Vehicle Dynamics Simulation

Side-slip angle estimation from experimental data

Kinematics and dynamics approach

Data analysis

Signal acquisition during an experimental test session

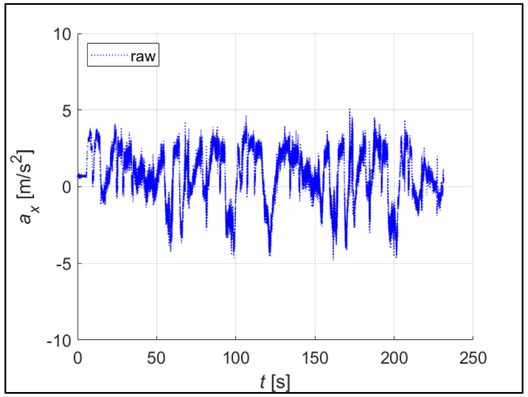
```
% Vehicle data
a = 1.19;           %[m] CoG to front axle
b = 1.38;           %[m] CoG to rear axle
l = a+b;            %[m] Wheelbase
tau = 1/12.5;       %[-] Steering ratio
J = 1800;           %[kg*m^2] Moment of inertia
m = 1446;           %[kg] Mass of the vehicle
```

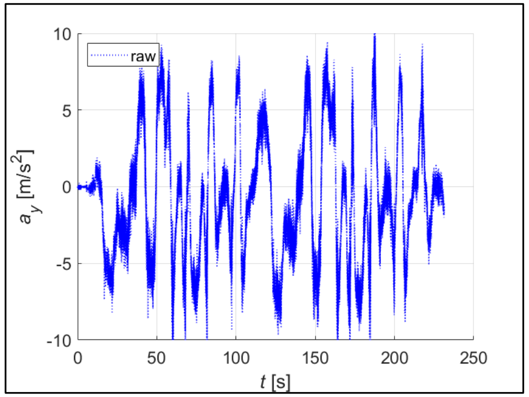
Data analysis

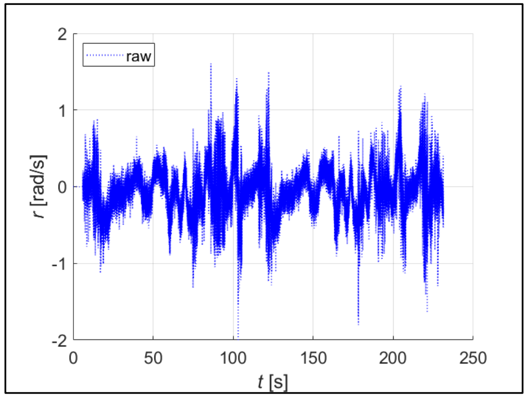
Acquired raw signals

- Longitudinal acceleration
- Lateral acceleration
