§1. LSZ reduction formula

*. why to study n-point correlation?

*. Experiment $\Longleftrightarrow$ Theoretical prediction?

§1.1 K-L spectrum representation

Scalar 1-particle propagator: $\langle\Omega|T\phi(x)\phi(y)|\Omega\rangle$

recall the 1-particle completeness relation

$$\mathbb{1} = \int\frac{d^3p}{(2\pi)^3}\frac{1}{2E_p}|p\rangle\langle p|$$

the completeness relation in the entire H-space is

$$\mathbb{1}_p = |\Omega\rangle\langle\Omega| + \sum_\lambda\int\frac{d^3p}{(2\pi)^3}\frac{1}{2\sqrt{p^2+m_\lambda^2}}|\lambda_p\rangle\langle\lambda_p|$$

For $x^0>y^0$ case:

$$\langle\Omega|\phi(x)\phi(y)|\Omega\rangle = \langle\Omega|\phi(x)\mathbb{1}_p\phi(y)|\Omega\rangle$$
$$= \sum_\lambda\int\frac{d^3p}{(2\pi)^3}\frac{1}{2\sqrt{p^2+m_\lambda^2}}\langle\Omega|\phi(x)|\lambda_p\rangle\langle\lambda_p|\phi(y)|\Omega\rangle$$

where

$$\langle\Omega|\phi(x)|\lambda_p\rangle = \langle\Omega|e^{iP\cdot x}\phi(0)e^{-iP\cdot x}|\lambda_p\rangle$$
$$= \langle\Omega|\phi(0)|\lambda_p\rangle e^{-ip\cdot x}\big|_{p^0=E_p}$$
$$= \langle\Omega|U^{-1}U\phi(0)U^{-1}U|\lambda_p\rangle e^{-ip\cdot x}\big|_{p^0=E_p} \quad (\text{def } U|\lambda_p\rangle = |\lambda_0\rangle)$$
$$= \langle\Omega|\phi(0)|\lambda_0\rangle e^{-ip\cdot x}\big|_{p^0=E_p}$$

Thus

$$\langle\Omega|\phi(x)\phi(y)|\Omega\rangle = \int\frac{d^4p}{(2\pi)^4}\frac{i}{p^2-m_\lambda^2+i\epsilon}e^{-ip\cdot(x-y)}\,|\langle\Omega|\phi(0)|\lambda_0\rangle|^2$$

Same as $x^0<y^0$ case. In conclusion:

K-L spectrum representation

$$\boxed{\langle\Omega|T\phi(x)\phi(y)|\Omega\rangle = \int_0^\infty\frac{dM^2}{2\pi}\rho(M^2)D_F(x-y;M^2)}$$

where $D_F(x-y;M^2)$ is the Feynman propagator with mass M

and the def. of positive spectrum density is:

