Group Theory

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1 Groups

1.1 Binary operations

Let G be a set. A *binary operation* $\circ$ on G is a map

$$G \times G \to G, \qquad (x, y) \to x \circ y$$

1.2 Cayley tables

A binary operation $\circ$ on a finite set G can be visualized using a *Cayley table*.

Example: $G = \{0, 1\}$ and $\circ \equiv$ multiplication.

$\circ$	0	1
0	0	0
1	0	1

1.3 Definition

A *group* $(G, \circ)$ is a tuple containing a set G and a binary operation $\circ$ where $\circ$ satisfies.

1. **Associative**: $\forall a, b, c \in G a \circ (b \circ c) = (a \circ b) \circ c$
2. **Identity**: $\exists e \,|\, \forall a \in G, ea = ae = a$
3. **Inverse**: $\forall a \in G \exists a^{-1} | a^{-1}a = aa^{-1} = e$