Loss Functions For Regression

Second Study Session

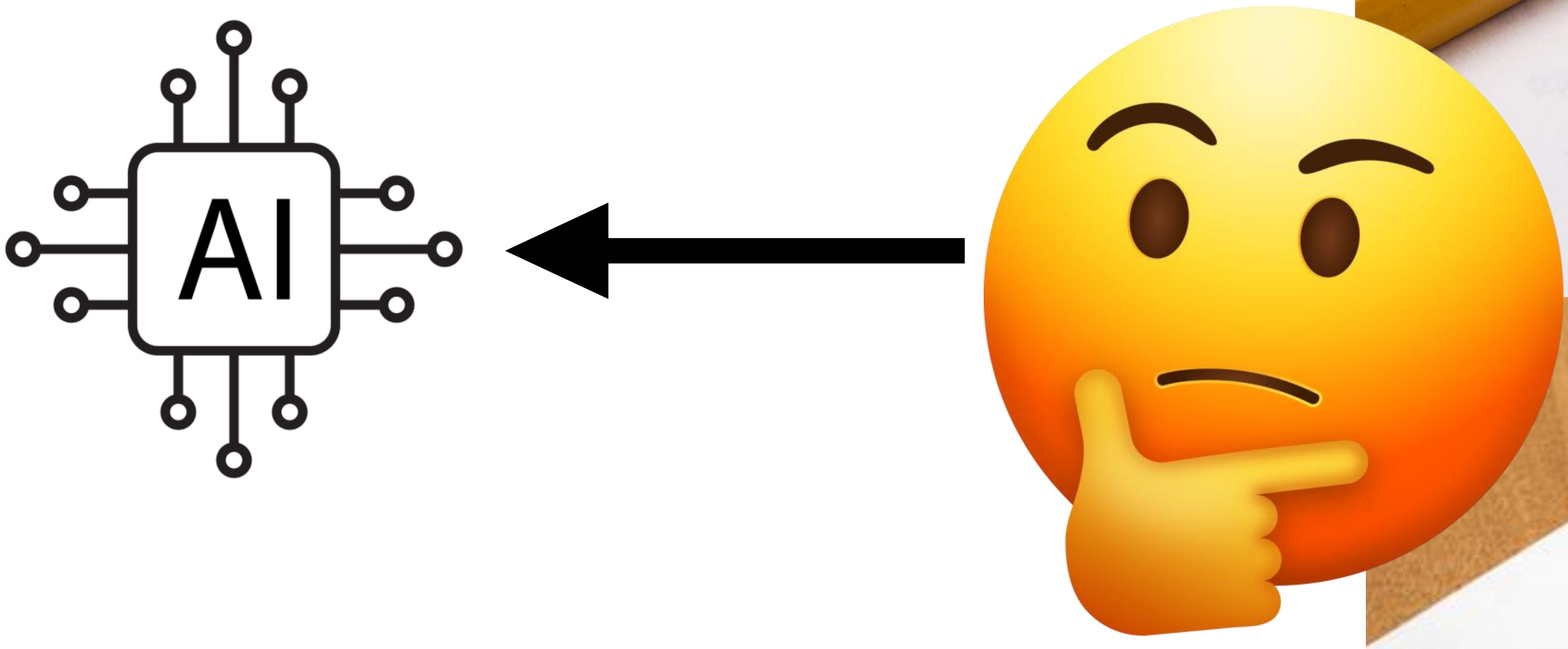
Lee JaeHyeong

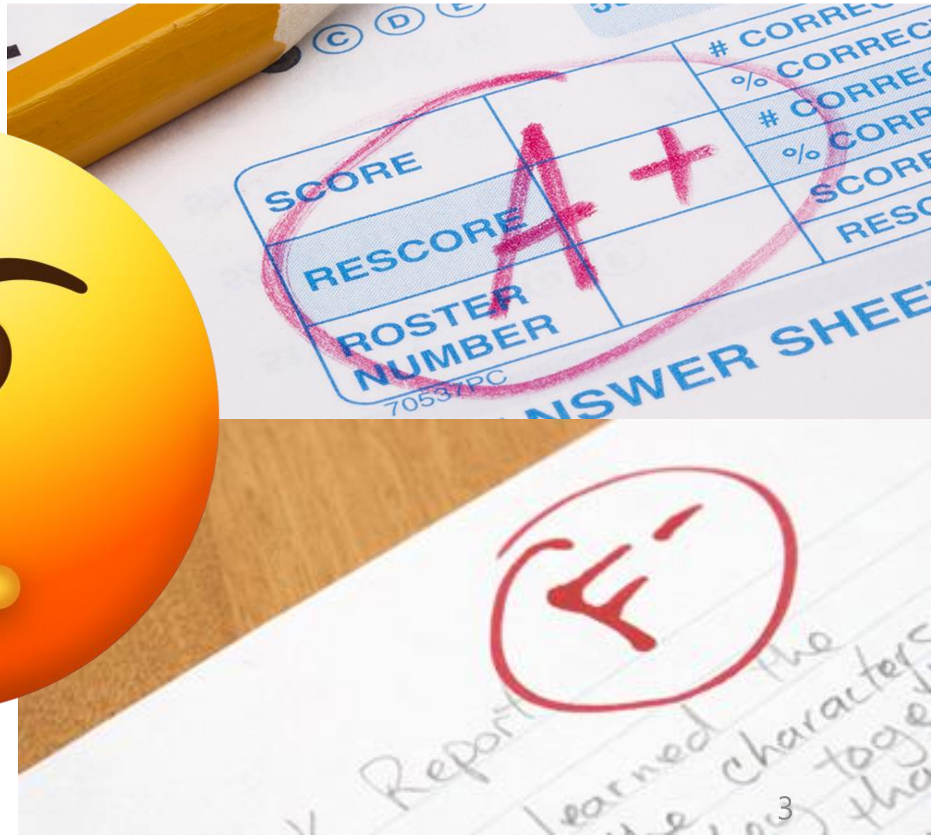
Context

- ✓ Loss Function?
- ✓ Regression vs Classification
- ✓ 3 Loss Functions for Regression
- ✓ Summary

