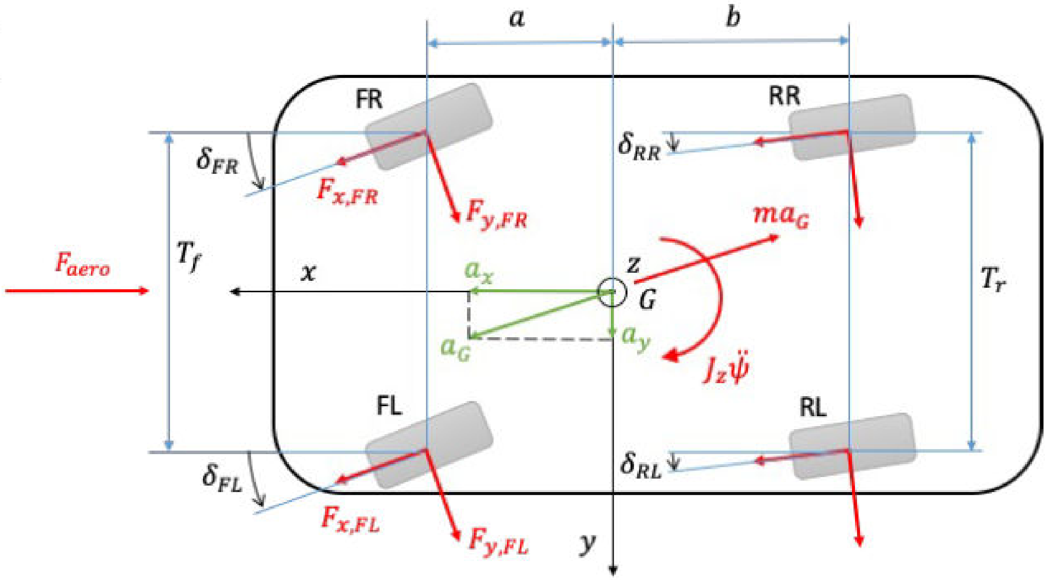
ES 08 - Lateral dynamics with 4 dof model

In this exercise two models will be build: one for the vehicle 3dof and the other one regarding the wheel dynamics. The sum of these two model group the 4dof mentioned in the title. The considered vehicle will have two wheel for each axle, adopting the "high speed cornering" model 8name adopted in the literature).

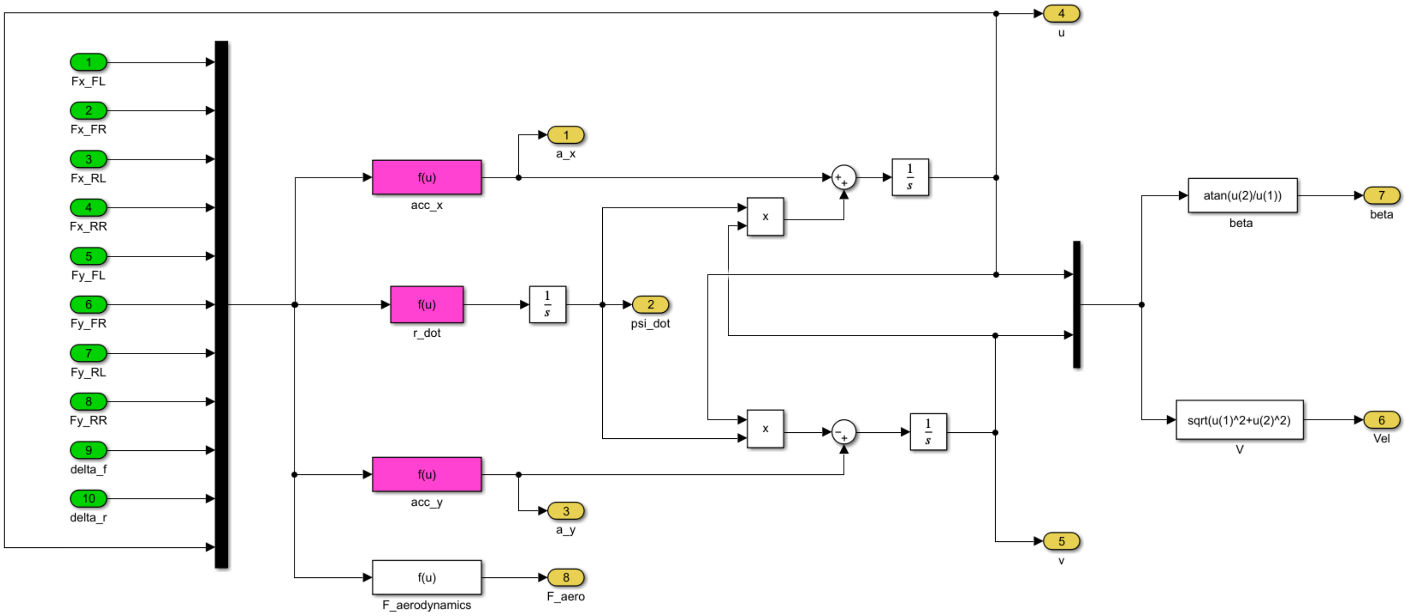


Firstly the vehicle 3dof will be build and tested, then the wheel dynamics will have its own focus as well.

