

# Category Theory

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## Contents

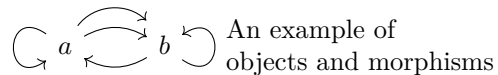
<b>1</b>	<b>Category</b>	<b>2</b>
1.1	Composition . . . . .	2
1.2	Identity . . . . .	2
1.3	Associativity . . . . .	2
<b>2</b>	<b>Homomorphism</b>	<b>3</b>
2.1	Isomorphisms . . . . .	3
2.2	Epimorphisms . . . . .	3
2.3	Monomorphisms . . . . .	3

# 1 Category

A category consists of *objects* and *morphism* or *arrows*.

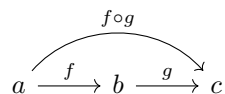
An arrow has a beginning and an ending, and it goes from one object to another.

Objects serve the purpose of marking the beginning and ending of a morphism.



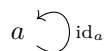
## 1.1 Composition

Composition is a property that says that if there is an arrow from  $a$  to  $b$ , and an arrow from  $b$  to  $c$ , there must exist an arrow from  $a$  to  $c$ .

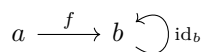


## 1.2 Identity

For every object there is an identity arrow.



The composition of an arrow with an identity is the arrow itself



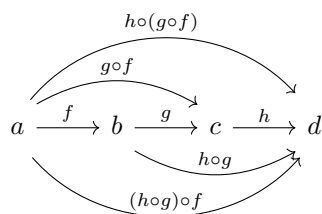
$$f \circ \text{id}_b = f$$

and also vice versa

$$\text{id}_b \circ f = f$$

## 1.3 Associativity

Compositions have the associative property



$$h \circ (g \circ f) = (h \circ g) \circ f$$

## 2 Homomorphism

An Homomorphism is a map between two structures of the same type.

### 2.1 Isomorphisms

A function  $f$  going from  $a$  to  $b$

$$f : a \rightarrow b$$

is invertible if there is a function  $g$  that goes from  $b$  to  $a$

$$g : b \rightarrow a$$

such that

$$\begin{aligned} g \circ f &= \text{id}_b \\ f \circ g &= \text{id}_a \end{aligned}$$

$$\begin{array}{ccc} & f & \\ a & \xrightarrow{\quad} & b \\ & \xleftarrow{\quad} & \\ & g & \end{array}$$

This is a *bijective* homomorphism and it's called isomorphism. An isomorphism is labelled  $\xrightarrow{\sim}$ .

### 2.2 Epimorphisms

$$a \xrightarrow{f} b \begin{array}{c} \xrightarrow{g_1} \\ \xrightarrow{g_2} \end{array} c$$

### 2.3 Monomorphisms