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Task1

A) Write a short answer

Give reasoning to the Eq. 1 in a general level: What are the phenomena contributing to the intensity? How do they affect the signal and why?

Response:

The intensity of a spin echo MRI image, as described in Equation 1, is determined by three primary physical phenomena: **proton density**, **T1 longitudinal relaxation**, and **T2 transverse relaxation**.

- **Proton Density (ρ):** This represents the concentration of hydrogen nuclei (protons) in a specific tissue slice. It acts as a baseline multiplier for the signal intensity; higher proton density generally leads to a higher potential signal because there are more spins available to contribute to the echo.
- **T1 Recovery (Longitudinal Relaxation):** This phenomenon is represented by the term ($1 - e - TR/T1$) and describes the process of the magnetization returning to its equilibrium state along the direction of the static magnetic field (the z-axis) after an excitation pulse. The Repetition Time (TR) is the interval between successive excitation pulses. If TR is short relative to the tissue's $T1$ value, the spins do not have enough time to fully recover their longitudinal magnetization before the next pulse, leading to a reduced signal.
- **T2 Decay (Transverse Relaxation):** Represented by the term ($e - TE/T2$), this describes the exponential decay of the signal in the transverse (xy) plane due to the loss of phase coherence among precessing spins. The longer the TE, the more the transverse magnetization decays before the signal is acquired.

B) Write a short answer

A hypothetical brain tumor has a lower concentration of water than the surrounding healthy tissue.
The $T1$ value of protons in the tumor is shorter than that of the protons in healthy tissue, but the $T2$ value of the tumor protons is longer. Which kind of weighting (values for TE and TR) should be introduced into the spin echo imaging sequence in order to ensure that there is contrast between the tumor and healthy tissue? Why?

Response:

A proton density–weighted spin echo sequence (long TR, short TE) would provide contrast because the tumor has a lower proton density than the surrounding tissue. By using long TR and short TE, T1 and T2 effects are minimized, so signal differences mainly reflect differences in proton density.

C) Write a short answer

A large concentration of superparamagnetic contrast agent is injected and accumulates in the tumor only. Which kind of weighting would now be optimal?

Response:

To detect a tumor with a high concentration of superparamagnetic contrast agent, the optimal setting is T2-weighting (Long TR and Long TE). According to the intensity equation, a shorter T2 value in the tumor causes the $e - TE/T2$ term to decrease rapidly. Consequently, the tumor will experience significant signal loss and appear much darker than the surrounding healthy tissue, which has a longer $T2$ and retains more signal at the same TE.

Task2

1)The strength of the frequency-encoding gradient in a 3-T MRI scanner is 40 mT/m. The sampling interval (dwell time) is 3 μs and the total readout time is 0.768 ms. Calculate the sampling step Δk in k-space (unit 1/m), the field of view (FOV; m), resolution (number of voxels) and the voxel size (m) in the frequency-encoding direction.

K-space formula with constant gradient:

$$k(t) = \gamma G t$$

$$\Delta k = (42.58 \times 106) \cdot 0.04 \cdot (3 \times 10 - 6) \Delta k = 5.11 \, m - 1$$

Field of View (FOV):

$$FOV = 1/\Delta k$$

$$FOV = 1/5.11 = 0.196 \, m \approx 19.6 \, cm$$

Resolution (number of voxels):

