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## Homework 2

### Exercise 1

Fill in the following table by using  $\checkmark$  when the set is closed, and  $\times$  when it is not:

$(-1, 2]$	$(-1, 1)$	$[-1, 1]$	$\mathbb{R} \setminus \{1\}$	$\{1, 2, 3\}$	$\mathbb{R} \setminus (0, 1)$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R} \setminus \mathbb{Q}$	$\mathbb{R}$
$\times$	$\times$	$\checkmark$	$\times$	$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\times$	$\times$

### Exercise 2

Fill in the following table by using  $\checkmark$  when the is a neighbourhood of 1, and  $\times$  when it is not:

$(-1, 2]$	$(-1, 1)$	$[-1, 1]$	$\mathbb{R} \setminus \{1\}$	$\{1, 2, 3\}$	$\mathbb{R} \setminus (0, 1)$	$\mathbb{Z}$	$\mathbb{Q}$	$\mathbb{R} \setminus \mathbb{Q}$	$\mathbb{R}$
$\checkmark$	$\checkmark$	$\checkmark$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$	$\times$

### Exercise 3

Which of the following sets either open or closed? Try to sketch some proofs.

$$A = \bigcup_{n \in \mathbb{N} \setminus \{1\}} \left(-1 + \frac{1}{n}, 1 - \frac{1}{n}\right), \quad B = \bigcup_{n \in \mathbb{N}} \left[-1 + \frac{1}{n}, 1 - \frac{1}{n}\right]$$

$$C = \bigcap_{n \in \mathbb{N} \setminus \{1\}} \left(-1 + \frac{1}{n}, 1 - \frac{1}{n}\right), \quad D = \bigcap_{n \in \mathbb{N}} \left[-1 + \frac{1}{n}, 1 - \frac{1}{n}\right]$$

$$E = \bigcup_{n \in \mathbb{N}} \left[-1 - \frac{1}{n}, 1 + \frac{1}{n}\right], \quad F = \bigcap_{n \in \mathbb{N}} \left(-1 - \frac{1}{n}, 1 + \frac{1}{n}\right)$$

$A$  - open  $\Rightarrow A = (-1 + 0^+, 1 - 0^-)$   $B = \{0\}$

$B$  - closed  $\Rightarrow B = (-1, 1)$

$C$  - open  $\Rightarrow C = \left(-\frac{1}{2}, \frac{1}{2}\right)$

$E$  - closed  $\Rightarrow E = [-2, 2]$

$F$  - open  $\Rightarrow F = (-1, 1)$



Exercise 4 Fill in the following table:

Nr.	A	int A	bd A	cl A	ext A	Izo A	A'
1	$(-\infty, -4] \cup (2, 5)$	$(-\infty, -4) \cup (2, 5)$	$\{-4, 5\}$	$(-\infty, -4] \cup [2, 5]$	$(-4, 2] \cup [5, \infty)$	$\emptyset$	$\text{cl } A$
2	$(-1, 9] \cup [10, \infty)$	$(-1, 9) \cup (10, \infty)$	$\{9, 10\}$	$[-1, 9] \cup [10, \infty)$	$(-\infty, -2) \cup (3, 10)$	$\emptyset$	$\text{cl } A$
3	$((-1, 9] \cup [10, 20)) \cap \mathbb{N}$	$(1, 20)$	$\{1, 20\}$	$[1, 20]$	$(-\infty, 2) \cup (3, 10)$	$\emptyset$	$\text{cl } A$
4	$\{1, 2, 3\}$	$\{2\}$	$\{1, 3\}$	$A$	$\mathbb{R} \setminus A$	$\emptyset$	$\text{cl } A$
5	$\mathbb{N}$	$A$	$\mathbb{Z}$	$\mathbb{Z}$	$\mathbb{R} \setminus \mathbb{N}$	$\emptyset$	$\mathbb{Z}$
6	$\mathbb{R} \setminus \{1, 2, 3\}$	$A$	$\emptyset$	$(-\infty, 2] \cup [3, \infty)$	$\{1, 2, 3\}$	$\emptyset$	$\text{cl } A$
7	$\mathbb{R} \setminus \mathbb{N}$	$(-\infty, 0) \cup \mathbb{R} \setminus \mathbb{Z}$	$\emptyset$	$(-\infty, 0]$	$\mathbb{N}$	$\emptyset$	$(-\infty, 0]$
8	$\mathbb{Z}$	$(-\infty, \infty)$	$\mathbb{Q}$	$\mathbb{Q}$	$\mathbb{R} \setminus \mathbb{Z}$	$\emptyset$	$\mathbb{Q}$
9	$\mathbb{R} \setminus \mathbb{Z}$	$A$	$\emptyset$	$\emptyset$	$\mathbb{Z}$	$\emptyset$	$\emptyset$
10	$\mathbb{Q}$	$\emptyset$	$\mathbb{R}$	$\mathbb{R}$	$\emptyset$	$\emptyset$	$\mathbb{R}$
11	$\mathbb{R} \setminus \mathbb{Q}$	$\emptyset$	$A$	$A$	$\mathbb{R} \setminus (\mathbb{R} \setminus \mathbb{Q})$	$A$	$\emptyset$
12	$\mathbb{R}$	$\mathbb{R}$	$\emptyset$	$\mathbb{R}$	$\emptyset$	$\emptyset$	$\mathbb{R}$