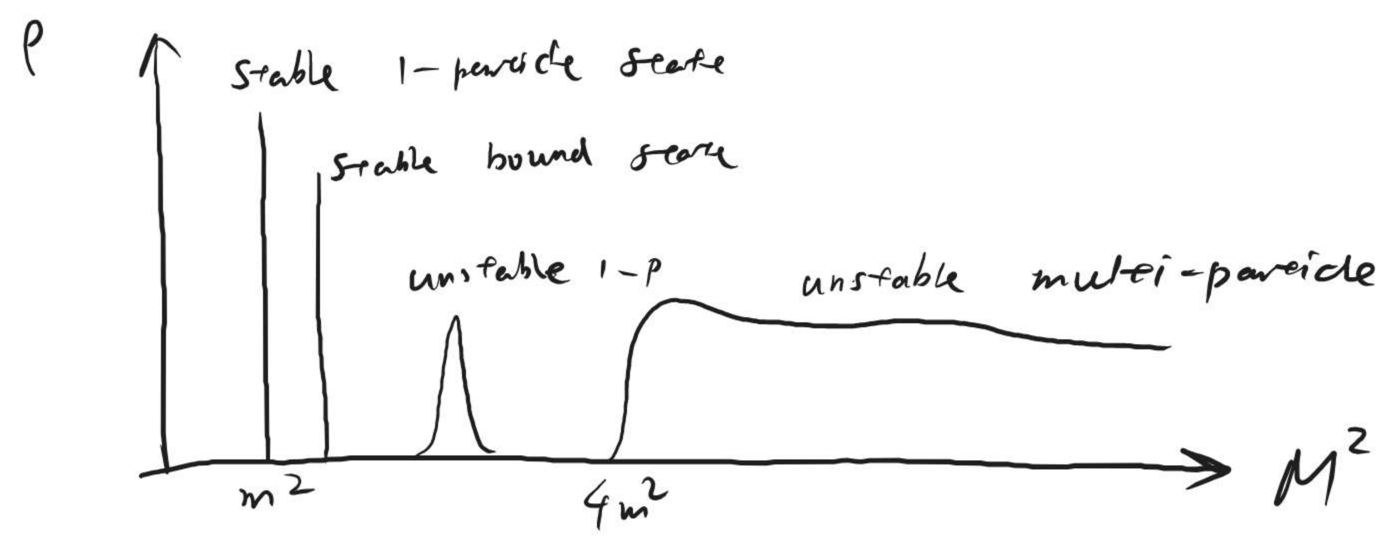
$$\rho(M^2) = \sum_\lambda 2\pi\delta(M^2-m_\lambda^2)\,|\langle\Omega|\phi(0)|\lambda_0\rangle|^2$$

K-L formula decomposes the 2-point correlation of interaction field theory into different eigenstate contributions

In momentum space:

$$\tilde{G}(p^2) \equiv \int\frac{d^4p}{(2\pi)^4}e^{ip\cdot(x-y)}\langle\Omega|T\phi(x)\phi(y)|\Omega\rangle$$

$$\boxed{\tilde{G}(p^2) = \int_0^\infty dM^2\rho(M^2)\frac{i}{p^2-M^2+i\epsilon}}$$



can be written as

$$\rho(M^2) = Z\cdot 2\pi\delta(M^2-m^2)$$
$$+ \text{ continuum when } M^2>4m^2$$

So

$$\tilde{G}(p^2) = \underbrace{\frac{iZ}{p^2-m^2+i\epsilon}}_{\text{pole in } p^2=m^2} + \underbrace{\int_{>4m^2}dM^2\rho(M^2)\Delta_F}_{\text{no poles}}$$

compared with the free scalar field:

$$G_f(p^2) = \frac{i}{p^2-m_0^2+i\epsilon}$$

* ① $\tilde{G}$ pole in <u>physical mass</u> $m=\sqrt{E^2-p^2}$

while G pole in <u>bare mass</u> m_0 from free $\mathcal{L}$

② $\tilde{G}$ has a residue $iZ = i|\langle\Omega|\phi(0)|\lambda_0\rangle|^2$, $0\leq Z\leq 1$

while G's is i, since $\langle 0|\phi(0)|p\rangle = 1$ in free case.

Z is also called Field-strength renormalization factor for scalar field

③ G only has one pole from the free particle

$\tilde{G}$ may also has poles from stable bound states, and the multi-particle states give <u>weaker</u> branch cut singularities. → on-shell

that is, when $p^2\sim m^2$, $\tilde{G}\sim G(m)$

Similarly for Dirac field.

$$\tilde{G}(p^2) = \frac{iZ_2}{\not{p}-m+i\epsilon} + \text{multi-particle term}$$

where $\sqrt{Z_2}\,u^{(s)}(p) = \langle\Omega|\psi(0)|p,s\rangle$

§1.1.5 Dispersion Relation

Using the identity $(\epsilon\to 0)$

$$\mathrm{Im}\frac{1}{p^2-m^2+i\epsilon} = \pi\delta(p^2-m^2)$$

Then

$$\boxed{\tilde{G}(p^2) = \frac{1}{\pi}\int_0^\infty dM^2\frac{\mathrm{Im}\,\tilde{G}(M^2)}{p^2-M^2+i\epsilon}}$$