# University of Chicago Booth School of Business

| Business 33840                      |   | John Huizinga |
|-------------------------------------|---|---------------|
| Macroeconomics                      | 2021-01-20  | Winter 2021   |
| Week 2 – 6/26 & 6/27                | (Target)  | QUIZ#         |
| CLASS NOTES I                       | ving and Covernment Dudget Deficits (I.6)   | 02            |
| <b>*</b>                            | ving and Government Budget Deficits (I.6) g & Wealth, Deficits & Debt (I.7) (later) | Q2            |
| The Government Debt                 | to GNP Ratio (I.8) (extra lecture? review session                                   | n?) <b>NO</b> |
| The Current Account an              | nd the Sectoral Balance Equation (I.9) (later)                                      |               |
| Real GDP and Inflation              | ı (I.10)  | <b>Q2</b>     |
| Nominal and Real Inter              | rest Rates (I.11)   | <b>Q2</b>     |
| Nominal and Real Budg               | get Deficits (I.12) (on your own, optional)   | NO            |
| CLASS NOTES II                      |   |               |
| Cobb – Douglas Produc               | ction Function (II.1)   | <b>Q2</b>     |
| Classical Model (II.2)              |   | Q2            |
| Endogenous and Exoge                | enous Variables (II.3)  | Q2            |
| N <sup>d</sup> – Key Equation (II.4 | 4)  | Q2            |
| <ul><li>Key Results</li></ul>       |   |               |
| – Picture   N <sup>d</sup> curve    |   |               |
| Constant Income Share               | es (II.5) (on your own, required)   | Q2            |

#### **I.6 Disposable Income, Saving and Government Budget Deficits**

The presence of the government in the economy means not all the income earned in a country will be available to the residents of that country to spend on consumption goods, spend on acquiring new capital or be saved.

The following notation will be used throughout these notes:

TRGP = Transfers by Government to Private Sector (including interest payments)

TRGF = Net Transfers by Government to Foreigners (including interest payments)

TRPF = Net Transfers by Private Sector to Foreigners

TA = Taxes paid by Private Sector to the Government

G<sup>C</sup> = Government Purchases of Consumption Goods

G<sup>I</sup> = Government Purchases of Investment Goods

YD = Private Disposable Income

 $S^P$  = Private Savings

 $S^G$  = Government Savings

 $S^{N}$  = National Savings

BD = Government Budget Deficit

Along with the following definitions:

$$G = G^{C} + G^{I}$$

$$YD = GNP + TRGP - TA$$

$$S^{P} = YD - C - TRPF$$

$$S^{G} = TA - G^{C} - TRGP - TRGF$$

$$BD = G + TRGP + TRGF - TA$$

$$S^{N} = S^{P} + S^{G}$$

Note 1: Not all spending is subtracted from disposable income to get private saving. Only consumption spending and transfers are subtracted. Investment spending is not subtracted. Likewise for the definition of government saving.

Note 2: Neither S<sup>P</sup> nor S<sup>G</sup> includes capital gains and losses on existing assets

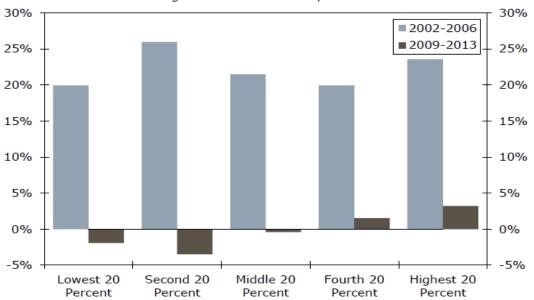
Note 3: If 
$$G = G^C$$
 then  $BD = -S^G$ 

Note 4: If the government earns profits from state-run enterprises those revenues should be added to  $S^G$  and subtracted from BD.

The following graphs (taken from John Silva and E. Nelson, "Income Growth: The Taxman Cometh", Wells Fargo Economics Group, October 20, 2014) show the importance of distinguishing between before tax and after tax income.

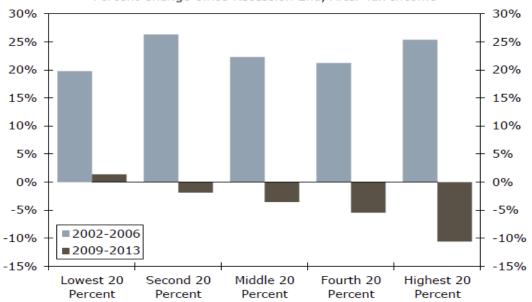
#### Income Growth During Economic Recoveries

Percent Change Since Recession End, Before-Tax Income



#### Income Growth During Economic Recoveries

Percent Change Since Recession End, After-Tax Income



The above graphs also touch on an important issue that will not be discussed much in this class: income distribution.

I highly encourage you to watch the video **Human Capital Investment, Inequality and Growth**, which is a talk Kevin Murphy (a Booth faculty member) gave to University alumni. You can find a link to the video on the class website; just click *Murphy on Inequality*.

A common tool for measuring inequality is the Gini coefficient; class notes have some details on this. Gini coefficients appear in an interesting article on the perceived tension between income growth and income inequality see John Paul Rathbone and Santiago Benedict Mander, **Chile business sees 'ghost' of Allende in free market reforms,** *Financial Times*, December 8, 2014.

For a recent interesting take on the world becoming more equal see Nicholas Eberstadt, **How the World is Becoming More Equal**, *The Wall Street Journal*, August 26, 2014.

#### **I.10 The Calculation of Real GDP and Inflation**

Virtually all variables in economics can be measured in *nominal* terms and in *real* terms. Nominal always means: Measured in Currency Real always measured: Measured in Goods

|         | Year 1 = Base Year |                | Yea            |                |                   |
|---------|--------------------|----------------|----------------|----------------|-------------------|
|         |                    | Nominal        |                | Nominal        | Real              |
|         | Price Quantity     | <u>GDP</u>     | Price Quantity | <u>GDP</u>     | <u>GDP</u>        |
|         | $P_1$ $Q_1$        | $P_1Q_1$       | $P_2$ $Q_2$    | $P_2Q_2$       | $P_BQ_2 = P_1Q_2$ |
| Tickets | \$5.00 100         | \$500          | \$6.00 140     | \$840          | \$700             |
| Beer    | \$2.00 1000        | \$ <u>2000</u> | \$3.50 1050    | \$ <u>3675</u> | \$ <u>2100</u>    |
|         |                    | \$2500         |                | \$4515         | \$2800            |

