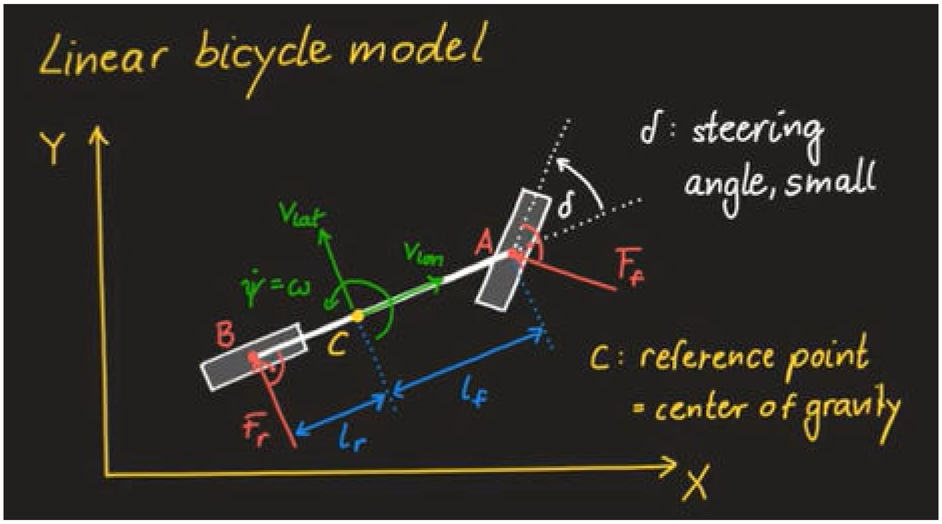
ES07 - Lateral dynamic: Single track model

The single track model, know also as the bicyle model, is the most simple model adopted for evaluate the lateral dynamic of vehicles. The approach of this course is to start from this simplified model and develop step by step more complex ones.



This model is based on some assumptions (note: 1 = front, 2 = rear):

- Radius of curvature is way bigger than the vehicle's length: $R >>> l$
- Small angles: δ_i α_i
- Rear driving wheel: $F_{xi} = 0$
- Limited lateral acceleration: $a_{lat} < 4m/s^2$

The aim of this exercise is to compute, plot and comment the dynamic behaviour for three different vehicle. Each one has its own values of a and b which will imply three different CoG. Therefore in this exercise the CoG will be the parameter which determines the vehicle behaviour.

In real cases however, the under/oversteering is determined by multiple factors, not only regarding the car geometrical characteristics but also regarding tyre parameters. Therefore it is not a trivial definition, for this reason depending on the focus of the application, the evaluation of the vehicle behaviour is set and modified accordingly.

The model is handled with a **state-space** approach.

Equation of motion $(V = const)$

$$\begin{cases} \dot{\beta} = \left(\frac{-C_F - C_R}{mV}\right)\beta + \left(\frac{-C_F a + C_R b - mV^2}{mV^2}\right)\dot{\psi} + \frac{C_F\delta_F + C_R\delta_R}{mV} \\ \ddot{\psi} = \left(\frac{-C_F a + C_R b}{J_Z}\right)\beta + \left(\frac{-C_F a^2 - C_R b^2}{J_Z V}\right)\dot{\psi} + \frac{(C_F a)\delta_F - (C_R b)\delta_R}{J_Z} \end{cases}$$

state space

$$\begin{cases} \dot{\boldsymbol{x}} = \boldsymbol{Ax} + \boldsymbol{Bu(t)} \\ \boldsymbol{y} = \boldsymbol{Cx} + \boldsymbol{Du(t)} \end{cases}$$

$$A = \begin{bmatrix} \frac{-C_F - C_R}{mV} & \frac{-C_F a + C_R b - mV^2}{mV^2} \\ \frac{-C_F a + C_R b}{J_Z} & \frac{-C_F a^2 - C_R b^2}{J_Z V} \end{bmatrix} \qquad \boldsymbol{B} = \begin{bmatrix} \frac{C_F}{mV} & \frac{C_R}{mV} \\ \frac{C_F a}{J_Z} & \frac{-C_R b}{J_Z} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \left(\frac{-C_R - C_F}{mV^2}\right) & \left(\frac{-C_F a + C_R b}{mV^3}\right) \\ -1 & -\frac{a}{V} \\ -1 & \frac{b}{V} \\ \frac{-C_R - C_F}{m} & \frac{-C_F a + C_R b}{mV} \end{bmatrix} \qquad \boldsymbol{D} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{C_F}{mV^2} & \frac{C_R}{mV^2} \\ 1 & 0 \\ 0 & 1 \\ \frac{C_F}{m} & \frac{C_R}{m} \end{bmatrix}$$

$$\boldsymbol{x} = \begin{bmatrix} \beta \\ \dot{\psi} \end{bmatrix} \qquad \boldsymbol{u(t)} = \begin{bmatrix} \delta_F \\ \delta_R \end{bmatrix} \qquad \boldsymbol{y} = \begin{bmatrix} \beta \\ \dot{\psi} \\ \rho \\ \alpha_F \\ \alpha_R \\ a_y \end{bmatrix}$$

The two states are the vehicle side slip angle and its yaw rate. The inputs are only the driving angle applied to the wheels. The velocity ahs been considered as an exogenous input, meaning that its calcualtion or observation is outside of the state-space. Simply the velocity is considered known.

The outputs are the states and other usefull dynamic parameters like the wheel side slip angle for both axle (α_f, α_r), lateral acceleration (a_y) and the radius of cruvature (ρ). The calculation of each of these output is here explained:

trajectory curvature, slip angles and lateral acceleration

$$\rho = \left(\frac{-C_R - C_F}{mV^2}\right)\beta + \left(\frac{-C_F a + C_R b}{mV^3}\right)\dot{\psi} + \frac{C_F\delta_F + C_R\delta_R}{mV^2}$$

$$\alpha_F = \delta_F - \beta - \frac{a}{V}\dot{\psi}$$

$$\alpha_R = \delta_R - \beta + \frac{b}{V}\dot{\psi}$$

$$a_y = V^2\rho = \left(\frac{-C_R - C_F}{m}\right)\beta + \left(\frac{-C_F a + C_R b}{mV}\right)\dot{\psi} + \frac{C_F\delta_F + C_R\delta_R}{m}$$

<u>All the mathematical evaluation will be done at steady-state condition</u>

Under/oversteering

As stated, the behaviour of the vehicle is depending on its capacity to follow a given trajectory. A mathematical reference for this behaviour is described by the understeering coefficent: the higher it is the more understeering the vehicle gets.

$$K_{US} = \frac{m}{lC_F C_R}(bC_R - aC_F)$$

Not only there is a direct influence of a and b, but the CoG location will have some influence over the cornering stiffness of each axle. Understeering factor depends on the difference between Cr and Cf. By changing the CoG position, it is necessary to study which of the two effects has a bigger influence.

NOTE: All the data is coming from the tyre exercise. In aprticular, the chart of the cornering stiffness against the vertical force is crucial in this exercise, since as stated the cornering stiffness has huge influence over the vehicle behaviour.

This initial part of the analysis is perfomed in `MATLAB`, once the vehicles characteristics are defined, some manoeuvres will be performed into Simulink environment.

PART 1: MATLAB - Define vehicle behaviour for three different CoG configurations

The exercise consider that there are only front tyres as steering ones. Moreover, the δ_f imposed is unitary (in radiants) meaning that the overall input of the system will be a simple matrix [1 0]. This has been done in order to simplify the model and the general discussion, since, for example, all the plots will represent the actual varibales tendency as function of time.

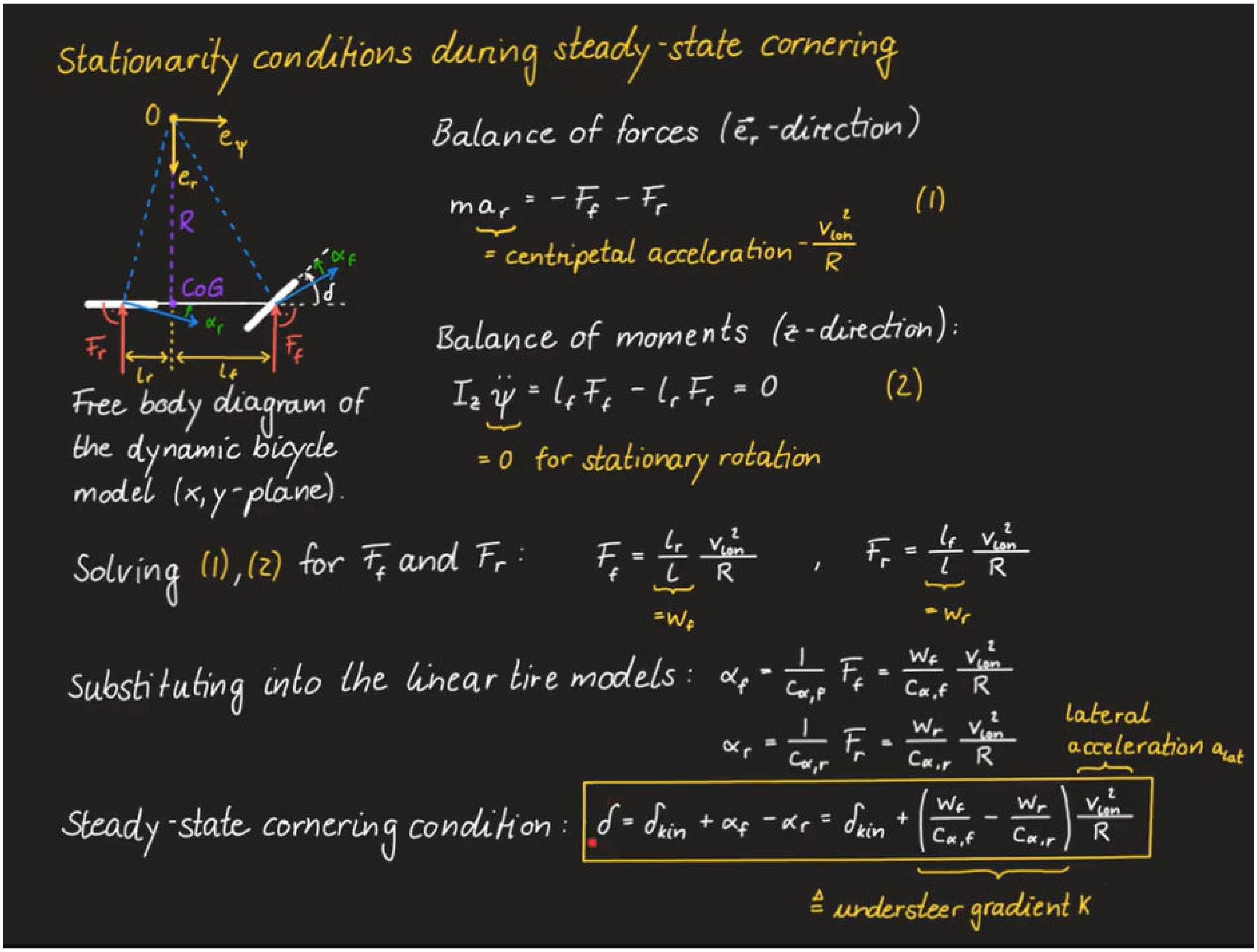
```
% Matlab reference file
% s_track_STUD

% Three cases analysed
% (1) a = L/2-10cm
% (2) a = L/2
% (3) a = L/2+10cm
```

Vehicle dynamics theory - recap

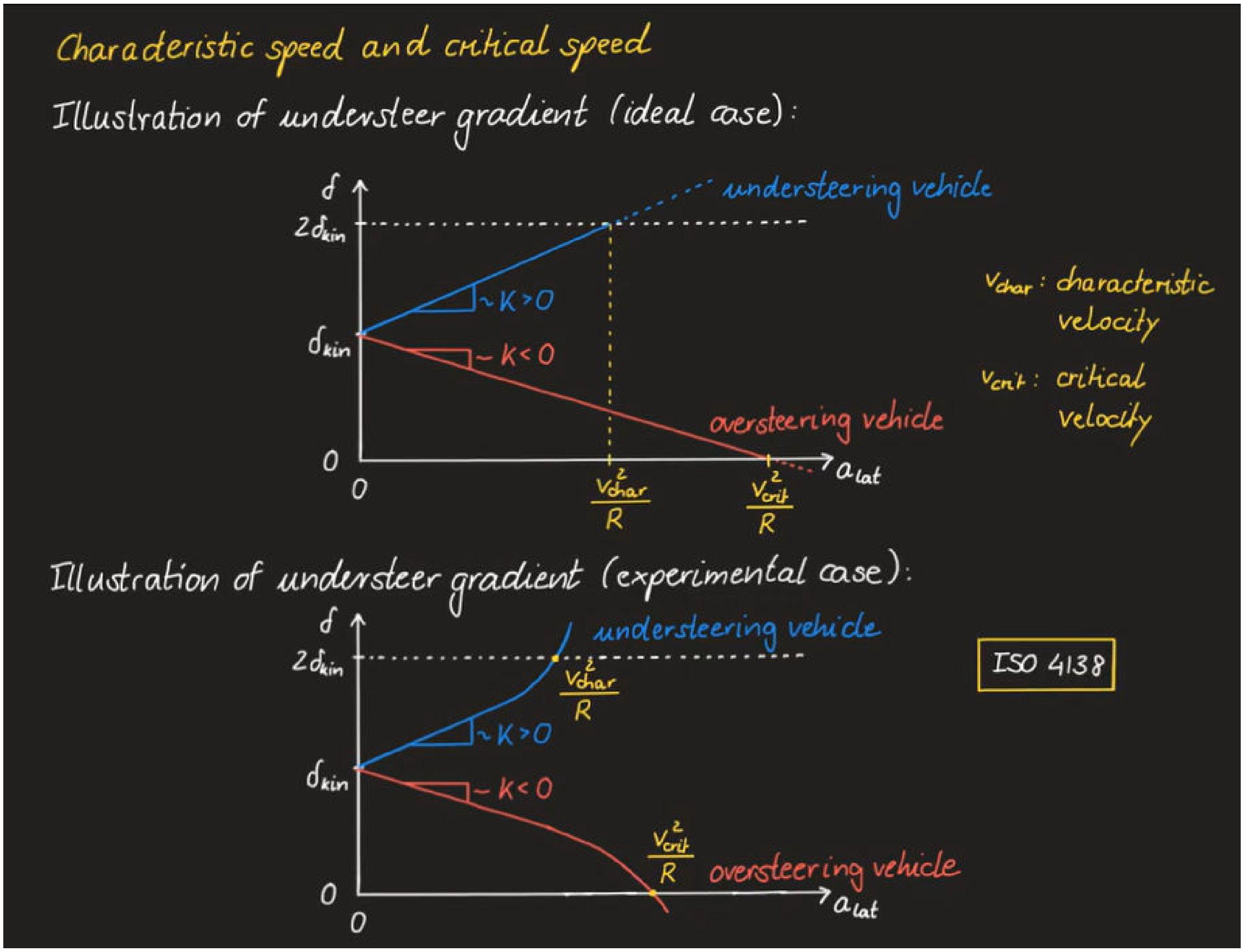
The under/oversteering behaviour is defined as the tendency og the vehicle, given a certain trajectory, its behaviour to have a larger or lower one respect to the one theoreticaly imposed. This is properly defined by the steering angle difference among the acutal steering angle to be imposed and the one needed for following the trajectory wanted.

