

Bag Model

$$\dot{z}_1 = z_2$$

4 states, all observable,
 $\dot{m}_{in} = u$ single control

$$\dot{z}_2 = \frac{A}{M} (P - P_b) - g$$

$$\dot{P}_b = \frac{\sigma RT}{Ad} (\dot{m}_{in} - \dot{m}_{escape}) \quad (\dot{V}_b = 0)$$

$$\dot{P} = \frac{\sigma RT}{A z_1} \left(\dot{m}_{escape} - \dot{m}_{out} - \frac{PA z_2}{RT} \right)$$

\dot{m}_{in} : input

$$\dot{m}_{escape} = \frac{KA(P_b - P)}{\mu L} \quad \text{incompressible Darcy flow, fix this}$$

$$\dot{m}_{out} = \frac{PLz_1}{\sqrt{RT}} \sqrt{\frac{2\sigma}{\sigma-1} \left(\left(\frac{P_b}{P} \right)^{\frac{2}{\sigma}} - \left(\frac{P_b}{P} \right)^{\frac{\sigma+1}{\sigma}} \right)}$$

At equilibrium

$$z_1^* = \text{ride height}$$

$$z_2^* = 0$$

$$\dot{m}_{in} = \dot{m}_{escape} = \dot{m}_{out} = Q$$

3 equations $(\dot{z}_2, \dot{P}_b, \dot{P}) = 0$ in 3 unknowns
 (Q, P_b, P)

Next step, explicit (possibly?) solutions
 for Q^*, P_b^*, P^*