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3. $$f(x) = 2^{x-1} - 4 + 1.5x \qquad x \in \mathbb{R}$$

(a) Show that the equation $f(x) = 0$ can be written as

$$x = \frac{1}{3}(8 - 2^x)$$

(2)

The equation $f(x) = 0$ has a root α, where $\alpha = 1.6$ to one decimal place.

(b) Starting with $x_0 = 1.6$, use the iteration formula

$$x_{n+1} = \frac{1}{3}(8 - 2^{x_n})$$

to calculate the values of x_1, x_2 and x_3, giving your answers to 3 decimal places.

(3)

(c) By choosing a suitable interval, prove that $\alpha = 1.633$ to 3 decimal places.

(2)

5. (i) The functions f and g are defined by

$$\mathrm{f} : x \rightarrow \mathrm{e}^{2x} - 5, \qquad x \in \mathbb{R}$$

$$\mathrm{g} : x \rightarrow \ln(3x - 1), \qquad x \in \mathbb{R},\ x > \frac{1}{3}$$

(a) Find f^{-1} and state its domain.

(3)

(b) Find fg(3), giving your answer in its simplest form.

(2)

(ii) (a) Sketch the graph with equation

$$y = |4x - a|$$

where a is a positive constant. State the coordinates of each point where the graph cuts or meets the coordinate axes.

(2)

Given that

$$|4x - a| = 9a$$

where a is a positive constant,

(b) find the possible values of

$$|x - 6a| + 3|x|$$

giving your answers, in terms of a, in their simplest form.

(5)

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Question 5 continued

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6. (a) Express $\sqrt{5}\cos\theta - 2\sin\theta$ in the form $R\cos(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$

State the value of R and give the value of α to 4 significant figures.

(3)

(b) Solve, for $-\pi < \theta < \pi$,

$$\sqrt{5}\cos\theta - 2\sin\theta = 0.5$$

giving your answers to 3 significant figures.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(4)

$$\mathrm{f}(x) = A\left(\sqrt{5}\cos\theta - 2\sin\theta\right) + B \qquad \theta \in \mathbb{R}$$

where A and B are constants.

Given that the range of f is

$$-15 \leqslant \mathrm{f}(x) \leqslant 33$$

(c) find the value of B and the possible values of A.

(4)

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Question 6 continued

Q6

(Total 11 marks)

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10. The curve C satisfies the equation

$$x\mathrm{e}^{5-2y} - y = 0 \qquad x > 0, \quad y > 0$$

The point P with coordinates $(2\mathrm{e}^{-1}, 2)$ lies on C.

The tangent to C at P cuts the x-axis at the point A and cuts the y-axis at the point B.

Given that O is the origin, find the exact area of triangle OAB, giving your answer in its simplest form.

(7)

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Question 10 continued

Q10

(Total 7 marks)

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11.

By writing $\sec\theta$ as $\dfrac{1}{\cos\theta}$, show that when $x = 3\sec\theta$,

$$\frac{dx}{d\theta} = 3\sec\theta\tan\theta$$

(2)

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12. (a) Show that

$$\cot x - \tan x \equiv 2\cot 2x, \qquad x \neq 90n°,\ n \in \mathbb{Z}$$

(4)

(b) Hence, or otherwise, solve, for $0 \leqslant \theta < 180°$

$$5 + \cot(\theta - 15°) - \tan(\theta - 15°) = 0$$

giving your answers to one decimal place.
[*Solutions based entirely on graphical or numerical methods are not acceptable.*]

(5)

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Question 12 continued

Q12

(Total 9 marks)

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14. Given that

$$y = \frac{(x^2 - 4)^{\frac{1}{2}}}{x^3} \qquad x > 2$$

(a) show that

$$\frac{dy}{dx} = \frac{Ax^2 + 12}{x^4(x^2 - 4)^{\frac{1}{2}}} \qquad x > 2$$

where A is a constant to be found.

(6)

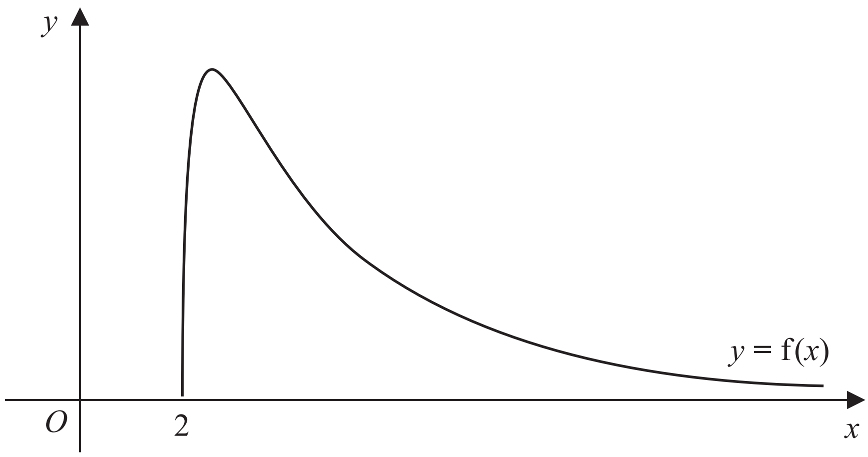

Figure 4

Figure 4 shows a sketch of part of the curve with equation $y = f(x)$ where

$$f(x) = \frac{24(x^2 - 4)^{\frac{1}{2}}}{x^3} \qquad x > 2$$

(b) Use your answer to part (a) to find the range of f.

(5)

(c) State a reason why f^{-1} does not exist.

(1)

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Question 14 continued

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Question 14 continued

Q14

(Total 12 marks)

TOTAL FOR PAPER: 125 MARKS

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