By steady state and stactionary equilibrim, this difference of delta can be easily computed:



In particular it is possible to state how if the steering angle increases with higher longitudinal speed an understeering vehicle is get, meanhwile if the steering angle decreases an oversteering vehicle is get.

Therefore, by plotting the difference of the steering angle versus the lateral acceleration, the corresponding slope of the line will be the ***understeering coefficent***:



This explanation was carried out on the steering angle since it is more near the common experience. Considering the beta variation instead, it is intresting to notice how it does not depend on the understeering coefficent, but only on characteristics concerning the rear part of the car. By a

$$\beta - \beta_0 = -\frac{m_R}{C_R} a_y = -K_\beta\, a_y$$

$$K_\beta = \frac{m_R}{C_R}$$

This linear characteristics can be considered true for small values of steering and side slip angles $(4m/s^2, 3° respectively)$ which is a limited portion of the real usage of a standard car. Considering higher acceleration fro example, the linear tendency transforms into a parabloic one, menaing thta the need of correcting the steering wheel is higher for higher longitudinal velocities (bottom chart of the previous slide presented).

The concept of <u>***stability***</u> is also related to the vehicle behaviour:

Stability vs. understeer / oversteer

Necessary and sufficient condition for stability: $c > 0$

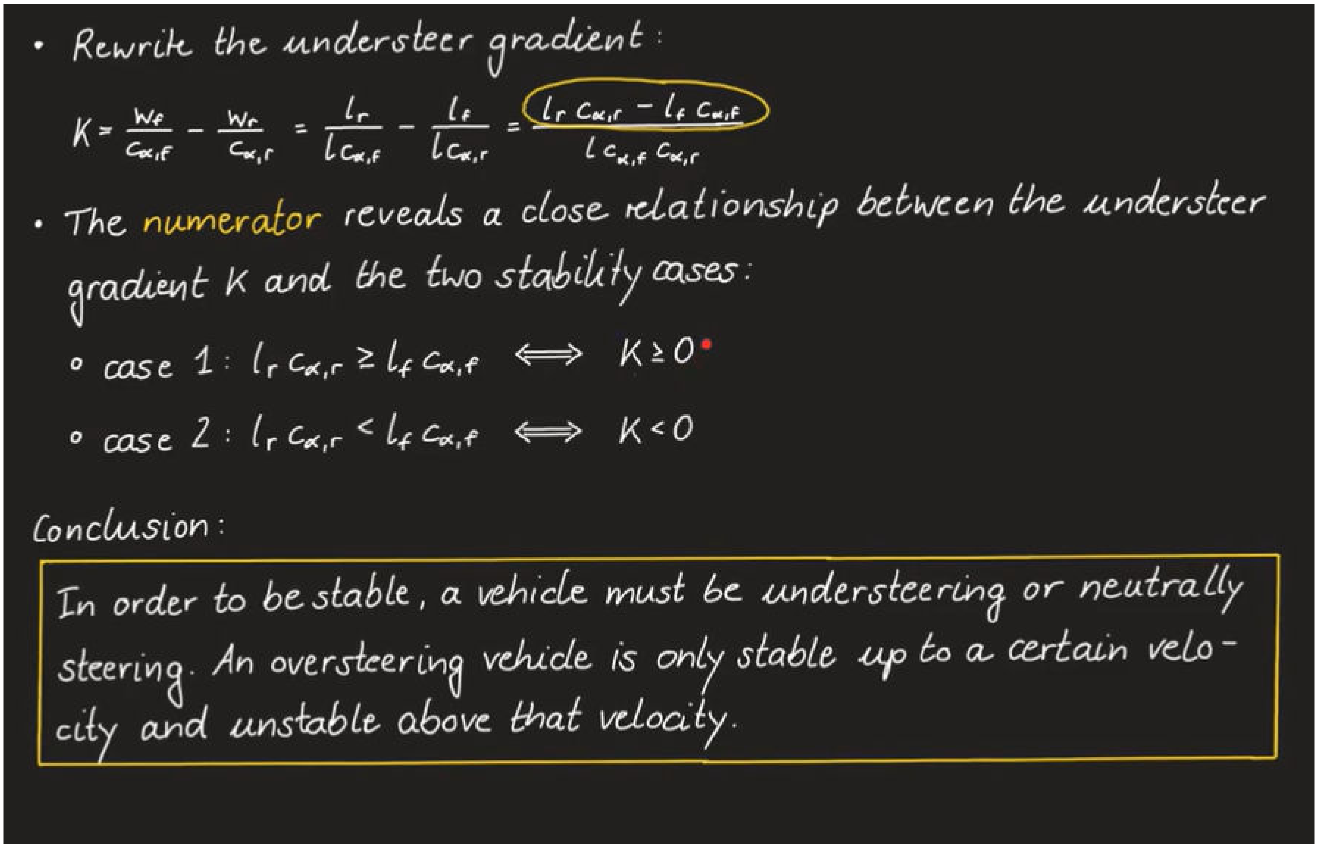
Explicit expression for the coefficient c: $c = \dfrac{m v_{lon}^2 (l_r C_{\alpha,r} - l_f C_{\alpha,f}) + C_{\alpha,f} C_{\alpha,r} l^2}{m I_z v_{lon}^2} > 0$

Observations:

- For any given set of parameters $m, I_z, l_r, l_f, l, C_{\alpha,r}, C_{\alpha,f} > 0$: If v_{lon} is sufficiently small, the vehicle is stable ($c > 0$).
- If $l_r C_{\alpha,r} \geq l_f C_{\alpha,f}$, the vehicle is stable ($c > 0$) for all longitudinal velocities v_{lon}.
- If $l_r C_{\alpha,r} < l_f C_{\alpha,f}$, the vehicle becomes unstable ($c \leq 0$) if the longitudinal velocity increases above a critical threshold.

Compare these observations to the understeer gradient: $K = \frac{w_f}{C_{\alpha,f}} - \frac{w_r}{C_{\alpha,r}}$

- The vehicle is understeering if $K > 0$, neutrally steering if $K = 0$, and oversteering if $K < 0$
- $w_f = \frac{l_r}{l}$ and $w_r = \frac{l_f}{l}$ represent the weight factor for the front / rear tire



The concept of stability can be deduced also from the ***root locus***. The stability of the system is a consequence of the pole graph. In a control context, for example, from the pole graph the information about the natural frequencies is obtained and therefore, it is possible determine whether the system is stable or unstable. Plotting the graphs of the poles gives information regarding natural frequencies and damping factors. The real part of the poles gives information about the natural frequencies while the imaginary part about the amplitude of the oscillations.

NOTE: The yaw rate is indicated with the letter "r".

State space calculation algorythm

```
% for count=1:length(vel)
%     V = vel(count);
%
%     A=[(-CF-CR)/(m*V),(-CF*a+CR*b-m*V^2)/(m*V^2);
%         (-CF*a+CR*b)/Jz,(-CF*a^2-CR*b^2)/(Jz*V)];
%     B=[CF/(m*V) CR/(m*V);
%         (CF*a/Jz) -(CR*b/Jz)];
%     C = [1,0
%         0,1
%         (-CR-CF)/(m*V^2),(-CF*a+CR*b)/(m*V^3)
%         -1, -a/V
%         -1, b/V
%         (-CR-CF)/(m),(-CF*a+CR*b)/(m*V)];
%     D = [0 0;
%         0 0;
%         CF/(m*V^2) CR/(m*V^2)
%         1 0
```

```
%          0 1
%          CF/m CR/m];
%
%      % Steady state response to delta_f=1, so to have gain = variable in the
%      % tendecy
%      u_r = [delta_f;0];
%      X_r(count,:) = -A^-1*B*u_r;
%      Y_r(count,:) = C*X_r(count,:)'+D*u_r;
%
%      % Outputs evaluation
%      beta_deltaf(count,:)    = Y_r(count,1)';
%      r_deltaf(count,:)       = Y_r(count,2)';
%      rho_deltaf(count,:)     = Y_r(count,3)';
%      ay_f(count,:)           = Y_r(count,4)';
%      ay_r(count,:)           = Y_r(count,5)';
%
%      % Rember that lateral acceleration is yaw rate*V
%      % ay_r(count,:) = X_r(count,2)'*V;
%
%      % state space dynamic system
%      G=ss(A,B,C,D);
%      % Wn, Z and P: natural frequencies, damping factors, poles of system G
%      [Wn,Z,P]=damp(G);
%      % builging matrices (k-th column)
%      POLES(count,:)=P;
%      ZETA(count,:)=Z;
%      FREQ(count,:)=Wn;
%      % determinant and trace of A
%      DET_A(count)=det(A);
%      TR_A(count)=trace(A);
% end
```

