Graphs

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1 Definition

A *graph* $G = (V, E)$ is a pair consisting of a set V (vertices) and a set E (edges). Every element in E is a distinct pair of vertices in V.

2 Degree

The *degree* of a vertex v, $deg(v)$ is defined as the numbers of edges that are incident on v.

The sum of the degrees of all vertices is equals to twice the number of edges.

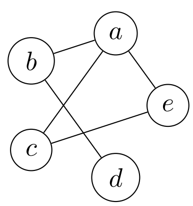
$$\sum_{v \in V} deg(v) = 2|E|$$

3 Paths

A *path* is a sequence of vertices connected by edges.
A path is a *cycle* if it starts and ends at the same vertex.
A path is *simple* if every vertex in the path is distinct.
A graph is *connected* if there is at least a path between any pair of vertices.

4 Adjacency Matrices

A finite graph can be represented by a square matrix $n \times n$ where n is the number of vertices.



$$\text{A=} \begin{array}{cc} & \begin{array}{ccccc} \text{a} & \text{b} & \text{c} & \text{d} & \text{e} \end{array} \\ \begin{array}{c} \text{a} \\ \text{b} \\ \text{c} \\ \text{d} \\ \text{e} \end{array} & \begin{pmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{pmatrix} \end{array}$$

Every row and column represents a vertice. 1 means that the two vertices are adjacent, 0 otherwise. The diagonal of this matrix will always e $0s$ since no vertice is adjacent to itself and $A = A^t$