ps6 solution

February 19, 2019

1. Multiple linear regression
(a)

In [19]:
```
#a
import pandas as pd
auto = pd.read_csv("auto.csv",na_values="?")
auto.describe()
```

Out[19]:

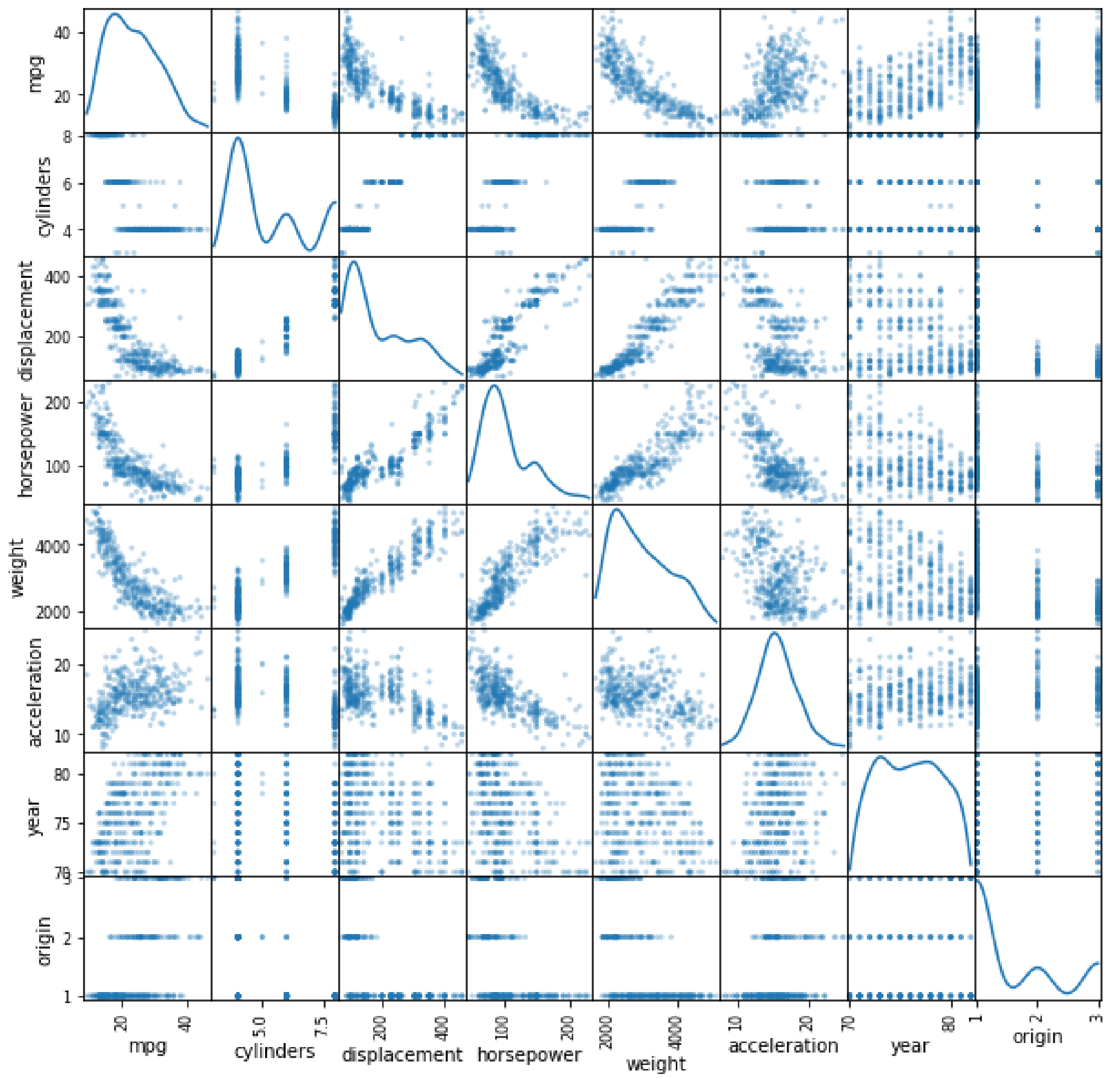
```
              mpg   cylinders  displacement  horsepower       weight  \
count  397.000000  397.000000    397.000000  392.000000   397.000000
mean    23.515869    5.458438    193.532746  104.469388  2970.261965
std      7.825804    1.701577    104.379583   38.491160   847.904119
min      9.000000    3.000000     68.000000   46.000000  1613.000000
25%     17.500000    4.000000    104.000000   75.000000  2223.000000
50%     23.000000    4.000000    146.000000   93.500000  2800.000000
75%     29.000000    8.000000    262.000000  126.000000  3609.000000
max     46.600000    8.000000    455.000000  230.000000  5140.000000

       acceleration        year      origin
count    397.000000  397.000000  397.000000
mean      15.555668   75.994962    1.574307
std        2.749995    3.690005    0.802549
min        8.000000   70.000000    1.000000
25%       13.800000   73.000000    1.000000
50%       15.500000   76.000000    1.000000
75%       17.100000   79.000000    2.000000
max       24.800000   82.000000    3.000000
```

