Value-Aware Digital Twin Synchronization via UAV Data Collection in Disaster Response

Lemma 1: If the given UAV trajectories $\boldsymbol{Q}$ and data gathering strategy $\boldsymbol{d}$ are two portions of a feasible solution to problem (10), the constraint (13) in problem (14) is redundant and the problem $\mathcal{P}_{1.1}$ is equivalent to the problem $\mathcal{P}_{1.2}$.

Proof Consider an auxiliary problem $\mathcal{P}_u$ which share the same objective function and constraints as problem (14), except the energy constraint (13). This problem optimizes transmission power $\mathcal{P}$ to maximize the accumulative data value, which is equivalent to minimizing the total hovering time across all the time slots. Since the data gathering strategy $\boldsymbol{d}$ are fixed, i.e., the amount of gathered data from each PT within each time slot is determined, the minimization of hovering time can be decoupled into N new subproblems corresponding to N time slots, in which the hovering time $\tau_j^h(t_k)$ of each UAV u_j is minimized together. The subproblem corresponding to the time slot t_k is formulated as.

$$\mathcal{P}1.2 \quad \min_{\boldsymbol{P}} \tau_1^h(t_k), \ldots, \min_{\boldsymbol{P}} \tau_K^h(t_k) \tag{1}$$

$$\text{s.t.}(1),(5),(6),(12) \tag{2}$$

Since the objective function is linear and all constraints are affine, each subproblem can admit an optimal solution. Denoted by $\boldsymbol{\tau}^{h,(*)} = \left\{\tau_j^{h,(*)}(t_k)\right\}_{j=1}^{K}$ the optimal solution of problem (1). Such optimal solutions within each time slot are also the optimal solution of problem $\mathcal{P}_u$. Now we analyze two properties of the sequence $\boldsymbol{\tau}^{h,(*)}(t_k)$ in two following cases.

Case (I): $\sum_{k=1}^{N} \tau_j^{h,(*)}(t_k) > C_j^{(3)}$. For any other feasible solution $\tau_j^h(t_k)$, we have $\tau_j^{h,(*)}(t_k) \leq \tau_j^h(t_k)$, so $\sum_{k=1}^{N} \tau_j^h(t_k) \geq \sum_{k=1}^{N} \tau_j^{h,(*)}(t_k) > C_j^{(3)}$ always holds. Then, the constraint (11) can not be met. Hence, no feasible solution exists for problem $\mathcal{P}1.1$ under the given $\boldsymbol{Q}$ and $\boldsymbol{d}$. This contradicts the assumption that $\boldsymbol{Q}$ and $\boldsymbol{d}$ are part of a feasible solution for problem (9). Thus, this case is ruled out.

Case (II): $\sum_{k=1}^{N} \tau_j^{h,(*)}(t_k) \leq C_j^{(3)}$. We show that the sequence $\boldsymbol{\tau}^{h,(*)}$ is also the optimal solution to the Problem $\mathcal{P}1.1$ by contradiction. Assume that the optimal solution of Problem $\mathcal{P}1.1$ is sequence $\boldsymbol{\tau}^h$. Note that since Problem $\mathcal{P}_u$ has not got constraint (11) than $\mathcal{P}1.1$, any feasible solution to Problem $\mathcal{P}1.1$ must also be feasible to Problem $\mathcal{P}_u$. Therefore, $\boldsymbol{\tau}^h$ is feasible for Problem $\mathcal{P}_u$. Since $\boldsymbol{\tau}^{h,(*)}$ is the unique optimal solution to Problem $\mathcal{P}_u$, there must exist at least one UAV hovering time $\tau_j^h(t_k)$ in $\boldsymbol{\tau}^h$ is greater than $\tau_j^{h,(*)}(t_k)$. This further implies that $\sum_{k=1}^{N} \tau_j^h(t_k) \geq \sum_{k=1}^{N} \tau_j^{h,(*)}(t_k)$. Then we have $C_i^{(1)}(t_k)\gamma^{-\sum_{l=1}^{N} \tau_j^h(t_l)} \leq C_i^{(1)}(t_k)\gamma^{-\sum_{l=1}^{N} \tau_j^{h,(*)}(t_l)}$, which means that $\boldsymbol{\tau}^{h,(*)}$ is more optimal than $\boldsymbol{\tau}^h$. This contradiction shows that $\boldsymbol{\tau}^{h,(*)}$ is optimal solution of both auxiliary Problem $\mathcal{P}_u$ and Problem $\mathcal{P}1.2$. This confirms that the energy constraint (13) is not active when $\boldsymbol{Q}$ and $\boldsymbol{d}$ are feasible to Problem $\mathcal{P}$, and is thus redundant to Problem $\mathcal{P}1.2$.

Lemma 2: The Problems $\mathcal{P}2.1$ and $\mathcal{P}2.2$ have the same optimal solution when the constraint 16(c) hold with equality.