a = L/2-10cm

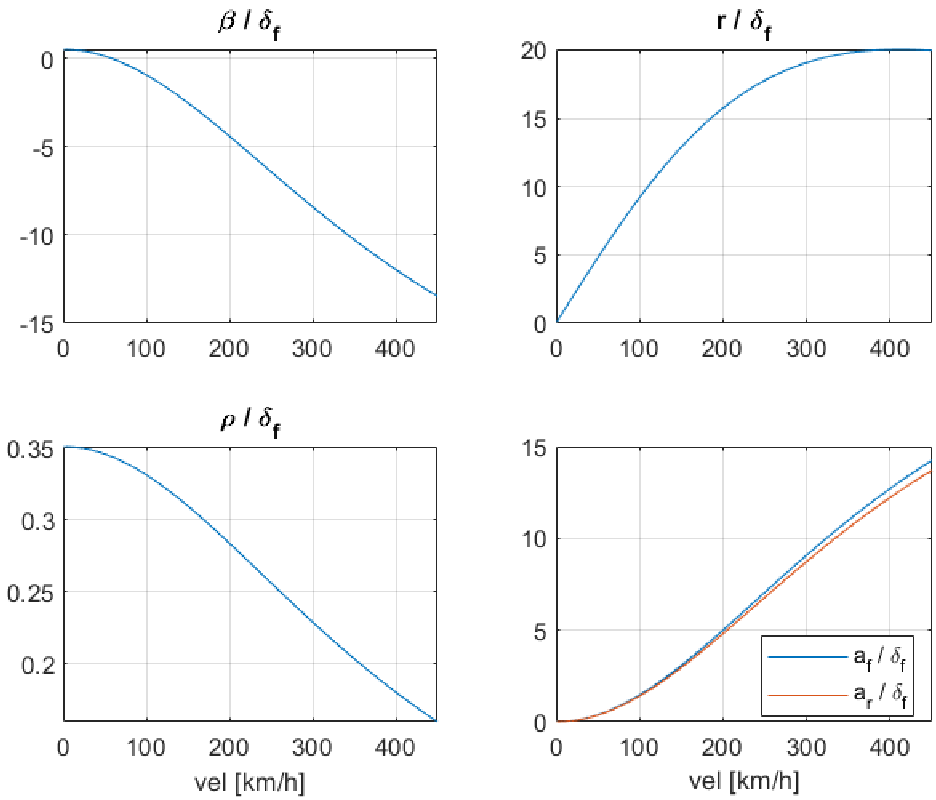
```
s_track_STUD
```

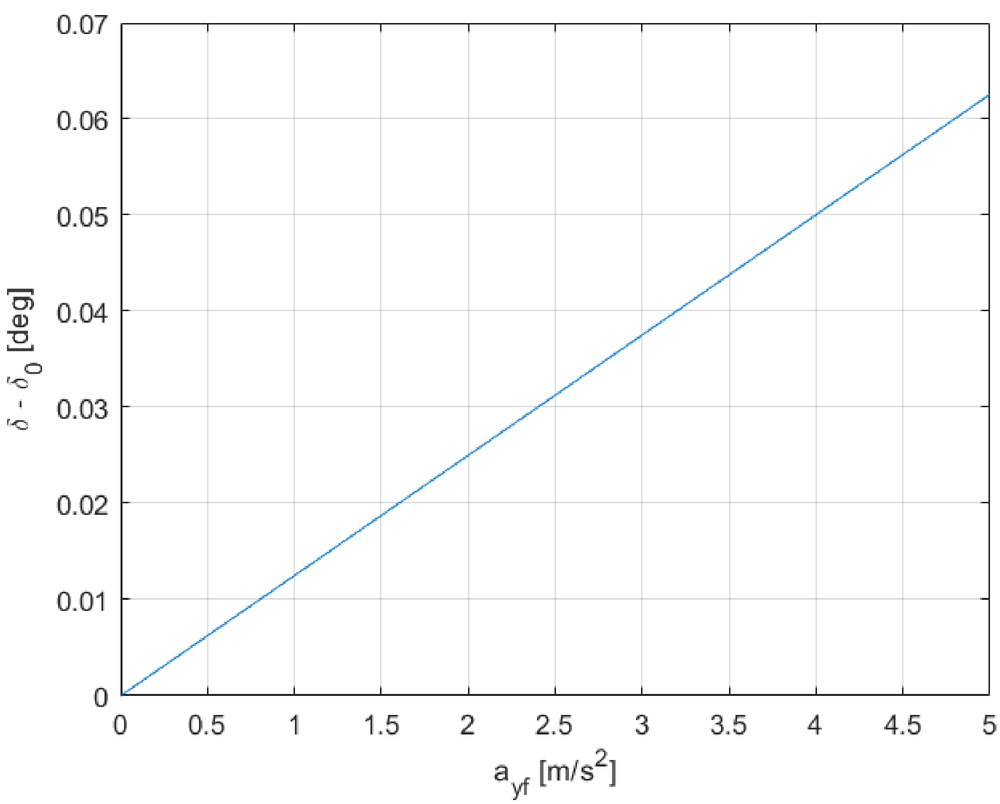
```
static load distribution (%F - %R): 54-46
axle cornering stiffness
CF = 187113.8666 N/rad
CR = 169035.7601 N/rad

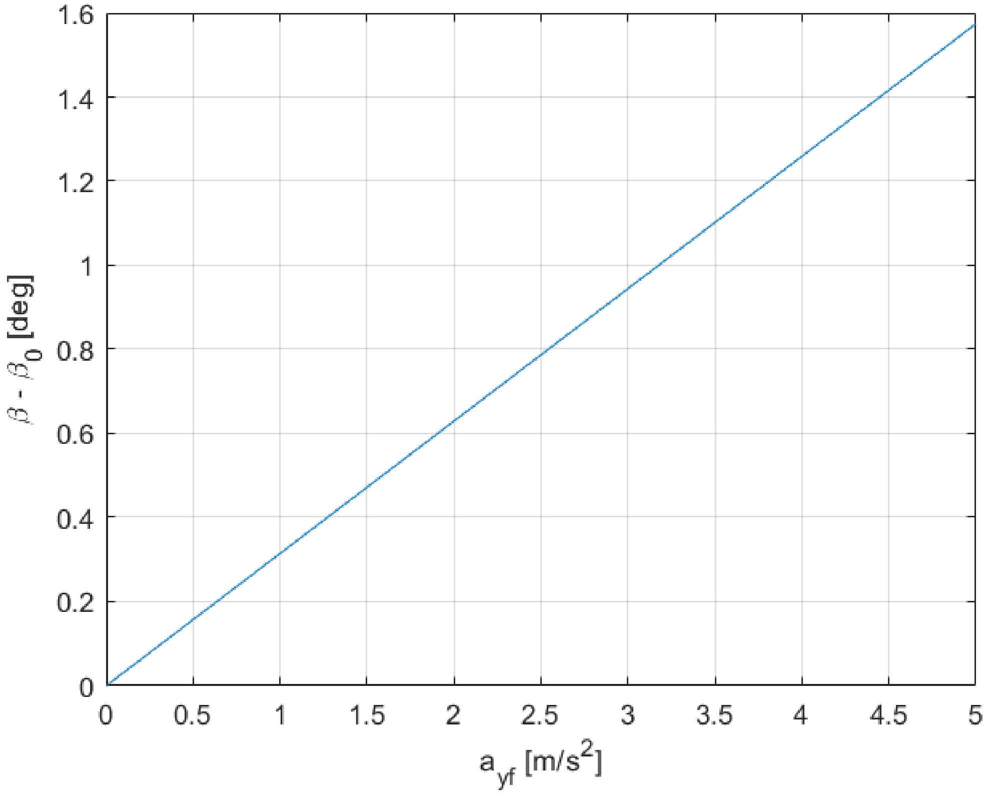
============ understeering ============

understeering gradient: K_US = 0.00021836 rad/(m/s^2)
slip agle gradient: K_beta = -0.0054942 rad/(m/s^2)
tangent speed: V_beta = 59.9773 km/h

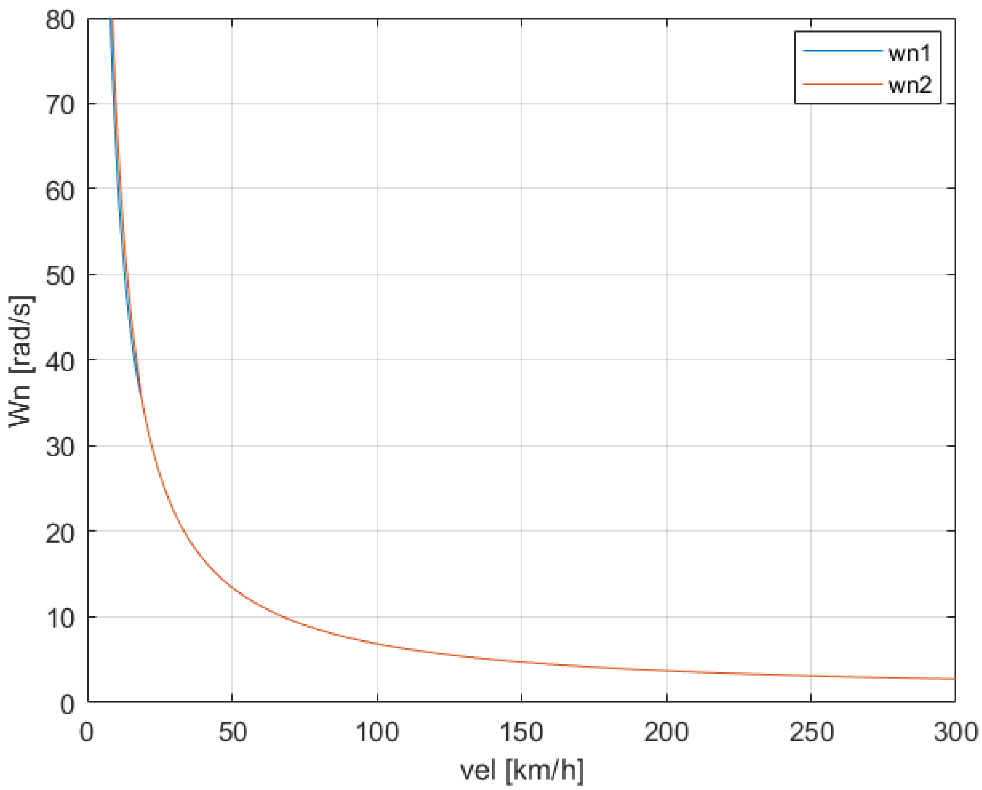

Single track analysis
```


β / δf
r / δf
ρ / δf
af / δf
ar / δf
vel [km/h]
vel [km/h]

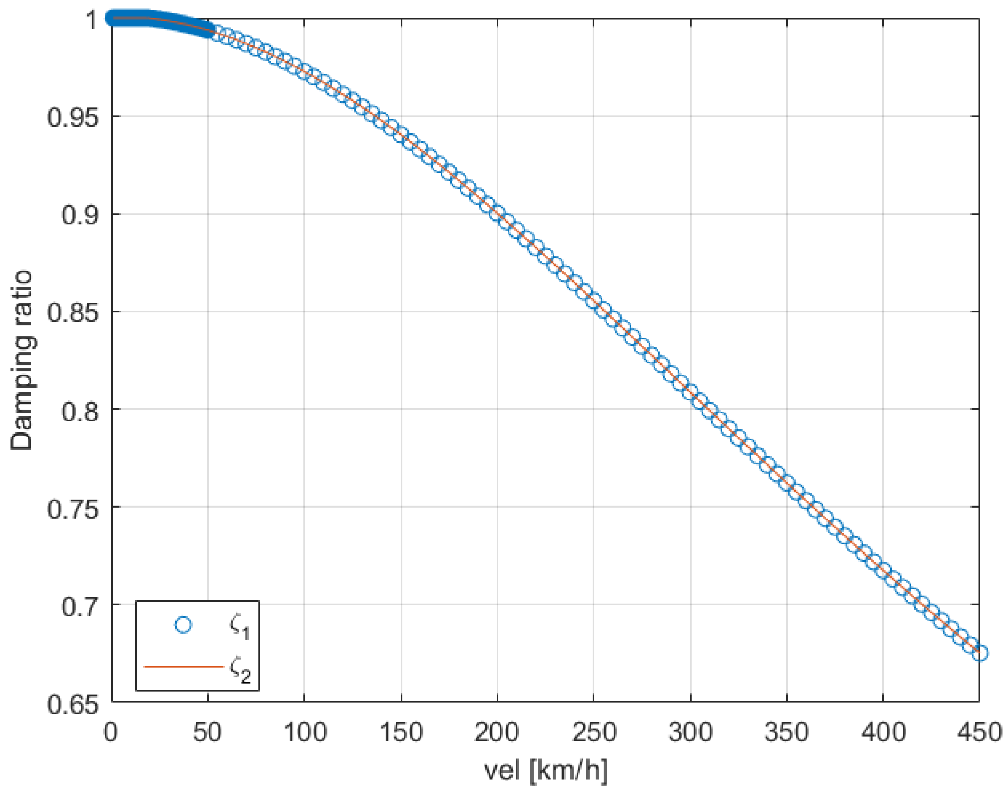

δ - δ0 [deg]
ayf [m/s2]

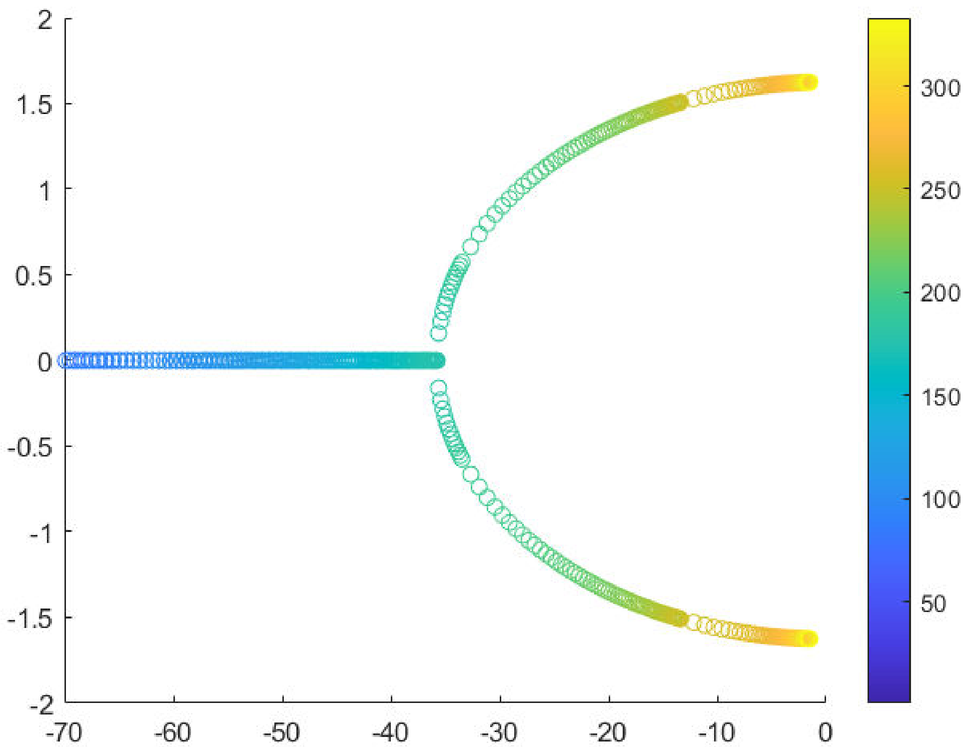


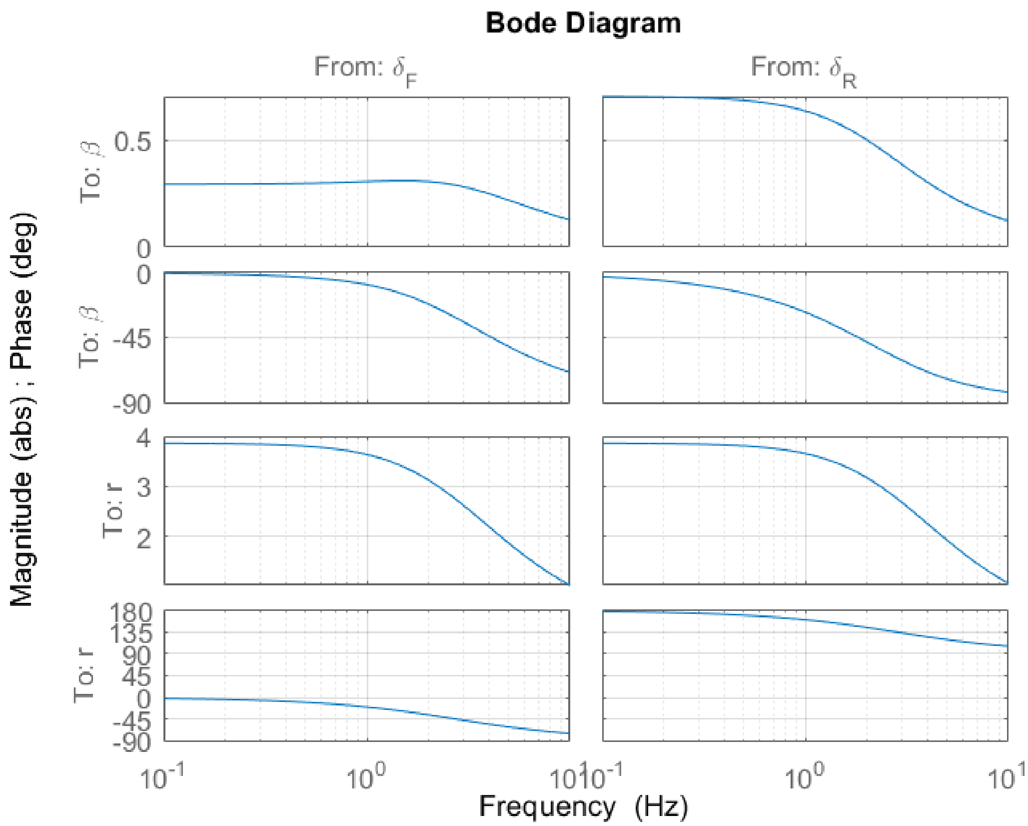
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
β - β_0 [deg]
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
a_{yf} [m/s^2]



wn1
wn2
80
70
60
50
40
30
20
10
0
Wn [rad/s]
0
50
100
150
200
250
300
vel [km/h]


Damping ratio
vel [km/h]
ζ_1
ζ_2





Analysis:

Since the vehicle is understeering, its steering angle will increase having higher velocities. Its side slip also will growth, however remebering the sign definition of side slips, this is a negative icrement. Following the same principle, the yaw rate increases as well. Instead, due to the cornering capacity loss, its radius or curvature decreases.

The understeering behaviour is the only one which swhows some difference in the side slip evolution. The physics behind is always realted to the difficulty in cornering: the front axle will tend to deviate more respect to the trajectory rather than the rear axle, implying higher side slip angle.

Regarding the understeering chart $(\delta - \delta_0 \;\; vs \;\; a_y)$, as stated in the theory part, the slope is positive since the understeering coefficent in this situation is positive = slope > 0. Sideslip angle follows the same idea.

Dynamic analysis: root locus dependent on vehicle speed

The root locus permit to appreciate the dynamic characteristics of the vehicle. Firstly it is important to observe that there is no eigenvalue which does not have not null imaginary part, in other words, the vehicle will always be stable. In particualr there is a constant tendecy: the higher the velocity the lower is the real part of the eigenvalue, which translates into lower natural frequency. This is confirmed by the natural frequency chart.

Regarding the imaginary part, an understeering vehicle shows that after a certian speed (which I think it is the characteristic speed) the vehicle shows a not null imaginary part. As stated, the imagianry part of a pole represent the oscillation when excited at the corresponding frequency, which is consequently related to the damping factor (for better understanding of the daming factor it has been explained in the 2DOF suspension part). The higher the damping factor, more overdamped the system will be, resuling in lower oscillation.

However, from the root locus it is possible to declare that higher the velocity more undamped the vehicle gets, allowing for higher oscillations (iamginary part abs value increases with velocity). This is confirmed by directly polotting the damping factor.

