UNIVERSITÀ DEGLI STUDI DI PADOVA
Dipartimento di Fisica e Astronomia "Galileo Galilei"
Corso di Laurea in Fisica

Optimal Control of Superconducting Qubit Gates

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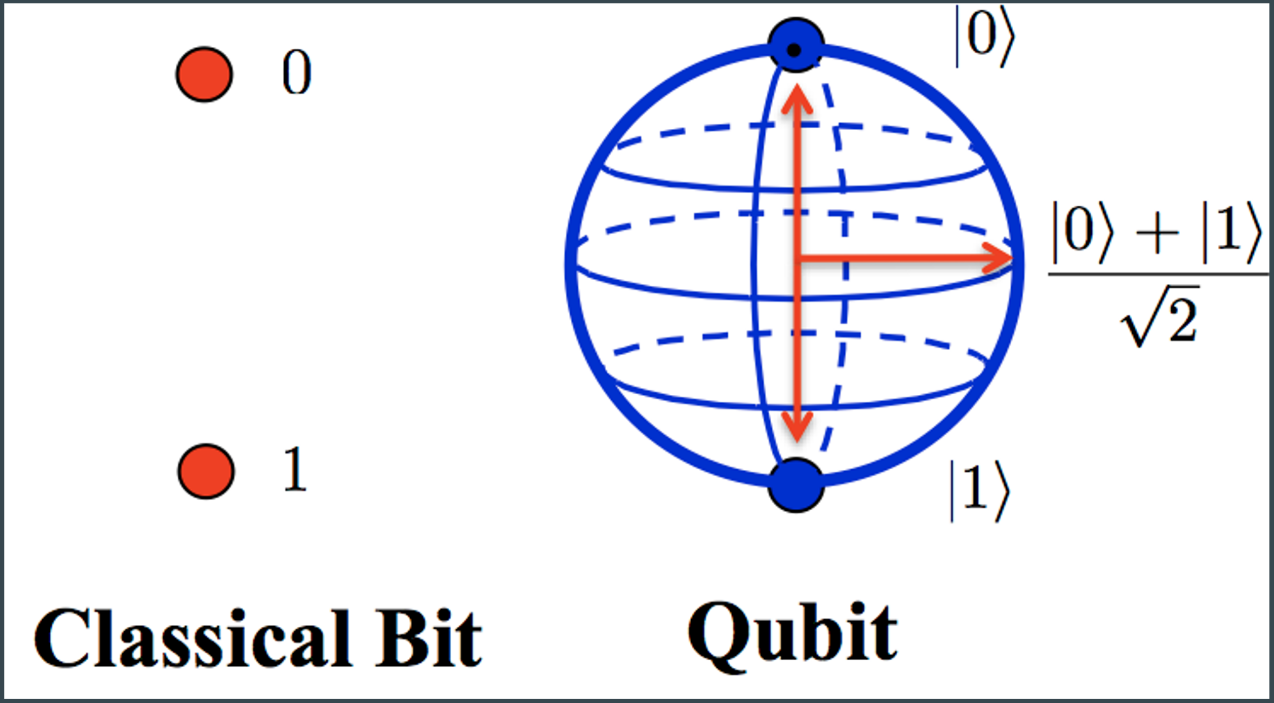
Relatore: Prof. Simone Montangero

Padova, 23/09/2021

Quantum Technologies



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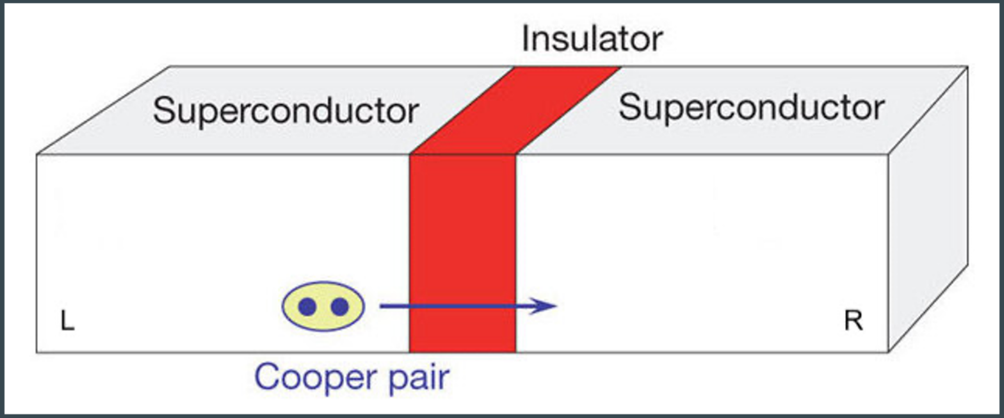
Gate $\mathcal{U}$: $\quad \mathcal{U}|\psi\rangle = |\chi\rangle \qquad |\psi\rangle, |\chi\rangle \in \mathcal{H}$

Superconducting Charge Qubits



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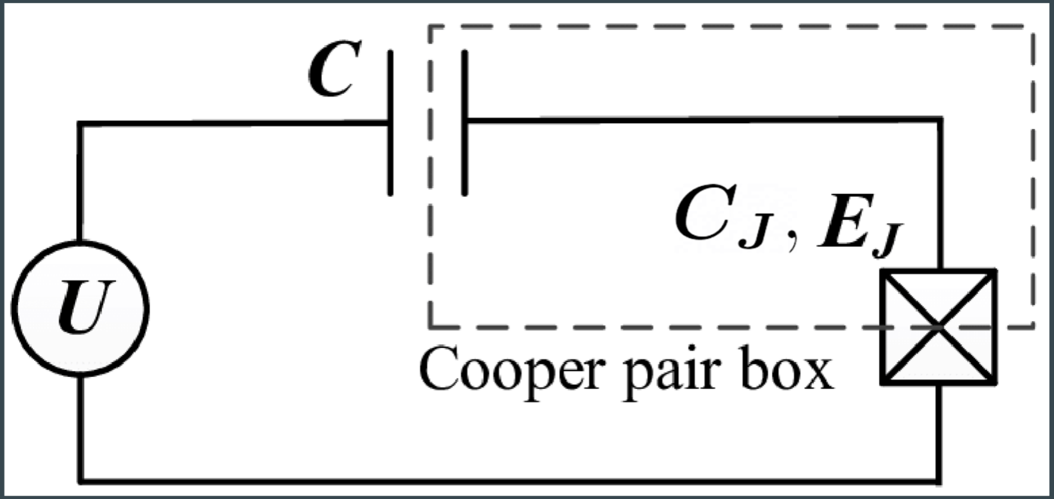
Josephson junction

