Integration

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1 Indefinite Integrals

1.1 Definition

Given a function $f(x)$, an **anti-derivative** or **primitive** is any function $F(x)$ such that

$$\frac{dF}{dx} = f(x)$$

The operator to find a primitive function is called the **indefinite integral**

$$\int f(x)\,dx = F(x) + C, \quad C \in \mathbb{R}$$

The function to integrate (integrand) is delimited by the integral symbol $\int$ and a differential of the variable of integration dx.
A function has infinitely many primitives, hence the $+C$ term. This essentially means that the derivative of a function is the same when the function is shifted up or down, the rate of change is the same. By reversing the process we don't know the up or down shift of the original function.

$$f(x) = \int \frac{df}{dx}\,dx + C$$

for some specific C.

1.2 Properties

If k is a constant

$$\int kf(x)\,dx = k\int f(x)\,dx$$

$$\int f(x) \pm g(x)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$$

1.3 Substitution Rule

Given an integral in the form

$$\int f(g(x))g'(x)\,dx$$

Let

$$u = g(x)$$

The differential of u is then

$$du = g'(x)dx$$

meaning that we can rewrite the integral as

$$\int f(u)\,du = F(u) + C = F(g(x)) + C$$

2 Integration By Parts

Starting from the product rule

$$\frac{d}{dx}\big(f(x)g(x)\big) = f'(x)g(x) + f(x)g'(x)$$

if we integrate both parts we get

$$f(x)g(x) + C = \int f'(x)g(x)\,dx + \int f(x)g'(x)\,dx$$
$$\int f(x)g'(x)\,dx = f(x)g(x) + C - \int f'(x)g(x)\,dx$$

Since the indefinite integral of $f'(x)g(x)$ is equal to some function plus an arbitrary constant, we can ignore the $+C$ term.

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$$