# tp-nn-cohen-stephen

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# 1 TP: MNIST with Neural Networks (NN)

### 1.1 Stephen Cohen

```
[23]: import numpy as np
  import tensorflow as tf
  import keras
  print("Using tensorflow version " + str(tf.__version__))
  print("Using keras version " + str(keras.__version__))
```

Using tensorflow version 2.16.1 Using keras version 3.3.3

### 1.2 Loading and preparing the MNIST dataset

Load the MNIST dataset made available by keras.datasets. Check the size of the training and testing sets.

Code:</div>

```
[24]: # The MNSIT dataset is ready to be imported from Keras into RAM
    # Warning: you cannot do that for larger databases (e.g., ImageNet)
    from keras.datasets import mnist
    (train_images, train_labels), (test_images, test_labels) = mnist.load_data()
```

The MNIST database contains 60,000 training images and 10,000 testing images. Using the pyplot package, visualize the first sample of the training set:

```
axes[i].set_title('Image '+str(i))

# Display the plot
plt.show()
```



The database contains images of handwritten digits. Hence, they belong to one of 10 categories, depending on the digit they represent. Reminder: in order to do multi-class classification, we use the softmax function, which outputs a multinomial probability distribution. That means that the output to our model will be a vector of size 10, containing probabilities (meaning that the elements of the vector will be positive sum to 1). For easy computation, we want to true labels to be represented with the same format: that is what we call **one-hot encoding**. For example, if an image **x** represents the digit 5, we have the corresponding one\_hot label (careful, 0 will be the first digit):

$$\mathbf{y} = [0, 0, 0, 0, 0, 1, 0, 0, 0, 0]$$

Here, you need to turn train and test labels to one-hot encoding using the following function:

Code:</div>

```
[26]: from keras.utils import to_categorical
    train_labels = to_categorical(train_labels, num_classes=10)
    test_labels= to_categorical(test_labels, num_classes=10)
```

Images are black and white, with size  $28 \times 28$ . We will work with them using a simple linear classification model, meaning that we will have them as vectors of size (784). You should then transform the images to the size (784) using the numpy function reshape.

Then, after casting the pixels to floats, normalize the images so that they have zero-mean and unitary deviation. Be careful to your methodology: while you have access to training data, you may not have access to testing data, and must avoid using any statistic on the testing dataset.

```
[27]: # Reshape images to vectors of pixels
img_rows, img_cols = train_images.shape[1], train_images.shape[2]
train_images = train_images.reshape((len(train_images),784))
```

## 2 First part: working with Numpy

Look at this cheatsheet for some basic information on how to use numpy.

#### 2.0.1 Defining the model

We will here create a simple, linear classification model. We will take each pixel in the image as an input feature (making the size of the input to be 784) and transform these features with a weight matrix  $\mathbf{W}$  and a bias vector  $\mathbf{b}$ . Since there is 10 possible classes, we want to obtain 10 scores. Then,

$$\mathbf{W} \in \mathbb{R}^{784 \times 10}$$
$$\mathbf{b} \in \mathbb{R}^{10}$$

and our scores are obtained with:

$$z = W^T x + b$$

where  $\mathbf{x} \in \mathbb{R}^{784}$  is the input vector representing an image. We note  $\mathbf{y} \in \mathbb{R}^{10}$  as the target one\_hot vector.

Here, you fist need to initialize W and b using np.random.normal and np.zeros, then compute z.

```
Code:</div>
```

```
[28]: # To avoid implementing a complicated gradient back-propagation,
# we will try a very simple architecture with one layer

def initLayer(n_input,n_output):

"""

Initialize the weights, return the number of parameters

Inputs: n_input: the number of input units - int
```

```
[29]: n_training = train_images.shape[0]
n_feature = img_rows*img_cols
n_labels = 10
W, b, nb_params = initLayer(n_feature, n_labels)
```

### 2.0.2 Computing the output

To obtain classification probabilities, we use the softmax function:

$$\mathbf{o} = softmax(\mathbf{z}) \text{ with } o_i = \frac{\exp(z_i)}{\sum_{i=0}^9 \exp(z_i)}$$

