Longitudinal dynamics

Choice of transmission ratios

In this exercise, a complete longitudinal car model is going to be analyzed for simulating the longitudinal dynamic behaviour of an electric car. The propulsion chosen is an electric for simplicity, in fact the exercised proposed can be dealed using only `MATLAB` tools, since the equations adopted are simple enough to avoid to develop a Simulink model.

The vehicle considered has 3 gears, which is very strange for an electric vehicle, but which is an intresting point to study since longitudinal dynamics is usually adopted for gear sizing.

Focus: compare different driving configurations.

```
close all; clear; clc
```

Electric motor characteristics

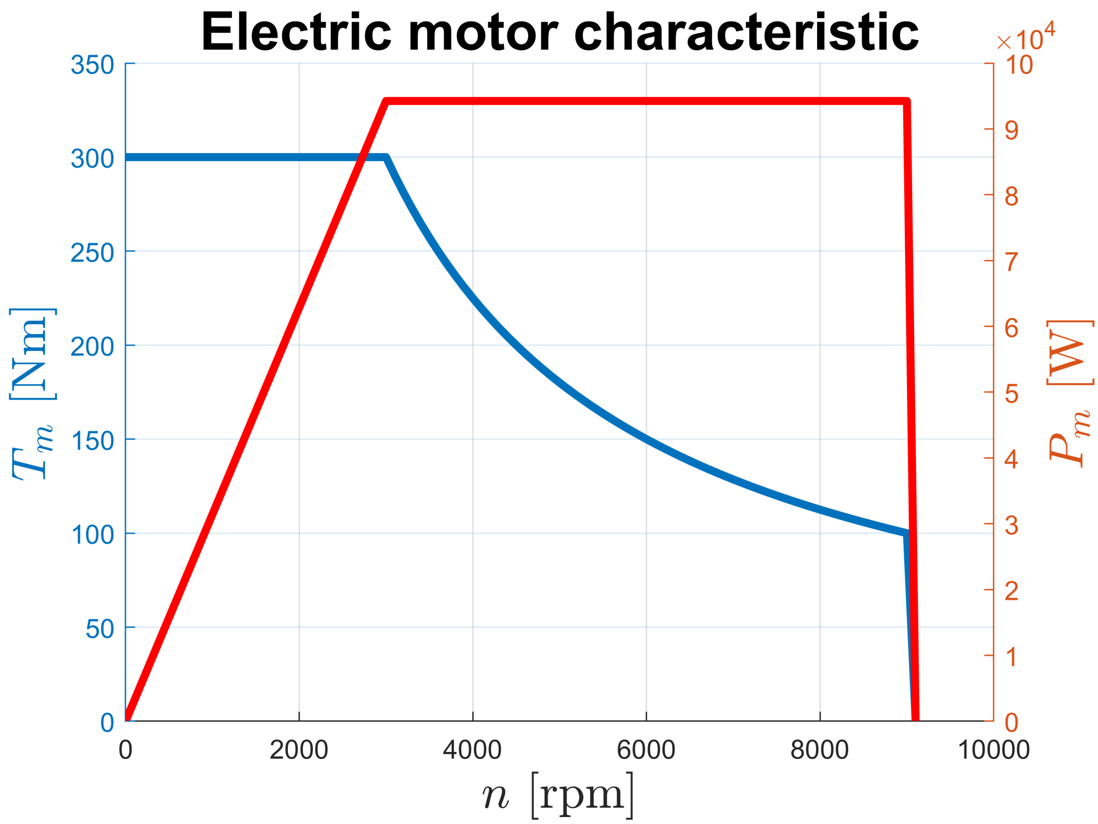
The motor map of an electric vehicle is given as data. Remember that the engine map of an electric vehicle is very simple, having a linear behaviour till the base speed, after that the power will be constant no matter the motor speed. This is due to the torque which is constant up the base speed a,d afterwards it decreases with an iperbola tendency. This "decay" is due to the limited amount of the voltage that the battery can provide, meanining that the battery can deliver a limited power whihc is always converted into a mechanical one ($P = \omega T$). If the rotational speed increases and having the power saturated to its maximum, it is natural that the torque delivered will decrease.

```
Pmax_mot = 300*3000*pi/30;  % [W] motor maximum power
nnom_mot = 3000;            %
nmax_mot = 9000;            % [rpm] motor maximum speed
disp(['Max motor power Pmax: ' num2str(Pmax_mot/1e3) ' [kW]'])
```

```
Max motor power Pmax: 94.2478 [kW]
```

