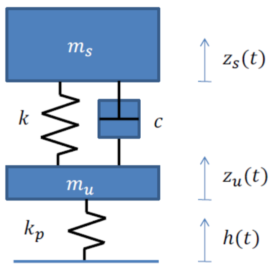
2DOF SUSPENSION

For the 1DOF the assumption of perfect rigid body for the tyre was done. Considering a 2DOF it means that the tyre has some deformability. This lead to two different movement, one for the sprung mass, another one for the unsprung mass. Therefore, two different absolute accelerations will be founded.

The scheme of the system is the following one, also with his dynamic equation:

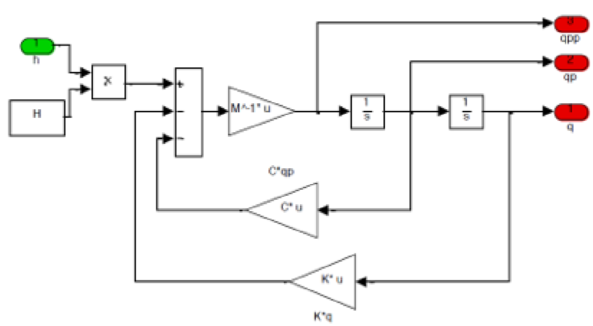


$$\begin{cases} m_s\ddot{z}_s = -c(\dot{z}_s - \dot{z}_u) - k(z_s - z_u) \\ m_u\ddot{z}_u = -c(\dot{z}_u - \dot{z}_s) - k(z_u - z_s) - k_p(z_u - h) \end{cases}$$

NOTE: The two forces in common between the two systems have to be equal and opposite for static and dynamic equilibium of the two masses.

In this lecture, considering the `MATLAB` properites, the matrix formualtion is adopted and used into a Simulink model.

$$M\ddot{q} + C\dot{q} + Kq = Hh$$



The data requested as inizialization is the following one.

```
warning('off','all')
clc
close all
clearvars

mtot = 1580;                                    %[kg]
mu = 160;                                       %[kg]
ks1 = 27*10^3; ks2 = 22.5*10^3; k1p = 4*274*10^3;   %[N/m]
c = 2664;   %value of 1DOF
ms = mtot-mu;
```

```
k = 2*(ks1+ks2);

c_opt = sqrt(ms*k/2*(k1p+2*k)/k1p);

M = [ms 0; 0 mu];

K = [k -k; -k k+k1p];

C = [c -c; -c c];

H = [0 k1p]';
```

Please note that for the natural frequencies evaluation the overall car system is consdiered. It would semplificate the 4*mass with 4*stiffness in the following procedure. This is done so to set a standard procedure which is indipendent on the overall number of suspension systems.

Being the springs in parallel, their contribution is summing, therefore the overall stiffness of the system is just the sum of frontal axle suspensions and rear ones.

Please note also that the computate of the optimal damping changes, since now there are more variables involved:

$$c_{ott} = \sqrt{\frac{m_s k}{2}\frac{k_p + 2k}{k_p}}$$

Natural frequencies and modes

Since out system is no longer a 1DOF, the natural frequency is no longer a trivial calculation. Furthemore, having two masses when they oscillated they can dispone themself in different way. To solve both questions, the mode analysis should be performed.

Every motion of a system can be represented as a linear combinations of modes. Each mode is the disposition of the system when subjected to an exictation with a frequency equal to the natural one. First mode takes palce at the first natural frequency (the smallest), the second to the second natural frequency and so on.

Having a 2DOF system, it is resonable to expect a behaviour where the two masses' movements are in phase and another one where they are in anti-phase.

NOTE: The natural frequencies are always computed with the undamped system indipendently on the DOF of the system, therefore, only the `M` and `K` will be considered in the eigenproblem. For a system of two matrices, the eigenvalues (λ) and eigen vector (v) are defined as the ones which permit to the following formula to be true:

$$Kv - M\lambda v = 0$$

The modes are the eigenvector founded. For convinience, all the eigenvectors are normalaised respect to the first one. Meanwhile the natural frequency is the square root of the eigenvalues. Be aware that the latter operation has to be done *element-by-element* (there is a certain difference between AA*AA and AA^0.5, having AA as a vector).

