ps4

February 6, 2019

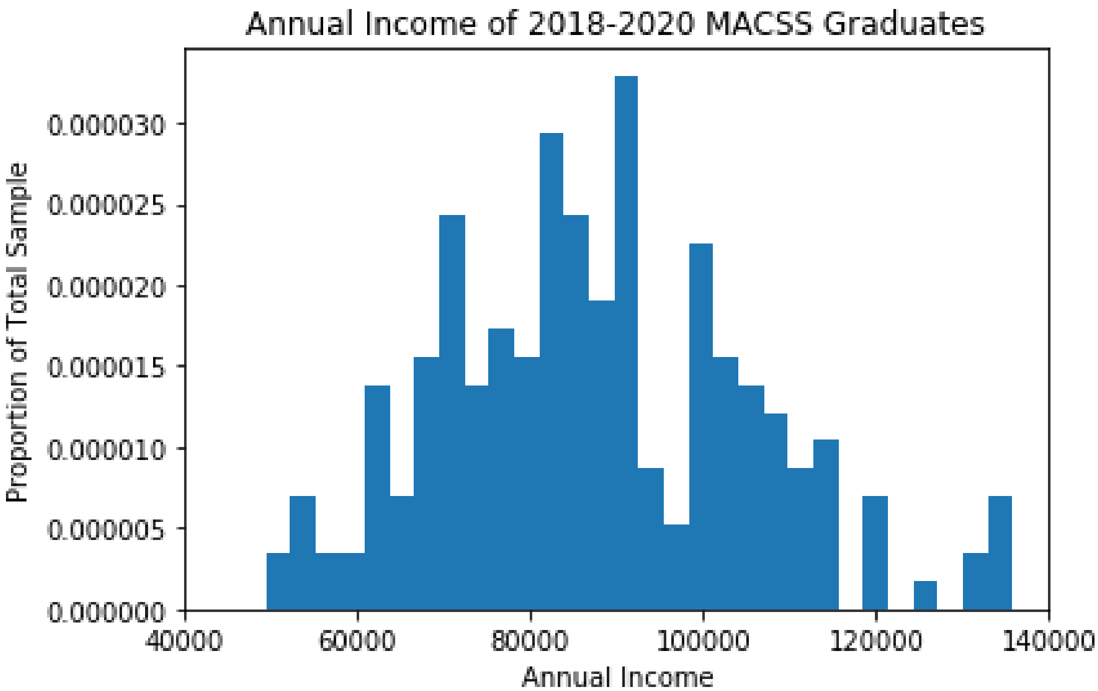
1.Some income data, lognormal distribution, and hypothesis testing

```
In [55]: import pandas as pd
         income = np.loadtxt("incomes.txt")
```

```
[ 51253.49715631 100630.32024137  83009.27613739  82882.10654304
  77338.29483892  81071.64131675  84760.04353269  74599.0967193
  94687.36110452  76720.48678222  84669.65776296  79707.04914362
 100026.64050846  89828.42639587  74006.05604302 103281.1855776
  88586.79236711 133631.92813961  91519.53047238 106863.33198279
  89622.32208316  63803.24097245  99116.670934   135865.02679613
  61344.18577082  99182.27630779  55038.04861413  71353.56078829
  90880.19896459  80950.37075039  84724.93806271  91236.88284731
  91628.21297882 105243.4671147   71500.93550933  56162.05440841
  78935.2939267   81097.09361259 130354.71406191 112159.02216504
  82259.96409801  75699.52604273  71699.30209176  93487.89133203
 120723.60773086 113045.98795795  70752.04409433 103679.41128338
  62517.62724779  70780.7133676  112840.45723451  65532.50301185
  69456.26059752  78471.04167764  76553.5638485   64774.04774462
  79461.70772595 108712.79343915 134832.20852612  52757.47937675
  83979.94687516  56981.25251453  52352.47669684  88008.83127479
  96516.7129408   92431.70598118  86036.04996776 106271.61349783
 106523.75491592  68018.94440642  69384.70794069  77796.96117204
 112145.95655858  89447.07822587 100330.97930044  76624.82196342
 102013.04734055  92429.01584718  82526.23206088  83772.14562675
  71897.49263019  67266.19673713 110148.79234709  62529.51330792
  60382.03294375 106394.42507683  84701.62739681  74337.45042844
  96396.44118743  71706.7848915   81868.62379825  79440.88332996
  60573.04791139 115447.28264921 102160.2852079  111413.54758781
  89836.98621301  85567.34476406 115565.2026309  109369.15454862
  82266.71210852  53918.32060906  68874.57038961  98621.62149733
  72833.87581358 134569.69980004  89494.58586974  81513.29859592
  71860.8012996   87372.47981208  82471.49409475  77345.58871429
  70506.45373879  79715.45369542 101475.35786055  91141.90918256
  84608.62472487  67173.95333009 112441.8933469   86479.68372484
 106205.5563009  120981.43440174 100496.85059862 115139.56548747
  74116.12458138  89903.73203272  88556.62918358  72892.56701538
```

