



# Breaking the Mass Sheet Degeneracy with Gravitational Waves Interference in Lensing events

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# Mass Sheet Degeneracy

# Mass Sheet Degeneracy

E. E. Falco, M. V. Gorenstein, and I. I. Shapiro, ApJ 289, L1 (1985)

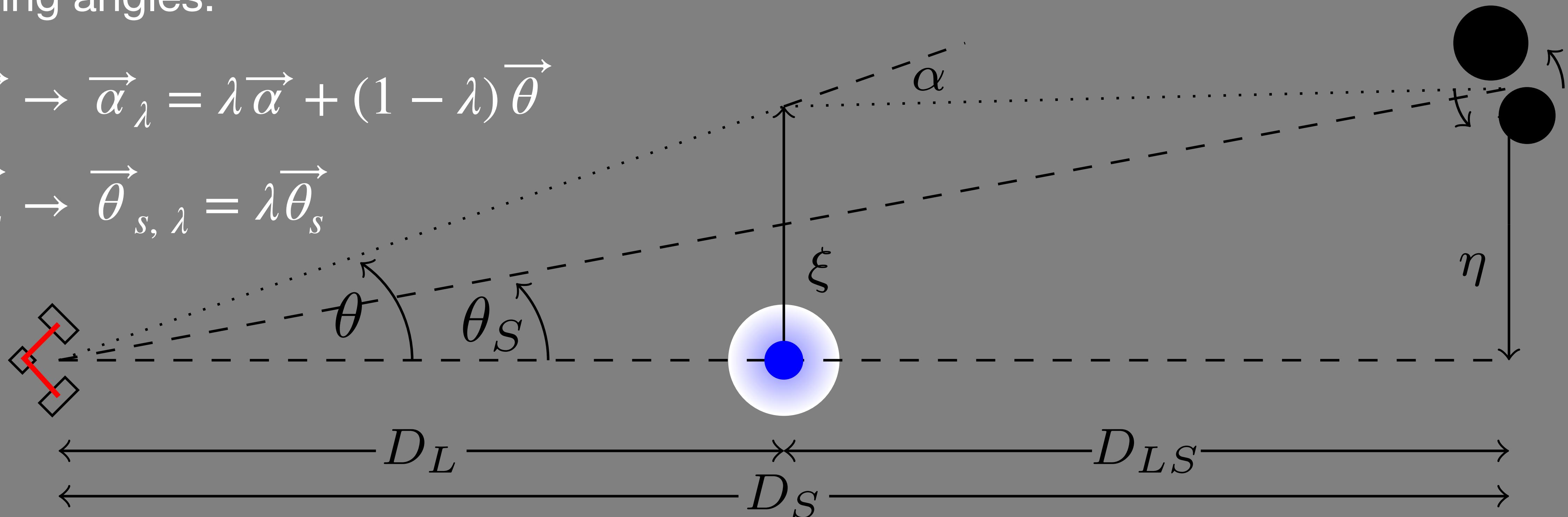
- Scalings of lens mass:

- $\kappa \rightarrow \kappa_\lambda = \lambda\kappa + (1 - \lambda)$

- Scaling angles:

- $\vec{\alpha} \rightarrow \vec{\alpha}_\lambda = \lambda\vec{\alpha} + (1 - \lambda)\vec{\theta}$

- $\vec{\theta}_s \rightarrow \vec{\theta}_{s,\lambda} = \lambda\vec{\theta}_s$



# MSD

## Why a problem?

- Observables are preserved!
- Problems: e.g. biased estimations of mass lens
- Biased estimation of cosmological parameter, e.g.  $H_0$

## Can we solve it?

- EM geometrical optics regime: multiple images; independent mass estimation of the lens (e.g. dynamics)
- EM wave optics regime: multiple lenses
- In **GW lensing**: 1 image and 1 lens can break MSD!

# Gravitational Waves Lensing

# Gravitational Lensing of Grav. Waves

- $\tilde{h}(f) \cdot F(f, \theta_s) = \tilde{h}_L(f)$

- $F(w, y) = -iwe^{iwy^2/2} \int_0^\infty dx x J_0(wx) \exp \left\{ iw \left[ \frac{1}{2}x^2 - \Psi(x) \right] \right\} \rightarrow F_\lambda$

- Where:

T. T. Nakamura and S. Deguchi, Progress of Theoretical Physics Supplement 133, 137 (1999).

- $w = \frac{1+z_L}{c} \frac{D_S D_L \theta_E^2}{D_{LS}} 2\pi f$

- $x = |\vec{x}| = |\vec{\theta}/\vec{\theta}_E|$

- $y = |\vec{y}| = |\vec{\theta}_s/\vec{\theta}_E|$

- $J_0$  - Bessel function of 0-th order

- $\Psi$  - dimensionless effective lensing potential

$$y_\lambda$$

$$\Psi_\lambda$$

NB: spherical symmetry!

# Lensed waveforms under mass-sheet transformation

Qualitative analysis

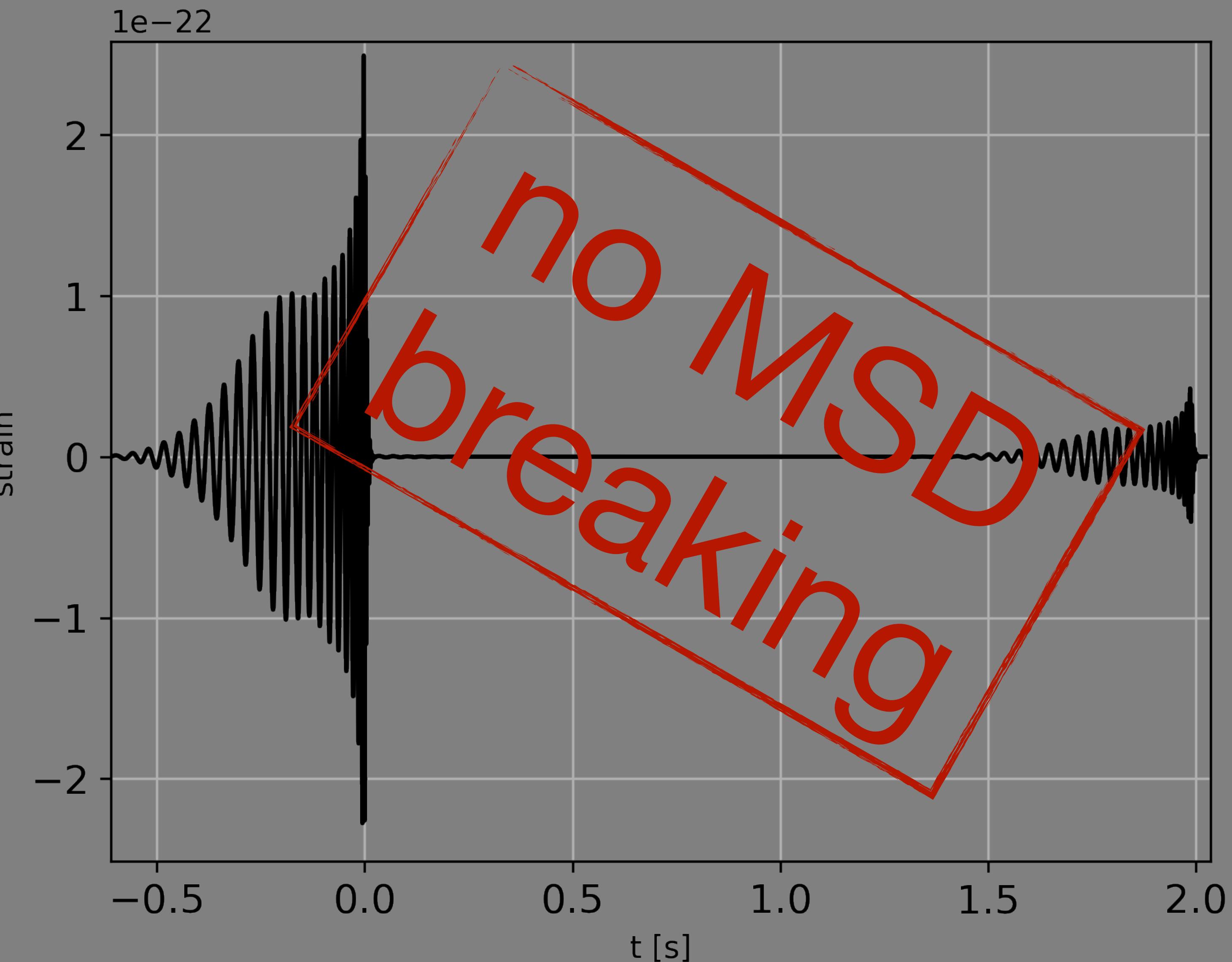
# Lensed GWs

3 regimes

- Geometrical Optics
  - $f \cdot \Delta t \gg 1$
  - $M_L > 10^5[(1 + z_L)f]^{-1}$

