

TAREFA BÁSICA – SISTEMAS LINEARES– ESCALONAMENTO (GAUSS)

Paola Martins - Sistemas Lineares

1a) $\begin{cases} 2x - y = 2 \\ -x + 3y = -3 \end{cases} \rightarrow D = \begin{vmatrix} 2 & -1 \\ -1 & 3 \end{vmatrix} = 6 - 1 = 5 //$

$D_x = \begin{vmatrix} 2 & -1 \\ -3 & 3 \end{vmatrix} = 6 - 3 \rightarrow D_x = 3$

$D_y = \begin{vmatrix} 2 & 2 \\ -1 & -3 \end{vmatrix} = -6 - (-2) \rightarrow -6 + 2 \rightarrow D_y = -4 //$

$x = \frac{3}{5}$ e $y = \frac{-4}{5}$

1b) $\begin{cases} 3x - y + z = 1 \\ 2x + 3z = -1 \\ 4x + y - 2z = 7 \end{cases} \rightarrow D = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ 4 & 1 & -2 \end{vmatrix} \begin{matrix} 9+4 \\ 2 \cdot 0 \\ 0 - 12+2 \end{matrix} \begin{matrix} 3-1 \\ 2 \cdot 0 \\ 4 \cdot 1 \end{matrix} = -10 \cdot 13 = -23$

$D_x = \begin{vmatrix} 1 & -1 & 1 \\ -1 & 0 & 3 \\ 7 & 1 & -2 \end{vmatrix} \begin{matrix} 0 \cdot 3-2 \\ -22-1 \\ 0-21-1 \end{matrix} \Rightarrow -22-1 = -23 \Rightarrow D_x = \frac{-23}{-23} = 1$

$D_y = \begin{vmatrix} 3 & 1 & 1 \\ 2 & 1 & 3 \\ 4 & 7 & -2 \end{vmatrix} \begin{matrix} 6+12+14 \\ 32-55 \\ 6+12+14 \end{matrix} \Rightarrow 32-55 = -23 \Rightarrow D_y = \frac{-23}{-23} = 1$

$D_z = \begin{vmatrix} 3 & -1 & 1 \\ 2 & 0 & 3 \\ 4 & 1 & -2 \end{vmatrix} \begin{matrix} 6-3-14 \\ 6-(-17) \\ 0 \cdot 4+2 \end{matrix} \Rightarrow 6-(-17) \rightarrow 6+17 = 23 \Rightarrow D_z = \frac{23}{-23} = -1$

3 $\begin{cases} x + 2y + z = 1 \\ 3x + y - 11z = -2 \\ 2x + 3y - z = 1 \end{cases} \rightarrow D = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -11 \\ 2 & 3 & -1 \end{vmatrix} \Rightarrow 36 - (-37) + 36 + 37 = 1$

$D_x = \begin{vmatrix} 1 & 2 & 1 \\ -2 & 1 & -11 \\ 1 & 3 & -1 \end{vmatrix} \Rightarrow -29 + 28 = -1$ $x = \frac{-1}{1} = -1$

$D_y = \begin{vmatrix} 1 & 1 & 1 \\ 3 & -2 & -11 \\ 2 & 1 & -1 \end{vmatrix} \Rightarrow -17 + 18 = 1$ $y = \frac{1}{1} = 1$

$D_z = \begin{vmatrix} 1 & 2 & 1 \\ 3 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix} \Rightarrow 2 - 2 = 0$ $z = \frac{0}{1} = 0$
 $a + b + c = -1 + 1 + 0 = 0$ letra C

2 $\begin{cases} 3x + 4y - z = 1 \\ 4x + 5y + 2z = 12 \\ x - 2y + 3z = 8 \end{cases} \rightarrow \begin{array}{l} \text{I} \quad 3x + 4y - z = 1 \\ \text{II} \quad 4x + 5y + 2z = 12 \\ \text{III} \quad x - 2y + 3z = 8 \end{array}$
 $\text{III} \cdot 4 \rightarrow 4x - 8y + 12z = 32$
 $\text{II} - \text{III} \rightarrow -4x - 5y - 2z = -12$
 $\text{IV} \quad 4x + 2y + 2z = 9$
 $-3y = -3 \Rightarrow y = 1$
 R: letra A

4 $\begin{cases} x + 2y - 3z = 29 \\ x + 3y + 2z = 4 \\ x - y - 2z = 8 \end{cases} \rightarrow D = \begin{vmatrix} 1 & 2 & -3 \\ 1 & 3 & 2 \\ 1 & -1 & -2 \end{vmatrix} \Rightarrow 1 + 15 = 16$

$D_x = \begin{vmatrix} 29 & 2 & -3 \\ 4 & 3 & 2 \\ 8 & -1 & -2 \end{vmatrix} \Rightarrow D_x = -130 + 146 = 16$ $x = \frac{16}{16} = 1$

$D_y = \begin{vmatrix} 1 & 29 & -3 \\ 1 & 4 & 2 \\ 1 & 8 & -2 \end{vmatrix} \Rightarrow 26 + 54 = 80$ $y = \frac{80}{16} = 5$

