

Fourier Analysis

Documentation

Paolo Bettelini
Scuola d'Arti e Mestieri di Trevano (SAMT)

Contents

1	Introduction	3
1.1	Abstract	3
1.2	Informations	4
1.3	Scope	4
2	Analysis	5
2.1	Requirements	5
2.1.1	Req-00	5
2.1.2	Req-01	5
2.1.3	Req-02	5
2.1.4	Req-03	6
2.1.5	Req-04	6
2.1.6	Req-05	6
3	Interactive Boxes	7
3.1	Description	7
3.2	Implementation	7
3.3	List of Functions	8
3.4	Injecting	8
3.5	Example	9
4	Website Structure	10
4.1	Dependency table	10
4.2	Sections	10
4.2.1	Fourier Analysis	10
4.2.2	Requirements	11
4.2.3	Introduction	11
4.2.4	Fourier Series vs Fourier Transform	12
4.2.5	Trigonometric Fourier Series	12
4.2.6	Trigonometric Fourier Series - C term	13
4.2.7	Trigonometric Fourier Series - Coefficients	13
4.2.8	Fourier Series - Conclusion	14
4.2.9	Main ideas - Complex plotting	14
4.2.10	Main ideas - Center of mass	15
4.2.11	Main ideas - Fourier Transform	15
4.2.12	A Simple Example	16
4.2.13	A Simple Example - Coefficients	16

4.2.14	A Simple Example - Conclusion	17
4.2.15	Exponential Fourier Series	17
4.2.16	Fast Fourier Transform	18
4.2.17	Conclusion	18
4.3	Interactive Boxes Implementations	19
4.3.1	Fourier Series 1D	19
4.3.2	Fourier Series 2D	19
4.3.3	Complex Plot	19
4.3.4	Center of mass	19
4.3.5	Fourier Transform	19

1 Introduction

1.1 Abstract

Fourier analysis is a method of defining periodic waveforms in terms of trigonometric functions. This branch of mathematics is widely used in signal processing, especially electronics, acoustics and communications. Many notorious algorithms have been developed thanks to Joseph Fourier. Operators such as the Fourier Transform are constantly used in the real world, without these discoveries the world would not be the same. Many software rely in Fourier Analysis, such as for instance Shazam, the famous service for identifying songs. Any audio spectrum visualized processes the signal using Fourier Transform, these are just a few of the many application of this analysis.

1.2 Informations

This is a project of the Scuola Arti e Mestieri di Trevano (SAMT) school under the following circumstances.

- **Section:** Computer Science
- **Year:** Third
- **Class:** Module 306
- **Supervisor:** Luca Muggiasca
- **Title:** Fourier Analysis
- **Start date:** 2021.09.09
- **Deadline:** 2021.12.23

and the following requirements

- **Documentation:** a full documentation of the work done
- **Changelog:** constant changelog for each work session
- **Source code:** working source code of the project

All the source code and documents can be found at <https://github.com/paolobettelini/fourier-series>.
The live version of the final product is available at <https://paolobettelini.github.io/fourier-series>.

1.3 Scope

The scope of this project is to create a website containing various explanations about Fourier Analysis.

2 Analysis

2.1 Requirements

2.1.1 Req-00

Req-00	
Name	Content
Priority	1
Version	2.0
Notes	none
Description	The website must contains a full explanation about Fourier Analysis.

2.1.2 Req-01

Req-01	
Name	Index
Priority	1
Version	2.0
Notes	none
Description	The website must contain an index of all the sections
Subrequirements	
Req-01.0	There must be a section about the topic introduction.
Req-01.1	There must be a section about the knowledge requirements.
Req-01.2	There must be a section about signal processing.
Req-01.3	There must be a section about the Fourier transform.
Req-01.4	There must be a section about the Fourier series.
Req-01.5	There must be a section about how to represent the Fourier series with epicycles.
Req-01.6	There must be a section about Fast Fourier Transform.

2.1.3 Req-02

Req-02	
Name	Responsiveness
Priority	1
Version	1.0
Notes	none
Description	The website must be responsive.