$$M_S = 60 M_\odot - z_S = 0.5$$

$$M_L = 10^4 M_\odot - z_L = 0.1 - y = 5$$



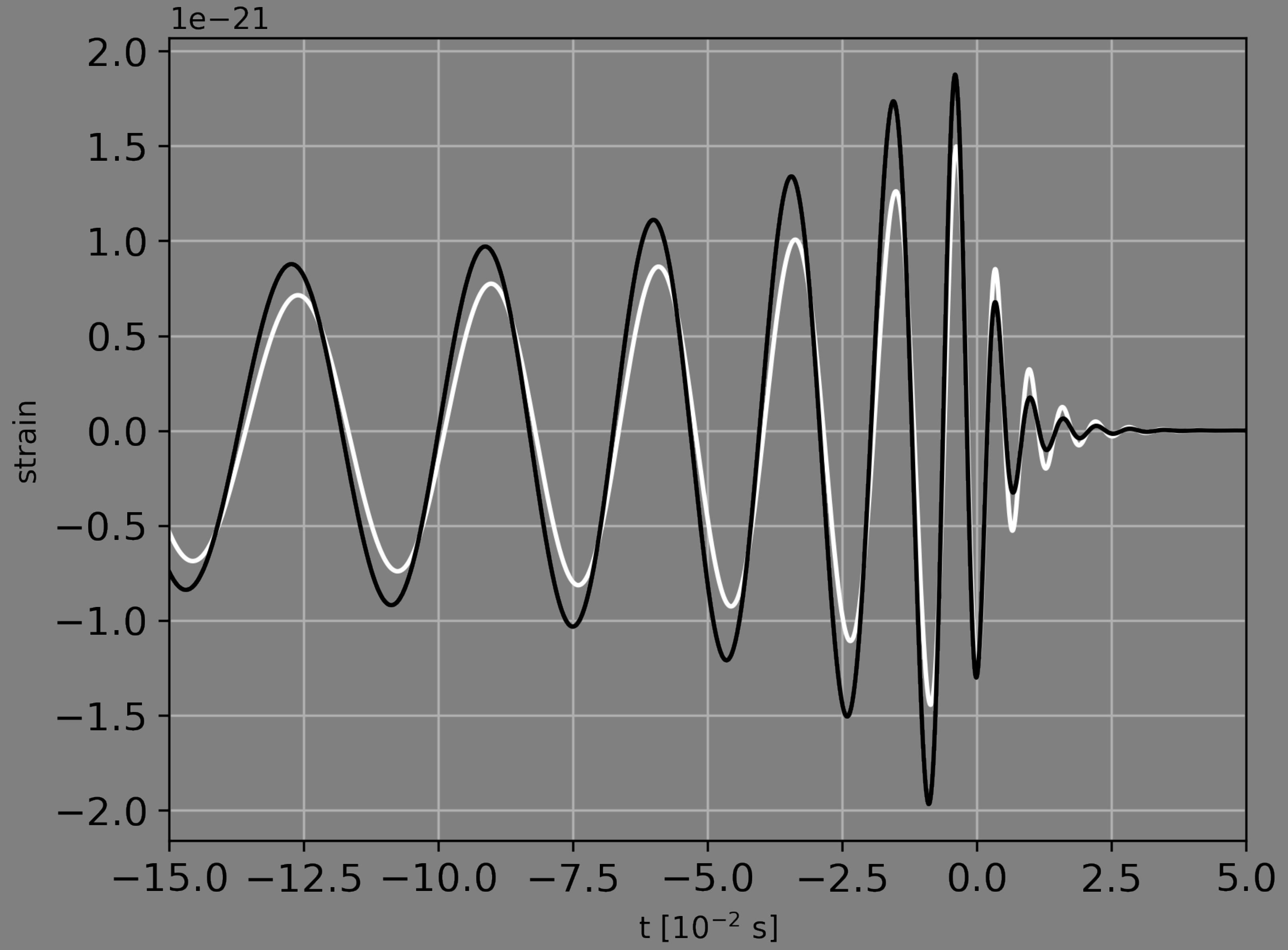
# Lensed GWs

3 regimes

- Wave Optics
  - $f \cdot \Delta t \lesssim 1$
  - $M_L \leq 10^5 [(1 + z_L)f]^{-1}$

