ps2_caiy

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1 PROBLEM SET 2

1.1 By YING CAI

1.1.1 Problem 1

In [123]:
```
from sympy import *
import numpy as np
```

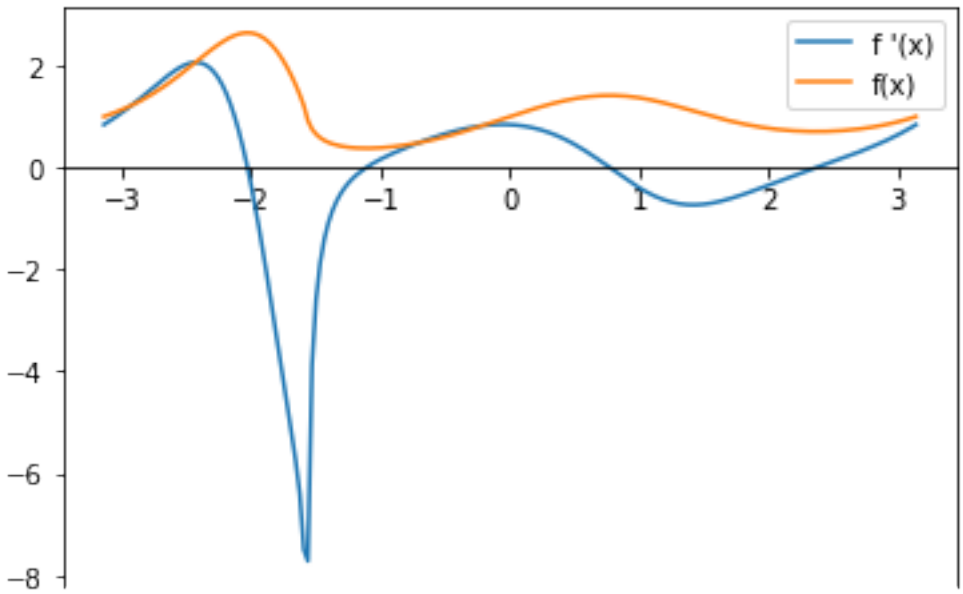
In [124]:
```
x = symbols('x')
diff((sin(x)+1)**(sin(cos(x))))
```

Out[124]:
```
(-log(sin(x) + 1)*sin(x)*cos(cos(x)) + sin(cos(x))*cos(x)/(sin(x) + 1))*
```

In [125]:
```
f_1 = lambdify(x, (sin(x)+1)**(sin(cos(x))))
```

In [126]:
```
f_dif = lambdify(x, diff((sin(x)+1)**(sin(cos(x)))))
```

In [127]:
```
x_points = np.linspace(-np.pi,np.pi,200)
f_calf = f_1(x_points)
f_caldif = f_dif(x_points)

import matplotlib.pyplot as plt
% matplotlib inline

plt.plot(x_points, f_caldif, label = "f '(x)")
plt.plot(x_points, f_calf, label = "f(x)")
ax = plt.gca()
ax.spines["bottom"].set_position('zero')
plt.legend()
plt.show()
```



