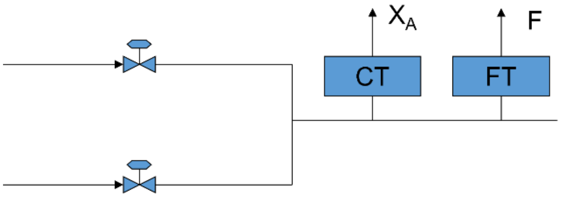
Q01

Problem 1: Blending Problem – Determine the Relative Gain Array Of the following problem



Such that :

$$\text{Steady State}: x_A = \frac{F_A}{F_A + F_B}, F = F_A + F_B$$

$$\begin{pmatrix} m_1 \\ m_2 \end{pmatrix} = \begin{pmatrix} F_A \\ F_B \end{pmatrix}; \quad \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} = \begin{pmatrix} x_A \\ F \end{pmatrix}$$

Ans01

[Q1]

$$m = \begin{bmatrix} F_A \\ F_B \end{bmatrix} \qquad c = \begin{bmatrix} x_A \\ F \end{bmatrix}$$

- For open loop :

We set the matrix, $G \Rightarrow \quad c = Gm$

Therefore, $G = \begin{bmatrix} \left(\frac{\partial y_1}{\partial m_1}\right)_{m_2=0} & \left(\frac{\partial y_1}{\partial m_2}\right)_{m_1=0} \\ \left(\frac{\partial y_2}{\partial m_1}\right)_{m_2=0} & \left(\frac{\partial y_2}{\partial m_2}\right)_{m_1=0} \end{bmatrix} = \begin{bmatrix} \frac{F_B}{(F_A+F_B)^2} & \frac{-F_A}{(F_A+F_B)^2} \\ 1 & 1 \end{bmatrix}$

- For close loop :

We can only make proposition and assume they still obey $y = Gm$.

Here, we can calculate the $\hat{g}_{11}$.

To calculate $\hat{g}_{11}$, we need to find $\left(\frac{\partial y_1}{\partial m_1}\right)_{y_2=0}$

$$\xrightarrow{\text{sub into}} \underset{y=Gm}{} \quad \begin{cases} y_1 = g_{11} m_1 + g_{12} m_2 \\ 0 = g_{21} m_1 + g_{22} m_2 \end{cases}$$

$$\Rightarrow \begin{cases} -\left(g_{21} / g_{22} \right) m_1 = m_2 \\ \hookrightarrow \; y_1 = g_{11} m_1 + g_{22} \left(\dfrac{-g_{21}}{g_{22}} \right) m_1 \end{cases}$$

$$\Rightarrow \; y_1 = \left(g_{11} - \frac{g_{12} g_{21}}{g_{22}} \right) m_1$$

$$\Rightarrow \; \hat{g}_{11} = g_{11} - \frac{g_{12} g_{21}}{g_{22}}$$

Now we can calculate one element of RGA

$$\lambda_{11} = \frac{\text{open loop relative gain}}{\text{close loop} \quad /\!/} = \frac{g_{11}}{\hat{g}_{11}}$$

$$= \frac{1}{1 - \dfrac{g_{12} g_{21}}{g_{11} g_{22}}} = \frac{1}{1 - \dfrac{\left(\frac{-F_A}{(F_A + F_B)^2} \right)}{\left(\frac{F_B}{(F_A + F_B)^2} \right)}} = \frac{1}{1 + \dfrac{F_A}{F_B}}$$

And we know that $\Lambda = \begin{bmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{bmatrix} = \begin{bmatrix} \lambda_{11} & 1 - \lambda_{11} \\ 1 - \lambda_{11} & \lambda_{11} \end{bmatrix}$

Therefore, RGA is : $\Lambda = \begin{bmatrix} \dfrac{F_B}{F} & \dfrac{F_A}{F} \\ \dfrac{F_A}{F} & \dfrac{F_B}{F} \end{bmatrix}$

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Q02

Problem 2: Consider the following two input- two output System, determine the relative gain array

- $y_1(s) = \frac{2}{3s+1} m_1(s) + \frac{5}{s+1} m_2(s)$
- $y_2(s) = \frac{4}{s+1} m_1(s) + \frac{1}{2s+1} m_2(s)$

Ans02

[Q2]

As the Q1, we follow the same process.

