Sequences

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Contents

1 Definition

A sequence denotes a series of indexed values. A sequence may be written as

$$\{a_n\} \qquad \{a_n\}_{n=1}^{\infty}$$

If $\lim_{n\to\infty} a_n$ exists and is finite we say that the sequence is *convergent*. If $\lim_{n\to\infty} a_n$ doesn't exist or is infinite we say that the sequence *diverges*.

If $f(x)$ is a function such that $f(n) = a_n$

$$\lim_{x\to\infty} f(x) = L \implies \lim_{n\to\infty} a_n = L$$

1.1 Properties

if $\{a_n\}$ and $\{b_n\}$ are convergent sequences, then

$$\lim_{n\to\infty}(a_n \pm b_n) = \lim_{n\to\infty} a_n \pm \lim_{n\to\infty} b_n$$
$$\lim_{n\to\infty} ca_n = c \lim_{n\to\infty} a_n$$
$$\lim_{n\to\infty}(a_n b_n) = \left(\lim_{n\to\infty} a_n\right)\left(\lim_{n\to\infty} b_n\right)$$

2 Squeeze Theorem

If $a_n \leq c_n \leq b_n$ for sufficiently large $n > N$ for some N and $\lim_{n\to\infty} a_n = \lim_{n\to\infty} b_n = L$ then $\lim_{n\to\infty} c_n = L$

3 Absolute Value

Note the following

$$-|a_n| \leq a_n \leq |a_n|$$

Then if we assume

$$\lim_{n\to\infty}(-|a_n|) = -\lim_{n\to\infty}|a_n| = 0$$

by the Squeeze Theorem we get

$$\lim_{n\to\infty} a_n = 0$$

We conclude that if $\lim_{n\to\infty}|a_n| = 0$ then $\lim_{n\to\infty} a_n = 0$.

4 Exponential sequence

The sequence $\{a^n\}_{n=0}^{\infty}$ converges iff $-1 < r \leq 1$

$$\lim_{n\to\infty} a^n = \begin{cases} 0 & \text{if } -1 < r < 1 \\ 1 & \text{if } r = 1 \end{cases}$$

5 Theorem

If $\lim_{n\to\infty} a_{2n} = L$ and $\lim_{n\to\infty} a_{2n+1} = L$ then $\{a_n\}$ is convergent and $\lim_{n\to\infty} a_n = L$.