The usual difficulty with the softmax function is the possibility of overflow when the scores  $z_i$  are already large. Since a softmax is not affected by a shift affecting the whole vector  $\mathbf{z}$ :

$$\frac{\exp(z_i - c)}{\sum_{i=0}^9 \exp(z_j - c)} = \frac{\exp(c) \exp(z_i)}{\exp(c) \sum_{i=0}^9 \exp(z_i)} = \frac{\exp(z_i)}{\sum_{i=0}^9 \exp(z_j)}$$

what trick can we use to ensure we will not encounter any overflow?

```
[31]: def softmax(z):
    """
    Perform the softmax transformation to the pre-activation values
    Inputs: z: the pre-activation values - numpy ndarray
    Outputs: out: the activation values - numpy ndarray
    """
    c = np.max(z)
    out = np.exp([zi - c for zi in z])/np.sum([zi-c for zi in z])
    return out
```

#### 2.0.3 Making updates

We define a learning rate  $\eta$ . The goal is to be able to apply updates:

$$\mathbf{W}^{t+1} = \mathbf{W}^t + \nabla_{\mathbf{W}} l_{MLE}$$

In order to do this, we will compute this gradient (and the bias) in the function update. In the next function updateParams, we will actually apply the update with regularization.

Reminder: the gradient  $\nabla_{\mathbf{W}} l_{MLE}$  is the matrix containing the partial derivatives

$$\left[\frac{\delta l_{MLE}}{\delta W_{ij}}\right]_{i=1..784,j=1..10}$$

**Remark**: Careful, the usual way of implementing this in python has the dimensions of **W** reversed compared to the notation of the slides.

Coordinate by coordinate, we obtain the following update:

$$W_{ij}^{t+1} = W_{ij}^t + \eta \frac{\delta l_{MLE}}{\delta W_{ij}}$$

Via the chain rule, we obtain, for an input feature  $i \in [0,783]$  and a output class  $j \in [0,9]$ :

$$\frac{\delta l_{MLE}}{\delta W_{ij}} = \frac{\delta l_{MLE}}{\delta z_j} \frac{\delta z_j}{\delta W_{ij}}$$

It's easy to compute that  $\frac{\delta z_j}{\delta W_{ij}} = x_i$ 

We compute the softmax derivative, to obtain:

$$\nabla_{\mathbf{z}} l_{MLE} = \mathbf{o} - \mathbf{y}$$

Hence,  $\frac{\delta l_{MLE}}{\delta z_{j}}=o_{j}-y_{j}$  and we obtain that

$$\frac{\delta l_{MLE}}{\delta W_{ij}} = (o_j - y_j) x_i$$

This can easily be written as a scalar product, and a similar computation (even easier, actually) can be done for **b**. Noting  $\nabla_{\mathbf{z}} l_{MLE} = \mathbf{o} - \mathbf{y}$  as grad in the following function, compute the gradients  $\nabla_{\mathbf{w}} l_{MLE}$  and  $\nabla_{\mathbf{b}} l_{MLE}$  in order to call the function updateParams.

Note: the regularizer and the weight\_decay  $\lambda$  are used in updateParams.

```
[32]: def update(eta, W, b, grad, X, regularizer, weight_decay):
          Perform the update of the parameters
          Inputs: eta: the step-size of the gradient descent - float
                 : W: the weights - ndarray
                 : b: the bias - ndarray
                 : grad: the gradient of the activations w.r.t. to the loss - list of \Box
       \hookrightarrow ndarray
                 : X: the data - ndarray
                 : regularizer: 'L2' or None - the regularizer to be used in \Box
       \hookrightarrow updateParams
                 : weight_decay: the weight decay to be used in updateParams - float
          Outputs: W: the weights updated - ndarray
                 : b: the bias updated - ndarray
          grad_w = np.array([np.array([grad[i]*X[j] for i in range(len(grad))]) for ju
       →in range(len(X))])
          grad_b = grad
          W = updateParams(W, grad_w, eta, regularizer, weight_decay)
          b = updateParams(b, grad_b, eta, regularizer, weight_decay)
          return W, b
```

The update rule is affected by regularization. We implement two cases: No regularization, or L2 regularization. Use the two possible update rules to implement the following function:

#### 2.0.4 Computing the Accuracy

Here, we simply use the model to predict the class (by taking the argmax of the output!) for every example in X, and count the number of times the model is right, to output the accuracy.