(b)

In [21]:
```
#b
from pandas.plotting import scatter_matrix
import matplotlib.pyplot as plt

%matplotlib inline
scatter_matrix(auto, alpha=0.3, figsize=(10, 10), diagonal="kde")
plt.show()
```



(c)

```
In [22]: #c
         auto.corr()
```

Out[22]:

	mpg	cylinders	displacement	horsepower	weight	\
mpg	1.000000	-0.776260	-0.804443	-0.778427	-0.831739	
cylinders	-0.776260	1.000000	0.950920	0.842983	0.897017	
displacement	-0.804443	0.950920	1.000000	0.897257	0.933104	
horsepower	-0.778427	0.842983	0.897257	1.000000	0.864538	
weight	-0.831739	0.897017	0.933104	0.864538	1.000000	
acceleration	0.422297	-0.504061	-0.544162	-0.689196	-0.419502	
year	0.581469	-0.346717	-0.369804	-0.416361	-0.307900	
origin	0.563698	-0.564972	-0.610664	-0.455171	-0.581265	

```
              acceleration      year    origin
mpg               0.422297  0.581469  0.563698
cylinders        -0.504061 -0.346717 -0.564972
displacement     -0.544162 -0.369804 -0.610664
horsepower       -0.689196 -0.416361 -0.455171
weight           -0.419502 -0.307900 -0.581265
acceleration      1.000000  0.282901  0.210084
year              0.282901  1.000000  0.184314
origin            0.210084  0.184314  1.000000
```

(d)

In [28]:
```python
#d.i.
import statsmodels.api as sm
auto['constant']=1
reg = sm.OLS(endog=auto['mpg'], exog=auto[['cylinders', 'displacement','horsepower','
result=reg.fit()
print(result.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                    mpg   R-squared:                       0.821
Model:                            OLS   Adj. R-squared:                  0.818
Method:                 Least Squares   F-statistic:                     252.4
Date:                Mon, 18 Feb 2019   Prob (F-statistic):          2.04e-139
Time:                        16:45:58   Log-Likelihood:                -1023.5
No. Observations:                 392   AIC:                             2063.
Df Residuals:                     384   BIC:                             2095.
Df Model:                           7
Covariance Type:            nonrobust
================================================================================
                   coef    std err          t      P>|t|      [0.025      0.975]
--------------------------------------------------------------------------------
cylinders       -0.4934      0.323     -1.526      0.128      -1.129       0.142
displacement     0.0199      0.008      2.647      0.008       0.005       0.035
horsepower      -0.0170      0.014     -1.230      0.220      -0.044       0.010
weight          -0.0065      0.001     -9.929      0.000      -0.008      -0.005
acceleration     0.0806      0.099      0.815      0.415      -0.114       0.275
year             0.7508      0.051     14.729      0.000       0.651       0.851
origin           1.4261      0.278      5.127      0.000       0.879       1.973
constant       -17.2184      4.644     -3.707      0.000     -26.350      -8.087
==============================================================================
Omnibus:                       31.906   Durbin-Watson:                   1.309
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               53.100
Skew:                           0.529   Prob(JB):                     2.95e-12
Kurtosis:                       4.460   Cond. No.                     8.59e+04
==============================================================================
```

```
Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 8.59e+04. This might indicate that there are
strong multicollinearity or other numerical problems.
```

Out[28]:

	mpg	cylinders	displacement	horsepower	weight	acceleration	year	\
0	18.0	8	307.0	130.0	3504	12.0	70	
1	15.0	8	350.0	165.0	3693	11.5	70	
2	18.0	8	318.0	150.0	3436	11.0	70	
3	16.0	8	304.0	150.0	3433	12.0	70	
4	17.0	8	302.0	140.0	3449	10.5	70	

	origin	name	constant
0	1	chevrolet chevelle malibu	1
1	1	buick skylark 320	1
2	1	plymouth satellite	1
3	1	amc rebel sst	1
4	1	ford torino	1

(i) From the above table, we can know that $\beta_0, \beta_2, \beta_4, \beta_6$ and β_7 are significant at the 1% level, which means displacement weight, year, and origin as well as intercept are statistically significant variables at the 1% level .

(ii) From the above table, we can know that β_1, β_3 and β_5 are not statistically significant at the 10% level, which means the variable cylinders, horsepower and acceleration are not statistically significant at the 10% level .

(iii) The estimated cofficient β_6 means that a vehicle model will travel about 0.7508 miles more per gallon on average if this vehicle model was produced 1 year later.

