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SAFFO: A SIFT based approach for digital anastylosis for fresco recOnstruction

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ABSTRACT

Anastylosis is an archaeological technique which focuses on the reconstruction of collapsed building and destroyed artworks, starting from the original pieces. Many digital approaches have been developed in the last decade, mainly based on 2D and 3D analysis of the structure of the fragments. These techniques aim at supporting the priceless work of the involved operators, mainly in the decision processes and in the resolution of positioning ambiguities. Techniques acting with this scope lie in the field of the digital anastylosis. In this paper we present SAFFO, a digital approach to 2D reconstruction of frescoes, based on the extraction of SIFT features from a painting. The approach appears to be very robust to false positives, resulting optimal in scenarios involving fragment sets containing spurious elements. The experiments have been performed on the DAFNE (Digital Anastylosis for Fresco challeNgE) dataset, which gathers more than 30 2D artworks and provides several tessellation for each. For its robustness against spurious fragments, SAFFO won the third place in the rank list of DAFNE Challenge 2019.

1. Introduction

Anastylosis is a technique which consists in restoring a historical artwork, like frescoes, statues or paintings, by attaching piece by piece all the original composing elements. This technique is mainly adopted by archaeologists for reconstructing buildings or structural parts of them [20], destroyed by either natural (earthquakes, tsunami, tornadoes) or artificial events (wars, terrorist attacks), and for which it was possible to find a sufficient quantity of remains. Unfortunately, it is really hard to faithfully reproduce a devastated artwork, since many of the original pieces may have literally been reduced to dust or because of the enormous number of fragments belonging to different artworks mixed together. Sometimes, the computer science and the artificial vision may come to help of these professional figures: digital anastylosis is a discipline which proposes to support on artworks' reconstruction the archaeologists, geodesists and all the operators involved. Albeit these tools do not mean to substitute human operators, whose professionalism, skills and knowledge are still priceless, they can provide remarkable support in decision making processes. Digital anastylosis has been often applied using virtual tools, aiming at modelling the 3D structure of an object and trying to recompose it using methodologies involving geometric optimizations [19], stereo imaging and photogrammetry techniques [7,16]. These technologies have quickly spread and have been often used, due to the fact that most artworks have a 3D structure, like statues, buildings, vases [2], so needing focused techniques which could take in consideration the dimensionality of the object. However, in the art history, there are plenty of artworks which develop in two dimensions, like paintings or whose region of interest can be projected onto a 2D space, like frescoes, tapestries and ancient documents. In this paper, a digital anastylosis approach for fresco reconstruction called SAFFO is described; this has participated to the DAFNE challenge (Digital Anastylosis of Frescoes challeNgE [5]), promoted by the Computer Vision and Multimedia Lab (CVML) of the University of Pavia, and won the third place price; an ad-hoc dataset has been built and released. SAFFO exploits the SIFT features by Lowe [11] for locating the proper position of a fragment into a fresco. The remaining of the work is organized as follows: Section 2 describes the state-of-the-art related to digital anastylosis; Section 3 describes SAFFO method while Section 4, with its subsections, presents and discusses the experimental analysis performed for DAFNE competition and extends the analysis to a broader collection of pictures in DAFNE dataset; Section 5 concludes the paper.