This having said, it is possible to solve the eigenproblem and evaluate the modes and the natural frequencies.

```
[vv,ww] = eig(K,M); % produces a diagonal matrix ww
    % of generalized eigenvalues and a full matrix vv
    % whose columns are the corresponding eigenvectors
    % so that kk*vv = mm*vv*ww --> (kk-mm*ww)*vv = 0
om = diag(ww).^0.5; % natural pulsations
    % eigenvector normalisation in order to have the
    % first element of phi = 1
f_nat = om/2/pi     % natural frequencies
```

```
f_nat = 2×1
    1.2722
   13.7598
```

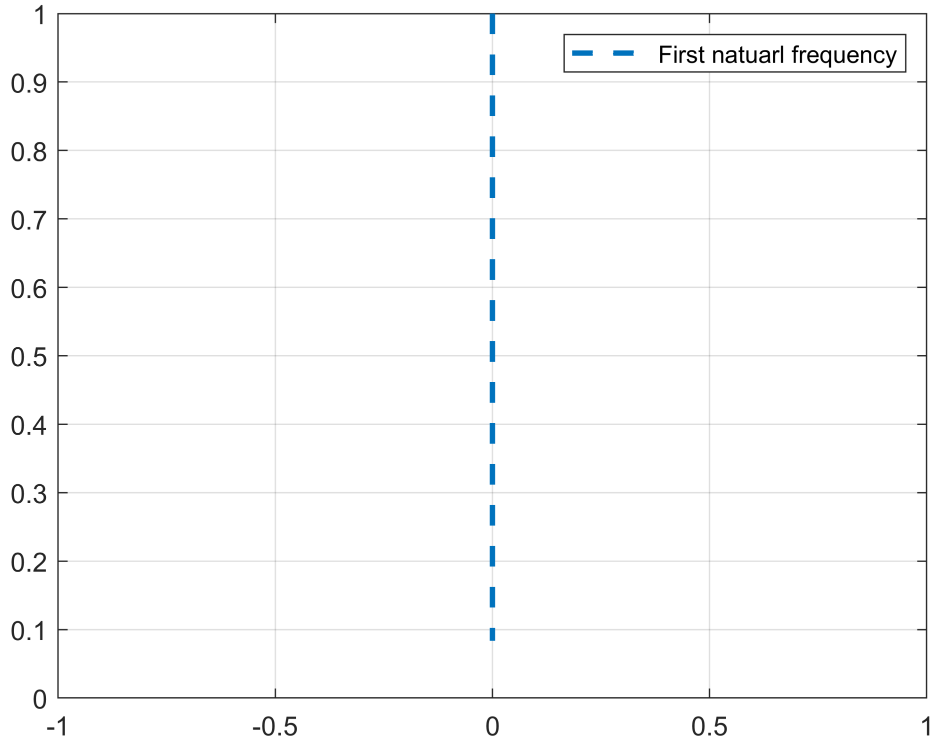
Remeber how for a car the frequency range for the natural frequencies are are 1Hz for the first one and below 15Hz for the second one (CBD explanation).

```
phi_1 = vv(1:2,1)./(vv(1,1));
phi_2 = vv(1:2,2)./(vv(1,2));
phi = [phi_1 , phi_2]
```

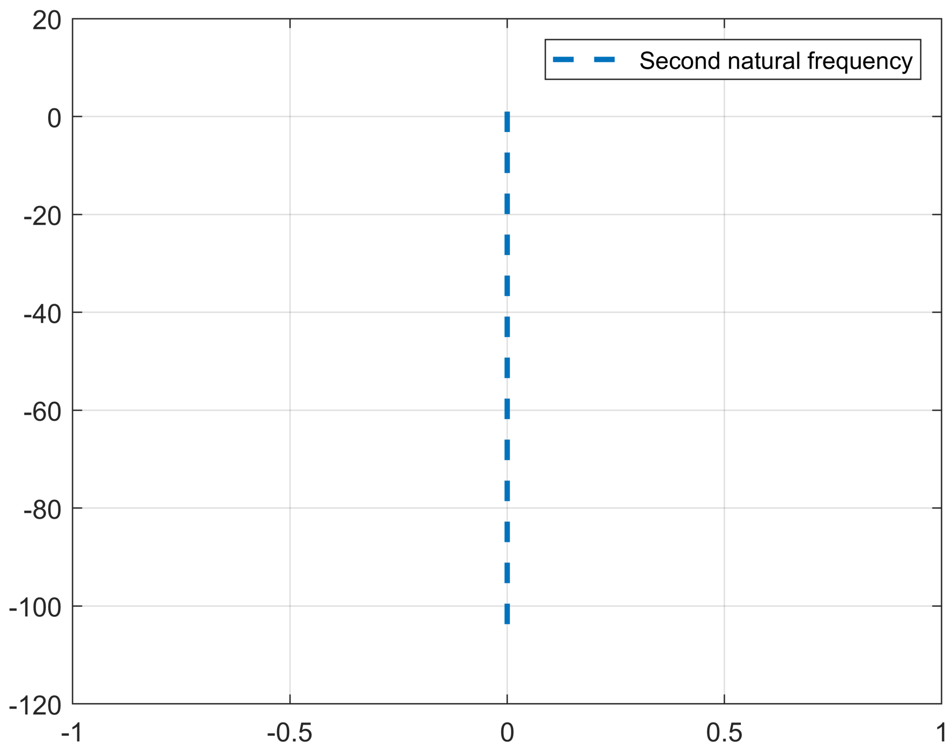
```
phi = 2×2
    1.0000    1.0000
    0.0836 -106.2111
```