Loss function?

How to say our model is working **good or bad**?
by which **criterion**?





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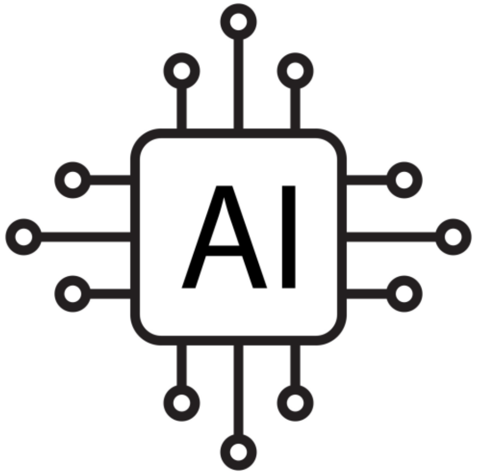
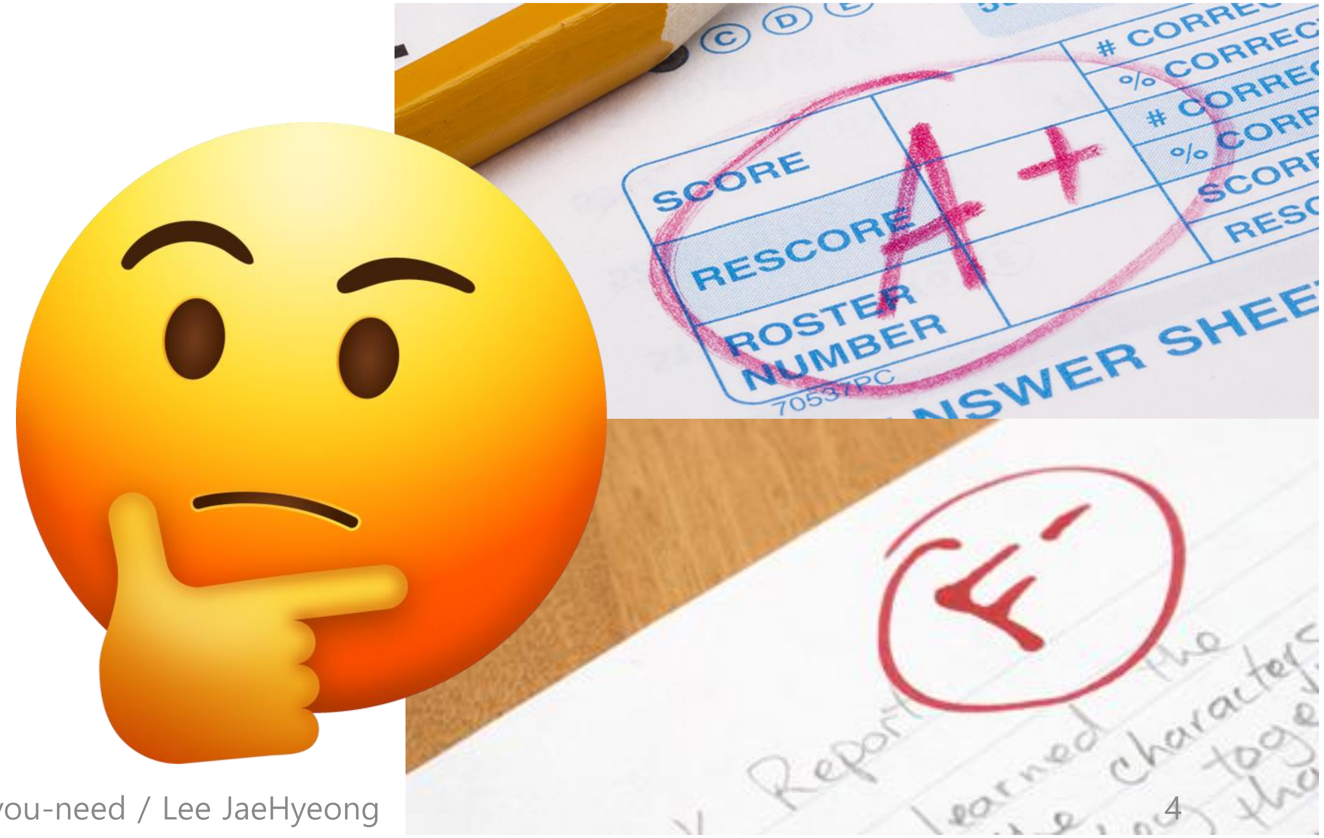
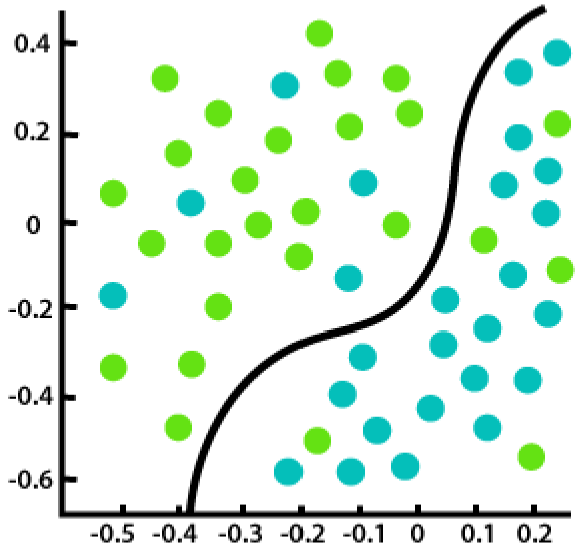
loss function !