Vehicle 3dof

Initially for the testing of this model, the forces will be overimposed as direct inputs. In further applications they will come from the wheels block (mostly from pacejka formulas). The modelling part for this exercise consinst on writing the acceleration formulas both in longitudinal and lateral directions. This was assest with a simple function block in Simulink with the help of generale note for checking the correctness of the formula. However this procedure is not perfectly clean and it too "mechanical", for future applications more adeguate solutions should be considered.

The model then presents in the following way, with the magenta color underling the model introduced:

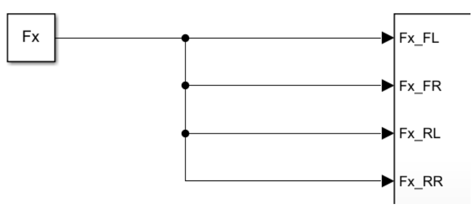


Once the model was set up, 6 questions have been poned in order to understand the characteristics and the behaviour of the vehicle.

Since the forces distribution changes between the scenarios, to spare time each time the model was modified accuratly and the simualtion was run. Therefore thuis livescript is more of a notebook than an actual script on the go.

```
warning off;
% latdyn_data -> reference main script
```

- **Apply a total longitudinal force of 2000 N, equally distributed on the 4 wheels, with null steering angles.**

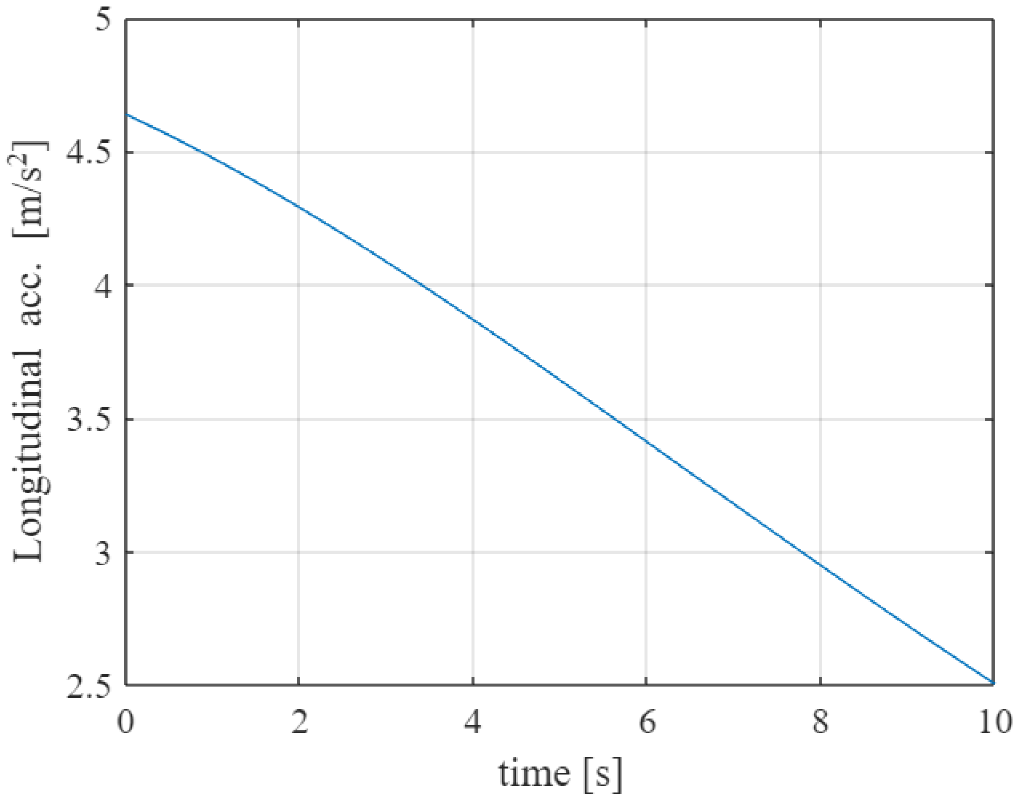


Note: modification needed into the model.

```
Fx=2000;                %[N]
Fy=000;                 %[N]
deltaf=0;               %[deg]
deltar=0;               %[deg]
vel_ini=80/3.6;         %[m/s]
latdyn_data;
```

<u>Question</u>: why longitudinal acceleration is not constant but decreases with time?

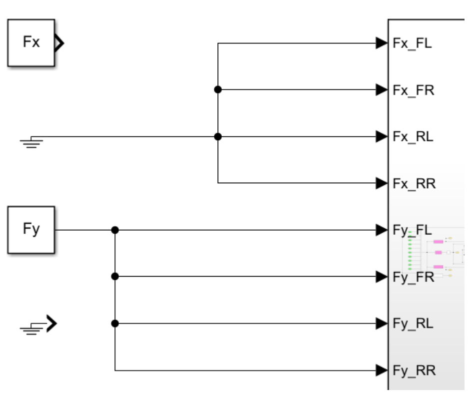
```
figure; plot(ax(:,1),ax(:,2));
xlabel('time [s]'); ylabel('Longitudinal acc. [m/s^2]'); grid on;
set(gca,'FontName','Times New Roman','FontSize',14);
```



This trend Is due to aerodynamics forces contribution. These forces depend on v^2, therefore they have parabolic tendency respect to time. Being force and acceleration directly related, here explained this tendency. Since the time of simualtion is only 10 seconds this is actually the inital part of the overaal longitudinal acceleraition decay, with higher time of simualiton it will decrase parabolically until it reaches the steady state at null acceleration.