Bode:

Frequency analysis perfomed in the bode diagram refers to a single velocity given as direct input. The intresting output of the Bode diagram is that there is a β magnitude shift bewteen passing from δ_f to δ_r, meanwhile regarding the yaw rate the shift is over the phase. This general rule "β=magnitude shift, r=phase shift" will be true no matter the undesteering coefficent values and the velocity set ass input. Regarding the latter influence instead, what changes is only that for higher velocities the difference is magnitude and phase will be increase accordingly as well.

a = L/2

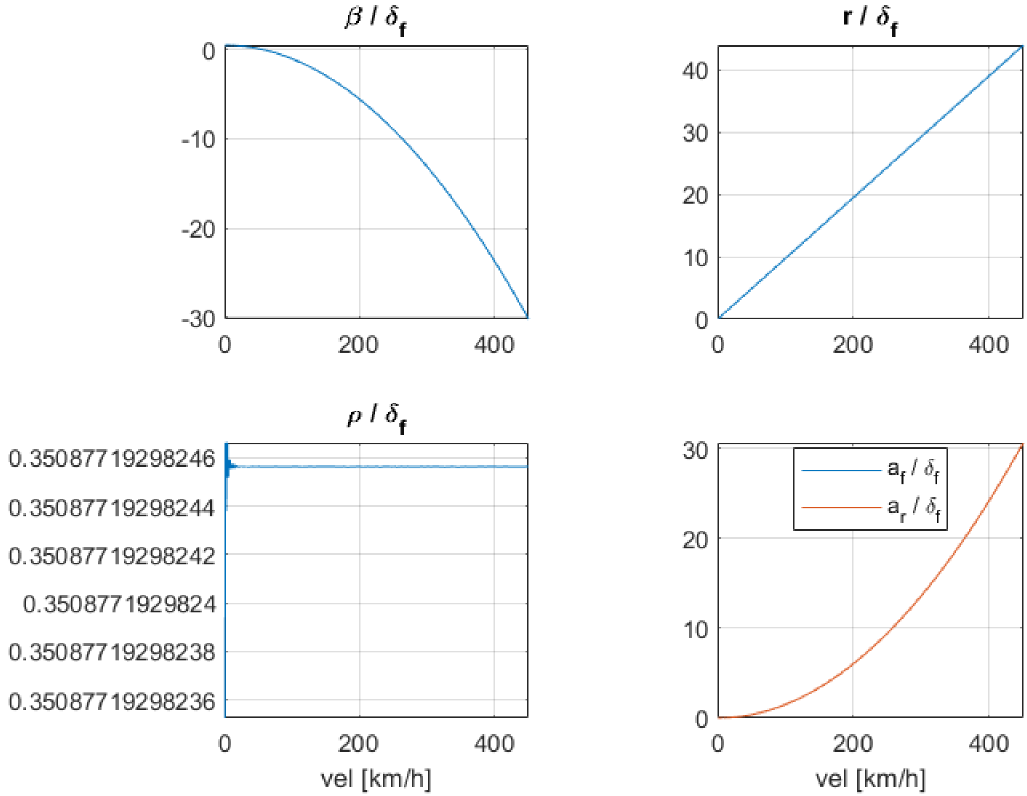
```
s_track_STUD
```

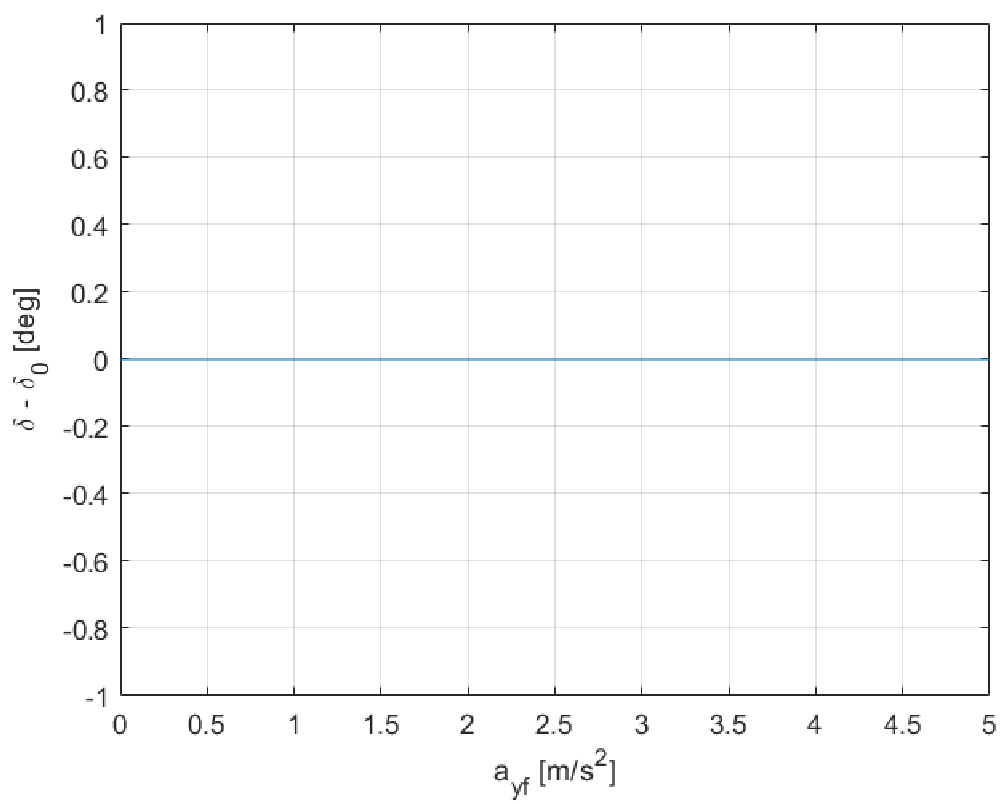
```
static load distribution (%F - %R): 50-50
axle cornering stiffness
CF = 178372.8905 N/rad
CR = 178372.8905 N/rad

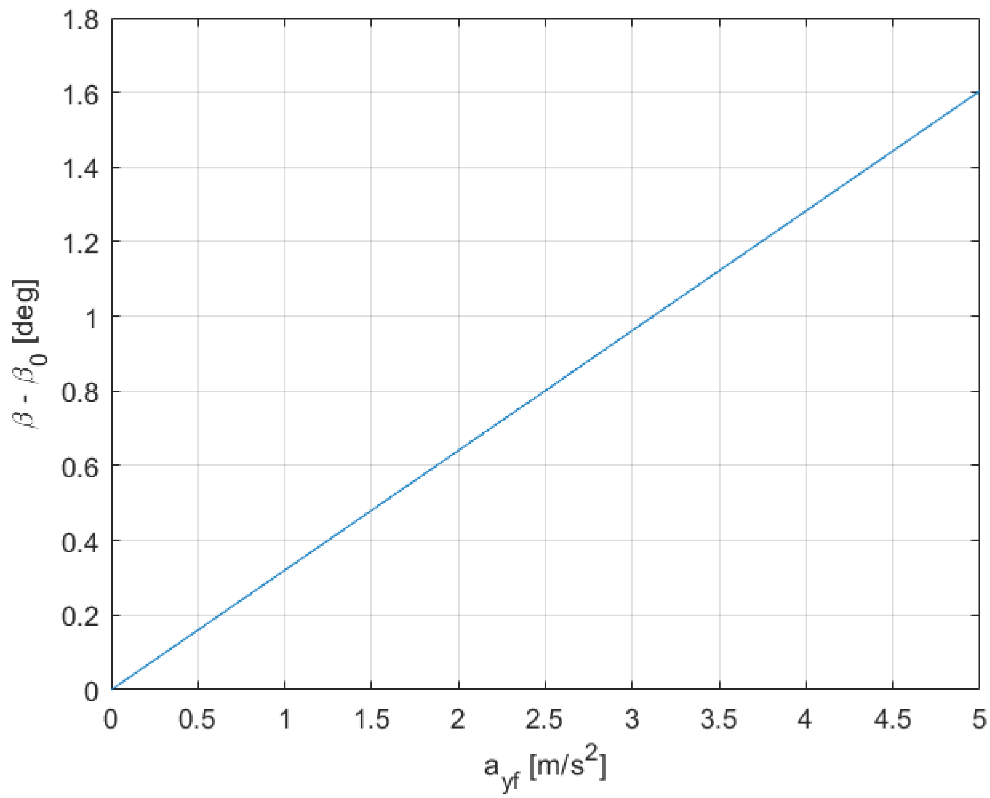
============ neutral vehicle ============

understeering gradient: K_US = 0 rad/(m/s^2)
slip agle gradient: K_beta = -0.0055995 rad/(m/s^2)
tangent speed: V_beta = 57.4295 km/h

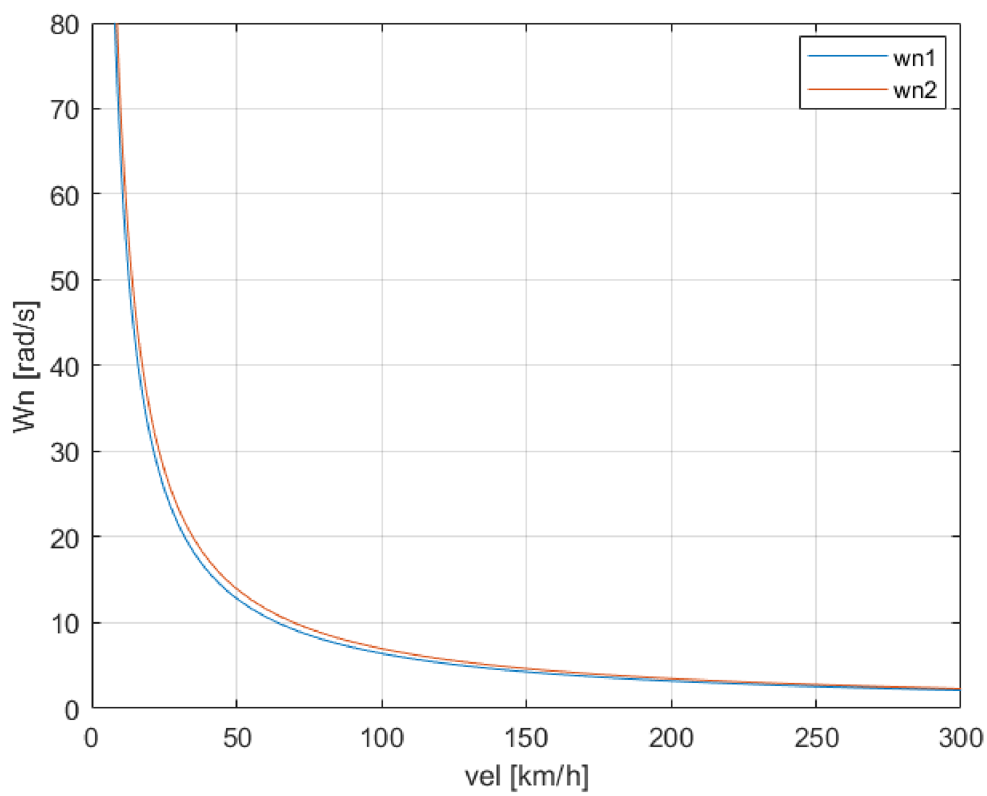

Single track analysis
```


β / δf
r / δf
ρ / δf
af / δf
ar / δf
vel [km/h]
vel [km/h]
0.350877192982346
0.350877192982344
0.350877192982342
0.35087719298224
0.350877192982338
0.350877192982336

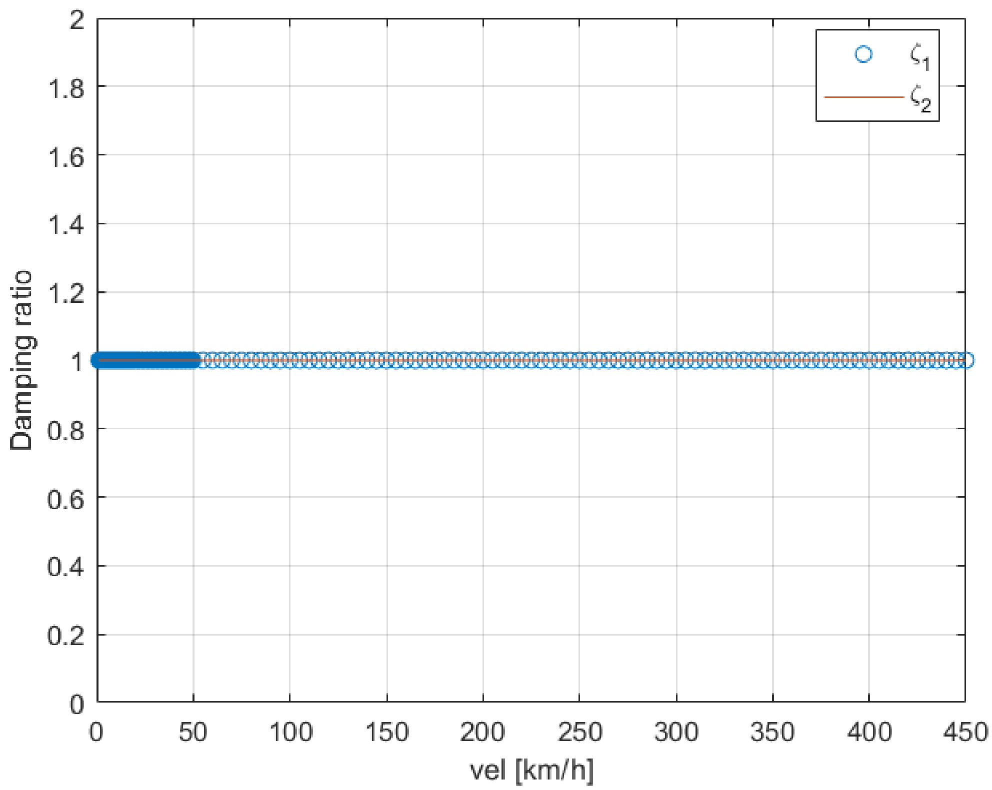

δ - δ0 [deg]
ayf [m/s2]



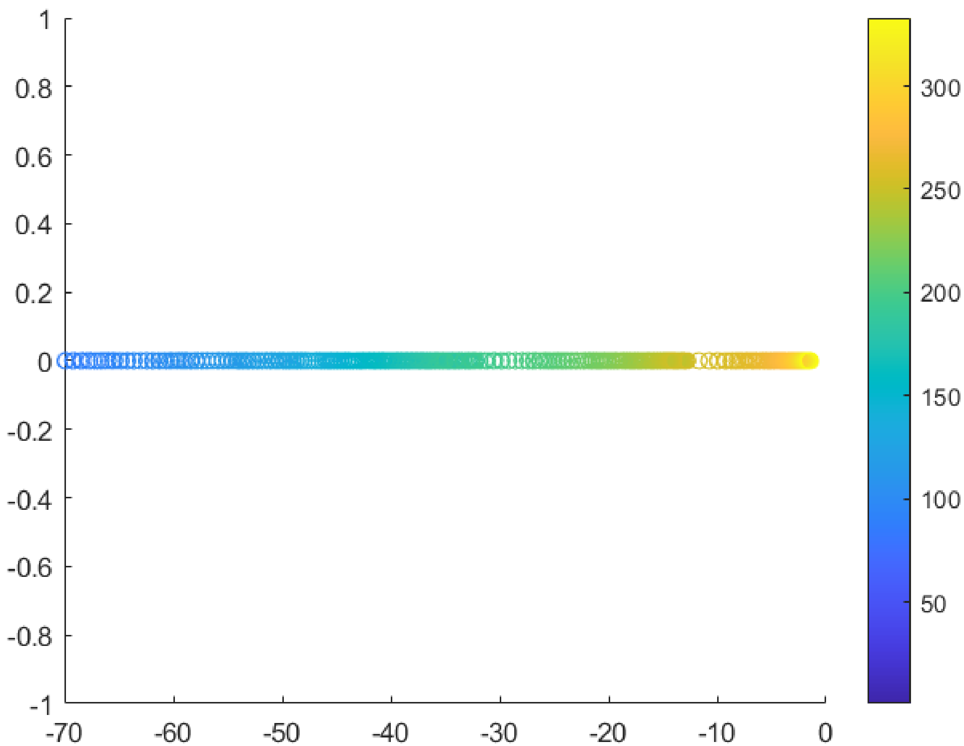
β - β_0 [deg]
a_{yf} [m/s^2]