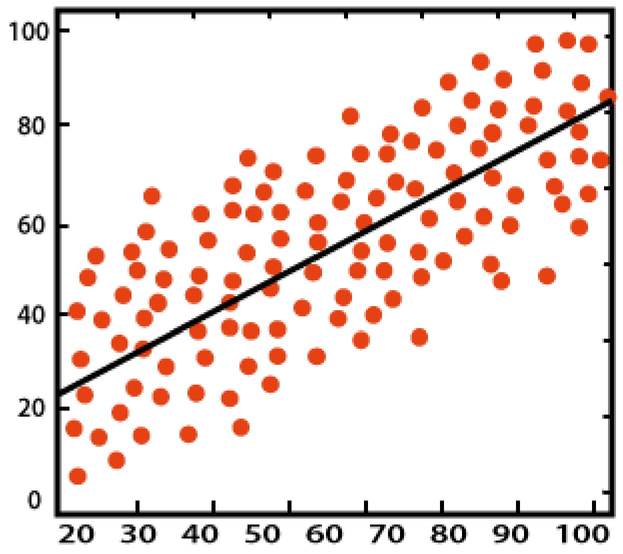


Regression vs Classification

- Classification
  - predict sample's class
- Regression
  - predict continuous value



Classification



Regression

3 Loss Functions for Regression

- MSE (L2 loss)
- MAE (L1 loss)
- Smooth Mean Absolute Error (Huber loss)

Mean Squared Error

$$\mathrm{MSE} = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$$

MSE = mean squared error
n = number of data points
Y_i = observed values
$\hat{Y}_i$ = predicted values

- L2 distance between prediction and ground truth becomes loss score.

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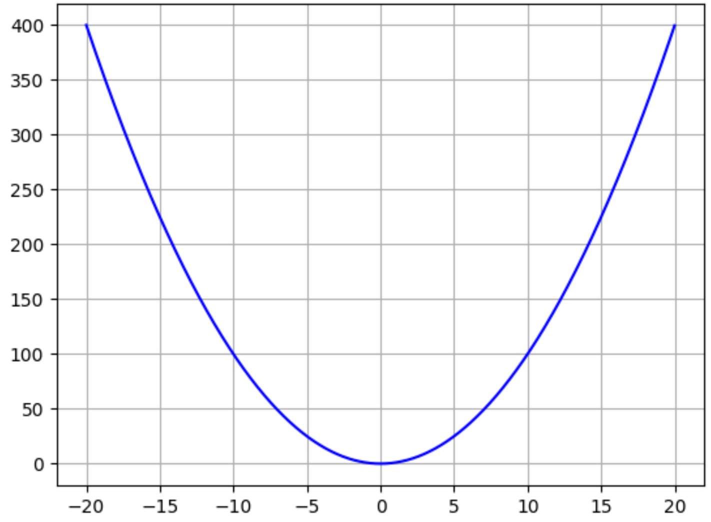

loss curve in terms of *"gt – pred"*

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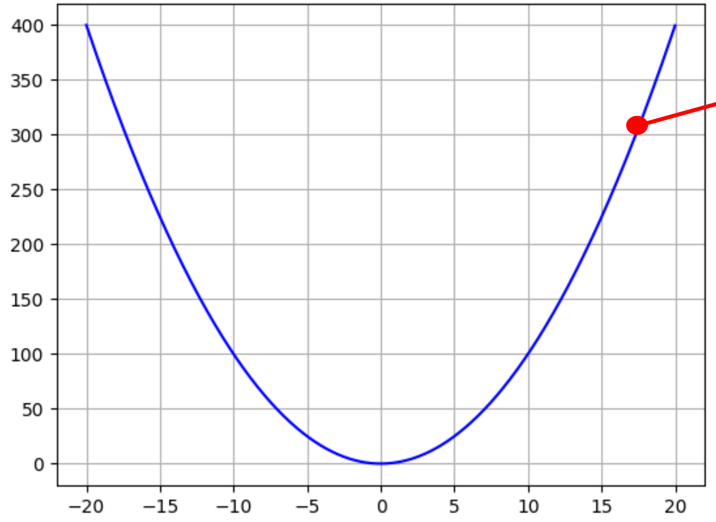

$$\frac{\partial MSE}{\partial pred_i} = \frac{1}{n}\sum_{i=1}^{n} -1 * 2(y_i - pred_i)$$