Code:</div>

```
[34]: def computeAcc(W, b, X, labels):
          Compute the loss value of the current network on the full batch
          Inputs: act_func: the activation function - function
                : W: the weights - list of ndarray
                : B: the bias - list of ndarray
                : X: the batch - ndarray
                : labels: the labels corresponding to the batch
          Outputs: loss: the negative log-likelihood - float
                 : accuracy: the ratio of examples that are well-classified - float
          11 11 11
            # Forward propagation
          z = np.array([forward(W, b, x) for x in X])
          # Compute the softmax and the prediction
          out = np.array([softmax(zl) for zl in z])
          pred = np.array([np.argmax(zl) for zl in z])
          pred= to_categorical(pred, num_classes=10)
          111
          for j in range(10):
            print(pred[j], labels[j], (labels[j]==pred[j]).all())
          # Compute the accuracy
          accuracy = np.sum([1 if (labels[i] == pred[i]).all() else 0 for i in_
       →range(len(labels))])
          return accuracy
      #train accuracy = computeAcc(W, b, train images, train labels)
      #print(train_accuracy)
```

### 2.0.5 Preparing training

The following hyperparameters are given. Next, we can assemble all the function previously defined to implement a training loop. We will train the classifier on **one epoch**, meaning that the model will see each training example once.

```
[35]: # Optimization
  eta = 0.01
  regularizer = 'L2'
  weight_decay = 0.0001
# Training
```

```
log_interval = 5000
```

```
[36]: # Data structures for plotting
     g_train_acc=[]
     g_valid_acc=[]
     #######################
     ### Learning process ##
      #########################
      #Took approximately 3 min to execute
     for j in range(n_training):
         # Getting the example
         X, y = train_images[j],train_labels[j]
         # Forward propagation
         z = forward(W, b, X)
          # Compute the softmax
         out = softmax(z)
         # Compute the gradient at the top layer
         derror = out - y # This is o - y
          # Update the parameters
         W, b = update(eta, W, b, derror, X, regularizer, weight_decay)
         if j % log_interval == 0:
              # Every log_interval examples, look at the training accuracy
             train_accuracy = computeAcc(W, b, train_images, train_labels)
              # And the testing accuracy
             test_accuracy = computeAcc(W, b, test_images, test_labels)
             g train acc.append(train accuracy)
              g_valid_acc.append(test_accuracy)
             result_line = str(int(j)) + " " + str(train_accuracy) + " " +
       str(test_accuracy) + " " + str(eta)
             print(result_line)
     g_train_acc.append(train_accuracy)
     g_valid_acc.append(test_accuracy)
     result_line = "Final result:" + " " + str(train_accuracy) + " " +
      str(test_accuracy) + " " + str(eta)
     print(result_line)
```

0 7969 1424 0.01 5000 43885 7340 0.01 10000 43432 7235 0.01 15000 43438 7252 0.01

```
20000 43394 7247 0.01

25000 43523 7275 0.01

30000 43558 7284 0.01

35000 43577 7289 0.01

40000 43603 7291 0.01

45000 43612 7290 0.01

50000 43620 7292 0.01

55000 43622 7294 0.01

Final result: 43622 7294 0.01
```

What can you say about the performance of this simple linear classifier?

```
Answer:</div>
```

On dirait que l'erreur est au minimum de 10% donc une accuracy de maximum 90% (On ne semble pas pouvoir dépasser ce seuil) De plus, le temps de calcul est très long (environ 3mn et 45s)