Real GDP Growth = (2800-2500)/2500 = .12 = 12%

Note: Ticket growth is (140-100)/100 = 0.4 = 40%

Beer growth is (1050-1000)/1000 = 0.05 = 5%

Share of Tickets in Base Year GDP is 1/5

Share of Beer in Base Year GDP is 4/5

(1/5)\*40% + (4/5)\*5% = 8% + 4% = 12%

Fixed Weight Index - Weights given by Base Year share of GDP

Real GDP also called constant dollar GDP, fixed dollar GDP, or GDP in \$1998

Year 1 GDP Deflator = (Nominal GDP/Real GDP) \* 100 = (2500/2500)\*100 = 100 Year 2 GDP Deflator = (Nominal GDP/Real GDP) \* 100 = (4515/2800)\*100 = 161.25

Inflation = % change in GDP deflator

Intuition: GDP deflator = P\*Q / Q = P and  $\%\Delta P = inflation$ 

For this economy inflation is = (161.25-100)/100 = .6125 = 61.25%

Note: Ticket inflation is (6-5)/5 = .20 = 20%

Beer inflation is (3.5-2)/2 = .75 = 75%

Share of Tickets in year 2 real GDP is 1/4

Share of Beer in year 2 real GDP is 3/4

(1/4)\*20% + (3/4)\*75% = 5% + 56.25% = 61.25%

Variable Weight Index - Weights given by Current Year share of real GDP

Note: There are also deflators for each component of GDP, i.e. a Consumption Deflator, an Investment Deflator, a Government Purchases Deflator . . .

In the U.S., the Fed uses the Consumption Deflator measure of inflation, also known as the PCE (Personal Consumption Expenditure) Deflator measure of inflation, as its target measure. It does not use inflation based on the CPI.

Chain Weighting: (a way to calculate real growth and get around the fixed weight index problem)

|         |                         |                      |                         | Year 1                     |                      | Year 2                     |                             |
|---------|-------------------------|----------------------|-------------------------|----------------------------|----------------------|----------------------------|-----------------------------|
|         | Price<br>P <sub>1</sub> | Price P <sub>2</sub> | Price<br>P <sub>A</sub> | Quantity<br>Q <sub>1</sub> | $\frac{GDP}{P_AQ_1}$ | Quantity<br>Q <sub>2</sub> | $\frac{\text{GDP}}{P_AQ_2}$ |
| Tickets | \$5.00                  | \$6.00               | \$5.50                  | 100                        | \$550                | 140                        | \$770.00                    |
| Beer    | \$2.00                  | \$3.50               | \$2.75                  | 1000                       | \$ <u>2750</u>       | 1050                       | \$2887.50                   |
|         |                         |                      |                         |                            | \$3300               |                            | \$3657.50                   |

Chain Weight Real GDP Growth = (3657.5-3300)/3300 = .1083 = 10.83% Fixed Weight Real GDP Growth = 12%

Why is the Chain Weight Real GDP growth lower? The key here is that the high growth industry (tickets) has a falling relative price, i.e. the price of tickets divided by the price of beer is lower in period 2 than in period 1.

The weight that is being used for tickets in the chain weight procedure is the share of tickets in year 1 GDP using  $P_A$ , while the weight being used for tickets in the fixed weight procedure is the share of tickets in year 1 GDP using  $P_1$ . With the relative price of tickets falling, ticket's share in the calculation of real GDP growth falls from 500/2500 = 1/5 using  $P_1$  to 550/3300 = 1/6 using  $P_A$ .

As a check: (1/6)\*40% + (5/6)\*5% = (40/6)% + (25/6)% = (65/6)% = 10.83%

Is it generally true that high growth industries have relative prices that are falling?

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The answer is yes.

Why? Patricia Pollard (See **Introducing Chain-Weight GDP Data**, National Economic Trends, Federal Reserve Bank of St. Louis, October 1, 1995) gives the following explanation:

"As prices change, people and businesses alter their purchases, substituting away from goods whose prices are rising relative to others and toward goods whose relative prices are declining. Thus, output increases most for goods whose relative prices are falling."

How can Pollard's analysis be summarized in 5 words?

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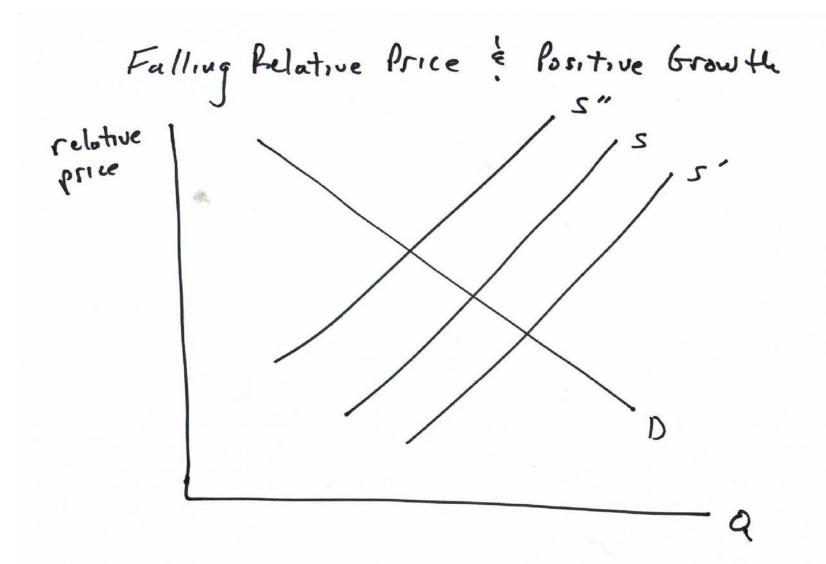
Pollard is saying, because demand curves slope down.

This may be correct. However, what determines prices and quantities?