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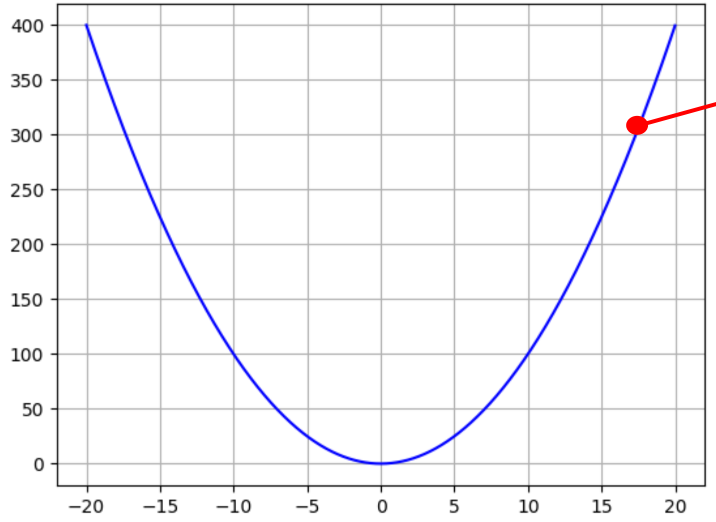

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high loss, large optimization steps,
low loss, small optimization steps.

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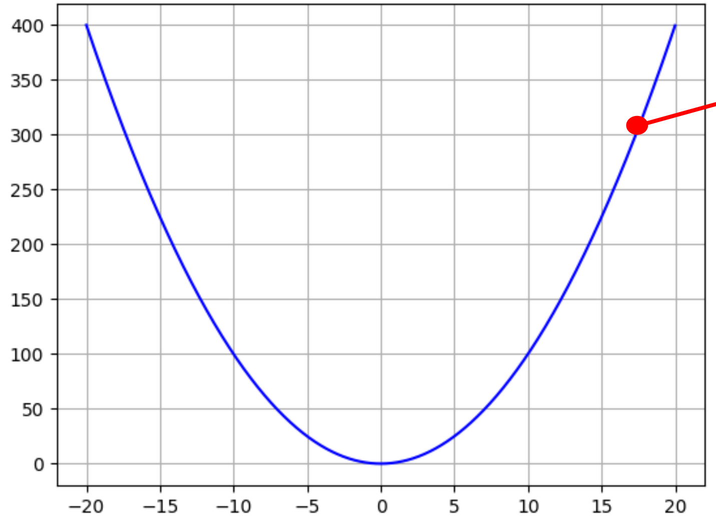

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sensitive to outliers!

loss curve in terms of *"gt – pred"*

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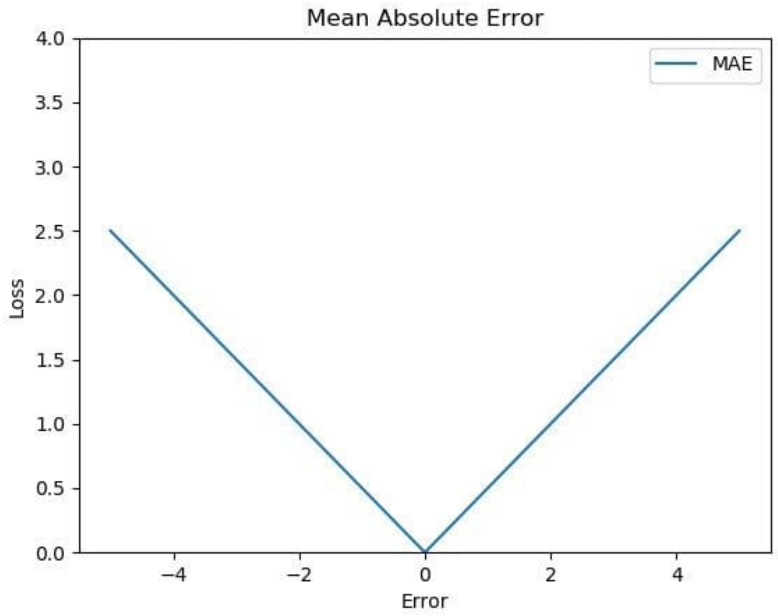

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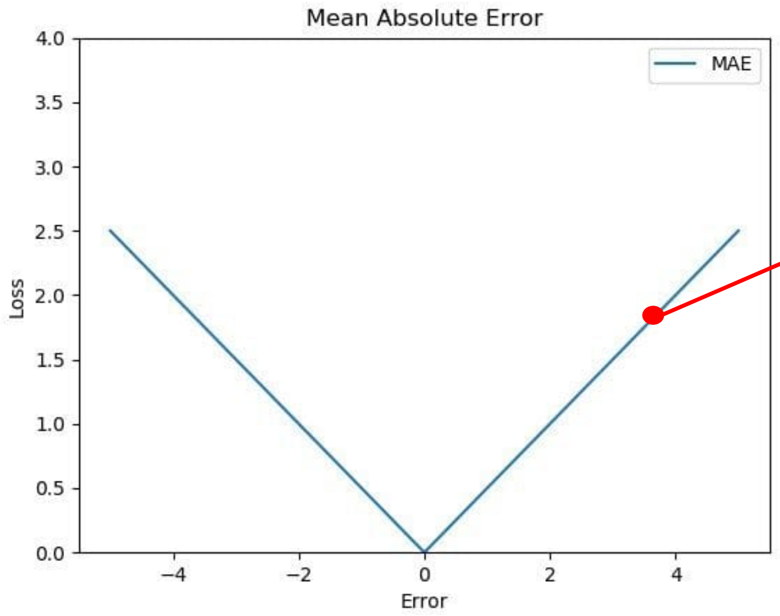