- As open loop : $y = Gm$

$$G = \begin{bmatrix} \frac{2}{3s+1} & \frac{5}{s+1} \\ \frac{4}{s+1} & \frac{1}{2s+1} \end{bmatrix}$$

- For close loop :

$$\begin{cases} y_1 = g_{11} m_1 + g_{12} m_2 \\ 0 = g_{21} m_1 + g_{22} m_2 \end{cases}$$

$$y_1 = \left(g_{11} - \frac{g_{12} g_{21}}{g_{22}} \right) m_1$$

$$\hat{g}_{11} = \left(g_{11} - \frac{g_{12} g_{21}}{g_{22}} \right)$$

$$\lambda_{11} = \frac{g_{11}}{\hat{g}_{11}} = \frac{1}{1 - \left(\frac{g_{12}\, g_{21}}{g_{11}\, g_{22}} \right)}$$

$$= \frac{1}{1 - \left(\frac{\left(\frac{5}{s+1}\right)\left(\frac{4}{s+1}\right)}{\left(\frac{2}{3s+1}\right)\left(\frac{1}{2s+1}\right)} \right)} = \frac{1}{1 - 10\left(\frac{(3s+1)(2s+1)}{(s+1)(s+1)} \right)}$$

$$= \frac{s^2+2s+1}{s^2+2s+1 - 10(6s^2+5s+1)} = \frac{s^2+2s+1}{-59s^2 - 58s - 9}$$

$$1 - \lambda_{11} = \frac{-59s^2 - 58s - 9 - (s^2+2s+1)}{-59s^2 - 58s - 9} = \frac{-60s^2 - 60s - 10}{-59s^2 - 58s - 9}$$

$$\Lambda = \begin{bmatrix} \lambda_{11} & 1-\lambda_{11} \\ 1-\lambda_{11} & \lambda_{11} \end{bmatrix}$$

$\divideontimes$

Q03

Problem 3

- Determine the interactive transfer functions of the hot-cold water system in the notes.
- What is the decoupling system of this system?
- Implement the decoupling system by SIMULINK.
- Compare the decoupling and original control system using SIMULINK.

Ans03-01

[Q3]

To decouple, we start from $y = Gm$

Now we want to change m to another controlled var, u.

$m = D u$, here we set $D = \begin{bmatrix} 1 & d_{12} \\ d_{21} & 1 \end{bmatrix}$

for simplicity.

directly effect

ORF $\Rightarrow$ $y = GD u$

if we see GD as a matrix $\hookrightarrow$ $\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} gd_{11} & gd_{12} \\ gd_{21} & gd_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

$\Rightarrow$ What we expect is that $\left[\begin{array}{l} y_1 \leftarrow u_1 \\ y_2 \leftarrow u_2 \end{array} \right.$ (1.1/2.2 pairing)

therefore,

$$y_1 = gd_{11} u_1 + gd_{12} u_2$$
$$y_2 = gd_{21} u_1 + gd_{22} u_2$$

$\rightsquigarrow$ $gd_{12} = 0$, $gd_{21} = 0$

$$\Rightarrow GD = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} 1 & d_{12} \\ d_{21} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} (g_{11} + g_{12} d_{21}) & (g_{11} d_{12} + g_{12}) \\ (g_{21} + g_{22} d_{21}) & (g_{21} d_{12} + g_{22}) \end{bmatrix}$$

$$gd_{12} = g_{11} d_{12} + g_{12} = 0 \qquad d_{12} = \left(\frac{-g_{12}}{g_{11}} \right)$$

$$gd_{21} = g_{21} + g_{22} d_{21} = 0 \qquad d_{21} = \left(\frac{-g_{21}}{g_{22}} \right)$$

