of the fixed income market)
  - 80% of mortgages securitized
  - Mortgage system no longer constrained by local imbalances
- Financial crisis: Agency problem in securitization
  - If a bank can sell its loan to someone else, it does not care about the quality of the loan → low approval standards
  - It only cares about the ability to sell the loan to someone else
- Slicing and repackaging → collaterized debt obligations (CDOs)
  - We can purchase pieces of different mortgage-backed securities and put them together to create CDOs
  - Idea: if pools are different enough, the CDO is less risky than the individual MBS pieces
  - Split the CDO into another set of tiers and sell these pieces

FICO Credit Score and the Likelihood of Low Documentation Mortgages

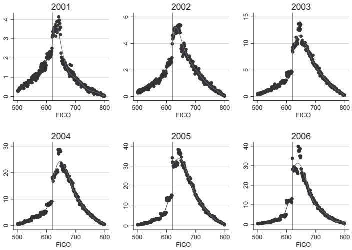


FIGURE II
Number of Loans (Low-Documentation)

Source: Amit Seru et al (2010): Did Securitization Lead to Lax Screening? Evidence from Subprime Loans.

- FICO scores above 620 easier to securitize (Fannie and Freddie rule of thumb)
- Mortgages right above the threshold indeed get screened less thoroughly

FICO Credit Scores and Loan Deliquencies

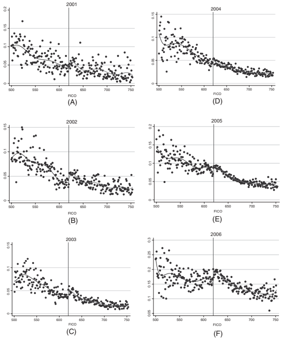


FIGURE VI
Annual Delinquencies for Low-Documentation Loans Originated in 2001–2006

Source: Amit Seru et al (2010): Did Securitization Lead to Lax Screening? Evidence from Subprime Loans.

- Loans with FICO scores just to the right of the 620 thresholds indeed more likely to be deliquent subsequently
- Evidence for lax screening due to securitization

Some Bond Terminology

- Callable bonds
  - The issuing firm has the right to retire the bonds before maturity
  - The firm exercises this option when financing gets cheaper:
    - when interest rates fall or credit worthiness improves
    - this option gives the firm more freedom, so callable bonds sell at a discount relative to non-callable bonds, i.e. they offer higher yields
- Straight Coupon Bond
  - Semiannual coupon with a fixed rate, principal (face value) paid only at terminal date
- Zero Coupon Bond
  - No coupon payments, makes a single payment at maturity
  - Also known as: discount bond or strip
- Perpetuity
  - Bonds that pay only interest, last forever

Pricing Fixed Income Securities

- Last week: **Price = Sum of Discounted Expected Future Cash Flows**
- For fixed income securities without default risk, cash flows are certain. The present value formula is:

$$P_0 = \frac{D_1}{1+r_1} + \frac{D_2}{(1+r_2)^2} + \frac{D_3}{(1+r_3)^3} + \cdots$$

  - We discount different payments at different discount rates: D_1 at rate r_1, D_2 at rate r_2, etc.
  - Time value of money is different depending on maturity
    - Long-term interest rate (per year) is usually higher than the short-term interest rate
    - These discount rates are determined by the market
- Pricing (safe) fixed income securities is simple: discount known cash flows at known discount rates

Pricing Fixed Income Securities: Example (I)

Question

- A two-year T-Note has a face value of \$1,000 and 10% annual coupon rate.
- The coupons are paid semi-annually.
- If the six-month, 1-year, 1.5-year, and 2-year rates are 4%, 4.5%, 4.8%, and 5% per year, compounded semi-annually, what is the price of this bond?

Pricing Fixed Income Securities: Example (II)

- This bond makes the following payments: \$50, \$50, \$50, and \$1,050
- If rates are compounded m times a year, the present value formula changes from

$$\mathrm{PV} = \frac{FV}{1+r} \quad \text{to} \quad \mathrm{PV} = \frac{FV}{\left(1+\frac{r}{m}\right)^m}$$