$D_z = \begin{vmatrix} 1 & 2 & 29 \\ 1 & 3 & 4 \\ 1 & -1 & 8 \end{vmatrix} \Rightarrow 3 - 99 = -96$ $z = \frac{-96}{16} = -6$
 $x + y + z = 1 + 5 + (-6) = 0$ letra D

5 $\begin{cases} x+y=5 \\ 0+2y+z=3 \\ 3x+2y+z=7 \end{cases} \rightarrow D = \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 3 & 2 & 1 \end{vmatrix} \Rightarrow 7-4=3$

$D_x = \begin{vmatrix} 5 & 1 & 0 \\ 3 & 2 & 1 \\ 7 & 2 & 1 \end{vmatrix} \Rightarrow 17-13=4$

$x = \frac{4}{3}$

$D_y = \begin{vmatrix} 2 & 5 & 0 \\ 0 & 3 & 1 \\ 3 & 7 & 1 \end{vmatrix} \Rightarrow (6+15)-(14) \Rightarrow 7$

$y = \frac{7}{3}$

$D_z = \begin{vmatrix} 2 & 1 & 5 \\ 0 & 2 & 3 \\ 3 & 2 & 7 \end{vmatrix} \Rightarrow (28+9)-(30+12) \Rightarrow -5$

$z = \frac{-5}{3}$

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6 $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \begin{bmatrix} x & 0 & 0 \\ 2x & y & 0 \\ -x & 2y & 2z \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix} \rightarrow \begin{cases} x=3 \\ 2x+y=7 \\ -x+2y+2z=-1 \end{cases}$

$D = \begin{vmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1 & 2 & 2 \end{vmatrix} \Rightarrow D=2$

$D_x = \begin{vmatrix} 3 & 0 & 0 \\ 7 & 1 & 0 \\ -1 & 2 & 2 \end{vmatrix} \Rightarrow D_x=6$

$x = \frac{6}{2} \Rightarrow x=3$

$D_y = \begin{vmatrix} 1 & 3 & 0 \\ 2 & 7 & 0 \\ -1 & -1 & 2 \end{vmatrix} \Rightarrow 14-12=2$

$y = \frac{2}{2} \Rightarrow y=1$

$D_z = \begin{vmatrix} 1 & 0 & 3 \\ 2 & 1 & 7 \\ -1 & 2 & -1 \end{vmatrix} \Rightarrow 11-11=0$

$z = \frac{0}{2} \Rightarrow z=0$

Letra E

ESCALONAMENTO

Paula Montina - Escalonamento (curso)

①
$$S = \begin{cases} 2y - z - 3z = -5 \\ x + 3y - z = 11 \\ x - 5z = 3 \end{cases}$$

$$\begin{array}{ccc|ccc|c} 1 & 0 & -1 & -2 & 1 & 0 & -5 & 3 \\ & 2 & -1 & -3 & 0 & 0 & -5 & -5 \\ & 1 & 3 & -1 & 0 & 0 & 11 & 0 \end{array}$$

$$\rightarrow \begin{array}{ccc|ccc|c} 3 & 0 & -1 & 7 & -11 & & & \\ & 0 & 3 & 4 & 8 & & & \end{array} \sim \begin{array}{ccc|ccc|c} & & & & & & & \\ & & & & & & & \\ & & & & & & & \end{array}$$

$$\begin{aligned} -y + 7(-1) &= -11 & x - 5(-1) &= 3 & 25z &= -25 \\ \boxed{-y = 4} & & \boxed{x = -2} & & z = \frac{-25}{25} \\ & & & & \boxed{z = -1} \end{aligned}$$

②
$$\begin{cases} x = 2y \\ 2y = 3z \\ x + y + z = 11 \end{cases} \rightarrow \begin{aligned} y &= \frac{x}{2} & z &= \frac{x}{3} \end{aligned}$$

$$x + \left(\frac{x}{2}\right) + \left(\frac{x}{3}\right) = 11 \rightarrow \text{mmc} = 6$$

$$x \cdot 6 + \left(\frac{x}{2}\right) \cdot 6 + \left(\frac{x}{3}\right) \cdot 6 = 11 \cdot 6$$

$$x \cdot 6 + 3x + 2x = 66 \rightarrow 11x = 66$$

$$x = \frac{66}{11} \Rightarrow \boxed{x = 6}$$

$$6 + 2 \cdot \left(\frac{x}{2}\right) + 3 \cdot \left(\frac{x}{3}\right)$$

$$6 + x + x \rightarrow \text{como } x \text{ é } 6, \text{ então:}$$

$$6 + 6 + 6 = \boxed{18}$$

letra B

03

$$\begin{cases} x + y + z = 0 \\ 2x - y - 2z = 1 \\ 6y + 3z = -12 \end{cases} \rightarrow \begin{array}{ccc|c|c} -2 & 1 & 1 & 1 & 0 \\ & 2 & -1 & -2 & 1 \\ & 0 & 0 & 3 & -12 \end{array} \rightarrow$$

