Physical Rendering

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1 Measurements

1.1 Radiant Flux

The radiant flux (or power) Φ is the total amount of energy passing through a surface per second and is measured in $[W]$ (watts) as $\frac{J}{s}$.

1.2 Irradiance

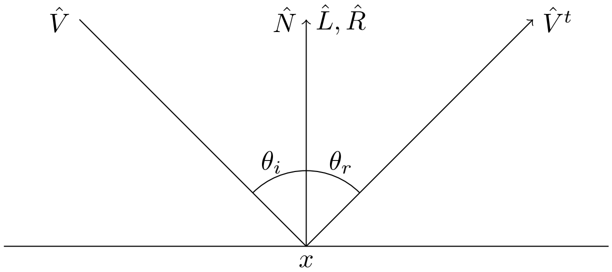
The irradiance E is the measurements of the radiant flux per *unit area* and is measured in $[W][M]^{-2}$ as $\frac{\Phi}{m^2}$.

1.3 Radiance

The radiance L is the irradiance per unit solid angle (steradian) and is measured in $[W][M]^{-2}[sr]^{-1}$ as $\frac{E}{sr}$.

2 Geometry

2.1 Fundamental vectors



- $\hat{V}$ direction towards the camera
- $\hat{N}$ surface normal
- $\hat{L}$ vector pointing torward the light source
- $\hat{R}$ reflected ray direction
- θ_i, θ_r incident and reflected angles

The reflected ray is given by $\hat{R} = \hat{L} - 2\hat{N}(\hat{L} \cdot \hat{N})$

2.2 Light attenuation

The amount of radiance on a point is given by

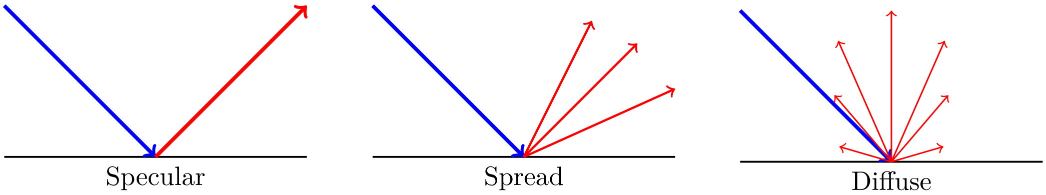
$$\Phi \left(\hat{L} \cdot \hat{N} \right) = \Phi \cdot \cos(\alpha)$$

where α is the angle between the two vectors.
This means that if the light source is perfectly above the point, then there is no light attenuation ($\alpha = 0$).

3 Materials

3.1 Types of materials

Different materials reflect incoming light to different directions and absord different amounts of it.



3.2 Bidirectional reflectance distribution function

The BRDF function (Bidirectional reflectance distribution function) is a probability distribution for the amount of light reflected in a certain direction.

$$f_r(\hat{w}, x, \hat{w}')$$

- $\hat{w}$ incoming ray direction
- x point of collision
- $\hat{w}'$ outgoing ray direction

This function follows the Helmholtz-reciprocity

$$\forall \hat{w}, x, \hat{w}', \quad f_r(\hat{w}, x, \hat{w}') = f_r(\hat{w}', x, \hat{w})$$

positivity

$$\forall \hat{w}, x, \hat{w}', \quad f_r(\hat{w}, x, \hat{w}') \geq 0$$

and energy conservation

$$\int_{\Omega} f_r(\hat{w}, x, \hat{w}') \cos\theta \, d\hat{w}' \leq 1$$

where Ω encloses the scene (usually a hemisphere).

3.3 Bidirectional transmittance distribution function

If the material can also transfer light through itself, we use the BTDF function (Bidirectional transmittance distribution function).

3.4 Bidirectional scattering distribution function

We use the BSDF or BxDF (Bidirectional scattering distribution function) to generalize both the BTDF and BRDF.