The values of the natural modes mean that for the first natural frequency (first column) the sprung mass moves of 1 and the unsprung of 0.0836 and they are in-phase. For the second mode, the sprung mass moves of 1 and the unsprung of -106.211.

```
x = linspace(0,0);
y = linspace(phi(1,1),phi(2,1));
figure
plot(x,y,'--','Linewidth',2)
grid on;
legend('First natuarl frequency')
```




```
y = linspace(phi(1,2),phi(2,2));
figure
plot(x,y,'--','Linewidth',2)
grid on;
legend('Second natural frequency')
```



Transfer function evaluation

Having a 2DOF system, the study of the system changes notably. In fact there are more variables which are of interest since they gives usefull information. In the case analysed, it is possible to indentify two of them: the sprung mass acceleration $\ddot{z}_s$ and the vertical force F_z . Each of them should be compared with a proper reference variable, in other words the denominator of each trasfer function is different ($\ddot{z}_s$ is refered to h and the vertical force F_z to a proper quantity $k_p h$). The why and how these two transfer function is going to be explained in the following chapter.

Comfort and handling

The transfer fucntions $\frac{\ddot{z}_s}{h}$ and $\frac{F_z}{k_p h}$ ar of interest since they represents respectively comfort and handling performances. In fact the sprung mass acceleration, which can be considered as the chassis' one, has some effects over the **comfort** since not only high accelerations lead to jumps in the cockpits, but also certain range of frequencies over the human body implies discomfort (4-10 Hz; CBD notes). The other trasnfer function is realted to **handling**; in fact since the vertical load is dorectly linke to the lateral force exerted on the ground by the tyre ($F_y = -C\alpha$; MVD notes), it is resonable to assume that the vertical load has a direct influence over the handling. Considering an oscillating vertical force: the lateral one is going to behave in the same way, which implies that the tyre struggles in turning the vehicle. Therefore, oscillating vertical forces implies that the vehicle would oscillates laterally when turning.

Receptance matrix

A 2DOF system surely permit to model efficently a suspension motion, however implies a certian degreee of compelxity in returns. In particular, the frequency responce cannot be evaluated in an easy way like in the 1DOF, where a simple Laplace transformation permit to compute it analytically. In fact in a 2DOF system it is necessary to perform some meccanical harmonic motion study to arrive to the conclusion that in order to evaluate the transfer function of a virbrating system with more than 1DOF it is necessary to evaluate the **receptance matrix**. By its own the receptance matrix is defined as the inverse of the **dynamic stiffness matrix**, which can be compued by analitical procedure as:

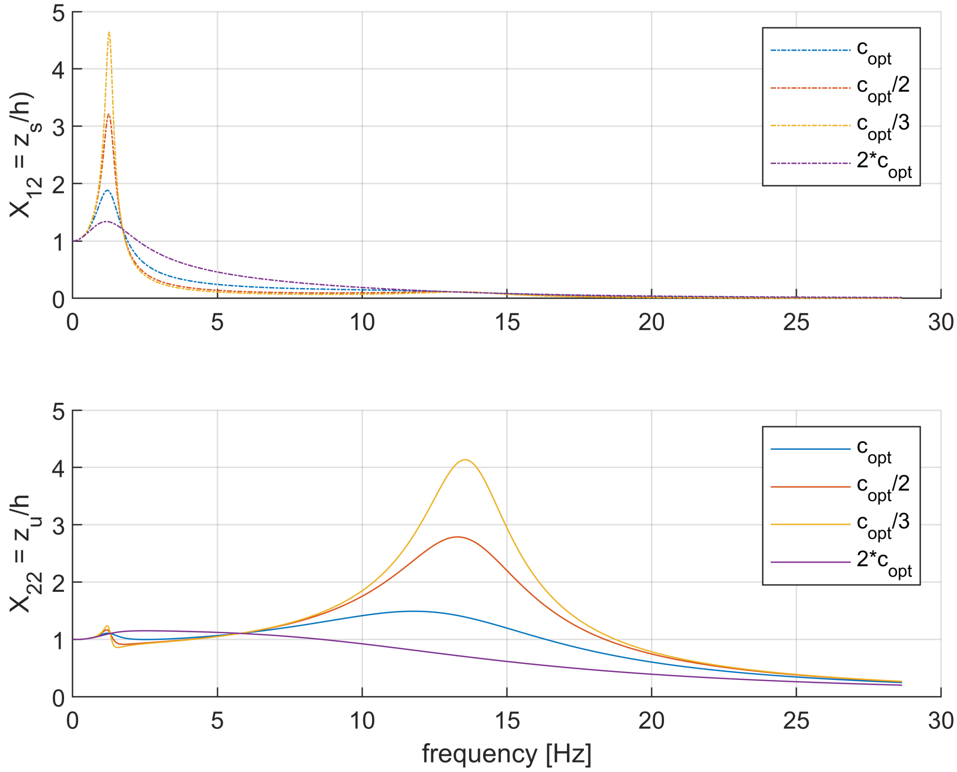
$$[K_{dyn}(\omega)] = ([K] + i\omega[C] - \omega^2[M])$$

