UNIVERSITÀ DEGLI STUDI DI PADOVA

Dipartimento di Fisica e Astronomia "Galileo Galilei"

Corso di Laurea in Fisica

Tesi di Laurea

Optimal Control of Superconducting Qubit Gates

Relatore

Prof. Simone Montangero

Laureando

Paolo Zinesi

Anno Accademico 2020/2021

Abstract

Quantum technologies promise to improve the performance of a whole range of computational processes. This is why researchers have been studying the physical implementation of such technologies for over 30 years.
One of the most well-established platforms to perform quantum calculations are superconducting qubits, in which Cooper pairs tunnel across insulating barriers that separate superconductors.
After introducing superconductivity theory, we simulate one- and two-qubit gates by controlling applied voltages and magnetic fields. Then, we show how an external noise source can alter the behavior of the quantum gate.
Finally, we apply optimal control theory to find the time-dependent voltage and magnetic fields that maximize the target gate's success probability. With this procedure we aim to compensate for the experimental imperfections via a general numerical approach.

Contents

Compiled on August 11, 2021

Introduction

Richard Feynman in 1982 proposed the quantum computer, a new kind of computer that can simulate quantum systems [1]. In his work, Feynman demonstrates that the exact simulation of quantum systems with universal classical computers is not always possible. In fact, a theory which reproduces quantum mechanical predictions needs to have a non-local structure [2]. Classical computers follow the rules of classical physics, which is a local theory, while quantum physics admit the existence of non-local correlations between entangled particles. So, to simulate some quantum states computers that follows quantum mechanical rule are needed.
Since the idea of Feynman, quantum technologies progressed considerably and a variety of dedicated software and hardware have been developed. Researchers found out that quantum mechanical properties, such as superposition and entanglement, can speed up some expensive tasks in certain algorithms. As an example, the Quantum Fourier Transform (QFT) algorithm [3] computes the discrete Fourier transform in polynomial time with respect to the input size, while the classical Fast Fourier Transform (FFT) algorithm needs exponential time.
The physical realization of a platform that can run such algorithms is the main challenge that quantum technologies have to overcome. Experimental imperfections and interactions with the environment affect the state of the system giving unexpected results. This is why the quality of the quantum hardware depends on both the number of qubits, the two-state quantum systems, and the error rate of the setup.
In this thesis we concentrate on superconducting qubits as a possible implementation of quantum computers. The superconducting qubits are based on devices called Josephson junctions, in which Cooper pairs of electrons tunnel across a thin insulating barrier that separates two superconductors. Under suitable conditions, the Josephson junction reduces to a two-state system that can be used for quantum computation [4]. Operations on a single qubit are performed using voltages and magnetic fields and the coupling between two qubits provides the necessary two-qubit operations.
Companies such as Google [5], IBM [6], Rigetti [7] and many others are developing commercial processors with more than 50 qubits using superconducting circuits. These processors allow to implement universal quantum computation and Google claimed in 2019 to have reached quantum supremacy using a 53-qubit processor [8]. D-Wave, on the other hand, announced in 2019 a 5000-qubit processor [9] specialized in quantum annealing but unable to support generic quantum computation. The investments and the results of these companies attract the interest of the scientific community and the superconducting technologies are constantly improved.
A solution to further reduce dephasing effects in the processor comes from the field of mathematical optimization. Quantum Optimal Control (QOC) [10–12] uses the tools of the mature Optimal Control Theory to find the time-dependent control pulses that minimize a specific cost function J. That function J depends on the goal of the optimization. With the optimal control pulses the target gates are obtained with greater probability.

In the first chapter we introduce the theory of superconductivity, starting from the early experimental observations and presenting the empirical London equations to explain them. Then, we present the basic concepts of the BCS theory and we describe the characteristics of the BCS ground state. From these results we present a derivation of the London equations and

a simple theoretical explanation of the Josephson effect.
In the second chapter we describe Josephson junctions and we show how to use them as qubits. Implementation of qubit gates in the noiseless approximation is presented, then a noise source is added to simulate the deviations of the gates from the expected behavior.
Finally, in the third chapter we use Quantum Optimal Control to implement gates that are robust against dephasing effects.

Chapter 1

Introduction to Superconductivity Theory

1.1 First experimental observations

Vanishing resistance below critical temperature, Meissner-Ochenselfed effect, ...

1.2 London equations

First and second London equations

1.3 BCS theory

Small attraction between two electrons[13], Cooper pairs, BCS ground state, energy gap and critical temperature, supercurrents, demonstration of London equations from BCS ground state+Ginzburg-Landau theory

1.4 Josephson effect

Cooper-pair tunneling, simple derivation without second quantization formalism

Chapter 2

Superconducting qubits

2.1 Josephson junctions

Advantages of superconducting qubits
Explanation of how it works, quantization parameters (n=excess of Cooper pairs), characteristic energy and capacitance of the setup

2.2 Charge qubits

Energy ranges, Hamiltonian in the box, Hamiltonian in the $|n>$ base, Hamiltonian at low temperatures H_Q, eigenstates of H_Q, time evolution described by H_Q for a fixed time T

2.3 Implementation of qubit gates

Without external noise sources the Hamiltonian of the system is exactly H_Q so its time evolution is described exactly by...