2.1.4 Req-03

Req-03	
Name	Introduction
Priority	1
Version	1.0
Notes	none
Description	The introduction section must contain an interactive Fourier series animation.
Subrequirements	
Req-03_0	The user must be able to draw an arbitrary path.
Req-03_1	The user drawn path is animated with a Fourier series, represented with epicycles.
Req-03_2	The interactive box must contains a timeline slider.
Req-03_3	The interactive box must contain a stop button.
Req-03_4	The interactive box must contain a resume button.
Req-03_5	The interactive box must contain a slider for the animation speed.

2.1.5 Req-04

Req-04	
Name	Interactiveness
Priority	1
Version	1.0
Notes	none
Description	The website must contain multiple interactive boxes.
Subrequirements	
Req-04_0	All the interactive boxes must follow the design described in Req-03.
Req-04_1	All the interactive boxes can contain optional settings.

2.1.6 Req-05

Req-05	
Name	Modularity
Priority	1
Version	1.0
Notes	none
Description	The interactive boxes must share the same base code.

2.2 Ga

3 Interactive Boxes

3.1 Description

InteractiveBoxes is a JavaScript library I wrote for canvas rendering based on the user input. The library injects its content into a HTML div element. The content consists of a canvas element, a stop/resume button and a range slider (the timeline), additional content is injected by the interactive box implementations. The user can interact with the timeline, pause and resume the animation or modify the input by simply drawing onto the canvas.

3.2 Implementation

To create an interactive box you need to create a class that extends `InteractiveBox.js`. The class of your custom interactive box must override some functions, otherwise you will get errors. You will also need to call the super constructor. Here are the declaration of those function in the `InteractiveBox.js` class and its constructor.

```
constructor(name, container, height, width) {
    ...
}

draw(ctx) {
    throw 'The function draw() has not been overwritten'
}

setPoints(points) {
    throw 'The function setPoints(points) has not been overwritten'
}

onTimeTravel(value) {
    throw 'The function onTimeTravel(value) has not been overwritten'
}
```

Overriding these functions will produce a class that looks like this

```
class MyCustomBox extends InteractiveBox {

    constructor(name, container, height, width) {
        super(name, container, height, width)

        // inject extra html, initialize variables, ...
    }

    draw(ctx) {
        this.clearCanvas();

        // draw function

        // update timeline
        this.setTime(...);
    }

    onTimeTravel(value) {
        // onTimeTravel function
    }

    setPoints(points) {
        // setPoints function
    }
}
```


3.3 List of Functions

Here is a list of public functions in `InteractiveBox.js`

Name	Description	Parameters	Returns
<code>constructor()</code>	Constructor	<ul style="list-style-type: none">• name the name of the box• container the div id• height the height of the canvas• width the width of the canvas	void
<code>pause()</code>	Pauses the animation	none	void
<code>resume()</code>	Resumes the animation	none	void
<code>toggle()</code>	Pauses or resumes the animation	none	void
<code>isPlaying()</code>	Returns true if the animation is playing	none	bool
<code>setTime()</code>	Updates the timeline, you should call this in the <code>draw()</code> function	<ul style="list-style-type: none">• value the time value $\in [0; 1]$	void
<code>clearCanvas()</code>	Clears the canvas	none	void
<code>draw()</code>	Called for each frame Must override!	<ul style="list-style-type: none">• ctx The canvas context	void
<code>onTimeTravel()</code>	Called when the user moves the timeline Must override!	<ul style="list-style-type: none">• value the time value $\in [0; 1]$	void
<code>setPoints()</code>	Called when the user draws a path Must override!	<ul style="list-style-type: none">• points array of $\{x,y\}$	void

3.4 Injecting

To inject the interactive box into the site we must create a div element to contain it.

```
<body>
  <!-- Here I place my MyCustomBox-->
  <div id="mycustombox-div">
  </div>
</body>
```

Then, in a JavaScript environment add the box to the div

```
new MyCustomBox('mycustombox1', 'mycustombox-div-box', 500, 500);
```

In order for everything to work you must include the `InteractiveBox.js` file, your `MyCustomBox.js` file and the InteractiveBoxes css stylesheet `boxes.css`.

Note: the name must be unique and the script must be executed after the body has loaded.

