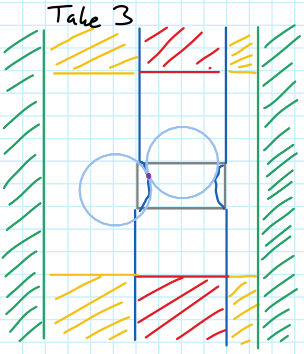
Take 3



$$\int_{\mathbb{R}^n} \frac{\chi_{E^c}-\chi_E}{|y-q|^{n+s}}\,dy \geq 0 \qquad \text{Contradiction } LHS<0$$

Ball-Sliding

$B_{r_0}(te_n) \subset E \quad \forall t > t_0$

$\partial B_{r_0}(t_0 e_n) \cap \partial E \neq \emptyset$

....

$= \{(x', x_n) \text{ s.d. } |x'-q'| > R+1\}$

$= \{(x', x_n) \text{ s.d. } |x'| < R, \ |x_n - q_n| > 2\mu\}$

$= \{(x', x_n) \text{ s.d. } |x'| \geq R, |x'-q'| \leq R+1, |x_n - q_n| > 2\mu\}$

$= \{(x', x_n) \text{ s.d. } |x'-q'| \leq R+1, \ |x_n - q_n| \leq 2\mu\}$

1. $\displaystyle \int \frac{\chi_{E^c}-\chi_E}{|y-q|^{n+s}} \overset{\subset E^c}{=} \int \frac{1}{|y-q|^{n+s}} = \int_{|y'|>2R} \frac{1}{|y|^{n+s}} \leq \int_{|y'|>R+1} \frac{1}{|y'|^{n+s}} \leq c(n)\int_{R+1}^{\infty} r^{-2-s} = -\frac{c(n)}{1+s}\left[\frac{1}{r^{1+s}}\right]_{R+1}^{\infty} = c(n,s)\,(R+1)^{-(1+s)} \leq R^{-(1+s)}$

2. $\displaystyle \int \frac{\chi_{E^c}-\chi_E}{|y-q|^{n+s}} \overset{\subset E}{=} -\int \frac{1}{|y-q|^{n+s}} \leq -c(n,s)\,\mu^{-s}$ Idea: Consider the ball with 2^{-n}

3. $\displaystyle \int \frac{\chi_{E^c}-\chi_E}{|y-q|^{n+s}} \leq \int \frac{1}{|y-q|^{n+s}} = \int_{\substack{|y'|\leq R+1 \\ |y_n|>2\mu}} \frac{1}{|y|^{n+s}} = \int_{|y_n|>2\mu}\int_0^{R+1} \frac{r^{n-2}}{(r^2+y_n^2)^{\frac{n+s}{2}}}\,dr\,dy \overset{\text{Sedrakyan's Lemma}}{\leq} \int_{|y_n|>2\mu}\int_0^{R+1} \frac{2\,r^{n-2}}{(r+y_n)^{n+s}}\,dr\,dy$

$\displaystyle \leq 4\int_0^{R+1}\left[\frac{r^{n-2}}{-(n+s)+1}(r+y_n)^{-(n+s)+1}\right]_{2\mu}^{\infty} = c(n,s)\int_0^{R+1}\frac{r^{n-2}}{(r+2\mu)^{n+s-1}}$

$\displaystyle \leq c(n,s)\int_0^{R+1}\frac{1}{(r+2\mu)^{1+s}} = -\frac{c(n,s)}{s}\left[(r+2\mu)^{-s}\right]_0^{R+1} = c(n,s)\left(\mu^{-s}-(R+\mu)^{-s}\right)$

Don't like this term, need better upper bound

3. $\displaystyle \int \frac{\chi_{E^c}-\chi_E}{|y-q|^{n+s}}\,dy \overset{\subset E^c}{=} \int \frac{1}{|y-q|^{n+s}}\,dy = \int_{\substack{R-1\leq|y'|\leq R+1 \\ |y_n|>2\mu}} \frac{1}{|y|^{n+s}} = c(n)\int_{R-1}^{R+1}\int_{2\mu}^{\infty}\frac{r^{n-2}}{(r^2+y_n^2)^{\frac{n+s}{2}}}$