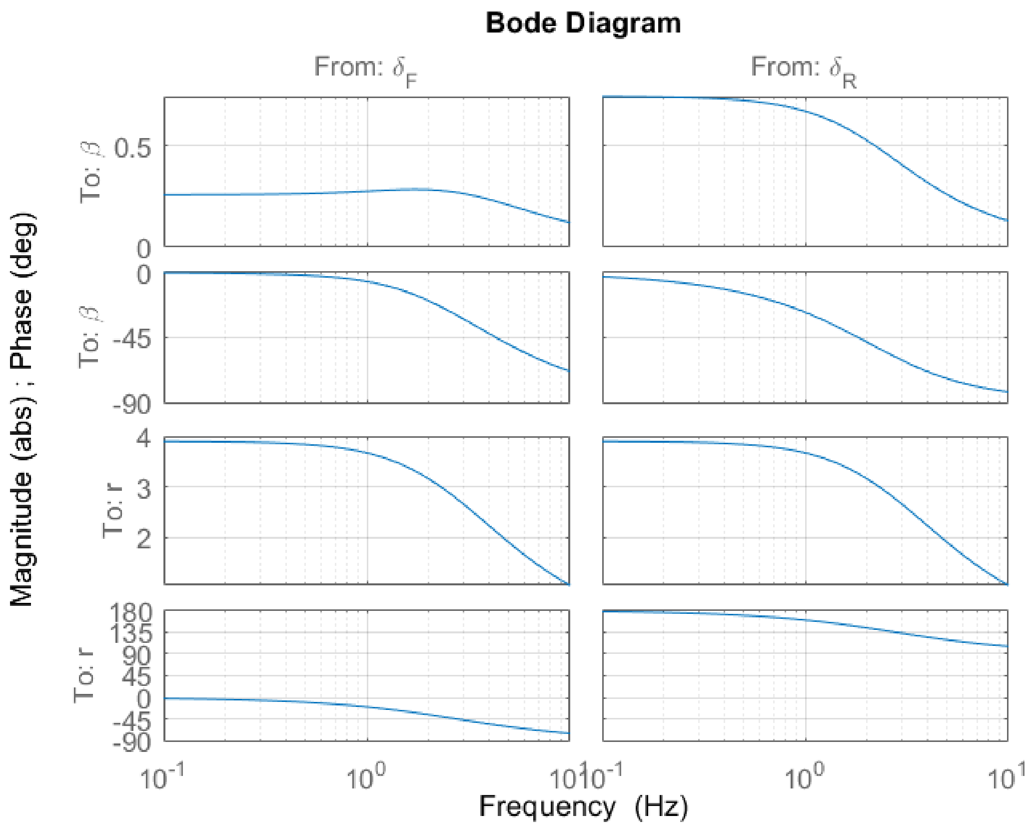


wn1
wn2
Wn [rad/s]
vel [km/h]



Damping ratio
vel [km/h]
ζ_1
ζ_2





Analysis:

The side slip angle of the vehicle shows a single flection point, meanwhile for an understeering there were two, suggesting that for neutral steering for considerable speed the side slip can be considered linear. Being neutral steering imply that the vehicle is able to follow perfectly the kinematic steering value. This imply a linear dependency of the yaw moment as well, meaning that the more speed the vehicle have, the yaw moment will be directly proportional. For the same principle, the radius of cornering will be null so as the understeering diagram.

Please note how for the radius of cornering some numerical errors produce some noise at very low speed.

The cornering capability of the axle is equal and for this reason the side slip angle front-rear can be perfecly superimpose without any difference.

As stated, the vehicle side slip angle difference does not depend on the understeering coefficent, the plot is equal to the understeering one.

Dynamic analysis:

The root locus shows always null imaginary part without even a single eigenvalue with positive (or null and multiplicity more than 1) real part, meaning that the system will be always stable. Not by chance, since the system is always stable its damping ratio will be unitary no matter the velocity adopted, meaning that the vehicle won't oscillate absorbing at the same time the lowest amoont of energy possible. The natural frequency shows the inverse proportionality respect to the velocity.

Bode diagram:

The same conclusion get in the previous vehicle can be translated into this case as well: β implies a magnitude shift from δ_f to δ_r, meanwhile the yaw rate shows a phase shift.

a = L/2+10cm

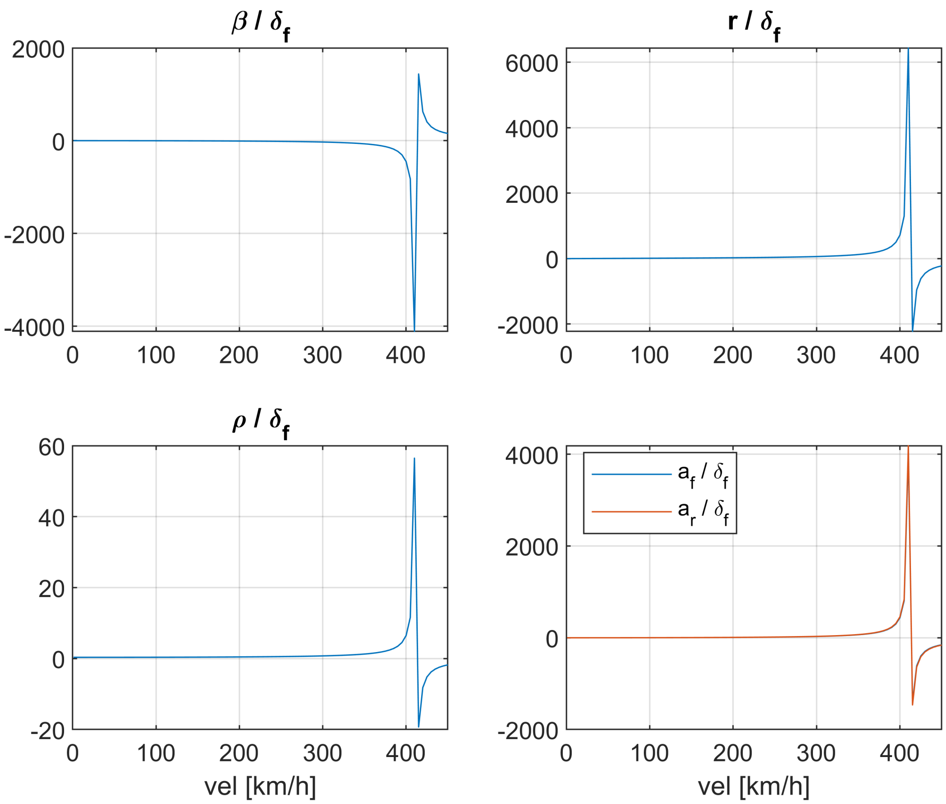
```
s_track_STUD
```

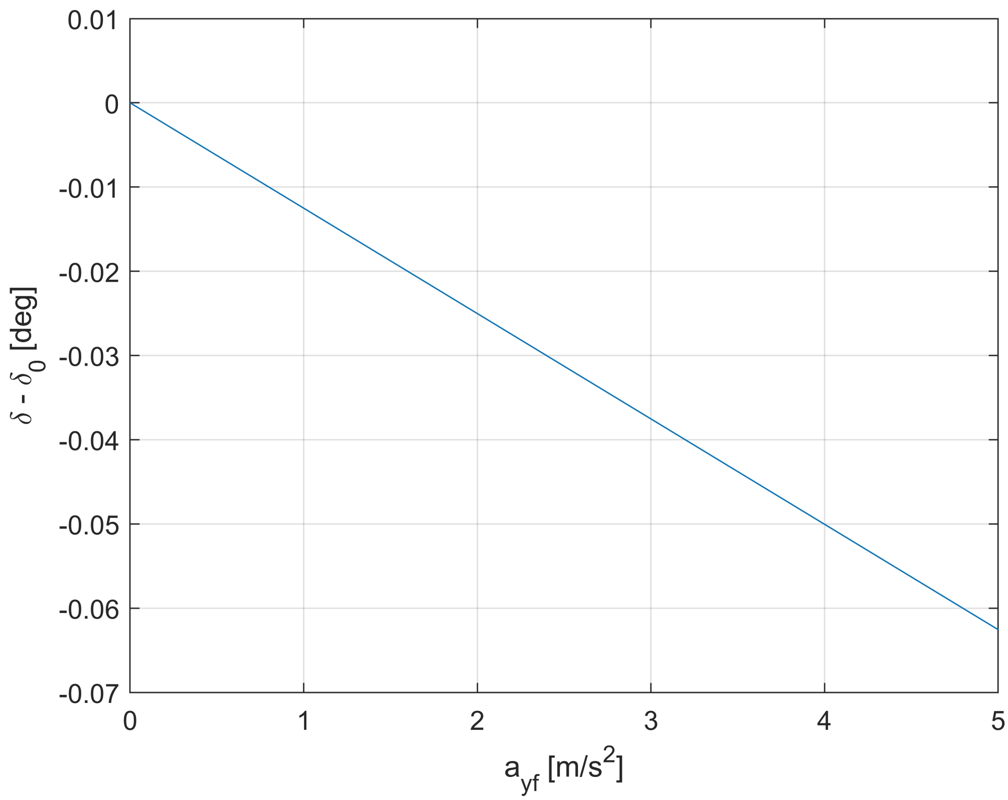
```
static load distribution (%F - %R): 46-54
axle cornering stiffness
CF = 169035.7601 N/rad
CR = 187113.8666 N/rad

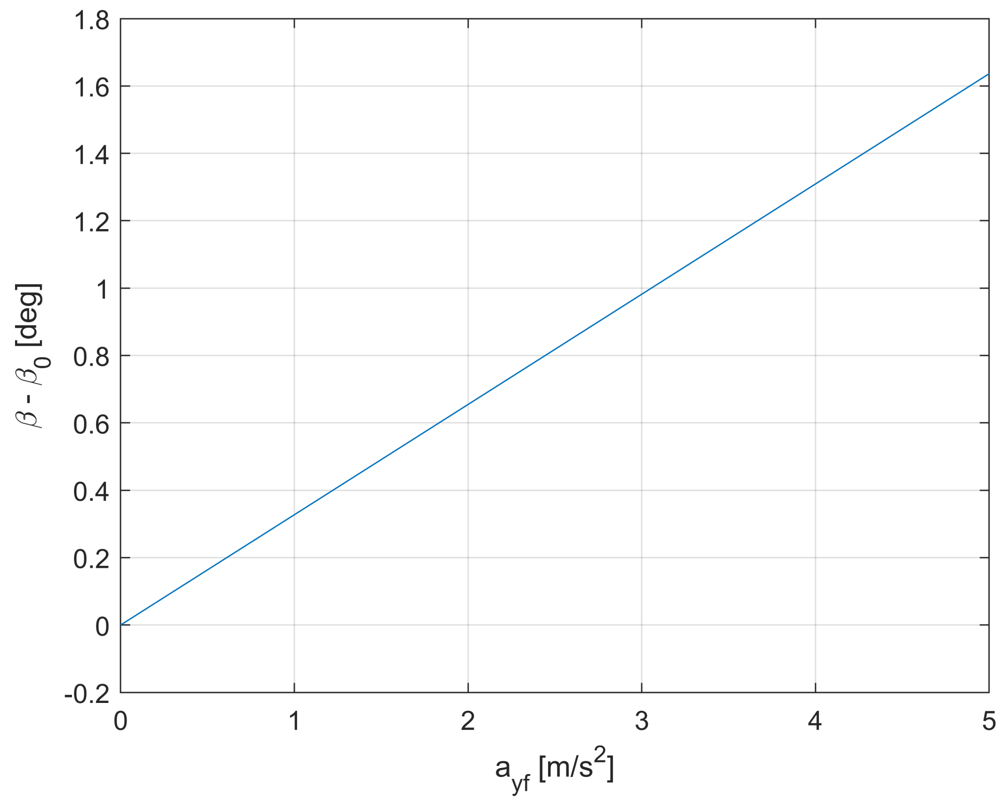
============ oversteering ============
critical speed: 411.3 km/h

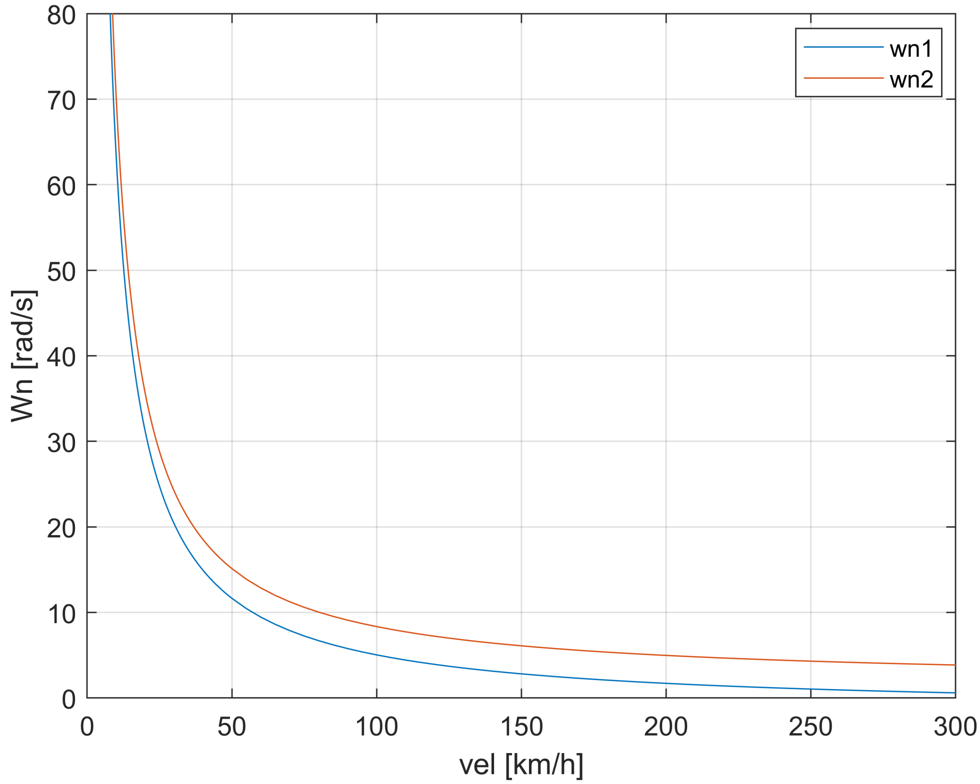
understeering gradient: K_US = -0.00021836 rad/(m/s^2)
slip agle gradient: K_beta = -0.0057125 rad/(m/s^2)
tangent speed: V_beta = 54.8273 km/h


Single track analysis
```