3.5 Example

Here is an example of interactive box where the path drawn by the user is progressively drawn on the canvas.

```
class Example extends InteractiveBox {

  #points = []; // The path to be drawn
  #counter = 0; // Drawing process

  constructor(name, container, height, width) {
    super(name, container, height, width)

    this.setPoints(this.#getDefaultPath());
  }

  onTimeTravel(value) {
    // Set counter accoring to value
    this.#counter = value * this.#points.length | 0;
  }

  setPoints(points) {
    this.#counter = 0; // Reset counter
    this.#points = points; // Update points
  };

  draw(ctx) {
    this.clearCanvas(); // Clear the canvas

    // Update counter and update timeline
    this.setTime(this.#counter++ / (this.#points.length - 1));
    if (this.#counter > this.#points.length) {
      this.#counter = 0; // Reset counter
    }

    ctx.beginPath();

    ctx.lineWidth = 2.0;
    ctx.strokeStyle = 'red';

    ctx.moveTo(this.#points[0].x, this.#points[0].y);
    for (var i = 1; i < this.#counter; i++) {
      ctx.lineTo(this.#points[i].x, this.#points[i].y);
    }

    ctx.stroke();
  };

  #getDefaultPath() {
    var circle = [];
    for (var i = 0; i < 100; i++) {
      circle[i] = {
        x: 250 + 50 * Math.cos(Math.PI * 2 / 100 * i),
        y: 250 + 50 * Math.sin(Math.PI * 2 / 100 * i)
      }
    }
    return circle;
  }
}
```

4 Website Structure

4.1 Dependency table

The website relies on various libraries, some of which are not stored locally. This means that the user will query third-party servers, thus the website will not work locally if you do not have a free internet connection.

Dependency table			
Name	Description	Stored	Version
Bootstrap (CSS)	Styling framework	Locally	4.0.0
Bootstrap (JS)	Styling framework	Locally	4.0.0
InteractiveBoxes	Canvas drawing	Locally	1.0
JQuery	Website Manipulation	Locally	3.6.0
Google Fonts	Fonts	Remotely	-
MathJax	LaTeX rendering	Remotely	3.x.x (latest)
Desmos	Graphic calculator	Remotely	1.6

4.2 Sections

The website is made up of several sections, each about a particular topic.

4.2.1 Fourier Analysis

What is Fourier analysis and where is it used.

This section contains the *FourierSeries2D* interactive box.

Fourier Analysis

Fourier analysis is the study of how a function can be represented as a sum of waves. Take a look at the animation playing at the side, a shape is being drawn using a chain of rotating circles of different sizes. You can even try drawing your own shape, it's interactive! This article will cover in detail how this animation works, and what math is behind it. The concepts that we'll discover are widely used in electronics, acoustics and communications. Operators such as the Fourier Transform are constantly used in the real world, without these discoveries the world would not be the same. Many software rely in Fourier Analysis, such as for instance Shazam, the famous service for identifying songs. Any audio spectrum visualized processes the signal using Fourier Transform, these are just a few of the many application of this analysis.

[LEARN MORE](#)

4.2.2 Requirements

What are the requirements to read the article.

Fourier Analysis

Fourier analysis is the study of how a function can be represented as a sum of waves. Take a look at the animation playing at the side, a shape is being drawn using a chain of rotating circles of different sizes. You can even try drawing your own shape, it's interactive! This article will cover in detail how this animation works, and what math is behind it. The concepts that we'll discover are widely used in electronics, acoustics and communications. Operators such as the Fourier Transform are constantly used in the real world, without these discoveries the world would not be the same. Many software rely in Fourier Analysis, such as for instance Shazam, the famous service for identifying songs. Any audio spectrum visualized processes the signal using Fourier Transform, these are just a few of the many application of this analysis.

[LEARN MORE](#)

The video player shows an animation of a shape being drawn using a chain of rotating circles of different sizes. The shape is a yellow musical note. The animation is set against a dark green background with a grid. The video player has a play button and a progress bar.

4.2.3 Introduction

Who was Joseph Fourier and what he had discovered.

