# Braking performance

Analysis of the braking performance for a vehicle, comparing unladen and laden mass.

```
clear; close all; clc; set(0,'DefaultAxesFontSize',18);
% vehicle data
m = 1325;           % [kg] - mass
L = 2.68;           % [m] - wheelbase
a = 1.15;           % [m] - front wheelbase (unladen)
h = 0.5;            % [m] - CoG height (unladen)
t = 1.51;           % [m] - track
r = 0.314;          % [m] - nominal radius
g = 9.81;           % [m/s^2] gravitational acceleration
m_U = m;                % [kg] unladen mass
m_L = 1730;             % [kg] laden mass
% CoG position
a_U = 1.15;         % [m] unladen vehicle
a_L = 1.36;         % [m] laden vehicle
A = [a_U a_L];
```

### Static load distribution

Front and Rear static loads are $F^0_{ZF} = mg\frac{b}{L}$ and $F^0_{ZR} = mg\frac{a}{L}$

```
M = [m_U m_L];
ZR(1) = m_U*a*g/L;
ZR(2) = (m_L*a_L)*g/L;
ZF = M*g - ZR;          % [N] front axle vertical force
```

### Dynamic load distribution

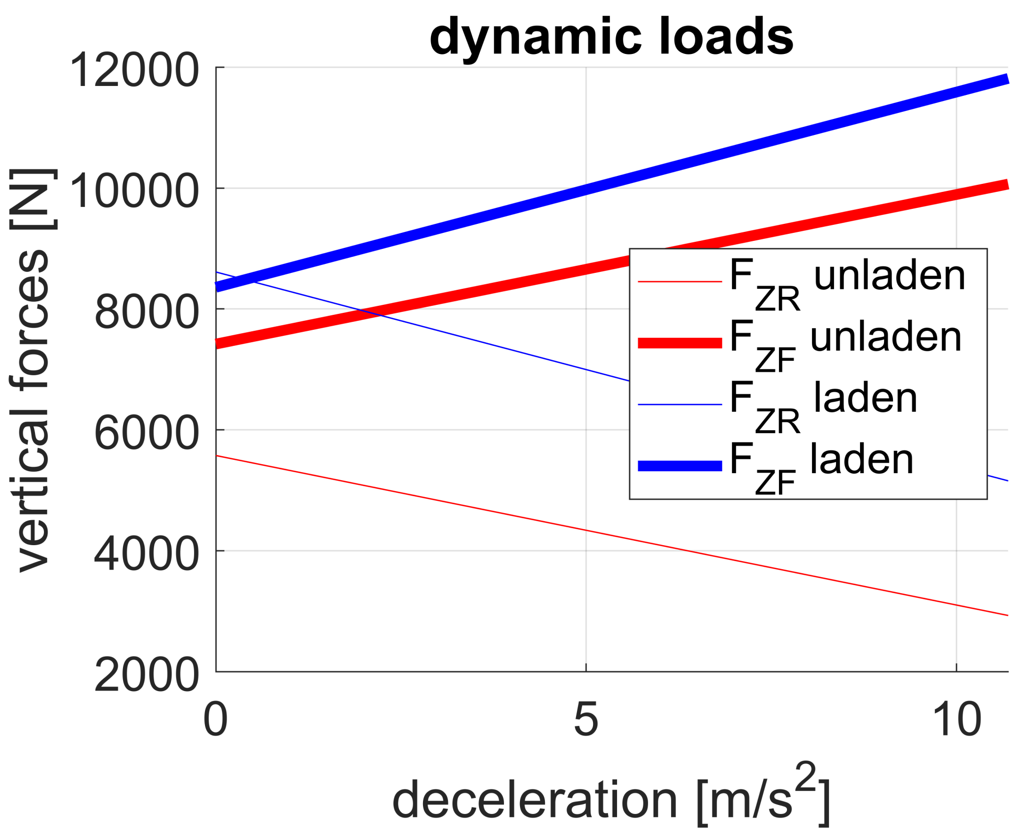
the modulus of the longitudinal load transfer is $\Delta F_Z = m|\ddot{x}|\frac{h}{L}$

