

## Attribution Statement:

Homework 10 by Parin Patel: I did this homework by myself, with help from the book and the professor. I used these websites to gather historical information related to aviation:

- <https://airandspace.si.edu/exhibitions/america-by-air/online/heyday/heyday11.cfm>
- [https://en.wikipedia.org/wiki/1955\\_in\\_aviation](https://en.wikipedia.org/wiki/1955_in_aviation)
- <https://www.fastcompany.com/3022215/what-it-was-really-like-to-fly-during-the-golden-age-of-travel>

## Exercises:

**2. Download and library the nlme package and use data ("Blackmore") to activate the Blackmore data set. Inspect the data and create a box plot showing the exercise level at different ages. Run a repeated measures ANOVA to compare exercise levels at ages 8, 10, and 12 using aov(). You can use a command like, myData <- Blackmore[Blackmore\$age <=12,], to subset the data. Keeping in mind that the data will need to be balanced before you can conduct this analysis, try running a command like this, table(myData\$subject,myData\$age)), as the starting point for cleaning up the data set.**

The findings of our repeated measure ANOVA for exercise levels at ages 8,10,and 12 showed that, on average, people this age got about 1 hour of exercise a week. This is based on the determined median value of 1. Additionally, as they get older, the more varied the amount of exercise they get becomes. This is because we see the 3<sup>rd</sup> and 4<sup>th</sup> quantiles increase as they get older. Same goes for the max. Finally, when we ran our aov function to analyze variance, we continued to see exercise increase with age. The results showed a p-value of 1.33e-11, which means there is a significance between age and the amount of exercise a person does. Also, based on our boxplot, we see that 15 is the age with the highest variance in exercise times. In addition, it has the highest median exercise amount. We start to see a overall dip in the median at the age of 18.

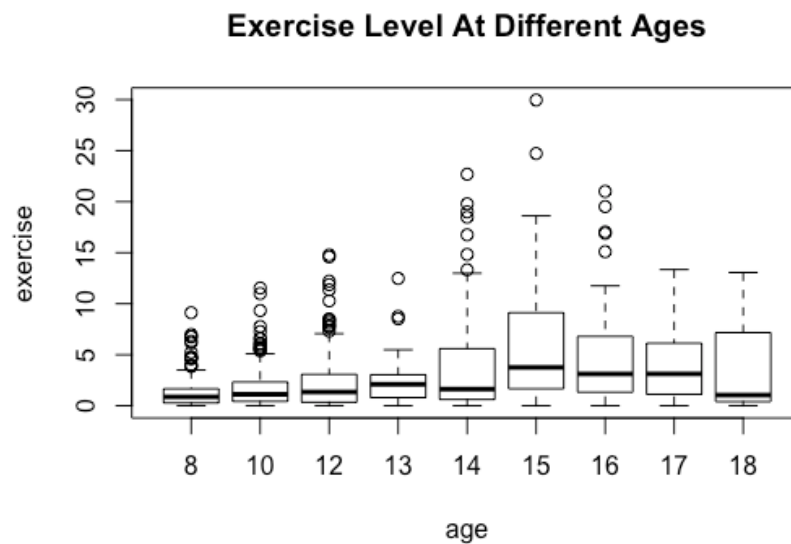
```
> str(Blackmore)
'data.frame':  945 obs. of  4 variables:
 $ subject : Factor w/ 231 levels "100","101","102",...: 1 1 1 1 1 2 2 2 2 2 ...
 $ age      : num  8 10 12 14 15.9 ...
 $ exercise: num  2.71 1.94 2.36 1.54 8.63 0.14 0.14 0 0 5.08 ...
 $ group    : Factor w/ 2 levels "control","patient": 2 2 2 2 2 2 2 2 2 2 ...
```

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```
> summary(Blackmore)
  subject      age      exercise
100      : 5   Min.   : 8.00   Min.   : 0.000
101      : 5   1st Qu.:10.00   1st Qu.: 0.400
105      : 5   Median :12.00   Median : 1.330
106      : 5   Mean   :11.43   Mean   : 2.531
107      : 5   3rd Qu.:14.00   3rd Qu.: 3.040
108      : 5   Max.   :18.00   Max.   :29.960
(Other):915
  group
control:359
patient:586
```