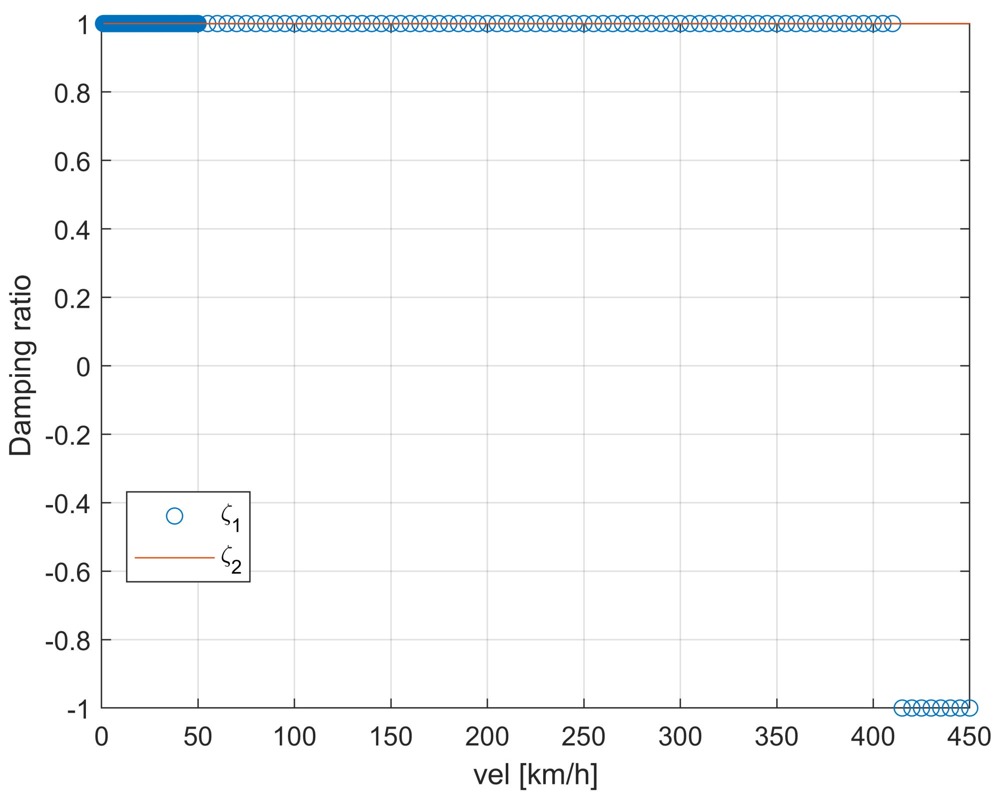



0.01
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
δ - δ0 [deg]
0
1
2
3
4
5
ayf [m/s2]

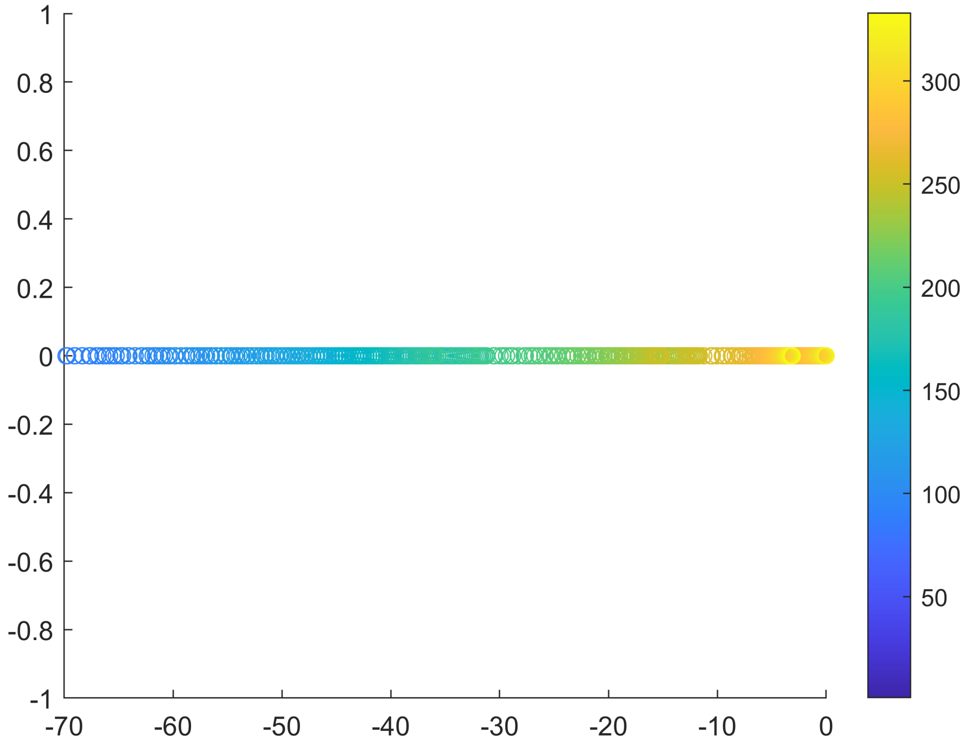

1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
-0.2
β - β0 [deg]
0
1
2
3
4
5
ayf [m/s2]



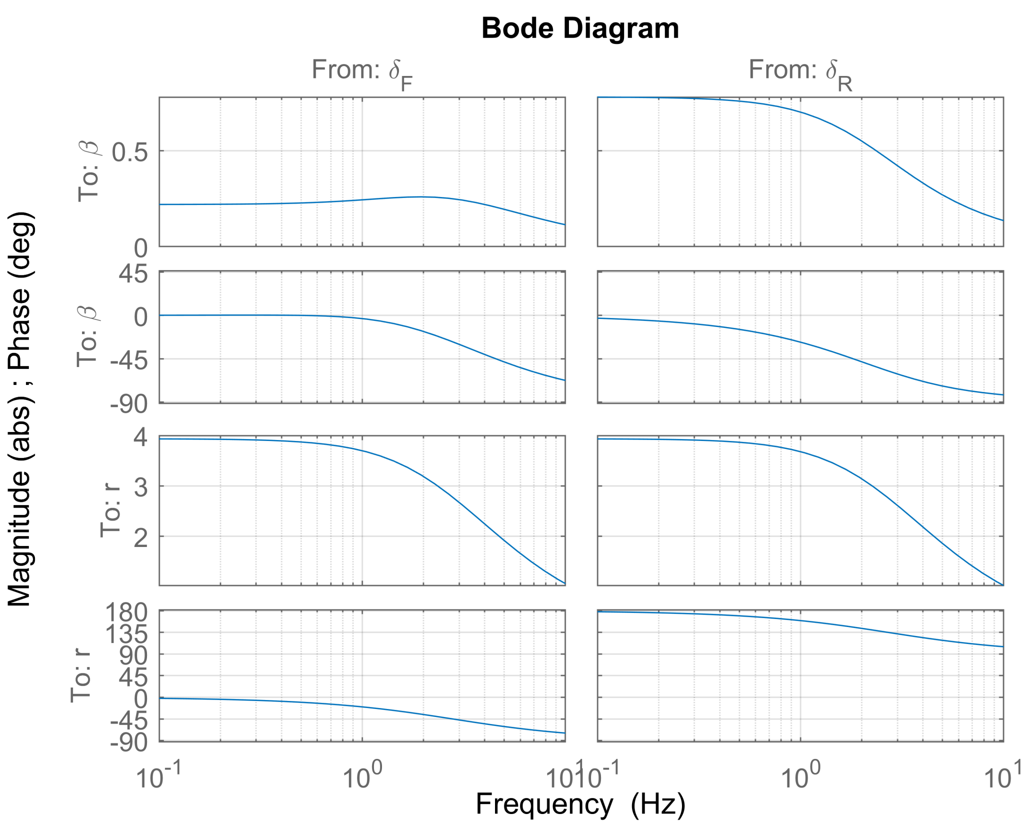
wn1
wn2
Wn [rad/s]
vel [km/h]



ζ_1
ζ_2
Damping ratio
vel [km/h]



1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
-70
-60
-50
-40
-30
-20
-10
0
300
250
200
150
100
50



Bode Diagram
From: δ_F
From: δ_R
To: β
To: r
Magnitude (abs) ; Phase (deg)
Frequency (Hz)

Analysis:

Of fundamental importance is the critical velcoity presence in an oversteering vehicle. This system is unstable after this velocity, therefore every quantity plotted present a vertical asindote.

Moreover, the understeering diagram show a negative slope. As explianed, this translates into the necessy to steer less than the kinematic sterring delta in order to keep the same trajectory. Vehicle side slip unaffected.

Dynamic analysis:

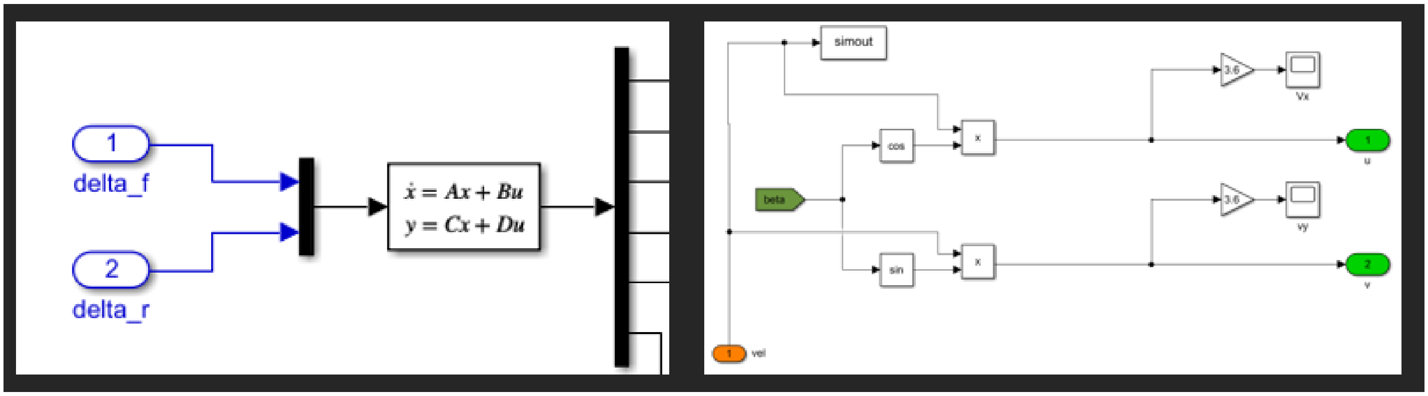
The root locus for velocities higher than the critical speed presents several eigenvalues in the origin of the chart, meaning instability. Considering the damping ratio this translates into a discontinuity point passing from critically damped into underdamped. Physically the vehicle will oscillates after certian velocity are reached.

Bode plot:

Nevertheless the instability, the Bode plot will follow always the same principle seen: β = magnitude shift, r = phase shift.

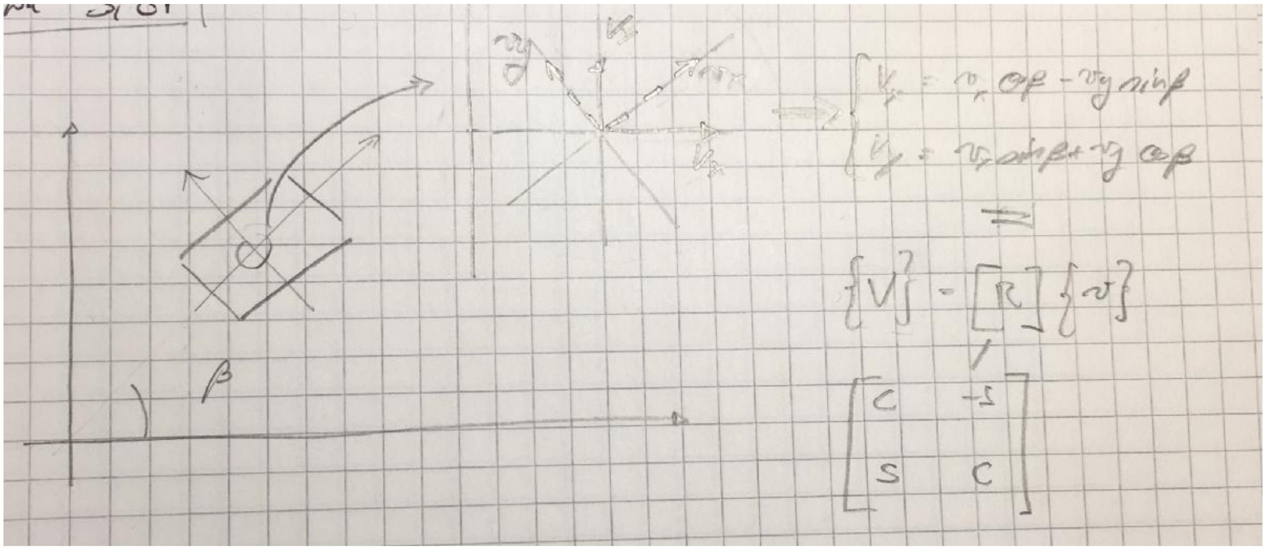
PART 2

Once completed the model with the requested blocks, the second part of this exercise is to run three simulations for the thre vehicle configurations in order to analyise the different performances. The first manoeuvre can be considered as a steady state anaylisis of the system, due to the slow and progessive input as well the low velocity. The second one, instead, can be interpetred as a transient responce of the system. The sudden steering input so as the high velocity put under critical conditon the vehicle.



The state space block features the A,B,C,D matrix defined in the script. Being all these matrix functions of velocoty, since in this simualtion the velocity is varying, the state space is defined at each time interval. In the MATLAB therefore the matrices definition is inside a *for* loop.

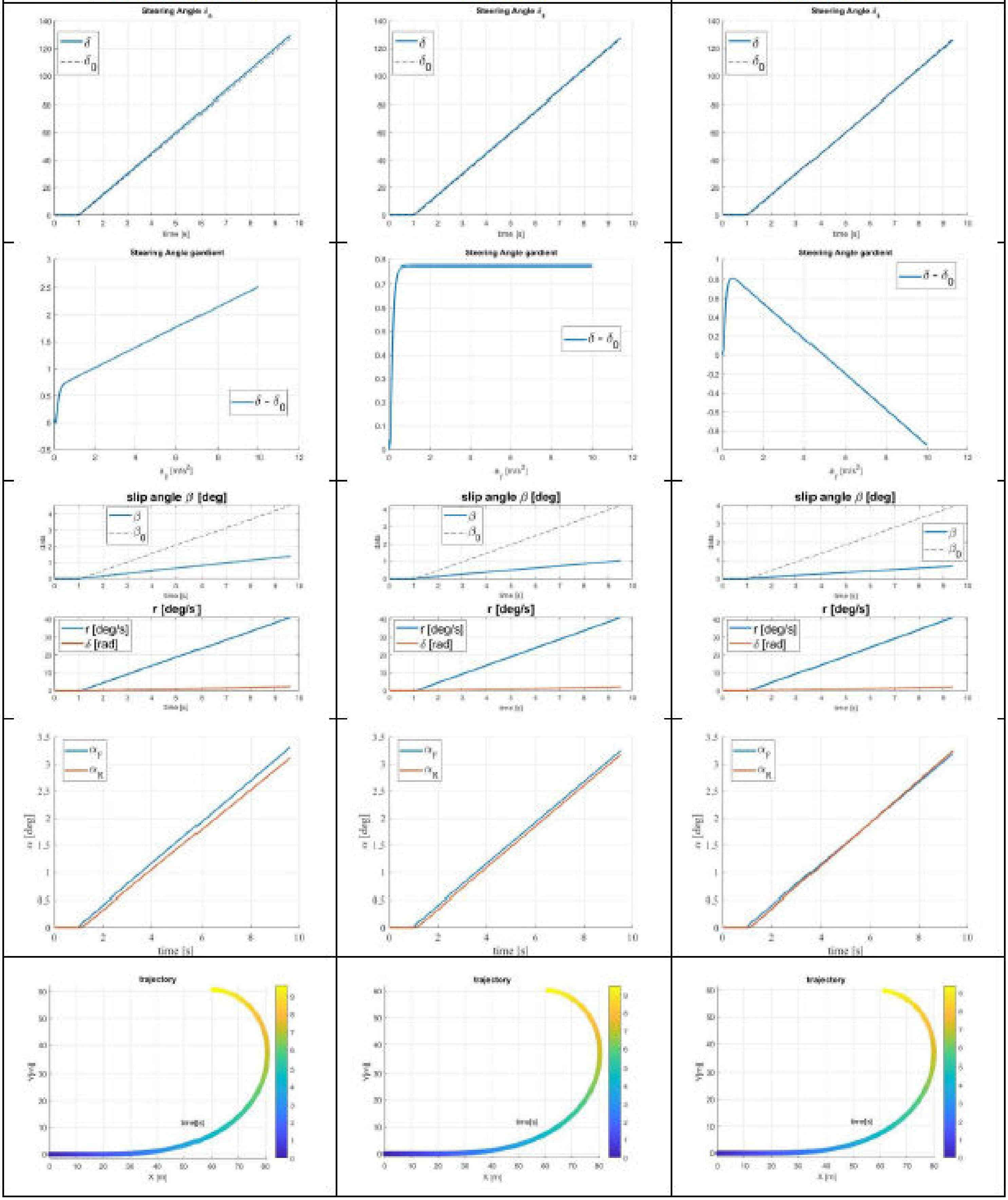
Regarding the transformaiton of the velocity components from the global reference system to the vehicle (local), a simple rotarion matrix can be applied. This matrix can be easily obtained by a simple geometrical equation, focusing the attention over the fact that in the x axis the two contributions reduce each other.



