

Complex Numbers

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Contents

1	Imaginary unit	2
1.1	Definition	2
1.2	Properties	2
2	Complex Numbers	3
2.1	Definition	3
2.2	Complex plane	3
2.3	Operations	3
2.3.1	Real part	3
2.3.2	Imaginary part	3
2.3.3	Absolute value	4
2.3.4	Conjugate	4

1 Imaginary unit

1.1 Definition

The imaginary unit or imaginary number i is a solution to the quadratic equation $x^2 = -1$ and is defined as

$$i^2 = -1$$

The equation $x^2 = -1$ has two solutions: i and $-i$, however, there is not any algebraic difference between these two solutions.

1.2 Properties

The imaginary number i has some amazing properties when it comes to exponentiation.

$$\left\{ \begin{array}{l} i^0 = +1 \\ i^1 = +i \\ i^2 = -1 \\ i^3 = -i \end{array} \right. \quad \left\{ \begin{array}{l} i^4 = +1 \\ i^5 = +i \\ i^6 = -1 \\ i^7 = -i \end{array} \right. \quad \dots$$

The multiplicative inverse of i is $-i$.

$$\frac{1}{i} = \frac{1}{i} \cdot \frac{i}{i} = \frac{i}{i^2} = -i$$

2 Complex Numbers

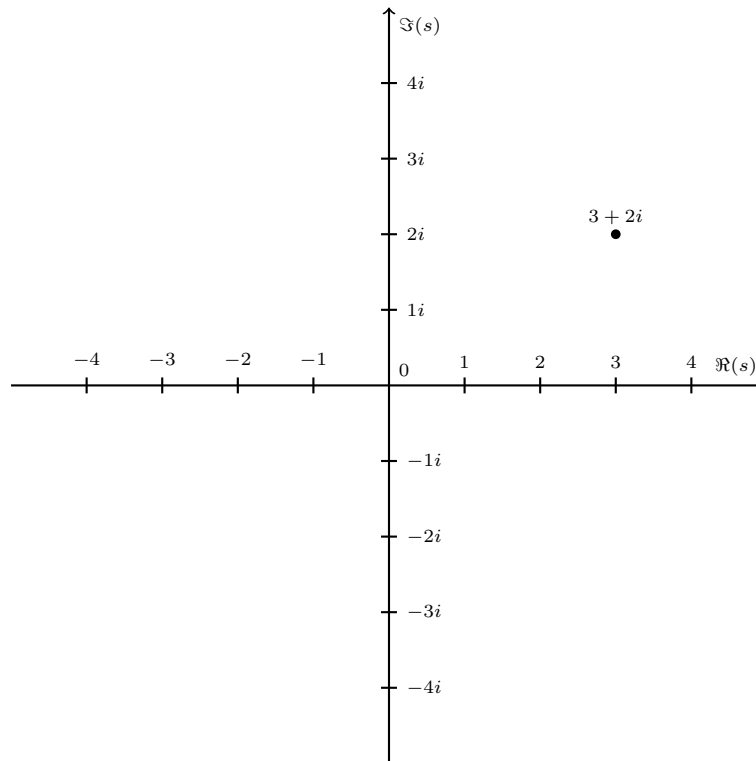
2.1 Definition

Complex numbers are numbers in the form $a + bi$, where $a, b \in \mathbb{R}$ and i is the imaginary unit. This set of numbers is called \mathbb{C} .

Since every number $n \in \mathbb{R}$ can be represented as a complex number in the form $n + 0i$, $\mathbb{R} \subset \mathbb{C}$.

2.2 Complex plane

We can represent each complex number on a plane, where the horizontal axis represent the real numbers \mathbb{R} and the vertical axis represents every scalar multiple of the imaginary unit i .



2.3 Operations

2.3.1 Real part

The real part of a complex number s is denoted by $\operatorname{Re}(s)$ or $\Re(s)$.

$$\operatorname{Re}(a + bi) = a$$

2.3.2 Imaginary part

The imaginary part of a complex number s is denoted by $\operatorname{Im}(s)$ or $\Im(s)$.

$$\operatorname{Im}(a + bi) = b$$

2.3.3 Absolute value

The absolute value of a complex number is its distance from the origin.

$$|a + bi| = \sqrt{a^2 + b^2}$$

2.3.4 Conjugate

The complex conjugate of a number $s = a + bi$ is denoted as s^* or \bar{s} . It is defined as

$$\overline{a + bi} = a - bi$$

Geometrically, s^* is the reflection about the real axis in the complex plane.

We also have the following trivial properties.

$$\begin{aligned}\overline{\bar{s}} &= s \\ \operatorname{Re}(\bar{s}) &= \operatorname{Re}(s) \\ \operatorname{Im}(\bar{s}) &= -\operatorname{Im}(s)\end{aligned}$$