```
> #Create ANOVA to compare exercise levels at ages 8, 10, and 12
> myData <- Blackmore[Blackmore$age <= 12,]
> table(myData$subject,myData$age)
```

```
 8 10 11.58 11.83 12
```

```

> is.factor(myData$age) #check if age is factor
[1] FALSE
> myData$ageFact<-as.factor(myData$age) #change age to factor
> list <- rowSums(table(myData$subject,myData$ageFact))==3
> list <- list[list == TRUE]
> list <- as.numeric(names(list))
Warning message:
NAs introduced by coercion
> summary(myData[myData$ageFact == 8,])
  subject      age      exercise      group      ageFact
100      : 1  Min.    :8  Min.    :0.000  control: 93    8      :231
101      : 1  1st Qu.:8  1st Qu.:0.280  patient:138   10      : 0
102      : 1  Median :8  Median :0.870                11.58: 0
103      : 1  Mean    :8  Mean    :1.259                11.83: 0
104      : 1  3rd Qu.:8  3rd Qu.:1.665                12      : 0
105      : 1  Max.    :8  Max.    :9.120
(Other):225

```

```

> summary(myData[myData$ageFact == 10,])
  subject      age      exercise      group      ageFact
100      : 1  Min.    :10  Min.    : 0.000  control: 92    8      : 0
101      : 1  1st Qu.:10  1st Qu.: 0.430  patient:137   10      :229
102      : 1  Median :10  Median : 1.120                11.58: 0
103      : 1  Mean    :10  Mean    : 1.746                11.83: 0
104      : 1  3rd Qu.:10  3rd Qu.: 2.330                12      : 0
105      : 1  Max.    :10  Max.    :11.540
(Other):223
> summary(myData[myData$ageFact == 12,])
  subject      age      exercise      group      ageFact
100      : 1  Min.    :12  Min.    : 0.000  control: 59    8      : 0
101      : 1  1st Qu.:12  1st Qu.: 0.340  patient:117   10      : 0
102      : 1  Median :12  Median : 1.330                11.58: 0
103      : 1  Mean    :12  Mean    : 2.307                11.83: 0
104      : 1  3rd Qu.:12  3rd Qu.: 3.272                12      :176
105      : 1  Max.    :12  Max.    :14.780
(Other):170

```

```

> myData <- myData[myData$subject %in% list,]
> summary(aov(exercise~ageFact+ Error(subject), data = myData))

Error: subject
          Df Sum Sq Mean Sq F value Pr(>F)
Residuals 165   1892    11.47

Error: Within
          Df Sum Sq Mean Sq F value    Pr(>F)
ageFact     2  102.7    51.35   27.04 1.33e-11 ***
Residuals 330   626.6     1.90