NOTE: the acceleration is constantly decreasing which does not mean that the vehicle is slowing down, but instaed that is accelerating less!

- **Apply a total later force of 2000 N, equally distributed on the 4 wheels, with null steering angles.**



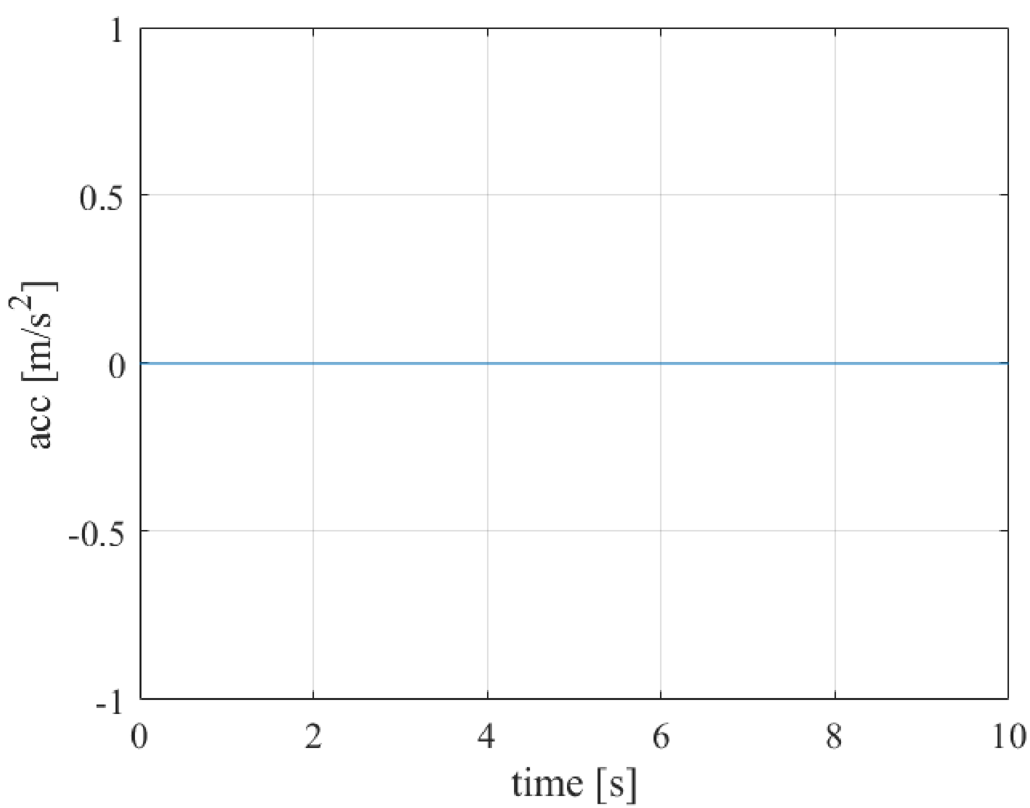
Model modification

```
Fx=000;                    %[N]
```

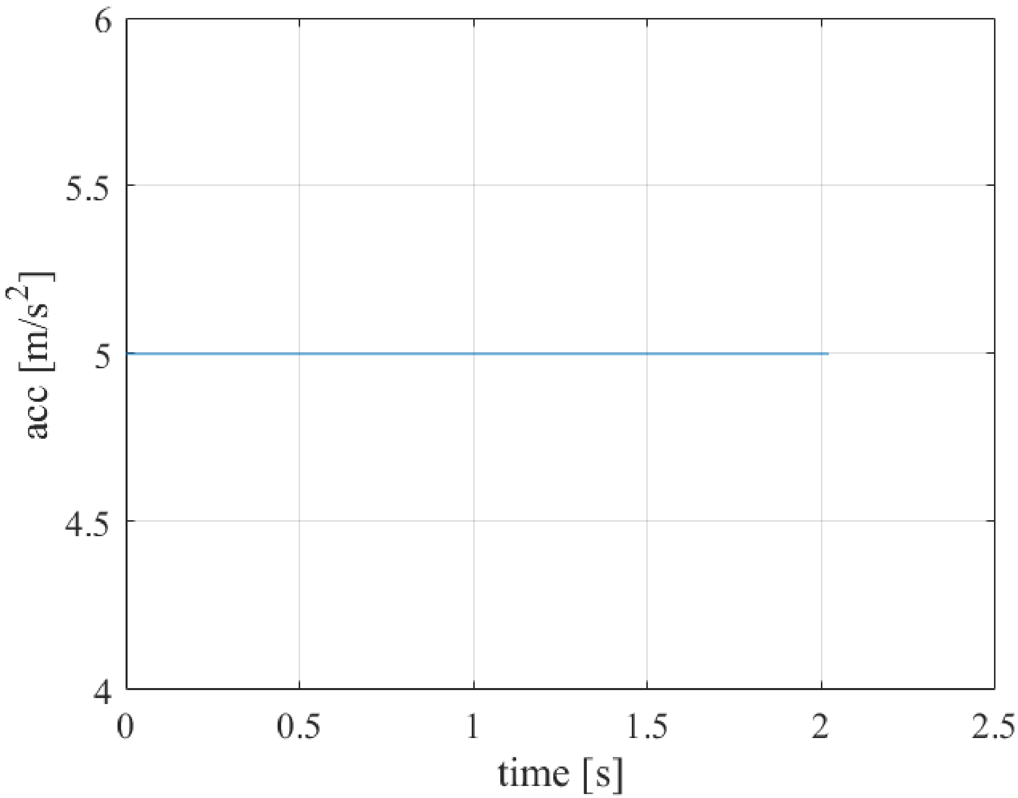
```
Fy=2000;                %[N]
deltaf=0;               %[deg]
deltar=0;               %[deg]
vel_ini=80/3.6;         %[m/s]
latdyn_data;
```

