Differential Equations

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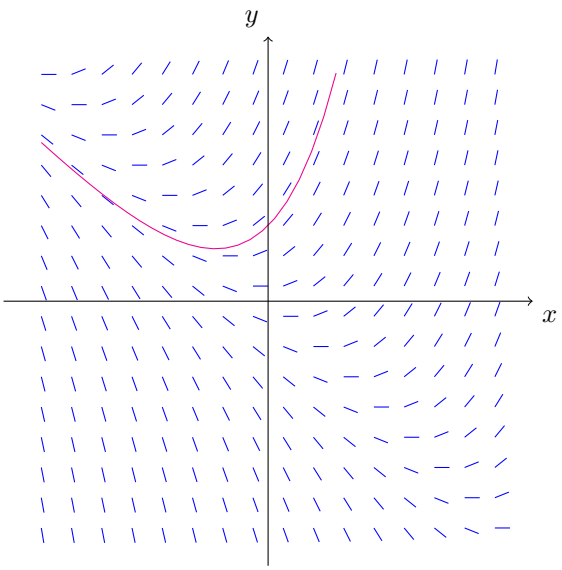
1 Definition

Differential equations are equations where the solution is a function or a set of functions.

2 Slope Field

A slope field or directional field is a field to visualize solutions to a first-order differential equation.



Slope field of $\frac{dy}{dx} = x + y$.

This field is obtained by picking points on the plane.
For each point (x, y) we know that the slope $(\frac{dy}{dx})$ is $x+y$.
This means that if a solution passes through (x, y), then its slope is $x + y$.
The red curve shows a solution.

3 Euler's Method

Euler's method is a technique for solving a first-order differential equation numerically given a point of the solution.
Starting at the known solution point A_0, we take small steps the direction of the slope field. As the length of the steps $s \to 0$ we approach the solution to the equation.
The angle of the slope is given by

$$\theta = \tan\left(\frac{dy}{dx}\right)$$

so each step gives the succession of points

$$A_n = A_{n_1} \cdot s\,(\cos(\theta), \sin(\theta))$$