Functions

Paolo Bettelini

Contents

1 Definition

Let $f \subset A \times B$. The set f is a function if

$$\forall x \in A \exists_{=1} y \in B \,|\, (x, y) \in f$$

2 Properties

2.1 Injectivity

Definition Injectivity

A function $f : A \to B$ is *injective* if

$$\forall a, b \in A, f(a) = f(b) \implies a = b$$

2.2 Surjectivity

Definition Surjectivity

A function $f : A \to B$ is *surjectiv* if

$$\forall b \in B \exists a \,|\, f(a) = b$$

2.3 Bijectivity

Definition Bijectivity

A function $f : A \to B$ is *bijective* if it has a one-to-one correspondence between each element of A and B.

Corollary Bijectivity properties

A function $f : A \to B$ is bijective iff it is both injective and surjective.

2.4 Invertibility

Definition Invertibility

A function f is invertible iff it is a bijection.

2.5 Continuity

Definition Continuity

A function f is continuous at a point c if

$$\lim_{c_0 \to c^+} f(c_0) = \lim_{c_0 \to c^-} f(c_0) = f(c)$$

A function f is continuous on an interval $[a; b]$ if it is continuous at each point $c \in [a; b]$

$$\forall c \in [a; b], \lim_{c_0 \to c^+} f(c_0) = \lim_{c_0 \to c^-} f(c_0) = f(c)$$

2.6 Periodic functions

A function f is periodic with a period T if

$$f(x) = f(x + kT), \quad k \in \mathbb{Z}$$

2.7 Odd functions

A function f is odd if

$$f(-x) = -f(x)$$

2.8 Even functions

A function f is even if

$$f(-x) = f(x)$$