BBH parameters

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1 l0 frame to j frame

That is, from cartesian coordinates to bilby ones. The parameters needed for this, in **L0 frame**, are:

- m_1 : mass of BH 1
- m_2 : mass of BH 2
- $s_{1,x}$: spin of BH 1 in x axis
- $s_{1,y}$: spin of BH 1 in y axis
- $s_{1,z}$: spin of BH 1 in z axis
- $s_{2,x}$: spin of BH 2 in x axis
- $s_{2,y}$: spin of BH 2 in y axis
- $s_{2,z}$: spin of BH 2 in z axis
- ι: inclination
- f_{ref} : reference frequency
- ϕ : phase

The parameters we want to get, in the **j frame**, are:

- $\theta_{j,n}$
- $\phi_{j,l}$
- tilt_1
- tilt_2
- $\phi_{1,2}$
- a_1
- a_2

See Table at pages 24-25 of [1] (doi) for the explanation of these parameters.

In the following, we write the definitions for these parameters from [2]. We use subscript m for BH number, $m \in [1, 2]$ and a for axis, $a \in [x, y, z]$, if not differently specified (e.g. $s_{m,a}$ is any of the spins of the 2 BHs in any of the 3 axes).

1.1 a_1 (a_2)

$$a_m = \sqrt{s_{m,x}^2 + s_{m,y}^2 + s_{m,z}^2} \tag{1}$$

1.2 $tilt_1$ ($tilt_2$)

$$tilt_m = \arccos\left(\frac{s_{m,z}}{a_m}\right) \tag{2}$$

1.3 $\theta_{j,n}$

$$\theta_{j,n} = \arccos\left(\hat{J}_x n_x + \hat{J}_y n_y + \hat{J}_z n_z\right) , \tag{3}$$

where

$$\hat{J}_a = \frac{J_a}{|J|} , \tag{4}$$

and

$$J_a = \tilde{s}_{1,a} + \tilde{s}_{1,a} \tag{5}$$

for $a \in [x, y]$ and

$$J_z \;=\; \tilde{s}_{1,z} + \tilde{s}_{1,z} + L_{mag} . \tag{6}$$

Also,

$$\tilde{s}_{m,a} = m_m^2 \cdot s_{m,a} \tag{7}$$

and

$$L_{mag} = (m_1 + m_2)^2 \cdot \frac{eta}{v_0}\left(1 + v_0^2\left(1.5\frac{eta}{6}\right)\right) , \tag{8}$$

with

$$eta = \frac{m_1 m_2}{(m_1 + m_2)^2} \tag{9}$$

and

$$v_0 = ((m_1 + m_2) M_{T,sun} \pi f_{ref})^{1/3} . \tag{10}$$

$M_{T,sun}$ is the geometrized nominal solar mass (see here).

1.4 $\phi_{1,2}$

This is different than the equation from lal. I don't get the right result with that, but with this, yes.

$$\phi_{1,2} = \phi_1 - \phi_2 \ , \tag{11}$$

where

$$\phi_m = \arctan 2\left(\frac{s_{m,y}}{a_m}, \frac{s_{m,x}}{a_m}\right) . \tag{12}$$

NB: this angle should be $\phi_{1,2} \in [0, 2\pi]$. Therefore, if $\phi_{1,2} < 0 \Rightarrow \phi_{1,2} = \phi_{1,2} + 2\pi$.

1.5 $\phi_{j,l}$

I still can't find the right equation for this.

1.6 other useful parameters

1.6.1 χ_{eff}

$$\chi_{eff} = \frac{a_1 \cos(tilt_1) + q \cdot a_2 \cos(tilt_2)}{1+q} \tag{13}$$

where $q = m_2/m_1$ is the mass ratio.

1.6.2 χ_p

$$\chi_p = \max(\chi_{1,p}, \frac{4q+3}{3q+4} \cdot q \cdot \chi_{2,p}) \ , \tag{14}$$

where $\chi_{n,p} = \sqrt{s_{n,x}^2 + s_{n,y}^2}$

References

[1] I M Romero-Shaw, C Talbot, S Biscoveanu, V D'Emilio, G Ashton, C P L Berry, S Coughlin, S Galaudage, C Hoy, M Hübner, K S Phukon, M Pitkin, M Rizzo, N Sarin, R Smith, S Stevenson, A Vajpeyi, M Arène, K Athar, S Banagiri, N Bose, M Carney, K Chatziioannou, J A Clark, M Colleoni, R Cotesta, B Edelman, H Estellés, C García-Quirós, Abhirup Ghosh, R Green, C-J Haster, S Husa, D Keitel, A X Kim, F Hernandez-Vivanco, I Magaña Hernandez, C Karathanasis, P D Lasky, N De Lillo, M E Lower, D Macleod, M Mateu-Lucena, A Miller, M Millhouse, S Morisaki, S H Oh, S Ossokine, E Payne, J Powell, G Pratten, M Pürrer, A Ramos-Buades, V Raymond, E Thrane, J Veitch, D Williams, M J Williams, and L Xiao. Bayesian inference for compact binary coalescences with bilby: validation and application to the first LIGO–Virgo gravitational-wave transient catalogue. *Monthly Notices of the Royal Astronomical Society*, 499(3):3295–3319, 09 2020.

[2] LAL team. SimInspiralTransformPrecessingWvf2PE. https://lscsoft.docs.ligo.org/lalsuite/lalsimulation/_l_a_l_sim_inspiral_8c_source.html#l03814, 20xx.