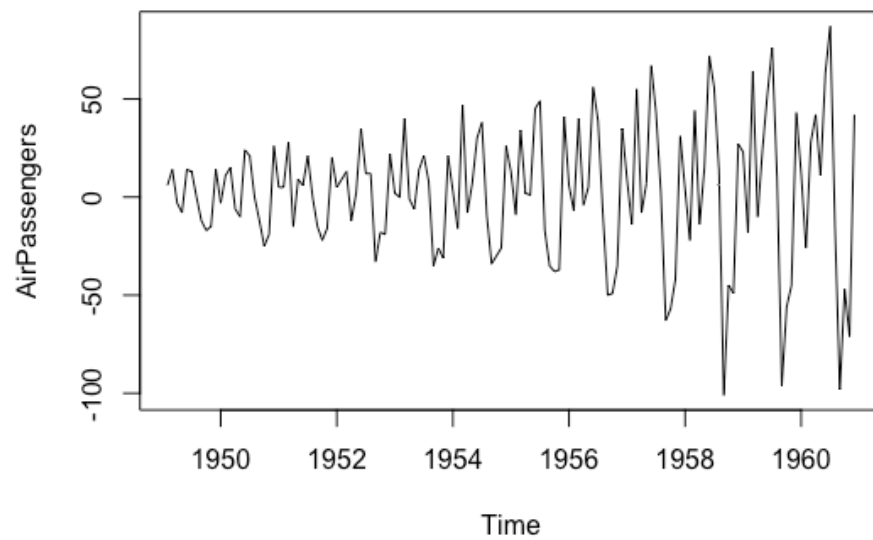
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
>

```

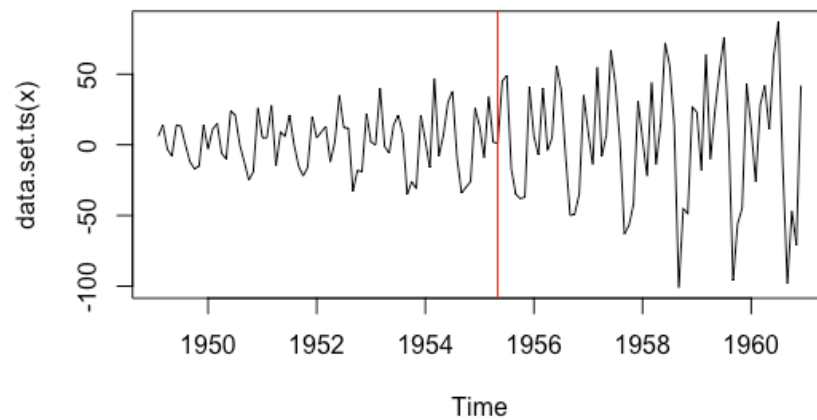
**5. Given that the AirPassengers data set has a substantial growth trend, use `diff()` to create a differenced data set. Use `plot()` to examine and interpret the results of differencing. Use `cpt.var()` to find the change point in the variability of the differenced time series. Plot the result and describe in your own words what the change point signifies.**

We used the `diff(AirPassengers)` function to determine the difference between the number of airline passengers, by month. This means that we compared the number of passengers at a certain time (month) against the number of passengers at a previous month to create a time series plot that plots the differences visually to show a trend over time. The red line we see in the second plot shows the changepoint in our dataset. This shows that the major shift in the data occurred in 1955, when there was an increase about 45,000 passengers from the numeric value of 0. This is likely because, based on a review of major events in aviation that year, the mid-1950s were considered the “Golden Age of Flying” due to the rise in the number of more affordable airline tickets and jumbo-jets.

**Plot of Differenced Data Set**



**Plot of Change Point in Variability**



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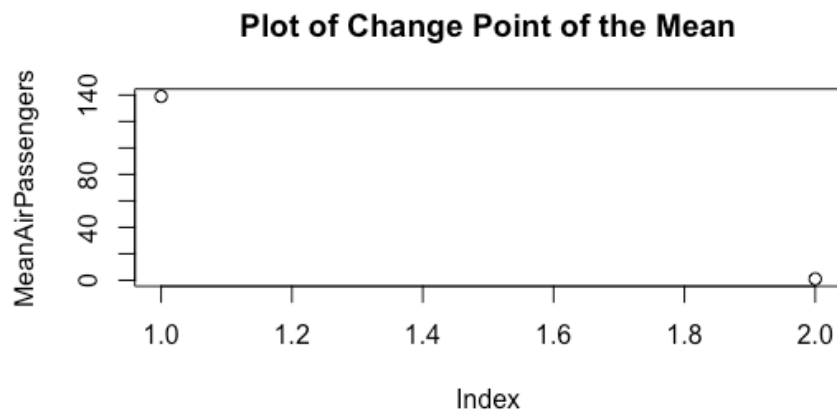
```
> #####Question 5:
> library("changepoint")
> data("AirPassengers")
> AirPassengers <- diff(AirPassengers) #create differenced data set.
> plot(AirPassengers,main="Plot of Differenced Data Set") #plot
>
> AP_Plot<-cpt.var(AirPassengers) #find change point in variability
> plot(AP_Plot, main="Plot of Change Point in Variability")
> AP_Plot
Class 'cpt' : Changepoint Object
  ~~~ : S4 class containing 12 slots with names
      cpttype date version data.set method test.stat pen.type pen.value minseglen cpts ncpts.max param.est

Created on : Fri Apr 26 15:58:35 2019

summary(.) :
-----
Created Using changepoint version 2.2.2
Changepoint type      : Change in variance
Method of analysis    : AMOC
Test Statistic        : Normal
Type of penalty       : MBIC with value, 14.88853
Minimum Segment Length : 2
Maximum no. of cpts   : 1
Changepoint Locations : 76
> |
```