$$\rightarrow \begin{array}{ccc|c|c} 2 & 0 & -3 & -4 & 1 \\ & 0 & 6 & 3 & -12 \end{array} \sim \begin{array}{ccc|c|c} & 0 & 0 & -5 & -10 \end{array}$$

$$x + (-9) + 2 = 0 \quad -y + (-4) \cdot 2 = 1$$

$$x = 7 \quad -y = 9 \cdot (-1) \quad -5z = -10$$

$$y = -9 \quad z = \frac{-10}{-5}$$

$$z = 2$$

04

A = Ali; B = bia; C = coco \Rightarrow Possuem R\$68,00

$B + \frac{20}{100} C = A \rightarrow 5B + C = 5A \rightarrow 5A - 5B - C = 0$

$C + \frac{20}{100} = A = 3 \cdot B \rightarrow \frac{C+A}{5} = 3B \rightarrow 5C + A = 15B$

$A - 15B + 5C = 0$

$$\begin{cases} A + B + C = 68 \\ 5A - 5B - C = 0 \\ A - 15B + 5C = 0 \end{cases}$$

$$D = \begin{array}{ccc|ccc} & & & -5 & 15 & 25 \\ 1 & 1 & 1 & 1 & 1 & 1 \\ 5 & -5 & -1 & 5 & -5 & \\ 1 & -15 & 5 & 1 & -15 & \end{array} \Rightarrow -101 - 35 = -136$$

CONTINUA NA PRÓXIMA

4) Continuação

D

$$DA = \begin{array}{ccc|ccc} & & & 0 & 1020 & 0 \\ 68 & 1 & 1 & 68 & 1 & \\ 0 & -5 & -1 & 0 & -5 & \Rightarrow -1700 - 1020 = \\ 0 & -15 & 5 & 0 & -15 & = 2720 // \end{array}$$

$$A = \frac{-2720}{-136} = 20$$

$$DB = \begin{array}{ccc|ccc} & & & 0 & 0 & 1700 \\ 1 & 68 & 1 & 1 & 68 & \\ 5 & 0 & -1 & 5 & 0 & \Rightarrow -68 - 1700 = \\ 1 & 0 & 5 & 1 & 0 & = -1768 // \end{array}$$

$$B = \frac{-1768}{-136} = 13$$

$$DC = \begin{array}{ccc|ccc} & & & 0 & -68 & 0 \\ 1 & 1 & 68 & 1 & 1 & \\ 5 & -5 & 0 & 5 & -5 & \Rightarrow -5100 + 340 = \\ 1 & -15 & 0 & 1 & -15 & = -4760 // \end{array}$$

$$C = \frac{-4760}{-136} = 35$$

Então: A = R\$ 20,00

B = R\$ 13,00 →

C = R\$ 35,00

$$C - A = 35 - 20 = 15$$

letra A

5)

$$A = \begin{array}{ccc|c} 0 & 3 & 4 & \\ 1 & 0 & 5 & \\ 2 & 1 & 0 & \end{array}$$

$$X = \begin{array}{c} x \\ y \\ z \end{array}$$

Alfeu = 134 reais →

Bento = 115 reais →

Cintia = 48 reais →

$$\begin{cases} 3y + 4z = 134 \\ x + 5z = 115 \\ 2x + y = 48 \end{cases}$$

$$\Rightarrow D = \begin{array}{ccc|ccc} & & & 0 & 0 & 0 \\ 0 & 3 & 4 & 0 & 3 & \\ 1 & 0 & 5 & 1 & 0 & \Rightarrow 34 // \\ 2 & 1 & 0 & 2 & 1 & \end{array}$$

$$D_x = \begin{array}{ccc|ccc} & & & 0 & 670 & 0 \\ 134 & 3 & 4 & 134 & 3 & \\ 115 & 0 & 5 & 115 & 0 & \Rightarrow 1180 - 670 = \\ 48 & 1 & 0 & 48 & 1 & = 510 // \end{array}$$

$$x = \frac{510}{34} = 15 //$$

$$D_y = \begin{array}{ccc|ccc} & & & 0 & 720 + 460 & 0 \\ 0 & 134 & 4 & 0 & 134 & \\ 1 & 15 & 5 & 1 & 15 & \Rightarrow 1532 - 920 = \\ 2 & 48 & 0 & 2 & 48 & = 612 // \end{array}$$

$$y = \frac{612}{34} = 18 //$$

$$D_z = \begin{array}{ccc|ccc} & & & 0 & 0 & 144 \\ 0 & 3 & 134 & 0 & 3 & \\ 1 & 0 & 15 & 1 & 0 & \Rightarrow 824 - 144 = \\ 2 & 1 & 48 & 2 & 1 & = 680 // \end{array}$$

$$z = \frac{680}{34} = 20 //$$

$$15 + 18 + 20 = 53 \text{ reais}$$

letra A.

