Torque

Paolo Bettelini

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1 Torque

The moment of a force or twisting moment is a physical quantity that allows to describe the twisting action of a force in relation to the point of application of the force with respect to an axis of rotation.

The unit of measure is the $N \cdot m$.

2 Definition

A rotational vector with respect to the plane formed by the vectors $\vec{R}$ and $\vec{F}$ is defined as $(\vec{R} \times \vec{F})$, so $\vec{M}$ is normal to the plane.

$$\vec{M} = \vec{R} \times \vec{F}$$

Using the 3-dimensional definition of the scalar product

$$\vec{M} = \begin{pmatrix} R_y \cdot F_z - R_z \cdot F_y \\ R_z \cdot F_x - R_x \cdot F_z \\ R_x \cdot F_y - R_y \cdot F_x \end{pmatrix}$$

Since two vectors are always coplanar, it is possible to choose a coplanar reference system

$$\vec{R} = \begin{pmatrix} R_1 \\ R_2 \\ 0 \end{pmatrix}, \quad \vec{F} = \begin{pmatrix} F_1 \\ F_2 \\ 0 \end{pmatrix}, \quad \vec{M} = \begin{pmatrix} 0 \\ 0 \\ R_1 \cdot F_2 - R_2 \cdot F_1 \end{pmatrix}$$

$\vec{M}$ can also be computed by $\vec{R} \cdot \vec{F} \cdot \sin(\alpha)$ where α is the angle between $\vec{R}$ and $\vec{F}$.

Sometimes we interpret the formula $|\vec{M}| = \vec{R} \cdot \vec{F}_\perp$ as the twisting effect of the force component perpendicular to $\vec{R}$, or $|\vec{M}| = b \cdot \vec{F}$ where b is the lever arm.

The sign convention for this measurement is

$$\begin{cases} +1, & \text{if the rotation is counterclockwise} \\ -1, & \text{if the rotation is clockwise} \end{cases}$$

3 Equilibrium conditions

For a system to be in equilibrium, the resulting force and twisting moments must be null

$$\begin{cases} \sum \vec{F}_j = 0 \\ \sum \vec{M}_j = 0 \end{cases}$$