  - Think as "the rate per period" and "the number of periods until we get the cash flow"
  - Here the six-month rates are: 2%, 2.25%, 2.4%, and 2.5%
- The price of the bond is

$$\begin{aligned} P &= \frac{C}{1+r_1} + \frac{C}{(1+r_2)^2} + \frac{C}{(1+r_3)^3} + \frac{FV+C}{(1+r_4)^4} \\ &= \frac{\$50}{1+0.02} + \frac{\$50}{(1+0.0225)^2} + \frac{\$50}{(1+0.024)^3} + \frac{\$1{,}000+\$50}{(1+0.025)^4} \\ &= \$1{,}094.66 \end{aligned}$$

Spot Rates

- We call these discount rates spot rates
  - If the three-year spot rate is 5%, we discount a year 3 cash flow to today at 5%
- **Important:** We use the same t-year spot rate to discount all year t cash flows
- We can compute spot rates by studying bond prices
- Zero-coupon bonds are the easiest for this because they make just one payment

$$P = \frac{FV}{(1+r_t)^t} \quad \Rightarrow \quad r_t = \left(\frac{FV}{P}\right)^{\frac{1}{t}} - 1$$

- **Example.** A three-year zero coupon bond with a FV of \$1,000 trades at \$868.79. What is the three-year spot rate?

$$r_t = \left(\frac{FV}{P}\right)^{\frac{1}{t}} - 1 = \left(\frac{\$1{,}000}{\$868.79}\right)^{\frac{1}{3}} - 1 = 4.8\%$$

Valuing a Bond using Discount Bonds (I)

Example

Suppose that you are offered a 5-year coupon bond with a face value of $FV = \$1{,}000$. An 8% coupon is paid annually. Suppose also that the zero-coupon bond prices up to 5 years are

Years to Maturity	1	2	3	4	5
Price	\$98	\$95	\$92	\$89	\$85

- Assume that each zero-coupon bond has a face value of \$100 and that everything is compounded annually
- What is the price of this bond?

Valuing a Bond using Discount Bonds (II)

- We could compute 1-, 2-, . . . , 5-year spot rates and use them to discount payments to present
  - This would work
- However, we can also think about zero-coupon bonds differently
  - What does it mean when the 4-year zero is worth \$89?
  - The cost of getting \$100 in year 4 is \$89 $\rightarrow$ the value of one year-4 dollar is 89 cents
- Compute the bond price by multiplying each payment by time-t dollar's value:

$$
\begin{aligned}
P &= \frac{B_1}{\$100}C + \frac{B_2}{\$100}C + \cdots + \frac{B_T}{\$100}(FV + C) \\
&= \frac{\$98}{\$100}\$80 + \frac{\$95}{\$100}\$80 + \frac{\$92}{\$100}\$80 + \frac{\$89}{\$100}\$80 + \frac{\$85}{\$100}(\$1{,}000 + \$80) \\
&= 0.98 \times \$80 + 0.95 \times \$80 + 0.92 \times \$8 + 0.89 \times \$80 + 0.85 \times \$1{,}080 \\
&= \$1{,}217.35
\end{aligned}
$$

- Check the equivalence!

Yield to Maturity (I)

- What we have done so far:
  - To price a coupon bond, we discount all cash flows to present at different discount rates for each period
  - These discount rates can be backed out from zero-coupon (strip) prices
- **Yield-to-maturity** answers the following question: "Instead of discounting the cash flows back at different rates, what unique rate (for all dates) would give us the same answer?"
- In an earlier example, the spot rates were $r_1 = 2.04\%$, $r_2 = 2.60\%$, $r_3 = 2.82\%$, $r_4 = 2.96\%$, and $r_5 = 3.30\%$ (check!)
  - We get the same price if we use these rates:

$$P = \frac{\$80}{1.0204} + \frac{\$80}{(1.0260)^2} + \cdots + \frac{\$1{,}080}{(1.0330)^5} = \$1{,}217.35$$

Yield to Maturity (II)

- If we pick one rate, say, $y = 3\%$, would we get the same price? Try it:

$$P^* = \frac{\$80}{1.03} + \frac{\$80}{(1.03)^2} + \cdots + \frac{\$1{,}080}{(1.03)^5} = \$1,228.99$$

- Pretty close, but not quite there
- Because the "trial" y gives too high a price, we need to increase the discount rate
- When we find the number y that gives the correct price, we have found the yield-to-maturity
  - It is the "average" yield you get per year if you buy the bond today
  - Formally: "yield to maturity = the rate of return realized over the life of the bond if all coupons are reinvested at an interest rate equal to the bond's yield to maturity."
  - For real investment projects knows as "Internal rate of return"

Yield to Maturity (III)