2.3.1 One-qubit gates

Rotation around x-axis, z-axis, NOT gate, Hadamard gate

2.3.2 Two-qubit gates

CNOT-gate

2.4 Noisy qubit gates

2.4.1 One-qubit gates

Perturbed Hamiltonian, example showing NOT gate

2.4.2 Two-qubit gates

Perturbed Hamiltonian, example showing CNOT-gate

Chapter 3

Optimal Control

...The error rate of a processor is reduced by minimizing external dephasing sources and by extending the phase coherence times. The former can never be completely eliminated because we need to interact with the system to couple the qubits and to read out the results....
....The cost function J depends on the physical quantities of the system and encodes the objective of the optimization. The maximum (or minimum) of J if found with gradient-based algorithms, in which the gradient of J is computed either analytically or numerically, and gradient-free algorithms...

Minimize $J_{gate} = 1 - \frac{1}{N_0^2} \left| Tr(U_t^\dagger U(T) \right|$, with $N_0 = dim\mathcal{H}$

Procedure: For the chosen control amplitude of every iteration propagate numerically $i\hbar\frac{\partial\eta}{\partial t} = H\eta$ from $t = 0$ to $t = T$. With $\eta(T)$ compute the value of J_{gate} and then with the minimization algorithm choose another amplitude and continue the loop. At the end a convergence test (even a weak one) is needed.

3.1 Optimization of one-qubit NOT gate

$$H(t) = H_c(t) + H_d = \frac{1}{2}\left[\epsilon(t)\sigma_z + \Delta(t)\sigma_x\right] + \alpha\sigma_z$$

3.2 Optimization of two-qubit CNOT gate

$$H(t) = H_c(t) + H_d = \frac{1}{2}\sum_{i=1,2}\left[\epsilon^{(i)}(t)\sigma_z^{(i)} + \Delta^{(i)}(t)\sigma_x^{(i)}\right] + E_{cc}(t)\sigma_z^{(1)}\sigma_z^{(2)} + \sum_{i=1,2}\alpha^{(i)}\sigma_z^{(i)}$$

Bibliography

[1] Richard P. Feynman. "Simulating physics with computers". In: *International Journal of Theoretical Physics* 21.6 (1982), pp. 467–488. ISSN: 1572-9575. DOI: 10.1007/BF02650179. URL: https://doi.org/10.1007/BF02650179.

[2] J. S. Bell. "On the Einstein Podolsky Rosen paradox". In: *Physics Physique Fizika* 1 (3 1964), pp. 195–200. DOI: 10.1103/PhysicsPhysiqueFizika.1.195. URL: https://link.aps.org/doi/10.1103/PhysicsPhysiqueFizika.1.195.

[3] Don Coppersmith. "An approximate Fourier transform useful in quantum factoring". In: *arXiv preprint quant-ph/0201067* (2002). URL: http://arxiv.org/abs/quant-ph/0201067.

[4] Yuriy Makhlin, Gerd Schön, and Alexander Shnirman. "Quantum-state engineering with Josephson-junction devices". In: *Rev. Mod. Phys.* 73 (2 2001), pp. 357–400. DOI: 10.1103/RevModPhys.73.357. URL: https://link.aps.org/doi/10.1103/RevModPhys.73.357.

[5] Davide Castelvecchi. "Quantum computers ready to leap out of the lab in 2017". In: *Nature* 541.7635 (2017), pp. 9–10. ISSN: 1476-4687. DOI: 10.1038/541009a. URL: https://doi.org/10.1038/541009a.

[6] Davide Castelvecchi. "IBM's quantum cloud computer goes commercial". In: *Nature* 543.7644 (2017), pp. 159–159. ISSN: 1476-4687. DOI: 10.1038/nature.2017.21585. URL: https://doi.org/10.1038/nature.2017.21585.

[7] Rigetti Computing. *Rigetti Computing introduces world's first scalable multi-chip quantum processor.* 2021. URL: https://www.globenewswire.com/news-release/2021/06/29/2255028/0/en/Rigetti-Computing-introduces-world-s-first-scalable-multi-chip-quantum-processor.html.

[8] Frank Arute et al. "Quantum Supremacy using a Programmable Superconducting Processor". In: *Nature* 574 (2019), 505–510. URL: https://www.nature.com/articles/s41586-019-1666-5.

[9] James Sanders. *D-Wave announces 5,000-qubit fifth generation quantum annealer.* 2019. URL: https://www.techrepublic.com/article/d-wave-announces-5000-qubit-fifth-generation-quantum-annealer/.

[10] J Werschnik and E K U Gross. "Quantum optimal control theory". In: *Journal of Physics B: Atomic, Molecular and Optical Physics* 40.18 (2007), R175–R211. DOI: 10.1088/0953-4075/40/18/r01. URL: https://doi.org/10.1088/0953-4075/40/18/r01.

[11] Steffen J. Glaser et al. "Training Schrödinger's cat: quantum optimal control". In: *The European Physical Journal D* 69.12 (2015), p. 279. ISSN: 1434-6079. DOI: 10.1140/epjd/e2015-60464-1. URL: https://doi.org/10.1140/epjd/e2015-60464-1.

[12] U. Boscain, M. Sigalotti, and D. Sugny. *Introduction to the Foundations of Quantum Optimal Control.* 2020. arXiv: 2010.09368 [quant-ph].

[13] H. Ibach and H. Lüth. *Solid-State Physics: An Introduction to Principles of Materials Science.* Advanced texts in physics. Springer Berlin Heidelberg, 2009. ISBN: 9783540938040. URL: https://books.google.it/books?id=qjxv68JFe3gC.