Fourier Analysis

Fourier analysis is the study of how a function can be represented as a sum of waves. Take a look at the animation playing at the side, a shape is being drawn using a chain of rotating circles of different sizes. You can even try drawing your own shape, it's interactive! This article will cover in detail how this animation works, and what math is behind it. The concepts that we'll discover are widely used in electronics, acoustics and communications. Operators such as the Fourier Transform are constantly used in the real world, without these discoveries the world would not be the same. Many software relay in Fourier Analysis, such as for instance Shazam, the famous service for identifying songs. Any audio spectrum visualized processes the signal using Fourier Transform, these are just a few of the many application of this analysis.

[LEARN MORE](#)

The video player shows an animation of a shape being drawn using a chain of rotating circles of different sizes. The shape is a yellow musical note. The animation is set against a dark green background with a grid. The video player has a play button and a progress bar.

4.2.4 Fourier Series vs Fourier Transform

What is the difference between the Furier series and the Fourier transform.

Fourier Analysis

Fourier analysis is the study of how a function can be represented as a sum of waves. Take a look at the animation playing at the side, a shape is being drawn using a chain of rotating circles of different sizes. You can even try drawing your own shape, it's interactive! This article will cover in detail how this animation works, and what math is behind it. The concepts that we'll discover are widely used in electronics, acoustics and communications. Operators such as the Fourier Transform are constantly used in the real world, without these discoveries the world would not be the same. Many software relay in Fourier Analysis, such as for instance Shazam, the famous service for identifying songs. Any audio spectrum visualized processes the signal using Fourier Transform, these are just a few of the many application of this analysis.

[LEARN MORE](#)

4.2.5 Trigonometric Fourier Series

Representing a periodic function using a sum of trigonometric functions.

Fourier Analysis

Fourier analysis is the study of how a function can be represented as a sum of waves. Take a look at the animation playing at the side, a shape is being drawn using a chain of rotating circles of different sizes. You can even try drawing your own shape, it's interactive! This article will cover in detail how this animation works, and what math is behind it. The concepts that we'll discover are widely used in electronics, acoustics and communications. Operators such as the Fourier Transform are constantly used in the real world, without these discoveries the world would not be the same. Many software relay in Fourier Analysis, such as for instance Shazam, the famous service for identifying songs. Any audio spectrum visualized processes the signal using Fourier Transform, these are just a few of the many application of this analysis.

[LEARN MORE](#)

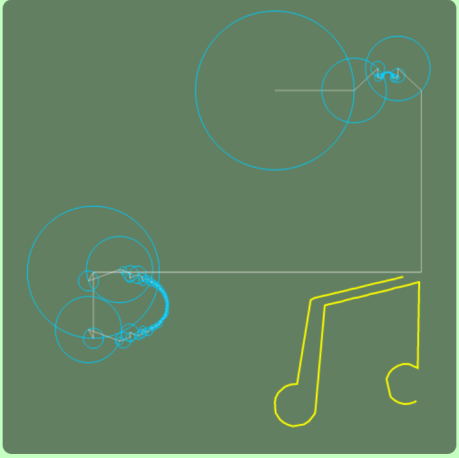
4.2.6 Trigonometric Fourier Series - C term

Finding the C term.

Fourier Analysis

Fourier analysis is the study of how a function can be represented as a sum of waves. Take a look at the animation playing at the side, a shape is being drawn using a chain of rotating circles of different sizes. You can even try drawing your own shape, it's interactive! This article will cover in detail how this animation works, and what math is behind it. The concepts that we'll discover are widely used in electronics, acoustics and communications. Operators such as the Fourier Transform are constantly used in the real world, without these discoveries the world would not be the same. Many software rely in Fourier Analysis, such as for instance Shazam, the famous service for identifying songs. Any audio spectrum visualized processes the signal using Fourier Transform, these are just a few of the many application of this analysis.