Question: why is lateral acceleration constant (differently from previous case?)

```
openfig("ayQ1.fig");
```

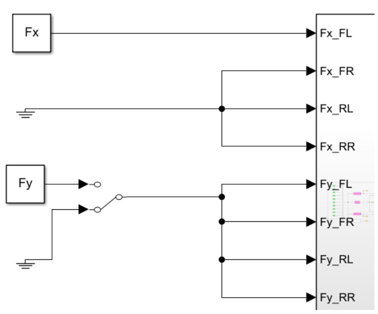



```
figure; plot(ay(:,1),ay(:,2)); xlabel('time [s]'); ylabel('acc [m/s^2]'); grid on;
set(gca,'FontName','Times New Roman','FontSize',14);
```



Also this effect is due to the aerodynamics forces. The model is structured with the hypothesis that the aerodynamics forces have influence only over the longitudinal dynamics. In other words, the aerodynamic contribution is present only on the ax formula.

- **Apply a longitudinal force of 1000 N only at the front left wheel (FL), with null steering angles.**

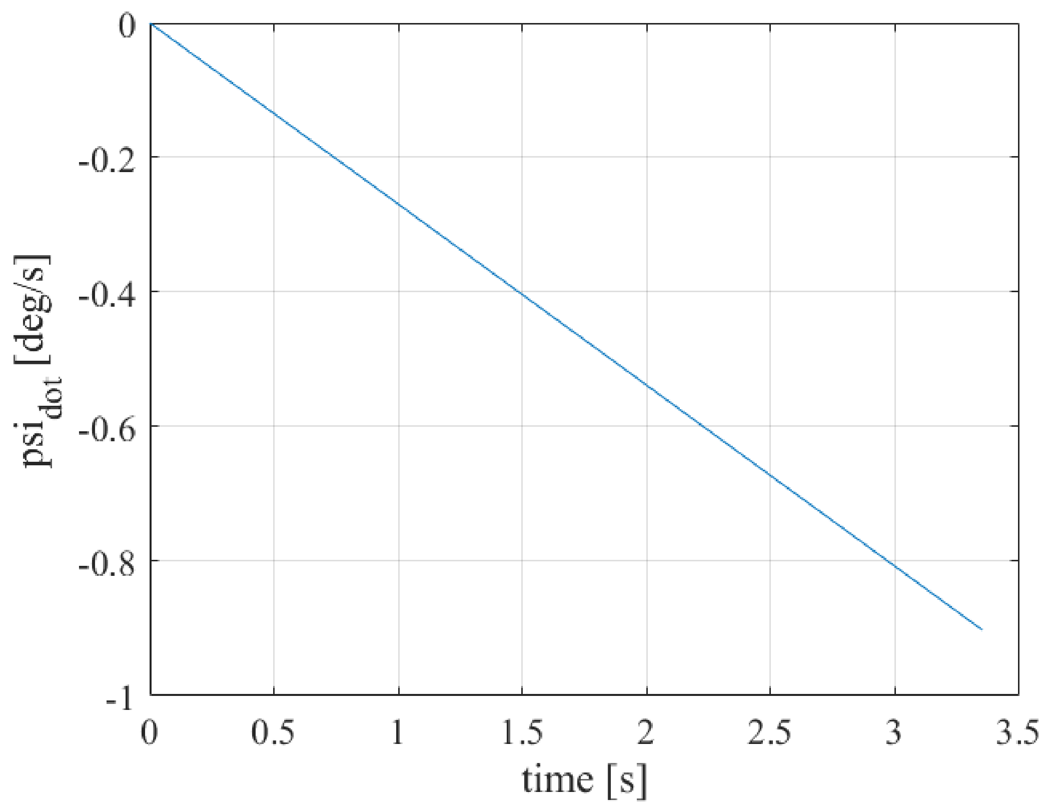


Model propely modified (manual switch never recomnended, howevere here it is usefull for passing from null to certain value rapidly).

```
Fx=1000;                %[N]
Fy=0000;               %[N]
deltaf=0;              %[deg]
deltar=0;              %[deg]
vel_ini=80/3.6;        %[m/s]
latdyn_data;
```