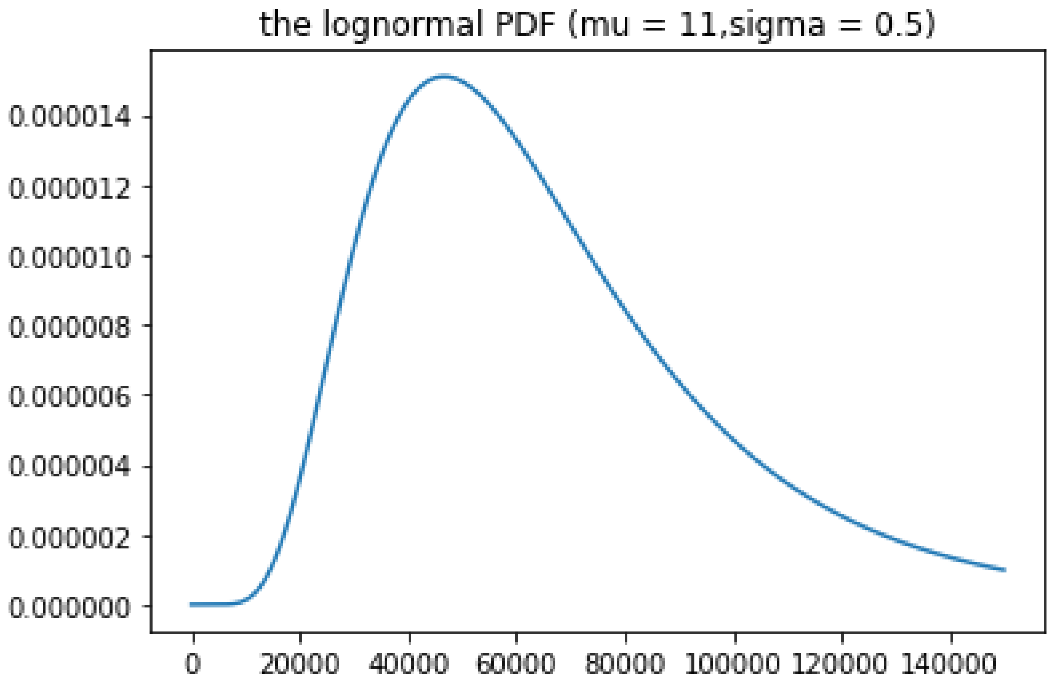
```
 76389.48408033 108885.33808596  49278.80193844  65585.29854
 79693.389294    73467.53823494  62375.11111402 126188.04662492
 82798.82771199  98574.18858555 104955.29009647  89982.91975939
 90586.34297755  98717.45268164 103249.08047696  84817.36068009
 92299.98553995  67700.42825536  86923.53139512  84979.67656441
 86775.28933212  70318.58330442  71056.64324088 119200.31288468
107878.23430231  99067.86118868 113874.25747567  87363.96061491
 61007.24503191  78825.59224855  81447.09172507  83765.07678131
120624.13289646  69716.15330603  91716.57641288  72161.00756676
 68527.43639097 107513.62585358  81465.96781549  94277.69505865
 90289.14896059  77817.45552848  67015.98816072 104942.59914713
 92465.52656145 130433.6344088   93048.60543387  87701.74185967
 61046.64052276  91784.42100065 101459.42358542  94593.75947595
 91409.58427076  62557.84947197  77838.40664978 102650.68476122
108567.45943204  96939.50585214 101945.11035163  84096.33721536
 99727.6089967  100007.51632221  86986.03510266  63510.70078076
 85019.89474873  70690.89103355  72377.94636608 108107.23017126
 92154.08621579  81684.90824755  83403.87501858 100597.17560947]
```