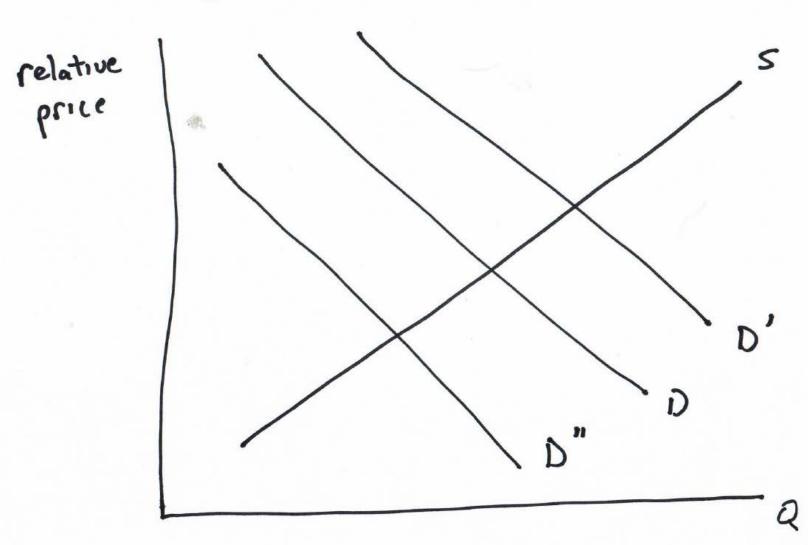
Prices and Quantities are jointly determined by the intersection of Supply and Demand curves!

A better answer than Pollard's is that demand curves slope down AND SUPPLY SHOCKS (SHIFTS OF THE SUPPLY CURVE) ARE MORE IMPORTANT THAN DEMAND SHOCKS (SHIFTS OF THE DEMAND CURVE).

This is clearly true for computers, though it is not true for health care.



# Rising Relative Price & Positive Growth



The CPI: The cost of purchasing a fixed bundle of consumption goods now relative to the cost in a base year.

Define the consumption bundle to be actual base year consumption

```
Period 1 CPI = (Current Cost/Base Year Cost) * 100 = (2500/2500)*100 = 100
Period 2 CPI = (Current Cost/Base Year Cost) * 100 = (4100/2500)*100 = 164
Note: 4100 = P_2 Q_1 = (6 \times 100) + (3.5 \times 1000)
```

Inflation = % change in CPI

For this economy CPI inflation = (164-100)/100 = .64 = 64%Note: Ticket inflation is (6-5)/5 = .20 = 20%Beer inflation is (3.5-2)/2 = .75 = 75%Share of Tickets in period 1 real GDP is 1/5Share of Beer in period 1 real GDP is 4/5 (1/5)\*20% + (4/5)\*75% = 4% + 60% = 64%CPI is Fixed Weight Index - Weights given by Base Year share of GDP!

Recall % change in Deflator = (161.25-100)/100 = .6125 = 61.25% Variable Weight Index - Weights given by Current Year share of real GDP

Measuring inflation with the change in CPI overstates inflation because consumers shift away from goods whose relative prices rise and this is not captured by a fixed weight index.

CPI inflation may also be overstated or understated due to improperly accounting for quality improvements in goods and the introduction of new goods (whose previous price was in essence infinity); however, this bias also applies to inflation measured by a variable weight deflator.

**The Core CPI**: The Core CPI omits food and energy prices, which are more volatile than other prices in the CPI. The rationale for this is to isolate the "permanent" or "trend" component of inflation.

The graphs below show the behavior of inflation in the U.S., the Euro Zone, the U.K. and Mexico, using both the Core CPI and CPI.

In all cases, inflation measured using the Core CPI is noticeably more stable.

Note: It is completely annoying that the graphs for the U.S. and Mexico are the reverse of the graphs for the U.K. and Eurozone, i.e. they reverse the black and blue lines.

However, careful examination of the graphs for Mexico and the U.K. reveals the following important concern: While the Core CPI measure of inflation does appear more stable, and therefore may be a better measure of trend or permanent inflation, it also appears that it may understate inflation; i.e. in Mexico core CPI inflation is lower than CPI inflation almost continuously throughout the seventeen year period, and in the U.K. it is essentially lower for all but the two year period from 2015–17.

A better measure of the permanent component of inflation would reduce the variability of inflation while maintaining the same mean.