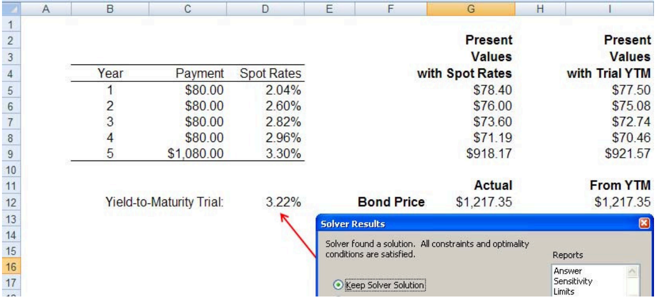
- How do we find yield-to-maturity?
  - Usually by trial and error
  - We can use Excel's Solver (under Data→Solver; activate in Excel 2013: **File** tab→**Add-Ins**→select **Excel Add-Ins** in **Manage** box→**Select** Solver...)
- If a coupon bond is trading at par (price = face value), yield-to-maturity is the same as the coupon rate
  - Example. If a 5-year bond with an annual coupon rate of 7.25% is trading at par (face value = price = $1,000), its yield to maturity is also 7.25%
  - Even for a non-par bond, the coupon rate is a benchmark:
    1. If the bond price is higher than the face value, the bond's yield to maturity must be lower than the coupon rate
    2. If the bond price is lower than the face value, the bond's yield to maturity must be higher than the coupon rate

Yield to Maturity (IV)

- Note: The yield to maturity is the interest rate which the bond pays if all coupons are reinvested (or rolled over) at that *same rate* until maturity
  - For bonds with default risk, expected yield-to-maturity is lower than the stated yield-to-maturity
  - If the bond is not held until maturity, the return is called the *holding period return*, which can be quite different from the original yield to maturity (it will include marking-to-market gains/losses)
  - Rates may change in the future, so the yield to maturity is not the same as the realized yield on the bond if you reinvest coupons at the existing (spot) rates

Yield to Maturity: Solving in Excel

- Pick a guess for yield-to-maturity
- Compute the bond price using this guess
- Use the solver to set the difference between the actual price and trial price to zero

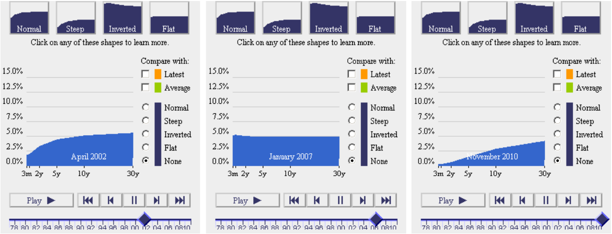


Year	Payment	Spot Rates	Present Values with Spot Rates	Present Values with Trial YTM
1	$80.00	2.04%	$78.40	$77.50
2	$80.00	2.60%	$76.00	$75.08
3	$80.00	2.82%	$73.60	$72.74
4	$80.00	2.96%	$71.19	$70.46
5	$1,080.00	3.30%	$918.17	$921.57

			Actual	From YTM
Yield-to-Maturity Trial:	3.22%	Bond Price	$1,217.35	$1,217.35

Yield Curve = A graphical presentation of spot rates

- Plot spot rates from three-month T-Bills to 30-year T-Bonds
- Normal yield curve is upward sloping: short-term bonds carry lower yields than long-term bonds
  - "In the absence of economic disruptions, investors who risk their money for longer periods expect to get a bigger reward than those who risk their money for shorter time periods"
- Inverted yield curve is downward sloping: short-term bonds carry higher yields than long-term bonds



Forward Rates

- Spot rates versus forward rates
  - **Spot rates** are the rates you get when you walk into the bank today and put your money away for t years
  - **Forward rates** are the rates you get when you walk into the bank today and agree to put your money away for t years at some time in the future
    "I'll get $10,000 in one year, and I want to put the money in a bank account at that time. I am afraid about what the rates will be in a year. So, I should lock the rate today."
- Forward rates are determined by spot rates
  - If spot and forward rates do not satisfy a certain relation, you can generate positive profits with no risk
  - This is the first example of a **no-arbitrage condition**
  - Definition of arbitrage: a zero-risk, zero-net investment strategy that generates a profit ("a free lunch")

Forward Rates: Example (I)

- The 1-year spot rate is 5% and the 2-year spot rate is 6%

  Question: What is the (one year) forward rate from year 1 to year 2?