Schematic of a charge qubit



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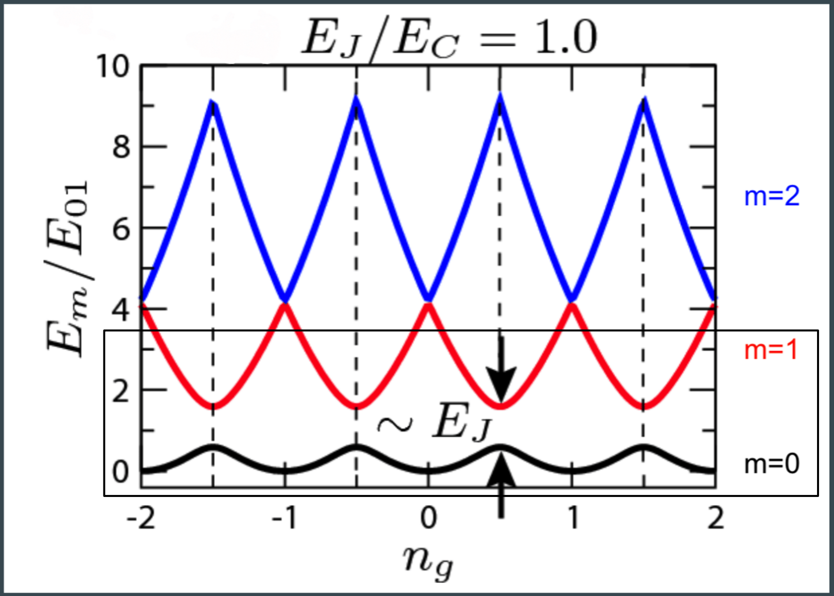
Tunable $n_g = \dfrac{CU}{2e}$

Superconducting Charge Qubits

Eigenenergies of the junction Hamiltonian

$$\boxed{E_J \ll E_C}$$

$E_J =$ Josephson energy
$E_C =$ electron charging energy



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Restricted Hamiltonian

$$\boxed{\hat{H}_{1Q} = -\frac{1}{2}B_z\hat{\sigma}_z - \frac{1}{2}B_x\hat{\sigma}_x}$$

Superconducting Charge Qubits

Corresponding two-qubit Hamiltonian

$$\hat{H}_{2Q} = \boxed{-\frac{1}{2}B_z^1\hat{\sigma}_z^1 - \frac{1}{2}B_x^1\hat{\sigma}_x^1} \boxed{-\frac{1}{2}B_z^2\hat{\sigma}_z^2 - \frac{1}{2}B_x^2\hat{\sigma}_x^2} \\ \boxed{+E_{CC}\,\hat{\sigma}_z^1\hat{\sigma}_z^2}$$

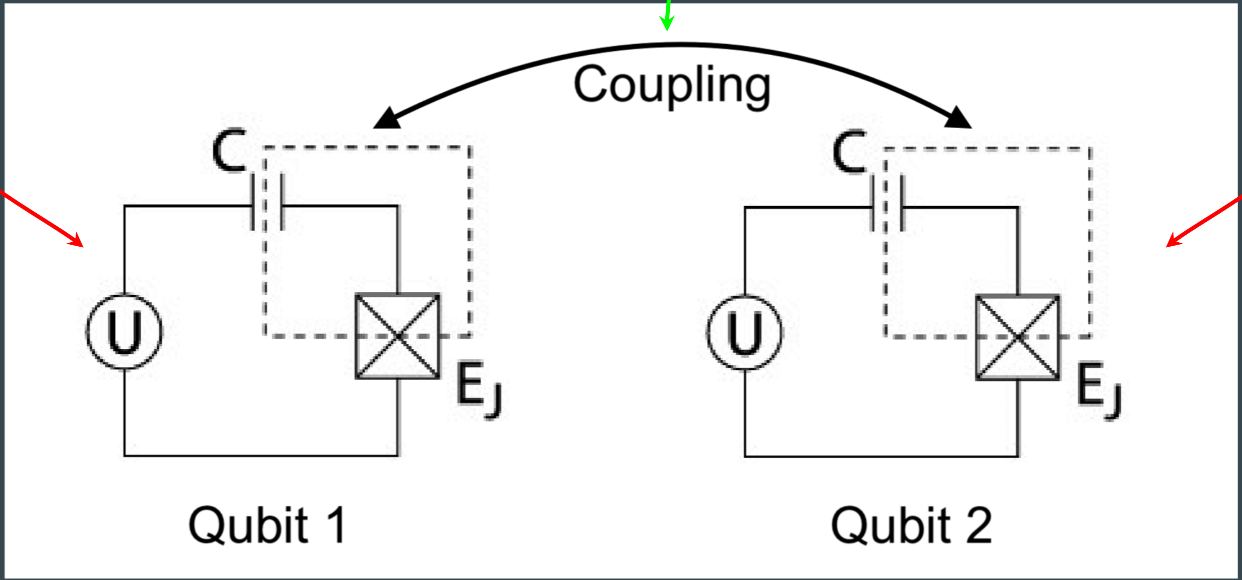


Image credits [Adapted]: Charge qubit, Wikipedia

Qubit Gate Implementation

$$i\hbar \frac{d}{dt}|\psi(t)\rangle = \hat{H}(t)|\psi(t)\rangle$$

Unitary state evolution, described by a time-dependent Hamiltonian

$\tau =$ fixed time of evolution

$$|\zeta_i\rangle \xrightarrow[\text{integration}]{\text{numerical}} |\zeta_i(\tau)\rangle$$

