Image enhancement

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October 3, 2023

Image enhancement

What do we mean by that?

Input: image
Output: processed image
Process an image so that the resulting image is more suitable for a *specific* task.

Examples of tasks

- the image is not well contrasted
- the image is corrupted by noise
- we want to detect particular areas of interest (segmentation)

Image enhancement

What do we mean by that?

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Taxonomy of methods

Two main classes

- spatial domain methods
  direct manipulation of image pixels
- frequency domain methods
  manipulation of the Fourier transform of the image

Spatial domain

What do we mean by that?

Spatial domain refers to the aggregate of pixels composing an image

Formally

Input: I
Output: I'

$$\mathrm{I}'_{xy} = T(\mathrm{I}_{xy})$$

The operator T may perform

- single-point processing
- local processing (over a neighbourhood)

Pixel neighbourhood

A neighbourhood of pixel (i, j) is (typically) a square of rectangular sub-image centred at pixel (i, j)

$$\begin{matrix} 2 & 2 & 2 & 2 & 2 \\ 2 & 1 & 1 & 1 & 2 \\ 2 & 1 & 0 & 1 & 2 \\ 2 & 1 & 1 & 1 & 2 \\ 2 & 2 & 2 & 2 & 2 \end{matrix}$$

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This definition implies the metric

$$d(p, q) = max(|i_p - i_q|, |j_p - j_q|)$$

Pixel neighbourhood

A neighbourhood of pixel (i, j) is a diamond centred at pixel (i, j)

$$\begin{array}{ccccc} & & 2 & & \\ & 2 & 1 & 2 & \\ 2 & 1 & 0 & 1 & 2 \\ & 2 & 1 & 2 & \\ & & 2 & & \end{array}$$

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$$d(p, q) = |i_p - i_q| + |j_p - j_q|$$

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$$
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2 & 2 & 1 & 2 & 2 \\
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 & 2 & 2 & 2 &
\end{array}
$$

This definition implies the metric

$$d(p, q) = \sqrt{(i_p - i_q)^2 + (j_p - j_q)^2}$$

Single point processing

The values in the output image are determined *only* from the value of the correspondent pixel in the input image by means of a *mapping function* T

$$T : \mathcal{G} \longrightarrow \mathcal{G}$$

$\mathcal{G}$ is the set of gray-level values

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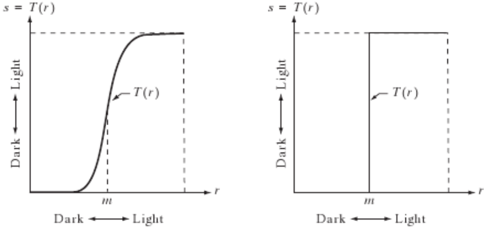

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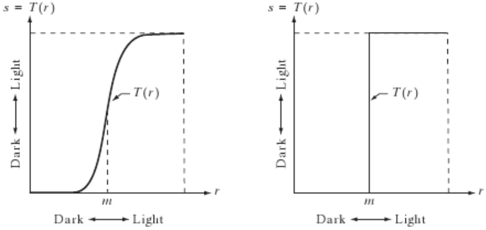

In general T is monotonically increasing

Image negative

If $\mathcal{G} = [g, G]$

$$\begin{array}{rccc} T: & \mathcal{G} & \longrightarrow & \mathcal{G} \\ & r & \longmapsto & G - (r - g) \end{array}$$

Image negative

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Enhances white or gray details embedded in dark areas

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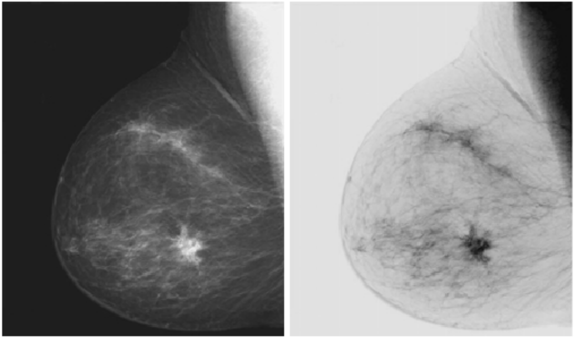

Raise to power transformation

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 & r & \longmapsto & cr^{\gamma}
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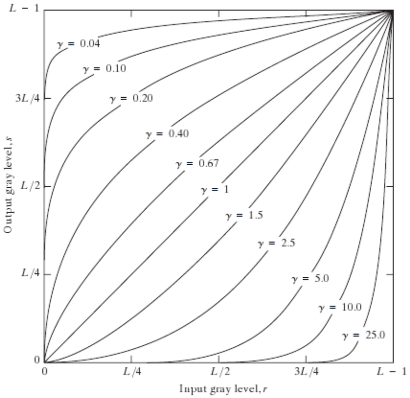


Raise to power transformation

Contrast enhancement

Raise to power transformation

Contrast enhancement

$c = 1,\ \gamma = .6$

Raise to power transformation

Contrast enhancement

$c = 1, \gamma = .4$

Raise to power transformation

Contrast enhancement

$c = 1,\ \gamma = .3$

Raise to power transformation

Contrast enhancement

Raise to power transformation

Contrast enhancement

$c = 1,\ \gamma = 3.0$

Raise to power transformation

Contrast enhancement

$c = 1, \ \gamma = 4.0$

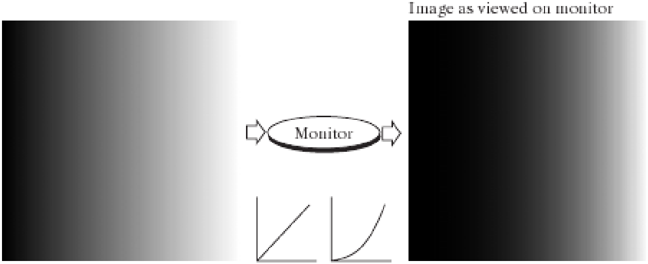
Raise to power transformation

Contrast enhancement

$c = 1, \gamma = 5.0$

Gamma correction

Some displays show a raise to power relation r^{γ} between original values and displayed values.
Target: make the relation look linear
Solution: apply to the image the $r^{1/\gamma}$ operator



Gamma correction

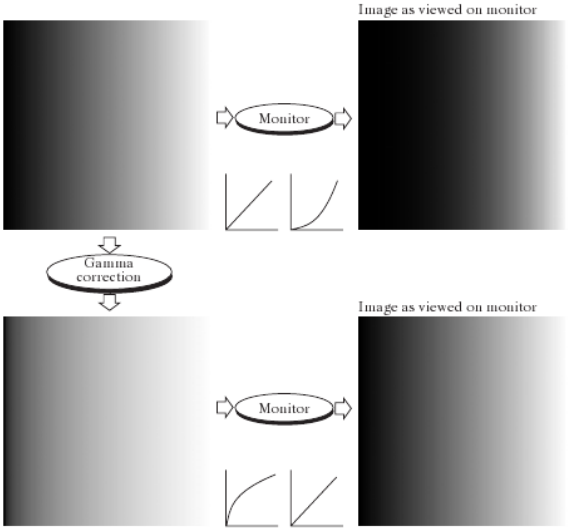


Image histogram

What it is

It is a discrete function $h : \mathcal{G} \longrightarrow \boldsymbol{N}$ where $h(r)$ is the number of pixels in the image having intensity r

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Normalised histogram

- $p(r) = h(r)/N$, where N is the number of pixels in the image
- $p(r)$ gives an estimate of the probability of gray level r
- $\sum_g^G p(r) = 1$

Image histogram

Dark image

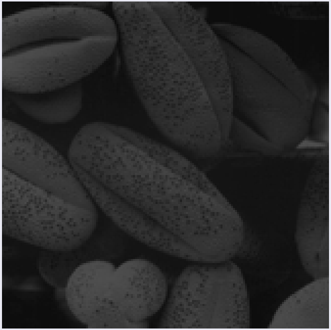
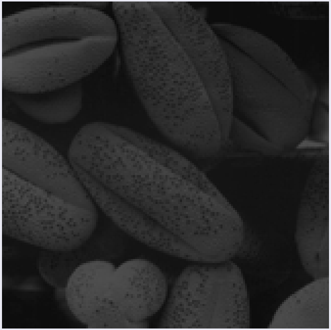


Image histogram

Dark image



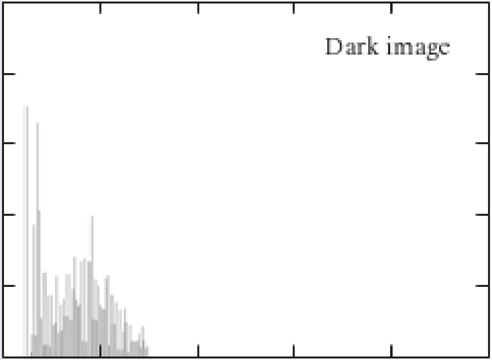


Image histogram

Bright image

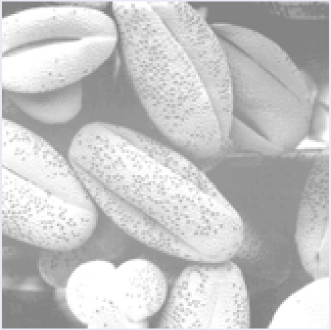


Image histogram

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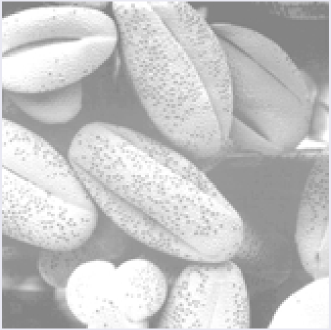