- Let r_1 and r_2 denote the spot rates and $f_{1,2}$ to denote the forward rate from year 1 to year 2
- You can put \$10,000 in a bank for two years in two ways:
  1. Invest the money for two years at the 2-year spot rate
  2. Invest the money for one year at the 1-year spot rate, and lock in the forward rate from year 1 to year 2
     - Roll the money over after one year
- The payoffs from these strategies are:
  - Payoff from Strategy 1: $\$10{,}000 * (1 + r_2)^2$
  - Payoff from Strategy 2: $\$10{,}000 * (1 + r_1)(1 + f_{1,2})$

Forward Rates: Example (II)

- The payoffs are:
  - Payoff from Strategy 1: $\$10{,}000 * (1 + r_2)^2$
  - Payoff from Strategy 2: $\$10{,}000 * (1 + r_1)(1 + f_{1,2})$
- If these payoffs are different, you can make a riskless profit:
  - If the payoff from strategy 2 is higher, borrow \$10,000 at the two-year spot rate and invest the proceeds in strategy 2
  - The year-2 payoff is now positive by assumption
- Equating the payoffs, we can solve for $f_{1,2}$:

$$(1+r_2)^2 = (1+r_1)(1+f_{1,2}) \quad \Rightarrow \quad f_{1,2} = \frac{(1+r_2)^2}{1+r_1} - 1 = \frac{(1+0.06)^2}{1+0.05} - 1 = 7.01\%$$

  - Thus, the forward rate from year 1 to year 2 is 7.01%
  - The same approach can be used to compute ALL forward rates

Understanding the Shape of the Yield Curve

1. Expectations hypothesis
   - Forward rates are expectations of what the spot rates are going to be in the future: for example, $f_{1,2} = \mathrm{E}[r_1]$ in one year
   - If the current 1-year spot rate is 5.25% and the forward rate from year 1 to year 2 is 5.5%, we expect the 1-year spot rate to increase by 25 basis points to 5.5% in a year
   - If the yield curve is upward sloping, the expectation hypothesis says that the spot rates are expected to increase
2. Liquidity preference theory
   - Most investors do not want to tie their capital for long periods of time (they need liquidity), so long term rates must compensate for the lack of liquidity
   - If so, $\text{forward rate} = \mathrm{E}[\text{spot rate}] + \text{liquidity premium}$
3. Market segmentation theory
   - Short- and long-term instruments are traded in separate markets with different types of investors, therefore it is hard to impose any structure on the yield curve

Interest Rate Risk

- Suppose you buy 10-year bonds with 5% annual coupons
- The face value of each bond is \$1,000 and they trade at par
- What happens if the rates increase by 5 basis points?
- Because the bond is trading at par,

$$P = \frac{\$50}{1.05} + \frac{\$50}{(1.05)^2} + \cdots + \frac{\$1{,}050}{(1.05)^{10}} = \$1{,}000.00$$

- Increase the rate by 0.05% and recompute the price

$$P_{\text{new}} = \frac{\$50}{1.0505} + \frac{\$50}{(1.0505)^2} + \cdots + \frac{\$1{,}050}{(1.0505)^{10}} = \$996.15$$

$\rightarrow$ you lose \$3.85 per bond

- Interest rate risk: Bond prices change as the rates change
- Important questions:
  1. How to measure interest rate risk?
  2. How to protect ("hedge") against interest rate risk?

Duration

- Duration is important for understanding interest rate risk
  - Duration = Average maturity of a bond's cash flows
- The longer a bond's duration, more sensitive its price is to interest rate changes

Computing duration

1. Compute bond's yield-to-maturity
2. Compute each cash flow's present value using y (important)
3. Weight payment dates by the present value of each cash flow, divided by the bond price

$$\text{Duration} = \frac{\text{PV}(\text{CF}_1)}{P} \times 1 + \frac{\text{PV}(\text{CF}_2)}{P} \times 2 + \cdots + \frac{\text{PV}(\text{CF}_T)}{P} \times T$$

Duration: Example

- Suppose a 5-year bond with annual 8% coupons trades at \$1,090. The bond's face value is \$1,000.
- What is the bond's duration?
  - The yield-to-maturity is the y that solves the following equation:

$$\$1{,}090 = \frac{\$80}{1+y} + \frac{\$80}{(1+y)^2} + \cdots + \frac{\$1{,}080}{(1+y)^5}$$