4 Rendering equation

The rendering equation tells us how much radiance is exiting a *surface point* in a given direction. Note that objects may also be emitting light.

$$\textit{Light exiting point} = \textit{Material emitted light} + \textit{Reflected incoming light}$$

Formally,

$$L_o(x, \vec{\omega}) = L_e(x, \vec{\omega}) + \int_{\Omega} L_i(x, \vec{\omega}) f_r(\vec{\omega}, x, \vec{\omega}') \cos\theta d\vec{\omega}'$$

- L_o outgoing radiance from point x in the direction $\vec{\omega}$
- L_e emitted radiance from point x in the direction $\vec{\omega}$ by the object itself
- Ω scene
- L_i incoming radiance to point x from the direction $\vec{\omega}$
- f_r BRDF
- $\cos\theta$ light attenuation

This value is difficult to compute. For each point, the light at that point depends on the incoming radiance of every other point, which also depends on the first point. This integral is infinite-dimensional because of the infinite bounces.

5 Phong reflection model

The phong reflection model is an approximation to the rendering equation.

5.1 Simplified ambient BRDF model

$$I_a = k_a I_a$$

- k_a ambient coefficient of an object (base color)
- I_a ambient intensity of the scene/light sources

5.2 Simplified diffuse BRDF model

$$I_d = k_d(\vec{L} \cdot \vec{N})$$

- k_d diffuse coefficient of an object
- $\vec{L}$ vector pointing to the light vector
- $\vec{N}$ surface normal

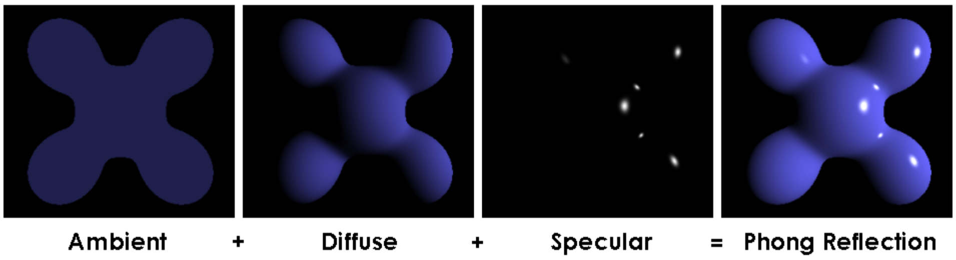
5.3 Simplified specular BRDF model

$$I_s = k_s(\vec{V} \cdot \vec{R})^n$$

- k_s specular coefficient of an object
- $\vec{V}$ direction towards the camera
- $\vec{R}$ reflected direction of the light ray
- $\vec{n}$ shininess factor

5.4 Result

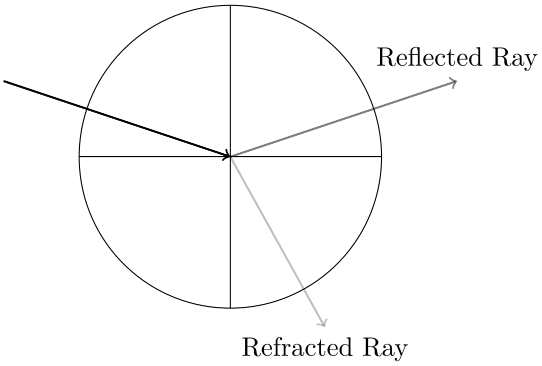
The sum of the following simplified models gives a good approximation to the rendering equation.



The diffuse and ambient values must be computed for each light in the scene. It is sufficient to consider only the light rays that hit the camera, meaning we can trace rays from the camera to the point, and assume that they're reflections from the light.

Note that the specular intensity cannot be precomputed as it is dependant on the camera position.

6 Reflection and refraction



ASSETS TO USE

The Fresnel Equation

$$R_s(\theta) = \left| \frac{n_1 \cos\theta - n_2 \sqrt{1 - \left(\frac{n_1}{n_2} \sin\theta\right)^2}}{n_1 \cos\theta + n_2 \sqrt{1 - \left(\frac{n_1}{n_2} \sin\theta\right)^2}} \right|$$

Snell's law

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{V_1}{V_2} = \frac{n_2}{n_1}$$

References

[1] K'aroly Zsolnai-Feh'er. *TU Wien Rendering Course*. https://users.cg.tuwien.ac.at/zsolnai/gfx/rendering-course/. 2018.