## 3 Second part: Autoencoder with Keras

#### 3.1 Autoencoder and PCA

First, we will try to connect the representation produced by Principal Component Analysis with what is learnt by a simple, linear, autoencoder. We will use the scikit-learn implementation of the PCA to obtain the two first components (hint: use the attribute .components\_), and visualize them:

```
Code:</div>
```

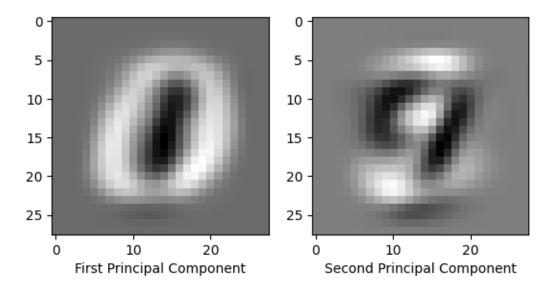
```
[37]: from sklearn.decomposition import PCA

# Let's find the first 2 PCA components
num_components = 2
pca = PCA(num_components).fit(train_images)

# Reshape so they resemble images and we can print them
eigen_mnist = pca.components_.reshape(2,28,28)

# Show the reshaped principal components
f, ax = plt.subplots(1,2)
ax[0].imshow(eigen_mnist[0], cmap='gray')
ax[0].set_xlabel('First Principal Component')
ax[1].imshow(eigen_mnist[1], cmap='gray')
ax[1].set_xlabel('Second Principal Component')
```

[37]: Text(0.5, 0, 'Second Principal Component')



```
[38]: # Print the variance explained by those components print(pca.explained_variance_) print(pca.explained_variance_ratio_)
```

[53.903347 39.41201 ] [0.0970372 0.07094979]

Comment on the visualization in relation to the variance explained by only keeping the two principal components:

Answer:</div>

On remarque que chacun des deux chiffres semblent expliquer 10% du data set global (au total 20% ici). Ce qui pourrait s'expliquer par le fait que chacun des 10 chiffres explique 10% des données (ce qui parait intuitif)

#### 3.1.1 Implementing the Autoencoder with Keras

```
[39]: from tensorflow.keras.models import Model
from tensorflow.keras.layers import Input, Dense
from tensorflow.keras.optimizers import SGD
```

Now, we will use Keras to implement the autoencoder. You can take a look at this cheatsheet for some basic commands to use keras.

In this first case, we implement a **simple linear autoencoder**. Build it in order to have the same capacity as the PCA decomposition (2 hidden dimensions!) we made just above.

```
[40]: # Input layer
input_layer = Input(shape=(784,))

# Encoding layer
latent_view = Dense(2, activation='linear', name='layer')(input_layer)

# Decoding layer
output_layer = Dense(784, activation='linear', name='output')(latent_view)

ae_model = Model(input_layer, output_layer, name='ae_model')
ae_model.summary()
```

Model: "ae\_model"

Layer (type)	Output Shape	Param #
<pre>input_layer_1 (InputLayer)</pre>	(None, 784)	0
layer (Dense)	(None, 2)	1,570
output (Dense)	(None, 784)	2,352

Total params: 3,922 (15.32 KB)

Trainable params: 3,922 (15.32 KB)

Non-trainable params: 0 (0.00 B)