torque and power characteristics: $P_m = T_m\omega_m$

```
w_mot = [0 1 nnom_mot nnom_mot:100:nmax_mot+100]*pi/30;
T_mot = [300 300 Pmax_mot./w_mot(3:end-1) 0];
P_mot = T_mot.*w_mot;
% plot data
figure(1); grid on; hold on
title('Electric motor characteristic','FontSize',20)
yyaxis left
plot(w_mot*30/pi,T_mot,'linewidth',3); ylim([0 350])
xlabel('$n$ [rpm]','interpreter','latex','FontSize',18);
ylabel('$T_m$ [Nm]','interpreter','latex','FontSize',18)
yyaxis right
plot(w_mot*30/pi,P_mot,'r','linewidth',3)
xlabel('$n$ [rpm]','interpreter','latex','FontSize',18);
ylabel('$P_m$ [W]','interpreter','latex','FontSize',18)
```



Vehicle data

```
m=1600;          % [kg] vehicle mass
Jz=2860;         % [kg*m^2] vehicle mass moment of inertia
a=1.1;           % [m] front wheelbase
b=1.5;           % [m] rear wheelbase
L=a+b;           % [m] wheelbase
Af=1.8;          % [m^2] drag area
rho=1.3;         % [kg/m^3] air density
Cx=0.28;         % [-] aerodynamic drag coefficient
Rfront=0.316;    % [m] raggio ruote ant a riposo
Rrear=0.316;     % [m] raggio ruote post a riposo
Kv=2.5e5;        % [N/m] rigidezza verticale ruote
R = 0.97*Rfront; % [m] rolling radius
Ir=.5;           % [kg*m^2] wheel mass moment of inertia
ha = 0.6;        % [m] pressure centre height
Hg=0.52;         % [m] CoG height
eta_t = 0.99;    % [-] transmission efficiency
g = 9.81;        % [m/s^2] gravity acceleration
mu = 1;          % [-] road friction coefficient
f0r = 0.0041;     % [Nm/N] tyre rolling resistance coefficient (torque)
f2r = 2.051e-7;  % [Nms/Nm^2] tyre rolling resistance quadratic coefficient (torque)
f0 = f0r/R;
k = f2r/R;
```

Matching vehicle and motor

In order to study the vehicle performance is necessary to comapre the power available from the propulsion systems with the power required t overcomes the motion resistance.

Motion resistance which can be described by both force or power, simply considering that $P_{res} = F_{res}V$. The motion resistance can be divided into three components:

- aerodynamic drag: $R_a = \frac{1}{2}\rho S C_x V^2$
- gradient resistance: $F_\alpha = mg \sin\alpha$
- rolling resistance: $F_r = mg\cos\alpha(f_0 + kV^2)$

The input of the system presented is the velocity, in fact the resistance force can be also written as: $F_{res} = A + BV^2$ (no linear dependecy!) where the two coefficents are also called **coast down** coefficents.

```
figure(2)
title('Matching required power and available power')
incl = 0;   % [%] inclination
v_vet = (0:1:220)/3.6;                  % vehicle speed vector
F_res_aero = 0.5*rho*Cx*Af*v_vet.^2;    % aerodynamic drag
w_ruota = v_vet/R;
alpha = atan(incl/100); % [rad] inclination angle
F_res_inc = m*g*sin(alpha);     % gradient resistance
F_res_rot = m*g*cos(alpha)*(f0r + f2r*w_ruota.^2)/R;  % rolling resistance
P_res = (F_res_aero+F_res_rot+F_res_inc).*v_vet;    % total necessary power

A = m*g*sin(alpha)+m*g*cos(alpha)*f0
```

```
A = 209.9491
```

```
B = 0.5*rho*Cx*Af + m*g*cos(alpha)*k
```

