Homework

1)

a) $x_n = \frac{4^n+6^n}{5^n}$; b) $x_n = \frac{(-1)^n}{n}$; c) $x_n = \frac{9^n}{n!}$; d) $x_n = \frac{n}{n^2+1}$

a) monotonicity: $\cancel{\frac{X_{n+1}}{X_n} = \frac{4^{n+1}+6^{n+1}}{5^{n+1}} \cdot \frac{5^n}{4^{n}+6^n}}$

$$\frac{4^n}{5^n} + \frac{6^n}{5^n} = \left(\frac{4}{5}\right)^n + \left(\frac{6}{5}\right)^n$$

$$\frac{X_{n+1}}{X_n} = \frac{\left(\frac{4}{5}\right)^{n+1} + \left(\frac{6}{5}\right)^{n+1}}{\left(\frac{4}{5}\right)^n + \left(\frac{6}{5}\right)^n} = \frac{\left(\frac{4}{5}\right)\left(\frac{4}{5}\right)^n + \left(\frac{6}{5}\right)\cdot\left(\frac{6}{5}\right)^n}{\left(\frac{4}{5}\right)^n + \left(\frac{6}{5}\right)^n} > 1 \Rightarrow$$

$\boxed{X_n \text{ is increasing.}}$ (1)

convergence:

$$\lim_{n\to\infty}\left[\underbrace{\left(\frac{4}{5}\right)^n}_{0} + \underbrace{\left(\frac{6}{5}\right)^n}_{\infty}\right] = \infty \quad (2)$$

boundedness: $LB(n) = 0$

$UB(n)$ = not bounded.

b) $x_n = \frac{(-1)^n}{n}$

– Not monotonic since it switches from negative to positive for each term.

– Boundedness: $|x_n| = \left|\frac{(-1)^n}{n}\right| = \frac{1}{n}$ – bounded

– Convergence: ?

3).

a) ~~$\lim_{n\to\infty} \frac{3^n+1}{5^n+1} = \frac{3}{5}$~~

$$x_n = \frac{3^n+1}{5^n+1} = \frac{3^n}{5^n} + \frac{1}{5^n+1} = \underbrace{\left(\frac{3}{5}\right)^n}_{<1} + \underbrace{\frac{1}{5^n+1}}_{\Rightarrow 0} \Rightarrow \lim_{n\to\infty} x_n = 0.$$

~~b)~~

a) $\lim_{n\to\infty} \frac{3^n+1}{5^n+1} = \underbrace{\frac{3^n}{5^n}}_{\to 0} + \underbrace{\frac{1}{5^n+1}}_{\Rightarrow 0} = 0$

b) $\lim_{n\to\infty} \left(\frac{9^n+(-3)^n}{9^{n-1}+3}\right) = \lim_{n\to\infty} \frac{9^{n-1}\cdot 9+(-3)^n}{9^{n-1}\left(1+\frac{3}{9^{n-1}}\right)} = ?$

c) $\left(\sin\frac{\pi}{10}\right)^n$; $\lim_{n\to\infty} \left(\sin\frac{\pi}{10}\right)^n = 0$,

d) $\lim_{n\to\infty}\left(\sqrt{4n^2+2n+1} - 2n\right) = \lim_{n\to\infty} \frac{\cancel{4n^2}+2n+1-\cancel{4n^2}}{\sqrt{4n^2+2n+1}+2n}$

$$= \lim_{n\to\infty} \frac{2n+1}{\sqrt{4n^2+2n+1}+2n} = \lim_{n\to\infty} \frac{\cancel{n}(2+\frac{1}{n})}{\cancel{n}\left(\sqrt{4+\underbrace{\frac{2}{n}}_{\to 0}+\underbrace{\frac{1}{n^2}}_{\to 0}}+2\right)} = \frac{2}{\sqrt{4}+2} = \frac{2}{4} = \frac{1}{2}.$$

e) $\lim\limits_{n\to\infty}\left(7+\dfrac{1-2n^3}{3n^4+2}\right)^2 \Rightarrow \cancel{\lim\limits_{n\to\infty}}$

$=\lim\limits_{n\to\infty}\left(7+\dfrac{n^3\left(\overset{\to 0}{\frac{1}{n^3}}-2\right)}{n^4\left(3+\frac{2}{n^4}\right)}\right)^2=\lim\limits_{n\to\infty}\left(\cancel{7}+\dfrac{-2}{3n}\right)^2$

$\Rightarrow\lim\limits_{n\to\infty}\left(7-\underset{\to 0}{\dfrac{2}{3n}}\right)^{2}=49.$

~~f) $\sqrt[3]{n^3+n+3}-\sqrt[3]{n^3+1}$~~

~~$\lim\limits_{n\to\infty}\sqrt[3]{n^3+n+3}-\sqrt[3]{n^3+1}$~~

g) $\lim\limits_{n\to\infty}\left(\dfrac{n^3+5n+1}{n^2-1}\right)^{\frac{1-5n^4}{6n^4+1}}$

$\lim\limits_{n\to\infty}\dfrac{n^3+5n+1}{n^2-1}=\cancel{\lim\limits_{n\to\infty}n^3\left(1+\frac{5n}{n^3}\right)}$

$=\lim\limits_{n\to\infty}\dfrac{n^3\left(1+\overset{\to 0}{\frac{5}{n^2}}+\overset{\to 0}{\frac{1}{n^3}}\right)}{n^2\left(1-\underset{\to 0}{\frac{1}{n^2}}\right)}=\infty$

$\lim\limits_{n\to\infty}\dfrac{1-5n^4}{6n^4+1}=\lim\limits_{n\to\infty}\dfrac{n^4\left(\overset{\to 0}{\frac{1}{n^4}}-5\right)}{n^4\left(6+\underset{\to 0}{\frac{1}{n^4}}\right)}=\dfrac{-5}{4}$

$\Rightarrow\lim\limits_{n\to\infty}(\infty)^{-\frac{5}{4}}=0$