### For the U.S.

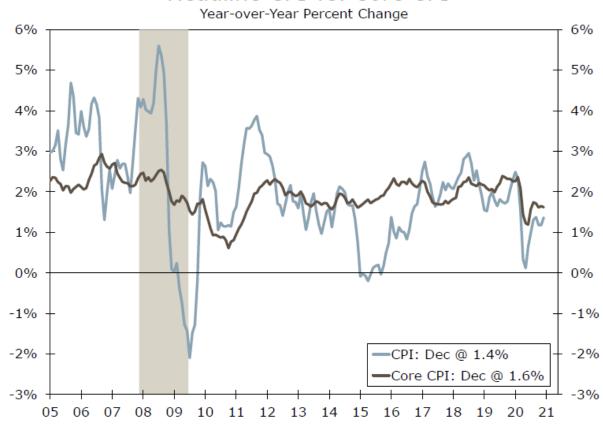
January 15, 2021

### **Economics Group**



Weekly Economic & Financial Commentary

#### Headline CPI vs. Core CPI

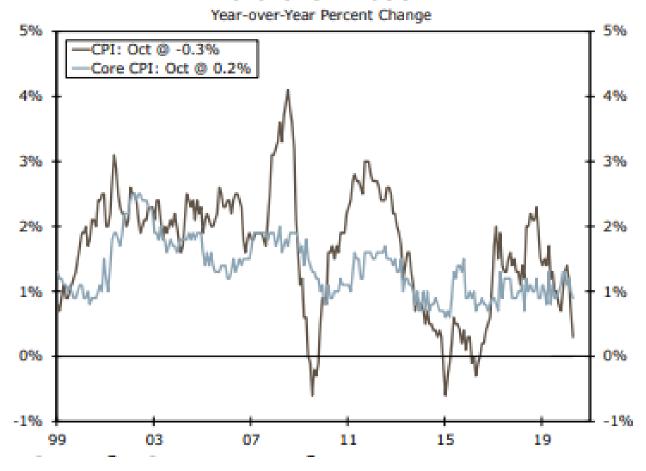




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### Eurozone Inflation

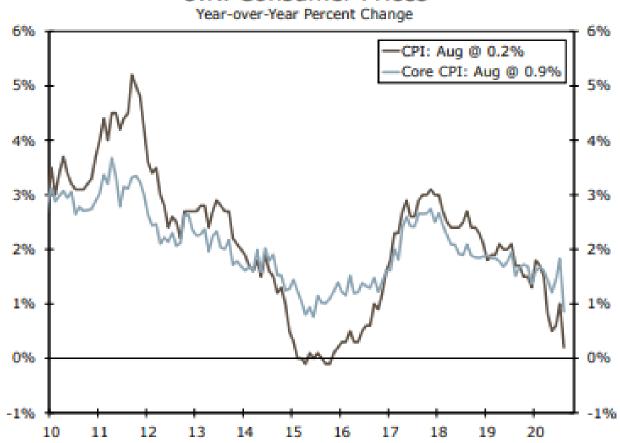




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### U.K. Consumer Prices

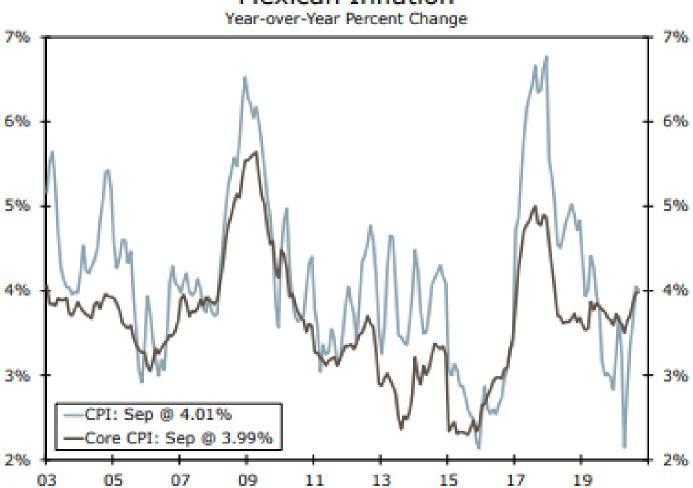


# WELLS SECURITIES

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## Mexican Inflation



## **I.11 Real and Nominal Interest Rates**

Every Asset has 4 returns

Notation (applied to a default-free one-period bond)

|                    | Nominal        | Real           |
|--------------------|----------------|----------------|
| Actual (Ex Post)   | i              | r              |
| Expected (Ex Ante) | i <sup>e</sup> | r <sup>e</sup> |

Nominal: Rate at which money is moved through time using this asset

Real: Rate at which goods are moved through time using this asset

To understand nominal and real interest rates we will focus on a simple asset: a one-period discount bond that pays one dollar for sure.

With a one-period bond, the nominal (real) return is the same as the nominal (real) interest rate; there are no capital gains or losses.

The goals of this analysis are (i) measure the nominal return on holding this bond (both actual and expected); (ii) measure the real return on holding this bond (both actual and expected); (iii) show the relationship between the nominal and real returns and the relationship between the bond price and the nominal interest rate.

#### Notation:

 $q_t$  = price of a one-period discount bond in period t

 $P_t$  = price level in period t

 $\pi_t$  = actual inflation rate from period t to period t+1

$$= (P_{t+1} - P_t) / P_t = (P_{t+1}/P_t) - 1$$

 $i_t$  = actual nominal return from period t to period t+1

 $r_t$  = actual real return from period t to period t+1

 $P_{t+1}^e$  = price level in period t+1, expected in period t

 $\pi^{e}_{t}$  = inflation rate from period t to period t+1, expected at period t

$$= (P_{t+1}^e - P_t) / P_t = P_{t+1}^e / P_t - 1$$

 $i^{e}_{t}$  = nominal return from period t to period t+1, expected at period t

 $r_t^e$  = real return from period t to period t+1, expected at period t

In what follows, the following simplifications are used; they follow directly from the above definitions of actual and expected inflation

$$1 + \pi_t = P_{t+1}/P_t$$
 so that  $1/(1 + \pi_t) = P_t/P_{t+1}$  and

$$1+\pi^{e}_{t}=P^{e}_{t+1}/P_{t}$$
 so that  $1/(1+\pi^{e}_{t})=P_{t}/P^{e}_{t+1}$ 

Actual Nominal Return: the rate at which money is transferred across time

\$1 period  $t \Rightarrow (1/q_t)$  bonds period  $t \Rightarrow \$(1/q_t)$  period t+1

$$i_t = [(1/q_t) - 1]/1 = 1/q_t - 1$$

$$1+i_t=1/q_t \tag{1}$$

The nominal interest rate and bond prices move in opposite directions.

#### Actual Real Return: the rate at which goods are transferred across time

1 good period  $t \Rightarrow P_t$  period  $t \Rightarrow (P_t/q_t)$  bonds period  $t \Rightarrow P_t$  period t+1

 $\Rightarrow P_t/(q_tP_{t+1}) \text{ goods period } t+1 = (1/q_t)(P_t/P_{t+1}) = (1+i_t)/(1+\pi_t) \text{ goods period } t+1$ 

$$r_t = [(1 + i_t \,)/(1 + \pi_t) \, \text{-} \, 1]/1.$$

Adding 1 to each side yields,

$$(1+r_t) = (1+i_t)/(1+\pi_t)$$
 or

$$(1+i_t) = (1+r_t)(1+\pi_t). \tag{2}$$

Sometimes this is expressed as

$$i_t = r_t + \pi_t. \tag{3}$$

or equivalently,

$$\mathbf{r}_{\mathsf{t}} = \mathbf{i}_{\mathsf{t}} - \boldsymbol{\pi}_{\mathsf{t}}. \tag{3'}$$

Equations (3) and (3') can be thought of as an approximation to equation (2), since  $r_t$   $\pi_t$  (which is the term included in equation (2) and excluded from equation (3)) will likely be a small number.

Alternatively, equations (3) and (3') can be viewed as an exact relation when interest rates and inflation rates are expressed as "continuously compounded" rates, as opposed to "simple" rates.

Simple rates are calculated as (end-start)/start).

Continuously compounded rates are calculated as ln (end/start), where ln represents the natural log.

Thus, continuously compounded inflation is defined as  $\ln{(P_{t+1}/P_t)}$ , the continuously compounded, actual, nominal interest rate is  $\ln{(1/q_t)}$ , and the continuously compounded, actual real interest rate is  $\ln{[(1/q_t)(P_t/P_{t+1})]}$ .

Note: ln(1) = 0.

Intuition behind  $r = i - \pi$ :

i = 10% => get 10% more units of money in the future if buy bond

 $\pi = 10\% \implies$  a given amount of money purchases 10% fewer goods in the future

Putting the two results together => if you buy bond today

then you can buy the same amount of goods in the future as you can buy now, not more and not less

$$=> r = i - \pi = 0\%$$

Equations (1) - (3) involve <u>actual</u> returns and inflation, however economic behavior is often governed by <u>expected</u> returns and inflation.