Steering ramp of 15°/s at 50 km/h

[CAMBIA DA SCREEN A PDF]

UNDERSTEERING	NEUTRAL	OVERSTEERING



Note: only intresting plot have been included, the others do not provide any particular interest, or they do not chnage among the three simulations.

First of all, the simualtion set is not a critical one. The input given is nearly quasi-static (low trasnient) and it is perfomred at low speed. These implications explain the absence of major changes. In fact the angle difference is not so relevant, just the understeering vehicle seems to present its characteristics pretty early.

The major change is in the understeering coefficent. As explianed in the theory, it has to be positive for an undertsteer, close to 0 for neutral and with negative slop for oversteer. This simualtion confirm the theoretical assumption done.

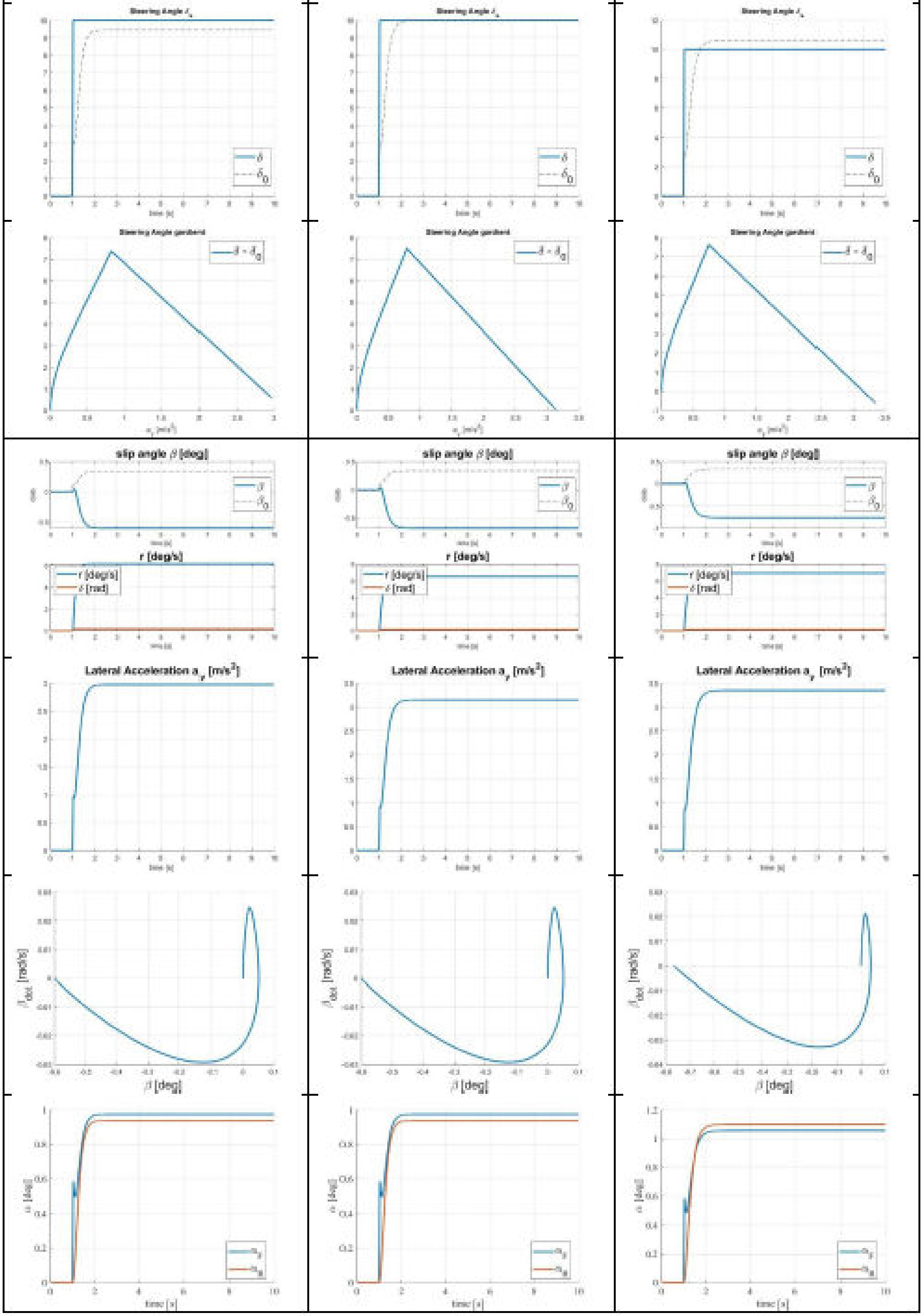
Another appreciable difference is on the axle side slips. In fact even at low velocity it is quite evident how for an understeering vehicle the front side slip is higher than the rear. This is due to the difficulty of the frontal axle to follow the given trajectory, in fact the frontal velocity will be more divergent from the given trajectory respect to the rear axle. On the contrary, the oversteering will present the opposite behaviour since the rear has more difficulty in following the trajectory. Moreover, having bigger side slip means that the lateral force of that axle increases. This is easily explained since in order to follow a given trajectory, an understeering vehicle has to extert more latral frontal force rspect to a neural one, meanwhile for an oversteering vehicle it is the rear which has to develop more lateral force in order to compensate its deviaton.

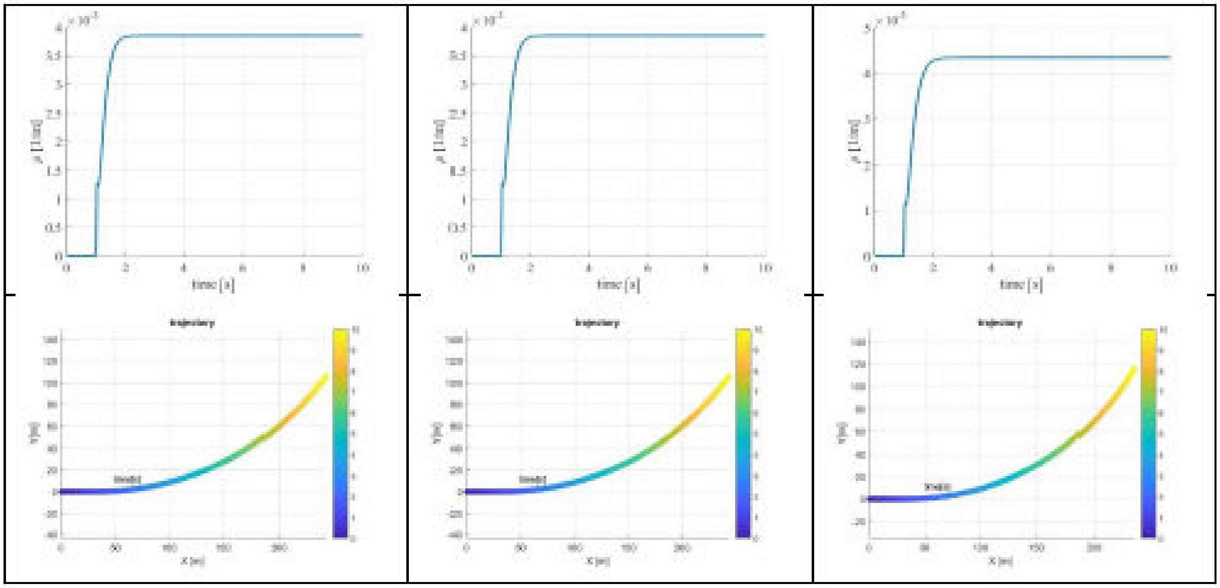
Steering step of 10° (400°/s) at 100 km/h

This simulation, respect to the previous one demands more from the vehicle stability. In fact, not only the steering wheel is instantaneous, provoking a sudden chnage to the system, but also it is performed at high speed, where the vehicle is more susceptible. It is also true that 10° for a steering angle is not so high, however resamble a not so rare scenario that could happen in a highway. Therefore it is a valid test for testing the vehicle stability in terms of forced responce.

The conclusion just given is true for resonable range of the lateral acceleration. For values above 4.5 m/s^2 the simualtion is ok numerically but not physical. Vehicle is non-linear, therefore above certain values the systems does not simulate correctly the real behaviour. Need for the following exercise model to properly simulate lateral dynamics.

UNDERSTEERING	NEUTRAL	OVERSTEERING





From these simulations it is possible to appreciate the differences among the three configurations. From common day expereince, udner/oversteering is defined over the steering angle adjustement to be done in order to correct the trajectory, meanwhile a more reliable and mathematical apporach prefers to define this behaviour over the undertsterring coefficent. Considering the results obtained from this simualtion it is possible to aprpeciate how the sterring angle approach is still valid. In fact, the understeering behaviour require higher steering angle in order to follow the trajectory, meanwhile the oversteering need less steering.

The reason for the strange plot of the understeering chart is due to the ideal kinematic delta angle. The latter, in fact, shows a minor chnage in the inital phase. For this reason, no matter the vehicle characteristics, the delta angle applied will be always higher than the kinematic one. However still some useful informations can be get from this chart. In fact the slope for and oversteering vehicle is bigger (in abs). This means that an oversteering vehicle has more capability in steering, meaning that the difference of delta will decrease with more ease.

Regarding the beta and yaw rate chart instead, it is possible to observe how after a transient phase the vehicle will arrive to a steady state. Differences can be appreciated in the steady state conditions reached by each vehicle: the oversteering one, always due to its cornering capacity, will have slightly higher yaw rate values and related lower beta (with sign). Following the same principle the oversteering vehicle shows higher lateral accelerations (more cornering capacity = follows closer trajectories = higher lateral acc.). Also the beta_dot vs beta chart can be more clear with this simulation.

Another consideration over the yaw rate is that being this a transient simualtion, an overshoot should be expected, however there is no. This is due to the linear model, which cannot represent correctly this behaviour.

The latetr can be interpetred like a sort of g-force diagram: for oversteering its shows higher acc. values (in abs) and stiff changes, confirming the hypothesis of more reactivity.

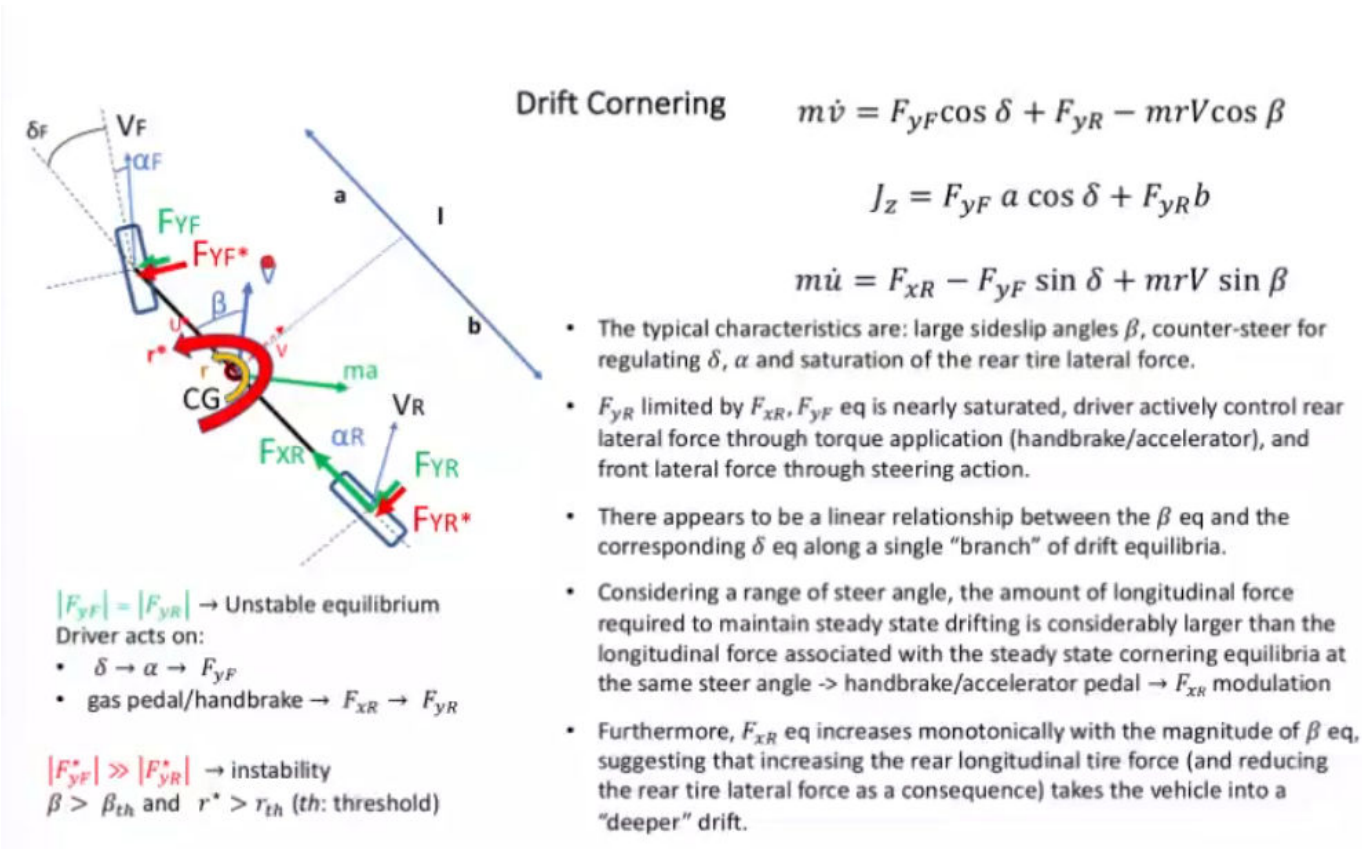
Regarding the alpha charts, the concept does not change respect to the previous simualtion: the more an axle has difficulties in turning, higher the lateral force for following a reference trajecotry have to be, therefore higher wheel side slip angles.