$|\zeta_i\rangle =$ basis state of the Hilbert space $\mathcal{H}$

$$\boxed{|\zeta_i(\tau)\rangle = \mathcal{U}(\tau)|\zeta_i\rangle} \quad i = 1, \ldots, \dim \mathcal{H}$$

$\mathcal{U}(\tau) =$ gate obtained numerically

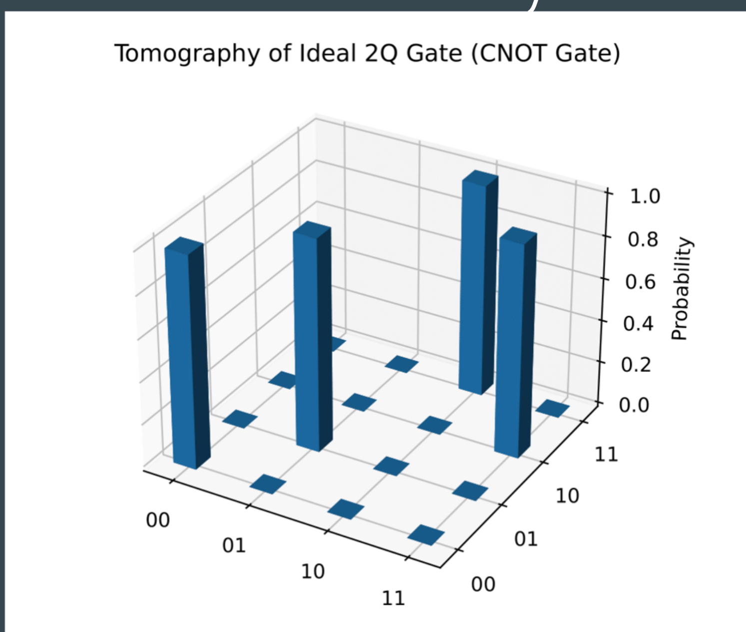
Qubit Gate Implementation: CNOT Gate

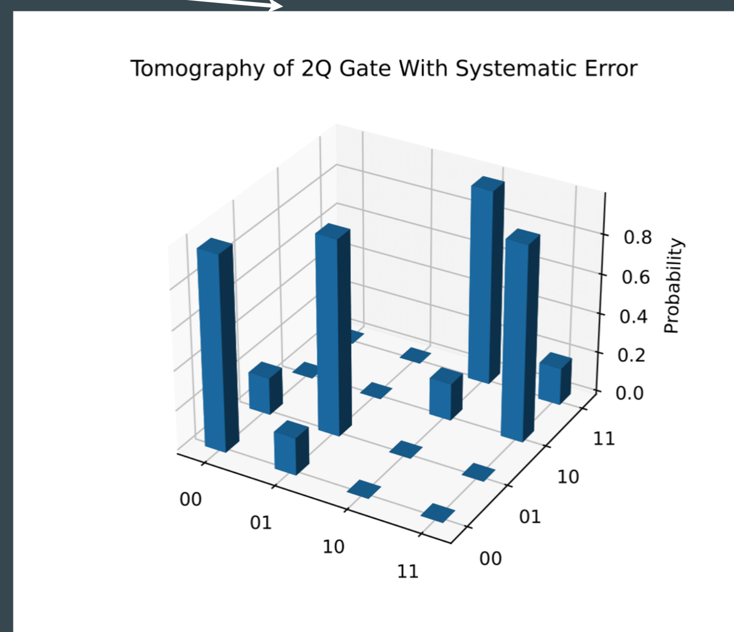
$$\boxed{\hat{H}_d = -\frac{1}{2}B_z^1\hat{\sigma}_z^1 - \frac{1}{2}B_z^2\hat{\sigma}_z^2 + E_{CC}\,\hat{\sigma}_z^1\hat{\sigma}_z^2}$$

$$\mathcal{U}_t = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

τ

$$+\beta\,\hat{\sigma}_z^1 + \beta\,\hat{\sigma}_z^2 \qquad (\beta = 0.02\,B_z^1)$$





$\hat{H}_d^*$

τ

Quantum Optimal Control

$$\boxed{\hat{H}(t) = \hat{H}_d^{*} + \hat{H}_c(t)}$$

$$\hat{H}_c(t) = \sum_l u_l(t)\, \hat{H}_{l,c}$$

$J =$ gate cost function (or infidelity) to minimize

$$J = J(\underbrace{u_1, \dots}_{})$$

$$J = 1 - \frac{1}{N_0^2}\left| Tr\left(\mathcal{U}_t^{\dagger}\, \underline{\mathcal{U}(\tau)}\right)\right|^2$$

$$|\zeta_i(\tau)\rangle = \mathcal{U}(\tau)|\zeta_i\rangle$$

$$N_0 = \dim \mathcal{H}$$

Quantum Optimal Control

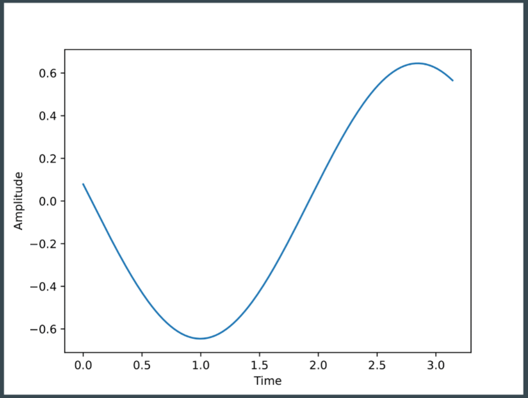
Chopped RAndom Basis (CRAB) algorithm applied to the basis of trigonometric functions

$$\text{random } \omega_i \in [0, \omega_{\max}] \qquad i = 1, \ldots, N_{be}$$

$$u(t) = \sum_{i=1}^{N_{be}} [A_i \sin(\omega_i t) + B_i \cos(\omega_i t)] \qquad A_i, B_i \in \mathbb{R}$$