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2. Digital anastylosis

Over recent years, the issues of digital anastylosis have been studied by proposing several methods and techniques to achieve feasible and effective solutions. Many of them have focused on the issue of archaeology to find two-dimensional or three-dimensional solutions to digital reconstruction, whereas some other works proposed classification methods, line in Rasheed and Nordin [15], to separate the fragments according to some similarity measurements. In both 2D and 3D cases, the common root of the problem is rebuilding the original artefact from a collection of pieces to be matched each other. Ideally, the collection of fragments is complete, meaning that the pieces totally recompose the original artefact perfectly. In practical scenarios, such a strong assumption sounds unreasonable. The reconstruction task is generally made harder for missing or corrupted fragments, small pieces, the effect of time passed which introduces erosion and affects color brightness and saturation. All these critical aspects make the digital anastylosis a challenging problem to fix. The direct implication of these issues are that fragments do not match perfectly and frescoes cannot be fully recomposed. Most of the works in the literature approach the problem at the same way the experts do. They look for pairwise matches between adjacent fragments by using color surface or texture properties, like the proposal of Smith et al. [17] that uses color and texture properties to classify two-dimensional fragments. Following a similar approach, Makridis and Daras [13] reconstructed ancient pottery objects considering the front and rear features of the pieces (in particular, information on color and local textures) to improve the classification accuracy. Alternatively, the work proposed by Leitao and Stolfi [9] exploited the boundary information, that is color information and geometry, of ceramic fragments to look for a matching like in a puzzle problem. Other effective work in the literature explored the use of color and boundary adjacency. Youguang et al. [21] addressed the problem of 2D image reconstruction by looking for contours matching, while the work by Tan et al. [18] started from the boundary analysis to predict the surrounding pixels of the pieces to overcome the presence of gaps and improve the probability of a matching among non-adjacent fragments. Under non generalist assumptions, the digital anastylosis can be seen as a clustering problem, meaning the problem of analysing the properties of the fragments and grouping them according some similarity measurements; at the same way happens in similar computer vision tasks, like image multi-object segmentation [22], object recognition [12] and biometrics [1]. Fuzzy logic has also been demonstrated effective in this context. Li-Ying and Ke-Gang [10] used fuzzy logic for clustering ceramic fragments according to surface texture properties. Hu et al. [8] proposed an inpainting quality assessment for digital image reconstruction. However, clustering approaches result particularly ineffective when dealing with complex paintings. Puzzle problems share some points with clustering approaches but it may often happen that very different fragments (in terms of visual or geometric features) have to be places beside each other. With classification or clustering solutions this constraint might be hard to implement. Further, in archaeological excavation sites some gaps among fragments reasonably exist, as well as the original artefact may not be totally covered by the findings. All these considerations put digital anastylosis into a more critical class of digital reconstruction problems which explains why fully automatic solutions are hard to achieve for realistic scenarios.

3. SAFFO

When dealing with the digital image reconstruction in the point of view of DAFNE challenge, more than a single issue has to be considered. A collection of fragments does not necessarily has to be completely used to reconstruct the original picture. Some pieces might miss as well as others might not belong to the reference painting. More details about the challenge and the DAFNE dataset are provided as Section 4.1. In this section the three main problems of the challenge are addressed, that are (i) the proper localization of the pieces belonging to the paintings, (ii) the avoidance of the spurious fragments and (iii) how to chose the right orientation of the fragments. The following subsections summarize these issues in accordance to the proposed low-level feature-based solution which is shown in Fig. 1.

3.1. Fragment location and spurious removal

Given a set of fragments $F = \{f_1, f_2, \cdots, f_n\}$ and the original image I, consisting in the fresco we aim at restoring, the SIFT features [11] are computed for each of the fragments. Let $KP^{f_i} = \{kp_1^{f_i}, kp_2^{f_i}, \cdots, kp_m^{f_i}\}$ be the m SIFT keypoints of a given fragment f_i, and $KP^I = \{kp_1^I, kp_2^I, \cdots, kp_r^I\}$ the r SIFT keypoints of the whole fresco, such that $r >> m$, look for the closest matches among features in KP^{f_i} and in KP^I. The number of chosen matches is always lower or equal than m, since for each keypoint kp^{f_i} in f_i at most one match is obtained. The matching operation is achieved by applying a *k-nearest neighbor* [4,6] matcher, with $k = 2$. Moreover, since the proportion between the fragment and the fresco still holds, given two keypoints in the fragment $kp_i(f_i)$ and $kp_j(f_i)$ and two keypoints in the fresco $kp_n(I)$ and $kp_o(I)$, their match is confirmed if the euclidean distance $d(kp_i(f_i), kp_j(f_i)) = d(kp_n(I), kp_o(I))$. In Fig. 2a, for a single fragment, all the keypoints and matches found on the related fresco are shown.

3.2. Rotating the fragment

Once a set of reliable matching keypoints is obtained, we need to properly rotate the fragment. Computing the homography mapping between matched features can be considered the proper solution to meet this requirement; at least four proper matches need to be found. The homography is as much accurate as many matching points are found. In presence of very small or corrupted pieces as well as of low detailed or less patterned fragments, forcing to find many matches can be counterproductive: this would cause the computation of a wrong homography matrix resulting in a wrong rotation degree. In the keypoint matching phase, three situations may happen:

- there are two or more matches between the fragment and the fresco: the fragment is likely to belong the fresco. It is possible to sort all the matches by means of a metric of confidence, which allows to get the two more secure matches, along with the related keypoints, in order to compute the rotation degree. An example is shown in Fig. 2a;
- there is only one match between the fragment and the fresco: it may be either correct or wrong. The processing does not go further for this kind of situation.
- no matches have been found between the fragment and the fresco image. In such cases the fragment is likely not to belong the fresco. There is still a low chance that the fragment actually belong to the fresco, but no matches have been detected. In Fig. 2a, an example of this situation is shown.