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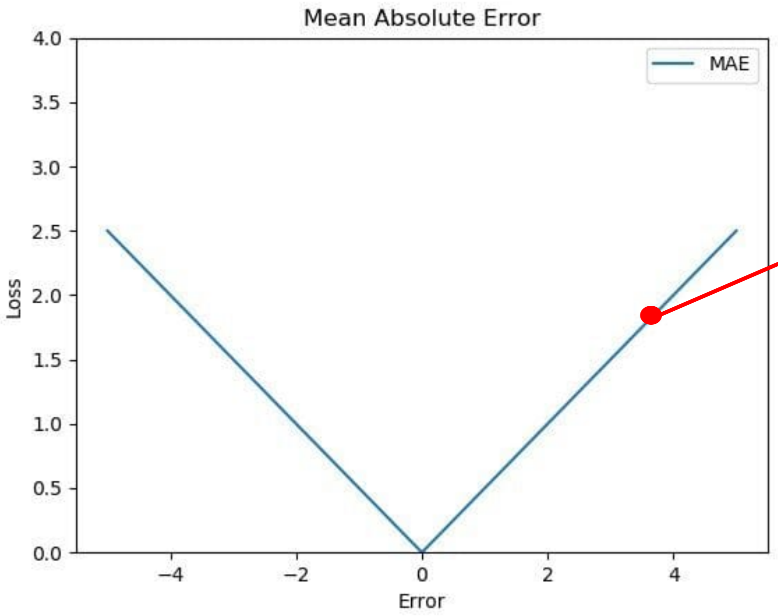

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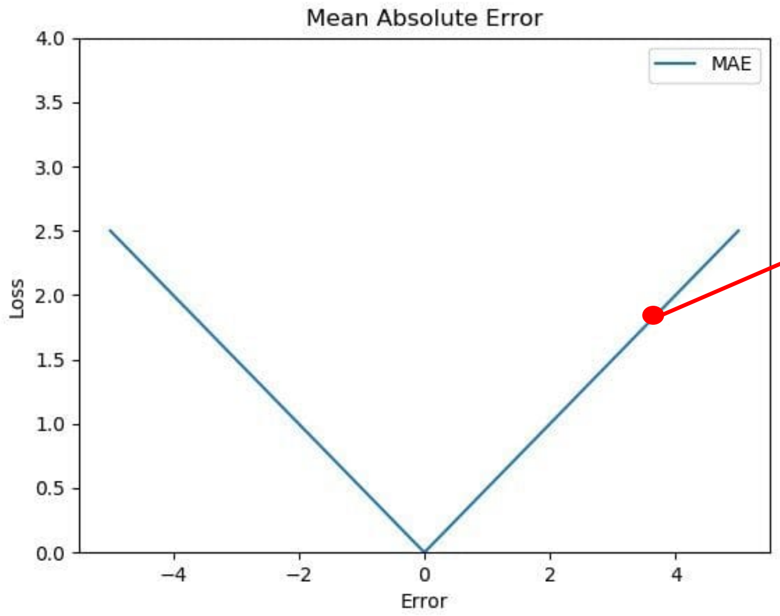

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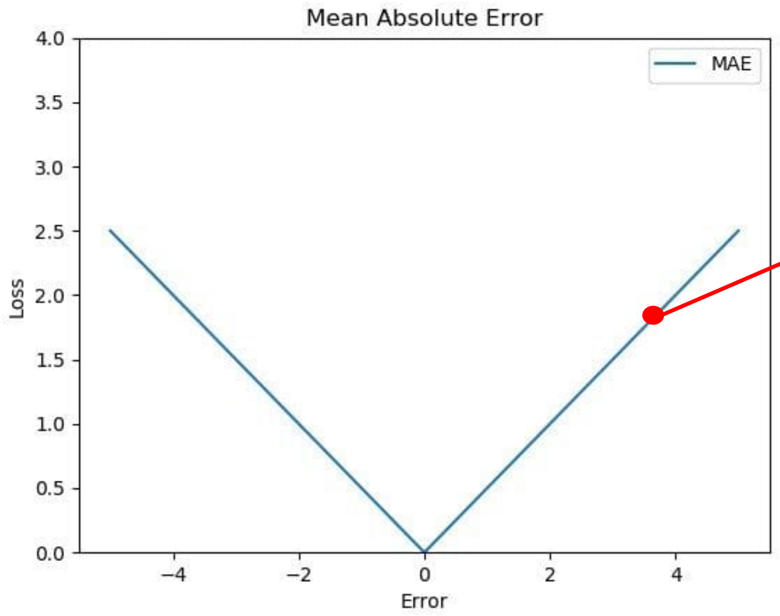

