Functions

Paolo Bettelini

Contents

1 Definition

A *function* is a relation f from a set A (domain) to a set B (codomain)

$$f : A \to B$$

2 Properties

2.1 Injectivity

A function $f : A \to B$ is *injective* iff

$$\forall a, b \in A, f(a) = f(b) \implies a = b$$

An element $a \in A$ can only be mapped to one element $b \in B$.

2.2 Surjectivity

A function $f : A \to B$ is *surjectiv* iff

$$\forall b \in B \exists a \,|\, f(a) = b$$

2.3 Bijectivity

A function $f : A \to B$ is *bijective* iff it has a one-to-one correspondence between each element of A and B. Every bijection is both surjective and injective.

2.4 Invertible

A function f is invertible iff it is a bijection.

2.5 Continuity

A function f is continuous at a point c iff

$$\lim_{c_0 \to c^+} f(c_0) = \lim_{c_0 \to c^-} f(c_0) = f(c)$$

A function f is continuous on an interval $[a; b]$ if it is continuous at each point $c \in [a; b]$

$$\forall c \in [a; b], \lim_{c_0 \to c^+} f(c_0) = \lim_{c_0 \to c^-} f(c_0) = f(c)$$

2.6 Periodic functions

A function f is periodic with a period T iff

$$f(x) = f(x + kT), \quad k \in \mathbb{Z}$$

2.7 Odd functions

A function f is odd iff

$$f(-x) = -f(x)$$

2.8 Even functions

A function f is even iff

$$f(-x) = f(x)$$