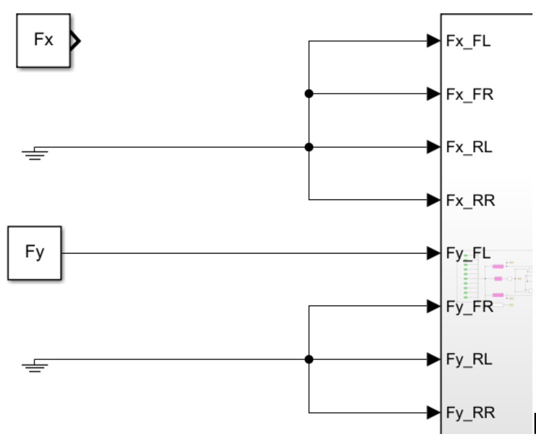
<u>Questions:</u> why does yaw rate increases (negative values)? Why is it increasing linearly?

```
figure; plot(yaw_rate(:,1),yaw_rate(:,2));xlabel('time [s]'); ylabel('psi_{dot} [deg/s]'); grid
set(gca,'FontName','Times New Roman','FontSize',14);
```



The linear behaviour is always due to the linear dependence of the yaw rate on the FL longitudinal force. The negative sign is due to the reference frame considered. It has been defined positive the turning toward the left curve, therefore having a bigger FL longitudinal implies that the vehicle will turn in the other direction, making it understeering in this case.

- **Apply a lateral force of 1000 N at the front left wheel, with null steering angles.**

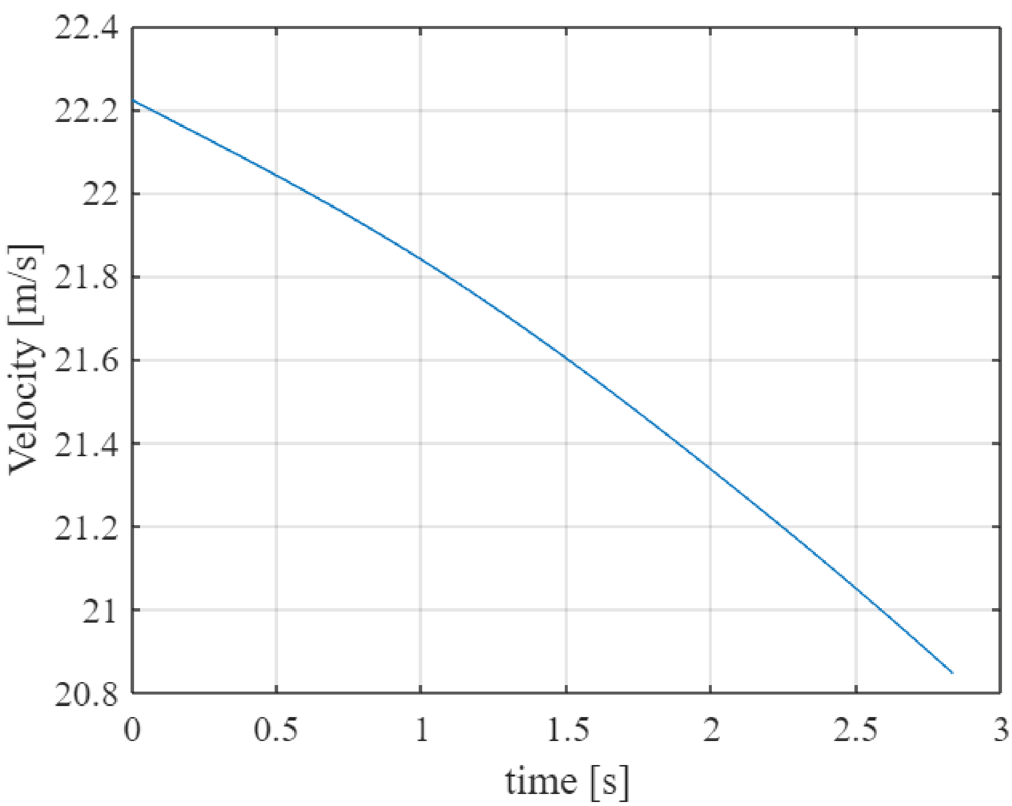


Model propely modified

```
Fx=0000;                %[N]
Fy=1000;                %[N]
deltaf=0;               %[deg]
deltar=0;               %[deg]
vel_ini=80/3.6;         %[m/s]
latdyn_data;
```

<u>Derive the time evolution of the vehicle speed</u>

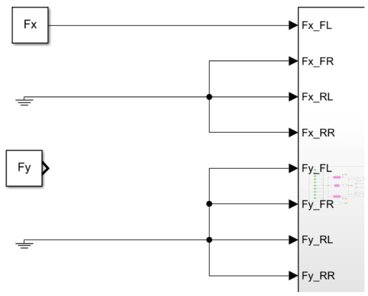
```
figure; plot(Vel(:,1),Vel(:,2));xlabel('time [s]'); ylabel('Velocity [m/s]'); grid on;
set(gca,'FontName','Times New Roman','FontSize',14);
```



As expected, the vehicle tend to corner more since the only lateral force applied is on the front. Moreover it will reduce its speed due to the aerodynamics resistance.