**6. Use `cpt.mean()` on the `AirPassengers` time series. Plot and interpret the results. Compare the change point of the mean that you uncovered in this case to the change point in the variance that you uncovered in Exercise 5. What do these change points suggest about the history of air travel?**

By plotting the `cpt.mean` of the airline passenger data, we get a confidence value of exactly 1. Our change point analysis from the previous exercise (#5) shows that there has been an event in 1955 that caused a surge in passengers in the time series data. Since our function used was `cpt.mean`, our change point analysis was focused in #6 on the mean. Our analysis in #5 was focused on the variance. From number 5's second change point line in the time series graph, we can see that the numeric value of the passengers (x) is 0 before it increases and then drops towards the end of 1995. Based on this, we can expect that our change point around the mean to be 0, which is the result of our analysis for #6. Therefore, the results of our overall findings suggests that there has been a change in the mean number of passengers over time.



```
> #####Question 6:
> # retrieve the confidence value
> MeanAirPassengers <- cpt.mean(AirPassengers, class = FALSE)
> MeanAirPassengers
      cpt conf.value
      139         1
> plot(MeanAirPassengers,main="Plot of Change Point of the Mean")
> MeanAirPassengers["conf.value"] #confidence value
conf.value
      1
>
```

**7. Find historical information about air travel on the Internet and/or in reference materials that sheds light on the results from Exercises 5 and 6. Write a mini-article (less than 250 words) that interprets your statistical findings from Exercises 5 and 6 in the context of the historical information you found.**

I briefly touched on this earlier in my answers, but the 1950's were considered the Golden Age of plane travel. According to the Smithsonian National Air and Space Museum, the 1950s was the age that saw the air travel become more common. The service became more widely spread and commonplace. For passengers, while tickets were still expensive compared to today's values, more passengers were able to buy them and travel to a increasing number of destinations. Airlines were becoming more comfortable and faster for public travel as well. They offered low-fare night coach services and had "red eye" specials. The piston-engine airplanes from the 1940's was replaced with modern day jet aircrafts. This also made flying more efficient. It was the age where airlines were increasing in the number of passengers they could fly and the distance they were flying them. This would explain the rise and change in passengers flying in our dataset. It was a change in an era that made air travel an important necessity.

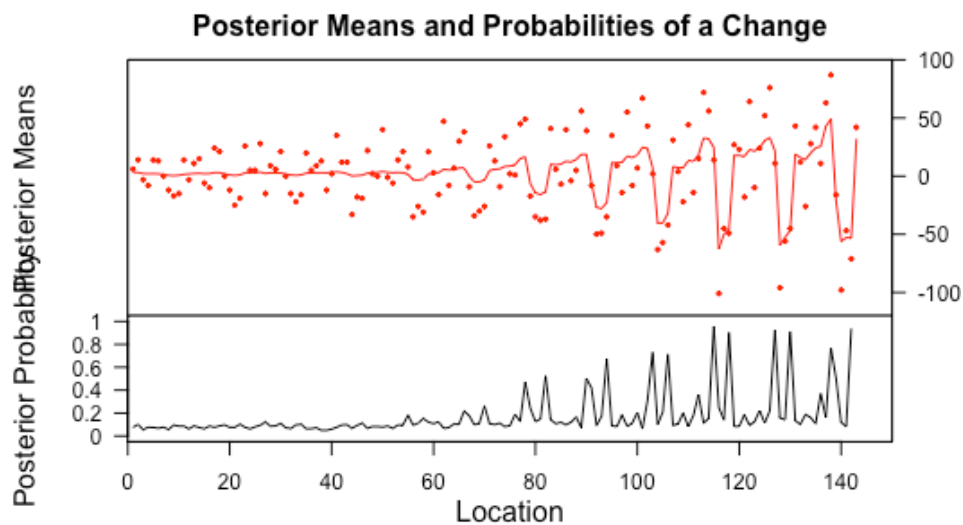
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**8. Use `bcp()` on the `AirPassengers` time series. Plot and interpret the results. Make sure to contrast these results with those from Exercise 6**

Our Bayesian approach to time series yielded similar results as our frequentist model in number 6. Based on the graph below, we can see that both the posterior probability (bottom line) and the posterior mean (top line) plots show a change in both probability and mean difference around the 76<sup>th</sup> location. This point, 76<sup>th</sup>, is the same change point we calculated in #6, when using the frequentist approach. We can then say with some certainty that these findings show that shift has indeed occurred in the dataset in and after the 76<sup>th</sup> location.



