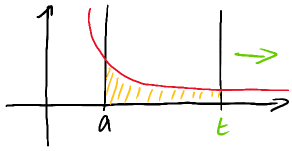
Uneigentlicher Integrationsbereich

↳ Unendliches Integral

$$\int_a^\infty f(x)\,dx = \lim_{t\to\infty}\int_a^t f(x)\,dx \qquad \int_{-\infty}^b f(x)\,dx = \lim_{t\to-\infty}\int_t^b f(x)\,dx$$



$$\int_{-\infty}^0 e^x\,dx = \lim_{t\to\infty}\int_t^0 e^x\,dx = \left[e^x\right]_t^0 = e^0 - e^t$$
$$= 1 - e^t \xrightarrow[t\to\infty]{} 1$$

$$\int_1^\infty \frac{1}{x}\,dx = \lim_{t\to\infty}\int_1^t \frac{1}{x}\,dx = \left[\ln(|x|)\right]_1^t = \ln(|t|) - \ln(1) \xrightarrow[t\to\infty]{} \infty$$

⇒ Integral existiert nicht

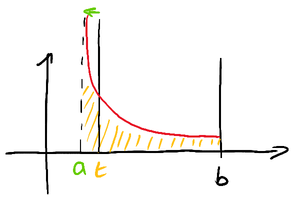
Integral über Polstelle

↳ $(a, b]$

$$\int_a^b f(x)\,dx = \lim_{t\to a}\int_t^b f(x)\,dx$$

$[a, b)$

$$\int_a^b f(x)\,dx = \lim_{t\to b}\int_a^t f(x)\,dx$$



$$\int_0^1 \frac{1}{\sqrt{x}}\,dx = \int_0^1 x^{-0,5}\,dx = \lim_{t\to 0}\int_t^1 x^{-0,5}\,dx$$
$$= \left[2x^{0,5}\right]_t^1 = 2\sqrt{1} - 2\sqrt{t} \xrightarrow[t\to 0]{} \underline{\underline{2}}$$

$$\int_0^1 \frac{1}{x}\,dx = \lim_{t\to 0}\int_t^1 \frac{1}{x}\,dx = \left[\ln(|x|)\right]_t^1 = \ln(1) - \ln(t)$$
$$= 0 - (-\infty) = \infty$$

⇒ Integral existiert nicht

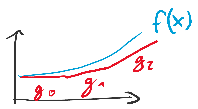
Differenzialgleichung

- Veränderung einer Grösse

$y' = 0$	$y' = y$
$y = C$	$y = C \cdot e^x$
$y' = (C') = 0$	$y' = C \cdot e^x = y$

Euler - Schritte

⇒ tangente folgen



$y' = x + y$, $x_0 = 0$ $\quad y_0 = 0 \quad h = 1$

Steigung	Berechnung
g_0: $F(x_0, y_0) = x_0 + y_0$	$x_1 = x_0 + h = 0 + 1 = 1$
	$y_1 = y_0 + h \cdot F(x_0, y_0) = 1 + 1\cdot(0 + 1) = 2$
g_1: $F(x_1, y_1) = x_1 + y_1$	$x_2 = x_1 + h = 1 + 1 = 2$
	$y_2 = y_1 + h \cdot F(x_1, y_1) = 2 + 1\cdot(1 + 2) = 5$

Separierbare Differentialgleichungen

$\Rightarrow y' = f(x) \cdot g(y)$

$\hookrightarrow (x^2 + \sin(x)) \cdot (e^y - y + 7)$

$y' = e^y - xe^y = (1-x)e^y$

$\Rightarrow f(x) = 1-x, \quad g(y) = e^y$

Autonome Differentialgleichungen

$y' = f(y)$

$\Rightarrow$ nur y

Rezept für die Lösung von separierbaren Differentialgleichungen

(I) $y' = \frac{\mathrm{d}y}{\mathrm{d}x} = f(x) \cdot g(y)$

(II) Trennung der Variablen: $\frac{\mathrm{d}y}{g(y)} = f(x) \cdot \mathrm{d}x$

(III) Integration auf beiden Seiten der Gleichung (falls möglich!)

$$\int \frac{\mathrm{d}y}{g(y)} = \int f(x)\,\mathrm{d}x$$

(IV) Auflösen nach y (falls möglich!)

$y' \cdot y^2 = \sin(x)$

I $y' = \frac{dy}{dx} = \frac{\sin(x)}{y^2}$

II $y^2 \cdot dy = \sin(x) \cdot dx$

III $\int y^2 \, dy = \int \sin(x) \, dx$

$\Rightarrow \frac{1}{3} y^2 = -\cos(x) + C$

IV $y = \sqrt[3]{-3\cos(x) + 3C}$

$x + y \cdot y' = 0, \quad y(3) = -4$

I $y' = \frac{dy}{dx} = \frac{-x}{y}$

II $y \, dy = -x \, dx$

III $\int y \, dy = -\int x \, dx$

$\Rightarrow \frac{1}{2} y^2 = -\frac{1}{2} x + C$

$x^2 + y^2 = \tilde{C} \quad \Rightarrow y = \pm\sqrt{\tilde{C} - x^2}$

$\Rightarrow -4 = -\sqrt{\tilde{C} - 9}$

$\Rightarrow 16 = \tilde{C} - 9 \Rightarrow \tilde{C} = 25$

$\Rightarrow y = -\sqrt{25 - x^2}$

Lineare Differentialgleichungen

$y' + f(x) \cdot y = g(x) \quad \Rightarrow f(x) =$ koeffizient von x

$y' = xy \quad \Rightarrow y' - xy = 0 \quad \Rightarrow f(x) = -x, \quad g(x) = 0$

$xy' + 2y = e^x \quad \Rightarrow y' + \frac{2}{x} y = \frac{e^x}{x} \quad \Rightarrow f(x) = \frac{2}{x}, \quad g(x) = e^x$

$\rightarrow$ linear weil y' und y nur 1. Potenz

nicht linear wenn $y' \cdot y$ oder y^n

$\rightarrow g(x) =$ Störglied

$\rightarrow$ Homogen wenn $g(x) = 0$