

# Netwon's Method

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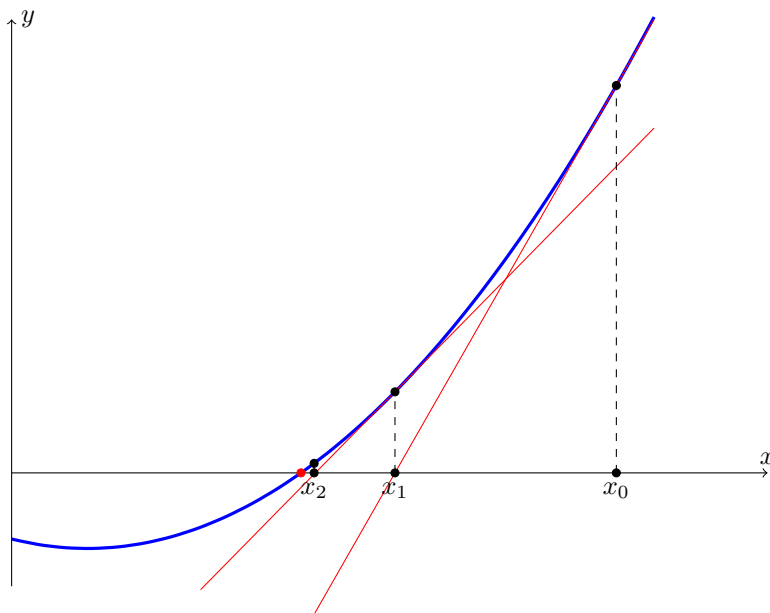
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# 1 Definition

The Newton's method is a numeric method to approximate the solution to equations in the form  $f(x) = 0$ . Starting from an initial approximation or guess  $x_0$ , the tangent line at  $x = x_0$  is given by

$$y = f(x_0) + f'(x_0)(x - x_0)$$



By projecting the root of the tangent onto the graph, we get another tangent whose root is closer to the root we are looking for. By doing this process recursively we may approach the solution.

The next approximation  $x_1$  occurs when the tangent of  $x_0$  is 0, so

$$\begin{aligned} 0 &= f(x_0) + f'(x_0)(x_1 - x_0) \\ x_1 - x_0 &= -\frac{f(x_0)}{f'(x_0)} \\ x_1 &= x_0 - \frac{f(x_0)}{f'(x_0)} \end{aligned}$$

as long as  $f'(x_0) \neq 0$ .

If  $x_n$  is an approximation of the solution to  $f(x) = 0$  and  $f'(x_n) \neq 0$  then a closer approximation is given by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$