Proof Assume that $\boldsymbol{\xi}^* = [\boldsymbol{d}^*, \boldsymbol{\lambda}^*]$ is an optimal solution to the Problem $\mathcal{P}2.2$, where the inequality 16(e) is strict for any $1 \leq i \leq n$ and $1 \leq k \leq N$, i.e., $\log_\gamma \lambda_{ik}^* < \log_\gamma S_i^{(1)} + \log_\gamma d_i^*(t_k) - \sum_{l=1}^{N} \tau_j^{h,(*)}(t_l)$. Then, we can construct a feasible solution $\boldsymbol{\xi} = [\boldsymbol{d}^*, \boldsymbol{\lambda}^* + \boldsymbol{\epsilon}]$ where at least one $\epsilon_{ik} > 0$ satisfies $\log_\gamma (\lambda_{ik}^* + \epsilon_{ik}) = \log_\gamma S_i^{(1)} + \log_\gamma d_i^*(t_k) - \sum_{l=1}^{N} \tau_j^{h,(*)}(t_l)$. Obviously, $\lambda_{ik}^* + \epsilon_{ik}$ is larger than λ_i^*, and hence the objective function is strictly larger than the that of $\boldsymbol{\xi}^*$, which contradicts the optimality of $\boldsymbol{\xi}^*$. Therefore, the optimal solution to the objective Problem $\mathcal{P}2.2$ can only be obtained when the constraint 16(c) holds with equality and Problems $\mathcal{P}2.1$ and $\mathcal{P}2.2$ have the same optimal solution.

Theorem 1: Given K UAVs with energy capacity E_c, the maximum transmission power P_{max}, each PT $v_i \in V$ with data size D_i and a positive parameter ε, the solutions obtained in each iteration by Algorithm 2 can converge to stable value.

Proof Recall that the original Problem $\mathcal{P}$ is solved via a block coordinate descent (BCD) framework, where each iteration sequentially optimizes transmission power in subproblem $\mathcal{P}1.1$, data gathering strategy in subproblem $\mathcal{P}2.1$, and UAV trajectory in subproblem $\mathcal{P}3.1$. In the following, we show that the accumulative data value increase monotonically in each subproblem, i.e., the solution obtained by Algorithm 2 eventually converges to a finite value.

Let $\boldsymbol{f}\left(\boldsymbol{P}^{(l)}, \boldsymbol{d}^{(l)}, \boldsymbol{Q}^{(l)}\right)$ be the optimal accumulated data value obtained by solving the original Problem $\mathcal{P}$ at the r-th iteration, and let $\boldsymbol{f}_X(\cdot)$ be the accumulated data valueo btained from the corresponding subproblem X.

In the transmission power optimization phase of the $(l+1)$-th round iterative, the Algorithm 2 solves Problem $\mathcal{P}1.3$ with fixed variables $\boldsymbol{d}^{(l)}$ and $\boldsymbol{Q}^{(l)}$, and obtains the solution $\boldsymbol{f}_{1.3}\left(\boldsymbol{P}^{(l+1)}, \boldsymbol{d}^{(l)}, \boldsymbol{Q}^{(l)}\right)$. Since the Problems $\mathcal{P}1.1$ and $\mathcal{P}1.3$ are equivalent, we have $\boldsymbol{f}(\cdot) = \boldsymbol{f}_{1.1}(\cdot) = \boldsymbol{f}_{1.3}(\cdot)$. Because the Algorithm 1 can obtain the optimal solution, we then have $\boldsymbol{f}_{1.3}\left(\boldsymbol{P}^{(l)}, \boldsymbol{d}^{(l)}, \boldsymbol{Q}^{(l)}\right) \leq \boldsymbol{f}_{1.3}\left(\boldsymbol{P}^{(l+1)}, \boldsymbol{d}^{(l)}, \boldsymbol{Q}^{(l)}\right)$, which implies $\boldsymbol{f}\left(\boldsymbol{P}^{(l)}, \boldsymbol{d}^{(l)}, \boldsymbol{Q}^{(l)}\right) \leq \boldsymbol{f}\left(\boldsymbol{P}^{(l+1)}, \boldsymbol{d}^{(l)}, \boldsymbol{Q}^{(l)}\right)$.

