Fourier Analysis

Paolo Bettelini

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1 Introduction

A function $f(x)$ is periodic if there is a positive number T (the period of f) such that

$$f(x+nT) = f(x) \quad \forall x \in D_f, n \in \mathbb{Z}$$

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$$f(x) = h + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2\pi nx}{T}\right) + b_n \sin\left(\frac{2\pi nx}{T}\right)$$

1.1 The h

We take the integral over the period $[t_0; t_0+T]$ on both sides

$$\int\limits_{t_0}^{t_0+T} f(x)\,dx = \int\limits_{t_0}^{t_0+T} h\,dx + \sum_{n=1}^{\infty}\left[\int\limits_{t_0}^{t_0+T} a_n \cos\left(\frac{2\pi nx}{T}\right)\,dx + \int\limits_{t_0}^{t_0+T} b_n \sin\left(\frac{2\pi nx}{T}\right)\,dx\right]$$

if you think about it, the integral over a period interval of a fuction such as $\sin(x)$ or $\cos(x)$ is 0.
If we consider $\sin(w_n x)$ or $\cos(w_n x)$ the function will make more full cycles in the span of the period T, all of which yield an area of 0.

$$\begin{aligned}\int\limits_{t_0}^{t_0+T} f(x)\,dx &= \int\limits_{t_0}^{t_0+T} h\,dx \\ &= h\int\limits_{t_0}^{t_0+T} dx \\ &= h[x]_{t_0}^{t_0+T}\end{aligned}$$

concluding that

$$h = \frac{1}{T}\int\limits_{t_0}^{t_0+T} f(x)\,dx$$

1.2 a_n

1.3 b_n

2 Complex form