$$N = Tro/\Delta t$$

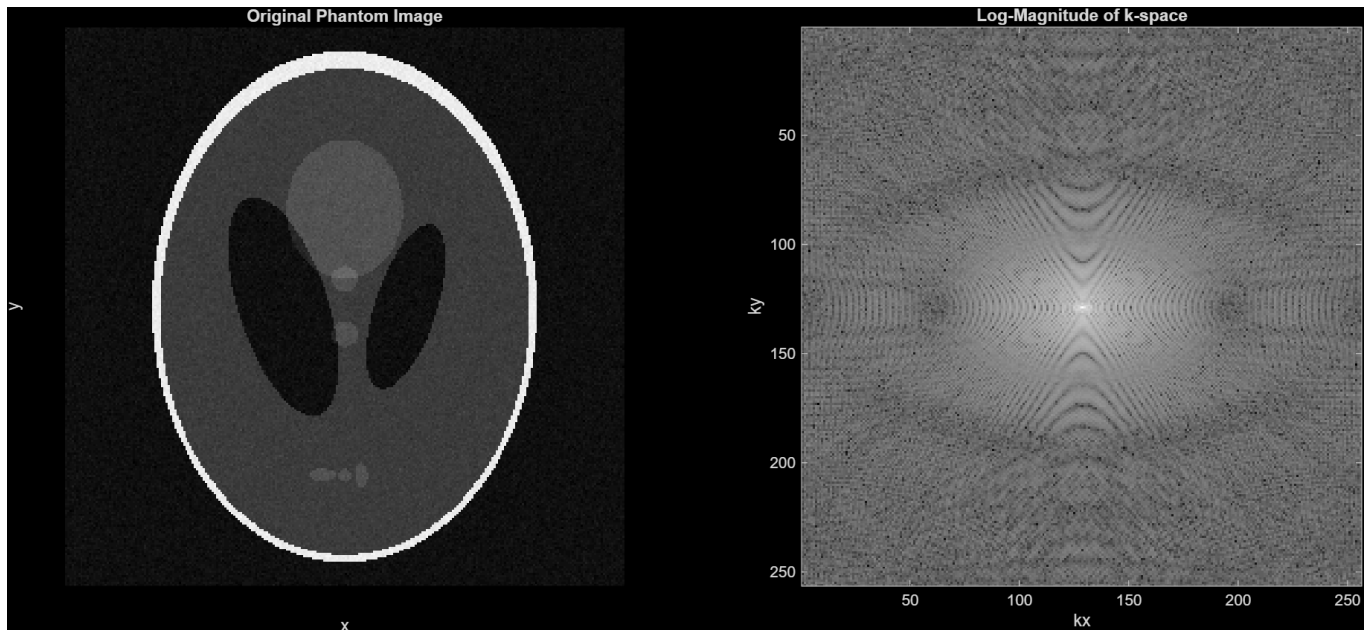$$N = 0.768 \times 10 - 3/3 \times 10 - 6 = 256$$

Voxel size (m):

$$\Delta x = FOV/N$$

$$\Delta x = 0.196/256 = 7.66 \times 10 - 4\,m \approx 0.77\,mm$$

2)Open the given MATLAB file E2_kspace.m and run the cell "2. Generate phantom". How would you describe the k-space image? How is the magnitude distributed (visually)?

The k-space image shows a strong concentration of signal magnitude at the center, with intensity gradually decreasing toward the periphery. The central region contains low spatial frequencies responsible for image contrast and coarse structures, while the outer regions contain high spatial frequencies related to edges and fine details. The distribution is symmetric, and noise is mainly visible in the outer regions.

3)Run the cell "3. Plot a couple of k-space points". What do the individual points in k-space represent? Are point 1 and point 2 the same?
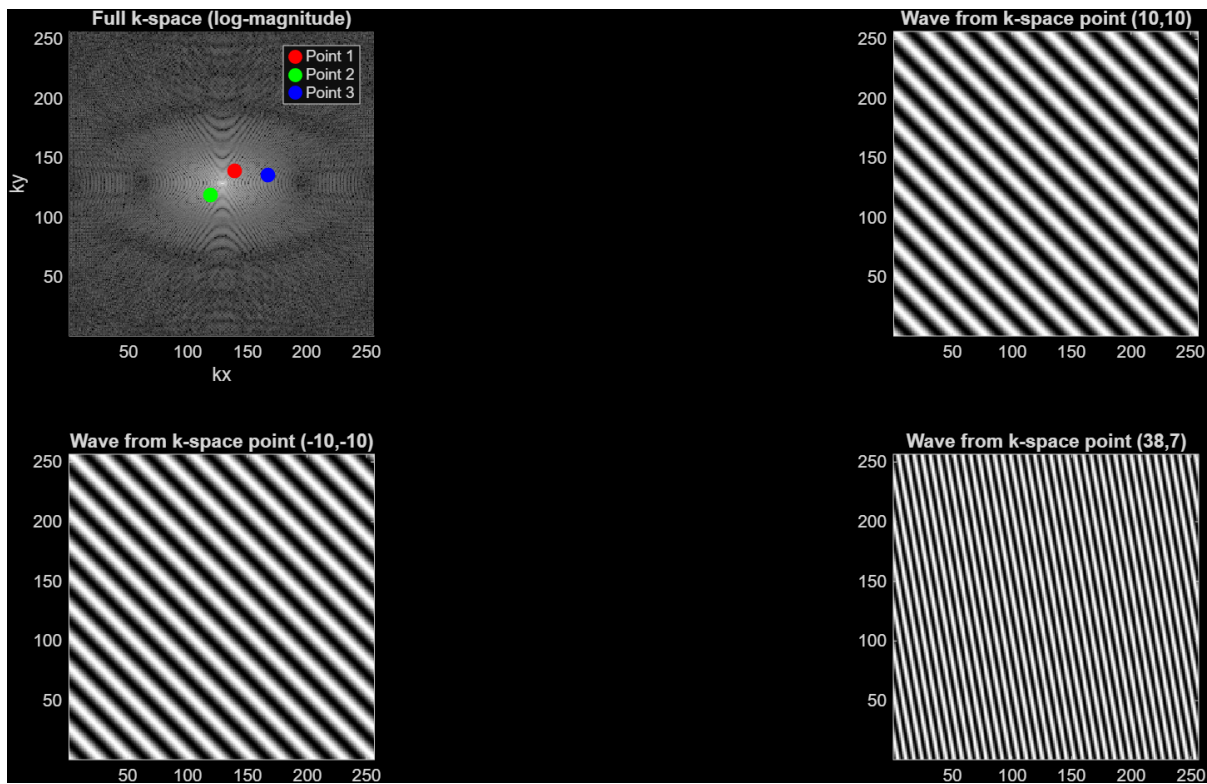
Each point in k-space represents a single spatial frequency component, corresponding to a sinusoidal wave with a specific orientation and spatial frequency.