- Yaw rate

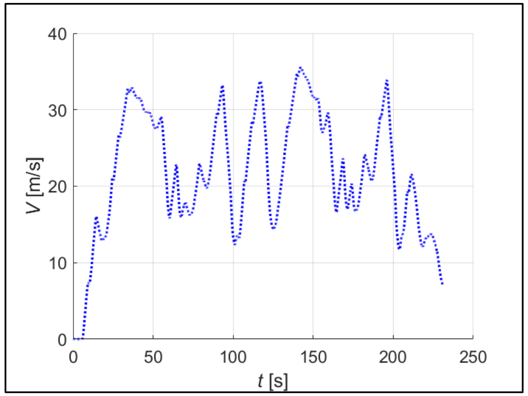
Inertial measurement unit



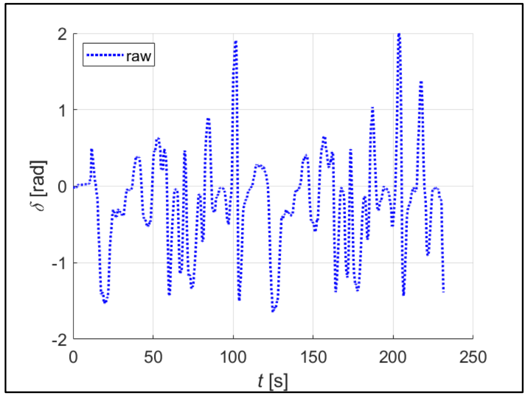




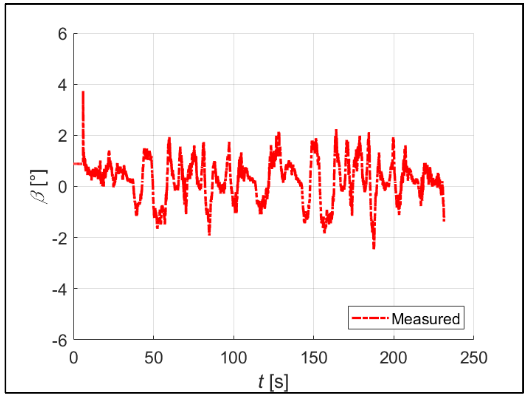
Data analysis
Acquired raw signals



Velocity



Steering-wheel angle



Side-slip angle

GNSS antenna

Potentiometer

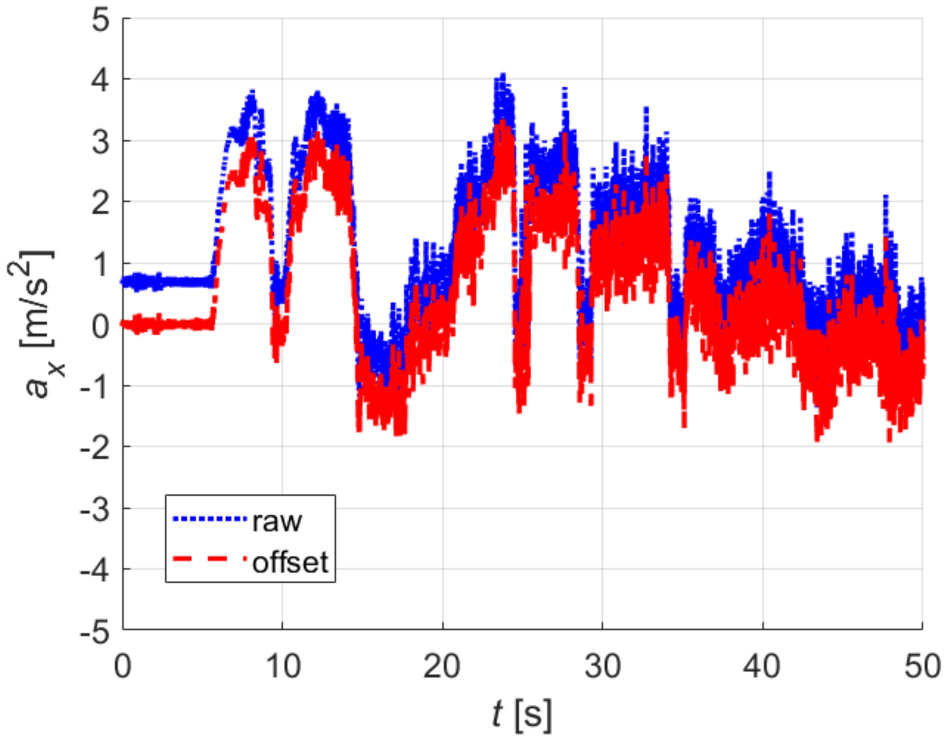
Optical sensor

Data analysis

Task: signal processing

Remove the initial offset from:

- Longitudinal acceleration
- Steering-wheel angle



Data analysis

Task: signal processing

Filter the following signals with a low-passband filter:

- Longitudinal acceleration
- Lateral acceleration
- Yaw rate

Filter characteristics:

