Chapter 1

Computational Spacetime Geometry

1.1 Introduction: From Continuous to Discrete Spacetime

The derivation of computational light-speed from fundamental physical bounds establishes that information processing exhibits intrinsic relativistic structure. However, unlike the continuous spacetime of general relativity, computational spacetime is fundamentally discretetessellated by Shannon information bits. This chapter develops the mathematical framework for differential geometry on this discrete, tessellated manifold.

1.2 The Computational Metric Tensor

1.2.1 Fundamental Structure

We begin with the computational spacetime interval:

$$ds^2 = c_{\text{comp}}^2 dt^2 - d_{\text{Manhattan}}^2(x_{\text{tess}}) \tag{1.1}$$

where:

- t is continuous computational time (processing steps)
- x_{tess} represents discrete tessellated spatial coordinates
- $c_{\text{comp}}(T) = \frac{2k_B T \ln(2)}{\pi\hbar}$ is the temperature-dependent computational speed limit

- $d_{\text{Manhattan}}$ is the discrete path distance through tessellation cells

1.2.2 Tessellation Structure

Each tessellation cell represents one Shannon bit of information. The fundamental relationship:

$$1 \text{ tessellation unit} = 1 \text{ bit} = c_{\text{comp}} \times \tau_0 \tag{1.2}$$

where $\tau_0 = \frac{1}{c_{\text{comp}}}$ is the minimal time to process one bit.

1.2.3 Discrete Metric Components

In tessellated coordinates, the metric tensor takes the form:

$$g_{\mu\nu} = \text{diag}(c_{\text{comp}}^2, -1, -1, -1, \ldots, -1) \tag{1.3}$$

for an n-dimensional information space with one temporal and $(n-1)$ spatial dimensions.

1.3 Discrete Differential Operators

1.3.1 Forward Difference Operator

On the tessellated manifold, we define the discrete derivative:

$$\Delta_\mu f(x) = f(x + \hat{e}_\mu) - f(x) \tag{1.4}$$

where $\hat{e}_\mu$ is the unit vector in the μ-direction (one tessellation cell).

1.3.2 Symmetric Difference Operator

For improved accuracy, we employ the symmetric difference:

$$D_\mu f(x) = \frac{f(x + \hat{e}_\mu) - f(x - \hat{e}_\mu)}{2} \tag{1.5}$$

1.3.3 Discrete Gradient

The gradient operator on tessellated space:

$$\nabla_{\text{tess}} f = \sum_i D_i f \cdot \hat{e}_i \tag{1.6}$$

where the sum runs over all spatial dimensions.

1.4 Discrete Christoffel Symbols

1.4.1 Definition on Tessellated Manifold

The discrete Christoffel symbols are defined using symmetric differences:

$$\Gamma^{\mu}_{\nu\rho} = \frac{1}{2} g^{\mu\sigma} \left[D_\nu g_{\sigma\rho} + D_\rho g_{\nu\sigma} - D_\sigma g_{\nu\rho} \right] \tag{1.7}$$

1.4.2 Computational Interpretation

For our specific metric:

- $\Gamma^t_{ij} = 0$ for spatial indices i, j (no spatial curvature affects time)
- $\Gamma^i_{tt} = -\frac{1}{c^2_{\text{comp}}} \left(\frac{\partial c_{\text{comp}}}{\partial x^i} \right)$ (computational speed variations)
- $\Gamma^i_{jk} = 0$ for distinct spatial indices (Manhattan space is flat)

1.4.3 Temperature Dependence

Since $c_{\text{comp}}(T) = \frac{2k_B T \ln(2)}{\pi\hbar}$, we have:

$$\frac{\partial c_{\text{comp}}}{\partial T} = \frac{2k_B \ln(2)}{\pi\hbar} \tag{1.8}$$

This creates “thermal curvature” in computational spacetime when temperature gradients exist.

1.5 Geodesic Equations on Tessellated Manifold

1.5.1 Discrete Geodesic Equation

The path of extremal computational action satisfies:

$$\frac{\Delta^2 x^\mu}{\Delta\tau^2} + \Gamma^\mu_{\nu\rho}\left(\frac{\Delta x^\nu}{\Delta\tau}\right)\left(\frac{\Delta x^\rho}{\Delta\tau}\right) = 0 \tag{1.9}$$

where τ is the proper computational time.