$$[R(\omega)] = [K_{dyn}]^{-1}$$

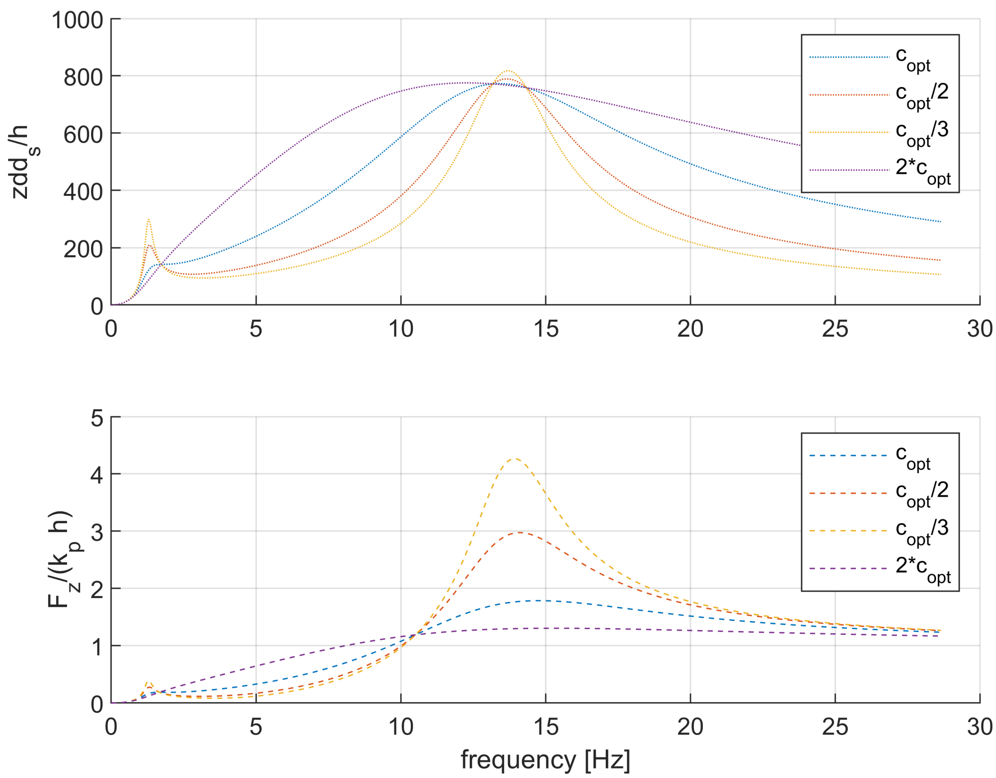
To facilitate this procedure, the code which compute the receptance matrix has been directly provided by the professor. It is of our interes to see the differences between handling and comfort for three values of damping.

Effects of damping variations

```
OM = [0:0.1:180].';
C_values = [c_opt c_opt/2 c_opt/3 2*c_opt];
C_names = {'c_{opt}' 'c_{opt}/2' 'c_{opt}/3' '2*c_{opt}'};

for count2=1:length(C_values)
    c = C_values(count2);
    C = [c -c;-c c];
    for count1=1:size(OM,1)
        w = OM(count1,1);
        Kdyn = (K + i*w.*C - w^2.*M);
        Rec = inv(Kdyn);
        X12(count1,1) = Rec(1,2)*H(2,1); % tf: zs/h
        X22(count1,1) = Rec(2,2)*H(2,1); % tf: zu/h
        Fz(count1,1) = k1p*(1-X22(count1,1)); % tf: Fz/h
    end
    figure(1);
    subplot(211); hold all;
    plot(OM/(2*pi),abs(X12),'-.','Displayname',C_names{count2});
    ylabel('X_{12} = z_s/h)'); legend('show'); grid on;

    subplot(212); hold all;
    plot(OM/(2*pi),abs(X22),'-','Displayname',C_names{count2});
    ylabel('X_{22} = z_u/h'); xlabel('frequency [Hz]')
    legend('show'); grid on;

    figure(2);
    subplot(211); hold all;
    plot(OM/(2*pi),OM.^2.*abs(X12),':','Displayname',C_names{count2});
    ylabel('zdd_s/h'); legend('show'); grid on;

    subplot(212); hold all;
    plot(OM/(2*pi),abs(Fz)./k1p,'--','Displayname',C_names{count2});
    ylabel('F_z/(k_p h)'); xlabel('frequency [Hz]')
    legend('show'); grid on;
end
```



