About assignments

- You may work together, but provide your own answers

  Identical answers == instant zero points

- Recommend that you look at/start the exercises before the exercise session

→ More time to concentrate on the difficult tasks in the exercise sessions

- Have additional information that might be useful for the other students? Share it with everyone/us!

Use of AI

- Feel free to use AI tools to **support** your learning
- Beware of hallucination / lack of substance

- **Please do not return AI-written reports**
  We're not here to check if LLMs can solve the exercises

- Please acknowledge the use of AI/LLMs in your reports

Task 1

A) Look for answers in the textbooks, articles, etc.

1-2 sentences should be enough

B) Carefully read the task description

i) The discrete definition of RMS might be useful

ii) RMS of PSD S(f): $RMS = \sqrt{\int_{\Delta f} S(f)\, df}$

Task 2

- The properties of vector products might be useful here

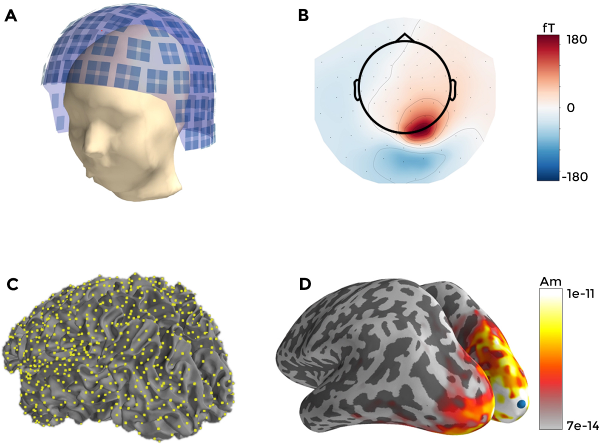
- Implementing the code:
  * Writing functions for recurring operations is a good coding practice
    Make a function for the magnetic field equation
  * In Matlab, one can get points on the unit sphere using the function 'sphere'. One can plot 3D surfaces (with colormaps) using 'surf'
  * In Python, maybe using mayavi for 3D plotting, and e.g. the trimesh package for generating a sphere mesh.

Task 3

- Imaging problems are often linear / they are solved with linear equations
- In MEG, one source modeling strategy is to distribute *n* sources on the surface of the brain
  - $\vec{y} = \mathbf{L}\vec{x}$

    $\vec{y} : \mathbb{R}^{m\,x\,1}$ - sensors / measurements
    $\mathbf{L} : \mathbb{R}^{m\,x\,n}$ - system matrix, lead field matrix, design matrix, coefficients matrix ...
    $\vec{x} : \mathbb{R}^{n\,x\,1}$ - sources

Distributed source imaging



Linear systems

- $\vec{y} = \mathbf{L}\vec{x}$
- Each sensor y_i "sees" each source x_i based on the model **L**

- Forward problem: we know $\vec{x}$ and **L**, and try to solve $\vec{y}$
  (**L** can be modelled based on tissue geometry and conductivities)

- Inverse problem: we know $\vec{y}$ and **L**, and try to solve $\vec{x}$

Linear systems

If m = n:

$$\hat{x} = \mathbf{L}^{-1}\vec{y}$$

However, **this is never* the case in medical imaging problems**:

In MEG, often 306 sensors, $m = 306$

Number of sources in thousands, $n = 10\ 000$

Usually $m < n$, $\mathbf{L}$ is not a square matrix

Underdetermined systems

Underdetermined system (when $m < n$) has infinitely many solutions.

$\rightarrow$ We must constrain the problem, or choose a preference for our solution (incorporating a *prior*)

$-$ One possible constraint is to choose a solution, which has the smallest possible norm, **minimum norm solution/estimate**

Problems of MNE

- Minimum norm solution is not *the correct* solution, but it is *a solution*.
- In order to satisfy the minimum norm prior, favours small and superficial sources
- The basic solution from 3A is never used as-is, need to regularize (3B; not trivial)

Task 3

A) Lagrange multipliers: Form the Lagrangian $\mathcal{L}$

- Find minimum of x by demanding that $\frac{\partial \mathcal{L}}{\partial x} = 0$ and $\frac{\partial \mathcal{L}}{\partial \lambda} = 0$.
- Can be solved with vectors/matrices or by using indices
- Remember to transpose: $(AB)C = C^T(AB)^T = C^T B^T A^T$

Vector derivative		
$f(\mathbf{x})$	$\rightarrow$	$\frac{\mathrm{d}f}{\mathrm{d}\mathbf{x}}$
$\mathbf{x}^T\mathbf{B}$	$\rightarrow$	$\mathbf{B}$
$\mathbf{x}^T\mathbf{b}$	$\rightarrow$	$\mathbf{b}$
$\mathbf{x}^T\mathbf{x}$	$\rightarrow$	$2\mathbf{x}$
$\mathbf{x}^T\mathbf{B}\mathbf{x}$	$\rightarrow$	$2\mathbf{B}\mathbf{x}$