Point 1 and point 2 are not the same point in k-space, but they are complex conjugates and produce spatial waves with identical magnitude and orientation.

4)Run the cell "4. Remove the center of k-space". Play around with the value of radius_center. What happens to the image and why?

radius_center = 30
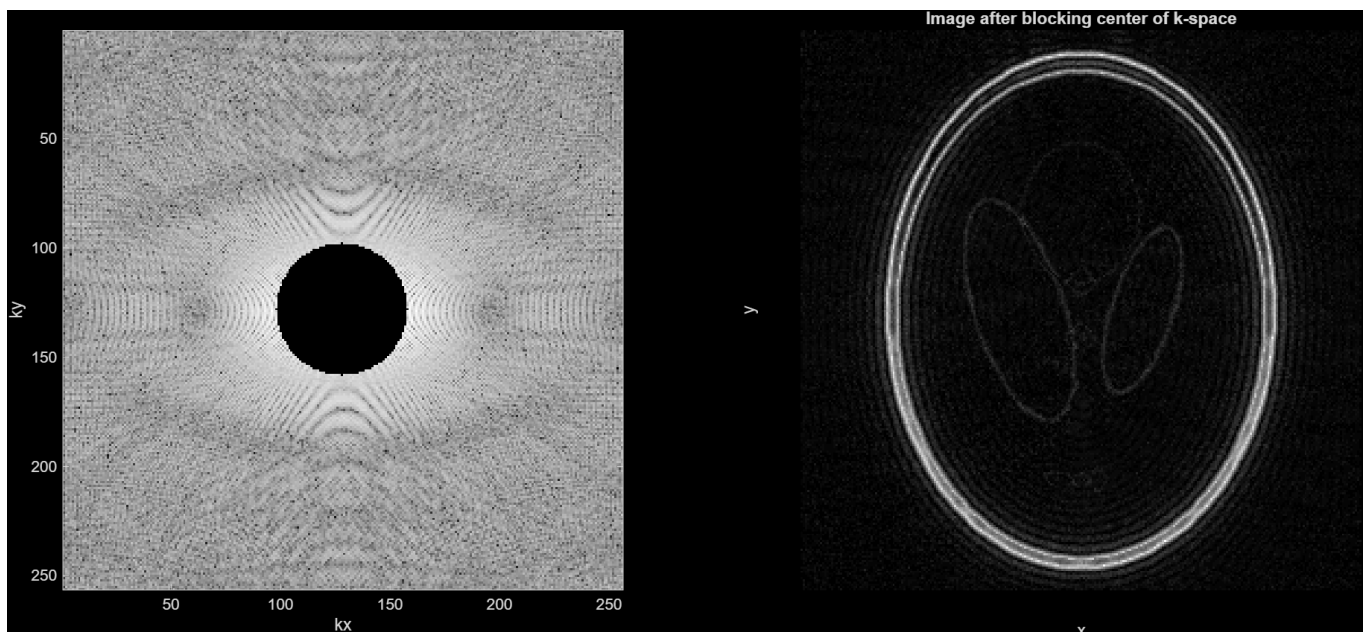
As the radius of the removed central region increases, more low-frequency information is lost, leading to a strong reduction of image contrast. The reconstructed image mainly contains edge information and appears unnatural, with only outlines of structures visible.