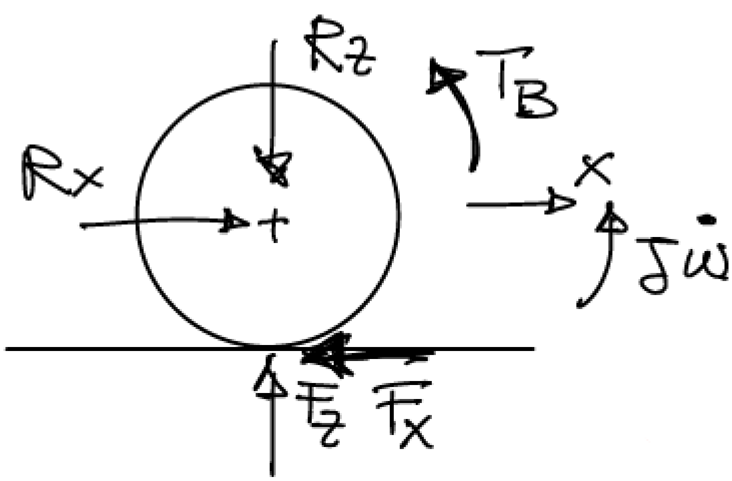
Front and Rear dynamic loads are $F_{ZF} = \frac{m}{L}(gb - \ddot{x}h)$ and $F_{ZR} = \frac{m}{L}(ga + \ddot{x}h)$

where the vehicle acceleration is negative, i.e.: $\ddot{x} < 0$

```
xdd = -(0:.1:1.1*g);            % [m/s^2] vehicle deceleration
% vertical forces (unladen)
ZR_da = M(1)*g*(a_U/L+h/L*xdd/g);
ZF_da = M(1)*g*(1-a_U/L-h/L*xdd/g);
% vertical forces (laden)
ZR_df = M(2)*g*(a_L/L+h/L*xdd/g);
ZF_df = M(2)*g*(1-a_L/L-h/L*xdd/g);
% plot longitudinal load transfer
figure(1)
hold on
plot(-xdd,ZR_da,'r')
plot(-xdd,ZF_da,'r','linewidth',4)
plot(-xdd,ZR_df,'b')
plot(-xdd,ZF_df,'b','linewidth',4)
xlabel('deceleration [m/s^2]')
```

```
ylabel('vertical forces [N]')
title('dynamic loads')
legend('F_{ZR} unladen','F_{ZF} unladen','F_{ZR} laden','F_{ZF} laden','Location','East')
grid on
```