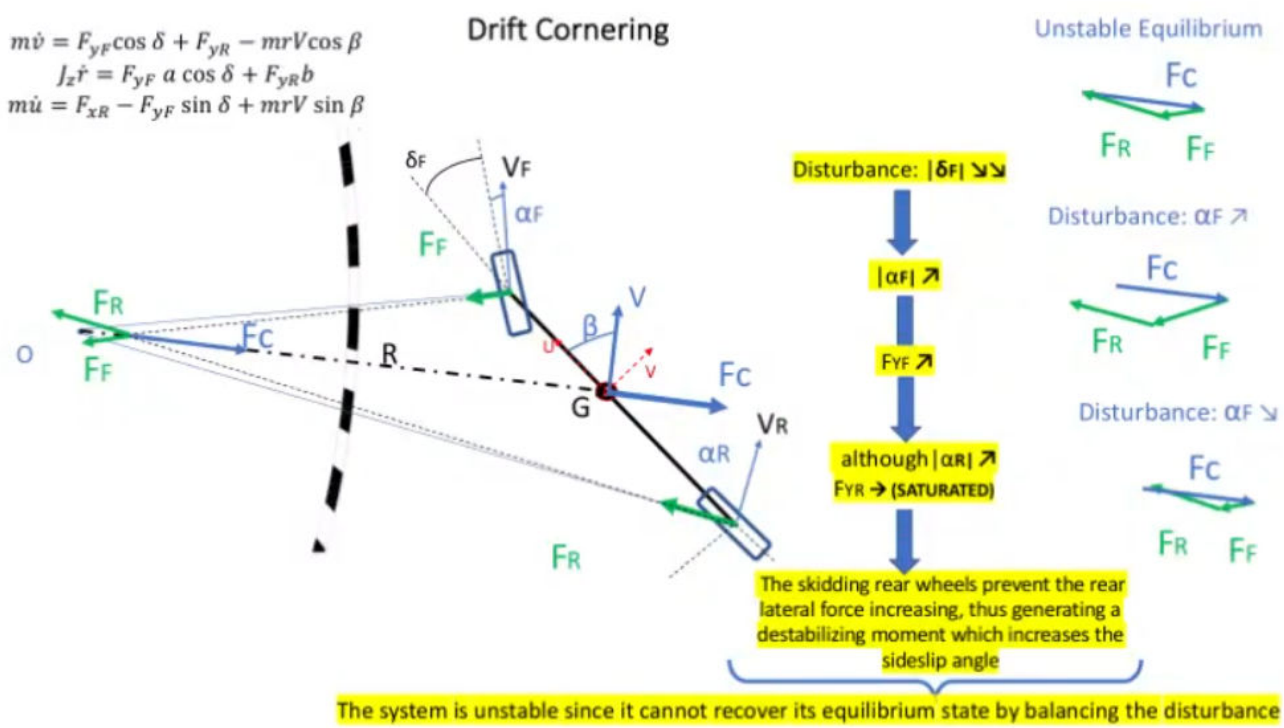
Following the same flow, the radius of curvature will be smaller higher for the vehicle which run a closer trajectory.

Finally a trajectory comparison can be done. There is not major changes in reality, probably this is due to the linearity of the model since it cannot represent dynamics above 4.5 m/s^2.

Drift manoeuvre

Drifting consinst on turning to a side but eltting the vehicle slide in the opposite direction. As common sense suggest, this is a incredibly non linear behaviour, however, it can be simualted even with a pretty similar model like a singel track. This beacause the physics behind it its know, however its evolution and control is pretty complex (even if nowadays autonomous control are able to perform it).





In real life, drifting is performed through gas pedal and braking, allowing the forces to be in a specific window capable to let the vehicle slide. However, the model proposed does not have any control over torque and braking, therefore to achive a drift manoeuvre a rear cornering stiffness with equal a nd b (neutral steering) is adopted.

Letting the model proposed to perform two simulations one at 6 km/h the other at 10 km/h permit to get to some intresting solutions. The critical speed for the model proposed becomes 8.2 km/h, meaning that it is expected to have a stable drifitng manoeuvre for velocity smaller than this one and a complete divergent drift for higher velocities.

```
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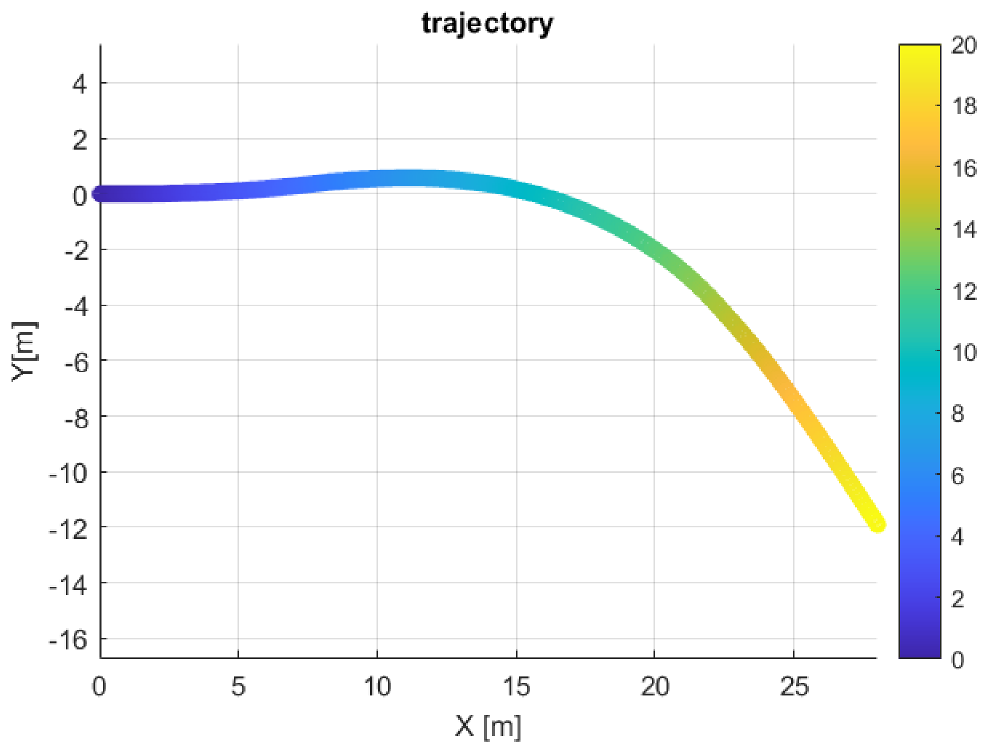
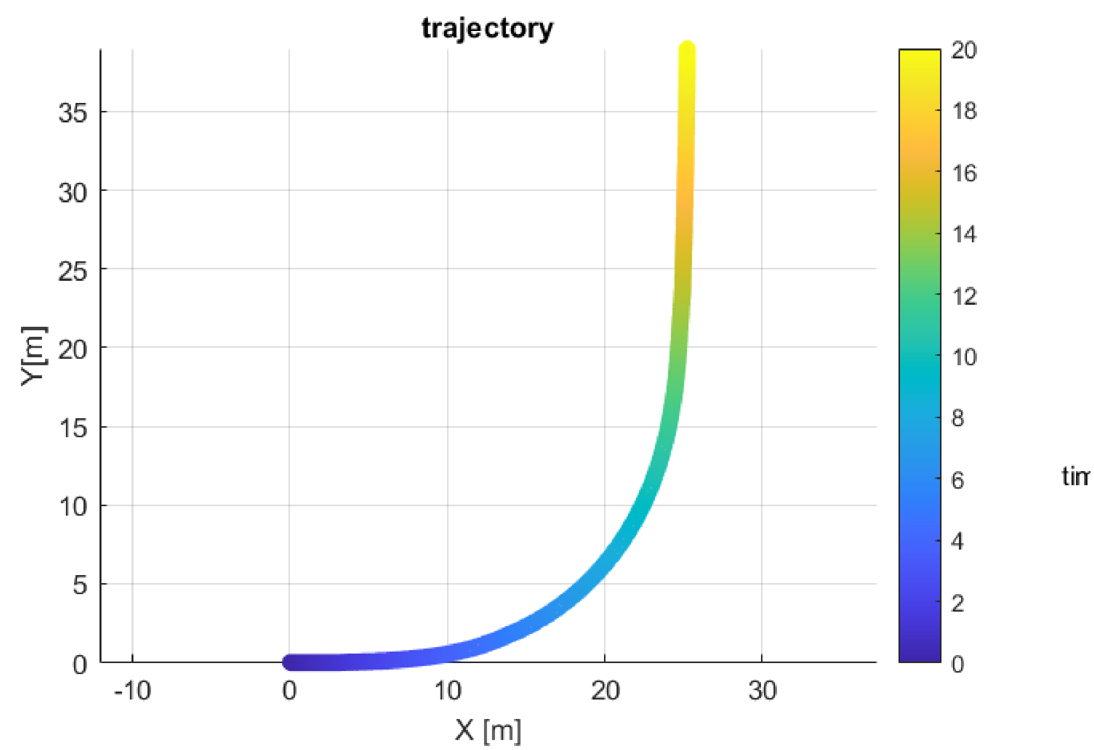
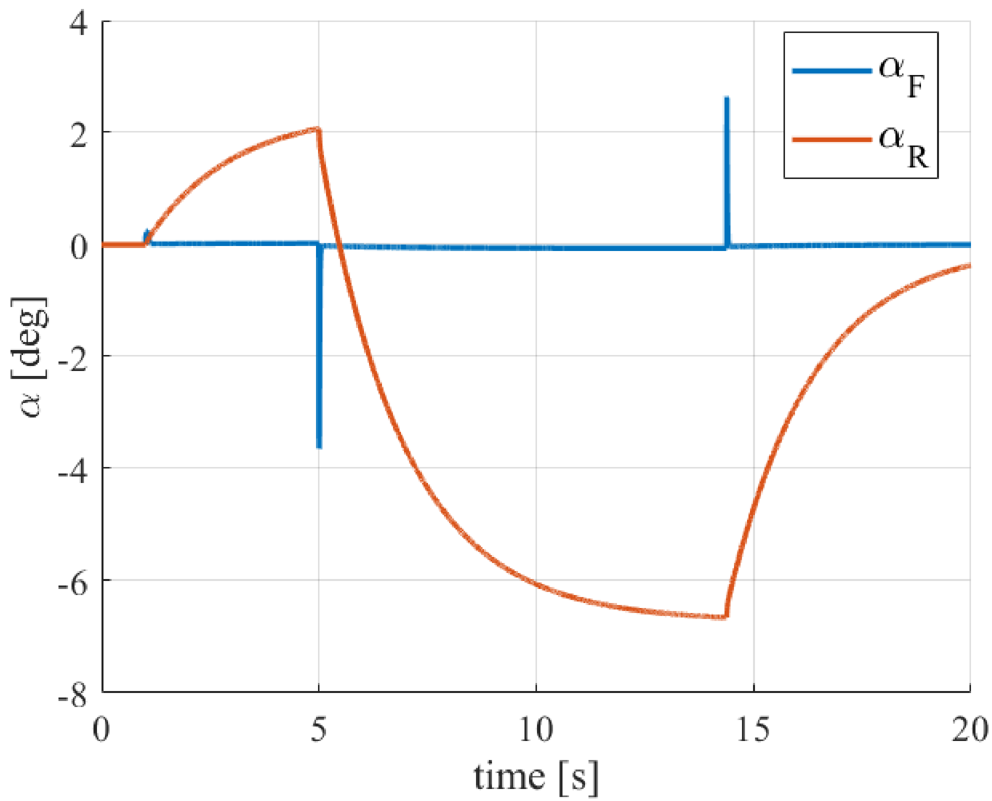
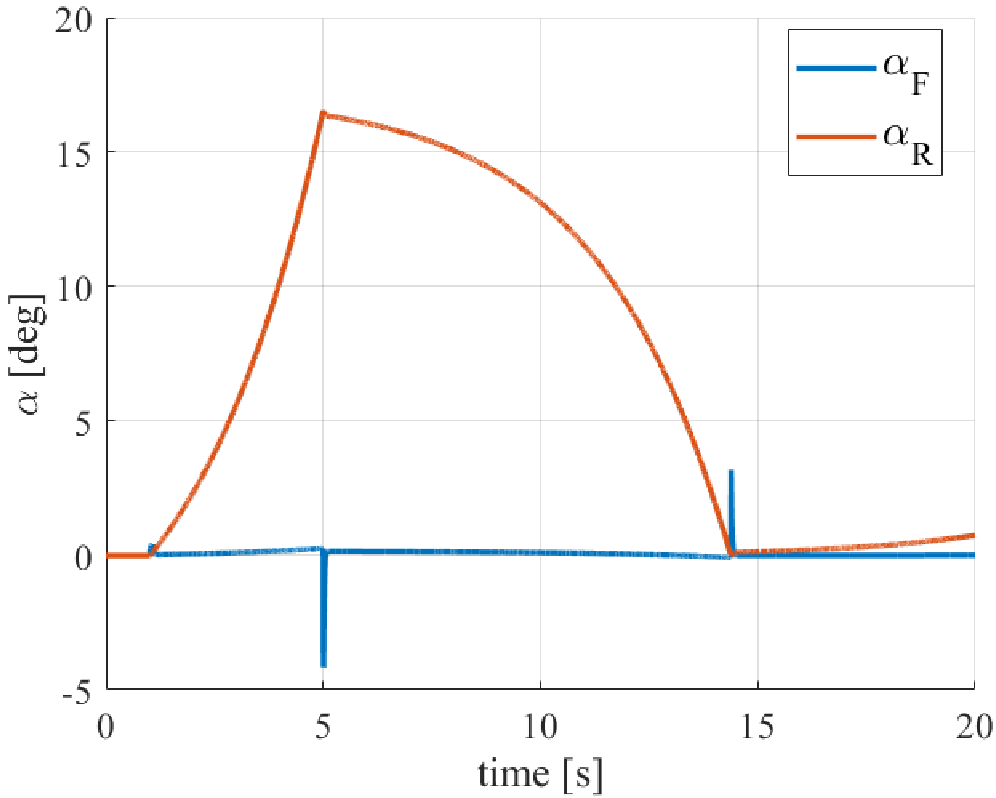




As expected, the results fulfill the premises. Moreover the drifitng performance is made possible thanks to the frontal side slip angle evolution. In fact for a complete drifting it raises a bit but then rapidly goes in the other direction, meaning that the rear axle is able to develop enough forces to countabalance the frotnal one even if they are poining toward different directions. Not by chance, the unstable manoeuvre shows an alpha rear that cannot change its sign, meaning that the car starts to slide in the opposite direction respect to the curve.

```
openfig('a6.fig'); openfig('a10.fig');
```





20
15
10
5
0
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α [deg]
0
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15
20
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