$r^2 \leq r^2+y_n^2$ $\displaystyle \leq c(n)\int_{R-1}^{R+1}\int_{2\mu}^{\infty}\frac{1}{(r^2+y_n^2)^{\frac{s+2}{2}}} \overset{\text{convexity}}{\leq} \int_{R-1}^{R+1}\int_{2\mu}^{\infty}\frac{1}{(r+y_n)^{s+2}} \leq c(n,s)\int_{R-1}^{R+1}\frac{1}{(r+\mu)^{s+1}} \leq c(n,s)\,(R-1+\mu)^{-s}$

4. $= S := \{(x', x_n) \text{ s.d. } |x-q'| \leq R+1, \ |x_n-q_n| \leq 2\mu\}$

Split S into 4 Parts

i) $S \cap B_{\Delta\mu} \cap B_{r_0}(z)$

ii) $S \cap B_{\Delta\mu} \cap B_{r_0}(\bar{z})$

Cancel as symmetric

iii) $S \cap (B_{\Delta\mu} \setminus (B_{r_0}(z) \cup B_{r_0}(\bar{z})))$

iv) $S \setminus B_{\Delta\mu}$

Choose Δ, μ s.t. $\Delta\mu \leq 1$

iv) $\displaystyle \int_{S\setminus B_{\Delta\mu}} \frac{\chi_{E^c}-\chi_E}{|y-q|^{n+s}} \leq \int_{\substack{\{\Delta\mu\leq|y'|\leq R+1, \\ |y_n|\leq 2\mu\}}} \frac{1}{|y'|^{n+s}} \leq c(n)\int_0^{2\mu}\int_{\Delta\mu}^{R+1}\frac{1}{r^{2+s}}\,dr \leq c(n,s)\,\mu\left((\Delta\mu)^{-(1+s)} - (R+1)^{-(1+s)}\right)$

$\displaystyle \leq c(n,s)\left(\frac{1}{\Delta^{1+s}\mu^s} - \frac{\mu}{R^{-(1+s)}}\right)$

iii) $\displaystyle \int \frac{\chi_{E^c}-\chi}{|y-q|^{n+s}} \leq \int_{P_{1,\Delta\mu}} \frac{1}{|y-q|^{n+s}} \leq C\,\Delta^{1-s}\mu^{1-s}$ If $R=r_0=1$ and $\lambda = \Delta\mu \leq 1$

else $\cdot)$ $\Delta\mu\leq 1$ $\displaystyle \int \frac{1}{|y-q|^{n+s}} \leq \int_{P_{r_0,\Delta\mu}} \frac{1}{|y|^{n+s}} + \int_{B_{\Delta\mu}\setminus B_{r_0\Delta\mu}} \frac{1}{|y|^{n+s}} \leq c(n,s)\,r_0^{-s}(\Delta\mu)^{1-s} + c(n,s)\left((r_0\Delta\mu)^{-s} - (\Delta\mu)^{-s}\right)$

$\cdot)$ $\Delta\mu > 1$ $\displaystyle \int \frac{1}{|y-q|^{n+s}} \leq \int_{P_{r_0,1}} \frac{1}{|y|^{n+s}} + \int_{B_{\Delta\mu}\setminus B_1} \frac{1}{|y|^{n+s}} \leq c(n,s)\,r_0^{-s} + c(n,s)\left(1-(\Delta\mu)\right)^{-s}$

Interessting r_0^{-s} $\Rightarrow$ stricter bound on μ, μ needs to be smaller

Thus in total

$R=r_0=1$ $\cdot)$ $\displaystyle\int_{\mathbb{R}} \frac{\chi_{E^c}-\chi_E}{|x-y|^{1+s}} \leq -c_1\mu^{-s} + C_0\left(R^{-(1+s)} + (R-1+\mu)^{-s} + \Delta^{-(1+s)}\mu^{-s} - \mu R^{-(1+s)} + \Delta^{1-s}\mu^{1-s}\right)$

$\displaystyle\leq -c_1\mu^{-s}\left(1 - \frac{C_0}{c_1}\left(R^{-(1+s)}\mu^s + \frac{\mu^s}{(R-1+\mu)^s} + \Delta^{-(1+s)} - \mu^{1+s}R^{-(1+s)} + \Delta^{1-s}\mu\right)\right.$

choose μ small $\leq -c_2\mu^{-s}$

If $R=r_0=1$ and $\Delta\mu\leq 1$ $\Rightarrow$ $\exists\,\mu_0$ s.t. $\forall\,\mu\leq\mu_0$ $E_\mu = E_0 \cup \Omega$

Now look at behavior of E for μ s.t. $\}\{$ but $]\,[$