**Expected Nominal Return**: rate at which money is expected to be transferred across time

\$1 period  $t \Rightarrow (1/q_t)$  bonds period  $t \Rightarrow$  expect  $(1/q_t)$  period t+1

$$i^e_t = [(1/q_t) - 1]/1 = 1/q_t - 1 = i_t$$
 (4)

Since the bond is riskless in its dollar payoff, the expected nominal return is the same as the actual nominal return.

Expected Real Return: rate at which goods are expected to be transferred across time

1 good period  $t \Rightarrow P_t$  period  $t \Rightarrow (P_t/q_t)$  bonds period  $t \Rightarrow \text{expect } P_t/q_t$  period t+1  $\Rightarrow \text{expect } P_t/(q_tP_{t+1}^e)$  goods period  $t+1 = \text{expect } (1/q_t)/(1+\pi_t^e)$  goods period t+1

$$r_{t}^{e} = [(1/q_{t})/(1+\pi_{t}^{e}) - 1]/1.$$

Adding 1 to each side, using (4) to substitute for  $(1/q_t)$ , and multiplying both sides by  $(1+\pi^e_t)$  yields,

$$(1+i^{e}_{t}) = (1+r^{e}_{t})(1+\pi^{e}_{t}). \tag{5}$$

This is often approximated by

$$i^e_t = r^e_t + \pi^e_t.$$
 (6)

or equivalently,

$$r_{t}^{e} = i_{t}^{e} - \pi_{t}^{e}.$$
 (6')

Thus, the relation between expected returns and expected inflation takes the same form as the relation between actual returns and actual inflation derived in equations (2), (3) and (3').

When comparing expected and actual real returns on an asset, in the general case where the expected nominal return can differ from the actual nominal return, the above equations can be combined to yield

$$r_{t} = r_{t}^{e} + (i_{t} - i_{t}^{e}) + (\pi_{t}^{e} - \pi_{t})$$
(8)

When the actual nominal return turns out to be higher than expected, actual real returns will turn out higher than expected real returns.

When inflation turns out to be higher than expected, actual real returns will turn out lower than expected real returns.

We have discussed measuring actual inflation. How one measure does expected inflation?

- 1. Survey people to see what they say they expect. *The Economist* publishes inflation forecasts from professional forecasters. A major concern with survey data is, do respondents take the surveys seriously enough to answer accurately?
- 2. Tie inflation to economic fundamentals. Use the past history of actual inflation, core inflation, or median inflation; augment this information with past history of money growth or possibly output growth. A major concern with this approach is, are people smart enough to build a reliable, stable forecasting equation?
- 3. Use the spread on nominal vs. inflation indexed securities (watch out for risk premia and liquidity premia) or CPI futures. See Simon Kwan, **Inflation Expectations: How the Market Speaks**, *Federal Reserve Bank of San Francisco Economic Letter*, October 3, August 2005 and Jens Christensen, **TIPS and the Risk of Deflation**, *Federal Reserve Bank of San Francisco Economic Letter*, October 25, August 2010. A major concern with this approach is, do people who actively participate in financial markets have the same expected inflation as people who do not participate?

Key Point: The expected real interest rate (r<sup>e</sup>) is the expected relative price of consumption today in terms of consumption tomorrow.

When r<sup>e</sup> increases, buying one unit of consumption today means expecting to give up more consumption in the future. Thus, a higher r<sup>e</sup> means consumption today is expected to be more expensive relative to consumption tomorrow.

Great Exam Question: True, False or Uncertain - An increase in expected inflation means people should buy now when things are cheap, buy before prices rise. They should shift consumption forward in time, consume more now and less in the future.

Great Exam Answer: Uncertain. It depends on what happens to the nominal interest rate.

If the nominal interest rate rises one-for-one with expected inflation, then the expected real interest rate doesn't change and the price of current consumption relative to future consumption is unaffected; there is no incentive to shift consumption.

If the nominal interest rate does not rise by this much then the expected real interest rate falls, current consumption is cheaper and there is an incentive to buy now.

If the nominal interest rate rises by more than expected inflation rises then the expected real rate rises, current consumption is more expensive, and there is an incentive to delay consumption.

Also, a change in the expected real interest rate is a change in a relative price. That means there are two effects on demand when the expected real interest rate changes.

What are they?

Great Exam Answer: Uncertain. It depends on what happens to the nominal interest rate.

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Also, a change in the expected real interest rate is a change in a relative price. That means there are two effects on demand when the expected real interest rate changes.

What are they? An income effect and a substitution effect.

The analysis in the question only covers the substitution effect; it ignores the income effect.

## CLASS NOTES II - THE PRODUCTION FUNCTION AND THE CLASSICAL MODEL OF THE LABOR MARKET

### **II.1 Cobb-Douglas Production Function**

One of the most important elements of economic analysis is the production function, which describes the amount of output that can be produced from a given amount of inputs. In macroeconomics the following production function, which involves a single kind of labor and a single kind of capital as the two inputs, is frequently used:

$$Y = A K^{\alpha} N^{1-\alpha}$$

Where Y = real output

K = real capital input, measured in units of capital

N = labor input, measured in hours worked

A = total factor productivity

 $\alpha$  = number between zero and one (usually about .33)

This production function is called a Cobb-Douglas production function, named after its originators: a University of Chicago economist (Douglas) and mathematician (Cobb).

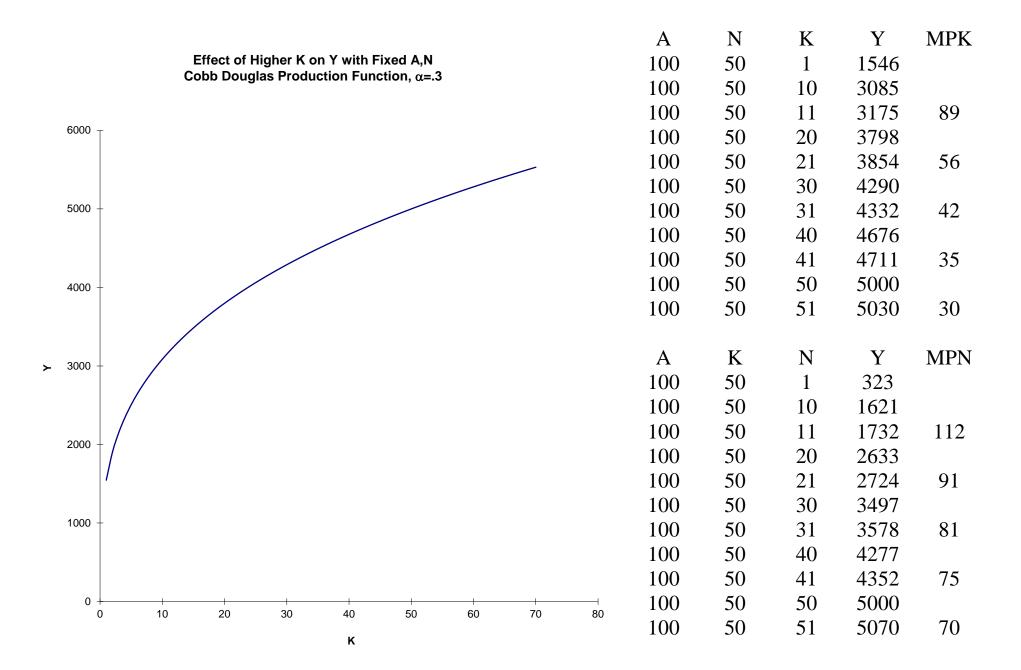
The function has several important properties.