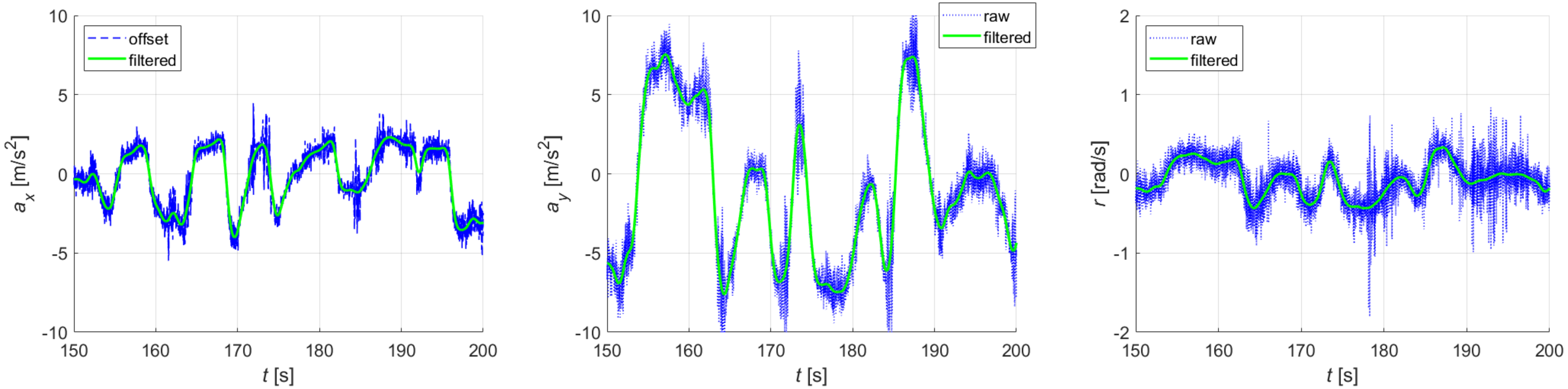
- use Matlab function “butter” and “filtfilt”
- Cut-off frequency equal to $0.6\ \mathrm{Hz}$
- Filter order equal to 2

Note that in the “butter” function, the normalisation of the cut-off frequency must be computed on the Nyquist frequency $f_N = f_{sampling}/2$

Data analysis

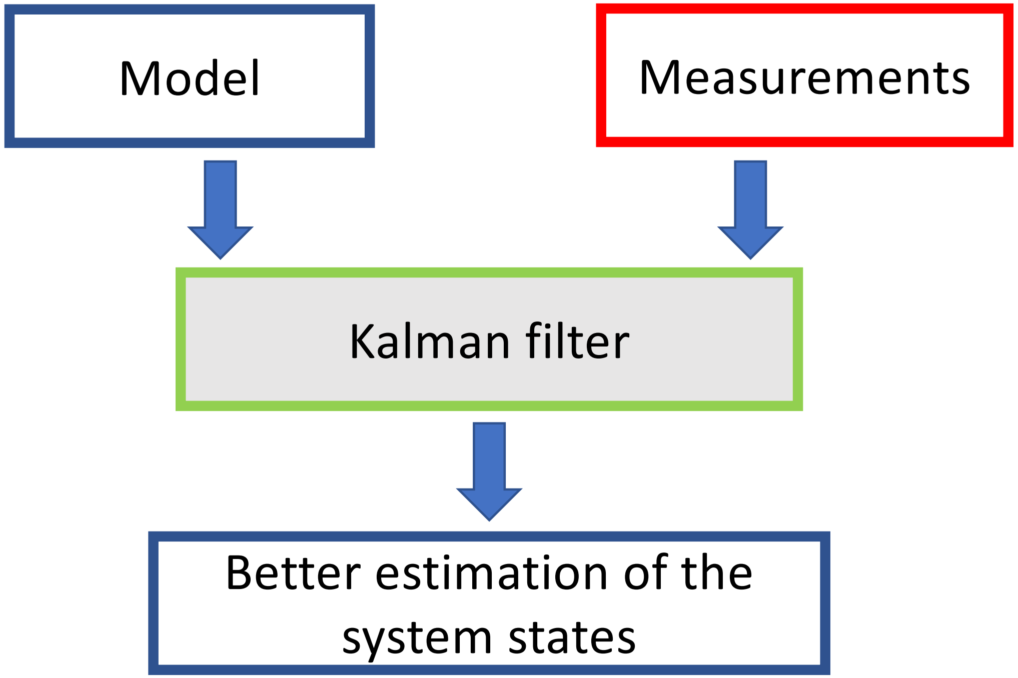
Task: signal processing

Filtering effect



Kalman filter introduction

Introduction



Kalman filter introduction

1. Definition of the model in the state space

$$\dot{\boldsymbol{x}}_f = \boldsymbol{A}_f \boldsymbol{x}_f + \boldsymbol{B}_f \boldsymbol{u}_f$$

$$\boldsymbol{y}_f = \boldsymbol{C}_f \boldsymbol{x}_f + \boldsymbol{D}_f \boldsymbol{u}_f$$

$\boldsymbol{x}_f$: state vector ($s\times1$)
$\boldsymbol{u}_f$: control input vector ($e\times1$)
$\boldsymbol{A}_f$: the state matrix ($s\times s$)
$\boldsymbol{B}_f$: the input matrix ($s\times e$)

