

# Vectors

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## 1 Definition

A vector is a geometric object that has a direction and a magnitude.

Vectors don't have an origin point and they can be represented in any position on the space space.

A vector can be expressed with its components

$$\vec{a} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

For an n-dimensional vector

$$\vec{a} = \begin{pmatrix} \vec{a}_1 \\ \vec{a}_2 \\ \vdots \\ \vec{a}_n \end{pmatrix}, \quad \vec{a} \in \mathbb{R}^n$$

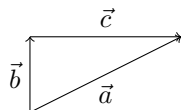
## 2 Addition

Vectors can be added together. This operation is commutative.

$$\vec{a} + \vec{b} = \vec{c}$$

### 2.1 Graphically

This plot shows how two vectors are added together in a two-dimensional space.



### 2.2 Using its components

The respective components of the vectors can be added to perform vector addition.

$$\vec{a} + \vec{b} = \begin{pmatrix} \vec{a}_1 \\ \vec{a}_2 \\ \vdots \\ \vec{a}_n \end{pmatrix} + \begin{pmatrix} \vec{b}_1 \\ \vec{b}_2 \\ \vdots \\ \vec{b}_n \end{pmatrix} = \begin{pmatrix} \vec{a}_1 + \vec{b}_1 \\ \vec{a}_2 + \vec{b}_2 \\ \vdots \\ \vec{a}_n + \vec{b}_n \end{pmatrix}$$

### 3 Scalar Product

A vector  $\vec{a}$  can be multiplied by a scalar value  $k$

$$k \cdot \vec{a} = \begin{pmatrix} k\vec{a}_1 \\ k\vec{a}_2 \\ \vdots \\ k\vec{a}_n \end{pmatrix}, \quad k \in \mathbb{R}$$

### 4 Linear Combination

A linear combination is a sum of two or more vectors, each with a coefficient.

$$\vec{c} = a \cdot \vec{a} + b \cdot \vec{b}, \quad a, b \in \mathbb{R}$$

### 5 Magnitude

To find the magnitude (or length) of a vector  $\vec{a}$  we can apply Pythagorean theorem.

$$||\vec{a}|| = \sqrt{a_x^2 + a_y^2}$$

For an n-dimensional vector.

$$||\vec{a}|| = \sqrt{a_1^2 + a_1^2 + \dots + a_n^2}, \quad \vec{a} \in \mathbb{R}^n$$

### 6 Vector given points

We can find a vector given two points  $A$  and  $B$ .

$$\begin{matrix} A(A_x; A_y) \\ B(B_x; B_y) \end{matrix}$$

The vector with direction going from  $A$  to  $B$  is given by

$$\begin{pmatrix} B_x - A_x \\ B_y - A_y \end{pmatrix}$$

For  $\mathbb{R}^n$

$$\begin{pmatrix} B_1 - A_1 \\ B_2 - A_2 \\ \vdots \\ B_n - A_n \end{pmatrix}$$

## 7 Unitary Vector

A unitary vector (or normalized vector) is a vector of magnitude 1.

$$||\hat{a}|| = 1$$

To normalize a vector it is sufficient to divide its components by its magnitude

$$\frac{\vec{a}}{||\vec{a}||} = \hat{a}$$

$\vec{a}$  can't be the *null* vector.

## 8 Basis

A set of vectors in a vector space is called a *basis* if every element of the vector space can be expressed as a linear combination of the *basis vectors*.

$$\mathcal{B} = \left\{ \vec{b}_1, \vec{b}_2, \dots, \vec{b}_n \right\}, \quad \forall \vec{b} \in \mathcal{B}, \vec{b} \in \mathbb{R}^n$$

### 8.1 Orthonormal basis

An Orthonormal basis is a basis in which each vector is orthogonal to each other and every vector is unitary.

$$\vec{a} = a \cdot \hat{i} + b \cdot \hat{j}, \quad a, b \in \mathbb{R}$$

1.  $\hat{i}$  è perpendicolare a  $\hat{j}$
2.  $\hat{i}$  e  $\hat{j}$  sono unitari
3.  $\hat{i} \neq a \cdot \hat{j}, \quad a \in \mathbb{R}$

## 9 Vettore posizione

Il vettore posizione è un vettore che parte dall'origine e va in un punto  $P$ . Questo vettore permette di rappresentare un punto nello spazio cartesiano con un vettore.