

(c)

(i)

The duration of bond A is

$$D_A = \frac{15/(1+10\%)}{118.95} \times 1 + \frac{15/(1+10\%)^2}{118.95} \times 2 + \frac{15/(1+10\%)^3}{118.95} \times 3 + \frac{15/(1+10\%)^4}{118.95} \times 4 + \frac{115/(1+10\%)^5}{118.95} \times 5 = 3.95$$

Similarly, we can find the duration of bond B is

$$D_B = \frac{15/(1+10\%)}{130.72} \times 1 + \frac{15/(1+10\%)^2}{130.72} \times 2 + \frac{15/(1+10\%)^3}{130.72} \times 3 + \dots + \frac{115/(1+10\%)^{10}}{130.72} \times 10 = 6.28$$

Suppose the proportion of bond A in the portfolio is  $x$  and the proportion of bond B is  $1 - x$ , then we want the duration of the portfolio to be 5 years.

$$\begin{aligned} x \times 3.95 + (1 - x) \times 6.28 &= 5 \\ x &= 0.549 \end{aligned}$$

So the proportion of bond A in the portfolio is 54.9% and the value of bond A in the portfolio is  $\$620901.16 \times 54.9\% = \$340874.7$ , which is  $\$340874.7/\$118.95 = 2865.7$  units of bond A.

Similarly, we should buy  $\$620901.16 \times (1 - 54.9\%)/\$130.72 = 2142.2$  units of bond B.

(ii)

Based on (i), we know the cash flow of the coupon of bond A is  $2865.7 \times \$15 = \$42985.5$ .

Similarly, the cash flow of the coupon of bond B is  $2142.2 \times \$15 = \$32133$ .

We can easily calculate the accumulated value of these cash flows at the end of the fifth year as

$$(\$42985.5 + \$32133) \times (1+r)^4 + \dots + (\$42985.5 + \$32133) \times (1+r) + (\$42985.5 + \$32133) + FV_A \times N_A + FV_B \times N_B + \frac{\$32133}{1+r} + \dots + \frac{\$32133}{(1+r)^5}$$

When  $r = 10\%$ , we have the future value in the fifth year as \$1,000,000.

If  $r = 9\%$ , we have the future value in the fifth year as \$1,000,347.

If  $r = 11\%$ , we have the future value in the fifth year as \$1,000,283.

Clearly, the three future values are close.