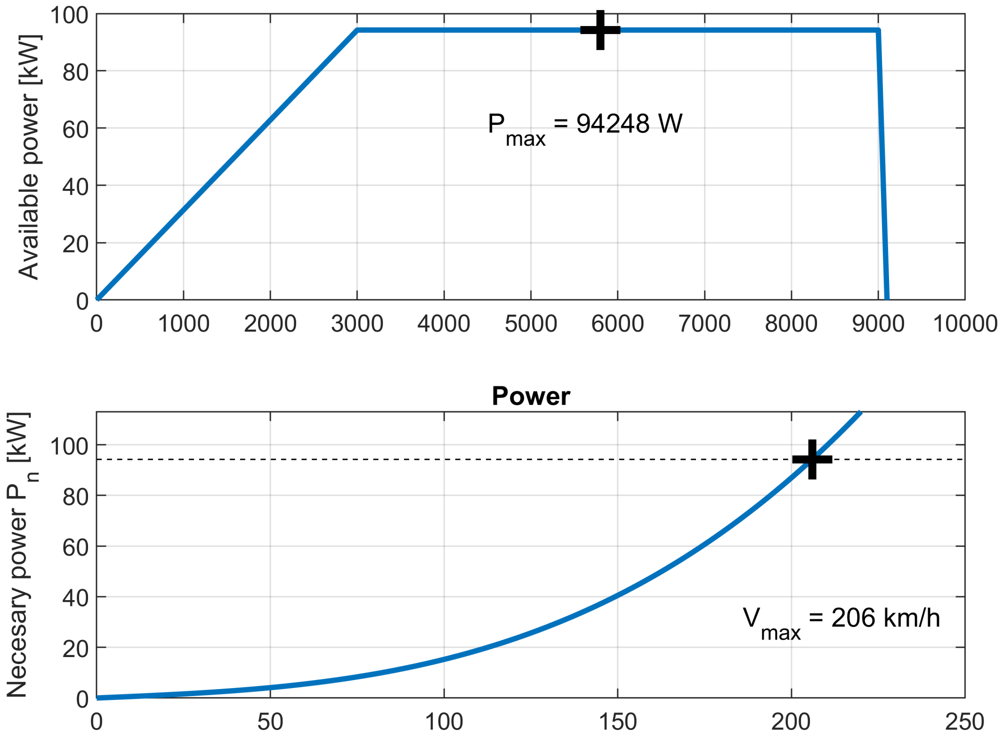
```
B = 0.3381
```

Once the resistance force has been evaluated, it is possible to compute the maximum velocity reachable by the vehicle. In fact, plotting the power available and the power necessary for moving, their intsersection in the maximum velocity which can be reached.

This is done automatically by the code provided by the professor.

```
subplot(212)

% Resistance Power
plot(v_vet*3.6,P_res/1000,'linewidth',2);
title('Power')
ylabel('Necesary power P_n [kW]'); grid on

% Power available from the EM
subplot(211)
plot(w_mot*30/pi,P_mot/1000,'linewidth',2); hold on
P_max = max(P_mot);
plot(w_mot(P_mot==P_max)*30/pi,P_max/1000,'+k','linewidth',3,'markersize',15)
text(mean(xlim)-500,mean(ylim)+10,['P_{max} = ',num2str(round(max(P_mot))),' W'])
```

```
ylabel('Available power [kW]'); grid on

% Plot the power at the maximum speed (or maximum power in general)
subplot(212); hold on
plot(xlim,P_max/1000*[1 1],'--k')
plot(v_vet((abs(P_res-P_max))<500)*3.6,P_max/1000,'+k','linewidth',3,'markersize',15)
v_max_pot = v_vet((abs(P_res-P_max))<500)*3.6;
text(v_max_pot-20,30,['V_{max} = ',num2str(round(v_max_pot)),' km/h'])
```