$\boldsymbol{y}_f$: measurement vector ($m\times1$)
$\boldsymbol{C}_f$: output matrix ($m\times s$)
$\boldsymbol{D}_f$: direct transmission matrix ($m\times e$)

2. Assignment of the process and measurement noise

$\boldsymbol{q}$: process noise ($s\times1$)
$\boldsymbol{r}$: measurement noise ($m\times1$)

$$\rightarrow \quad \boldsymbol{Q} = \begin{bmatrix} \boldsymbol{q}_1 & \cdots & \boldsymbol{0} \\ \vdots & \ddots & \vdots \\ \boldsymbol{0} & \cdots & \boldsymbol{q}_s \end{bmatrix} \quad \boldsymbol{R} = \begin{bmatrix} \boldsymbol{r}_1 & \cdots & \boldsymbol{0} \\ \vdots & \ddots & \vdots \\ \boldsymbol{0} & \cdots & \boldsymbol{r}_m \end{bmatrix}$$

$_f$ subscript stands for filter

Kalman filter algorithm

3. Discretisation

$$\dot{\boldsymbol{x}}_{f,k} = \frac{\boldsymbol{x}_{f,k} - \boldsymbol{x}_{f,k-1}}{\Delta t}$$

$$\boldsymbol{x}_{f,k} = \Delta t \boldsymbol{A}_f \boldsymbol{x}_{f,k-1} + \boldsymbol{x}_{f,k-1} + \Delta t \boldsymbol{B}_f \boldsymbol{u}_{f,k-1} = (\Delta t \boldsymbol{A}_f + \boldsymbol{I})\boldsymbol{x}_{f,k-1} + \Delta t \boldsymbol{B}_f \boldsymbol{u}_{f,k-1}$$

$$\boldsymbol{x}_{f,k} = \boldsymbol{A}_{f,d} \boldsymbol{x}_{f,k-1} + \boldsymbol{B}_{f,d} \boldsymbol{u}_{f,k-1}$$

$$\boldsymbol{y}_{f,k} = \boldsymbol{C}_f \boldsymbol{x}_{f,k} + \boldsymbol{D}_f \boldsymbol{u}_{f,k}$$

$_d$ subscript stands for <u>d</u>iscretised

Kalman filter algorithm

3. Generic Kalman equation

Prediction phase (a priori estimation)

$$\bar{\boldsymbol{x}}_{f,k} = \boldsymbol{A}_d \boldsymbol{x}_{f,k-1} + \boldsymbol{B}_d \boldsymbol{u}_{f,k-1}$$

$$\bar{\boldsymbol{P}}_{f,k} = \boldsymbol{A}_d \boldsymbol{P}_{f,k-1} \boldsymbol{A}_d^T + \boldsymbol{Q}$$

Measurement update (a posteriori estimation)

$$\boldsymbol{K}_{f,k} = \bar{\boldsymbol{P}}_{f,k} \boldsymbol{C}_f^T (\boldsymbol{C}_f \bar{\boldsymbol{P}}_{f,k} \boldsymbol{C}_f^T + \boldsymbol{R})^{-1}$$

$$\boldsymbol{x}_{f,k} = \bar{\boldsymbol{x}}_{f,k} + \boldsymbol{K}_{f,k} \left(\boldsymbol{y}_k - \boldsymbol{C}_f \bar{\boldsymbol{x}}_{f,k} - \boldsymbol{D}_f \boldsymbol{u}_{f,k} \right)$$

$$\boldsymbol{P}_{f,k} = \bar{\boldsymbol{P}}_{f,k} - \boldsymbol{K}_{f,k} \boldsymbol{C} \bar{\boldsymbol{P}}_{f,k}$$

$\bar{\ }$ stands for *a priori*

Kalman filter: kinematic approach

$$\dot{\boldsymbol{u}} = \boldsymbol{r}\boldsymbol{v} + \boldsymbol{a}_x$$
$$\dot{\boldsymbol{v}} = -\boldsymbol{r}\boldsymbol{u} + \boldsymbol{a}_y$$