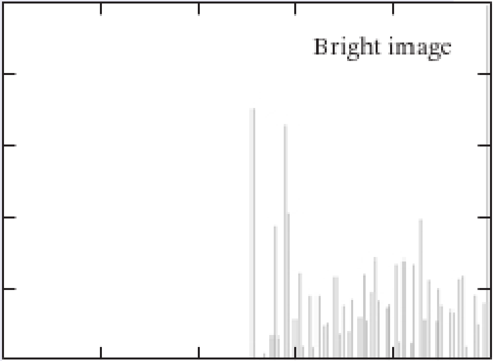

Image histogram

Low-contrast image

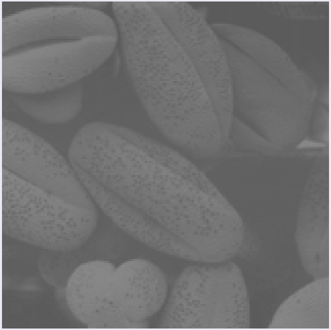

Image histogram

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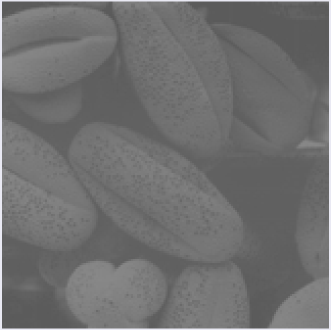


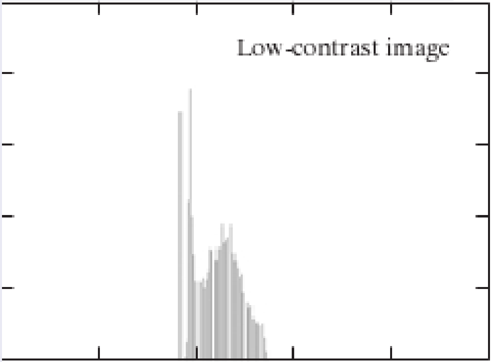
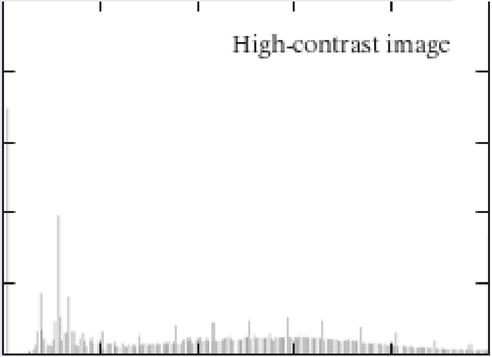


Image histogram

High-contrast image

Image histogram

High-contrast image



Thresholding

What it is

An easy way to segment out regions of interest from the background.

Thresholding

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An easy way to segment out regions of interest from the background.

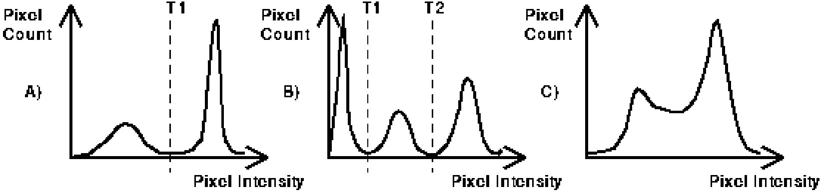
How it works

Input: I, k
Output:I$'$, binary image

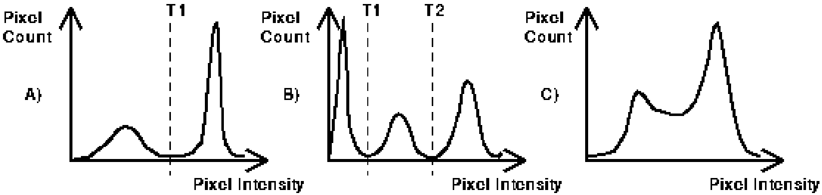
$$\mathrm{I}'_{xy} = \begin{cases} 0 & \mathrm{I}_{xy} \leq k \\ 1 & \mathrm{I}_{xy} > k \end{cases}$$

Thresholding

Thresholding



Thresholding

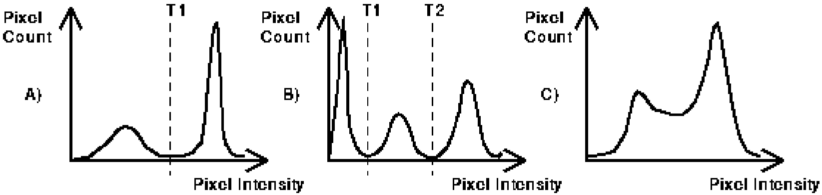


Case A

Input: I, T_1
Output:I$'$, binary image

$$I'_{xy} = \begin{cases} 0 & I_{xy} \leq T_1 \\ 1 & I_{xy} > T_1 \end{cases}$$

Thresholding

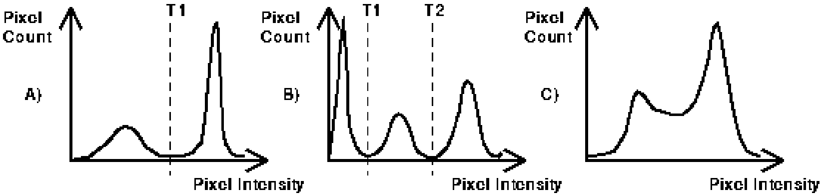


Case B

Input: I, T_1, T_2
Output:I′, binary image

$$I'_{xy} = \begin{cases} 0 & I_{xy} < T_1 \vee I_{xy} > T_2 \\ 1 & T_1 < I_{xy} < T_2 \end{cases}$$

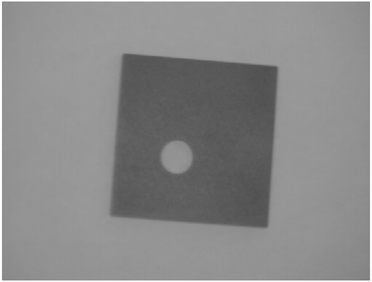
Thresholding

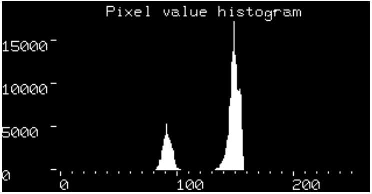


Case C

No way!!

Thresholding





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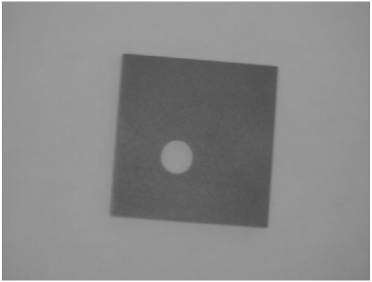
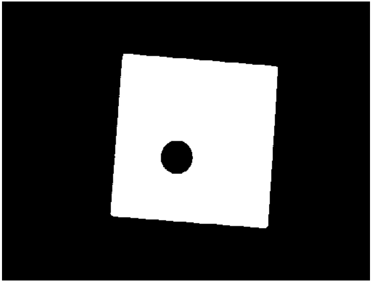


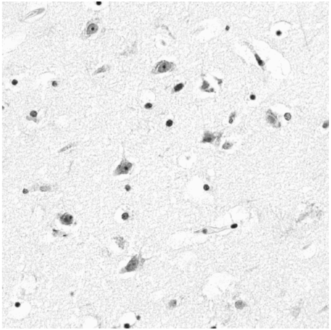


$T_1 = 120$

Thresholding

A slice of brain tissue containing

- nervous cells : the large gray blobs, with darker circular nuclei
- glia cells: isolated, small, black circles



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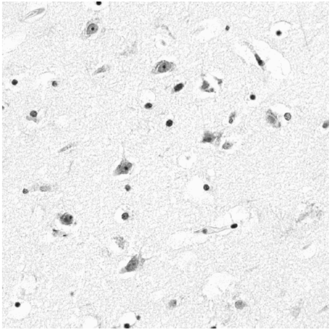

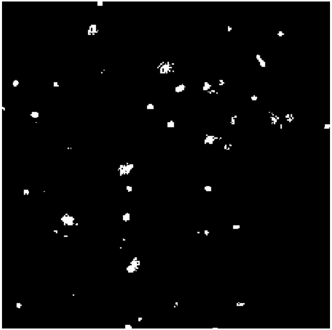

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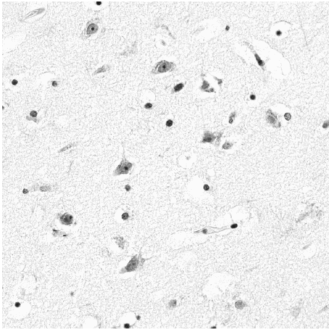
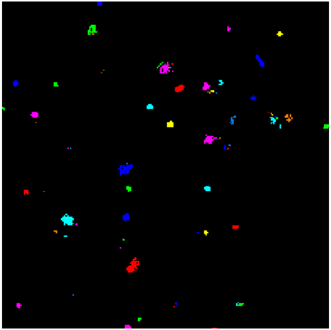
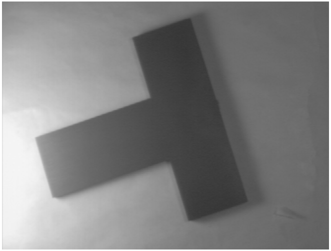
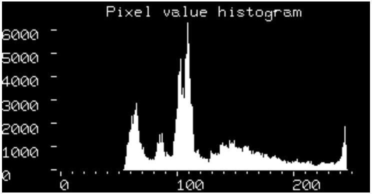