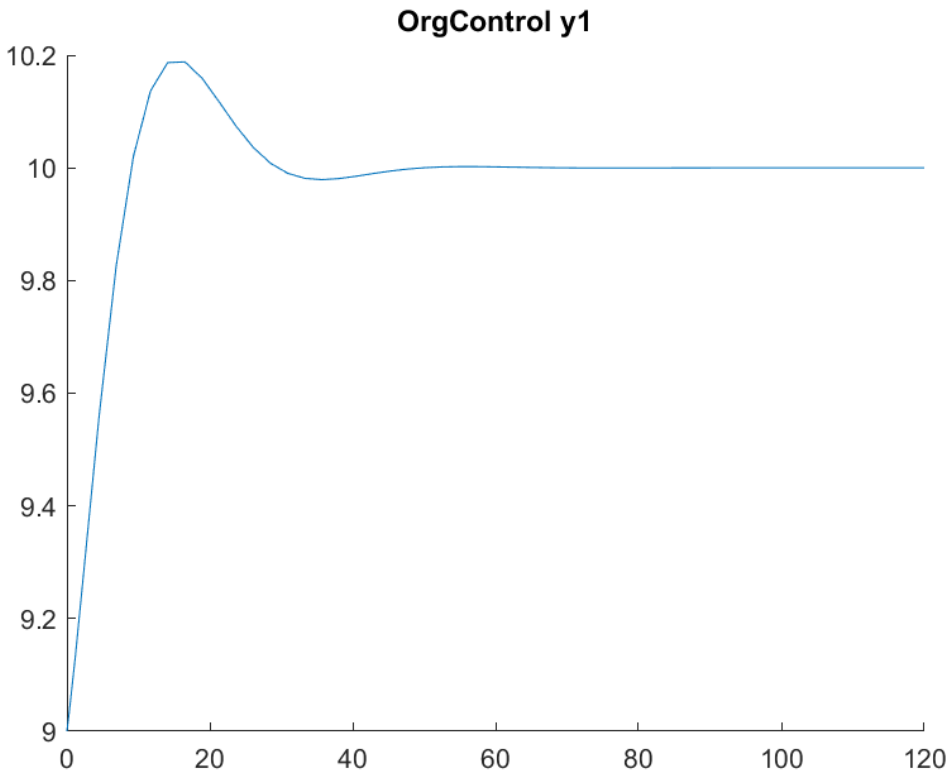
$$D = \begin{bmatrix} 1 & \left(\frac{-g_{12}}{g_{11}} \right) \\ \left(\frac{-g_{21}}{g_{22}} \right) & 1 \end{bmatrix}$$

Ans03-02

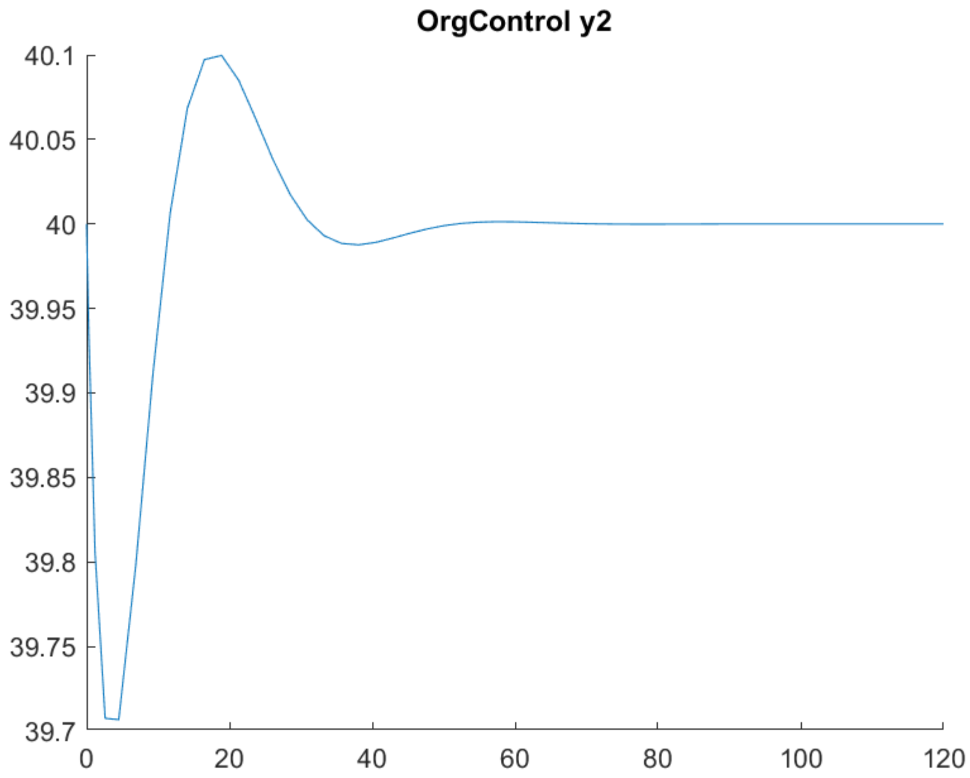
Original control

- First we see the original PID control to the process with no decoupling.