In order to measure the confidence of the keypoints, the Hellinger distance [3] is used. Once two reliable keypoint matches are found, the rotation approach takes place. As shown in Fig. 2b, firstly, the angle between the two keypoints on the fragment is computed; similarly, the same approach is followed for computing the angle between the keypoints on the fresco image. let x and y be these angles, respectively for the fragment and for the fresco. The rotation angle is the difference $x - y$. The fragment location,

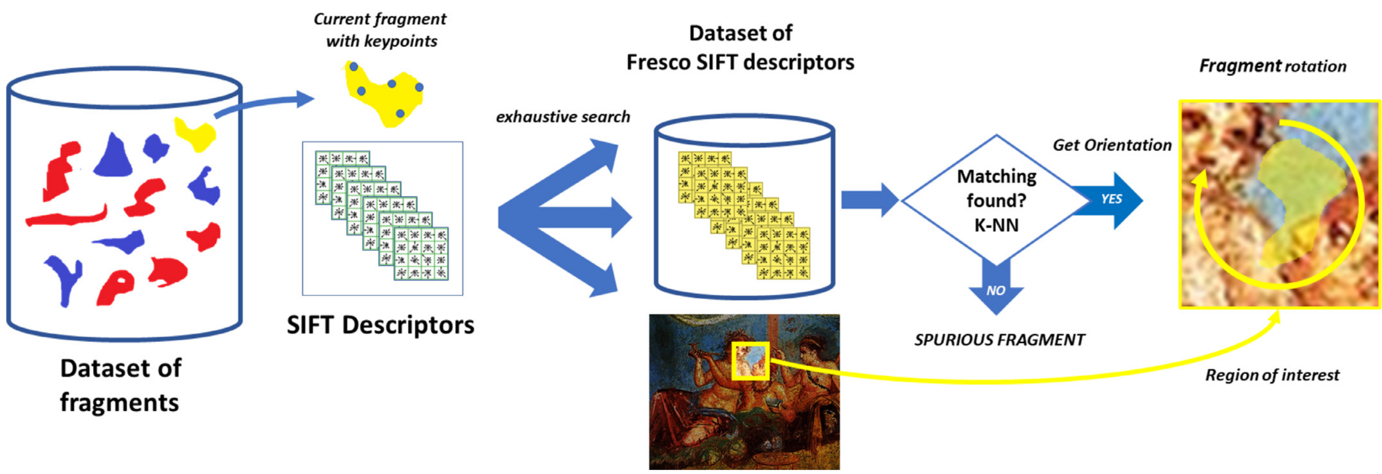
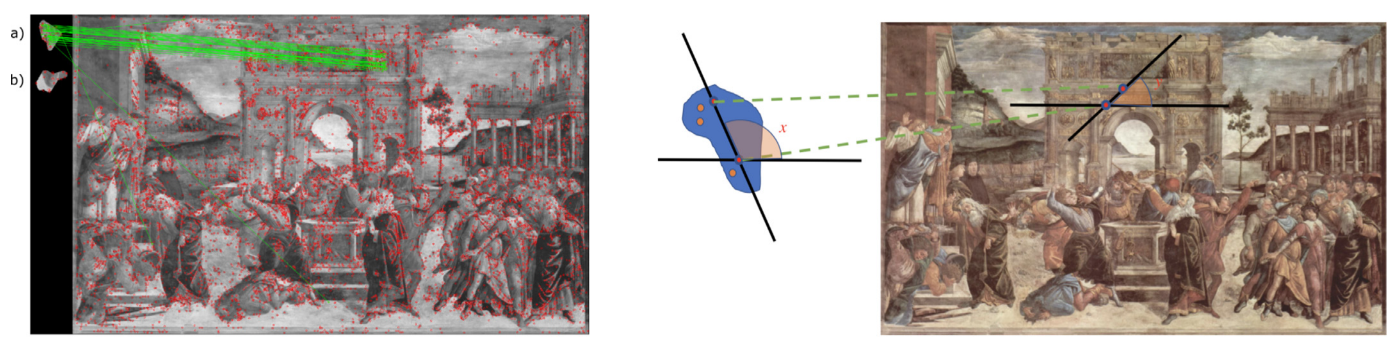