What loss shoud we use? Choose the usual one and import it directly from Keras. You can use a simple SGD optimizer, and then compile the model; finally, train it to rebuild images from the original examples.

```
[42]: from tensorflow.keras.losses import MeanSquaredError
from tensorflow.keras.optimizers import SGD

loss = MeanSquaredError()

# Utilisez 'learning_rate' au lieu de 'lr'
optimizer = SGD(learning_rate=1e-1)

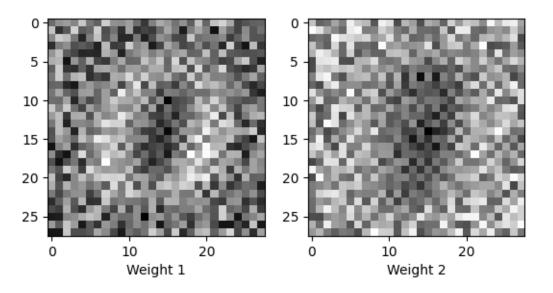
ae_model.compile(optimizer=optimizer, loss=loss)
```

```
Epoch 1/10
469/469
                    1s 2ms/step -
loss: 0.8566 - val_loss: 0.6748
Epoch 2/10
469/469
                    1s 1ms/step -
loss: 0.6621 - val_loss: 0.6272
Epoch 3/10
469/469
                    1s 1ms/step -
loss: 0.6271 - val_loss: 0.6195
Epoch 4/10
469/469
                    1s 1ms/step -
loss: 0.6196 - val_loss: 0.6174
Epoch 5/10
469/469
                    1s 2ms/step -
loss: 0.6175 - val_loss: 0.6165
Epoch 6/10
469/469
                    1s 1ms/step -
loss: 0.6179 - val_loss: 0.6159
Epoch 7/10
469/469
                    1s 1ms/step -
loss: 0.6160 - val_loss: 0.6156
Epoch 8/10
469/469
                    1s 1ms/step -
loss: 0.6172 - val_loss: 0.6155
Epoch 9/10
469/469
                    1s 2ms/step -
loss: 0.6179 - val_loss: 0.6151
Epoch 10/10
469/469
                    1s 2ms/step -
loss: 0.6153 - val_loss: 0.6149
```

Assuming that the name of your layer (obtained through the command model.summary()) is 'layer', here is the way to obtained the weights. Visualize the weights of the encoder and compare them to the two components obtained through the PCA.

```
[51]: weights, bias = ae_model.get_layer('layer').get_weights()
weights = [[x[0] for x in weights],[x[1] for x in weights]]
```

## [54]: Text(0.5, 0, 'Weight 2')



Now, visualize the images rebuilt by the network!

```
[55]: # Select a few images at random: look from n
n = np.random.randint(0,len(test_images)-5)

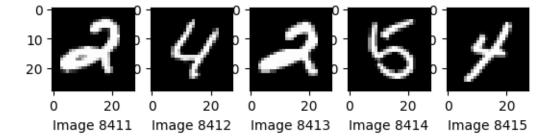
# Plot a few images from n
f, ax = plt.subplots(1,5)
for i,a in enumerate(range(n,n+5)):
    ax[i].imshow(train_images[a].reshape((28,28)), cmap='gray')
    ax[i].set_xlabel('Image '+str(a))
```

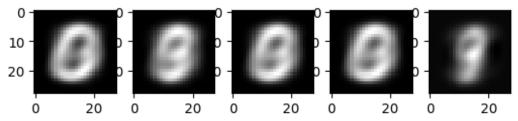
```
# Get the prediction from the model

# ... and plot them
f, ax = plt.subplots(1,5)
for i,a in enumerate(range(n,n+5)):
    ax[i].imshow((ae_model.predict(train_images[a].reshape(1,784)).
    reshape((28,28))), cmap='gray')
    ax[i].set_xlabel('Predicted Image '+str(a))

plt.show()
```

1/1	0s	33ms/step
1/1	0s	18ms/step
1/1	0s	18ms/step
1/1	0s	15ms/step
1/1	0s	16ms/step





