Ray-Sphere Intersection

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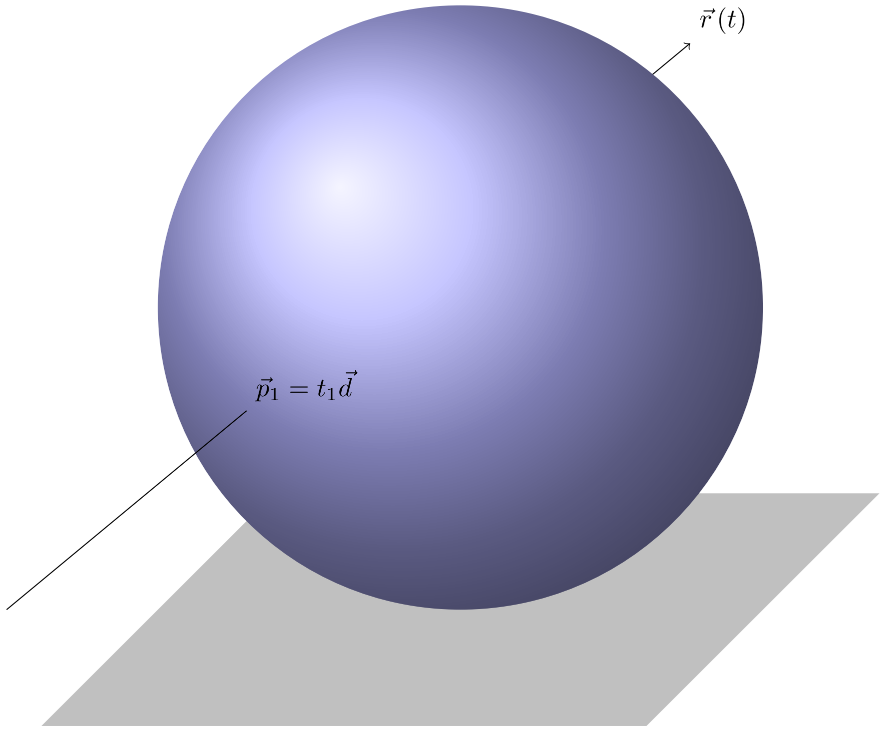
Abstract

Intersection between a ray starting from an origin and a sphere.
This computation is often used in rendering techniques such as ray tracing.

Contents

1 Derivation



We are given:

- The ray origin $\vec{o}$
- The ray direction $\vec{d}$ such that $||\vec{d}|| = 1$
- The center of the sphere $\vec{c}$
- The radius of the sphere r

The position of the ray after travelling t distance is given by

$$\vec{r}(t) = \vec{o} + t\vec{d}$$

The equation of the sphere is given by

$$(\vec{p} - \vec{c})^2 = r^2$$

where $\vec{p}$ is a point on the surface of the sphere.
We want to find the distance t for which the ray intersects with the surface of the sphere.

$$\vec{p} = \vec{o} + t\vec{d}$$

We substitute the definition of $\vec{p}$ into the sphere equation.

$$\left(\vec{o} + t\vec{d} - \vec{c}\right)\left(\vec{o} + t\vec{d} - \vec{c}\right) = r^2$$

$$\underbrace{\left(\vec{d} \cdot \vec{d}\right)}_{\text{A}} t^2 + \underbrace{2\vec{d}\left(\vec{o} - \vec{c}\right)}_{\text{B}} t + \underbrace{\left(\vec{o} - \vec{c}\right)^2 - r^2}_{\text{C}} = 0$$

We can then rewrite the equation as

$$At^2 + Bt + C = 0$$

Using the quadratic formula

$$\vec{t}_{1,2} = \frac{-B \pm \sqrt{\Delta}}{2A}$$

Where $\Delta = B^2 - 4AC$
The points of collision are $\vec{p}_1 = \vec{r}(t_1)$ and $\vec{p}_2 = \vec{r}(t_2)$.
The points of collision could be 2, 1 or none.

$$\begin{cases} 2, & \text{if } \Delta < 0 \\ 1, & \text{if } \Delta = 0 \\ 0, & \text{if } \Delta > 0 \end{cases}$$

If t_1 or t_2 is negative, the intersects is behind the ray (or camera).
If t_1 or t_2 is 0, the collision is the ray or camera itself.
Discard those points if you are rendering.