Fig. 1. Overall flowchart of SAFFO. From the left, the dataset of fragments with original and spurious, blue and red respectively. From the collection a piece is randomly selected (yellow) and the SIFT descriptors are computed for it. An exhaustive search is performed on the collection of SIFT descriptors of the whole original picture. If a match is found, the rotation is computed to find the best fit of the fragment on the region of interest extracted from the fresco. In case such a match is not found, the piece is marked as spurious and discarded.



(a) *Punishment of Rebels* by Botticelli.

(b) Rotation matching.

Fig. 2. On the left, two fragments of the fresco *Punishment of Rebels* by Botticelli, and the whole fresco. The red dots are the SIFT keypoints detected and the green lines associate a keypoint in the fragment on the top with a keypoint in the fresco. The second fragment, even if similar in appearance to the one on the top, does not allow to find a match with the reference image. On the right, the fragment we aim at pose into the fresco is shown. The orange dots are keypoints with proper matches on the fresco, whereas the red ones are the keypoints whose matches with the related keypoints on the fresco are the more reliable. Therefore, these are used to rotate the fragment. On the rightmost part of the image, the fresco is shown.

along with the rotation degree are reported in the solution file. The fragment is then properly applied on the fresco. The code of SAFFO is available on github [14].

4. Experimental results

This section discusses the experimental results achieved on DAFNE Dataset linked to DAFNE Challenge (Digital Anastylosis of Frescoes challeNgE) which has been launched in 2019 by the Computer Vision and Multimedia Lab of University of Pavia[1] [5]. As discussed in sections above, the DAFNE challenge is a fully supervised problem where the goal is looking for matching among several fragments of a fresco, which can be very damaged or very small and poorly patterned. More details about the challenge and the dataset are in following Section 4.1).

The experimentation focuses the attention on reducing as much as possible the placement of fragments on the wrong location as well as avoiding that spurious pieces are chosen to belong to the wrong fresco. The consequence of such a choice impacts on the total amount of right pieces to match with the reference fresco because we adopted very strict constraints of matching to be sure to avoid spurious and placement errors. SAFFO, however, achieved the third place in the rank list of participants to DAFNE Challenge, thus confirming the effectiveness of the simple solution over the others. The rest of this section presents and discusses the results achieved on more than 500 different tessellations of the frescoes in DAFNE dataset, emphasizing the strenghts and the limitations of the SAFFO approach.

4.1. DAFNE: The challenge and the dataset

The DAFNE Challenge shed light on the problem oh heritage conservation, to highlight the crucial impact that destructive phenomena such as wars or earthquakes as well as vandalism acts can have on the conservation and to avoid that cultural heritage would otherwise be lost forever. In particular, DAFNE challenge focuses on reconstruction of frescoes from collection of fragments. The main idea can be easily linked to puzzle solving problems, where fragments have to be matched each other to form the original look of the reference painting. However, the point of view of DAFNE challenge is that the collection of pieces does not completely reassemble the drawing, simulating the erosion effect that time and

[1] (https://vision.unipv.it/DAFchallenge/DAF-notice.html)

Fig. 3. The reconstruction results obtained on the test-set of DAFNE Challenge 2019. From the left, the reconstructed fresco by Federico Zuccari "The Cardinal's hat conferred on Saint Charles Borromeo", by Domenichino "The Virgin and the Unicorn" and by Andrea di Bonaiuto "Allegory of the Active and Triumphant Church and the Dominican order".

disruptive events typically introduce on the findings in an excavation site. Moreover, the tessellation of the fresco into pieces is mixed with spurious fragments that do not belong to the reference picture but they can probably misplaced in a region of the fresco, thus introducing an error in the reconstruction process. To further simulate real scenarios, the tessellation is such that it does not cover the whole surface of the original fresco; some gaps of variable size among pieces are expected. All these aspects make DAFNE challenge closer to the real working environment of archaeologists and expert of art reconstruction/restoration but it also introduces difficulties that makes less trivial looking for a fully automatic computer-based solution. In the challenge version, DAFNE dataset has been divided into two sets, training and testing set, to meet the needs of automated learning approaches. The former consists in 62 frescoes, each fragmented in 18 different ways, (for a total of 1.116 configurations) while the testing dataset includes three frescoes. On the three paintings composing the test-set, the ranking list has been developed and winning solutions have been chosen by the DAFNE organizing committee (our solution won the third prize). For completeness of this experimental discussion, the results achieved on the test-set as well as the visual results are presented in Fig. 3. However, since SAFFO does not develop a learning process but, rather, works with low-level features, a significantly wider collection of frescoes and tessellations have been considered in our experimental design. A total of 500 different tessellations of the pictures have been considered; the original picture of the fresco as well as the ground truth were available for each of them.