[LEARN MORE](#)



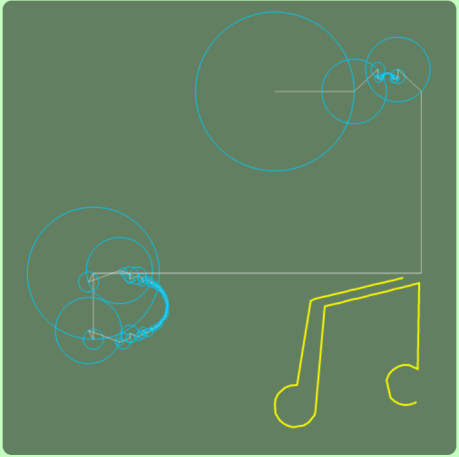
4.2.7 Trigonometric Fourier Series - Coefficients

Finding the coefficients a_n and b_n .

Fourier Analysis

Fourier analysis is the study of how a function can be represented as a sum of waves. Take a look at the animation playing at the side, a shape is being drawn using a chain of rotating circles of different sizes. You can even try drawing your own shape, it's interactive! This article will cover in detail how this animation works, and what math is behind it. The concepts that we'll discover are widely used in electronics, acoustics and communications. Operators such as the Fourier Transform are constantly used in the real world, without these discoveries the world would not be the same. Many software rely in Fourier Analysis, such as for instance Shazam, the famous service for identifying songs. Any audio spectrum visualized processes the signal using Fourier Transform, these are just a few of the many application of this analysis.

[LEARN MORE](#)



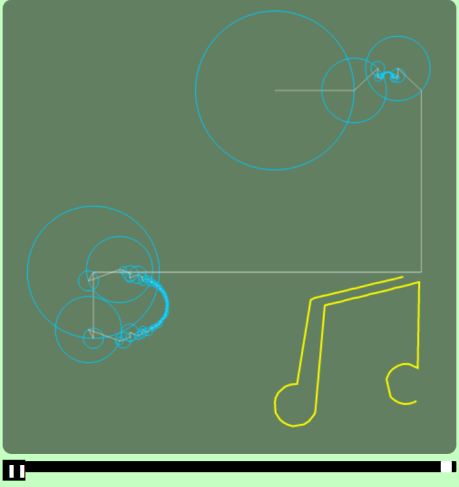
4.2.8 Fourier Series - Conclusion

Conclusion on the last chapters.

Fourier Analysis

Fourier analysis is the study of how a function can be represented as a sum of waves. Take a look at the animation playing at the side, a shape is being drawn using a chain of rotating circles of different sizes. You can even try drawing your own shape, it's interactive! This article will cover in detail how this animation works, and what math is behind it. The concepts that we'll discover are widely used in electronics, acoustics and communications. Operators such as the Fourier Transform are constantly used in the real world, without these discoveries the world would not be the same. Many software rely in Fourier Analysis, such as for instance Shazam, the famous service for identifying songs. Any audio spectrum visualized processes the signal using Fourier Transform, these are just a few of the many application of this analysis.

[LEARN MORE](#)



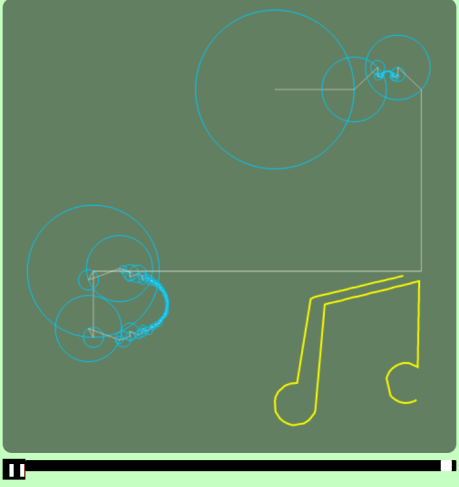
4.2.9 Main ideas - Complex plotting

Plotting a function around the origin in the complex plane using Euler's identity.

Fourier Analysis

Fourier analysis is the study of how a function can be represented as a sum of waves. Take a look at the animation playing at the side, a shape is being drawn using a chain of rotating circles of different sizes. You can even try drawing your own shape, it's interactive! This article will cover in detail how this animation works, and what math is behind it. The concepts that we'll discover are widely used in electronics, acoustics and communications. Operators such as the Fourier Transform are constantly used in the real world, without these discoveries the world would not be the same. Many software rely in Fourier Analysis, such as for instance Shazam, the famous service for identifying songs. Any audio spectrum visualized processes the signal using Fourier Transform, these are just a few of the many application of this analysis.