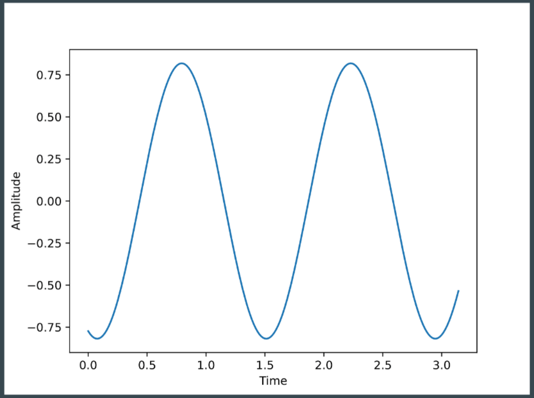
$\hookrightarrow$ <u>optimization</u>

$u(t) =$



$i = 1$

$+$

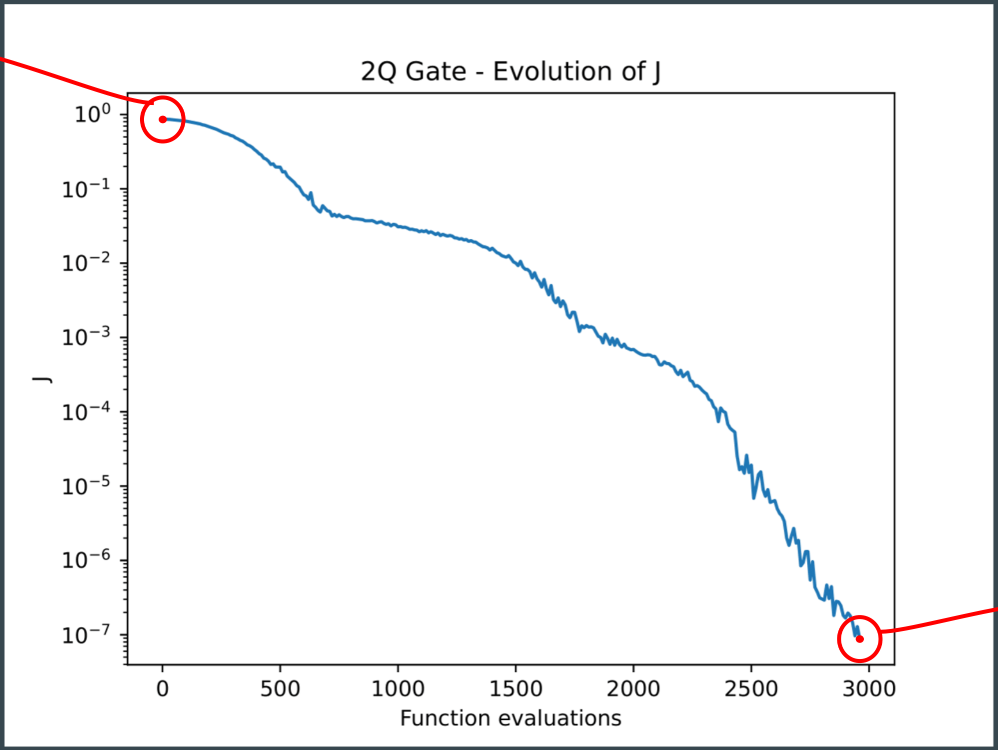


$i = 2$

$+ \ \ldots$

Optimization of Real 2Q Gate (CNOT Gate)

Random pulses



Optimal pulses

$$\hat{H}_c(t) = \boxed{u_x(t)}\left(\hat{\sigma}_x^1 + \hat{\sigma}_x^2\right) + \boxed{u_{zz}(t)}\,\hat{\sigma}_z^1\hat{\sigma}_z^2$$

$$J_0 = 0.87 \xrightarrow[\text{optimization}]{} J_{\text{opt}} = 8.7 \cdot 10^{-8}$$

Conclusions

- Josephson junctions can be used as qubits in quantum computation.
- The charge qubit setup allows to implement the CNOT gate.
- Quantum optimal control can reduce the infidelity of an implemented gate by adding a time-dependent control term to the system Hamiltonian.