4.2. Results and discussions

In Fig. 3, the reconstruction of the frescoes from the testing set for DAFNE challenge is reported. The matching obtained by using SIFT features achieved accurate results, therefore the application over the alpha channel of the fresco yields a perfect match. The number of fragments wrongly positioned is minimum, as well as those with a wrong rotation degree. Finally, almost none of the spurious fragment has been located on the fresco. It must be considered that during DAFNE challenge, and in particular during the testing stage of the competition, the information about spurious fragments was not given. This added a double level of difficulties, because discriminating among false negatives and spurious fragments is less trivial then expected. By visually inspecting the results presented in Fig. 3 it can be noticed that the pieces that have been selected to belonging to the reference image show a very good match. Our strict constraints for a successfully match introduced, of course, some false negatives but ensured a false positive rate equal to zero; a contribution that brought SAFFO at the third place in the rank list of participants to DAFNE challenge [14].

Table 1
The table reports the results achieved by SAFFO at DAFNE challenge and provide a comparison with other participants, the best performing algorithms in the rank list of the challenge.

		Authentic fragments		spurious, misplaced
	#frags	correct	missing	
F. Zuccari - The Cardinal's hat conferred on Saint Charles Borromeo				
Algorithm 1	943	943	0	0
Algorithm 2	938	933	5	5
SAFFO	490	490	*453*	**0**
Algorithm 3	1140	920	23	220
Domenichino - A Virgin with a Unicorn				
Algorithm 1	474	473	1	1
Algorithm 2	474	473	1	1
Algorithm 3	577	470	4	107
SAFFO	231	231	*243*	**0**
A. Di Bonaiuto - Active and Triumphant Church and the...				
Algorithm 1	1738	1738	0	0
Algorithm 2	1728	1726	12	2
SAFFO	1091	1091	*647*	**0**
Algorithm 3	1836	1470	268	366

A comparative analysis of SAFFO and other algorithms, the best performing ones, is provided in Table 1[2] It can be further confirmed that the strength of SAFFO is in its robustness to spurious or misplaced fragments, which are always equal to zero. On the other hand, such a strict approach is counterproductive in terms of missing authentic fragments, as can be seen in Table 1.

After the results achieved on DAFNE challenge, we extended the experimentation to a broader collection of pictures in DAFNE dataset. The objective was that of assessing the effectiveness of SAFFO on different tessellations of the frescoes, ranging from few/big fragments up to a lot of/small pieces. The experimentation was therefore carried out on a subset of 540 images: 30 different frescoes, each with 18 different fragmentation. Each fragmentation differs from the others on the size and the number of pieces but each of them contains both fragments that belong to the fresco and spurious ones. Different fragmentation contain a variable number of very small and less-patterned pieces that are hard to identify to belong or not to the reference fresco as well as to identify the exact position and the rotation of the fragment to achieve a good match. These are the main factors that make this problem not trivial to solve.

[2] The results are courtesy of DAFNE Challenge Committee. Therefore, authors' names cannot be revealed since they publish their results.

Table 2
Table summarising the results achieved on DAFNE dataset in terms of mean results and on highest and lowest parameter partitions of the frescoes.