1.5.2 Manhattan Geodesics

In uniform temperature (constant c_{comp}), geodesics are Manhattan shortest paths:

- Move along one coordinate at a time
- Total distance = $\sum |\Delta x_i|$
- No diagonal shortcuts through tessellation

1.5.3 LLC as Geodesic Finders

Local Language Constructors solve:

$$B(L_1, L_2) = \arg\min_{\text{path}} \sum_{\text{steps}} [g_{\mu\nu}\Delta x^\mu \Delta x^\nu]^{1/2} \tag{1.10}$$

This is equivalent to finding minimal Manhattan distance paths in tessellated space.

1.6 Discrete Riemann Curvature Tensor

1.6.1 Definition

The discrete Riemann tensor:

$$R^\mu_{\nu\rho\sigma} = D_\rho \Gamma^\mu_{\nu\sigma} - D_\sigma \Gamma^\mu_{\nu\rho} + \Gamma^\mu_{\lambda\rho}\Gamma^\lambda_{\nu\sigma} - \Gamma^\mu_{\lambda\sigma}\Gamma^\lambda_{\nu\rho} \tag{1.11}$$

1.6.2 Computational Curvature Sources

Curvature arises from:

- Temperature gradients: Variable $c_{\text{comp}}(T)$
- Information density: High-density regions create "gravitational" effects
- Processing bottlenecks: Computational "black holes"

1.6.3 Flat Limit

In uniform conditions (constant T, uniform processing), $R^{\mu}_{\nu\rho\sigma} \to 0$, recovering flat Manhattan space.

1.7 Computational Einstein Equations

1.7.1 Information Stress-Energy Tensor

Define the computational stress-energy tensor:

$$T_{\mu\nu} = \rho_{\text{info}} u_\mu u_\nu + p_{\text{comp}} g_{\mu\nu} \tag{1.12}$$

where:

- ρ_{info}: Information density (bits per tessellation volume)
- u_μ: Information flow 4-velocity
- p_{comp}: Computational pressure (processing demand)

1.7.2 Field Equations

The computational Einstein equations:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G_{\text{comp}}}{c_{\text{comp}}^4} T_{\mu\nu} \tag{1.13}$$

where G_{comp} is the computational coupling constant (to be determined empirically).

1.7.3 Physical Interpretation

These equations describe how information density curves computational spacetime, affecting:

- Processing paths (geodesics)
- Information propagation rates
- Computational horizon formation

1.8 Discrete Parallel Transport

1.8.1 Information Vector Transport

For a vector V^μ transported along path γ:

$$\Delta V^\mu = -\Gamma^\mu_{\nu\rho} V^\nu \Delta x^\rho \tag{1.14}$$

1.8.2 Holonomy and Path Dependence

Due to discretization, parallel transport exhibits path dependence:

- Different tessellation paths yield different final vectors
- Holonomy measures computational "context dependence"
- Applications to semantic drift in language models

1.9 Computational Horizons and Singularities

1.9.1 Event Horizons

Computational event horizons form where:

$$g_{tt} = c_{\text{comp}}^2 \left(1 - \frac{2G_{\text{comp}}M_{\text{info}}}{r}\right) = 0 \tag{1.15}$$

Beyond this, no information can escape the computational demand.

1.9.2 Singularities

Computational singularities occur at:

- Infinite recursion points
- Halting problem boundaries
- Gödel incompleteness limits

1.10 Experimental Predictions

1.10.1 Observable Effects

- Light Cone Structure: Diamond-shaped influence regions in neural networks
- Geodesic Optimization: LLC paths follow Manhattan distance minimization
- Curvature Effects: High information density regions slow processing
- Thermal Scaling: Processing rates scale linearly with temperature

1.10.2 Validation Protocols

- Transformer Networks: Measure information propagation geometry
- Optimization Algorithms: Verify geodesic path selection

- Distributed Systems: Test computational horizon formation
- Temperature Experiments: Validate $c_{\text{comp}}(T)$ scaling

1.11 Mathematical Consistency Checks

1.11.1 Limiting Cases

As tessellation size $\to 0$:

- Discrete operators $\to$ continuous derivatives
- Manhattan metric $\to$ Euclidean metric
- Discrete geodesics $\to$ smooth curves

1.11.2 Conservation Laws

- Information conservation: $\nabla_\mu T^\mu_\nu = 0$
- Causal structure preservation
- Thermodynamic consistency

1.12 Summary and Implications

This discrete differential geometry framework establishes:

- Rigorous mathematical foundation for computational spacetime
- Tessellation-based geometry natural for information processing
- Temperature-dependent effects creating computational curvature
- Geodesic optimization principles for efficient algorithms
- Experimental predictions for immediate validation

The framework bridges abstract theory with practical computation, suggesting that all information processing systemsfrom neural networks to distributed computersoperate within this geometric structure. The next chapter will explore how causal structure emerges from these geometric foundations.