[[Q01]]

Sol $G(s) = \frac{Y(s)}{U(s)} = \frac{e^{-s}}{(s+1)^2}$

$$\mathcal{Z}\left[H_{ZOH}(s)\, G(s) \right] = \mathcal{Z}\left[\frac{1-e^{-sT}}{s} \frac{e^{-s}}{(s+1)^2} \right]$$

$$= (1-z^{-1})(z^{-T})\, \mathcal{Z}\left[\frac{1}{s(s+1)^2} \right]$$

$$\Rightarrow \frac{1}{s(s+1)^2} = \frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s+1)^2}$$

$$\begin{array}{lllll} A(s+1)^2 & \Rightarrow & As^2 & + 2As & + A \\ B\, s(s+1) & & Bs^2 & + Bs & \\ C\, s & & & Cs & \\ \hline & & 0 & 0 & 1 \end{array}$$

$$\left| \begin{array}{l} A = 1 \\ B = -1 \end{array} \right. \qquad \begin{array}{l} 2(1) + (-1) + C = 0 \\ C = -1 \end{array}$$

$$\Rightarrow \frac{1}{s} + \frac{-1}{(s+1)} + \frac{-1}{(s+1)^2}$$

$$= (1-z^{-1})(z^{-T})\, \mathcal{Z}\left[\frac{1}{s} + \frac{-1}{(s+1)} + \frac{-1}{(s+1)^2} \right]$$

$$= (1-z^{-1})(z^{-T}) \left[\frac{1}{1-z^{-1}} + (-1)\frac{1}{1-e^{-T}z^{-1}} + (-1)\frac{T e^{-T} z^{-1}}{(1-e^{-T}z^{-1})^2} \right]$$

$$\mathcal{Z}\left[\frac{1}{s}\right] = \mathcal{Z}[1] = \left[\frac{1}{1-z^{-1}}\right] \quad \textcircled{1}$$

$$\mathcal{Z}\left[\frac{-1}{(s+1)}\right] = \mathcal{Z}\left[(-1)e^{-t}\right] = \left[(-1)\frac{1}{1-e^{-T}z^{-1}}\right] \quad \boxed{2}$$

$$\left[(-1)e^{-nT}\right]$$

$$\mathcal{Z}\left[\frac{-1}{(s+1)^2}\right]$$

$$= \mathcal{Z}\left[(-1)\,t\,e^{-t}\right]$$

$$= \mathcal{Z}\left[(-1)(nT)e^{-nT}\right]$$

$$= (-1)\left[T\frac{e^{-T}z^{-1}}{(1-e^{-T}z^{-1})^2}\right] \quad \boxed{3}$$

	$n=0$	$n=1$	$\cdots$	$n=n$	
①	$e^{-(0)T}z^{-(0)}$	$e^{-T}z^{-1}$		$e^{-nT}z^{-n}$	
① $(e^{-T}z^{-1})$		$e^{-T}z^{-T}$		$e^{-(nT)}z^{-n}$	$e^{-(n+1)T}z^{-(n+1)}$

$$\textcircled{1}\left(1-e^{-T}z^{-1}\right) = 1 - \underset{0}{f}$$

$$\textcircled{1} =$$

	$n=0$	$n=1$		$n=n$
①	$(0T)e^{-0T}z$	$(Te^{-T})z^{-1}$	$\cdots$	$(nTe^{-nT})z^{-n}$
① $e^{-T}z^{-1}$		$(0Te^{-T})$	$\cdots$	$(n-1)Te^{-nT})$
				$(nTe^{-(n+1)T})$

$$\textcircled{1} - \textcircled{1}e^{-T}z^{-1} = 0 + Te^{-T}z^{-1} + \cdots\; Te^{-nT}z^{-1}$$

$$+ \underset{0}{(nT)}\,e^{-(n+1)T}z^{-(n+1)}$$

$$= T\left(e^{-T}z^{-1} + \cdots\; e^{-nT}z^{-n}\right)$$

$$= T\left(\frac{1}{1-e^{-T}z^{-1}} - 1\right)$$

$$\textcircled{1}\left(1-e^{-T}z^{-1}\right) = T\frac{1-(1-e^{-T}z^{-1})}{1-e^{-T}z^{-1}}$$

$$= T\frac{e^{-T}z^{-1}}{1-e^{-T}z^{-1}}$$

$$\textcircled{1}\left(\frac{\times z^2}{\times z^2}\right) = \frac{ze^{-T}}{(z-e^{-T})^2}$$

$$\textcircled{1} = T\frac{e^{-T}z^{-1}}{(1-e^{-T}z^{-1})^2}$$