$$M_S = 100 M_\odot - z_S = 0.1$$

$$M_L = 100 M_\odot - z_L = 0.01$$



Unlensed



Lensed

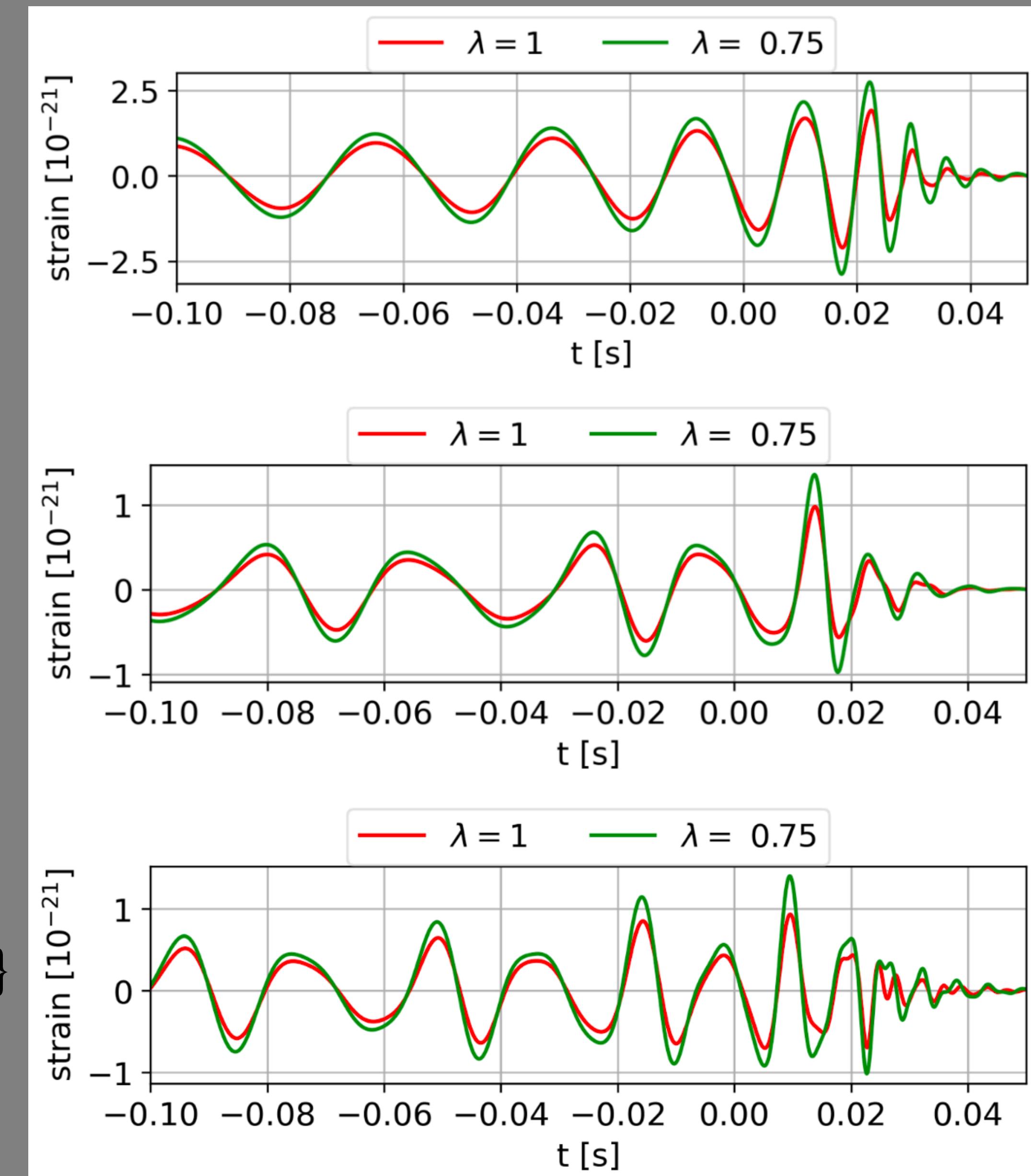
# Lensed GWs

$$q = \frac{m_2}{m_1} = 1$$

$$q = 0.1$$

$$q = 0.1 \text{ & } s_{1,2;z} = \{0.7, 0.2\}$$

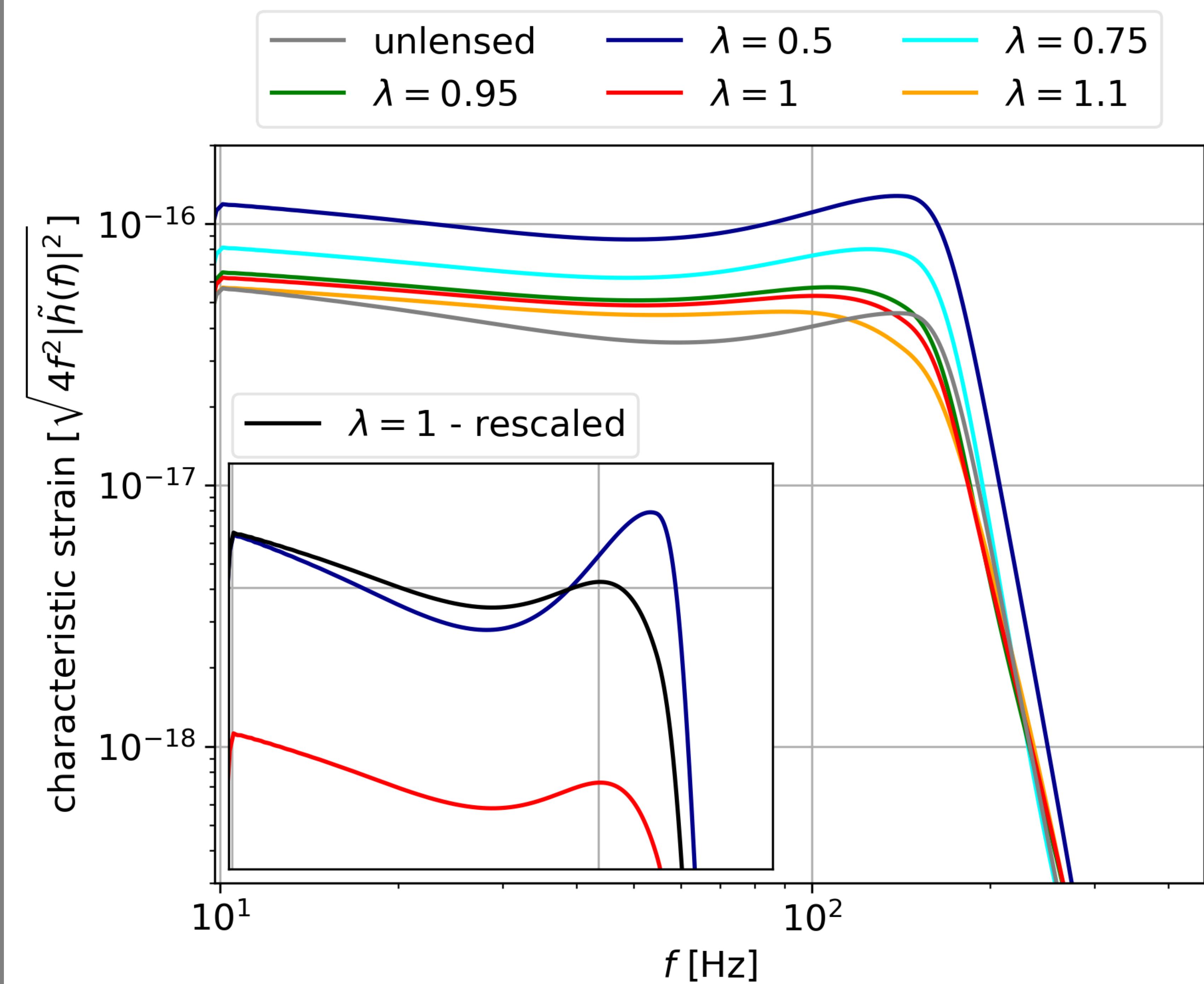
# Wave optics



# Lensed GWs

Wave optics

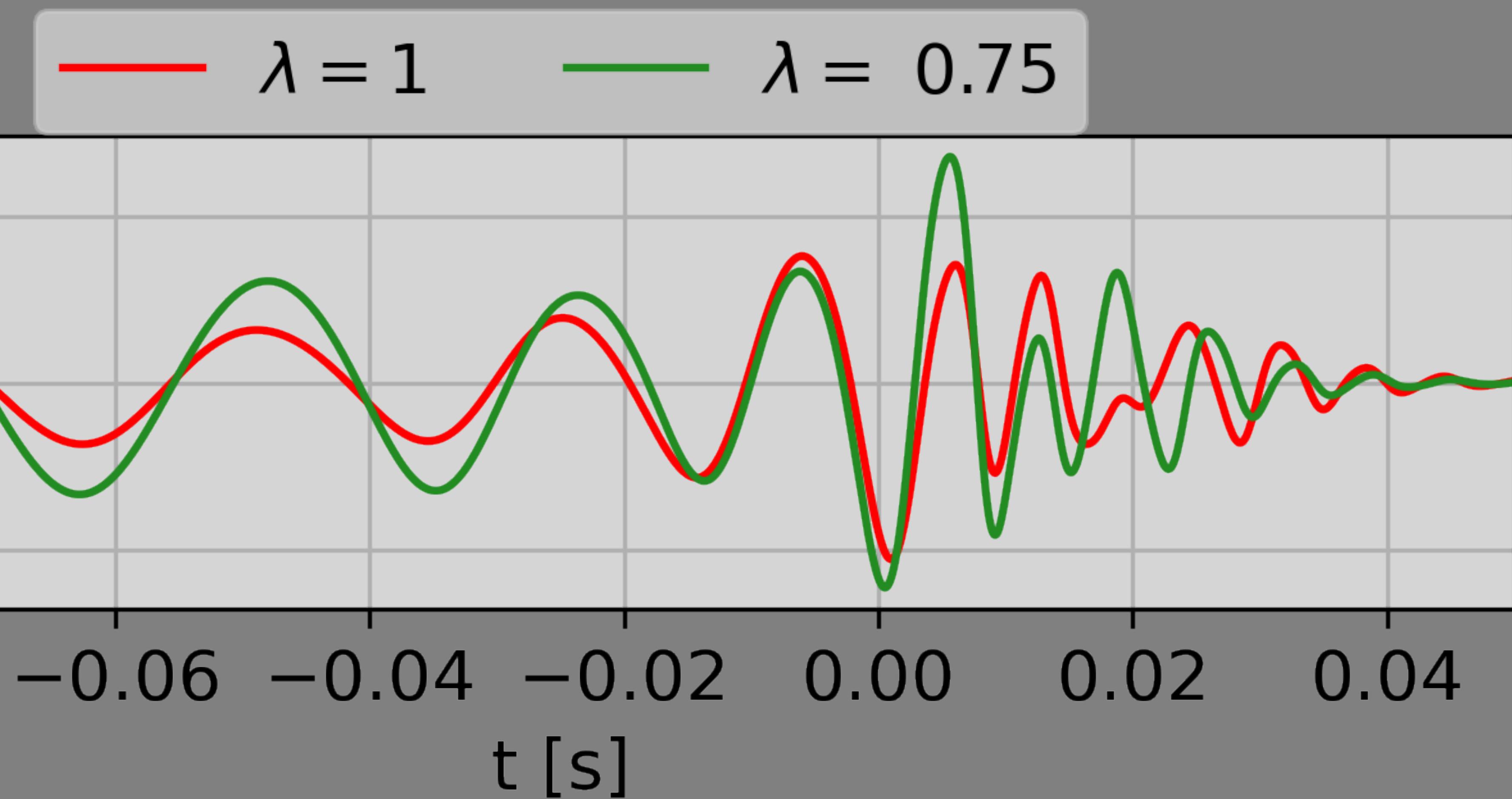
$$q = 1$$



# Lensed GWs

3 regimes

Interference regime:  $f \cdot \Delta t \approx 1$



$$M_L = 500 M_\odot \text{ & } y = 1 \text{ & } z_L = 0.01$$

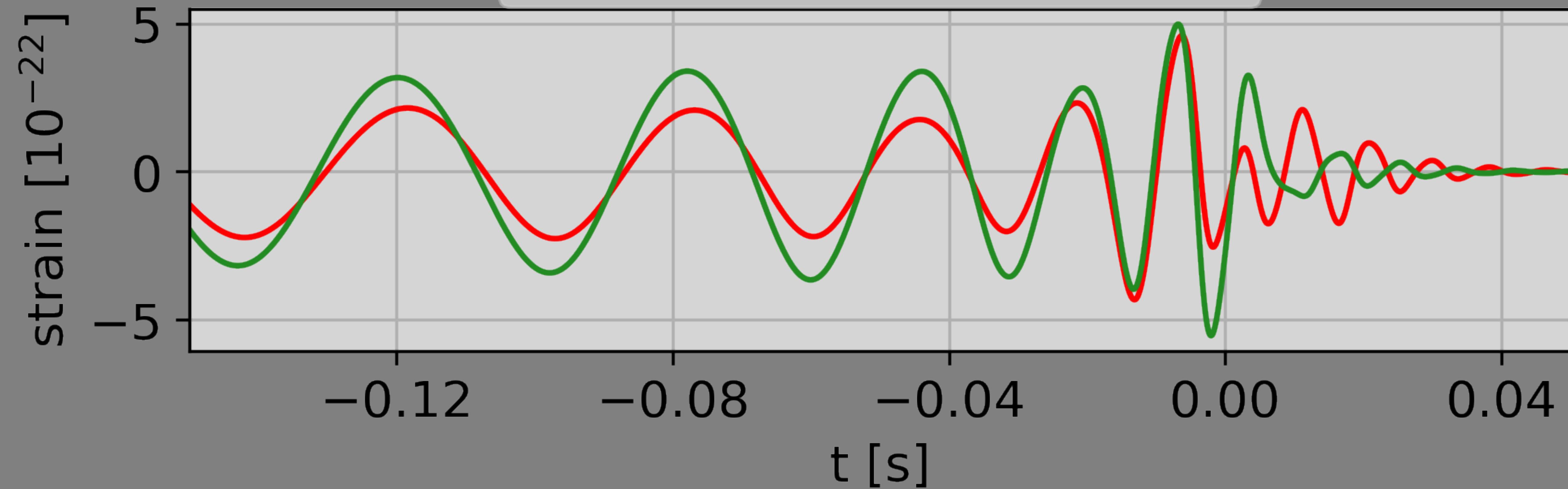
$$M_S = 100 M_\odot \text{ & } q = 1 \text{ & } z_S = 0.1$$