X12 = zs/h
X22 = zu/h
copt
copt/2
copt/3
2*copt
frequency [Hz]



zdds/h
Fz/(kp h)
copt
copt/2
copt/3
2*copt
frequency [Hz]

The first two charts represent the displacement filter effects. The conclusion are the same done in the 1DOF system: an underdamped system would present amplification effetcs at the natural frequencies. Therefore, considering the displacement filtering effects, an overdamped system shows better performances.

The last two plots shows more intreting results.

Regarding **comfort** performances (third chart), having underdamped system is the best choice; Considering **handling** performance instead, the overdamped shows the best behaviour. This means that comfrot and handling are two opposite behaviours: a car cannot be perfectly balaned in both, therefore a compromise between them has to be founded for convenctional cars.

Considering more extreme situations intead, like a F1 car, this explain why thier suspesions are so stiff: they are exploiting the best handling performance possilbe from the damper. Not by chace the F1 cars are well know for being uncomfortable to drive since there is a nearly direct connection between wheel movement and chassis one. See for example the helmet movements when a car moves into a curb: it feels all over it! These uncomfort behaviour also partially explain the tireness of motorsport on 4 wheels: riding very uncomfortable cars sucks off all the energy from a body!

Please note that the best performance mentioned is considered as the one which shows lower peak and a more flat behaviour, which translates into having a best filtering effect.

NON LINEAR DAMPERS

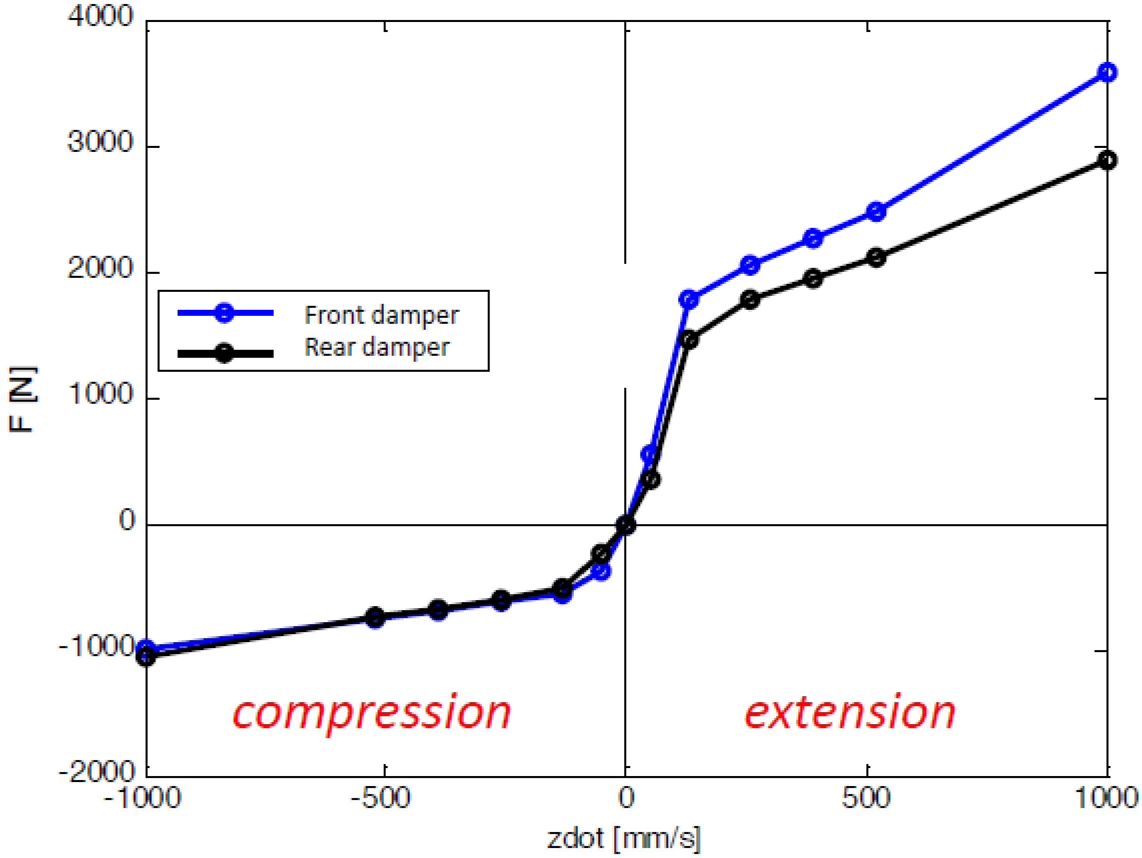
A further step toward a realistic simulation is to consider a non linear damper behaviour. In fact, the damper exert different forces values during compression and extension, with the extension one being usually bigger 3-4 times more than in compression.