Thank you for your attention

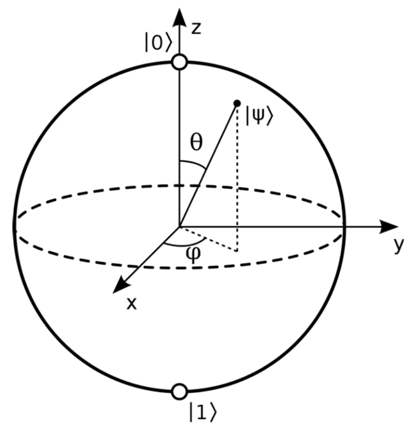
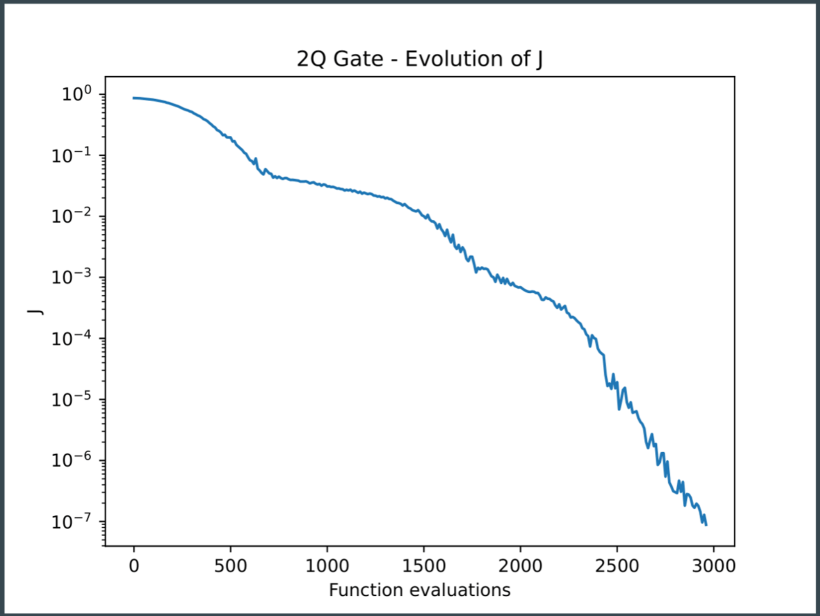
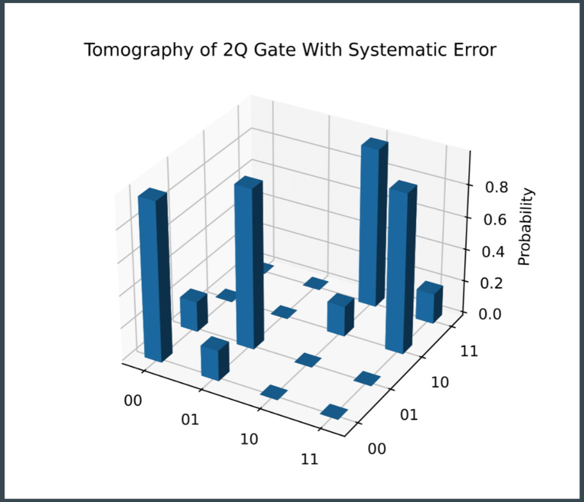


Image credits: Qubit, Wikipedia



$$J = 1 - \frac{1}{N_0^2}\left|Tr\left(\mathcal{U}_t^{\dagger}\,\mathcal{U}(\tau)\right)\right|^2$$



With support from P. Rembold and M. Rossignolo

Backup slides

Superconducting Charge Qubits

$$\boxed{\hat{H}_{1Q} = 4E_C(\hat{n} - n_g)^2 - E_J \cos\hat{\Theta}}$$

$\hat{n}$ = number operator of excess Cooper-pair charges

$\hat{\Theta}$ = phase of the superconducting order parameter $\longrightarrow \left[\hat{\Theta}, \hat{n}\right] = i$

$$E_C = \frac{e^2}{2(C + C_J)}$$

$$n_g = \frac{CU}{2e}$$

$$E_J = \frac{\hbar}{2e} I_c$$

I_c = critical current of the junction

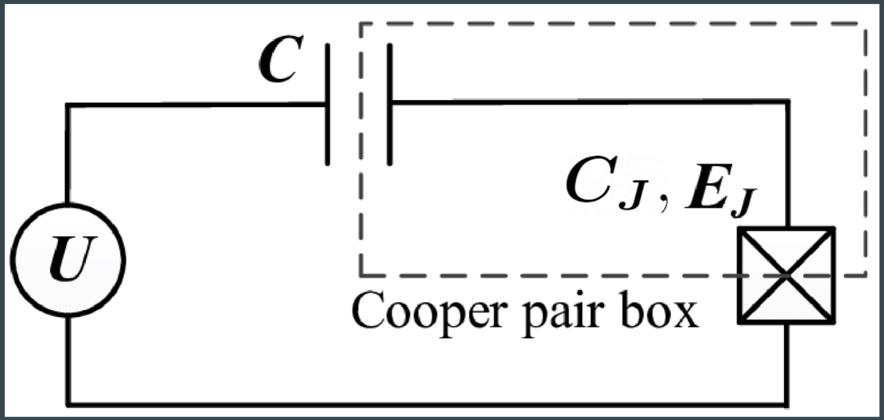


Image credits [Adapted]: PRA 87, 022324 (2013)

Superconducting Charge Qubits

$$\boxed{\hat{H}_{1Q} = 4E_C(\hat{n} - n_g)^2 - E_J \cos\hat{\Theta}}$$

$$\left.\begin{array}{l} \left.\begin{array}{l} C_J \leq 10^{-15}\,\mathrm{F} \\ C \ll C_J \end{array}\right\} \longrightarrow E_C/k_B \geq 1\,\mathrm{K} \quad \text{1)} \\ \\ \text{2)}\ E_J/k_B \sim 100\,\mathrm{mK} \end{array}\right\} E_C/E_J \sim 10$$

3) Superconductors with energy gap $\Delta \gg E_C$

$$\boxed{\hat{H}_{1Q} = -\frac{1}{2}B_z\hat{\sigma}_z - \frac{1}{2}B_x\hat{\sigma}_x}$$