## Appendix I: Source Code

```
##Question 2:
```

```
library("nlme")
```

```
library("carData")
```

```
data("Blackmore")
```

```
Blackmore <- data.frame(Blackmore)
```

```
str(Blackmore) #decimal place noticed. Lets remove by rounding.
```

```
Blackmore$age <- round(Blackmore$age)
```

```
summary(Blackmore)
```

```
boxplot(exercise~age, data = Blackmore, main="Exercise Level At Different Ages")#get boxplot
```

```
#Create ANOVA to compare exercise levels at ages 8, 10, and 12
```



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```
myData <- Blackmore[Blackmore$age <= 12,]
table(myData$subject,myData$age)
is.factor(myData$age) #check if age is factor
myData$ageFact<-as.factor(myData$age) #change age to factor
list <- rowSums(table(myData$subject,myData$ageFact))==3
list <- list[list == TRUE]
list <- as.numeric(names(list))
```

```
summary(myData[myData$ageFact == 8,])
summary(myData[myData$ageFact == 10,])
summary(myData[myData$ageFact == 12,])
```

```
myData <- myData[myData$subject %in% list,]
summary(aov(exercise~ageFact+ Error(subject), data = myData))
```

#####Question 5:

```
library("changepoint")
data("AirPassengers")
AirPassengers <- diff(AirPassengers) #create differenced data set.
plot(AirPassengers,main="Plot of Differenced Data Set") #plot
```

```
AP_Plot<-cpt.var(AirPassengers) #find change point in variability
plot(AP_Plot, main="Plot of Change Point in Variability")
```

#####Question 6:

```
# retrieve the confidence value
MeanAirPassengers <- cpt.mean(AirPassengers, class = FALSE)
plot(MeanAirPassengers,main="Plot of Change Point of the Mean")
MeanAirPassengers["conf.value"] #confidence value
```

###Question 8:

```
library("bcp")
bcpAirPassengers <- bcp(as.vector(AirPassengers))
plot(bcpAirPassengers)
```

## Appendix II: Output