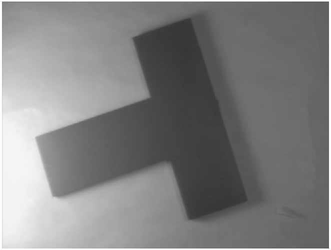
Connected components

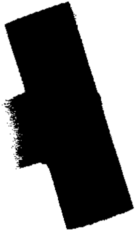
Thresholding





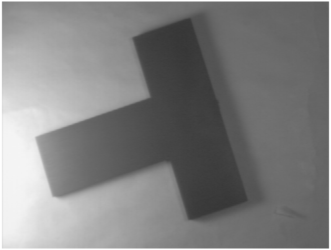
Thresholding

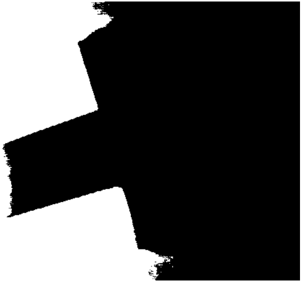




$T_1 = 80$

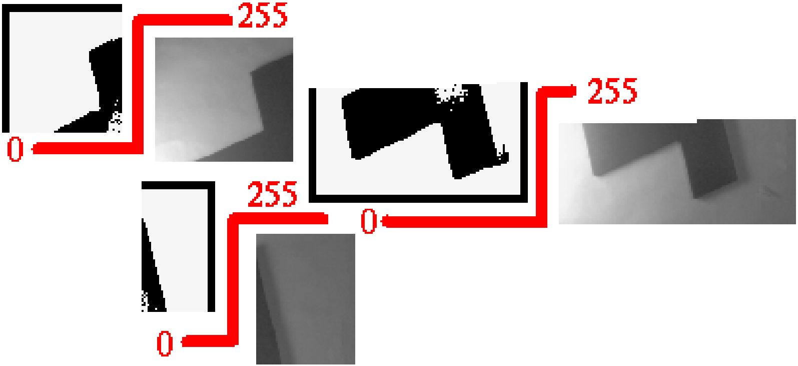
Thresholding





$T_1 = 120$

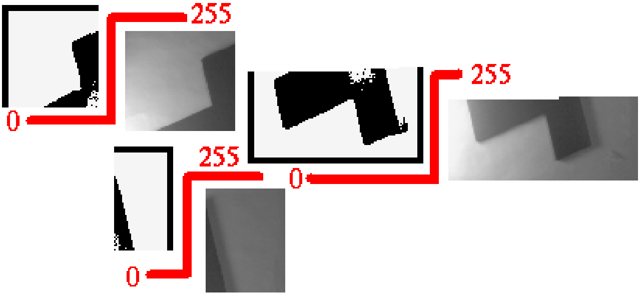
Adaptive thresholding



Adaptive thresholding

A different threshold is computed for each pixel.
Assumption: local uniform illumination.
Two main approaches:

- Chow and Kaneko approach
- Local thresholding



Adaptive thresholding

Chow and Kaneko

- divide an image into an array of overlapping sub-images
- compute the histogram for each sub-image
- find the optimum threshold for each sub-image
- find the threshold for each pixel by interpolation

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Drawback:

- it is computational expensive
- might be not suitable for real-time applications

Adaptive thresholding

How do we compute the threshold value?

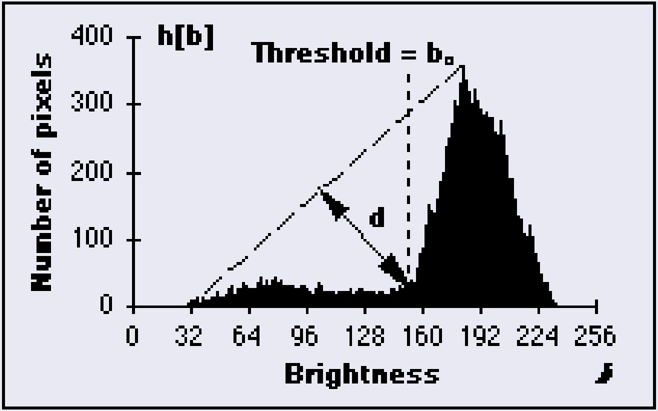
Isodata algorithm:

Assume $\mathcal{G} = [0, 2^L]$
$T_0 = 2^{L-1}$
$k = 0$
repeat
  compute background B_k and foreground F_k
  f_k = mean(F_k)
  b_k = mean(B_k)
  k = k+1
  $T_k = (f_k + b_k)/2$
until $T_k = T_{k-1}$

Adaptive thresholding

How do we compute the threshold value?

Triangle algorithm



Adaptive thresholding

How do we compute the threshold value?

Triangle algorithm

find the histogram peak r_{max}
find the histogram minimum r_{min}
find the straight line R between $(r_{min}, h(r_{min}))$
  and $(r_{max}, h(r_{max}))$
for each $r_{min} < r < r_{max}$ compute d((r,h(r), R)
take as threshold r_0 having the maximal distance

Adaptive thresholding

Local thresholding

Statistically examine the intensity values of the local neighbourhood of each pixel.

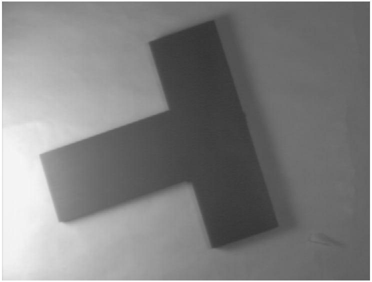
- $T = \text{mean}$
- $T = \text{median}$
- $T = (\max + \min)/2$
- $T = \text{mean} - C$

What about the size?

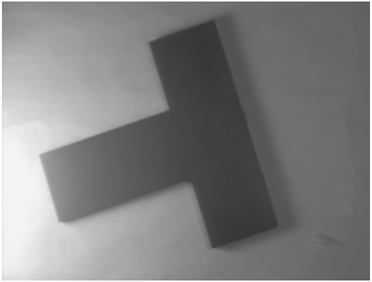
- large enough to cover sufficient foreground and background pixels
- not so large to violate the assumption of approximately uniform illumination

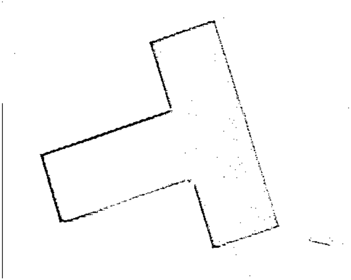
Computationally less intensive than Chow and Kaneko

Adaptive thresholding



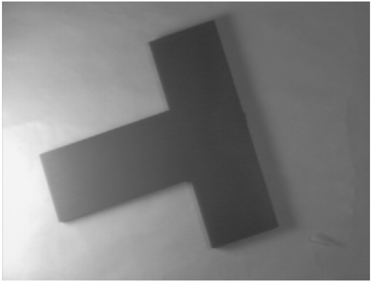
Adaptive thresholding

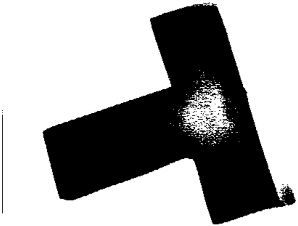




7×7, mean - C $C = 4$

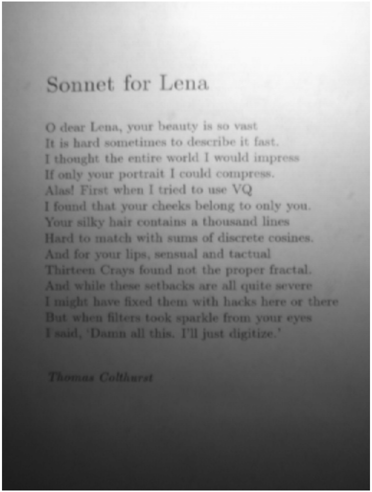
Adaptive thresholding



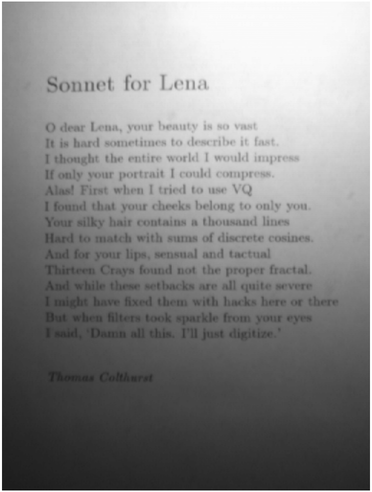


140×140, mean - C $C = 8$

Adaptive thresholding



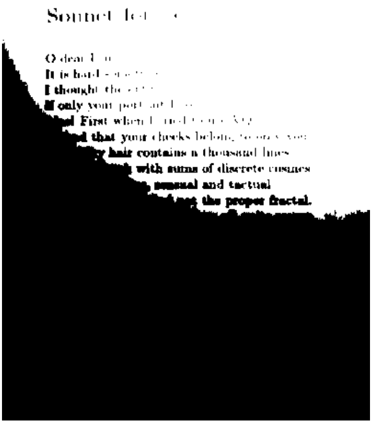
Adaptive thresholding



Sonnet for Lena

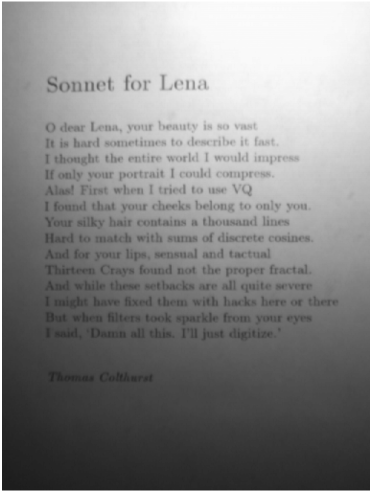
O dear Lena, your beauty is so vast
It is hard sometimes to describe it fast.
I thought the entire world I would impress
If only your portrait I could compress.
Alas! First when I tried to use VQ
I found that your cheeks belong to only you.
Your silky hair contains a thousand lines
Hard to match with sums of discrete cosines.
And for your lips, sensual and tactual
Thirteen Crays found not the proper fractal.
And while these setbacks are all quite severe
I might have fixed them with hacks here or there
But when filters took sparkle from your eyes
I said, 'Damn all this. I'll just digitize.'