In [62]:
```
#a
import matplotlib.pyplot as plt
%matplotlib inline
num_bins = 30
count,bins,ignored = plt.hist(income, num_bins,normed=True)
plt.title("Annual Income of 2018-2020 MACSS Graduates")
plt.xlabel("Annual Income")
plt.ylabel("Proportion of Total Sample")
plt.xlim([40000, 140000])
plt.show()
```



In [70]: #b

```
import numpy as np
x = np.linspace(1e-4,150000,100000)
def lognorm(x,mu,sig):
    return 1/(x*sig * np.sqrt(2 * np.pi))*np.e**(-(np.log(x) - mu)**2 / (2

plt.plot(x,lognorm(x,mu=11,sig=0.5))
plt.title("the lognormal PDF (mu = 11,sigma = 0.5)")
plt.show()
```




```
In [71]: import scipy.stats as sts

         def trunc_lognorm_pdf(x, mu, sig, cut_lb, cut_ub):

             if cut_ub == 'None' and cut_lb == 'None':
                 prob_notcut = 1.0
             elif cut_ub == 'None' and cut_lb != 'None':
                 prob_notcut = 1.0 - sts.lognorm.cdf(cut_lb, sig, scale=np.exp(mu))
             elif cut_ub != 'None' and cut_lb == 'None':
                 prob_notcut = sts.lognorm.cdf(cut_ub, sig,scale=np.exp(mu))
             elif cut_ub != 'None' and cut_lb != 'None':
                 prob_notcut = (sts.lognorm.cdf(cut_ub, sig,scale=np.exp(mu)) -
                                sts.lognorm.cdf(cut_lb, sig,scale=np.exp(mu)))

             pdf_vals = ((1/(x*sig * np.sqrt(2 * np.pi)) *
                         np.exp( - (np.log(x) - mu)**2 / (2 * sig**2))) /
                         prob_notcut)

             return pdf_vals

         def loglike_trunclognorm(x, mu, sig, cut_lb, cut_ub):

             pdf_vals = trunc_lognorm_pdf(x, mu, sig, cut_lb, cut_ub)
             ln_pdf_vals = np.log(pdf_vals)
             loglike_val = ln_pdf_vals.sum()
```