(e)

(i) The three variables that look most likely to have a nonlinear relationship with mpg_i I think are displacement, horsepower and weight.

In [35]:

```
#i.
import numpy as np
auto['horsepower^2']=np.square(auto['horsepower'])
auto['displacement^2']=np.square(auto['displacement'])
auto['weight^2']=np.square(auto['weight'])
auto['acceleration^2']=np.square(auto['acceleration'])
reg2 = sm.OLS(endog=auto['mpg'], exog=auto[['constant','cylinders', 'displacement','ho

result2=reg2.fit()
print(result2.summary())
```

```
                            OLS Regression Results
==============================================================================
```

```
Dep. Variable:                    mpg   R-squared:                       0.870
Model:                            OLS   Adj. R-squared:                  0.866
Method:                 Least Squares   F-statistic:                     230.2
Date:                Tue, 19 Feb 2019   Prob (F-statistic):          1.75e-160
Time:                        21:46:47   Log-Likelihood:                -962.02
No. Observations:                 392   AIC:                             1948.
Df Residuals:                     380   BIC:                             1996.
Df Model:                          11
Covariance Type:            nonrobust
=================================================================================
                    coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------
constant         20.1084      6.696      3.003      0.003       6.943      33.274
cylinders         0.2519      0.326      0.773      0.440      -0.389       0.893
displacement     -0.0169      0.020     -0.828      0.408      -0.057       0.023
horsepower       -0.1635      0.041     -3.971      0.000      -0.244      -0.083
weight           -0.0136      0.003     -5.069      0.000      -0.019      -0.008
acceleration     -2.0884      0.557     -3.752      0.000      -3.183      -0.994
year              0.7810      0.045     17.512      0.000       0.693       0.869
origin            0.6104      0.263      2.320      0.021       0.093       1.128
horsepower^2      0.0004      0.000      2.943      0.003       0.000       0.001
displacement^2 2.257e-05   3.61e-05      0.626      0.532   -4.83e-05    9.35e-05
weight^2       1.514e-06   3.69e-07      4.105      0.000    7.89e-07    2.24e-06
acceleration^2    0.0576      0.016      3.496      0.001       0.025       0.090
==============================================================================
Omnibus:                       33.614   Durbin-Watson:                   1.576
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               77.985
Skew:                           0.438   Prob(JB):                     1.16e-17
Kurtosis:                       5.002   Cond. No.                     5.13e+08
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
[2] The condition number is large, 5.13e+08. This might indicate that there are
strong multicollinearity or other numerical problems.
```

(ii)The adjusted R-squared of the new model is 0.866. It's higher than the adjusted R-squared from part (d), which is 0.818.

(iii) Compared to those of the previous model, the statistical significance of the $displacement_i$ variable coefficient and the coefficient on its squared term both decreased and they are not significant at the 10% level.

(iv) The statistical significance of the coefficient on cylinders decreases and is not siginificant at the 10% level.

(f)

```
In [40]: #f
         print("the predicted mpg will be:",result2.predict(exog=[1,6,200,100,3100,15.1,99,1,10
```

```
the predicted mpg will be: [38.7321111]
```

2. Classication problem: KNN by hand and in Python

(a)

```
In [43]: data=pd.DataFrame({"X1":[0,2,0,0,-1,1], "X2":[3,0,1,1,0,1],
                            "X3":[0,0,3,2,1,1],"Y":["Red","Red","Red","Green","Green","Red"]})
         data["distance"]=np.sqrt(data["X1"]**2+data["X2"]**2+data["X3"]**2)
         data
```

Out[43]:

	X1	X2	X3	Y	distance
0	0	3	0	Red	3.000000
1	2	0	0	Red	2.000000
2	0	1	3	Red	3.162278
3	0	1	2	Green	2.236068
4	-1	0	1	Green	1.414214
5	1	1	1	Red	1.732051

(b)

The nearest neighbor of our test point(0,0,0) is the 5th observation, which is green. Therefore, our KNN oprediction with K = 1 is green.

(c)

The 3 nearest neighbor of our test point(0,0,0) are the 5th, 6th and 2nd observations. 2 of the 3 observations are red. Therefore, our KNN oprediction with K = 3 is red.

(d)

A highly nonlinear Bayes boundary would suggest that there is less advantage to generalizing further due to high variance, so the best value for K would be small.