Thomas Colthurst



threshold

Adaptive thresholding



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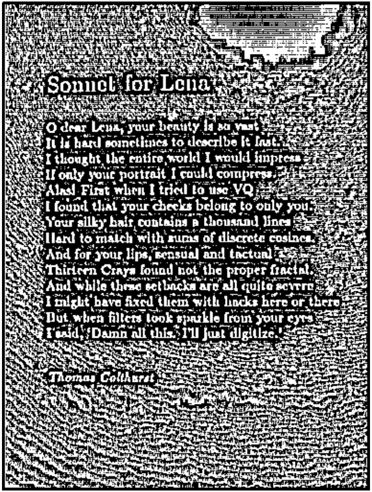
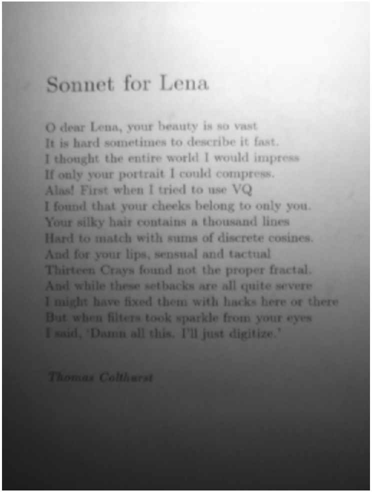
I thought the entire world I would impress
If only your portrait I could compress.
Alas! First when I tried to use VQ
I found that your cheeks belong to only you.
Your silky hair contains a thousand lines
Hard to match with sums of discrete cosines.
And for your lips, sensual and tactual
Thirteen Crays found not the proper fractal.
And while these setbacks are all quite severe
I might have fixed them with hacks here or there
But when filters took sparkle from your eyes
I said, 'Damn all this. I'll just digitize.'

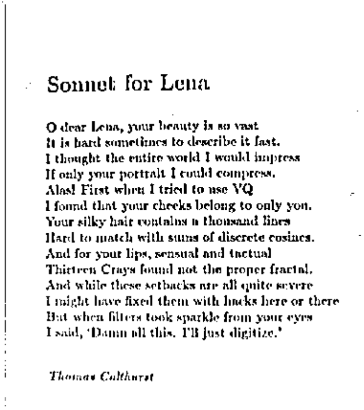
Thomas Colthurst



7×7 Mean

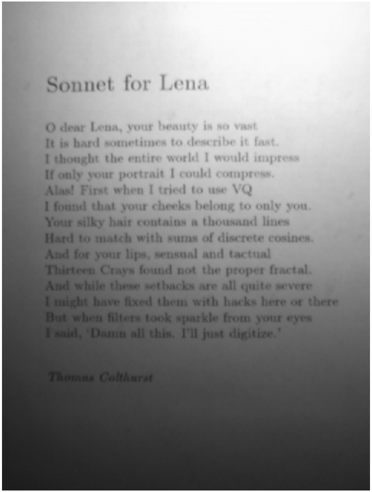
Adaptive thresholding





7×7 Mean - C $C = 7$

Adaptive thresholding



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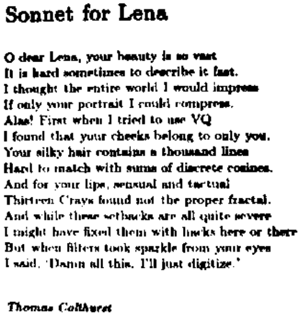

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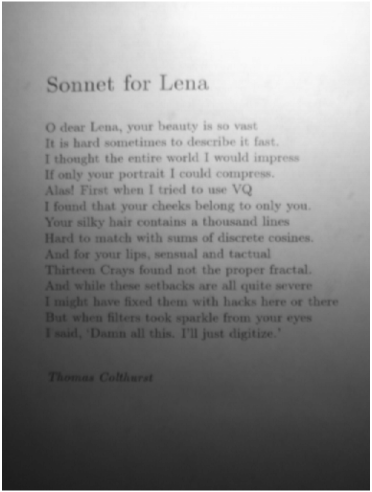
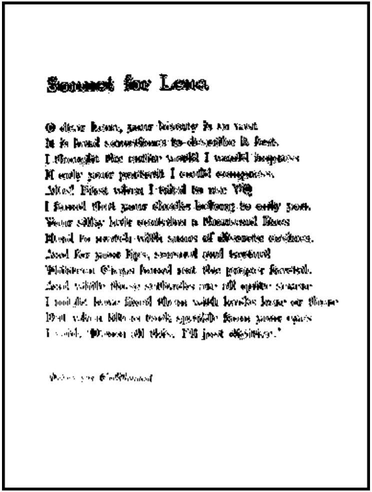
75×75 Mean - C $C = 10$

Adaptive thresholding



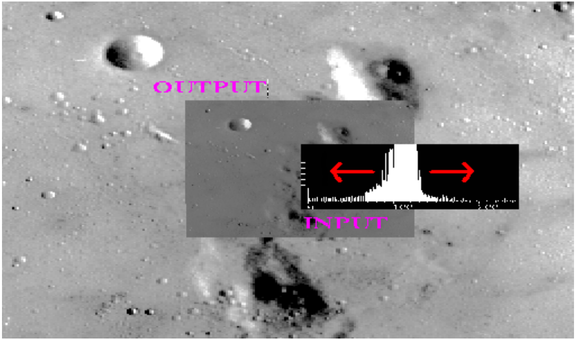


7×7 Median - C $C = 4$

Contrast stretching

What it is

Simple technique that attempts to improve the contrast *stretching* the range of intensities $[m, M]$ to span a different (larger) range of values $[m', M']$



Contrast stretching

How it works

$$T(r) = (r - m)\frac{M' - m'}{M - m} + m'$$

Contrast stretching

How it works

$$T(r) = (r - m)\frac{M' - m'}{M - m} + m'$$

If $m' = 0$ and $M' = 255$

$$T(r) = 255\frac{r - m}{M - m}$$

Contrast stretching

Outliers

- a single outlying pixel (very low or very high) can affect the performance
- m and M are not taken as the minimum and the maximum
  - $m = n$th percentile and $M = N$th percentile
  - Cutoff fraction of the histogram peak

Contrast stretching

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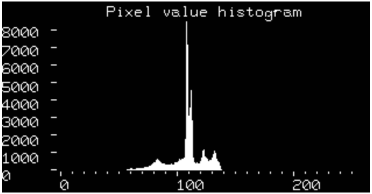
Cutoff fraction

1. Compute histogram h of input image I
2. Find the histogram peak p
3. Decide a cutoff fraction $0 < c < 1$
4. $m = \min\{r : h(r) \geq cp, h(r) > 0\}$
5. $M = \max\{r : h(r) \geq cp, h(r) > 0\}$

