Fundamentals of Quantum Physics

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1 The Probabilistic Nature of Qubits

A qubit is comprable to a bit in the "classical" world, but it exists on a sub-atomic level.
When a qubit is measured, its state will either be a "1" or a "0".
The crucial aspect is that before the measurement, a qubit is in a *superposition* of both states.

For example, a given qubit $|\Psi\rangle$ can be represented as

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

which means a linear combination of the two states $|0\rangle$ and $|1\rangle$.

The coefficients α and β represent the probability of the qubit collapsing into one of the two states when measured. The probability of the qubit collapsing into $|0\rangle$ is $|\alpha|^2$, while the probability of collapsing into $|1\rangle$ is $|\beta|^2$.
Since there is 100% chance of the qubits collapsing into one of the two states, α and β must satisfy the following requirement:

$$|\alpha|^2 + |\beta|^2 = 1$$

A uniform superposition of the two states looks like this:

$$|\Psi\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}$$

which means that we have 50% probability of the state collapsing into a $|0\rangle$ or $|1\rangle$ since $\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$.

When the qubit is measured, the superposition is destroyed, leaving it in a "classical" binary state.

You mght have noticed that I used the absolute value in $|\alpha|^2$ or $|\beta|^2$, which doesn't usually make sense since squaring the value is already going to give us a positive result.
This is because the coefficients α and β can also be complex numbers. The absolute value of a complex number is defined as its distance from the origin.

2 Types of Algorithms

2.1 Deterministic

Example: the quantum circuit is run and we measure the output.

2.2 Probabilistic

Example: The more we run the circuit, the higher the chance of getting the right or a more precise result.