# Lensed GWs

3 regimes

Interference regime:  $f \cdot \Delta t \approx 1$

—  $\lambda = 1$  —  $\lambda = 0.75$



$$M_L = 500 M_\odot \text{ & } y = 1 \text{ & } z_L = 0.01$$

$$M_S = 100 M_\odot \text{ & } q = 1 \text{ & } z_S = 0.5$$

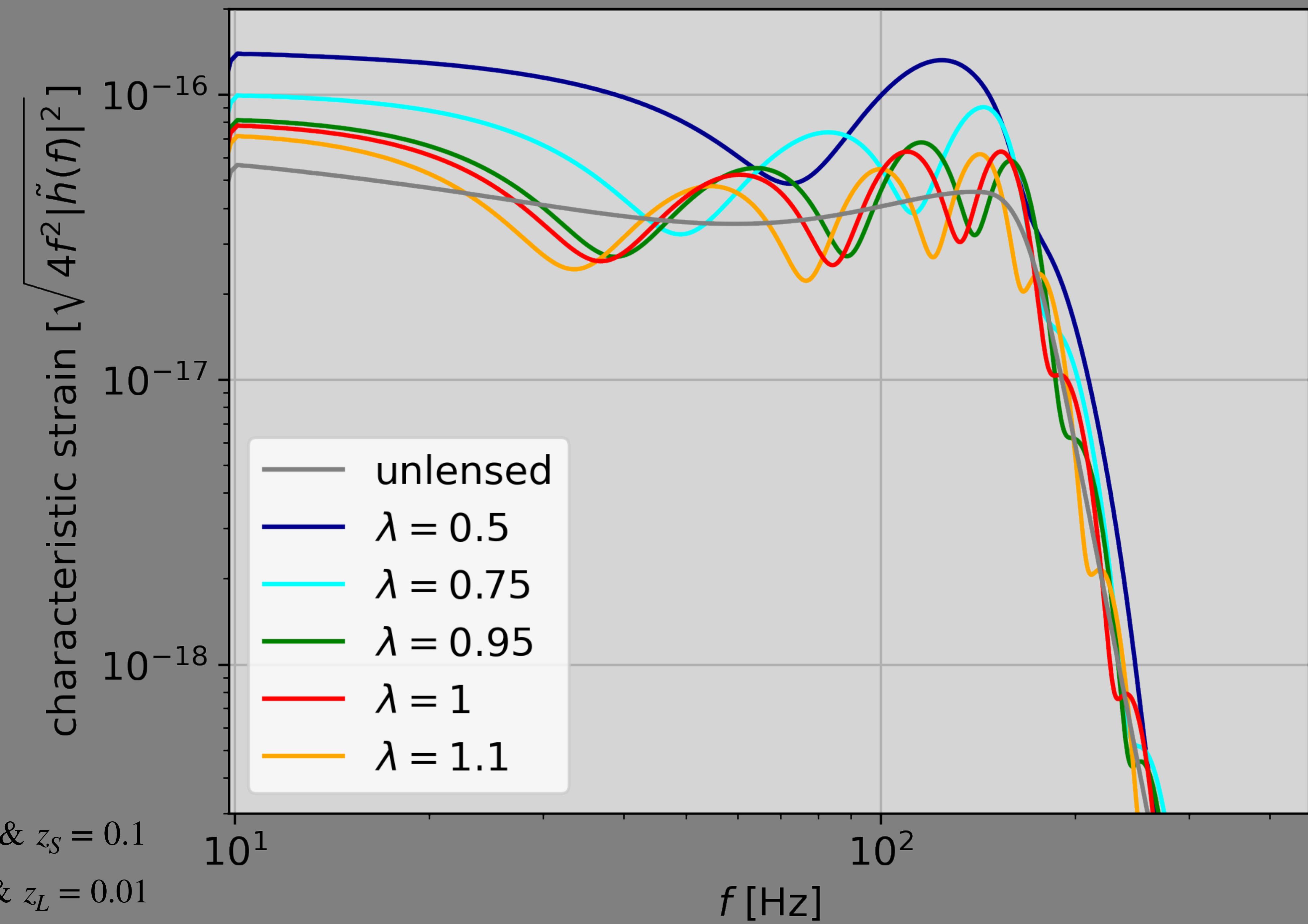
# Lensed GWs

Interference  
regime

$$\circ f \cdot \Delta t \approx 1$$

$$M_S = 100 M_\odot \text{ & } q = 1 \text{ & } z_S = 0.1$$

$$M_L = 500 M_\odot \text{ & } y = 1 \text{ & } z_L = 0.01$$



# **S/N - template matching**

Quantitative analysis

# Signal-to-Noise ratio

$$\rho = \frac{(s | h_T)}{\sqrt{(h_T | h_T)}} \approx \frac{(h | h_T)}{\sqrt{(h_T | h_T)}}$$

- $s(t) = h(t) + n(t)$

- Inner product:

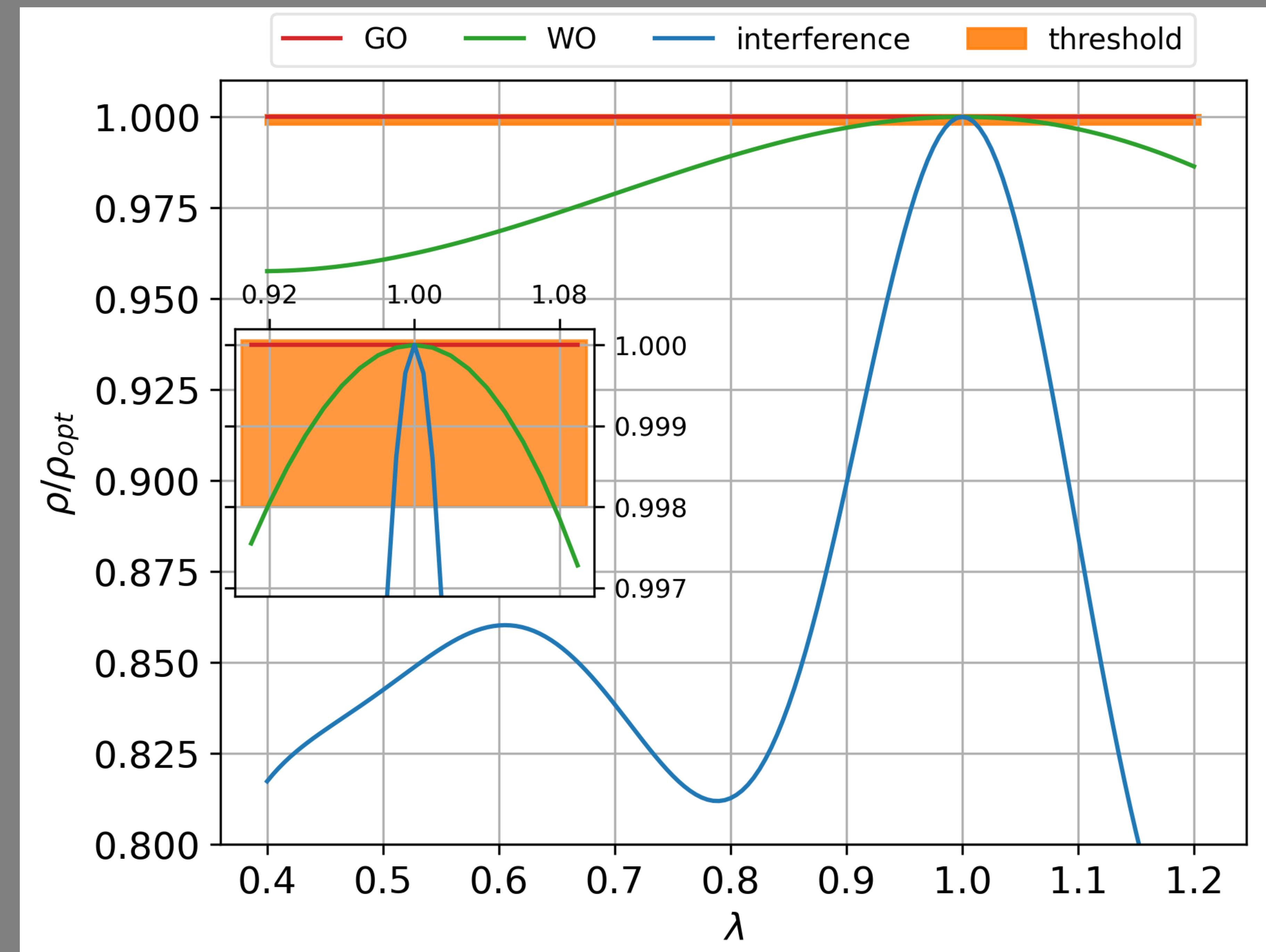
$$(a | b) = 4 \operatorname{Re} \left[ \int_0^\infty \frac{\tilde{a}(f) \cdot \tilde{b}^*(f)}{S_n(f)} df \right]$$