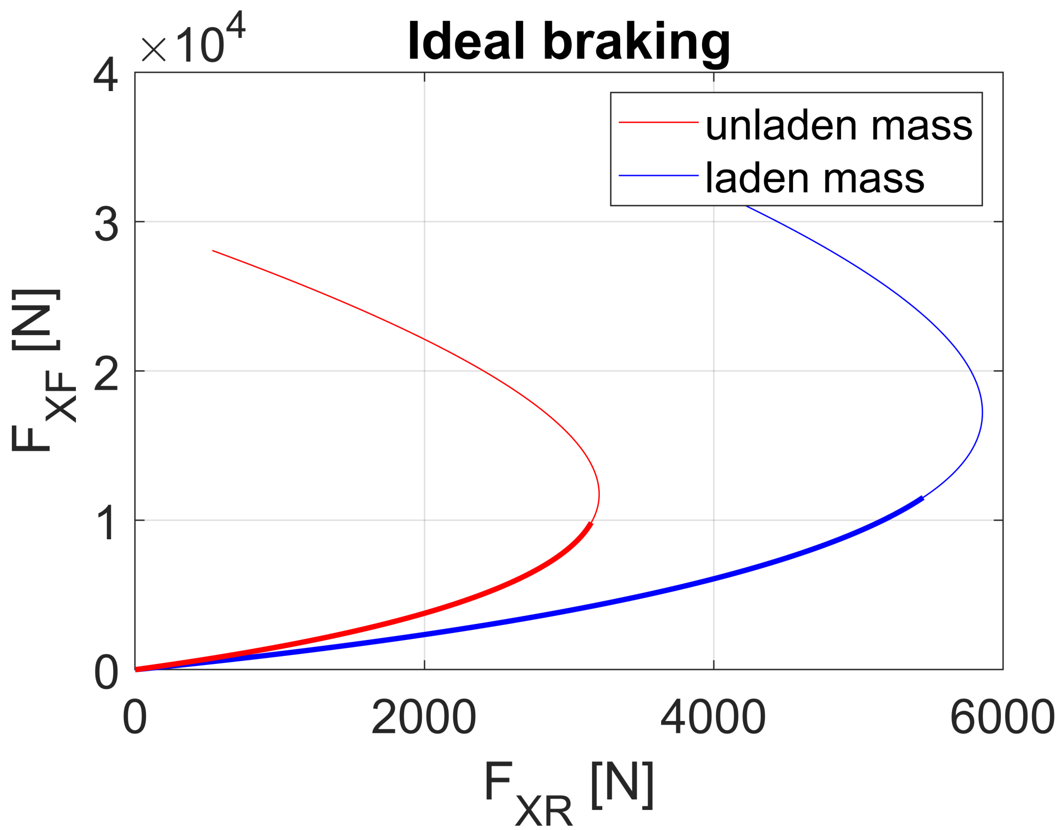

The acceleration implies a longitudinal load transfer, which will have some consequencies over the braking capability. In particualr, higher is the decelleration, higher is the vertical front force (comparison among the two red lines). Moreover, higher is the mass of the vehicle (laden case), higher will be the forces involved.

### Ideal braking

Ideal braking corresponds to the **maximum achievable braking effort**, i.e., when front and rear wheels are braked at the maximum limit ($\ddot{x}_{\max} = -\mu g$) **without slipping**:

$F_{XF} = \mu \frac{mg}{L}(b + \mu h)$ and $F_{XR} = \mu \frac{mg}{L}(a - \mu h)$

```
mu_0=(0:.01:2.2);
mu_1=(0:.01:1);     % Mhu rarely goes above 1, refer to this vector for future considerations

% unladen. Forces written as function of mhu
Fxf_U=@(mu) mu.*M(1).*g.*(L-a_U+mu.*h)./L;
Fxr_U=@(mu) mu.*M(1)*g.*(a_U-mu.*h)./L;

clear figure(2) % clean the previous plot and draw on "white board"
figure(2)
plot(Fxr_U(mu_0),Fxf_U(mu_0),'r')
hold all

% laden [laden -> pieno carico]
Fxf_L=@(mu) mu.*M(2).*g.*(L-a_L+mu.*h)./L;
Fxr_L=@(mu) mu.*M(2).*g.*(a_L-mu.*h)./L;
plot(Fxr_L(mu_0),Fxf_L(mu_0),'b')
hold all


plot(Fxr_L(mu_1),Fxf_L(mu_1),'b','linewidth',2)
hold all

plot(Fxr_U(mu_1),Fxf_U(mu_1),'r','linewidth',2)
hold all

legend({'unladen mass','laden mass'})

xlabel('F_X_R [N]')
ylabel('F_X_F [N]')
title('Ideal braking')
grid on
```

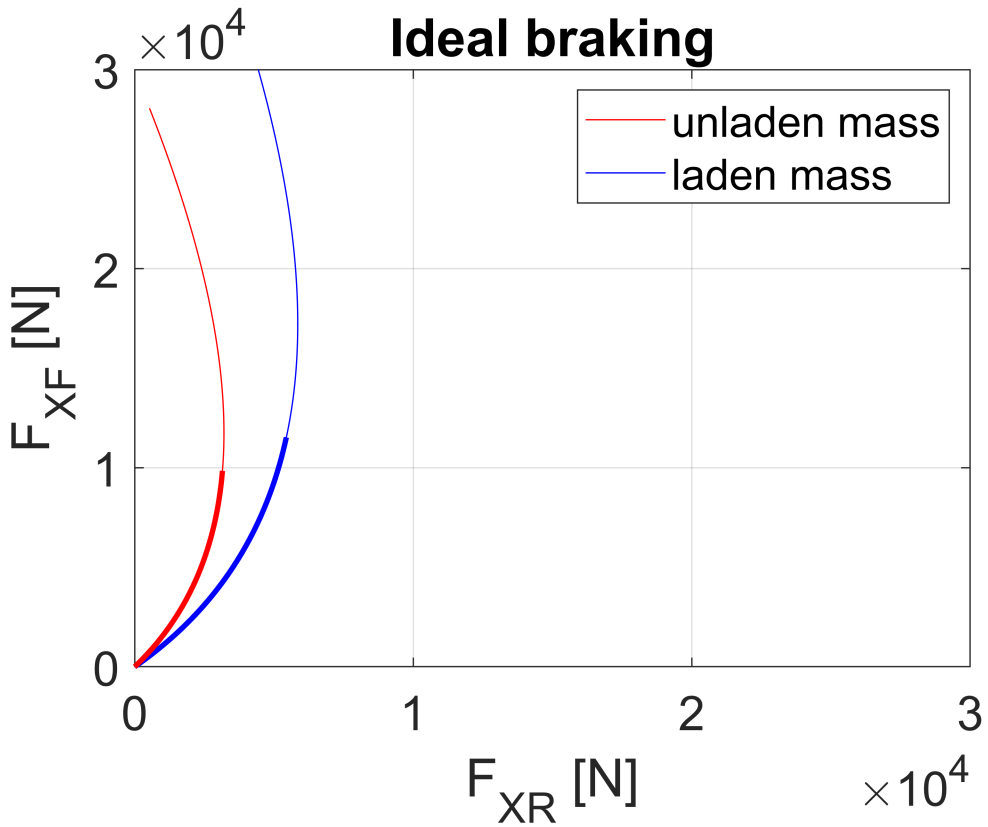



By considering the equation for front and rear axle in ideal braking as a parametric fomrula, the braking parabola is get.