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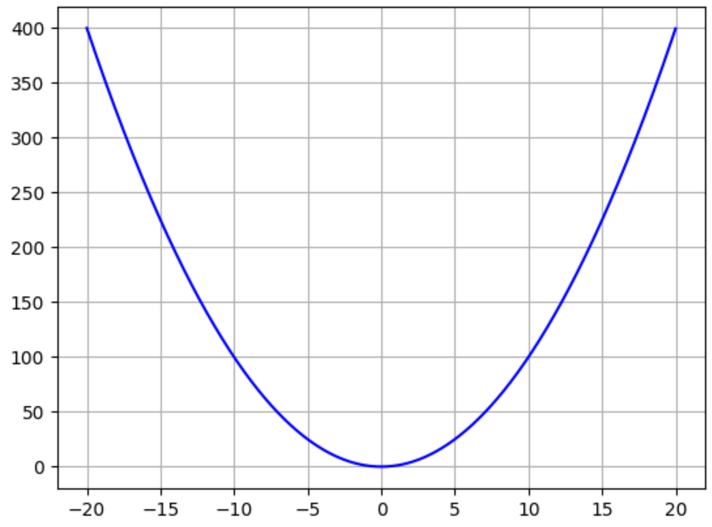
optimization steps aren't related with loss value !
MAE gives same weight for all samples!
-> more robust to outliers

loss curve in terms of *"gt - pred"*

Comparison

MSE Loss



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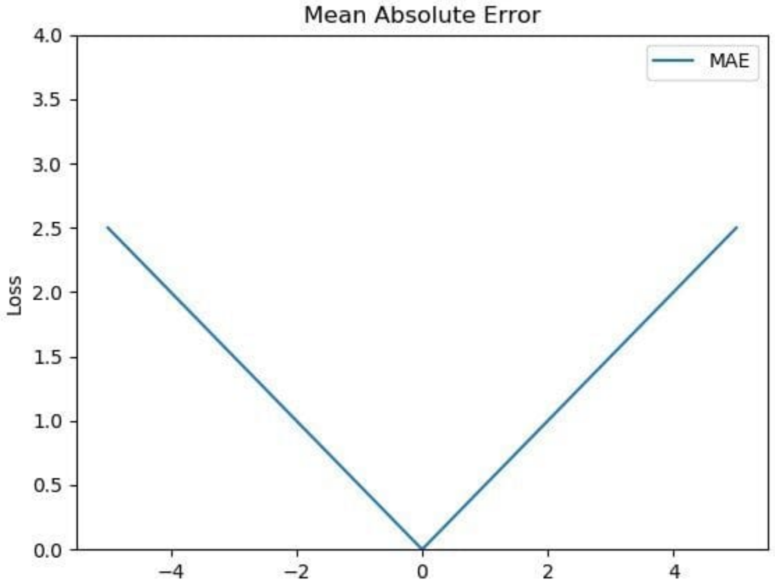
MAE Loss



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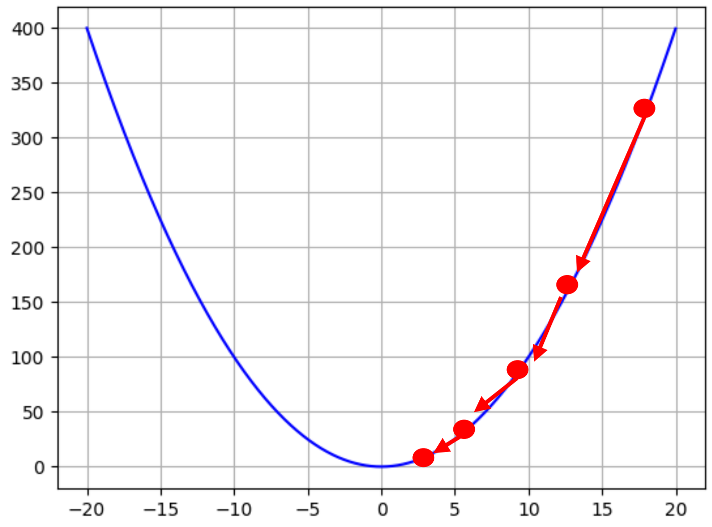

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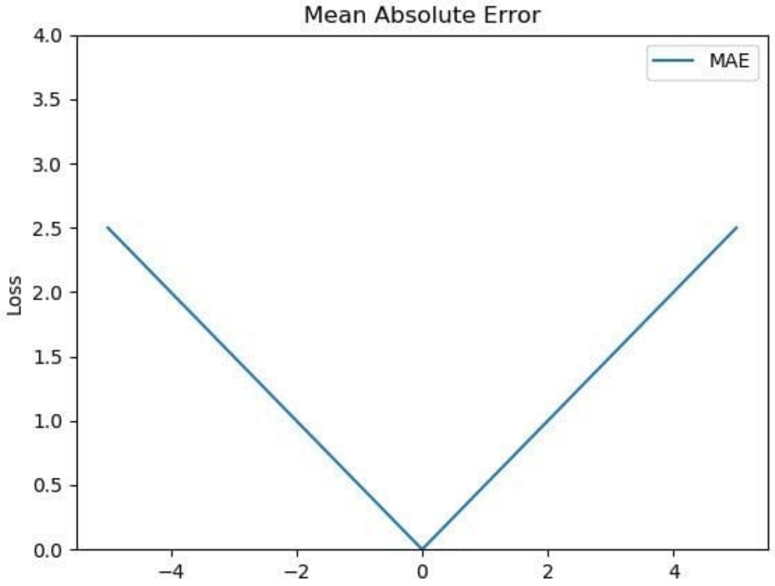

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high loss -> gradient scaled -> bigger optim steps

