Introduction

Richard Feynman in 1982 proposed the quantum computer, a new kind of computer that can simulate quantum systems [1]. In his work, Feynman demonstrates that the exact simulation of quantum systems with classical computers is not always possible. Classical computers can't simulate entangled states using only local correlations, as is stated in a more general form in J. S. Bell's theorem [2]. Since the idea of Feynman, quantum technologies progressed considerably and a variety of dedicated software and hardware have been developed. Researchers found quantum algorithms that can outperform their classical counterparts by exploiting quantum mechanical properties such as superposition and entanglement. As an example, the Quantum Fourier Transform (QFT) algorithm [3] computes the discrete Fourier transform in polynomial time with respect to the input size, while the classical Fast Fourier Transform (FFT) algorithm needs exponential time.
The physical realization of a platform that can run such algorithms is the main challenge that quantum technologies have to overcome. Experimental imperfections and interactions with the environment affect the state of the system and the results might be wrong. This is why the quality of the quantum hardware depends on both the number of qubits, the two-state quantum systems, and the error rate of the setup.
In this thesis we concentrate on superconducting qubits as a possible implementation of quantum computers. The superconducting qubits consist on devices called Josephson junctions, in which Cooper pairs of electrons tunnel across a thin insulating barrier that separates two superconductors. Under suitable conditions, the Josephson junction reduces to a two-state system that can be used for quantum computation [4]. Operations on a single qubit are performed using voltages and magnetic fields and the coupling between two qubits provides the necessary two-qubit operations. There exist different versions of the superconducting qubit: phase, charge and flux qubits. Every version exploit different aspects of the Josephson junction and the experimental implementation is therefore different. The low temperature of the setup, in the range of tens of mK, and the high coherence of the Cooper pair tunneling limit the dephasing effects.
Companies such as Google [5], IBM [6], Rigetti [7] and many others are developing commercial processors with more than 50 qubits using superconducting circuits. These processors allow to implement universal quantum computation and Google claimed in 2019 to have reached quantum supremacy using a 53-qubit processor [8]. D-Wave, on the other hand, announced in 2019 a 5000-qubit processor [9] specialized in quantum annealing but unable to support generic quantum computation. The investments and the results of these companies attract the interest of the scientific community and the superconducting technologies are constantly improved.
The error rate of a processor is reduced by minimizing external dephasing sources and by extending the phase coherence times. The former can never be completely eliminated because we need to interact with the system to couple the qubits and to read out the results. A solution to further reduce dephasing effects in the processor comes from the field of mathematical optimization. Quantum Optimal Control (QOC) [10, 11] uses the tools of the mature Optimal Control Theory to find the time-dependent control pulses that maximize a specific cost function J.