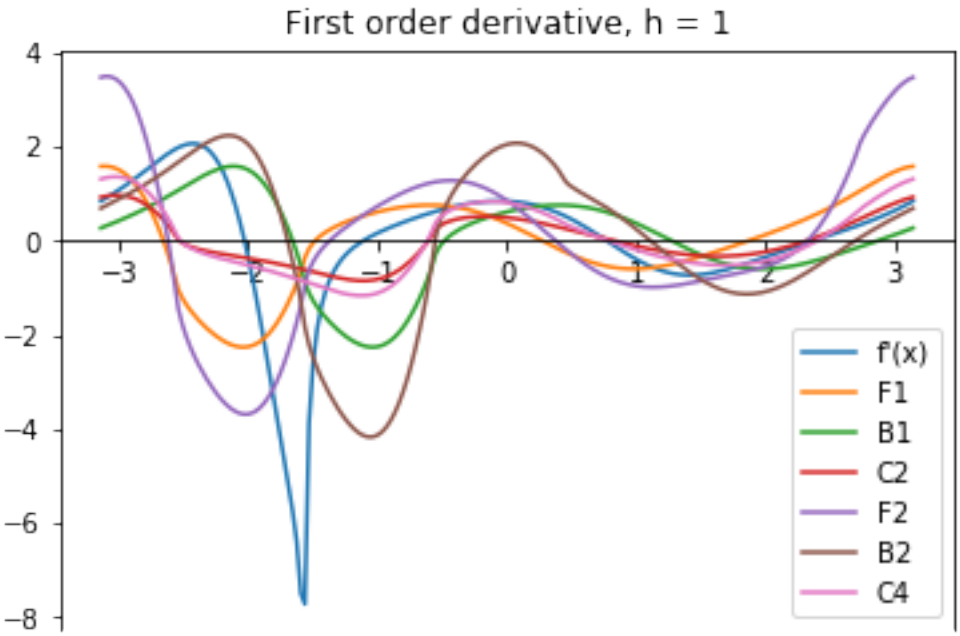
1.1.2 problem 2

```
In [128]: def dif_F1(f,x,h):
              return((f(x + h) - f(x))/h)

          def dif_B1(f,x,h):
              return ((f(x) - f(x - h))/h)

          def dif_C2(f,x,h):
              return ((f(x + h) - f(x - h))/(2.0*h))

          def dif_F2(f,x,h):
              return (-3*f(x) + 4.0*f(x + h) - f(x + 2*h))/(2.0*h)

          def dif_B2(f,x,h):
              return( (3.0*f(x) - 4.0*f(x - h) + f(x - 2*h))/(2.0*h))

          def dif_C4(f,x,h):
              return ((f(x - 2*h) - 8.0*f(x - h) + 8*f(x + h) - f(x + 2*h))/(12.0*h
```

```
In [129]: h = 1
          f = f_1
          % matplotlib inline
          plt.plot(x_points, f_caldif, label="f'(x)")
          plt.plot(x_points, dif_F1(f, x_points, h), label='F1')
          plt.plot(x_points, dif_B1(f, x_points, h), label='B1')
          plt.plot(x_points, dif_C2(f, x_points, h), label="C2")
```

```
plt.plot(x_points, dif_F2(f, x_points, h), label='F2')
plt.plot(x_points, dif_B2(f, x_points, h), label='B2')
plt.plot(x_points, dif_C4(f, x_points, h), label='C4')
ax = plt.gca()
ax.spines["bottom"].set_position('zero')
plt.legend()
plt.title("First order derivative, h = 1")
```

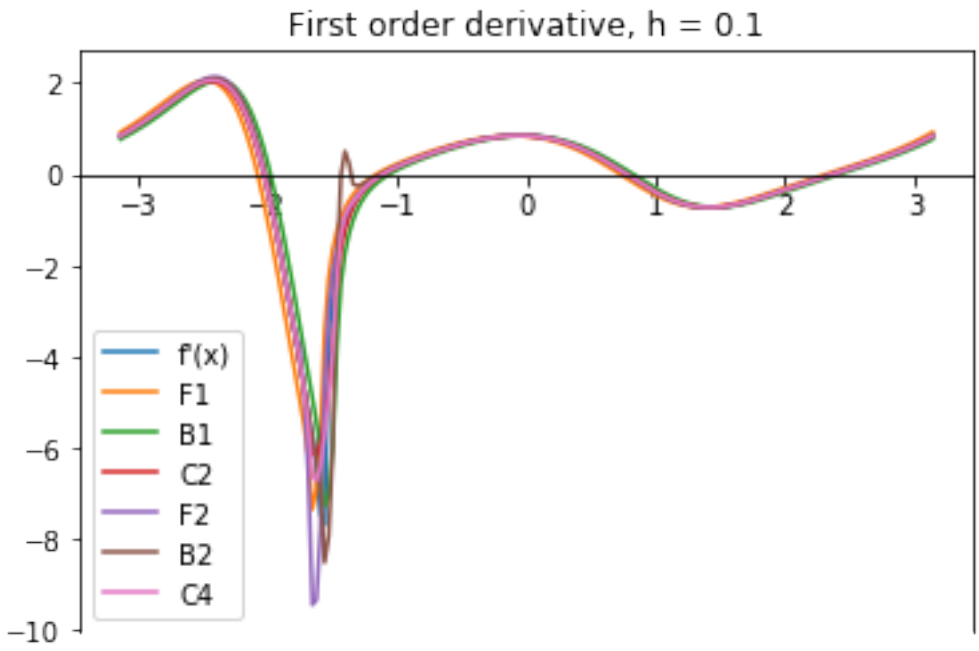
```
Out[129]: Text(0.5,1,u'First order derivative, h = 1')
```




```
In [130]: h = 0.1
```

```
% matplotlib inline
plt.plot(x_points, f_caldif, label="f'(x)")
plt.plot(x_points, dif_F1(f, x_points, h), label='F1')
plt.plot(x_points, dif_B1(f, x_points, h), label='B1')
plt.plot(x_points, dif_C2(f, x_points, h), label="C2")
plt.plot(x_points, dif_F2(f, x_points, h), label='F2')
plt.plot(x_points, dif_B2(f, x_points, h), label='B2')
plt.plot(x_points, dif_C4(f, x_points, h), label='C4')
ax = plt.gca()
ax.spines["bottom"].set_position('zero')
plt.legend()
plt.title("First order derivative, h = 0.1")
```

```
Out[130]: Text(0.5,1,u'First order derivative, h = 0.1')
```