5)Run the cell "5. Remove the edges of k-space". Play around with the value of radius_edges. What happens to the image and why? What is the artefact seen near the edges called?

radius_edges = 50
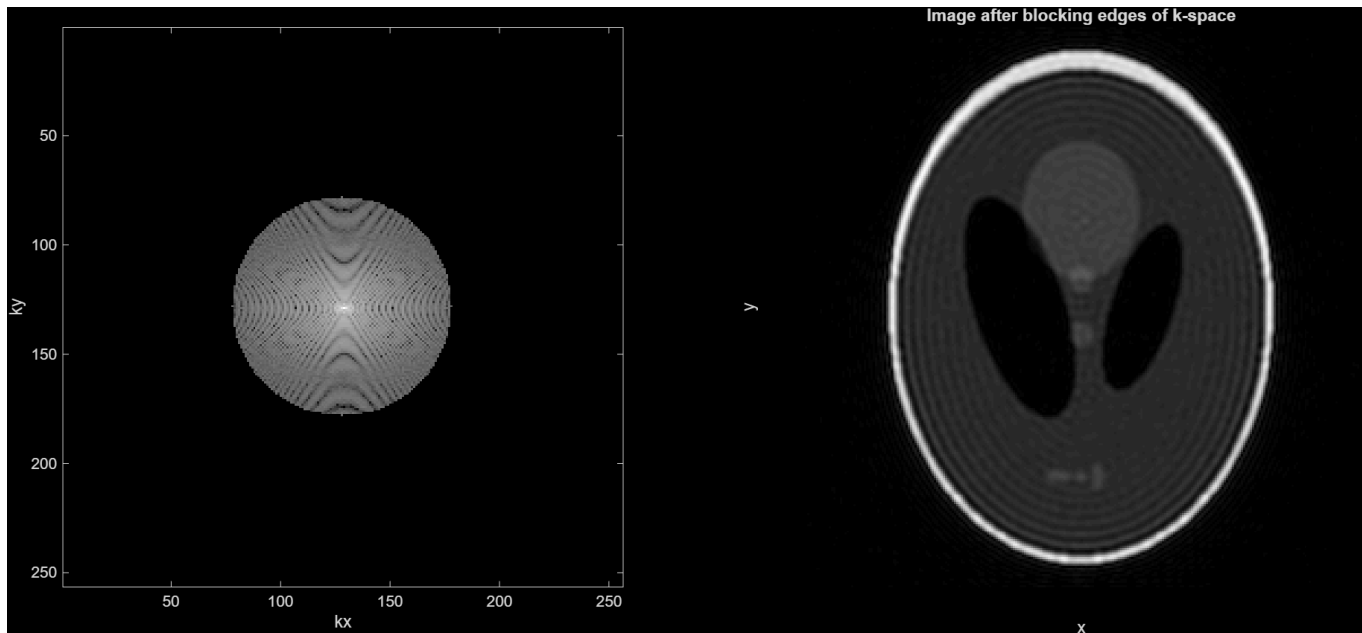
By decreasing the radius_edges parameter, the portion of k-space maintained around the origin (zero frequencies) is narrowed. Consequently, the reconstructed image loses sharpness, becoming progressively more blurred. Conversely, by increasing the radius, the image recovers fine details and sharp edges. The artifact visible in the form of oscillations is called the *truncation artifact.

