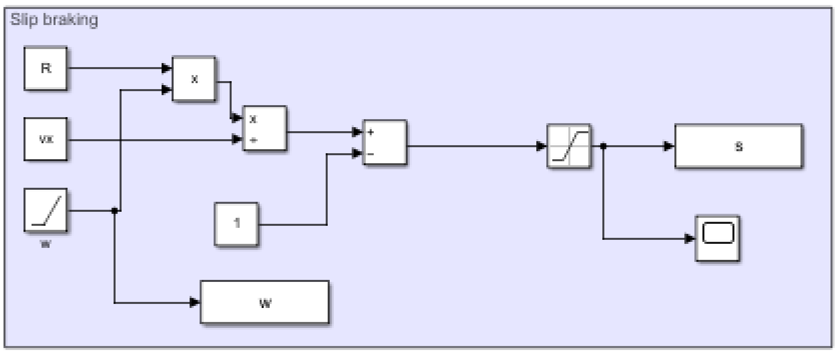
# Tyre transient model and wheel rotational dynamics

The aim of this exercise is to properly develop a model for the tyre dynamics considering the slip for both traction and braking conditon and the contribution of the relaxation length.

## Slip (braking)

First thing first, it is necessary to develop the slip block for the simulink model. The slip is defined as:

$s = \dfrac{\Omega - \Omega_0}{\Omega_0}$ which can be written also with the velocity: $s = \dfrac{V - V_x}{V_x}$ just dividing everything for the rolling radius (with the hypothesis that the effective radius and the loaded one are equal). The interpetration of the slip is simply "how far the tyre is from pure rolling". Having the formual defined, the corresponding Simlunk model can be developed as the following one:



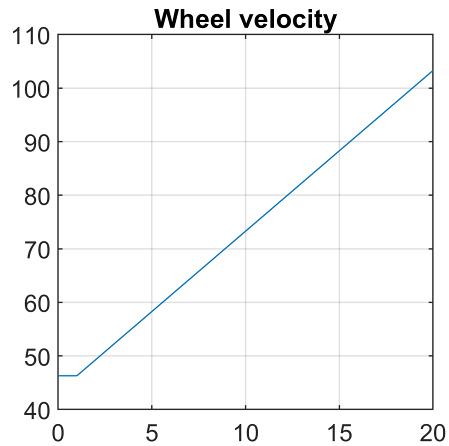

Which can be tested with the required conditions:

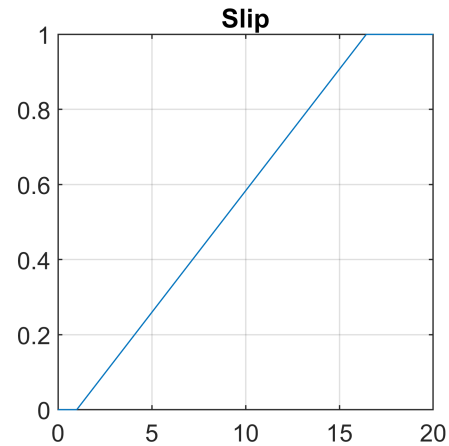
```
close all; clc; warning('off','all');
% Braking condition
Ts = 20;                    %[s]
vx = 50/3.6;                %[m/s]
R = 0.3;                    %Effective wheel radius [m]
w_slope = 3;               %[rad/s^2]

% Longitudinal force
Fx_max = 5*10^3;
Ck = 5000/0.1; %deduced form the graph on the slide
s_max = 10/100;
s_slope = s_max-0/(Ts-0);

open('sim_SlipBlock'); sim('sim_SlipBlock')

figure
subplot(1,2,1);
plot(w.time,w.data(:,1)); grid on;
title('Wheel velocity'); axis square;
subplot(1,2,2);
plot(s.time,s.data(:,1)); grid on;
title('Slip'); axis square
```

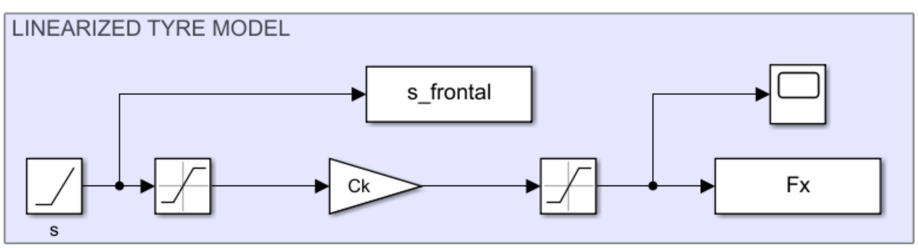


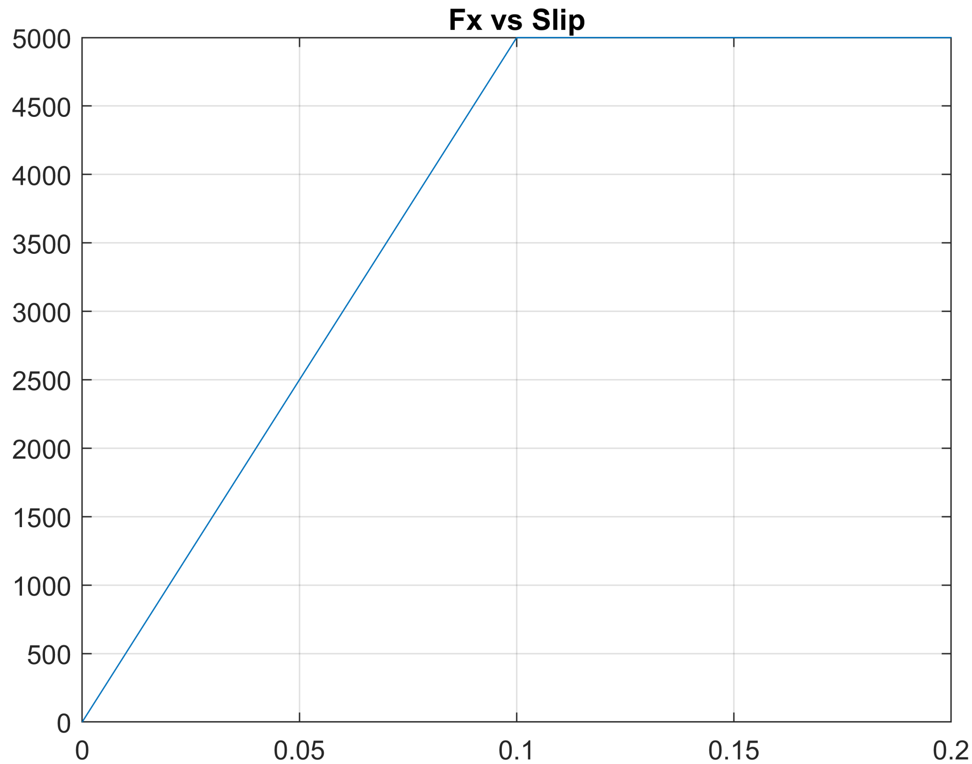

The situation presented represents a locking phenomena which occurs since it has been imposed a constant deceleration. Please note how for convection the slip has been saturated to -1 since values bigger than this mean have no physical sense.

## Linearized tyre model

The longitudinal force applied by the wheel is going to be assumed to be linear with a saturation effect for high values of the slip. Therefore the equation is trival as the Simulink model:




```
figure
plot(s_frontal.data(:,1),Fx.data(:,1)); grid on;
xlim([0 0.2]); title('Fx vs Slip');
```



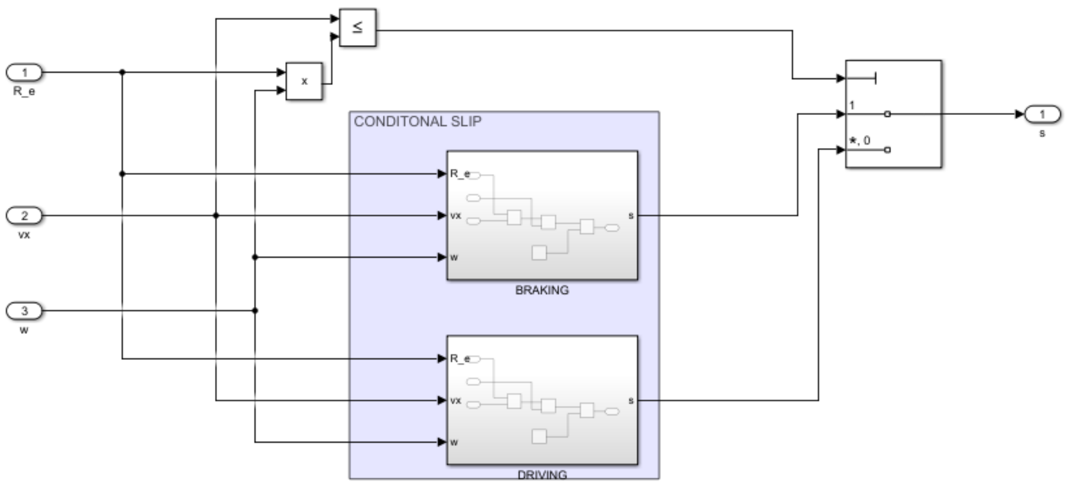

The following model tries to resamble the behaviour observed in the previous exercise, where the peak has been neglected but there is an horizontal asymptot.

## Conditional slip

However, it has been assest that if the traction situation is considered instead, the slip formual adopted wolud give some numerical problems. In fact, the effective radius when the wheel is locked due to braking theoretically it should rise to infinite values, meanwhile in the wheel is drifting, its velocity is null making the ratio going to minus infinite. For this reason, it is resonable to separate the two slipping formula depending on the condition given: braking or traction.

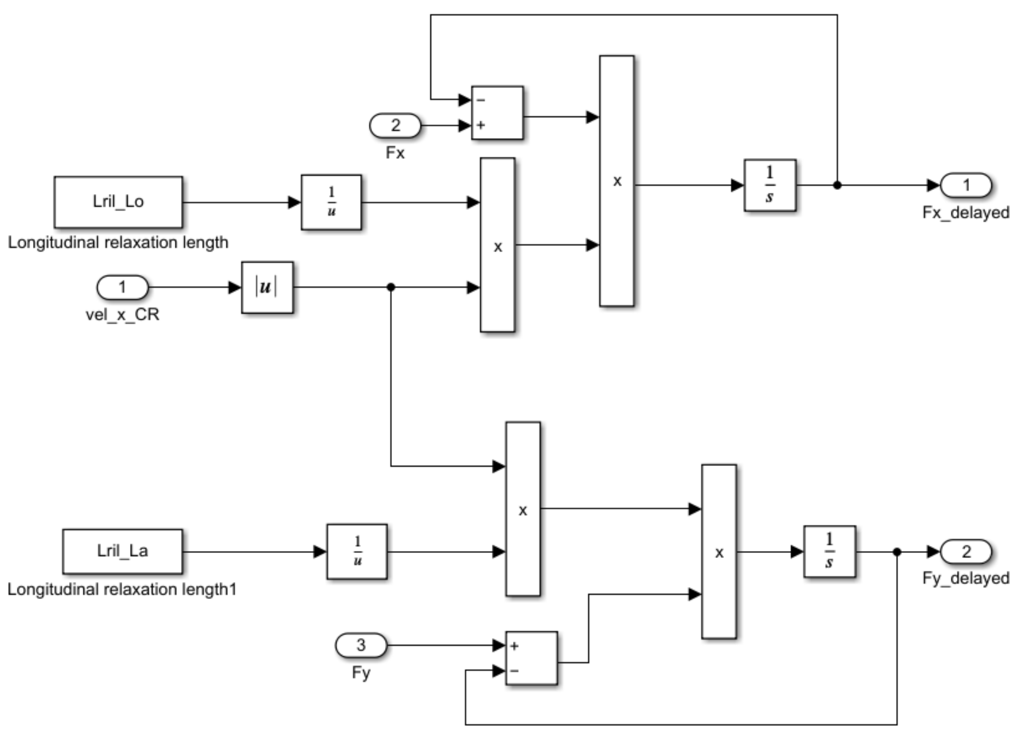
How to distinguish the two? Simply considering that if the velocity of the vehicle is higher than the velocity of the wheels then the vehicle is experiencing a braking condition *(Take as reference braking over ice, vehicle is going foward but the wheel is blocked!)*.

IF w*Re <= Vx then braking condition (wheel slows down faster than vehicle)




## Relaxation length

Relaxation length is a property of pneumatic tires that describes the delay between when a slip angle is introduced and when the cornering force reaches its steady-state value. In the model proposed, it has been developed as a simple delay effect using the Laplace domain: $\frac{1}{1+\tau s}$.




To run the model, it is necessary firstly to lunch the following file which has some wheel data.

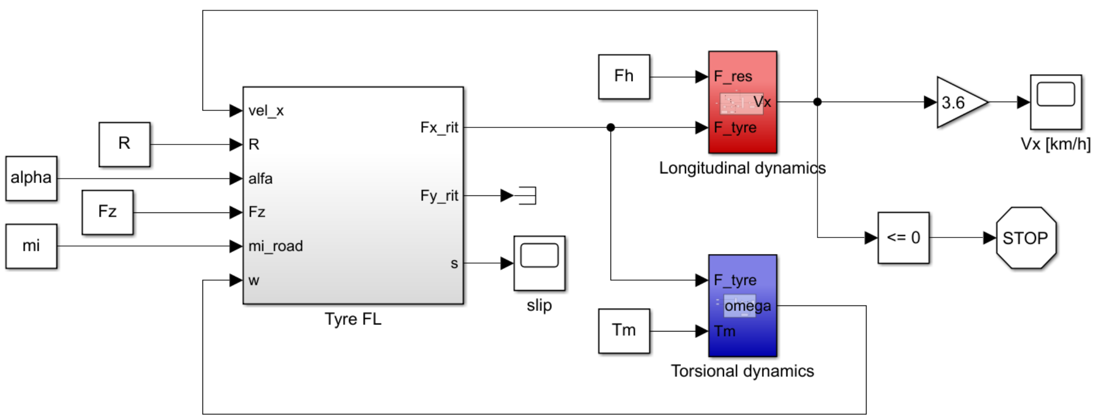
```
Es02_single_wheel_dynamics
```

```
Longitudinal dynamics
Initial speed: 20 km/h
Road slope: 4%
```

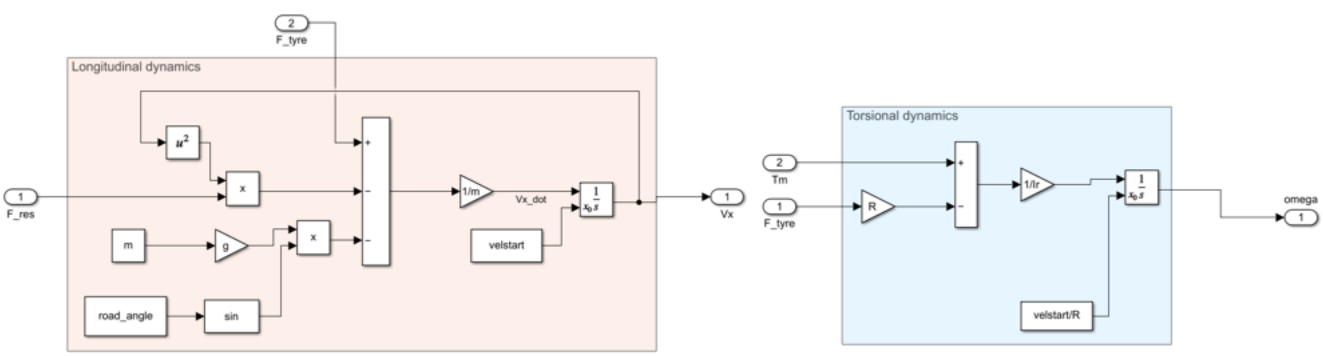
Also the **radiar tyre deformation** is a phenomena which occurs in normal tyres. The deformation of the wheel implies a change in the arm, which implies torque. In the simulation, it has been assumed that there is no change in the wheel radius (remeber the consideration that efective rolling radius and loaded one are equals).

## Single wheel dynamics

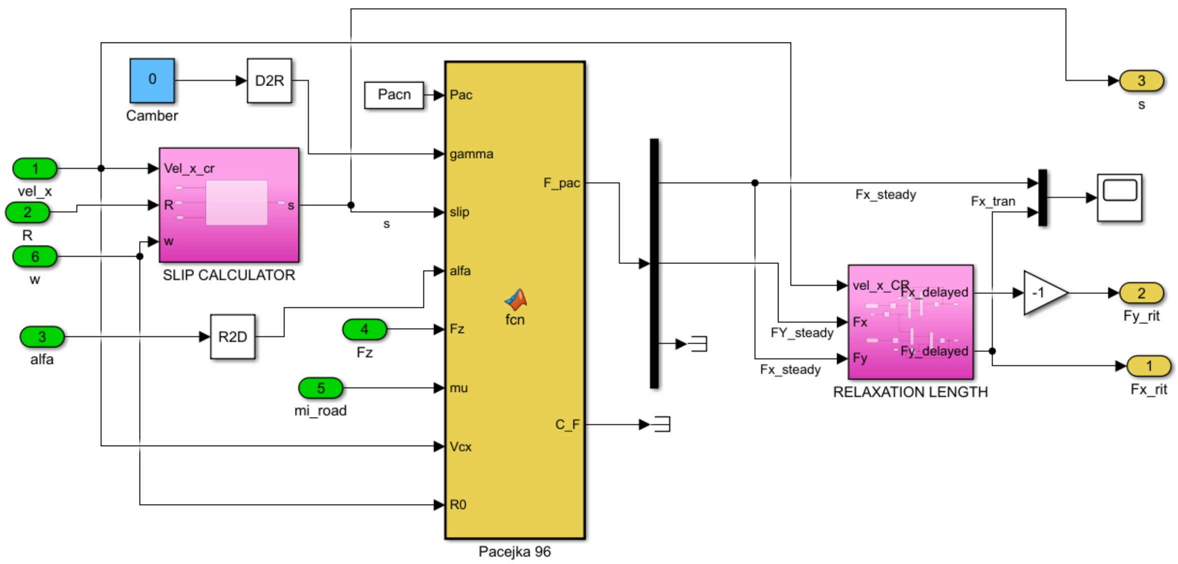
Each effect has been developed so to be inlcuded into a more general tyre model. Firstly the wheel model presents as follows:




Where the two coloured blocks have been developed during the exercises consdiering the dynamic motion longitudinally and rotationally of the wheel, developed in Simulink in the following way:




Meanwhile the previous developed blocks have been introduced inside the `Tyre FL` block:



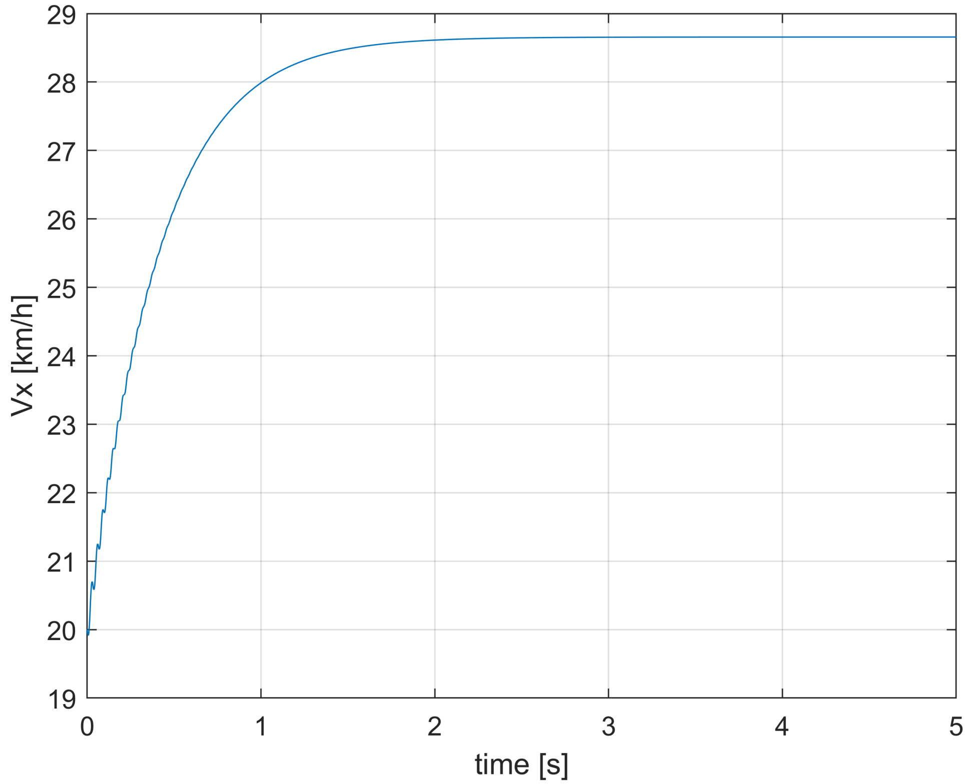

The model can be run, since the useful simulation settings are included into the previous launched `MATLAB` file.

The condition set for this simualtion is that the wheel has been putted over a benchmark (with an incliantion of 2.2906°) whose grip coefficent is 0.9, which is nearly perfect adherence. The vertical force applies is of 4500 N, a standard value for a single wheel in passanger cars, with no side slip applied. Driving torque of 100 Nm also suggest a vary ideal situation.

```
sim('wheel_qc_VDS_base')
```

With this premises, the wheel develops enough force to actually accelerate thanks to the great torque applied and stabilised it velocity value around 28.6 km/h.

```
figure(5)
plot(Vx(:,1),Vx(:,2)); grid on; xlabel('time [s]'); ylabel('Vx [km/h]'); hold on;
```

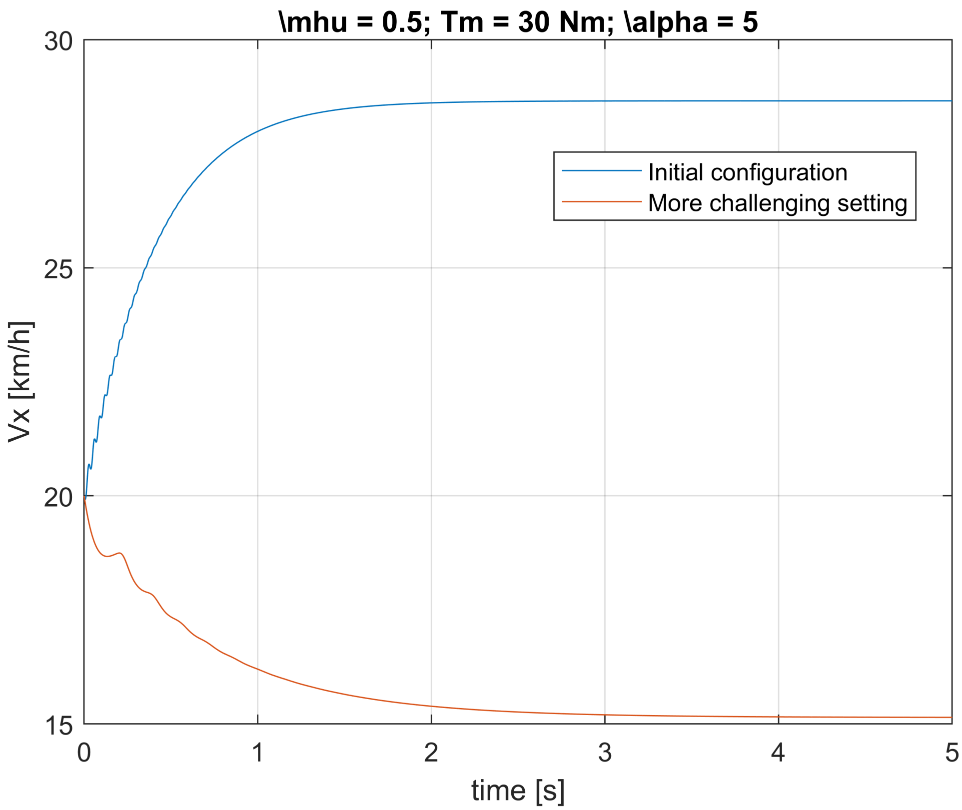



## Further analysis

It is of our interest to study how the model react to some changes. Firstly, it is intresting to modify the wheel side slip, putting it different from 0. Moreover, the grip will be dimished so to make more difficult the traction condition.

```
alpha   = deg2rad(5); % [rad] sideslip angle
Tm      = 30; % [Nm] driving torque at the wheel
mi      = 0.5; % road friction coefficient

sim('wheel_qc_VDS_base')
figure(5)
plot(Vx(:,1),Vx(:,2)); grid on; xlabel('time [s]'); ylabel('Vx [km/h]');
title('\mhu = 0.5; Tm = 30 Nm; \alpha = 5')
legend('Initial configuration','More challenging setting','location','best')
```




As expected, the wheel has more difficulties in its motion and it slows down. However the slips values never exceeds values of

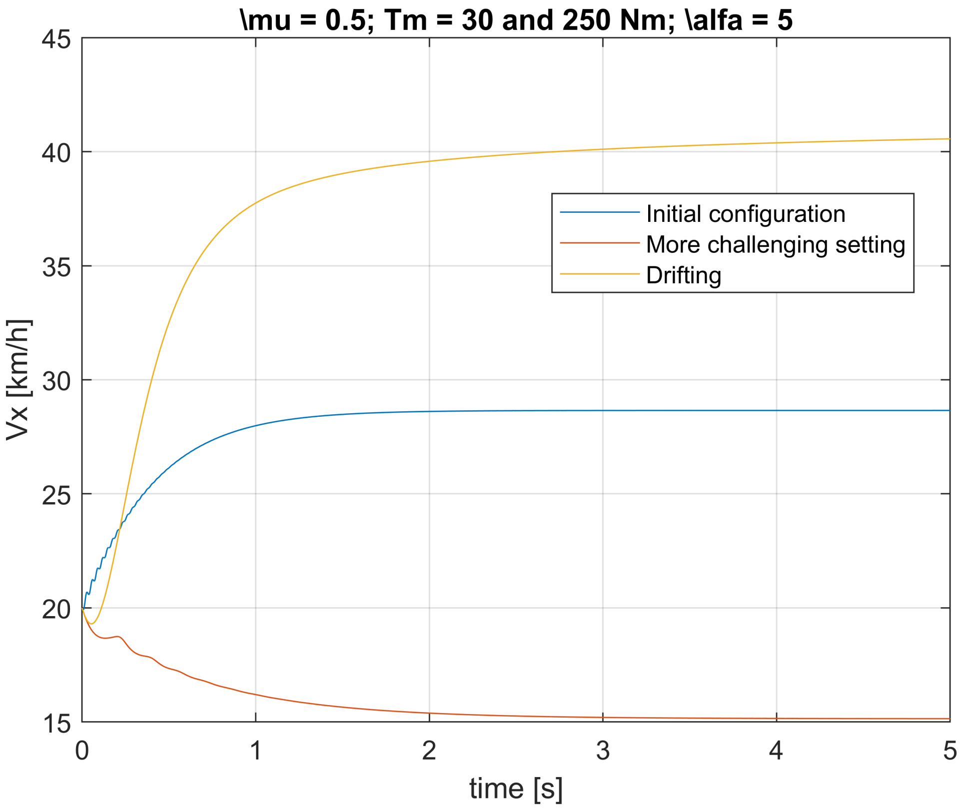
```
disp(num2str(max(slip(:,2))*100))
```

```
15.9179
```

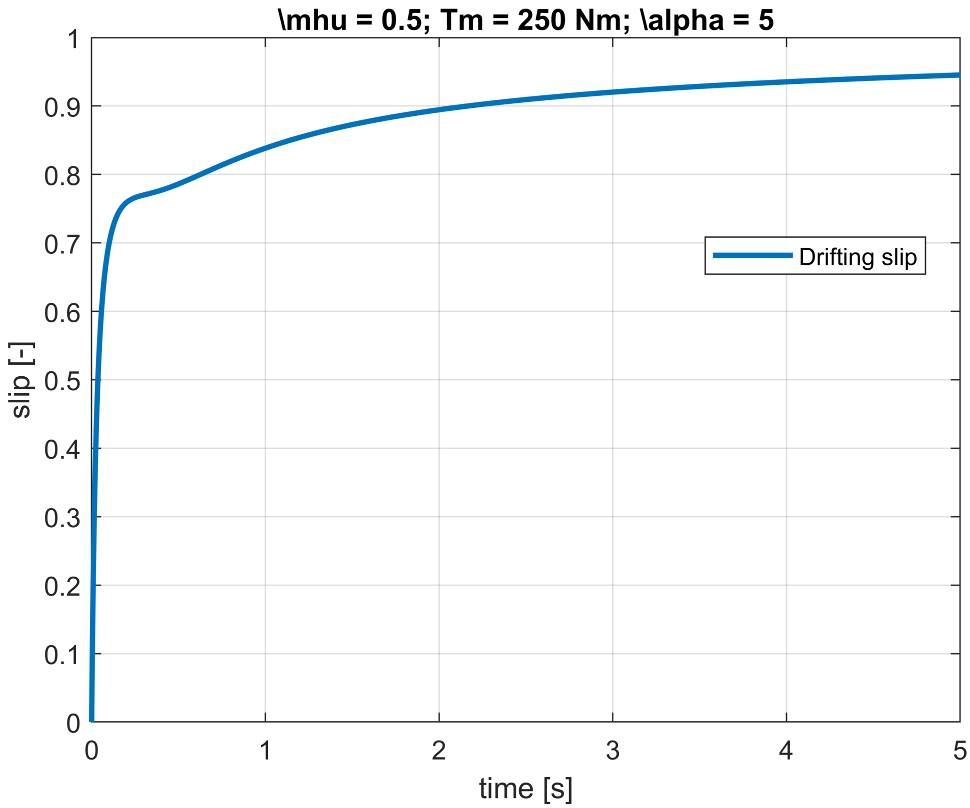
Suggesting that the even in this more difficult condition the tyre overall stays within small values.

To test really the model adopted a **drifting** condition can be imposed. It seems like a flex, by a certain point of view it is, however it could be usefull to observe how the model reacts to drastic conditions.

```
alpha   = deg2rad(5); % [rad] sideslip angle
Tm      = 250; % [Nm] driving torque at the wheel
mi      = 0.5; % road friction coefficient
sim('wheel_qc_VDS_base')
figure(5)
plot(Vx(:,1),Vx(:,2)); grid on; xlabel('time [s]'); ylabel('Vx [km/h]');
title('\mu = 0.5; Tm = 30 and 250 Nm; \alfa = 5')
legend('Initial configuration','More challenging setting','Drifting','location','best')
```




```
figure(6)
plot(slip(:,1),slip(:,2),'Linewidth',2); grid on; xlabel('time [s]'); ylabel('slip [-]');
title('\mhu = 0.5; Tm = 250 Nm; \alpha = 5')
legend('Drifting slip','location','best')
```




Here, the torque applied to the wheel has been increased up to unfeasible value, just to test how the model will react. In fact, it is possible to study how the torque is so high that initially the wheel cannot make contact. In fact the velocity decreases but the slip exponentially goes nearly to 1 suggesting that the wheel is actually drifting. But as soon as the wheel makes contact again (visible from the change of the concavity in the slip chart) some grip is making contact and the wheel will accelerate immediatly.

Whai if we test this model with negatove torque values assuming a braking action? Well, it does not work!

The wheel torsional dynamics has been developed only considering traction, in fact the model's corresponding equation does not consider any breaking condition since it does not include any interia contribution, so as the fact that there is a change in the rotation directions (contact force in traction counterbalance the driving torque, meanwhile in breaking it opposes to the breaking torque).