	Fresco Name	# of fragments	# authentic fragments	FP	FN	Median matching (Jaccard)
Highest rate true fragments	Mantegna Camera picta	163	112 (68.71%)	0	51	96
Lowest rate true fragments	Giotto Massacre of the Innocents	1500	8 (0.53%)	0	1492	93
Largest fragments set	Fra Angelico Cristo deriso	2013	199 (9.46%)	0	1904	95
Smallest fragments set	Vasari Paul III Farnese	85	38 (44.71%)	0	47	93
Highest pixel error	Signorelli Come Benedetto esorcizza il demonio	580	280 (49.98%)	0	293	98
Lowest pixel error	Tiepolo Banchetto di Cleopatra	281	40 (14.23%)	0	241	86
Mean value	–	*483.37*	*108.21(26.46%)*	*0*	*385.15*	*94.12*

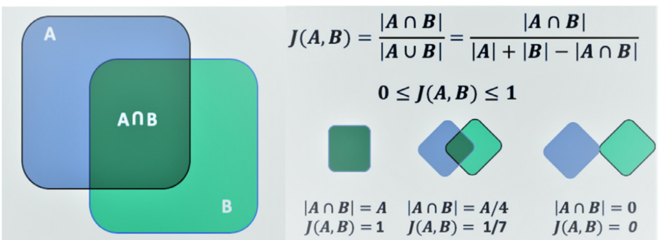


Fig. 4. The Jaccard metric; in the figure above, A is the ground truth provided for a single fragment, while B is the estimated position.

Fig. 5. (left) Giotto - Massacre of the Innocents in one of tessellation in DAFNE dataset where the lowest percentage of original fragments of the total is achieved. (right) Raffaello - Trionfo-di-Galatea in a tessellation where the highest matching score is achieved.

The Jaccard index, also known as Intersection Over Union, has been chosen as a metric to define the correctness of the positioning of the fragments and to quantify the precision of the proposed method. The Jaccard index is a statistic used for gauging the similarity and diversity of sample sets. The Jaccard coefficient measures similarity between finite sample sets, and is defined as the size of the intersection divided by the size of the union of the sample sets. The Fig. 4 shows how the Jaccard index works. Let A be the position of fundamental truth and B the estimated position, $J(A, B)$ is the Jaccard index summarising how well B overlaps on A. Considering the amount of different tessellations and the very high variance of the parameters of each, it is quite hard to provide a unique aggregate quantitative information about the accuracy and precision of the proposed method. In fact, we achieved an average Jaccard index of similarity over all experimental samples of 94.12%. Even if it can be considered an encouraging result, it could not be considered an overall reliable measure of the quality achieved on our proposed solution. Therefore, we explored in more details the DAFNE Dataset. To mention some of the different parameters of the DAFNE collection, there are some fragmentation with a percentage of original fragments on the total that is lower than 5% (the lowest of such a percentage is reached on tessellation named *Giotto_MassacreoftheInnocents_2024x2047_2019-2-16_16,13,48* with a ratio of 8 positive fragments in a collection of 1500 pieces). In this configuration the result obtained is shown in Fig. 5. A false positive rate of 0 has been achieved even in this critical case and 7 of 8 original fragments have been properly placed with the best fragments placed at 93.53% of matching with the ground-truth and a worst match of 93.50%. Similarly, but from an opposite viewpoint, the fresco by Raffaello "Trionfo di Galatea" is one for which the highest matching has been achieved for a mean percentage of 97.11% on a percentage of fragments of 37.86% of 486 fragments. Even in this case, the false positive rate is equal to 0. Due to the variability of conditions above mentioned in DAFNE dataset, Table 2 summarise the significant results achieved on specific frescoes, in particular those with the highest and lowest percentage of fragments belonging to the painting, those with the highest and lowest number of total fragments and those with the highest and lowest average percentage of fragments correctly identified.

In addition, the last row shows the average of the results on all the subset frescoes. The results are reported in terms of total number of fragments, number of fragments belonging to the painting (and the percentage with respect to the total number of fragments), false positives (i.e. the number of spurious fragments inserted in the fresco), false negatives (the number fragments that have not been included in the fresco because they were incorrectly identified as spurious), and the average percentage of the Jaccard index. As can be seen from the results in the Table 2 this method again obtains 0 spurious fragments for each fresco, therefore a fragment that is not part of it is never inserted in the fresco; a result that further confirms the strength of the method and explain its rank in DAFNE competition. However, the percentage of the correct fragments is relatively low 26.4% but these are correctly inserted at 94.12% as indicated by the Jaccard index. It can also be noticed that the fresco with the lowest percentage of inserted fragments *Giotto - Massacre of the Innocents* and the lowest percentage of well-inserted pieces *Tiepolo - Cleopatra's Banquet* still have an average Jaccard index higher than 90%.