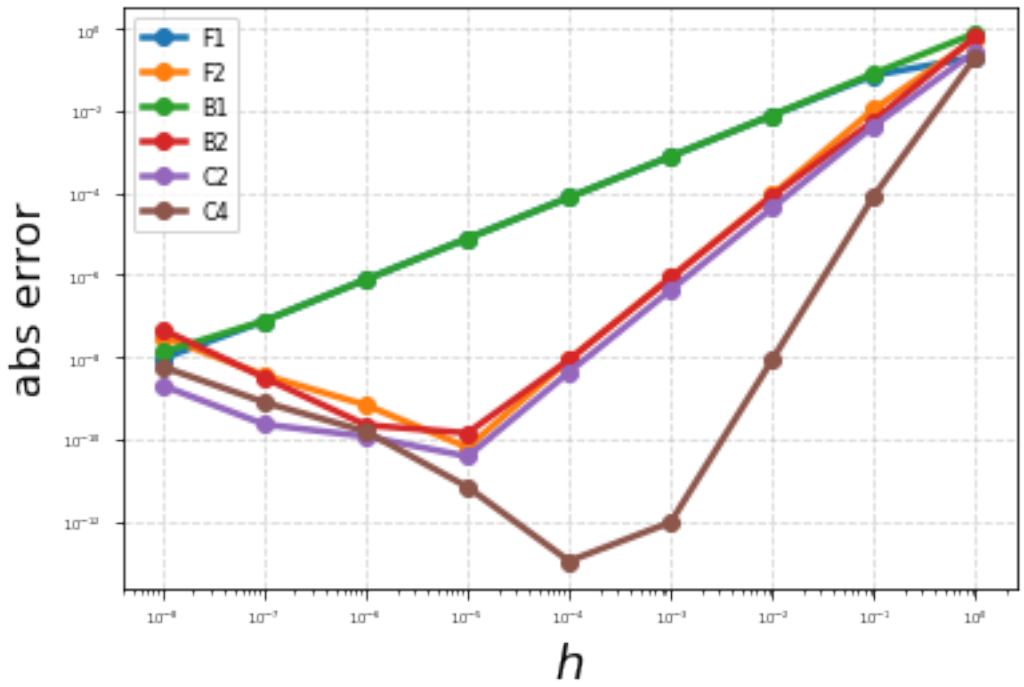
1.1.3 problem 3

```
In [131]: def ae_F1(h):
              x = 1.0
              f = f_1
              return abs(f_dif(x) - dif_F1(f,x,h))

          def ae_F2(h):
              x = 1.0
              f = f_1
              return abs(f_dif(x) - dif_F2(f,x,h))

          def ae_B1(h):
              x = 1.0
              f = f_1
              return abs(f_dif(x) - dif_B1(f,x,h))

          def ae_B2(h):
              x = 1.0
              f = f_1
              return abs(f_dif(x) - dif_B2(f,x,h))

          def ae_C2(h):
              x = 1.0
              f = f_1
```

```
        return abs(f_dif(x) - dif_C2(f,x,h))

    def ae_C4(h):
        x = 1.0
        f = f_1
        return abs(f_dif(x) - dif_C4(f,x,h))
```