$$\underbrace{\begin{Bmatrix} \dot{\boldsymbol{u}} \\ \dot{\boldsymbol{v}} \end{Bmatrix}}_{\dot{\boldsymbol{x}}_f} = \underbrace{\begin{bmatrix} \boldsymbol{0} & \boldsymbol{r} \\ -\boldsymbol{r} & \boldsymbol{0} \end{bmatrix}}_{\boldsymbol{A}_f} \underbrace{\begin{Bmatrix} \boldsymbol{u} \\ \boldsymbol{v} \end{Bmatrix}}_{\boldsymbol{x}_f} + \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{\boldsymbol{B}_f} \underbrace{\begin{Bmatrix} \boldsymbol{a}_x \\ \boldsymbol{a}_y \end{Bmatrix}}_{\boldsymbol{u}_f}$$

hp: the velocity measured by the GNSS antenna can be assumed to be the longitudinal component of the velocity (the lateral velocity is small)

$$\underbrace{\{\boldsymbol{u}_{\text{exp}}\}}_{\boldsymbol{z}_f} = \underbrace{[1 \quad 0]}_{\boldsymbol{C}_f} \underbrace{\begin{Bmatrix} \boldsymbol{u} \\ \boldsymbol{v} \end{Bmatrix}}_{\boldsymbol{x}_f}$$

Kalman filter: kinematic approach

Tasks:

1. Build matrices $\boldsymbol{A}_f$, $\boldsymbol{B}_f$ and $\boldsymbol{C}_f$
2. Discretise the problem in order to achieve matrices $\boldsymbol{A}_{f,d}$ and $\boldsymbol{B}_{f,d}$
3. Build the matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$

$$q_1 = q_2 = 10^{-4}$$
$$r_1 = 10^{-10}$$

4. Implement the 5 Kalman equations within a *for* cycle

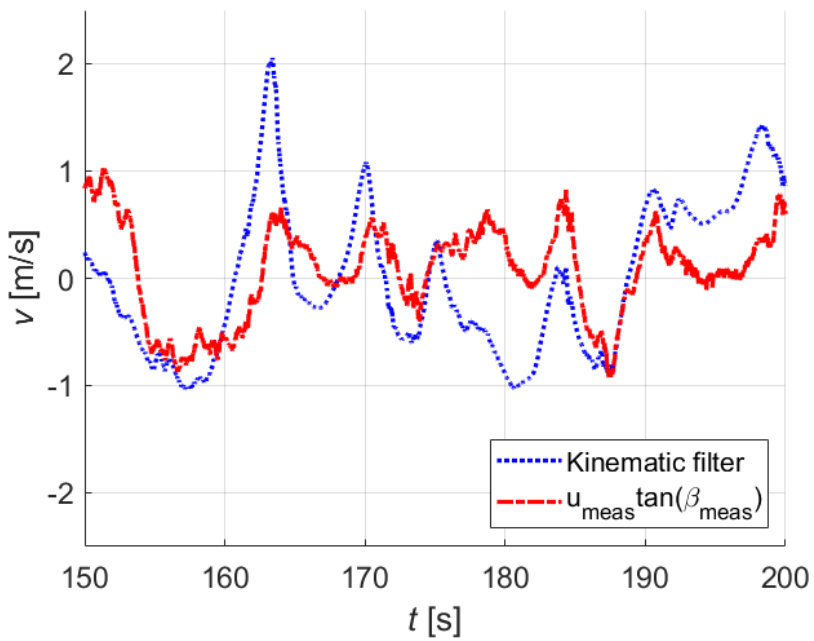
$$\begin{cases} \text{for } k = 1 : size(t, 2) \\ \dots \\ end \end{cases}$$

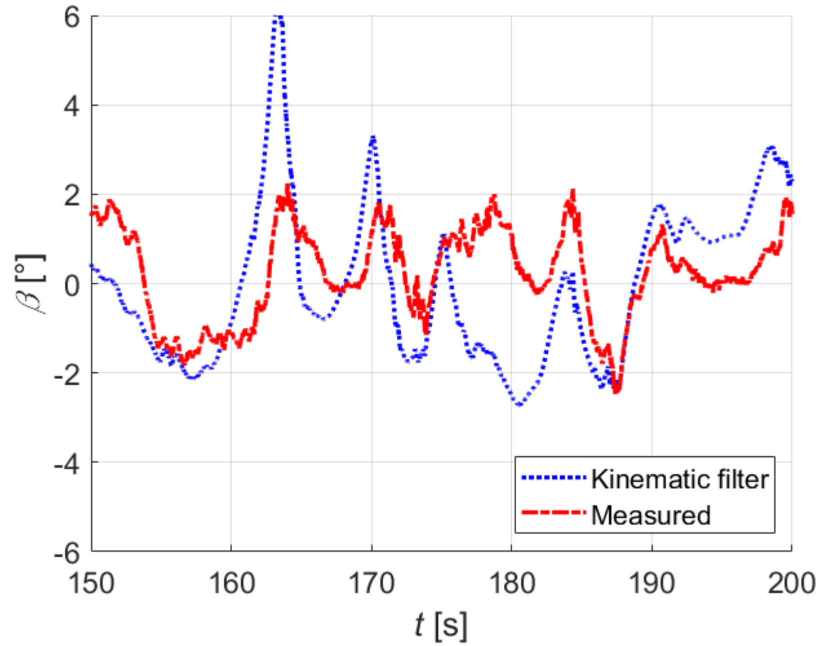
5. Compare the experimental results to the estimated magnitudes

