April 10, 2025

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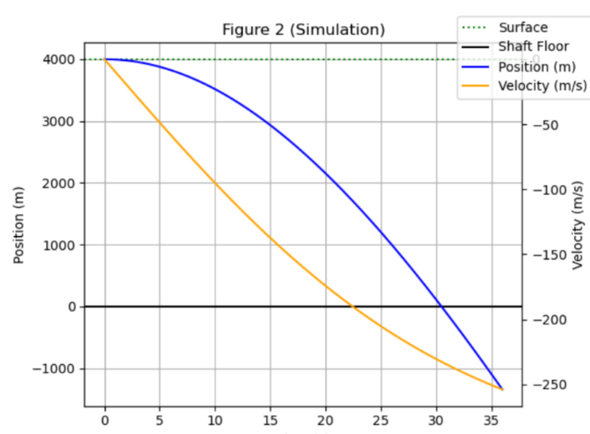
I: INTRODUCTION

The Equatorial Sur mining company claims many achievements to its name as we progress into the 21st century. Its global yields of rare-Earth minerals, successful operations, and clean practices on three continents notwithstanding, the organization’s namesake- Caverne Brillante de l’Écliptique- is truly its crown jewel. Boasting a main central depth of 4km and an ever-expanding working area, the company requires that precise measurements be taken of the main shaft to ensure growth proceeds safely in all stages henceforth. Using Python and computer modeling, depth measurements will be estimated dynamically via freefall calculations using a test mass of 1kg dropped down the center of the main shaft for simplicity in calculations. Thus, the numerical value for mass can be effectively disregarded through the process.

II: FALL DURATION

Freefall time of the test mass is initially evaluated using a kinematic equation freefall: taking the acceleration constant due to Earth’s gravity plus the drag coefficient alpha times velocity squared. This equation is set up as an initial value differential equation, and solved for position and velocity values using the **solve_IVP** program.

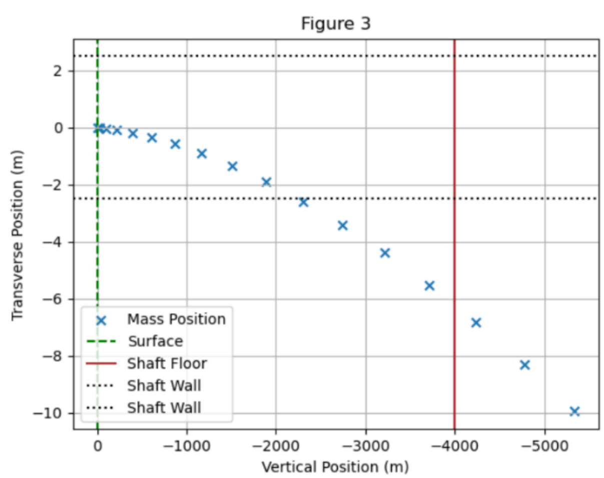
Omitting all external factors in a basic case, the test mass will take 28.6 seconds. to reach the bottom of the main shaft at 4km deep. Adjusting the simulation to real-world conditions, the drag coefficient is adjusted to reflect actual values, and freefall time to the bottom of the shaft is found to be 30.5 seconds.



III: KINEMATIC DEPTH MEASUREMENT COMPLICATIONS

While convenient, this kinematic method carries complications. The Coriolis force is an imaginary force, such that it affects objects without imparting them with a direct, classical force. Instead, in the mid-to-upper latitudes, effects of the Coriolis force can be most easily seen in the flightpaths of airliners: taking apparent curved flight paths to their destinations to compensate for Earth's rotation, and in the formation of cyclonic storms: where in the northern hemisphere, wind deflects to its right around low pressure airmasses, and to its right from high pressure centers. While laterally negligible at low latitudes, the Coriolis force due to the Earth's sidereal motion has a large vertical component and will affect objects travelling vertically. While l'Écliptique has the deepest central shaft in this hemisphere, it is also among the narrowest for its depth.

The accelerative component of the Coriolis force is used to set up a differential equation once more and solved with **SOLVE_IVP**, this time to plot transverse (horizontal) position and vertical position, accounting for location of the shaft walls. If the test mass is dropped from the center of the 5m shaft, the mass will strike the shaft wall in 21.9 seconds under idealized conditions. The mass dropped from the same location plus drag strikes the shaft wall in 22.3 seconds, complicating the kinematic method for precise measurements of l'Écliptique.

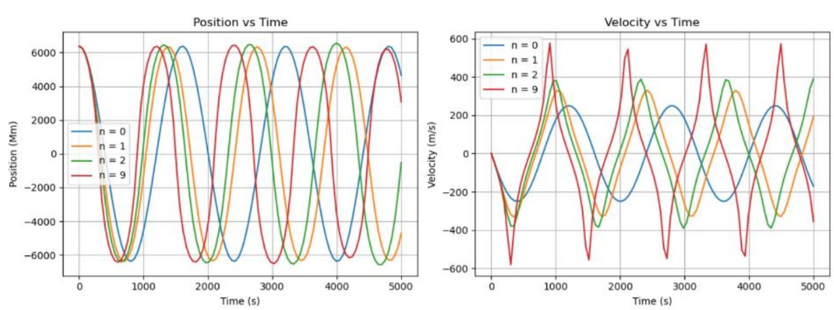


IV: CROSSING TIMES

Questions arise of the future for this historic mine as expansion lies on the horizon, not the least of which is how deep the company's drive for innovation runs. In an effort to be taken seriously, the same kinematic method is employed to evaluate a test mass dropped into a hypothetical trans-Earth tunnel under idealized conditions neglecting drag and Coriolis force. The initial method using **solve_IVP** is virtually the same for this step, apart from scale. For nonhomogeneous Earth simulations, the **integrate.quad** function is used to find normalized Earth density, mass per that density, and those values are used to create a differential equation for vertical kinematic motion.

Assuming a homogeneously dense Earth, core passage of a mass's journey through the Earth will occur at roughly 360.4 seconds. Nonhomogeneous Earth simulations show promise for shorter core passage time but require further examination. Trans-lunar tunnels show similar results using the same methods, with a core passage time of 841.3 seconds.

Figure 7: Gravitational Oscillation Through Nonuniform-Density Earth



V: FUTURES

The kinematic depth measurement technique is a purely physical but inherently flawed method in a practical sense but can be improved by modeling with additional parameters. Considering factors to supplement those used herein such as spin and shape of the test mass, angle or inclination of its release, gravitational anomalies, and effects of a non-spherical Earth will increase the accuracy and validity of kinetic modeling of this historic mine.