Functions

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Contents

1　Definition

A *function* is a relation f from a set A (domain) to a set B (codomain)

$$f : A \to B$$

2　Properties

2.1　Injectivity

> **Definition** Injectivity
>
> A function $f : A \to B$ is *injective* if
>
> $$\forall a, b \in A, f(a) = f(b) \implies a = b$$

2.2　Surjectivity

> **Definition** Surjectivity
>
> A function $f : A \to B$ is *surjectiv* if
>
> $$\forall b \in B \exists a \mid f(a) = b$$

2.3　Bijectivity

> **Definition** Bijectivity
>
> A function $f : A \to B$ is *bijective* if it has a one-to-one correspondence between each element of A and B.

> **Corollary** Bijectivity properties
>
> A function $f : A \to B$ is bijective iff it is both injective and surjective.

2.4　Invertibility

> **Definition** Invertibility
>
> A function f is invertible iff it is a bijection.

2.5　Continuity

> **Definition** Continuity
>
> A function f is continuous at a point c if
>
> $$\lim_{c_0 \to c^+} f(c_0) = \lim_{c_0 \to c^-} f(c_0) = f(c)$$

A function f is continuous on an interval $[a; b]$ if it is continuous at each point $c \in [a; b]$

$$\forall c \in [a; b], \lim_{c_0 \to c^+} f(c_0) = \lim_{c_0 \to c^-} f(c_0) = f(c)$$

2.6 Periodic functions

A function f is periodic with a period T if

$$f(x) = f(x + kT), \quad k \in \mathbb{Z}$$

2.7 Odd functions

A function f is odd if

$$f(-x) = -f(x)$$

2.8 Even functions

A function f is even if

$$f(-x) = f(x)$$