Note that matrix $\boldsymbol{A}_f$ is not constant and must be computed at each time step k since it contains vector $\boldsymbol{r}$

Kalman filter: kinematic approach

Results





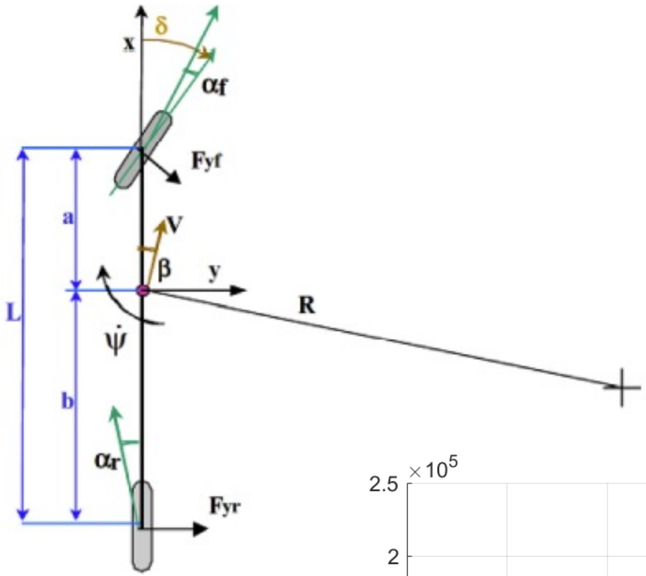
Kalman filter: dynamic approach

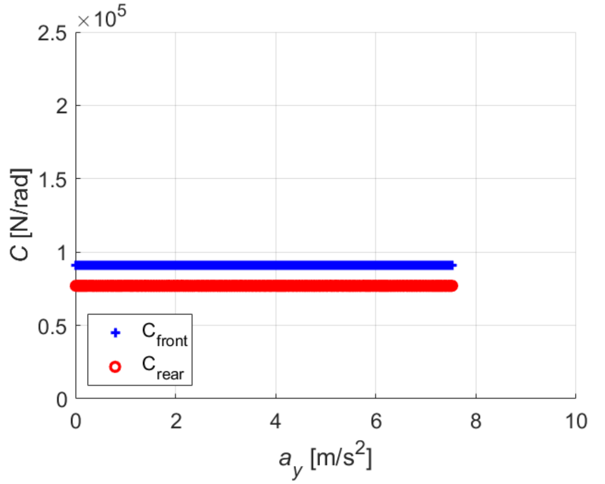
Fundamental equations

$$m(\dot{v} + ur) = F_{y,f} + F_{y,r}$$
$$J\dot{r} = F_{y,f}a - F_{y,r}b$$

$$F_{y,f} = C_f \alpha_f$$
$$F_{y,r} = C_r \alpha_r$$

$$\alpha_f = \tau_{sw}\delta - \frac{v+ra}{u}$$
$$\alpha_r = -\frac{v-rb}{u}$$
$$\begin{cases} C_f = 91000[\text{N/rad}] \\ C_r = 77000\ [\text{N/rad}] \end{cases}$$