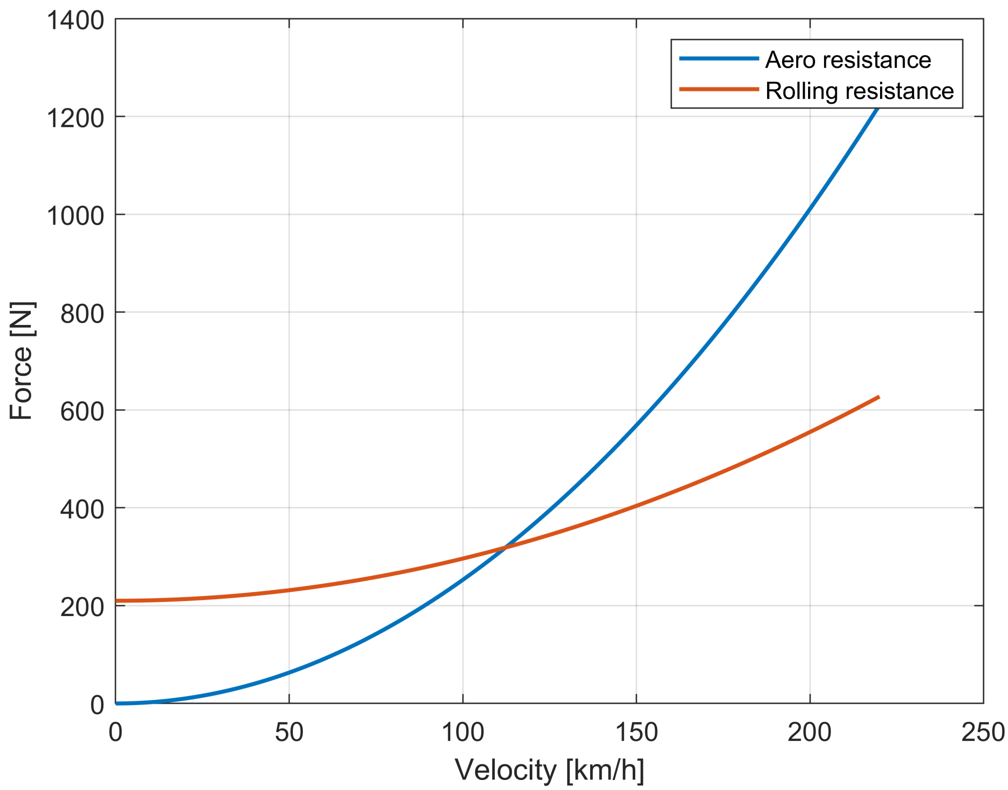


In this simualtion then, the maximum velocity correspond by 206 km/h which is considerable. Consider that the data are not so different for a VW e-Golf car (100 kWh of power). 2L Diesel VW Golf has 202 km/h as maximum speed, having a reference.

```
% figure %control that the acutal P_res computed is the correct one
% plot(v_vet,F_res.*v_vet)
```

Also important to study is the **characteristic velocity** which the velocity for which the aerodynamic resistance becomes the most important factor in the resistive force. It can be evaluated by simply plotting the two resistance contribution and pointing out the intersecton point.

```
F_res = A+B.*(v_vet).^2;
figure(3); plot(v_vet*3.6,F_res_aero,'LineWidth',1.5);
xlabel('Velocity [km/h]'); ylabel('Force [N]');
grid on; hold on;
plot(v_vet*3.6,F_res_rot,'LineWidth',1.5);
legend('Aero resistance','Rolling resistance')
```

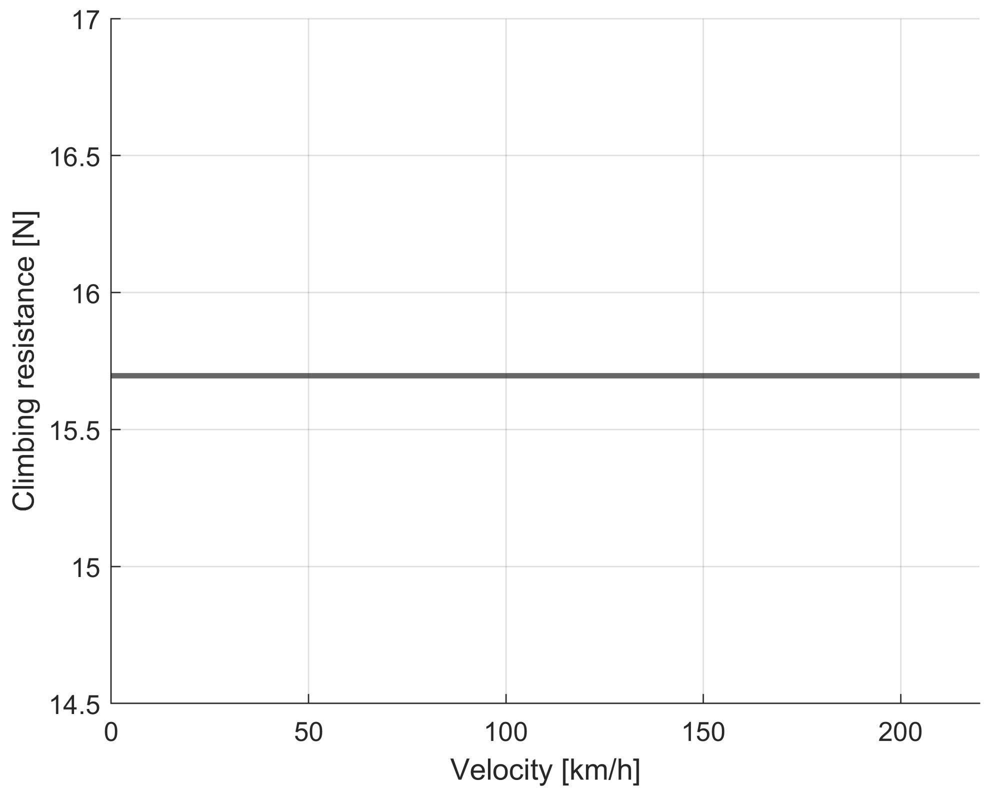



```
Vcar = 113; %[km/h] from the chart
```