6)Run the cell "6. Partial Fourier". Reflect on the exercise 3 above. Why does mirroring the k-space allow us to reconstruct a good-quality image? What advantages and disadvantages do the partial Fourier methods have?
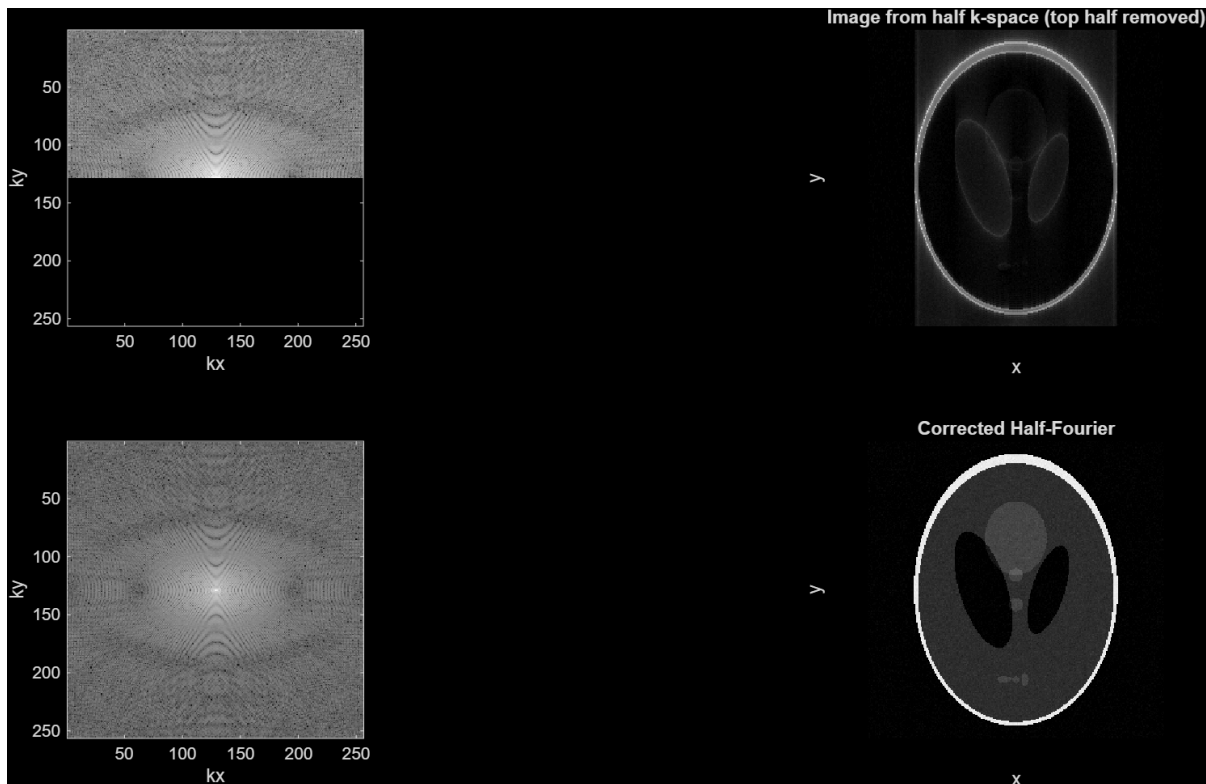
Mirroring the k-space allows us to reconstruct a good-quality image because it exploits the **Hermitian symmetry** of the k-space. As seen in point 3, Point 1 and point 2 are complex conjugates, so the information collected is redundant and we can sample only slightly more than half of the k-space and mathematically 'reconstruct' the missing part. This process allows us to almost halve the scan time or reduce the echo time (TE) without losing spatial resolution. But there are also some disadvantages: as we collect fewer raw data points, the Signal-to-Noise Ratio decreases by a factor of approximately $\sqrt{2}$ and the reconstruction is highly sensitive to phase errors caused by patient motion or magnetic field inhomogeneities, which can lead to artifacts if the symmetry is not perfectly maintained.

Task3

A) Calculate

The IEC 601-1 standard defines the maximum permitted exposure for optical devices as $9000 \, \frac{\text{W}}{\text{m}^2\text{sr}}$. What is the maximum power at which we can input light into the subject's scalp from one source, when the fiber tip is at 3 mm distance from the subject? The subject's scalp is assumed to be flat.

Radiance S is dependent on optical power P, size of illuminated area A, and solid angle Ω:

$$S = \frac{P}{A\Omega}$$

Response:

From Figure 1, we can see that the optical fiber has refractive indices of $n_1 = 1.48$ and $n_2 = 1.46$

The Numerical Aperture (NA) is defined as:

$$NA = \sqrt{n_1^2 - n_2^2} = \sqrt{1.48^2 - 1.46^2} \approx 0.2425$$

Since the light exits into the air $(n_{air} = 1)$, the maximum exit angle ϕ_{max} is calculated as:

$$\sin(\phi_{max}) = \frac{NA}{n_{air}} = 0.2425 \implies \phi_{max} \approx 14.03°$$

The solid angle Ω for a cone with a half-angle of ϕ_{max} is given by:

$$\Omega = 2\pi(1 - \cos(\phi_{max})) \approx 2\pi(1 - 0.970) \approx 0.187 \text{ sr}$$