**NOTE:** As stated, this chart is obtained assuming that $\ddot{x} = -\mu g$ but this is the MAXIMUM DECELERATION possible, not the general deceleration. Therefore the ideal braking parabola should be taken into consideration only during the extreme condition of best possible braking performance (emergy braking, not a general braking for traffic light, for example).

```
clear figure(3)
figure(3)

% Mhu references values (up to 2.2)
plot(Fxr_U(mu_0),Fxf_U(mu_0),'r') % Unloaded
hold all

plot(Fxr_L(mu_0),Fxf_L(mu_0),'b') % Loaded
hold all

% mhu up to 1
plot(Fxr_L(mu_1),Fxf_L(mu_1),'b','linewidth',2) % loaded
hold all

plot(Fxr_U(mu_1),Fxf_U(mu_1),'r','linewidth',2) % unloaded
hold all

legend({'unladen mass','laden mass'})
```

```
xlim([0 3e04]); ylim([0 3e04])
xlabel('F_X_R [N]')
ylabel('F_X_F [N]')
title('Ideal braking')
grid on
```




Respect to the previous chart, some rescaling was perfomed.

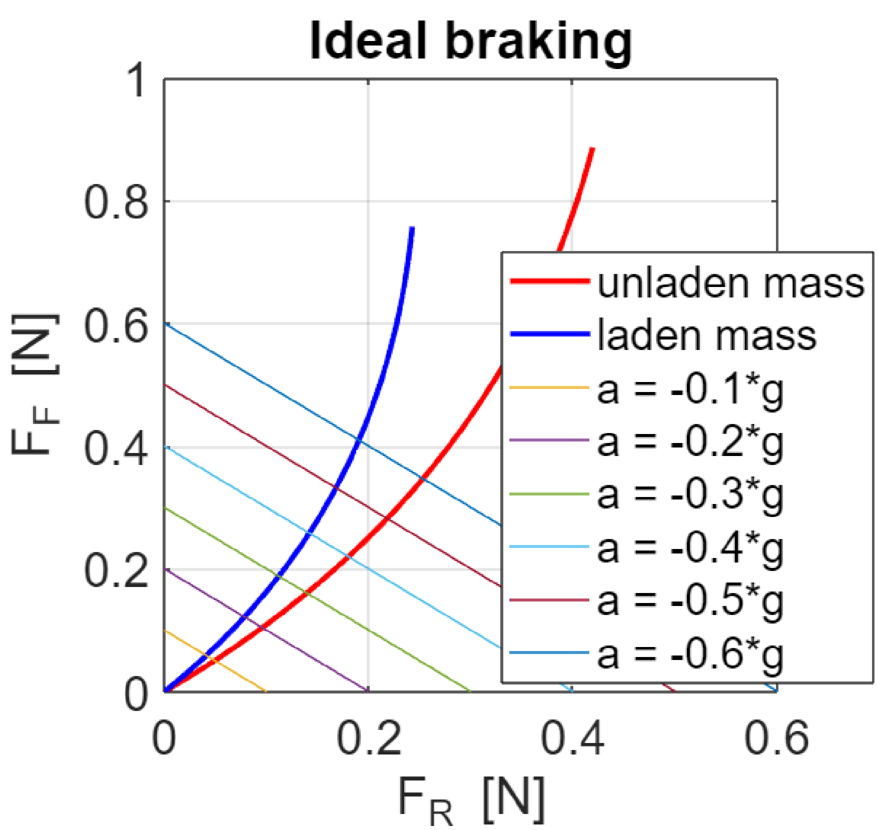
## Forces normalized with weight

Constant deceleration curves, with *normalized* forces:

$\bar{F}_{XF} = \frac{F_{XF}}{mg}$ and $\bar{F}_{XR} = \frac{F_{XR}}{mg}$

```
% constant acceleration lines
a = -0.1*g;

% Laden
F_xf_NORM = Fxf_L(mu_1)/m/g;
F_xr_NORM = Fxr_L(mu_1)/m/g;

figure(4)
plot(F_xr_NORM,F_xf_NORM,'r','linewidth',2) % loaded
hold all

% Unladedn
```

```
F_xf_NORM = Fxf_U(mu_1)/m/g;
F_xr_NORM = Fxr_U(mu_1)/m/g;

figure(4)
plot(F_xr_NORM,F_xf_NORM,'b','linewidth',2) % unloaded
hold all

for a=0.1:0.1:0.6
    figure(4)
    plot([a 0],[0 a])
end

legend('unladen mass','laden mass','a = -0.1*g','a = -0.2*g','a = -0.3*g','a = -0.4*g','a = -0.

xlabel('F_X_R [N]')
ylabel('F_X_F [N]')
title('Ideal braking')
grid on
axis('square')
```




Constant friction coefficient lines for ***Front axle***: $\bar{F}_{XF} = \mu \dfrac{b + h\bar{F}_{XR}}{L - \mu h}$

```
% F_xf_NORM = @(mu)  mu_1*(L-a+h*F_xf_NORM)/(L-mu_1*h);
```

Constant friction coefficient lines for ***Rear axle***: $\bar{F}_{XR} = \dfrac{a}{h} - \dfrac{L + \mu h}{\mu h}\bar{F}_{XR}$

```
% F_xr_NORM = @(mu)  a/h-(L+mu_1*h)/(mu_1*h)*F_xr_NORM;
```

Deceleration lines usefull to take as reference for comforto or general deceleration performance.

## Ideal traction/braking parabola

the *normalized* parametric equations for the braking parabola are:

$$\bar{F}_{XF} = -\frac{\mu}{L}(b - \mu h)$$

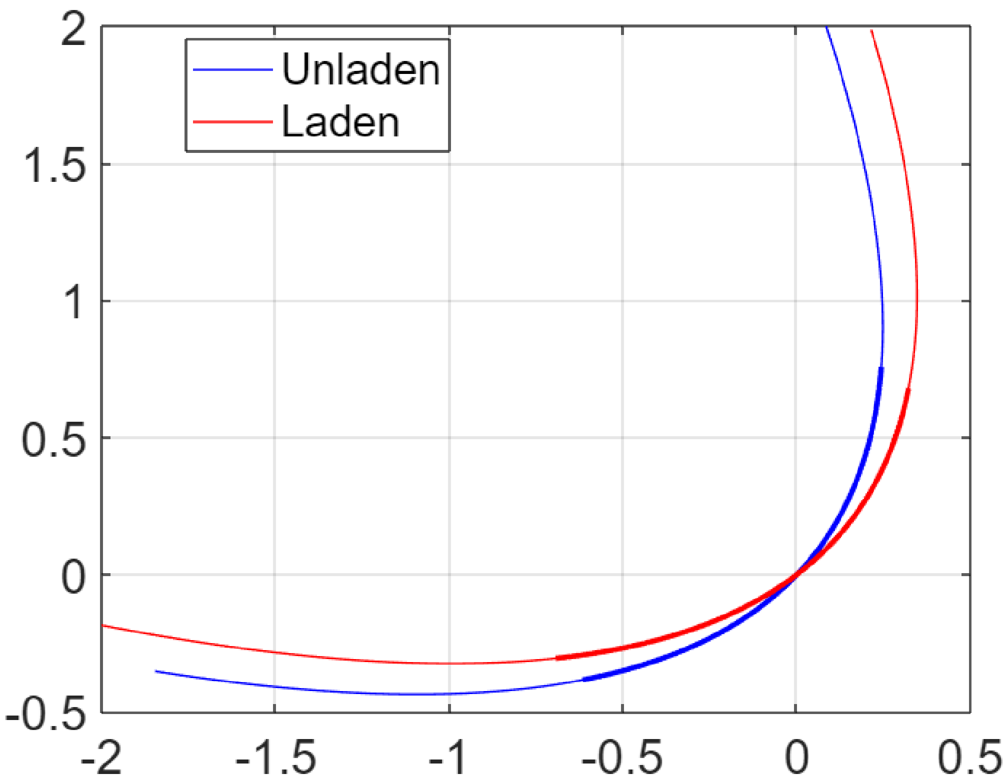
$$\bar{F}_{XR} = -\frac{\mu}{L}(a + \mu h)$$

```
%% Mhu_0 %%%%%%%%%%%%%%%
mu_total = (-2.2:.01:2.2);
```