Contrast stretching

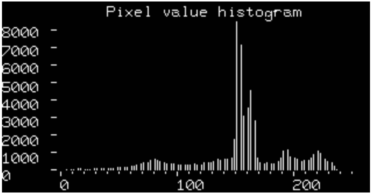
Contrast stretching



Contrast stretching

$m = 79, M = 136$

Contrast stretching

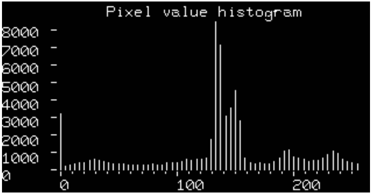


$m = 79$, $M = 136$

Contrast stretching

$c = 0.03$

Contrast stretching

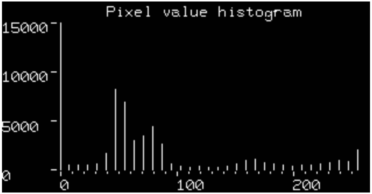


$c = 0.03$

Contrast stretching

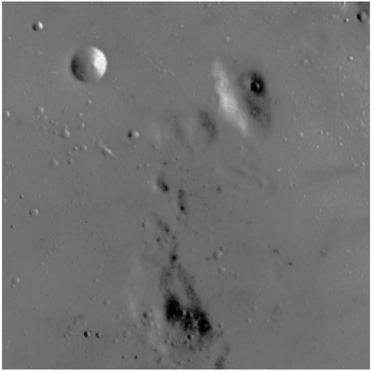
$c = 0.125$

Contrast stretching

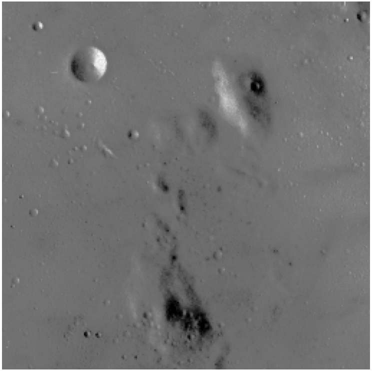


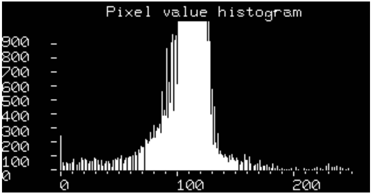
$c = 0.125$

Contrast stretching

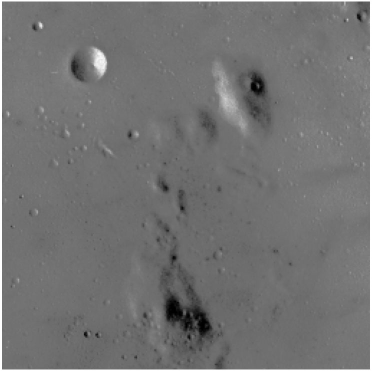


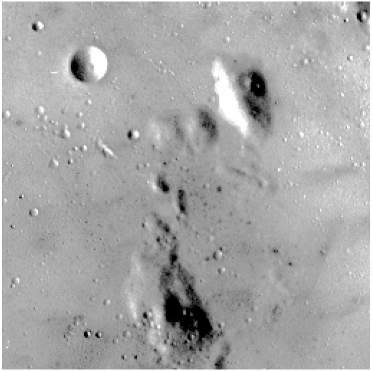
Contrast stretching





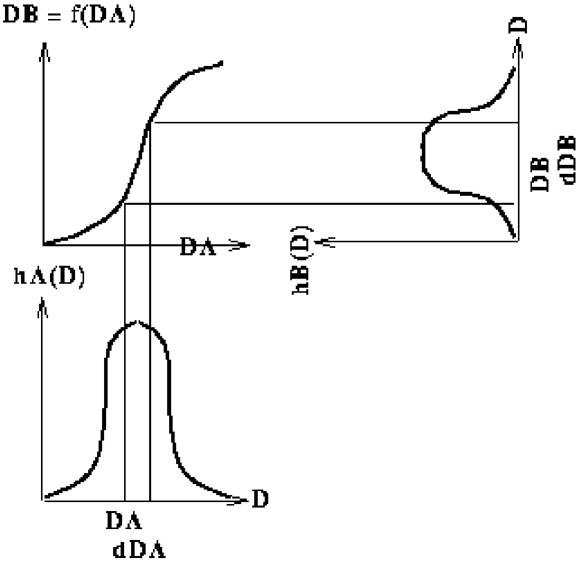
Contrast stretching





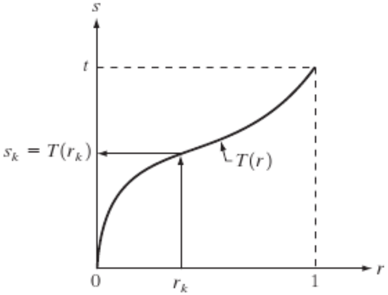
m = 1st M = 99th percentile

Histogram transformation



Histogram transformation

- $0 \le r \le G$
- $\mathrm{I}'_{xy} = T(\mathrm{I}_{xy})$
- $T(r) = s$ such that
  - monotonically increasing
  - $0 \le T(r) \le G$
- $p_{\mathrm{I}}(r)$ and $p_{\mathrm{I}'}(s)$ the gray levels probability distributions



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- $p_{\mathrm{I}}(r)$ and $p_{\mathrm{I}'}(s)$ the gray levels probability distributions

Relation between $p_{\mathrm{I}}(r)$ and $p'_{\mathrm{I}}(s)$

If $T(r)$ continuous and differentiable

$$p_{\mathrm{I}'}(s) = p_{\mathrm{I}}(r)\frac{dr}{ds}$$

Histogram equalisation

Transformation function

$$T(r) = G \int_0^r p_\mathsf{I}(w)\, dw$$

Histogram equalisation

Transformation function

$$T(r) = G \int_0^r p_{\mathrm{I}}(w)\, dw$$

- $T(r)$ is the cumulative distribution function
- T is monotonically increasing
- $T(0) = 0$ and $T(G) = G \implies 0 \leq T(r) \leq G$

Histogram equalisation

Transformation function

$$T(r) = G \int_0^r p_{\mathrm{I}}(w)\, dw$$

The new pdf $p_{\mathrm{I}'}(s)$

$$p_{\mathrm{I}'}(s) = p_{\mathrm{I}}(r) \frac{dr}{ds}$$

Histogram equalisation

Transformation function

$$T(r) = G \int_0^r p_{\mathsf{I}}(w)\, dw$$

The new pdf $p_{\mathsf{I}'}(s)$

$$\frac{ds}{dr} = \frac{dT(r)}{dr}$$

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Transformation function

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The new pdf $p_{\mathsf{I}'}(s)$

$$\frac{ds}{dr} = \frac{dT(r)}{dr} = G \frac{d}{dr} \int_0^r p_{\mathsf{I}}(w)\, dw$$

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The new pdf $p_{\mathsf{I}'}(s)$

$$\frac{ds}{dr} = \frac{dT(r)}{dr} = G \frac{d}{dr} \int_0^r p_\mathsf{I}(w)\, dw = G p_\mathsf{I}(r)$$

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$$p_{\mathrm{I}'}(s) = p_{\mathrm{I}}(r) \frac{dr}{ds}$$

Histogram equalisation

Transformation function

$$T(r) = G \int_0^r p_{\mathrm{I}}(w)\, dw$$

The new pdf $p_{\mathrm{I}'}(s)$

$$p_{\mathrm{I}'}(s) = p_{\mathrm{I}}(r) \frac{1}{G p_{\mathrm{I}}(r)}$$

Histogram equalisation

Transformation function

$$T(r) = G \int_0^r p_{\mathsf{I}}(w)\, dw$$

The new pdf $p_{\mathsf{I}'}(s)$

$$p_{\mathsf{I}'}(s) = \frac{1}{G}$$

Histogram equalisation

Transformation function

$$T(r) = G \int_0^r p_{\mathrm{I}}(w)\, dw$$

The new pdf $p_{\mathrm{I}'}(s)$

$$p_{\mathrm{I}'}(s) = \frac{1}{G}$$

$p_{\mathrm{I}'}$ is a uniform probability distribution function

Histogram equalisation

In the discrete case

$$p_{\mathrm{I}}(r) = \frac{h(r)}{N}$$

Histogram equalisation

In the discrete case

$$T(r) = G \sum_{0}^{r} p_{\mathrm{I}}(i)$$

Histogram equalisation

In the discrete case

$$T(r) = \frac{G}{N} \sum_{0}^{r} h(i)$$

Histogram equalisation

In the discrete case

$$T(r) = \frac{G}{N} \sum_{0}^{r} h(i)$$

It is trivial that

- T is monotonically increasing
- $0 \leq T(r) \leq G$

What actually does happen

I such that

- $G = 7$
- size 64×64 (i.e. $N = 4096$)
- histogram as

r	0	1	2	3	4	5	6	7
$h(r)$	790	1023	850	656	329	245	122	81
$p_I(r)$	0.19	0.25	0.21	0.16	0.08	0.06	0.03	0.02

What actually does happen

Original histogram

r	0	1	2	3	4	5	6	7
$h(r)$	790	1023	850	656	329	245	122	81
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Function T

r	0	1	2	3	4	5	6	7
$T(r)$	1.33	3.08	4.55	5.67	6.23	6.65	6.86	7.00
s	1	3	5	6	6	7	7	7

What actually does happen

Original histogram

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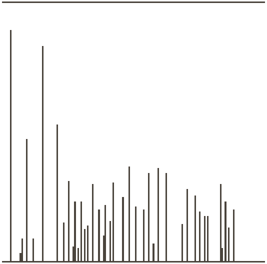
$T(r)$	1.33	3.08	4.55	5.67	6.23	6.65	6.86	7.00
s	1	3	5	6	6	7	7	7

Equalised histogram

s	0	1	2	3	4	5	6	7
$h(s)$	0	790	0	1023	0	850	985	448
$p_{I'}(s)$	0	0.19	0	0.25	0	0.20	0.24	0.10

Histogram equalisation

Histogram equalisation



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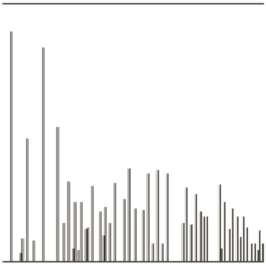

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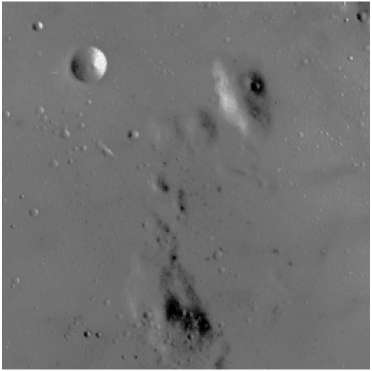

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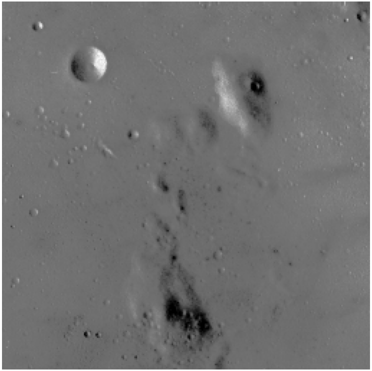

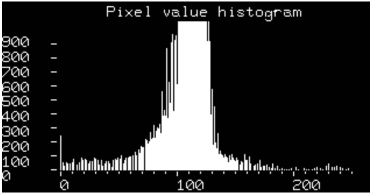

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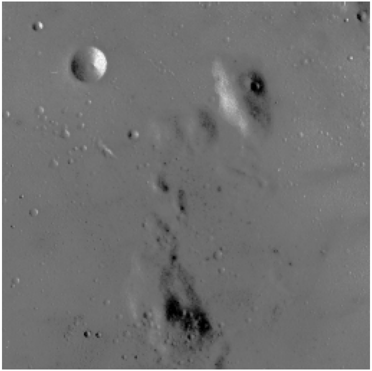

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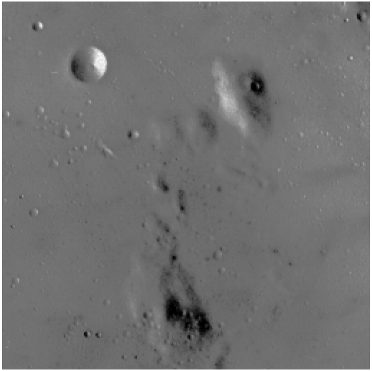