[LEARN MORE](#)



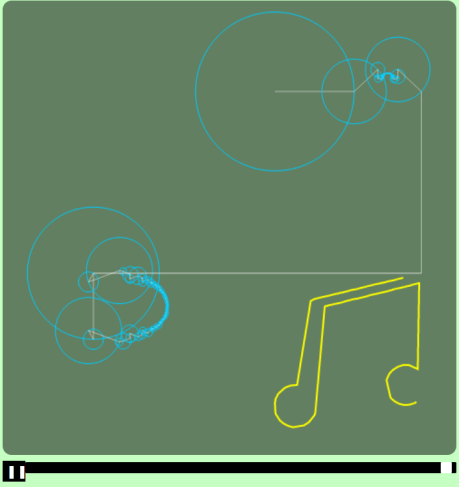
4.2.10 Main ideas - Center of mass

Computing the center of mass of $f(t)e^{-2\pi ti\xi}$

Fourier Analysis

Fourier analysis is the study of how a function can be represented as a sum of waves. Take a look at the animation playing at the side, a shape is being drawn using a chain of rotating circles of different sizes. You can even try drawing your own shape, it's interactive! This article will cover in detail how this animation works, and what math is behind it. The concepts that we'll discover are widely used in electronics, acoustics and communications. Operators such as the Fourier Transform are constantly used in the real world, without these discoveries the world would not be the same. Many software relay in Fourier Analysis, such as for instance Shazam, the famous service for identifying songs. Any audio spectrum visualized processes the signal using Fourier Transform, these are just a few of the many application of this analysis.

[LEARN MORE](#)



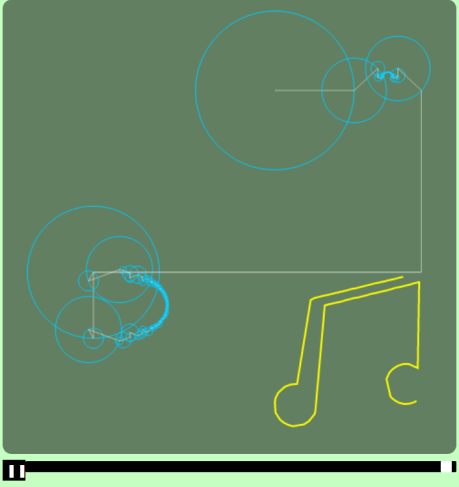
4.2.11 Main ideas - Fourier Transform

What is the Fourier transform operator.

Fourier Analysis

Fourier analysis is the study of how a function can be represented as a sum of waves. Take a look at the animation playing at the side, a shape is being drawn using a chain of rotating circles of different sizes. You can even try drawing your own shape, it's interactive! This article will cover in detail how this animation works, and what math is behind it. The concepts that we'll discover are widely used in electronics, acoustics and communications. Operators such as the Fourier Transform are constantly used in the real world, without these discoveries the world would not be the same. Many software relay in Fourier Analysis, such as for instance Shazam, the famous service for identifying songs. Any audio spectrum visualized processes the signal using Fourier Transform, these are just a few of the many application of this analysis.

[LEARN MORE](#)



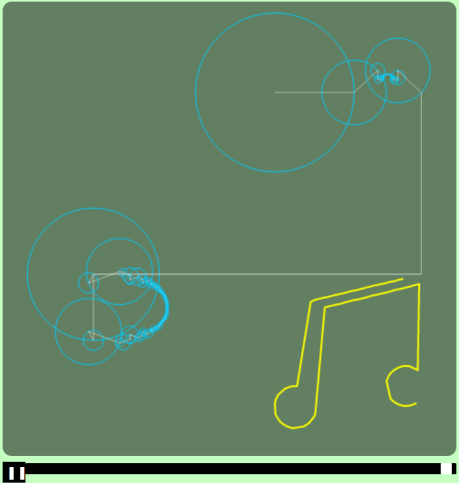
4.2.12 A Simple Example

Computing the Fourier series of a simple function.