The physical reason is that having 2DOF it has been shown that the motion of the system is no longer trivial. In particular, there is a focus over the handling and comfort performances of the system, and being them two complementar phenomena, it is necessary that for certain range of frequencies one is prefered to the other. Being more specific, depending on the velocity applied to the damper two different task are set:

- Low speed: controls roll and pitch motion (***handling***)
- High speed: controls the vertical motion pf the sprung mass, due to road irregularities (***ride comfort***)

In order to get this behaviour, the damping shoulde be high at low frequencies (since it has to filter as much as possibile unwangted vibrations) and low at high frequencies (since it has to allow optimal trasmissibility car-gorund force).

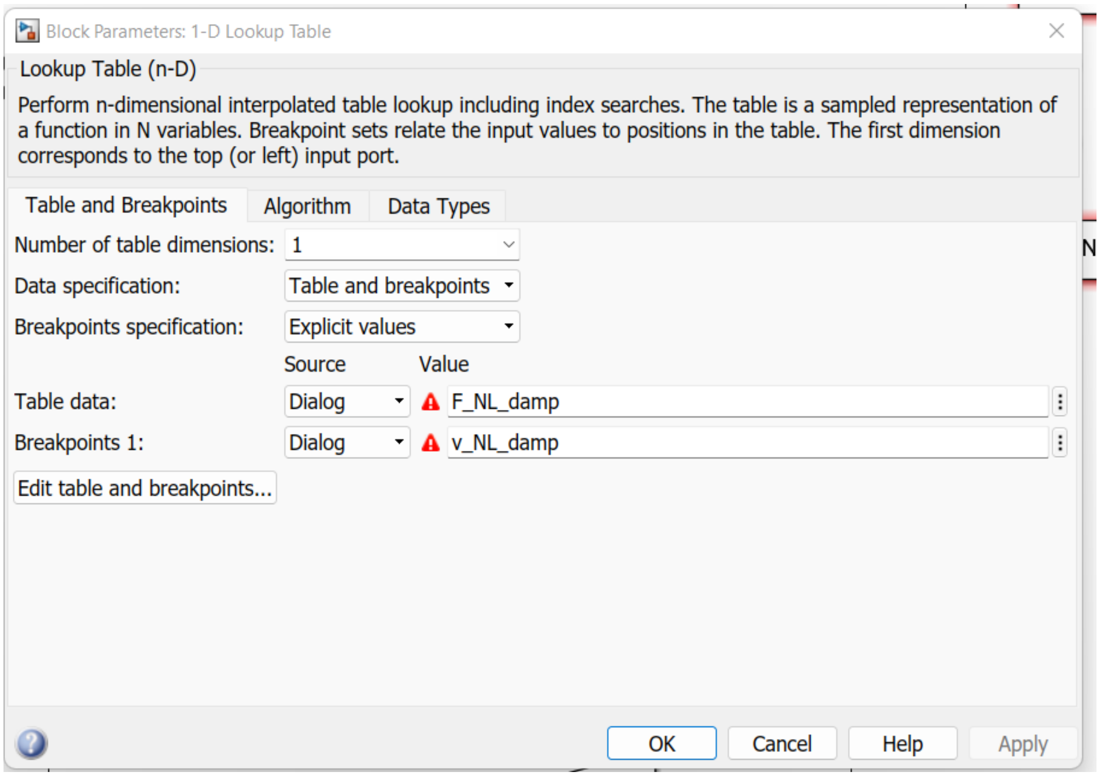
This new feature is represented by the following table:



Where it can be easily implemented in the previous model by the use of a lookup table. Please note that the force is modelled in this way, where the vector [1;-1] is due to the fact that sprung and unsprung mass exeter an equal an opposite force between them.

$$\begin{cases} m_s\ddot{z}_s = -F_{damp} - k(z_s - z_u) \\ m_n\ddot{z}_u = F_{damp} - k(z_u - z_s) - k_p(z_u - h) \end{cases}$$

On Simulink the implementation of a lookup table wants the following declaration



Where the two vecotrs have been defined as follows:

```
v_NL_damp = [-1000 -520 -390 -260 -130 -50 0 50 130 260 390 520 1000]/1000;
F_NL_damp = 4*[-672 -493 -450 -400 -351 -199 0 301 1068 1260 1386 1514 2130];
```

NOTE: The velocity has been expressed in m/s and the damping values in the table are refered for a single damper. Since it is suggest to consider the overall car system, this vector has to be multiplied by each suspension system.