  - We can use Excel's solver to find $y = 5.871\%$
  - The present values of the payments are \$75.56, \$71.37, \$67.42, \$63.68, and \$811.97.
  - The "weights" for computing duration are then \$75.56 / \$1,090 $= 0.0693, 0.0655, \ldots,$ and 0.7449
  - The final step is to use these weights to compute duration:

$$\text{Duration} = 0.0693 \times 1 + 0.0655 \times 2 + \cdots + 0.7449 \times 5 = 4.34 \text{ years}$$

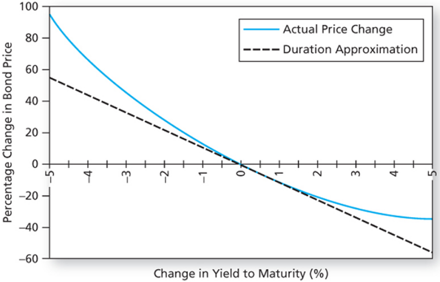
Duration and Interest Rate Risk: Some Math

- The basic pricing equation is
$$P = \frac{C}{1+y} + \frac{C}{(1+y)^2} + \cdots + \frac{FV+C}{(1+y)^T}$$
- Differentiating both sides with respect to y gives:
$$\frac{dP}{dy} = -\frac{C}{(1+y)^2} - 2\frac{C}{(1+y)^3} - \cdots - T\frac{FV+C}{(1+y)^{T+1}}$$
- Taking $-\frac{1}{1+y}$ out as the common factor, we get
$$\begin{aligned} \frac{dP}{dy} &= -\frac{1}{1+y}\left(\frac{C}{1+y} + 2\frac{C}{(1+y)^2} + \cdots + T\frac{FV+C}{(1+y)^T}\right) \\ &= -\frac{1}{1+y} \times \text{Duration} \times P \end{aligned}$$
- If we solve for the price change, ΔP, we get
$$\Delta P \approx -\frac{1}{1+y} \times \text{Duration} \times P \times \Delta y$$

Duration-based Approximation vs Exact Price Change

- The equation $\Delta P \approx -\frac{1}{1+y} \times \mathrm{Dur} \times P \times \Delta y$ is a linear approximation
- The approximation gradually worsens as the magnitude of the yield change increases

Convexity of 30-year maturity, 8% coupon bond; initial yield-to-maturity = 8%



Source: Bodie, Kane, and Marcus, 10th Edition, p.528

Duration and Price Change (I)

- The longer the duration, the more sensitive the bond price is to rate changes
- Consider getting \$1 million tomorrow or in 10 years. In which case do rate changes influence the present value more?

Example

A 10-year bond with 8% annual coupons and a face value of \$1,000 is trading at par

- Question 1. What is the approximate price change (using duration) when the yield goes up by (a) 5 basis points or by (b) 50 basis points?
- Question 2. What is the exact price change when the yield goes up by these amounts?

Duration and Price Change (II)

- We first compute duration:
  - The yield-to-maturity is 8% (par bond)
  - The PVs of the ten cash flows are \$74.07, \$68.59, ..., \$500.25
  - The duration of the bond is then 7.2469 years
- We can now compute the approximate price changes:

$$
\begin{aligned}
\Delta P &\approx -\frac{1}{1.08} \times 7.2469 \times \$1{,}000 \times 0.05\% = -\$3.36 \\
\Delta P &\approx -\frac{1}{1.08} \times 7.2469 \times \$1{,}000 \times 0.5\% = -\$33.55
\end{aligned}
$$

- Recompute the price using 8.05% or 8.5% as the discount rate:
  - If the increase is 0.05%, the price change is $-$\$3.35
  - If the increase is 0.5%, the price change is $-$\$32.81

$\rightarrow$ The first approximation is very good, the second one is a bit off the mark

Some Properties of Duration

- If we own two bonds, the duration of this bond portfolio is the weighted average of the individual duration
- This result extends to many portfolios with multiple bonds
- Because duration is a weighted-average of payment dates, we have the following results:
  1. The duration decreases as the coupon rate increases
  2. The duration decreases as the yield-to-maturity increases
- In both cases, the present value of the short term payments increases relative to the long term payments $\rightarrow$ duration decreases

Using Duration: Immunization (I)

- Immunization refers to strategies used by investors to shield their overall financial status from exposure to interest rate fluctuations

Example

- The yield curve is flat at 6%
- You bought 100 bonds with a face value of \$1,000, an annual coupon of 4%, and maturity of 5 years
- You are thus exposed to interest rate risk
- If you have access to 10-year zero-coupon bonds (each of them with $FV = \$1{,}000$), how <u>many</u> such bonds should you buy or sell to hedge the interest rate risk?