Fourier Analysis

Fourier analysis is the study of how a function can be represented as a sum of waves. Take a look at the animation playing at the side, a shape is being drawn using a chain of rotating circles of different sizes. You can even try drawing your own shape, it's interactive! This article will cover in detail how this animation works, and what math is behind it. The concepts that we'll discover are widely used in electronics, acoustics and communications. Operators such as the Fourier Transform are constantly used in the real world, without these discoveries the world would not be the same. Many software rely in Fourier Analysis, such as for instance Shazam, the famous service for identifying songs. Any audio spectrum visualized processes the signal using Fourier Transform, these are just a few of the many application of this analysis.

[LEARN MORE](#)



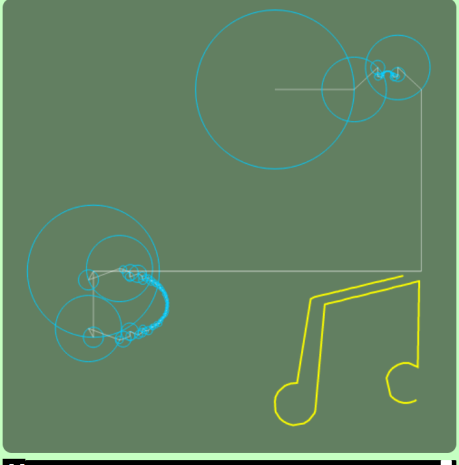
4.2.13 A Simple Example - Coefficients

Finding the coefficients of the Fourier series.

Fourier Analysis

Fourier analysis is the study of how a function can be represented as a sum of waves. Take a look at the animation playing at the side, a shape is being drawn using a chain of rotating circles of different sizes. You can even try drawing your own shape, it's interactive! This article will cover in detail how this animation works, and what math is behind it. The concepts that we'll discover are widely used in electronics, acoustics and communications. Operators such as the Fourier Transform are constantly used in the real world, without these discoveries the world would not be the same. Many software rely in Fourier Analysis, such as for instance Shazam, the famous service for identifying songs. Any audio spectrum visualized processes the signal using Fourier Transform, these are just a few of the many application of this analysis.

[LEARN MORE](#)



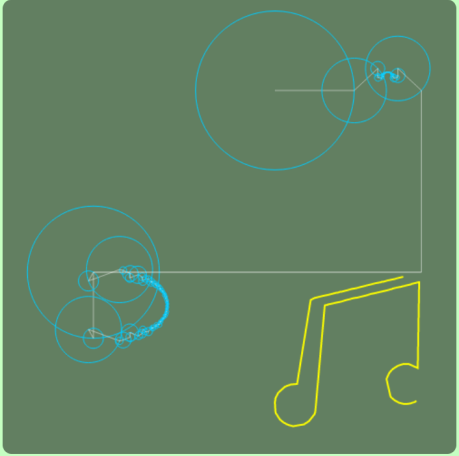
4.2.14 A Simple Example - Conclusion

Demonstrating the Fourier series by plotting it.

Fourier Analysis

Fourier analysis is the study of how a function can be represented as a sum of waves. Take a look at the animation playing at the side, a shape is being drawn using a chain of rotating circles of different sizes. You can even try drawing your own shape, it's interactive! This article will cover in detail how this animation works, and what math is behind it. The concepts that we'll discover are widely used in electronics, acoustics and communications. Operators such as the Fourier Transform are constantly used in the real world, without these discoveries the world would not be the same. Many software rely in Fourier Analysis, such as for instance Shazam, the famous service for identifying songs. Any audio spectrum visualized processes the signal using Fourier Transform, these are just a few of the many application of this analysis.

[LEARN MORE](#)



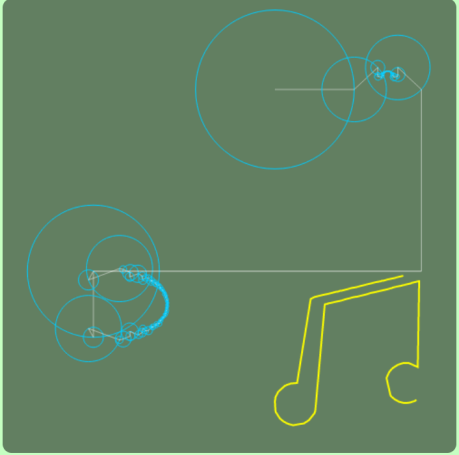
4.2.15 Exponential Fourier Series

Defining the Fourier series using Euler's Identity.