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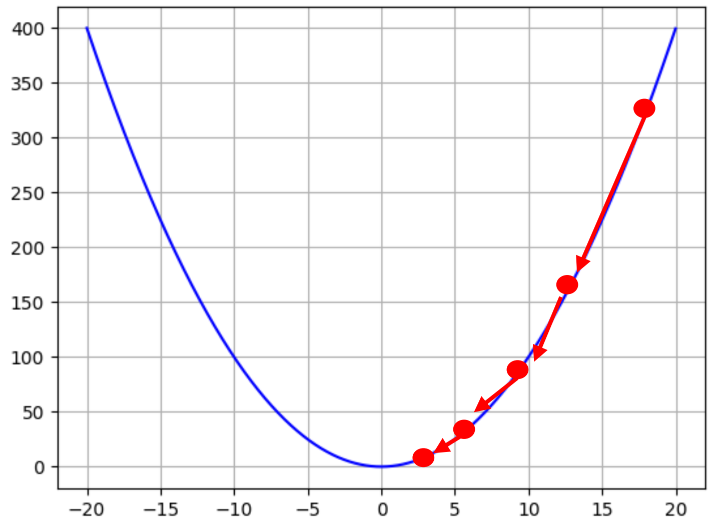

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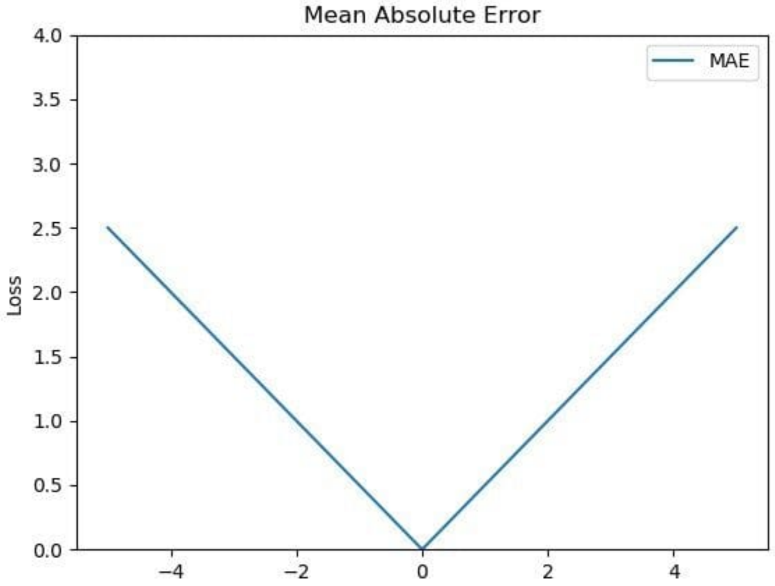

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faster convergence / stable optimization when loss is low

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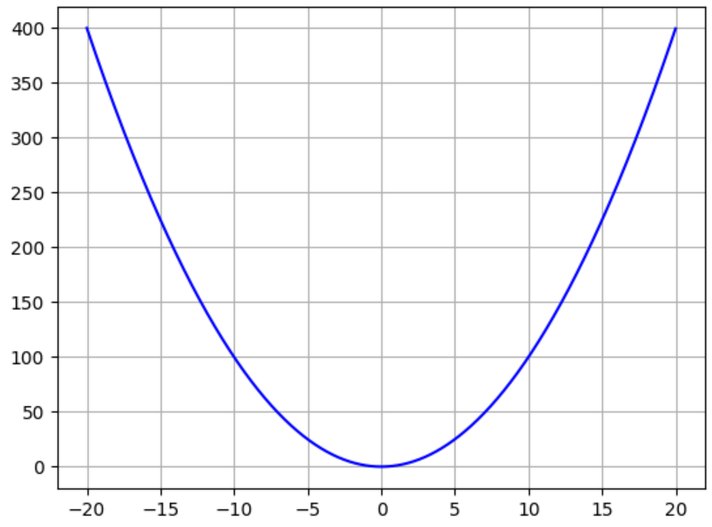

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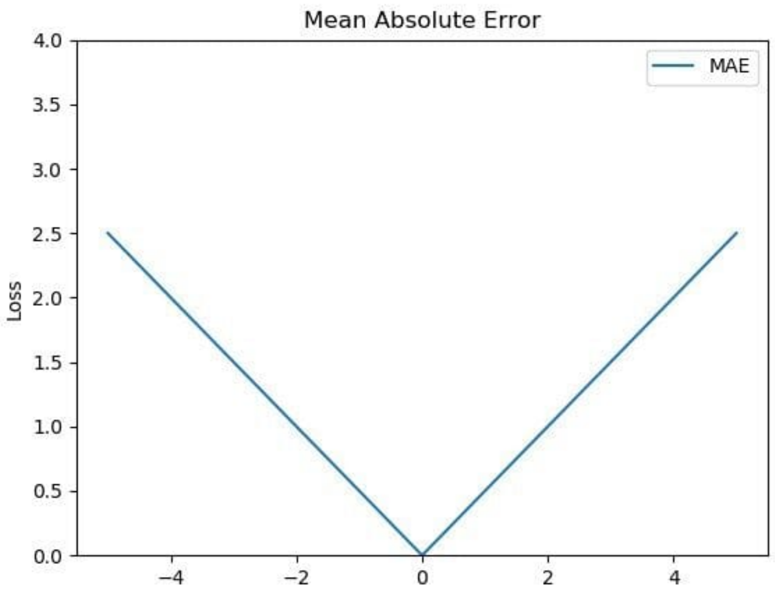