In [132]:
```
h_logpoints = np.logspace(-8,0,9)
```

In [133]:
```
fig = plt.figure(figsize=(6,4))
plt.plot(h_logpoints, ae_F1(h_logpoints),marker='o', linewidth=2.5, label
plt.plot(h_logpoints, ae_F2(h_logpoints),marker='o', linewidth=2.5,label=
plt.plot(h_logpoints, ae_B1(h_logpoints),marker='o',linewidth=2.5,label='
plt.plot(h_logpoints, ae_B2(h_logpoints),marker='o',linewidth=2.5, label=
plt.plot(h_logpoints, ae_C2(h_logpoints),marker='o',linewidth=2.5, label=
plt.plot(h_logpoints, ae_C4(h_logpoints),marker='o',linewidth=2.5, label=
plt.xscale('log');plt.yscale('log')
plt.xlabel('$h$', fontsize=18);plt.ylabel('abs error', fontsize=16)

plt.tick_params(axis="both", labelsize=5)
plt.legend(loc='upper left', fontsize=8)
plt.grid(ls="--", alpha=0.5)
plt.show()
```



1.1.4 problem 4

```
In [134]: plane = np.load(r"C:\Users\chxzh\Desktop\persp-model-econ_W19-master\plar

In [135]: print plane

[[  7.    56.25  67.54]
 [  8.    55.53  66.57]
 [  9.    54.8   65.59]
 [ 10.    54.06  64.59]
 [ 11.    53.34  63.62]
 [ 12.    52.69  62.74]
 [ 13.    51.94  61.72]
 [ 14.    51.28  60.82]]


In [136]: import pandas as pd
          position = pd.DataFrame(columns=list('txy'))

In [137]: for i in range(8):
              a = np.deg2rad(plane[i][1])
              b = np.deg2rad(plane[i][2])
              r = pd.DataFrame([[i+7,a*np.tan(b)/(np.tan(b)-np.tan(a)),a*np.tan(b)*
              position = position.append(r)

In [138]: position = position.set_index('t')
          print position

           x         y
t
7   2.574675  3.853274
8   2.628297  3.828488
9   2.681703  3.801556
10  2.737160  3.775686
11  2.790344  3.748993
12  2.839398  3.725894
13  2.897326  3.700414
14  2.949213  3.678587


In [139]: def speed(t):
              if t in range(8,14):
                  dx_t = (position.loc[t + 1][0] - position.loc[t - 1][0])/2.0
                  dy_t = (position.loc[t + 1][1] - position.loc[t - 1][1])/2.0

              elif t == 7:
                  dx_t = (position.loc[t + 1][0] - position.loc[t][0])
                  dy_t = (position.loc[t + 1][1] - position.loc[t][1])
```

```
            else:
                dx_t = (position.loc[t][0] - position.loc[t - 1][0])
                dy_t = (position.loc[t][1] - position.loc[t - 1][1])

            return (dx_t**2 + dy_t**2)**0.5
```

```
In [140]: df_speed = pd.DataFrame(columns = ["t","speed"])
          for t in range(7,15):
              r = pd.DataFrame([[t,speed(t)]], columns = ["t","speed"])
              df_speed = df_speed.append(r)
          df_speed = df_speed.set_index("t")
```

```
In [141]: print df_speed
```

```
       speed
t
7   0.059073
8   0.059434
9   0.060497
10  0.060344
11  0.056859
12  0.058747
13  0.059785
14  0.056291
```

1.1.5 problem 5

```
In [142]: def Japprox(f,vx,h = 0.1):
              dim_f = len(f)
              dim_x = len(vx)
              I = np.identity(dim_x)
              Jmatrix = zeros(dim_f, dim_x)
              for i, sub_f in enumerate(f):
                  for j,s in enumerate(vx):
                      func = lambdify((x,y),sub_f,"numpy")
                      forward = vx + h*I[:,j]
                      back = vx - h*I[:,j]
                      Jmatrix[i,j] = (func(forward[0],forward[1])-func(back[0],back
              return Jmatrix
```

