1 Fractal Knowledge Graph Specification

A fractal knowledge graph represents an idea as a subgraph with multi-resolution, hierarchical structure, constrained by the Bekenstein bound.

[Fractal Knowledge Graph] A fractal knowledge graph is a tuple $G = (V, E, \mathcal{F})$, where:

- V: Nodes representing objects at fractal levels $F(v) \in \mathbb{N}_0$.
- E: Directed edges (v_1, v_2) with proof distances $d_{\text{proof}}(v_1, v_2) = \log \left| \frac{\pi(v_1)}{\pi(v_2)} \right|$, where $\pi(v) = \prod_i p_i^{n_i}$ is the prime signature.
- $\mathcal{F}$: Fractal hierarchy, where each $v \in V$ has a subgraph $G_v = (V_v, E_v)$ with $F(G_v) = F(v)$.

[Resolution, Density, Granularity] For a subgraph $G_{\text{sub}} = (V_{\text{sub}}, E_{\text{sub}})$:

- Resolution: $R(G_{\text{sub}}) = \frac{|V_{\text{sub}}|}{|E_{\text{sub}}|} \cdot \log |O_{\text{sub}}|$.
- Density: $\rho(G_{\text{sub}}) = \frac{|\{e \in E_{\text{sub}} : e \text{ is a valid proof}\}|}{|V_{\text{sub}}| \cdot |E_{\text{sub}}|}$.
- Granularity: $\Gamma(G_{\text{sub}}) = \frac{1}{|V_{\text{sub}}|} \sum_{v \in V_{\text{sub}}} \Gamma(v)$, where $\Gamma(v) = \min\{k : v \in \mathcal{P}^{(k)}(S)\}$.

[Fractal Level and Dimension]

- Fractal level: $F(G_{\text{sub}}) = \max_{v \in V_{\text{sub}}} \Gamma(v)$.
- Fractal dimension: $D_f = \lim_{\epsilon \to 0} \frac{\log N(\epsilon)}{\log(1/\epsilon)}$, where $N(\epsilon)$ is the number of subgraphs covering G at resolution ϵ.

[Connectivity Bound] A subgraph G_{sub} is connected if:

$$|E_{\text{sub}}| \leq \kappa \cdot \sum_{v \in V_{\text{sub}}} \deg(v_{\text{ext}}),$$

where $\deg(v_{\text{ext}})$ is the external degree, and κ depends on D_f.

[Event Propagation] An event at node v with fractal level $F(v)$ updates G_{sub} with density change:

$$\Delta\rho(G_{\text{sub}}) = \rho(G'_{\text{sub}}) - \rho(G_{\text{sub}}).$$

Propagation is constrained by d_{proof} and the Bekenstein bound $S \leq \frac{A}{4\ell_P^2} \ln 2$.