- $S_n(f)$  - (single-sided) power spectral density (L1-O3-LIGO)

Confidence region:  $\Delta\chi^2 \approx 2\rho_{opt}^2 \left[ 1 - \frac{\rho}{\rho_{opt}} \right]$        $3\sigma \rightarrow \Delta\chi^2 \approx 11.8$

# S/N

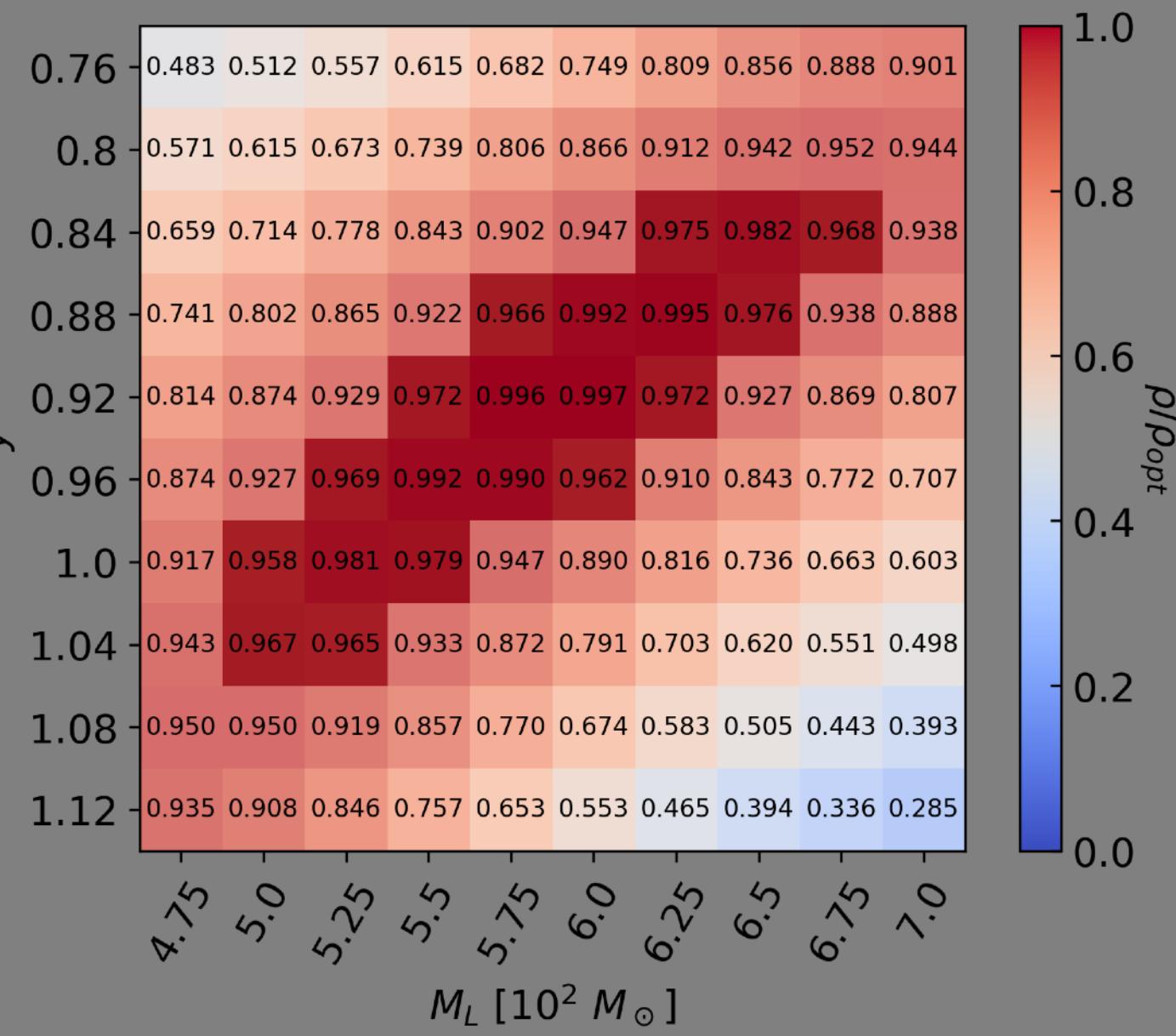
- $M_S = 100 M_\odot$
- $z_S = 0.1$
- $z_L = 0.01$
- $3\sigma \rightarrow \Delta\chi^2 \approx 0.998$
- GO  
 $M_L = 500 M_\odot$   
 $y = 10$
- Int.  
 $M_L = 500 M_\odot$   
 $y = 1$
- WO  
 $M_L = 100 M_\odot$   
 $y = 1$