(e)

```
In [45]: from sklearn.neighbors import KNeighborsClassifier

         knn = KNeighborsClassifier(n_neighbors = 2)
         neighbor=knn.fit(data[["X1","X2","X3"]], data["Y"])
         print("The KNN estimate of the test point (1,1,1) with K = 2 is",
               neighbor.predict([(1,1,1)]))
```

```
The KNN estimate of the test point (1,1,1) with K = 2 is ['Green']
```

3. Multivariable logistic (logit) regression

```
In [48]: auto['mpg_high']=np.where(auto['mpg']>auto['mpg'].median(),1,0)
```

(a)

```
In [52]: logit = sm.Logit(auto['mpg_high'], auto[['constant','cylinders','displacement','horsep
         result = logit.fit()
         print(result.summary())
```

```
Optimization terminated successfully.
         Current function value: 0.189320
         Iterations 9
                           Logit Regression Results
==============================================================================
Dep. Variable:               mpg_high   No. Observations:                  392
Model:                          Logit   Df Residuals:                      384
Method:                           MLE   Df Model:                            7
Date:                Tue, 19 Feb 2019   Pseudo R-squ.:                  0.7265
Time:                        22:26:57   Log-Likelihood:                -74.213
converged:                       True   LL-Null:                       -271.30
                                        LLR p-value:                 4.235e-81
================================================================================
                   coef    std err          z      P>|z|      [0.025      0.975]
--------------------------------------------------------------------------------
constant       -22.7150      6.140     -3.700      0.000     -34.749     -10.681
cylinders       -0.0633      0.437     -0.145      0.885      -0.919       0.792
displacement    -0.0002      0.013     -0.017      0.987      -0.026       0.025
horsepower      -0.0399      0.025     -1.618      0.106      -0.088       0.008
weight          -0.0048      0.001     -3.935      0.000      -0.007      -0.002
acceleration    -0.0178      0.141     -0.126      0.899      -0.294       0.258
year             0.5196      0.084      6.169      0.000       0.355       0.685
origin           0.4990      0.360      1.385      0.166      -0.207       1.205
================================================================================

Possibly complete quasi-separation: A fraction 0.18 of observations can be
perfectly predicted. This might indicate that there is complete
quasi-separation. In this case some parameters will not be identified.
```

The regressors that have coefficients that are statistically significant at the 5% level include weight and year.

(b)

```
In [57]: from sklearn.model_selection import train_test_split
         auto.dropna(inplace=True)
         X_train, X_test, y_train, y_test = train_test_split(auto[['constant','cylinders','dis
```

(c)

```
In [63]: from sklearn.linear_model import LogisticRegression

         logit2 = LogisticRegression(random_state=10).fit(X_train, y_train)
         b0,b1,b2,b3,b4,b5,b6,b7=logit2.coef_[0]
         coef = pd.DataFrame({"coefficient":['constant','cylinders','displacement','horsepower
         print(coef)
```

```
    coefficient  estimate
0      constant -0.051614
1     cylinders -0.372175
2  displacement  0.005731
3    horsepower -0.048376
4        weight -0.005011
5  acceleration -0.239298
6          year  0.306803
7        origin -0.003683
```

```
C:\ProgramData\Anaconda3\lib\site-packages\sklearn\linear_model\logistic.py:433: FutureWarning
  FutureWarning)
```

(d)

In [67]:
```python
y_pred = logit2.predict(X_test)
print("the predicted values of mpg_high for the test set is:\n",y_pred)
```

```
the predicted values of mpg_high for the test set is:
 [1 1 0 1 0 0 0 0 1 1 0 0 1 1 0 0 0 1 0 0 0 1 0 0 0 0 0 1 0 0 0 0 1 0 0 1 1
 0 0 1 0 0 1 0 0 0 0 1 0 1 1 0 0 0 0 1 0 1 1 1 1 1 0 0 1 1 0 1 0 0 0 1 1 1
 1 0 0 0 1 1 1 0 0 0 0 1 0 0 1 1 1 0 1 1 0 1 1 0 0 1 1 1 1 0 0 1 1 0 1 0 0
 0 0 1 0 0 1 0 0 1 0 1 1 1 0 0 0 1 0 1 0 1 1 0 1 1 1 0 1 1 1 0 0 1 1 1 1 1
 0 1 0 1 0 0 1 0 1 1 1 0 1 1 1 1 1 0 0 1 0 1 0 0 1 0 1 0 1 0 0 1 0 1 0 1 1
 1 1 0 1 1 0 1 1 0 1 0]
```

In [68]:
```python
from sklearn.metrics import confusion_matrix
confusion_matrix = confusion_matrix(y_test, y_pred)
confusion_matrix
```

Out[68]:
```
array([[89, 16],
       [10, 81]], dtype=int64)
```

In [69]:
```python
from sklearn.metrics import classification_report
print(classification_report(y_test, y_pred))
```

```
              precision    recall  f1-score   support

           0       0.90      0.85      0.87       105
           1       0.84      0.89      0.86        91

   micro avg       0.87      0.87      0.87       196
   macro avg       0.87      0.87      0.87       196
weighted avg       0.87      0.87      0.87       196
```

The F1-scores are almost the same. Therefore, this model predicts equally well on low mpg and high mpg.

```
In [ ]:
```