Semantic Probability Chains and Bayesian Equivalence

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1 Introduction

We develop here a framework called **Semantic Probability Chains (SPC)**, combining graph theory, conditional probabilities, and Bayesian inference. SPC explicitly represents states as weighted attribute cliques, transformations as constrained operations, and transitions as probabilistic updates analogous to Bayesian inference.

2 Foundations

2.1 Cliques as Attribute States

A clique C is formally defined as a set of weighted attributes:

$$C = \{(a_i, w_i)\}_{i=1}^{n}, \quad \text{where } w_i \in [0,1], \quad \sum_i w_i \leq 1.$$

Attributes a_i represent informational or empirical properties. The weight w_i quantifies the probabilistic or semantic significance.

2.2 Transformations as Constrained Operations

We define transformations explicitly from clique C_i to clique C_{i+1}:

$$T : C_i \mapsto C_{i+1}, \quad T \in \{T_1, T_2, \ldots, T_k, \ldots\}$$

These operations form at least a **semigroup** structure with:

- **Closure**: Every operation produces a valid clique.
- **Associativity**: $(T_j \circ T_k)(C_i) = T_j(T_k(C_i))$.

Potential future extensions include inverse transformations, making it a group structure.

3 Probabilistic Structure

Each transformation T_k has a conditional probability weight:

$$P(C_{i+1}|C_i) = w(T_k|C_i), \quad \sum_k w(T_k|C_i) = 1.$$

Weights derive explicitly from empirical frequencies, theoretical considerations, or computational outputs.

4 Frustums as Conditional Probability Spaces

The **Frustum** between cliques explicitly encodes attribute intersections and conditional probabilities:

$$\text{Frustum}(C_i, C_{i+1}) = C_i \cap C_{i+1}, \quad P(\text{Frustum}(C_i, C_{i+1})) = P(C_{i+1}|C_i).$$

5 Semantic Chains as Bayesian Networks

The SPC forms explicitly a Directed Acyclic Graph (DAG):

$$C_0 \xrightarrow{T_0} C_1 \xrightarrow{T_1} C_2 \ldots \xrightarrow{T_{N-1}} C_N.$$

This explicitly matches Bayesian inference:

$$P(C_N|C_0) = \prod_{i=0}^{N-1} P(C_{i+1}|C_i).$$

6 Empirical Confirmation as Bayesian Updating

Empirical traversals of the graph (hypothesis confirmations) explicitly represent Bayesian updates from hypotheses to theorems:

$$P(L|H, \text{empirical confirmations}) \to 1.$$

Thus, repeated empirical observations explicitly transform uncertain hypotheses into confirmed a priori theorems.

7 Illustrative Example

Consider a scenario with initial clique:

$$C_{\text{initial}} = \{(\text{Condition A}, 0.7), (\text{Condition B}, 0.3)\}.$$

Empirical testing (transformation) updates the state:

$$C_{\text{initial}} \xrightarrow{T_{\text{empirical}}} C_{\text{confirmed}} \cdots \to C_{\text{theorem}}.$$

Each step explicitly updates probabilities consistent with Bayesian logic.

8 Future Directions

Future work explicitly includes:

1. Exploration of inverse transformations for quantum mechanical analogies.

2. Rigorous mathematical group theory expansion.

3. Numeric simulations for deeper insights.

9 Conclusion

The SPC framework developed here demonstrates explicit equivalence to Bayesian inference. Attributes as weighted cliques, transformations as semigroup operations, and explicit probabilistic structures form a rigorous foundation for further theoretical and practical exploration.