The traction is simply considered by using a negative mhu which implies that the force is generating a rotative torque.

```
% Unladen
F_xf_NORM =@(mu) -mu./L.*(L-a_U-mu.*h);
F_xr_NORM =@(mu) -mu./L.*(a_U+mu.*h);

figure(5)
plot(F_xr_NORM(mu_total),F_xf_NORM(mu_total),'b','linewidth',0.5)
xlim([-2 0.5]); ylim([-0.5 2]);
hold on; grid on

% Laden
F_xf_NORM =@(mu) -mu./L.*(L-a_L-mu.*h);
F_xr_NORM =@(mu) -mu./L.*(a_L+mu.*h);

figure(5)
plot(F_xr_NORM(mu_total),F_xf_NORM(mu_total),'r','linewidth',0.5)
xlim([-2 0.5]); ylim([-0.5 2]);
hold on; grid on;

%% Mhu_1 %%%%%%%%%%%%%%%
mu_reference= (-1:.01:1);

% Unladen
F_xf_NORM =@(mu) -mu./L.*(L-a_U-mu.*h);
F_xr_NORM =@(mu) -mu./L.*(a_U+mu.*h);

figure(5)
plot(F_xr_NORM(mu_reference),F_xf_NORM(mu_reference),'b','linewidth',2)
xlim([-2 0.5]); ylim([-0.5 2]);
hold on; grid on

% Laden
F_xf_NORM =@(mu) -mu./L.*(L-a_L-mu.*h);
F_xr_NORM =@(mu) -mu./L.*(a_L+mu.*h);
```

```
figure(5)
plot(F_xr_NORM(mu_reference),F_xf_NORM(mu_reference),'r','linewidth',2)
xlim([-2 0.5]); ylim([-0.5 2]);

legend('Unladen','Laden','location','best'); ylabel('F_{XF}'); xlabel('F_{XR}');
hold on; grid on
```




Up to now the mhu was set only as positive, in order to have the Fx which generates the rotaion of the wheel and that opposes to the braking traction. If the mhu is set negative, considering that the Fx changes its side, the traction part is added.

## Braking bias ()

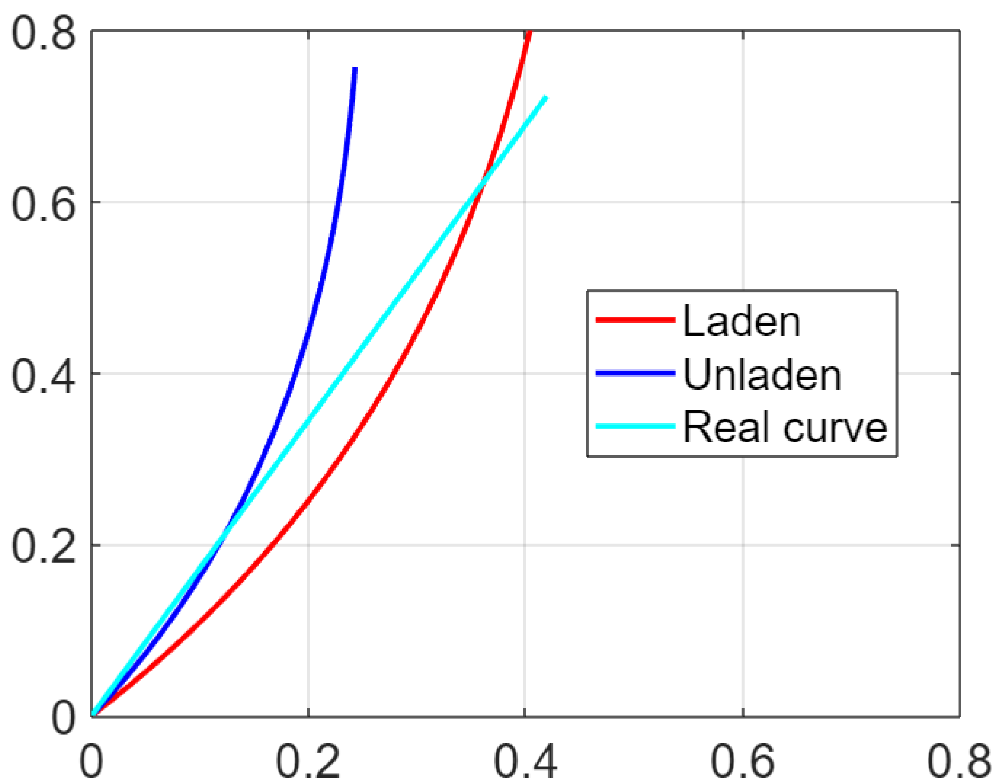
define $\varphi = \frac{F_{XR}}{F_X} = \frac{F_{XR}}{F_{XF} + F_{XR}}$

