\5. A six-month zero-coupon bond with face value 99.46, a one-year zero- coupon bond sells for 90.50. Suppose a new coupon bond, making semi-annual coupon payments, is issued today with face value $100, maturity of 18 months, and a semi-annual coupon payment of 9% (the 9% is expressed as an annual rate).

(a) Calculate the no-arbitrage price of the coupon bond today.

For the six-month zero-coupon bond, the yield to maturity $r\_1 = \frac{$100}{$99.46}-1=0.0054$. Similarly, the yield to maturity of the one-year zero- coupon bond $r\_2 = (\frac{$100}{$97.23})^{\frac{1}{2}}-1 = 0.0141$ and the yield to maturity of the 18-month zero- coupon bond $r\_3 = (\frac{$100}{$90.50})^{\frac{1}{3}}-1 = 0.0338$.

So we have the non-arbitrage price of the new coupon

$$P = \frac{4.5}{1+0.0054}+\frac{4.5}{(1+0.0141)^2}+\frac{100+4.5}{(1+0.0338)^3}=$103.4332$$

(b) Calculate the implied forward rates in this economy.

Forward rate after one year .

Forward rate after 18 months

(c) If the liquidity preference theory is correct and there exists a liquidity premium of 0.5% per period, what is the market’s expectation of the price the bond will sell for in one year? 1 year = 2 periods here.

If the liquidity preference theory holds and the liquidity premium is 0.5%, the forward rate of each period is:

$$f\_{2,3} = 0.0744-0.005 = 0.0694 \\
f\_{1,2} = 0.0229-0.005 = 0.0179 \\
f\_{0,1} = 0.0054-0.005 = 0.0004$$

So the market price is:

$$Market Price = \frac{FV+CP}{(1+f\_{2,3})(1+f\_{1,2})(1+f\_{0,1})}=\frac{100+4.5}{(1+0.0694)(1+0.0179)(1+0.0004)}=$95.96$$