Logic

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1 Boolean Algebra

$$
\begin{aligned}
x \vee 0 &= x \\
x \wedge 0 &= 0 \\
x \vee 1 &= 1 \\
x \wedge 1 &= x \\
x \vee x &= x \\
x \wedge x &= x \\
x \wedge (x \vee y) &= x \\
x \vee (x \wedge y) &= x \\
x \wedge \neg x &= 0 \\
x \vee \neg x &= 1 \\
\neg x \wedge \neg y &= \neg(x \vee y) \\
\neg x \vee \neg y &= \neg(x \wedge y)
\end{aligned}
$$

2 Proof theory

2.1 k-ary Boolean function

A k-ary Boolean function is a mapping from $\{T, F\}^k \to \{T, F\}$

2.2 0-ary Boolean function

The 0-ary Boolean function are the *verum* ($\top$) and *falsum* ($\bot$). The represent respectively the True value and the False value.

2.3 Biconditional logical connective

A biconditional logical connective (written as *iff* or *xnor*) is the relation of equivalence between two statements P and Q. The relation $P \iff Q$ is both a sufficient condition and a necessary condition.

$$P \iff Q \equiv (P \implies Q) \wedge (P \impliedby Q)$$