Theory of Computation

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Contents

1 Fields of Study

1.1 Complexity Theory

Classify problems according to their degree of ”difficulty”.

1.2 Computability Theory

Classify problems as being solvable or unsolvable.

1.3 Automata Theory

Compare different computation models.

2 Alphabet

An *alphabet* is a finite set of *symbols*. For example: $\{a, b, c, \cdots, z\}$
The set $\{0, 1\}$ is the binary set. The set $\{0, 1\}^*$ is the set of all binary strings (union of all n-permutations of $\{0, 1\}$ and an empty string). In general, if Σ is an alphabet Σ^* is the set of all strings over Σ

$$\Sigma^* = \lambda \cup \bigcup_{n \in \mathbb{N}} \Sigma^n$$

where λ is the empty string. Note that $\lambda \neq \varnothing \neq \{\lambda\}$.
The length of a string w is denoted as $|w|$.

A set of strings is called a *language*.

3 Finite automaton

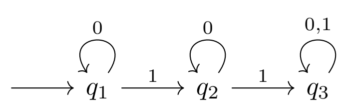
A finite automaton is a machine which process a string symbol by symbol from left to right. The automaton is in one of his *states* after processing a symbol. The machine might terminate in an *accept state* or not.

A finite automaton $M = (Q, \Sigma, \delta, q, F)$

- Q is a finite set of *states*
- Σ is an alphabet
- $\delta : Q \times \Sigma \to Q$ is the *transition function*
- q is an element of Q called the *start state*
- F is a subset of Q which contains the *accept states*

The transition function is the logical components, it determines in which state the machine will be after processing a symbol at any state.

The following automaton processes a binary string. The start state is q_1 and the only accept state is q_3. The program moves to the next state only if the symbol is 1, so it will reach q_3 only if the input strings contains at least two 1s.



The language of M, denoted $L(M)$ is the set of all accepted strings by M.