Climing resistance for 10% of inclination

The climbing force is a constant contribution since it depends only on the road inclination. Therefore a constant line will result as a plot.

```
incl10 = 10/100;
alpha10 = atan(incl10/100);       % [rad] inclination angle
F_res_inc10 = m*g*sin(alpha10);   % gradient resistance
figure(4)
yline(F_res_inc10,'LineWidth',2)
xlim([v_vet(1)*3.6 v_vet(length(v_vet))*3.6])
xlabel('Velocity [km/h]'); ylabel('Climbing resistance [N]'); grid on;
```



Gear ratios

The gear ratio are choosen by a progressive method. In particular, the first one is evaluated considering the maximum climbing performance, set to an incliantion of 100% which is 45° of road inclination.

To determine the *final transmission ratio* τ_F, it has been assumed a minimum gear ratio $\tau_{G3} = 1$; computation of maximum speed (intersection of available and necessary power) at the maximum motor speed, in order to exploit the motor power to accelerate the vehicle from velocities lower than the maximum: $\tau_F = \frac{\omega_{v,\max} R_e}{v_{\max} \tau_{G3}} = \frac{\omega_{\max} R_e}{v_{\max} \tau_{G3}}$

```
% final transmission ratio
v_max = v_max_pot/3.6;
w_max = nmax_mot*2*pi/60; % At maximum speed the EM is at w_max!
tau_F_initial = w_max*R/(v_max)
```

```
tau_F_initial = 5.0485
```

Since the computed value of τ_F is quite large, assume $\tau_F = 3.8$ (given by the professor as data) and compute the values of all the gearbox ratios, including τ_{G3}.

```
tau_F = 3.8;
tau_G3 = w_max*R/v_max/tau_F;
```

Compute *first gear ratio* based on maximum road slope: $\tau_{GI} = \frac{mg(\sin \alpha_{max} + f_0 \cos \alpha_{\max}) R_e}{\eta_t \tau_F T_{m,\max}}$

```
incl_max = 100;   % [%] inclination
alpha_max = atan(incl_max/100); % [rad] inclination angle
tau_G1 = m*g*(sin(alpha_max)+f0*cos(alpha_max))*R/(tau_F*eta_t*max(T_mot));
```

NOTE: the taus with the roman pedix are the tau which includes the transission ratio from the engine directly to the wheels, therefore they include the differential ratio: $\tau_{ROMAN} = \tau_F * \tau_{Gi}$

```
tau_I   = tau_G1*tau_F;
tau_III = tau_G3*tau_F;
```

Intermediate ratios: based on geometric progression of ratio $r = \sqrt{\tau_{III}/\tau_I}$, so $\tau_{II} = r\tau_I$

```
r = sqrt(tau_III/tau_I);
tau_II = r*tau_I;
tau_G2 = tau_II/tau_F;
disp(['First gear ratio: ', num2str(tau_G1), ' Second gear ratio: ', num2str(tau_G2),' Third ge
```

```
First gear ratio: 3.0547 Second gear ratio: 2.0145 Third gear ratio: 1.3286
```

- What is the gear ratio that allows to reach the maximum vehicle speed? It is the same for ICE? A check over the minimal motor speed has to be done?

It the case proposed the highest gear permit to obtain the maximum speed. However in convenctional ICE cars, the last gear is set for optimizing fuel consumption menahwile the penultimate gear is the one set for reaching the maximum velocity, in order to facilitate the overtaking in highway (passing from best fuel consumption condition, downshifting so to reach the maximum speed).

There is no need to check the motor minimum speed since the electric motor has the priviledge to be auto-starting, not by change the engine map shows that the torque have a certain value for small rpm, which is something that with ICE does not occur. A classi ICE engine map start at some rpm which are not null, underlining the necessity of a starting mechanism.