For $n_g \in [0, 1]$
$B_z = 4E_C(1 - 2n_g)$
$B_x = E_J$

Superconducting Charge Qubits

Eigenenergies of the junction Hamiltonian

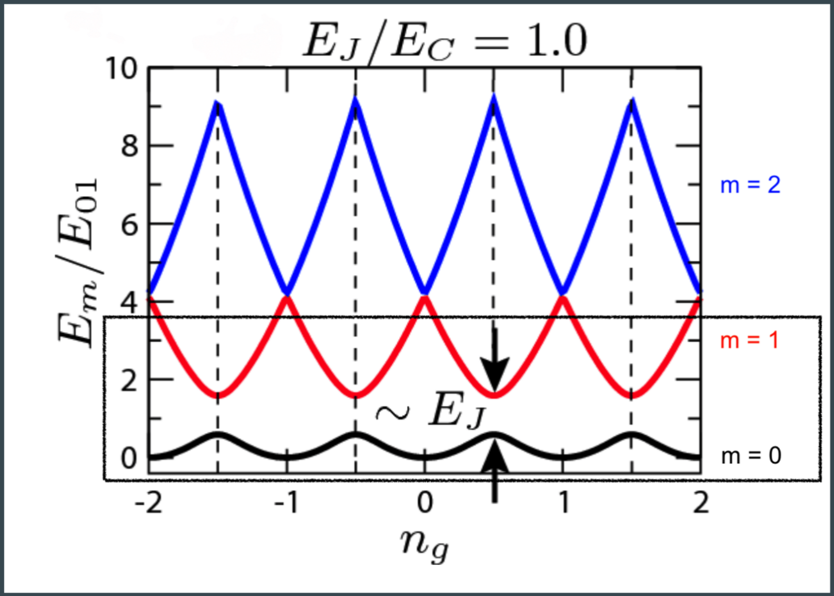


Image credits [Adapted]: PRA 76, 042319 (2007)

$$\boxed{E_J \ll E_C}$$

$E_J =$ Josephson energy
$E_C =$ electron charging energy

$$n_g = \frac{CU}{2e} \longrightarrow \underline{\text{tunable}}$$

Junction Hamiltonian restricted to the states $|0\rangle$ and $|1\rangle$

$B_z = B_z(E_C, n_g)$
$B_x = E_J$

$$\boxed{\hat{H}_{1Q} = -\frac{1}{2}B_z\hat{\sigma}_z - \frac{1}{2}B_x\hat{\sigma}_x}$$

Qubit Gate Implementation: x- and z- Rotations

$$\boxed{\hat{H}_x = -\frac{1}{2} B_x \hat{\sigma}_x}$$

$$\boxed{\hat{H}_z = -\frac{1}{2} B_z \hat{\sigma}_z}$$

$$\tau = \frac{\hbar\alpha}{B_x}$$

$$\mathcal{U}_x(\alpha) = \begin{pmatrix} \cos\frac{\alpha}{2} & i\sin\frac{\alpha}{2} \\ i\sin\frac{\alpha}{2} & \cos\frac{\alpha}{2} \end{pmatrix}$$

$$\tau = \frac{\hbar\beta}{B_z}$$

$$\mathcal{U}_z(\beta) = \begin{pmatrix} e^{i\beta/2} & 0 \\ 0 & e^{-i\beta/2} \end{pmatrix}$$

Qubit Gate Implementation: NOT and Hadamard Gates

$$i\,\mathrm{NOT} = \mathcal{U}_x(\pi)$$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$H \propto \mathcal{U}_x\left(\frac{\pi}{4}\right)\mathcal{U}_z\left(\frac{\pi}{4}\right)\mathcal{U}_x\left(\frac{\pi}{4}\right)$$

Qubit Gate Implementation: CNOT Gate

(Perfect) Hadamard gate applied to the qubit 2

$$\mathrm{CNOT} \propto H^2 \Big[\underbrace{\mathcal{U}_z^1\left(-\frac{\pi}{2}\right)\mathcal{U}_z^2\left(-\frac{\pi}{2}\right)} \underbrace{\exp\left(i\frac{\pi}{4}\hat{\sigma}_z^1\hat{\sigma}_z^2\right)} \Big] H^2$$

$$\hat{H} = -\frac{1}{2}B_z^1\hat{\sigma}_z^1 - \frac{1}{2}B_z^2\hat{\sigma}_z^2$$

with $B_z^1 = B_z^2 < 0$

for $\tau_2 = \dfrac{\hbar\pi}{2(-B_z^1)}$

$$\hat{H} = E_{CC}\,\hat{\sigma}_z^1\hat{\sigma}_z^2$$

with $E_{CC} < 0$

for $\tau_1 = \dfrac{\hbar\pi}{4(-E_{CC})}$

Quantum Optimal Control

$$u(t) = \sum_{i=1}^{N_{be}} [A_i \sin(\omega_i t) + B_i \cos(\omega_i t)]$$

$$\omega_i = \frac{2\pi}{\tau}(i - r_i) \qquad \begin{cases} i = 1, \ldots, N_{be} \\ \text{random } r_i \in [0,1] \\ \tau = \text{ fixed time of evolution} \end{cases}$$

$$\omega_{\max} = \frac{2\pi N_{be}}{\tau}$$

$$A_i, B_i \in \mathbb{R}$$

Optimization using the Nelder-Mead method

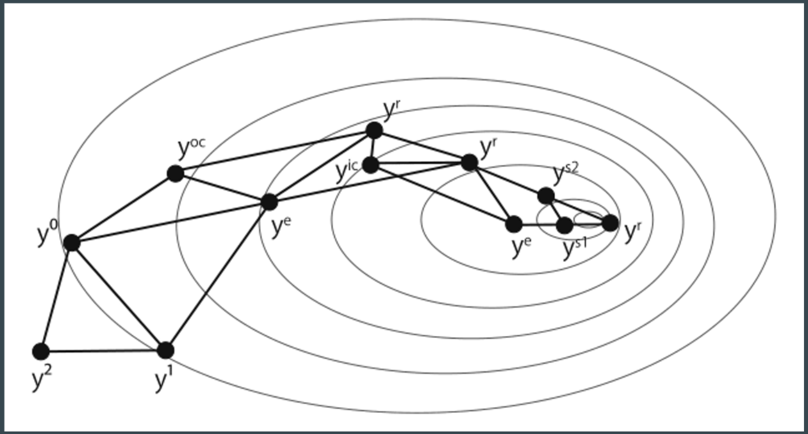


Image credits: IPSJ T Comput Vis Appl 9, 20 (2017)