- **Apply a longitudinal force of 1000 N only at the Front Left wheel, with front steering wheel angle of 45°**

Reminder: Simulink adot the equation in radiants = conversion needed

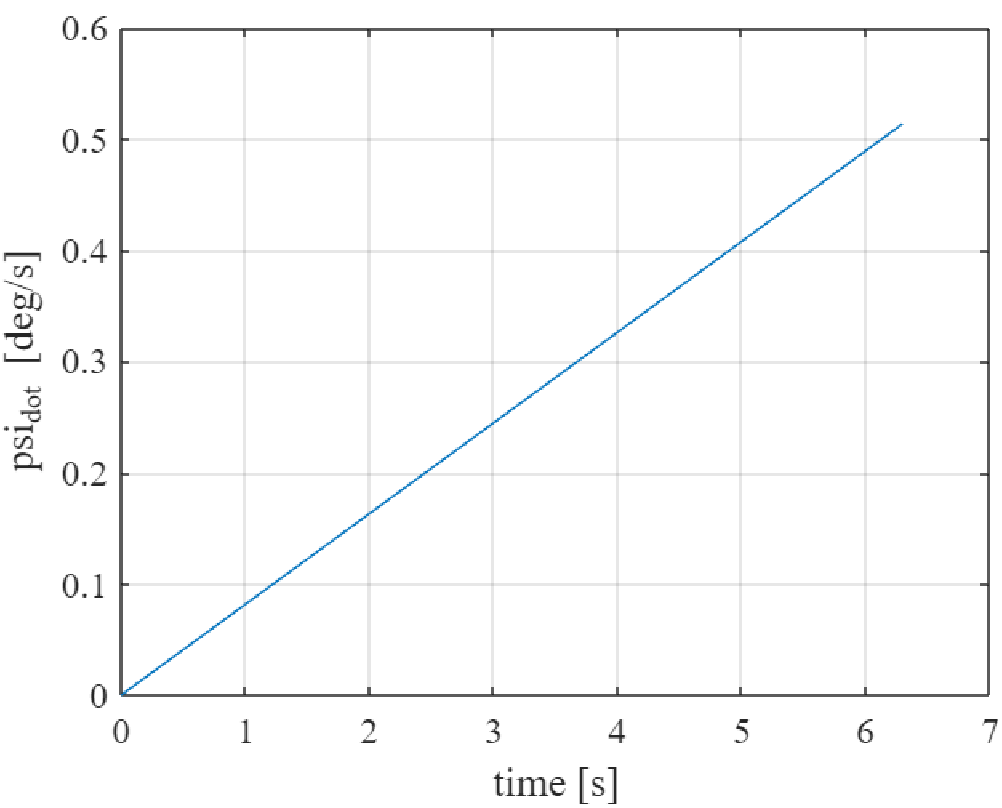


Modification done

```
Fx=1000;                %[N]
Fy=0000;                %[N]
deltaf=deg2rad(45);     %[rad]
deltar=0;               %[deg]
vel_ini=80/3.6;         %[m/s]
latdyn_data;
```

It is expected that the vehicle will show a constant positive yaw rate growth, meaning that the vehicle is turning. This can be explained since the front left is exerting a yaw moment toward the curve in addition with a constant steering frontal axle, which will help in the cornering action.

```
figure; plot(yaw_rate(:,1),yaw_rate(:,2));xlabel('time [s]'); ylabel('psi_{dot} [deg/s]'); gric
set(gca,'FontName','Times New Roman','FontSize',14);
```



- **Derive the expression of the front steering wheel angle that leads to null yaw acceleration when a force F_{xFL} is applied. Then compute the numerical value and verify the result using Simulink model**

Expression of δ_f having only a F_{x_FL} applied in order to reach null yaw acc.

$$\ddot{\psi} = f(F_{xi}, F_{yi}) \rightarrow \text{only } F_{x\_FL} \text{ term.}$$

$$\ddot{\psi} = \frac{1}{J_z}\left\{ \frac{T_f}{2}(-F_{x\_FL})\cos\delta_f + a(F_{x\_FL})\sin\delta_f \right\}$$

Hp: linearization of trigonometric formula not needed $\cos(\delta_f) \neq 1$ $\sin\delta_f \neq \delta_f$

$$\cancel{J_z}\,\underbrace{\ddot{\psi}}_{=0} = F_{x\_FL}\cdot a\sin\delta_f - \frac{T_f}{2}F_{x\_FL}\cos\delta_f$$