Using Duration: Immunization (II)

- Given that the yield curve is flat, the price of our bond is easy to compute. It is

$$P = \frac{\$40}{1+0.06} + \frac{\$40}{(1+0.06)^2} + \cdots + \frac{\$1{,}040}{(1+0.06)^5} = \$915.75$$

- The duration of this bond is 4.611 years
- The duration of each zero-coupon bond is, of course, 10 years
- We'll work with the (approximate) price change formula:

$$\Delta P \approx -\frac{1}{1+y} \times \mathrm{Duration} \times P \times \Delta y$$

- Letting subscript 'old' to denote our current portfolio and 'new' a portfolio of zeros, we can try to find a value P_{new} such that the sum of price changes is zero:

$$\begin{aligned} \Delta P_{\mathrm{old}} + \Delta P_{\mathrm{new}} \quad &\approx \quad -\frac{1}{1+y} \times \mathrm{Dur}_{\mathrm{old}} \times P_{\mathrm{old}} \times \Delta y \\ &\quad - \frac{1}{1+y} \times \mathrm{Dur}_{\mathrm{new}} \times P_{\mathrm{new}} \times \Delta y = 0 \end{aligned}$$

Using Duration: Immunization (III)

- This simplifies to:

$$\mathrm{Dur}_{\mathrm{old}} \times P_{\mathrm{old}} + \mathrm{Dur}_{\mathrm{new}} \times P_{\mathrm{new}} = 0 \quad \rightarrow \quad P_{\mathrm{new}} = -P_{\mathrm{old}} \times \frac{\mathrm{Dur}_{\mathrm{old}}}{\mathrm{Dur}_{\mathrm{new}}}$$

- This formula says that the market value of the new position, P_{new} has to equal minus the market value of the old position, P_{old}, times the ratio of the durations
- The market value of our existing position is $100 \times \$915.75 = \$91{,}575$
- The durations are 4.611 (old) and 10 (new)
- Hence, the formula tells us to get a position in 10-year zeros of

$$P_{\mathrm{new}} = -\$91{,}575 \times \frac{4.611}{10} = -\$42{,}222$$

- We need to <u>sell</u> \$42,222 worth of zero coupon bonds

Using Duration: Immunization (IV)

- We often think about hedging in terms of how <u>many</u> bonds to buy or sell
  - The zeros have a face value of \$1,000, so each has a price of $\frac{\$1{,}000}{(1+0.06)^{10}} = \$558.39$
  - Thus, we need to sell $\frac{\$42{,}222}{\$558} = 75.6$ of these zero coupon bonds
- Does this strategy immunize our bond portfolio?
  - Suppose the rates increase by 1%
  - The value of the 'old' portfolio changes approximately by $-\frac{1}{1+0.06} \times 4.611 \times \$91{,}575 \times 1\% = -\$3{,}983.18$
  - The value of the 'new' portfolio changes approximately by $-\frac{1}{1+0.06} \times 10 \times (-\$42{,}222) \times 1\% = +\$3{,}983.18$
  - These changes exactly offset each other
- **Note:** the hedge is still imperfect because it relies on the duration-based approximation

Real Life Immunization Scenarios

- Banks insulate their portfolios from interest rate fluctuations
  - Banks' assets have long durations: mainly long-term loans
  - Banks' liabilities have short durations: short-term deposits
  - When the interest rates increase, the value of the assets decreases more than the value of the liabilities
  - A bank's net worth could be wiped out if the duration discrepancy is big enough
- The opposite is true for pension funds: liabilities (promised payments to retirees) have longer maturities than the assets
- An institution that wants to decrease interest rate exposure needs to
  1. Figure out the durations of the (existing) assets and liabilities
  2. Buy or sell some other bonds to hedge the risk

  The institution can accomplish this in many different ways, using different instruments

What We Learned Today

- We learned about different bond issuers, default risk, and how to price bonds
- We studied yield to maturity, forward rates, and their relationship to the yield curve
- We examined the interest rate risk of bonds and saw that duration proxies interest rate risk
- Duration measures the sensitivity of bond price to changes in interest rates
- We can exploit the fact that the duration of a portfolio equals the sums of the duration of the constituents to hedge interest rate risk