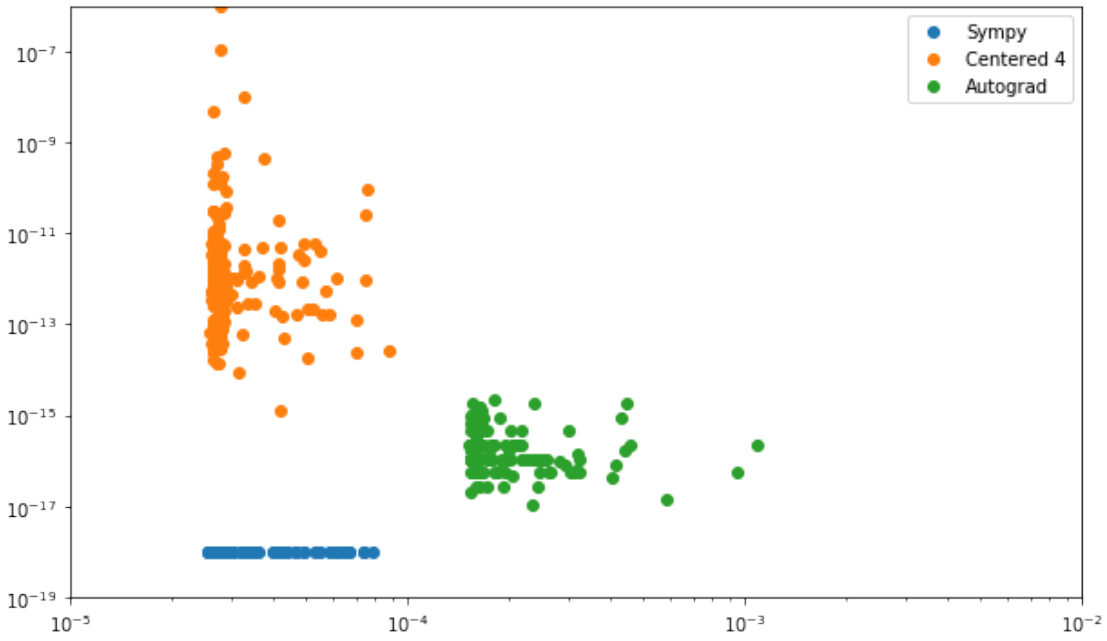
```
In [143]: import random
          x = symbols('x')
          y = symbols('y')
          f_test = [x**2,x**3-y]
          x_test = [random.randint(1,10),random.randint(1,10)]
          print f_test,x_test,Japprox(f_test,x_test)
```

```
[x**2, x**3 - y] [5, 6] Matrix([[9.99999999999996, 0.0], [75.0099999999998, -0.9999
```

1.1.6 problem 7

```
In [144]: import time
          import autograd.numpy as np
          from autograd import grad

          def f_1(x):
              return ((np.sin(x)+1)**(np.sin(np.cos(x))))

          def f_dif(x):
              return (-np.log(np.sin(x) + 1)*np.sin(x)*np.cos(np.cos(x)) + np.sin(n

          ag_dif = grad(f_1)

          def appr_time(n):

              time_sp = np.zeros(n)
              time_c4 = np.zeros(n)
              time_ag = np.zeros(n)
              abser_sp = (1e-18)*np.ones(n)
              abser_c4 = np.zeros(n)
              abser_ag = np.zeros(n)
              for i in range(n):

                  x = np.random.uniform(low=-np.pi, high=np.pi)
                  sp_start = time.clock()
                  val_sp = f_dif(x)
                  sp_end = time.clock()
                  time_sp[i] = sp_end - sp_start

                  c4_start = time.clock()
                  val_c4 = dif_C4(f_1,x,h = 0.001)
                  c4_end = time.clock()
                  time_c4[i] = (c4_end - c4_start)
                  abser_c4[i] = abs(val_c4 - val_sp)

                  ag_start = time.clock()
                  val_ag = ag_dif(x)
                  ag_end = time.clock()
                  time_ag[i] = ag_end - ag_start
                  abser_ag[i] = abs(val_ag - val_sp)

              return time_sp,time_c4,time_ag,abser_sp,abser_c4,abser_ag

In [156]: time_sp,time_c4,time_ag,abser_sp,abser_c4,abser_ag = appr_time(200)
```

```
fig = plt.figure(figsize=(10,6))
plt.scatter(time_sp,abser_sp,label = "Sympy")
plt.scatter(time_c4,abser_c4,label = "Centered 4")
plt.scatter(time_ag,abser_ag,label = "Autograd")
plt.yscale('log');plt.xscale('log')
plt.legend()
plt.xlim(1.0e-5,1.0e-2);plt.ylim(1.0e-19,1.0e-6)
plt.show()
```



In [ ]: