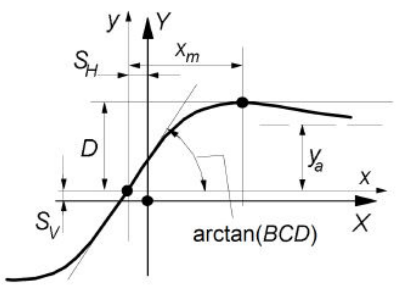
TYRE MODELS

Tyre highly effects the car responces since they are the connection point between vehicle forces and ground. Furthemore, tyres are very complex systems, highly non linear and with tons of parameters. Not by chance, the first academic who was able to develop a function for describing the tyre forces ascended into the Olimpus of engineering: Pacejka and his work made a considerable historic chnage to the automitive industry (in Formula1 they still make references to Pacejka models to be clear!).

Pacejka formulation is a formual which permit to evaluate the longitudinal force as a function of the slip (s, σ or k), tyre slip angle (α) and tyre characteristics ($Cornering\ stiffiess\ C$). It has its limit as a formulation, like the incapacity to link directly longitudinal and lateral froces (fot this the elliptical model ahs to be adopted).

A general representation of the Pacejka formualtion is:



Where it is possible to distinguish the important parameters:

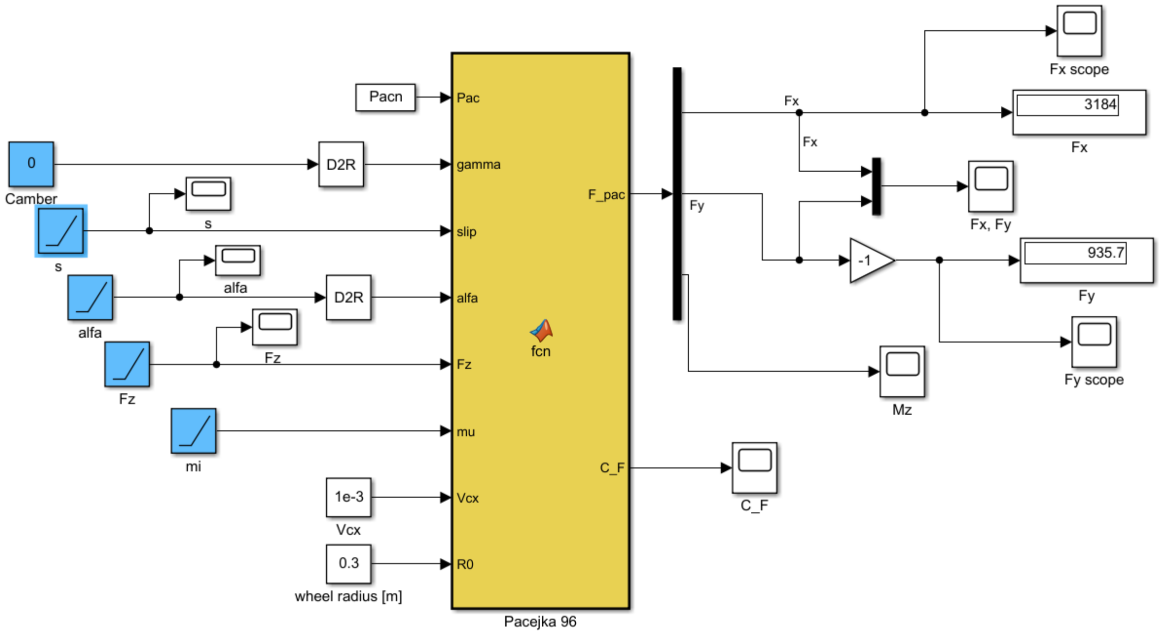
- X and Y can be whichever input (s,α) and output (F_x, F_y, etc...)
- B: Stiff factor.
- C: shape factor.
- D: peak value
- Product BCD: Slope of the curve close to the origin which permit to linearize the system around the origin (think of it like a Taylor expansion!). Very usefull for understading the behaviour of the overall system, also considering that in standard commercial vehicle the slip rarely overcome values of 20%.

Investigation:

The system is modeled with a scratch model given by the professor, from which it is possible to obtain sevral information. Aim of this exercise is to run several simualtions and to investigate the results.

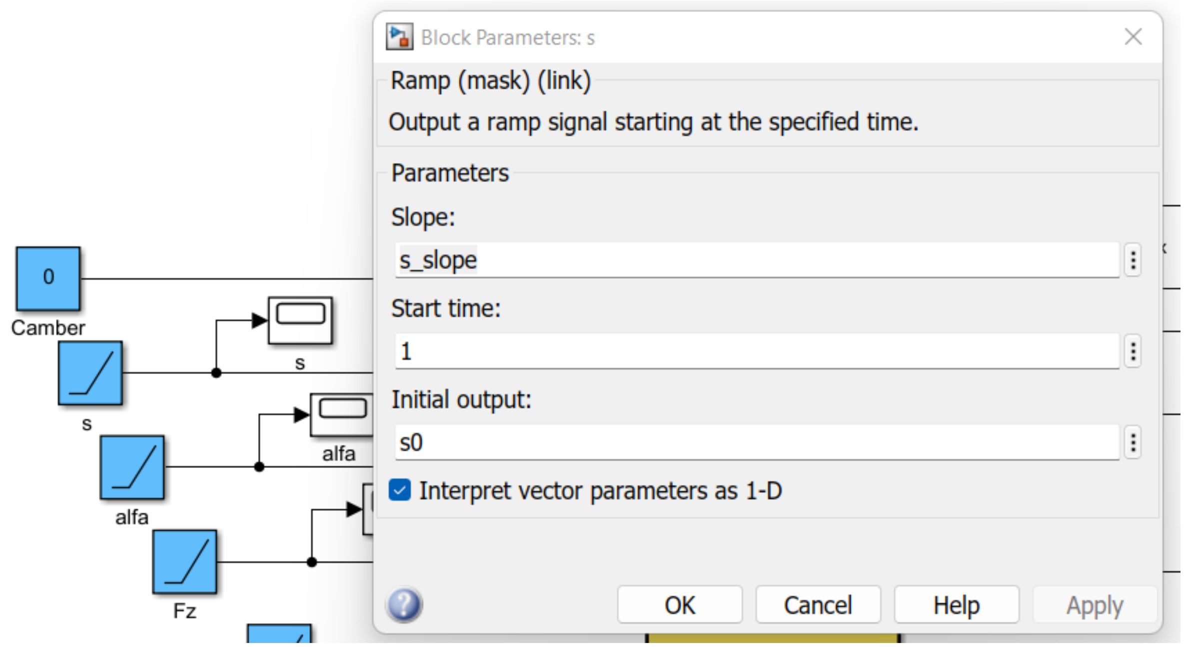
```
% Usefull set ups
close all; clc; clear; warning('off','all');

% Initialisation
INI_Tyre_model
nome_modello_simulink = 'Tyre_Pac96';
open(nome_modello_simulink)
```

The general overview of the model is here proposed. The task is to develop the charts of tyre-ground contact in function of slip, wheel side slip angle, friction coefficent and vertical force. The Pacejka block requires all this variables as input so as all the peculiar parameters value of the model. They are stored in the `INI_tyre_model` script and they are derived from experimental observation.



Regarding our simulation setting, there are several dependecy for each force. Only come of them are of our interest but also what is important is to see also the depency over varying some fixed value like the friction coefficent and the vertical load. For sake of semplicity, here the variable for the plot is going to be called *argument* meanwhile the parameter which changes quantumly will be declared as *variable*. In other words, to be more clear, the argument is going to vary linearly along the execution of the simulation, meanwhile the variable will change between each simualtion. In the for cycle the simulation will be launched and at the end of it the variable will change.

To model the linear variation of the argument, the ramp signal can be adopted in Simulink like in the example here shown (taken from the first point where the slip is the argument and the vertical laod is the variable):



Therefore, in the `MATLAB` script, whenever a certain argument is set, it is necessary to specify that it has a certian slope which has been computed manually of on a paper, in order to obtain the same results presented in the professor slides.

With this procedure set, all the following points can be calculated, therefore from now on only the theoretical discussion is going to be done. Furthermore on the slides some theoretical questions are proposed and my personal answer is going to be given here.

Of course I want to remeber that I'm not a professor neither a Ph.D but just a collegue of yours, which means that some incorrectness could be present so as unclear explanations.

Variation vertical load

1. Fx = f(s)

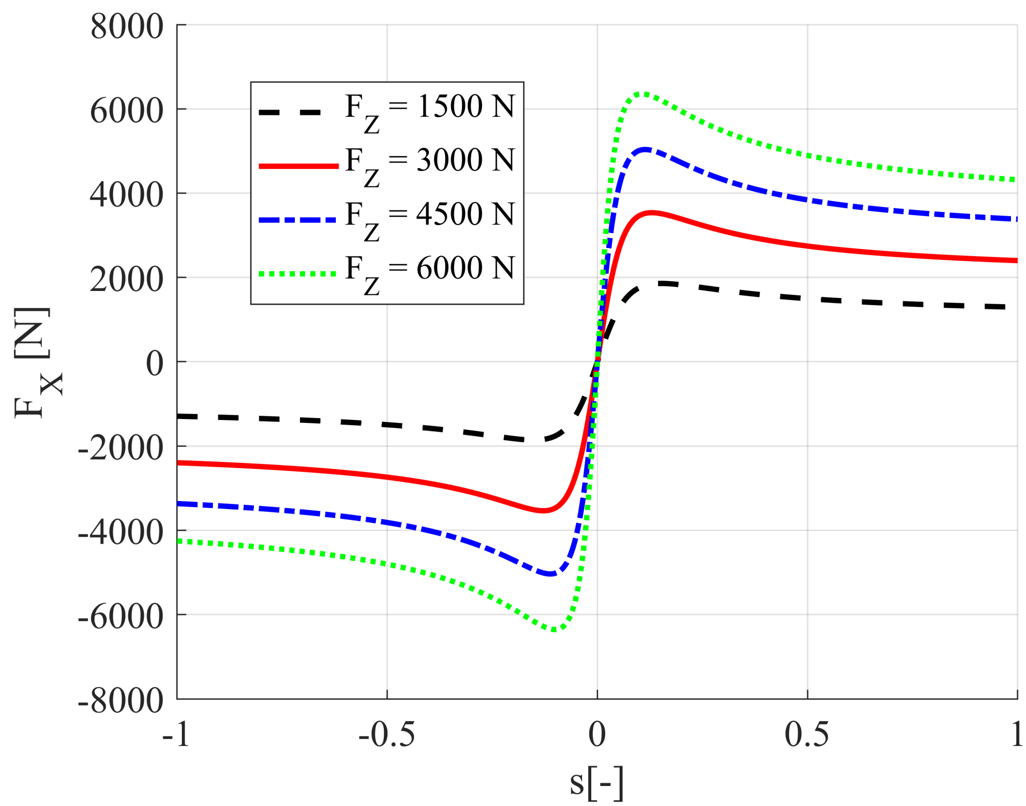
Variables:

```
% friction coefficient
Ts = 11;
mu_slope = 0;
mu0 = 1;
alfa0 = 0;
alfa_slope = 0;
Fz_vet = [1500 3000 4500 6000];
Fz_slope = 0;
s0 = -1;
s_slope = 0.2;
```

Plots settings and simulation runs:

```
F_Size = 14; % FontSize
plotcol = {'--k','-r','-.b',':g','.m','-k'};

% Graphics setting
```

```
plotleg = {'F_Z = 1500 N','F_Z = 3000 N','F_Z = 4500 N','F_Z = 6000 N'};
nome_fig = 'Fx-s';
figure('Name',nome_fig,'NumberTitle','off','PaperType','A4')
hold all; grid on

% For cycle (varying the requested parameter., e.g., vertical load)
for cont1 = 1: length(Fz_vet)
    Fz0 = Fz_vet(cont1);
    % execute simulink model
    string_sim = strcat({'sim '},{nome_modello_simulink});
    eval(string_sim{1})
    plot(s(:,2),Fx(:,2),plotcol{cont1},'Linewidth',2);
end

% Plot formatting and saving
legend(plotleg,'Location','best')
set(gca,'FontName','Times New Roman','FontSize',F_Size)
xlabel('s[-]'); ylabel('F_X [N]')
```




```
% print -dpng Fx_s.png; saveas(gcf, 'Fx_s', 'fig')command for saving
% figures as png
```

The Pacejka model permit to define the traction (or braking) force as a s-shaped function. As a convection, the negative slip is considered as the breaking one and it is possibile to assest that during traction or breaking the developing of contact forces is equal. In fact for the tyre-ground contact, what changes between the two conditions is only the direction of the moment which is giving the movement (during traction the axle is spinning the wheels, meanwhile during breaking the moment of intertia so as the longitudinal force moment are counteracting th ìe breaking torue).

For low values of the slip, it is possible to assest how the linearization holds and this is true till the maximum is reached, which, reading with the matlab ruler, is around $s = 0.13$. Please note how the maximum value of the longitudinal force is obtained always for the same value of slip indipendently on the vertical force applied. The maximum point of grip is the reference for both traction and breaking, but it nearly impossible to reach it exactly, this is due to a very fundamental characteristic of the tyre model: it is very unstable!

Imagine a ball running a surface which has the same shape of the chart obtained, in order to not fall the ball should be in a very specific equilibrium condition coincident with the maximum value, but it could take nothing that the ball start again to run somewhere. If it fell off the linear condition, it is true that the model could be simplified, but the longitudinal force drops! Also the other direction is not the ideal one since the slip increases too much, but at least here the longitudinal force it does not decrease too much, which could be acceptable for traction (but remember that the slip is a bit too much) but in breaking not at all. The latter is due to the fact that the longitudinal force counteract the breaking torque, so if the tyre-ground force is quite big, the breaking torque to be applied should be quite big as well.

Furthemore, please remember that all this conditions (traction, breaking or manoeuvres in general) and not done in steady state, which means that it is resonable to imagine a ball running all over, up and down over the curve during a simple corner. From a physical point of view than it is resonable to assume that the tyre could reach easily rolling motion or very very high slip conditions unstable condition.

To be honest, pure rolling condition is not at all easy to achive during transient due to the contact patch deformations. However during stable condition like running on an highway the slip does not increases so much, permitting to linearize with ease the tyre, but of course the same cannot be sayed when breaking on a downhill or for a Formula1 car.

The vertical force has a proportional influence over the longitudinal force, not by chance their correlation is linked trough the adherence coefficent μ, defined exactly as the normalaised coefficent which connects these two quantities. From a physical point of view, it means that having more weight on a certian wheel will implies that that tyre would develop more longitudinal force. Therefore during traction this is welcomed as a behaviour meanwhile with braking depends, since it could imply locking phenomena and on the rear axle this has to be avoided since it could imply oversteering and consequently unstability.

2.Fy = f(alpha)

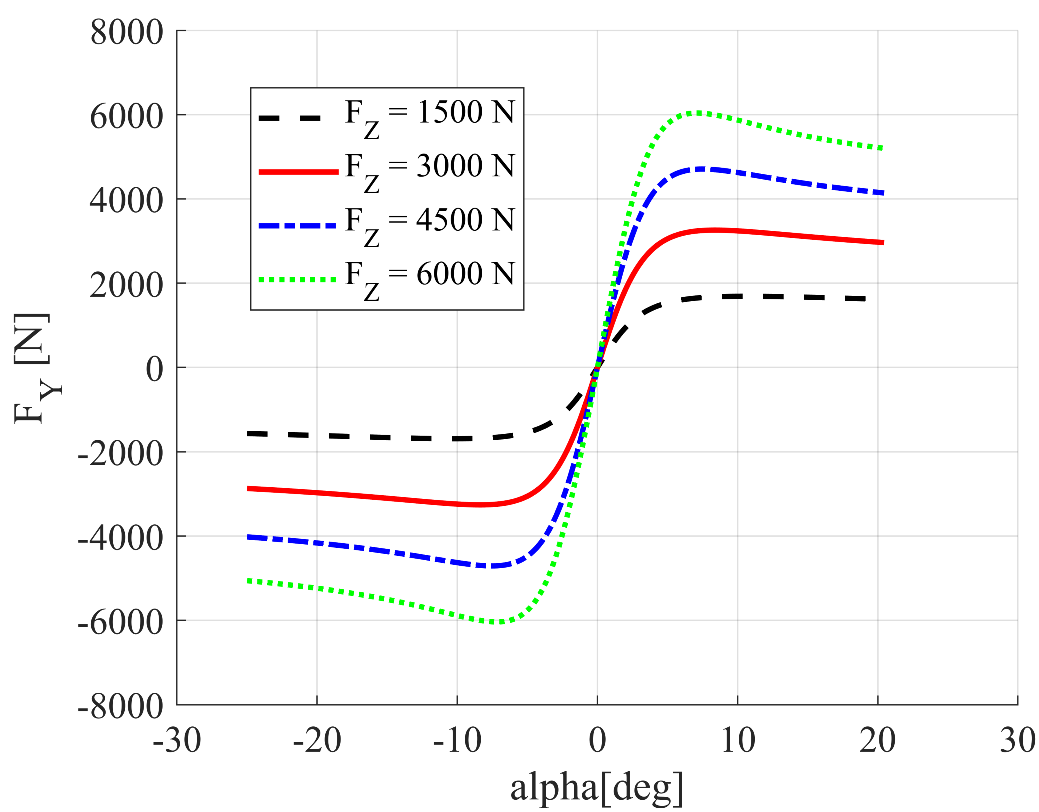
Variables:

```
% friction coefficient
Ts = 11;
mu_slope = 0;
mu0 = 1;
alfa0 = -25;
alfa_slope = (25--25)/Ts;
Fz_vet = [1500 3000 4500 6000];
Fz_slope = 0;
s0 = 0;
```

```
s_slope = 0;
F_Size = 14; % FontSize
plotcol = {'--k','-r','-.b',':g','.m','-k'};
```

Plots:

```
% Graphics
plotleg = {'F_Z = 1500 N','F_Z = 3000 N','F_Z = 4500 N','F_Z = 6000 N'};
nome_fig = 'Fy-alpha';
figure('Name',nome_fig,'NumberTitle','off','PaperType','A4')
hold all; grid on
% For cycle (varying the requested parameter., e.g., vertical load)
for cont1 = 1: length(Fz_vet)
    Fz0 = Fz_vet(cont1);
    % execute simulink model
    string_sim = strcat({'sim '},{nome_modello_simulink});
    eval(string_sim{1})
    plot(alfa(:,2),Fy(:,2),plotcol{cont1},'Linewidth',2);
end

% Plot formatting and saving
legend(plotleg,'Location','best')
set(gca,'FontName','Times New Roman','FontSize',F_Size)
xlabel('alpha[deg]'); ylabel('F_Y [N]')
```



The tendency of the lateral force as function of the wheel side slip angle could resamble the one of the longitudinal force with the slip, however some differences are present.

Firstly, negative side slip angles have nothing to do with breaking, therefore the symmetry respect to the origin meaning changes since hereit can be interpetred as the fact that the tyre extert the same behaviour no matter if the corner is turning right or left.

The linearization can hold for a very tiny region, since the slope changes quite a lot in the origin neighborhood. Moreover the maximum is no loger so easy to distinguish as it was for the previous case, since for low value of vertical force it seems that the maximum coincide with the asymptotic value.

However the general comments done in the previous chart over the concept of stability can be copy and pasted here.

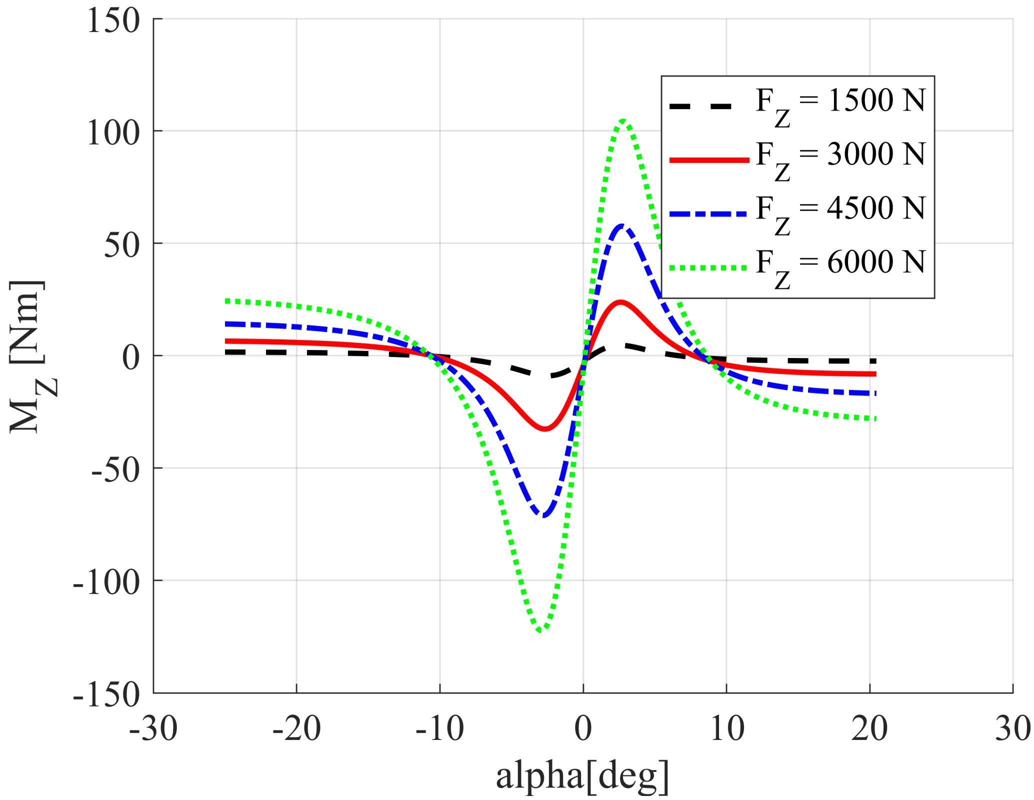
3.Mz = f(alpha)

Variables:

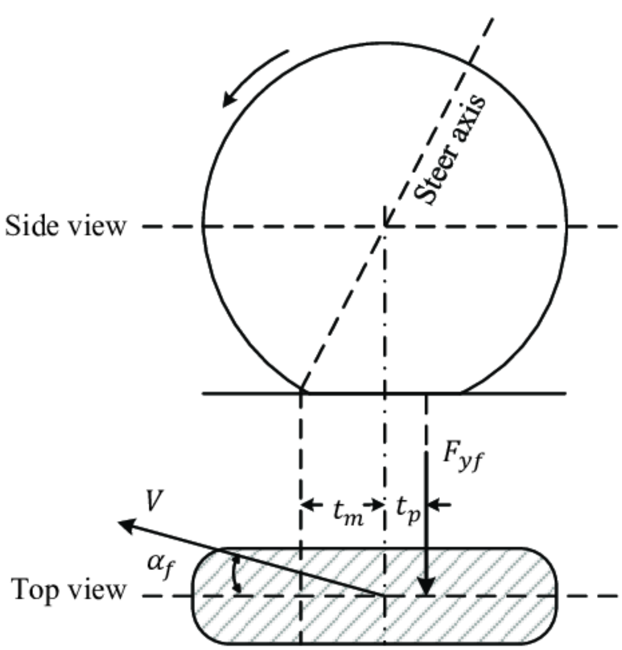
```
% friction coefficient
Ts = 11;
mu_slope = 0;
mu0 = 1;
alfa0 = -25;
alfa_slope = (25--25)/Ts;
Fz_vet = [1500 3000 4500 6000];
Fz_slope = 0;
s0 = 0;
s_slope = 0;
F_Size = 14; % FontSize
plotcol = {'--k','-r','-.b',':g','.m','-k'};
```

Plots:

```
% Graphics
plotleg = {'F_Z = 1500 N','F_Z = 3000 N','F_Z = 4500 N','F_Z = 6000 N'};
nome_fig = 'Fx-s';
figure('Name',nome_fig,'NumberTitle','off','PaperType','A4')
hold all; grid on
% For cycle (varying the requested parameter., e.g., vertical load)
for cont1 = 1: length(Fz_vet)
    Fz0 = Fz_vet(cont1);
    % execute simulink model
    string_sim = strcat({'sim '},{nome_modello_simulink});
    eval(string_sim{1})
    plot(alfa(:,2),Mz(:,2),plotcol{cont1},'Linewidth',2);
end

% Plot formatting and saving
legend(plotleg,'Location','best')
set(gca,'FontName','Times New Roman','FontSize',F_Size)
xlabel('alpha[deg]'); ylabel('M_Z [Nm]')
```



M_z can be interpetred as the self aligning torque of a tyre. In fact when a side slip angle occurs also some slippage occurs at the tyre ground contact. With some slip, the pressure distribution on the patch will chnage and with it the lateral force applied to the tyre will move as well. Considering that it has been assumed that the wheel rotates around its centre, this lateral force movement implies that the application point of the force will be different frm the centre, resulting in the end in a moment.



Considering the example shown in the picture, the moment created will let the tyre to align its velocity to its longitudinal direction, from here the name *self alignement*. Making reference to every day conditions, what happens if you impone a certian steering angle (and consequently a wheel side slip) and take off your hand from the steering wheel? It will return straight and this is due to the self alignment torque!

However, what happens if the side slip angle increases a lot? That the lateral force arrives to a point where pass from the rear of the wheel to the front, what happens next? The moment now does not align the wheel to the velocity vector but instead it will disalign them: it wont be anymore a self alignement but a divergent one.

All this is perfectly show from the chart obtained, where the latter consideration explains the physical meaning behind the torque crossing the x-axle at a certain side slip value. Also here, the symmetry respect to the origin resambles the same behaviour between left and right wheel. In the end, please note also how the maximums have the same value of slip.

Variation friction coefficent mhu

4.Fx = f(s)

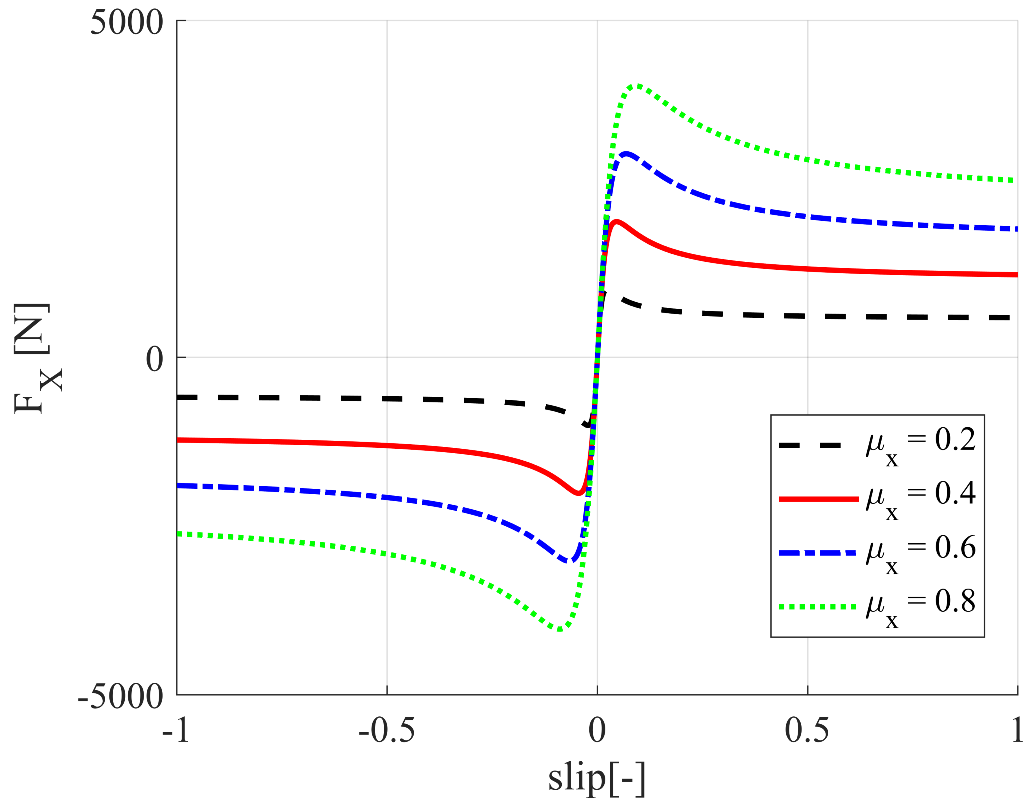
Variables:

```
% friction coefficient
Ts = 11;
alfa0 = 0;
alfa_slope = 0;
mhu_vet = [0.2 0.4 0.6 0.8];
mhu_slope = 0;
Fz0 = 4500;
Fz_slope = 0;
s0 = -1;
s_slope = 0.2;
F_Size = 14; % FontSize
plotcol = {'--k','-r','-.b',':g','.m','-k'};
```

Plots:

```
% Graphics
plotleg = {'\mu_x = 0.2','\mu_x = 0.4','\mu_x = 0.6','\mu_x = 0.8'};
nome_fig = 'Fx-s';
figure('Name',nome_fig,'NumberTitle','off','PaperType','A4')
hold all; grid on
% For cycle (varying the requested parameter., e.g., vertical load)
for cont1 = 1: length(mhu_vet)
    mu0 = mhu_vet(cont1);
    % execute simulink model
    string_sim = strcat({'sim '},{nome_modello_simulink});
    eval(string_sim{1})
    plot(s(:,2),Fx(:,2),plotcol{cont1},'Linewidth',2);
end

% Plot formatting and saving
legend(plotleg,'Location','best')
set(gca,'FontName','Times New Roman','FontSize',F_Size)
```

```
xlabel('slip[-]'); ylabel('F_X [N]')
```



The tendecy of changing the vertical load and the friction coefficent is the same, suggesting that the conclusion done before over the proportional link between vertical and longitudinal force trough the adherence coefficent is correct.

However the major chnage respect to before, is that here the abscissa value for the peak of the longitudinal force changes! For the smaller value of the grip coefficent it is around 0.02, meanwhile for values around the unitary one the slip value is about 0.08.

What is intresting to note is that the peak values varying mhu is smaller than the one obtained changing the vertical load, letting us to conclude that for the same variation of μ and F_z, a grip loss implies a bigger decrease of the longitudinal force respect to varying the vertical load.

Q1. when driving in the linear field (normal driving), can the driver notice different friction conditions, e.g., with passing from 1 to 0.6?

If its nearly impossible to appreciate the difference from the chart, from the car it is even more difficult! Therefore no, in fact the linearization could be applied to regardless the value of mhu.

Q.2 how is it possible to assess the actual friction conditions?

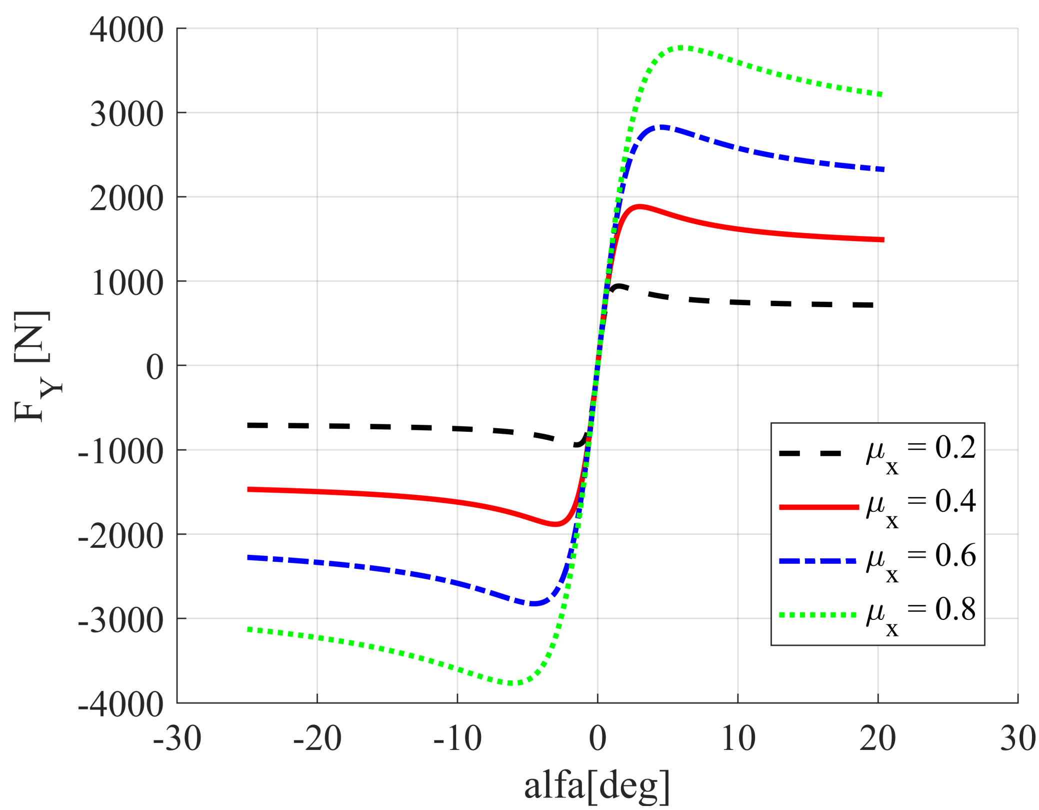
One idea is to make reference to the slip valiie of the maximum, since as said it shift depending on the grip coefficent. Another approach, maybe way less refined, could be to lock voluntary the wheel and study the value of the longitudinal force.

5.Fy = f(alpha)

```
% friction coefficient
Ts = 11;
alfa0 = -25;
alfa_slope = (25--25)/Ts;
mhu_vet = [0.2 0.4 0.6 0.8];
mhu_slope = 0;
Fz0 = 4500;
Fz_slope = 0;
s0 = 0;
s_slope = 0;
F_Size = 14; % FontSize
plotcol = {'--k','-r','-.b',':g','.m','-k'};
```

Plots:

```
% Graphics
plotleg = {'\mu_x = 0.2','\mu_x = 0.4','\mu_x = 0.6','\mu_x = 0.8'};
nome_fig = 'Fy-alpha';
figure('Name',nome_fig,'NumberTitle','off','PaperType','A4')
hold all; grid on
% For cycle (varying the requested parameter., e.g., vertical load)
for cont1 = 1: length(mhu_vet)
    mu0 = mhu_vet(cont1);
    % execute simulink model
    string_sim = strcat({'sim '},{nome_modello_simulink});
    eval(string_sim{1})
    plot(alfa(:,2),Fy(:,2),plotcol{cont1},'Linewidth',2);
end

% Plot formatting and saving
legend(plotleg,'Location','best')
set(gca,'FontName','Times New Roman','FontSize',F_Size)
xlabel('alfa[deg]'); ylabel('F_Y [N]')
```



Respect to having changes over the vertical load, the adherence coefficent variation permit to appreciate way more the linearization for small value of slip. In fact, here as for the longitudinal force, it is possible observe how the curves overlaps between them.

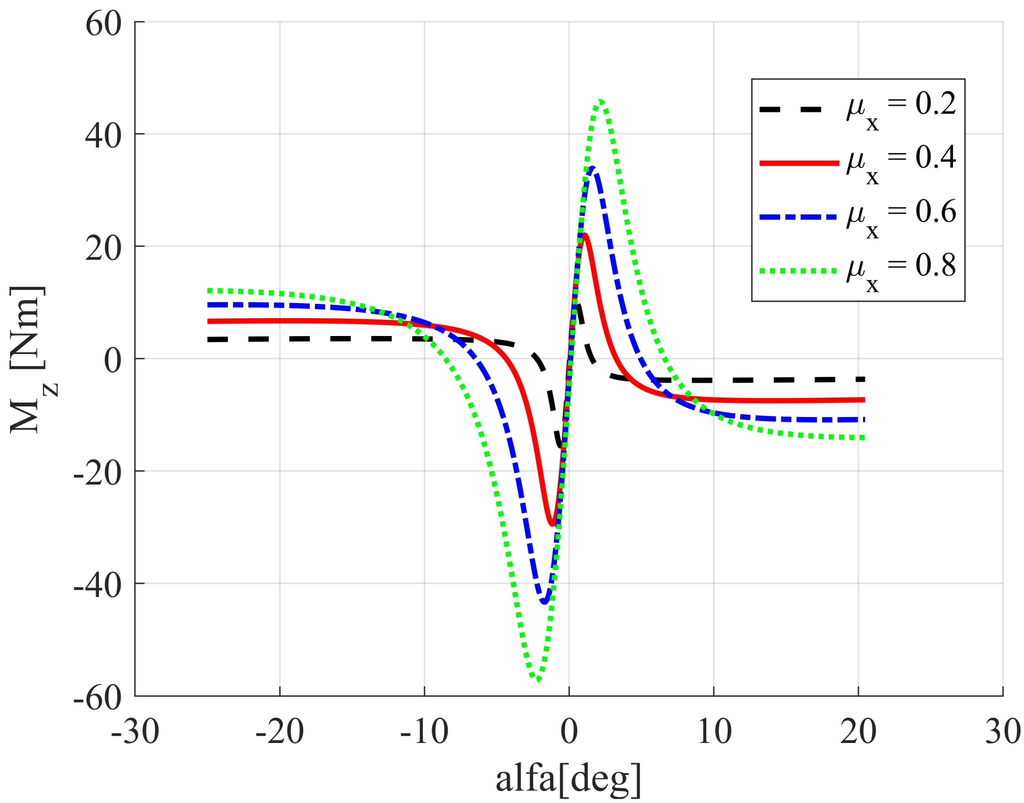
6.Mz = f(alpha)

```
% friction coefficient
Ts = 11;
alfa0 = -25;
alfa_slope = (25--25)/Ts;
mhu_vet = [0.2 0.4 0.6 0.8];
mhu_slope = 0;
Fz0 = 4500;
Fz_slope = 0;
s0 = 0;
s_slope = 0;
F_Size = 14; % FontSize
plotcol = {'--k','-r','-.b',':g','.m','-k'};
```

Plots:

```
% Graphics
plotleg = {'\mu_x = 0.2','\mu_x = 0.4','\mu_x = 0.6','\mu_x = 0.8'};
nome_fig = 'Mz-alpha';
figure('Name',nome_fig,'NumberTitle','off','PaperType','A4')
hold all; grid on
% For cycle (varying the requested parameter., e.g., vertical load)
```

```
for cont1 = 1: length(mhu_vet)
    mu0 = mhu_vet(cont1);
    % execute simulink model
    string_sim = strcat({'sim '},{nome_modello_simulink});
    eval(string_sim{1})
    plot(alfa(:,2),Mz(:,2),plotcol{cont1},'Linewidth',2);
end
% Plot formatting and saving
legend(plotleg,'Location','best')
set(gca,'FontName','Times New Roman','FontSize',F_Size)
xlabel('alfa[deg]'); ylabel('M_z [Nm]'); warning('off','all');
```



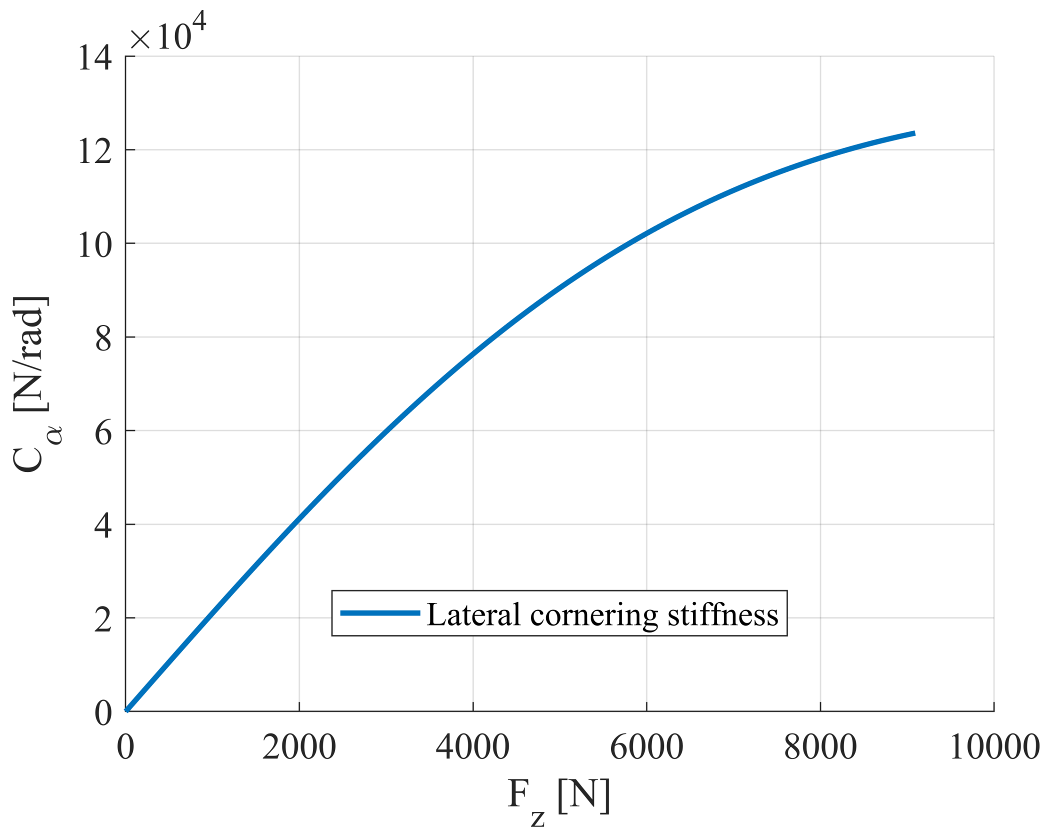
The major chnages between μ and F_z variation are the same studied for the lateral force. Tha peak values now occurs at different values of the slip and their peaks reach smaller values.

7.C_alpha = f(Fz)

```
Ts = 11;
mu_slope = 0;
mu0 = 1;
alfa0 = 0;
alfa_slope = 0;
Fz0 = 0;
Fz_slope = 10000/Ts;
s0 = 0;
s_slope = 0;
```

Plots:

```
% Graphics
plotleg = {'Lateral cornering stiffness'};
nome_fig = 'C_{\alpha}-Fz';
figure('Name',nome_fig,'NumberTitle','off','PaperType','A4')
hold all; grid on
% execute simulink model
string_sim = strcat({'sim '},{nome_modello_simulink});
eval(string_sim{1})
plot(Fz(:,2),-C_F(:,3),'Linewidth',2); % NOTE THE SIGN CHANGE

% Plot formatting and saving
legend(plotleg,'Location','best')
set(gca,'FontName','Times New Roman','FontSize',F_Size)
xlabel('F_z [N]'); ylabel('C_{\alpha} [N/rad]'); warning('off','all');
```



NOTE: The cornering stiffness is defined by $F_y = -C_\alpha \alpha$, suggesting that α and F_y have opposite sign.

This is imposed since the car when turning, to counteract the centrifugal force of the sprung mass, it has to develop some forces toward the bend of the curve. Furthermore, considering the wheel body diagram, the velocity imposed by the vehicle will follow the curve, meaning that α will be in the quadrant opposite to the curve. If F_y would be defined with the same sign convenction of the α, then it would point toward the outside of the curve and the equilibrium could not be achived. Being C_α a stiffness, it would make no sense physically to define it negative. From here the conclusion to imposing "manually" that the wheel side slip angle and lateral force have opposite sign.

However, for semplicity, the Pacejka model used automatically takes the absolute value of F_y. For this reason, by mathematical computation, the obtianed C_α is negative (due to Pacejka model itself), therefore it is necessary to impose manually the negative sign, in order to satisfy the sign convenction (make also reference to the figure previously shown in point 3).

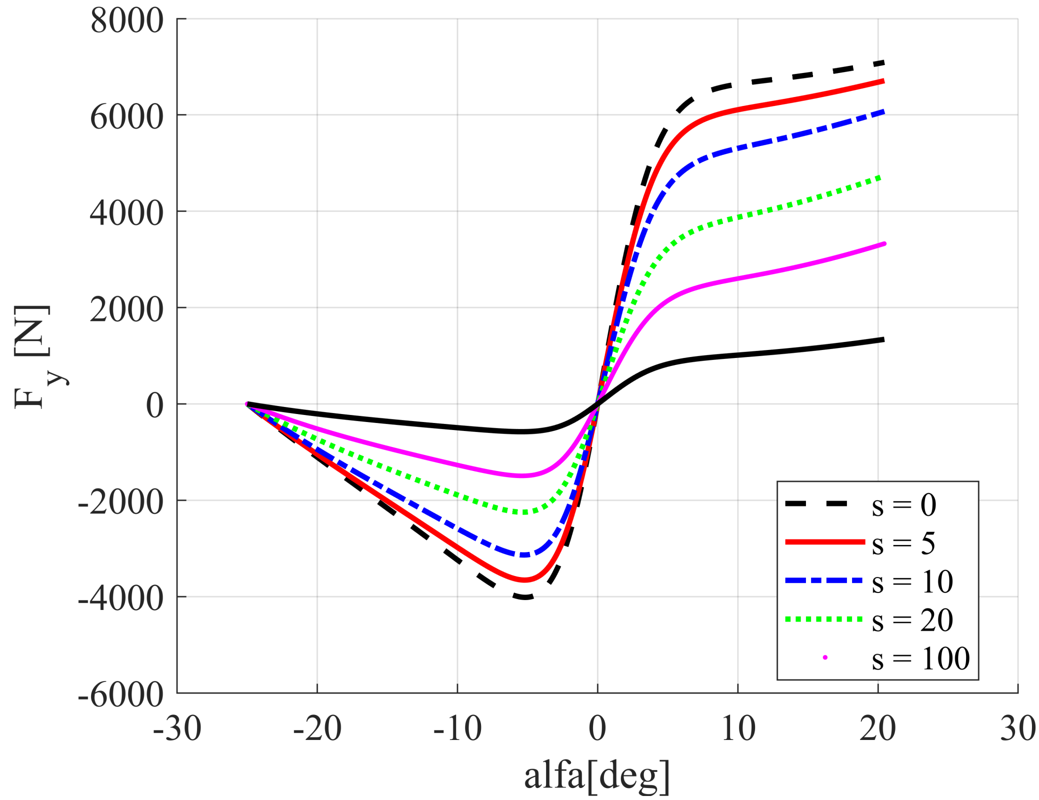
8.Combined slip: Fy = f(alpha), s varying

The combined slip situation is way more realistic, since it is pretty rare that in a manoeuvre only the slip changes, ususally both side slip and slip varies.

```
Ts = 11;
mu_slope = 0;
mu0 = 1;
alfa0 = -25;
alfa_slope = (25--25)/Ts;
Fz0 = 0;
Fz_slope = 10000/Ts;
s_vet = [0 5 10 20 35 100]/100;
s_slope = 0;
```

Plots:

```
% Graphics
plotleg = {'s = 0','s = 5','s = 10','s = 20','s = 100'};
nome_fig = 'Fy-alpha';
figure('Name',nome_fig,'NumberTitle','off','PaperType','A4')
hold all; grid on
% For cycle (varying the requested parameter., e.g., vertical load)
for cont1 = 1: length(s_vet)
    s0 = s_vet(cont1);
    % execute simulink model
    string_sim = strcat({'sim '},{nome_modello_simulink});
    eval(string_sim{1})
    plot(alfa(:,2),Fy(:,2),plotcol{cont1},'Linewidth',2);
end
% Plot formatting and saving
legend(plotleg,'Location','best')
set(gca,'FontName','Times New Roman','FontSize',F_Size)
xlabel('alfa[deg]'); ylabel('F_y [N]'); warning('off','all');
```



In the combined slip situation it is possibile to assest how there is a relevant influence of the slip over the lateral force development.

In particular it is visibile how the maximum is no longer so easy to define, in fact it seems that the F_y will increase accordingly with α. The comparision with the single-side-slip-variation shows that where there was a peak, here there is an inflection point, which also is obtained for smaller values of α having higher slips. Also, for the same value of the wheel side slip, having an higher slip implies a reduction of the lateral force value, suggesting that having slippage saturates the lateral force development capacity of the wheel.

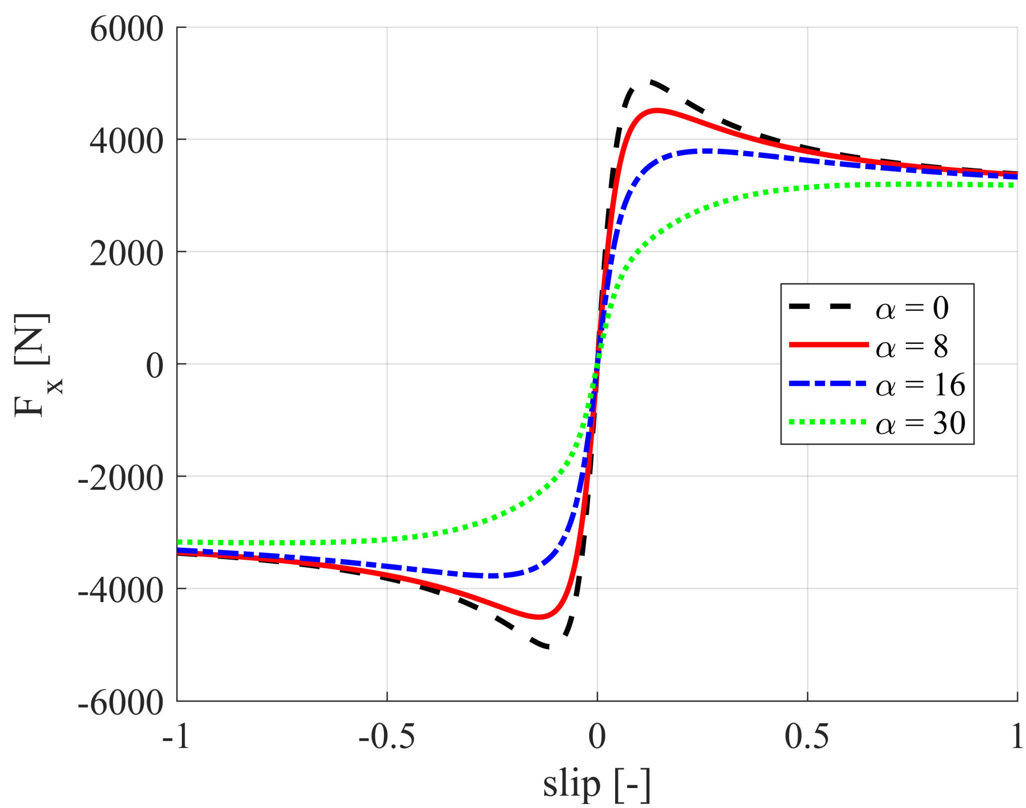
9.Combined slip: Fx = f(s), α varying

```
Ts = 11;
mu_slope = 0;
mu0 = 1;
alfa_vet = [0 8 16 30];
alfa_slope = 0;
Fz0 = 4500;
Fz_slope = 0;
s0 = -1;
s_slope = 0.2;
```

Plots:

```
% Graphics
plotleg = {'\alpha = 0','\alpha = 8','\alpha = 16','\alpha = 30'};
```

```
nome_fig = 'Fx-s';
figure('Name',nome_fig,'NumberTitle','off','PaperType','A4')
hold all; grid on
% For cycle (varying the requested parameter., e.g., vertical load)
for cont1 = 1: length(alfa_vet)
    alfa0 = alfa_vet(cont1);
    % execute simulink model
    string_sim = strcat({'sim '},{nome_modello_simulink});
    eval(string_sim{1})
    plot(s(:,2),Fx(:,2),plotcol{cont1},'Linewidth',2);
end

% Plot formatting and saving
legend(plotleg,'Location','best')
set(gca,'FontName','Times New Roman','FontSize',F_Size)
xlabel('slip [-]'); ylabel('F_x [N]'); warning('off','all');
```



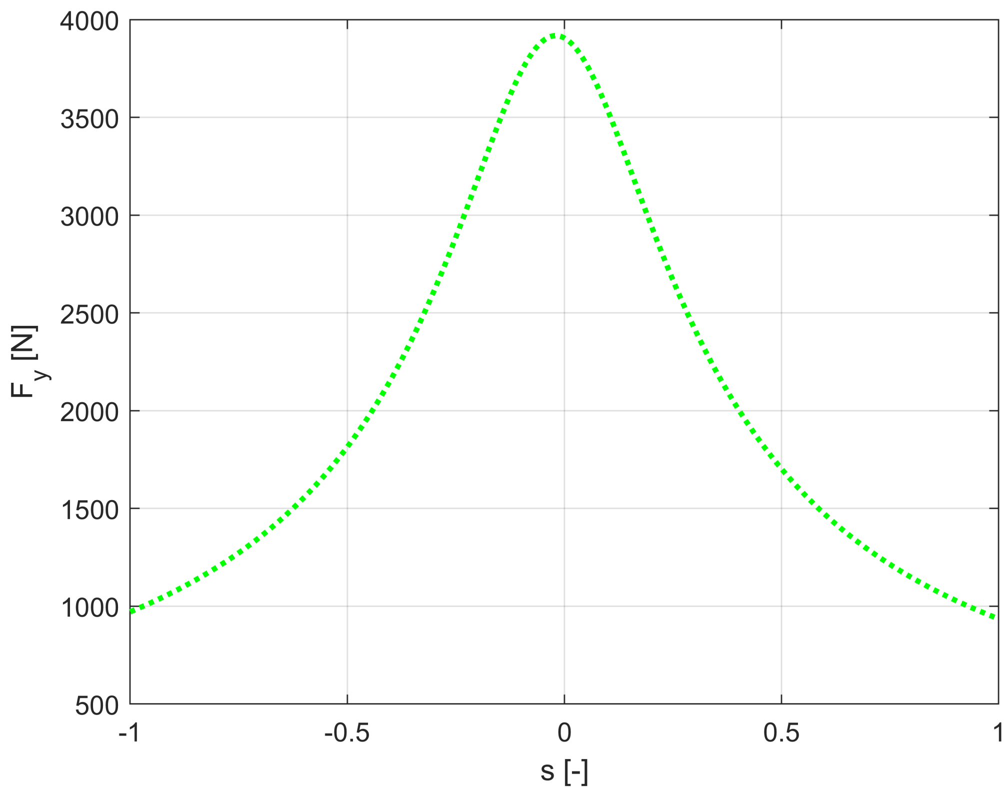
The combined slip for the longitudinal force have the effect to not only gradually eliminate the maximum for higher side slip, (with $\alpha = 30$ there is no longer a maximum but an asymptot) but also implies a sort of saturation. In facting, having only the slip varying, there was a certain difference in the longitudinal force values for different amount of vertical load and grip coefficent. Here meanwhile, no matter the value of the side slip, the tyre for high values of slip will extert the same amount of longitudinal force.

Q1. for a Front 2WD vehicle, what happens during cornering, when a high driving torque is applied to the wheels? Does the vehicle follow a larger or smaller trajectory?

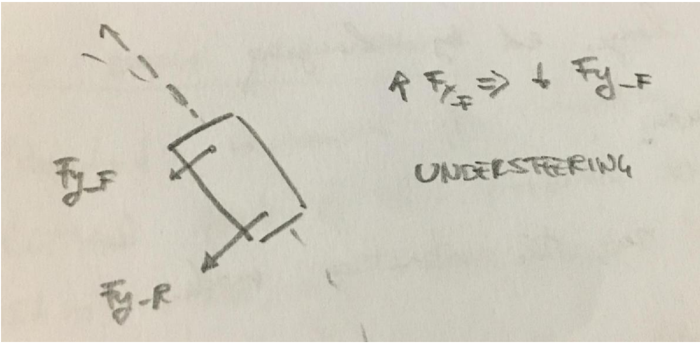
The vehicle will shows an understeering behaviour. To be more precise, this kind of behaviour can be seen more when considering the lateral dynamics, where there is a specific analysis for evaluating how the vehicle behaves when turning.

However, it is possible to get a general idea over this phenomena also considering the tyre characteristics. Having traction on the frontal axle, implies that the front wheels will increase their F_x meanwhile the F_y will dimish. This can be assest not onyl from the elliptical model, but also by plotting the lateral force as function of the slip.

```
figure
plot(s(:,2),Fy(:,2),plotcol{cont1},'Linewidth',2); grid on;
ylabel('F_y [N]'); xlabel('s [-]');
```

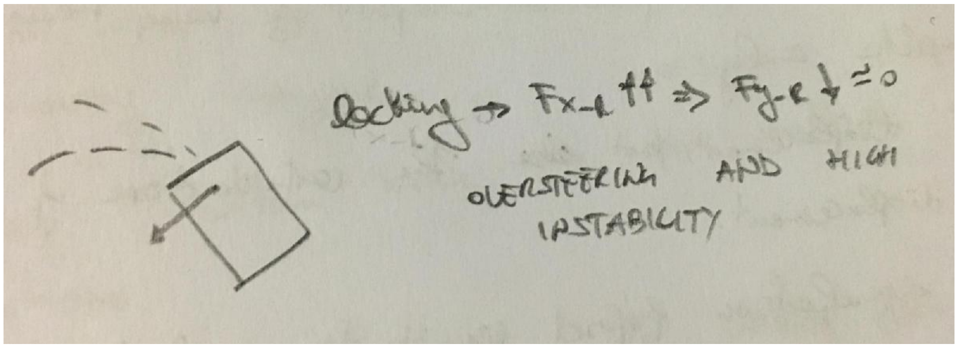


If the lateral force dimish, then the vehicle will make more effort to maintain the trajectory, increasing therefore the overall path.



Q2. and what happens if the hand-brake is activated, thus locking the rear wheels?

Locking the rear wheels implies that the slip becames unitary, therefore the lateral force exeter by the rear wil be pratically null, since the front will still developing some lateral forces, the vehicle will curve too much, becoming oversteering and afterward unstable.



10.Adherence ellipse

The elliptical model

```
disp('Tried to plot it, unsuccessfully :(')
```

```
Tried to plot it, unsuccessfully :(
```