- Org Control to variable (y1)



- Org Control to variable (y2)



Approximate ODE process with transfer function

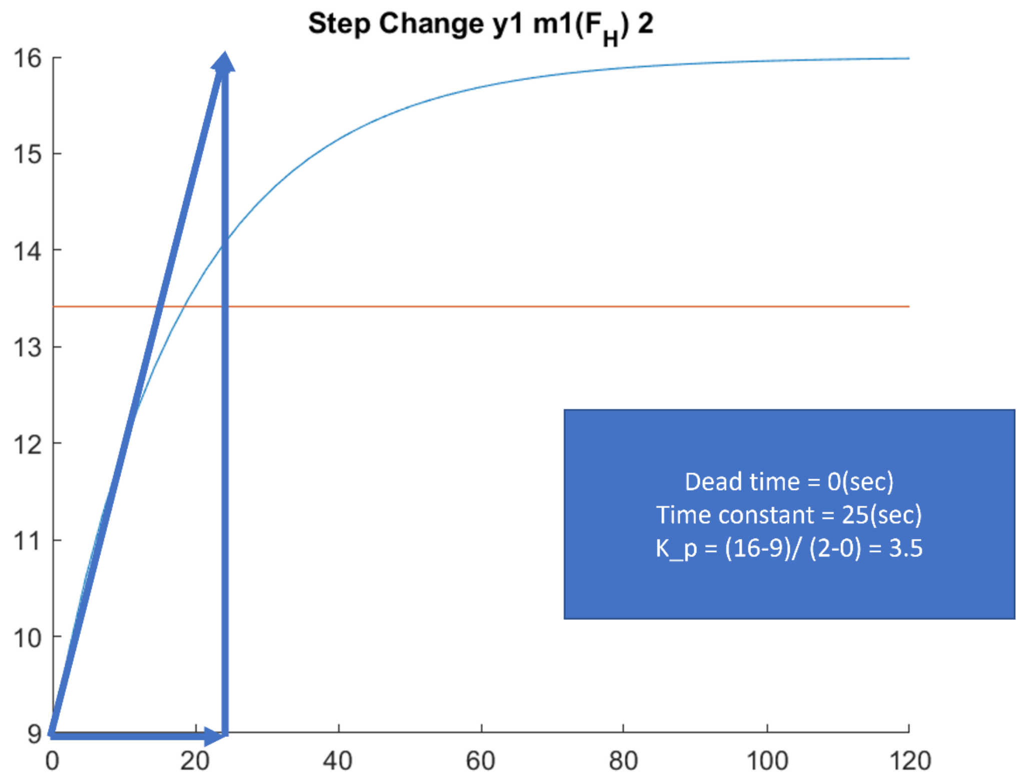
- We could approximate ethe ODE process to transfer function with FOPDT method.

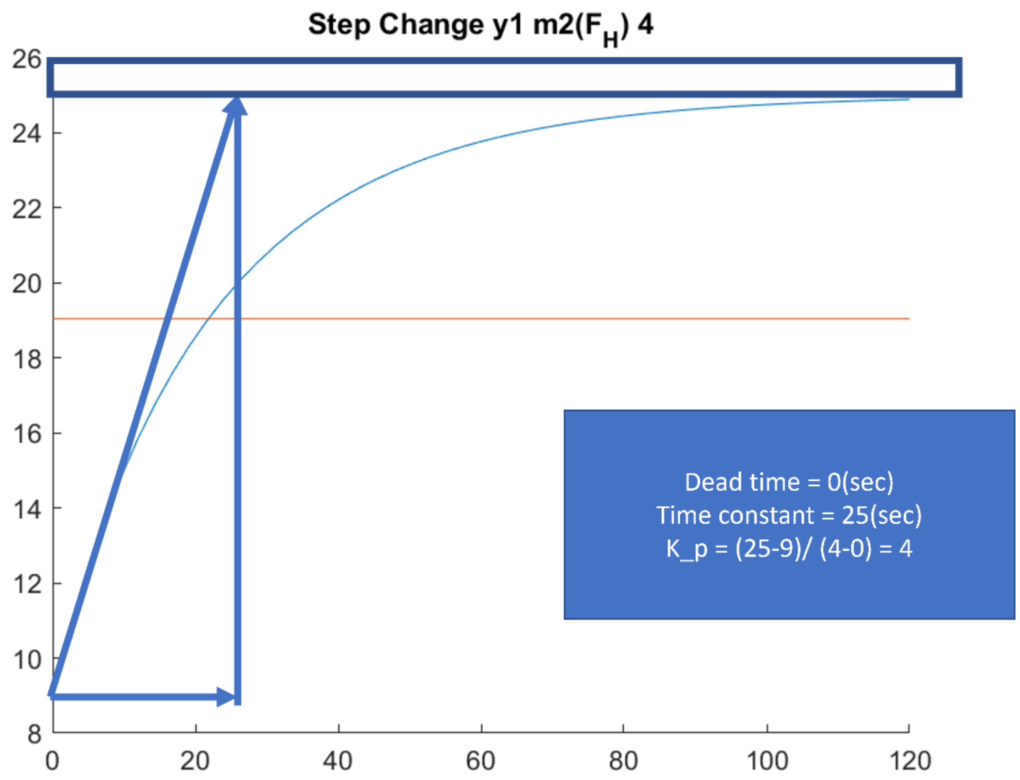
According to the formula:

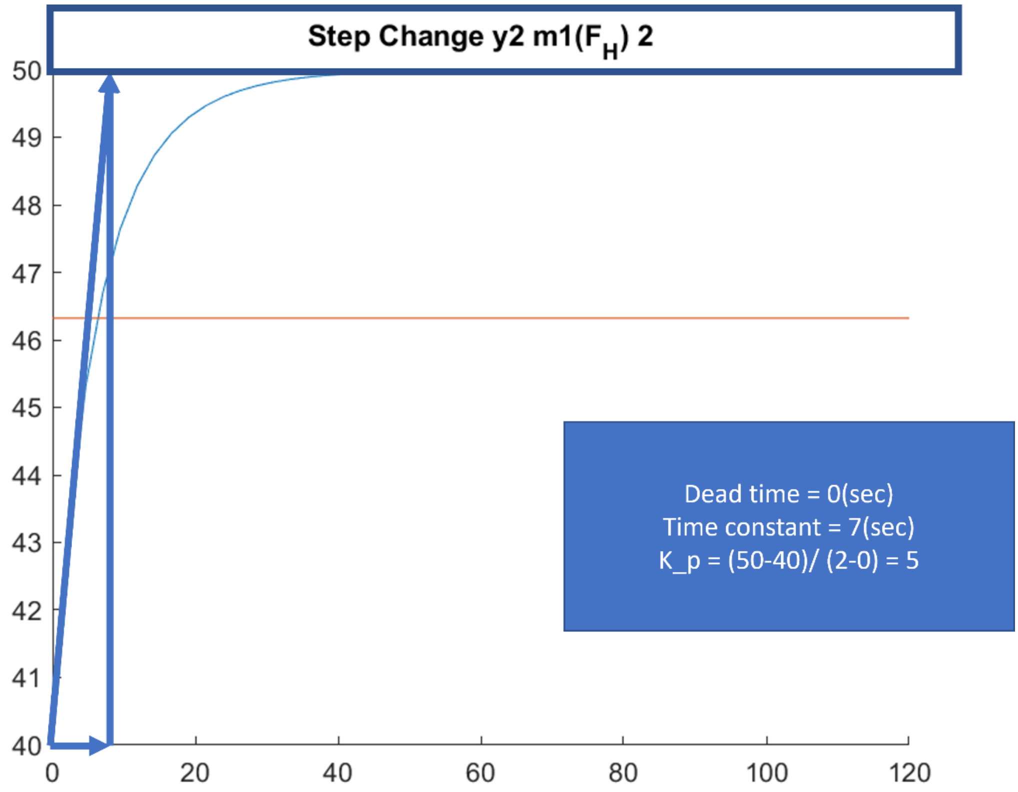
$$Transfer\ func = \frac{\Delta y}{\Delta u} \simeq \frac{Ke^{-\theta_s}}{\tau s + 1}$$