- (1) If markets are competitive, labor will be paid the fraction 1- $\alpha$  of output and capital paid the fraction  $\alpha$ . This will be demonstrated later in Section II.5 of these notes.
- (2) Constant returns to scale: if you double all inputs, output will be doubled
- (3) Decreasing marginal product of labor (i.e. decreasing MPN) and decreasing marginal product of capital (i.e. decreasing MPK): Holding the amount of other inputs constant, each additional unit of labor input creates a smaller increase in output; likewise for each additional unit of capital.

See the graph and table on the following page.

The fact that the marginal product of labor decreases with increased labor input will be a key factor in the model of labor demand we will use.

The fact that the marginal product of capital decreases with increased capital input is important not only for the model of demand for capital we will use, but also because it helps one evaluate a common misperception about economic growth - the misperception that higher investment is the key to eternal economic growth. (See Class Notes VIII)



The exact formulas for the marginal product of labor and marginal product of capital in the case of the Cobb-Douglas production function are:

$$MPN = (1-\alpha) A K^{\alpha} N^{-\alpha} = (1-\alpha) A K^{\alpha} / N^{\alpha} = (1-\alpha) A (K/N)^{\alpha}$$

 $MPN = \partial Y/\partial N$ 

$$MPK = \alpha \ A \ K^{\alpha-1} N^{1-\alpha} = \alpha \ A \ N^{1-\alpha} / K^{1-\alpha} = \alpha \ A \ (N/K)^{1-\alpha}$$

 $MPK = \partial Y/\partial K$ 

Examination of these formulas shows:

- a) Labor and Capital are complements, not substitutes. Capital makes labor more productive, and the reason why the marginal product of labor falls with increased labor is that the capital per unit of labor is falling. Likewise, labor makes capital more valuable.
- b) Total Factor productivity helps all factors equally.

How should we think about TFP?

#### It will include

- (i) The quality of labor and capital; N and K measure the quantity. If the quality of inputs goes up we expect output to go up for the same quantity of inputs.
- (ii) The efficiency with which labor and capital are utilized and combined.
- (iii) Omitted inputs; e.g. energy.

## **II.2 The Classical Economic Model**

Models are mathematical descriptions of simplified economies.

Math forces us to use logic instead of potentially faulty intuition; it allows use to prove results; it enables us to discover results that would be missed if we relied on intuition.

Simplified economies allow us to understand what is going one; they allow us to develop valid intuition.

Models describe simplified economies by use of assumptions, which come in two types: Critical and Simplifying

Critical: Change the assumption and the model works very differently

Simplifying: Change the assumption and the model works basically the same, but it becomes more complicated and less valuable for developing intuition.

The model developed in this class is called a Classical Model.

Classical Models typically have at least three things in common:

- (1) Agents maximize. For example, firms maximize profits and households maximize utility. (Critical)
- (2) Wages, prices, interest rates, etc. are flexible enough to ensure that markets clear, i.e. supply is equilibrated with demand. (Critical)
- (3) Firms are perfectly competitive. That is, they take wages and prices as given. They have no monopoly power. (Simplifying)

## Other Assumptions of the model developed in class

The model we will work with is for a one-good economy. That is, the economy produces a single good, which can be either consumed, or through investment, be added to the capital stock. The assumption of a one good economy is essentially the assumption that the relative price of different goods is fixed. It also means there are no intermediate goods.

The initial model will be for a closed economy, i.e. one that doesn't trade with the rest of the world. Thus, we will have real GDP and real GNP being equal, and will use Y to represent both of them.

The economy will be comprised of households, firms, and the government.

Households sell labor to firms, who produce output using capital and labor. Households own the firms and receive the profits. All households are the same and all firms are the same.

The model we will begin with will have the amount of capital fixed, so firms only have the ability to change labor input. Models in which the amount of capital is not fixed are included in Class Notes VIII. These notes cover a field of economics known as Growth Theory.

In this model the government will

- (i) collect income taxes, value added/sales taxes, and corporate profits taxes
- (ii) use revenue from these taxes to make transfer payments to households
- (iii) use revenue from these taxes to purchase goods and services
- (iv) have the ability to run budget deficits and issue debt.

The government does not hire labor or produce any goods.

The model we work with will abstract from the effects of uncertainty. That is, we will not assume people can forecast the future with perfect certainty, but neither will we assume that people will alter their behavior as the future becomes more or less uncertain. A world in which expected inflation is  $5\% \pm 1\%$  is the same as a world in which expected inflation is  $5\% \pm 3\%$ .

