Linear Algebra

Paolo Luciano Rivera

2021

1 Definitions

1.1 Vectors and Vector Spaces

- The *span* of $\mathbf{u}$ and $\mathbf{y}$ is the set of all *linear combinations*:
$$a\mathbf{u} + b\mathbf{v}$$
- When $\mathbf{u}$ is a linear combination of the others basis vectors, then it's *linearly dependent*
- The basis $\mathbf{B}$ of a *vector space* V is a set of *linearly independent* vectors that *span* the full space

2 Linear Algebra Intuition

Notes from the 3B1B video series.

2.1 Vectors and Vector Spaces

- Vectors $\mathbf{x} = (x_1, \dots, x_n)$ can be viewed interchangeably as **ordered lists** or as objects with length and direction, i.e. an arrow
- Vectors are points in L.A., sometimes it's easier to think of them as arrows, sometimes as points (when you have many), sometimes as lists (when you're programming)
- Vectors live in a **vector space** V that has certain properties
- in Linear Algebra (L.A.) vectors always start at the origin $\mathbf{0} = (0, \dots, 0)$
- Vectors are almost always represented as column vectors
- Vectors have a sum $+$ and a scalar multiplication $\cdot$
- Vector sum can be viewed as a translation to the vector space, similarly to how real numbers translate the $\mathbb{R}$
- Scalar multiplication can be viewed as *scaling* the vector by the magnitude of the scalar.
- Any vector $\mathbf{x}$ can be viewed as the *linear combination* of the basis vectors (on a given space). e.g. for $\mathbb{R}^2$
$$\mathbf{x} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} = (3, -2)$$
- When you think of a vector space, you have to think about the basis of it $\mathbf{B}$, in the previous example $\hat{\mathbf{i}}$ and $\hat{\mathbf{j}}$ *span* $\mathbb{R}^2$

3 Useful Properties

3.1 Matrices

- $2x2$ **matrices** $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$
  - **Inverse** If $\det(A) = ad - cd \neq 0 \Rightarrow A^{-1} = \frac{1}{ad-cd} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$