Check vehicle performance

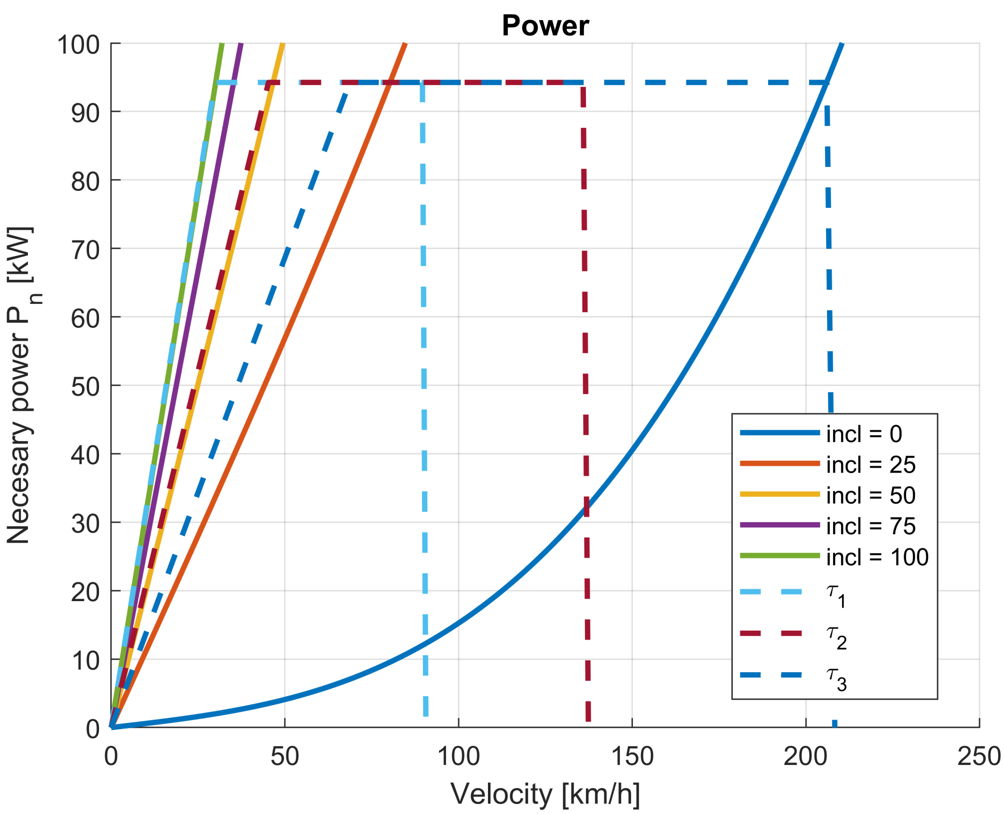
To check the vehicle performance in a wide operating condition, it could be usefull to study also the power curve at variable road inclination. It can be easily done by computing the aerodynamic and rolling resistance contribution for a set of road inclinations.

```
incl_vet = [0:25:100];

for count1=1:length(incl_vet)
    incl = incl_vet(count1);   % [%] inclination
    F_res_aero = 0.5*rho*Cx*Af*v_vet.^2;    % aerodynamic drag
    alpha = atan(incl/100); % [rad] inclination angle
    F_res_inc = m*g*sin(alpha);     % gradient resistance
    F_res_rot = m*g*cos(alpha)*(f0r + f2r*w_ruota.^2)/R;  % rolling resistance
    P_res = (F_res_aero+F_res_rot+F_res_inc).*v_vet;

    % Resistance Power
    figure(5); hold on;
```

```
    plot(v_vet*3.6,P_res/1000,'linewidth',2);
    ylim([0 100]);
    xlim([0 200]);
    title('Power'); ylabel('Necesary power P_n [kW]'); xlabel('Velocity [km/h]');grid on
end
legend('incl = 0','incl = 25','incl = 50','incl = 75','incl = 100')
```

Plot for differents gear ratios:

```
tau_vet = [tau_G1 tau_G2 tau_G3];
for count=1:length(tau_vet)
    v_veh = w_mot*R/tau_vet(count)/tau_F*3.6; %[km/h]
    figure(5); hold on;
    plot(v_veh,P_mot/1000,'--','LineWidth',2);
    ylim([0 100]);
    xlim([0 250]);
end
legend('incl = 0','incl = 25','incl = 50','incl = 75','incl = 100','\tau_1','\tau_2','\tau_3',
```



This plot permit to study the vehicle perfomances in terms of climbing and top speed.