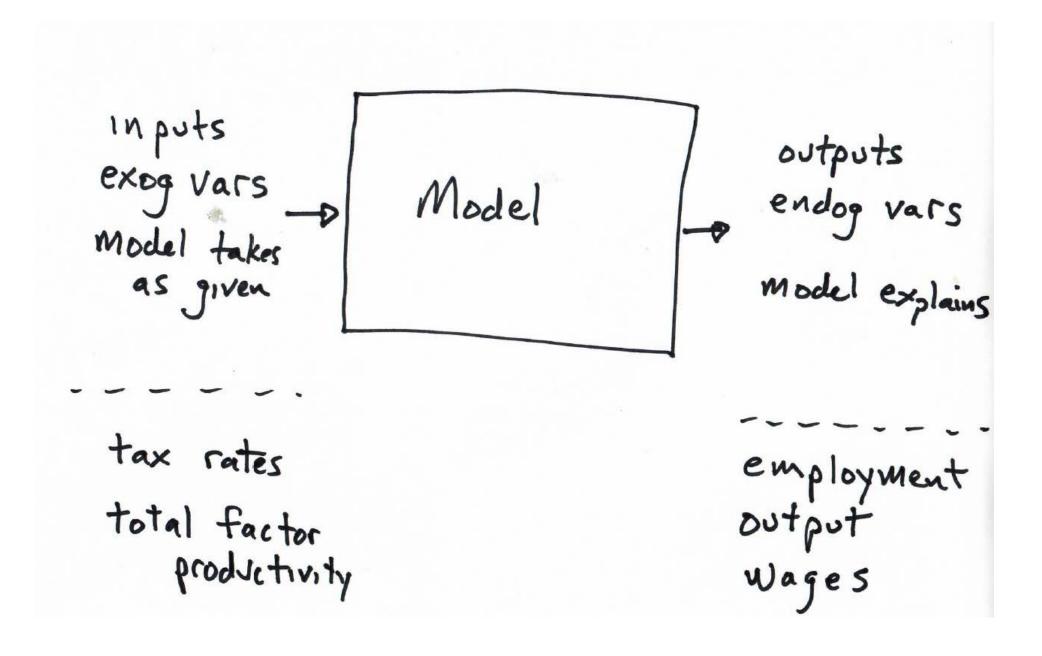
# **II.3 Endogenous and Exogenous Variables**

In any model, it is important to distinguish between endogenous and exogenous variables.

Think of exogenous variables as inputs to a model and endogenous variables as outputs.

An exogenous variable is one which the model takes as given.

An endogenous variable is one that a model can explain.



It is important to distinguish between what variables are exogenous to the particular agents in the model, as distinct from what variables are exogenous to the model itself.

For example, with perfectly competitive firms who take wages and prices as given, wages and prices are exogenous to the individual firm. Wages and prices can be endogenous to the model, however.

# **II.4 Labor Demand**

#### New Notation:

 $N^d$  = labor demand

W = nominal wage

P = price of output received by firm

W/ P = real wage

W has units \$ / hour

P has units \$ / good

W/P has units 
$$\frac{\$/_{\text{hour}}}{\$/_{\text{good}}} = \frac{\text{goods}}{\text{hour}}$$

$$W = $12 / hour$$
  $P = $4 / good$   $W/P = 3 goods / hour$ 

### **Key equation: W/P = MPN**

This equation must hold if firms are maximizing profits.

It states that the marginal cost of hiring an additional unit of labor (W/P) is set equal to the marginal benefit of hiring an additional unit of labor (MPN). Both marginal cost and marginal benefit are measured in the same units, goods.

Note: This equation could also be done using marginal cost and marginal benefit measured in currency, i.e. W = P MPN. Obviously this is the same as the key equation above.

#### Intuition:

If W/P > MPN, marginal cost exceeds marginal benefit

profit maximization implies hire less labor

with less labor MPN rises

once MPN rises enough to equal W/P, stop

If W/P < MPN, marginal benefit exceeds marginal cost

profit maximization implies hire more labor

with more labor MPN falls

once MPN falls enough to equal W/P, stop

Firms adjust MPN by adjusting N; they adjust until W/P = MPN.

**Key equation: W/P = MPN** 

#### **Key results:**

Three variables affect N<sup>d</sup>, they are W/P, A, and K.

To isolate these effects change one variable at a time, holding all other variables constant.

## Holding A and K constant, an increase in W/P decreases the amount of labor demanded

Math:

Before W/P increases the key equation holds, W/P = MPN.

After W/P increases, at current N<sup>d</sup>, the LHS of the key equation is now above the RHS; marginal cost exceeds marginal benefit. To bring the two sides of the equation back into balance, N<sup>d</sup> must decrease and thereby increase MPN.

#### Intuition:

Prior to an increase in the real wage, the last unit of labor hired by a profit maximizing firm was just productive enough to cover its cost, the real wage. Now, with a higher real wage, that last unit of labor is not productive enough to cover its cost and profits can be increased if that unit of labor is not hired. Labor demand falls.

#### Holding K and W/P constant, an increase in A increases the amount of labor demanded

Math:

Before A increases the key equation holds, W/P = MPN.

After A increases, at current N<sup>d</sup>, the RHS of the key equation is now above the LHS; marginal benefit exceeds marginal cost. To bring the two sides of the equation back into balance, N<sup>d</sup> must increase and thereby decrease MPN.

#### **Intuition:**

Prior to an increase in productivity, the last unit of labor hired by a profit maximizing firm was just productive enough to cover its cost, the real wage. Now, with a higher productivity, that last unit of labor is more than productive enough to cover its cost and profits can be increased by hiring an additional unit of labor. Labor demand increases.

What's wrong with the following logic? With higher productivity, firms need less labor to produce the same amount of goods, so labor demanded should fall.

Hint: What is the implied objective of the firm in this statement and how does it compare to the objective assumed in our model?

What does the above analysis tell you about a centrally planned economy where output of different goods is determined first, and then labor is allocated to ensure that production goals are met?

In particular, if one industry has an increase in productivity and another has a decrease in productivity then what reallocation of labor is likely to happen? Does it make intuitive sense that this is a good system?

The beauty, the power, the sheer magnificence of competitive markets and people acting in their own best interest is that such a system can deliver efficient social outcomes and does not require an omniscient benevolent central planner!

### Holding A and W/P constant, an increase in K increases the amount of labor demanded

Math:

Before K increases the key equation holds, W/P = MPN.

After K increases, at current N<sup>d</sup>, the RHS of the key equation is now above the LHS; marginal benefit exceeds marginal cost. To bring the two sides of the equation back into balance, N<sup>d</sup> must increase and thereby decrease MPN.

#### Intuition:

Prior to an increase in the capital stock, the last unit of labor hired by a profit maximizing firm was just productive enough to cover its cost, the real wage. Now, with a higher capital stock that unit of labor is more productive, so that last unit of labor is more than productive enough to cover its cost and profits can be increased by hiring an additional unit of labor. Labor demand increases.

# <u>Picture = Labor Demand Curve</u>:

Three variables effect labor demand. Only one can be placed on the vertical axis.

How is the variable that is placed on the vertical axis selected?

In general?

# <u>Picture = Labor Demand Curve</u>:

Three variables effect labor demand. Only one can be placed on the vertical axis.

How is the variable that is placed on the vertical axis selected?

In general? The variable that clears the market, i.e. the variable that adjusts to equilibrate supply and demand, appears on the vertical axis.

In this case?