Fourier Analysis

Fourier analysis is the study of how a function can be represented as a sum of waves. Take a look at the animation playing at the side, a shape is being drawn using a chain of rotating circles of different sizes. You can even try drawing your own shape, it's interactive! This article will cover in detail how this animation works, and what math is behind it. The concepts that we'll discover are widely used in electronics, acoustics and communications. Operators such as the Fourier Transform are constantly used in the real world, without these discoveries the world would not be the same. Many software rely in Fourier Analysis, such as for instance Shazam, the famous service for identifying songs. Any audio spectrum visualized processes the signal using Fourier Transform, these are just a few of the many application of this analysis.

[LEARN MORE](#)



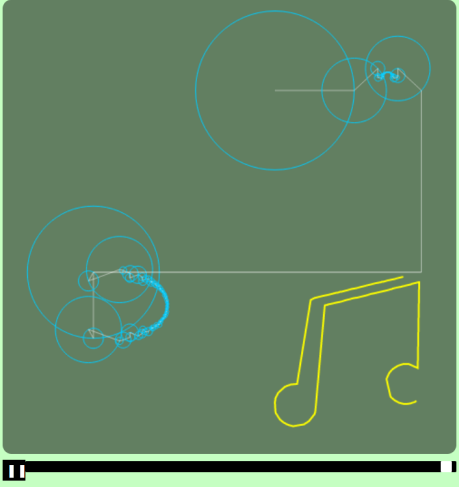
4.2.16 Fast Fourier Transform

What is the Fast Fourier Transform algorithm.

Fourier Analysis

Fourier analysis is the study of how a function can be represented as a sum of waves. Take a look at the animation playing at the side, a shape is being drawn using a chain of rotating circles of different sizes. You can even try drawing your own shape, it's interactive! This article will cover in detail how this animation works, and what math is behind it. The concepts that we'll discover are widely used in electronics, acoustics and communications. Operators such as the Fourier Transform are constantly used in the real world, without these discoveries the world would not be the same. Many software rely in Fourier Analysis, such as for instance Shazam, the famous service for identifying songs. Any audio spectrum visualized processes the signal using Fourier Transform, these are just a few of the many application of this analysis.

[LEARN MORE](#)



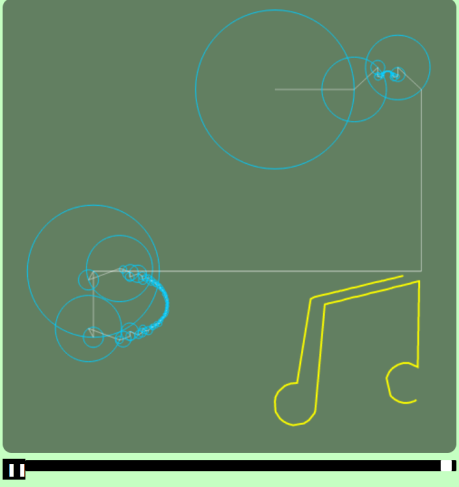
4.2.17 Conclusion

How the animation in Chapter. 1 works.

Fourier Analysis

Fourier analysis is the study of how a function can be represented as a sum of waves. Take a look at the animation playing at the side, a shape is being drawn using a chain of rotating circles of different sizes. You can even try drawing your own shape, it's interactive! This article will cover in detail how this animation works, and what math is behind it. The concepts that we'll discover are widely used in electronics, acoustics and communications. Operators such as the Fourier Transform are constantly used in the real world, without these discoveries the world would not be the same. Many software rely in Fourier Analysis, such as for instance Shazam, the famous service for identifying songs. Any audio spectrum visualized processes the signal using Fourier Transform, these are just a few of the many application of this analysis.

[LEARN MORE](#)



4.3 Interactive Boxes Implementations

4.3.1 Fourier Series 1D

4.3.2 Fourier Series 2D

4.3.3 Complex Plot

4.3.4 Center of mass

4.3.5 Fourier Transform