i.e., $\beta_P = \frac{1-\varphi}{\varphi}$

the real braking force will follow distribution $F_{XF} = \beta_P F_{XR}$

```
% Recreating the ideal braking curve as seen in figure 4

% Laden
F_xf_NORM = Fxf_L(mu_1)/m/g;
F_xr_NORM = Fxr_L(mu_1)/m/g;
```

```
figure(6)
plot(F_xr_NORM,F_xf_NORM,'r','linewidth',2) % laden
hold all

% Unladen
F_xf_NORM = Fxf_U(mu_1)/m/g;
F_xr_NORM = Fxr_U(mu_1)/m/g;

figure(6)
plot(F_xr_NORM,F_xf_NORM,'b','linewidth',2) % unloaded
hold all

% Laden or unladen it doesn change the real curve!
phi = Fxr_L(mu_1)/(Fxf_L(mu_1)+Fxr_L(mu_1));
beta_p = (1-phi)/phi;
F_xf_real_NORM = beta_p*Fxr_L(mu_1)/m/g;

figure(6)
plot(Fxr_L(mu_1)/m/g,F_xf_real_NORM,'c','linewidth',2);
xlim([0 0.8]); ylim([0 0.8]);
xlabel('Fx_R'); ylabel('Fx_F');
legend('Laden','Unladen','Real curve','location','best')
grid on; hold on;
```

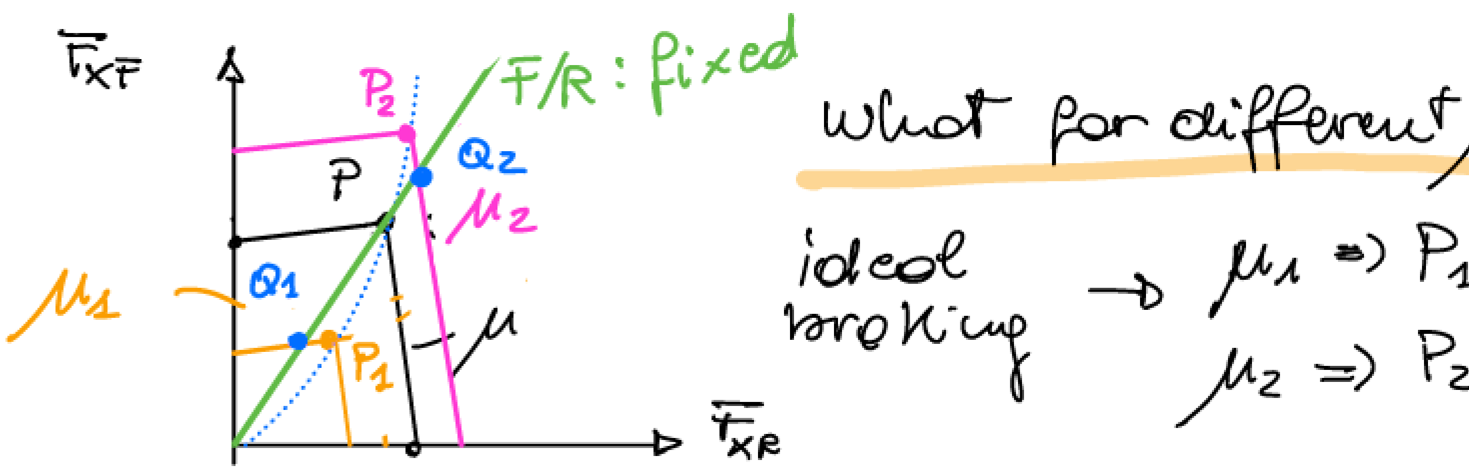



Real cars due to mechanical constrains can perform braking only with a fixed braking bias. This translates into a constant line in the braking parabola. There would be an intersection point between the ideal braking and the fixed ratio line. This is the best possible condition, since the car would perform a braking with maximum deceleration possibile without creating slip in both axle. To achive this condition, a certain value of mhu is necessary.