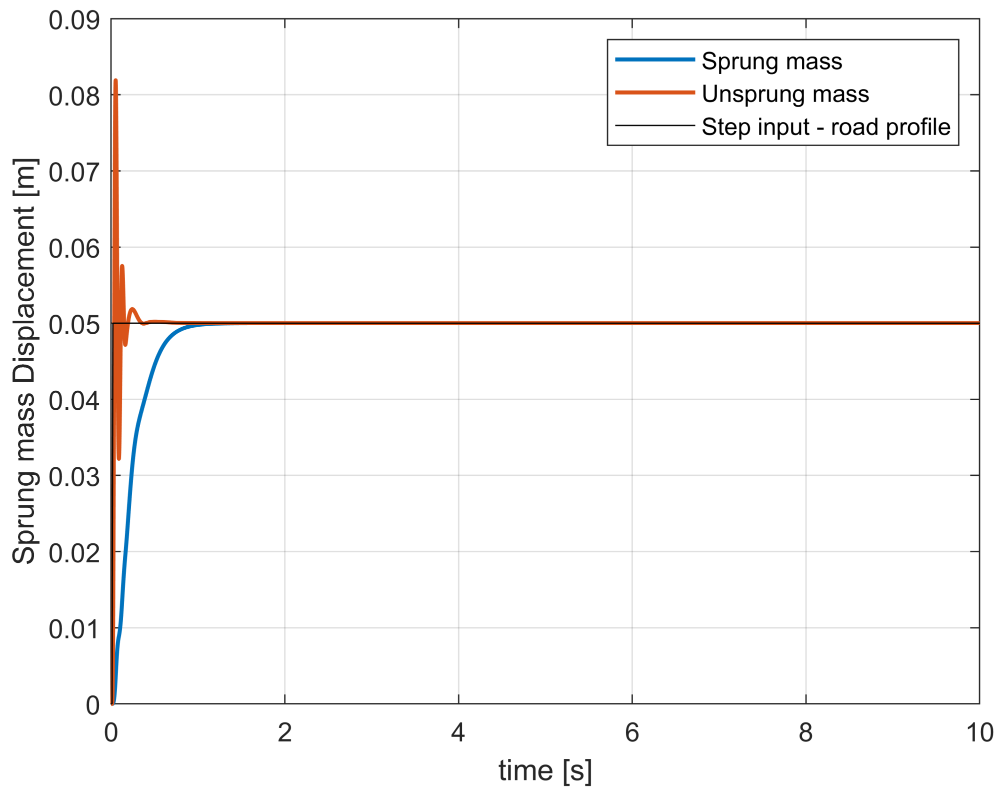
Then, it is now possible to load the data needed and launch the simulation on Simlunk with the new damper charcteristics. Firstly the step responce is analysed, then the TF.

```
clc;
dati = 'Dati_Necessari'; open(dati); run(dati);
Ts = 10; choice2 = 1;
nfile = 'sim_NONLINEAR_2DOF'; open(nfile); sim(nfile)
```

Step results

```
figure(3)
plot(time,zs_NL,'LineWidth',1.5) %z from simulink is in m!!!
xlabel('time [s]'); xlim([0 Ts]);
ylabel('Sprung mass Displacement [m]')
grid on
hold on

figure(3)
plot(time,zu_NL,'LineWidth',1.5)
xlabel('time [s]'); xlim([0 Ts]);
ylabel('Unprung mass Displacement [m]')
```

```
figure(3)
plot(time,road_profile_NL,'k','LineWidth',0.5) %z from simulink is in m!!!
xlim([0 3])
xlabel('time [s]'); xlim([0 Ts]);
ylabel('Sprung mass Displacement [m]')
grid on; hold on;
legend('Sprung mass','Unsprung mass','Step input - road profile'); grid on; hold on
```



Here it is possible to see how the non linear damper shows a very damped responce for the sprung mass. In fact, since here the wanted responce is to block as much as possible the accelerations coming from the ground so to have comfortability, it is resonable that the system is *overdamped* at low frequencies.