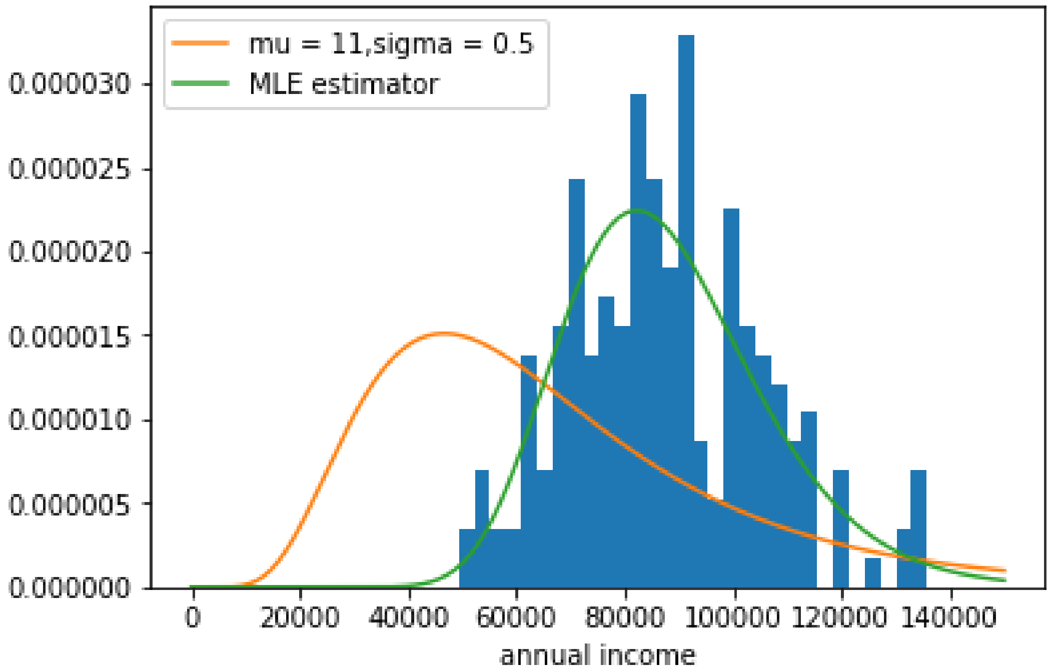
```
        return loglike_val

    print(' the log likelihood value is:', loglike_trunclognorm(income, mu=11,
```

```
 the log likelihood value is: -2379.120591931827
```

```
In [86]: #c
         import scipy.optimize as opt
         mu_init=11
         sig_init=0.5
         para_init=np.array([mu_init,sig_init])
         mle_args=(income,0,150000)

         def crit(parmts,*args):
             mu,sig=parmts
             x,cut_lb,cut_ub=args
             LL=loglike_trunclognorm(x,mu,abs(sig),cut_lb,cut_ub)
             neg_ll_val=-LL
             return neg_ll_val


         result=opt.minimize(crit,para_init,args=(mle_args))
         mu_est,sig_est=result.x

         print("the estimated mu is:",mu_est)
         print("the estimated sigma is:",sig_est)
         print("the log likelihood value is:",loglike_truncnorm(income,mu_est,sig_e
         print("the variance-covariance matrix is:",result.hess_inv)
```

```
the estimated mu is: 11.361699976140056
the estimated sigma is: 0.21174326472241192
the log likelihood value is: None
the variance-covariance matrix is: [[0.00032821 0.00066662]
 [0.00066662 0.00147221]]
```

```
In [89]: num_bins = 30
         count,bins,ignored = plt.hist(income, num_bins,normed=True)
         plt.plot(x,lognorm(x,mu=11,sig=0.5),label="mu = 11,sigma = 0.5")
         plt.plot(x,lognorm(x,mu=mu_est,sig=sig_est),label="MLE estimator")
         plt.xlabel("annual income")
         plt.legend()
         plt.show()
```




```
In [93]: #d
         mu_new, sig_new = np.array([11, 0.5])
         log_lik_h0 = loglike_trunclognorm(income, mu_new, sig_new, 0, 150000)
         print('hypothesis value log likelihood', log_lik_h0)
         log_lik_mle = loglike_trunclognorm(income, mu_est, sig_est, 0, 150000)
         print('MLE log likelihood', log_lik_mle)
         LR_val = 2 * (log_lik_mle - log_lik_h0)
         print('likelihood ratio value', LR_val)
         pval_h0 = 1.0 - sts.chi2.cdf(LR_val, 2)
         print('chi squared of H0 with 2 degrees of freedom p-value = ', pval_h0)
```

```
hypothesis value log likelihood -2379.120591931827
MLE log likelihood -2240.934337511636
likelihood ratio value 276.3725088403826
chi squared of H0 with 2 degrees of freedom p-value =  0.0
```

p-value is very close to zero, therefore we can reasonably reject the null hypothesis that the data comes from is the model in which mu=11,sigma=0.5

```
In [95]: #e
         print("the probability that I will earn more than $100,000:",1-sts.lognorm
         print("the probability that I will earn less than $75,000:", sts.lognorm.c
```

```
the probability that I will earn more than $100,000: 0.23755402258976566
the probability that I will earn less than $75,000: 0.2596439222572218
```