The light exits the fiber and diverges over the distance d. The radius of the illuminated spot on the scalp (r) is:

$$r = \frac{w}{2} + d \cdot \tan(\phi_{\max})$$

$$r = 0.05 \text{ mm} + 3 \text{ mm} \cdot \tan(14.03°) \approx 0.05 + 0.7496 \approx 0.7996 \text{ mm}$$

The area A in square meters is:

$$A = \pi \cdot r^2 = \pi \cdot (0.7996 \times 10^{-3} \text{ m})^2 \approx 2.0086 \times 10^{-6} \text{ m}^2$$

Using the radiance formula $S = \frac{P}{A\Omega}$, we solve for P:

$$P = S \cdot A \cdot \Omega$$

$$P = 9000 \frac{\text{W}}{\text{m}^2\text{sr}} \cdot (2.0086 \times 10^{-6} \text{ m}^2) \cdot (0.1872 \text{ sr})$$

$$P \approx 3.385 \times 10^{-3} \text{ W} \approx \mathbf{3.39 \, mW}$$

B) Calculate

Calculate the optical power detected by each detector in the array (figure 2 a), and the corresponding signal-to-noise ratio for the duration (appr. 10 ms) of one pulse when source S (inside red box) is emitting. In Figure 2A, the distance between rows and columns is 1 cm, and the optical power in the tissue attenuates 60 dB/cm in the first 1 cm, and 20 dB/cm further away. You may assume photon shot noise to be the only significant noise component. Photon shot noise is defined as $n = \sqrt{\eta N}$, where $\eta = 0.1$ is the detector quantum efficiency, and N the number of incident photons at the detector. Assume a light wavelength of 800 nm.

Response:

Immediate neighbors (4 detectors): $d = 1$ cm
Diagonal neighbors (4 detectors): $d = \sqrt{1^2 + 1^2} = \sqrt{2} \approx 1.41$ cm

Total Attenuation (L):

- For $d = 1$ cm: $L = 60$ dB
- For $d = 1.41$ cm: $L = 60 + (1.41 - 1) \times 20 \approx 60 + 8.2 = 68.2$ dB

Detected Optical Power (P_{det})
Using the maximum input power $P_{in} \approx 3.39$ mW calculated in Part A:

- **For 1 cm distance:** $P_{det} = P_{in} \cdot 10^{(-60/10)} = 3.39$ mW $\cdot\, 10^{-6} \approx 3.39$ nW
- **For 1.41 cm distance:** $P_{det} = P_{in} \cdot 10^{(-68.2/10)} = 3.39$ mW $\cdot\, 10^{-6.82} \approx 0.51$ nW

Number of Incident Photons (N)
The energy of a single photon at $\lambda = 800$ nm is $E_{ph} = \frac{hc}{\lambda}$:

$$E_{ph} = \frac{(6.626 \times 10^{-34} \text{ Js}) \cdot (3 \times 10^8 \text{ m/s})}{800 \times 10^{-9} \text{ m}} \approx 2.48 \times 10^{-19} \text{ J}$$

For a pulse duration $\Delta t = 10$ ms, the number of incident photons is $N = \frac{P_{det} \cdot \Delta t}{E_{ph}}$:

- **At 1 cm:** $N \approx \frac{3.39 \times 10^{-9} \cdot 0.01}{2.48 \times 10^{-19}} \approx 1.37 \times 10^8$ photons
- **At 1.41 cm:** $N \approx \frac{0.51 \times 10^{-9} \cdot 0.01}{2.48 \times 10^{-19}} \approx 2.06 \times 10^7$ photons

Signal-to-Noise Ratio (SNR)
With quantum efficiency $\eta = 0.1$ and shot noise $n = \sqrt{\eta N}$, the SNR is:

$$SNR = \frac{\text{Signal}}{\text{Noise}} = \frac{\eta N}{\sqrt{\eta N}} = \sqrt{\eta N}$$

- **SNR at 1 cm:** $\sqrt{0.1 \cdot 1.37 \times 10^8} \approx \sqrt{1.37 \times 10^7} \approx$ **3701** (or ≈ 71.4 dB)
- **SNR at 1.41 cm:** $\sqrt{0.1 \cdot 2.06 \times 10^7} \approx \sqrt{2.06 \times 10^6} \approx$ **1435** (or ≈ 63.1 dB)