Where

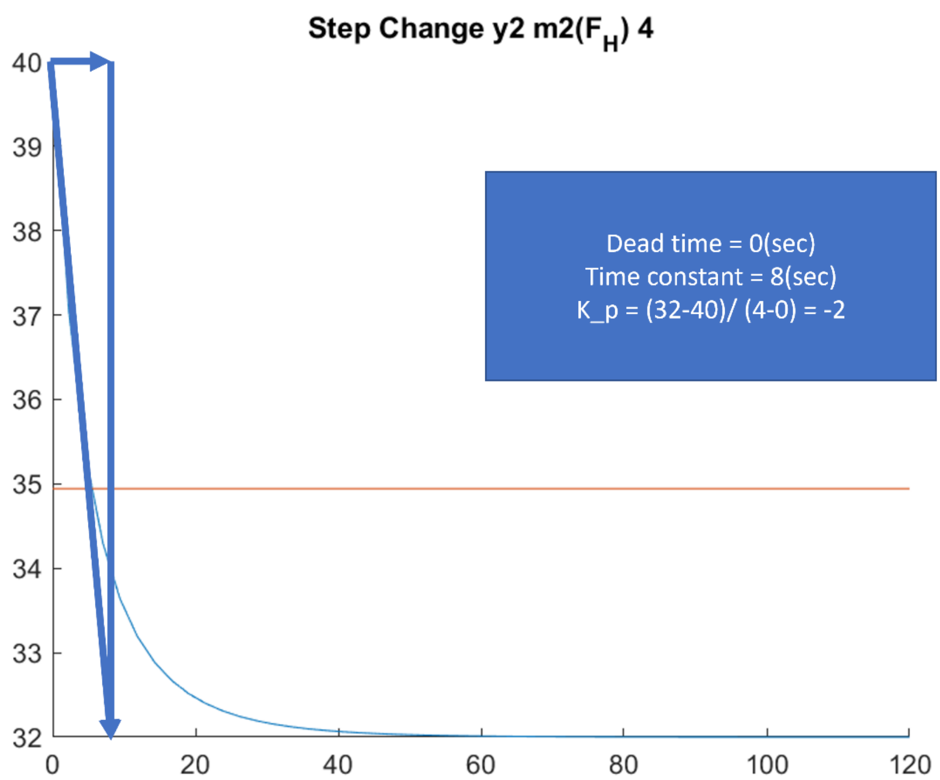
K	Change of CV divide by change of MV
τ	Time constant
θ_s	Process time delay


Step Change y1 m1(F_H) 2
16
15
14
13
12
11
10
9
0
20
40
60
80
100
120
Dead time = 0(sec)
Time constant = 25(sec)
K_p = (16-9)/ (2-0) = 3.5


Step Change y1 m2(F_H) 4
26
24
22
20
18
16
14
12
10
8
0
20
40
60
80
100
120
Dead time = 0(sec)
Time constant = 25(sec)
K_p = (25-9)/ (4-0) = 4



Step Change y2 m1(F_H) 2
50
49
48
47
46
45
44
43
42
41
40
0
20
40
60
80
100
120
Dead time = 0(sec)
Time constant = 7(sec)
K_p = (50-40)/ (2-0) = 5



Step Change y2 m2(F_H) 4
40
39
38
37
36
35
34
33
32
0
20
40
60
80
100
120
Dead time = 0(sec)
Time constant = 8(sec)
K_p = (32-40)/ (4-0) = -2

- Therefore, with a little bit adjustment, we can get the transfer function which can almost describe the original ODE process.
- Transfer function: (may different from different people)

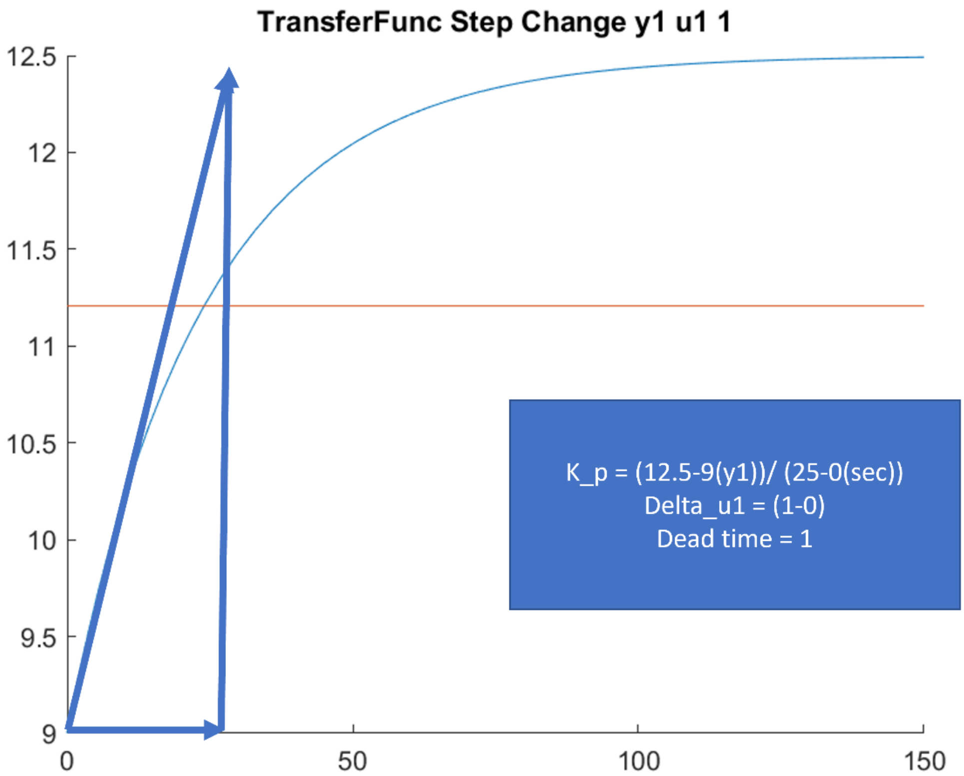
g_{11}	$g_{11} = \dfrac{1\exp(-0)}{25s+1}$
g_{12}	$g_{12} = \dfrac{1\exp(-0)}{25s+1}$
g_{21}	$g_{21} = \dfrac{5\exp(-0)}{7s+1}$
g_{22}	$g_{22} = \dfrac{-2\exp(-0)}{8s+1}$