The proposed method is particularly restrictive and does not place a fragment if the conditions are not all satisfied (false positive at the fragment level equal to zero). This has an advantage in preventing the positioning of spurious pieces, but in another point of view the proposed method is also very rigorous and could not place a correct match if the matching parameters are low or not marginally satisfied. This justifies the low mean percentage of fragments inserted in the frescoes. Figs. 6 and 7 highlight the percentage of fragments positioned in the image and the comparison with ground truth. The histogram in the figure shows the matching percentage of overlapping of pieces with the ground-truth (in other words, the Jaccard value for each fragment). Such overlapping of the predicted position of fragments with the ground truth masks is shows on the last picture in Figs. 6 and 7 (pixels that do not match the ground-truth are highlighted in red). The two frescoes

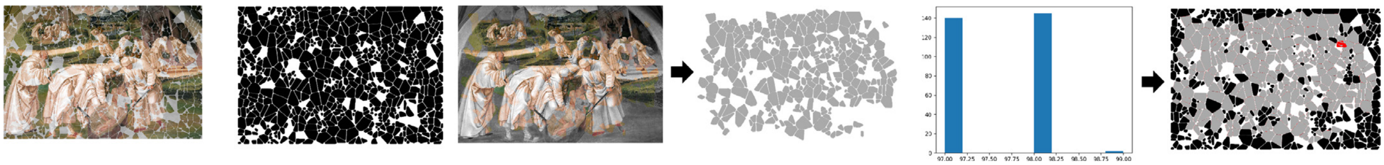


Fig. 6. Reconstruction of *Come Benedetto esorcizza diavolo* by Signorelli, and comparison with the ground truth.

Fig. 7. Reconstruction of *Banquet of Cleopatra* by Tiepolo, and comparison with the ground truth.

are those with the highest and lowest median value in the Jaccard index. In Fig. 6 there is the fresco *"How Benedict exorcises the devil"* by Signorelli, the example of reconstruction with the highest percentage of Jaccard's index. The histogram in the figure shows that most of the fragments have 96% and 98% of the Jaccard index percentage. In Fig. 7 we have the fresco *"Baquet of Cleopatra"* by Tiepolo and it is the example of reconstruction with the highest percentage of Jaccard index. The histogram in the figure shows that most of the fragments have 86% of Jaccard index percentage.

5. Conclusions

Digital anastylosis can represent a valid support for experts and archaeologists when working in original excavation site, where findings are often damaged or disrupted into pieces. The computer-assisted reconstruction of a fragmented historical object is undoubtedly an unsupervised task. On the other hand, unlike what happens in practice, the literature has explored this task under several assumptions that do not comply with the true conditions of recomposing an ancient object found in an excavation mission. One of this example, is represented by the DAFNE competition, which has proposed a challenge on the reconstruction of frescoes configuring the problem like a supervised task. The original reference fresco is made available, along with the ground truth information (position and rotation) of the fragments, but the reconstruction is made hard from the presence of spurious fragment among original pieces belonging to a specific fresco. Moreover, the fragments often do not match exactly to each other and they do not totally recompose the original fresco. The method proposed in this paper, named SAFFO, starts from a simple assumption, considering that the problem of digital reconstruction can be considered reliable if it minimizes the presence of false positive matches (i.e., the presence on the reconstructed fresco of pieces that are wrongly places or, even worse, that do not belong to the original fresco). The experimental results achieved on a template matching proposal based on the SIFT features extracted from each fragment, suggest that the method is very robust, since besides positioning an acceptable percentage of fragments, it never gets wrong, by positioning a spurious element in the wrong fresco. Even in presence of very odd conditions, where the percentage of original fragments of the total is very low or when the pieces are not patterned or very small, the proposed solution reduces the error at the minimum. The strict constraint to accept a match implies a reduced true positive rate as side effect, but experimental results carried out on a wide collection of different tessellations of 30 different frescoes in DAFNE dataset shows that on average the proposed solution is effective and accurate. As said above, in practical scenarios the computer assisted procedure at digital anastylosis should reasonable considered as an unsupervised task. In this point of view, the DAFNE dataset would not consider the presence of a reference image, along with the original position of the fragment. Challenging issues would arise from such a version of the problem, up to make possible to demonstrate that it is reducible to a NP-complete problem. Generative methods, like Hidden Markov Model (HMM) or Generative Adversarial Networks (GAN), to mention a few of them, that can predict the surrounding pixels of the fragments thus contributing to increase the probability of a match between pieces separated by a gap could be explored to demonstrate the feasibility of a blind version of DAFNE challenge.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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