2.Linear regression and MLE

```
In [106]: sick=pd.read_csv("sick.txt")

In [110]: #a
          def norm_pdf(xvals, sig):
              sig=abs(sig)
              pdf_vals = (1/(sig*np.sqrt(2*np.pi)))*np.exp(-(xvals)**2 / (2*sig**2)
              return pdf_vals

          def log_lik_norm(y, x1, x2, x3, b0, b1, b2, b3, sig):

              err=y-b0-b1*x1-b2*x2-b3*x3
              pdf_vals = norm_pdf(err, sig)
              ln_pdf_vals = np.log(pdf_vals)
              log_lik_val = ln_pdf_vals.sum()

              return log_lik_val

          def crit2(params,*args):

              b0, b1, b2, b3, sig = params
              y, x1, x2, x3 = args
              log_lik_val = log_lik_norm(y, x1, x2, x3, b0, b1, b2, b3, sig)
              neg_log_lik_val = -log_lik_val

              return neg_log_lik_val

          b0_init, b1_init, b2_init, b3_init, sig_init = (0.3,0.3,0.2,0.1,1)
          y=sick['sick']
          x1, x2, x3 = sick['age'], sick['children'], sick['avgtemp_winter']
          params_init = np.array([b0_init, b1_init, b2_init, b3_init, sig_init])
          results = opt.minimize(crit2, params_init,(y, x1, x2, x3))
          b0_est, b1_est, b2_est, b3_est, sig_est = results.x
          LL_est=-results.fun
          print('beta 0=', b0_est)
          print('beta 1=', b1_est)
          print('beta 2=', b2_est)
          print('beta 3=', b3_est)
          print('sigma=', sig_est)
          print("Value of the log likelihood function:",LL_est)
          print(' the estimated variance covariance matrix of the estimates:', resu

C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:11: RuntimeWarning
  # This is added back by InteractiveShellApp.init_path()
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:663: RuntimeW
  grad[k] = (f(*((xk + d,) + args)) - f0) / d[k]
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:11: RuntimeWarning
```

```
  # This is added back by InteractiveShellApp.init_path()
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:11: RuntimeWarning
  # This is added back by InteractiveShellApp.init_path()
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:663: RuntimeW
  grad[k] = (f(*((xk + d,) + args)) - f0) / d[k]
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:11: RuntimeWarning
  # This is added back by InteractiveShellApp.init_path()
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:11: RuntimeWarning
  # This is added back by InteractiveShellApp.init_path()
C:\ProgramData\Anaconda3\lib\site-packages\scipy\optimize\optimize.py:663: RuntimeW
  grad[k] = (f(*((xk + d,) + args)) - f0) / d[k]
C:\ProgramData\Anaconda3\lib\site-packages\ipykernel_launcher.py:11: RuntimeWarning
  # This is added back by InteractiveShellApp.init_path()


beta 0= 0.2516463718310378
beta 1= 0.012933350779986119
beta 2= 0.40050203860631045
beta 3= -0.009991673217542055
sigma= 0.003017683026813868
Value of the log likelihood function: 876.8650464335163
 the estimated variance covariance matrix of the estimates: [[ 8.05391939e-07  9.71
  -4.74676588e-09]
 [ 9.71035750e-09  3.60002497e-09 -3.15149964e-08 -2.31661314e-09
   3.56176950e-10]
 [-1.82193863e-07 -3.15149964e-08  3.15114478e-07  2.08224080e-08
  -5.92877946e-09]
 [-2.00286870e-08 -2.31661314e-09  2.08224080e-08  1.79792868e-09
  -1.24479835e-10]
 [-4.74676588e-09  3.56176950e-10 -5.92877946e-09 -1.24479835e-10
   2.08654310e-08]]
```

```
In [112]: #b
          LL_new=log_lik_norm(y,x1,x2,x3,b0=1,b1=0,b2=0,b3=0,sig=0.1)
          print('hypothesis value log likelihood', LL_new)
          print('MLE log likelihood', LL_est)
          LR_val=2*(LL_est-LL_new)
          print('likelihood ratio value', LR_val)
          pval_h0 = 1.0 - sts.chi2.cdf(LR_val, 5)
          print('chi squared of H0 with 5 degrees of freedom p-value = ', pval_h0)
```

```
hypothesis value log likelihood -2253.700688042125
MLE log likelihood 876.8650464335163
likelihood ratio value 6261.131468951283
chi squared of H0 with 5 degrees of freedom p-value =  0.0
```

The likelihood that age, number of children and average winter temperature have no effect on the number of sick days is very close to zero.