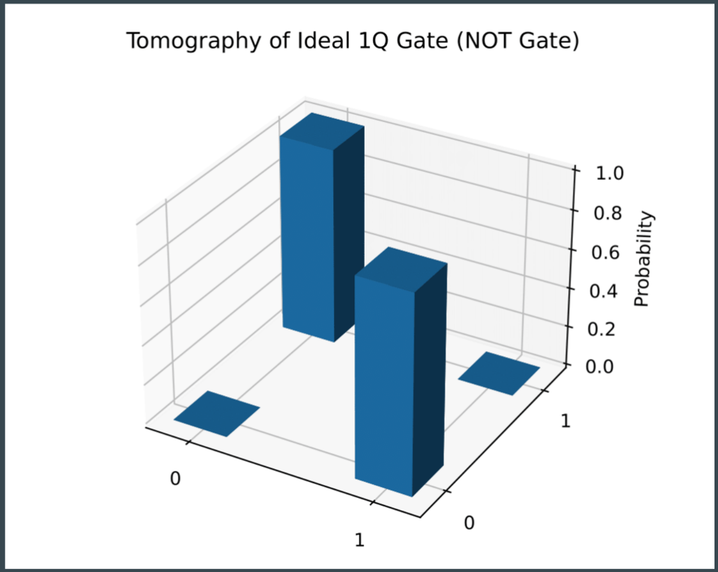
Qubit Gate Implementation: NOT Gate

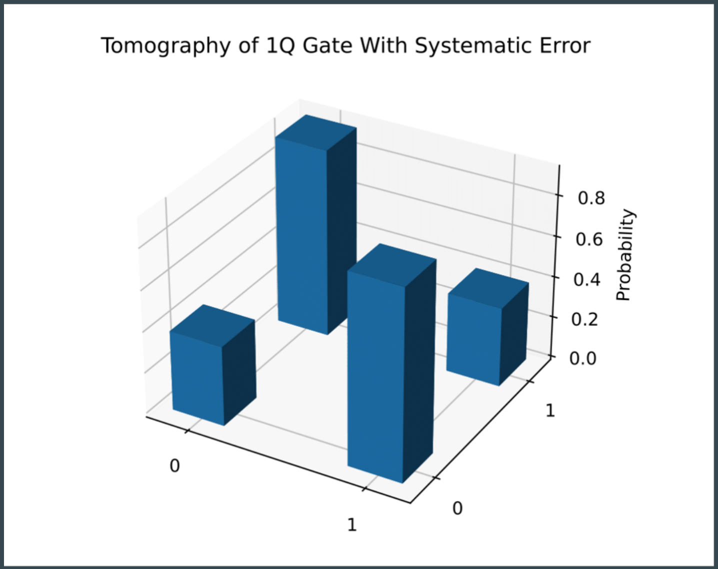
$$\boxed{\hat{H}_d = -\frac{1}{2} B_x \hat{\sigma}_x} \qquad \mathcal{U}_t = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\tau = \frac{\hbar \pi}{B_x}$$

$$+\beta\, \hat{\sigma}_z$$

$$(\beta = 0.2\, B_x)$$



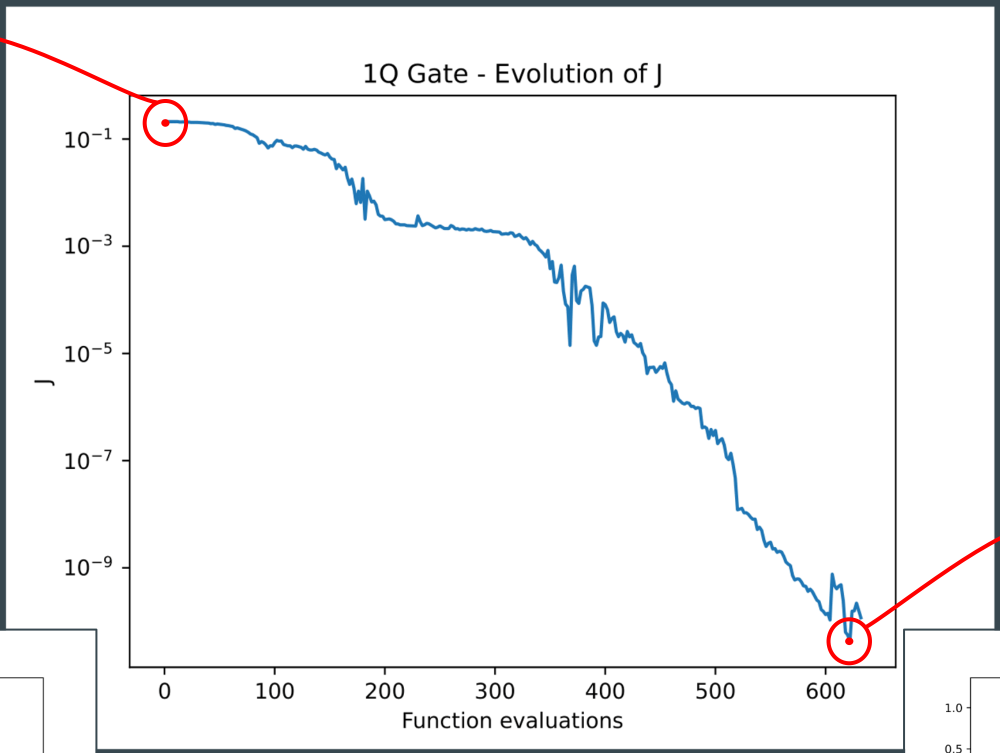


$$\hat{H}_d^{*}$$

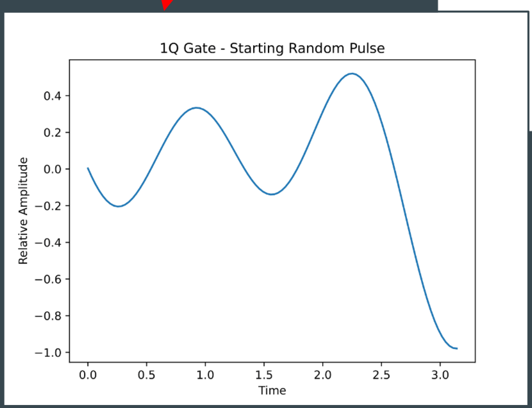
$$\tau$$

Optimization of Real 1Q Gate (NOT Gate)