Inclination of the real plant is based on the geometrical design of the calipper and it is unaffected by the mass. The slope of the real braking is designed over the effective radius and the area (mostly the radius -> rear discs brakes are smaller than the front ones).

$$T_b = \mu_d * P_{oil} * A_{pedal} * r_e$$

Moreover, if the mhu is not the desired one, the working point will shift upward or downwards accordingly. Making reference to this chart:




For the working point Q1 the front axle will lock, meanwhile for the working point Q2 the rear axle will lock. The latter has to be completely avoided, since having the rear axle locked before the front one will imply instability (oversteering and possible spins).

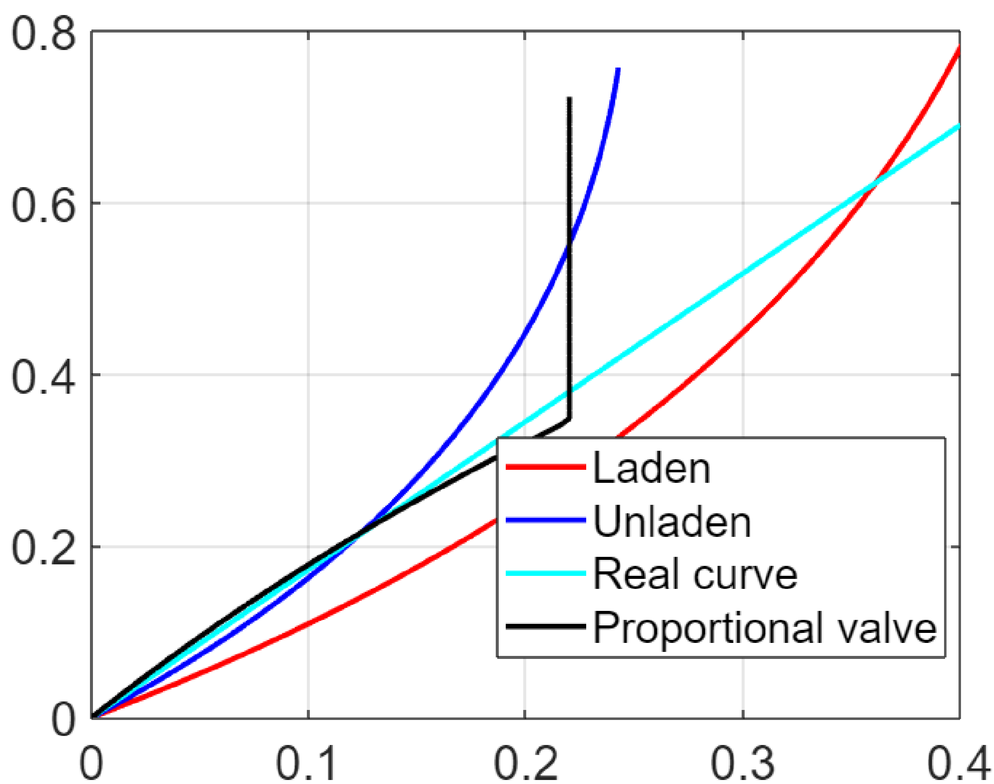
To avoid that in extreme condition the working point goes above the ideal braking parabola, a proportional valve is adopted.

## Rear proportional valve

Introducing a pressure limiting valve for the real calipers, the rear axle force will not grow larger than the design treshold ($\bar{F}_{XR} = 0,22$)

```
Fxr_L_PV = zeros(length(mu_1),1);
for count=1:length(mu_1)
    mu   = mu_1(count);
    Fxf_NORM = mu*(L-a_U+mu*h)/L;

    if Fxf_NORM <= 0.22
        Fxr_L_PV(count)    =   Fxf_NORM;
    else
        Fxr_L_PV(count)   =   0.22;
    end
```

```
end

figure(6)
plot(Fxr_L_PV,F_xf_real_NORM,'k','linewidth',2);
xlabel('Fx_R'); ylabel('Fx_F');
xlim([0 0.4]); ylim([0 0.8]);
legend('Laden','Unladen','Real curve','Proportional valve','location','best')
```




Working principle of the proportional valve is pretty simple: the braking system of the car should never cross the laden ideal braking curve. To assest this, a pressure limiting valve on the rear part of the plant is mounted, preventing the oil pressure to grow too much (60 bars). Considering that the braking torque is directly related to the oil pressure ($T_b = \mu_d * P_{oil} * A_{pedal} * r_e$), lowering the pressure will have as consequence to reduce the braking torque. Lower the braking torque, lower will be the longitudinal force acting on the wheel.

In the end, considering the ideal braking parabole, the ratio is set so to not increase at all the rear forces. In othere words a constant rear normalised force part is present.