Logic

Paolo Bettelini

Contents

1 Boolean Algebra

$$
\begin{aligned}
x \vee 0 &= x \\
x \wedge 0 &= 0 \\
x \vee 1 &= 1 \\
x \wedge 1 &= x \\
x \vee x &= x \\
x \wedge x &= x \\
x \wedge (x \vee y) &= x \\
x \vee (x \wedge y) &= x \\
x \wedge \neg x &= 0 \\
x \vee \neg x &= 1 \\
\neg x \wedge \neg y &= \neg(x \vee y) \\
\neg x \vee \neg y &= \neg(x \wedge y)
\end{aligned}
$$

2 Necessity and sufficiency

2.1 Sufficiency

2.2 Necessity

2.3 Biconditional logical connective

A biconditional logical connective (written as *iff* or *xnor*) is the relation of equivalence between two statements P and Q. The relation $P \iff Q$ is both a sufficient condition and a necessary condition.

$$P \iff Q \equiv (P \implies Q) \land (P \impliedby Q)$$

3 Proof theory

3.1 k-ary Boolean function

A k-ary Boolean function is a mapping from $\{T, F\}^k \to \{T, F\}$

3.2 0-ary Boolean function

The 0-ary Boolean function are the *verum* ($\top$) and *falsum* ($\perp$) connectives. The represent respectively the True value and the False value.

3.3 Propositional variable

A *propositional variable* is an input boolean variable.

3.4 Propositional formula

A *propositional formula* is a formula which has a unique truth value given all variables.

3.5 Truth assignment

A *truth assignment* is a function which maps a set of propositional variables $V = \{p_1, p_2, \ldots, p_n\}$ to a boolean value

$$\tau : V \to \{T, F\}$$

A formula A involving the variables $V = \{p_1, p_2, \ldots, p_n\}$ defines a k-ary boolean function $f_A(x_1, x_2, \ldots, x_n)$ where $x_n = \tau(p_n)$.

3.6 Language

A *language* L is a set of connectives which may be used to describe an L-formula.

A language L is *complete* iff every k-ary boolean functions can be defined by an L-formula.

3.7 Tautology

A propositional formula A is a *tautology* $\models A$ if its k-ary boolean function f_A is always T.

3.8 Satisfiability

A propositional formula A is *satisfiable* if f_A is T for some input.