Problem 1

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Vectors **A**, **B**, and **C** at a point are given by

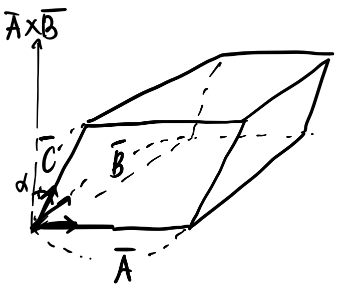
$$\mathbf{A} = A_1\mathbf{a}_1 + A_2\mathbf{a}_2 + A_3\mathbf{a}_3$$

$$\mathbf{B} = B_1\mathbf{a}_1 + B_2\mathbf{a}_2 + B_3\mathbf{a}_3$$

$$\mathbf{C} = C_1\mathbf{a}_1 + C_2\mathbf{a}_2 + C_3\mathbf{a}_3 \ .$$

Prove that

$$\mathbf{A}\cdot\mathbf{B}\times\mathbf{C} = \mathbf{B}\cdot\mathbf{C}\times\mathbf{A} = \mathbf{C}\cdot\mathbf{A}\times\mathbf{B} \ .$$



$V = |\bar{A}\times\bar{B}|\cdot|\bar{C}\cdot\cos\alpha|$ (α는 $\bar{C}$와 $\bar{A}\times\bar{B}$가 이루는 각)

$= \bar{C}\cdot\bar{A}\times\bar{B}$ 이고, 밑변을 잡는 방법에 따라

$A\cdot B\times C$, $B\cdot C\times A$, $C\cdot A\times B$는 전부 같은 입체의 부피를 의미하므로 같다.

Problem 2

Let $\mathbf{A} = \mathbf{a}_1 + \alpha\,\mathbf{a}_2 + \mathbf{a}_3$ and $\mathbf{B} = \alpha\,\mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3$. **A** and **B** at a point are normal to each other.

Find α .

$\bar{A}$와 $\bar{B}$가 수직이므로 $\bar{A}\cdot\bar{B} = 0$을 만족한다.

$\therefore\ \alpha + \alpha + 1 = 0,\quad \alpha = -\frac{1}{2}$

Problem 3

Given vectors $\mathbf{A} = 2\mathbf{a}_1 + 5\mathbf{a}_3$ and $\mathbf{B} = \mathbf{a}_1 - 3\mathbf{a}_2 + 4\mathbf{a}_3$ at a point , Find $|\mathbf{A} \times \mathbf{B}| + \mathbf{A} \cdot \mathbf{B}$.

$$\bar{A} \times \bar{B} = \det \begin{vmatrix} a_1 & a_2 & a_3 \\ 2 & 0 & 5 \\ 1 & -3 & 4 \end{vmatrix} = a_1 \cdot 15 - a_2(3) + a_3(-6)$$
$$= 15a_1 - 3a_2 - 6a_3.$$

$|\bar{A} \times \bar{B}| = \sqrt{225+9+36} = \sqrt{270}$, $\bar{A} \cdot \bar{B} = 2 + 20 = 22$. $|\bar{A} \times \bar{B}| + \bar{A} \cdot \bar{B} = 3\sqrt{30} + 22$

Problem 4

Find the expression for the differential length vector $d\mathbf{l}$ along the curve $x = y = z^2$ at the point (1,1,1) in terms of dz.

$d\bar{l} = dx\,\bar{a}_x + dy\,\bar{a}_y + dz\,\bar{a}_z$. 이고,

(1,1,1) 에서 $x=y=z^2$ 은 $dx=dy=2dz$를 만족한다. $d\bar{l}$을 dz 에 대해 정리하면,

$d\bar{l} = dz(2\bar{a}_x + 2\bar{a}_y + \bar{a}_z)$

Problem 5

Find the expression for the unit vector normal to the curve

$x = y^2 = z^3$ and the line $x = y = z$ at the point (1,1,1).

$l_1 : x = y^2 = z^3$, (1,1,1)에서 $dx = 2dy = 3dz$를 만족. $d\bar{l_1} = dz(3\bar{a_x} + \frac{3}{2}\bar{a_y} + \bar{a_z})$

$l_2 : x = y = z$, (1,1,1) 에서 $dx = dy = dz$를 만족. $d\bar{l_2} = dz(\bar{a_x} + \bar{a_y} + \bar{a_z})$

$$d\bar{l_1} \times d\bar{l_2} = \begin{vmatrix} a_x & a_y & a_z \\ 3dz & \frac{3}{2}dz & 1dz \\ 1dz & 1dz & 1dz \end{vmatrix} = a_x \cdot \frac{1}{2} d^2 z^2 - a_y \cdot 2d^2 z^2 + a_z \cdot \frac{3}{2} \cdot d^2 z^2$$

$$= d^2 z^2 \left(\frac{1}{2}\bar{a_x} - 2\bar{a_y} + \frac{3}{2}\bar{a_z} \right)$$

$$|d\bar{l_1} \times d\bar{l_2}| = \sqrt{\frac{1}{4}d^4 z^4 + 4d^4 z^4 + \frac{9}{4}d^4 z^4} = d^2 z^2 \cdot \frac{\sqrt{26}}{2}$$

$$a_n = \frac{\frac{1}{2}\bar{a_x} - 2\bar{a_y} + \frac{3}{2}\bar{a_z}}{\frac{\sqrt{26}}{2}} = \frac{\bar{a_x} - 4\bar{a_y} + 3\bar{a_z}}{\sqrt{26}}$$

Problem 6

Find the unit vector normal to the surface defined by

$$x^2 + y^2 + 4z^2 = 8$$

at the point (2,0,1).

$l_1 : x^2+4z^2 = 8,\ y=0$, $(2,0,1)$에서 $dx = -2dz,\ dy=0$ 만족. $d\bar{l_1} = -2dz\,\bar{a_x} + dz\,\bar{a_z}$

l_2 $x^2+y^2=4,\ z=1$, $(2,0,1)$에서 $dx=0,\ dz=0$. 만족. $d\bar{l_2} = dy\,\bar{a_y}$

$$d\bar{l_1} \times d\bar{l_2} = \det\begin{vmatrix} \bar{a_x} & \bar{a_y} & \bar{a_z} \\ -2dz & 0 & dz \\ 0 & dy & 0 \end{vmatrix} = -\bar{a_x}\cdot dy\,dz + \bar{a_z}\cdot(-2dy\,dz)$$

$$a_n = \frac{-\bar{a_x}\cdot dy\cdot dz - 2dy\cdot dz\cdot \bar{a_z}}{\sqrt{dy^2dz^2 + 4dy^2dz^2}} = \frac{-\bar{a_x} - 2\bar{a_z}}{\sqrt{5}}$$