Predicted Imager@dilt@ed Imager@dilt@ed Imager@dilt@ed Imager@dilt@ed Image 8-

Do the same ( = build a new model) with a latent dimension that is largely higher than 2. Compare the visualizations and the images that are rebuilt.

```
Code:</div>
```

```
[56]: input_layer = Input(shape=(784,)) #Input
latent_view = Dense(64, activation='linear')(input_layer) #Encoder
```

```
output_layer = Dense(784, activation='linear')(latent_view) #Decoder
ae_model = Model(input_layer, output_layer, name='ae_model')
ae_model.summary() #Affichage
optimizer = SGD(learning_rate=1e-1)
ae_model.compile(optimizer='adam', loss='mean_squared_error')
batch_size = 128
epochs = 10
# No noise here - we want to train a simple auto-encoder and compare visually_
 ⇔with PCA
history = ae_model.fit(train_images,
                       train_images,
                       epochs=epochs,
                       batch_size=batch_size,
                       verbose=1,
                       shuffle=True,
                       validation_data=(test_images,test_images))
n = np.random.randint(0,len(test_images)-5)
f, ax = plt.subplots(1,5)
for i,a in enumerate(range(n,n+5)):
    ax[i].imshow(train_images[a].reshape((28,28)), cmap='gray')
    ax[i].set_xlabel('Image '+str(a))
f, ax = plt.subplots(1,5)
for i,a in enumerate(range(n,n+5)):
    ax[i].imshow((ae_model.predict(train_images[a].reshape(1,784)).
 →reshape((28,28))), cmap='gray')
    ax[i].set_xlabel('Predicted Image '+str(a))
plt.show()
```

### Model: "ae\_model"

Layer (type)	Output Shape	Param #
<pre>input_layer_2 (InputLayer)</pre>	(None, 784)	0
dense (Dense)	(None, 64)	50,240

```
dense_1 (Dense) (None, 784) 50,960
```

Total params: 101,200 (395.31 KB)

Trainable params: 101,200 (395.31 KB)

Non-trainable params: 0 (0.00 B)

Epoch 1/10

469/469 2s 2ms/step - loss: 0.4366 - val\_loss: 0.1188

Epoch 2/10

469/469 1s 2ms/step - loss: 0.1146 - val\_loss: 0.1014

Epoch 3/10

469/469 1s 2ms/step -

loss: 0.1033 - val\_loss: 0.0990

Epoch 4/10

469/469 1s 2ms/step -

loss: 0.1012 - val\_loss: 0.0983

Epoch 5/10

469/469 1s 2ms/step -

loss: 0.1009 - val\_loss: 0.0979

Epoch 6/10

469/469 1s 2ms/step -

loss: 0.1005 - val\_loss: 0.0976

Epoch 7/10

469/469 1s 2ms/step -

loss: 0.1004 - val\_loss: 0.0973

Epoch 8/10

469/469 1s 2ms/step -

loss: 0.0997 - val\_loss: 0.0969

Epoch 9/10

469/469 1s 2ms/step -

loss: 0.0997 - val\_loss: 0.0967

Epoch 10/10

469/469 1s 2ms/step -

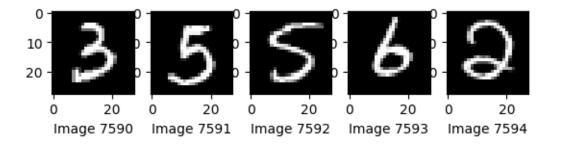
loss: 0.0995 - val\_loss: 0.0967

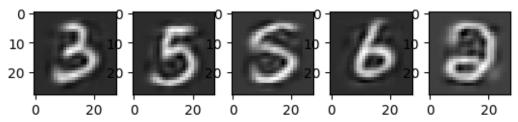
1/1 0s 29ms/step 1/1 0s 30ms/step

1/1 0s 19ms/step

1/1 0s 20ms/step

1/1 0s 22ms/step





Predicted Imag@r@di@Ded Imag@r@di@Led Imag@r@di@Led Imag@r@di@Bed Image 7!