```
> ##Question 2:
> library("nlme")
> library("carData")
```

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>

>

> data("Blackmore")

> Blackmore <- data.frame(Blackmore)

> str(Blackmore) #decimal place noticed. Lets remove by rounding.

'data.frame': 945 obs. of 4 variables:

\$ subject : Factor w/ 231 levels "100","101","102",...: 1 1 1 1 1 2 2 2 2 2 ...

\$ age : num 8 10 12 14 15.9 ...

\$ exercise: num 2.71 1.94 2.36 1.54 8.63 0.14 0.14 0 0 5.08 ...

\$ group : Factor w/ 2 levels "control","patient": 2 2 2 2 2 2 2 2 2 2 ...

> Blackmore\$age <- round(Blackmore\$age)

> summary(Blackmore)

	subject	age	exercise
100	: 5	Min. : 8.00	Min. : 0.000
101	: 5	1st Qu.:10.00	1st Qu.: 0.400
105	: 5	Median :12.00	Median : 1.330
106	: 5	Mean :11.43	Mean : 2.531
107	: 5	3rd Qu.:14.00	3rd Qu.: 3.040
108	: 5	Max. :18.00	Max. :29.960

(Other):915

group

control:359

patient:586

>

> boxplot(exercise~age, data = Blackmore, main="Exercise Level At Different Ages")#get  
boxplot

> #Create ANOVA to compare exercise levels at ages 8, 10, and 12

> myData <- Blackmore[Blackmore\$age <= 12,]

> table(myData\$subject,myData\$age)

8 10 12

100 1 1 1

101 1 1 1

102 1 1 1

103 1 1 1

104 1 1 1

105 1 1 1

106 1 1 1

107 1 1 1

108 1 1 1

109 1 1 0

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110 1 1 1  
111 1 1 1  
112 1 1 1  
113 1 1 1  
114 1 1 1  
115 1 1 1  
116 1 1 1  
117 1 1 1  
118 1 1 1  
119 1 1 1  
120 1 1 1  
121 1 1 1  
122 1 1 1  
123 1 1 1  
124 1 1 1  
125 1 1 1  
126 1 1 1  
127 1 1 1  
128 1 1 0  
129 1 1 1  
130 1 1 1  
132 1 1 0  
133 1 1 1  
134 1 1 1  
135 1 1 1  
136 1 1 1  
137 1 1 1  
138 1 1 1  
139 1 1 1  
140 1 1 1  
141 1 1 1  
142 1 1 1  
143 1 1 1  
144 1 1 1  
145 1 1 1  
146 1 1 1  
147 1 1 0  
148 1 1 1  
149 1 1 1  
150 1 1 1  
151 1 1 1  
152 1 1 1  
153 1 1 1  
154 1 1 1  
155 1 1 0  
156 1 1 1

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157 1 1 1  
158 1 1 1  
159 1 1 1  
160 1 1 1  
161 1 1 1  
162 1 1 0  
163 1 1 1  
164 1 1 1  
165 1 1 1  
166 1 1 1  
167 1 1 1  
168 1 0 1  
169 1 1 0  
170 1 1 1  
171 1 1 1  
172 1 1 1  
173 1 1 1  
174 1 1 0  
175 1 1 1  
176 1 1 1  
177 1 1 1  
178 1 1 1  
179 1 1 1  
180 1 1 1  
181 1 1 1  
182 1 1 1  
183 1 1 1  
184 1 1 0  
185 1 1 1  
186 1 1 1  
187 1 1 1  
188 1 1 1  
189 1 1 1  
190 1 1 1  
192 1 1 1  
193 1 1 0  
194 1 1 1  
195 1 1 1  
196 1 1 0  
198 1 1 1  
199 1 1 1  
200 1 1 1  
201 1 1 0  
202 1 1 1  
203 1 1 1  
204 1 1 1

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205 1 1 1  
206 1 1 1  
207a 1 1 1  
207b 1 1 1  
208 1 1 1  
209 1 1 1  
210 1 1 1  
211 1 1 0  
212 1 1 1  
213 1 1 1  
214 1 1 1  
215 1 1 1  
216 1 1 0  
217 1 1 0  
218 1 1 1  
219 1 1 1  
220 1 1 1  
221 1 1 1  
222 1 0 1  
223 1 1 0  
224 1 1 1  
225 1 1 1  
226 1 1 0  
227 1 1 0  
228 1 1 1  
229a 1 1 1  
229b 1 1 1  
230 1 1 1  
231 1 1 1  
232 1 1 1  
233 1 1 1  
234 1 1 0  
235 1 1 1  
236 1 1 1  
237 1 1 1  
238 1 1 1  
239 1 1 0  
240 1 1 1  
241 1 1 0  
242 1 1 1  
243 1 1 1  
244 1 1 1  
245 1 1 1  
246 1 1 1  
247 1 1 1  
248 1 1 0

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249 1 1 1  
250 1 1 1  
251 1 1 1  
252 1 1 1  
253 1 1 1  
254 1 1 1  
255 1 1 1  
255b 1 1 1  
256 1 1 1  
257 1 1 1  
258 1 1 0  
259 1 1 1  
260 1 1 0  
261 1 1 0  
262 1 1 0  
263 1 1 0  
264 1 1 0  
265 1 1 0  
266 1 1 0  
267 1 1 1  
268 1 1 1  
269 1 1 0  
270 1 1 1  
271 1 1 1  
272 1 1 0  
273a 1 1 1  
273b 1 1 1  
274 1 1 0  
275 1 1 1  
276 1 1 1  
277 1 1 1  
278 1 1 1  
279a 1 1 0  
279b 1 1 1  
280a 1 1 1  
280b 1 1 1  
281 1 1 1  
282 1 1 1  
283 1 1 0  
284 1 1 1  
285 1 1 0  
286 1 1 1  
300 1 1 1  
301 1 1 1  
302 1 1 1  
303 1 1 1

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```
304 1 1 1
305 1 1 1
306 1 1 1
307 1 1 1
308 1 1 1
309 1 1 1
310 1 1 1
311 1 1 1
312 1 1 1
313 1 1 1
314 1 1 1
315 1 1 0
316 1 1 0
317 1 1 1
318 1 1 1
319 1 1 1
320 1 1 1
321 1 1 1
322 1 1 0
323 1 1 1
324 1 1 0
325 1 1 1
326 1 1 1
327 1 1 1
328 1 1 1
329 1 1 1
330 1 1 1
331 1 1 1
332 1 1 1
333 1 1 1
334 1 1 1
335 1 1 1
336 1 1 1
337 1 1 0
338 1 1 1
340 1 1 1
341 1 1 0
```

```
> is.factor(myData$age) #check if age is factor
```

```
[1] FALSE
```

```
> myData$ageFact<-as.factor(myData$age) #change age to factor
```

```
> list <- rowSums(table(myData$subject,myData$ageFact))==3
```

```
> list <- list[list == TRUE]
```

```
> list <- as.numeric(names(list))
```

