Financial Statistics: Time Series, Forecasting, Mean Reversion, and High Frequency Data

FINM 33170 and STAT 33910 Winter 2021

HW 4, due Friday 12 February, 2021.

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1. **Currencies (continued)**

   (a) Determine which of the currencies are stationary according to the Dickey-Fuller test (at the 5 % level). (You can do this with the lm command for each currency, or try to put lm into a loop.)

   (b) Plot each of these log exchange rates (or exchange rates), and use what you have learned in the course and your general knowledge of world events. Which of the conclusions from the Dickey-Fuller test may be inaccurate or lead to wrong trading strategies, and why?

2. **Betting on Danish Kroner**

   (a) Investigate whether DKK (Danish Kroner) and EUR (Euros) are cointegrated: Carry out the regression of log DKK on log EUR, find a cointegrating vector, and discuss whether the residual series seems stationary (using plot and test).

   (b) We **assume** in the following that the two series are cointegrated, with the cointegrating vector $(1, -\hat{\gamma})$ from the regression above. (This may or may not be consistent with the conclusion in the previous question.) If you have not been able to find $\hat{\gamma}$, then assume that $\hat{\gamma} = 1$ (which is close enough)

   Denote the exchange rate at day i by p_i. The superscript indicates the currency: the log price of EUR at day 20 is $\log(p^{EUR})_{20}$, the log price of DKK at day 20 is $\log(p^{DKK})_{20}$.

   Suppose you are asked to trade the pair (DKK, EUR). Specifically, you need to form a mean-reverting portfolio by being long on DKK and simultaneously short EUR. Use the Let C_1 be the sample mean of $\log(p^{DKK})_i - \hat{\gamma} \log(p^{EUR})_i$ $i = 1, 2, \cdots, 2758$. Also let C_2 be **one half the sample standard deviation of this series.** (You can get these numbers from the previous analysis, or by direct computation in R.)

   Consider two strategies:

   <u>Basic Strategy:</u>

   At the first time when $\log(p^{DKK}) - \hat{\gamma} \log(p^{EUR}) \geq C_1 + C_2$, short $\frac{1}{p^{DKK}}$ units of DKK

and go long $\frac{\hat{\gamma}}{p^{EUR}}$ units of EUR,
At the first time when $\log(p^{DKK}) - \hat{\gamma}\log(p^{EUR}) \leq C_1 - C_2$, liquidate the earlier position and take the opposite position. Finally liquidate if/when $\log(p^{DKK}) - \hat{\gamma}\log(p^{EUR}) \geq C_1 + C_2$ again.
Reinforcing Strategy:
Whenever $\log(p^{DKK}) - \hat{\gamma}\log(p^{EUR}) \geq C_1 + C_2$, short $\frac{1}{p^{DKK}}$ unit of DKK and long $\frac{\hat{\gamma}}{p^{EUR}}$ units of EUR,
Whenever $\log(p^{DKK}) - \hat{\gamma}\log(p^{EUR}) \leq C_1 - C_2$, liquidate and take the opposite position.
For both strategies, do nothing if $\log(p^{DKK}) - \hat{\gamma}\log(p^{EUR}) \in [C_1 - C_2, C_1 + C_2)$.

Suppose each trade costs 0.01 cents, and that interest differentials are negligible. Follow both the basic strategy and the reinforcing strategy from Jan 1, 2007, to to Dec 22, 2017, using R. What is the profit/loss at the end?

(c) (Bonus question) Instead of using $C_1 \pm C_2$, find the optimal thresholds to enter and exit (liquidate) the trade (the tresholds that maximize your profit)

3. **Conditional Expectations**

(An excursion to the theoretical side. We will not have many of these, but I though I should give you a sense of what it might involve.)

We assume that we have given a measurable space $(\Omega, \mathcal{F})$, with a probability distribution P on $\mathcal{F}$.

(a) (Uniqueness.) Let X be a random variable (on $(\Omega, \mathcal{F})$), with $E|X| < +\infty$. Let $\mathcal{A}$ be a sigma-field on Ω, with $\mathcal{A} \subseteq \mathcal{F}$. Let the conditional expectation $E(X|\mathcal{A})$ be as defined in Theorem 2.5 on p. 122 in the notes. (The "official" definition.) Assume that both Z_1 and Z_2 satisfy the criteria for being $E(X|\mathcal{A})$. Show that $P(Z_1 = Z_2) = 1$.

(b) (Bonus question: What about martingales?) Read forward to the definition of a martingale. Suppose that $(M_t^{(1)})_{0\leq t\leq T}$ and $(M_t^{(2)})_{0\leq t\leq T}$ are both martingales with respect to filtration $(\mathcal{F}_t)_{0\leq t\leq T}$ and probablity distribution P. (We assume that $\mathcal{F}_T \subseteq \mathcal{F}$). Assume that $M_T^{(1)} = M_T^{(2)}$. Determine whether (or not) it is necessarily true that $P(M_t^{(1)} = M_t^{(2)}$ for all $t \in [0, T]) = 1$? [Hint 1: This is not a question that requires background in measure-theoretic probability. It does require having had some exposure to some fairly basic probability, plus some possibly hard thinking. A correct argument is only a few lines. You may wish to use that a Wiener process is a martingale. Hint 2: For the purposes of this question, we have not assumed that martingales are right continuous with left limits. Hint 3: Ignore Hint 2 unless you already know what it refers to.]