

# Mass-Sheet degeneracy in Gravitational Wave Lensing

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## 1. Mass-Sheet Degeneracy

The MSD is a well-known problem in lensing of light.  
It consists on [1]:

Scaling of lens mass

( $\kappa$  - surface mass density)

$$\kappa \rightarrow \kappa_s = \lambda \kappa + (1 - \lambda)$$

Scaling of angles

$$\vec{a} \rightarrow \vec{a}_s = \lambda \vec{a} + (1 - \lambda) \vec{\theta}$$

$$\vec{\theta}_S \rightarrow \vec{\theta}_{S,\lambda} = \lambda \vec{\theta}_S$$

## 2. Problem

Observables are preserved!

Biased estimations of lens parameters

Biased estimation of cosmological parameter

## 3. Method

Exploiting the work done in [3], we:

1. Rewrite the amplification factor [2] to include the MSD parameter:  $\tilde{h}_L(f) = \tilde{h}(f) \cdot F(f, y, \lambda)$
2. Simulate a BBH event over time
3. Study the event through:
  - 1. MSD parameter fixed
  - 2. MSD parameter free

Parameter Value

Parameter	Value
$M$	71.78
$q$	0.94
$d_L$ [Mpc]	1500
$\cos \theta_{IN}$	0.95
$M_*$ [ $M_\odot$ ]	700
$y$	1.8
$\lambda$	0.5
detectors	H1,L1,V1
optimal S/N	78
soft approx.	IMRPhenomXPN



acknowledgements



## 4. Analysis

- In the "standard" PE analysis, the lens mass and the source distance are biased, as we can see from the first plot (bottom left).
- When we add the MSD parameter  $\lambda$ , there  $M_{Lx} - y$  correlation is gone and we retrieve correctly the lens and source parameters.
  - Moreover, bla bla bla



## 5. Conclusions

Studying the MSD is important to:

1. set correct uncertainties of lens parameters
2. characterise lens model
3. astrophysical and cosmological studies

In this work we were able to properly recognise an external presence of mass and reconstruct the lens parameters correctly.

## Bibliography

- [1] E. E. Falco, M. V. Gorenstein, and L. I. Shapiro, *ApJ* 269, L1 (1982)
- [2] T. I. Nakamura and S. Deguchi, *Progress of Theoretical Physics Supplement* 123, 137 (1996)
- [3] P. Cremonese, J.M. Ezquiaga, M. Sotano, *Phys. Rev. D* 104 (2021) 2, 023503