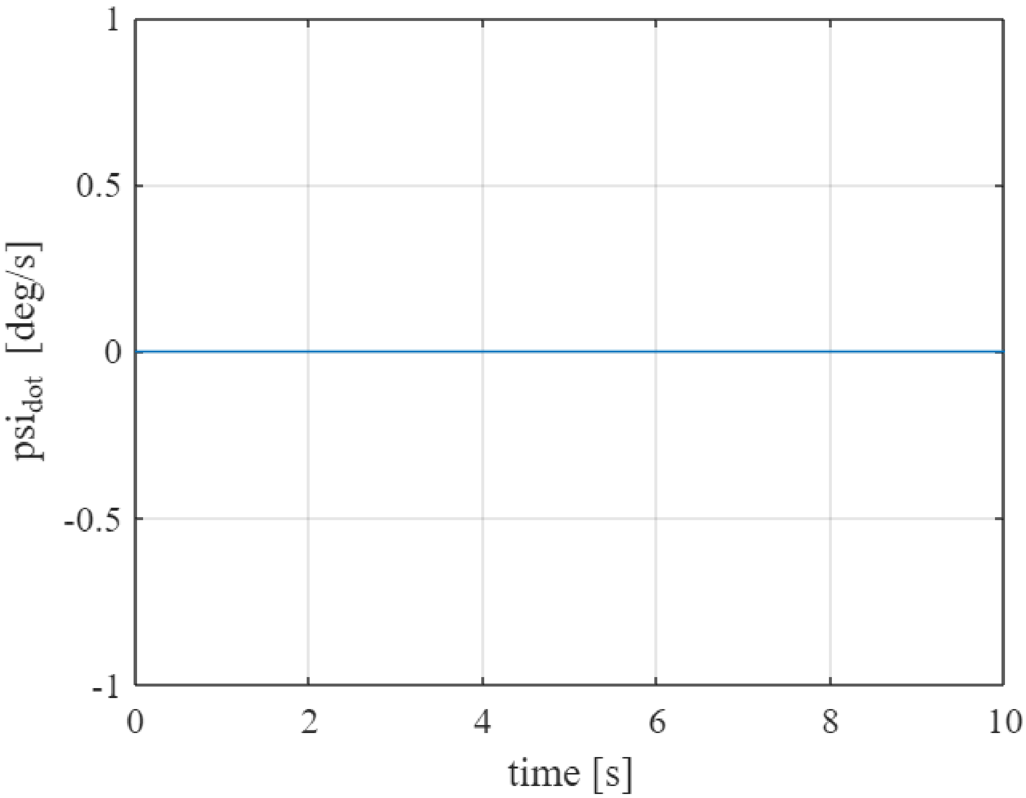
$\cos\delta_f \neq 0$

$$0 = \cancel{F_{x\_FL}}\cdot a\,\tan\delta_f - \frac{T_f}{2}\cancel{F_{x\_FL}}$$

$$\delta_f = \mathrm{atan}\left(\frac{T_f}{2a}\right)$$

```
Fx=1000;                %[N]
Fy=0000;                %[N]
deltaf = atan(Tf/(2*a));%[deg]
deltar=0;               %[deg]
vel_ini=80/3.6;         %[m/s]
latdyn_data;

figure; plot(yaw_rate(:,1),yaw_rate(:,2));xlabel('time [s]'); ylabel('psi_{dot} [deg/s]'); gric
set(gca,'FontName','Times New Roman','FontSize',14);
```

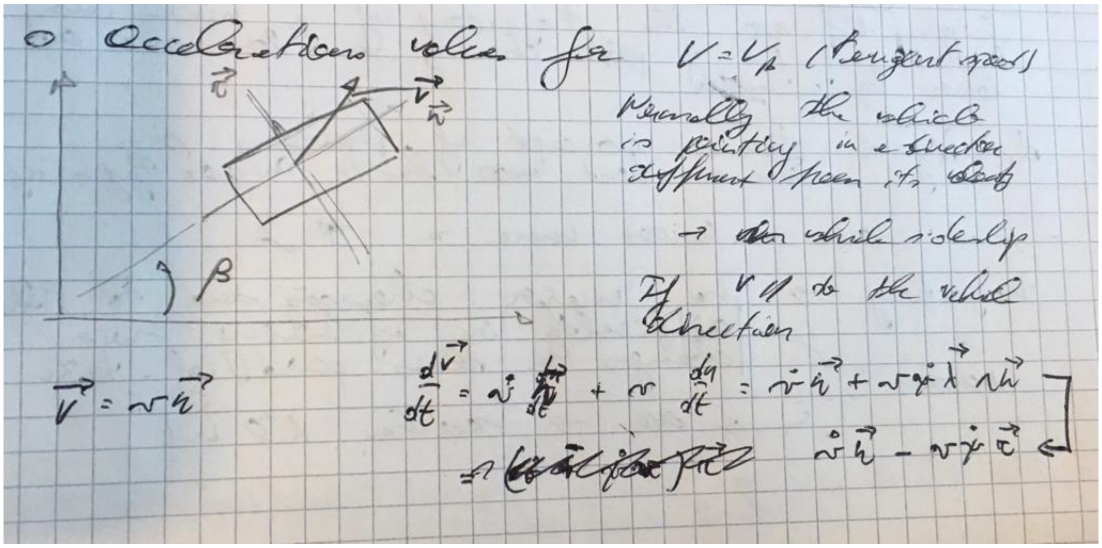


The formula obtained is correct. The simualtion run is a steady state lateral acceleration cornering, therefore every other quantities are constantly increasing with excpetion of the longitudinal acceleration which is decreasing instaed.