- Now we have the transfer function for each controlled variable, and manipulated variable. Decoupling can be done by the above formula. In this case, we assume that the decoupling matrix, D, is

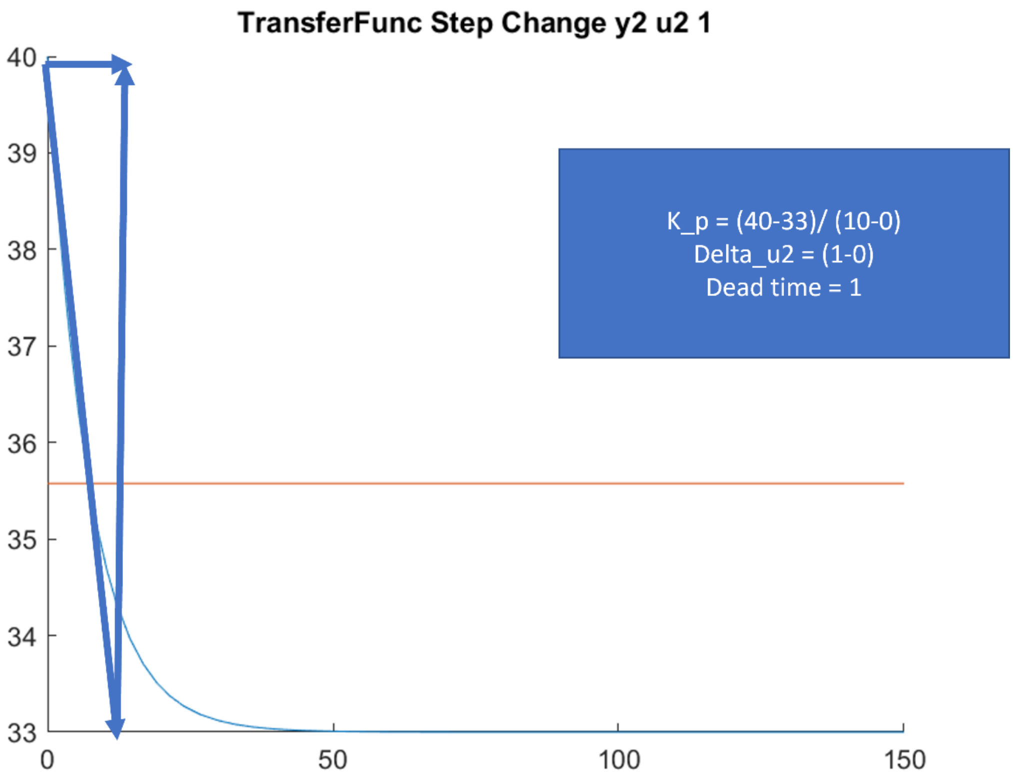
$$D = \begin{bmatrix} 1 & d_{12} \\ d_{21} & 1 \end{bmatrix}$$

- Therefore, we can draw the reaction curve for each decoupling manipulated variable, u_*, and determine the coefficient of PID controller.