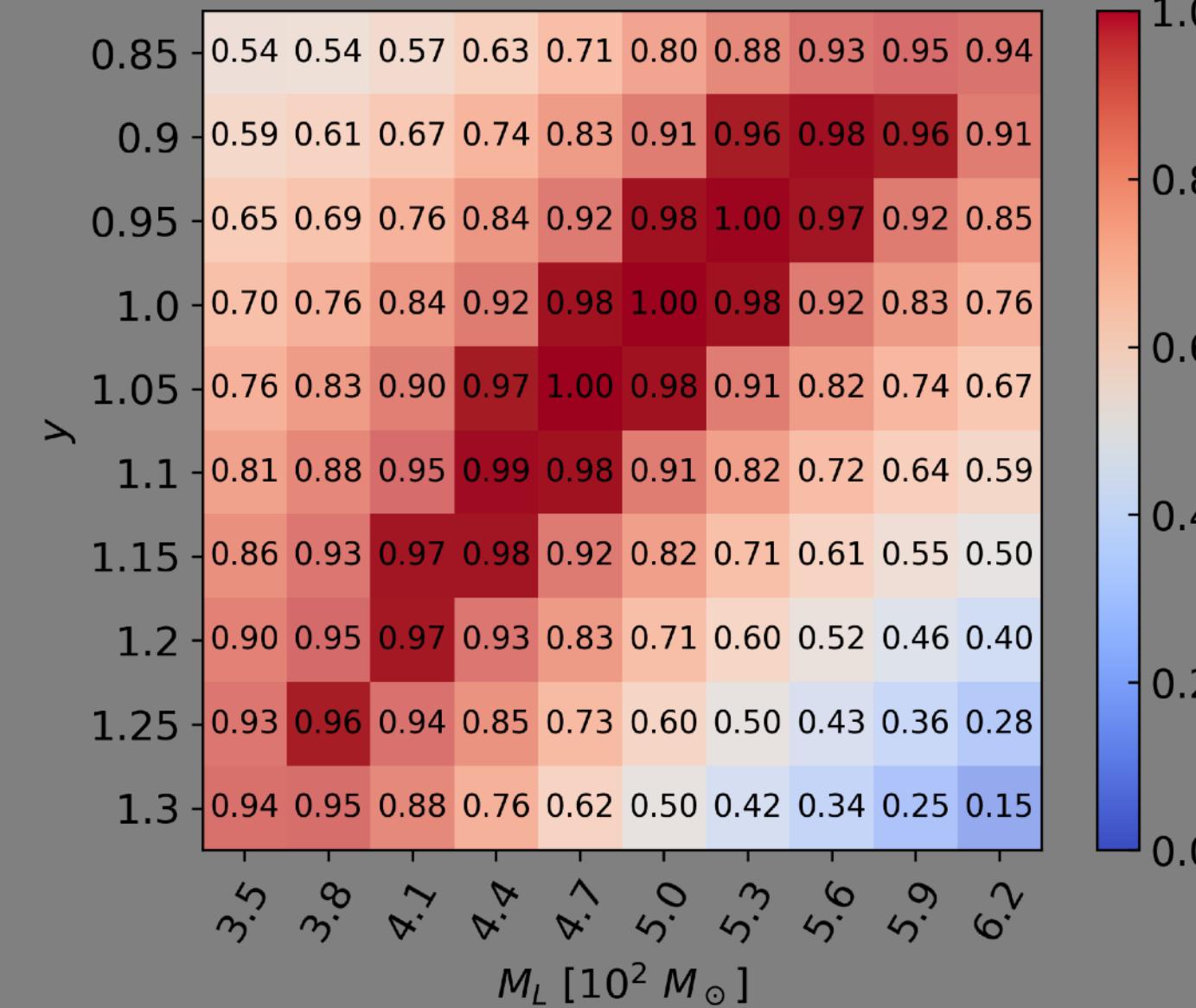
**S/N**

$$\lambda_{min} = 0.93$$

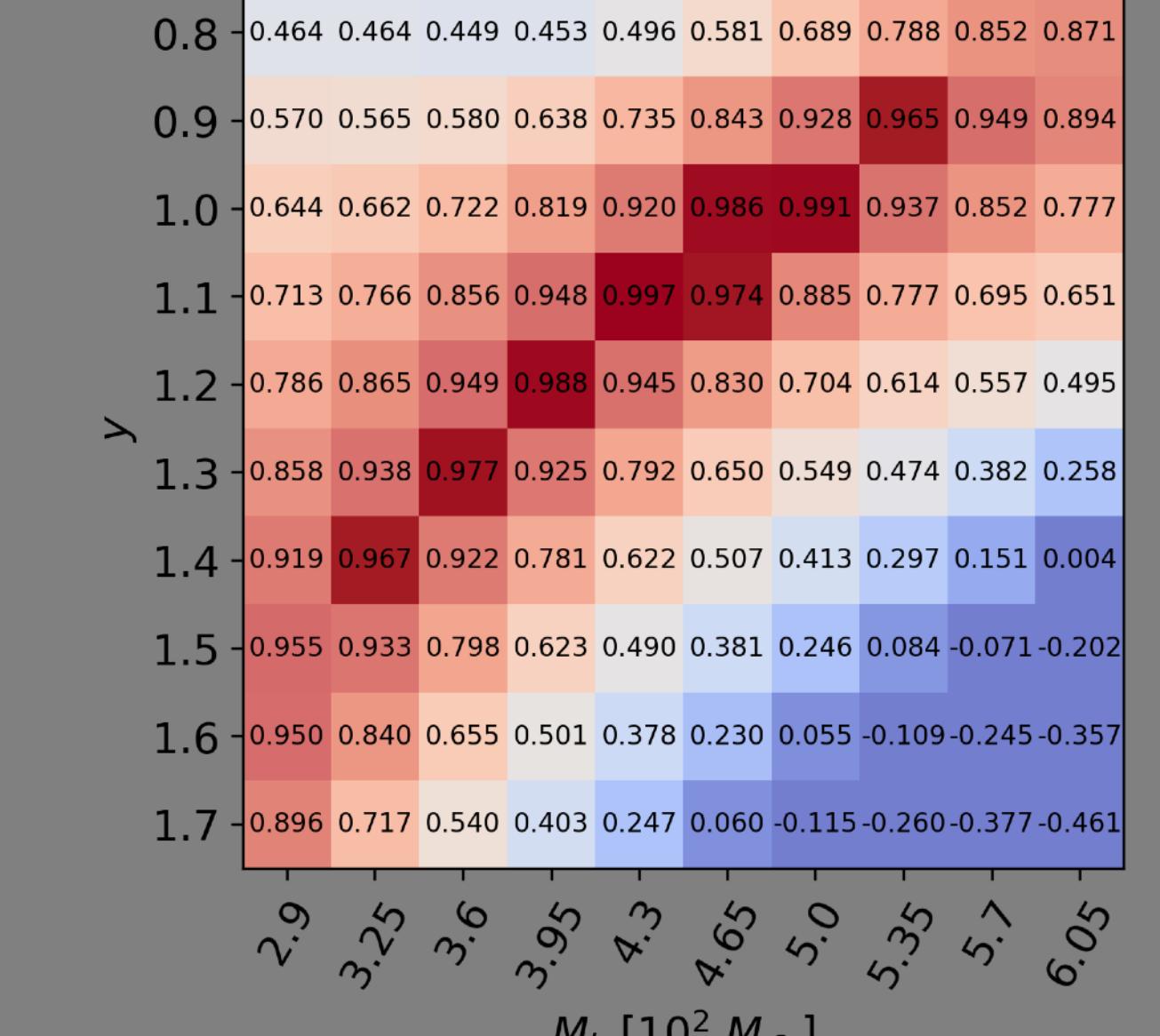
$\rho_{opt}$  11



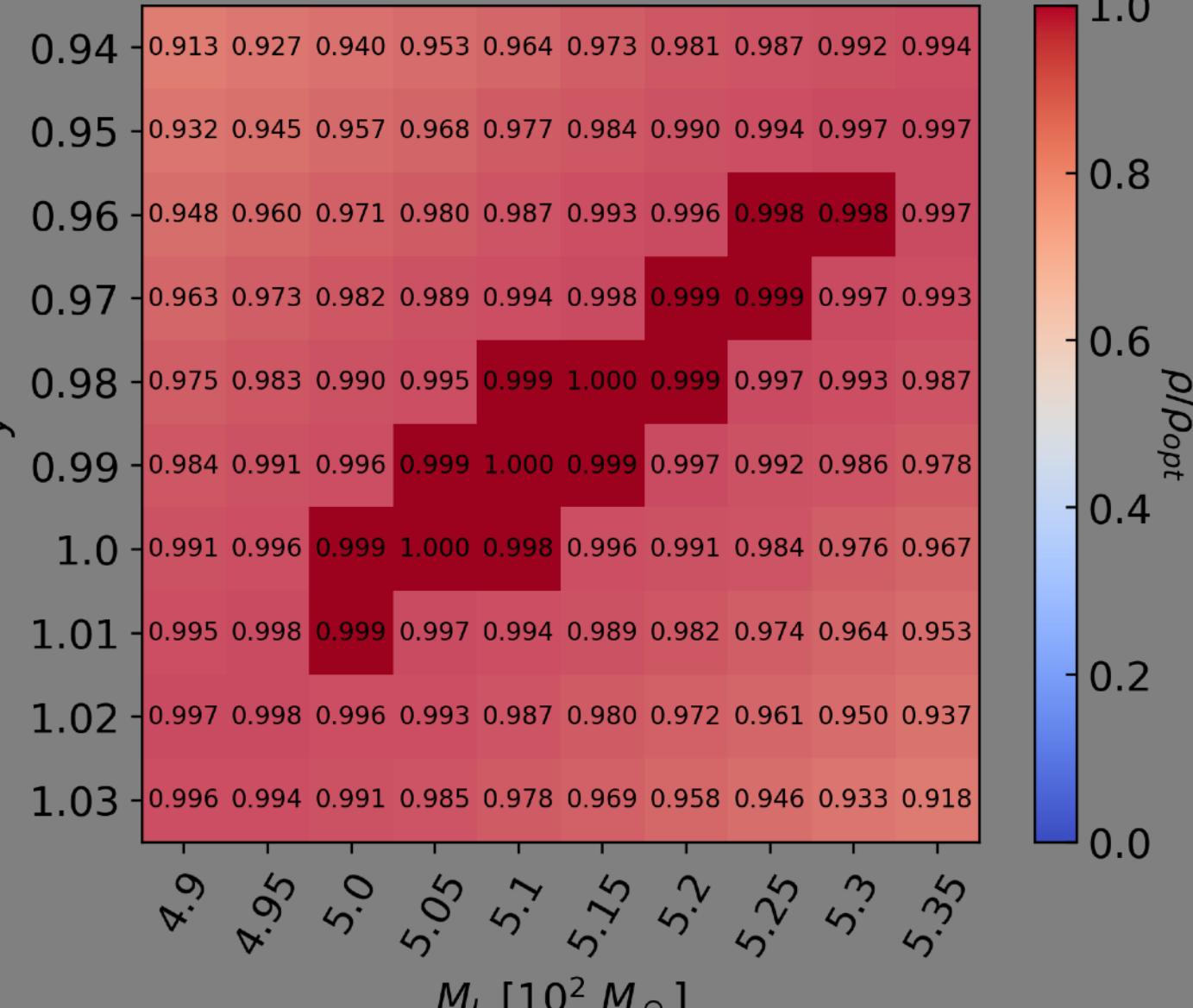
$$\lambda = 1$$



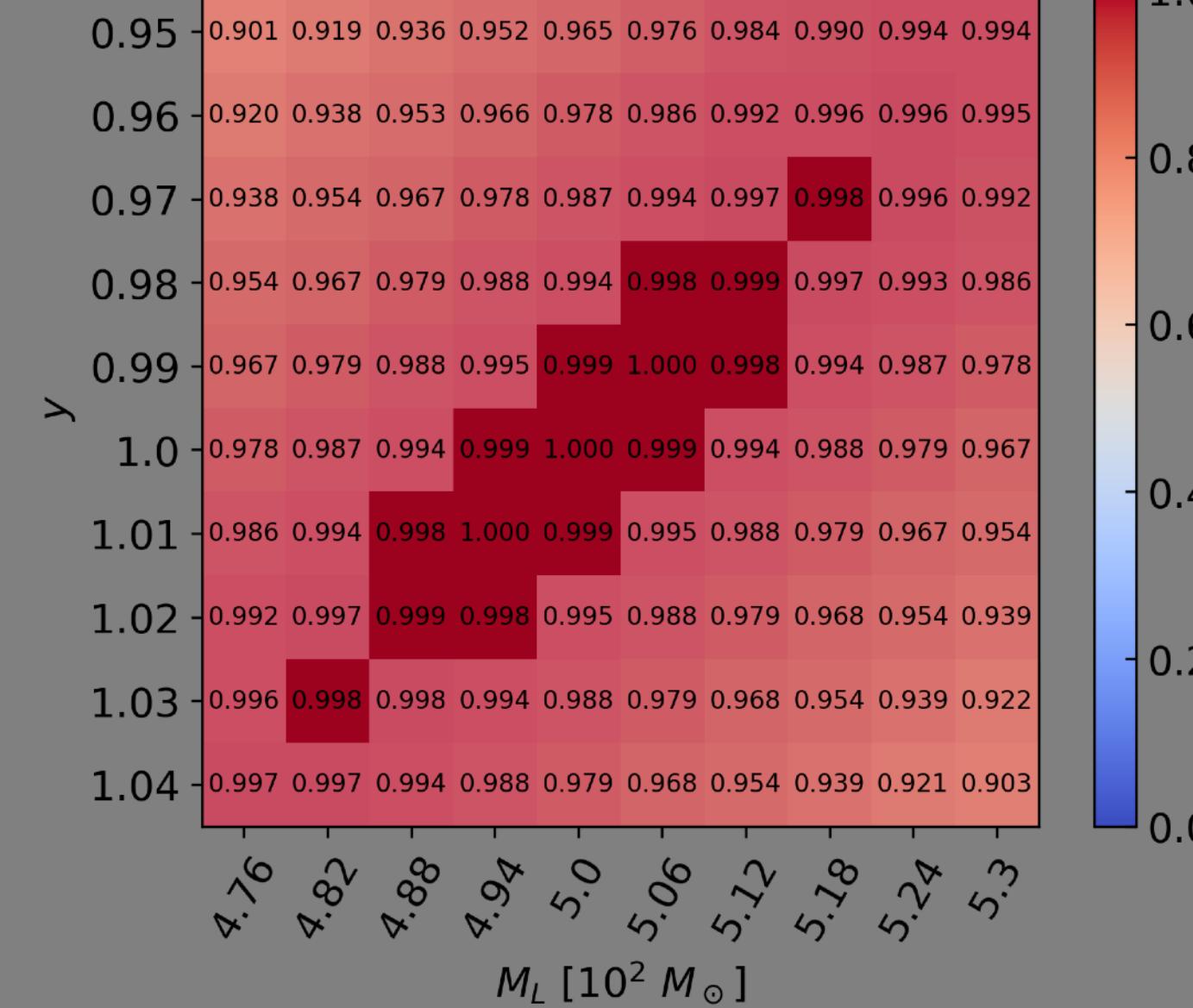
$$\lambda_{max} = 1.03$$



$\rho_{opt}$  55



$$\lambda = 1$$



$$\lambda_{min} = 0.99$$

$$\lambda_{max} = 1.01$$

Interference  
regime

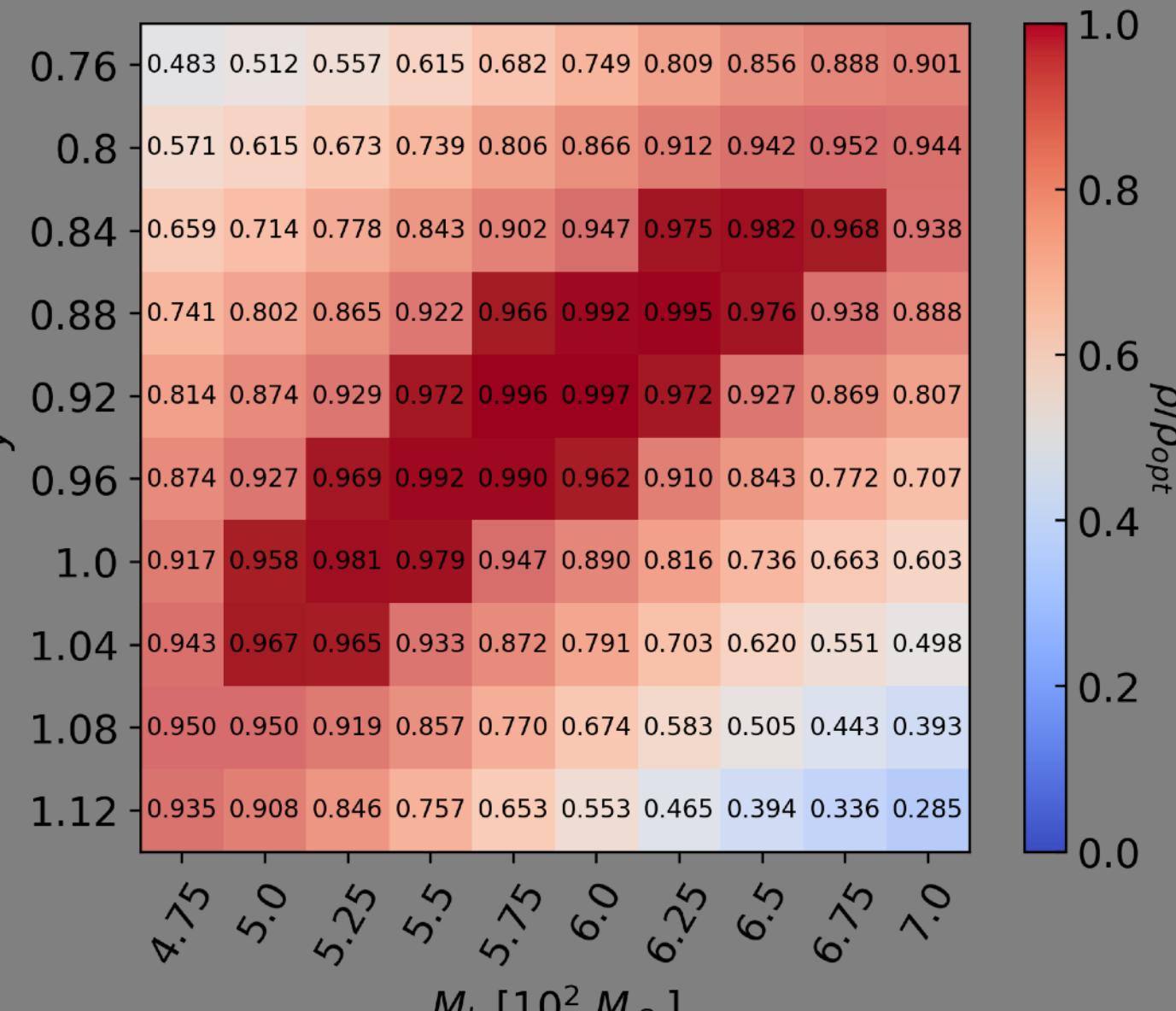
Signal

$M_L = 500$   