Warning message:

NAs introduced by coercion

```
>
```

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```
> summary(myData[myData$ageFact == 8,])
  subject    age  exercise    group
100 : 1  Min.   :8  Min.   :0.000 control: 93
101 : 1 1st Qu.:8 1st Qu.:0.280 patient:138
102 : 1 Median :8 Median :0.870
103 : 1 Mean   :8 Mean   :1.259
104 : 1 3rd Qu.:8 3rd Qu.:1.665
105 : 1 Max.   :8 Max.   :9.120
(Other):225
ageFact
8 :231
10: 0
12: 0
```

```
> summary(myData[myData$ageFact == 10,])
  subject    age  exercise    group
100 : 1  Min.  :10  Min.   :0.000 control: 92
101 : 1 1st Qu.:10 1st Qu.: 0.430 patient:137
102 : 1 Median :10 Median : 1.120
103 : 1 Mean   :10 Mean   : 1.746
104 : 1 3rd Qu.:10 3rd Qu.: 2.330
105 : 1 Max.   :10 Max.   :11.540
(Other):223
ageFact
8 : 0
10:229
12: 0
```

```
> summary(myData[myData$ageFact == 12,])
  subject    age  exercise    group
100 : 1  Min.  :12  Min.   :0.000 control: 68
101 : 1 1st Qu.:12 1st Qu.: 0.350 patient:121
102 : 1 Median :12 Median : 1.350
103 : 1 Mean   :12 Mean   : 2.289
104 : 1 3rd Qu.:12 3rd Qu.: 3.080
105 : 1 Max.   :12 Max.   :14.780
(Other):183
ageFact
8 : 0
10: 0
```



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```
>  
> myData <- myData[myData$subject %in% list,]  
> summary(aov(exercise~ageFact+ Error(subject), data = myData))
```

```
Error: subject  
      Df Sum Sq Mean Sq F value Pr(>F)  
Residuals 176  1931   10.97
```

```
Error: Within  
      Df Sum Sq Mean Sq F value  Pr(>F)  
ageFact   2  105.2   52.60  27.82 6.09e-12 ***  
Residuals 352  665.7    1.89
```

```
---  
Signif. codes:  
0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
>  
> #####Question 5:  
> library("changepoint")  
> data("AirPassengers")  
> AirPassengers <- diff(AirPassengers) #create differenced data set.  
> plot(AirPassengers,main="Plot of Differenced Data Set") #plot  
>  
> AP_Plot<-cpt.var(AirPassengers) #find change point in variability  
> plot(AP_Plot, main="Plot of Change Point in Variability")  
>  
>  
>  
> #####Question 6:  
> # retrieve the confidence value  
> MeanAirPassengers <- cpt.mean(AirPassengers, class = FALSE)  
> plot(MeanAirPassengers,main="Plot of Change Point of the Mean")  
> MeanAirPassengers["conf.value"] #confidence value  
conf.value  
      1  
>  
>  
> ###Question 8:  
> library("bcp")  
> bcpAirPassengers <- bcp(as.vector(AirPassengers))  
> plot(bcpAirPassengers)
```