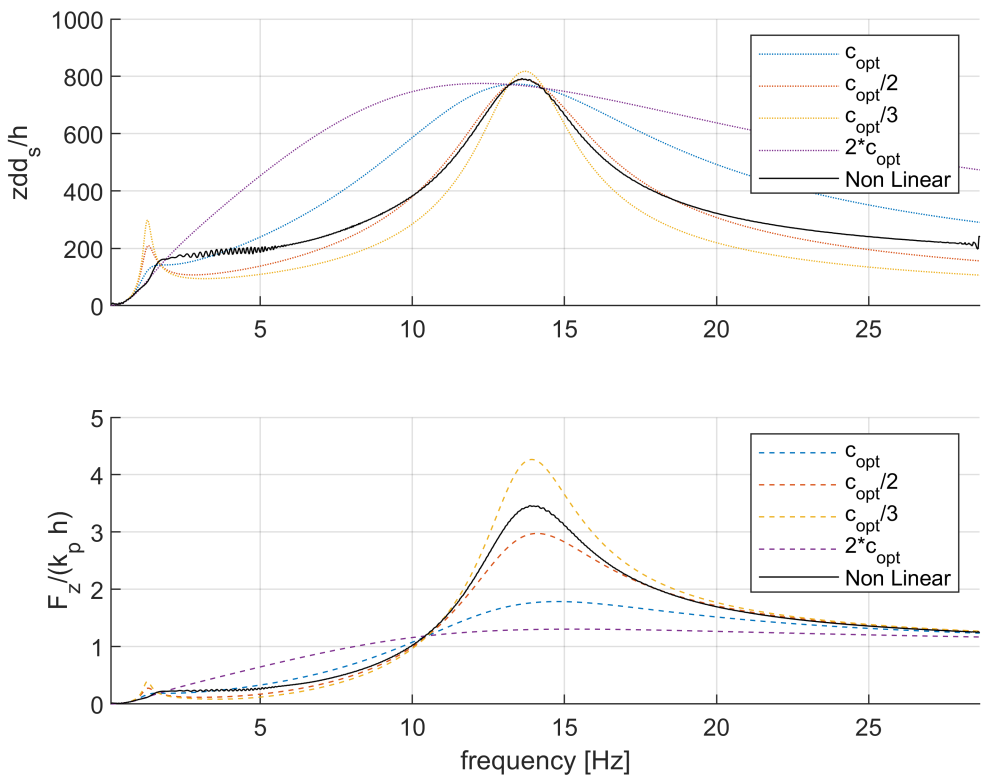
Transfer function (chirp input)

Having commented the step input responce, the trasnfer function could give more informations. Since the system is no longer linear, the TF has to be estimated by the command `tfestimate`.

Please note that since the system is not linear, the TF depends also on the amplitude of the input signal. For this reason the h0 has been defined.

```
% Parameters change to set the chirp input

Ts = 100; choice2 = 3;
w_in = 0; w_end = 180;      %[rad/s] Values given by the slides
f_in=0.1;                   %[Hz] To avoid possible simualtions errors
f_end=w_end/2/pi;           %[Hz]
```

```
h0 = 2/100;                    %[m] This is teh amplitude of thh chirp signal

nfile = 'sim_NONLINEAR_2DOF'; open(nfile); sim(nfile)

[G_zdds_h_NL,F] = tfestimate(road_profile_NL,zdd_NL.data(:,1),[],[],[],f_sampling); %zs is the

relative_disp_NL = road_profile_NL-zu_NL;
G_Fz_kph_NL = tfestimate(k1p*road_profile_NL,k1p*relative_disp_NL);

f = linspace(f_in,f_end,length(G_zdds_h_NL));

figure(2);
subplot(211); hold all;
plot(F,abs(G_zdds_h_NL),'k','LineWidth',0.5,'Displayname','Non Linear');
xlim([f_in f_end])
ylabel('zdd_s/h'); legend('show'); grid on;

subplot(212); hold all;
plot(F,abs(G_Fz_kph_NL),'k','LineWidth',0.5,'Displayname','Non Linear');
xlim([f_in f_end])
ylabel('F_z/(k_p h)'); xlabel('frequency [Hz]')
legend('show'); grid on;
```



As expected from the theory, the non linear damper shows overdamped behaviour for low frequencies, so to have acceptable comfort performance and underdamped one at high frequencies. The typology of damping can be directly deduce form the chart. Not only there is the c_{opt} transfer fucntion, but it can be see from the non linear tendency. In fact, having low values of the transfer fucntions means that the system is "rigid", therefore there is a filtering effect for which at a given input the output is not going to be much different. On the other way if the TF values are big: it means that the system can be easily exicted and that even for low values of input the output is going to be considerably big.