

# Limits

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# 1 Definition

A limit is usually used to describe the behavior of a function as its argument approaches a given value.

The limit towards a certain value  $c$  within a function can be approached both from the right and from the left.

The limit in a general sense exists if the value approached from both sides is the same and well-defined.

We define the limit of  $x$  approaching  $c$  from the left within the function  $f(x)$  as

$$\lim_{x \rightarrow c^-} f(x)$$

We define the limit of  $x$  approaching  $c$  from the right within function  $f(x)$  as

$$\lim_{x \rightarrow c^+} f(x)$$

We define the limit of  $x$  approaching  $c$  within function  $f(x)$  as

$$\lim_{x \rightarrow c} f(x)$$

Formally, given a function  $f : D \rightarrow \mathbb{R}$  the limit  $L = \lim_{x \rightarrow c} f(x)$  exists if given an arbitrary small  $\epsilon > 0$  there is another number  $\delta > 0$  such that

$$|f(x) - L| < \epsilon, \quad \forall x \in D \text{ where } 0 < |x - c| < \delta$$