Grover's Algorithm

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Contents

1 Introduction

Given a list of N element an item ω with a unique properties, on average we will need to check $\frac{N}{2}$ elements before finding ω. This classical computation is $O(N)$ in time complexity.

Grover's algorithm reduces this time complexity to $O(\sqrt{N})$, meaning that if we have a list of size 100 it will take 10 steps to find ω instead of 50 on average.

This quantum algorithm uses amplitude amplification of a superposition to have a near perfect probability of finding ω.

2 Algorithm

$$U_\omega |x\rangle = \begin{cases} -|x\rangle, & \text{if } x = \omega \\ +|x\rangle, & \text{if } x \neq \omega \end{cases}$$

$$U_\omega = \begin{bmatrix} (-1)^{f(0)} & 0 & \cdots & 0 \\ 0 & (-1)^{f(1)} & \cdots & 0 \\ \vdots & 0 & \ddots & \vdots \\ 0 & 0 & \cdots & (-1)^{f(2^n-1)} \end{bmatrix}$$