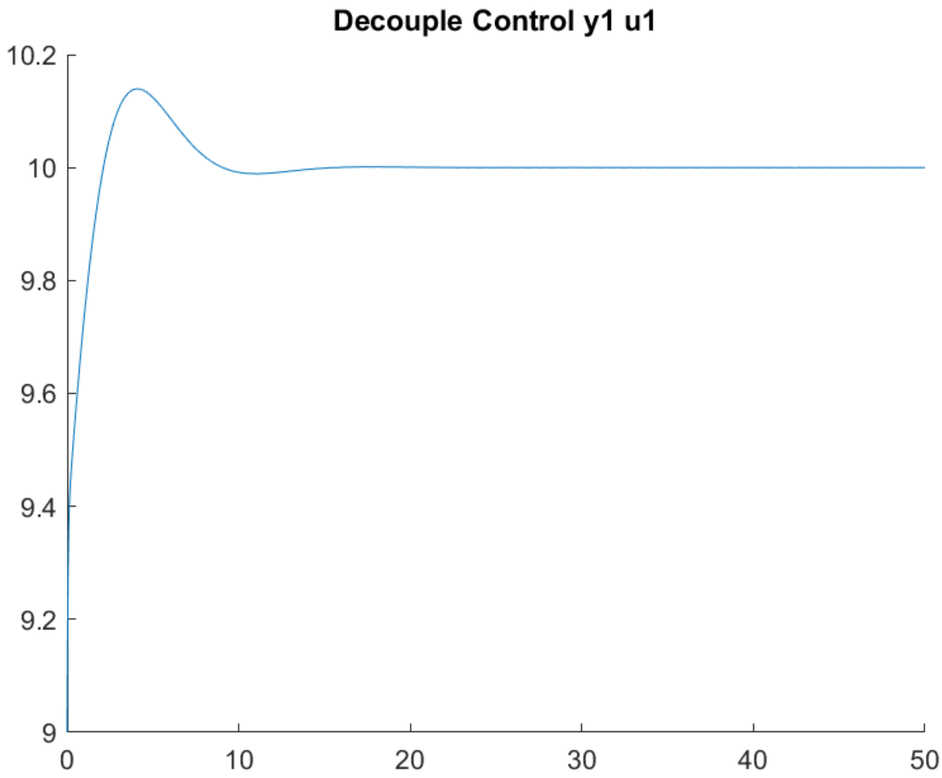
- Use Decouple variable, u_1, to control y1, h.

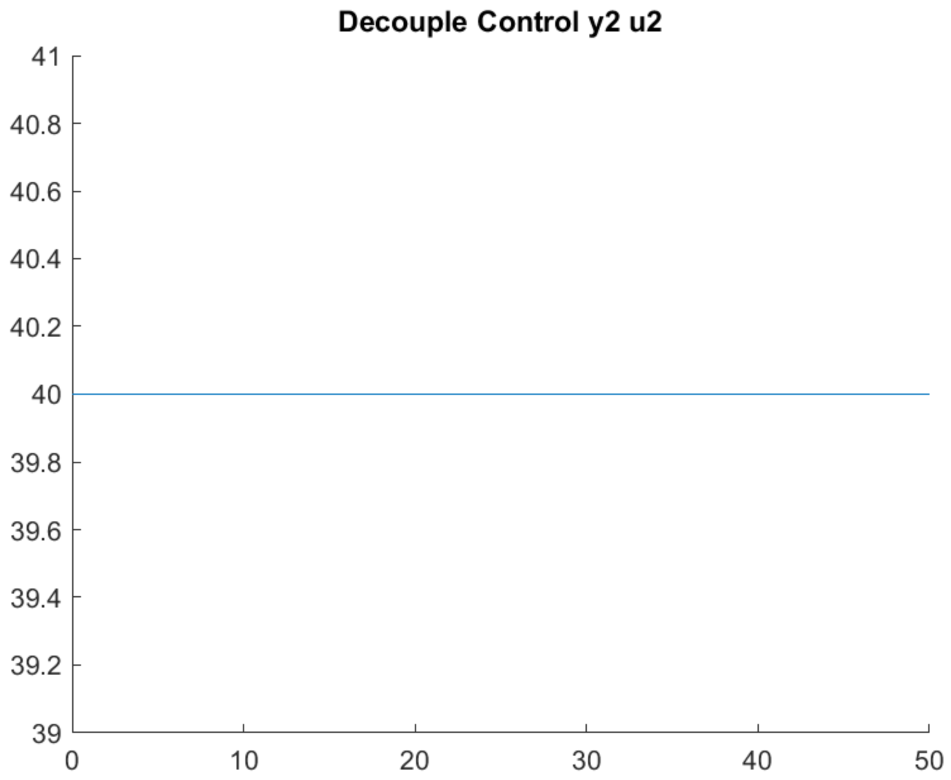


- Use Decouple variable, u_2, to control y2, T_{out}.



- Finally, we can implement the control system with decouple.
- We can see the control system is obvious better than the original control system.





Decouple Control y2 u2
41
40.8
40.6
40.4
40.2
40
39.8
39.6
39.4
39.2
39
0
10
20
30
40
50