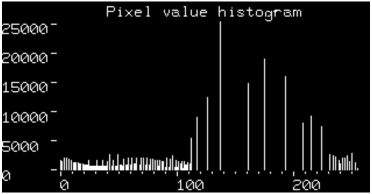

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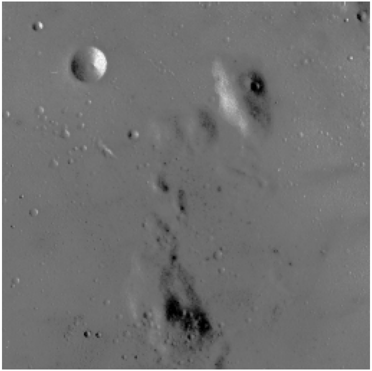

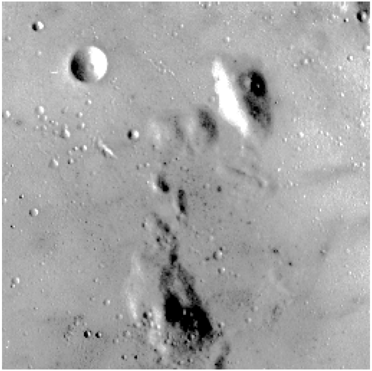

Contrast stretching

Histogram equalisation

Histogram equalisation

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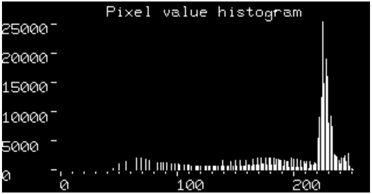

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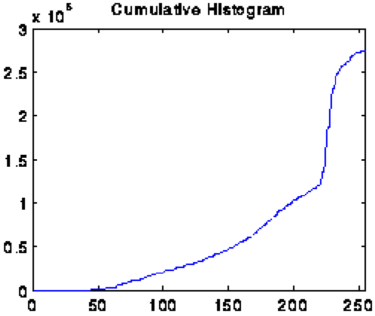
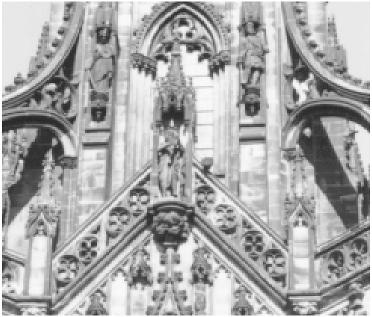
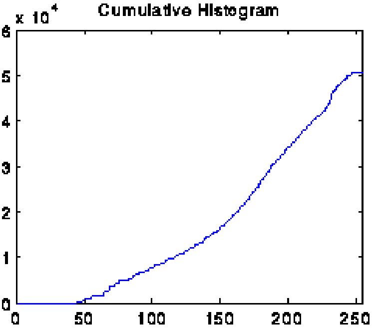


Cumulative histogram

Histogram equalisation

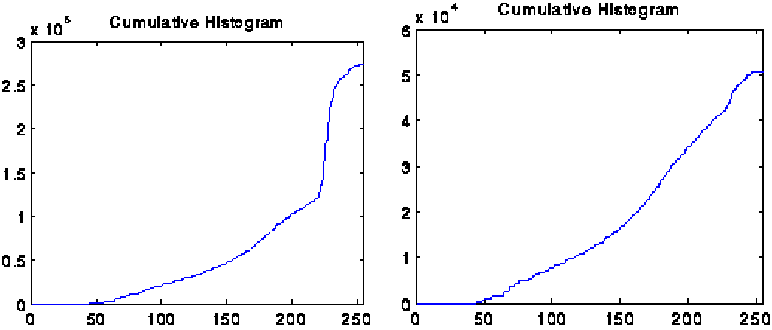
Histogram equalisation





Cumulative histogram

Histogram equalisation



Histogram equalisation

Histogram matching

What it is

Shape the histogram of the output image in order to highlight certain intensity levels in an image

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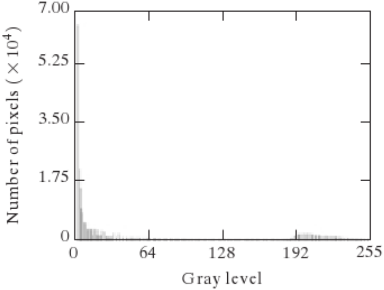

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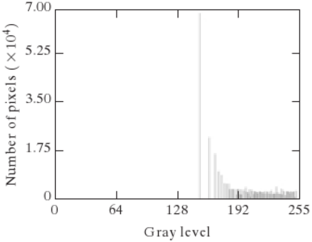

Histogram matching

What it is

Shape the histogram of the output image in order to highlight certain intensity levels in an image

$$T(r) = G\int_0^r p_{\text{I}}(w)\,dw$$

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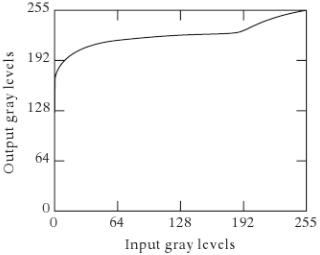

Histogram matching

Ingredients

- $p_{\mathsf{I}}(r)$ gray levels probability distribution of the input image
- $p_{\mathsf{I}'}(s)$ gray levels probability distribution of the output image
- Target: determine the transformation $T : \mathcal{G} \longrightarrow \mathcal{G}$ such that I' has gray levels probability distribution $p_{\mathsf{I}'}(s)$

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$$F(z) = G \int_0^z p_{\mathrm{I}'}(w)\, dw$$

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$$s = H(r) = G \int_0^r p_{\mathrm{I}}(w)\, dw$$

$$F(z) = G \int_0^z p_{\mathrm{I}'}(w)\, dw$$

$$z = T(r) = F^{-1}(s) = F^{-1}(H(r))$$

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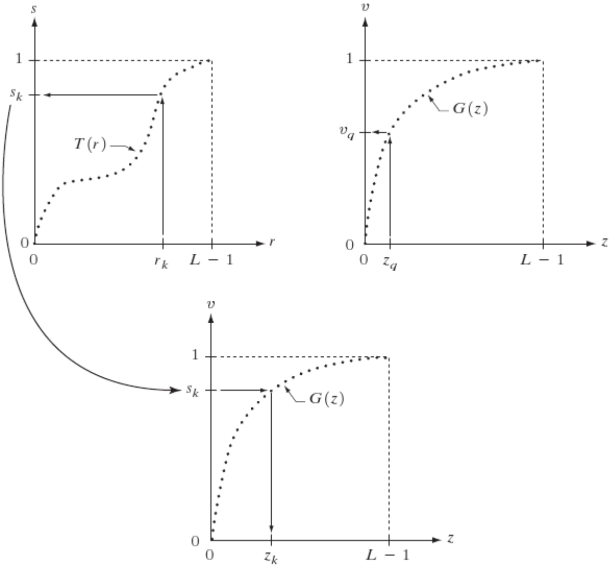
How it works

1. Determine H that transform original distribution p_{I} into a uniform distribution
2. Determine F that transform target distribution p_{I} into a uniform distribution
3. Take $T = F^{-1}H$

Histogram matching



Histogram matching

The discrete case

- $p_{\mathrm{I}}(r) = \frac{h_{\mathrm{I}}(r)}{N}$
- $H(r) = G \sum_0^r p_{\mathrm{I}}(i) = \frac{G}{N} \sum_0^r h_{\mathrm{I}}(i)$
- $v = F(z) = G \sum_0^z p_{\mathrm{I}'}(j)$
- $z = F^{-1}(H(r))$

Histogram matching

The discrete case

- $p_{\mathrm{I}}(r) = \frac{h_{\mathrm{I}}(r)}{N}$
- $H(r) = G \sum_0^r p_{\mathrm{I}}(i) = \frac{G}{N} \sum_0^r h_{\mathrm{I}}(i)$
- $v = F(z) = G \sum_0^z p_{\mathrm{I}'}(j)$
- $z = F^{-1}(H(r))$

Problem

For each r determine z such that $F(z) = s$

Histogram matching

Solution

$$T(r) = \min_{Z} |F(z) - s|$$

Histogram matching

Procedure

1. Compute the histogram h_{I} from input image I
2. Compute the vector $H(r) = s = \frac{G}{N} \sum_0^r h_{\mathrm{I}}(r)$
3. Compute the vector $V(s) = \min_z |F(z) - s|$
4. For each pixel $\mathrm{I}_{xy} = r_{xy}$ take $\mathrm{I}'_{xy} = V(H(r_{xy}))$

Histogram matching

Implementation

```
for each s = 0 to G do
  go = true;
  z = 0;
  while go do
    if F(z) − s ≥ 0 then
      if |F(z) - s| < |F(z-1) - s| then V(s) = z
      if |F(z) - s| > |F(z-1) - s| then V(s) = z − 1
      go = false;
    else
      z = z+1
  end
end
```

Histogram matching

Implementation

```
for each sϵ{s = T(r)} do
  go = true;
  z = 0;
  while go do
    if F(z) − s ≥ 0 then
      if |F(z) - s| < |F(z-1) - s| then V(s) = z
      if |F(z) - s| > |F(z-1) - s| then V(s) = z − 1
      go = false;
    else
      z = z+1
  end
end
```

Numerical example

I such that

- $G = 7$
- size 64×64 (i.e. $N = 4096$)
- histogram as

r	0	1	2	3	4	5	6	7
$h(r)$	790	1023	850	656	329	245	122	81
$p_{\mathsf{I}}(r)$	0.19	0.25	0.21	0.16	0.08	0.06	0.03	0.02

- specified histogram as

z	0	1	2	3	4	5	6	7
$p_{\mathsf{I}'}(z)$	0.00	0.00	0.00	0.15	0.20	0.30	0.20	0.15

Numerical example

Original histogram

r	0	1	2	3	4	5	6	7
$h(r)$	790	1023	850	656	329	245	122	81
$p_I(r)$	0.19	0.25	0.21	0.16	0.08	0.06	0.03	0.02