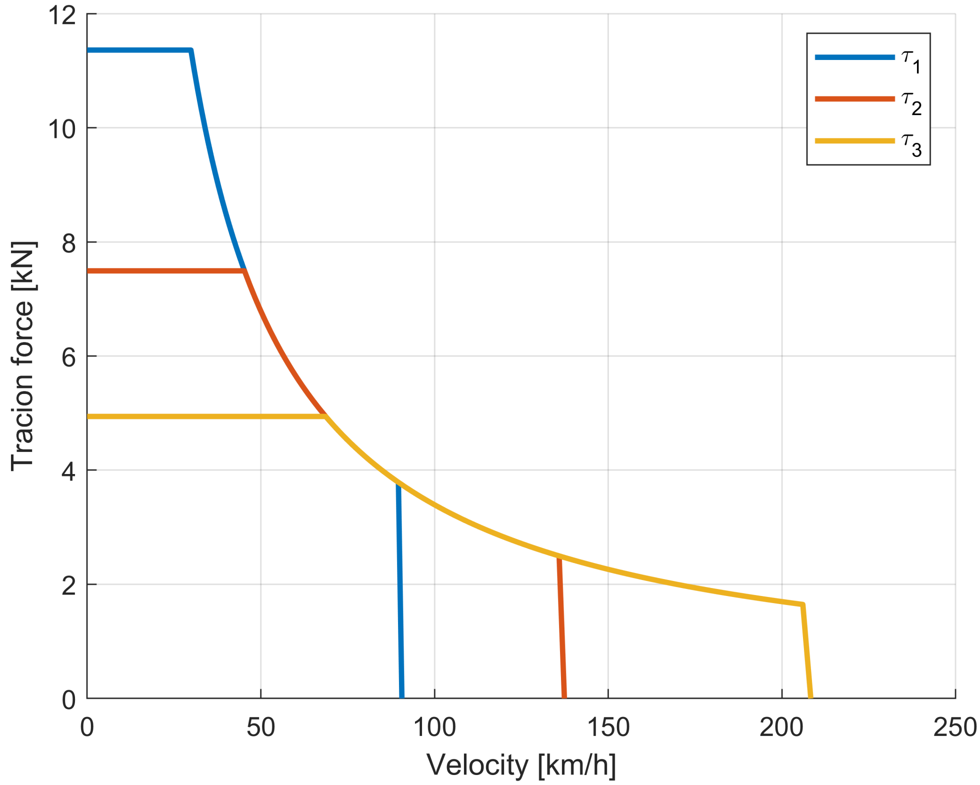
Please be aware that all this considerations do not take into account the driving wheel and load transfer effects, which they cannot be neglected when doing the maximum slope calcualtion. Therefore, this evaluation is a simple reference for further investigation, something like "this is what our car can theoretically develop, but what it can actually do is completely another story".

Regarding climbing, having used the incl=100 as the design condition for the first gear, available power and required one are overlaying. Physically means that the vehicle barely can climb up. Remember that this have been proposed with $\mu = 1$, in other words, in real applications the vehicle cannot climb an inclination of 100 unless it has ideal conditions. Nevertheless, the overall performance is considerable: the electric motor have as one of its advantage the capacity to deliver quickly high torque values. With a second gear in fact, the vehicle seems capable to climb an inclination of 50%, corresponding to a $26.5651°$ road angle, which is basically covering all possible uphill that a car can face (take as reference that the maximum inclinantion in all Stelvio is 14% and the car proposed can do it even with the third gear -theoretically-).

Considering maximum speed, it can be pointed out as the intersection between available power and requested power having the third gear selected. The maximum speed rechable with a certain gear selected also can be evaluated as the intersection between the available power and the power requested.

Tracion force vs speed for each gear

```
for count=1:length(tau_vet)
    v_veh = w_mot*R/tau_vet(count)/tau_F*3.6; %[km/h]
    figure(6); hold on;

    F_x = T_mot*tau_vet(count)*tau_F/R; %Fx comes from the torque
    % Not from P_mot (differently from how written in the slides)

    plot(v_veh,F_x/1000,'LineWidth',2);
end
grid on;
xlabel('Velocity [km/h]'); ylabel('Tracion force [kN]');
legend('\tau_1','\tau_2','\tau_3','Location','best')
```



From an experimental point of view it is easy to assest that having a lower gear permit to increase the traction torque (thus traction forces). The reason why this happens is simply described here: the traction force is directly proportional to the trasmission ratio, therefore the highe it is the bigger the force developed.

Always remeber that the trasmission ratio defined in this exercise is $\tau = \frac{\omega_{in}}{\omega_{out}}$ starting from the engine toward the wheels:

in = something arriving from engine side (omega_in) -> Then there is the reduction (/tau) -> out = something linked to the wheels (omega_out = omega_in/tau and omega_in is bigger than omega_out).

Compute the maximum inclination angle that can be climbed by a FWD, RWD or AWD vehicle.

Define torque ratio $K_T = \frac{T_F}{T_R}$. In ideal traction condition this value should vary since the ratio between front and rear vertical load varies in transient manoeuvres.

