Limits

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1 Definition

A limit is usually used to describe the behavior of a function as its argument approaches a given value.

The limit towards a certain value c within a function can be be approached both from the right and from the left.
The limit in a general sense exists if the value approached from both sides is the same and well-defined.

We define the limit of x approaching c from the left within the function $f(x)$ as

$$\lim_{x \to c^-} f(x)$$

We define the limit of x approaching c from the right within function $f(x)$ as

$$\lim_{x \to c^+} f(x)$$

We define the limit of x approaching c within function $f(x)$ as

$$\lim_{x \to c} f(x)$$

Formally, given a function $f : D \to \mathbb{R}$ the limit $L = \lim_{x \to c} f(x)$ exists if given an arbitrary small $\epsilon > 0$ there is another number $\delta > 0$ such that

$$|f(x) - L| < \epsilon, \quad \forall x \in D \text{ where } 0 < |x - c| < \delta$$

2 Properties

If the limit exists

$$\lim_{x \to c} f(g(x)) = f(\lim_{x \to c} g(x))$$

3 Continuity

A function f is continuous at a point c iff

$$\lim_{c_0 \to c^+} f(c_0) = \lim_{c_0 \to c^-} f(c_0) = f(c)$$

A function f is continuous on an interval $[a; b]$ iff

$$\forall c \in [a; b], \lim_{c_0 \to c^+} f(c_0) = \lim_{c_0 \to c^-} f(c_0) = f(c)$$