Differentiation

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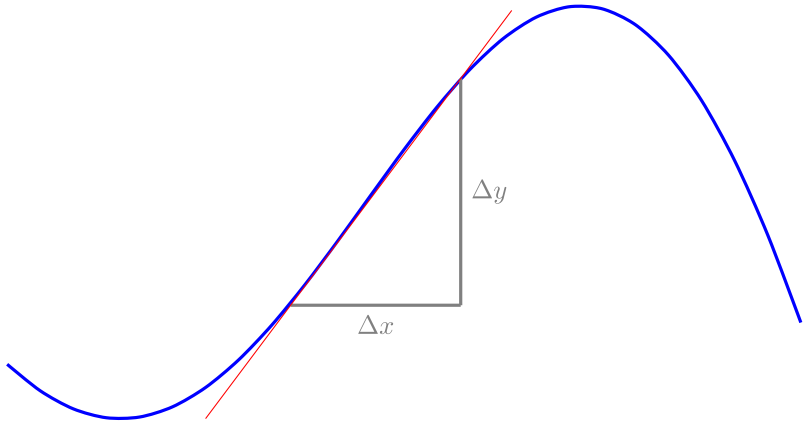
1 Definition



$$\lim_{h \to 0^+} f(x) = \frac{f(x+h) - f(x)}{h}$$

2 Chain Rule

2.1 Definition

If z depends on y, and y depends on x, then z also depends on x.

$$\frac{dz}{dx} = \frac{dz}{dy} \cdot \frac{dy}{dx}$$

2.2 Proof

Assuming that z and y are differentiable in x

$$\begin{aligned}
\frac{dz}{dx} &= \lim_{\Delta x \to 0} \frac{\Delta z}{\Delta x} = \lim_{\Delta x \to 0} \frac{\Delta z}{\Delta y} \cdot \frac{\Delta y}{\Delta x} \\
&= \left(\lim_{\Delta x \to 0} \frac{\Delta z}{\Delta y} \right) \left(\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} \right) \\
&= \left(\lim_{\Delta x \to 0} \frac{\Delta z}{\Delta y} \right) \cdot \frac{dy}{dx}
\end{aligned}$$

As $\Delta x \to 0$ also $\Delta y \to 0$, so we can replace Δx with Δy

$$\begin{aligned}
\frac{dz}{dx} &= \left(\lim_{\Delta y \to 0} \frac{\Delta z}{\Delta y} \right) \cdot \frac{dy}{dx} \\
&= \frac{dz}{dy} \cdot \frac{dy}{dx}
\end{aligned}$$

3 Rules for differentiation

$$\frac{d}{dx}(n) = 0$$

$$\frac{d}{dx}(x^n) = nx^{n-1}, \quad n \in \mathbb{R}^*$$

$$\frac{d}{dx}\left(n \cdot f(x)\right) = n\frac{d}{dx}\left(f(x)\right)$$

$$\frac{d}{dx}(f+g) = f' + g'$$

$$\frac{d}{dx}(f \cdot g) = g'f + gf'$$

$$\frac{d}{dx}(f(g(x))) = f'(g(x)) \cdot g'(x)$$

$$\frac{d}{dx}(f^g) = f^g\left(\frac{f'g}{f} + g' \ln f\right)$$