Kalman filter: dynamic approach

Equation linearisation

$$\dot{v} = \frac{C_f}{m}\left(\tau_{sw}\delta - \frac{v + ra}{u}\right) + \frac{C_r}{m}\left(\frac{r\,b - v}{u}\right) - u\,r$$
$$\dot{r} = \frac{C_f a}{J}\left(\tau_{sw}\delta - \frac{v + ra}{u}\right) - \frac{C_r\,b}{J}\left(\frac{rb - v}{u}\right)$$

$$f(x)\,f(y) \cong f(x_0)\,f(y_0) + f'(x_0)(x - x_0)f(y_0) + f'(y_0)(y - y_0)f(x_0)$$

$$\frac{r}{u} \cong \frac{r_0}{u_0} + \frac{r - r_0}{u_0} - \frac{r_0(u - u_0)}{u_0^2}$$
$$u\,r \cong u_0 r_0 + (u - u_0)\,r_0 + (r - r_0)\,u_0$$
$$\frac{v}{u} \cong \frac{v_0}{u_0} + \frac{v - v_0}{u_0} - \frac{v_0(u - u_0)}{u_0^2}$$

Kalman filter: dynamic approach

States, measurements and covariances

$$\boldsymbol{x}_f = \begin{Bmatrix} v \\ r \end{Bmatrix}$$

$$y_f = r$$

$$\boldsymbol{u}_f = \begin{Bmatrix} u \\ \delta \end{Bmatrix}$$

$$\boldsymbol{C} = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$\boldsymbol{Q} = \begin{bmatrix} 0.5 \cdot 10^{-2} & 0 \\ 0 & 0.5 \cdot 10^{-2} \end{bmatrix}$$

if $a_y \leq 4\ m/s^2$

$$\boldsymbol{R} = [10^{-1}]$$

if $a_y > 4\ m/s^2$

$$\boldsymbol{R} = [10^{-6}]$$

Kalman filter: dynamic approach

Linearised matrices

$$\boldsymbol{x}_{f,k} = \boldsymbol{A}_{f,d}\boldsymbol{x}_{f,k-1} + \boldsymbol{B}_{f,d}\boldsymbol{u}_{f,k-1} + \boldsymbol{Z}_{f,d}$$

$$\boldsymbol{A}_{f,d} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \Delta t + \boldsymbol{I}$$

$$A_{11} = -\frac{C_f + C_r}{m\,u_0}$$

$$A_{12} = -\frac{C_f a - C_r b}{m\,u_0} - u_0$$

$$A_{21} = -\frac{C_f a - C_r b}{J\,u_0}$$

$$A_{22} = -\frac{C_f a^2 + C_r b^2}{J\,u_0}$$

$$\boldsymbol{B}_{f,d} = \begin{bmatrix} B_{11} & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \Delta t$$

$$B_{11} = \frac{C_f v_0 + C_f r_0 a - C_r r_0 b + C_r v_0}{m\,u_0^2} - r_0$$

$$B_{12} = \frac{C_f \tau_{sw}}{m}$$

$$B_{21} = \frac{C_f v_0 a + C_f r_0 a^2 + C_r r_0 b^2 - C_r v_0 b}{J\,u_0^2}$$

$$B_{22} = \frac{C_f \tau_{sw} a}{J}$$

Kalman filter: dynamic approach

Linearised matrices

$$\boldsymbol{x}_{f,k} = \boldsymbol{A}_{f,d}\boldsymbol{x}_{f,k-1} + \boldsymbol{B}_{f,d}\boldsymbol{u}_{f,k-1} + \boldsymbol{Z}_{f,d}$$

$$\boldsymbol{Z}_{f,d} = \begin{bmatrix} Z_1 \\ Z_2 \end{bmatrix} \Delta t$$

$$Z_1 = -\frac{C_f v_0 + C_f r_0 a - C_r r_0 b + C_r v_0}{m\, u_0} + r_0 u_0$$

$$Z_2 = -\frac{C_f v_0 a + C_f r_0 a^2 + C_r r_0 b^2 - C_r v_0 b}{J\, u_0}$$

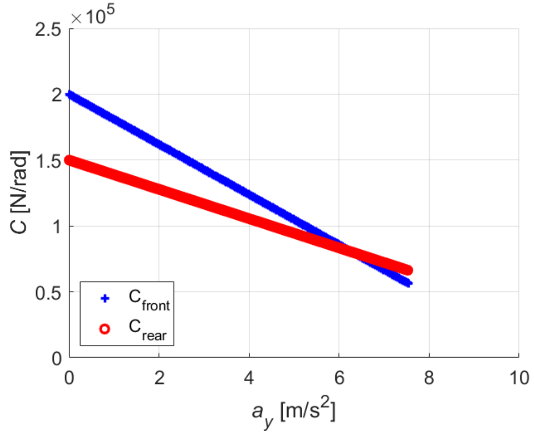
r_0, v_0 and u_0 are the magnitudes at the previous time step