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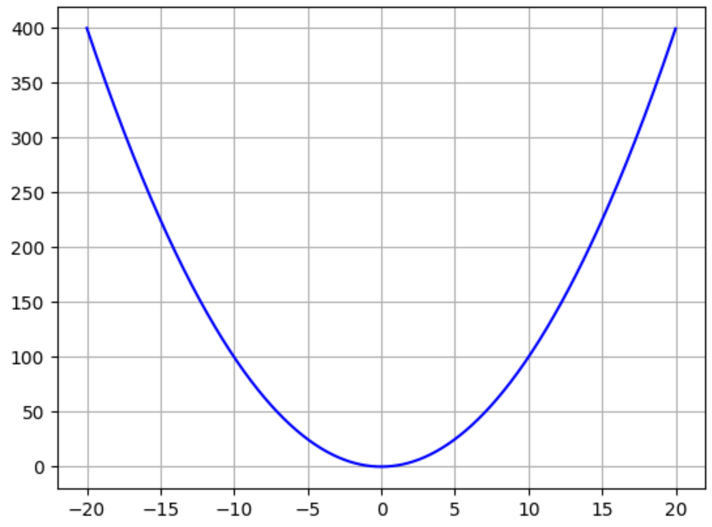

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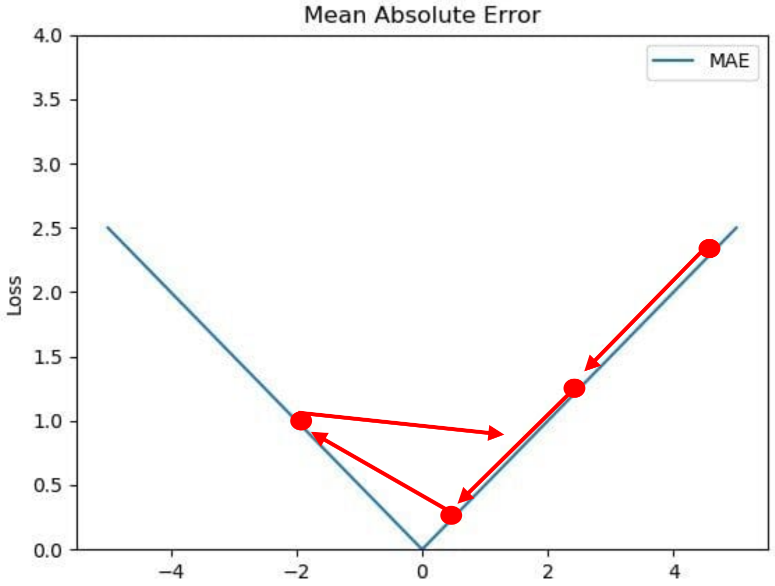

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gradient <-> loss value are independent

can bounce when loss becomes 0 !

Huber Loss

$$L_\delta(y, f(x)) = \begin{cases} \frac{1}{2}(y - f(x))^2 & \text{for} |y - f(x)| \le \delta, \\ \delta\, |y - f(x)| - \frac{1}{2}\delta^2 & \text{otherwise.} \end{cases}$$

- Act like MAE when residuals are larger than delta.
- Act like MSE when residuals are smaller then delta.
- robust to outliers, no bouncing when loss ~ 0 !

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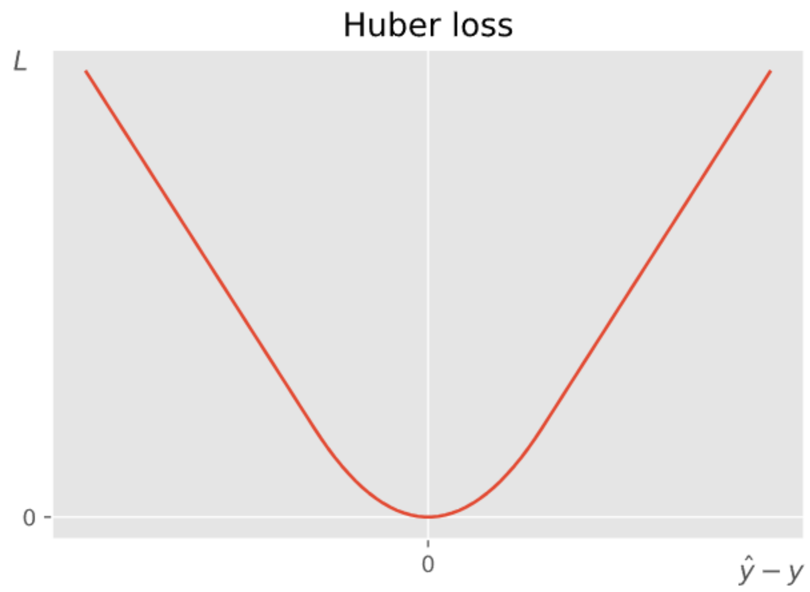

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have to find hyper-parameter delta by training.

Takeaways

1. When outliers are just corrupted data, and we are focusing on general regression, use MAE.

2. When outliers are important feature that need to be considered, use MSE.

3. MAE can make bounce when loss becomes smaller, use Huber Loss instead. (MAE + MSE)

Thank You !