Note: Longitudinal load transfer (acceleration and aerodynamic force) are neglected in the next steps. The maximum slope at almost zero speed (Ra=0) and zero acceleration (ax=0) is computed, all the available power is used to overcome the road slope, no further power for vehicle acceleration and to increase the speed remains.

In real application the ratio between the two traction torque is constant and depending on the vertical load on teh two axles, two different condition could be reached, each one with a specific axle slipping.

- front axle critical: $i_{\max}\big|_F = \tan\alpha_{\max}\big|_F = \dfrac{\mu b(1+1/K_T)}{L+\mu h_G(1+1/K_T)}$
- rear axle critical: $i_{\max}\big|_R = \tan\alpha_{\max}\big|_R = \dfrac{\mu a(1+K_T)}{L-\mu h_G(1+K_T)}$

These formulas contains also FWD and RWD since they are two limit values. They can be found simply letting $1/K_T = 0$ in the first formula and $K_T = 0$ in the second formula. The best K_T has been choosen as the one which allow the maximum climbing alpha (0.6 from this simulation).

Calculation of the maximum slope for FWD and RWD:

```
i_max = mu*b/(L+mu*Hg);
incl_FWD = i_max * 100;
alpha_max_FWD = atan(i_max)*360/2/pi;
X_FWD = ['Maximum angle for FWD is : ',num2str(alpha_max_FWD),' deg'];
disp(X_FWD)
```

```
Maximum angle for FWD is : 25.6768 deg
```

```
i_max = mu*a/(L-mu*Hg);
incl_RWD = i_max * 100;
alpha_max_RWD = atan(i_max)*360/2/pi;
X_RWD = ['Maximum angle for RWD is : ',num2str(alpha_max_RWD),' deg'];
disp(X_RWD)
```

```
Maximum angle for RWD is : 27.872 deg
```

Calculation of the maxium slope for variable K_t.

```
Kt = 0.6;

F_zf = m*g*(b/L*cos(alpha_max*2*pi/360)-Hg/L*sin(alpha_max*2*pi/360));
F_zr = m*g*cos(alpha_max*2*pi/360)-F_zf;

if F_zr/F_zf >= 1/Kt
    disp('Frontal axle critical')
    i_max = mu*b*(1+1/Kt)/(L+mu*Hg*(1+1/Kt));
    incl = i_max * 100;
    X = ['inclination: ', num2str(incl), '%'];
    disp(X)
    alpha_max = atan(i_max)*360/2/pi
else
    disp('Rear axle critical')
    i_max = mu*a*(1+Kt)/(L-mu*Hg*(1+Kt));
    incl = i_max * 100;
    X = ['inclination: ', num2str(incl), '%'];
    disp(X)
    disp(['Maximum angle for AWD with Kt = ', num2str(Kt),': ',num2str(atan(i_max)*360/2/pi),'
end
```

```
Rear axle critical
inclination: 99.5475%
```

```
Maximum angle for AWD with Kt = 0.6: 44.8701 deg
```

- What if the center of gravity is moved in the middle of the two axles? Impose e.g. a=b=1.3m.

```
a = 1.3; b = a; % [m]
i_max = mu*b/(L+mu*Hg);
incl_FWD = i_max * 100;
alpha_max_FWD = atan(i_max)*360/2/pi;
X_FWD = ['Maximum angle for FWD with 50:50 mass distribution is : ',num2str(alpha_max_FWD),' de
disp(X_FWD)
```

```
Maximum angle for FWD with 50:50 mass distribution is : 22.6199 deg
```

```
i_max = mu*a/(L-mu*Hg);
incl_RWD = i_max * 100;
alpha_max_RWD = atan(i_max)*360/2/pi;
X_RWD = ['Maximum angle for RWD with 50:50 mass distribution is : ',num2str(alpha_max_RWD),' de
disp(X_RWD)
```

```
Maximum angle for RWD with 50:50 mass distribution is : 32.0054 deg
```

Regarding the RWD, for the same road inclination, the moment due to the load transfer remain the same. Since the rear axle has a lower arm, to counterbalance this load transfer the vertical force acting on the rear wheels must increase. Higher vertical force implies higher longitudinal force (= traction force) resulting in a better grip. This permit to reach higher values of road angle.

Viceversa for the FWD, since increasing a means that the CoG is moved toward the rear, as a consequence a lightening on the front axle is obtained. Less vertical force means less grip and therefore the critical condition gets easier to reach.

MAZZAMAURIELLO QUESTION: Which is the maximum value of inclination possibile? And of alpha? Which is the formula that link the two quantities together?

MAZZAMAURIELLO QUESTION: Which traction system provide the best performance in climbing? Why?