#### 3.1.2 Bonus: De-noising Autoencoder

Now, we can implement a **de-noising autoencoder**. The following function will transform an array of images by adding it random noise. Create a new autoencoder model, this time with **more layers** and **non-linear activations** (like the ReLU) and train it to rebuild the de-noised images. Display some testing images, with noise, and re-built.

```
def noise(array):
    """
    Adds random noise to each image in the supplied array.
    """
    noise_factor = 0.4
    noisy_array = array + noise_factor * np.random.normal(
        loc=0.0, scale=1.0, size=array.shape
    )
    return noisy_array
```

```
[58]: # Create a copy of the data with added noise
noisy_train_images = noise(train_images)
noisy_test_images = noise(test_images)
```

```
[66]: # Visualize some of the images with noise against the originals
```

```
# Build a new model with more layers and Relu activations
# Compile it but here, use noised data as inputs !
# Visualize the images rebuilt by the model !
input_layer = Input(shape=(784,))
latent_view = Dense(256, activation='relu')(input_layer)
latent_view_encore = Dense(128, activation='sigmoid')(latent_view)
latent_view_toujours = Dense(64, activation='linear')(latent_view_encore)
output_layer = Dense(784, activation='linear')(latent_view_toujours)
noise_ae_model = Model(input_layer, output_layer, name='ae_model')
noise_ae_model.summary()
optimizer = SGD(learning_rate=1e-1)
noise ae model.compile(optimizer='adam', loss='mean squared error')
batch size = 128
epochs = 10
history = noise_ae_model.fit(noisy_train_images,
                       train_images,
                       epochs=epochs,
                       batch_size=batch_size,
                       verbose=1,
                       shuffle=True,
                       validation_data=(noisy_test_images,test_images))
n = np.random.randint(0,len(test_images)-5)
f, ax = plt.subplots(1,5)
for i,a in enumerate(range(n,n+5)):
   ax[i].imshow(train_images[a].reshape((28,28)), cmap='gray')
   ax[i].set_xlabel('Image '+str(a))
```

```
f, ax = plt.subplots(1,5)
for i,a in enumerate(range(n,n+5)):
    ax[i].imshow((noise_ae_model.predict(train_images[a].reshape(1,784)).
    reshape((28,28))), cmap='gray')
    ax[i].set_xlabel('Predicted Image '+str(a))

plt.show()
```

Model: "ae\_model"

Layer (type)	Output Shape	Param #
<pre>input_layer_10 (InputLayer)</pre>	(None, 784)	0
dense_22 (Dense)	(None, 256)	200,960
dense_23 (Dense)	(None, 128)	32,896
dense_24 (Dense)	(None, 64)	8,256
dense_25 (Dense)	(None, 784)	50,960

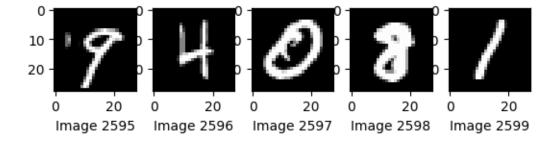
Total params: 293,072 (1.12 MB)

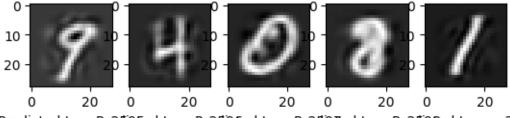
Trainable params: 293,072 (1.12 MB)

Non-trainable params: 0 (0.00 B)

Epoch 1/10 469/469 3s 4ms/step loss: 0.4771 - val\_loss: 0.1982 Epoch 2/10 469/469 1s 3ms/step loss: 0.1880 - val\_loss: 0.1547 Epoch 3/10 469/469 1s 3ms/step loss: 0.1526 - val\_loss: 0.1374 Epoch 4/10 469/469 1s 3ms/step loss: 0.1372 - val\_loss: 0.1277 Epoch 5/10

469/469 2s 3ms/step loss: 0.1290 - val\_loss: 0.1217 Epoch 6/10 469/469 1s 3ms/step loss: 0.1235 - val\_loss: 0.1188 Epoch 7/10 469/469 1s 3ms/step loss: 0.1204 - val\_loss: 0.1169 Epoch 8/10 469/469 1s 3ms/step loss: 0.1182 - val\_loss: 0.1152 Epoch 9/10 469/469 2s 3ms/step loss: 0.1167 - val\_loss: 0.1148 Epoch 10/10 469/469 2s 3ms/step loss: 0.1158 - val\_loss: 0.1144 Os 49ms/step 1/1 1/1 Os 16ms/step 1/1 0s 23ms/step 1/1 Os 14ms/step 1/1 Os 23ms/step





Predicted Imag@r@රෝ@fed Imag@r@

Assuming that we normalize the images to be in the 0-1 range, what other loss function could we use ?

# Answer:</div>

On pourrait utiliser la fonction de perte binaire entropique.