Numerical example

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Function H

r	0	1	2	3	4	5	6	7
$H(r)$	1.33	3.08	4.55	5.67	6.23	6.65	6.86	7.00
s	1	3	5	6	6	7	7	7

Numerical example

Original histogram

r	0	1	2	3	4	5	6	7
$h(r)$	790	1023	850	656	329	245	122	81
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s	1	3	5	6	6	7	7	7

Function F

z	0	1	2	3	4	5	6	7
$F(r)$	0.00	0.00	0.00	1.05	2.45	4.55	5.95	7.00
v	0	0	0	1	2	5	6	7

Numerical example

Function H

r	0	1	2	3	4	5	6	7
$H(r)$	1.33	3.08	4.55	5.67	6.23	6.65	6.86	7.00
s	1	3	5	6	6	7	7	7

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$F(r)$	0.00	0.00	0.00	1.05	2.45	4.55	5.95	7.00
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Numerical example

Function H

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$H(r)$	1.33	3.08	4.55	5.67	6.23	6.65	6.86	7.00
s	1	3	5	6	6	7	7	7

Function F

z	0	1	2	3	4	5	6	7
$F(r)$	0.00	0.00	0.00	1.05	2.45	4.55	5.95	7.00
v	0	0	0	1	2	5	6	7

Function T

r	0	1	2	3	4	5	6	7
z	3	4	5	6	7	7	7	7

Numerical example

Original histogram

r	0	1	2	3	4	5	6	7
$p_I(r)$	0.19	0.25	0.21	0.16	0.08	0.06	0.03	0.02

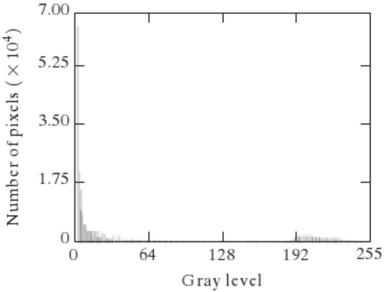
Specified histogram

z	0	1	2	3	4	5	6	7
$p_{I'}(z)$	0.00	0.00	0.00	0.15	0.20	0.30	0.20	0.15

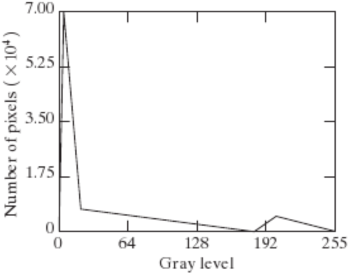
Real histogram

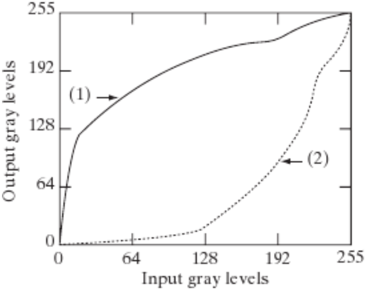
z	0	1	2	3	4	5	6	7
$p_{I'}(z)$	0.00	0.00	0.00	0.19	0.25	0.21	0.24	0.11

Histogram matching

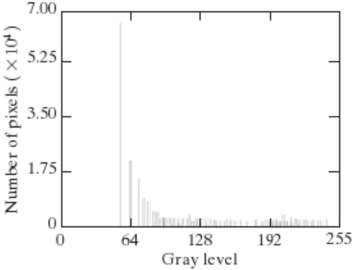


Histogram matching





Histogram matching



Local enhancement

for $i = r_i$ to m step $2r_i$
  for $j = r_j$ to n step $2r_j$
    $N(i,j) = \{\mathsf{I}_{yx} : i - r_i \leq y \leq i + r_i, j - r_j \leq j + r_j\}$
    compute T_{ij} via histogram equalisation or matching on $N(i,j)$
    $\mathsf{I}'_{ij} = T_{ij}(\mathsf{I}_{ij})$
  end
end

Local enhancement

```
for i = r_i to m step 2r_i
  for j = r_j to n step 2r_j
    N(i,j) = {I_yx : i - r_i <= y <= i + r_i, j - r_j <= j + r_j}
    compute T_ij via histogram equalisation or matching on N(i,j)
    I'_ij = T_ij(I_ij)
  end
end
```

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    $N(i,j) = \{\mathrm{I}_{yx} : i - r_i \leq y \leq i + r_i, j - r_j \leq j + r_j\}$

    compute T_{ij} via histogram equalisation or matching on $N(i,j)$

    $\mathrm{I}'_{ij} = T_{ij}(\mathrm{I}_{ij})$

  end

end

Implementation issue

How do we compute the histograms?