In the data gathering strategy optimization phase of the $(l+1)$-th iterative, with $\boldsymbol{P}^{(l+1)}$ and $\boldsymbol{Q}^{(l)}$ fixed, Algorithm 1 solves Problem $\mathcal{P}2.3$ and obtains the solution $\boldsymbol{f}_{2.3}\left(\boldsymbol{P}^{(l+1)}, \boldsymbol{d}^{(l+1)}, \boldsymbol{Q}^{(l)}\right)$. Since Problems $\mathcal{P}2.1$ and $\mathcal{P}2.2$ are equivalent, we have $\boldsymbol{f}(\cdot) = \boldsymbol{f}_{2.1}(\cdot) = \boldsymbol{f}_{2.2}(\cdot)$. Given that the relaxation in $\mathcal{P}2.2$ is tight at the local variables, we have $\boldsymbol{f}\left(\boldsymbol{P}^{(l+1)}, \boldsymbol{d}^{(l)}, \boldsymbol{Q}^{(l)}\right) = \boldsymbol{f}_{2.2}\left(\boldsymbol{P}^{(l+1)}, \boldsymbol{d}^{(l)}, \boldsymbol{Q}^{(l)}\right) \leq \boldsymbol{f}_{2.3}\left(\boldsymbol{P}^{(l+1)}, \boldsymbol{d}^{(l)}, \boldsymbol{Q}^{(l)}\right)$. The Algorithm 1 can get optimal solution of convex Problem $\mathcal{P}2.3$, so $\boldsymbol{f}\left(\boldsymbol{P}^{(l+1)}, \boldsymbol{d}^{(l)}, \boldsymbol{Q}^{(l)}\right) \leq \boldsymbol{f}_{2.3}\left(\boldsymbol{P}^{(l+1)}, \boldsymbol{d}^{(l)}, \boldsymbol{Q}^{(l)}\right) \leq \boldsymbol{f}_{2.3}\left(\boldsymbol{P}^{(l+1)}, \boldsymbol{d}^{(l+1)}, \boldsymbol{Q}^{(l)}\right)$ always holds. Since the constraint 16(a) is relaxed to a convex upper approximation constraint 17, all solutions to Problem $\mathcal{P}2.3$ are feasible for Problem $\mathcal{P}2.2$. Then, we further have $\boldsymbol{f}\left(\boldsymbol{P}^{(l+1)}, \boldsymbol{d}^{(l)}, \boldsymbol{Q}^{(l)}\right) \leq \boldsymbol{f}_{2.3}\left(\boldsymbol{P}^{(l+1)}, \boldsymbol{d}^{(l+1)}, \boldsymbol{Q}^{(l)}\right) \boldsymbol{f}_{2.2}\left(\boldsymbol{P}^{(l+1)}, \boldsymbol{d}^{(l+1)}, \boldsymbol{Q}^{(l)}\right)$. Hence, we conclude $\boldsymbol{f}_{2.2}\left(\boldsymbol{P}^{(l+1)}, \boldsymbol{d}^{(l+1)}, \boldsymbol{Q}^{(l)}\right) = \boldsymbol{f}\left(\boldsymbol{P}^{(l+1)}, \boldsymbol{d}^{(l+1)}, \boldsymbol{Q}^{(l)}\right)$, we thus have $\boldsymbol{f}\left(\boldsymbol{P}^{(l+1)}, \boldsymbol{d}^{(l)}, \boldsymbol{Q}^{(l)}\right) \leq \boldsymbol{f}\left(\boldsymbol{P}^{(l+1)}, \boldsymbol{d}^{(l)}, \boldsymbol{Q}^{(l)}\right)$.

Similarly, in the trajectory optimization phase, we obtain $\boldsymbol{f}\left(\boldsymbol{P}^{(l+1)}, \boldsymbol{d}^{(l+1)}, \boldsymbol{Q}^{(l)}\right) \leq \boldsymbol{f}\left(\boldsymbol{P}^{(l+1)}, \boldsymbol{d}^{(l+1)}, \boldsymbol{Q}^{(l+1)}\right)$.

By combining the above inequalities, we conclude that the accumulative data value improves monotonically at each round of iteration. the details are shown as follows.

$$\begin{aligned} \boldsymbol{f}\left(\boldsymbol{P}^{(l)}, \boldsymbol{d}^{(l)}, \boldsymbol{Q}^{(l)}\right) \leq & \boldsymbol{f}\left(\boldsymbol{P}^{(l+1)}, \boldsymbol{d}^{(l)}, \boldsymbol{Q}^{(l)}\right) \\ \leq & \boldsymbol{f}\left(\boldsymbol{P}^{(l+1)}, \boldsymbol{d}^{(l+1)}, \boldsymbol{Q}^{(l)}\right) \\ \leq & \boldsymbol{f}\left(\boldsymbol{P}^{(l+1)}, \boldsymbol{d}^{(l+1)}, \boldsymbol{Q}^{(l+1)}\right). \end{aligned} \tag{3}$$

It can be seen that the accumulative data value $\boldsymbol{f}(\boldsymbol{P}, \boldsymbol{d}, \boldsymbol{Q})$ is bound for any given $\boldsymbol{P}^{(0)}, \boldsymbol{d}^{(0)}, \boldsymbol{Q}^{(0)}$. The ascending sequence $\left\{\boldsymbol{f}\left(\boldsymbol{P}^{(l)}, \boldsymbol{d}^{(l)}, \boldsymbol{Q}^{(l)}\right)\right\}_{r=0}^{\infty}$ converges to a finite value. Since the iterations are terminated when $\|\boldsymbol{Q}^{(l+1)} - \boldsymbol{Q}^{(l)}\|_2 + \|\boldsymbol{P}^{(l+1)} - \boldsymbol{P}^{(l)}\|_2 + \|\boldsymbol{d}^{(l+1)} - \boldsymbol{d}^{(l)}\|_2 \leq \varepsilon$ is satisfied, Algorithm 1 is guaranteed to converge to a stable solution within a finite number of iterations.