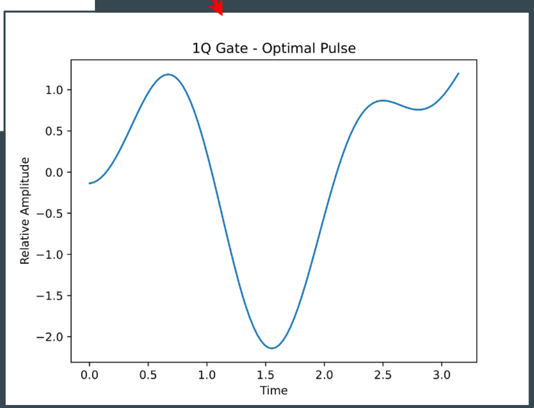
Random pulse



Optimal pulse



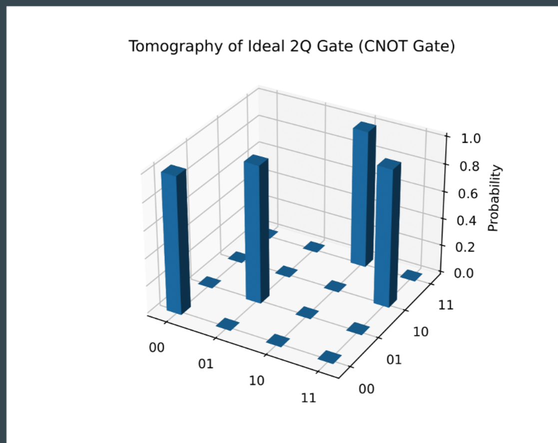
$$\hat{H}_c(t) = \boxed{u_x(t)}\,\hat{\sigma}_x$$

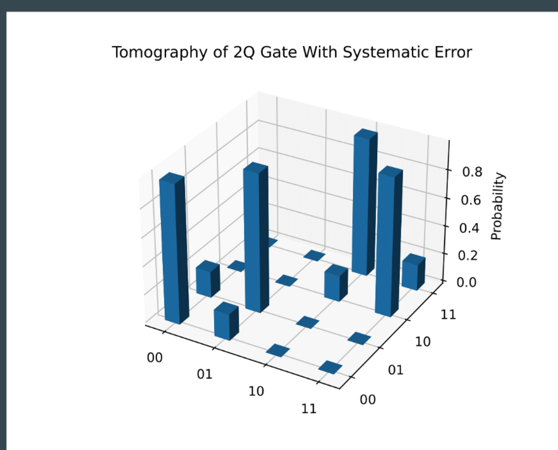


$$J_0 = 0.21 \xrightarrow{\text{optimization}} J_{\text{opt}} = 2.8 \cdot 10^{-11}$$

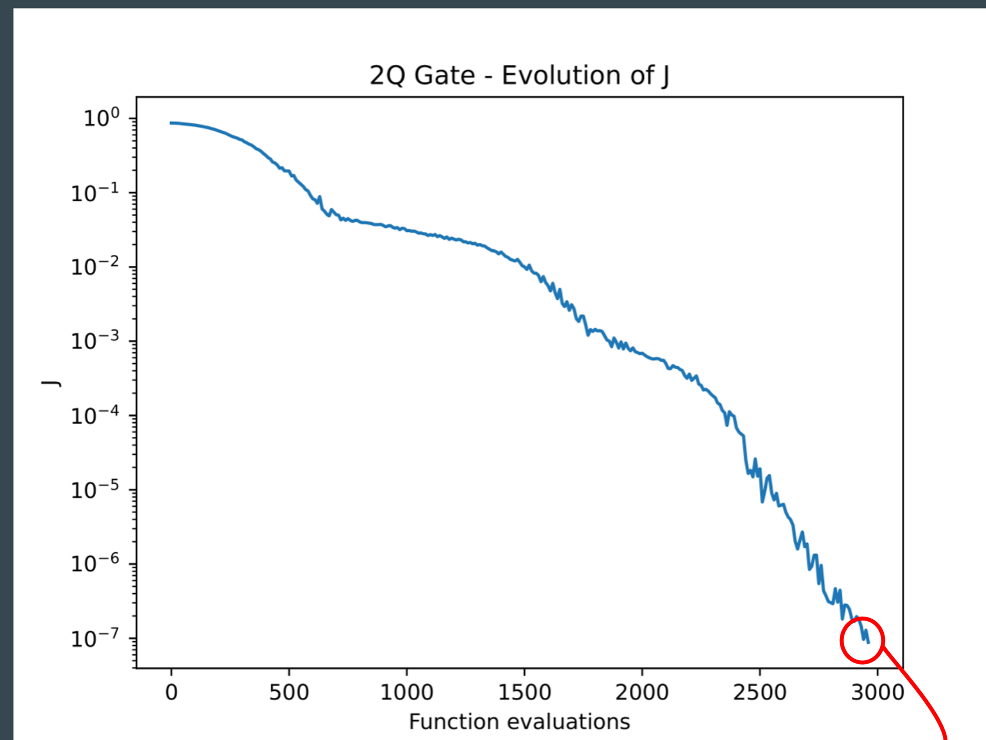
Implementation of Real 2Q Gate (CNOT Gate)

$$J = 1 - \frac{1}{N_0^2} \left| Tr\left(\mathcal{U}_t^\dagger \mathcal{U}(\tau)\right) \right|^2$$





$$+\beta\,\hat{\sigma}_z^1$$
$$+\beta\,\hat{\sigma}_z^2$$



optimization

$$J_{\text{opt}} = 8.7 \cdot 10^{-8}$$