Wheel kinematics

The model develop makes reference to the frontal left wheel only, but the procedure in equal for the other tyres, therefore it can be applied also to the other three, being aware on the differences in the fomulas. This model gives as output the two veloclity components refering to each wheel reference frame, having as input the component of the velocity from the vehicle reference system. Therefore it is fundamental to make reference to the fact that the velocity have two components into a non-inertial reference frame. Some transformations are needed.

Question: when driving on a turn at = & (tangent speed), what is the acceleration?



The acceleraation lose two terms but still has two components among the two axis.

Remeber: The derivative of a versor is the velocity at which it is rotating $(\dot{\psi})$ multiplied scalarly $(*)$ with the vectorial product of the direction around which it is rotating $(\vec{\lambda})$ for the versor direction $(\vec{n})$.

However in general terms the acceleration has the following formula:

$$\vec{a} = a_x\vec{\tau} + a_y\vec{n} = \left(\dot{u} - \dot{\psi}v\right)\vec{\tau} + \left(\dot{v} + \dot{\psi}u\right)\vec{n}$$

From which the following quantities can be deduced:

$$u = \int \left(a_x + \dot{\psi}v\right) dt \qquad v = \int \left(a_y - \dot{\psi}u\right) dt$$

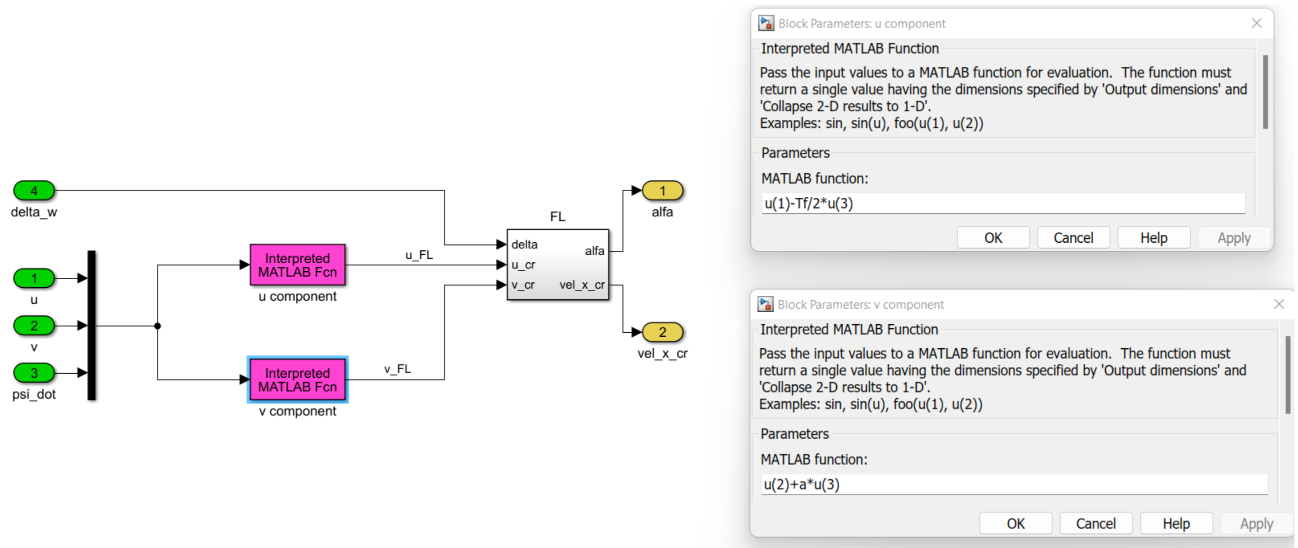
$$\beta = \arctan\frac{v}{u} \qquad V = \sqrt{u^2 + v^2}$$

That's exactly what happens in our model, being aware on the fact that for each tyre there is a correction factor due to the misalignement of the wheels respect to the car body:

$$u_{FL} = u - \frac{T_F}{2}\dot{\psi} \qquad v_{FL} = v + a\,\dot{\psi}$$

$$\beta_{FL} = \text{atan}\left(\frac{v_{FL}}{u_{FL}}\right) \qquad \alpha_{FL} = \beta_{FL} - \delta_{FL}$$

Therefore final model will presents like this:



- **Compute the longitudinal velocity of the wheel centers and the slip angles when VG = 100 km/h; = 3° ; ˙ = 20°/s**

No informaiton over delta_FL; Assumed equal to 65 deg (presumed common sense of engineering).

```
Vg = 100/3.6; beta = deg2rad(3); psi_dot = deg2rad(20); delta_FL = deg2rad(65);
u_FL = -Vg*Tf/2*psi_dot;    % tau direction
v_FL = Vg*a*psi_dot;        % n direction
beta_FL = atan(v_FL/u_FL);
Vx_FL = sqrt(u_FL^2+v_FL^2);
alpha_FL = beta_FL + delta_FL;
disp(['Longitudinal velocity for the FL trye = ',num2str(Vx_FL*3.6),'km/h']);
```

```
Longitudinal velocity for the FL trye = 46.8698km/h
```

```
disp(['Side slip angle for the FL trye = ',num2str(rad2deg(alpha_FL)),'deg']);
```

```
Side slip angle for the FL trye = 9.992deg
```