# <u>Picture = Labor Demand Curve</u>:

Three variables effect labor demand. Only one can be placed on the vertical axis.

How is the variable that is placed on the vertical axis selected?

In general? The variable that clears the market, i.e. the variable that adjusts to equilibrate supply and demand, appears on the vertical axis.

In this case? The real wage; not the nominal wage.

- (1) With the real wage on the vertical axis, the labor demand curve is downward sloping.

  See FIGURE II-2
- (2) An increase in total factor productivity shifts the labor demand curve to the right.

  See FIGURE II-3
- (3) An increase in the capital stock shifts the labor demand schedule to the right.

See FIGURE II-3

FIGURE II-4 is a shorthand way to summarize FIGURE II-3

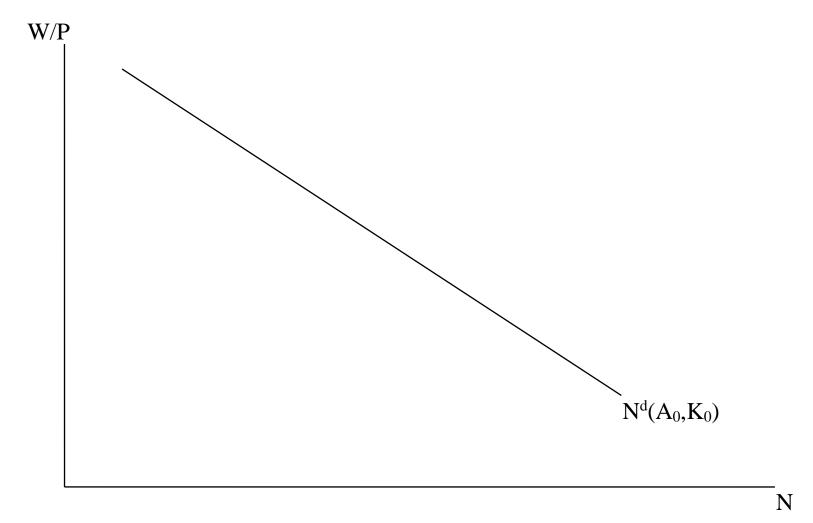


FIGURE II-2

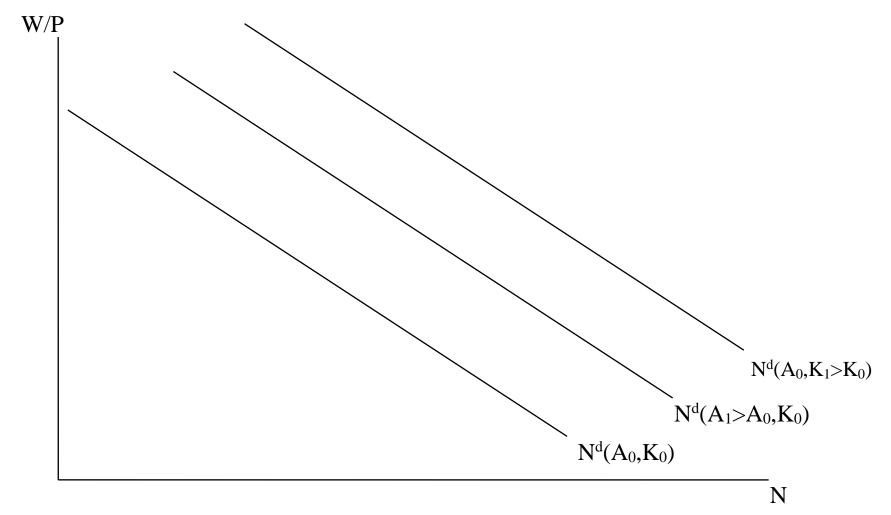


FIGURE II-3

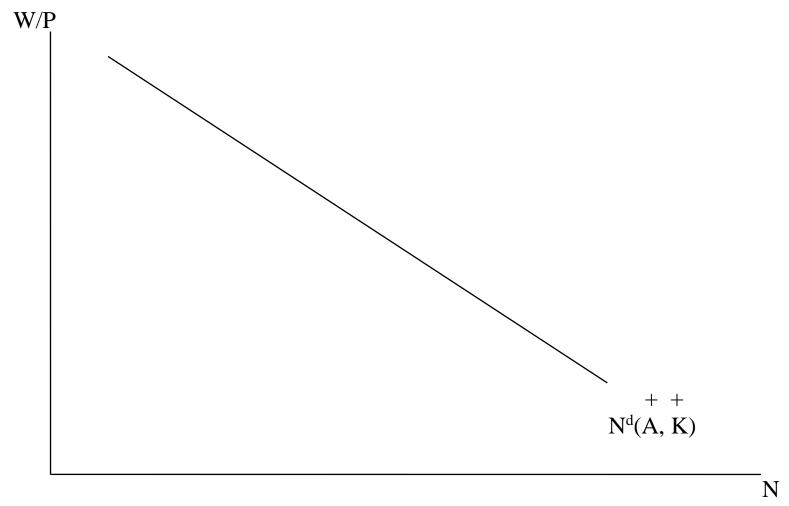


FIGURE II-4

## **II.5 Constant Income Shares**

The discovery of the Cobb-Douglas production function

Find Y(K,N) such that

$$W/P * N = constant * Y$$
 empirical fact

i.e. find Y(K,N) such that

$$MPN * N = constant * Y$$
 economic theory  $(W/P = MPN)$ 

i.e. find Y(K,N) such that

$$\frac{\partial Y}{\partial N} * N = \text{constant} * Y \qquad \text{math fact} \qquad (MPN = \frac{\partial Y}{\partial N})$$

Solution:  $Y = A K^{\alpha} N^{1-\alpha}$ 

Math:  $N(W/P) = N MPN = N (1-\alpha) A (K/N)^{\alpha} = (1-a) A K^{\alpha} N^{1-\alpha} = (1-\alpha) Y$ 

Intuition: As relative supplies of N and K vary, their marginal products, and therefore their compensation per unit input, move in exactly the right way to keep constant income shares.

 $N \uparrow \text{ with A and K fixed } => Y \uparrow$ 

$$=> MPN = W/P \downarrow$$

and the fall in W/P combined with the rise in Y is just balanced so that

$$\% \uparrow (N * W/P) = \% \uparrow Y$$

And thus the ratio of N \* W/P to Y remains unchanged

Likewise for K↑

As TFP changes, the marginal products of both N and K vary in equal proportion, and thus so will their compensation.