Kalman filter: dynamic approach

Task: implement different cornering function of the lateral acceleration

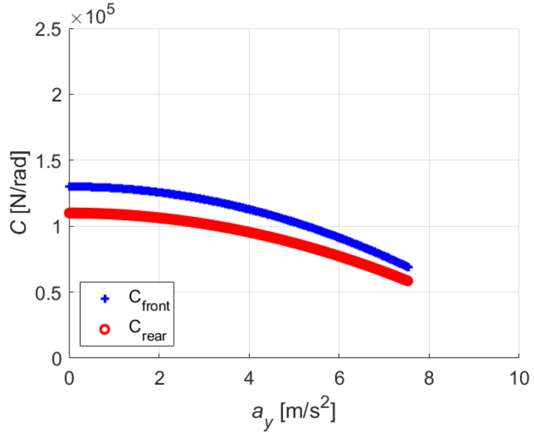
1. Linearly decrease

$$\begin{cases} C_f = 200000 - 19050\,|a_y| \\ C_r = 150000 - 11110\,|a_y| \end{cases}$$



2. Parabolic decrease

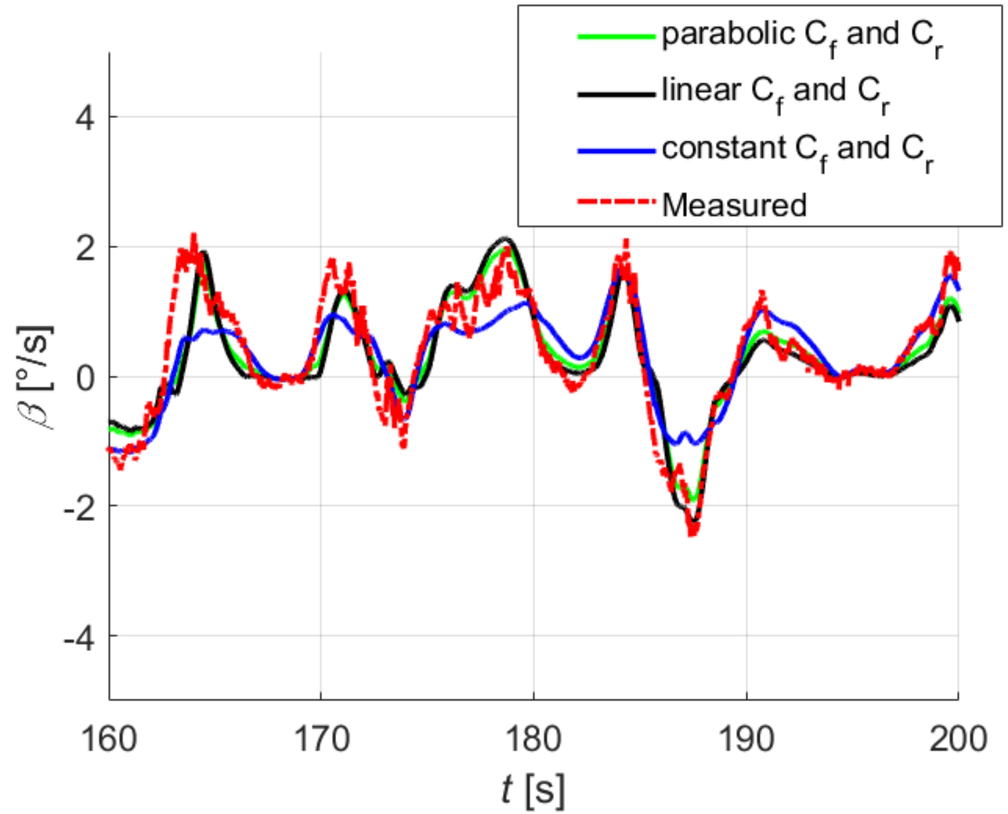
$$\begin{cases} C_f = 130000 - 1075\,|a_y|^2 \\ C_r = 110000 - 910\,|a_y|^2 \end{cases}$$



Kalman filter: dynamic approach

Task: implement different cornering function

4. Compare the results in terms of side-slip angle



Kalman filter: dynamic approach

Other results (considering a parabolic decrease of the cornering stiffnesses