

Functions

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1 Surjectivity

A function $f : D_f \rightarrow I_f$ is said to be **surjective** if

$$\forall y \in I_f, \exists x \in D_f | f(x) = y$$

2 Injectivity

A function $f : D_f \rightarrow I_f$ is said to be **injective** if

$$\forall x_1, x_2 \in D_f, f(x_1) = f(x_2) \Rightarrow x_1 = x_2$$

3 Bijectivity

A function is said to be **bijective** iff it's both injective and surjective.

4 Continuity

A function f is continuous at a point c iff

$$\lim_{c_0 \rightarrow c^+} f(c_0) = \lim_{c_0 \rightarrow c^-} f(c_0) = f(c)$$

A function f is continuous on an interval $[a; b]$ iff it is continuous at each point $c \in [a; b]$

$$\forall c \in [a; b], \lim_{c_0 \rightarrow c^+} f(c_0) = \lim_{c_0 \rightarrow c^-} f(c_0) = f(c)$$

5 Periodic functions

A function f is periodic with a period T iff

$$f(x) = f(x + kT), \quad k \in \mathbb{Z}$$

6 Odd functions

A function f is odd iff

$$f(-x) = -f(x)$$

7 Even functions

A function f is even iff

$$f(-x) = f(x)$$

8 Intermediate value Theorem

A function f continuous on an interval $[a; b]$ will take every value in the interval $[f(a); f(b)]$.

9 Bolzano's Theorem

If $f(x)$ is continuous on $[a; b]$ and $f(a) \cdot f(b) < 0$ then there is a root.

$$f(a) \cdot f(b) < 0 \implies \exists c \in [a; b] \mid f(c) = 0$$

10 Weierstrass Theorem

If $f(x)$ is continuous in $[a; b]$ then the function will a maxima and a minima.