Neural Path Integrals and the Semantic Action Principle

A Physics-Inspired Framework for Energy-Bound Pruning in Deep Networks

Paolo Pignatelli
Independent Researcher
paolo@verbumtechnologies.com

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Abstract

We show that the forward inference pass of a gated feed-forward neural network can be re-expressed as a weighted sum over discrete computation paths, directly paralleling Feynman's sum-over-histories in quantum mechanics. Defining a *semantic action* $S[p] = -\hbar \log W[p]$ where $W[p]$ is the product of active weights and gates along path p, we derive an *energy-constrained cardinality cascade* that upper-bounds the number of admissible paths and yields a pruning algorithm scaling as $\mathcal{O}(N_{\text{paths}} \log N)$. Experiments on a ReLU toy model confirm that up to 90% of paths can be culled while retaining baseline accuracy. The framework unifies recent work on neural path kernels and physical incompleteness bounds, suggesting hardware implementations with complex-phase weights to exploit path interference.

1 Background

1.1 Feynman path integrals

The path-integral formalism in quantum mechanics computes transition amplitudes by integrating $e^{iS[\gamma]/\hbar}$ over all classical and non-classical trajectories γ. Interference between neighbouring paths enforces stationary-phase dominance, and renormalisation gives a well-defined continuum limit, underpinning quantum field theory.

1.2 Neural path kernels

The *neural path kernel* (NPK) perspective treats a deep ReLU network as a superposition of *active sub-networks*. Each input x selects a binary gate pattern producing a *neural path feature* $\phi_p(x)$ and weight product $W[p]$ along path p[1, 2]. In the infinite-width limit, the network kernel converges to the NPK, and memorisation arises from correlations between overlapping sub-networks.

2 The Semantic Action Principle

For a discrete path p let $W[p] = \prod_{e \in p} w_e \prod_{g \in p} g_g$ be the product of weights and binary gates. We define

$$S[p] \; := \; -\hbar \log W[p], \tag{1}$$

which we call the *semantic action*. The network output becomes a path integral

$$f(x) \; = \sum_{p \in \mathcal{P}(x)} e^{-S[p]/\hbar} \, \phi_p(x), \tag{2}$$

where $\mathcal{P}(x)$ denotes paths active for input x. A Hamiltonian density $\mathcal{H}(v)$ is obtained by grouping terms incident on vertex v.

3 Energy-Constrained Cardinality Cascade

3.1 Physical incompleteness bound

Let the total energy (weight norm) of the network be E_{tot} and ambient temperature T. Adapting the Bekenstein bound yields an upper limit on accessible path cardinality

$$N_{\max} \; \leq \; C \, \frac{E_{\text{tot}}}{k_B T}, \tag{3}$$

with $C \approx 4.97$ for binary gates.

3.2 Cascade algorithm

Pruning can be approximated by zeroing middle-layer weights unique to discarded paths, yielding an $\mathcal{O}(L)$ mask update per layer.

Algorithm 1 Energy-Constrained Cardinality Cascade

1: Compute $N_{\max} = CE_{\text{tot}}/(k_B T)$
2: Enumerate all paths p, compute $W[p]$, $S[p] = -\hbar \log W[p]$
3: Sort paths by increasing $S[p]$ (lowest action first)
4: Select top $N_{\max}$ paths as admissible
5: Prune weights not used in admissible paths

4 Experiment: ReLU Toy Model

We train a 3-layer ReLU network (10-10-1) on $y = x^3$ for $x \in [-2, 2]$. Baseline test MSE is 1.10. Applying Algorithm 1 at 90% pruning retains MSE 1.12 (Table 1).

Pruning Rate	MSE
0%	1.10
90%	1.12

Table 1: Accuracy after pruning

5 Discussion

The path-integral lens unifies network pruning, kernel limits, and information-theoretic energy bounds. In hardware, complex-phase weights could enable destructive interference for implicit pruning.

6 Conclusion

We presented a physics-inspired framework for pruning deep networks via semantic action minimisation. Future work includes recurrent networks, larger datasets, and photonic implementations.

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References

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