 $y = 1$

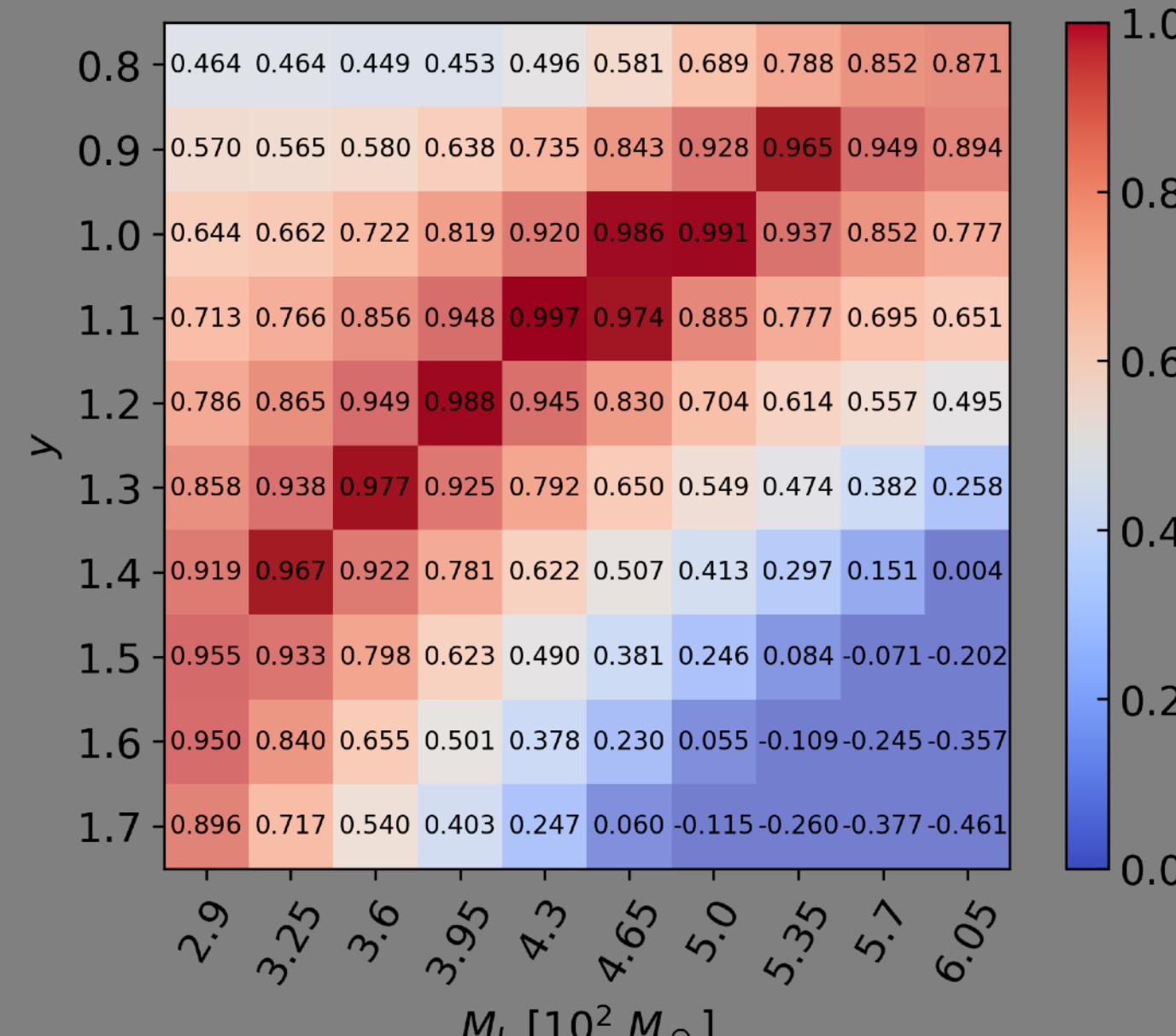
**S/N**

$$\lambda_{min} = 0.93$$

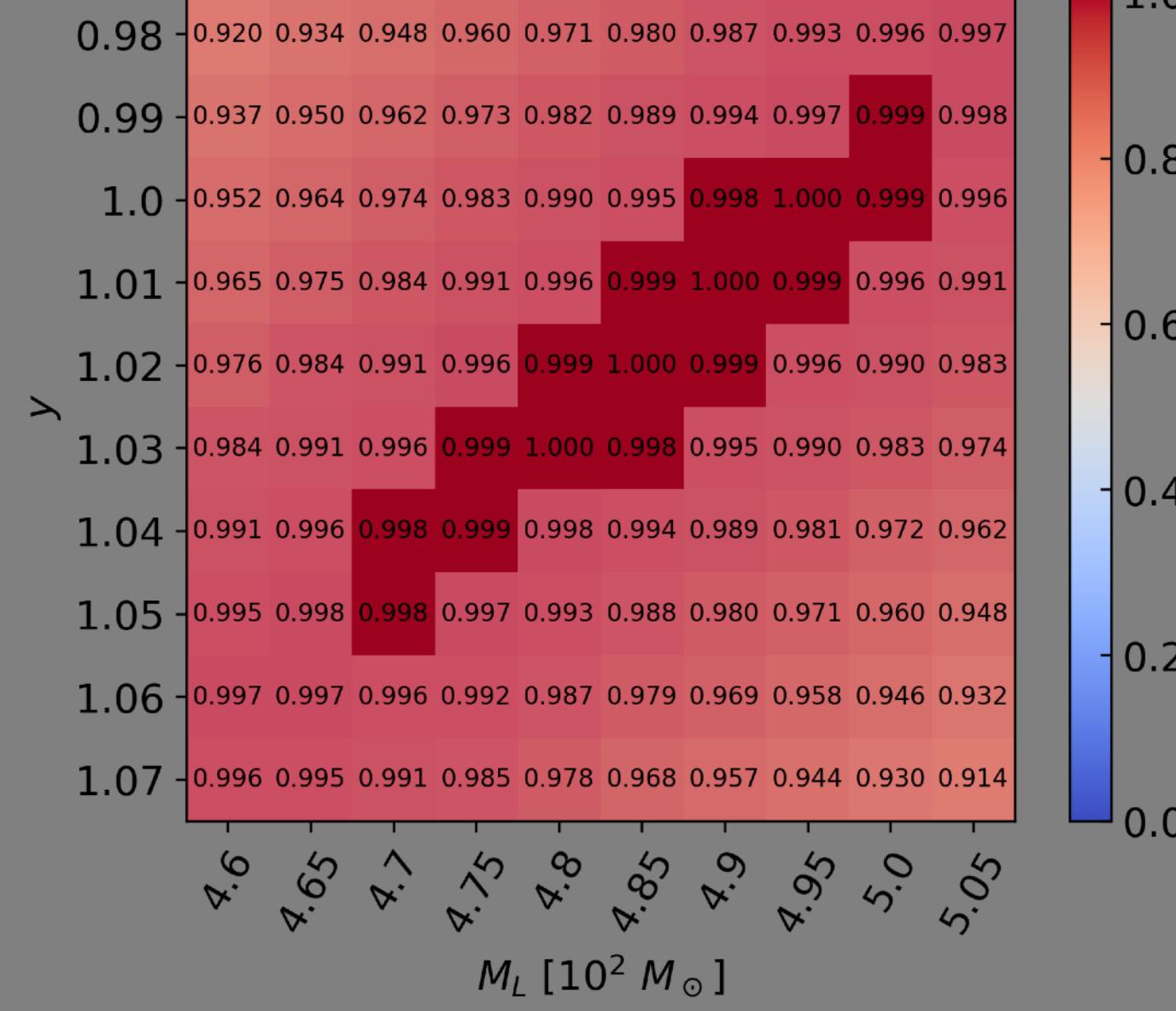
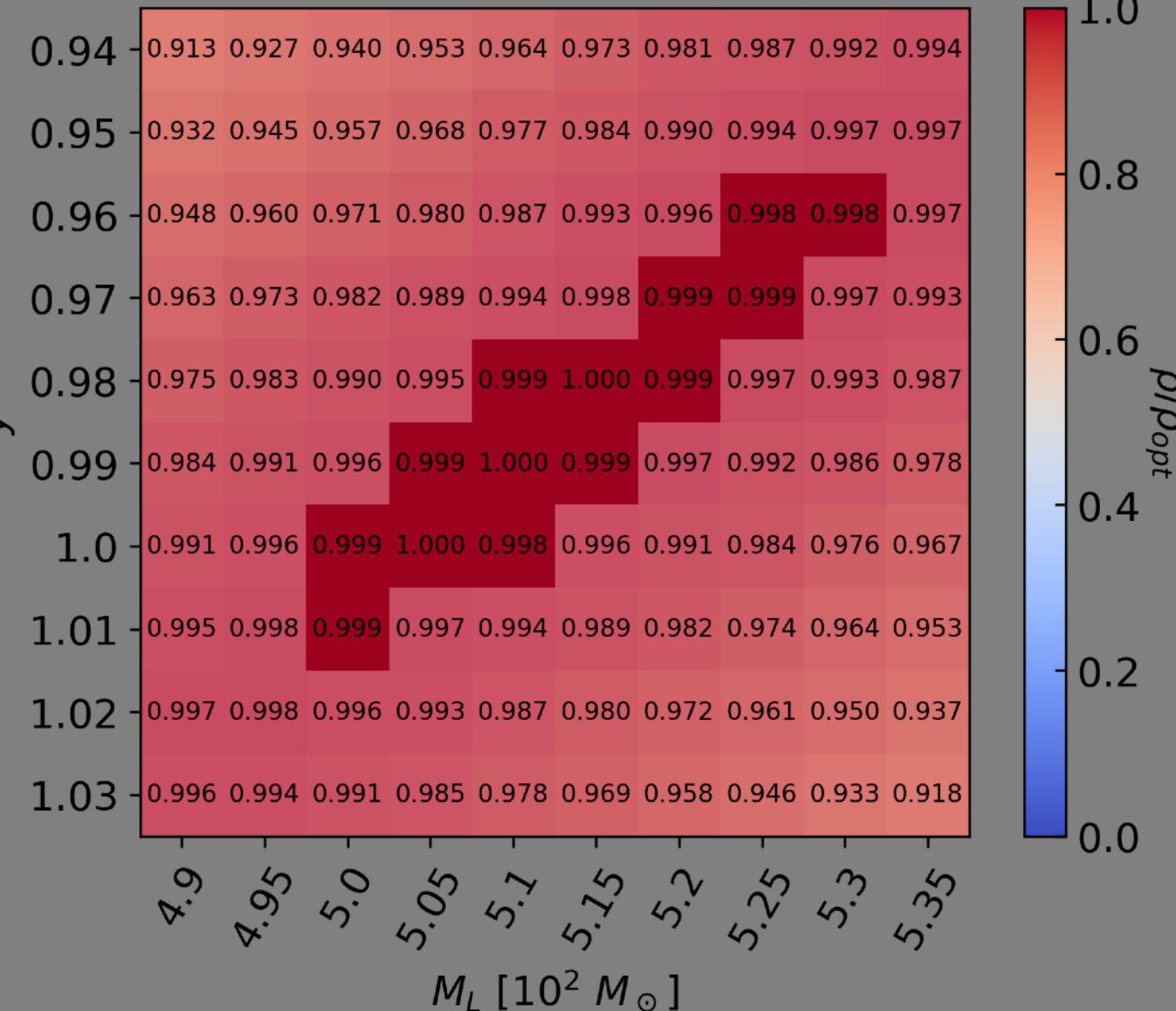
$\rho_{opt} \approx 1.1$



$$\lambda_{max} = 1.03$$



$\rho_{opt} \approx 5.5$



$$\lambda_{min} = 0.99$$

$$\lambda_{max} = 1.01$$

Interference regime

$\Delta y < 40\%$   
 $\Delta M_L \approx 35\%$

$\Delta M_L \approx 12 - 20\%$

$\Delta y \approx 5\%$   
 $\Delta M_L \approx 6\%$

# Conclusions

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# Conclusions

1. We analysed how MSD behave in GW lensing
2. In the GO regime it can not be broken
3. In WO can be broken in some cases
4. In interference regime is broken
5. How well it is broken depends on the strength of the signal and sensitivity of detectors. Nowadays we might have up to  $\Delta y \approx 5\%$  and  $\Delta M \approx 6\%$

*More info*