Physical Rendering

Paolo Bettelini

Contents

1 Measurements

1.1 Radiant Flux

The radiant flux (or power) Φ is the total amount of energy passing through a surface per second and is measured in $[W]$ (watts) as $\frac{J}{s}$.

1.2 Irradiance

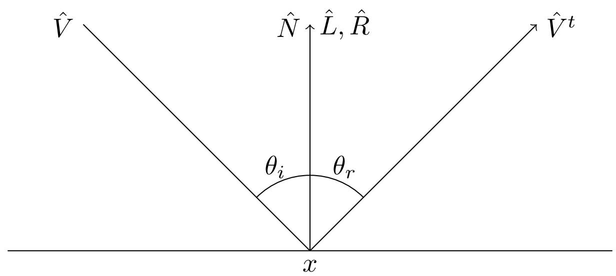
The irradiance E is the measurements of the radiant flux per *unit area* and is measured in $[W][M]^{-2}$ as $\frac{\Phi}{m^2}$.

1.3 Radiance

The radiance L is the irradiance per unit solid angle (steradian) and is measured in $[W][M]^{-2}[sr]^{-1}$ as $\frac{E}{sr}$.

2 Geometry

2.1 Fundamental vectors



- $\hat{V}$ direction towards the camera
- $\hat{N}$ surface normal
- $\hat{L}$ vector pointing torward the light source
- $\hat{R}$ reflected ray direction
- θ_i, θ_r incident and reflected angles

The reflected ray is given by $\hat{R} = \hat{L} - 2\hat{N}(\hat{L} \cdot \hat{N})$

2.2 Light attenuation

The amount of radiance on a point is given by

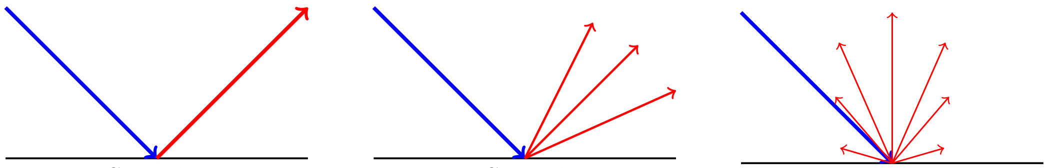
$$\Phi \left(\hat{L} \cdot \hat{N} \right) = \Phi \cdot \cos(\alpha)$$

where α is the angle between the two vectors.
This means that if the light source is perfectly above the point, then there is not light attenuation ($\alpha = 0$).

3 Materials

3.1 Types of materials

Different materials reflect incoming light to different directions and absord different amounts of it.



Specular Spread Diffuse

3.2 Bidirectional reflectance distribution function

The BRDF function (Bidirectional reflectance distribution function) is a probability distribution for the amount of light reflected in a certain direction.

$$f_r(\hat{w}, x, \hat{w}^t)$$

- $\hat{w}$ incoming ray direction
- x point of collision
- $\hat{w}^t$ outgoing ray direction

This function follows the Helmholtz-reciprocity

$$\forall \hat{w}, x, \hat{w}^t, \quad f_r(\hat{w}, x, \hat{w}^t) = f_r(\hat{w}^t, x, \hat{w})$$

positivity

$$\forall \hat{w}, x, \hat{w}^t, \quad f_r(\hat{w}, x, \hat{w}^t) \geq 0$$

and energy conservation

$$\int_\Omega f_r(\hat{w}, x, \hat{w}^t) \cos\theta \, d\hat{w}^t \leq 1$$

where Ω encloses the scene (usually a hemisphere).

3.3 Bidirectional transmittance distribution function

If the material can also transfer light through itself, we use the BTDF function (Bidirectional transmittance distribution function).

3.4 Bidirectional scattering distribution function

We use the BSDF (Bidirectional scattering distribution function) to generalize both the BTDF and BRDF.

4 Rendering equation

The rendering equation tells us how much radiance is exiting a *surface point* in a given direction

$$\textit{Light exiting point} = \textit{Material emitted light} + \textit{Reflected incoming light}$$

Formally,

$$L_o(x, \vec{\omega}) = L_e(x, \vec{\omega}) + \int_\Omega L_i(x, \vec{\omega}) f_r(\vec{\omega}, x, \vec{\omega}^t) \cos\theta d\vec{\omega}^t$$

- L_o exiting radiance
- L_o emitted radiance
- Ω scene
- L_i incoming radiance
- f_r BRDF
- $\cos\theta$ light attenuation

This value is difficult to compute. For each point, the light at that point depends on the incoming radiance of every other point, which also depends on the first point. This integral is infinite-dimensional because of the infinite bounces.

ASSETS TO USE

The Fresnel Equation

$$R_s(\theta) = \left| \frac{n_1 \cos\theta - n_2 \sqrt{1 - \left(\frac{n_1}{n_2}\sin\theta\right)^2}}{n_1 \cos\theta + n_2 \sqrt{1 - \left(\frac{n_1}{n_2}\sin\theta\right)^2}} \right|$$

Snell's law

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{V_1}{V_2} = \frac{n_2}{n_1}$$

References

[1] K'aroly Zsolnai-Feh'er. *TU Wien Rendering Course*. https://users.cg.tuwien.ac.at/zsolnai/gfx/rendering-course/. 2018.