Set Theory

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Contents

1 Definitions

1.1 Subset

If A and B are sets, then A is a *subset* of B ($A \subseteq B$), iff all the elements of A are also in B.

1.2 Empty Set

The empty set $\emptyset$ is a subset of all other sets.

1.3 Power Set

If B is a set, then the *power set* $\mathcal{P}(B)$ is defined as the set of all subsets of B

$$\mathcal{P}(B) = \{A \,|\, A \subseteq B\}$$

Note that $\emptyset \subseteq \mathcal{P}(B)$ and $B \in \mathcal{P}(B)$

1.4 Union

If A and B are sets, then their *union* is

$$A \cup B = \{x \,|\, c \in A \vee X \in B\}$$

1.5 Intersection

If A and B are sets, then their *intersection* is

$$A \cap B = \{x \,|\, x \in A \wedge x \in B\}$$

1.6 Difference

If A and B are sets, then their *difference* is

$$A \backslash B = \{x \,|\, x \in A \wedge x \notin B \vee x \in B \wedge \notin A\}$$

1.7 Cartesian Product

If A and B are sets, then their *cartesian product* is

$$A \times B = \{(x, y) \,|\, x \in A \wedge y \in B\}$$

1.8 Complement

If A is a set, its *complement* is

$$\bar{A} = \{x \,|\, x \notin A\}$$

1.9 Binary Relation

If A and B are sets, a function $f : A \to B$ is a *binary relation* R

$$R = \{(a, b) \,|\, f(a) = b\}$$

Note that $R \subseteq A \times B$

1.10 Closure

A binary operation R for $f : A \to B$ is closed iff

$$\forall a \in A, \forall b \in B, (a, b) \in R$$

1.11 Injection

A function $f : A \to B$ is *injective* iff

$$\forall a, b \in A \,|\, a \neq b, f(a) \neq f(b)$$

1.12 Surjectivity

A function $f : A \to B$ is *surjectiv* iff

$$\forall b \in B \exists a \,|\, f(a) = b$$

1.13 Bijectivity

A function $f : A \to B$ is *bijective* iff it is both surjective and injective.

1.14 Reflexive relation

A binary relation R for $f : A \to B$ is *reflexive* iff

$$\forall a \in A, (a, a) \in R$$

1.15 Symmetric relation

A binary relation R for $f : A \to B$ is *symmetric* iff

$$\forall (a, b) \in R, (b, a) \in R$$

1.16 Transitive relation

A binary relation R for $f : A \to B$ is *transitive*

$$\forall a, b, c \in A, (a, b) \in R \land (b, c) \in R \implies (a, c) \in R$$