Local enhancement

```
for i = r_i to m step 2r_i
  for j = r_j to n step 2r_j
    N(i,j) = {I_yx : i - r_i ≤ y ≤ i + r_i, j - r_j ≤ j + r_j}
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```

for $i = r_i$ to m step $2r_i$
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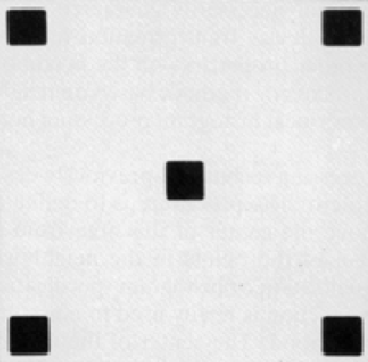
    $N(i,j) = \{\mathrm{I}_{yx} : i - r_i \leq y \leq i + r_i, j - r_j \leq j + r_j\}$
    compute T_{ij} via histogram equalisation or matching on $N(i,j)$
    $\mathrm{I}'_{ij} = T_{ij}(\mathrm{I}_{ij})$
  end
end

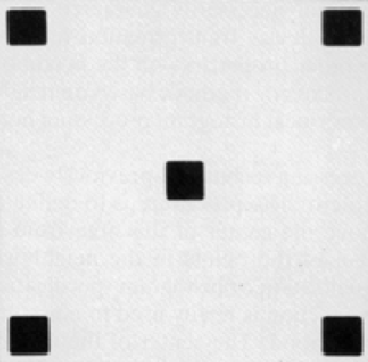
Implementation issue

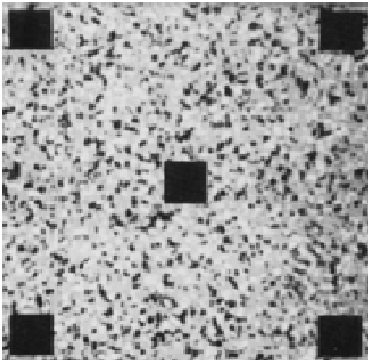
How do we compute the histograms?
Hint: only one column changes for each inner cycle, and only one row for each outer cycle.

Local enhancement



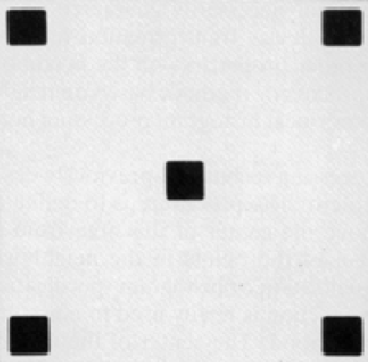
Local enhancement

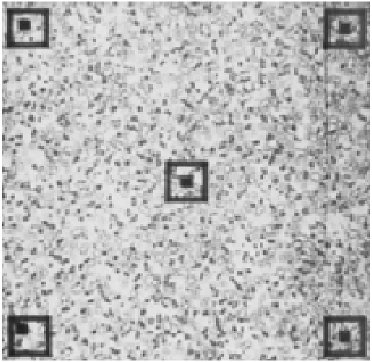




Hist-eq

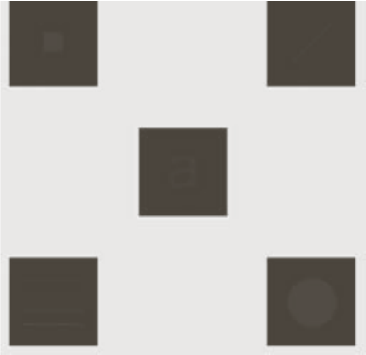
Local enhancement



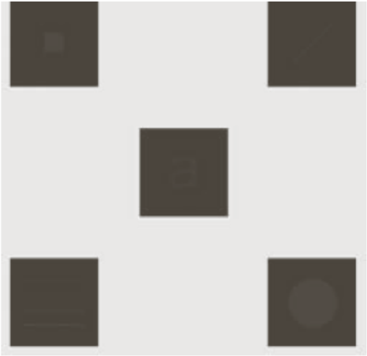


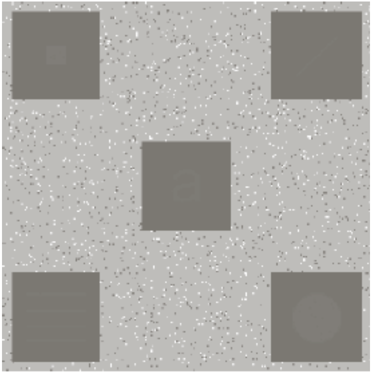
Loc-enh 7×7

Local enhancement



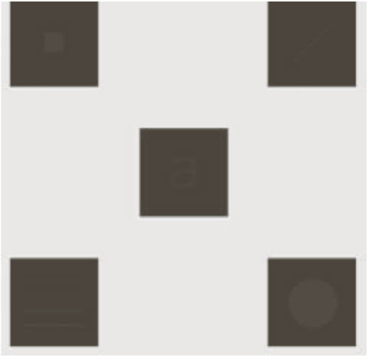
Local enhancement





Hist-eq

Local enhancement



Loc-enh 3×3

Histogram statistics

Ingredients

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- $$\mu = \sum_{0}^{G} r_i p_{\mathrm{I}}(r)$$

Histogram statistics

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- $$\mu = \sum_{0}^{G} r_i p_{\mathrm{I}}(r)$$

- $$m_n = \sum_{0}^{G} (r_i - \mu)^n p_{\mathrm{I}}(r)$$

Histogram statistics

Ingredients

- $$\mu = \sum_{0}^{G} r_i p_{\mathrm{I}}(r)$$
- $$m_n = \sum_{0}^{G} (r_i - \mu)^n p_{\mathrm{I}}(r)$$
- $$\sigma^2 = \sum_{0}^{G} (r_i - \mu)^2 p_{\mathrm{I}}(r)$$

Histogram statistics

Statistics can be used:

- globally: measured over the entire image
- locally: measured over pixel neighbourhoods

Histogram statistics

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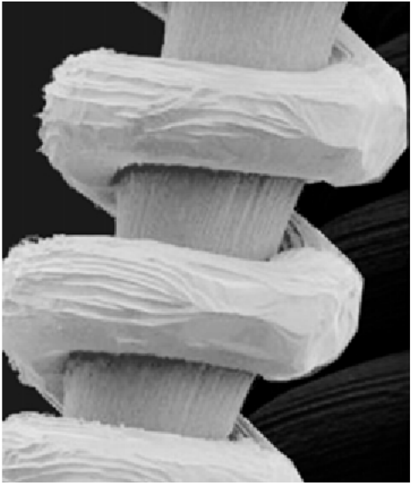
- globally: measured over the entire image
- locally: measured over pixel neighbourhoods

Local measurement

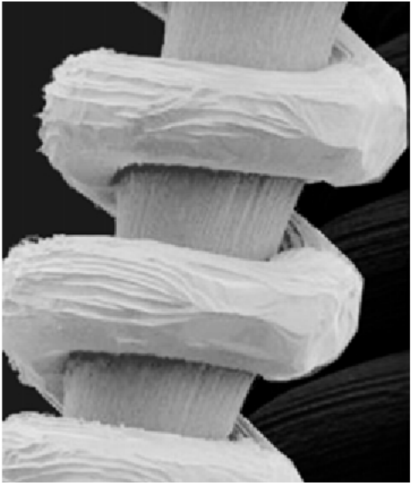
For each pixel (i, j)

1. $N(i,j) = \{\mathrm{I}_{yx} : i - r_i \leq y \leq i + r_i \, , \, j - r_j \leq j + r_j\}$
2. Compute $p_{\mathrm{I}(N(i,j)}$
3. $\mu = \sum_0^G r_i p_{\mathrm{I}(N(i,j))}(r)$
4. $m_n = \sum_0^G (r_i - \mu)^n p_{\mathrm{I}(N(i,j))}(r)$
5. $\sigma^2 = \sum_0^G (r_i - \mu)^2 p_{\mathrm{I}(N(i,j))}(r)$

Histogram statistics

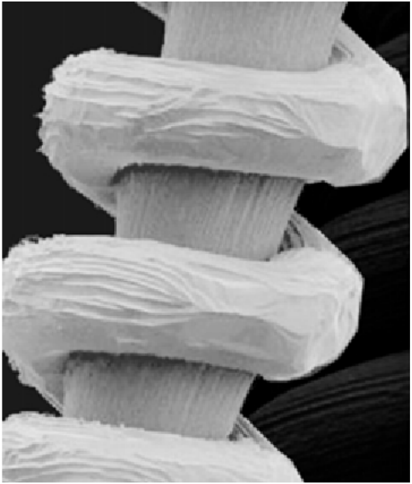


Histogram statistics



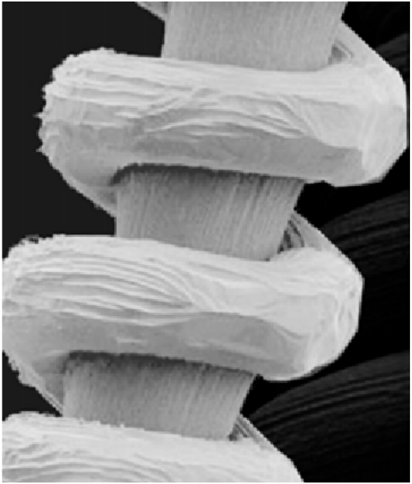
Task: enhance the dark filament on the right side of the image

Histogram statistics



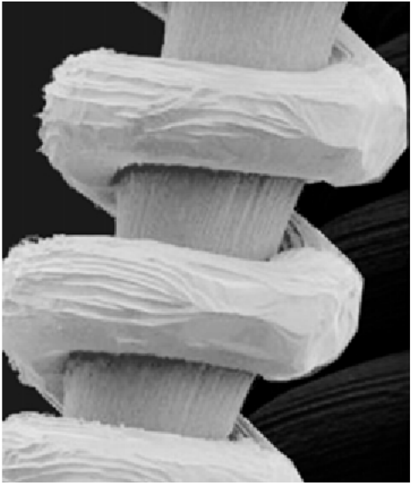
- enhance only dark areas

Histogram statistics



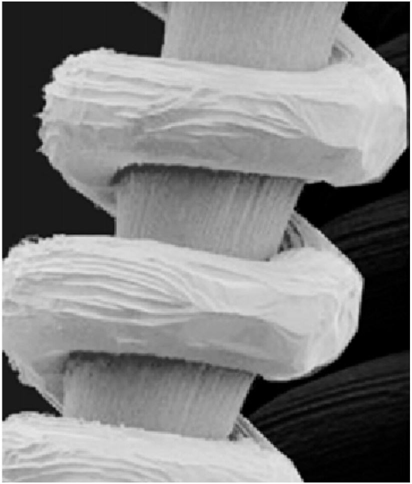
- enhance only dark areas
- discriminate dark areas from bright areas

Histogram statistics



- enhance only dark areas
- discriminate dark areas from bright areas
- determine areas with low contrast

Histogram statistics

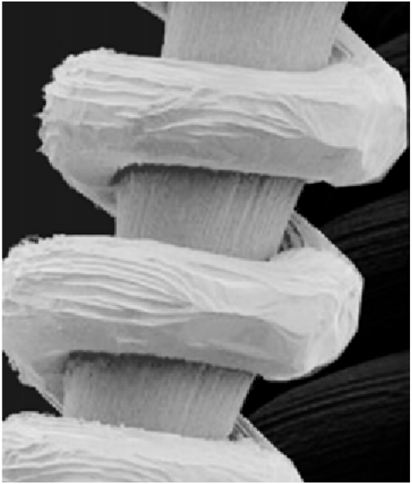


- enhance only dark areas
- discriminate dark areas from bright areas
- determine areas with low contrast
- discriminate low contrast areas from flat areas

Histogram statistics

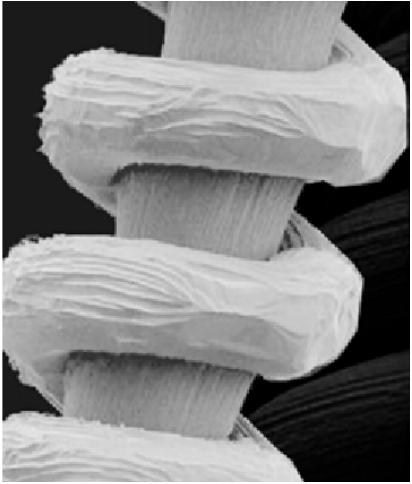
$$\mathrm{I}'(i,j) = \begin{cases} E \cdot \mathrm{I}(i,j) & \mu_{N(ij)} \leq k_0 \mu_{\mathrm{I}} \wedge k_1 \sigma_{\mathrm{I}} \leq \sigma_{N(i,j)} \leq k_2 \sigma_{\mathrm{I}} \\ \mathrm{I}(i,j) & \textit{otherwise} \end{cases}$$

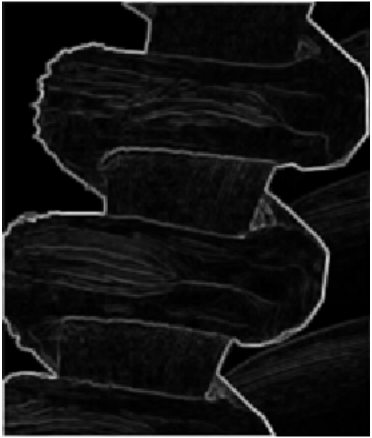
Histogram statistics



Local means

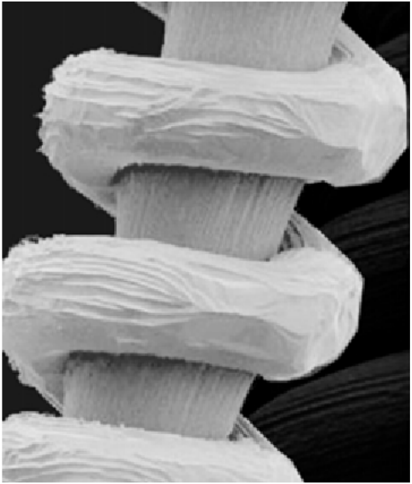
Histogram statistics





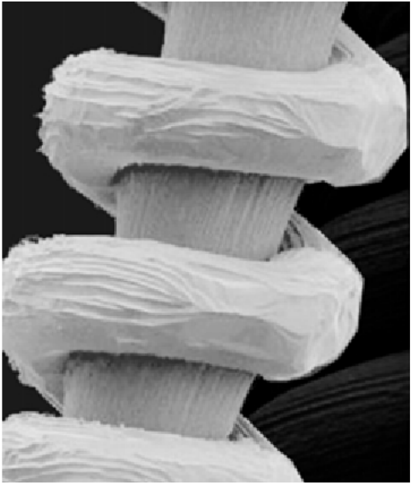
Local standard deviations

Histogram statistics



Multiplication constants

Histogram statistics



Enhanced image