Zusammenfassung generell

31 May 2024 15:14

Integrationstafel :

$x^{-n} = \frac{1}{x^n}$ $\Rightarrow \frac{1}{\sqrt[a]{x}} = x^{-\frac{1}{a}}$

$\sqrt[n]{x^m} = x^{\frac{m}{n}}$

$(x^n)^m = x^{n \cdot m}$

$x^n \cdot x^m = x^{n+m}$

$\frac{x^n}{x^m} = x^{n-m}$

$4! = 1 \cdot 2 \cdot 3 \cdot 4$

$n! = 1 \cdot 2 \cdot 3 \cdot \ldots \cdot n$

$\log(a^n) = n \cdot \log(a)$

$\log_b(a) = \frac{\log(a)}{\log(b)}$

$\ln(x \cdot y) = \ln(x) + \ln(y)$

$\ln(1) = 0$, $\ln(0) = -\infty$, $\ln(e) = 1$

$a = \ln(x) \Rightarrow e^a = x$, $e^{\ln(x)} = x$

$(a+b)^4 = a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$

$(a+b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

$e^{ax} = b \Rightarrow ax = \ln(b)$

$0! = 1$

$x^{-\infty}, x > 1 = 0$

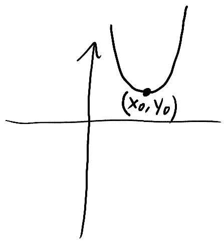
$x^{-\infty}, x < 1 = -\infty$

Trionometrischer Pythagoras x§

$\sin^2(x) + \cos^2(x) = 1$

Scheitelpunktform $y = a(x - x_0)^2 + y_0$

Parabel $y = ax^2 + b$



sin cos tan Chart S.91...

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RAD!

θ	0° (or) 0	30° (or) $\frac{\pi}{6}$	45° (or) $\frac{\pi}{4}$	60° (or) $\frac{\pi}{3}$	90° (or) $\frac{\pi}{2}$
sin θ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan θ	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Not Defined

Ableiten

$f(x) = \frac{df}{dx}$

$f(x) = 2x^5 - 4x^3 - 2$

$f'(x) = 2 \cdot 5x^{5-1} - 4 \cdot 3x^{3-1} - 2 \cdot 0x$

$= 10x^4 - 12x^2$

- $(\sin(x))' = \cos(x)$
- $(\cos(x))' = -\sin(x)$
- $(e^x)' = e^x$
- $(a^x)' = a^x \cdot \ln(a)$
- $(\ln(x))' = \frac{1}{x}$
- $(\log_a(x))' = \frac{1}{x \cdot \ln(a)}$

(S. 132)

- Taktor regel :

$(c \cdot f)' = c \cdot f'(x) = c \cdot k \cdot x^{k-1}$

Bsp: $4x^3 = 4 \cdot 3x^{3-1} = 12x^2$

$(e^{a \cdot x})' = a \cdot e^{a \cdot x}$

$(\ln(|x|))' = \frac{1}{x}$ $\Rightarrow \int \frac{1}{x} = \ln(|x|) + c$

$(\sin(x^a))' = a \cdot x^{(a-1)} \cdot \cos(x^a)$

Summenregel :

$(f+g)'(x) = f'(x) + g(x)'$

- Produktregel :

$(u \cdot v)'(x) = u'(x) \cdot v(x) + u(x) \cdot v'(x)$

Bsp: $f(x) = (3x^3 + x^2)(4x^2 + 1)$

$u = 3x^3 + x^2$ $v = 4x^2 + 1$

- Quotientenregel :

$\left(\frac{u}{v}\right)'(x) = \frac{u'(x) \cdot v(x) - u(x) \cdot v'(x)}{(v(x))^2}$

- Kettenregel :

$(f \circ g)'(x) = f'(u) \cdot u'(x)$ $\left(\frac{1}{(1-7x)}\right)' = (1-7x)^{-1} \Rightarrow u = 1-7x \Rightarrow u' = -7$

$\Rightarrow -1(u)^{-2} = -1(1-7x)^{-2} \cdot (-7) = 7(1-7x)^{-2}$

Grenzwerte

(1) $\lim_{n\to\infty}(c \cdot a_n) = c \cdot \lim_{n\to\infty} a_n$

(2) $\lim_{n\to\infty}(a_n + b_n) = \lim_{n\to\infty}(a_n) + \lim_{n\to\infty}(b_n)$, $\quad \lim_{n\to\infty}(a_n - b_n) = \lim_{n\to\infty}(a_n) - \lim_{n\to\infty}(b_n)$

(3) $\lim_{n\to\infty}(a_n \cdot b_n) = \lim_{n\to\infty}(a_n) \cdot \lim_{n\to\infty}(b_n)$

(4) $\lim_{n\to\infty}(\frac{a_n}{b_n}) = \frac{\lim_{n\to\infty}(a_n)}{\lim_{n\to\infty}(b_n)}$, falls $\lim_{n\to\infty}(b_n) \neq 0$ und $b_n \neq 0$ für alle n.

(5) $\lim_{n\to\infty}(a_n^{\,k}) = (\lim_{n\to\infty}(a_n))^k$ für $k \in \mathbb{N} \setminus \{0\}$

(6) $\lim_{n\to\infty}(\sqrt[k]{a_n}) = \lim_{n\to\infty}(a_n^{1/k}) = (\lim_{n\to\infty}(a_n))^{1/k} = \sqrt[k]{\lim_{n\to\infty}(a_n)}$ für $a_n \geq 0$

- $\lim_{n\to\infty}(1/n) = \lim_{n\to\infty}(1/(n+1)) = 0$, $\quad \lim_{n\to\infty}(c) = c$
- $\frac{\text{"Polynom"}}{\text{"Polynom"}} \xrightarrow[n\to\infty]{} \frac{\infty}{\infty}$, $\quad$ "$\infty - \infty$" $\Rightarrow$ unbestimmt
- $\lim_{n\to\infty}(1 + 1/n)^n = e$, $\quad \lim_{n\to\infty}(1 + k/n)^{mn} = e^{m\cdot k}$, $\quad \lim_{x\to 0}\left(\frac{\sin(x)}{x}\right) = 1$

$\lim_{x\to\infty}(e^x) = \infty \;/\; \lim_{x\to\infty}(e^{-x}) = 0$

Spezielle Integrationen

S. 476

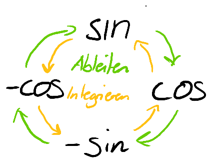
$\int \frac{1}{x}\,dx = \ln(|x|) + c$

$\int u^k\,du = \frac{u^{k+1}}{k+1} + c$

$\int e^{ax}\,dx = \frac{e^{ax}}{a} + c$

$\int a\,dx = \frac{ax}{a} + c$

$\int_a^b f(x) = (-1)\int_b^a f(x)$



$\boxed{+c}$ nicht vergessen